Portfolio Selection:
Basic Universal & Mean-Variance Efficient Portfolios

STAT GR 5261 Statistical Method in Finance

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Professor Zhiliang Ying

Members:
Junyi Mao, jm5038
Yiqi Lei, yl4353
Jiahang Li, jl5531
Wozhi Liu, wl2748
Ziqin Zhao, zz2709
Hanbo Jiao, hj2524

Department of Statistics, Columbia University

1. Introduction

Both individual investors and institutional investors are facing the biggest challenge of "how to invest to meet future needs". Although the investment goals of investors are not the same, no matter what the goals are, everyone faces the same challenges. These challenges are not limited to choosing which asset classes to invest in. The final decision on how to invest is also a difficult one. A large number of practices have proved that the effect of investing in a portfolio is much better than that of investing in a single security or evaluating each security separately. A portfolio is a collection of stocks, bonds, and financial derivatives held by investors or financial institutions. In this project, we will talk about two portfolio strategies, Universal Portfolios, and Mean-Variance Efficient Portfolios.

The sources of our data sets are from public information. The data we used in this project was chosen from 5 stocks which are all leading companies in different industries so that we can avoid "Black Swan" events, and we got the data from Yahoo Finance (https://finance.yahoo.com). Meanwhile, we determined 10-year US T-Note futures as the risk-free asset. Here are the names and abbreviations of 5 stocks we selected:

- AAPL: Apple
- BA: Boeing
- CVS: CVS Healthcare Corp
- JPM: JP Morgan Chase
- KO: Coca-Cola Company

The adjusted closing price is from April 13, 2015, to April 9, 2020. Although we once considered deleting the data in 2020, we finally decided to keep it because these five stocks were not significantly affected by the Covid-19 pandemic.

2. Main Objectives and Planning

In this project, we would compare the performances of two portfolio selection strategies in the current stock market, Universal Portfolios, and Mean-Variance Efficient Portfolios.

- Universal Portfolio
    - Objective: To find the return of best possible asset allocation
    - Approach: Price-weighted Portfolio, Equal-weighted Portfolio, Trading Volume-weighted Portfolio

    - Assumption: No statistical assumption about distribution of asset price
- Mean-Variance Efficient Portfolio
    - Objective: To minimize the variance of portfolio returns
    - Approach: Global minimum portfolio and tangency portfolio
    - Assumption: Assuming asset returns are independent and identically distributed random vectors

Both strategies use returns from previous investment periods to build the portfolio for the current investment period. Then we compare those strategies to find the best strategy for the future investment.

3. Methods of Analysis

I. Universal Portfolio

When investing in the capital market, we always consider a sequence of algorithms for building up our portfolio and adjust it progressively. Normally, we believe that we have a good estimation of future market performance, so that we could optimize the portfolio based on our goal of return and our preference of risks. In practice, we could utilize the universal portfolio approach to achieve our purpose. We will introduce three types of universal portfolio approaches in the following paragraphs.

Price-weighted portfolio:

The main idea of a price-weighted portfolio is weighting each stock within a portfolio based on its price. The approach is investing in each stock in proportion to Adj_close price on the first day, then rebalance the portfolio in terms of price changes before every trading day. In other words, if a stock price increases during the day, it will take more proportion of the portfolio on the next trading day. In this way, the price weighted portfolio is an aggressive approach because the investors believe that a stock outperform today will also be outstanding on the next trading day and vice versa. So, definitely, such a portfolio is price dominant. One pro is that such a portfolio may generate more return, but the con is that It will drive the price of a stock far and far from its true value which leads to more risk.

Equal-weighted portfolio:

For an equal-weighted portfolio, we give the same weight or importance to each stock. The approach is investing the same amount of money on each stock on the first day, then rebalancing the portfolio before each trading day to ensure the equal distribution. In contrast to price weighted portfolio, the equal-weighted portfolio is more conservative because it supposes today's outperforming stock will be less important on the coming day by assigning less

proportion to it. In this way, the equal-weighted approach will drag stocks' price back to its true value. So it is mean-reversion oriented.

Trading volume-weighted portfolio:

For trading volume-weighted, we weigh each stock based on its trading volume, which is calculated by closing price times its volume. The approach is similar to price weighted, but the difference is we rebalance the portfolio based on trading volume instead of merely closing price. In other words, in such an approach, we not only consider stocks' price, but also its trading volume. Trading volume weights only requires intraday databases, but not other complex databases. This is one pro of it.

Compare the three approaches:

Making the three graphs together and comparing them, we notice that the price weighted line outperforms most of the time up to Dec 2019. This is because AAPL and Boeing dominate the portfolio, and their stock price increases a lot. Their price almost tripled during the period. This makes price weighted portfolios generate more return. However, after Dec 2019, an equal-weighted portfolio generates more return. This is because AAPL and Boeing stock price began to crash down, especially Boeing.

II. Mean-variance Optimization

Investors have various preferences of risks and returns. According to Markowitz's Modern Portfolio Theory, the main goal of our mean-variance optimization strategy is to maximize the expected return based on a given level of market risk, or to minimize the risk at given return.

Global minimum portfolio:

We first consider the situation without a risk-free asset available. We achieve our goal by solving the optimization problem with two constraints. Mathematically, we are going to

solve $\{w_i, \text{i=1, … , n}\}$ satisfying $\quad min \sum_{i,j} w_i w_j \sigma_{ij}$

constrained by $\quad \sum_i w_i = 1 \quad \sum_i w_i \mu_i = \mu_p$

where there are n assets (five stocks in our case), for asset i, R_i = return of asset i, w_i = weight of asset i, $\sigma_{ij} = \text{Cov}(R_i, R_j)$, $\mu_i = \text{E}[R_i]$, μ_p = portfolio return.

Using this, for each portfolio return, we have a corresponding optimal (minimum variance) portfolio with specific weights and portfolio variance. Various choices of portfolio return enable us to draw a curve called the efficient frontier, which will be shown in our results. We can find

the global minimum point who has the smallest variance among all portfolios, and then the return of the global minimum portfolio can be calculated.

Tangency portfolio (Sharp optimal portfolio):

Then, we consider the situation when a risk-free asset is available for investment. By the two-fund theorem, any linear combinations of two minimum variance portfolios are also minimum variance portfolios. Then the problem is formulated as

solving $\{ w_i$, i=1, … , n$\}$ $\quad min \sum_{i,j} w_i w_j \sigma_{ij}$

constrained by: $\quad \sum_i w_i \mu_i + (1 - \sum_i w_i) \mu_f = \mu_p$

where there μ_f is the risk-free return.

When the solution of $\{ w_i$, i=1, … , n$\}$ are scaled so that their sum equals 1, they generate a portfolio called the tangency portfolio or market portfolio. Let μ_M, σ_M denote the expected return and standard deviation of the market portfolio, then our portfolio consisting of both risky assets and risk-free asset satisfies $\alpha \, \mu_M$ + (1-α) $r_f = \mu_p$ and $\alpha \, \sigma_M = \sigma_p$, $\alpha = \sum_i w_i$. Now, our efficient frontier becomes a straight line with μ_f as the y-intercept and is tangent to the efficient frontier without the risk-free asset.

The Sharpe Ratio is defined as the expected excess return of a market portfolio divided by the standard deviation of the portfolio, ie. $\frac{\mu_M - \mu_f}{\sigma_M}$. The tangent point we find maximizes the sharpe ratio, and this sharpe ratio is also the slope of the straight-line efficient frontier. The market portfolio at this point is the sharpe optimal portfolio.

Moreover, in our project, we also consider situations with or without short-selling. When short selling is allowed, we use the Lagrange multiplier method to get the formulas with n+2 variables $\{ w_1, ..., w_n, \lambda_1, \lambda_2 \}$ such that $\sum_{j=1}^{n} w_j \sigma_{ij} - \lambda_1 \mu_i - \lambda_2 = 0$, i=1, …, n.

Besides these two specific points, investors can choose freely along the efficient frontiers with desired portfolio returns and variances.

4. Results

I. Basic Universal Portfolio

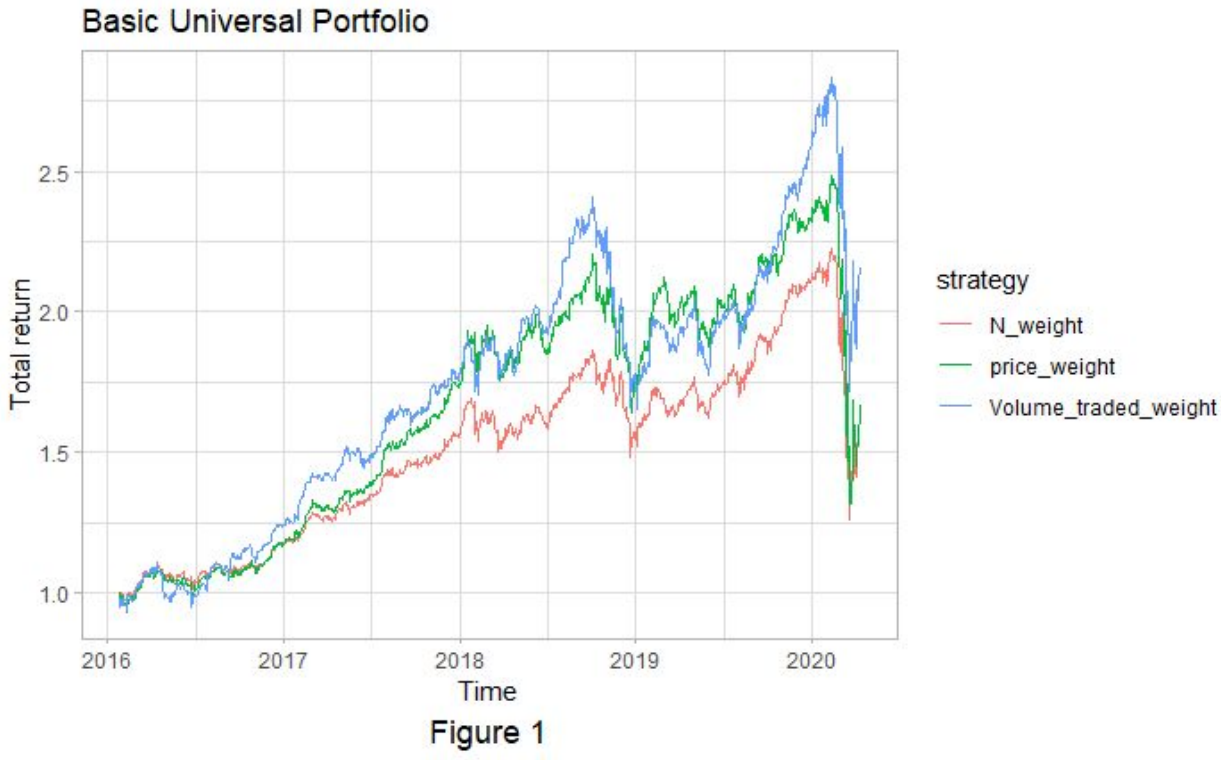

Figure 1: Daily Portfolio Return of Basic Universal Portfolio

II. Mean-variance Optimization

We present the performance of the Mean-variance optimization portfolio strategy by using daily adjusted closing price data from April 13, 2015, to April 9, 2020, for five stocks (AAPL, BA, CVS, JPM, KO)

a. Portfolio Return Over Time (with Short-Selling)

Figure 2: Daily Portfolio Return of GMP and Tangency Strategy in Short Selling Market

It shows a similar trend between GMP and Tangency portfolios. Both returns are increasing as time goes on. When short selling is allowed in the market, the portfolio returns between the two methods are not much different.

The expected daily return of GMP should be lower than the tangency portfolio. However, the performance of GMP is better. It might be the result of the higher expected risk of the tangency portfolio.

b. Portfolio Return Over Time (No Short-Selling)

Figure 3: Daily Portfolio Return of GMP and Tangency Strategy without Short Selling

Due to more restrictions (No Short Sell Market), the return of the tangency portfolio has been significantly reduced. At the same time, the gap between GMP and tangency is obviously increasing as time goes on. In this situation, the global minimum portfolio performs better.

c. Portfolio Return Over Time (no Short-Selling and with Short-Selling)

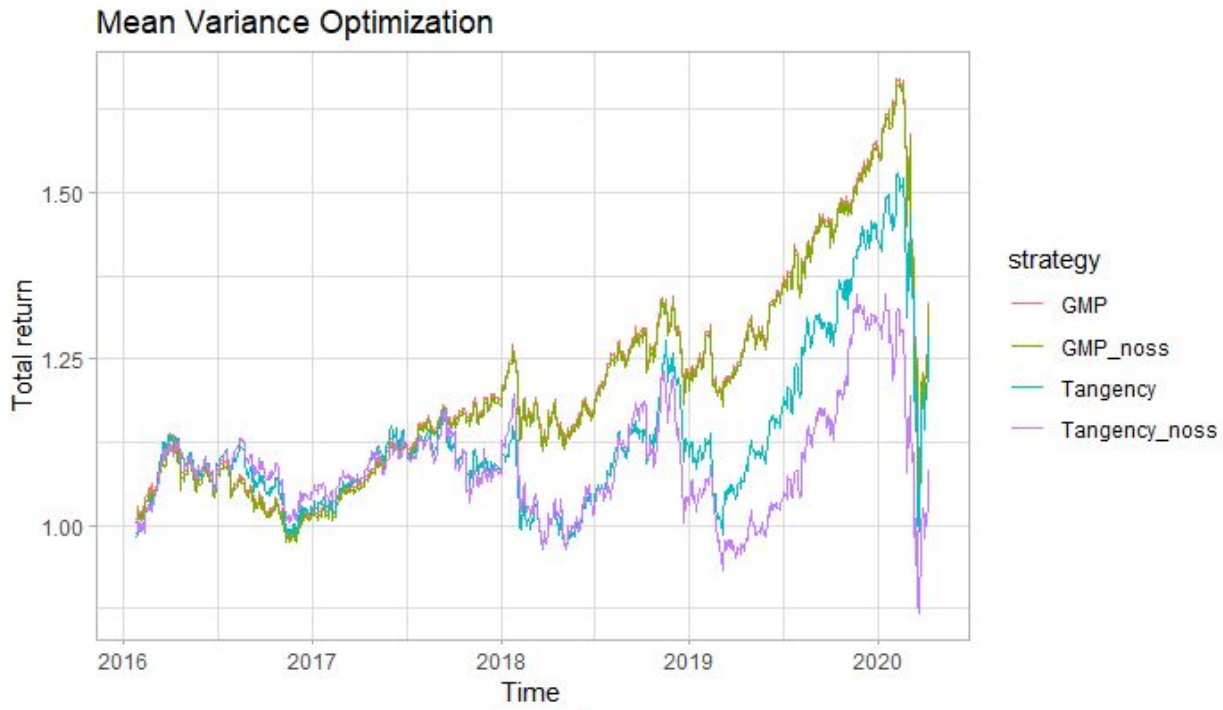

Figure 4: Daily Portfolio Return of GMP and Tangency Strategy with SS and without SS

At most of the trading days, the GMP strategy with short-selling has the highest rate of return

d. Efficient Frontier (with Short-Selling)

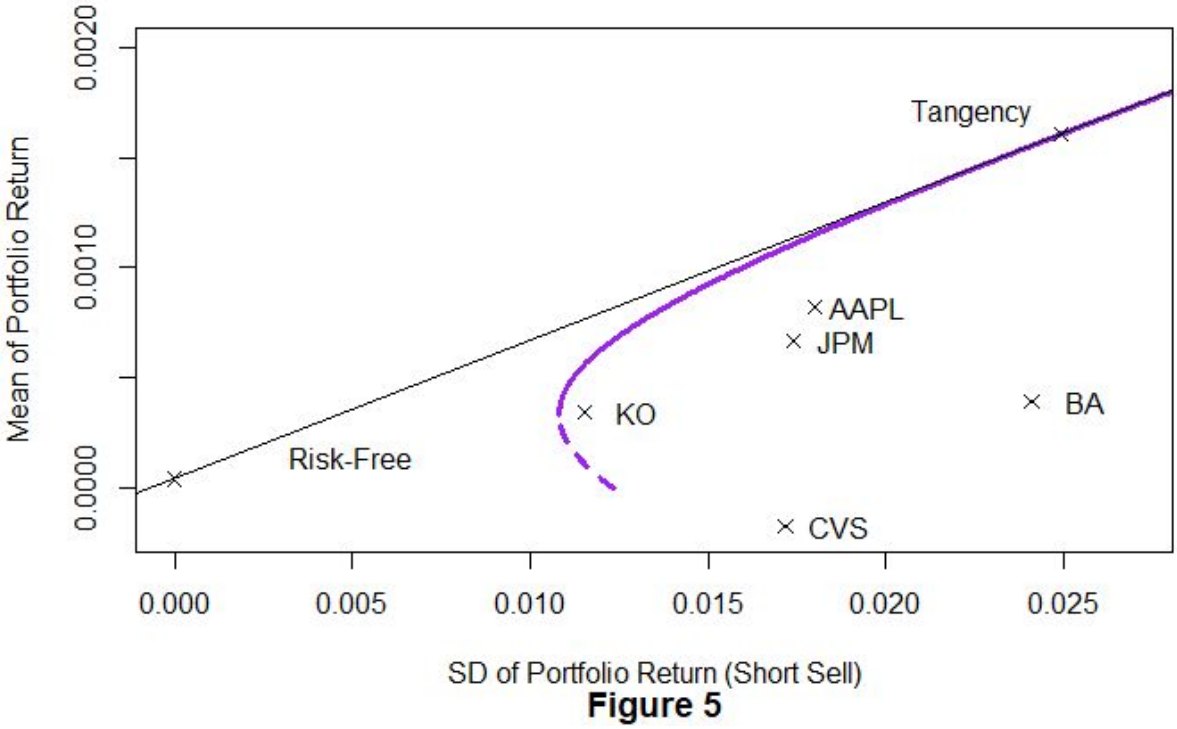

Figure 5

Figure 5: A plot of the mean-variance efficient frontier and buy-and-hold portfolios for AAPL, BA, CVS, JPM, KO (short-selling is allowed)

Points on the efficient frontier indicate optimal portfolios that offer the highest expected return for a defined level of risk, or the lowest risk for a given level of expected return. The straight line is the Capital Market Line and its intercept with the efficient frontier makes the most efficient portfolio, the tangency portfolio. The slope is the Sharpe-ratio of the market portfolio. The tangent point shows the most efficient portfolio, the tangency portfolio.

e. Efficient Frontier (without Short-Selling)

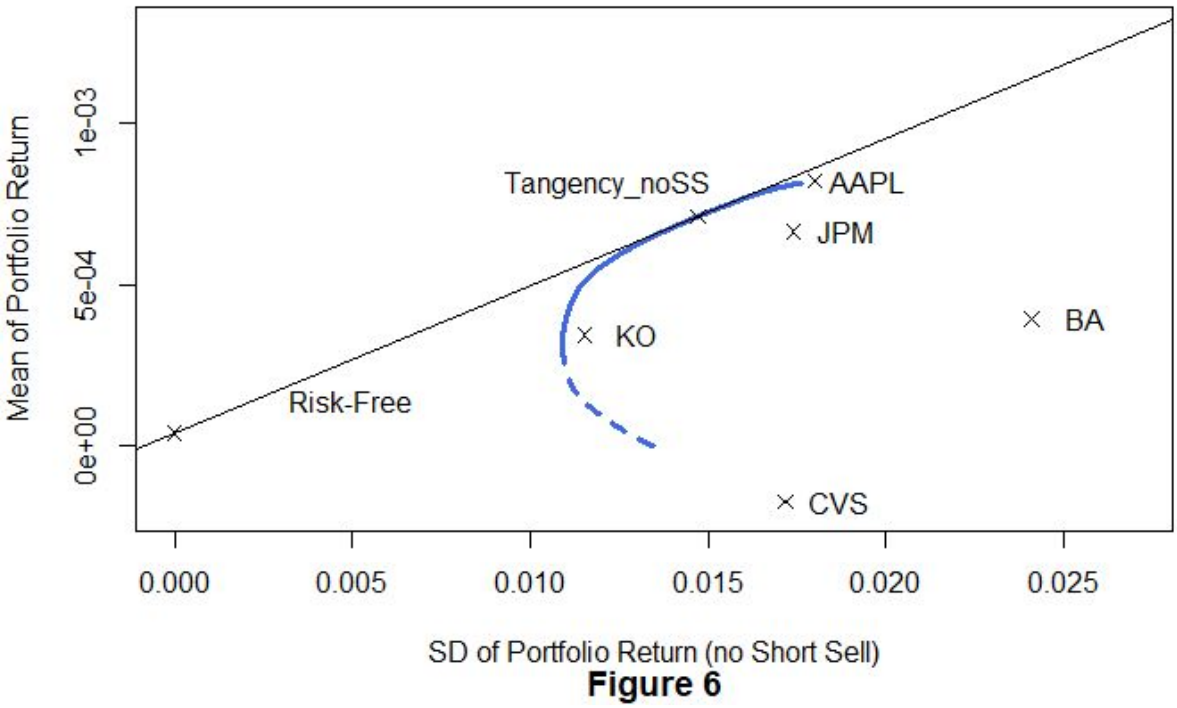

Figure 6

Figure 6: A plot of the mean-variance efficient frontier and buy-and-hold portfolios for AAPL, BA, CVS, JPM, KO (short-selling is not allowed)

Similar to Figure 5, Points on the efficient frontier indicate optimal solutions for our portfolio selection. The tangent point shows the most efficient portfolio, the tangency portfolio.

III. Comparison

Figure 7: The summary plot of portfolio returns.

From figure 7, the Basic Universal Portfolio performs better than the Mean-variance Optimization Portfolio. It might be because the mean-variance optimization portfolio is more conservation.

5. Conclusion

Universal portfolio and mean-variance efficient portfolios are two investment strategies. The universal portfolio needs to rebalance its portfolio daily. The mean-variance efficient portfolio attempts to minimize its variance while maintaining a desired return. From the plot shown in the comparison part, the trading volume-weighted strategy seems to make the largest return all over the time and it also outperformed the market, since the S&P 500's return is 48% at that period. Even in the Coronavirus period, it still outperformed the market. Therefore, we can conclude that if investors want to make investment in the stock market, the trading volume-weighted strategy seems the best selection for them.