Digital 3D Geometry Processing
Exercise 11 - Solving Laplace Equatuions II

Handout date: 26.11.2018

Submission deadline: 06.12.2018, 23:00 h

Note

Copying of code (either from other students or from external sources) is strictly prohibited! Any violation of this rule will lead to expulsion from the class.

What to hand in

A .zip compressed file renamed to `Exercise`*n*`-GroupMemberNames.zip` where *n* is the number of the current exercise sheet. It should contain:

- Hand in **only** the files you changed (headers and source). It is up to you to make sure that all files that you have changed are in the zip.
- A `readme.txt` file containing a description on how you solved each exercise (use the same numbers and titles) and the encountered problems.
- Other files that are required by your `readme.txt` file. For example, if you mention some screenshot images in `readme.txt`, these images need to be submitted too.
- Submit your solutions to Moodle before the submission deadline.

Goal

In this exercise you will implement deformation of the triangular mesh. Given a set of fixed vertices and a set of shifted vertices, the goal is to compute smooth displacements of the remaining vertices.

User Interface

You do not need to implement anything for this part. The framework provided with Exercise 11 offers an interface for selecting fixed vertices, shifted vertices and displacement vector. While describing the interface we refer to Figure 1.

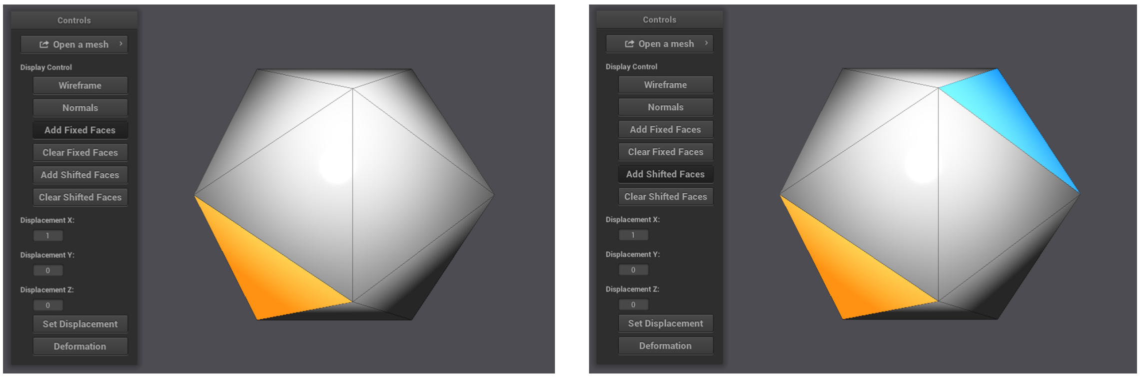

Figure 1: Generating Edges at Isolines of Harmonic Functions.

- To specify fixed vertices, toggle `Add Fixed Faces` button on the menu. To select a face, right click on it, while holding `control` key. All the vertices of the face are added to the fixed vertices. The selected fixed faces are highlighted in yellow. When you finish specifying the fixed faces un-toggle the button `Add Fixed Faces` on the menu.

- To specify shifted vertices, toggle `Add Shifted Faces` button on the menu. To select a face, right click on it, while holding `control` key. All the vertices of the face are added to the shifted vertices. The selected fixed faces are highlighted in blue. When you finish specifying the fixed faces un-toggle the button `Add Shifted Faces` on the menu.

- To select a different set of fixed vertices or shifted vertices, press the button `Clear Fixed Faces` or `Clear Shifted Faces`.

- **Important Note:** Make sure that you do not select the same vertex both as fixed and shifted. These constraints are mutually contradicting. Verify that only one of the buttons is toggled at any time: `Add Fixed Faces` or `Add Shifted Faces`.

- To specify the shift vector, enter its X, Y and Z components in the windows `Displacement X`, `Displacement Y` and `Displacement Z` correspondingly. Press the button `Set Displacement` to see the displacement vector. To edit the displacement vector, enter the new values in the windows and press `Set Displacement` again.

Solving Laplace Equations

To compute the smooth displacements of the unconstrained vertices, solve a linear system $\mathbf{L}^2\mathbf{x} = \mathbf{b}$, where $\mathbf{L}^2$ is squared Discrete Laplace-Beltrami matrix, $\mathbf{x}$ are the unknown values of the displacements, and $\mathbf{b}$ is equal to 0 for all vertices except for the ones for which the values of displacements $\mathbf{x}_i$ are constrained.

In order to obtain a smooth mesh deformation, one needs to constrain the displacements of the two sets of vertices. The displacements of the selected fixed vertices should be set to zero and the displacements for the selected shifted vertices should be equal to the specified displacement vector. To constrain the displacement $\mathbf{x}_i$, replace the i-th row of $\mathbf{L}^2$ by $[0,...0,1,0...0]$ with 1 at position i.

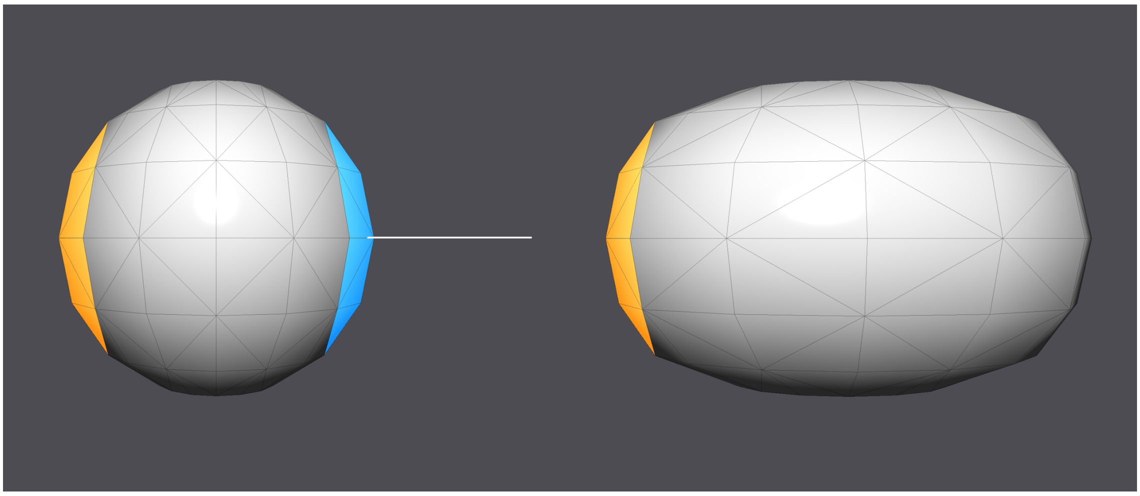

Figure 2: Deforming a sphere.

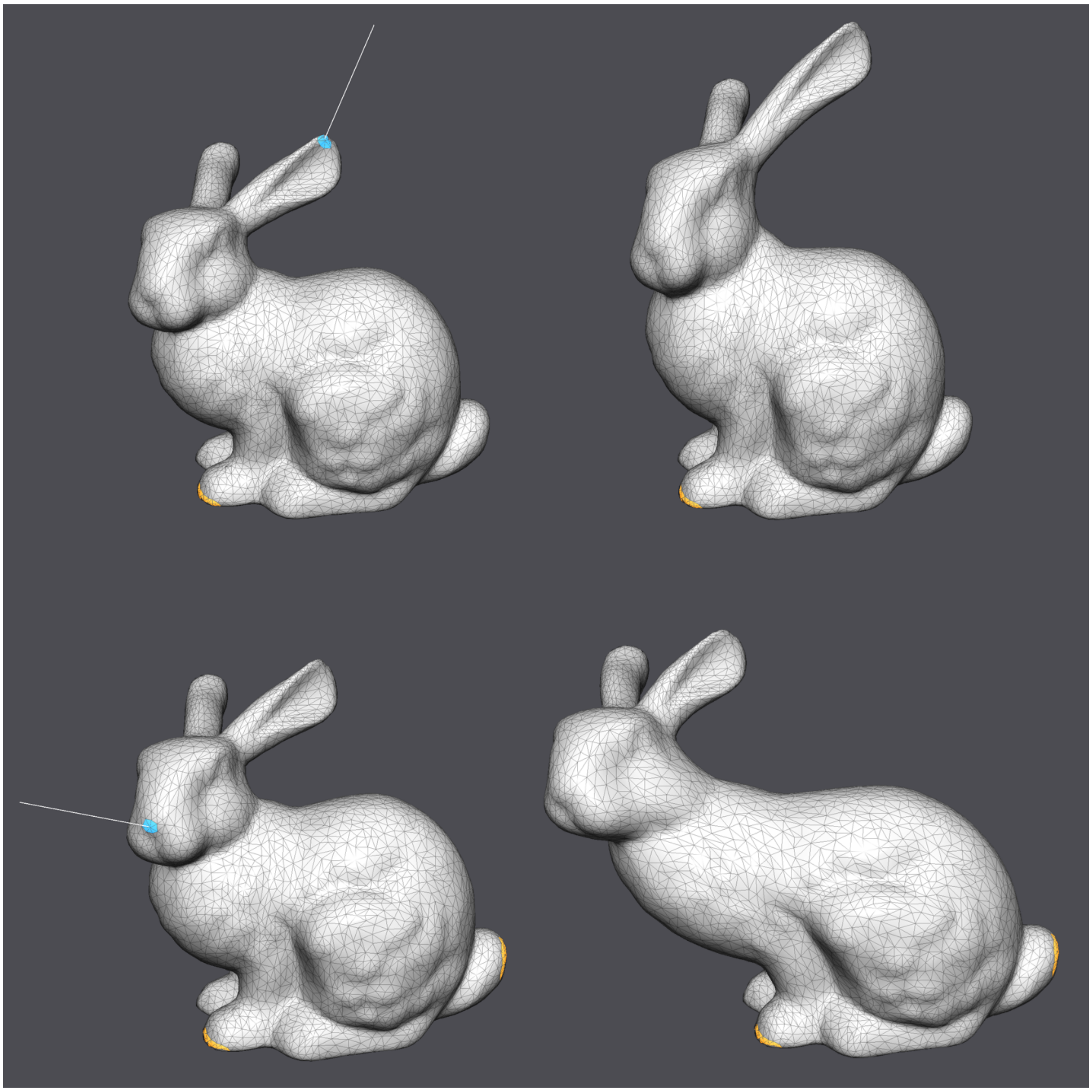

Figure 3: Deforming a bunny.

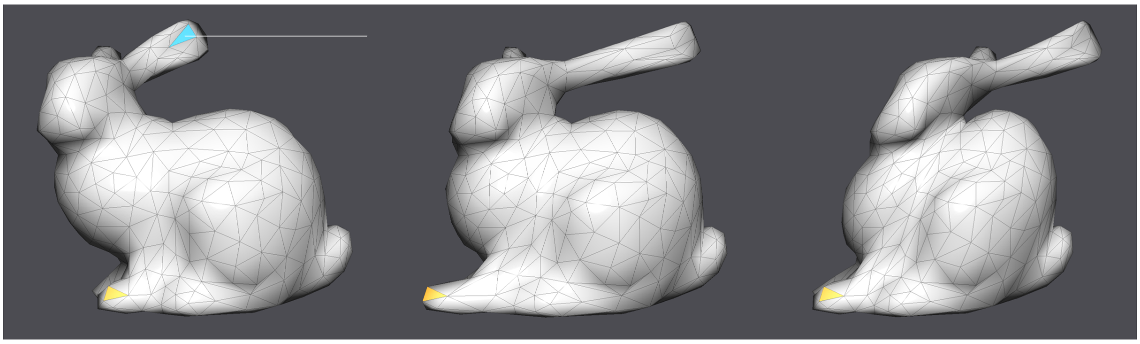

Figure 4: $\mathbf{Lx} = \mathbf{b}$ (middle) versus $\mathbf{L}^2\mathbf{x} = \mathbf{b}$ (right).

Hint: Once you set up the **sparse** squared Discrete Laplace-Beltrami matrix, Eigen library only allows to traverse it column-wise. Since the matrix $\mathbf{L}^2$ is symmetric, edit the columns $\mathbf{A}$ to set the constraints and transpose the result.

The displacements are computed independently for X, Y and Z axis. Implement the linear solve for one axis in the function `deformation_axis()`.

Comparing $\mathbf{Lx} = \mathbf{b}$ and $\mathbf{L}^2\mathbf{x} = \mathbf{b}$

Replace squared Discrete Laplace-Beltrami matrix $\mathbf{L}^2$ by Discrete Laplace-Beltrami matrix $\mathbf{L}$ (Figure 4). Comment on the difference in the results.

Comparing Different Laplacian Weights

Replace the cotangent weight used in Discrete Laplace-Beltrami matrix L by the uniform weight. Comment on the difference in results.

Comparing with Physical Deformation

Comment on how the computed deformations differ from a real physical material, e.g. an elastic rubber membrane or a thin metal sheet. Create your own example shapes and deformations where those differences would be easily explainable. **Please upload your (.obj/.stl) model as well as images of its deformations in the submission.**