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1D elasticity plasticity using Machine Learning

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1. Introduction

A nueral network constitutive model for 1D cyclic plasticity has been proposed by Teranishi [2022]. He proposes a neural network-based constitutive model for uniaxial cyclic plasticity, integrating a feed-forward neural network with a return mapping algorithm. The model demonstrates improved computational efficiency, reducing calculation time by 70%, and maintains accuracy under random loading conditions, making it suitable for large-scale structural simulationsâ.

2. Continum model

2.1 Analytical model

We restrict ourselves to 1D and define the total elastistic strain ε and it's decomposition into elastic ε_e and plastic parts ε_p by

$$\varepsilon = u1x$$

$$\varepsilon = \varepsilon_e + \varepsilon_p$$

The cauchy stresses with Young's modulus E are

$$\sigma = E\,(\varepsilon - \varepsilon_p)$$

and the yield criteria is defined by

$$\phi = |\sigma| - \beta \leq 0 \quad \text{with} \quad \beta = \gamma_o + h\alpha$$

where γ_o is the initial yield strength. h is the linear hardening parameter and α is the accumlated equivalent elasto plastic strain variable.

Associated flow rule with

$$\mathcal{D}^p = \sigma : \dot{\varepsilon}^p - \beta\dot{\alpha}$$

$$\mathcal{L}^p(\boldsymbol{\sigma}, \boldsymbol{\beta}, \lambda) = -\mathcal{D}^p(\boldsymbol{\sigma}, \boldsymbol{\beta}) + \lambda\phi(\boldsymbol{\sigma}, \boldsymbol{\beta})$$

optimization conditions

$$\partial_\sigma \mathcal{L}^p = -\dot{\varepsilon}^p + \lambda\,\partial_\sigma\phi = 0 \quad \Longrightarrow \quad \dot{\varepsilon}^p = \lambda\,\partial_\sigma\phi$$

$$\partial_\beta \mathcal{L}^p = \dot{\alpha} + \lambda\,\partial_\beta\phi = 0 \quad \Longrightarrow \quad \dot{\alpha} = \lambda\,\partial_\beta\phi$$

$$\partial_\gamma \mathcal{L}^p = \phi = 0$$

Evaluation of $\dot{\phi} = 0$

$$\dot{\phi} = \partial_\sigma\,\phi\,\dot{\sigma}$$

with

$$\dot{\sigma} = E(\dot{\epsilon} - \dot{\varepsilon_p}) \quad and \quad \dot{\beta} = h\dot{\alpha} \tag{1}$$

we get

$$\dot{\phi} = \partial_\sigma\,\phi\,\mathbb{E}\,(\dot{\epsilon} - \lambda\partial_\sigma\phi) \quad and \quad -\lambda h = 0$$

Using the defination $sign(\sigma) = \frac{\sigma}{|\sigma|} =: n \quad yields$

$$nE\dot{\epsilon} - \lambda nEn - \lambda h = 0 \implies \lambda = \frac{nE\dot{\epsilon}}{nEn + h}$$

Inserting λ into equation (1)

$$\dot{\sigma} = E(\dot{\epsilon} - \lambda n) = E(\dot{\epsilon} - \frac{nnE}{nEn + n}\dot{\epsilon})$$

$$\dot{\sigma} = (E - \frac{EnnE}{nEn + h})\dot{\epsilon} = \mathbf{C}^{ep}\dot{\boldsymbol{\varepsilon}}$$

2.2 Algorithmic

Integration

$$\varepsilon^p_{n+1} = \varepsilon^p_n + \int\limits_{\Delta t} \dot{\varepsilon}^p \, dt = \dot{\varepsilon}^p_{n+1}\Delta t = \varepsilon^p_n + \Delta\varepsilon^p$$

$$\alpha^p_{n+1} = \alpha_n + \int\limits_{\Delta t} \dot{\alpha} \, dt = \dot{\alpha}_{n+1}\Delta t = \alpha_n + \Delta\alpha$$

Evaluation of the yield function by $\epsilon = \epsilon_{n+1}$ given:

$$\phi = |\sigma| - \beta = |E(\epsilon - \epsilon^p_n - \Delta\varepsilon^p)| - (\beta_n + h\Delta\alpha)$$

Define

$$\phi^{\text{trial}} = |\sigma^{\text{trial}}| - \beta_n \quad with \quad \sigma^{\text{trial}} = E(\epsilon - \varepsilon^p_n)$$

If $\phi^{\text{trial}} > 0$, solve for plastic values: $n^{\text{trial}} \equiv \text{sign}(\sigma^{\text{trial}})$

$$\phi = |\sigma^{\text{trial}} - E\Delta\varepsilon^p| - \beta_n - h\Delta\alpha$$
$$\phi = |\sigma^{\text{trial}} - E\Delta\gamma\, n| - \beta_n - h\Delta\gamma$$
$$\phi = ||\sigma^{\text{trial}}|n^{\text{trial}} - E\Delta\gamma\, n| - \beta_n - h\Delta\gamma$$

with $n = n^{\text{trial}}$ and $\delta\gamma = \lambda\Delta t$

$$\phi_{n+1} = ||\sigma^{\text{trial}}| - E\Delta\gamma\, n| - \beta_n - h\Delta\gamma$$
$$\phi_{n+1} = ||\sigma^{\text{trial}}| - \beta_n(E + h)\Delta\gamma$$
$$\phi_{n+1} = \phi^{\text{trial}} - \beta_n(E + h)\Delta\gamma \tag{3}$$

yields for $\phi_{n+1} = 0$

$$\Delta\gamma = \frac{\phi^{\text{trial}}}{E + h}, \quad \Delta\gamma \geq 0$$

Update of variables

$$\epsilon^p_{n+1} = \varepsilon^p_n + \Delta\gamma n^{\text{trial}} \quad \text{with} \quad n^{\text{trial}} = \text{sign}(\sigma^{\text{trial}})$$
$$\sigma_{n+1} = E(\epsilon_{n+1} - \epsilon_{n+1} - \varepsilon^p_{n+1})$$
$$\alpha_{n+1} = \alpha_n + \Delta\gamma$$

Algorithmic box:

Given: E, y_0, h, ε_{n+1}, ε^p_n, α^n
Compute:

$$\sigma^{\text{trial}} = E(\varepsilon_{n+1} - \varepsilon^p_n)$$

$$\beta^{\text{trial}} = \beta_n$$

$$\phi^{\text{trial}} = |\sigma^{\text{trial}}| - \beta^{\text{trial}}$$

If $\phi \leq 0$: Elastic step
Else: $\varepsilon^p_{n+1} = \varepsilon^p_n$; $\alpha_{n+1} = \alpha_n$

$$\Delta\gamma = \frac{\phi^{\text{trial}}}{E + h}$$

$$\varepsilon^p_{n+1} = \varepsilon^p_n + \Delta\gamma n^{\text{trial}} \quad \text{with} \quad n^{\text{trial}} = \text{sign}(\sigma^{\text{trial}})$$

$$\sigma_{n+1} = E(\varepsilon_{n+1} - \varepsilon^p_{n+1})$$

$$\alpha_{n+1} = \alpha_n + \Delta\gamma$$

References

M. Teranishi. Neural network constitutive model for uniaxial cyclic plasticity based on return mapping algorithm. In *Mechanics research communications*, volume 119, pages 103815. Elsevier, 2022.

A Appendix

A.1 Inkscape handling and export

In sub-directory `make_figures` the inkscape-template file `template_tm.svg` can be found. It includes predefined arrows indicating forces, coordinate systems, velocities, etc. corresponding to the "Technische Mechanik" books.

Thus, open the above mentioned file an start drawing your sketch, see Figure 1. All text should be written, as you do it in LaTeX. To add the template permanently to the list of templates directly in inkscape unter "Datei → Vorlagen..." copy the file `template_tm.svg` to following directoy depending on your operating system:

- For Linux and Mac it is `~/.config/inkscape` , where ˜ is representing the path to user's home-directory
- For Windows it is `%APPDATA%\\Inkscape` (just use it in Explorer as is)

Figure 1: Example of drawing in inkscape

To keep an editable picture, please save it to an inkscape-SVG file via "Datei → Speichern unter..." or "File → Save as...“ and choose "Inkscape-SVG (*.svg)" as file format.

For the export to a file format, which can be include to a LaTeX document while the text in the picture is interpreted by the LaTeX processor, please use "Datei → Kopie speichern ..." or "File → Save a copy“. There you should choose "Encapsulated Postscript (*.eps)“ as file format and enter a filename. After a click on "Speichern“ or "Save“ an additional window with save options is shown. Please active/deactivate the option as shown in Figure 2 and click "OK“. Choosing "Seitengröße von Dokument nutzen“ will cause a (wrong) scaling of all graphic objects during LaTeXprocessing.

After this two files will be saved. One file with the extension "eps“ including all graphic objects and another file with the extension "eps_tex“ including all text objects. Please see Figure **??** regarding the LaTeX-include.

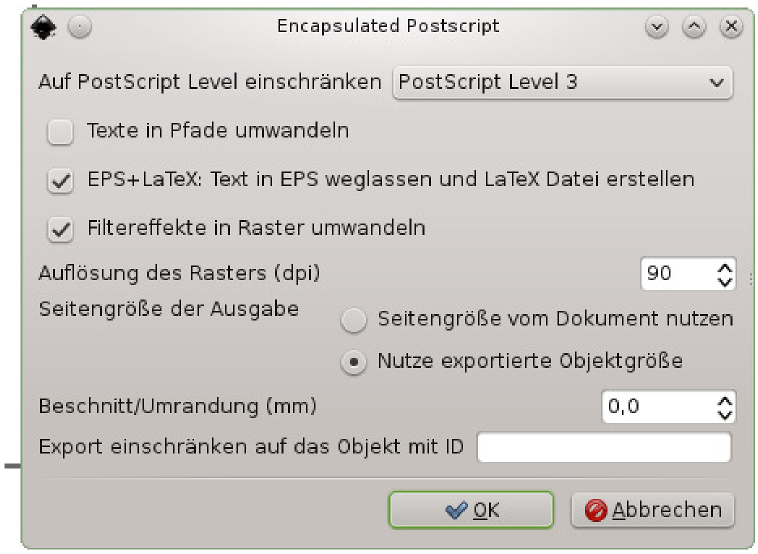

Figure 2: Example of drawing in inkscape