Worksheet 8a: Support Vector Machines

COMP90051 Statistical Machine Learning

Semester 1, 2021

Exercise 1. Let K_1 and K_2 be valid kernels on a vector space $\mathcal{X}$ where $c > 0$ be a constant and $f()$ be a real-valued function on $\mathcal{X}$. Prove that the following new kernels are also valid:

(i) $K(u, v) = cK_1(u, v)$

(ii) $K(u, v) = K_1(u, v) + K_2(u, v)$

(iii) $K(u, v) = f(u)K_1(u, v)f(v)$

You may find the following theorem helpful.

Theorem 1 (Mercer's theorem). *A symmetric function $K : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ is a valid kernel on $\mathcal{X}$ if the Gram matrix*

$$\mathbf{K} = \begin{pmatrix} K(x_1, x_1) & \cdots & K(x_1, x_n) \\ \vdots & \ddots & \vdots \\ K(x_n, x_1) & \cdots & K(x_n, x_n) \end{pmatrix}$$

is positive semi-definite for any finite sequence $x_1, \ldots, x_n \in \mathcal{X}$.