SHEET 3

1. Evaluate the integral
$$\int_C Re(z)\ dz$$
where C is the line segment from 0 to $1+i$

2. Find the value of the integral
$$\oint_C \frac{1}{z-2}\ dz$$
around:
   (a) The circle $|z-1|=5$
   (b) The circle $|z-2|=4$
   (c) The square with vertices at the points $2\pm 2i$ and $-2\pm 2i$

3. Evaluate the integral
$$\oint_C |z|\,\bar{z}\ dz$$
where C is the closed contour consisting of the upper semi-circle $|z|=1$ and the line segment $y=0$ for $-1\leq x\leq 1$

4. Find the value of
$$\int_C \bar{z}\ dz$$
where C is the parabola $y=x^2$ from $-1+i$ to $1+i$

5. Let C denote the arc of the circle $|z|=2$ from $z=2$ to $z=2i$ that lies in the first quadrant. Show that:
$$\left|\int_C \frac{z-2}{z^4+1}\ dz\right| \leq \frac{4\pi}{15}$$

6. Show that if C is the boundary of the triangle with vertices at the points 0, $3i$, and -4 oriented in the counterclockwise direction, then
$$\left|\int_C (e^z-\bar{z})\ \ dz\right| \leq 60$$

7. Let C denote the positively-oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate each of the following integrals:

   (a) $\displaystyle\oint_C \frac{e^{-z}}{z - \frac{\pi}{2}}\, dz$

   (b) $\displaystyle\oint_C \frac{z}{2z+1}\, dz$

   (c) $\displaystyle\oint_C \frac{\tan\left(\frac{z}{2}\right)}{(z-1)^2}\, dz$

8. Evaluate the integral

$$\oint_C \frac{\cos \pi z}{z^2 - 1}\, dz$$

   around:

   (a) Rectangle with vertices at $2 \pm i$, $-2 \pm i$

   (b) Rectangle with vertices at $\pm i$, $2 \pm i$

9. Evaluate the integral

$$\oint_C \frac{z^2}{z^2 + 4}\, dz$$

   where C is the square with vertices at ± 2, $\pm 2 + 4i$

10. Find the value of the integral

$$\oint_C \frac{1}{z+2}\, dz$$

    where C is the unit circle $|z| = 1$

    And hence, deduce that

$$\int_0^{\pi} \frac{1 + 2\cos\theta}{1 + 4\cos\theta} d\theta = 0$$

11. Show that

$$\oint_C \frac{e^{zt}}{(z^2+1)}\, dz = \pi i \left(\sin t - t\cos t\right)$$

    where $t > 0$ and C is the circle $|z| = 3$

12. Evaluate the integral

$$\oint_C \frac{(3i - z)}{(z - i)(z + 2i)}\, dz$$

where C is the circle:

(a) $x^2 + y^2 = 9$

(b) $|z + 1| = 2$

13. Using Cauchy's integral formula, integrate the following counterclockwise:

(a) $\displaystyle\oint_C \frac{z + 2}{z - 2}\, dz$ where $C : |z - 1| = 2$

(b) $\displaystyle\oint_C \frac{e^{3z}}{3z - i}\, dz$ where $C : |z| = 1$

(c) $\displaystyle\oint_C \frac{\sinh \pi z}{z^2 - 3z}\, dz$ where $C : |z| = 1$

(d) $\displaystyle\oint_C \frac{\tan z}{z - i}\, dz$ where C is the triangle with vertices 0 and $\pm 1 + 2i$