

# Determining the Position of an Accelerometer Sensor using Linear Regression

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## Abstract

This research describes a repeatable method of locating an accelerometer sensor in any device that fits within an 8.5" by 11" area. This method is useful since most mobile phone manufacturers do not specify the location of the accelerometer sensors in their devices and this specification is necessary for accurate motion tracking and similar applications. An accelerometer device was taped to the top left corner of a 3D-printed 8.5" by 11" frame, which was placed onto a turntable. After recording acceleration data at 78RPM for 10 seconds, the turntable was stopped, the device was moved 1cm to the right, and the turntable was restarted. This was repeated until the device reached the top right corner of the frame, after which the device was replaced at the top left corner and followed a similar process but moving downwards until it reached the bottom left corner instead. These recordings were then used in a linear regression algorithm to determine the radius of rotation, and therefore the position of the sensor. The results showed that the calculated location was inaccurate by 0.49cm in the horizontal direction and 0.03cm in the vertical direction. This method could be improved with the manufacturing of a more stable spinning frame to minimize wobbling which caused unwanted accelerations. This method is a simple, fast, and viable method of locating a sensor that may prove crucial for fields spanning from mobile application development to medical supply manufacturing which require an accurate positioning of an accelerometer sensor.

## Introduction

In a world where an increasing number of devices require motion-tracking sensors, from mobile phones to fitness trackers<sup>1</sup> to medical applications<sup>2</sup>, these motion-tracking sensors, specifically accelerometers, are becoming crucial in the development of today's and tomorrow's electronics. An accelerometer is a very small sensor, usually less than 2mm in length<sup>3</sup>, that tracks its acceleration in all 3 directional axes. While an accelerometer sensor is a core electronic in a mobile device, many mobile phone manufacturers do not indicate the location of this sensor on the part diagrams of their devices. This research was aimed at solving this inconsistency. Past experiments have performed very few trials aimed at obtaining the location of an accelerometer sensor<sup>4</sup>. Therefore, the objective of this research was to determine if linear regression would be an accurate and viable method of determining the location of an accelerometer sensor, since linear regression demands a large amount of trials to obtain results. It was hypothesised that this method would be an accurate and viable one because it required many trials to calculate its results, which increases the reliability of one's data. If this hypothesis would be true, then the linear regression-based experiment would result in an error margin of 2mm, which is the average length of the sensor itself, proving its relative accuracy to the size of the average accelerometer.

## Method

### 1. Data Collection

The designed experiment began with the creation of the frame on which an accelerometer was placed. A 16-inch disk was cut out of whiteboard material and a hole was cut into the center of this disk. A frame of dimensions 8.5" by 11" was 3D printed and placed onto the disk, ensuring that the center of the frame was lined up with the center of the disk. The item containing the disk with the frame atop it was referred to as the SpinFrame for simplicity. Next, the hole in the center of the disk was placed onto the spindle of a turntable capable of rotating at 78RPM.

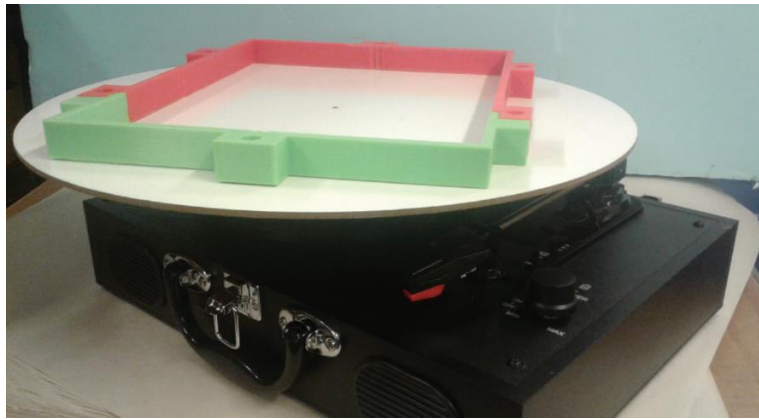


Figure 1: Turntable with SpinFrame attached<sup>5</sup>.

An accelerometer-containing device, namely the PocketLab Voyager, was taped in the top left corner of the frame on the SpinFrame. It was ensured that the device's local axes were aligned with those of the frame. Considering the frame in 2 dimensions, the device's positive  $x$  axis was aligned to the right of the device, and its positive  $y$  axis was facing forwards from the device. The PocketLab Voyager device was chosen because the location of the accelerometer sensor inside the device was known by observing its circuit diagram, and because of its small and light form factor. The size of its accelerometer sensor was slightly larger than the average sensor at 5mm, though this was only important when the error was ultimately calculated<sup>4</sup>. The PocketLab Voyager was connected using Bluetooth to an experimenter's laptop for data collection. The turntable was switched on, and after reaching its maximum rotational speed of 78RPM, acceleration data was recorded for approximately 10 seconds. The recording was then stopped, followed by the turntable. The accelerometer-containing device was then moved 1cm to its right, ensuring that its local axes remained pointing in the same direction. This process was repeated until the device reached the top right corner of the SpinFrame. The device was then replaced at the top left corner of the frame, and similar steps were repeated, except the device was moved 1cm downwards between runs instead of 1cm rightwards. This process was repeated until the device reached the bottom left of the frame, at which point the data collection was complete.

## 2. Data conversion

In order to use linear regression as a means of analyzing the obtained data, one must stop to consider the theory behind the physics involved in the experiment. The centripetal acceleration of an object in rotational motion is constant while its rotational velocity is constant, like the accelerometer sensor on the SpinFrame. Knowing this, the centripetal acceleration was the average recorded acceleration and the angular velocity was always 78RPM. With these, the radius of each run was calculated using the equation:

$$\vec{R} = -\frac{\vec{a}}{\omega^2} \quad \text{Eq. 1}$$

where  $\vec{R}$  was the radius from the center of rotation to the accelerometer sensor,  $\vec{a}$  was the centripetal acceleration during a given run, and  $\omega$  was the angular velocity during a given run. All vectors used in this experiment belong to the 2D plane of the SpinFrame, where the center of rotation is the origin.

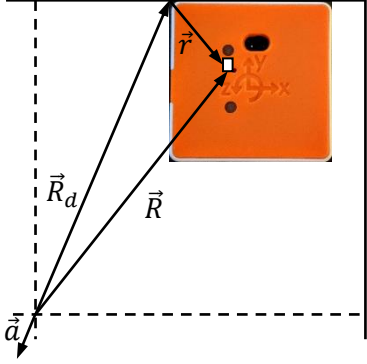


Figure 2: Vector diagram representing the relationships between various vectors used in locating the accelerometer sensor (depicted as the small rectangle) on the PocketLab Voyager. These vectors belong to the 2D plane which represents the SpinFrame.

To obtain the position of the accelerometer sensor with reference to a fixed point on the system, the position vector  $\vec{R}_d$  was introduced. This vector, as seen in Figure 2, described the position of the top left corner of the accelerometer device with respect to the origin. The final vector,  $\vec{r}$ , described the position of the accelerometer sensor with respect to the top left corner of the device, and was therefore the vector that would ultimately need to be found. The vector equation describing the relationship between the vectors  $\vec{R}$ ,  $\vec{R}_d$  and  $\vec{r}$ , as depicted in Figure 2, was:

$$\vec{R} = \vec{R}_d + \vec{r} \quad \text{Eq. 2}$$

Rearranging to isolate  $\vec{r}$ , the equation became:

$$\vec{r} = \vec{R} - \vec{R}_d \quad \text{Eq. 3}$$

$\vec{R}_d$  was then substituted from Equation 1:

$$\vec{r} = -\frac{\vec{a}}{\omega^2} - \vec{R}_d \quad \text{Eq. 4}$$

This equation relates the recorded acceleration data to the position of the accelerometer sensor with respect to the top left corner of the accelerometer device. Having derived this equation, the last step was to utilize linear regression to obtain the final results.

### 3. Linear Regression

Linear regression requires an independent and a dependent variable to function. In this case, the independent variable was  $\vec{R}$ , and the dependent variable was  $\vec{r}$ . Since the experiment was first performed by varying the horizontal position of the device on the SpinFrame, the first linear regression compared the horizontal component of  $\vec{R}$ , labelled  $R_x$ , to the horizontal component of  $\vec{r}$ , similarly labelled  $r_x$ . The second linear regression was in the vertical direction rather than the horizontal direction, meaning it compared the vertical component of  $\vec{R}$ , labelled  $R_y$ , to the vertical component of  $\vec{r}$ ,  $r_y$ .

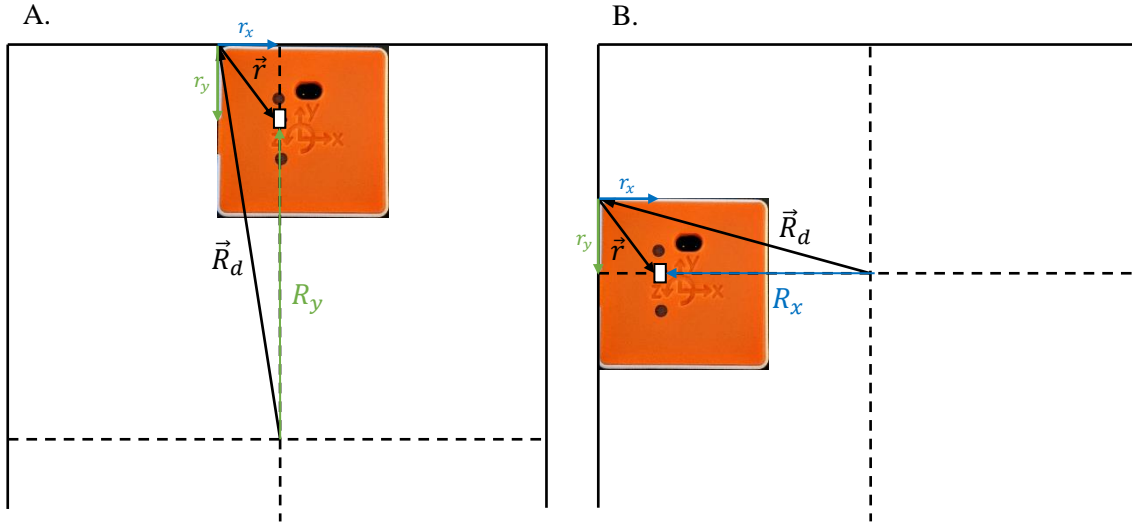


Figure 3: Vector diagrams representing the locations at which the accelerometer sensor of the PocketLab Voyager would line up with one of the SpinFrame's two axes. The diagrams also depict the horizontal and vertical components of  $\vec{r}$  and of  $\vec{R}$ . (A) The accelerometer sensor is lined up with the vertical axis of the SpinFrame. (B) The accelerometer sensor is lined up with the horizontal axis of the SpinFrame.

The goal of the linear regression was to obtain the  $\vec{r}$  value of the obtained linear trend line when  $\vec{R}$  is equal to 0. Figure 3A depicts where  $R_x = 0$ , and Figure 3B depicts where  $R_y = 0$ . This did not occur during the experiment and it did not need to, seeing as this point would be calculated by plugging in  $R_x = 0$  or  $R_y = 0$  to the linear equations obtained from the linear regression. The obtained  $r_x$  and  $r_y$  values described, respectively, the horizontal or vertical distances from the top left of the accelerometer device to the accelerometer sensor. These were then compared to the schematic of the PocketLab Voyager to draw conclusions.

## Results

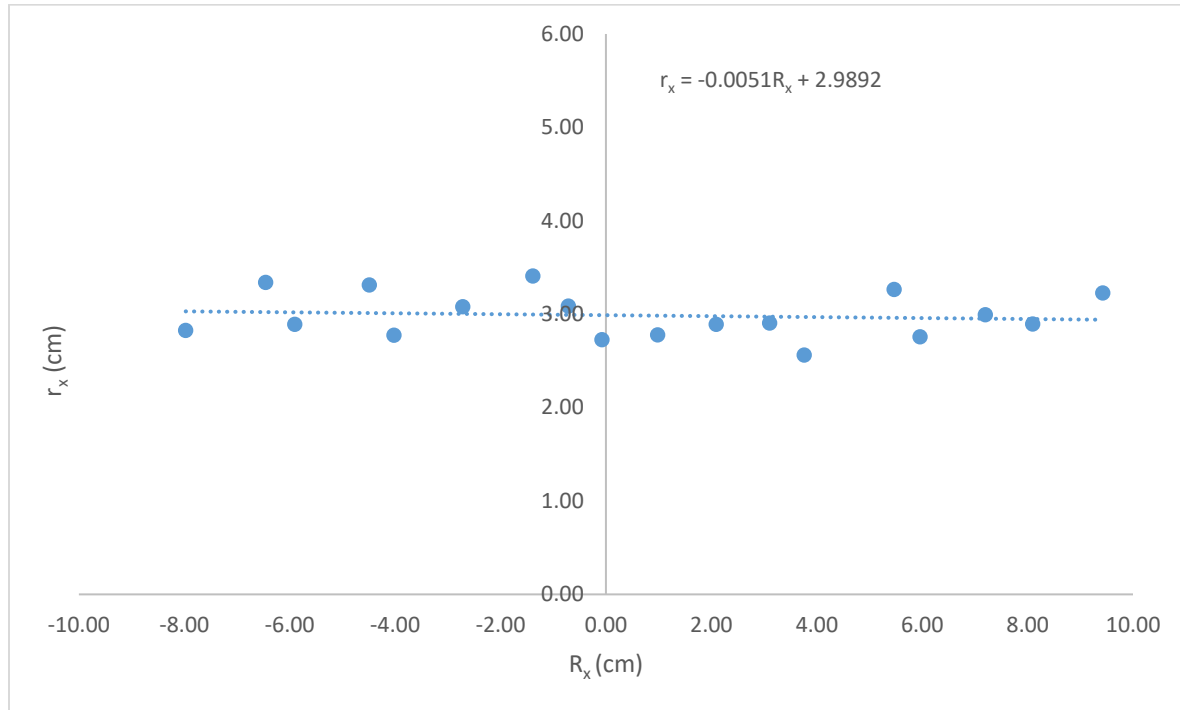


Figure 4: The horizontal distance from the top left corner of the accelerometer device to the accelerometer sensor ( $r_x$ ) compared to the horizontal distance from the center of rotation to the sensor ( $R_x$ ).

This graph was the result of the first linear regression, in which the accelerometer device was being moved horizontally along the edge of the SpinFrame. It compares  $r_x$ , the distance from the top left of the PocketLab Voyager to the accelerometer sensor, to  $R_x$ , the distance from the center of rotation to the accelerometer sensor. Each point on the graph was the result of one run during experimentation. This graph describes the linear relation between the two position vectors, as described by the linear trend line equation on the graph. The estimated horizontal position at  $R_x = 0$  was 2.99cm from the top left corner of the device, while the schematic position was 2.50cm with respect to the top left corner<sup>4</sup>. Using this, the horizontal error was 0.49cm, or 19.4%.

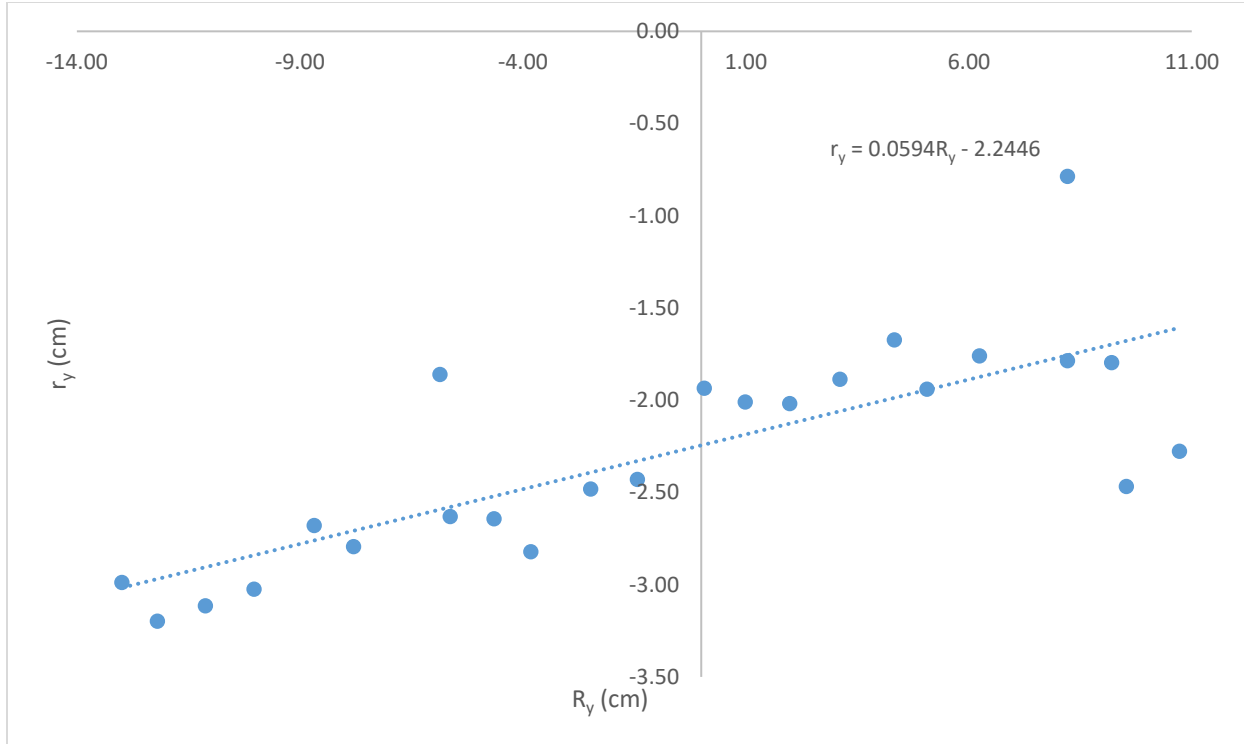


Figure 5: The vertical position of the accelerometer sensor with respect to the top left corner of the accelerometer device ( $\vec{r}_y$ ) as a function of the vertical position of the accelerometer sensor with respect to the center of rotation ( $\vec{R}_y$ ).

This graph was the result of the second linear regression, and is very similar to Figure 4 in terms of function. It describes the vertical distance of the accelerometer sensor instead of Figure 4's horizontal distance, as this graph's data was obtained while varying the vertical position of the accelerometer device during the experiment, rather than varying its horizontal position. This graph has more data points than Figure 4, simply because there were more runs necessary for the accelerometer device to reach the bottom left corner of the SpinFrame seeing as the SpinFrame had dimensions 8.5" by 11". The estimated vertical position when  $R_y = 0$  was 2.24cm downwards from the top left corner of the device, while the schematic position was 2.10cm downwards from the same point of reference<sup>4</sup>. This resulted in a vertical error of 0.14cm, or 6.67%.

## Discussion

The linear regression method resulted in a horizontal error of 0.49cm, or 19.4%, and a vertical error of 0.14cm, or 6.67%. Though this error is significantly larger than the 2mm of error described in the initial prediction, the size of the PocketLab Voyager's accelerometer sensor was found to be on the order of 5mm rather than the expected 2mm. Therefore, the estimated position of the accelerometer sensor may have less error than one might expect, since the position may lie within the bounds of the 5mm sensor. Considering this fact, the hypothesis was proven, but there were still inaccuracies that can be improved through various changes to the experiment.

The error was most likely due to the visible wobble of the SpinFrame while it rotated on the turntable. This would have resulted in error because there were accelerations that were recorded by the accelerometer sensor that were not caused by its rotation. This would cause the magnitude and direction of the centripetal acceleration,  $\vec{a}$ , to differ from their expected values. The inaccurate acceleration would directly affect  $\vec{R}$  because of their relation described in Equation 1, affecting the final results. The wobble may have occurred as a result of imperfections in the center of mass of the system. It is possible that, because the frame is rectangular rather than square-shaped, the additional mass of the frame combined with the mass of the accelerometer device near the far edge of the frame would have resulted in the center of mass being located between the center of rotation and the edge of the frame. Having a center of mass located elsewhere from the center of rotation will cause gravity to affect its new location. The force of gravity on this point would cause the wobbling of the SpinFrame. This would also explain why there is more error when the device was moved horizontally on the edge of the frame than when it was being moved vertically. When it was being moved horizontally, one side of the device was always in contact with the edge of the frame that was the furthest from the center of rotation, thus bringing the center of mass closer to the edge, while when it was being moved vertically, it was being shifted along the edge of the frame that was significantly closer to the center of rotation, meaning the center of mass was still closer to the device, but to a lesser degree.

To fix this source of error, one can increase the mass of the base of the SpinFrame, such that it is significantly more massive than the accelerometer device and the 3D-printed frame. By doing so, one can ensure that the center of mass is unaffected by the location of the accelerometer device and the 3D-printed frame, allowing the center of mass to remain at the center of rotation during the entire experiment.

To confirm the reliability of the linear regression method, the method should be repeated with various devices that both contain accelerometer sensors and have circuit diagrams available in order to compare the estimated location of the accelerometer sensor to the schematic's location. The method was only tested using the PocketLab Voyager sensor as a result of time limitations. Ideally, this would be done after a solution to the aforementioned sources of error would be found and implemented, to confirm that the linear regression method itself is reliable.

After the accuracy and reliability of the method would be proven through the methods previously mentioned, this method can be used on a larger scale. In its current form, the SpinFrame has a frame of dimensions 8.5" by 11", limiting the size of devices that can be used within its



bounds. Despite being more cumbersome, the SpinFrame can be made larger to fit wider and longer devices, and the frame itself can be made into various shapes instead of its current legal paper dimensions. Seeing as these dimensions were originally chosen to accommodate a pedagogical experiment that required a sheet of paper being placed within the frame, these dimensions need not be repeated in future versions of the SpinFrame for the use of this method. One use for this method is in the medical field. Medical devices are being developed that use accelerometers, such as fall detectors, devices that track one's balance control, and devices that track energy expenditure<sup>1</sup>. Having precise body-attached accelerometer sensors would allow these devices to target specific muscles, or other locations, of small surface area. The described linear regression method may assist these fields by minimizing the region that an accelerometer sensor may be found in, thus improving their ability to place a sensor on such a small surface area. This method may also be used by engineers of wearable technology such as fitness trackers<sup>2</sup> to improve the device's accuracy in a similar fashion to the aforementioned medical devices, and by mobile developers to track precise movements and rotations of mobile devices.

## Conclusion

This paper describes a method of locating an accelerometer sensor in a device using linear regression. The method was performed using a PocketLab Voyager device and linear regression estimated the position of the accelerometer sensor to be at (2.99, 2.24) cm with reference to the top left corner of the device, while the schematic of the device confirmed the sensor's position to be at (2.50, 2.10) cm from the same point of reference. Considering the size of the PocketLab Voyager's accelerometer sensor is on the order of 5mm, the method was deemed accurate, but could be improved by increasing the mass of the SpinFrame's base to ensure its center of mass is located at its center of rotation thus minimizing wobble, and its reliability could be improved by testing the method on various devices with known accelerometer locations. With these changes, the method of locating an accelerometer sensor in a device using linear regression can be useful in medical fields, fitness tracking devices, and mobile development as a means of maximizing precision and reliability of these products.

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