SHEET 4

1. Obtain the Maclaurin series representation of

$$z\cosh(z^2) = \sum_0^\infty \frac{z^{4n+1}}{(2n)!}$$

2. Obtain the Taylor series representation of e^z on the form

$$e^z = e\sum_0^\infty \frac{(z-1)^n}{n!}, \quad |z-1| < \infty$$

3. Find the Maclaurin series of

$$f(z) = \frac{z}{z^2+9}$$

4. Find the Laurent series of the following functions about their isolated singularities

   (a) $f(z) = \dfrac{e^{2z}}{(z-1)^3}$

   (b) $f(z) = (z-3)\sin\left(\dfrac{1}{z+2}\right)$

   (c) $f(z) = \dfrac{z-\sin z}{z^3}$

5. Find a series representation for the following functions in the indicated domains

   (a) $f(z) = \dfrac{1}{z(z-3)}$ A. $0 < |z| < 3$ B. $3 < |z| < \infty$

   (b) $f(z) = \dfrac{1}{z(1+z^2)}$ A. $0 < |z| < 1$ B. $1 < |z| < \infty$

   (c) $f(z) = \dfrac{(z+1)}{(z-1)}$ A. $|z| < 1$ B. $1 < |z| < \infty$

   (d) $f(z) = \dfrac{1}{z^2(1-z)}$ A. $0 < |z| < 1$ B. $1 < |z| < \infty$

6. Find the radius & circle of convergence for the following series:

(a) $\displaystyle\sum_{0}^{\infty} \frac{1}{(1-2i)^{k+1}}(z-2i)^k$

(b) $\displaystyle\sum_{0}^{\infty} (1+3i)^k(z-i)^k$

(c) $\displaystyle\sum_{0}^{\infty} \frac{(z-2i)J^k}{k^k}$

(d) $\displaystyle\sum_{0}^{\infty} \left(\frac{2-i}{1+5i}\right) z^k$

(e) $\displaystyle\sum_{0}^{\infty} \frac{2^k}{k(k+1)} z^{2k+1}$

(f) $\displaystyle\sum_{0}^{\infty} \frac{(2k)!}{(k!)^2}(z-3i)^k$