SHEET 5

1. Determine the location & order of the zeros for the following functions:
   (a) $(z+16i)^3$
   (b) $\cot z$
   (c) $\cos z$
   (d) $\left(3z^2+1\right)e^{-z}$
   (e) $\left(z^2-1\right)^2(z+4)$

2. Find all the singularities and the corresponding residues of the following:
   (a) $\dfrac{1}{z^2+4}$
   (b) $\dfrac{\cos z}{z^3}$
   (c) $\dfrac{z^2+1}{z^2-2}$
   (d) $\tan z$
   (e) $\dfrac{z^2}{z^4-1}$
   (f) $\dfrac{2z+1}{z^2-z-2}$

3. Show that all singular points of each of the following functions are poles. Also, determine the order of each pole and the residue of the function at the pole.
   (a) $\tan z$
   (b) $\dfrac{1-e^{2z}}{z^4}$
   (c) $\dfrac{e^z}{z^2+\pi^2}$
   (d) $\dfrac{1-\cosh z}{z^3}$

4. Show that
   (a) $\underset{z=-1}{Res}\dfrac{z^{\frac{1}{4}}}{z+1}=\dfrac{1+i}{\sqrt{2}} \qquad |z|>0\ , \quad 0<\arg z<2\pi$
   (b) $\underset{z=i}{Res}\dfrac{\operatorname{Log} z}{\left(z^2+1\right)^2}=\dfrac{\pi+2i}{8}$

(c) $\underset{z=\pi i}{Res} \dfrac{e^z}{\sinh z} + \underset{z=-\pi i}{Res} \dfrac{e^z}{\sinh z} = -2\cos\pi$

5. Use Cauchy's residue theorem to evaluate the following:

   (a) $\displaystyle \oint_C \frac{\sin \pi z}{z^4}\, dz$

   where C is the circle $|z-i| = 2$, described in the positive sense.

   (b) $\displaystyle \oint_C \frac{1}{(z-1)^3(z+4)}\, dz$

   where C is the circle $|z| = 3$, described in the positive sense.

   (c) $\displaystyle \oint_C \frac{z+10}{6z^2+5z+1}\, dz$

   where C is the circle:

   i. $|z| = 1$
   ii. $|z| = 2$
   iii. $|z| = 3$

   (d) $\displaystyle \oint_C e^{-\frac{1}{z}} \sin \frac{1}{z}\, dz$

   where C is the circle $|z| = 1$, described in the positive sense.

   (e) $\displaystyle \oint_C \frac{e^{-z}}{(z-1)^5}\, dz$

   where C is the circle $|z| = 2$, described in the positive sense.

6. Find the value of

$$\oint_C \frac{3z^2+2}{(z-1)(z^2+9)}\, dz$$

   taken counterclockwise around the circle:

   (a) $|z-2| = 2$

   (b) $|z| = 4$

7. If C is the circle $|z| = 1$ described in the positive sense, show that:

   (a) $\displaystyle \oint_C \frac{e^{-z}}{z^2}\, dz = -2\pi i$

   (b) $\displaystyle \oint_C z e^{\frac{1}{z}}\, dz = \pi i$