Massey University
SINGAPORE

280371 PROCESS ENGINEERING OPERATIONS

Drying

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Useful resources :

Transport Processes and Unit Operations
Christie J. Geankoplis
Prentice Hall; 3rd Edition, 1993

Drying Phenomena: Theory and Applications
https://onlinelibrary.wiley.com/doi/book/10.1002/9781118534892

Drying Lecture 1 Introduction

meat

rice

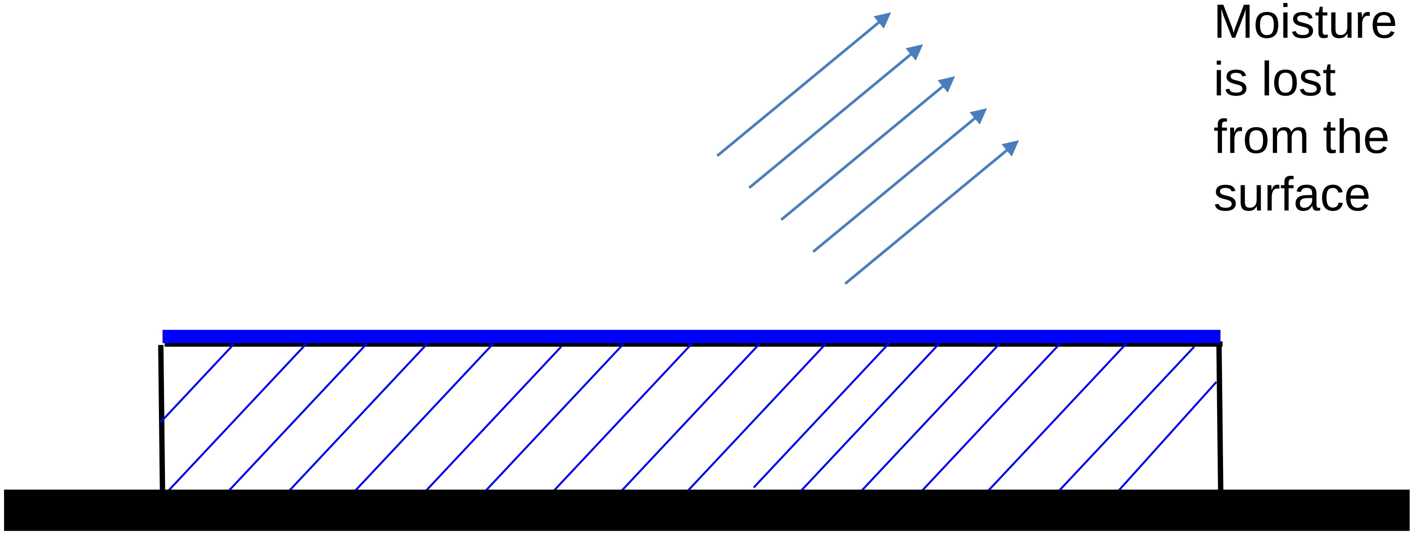
raisins

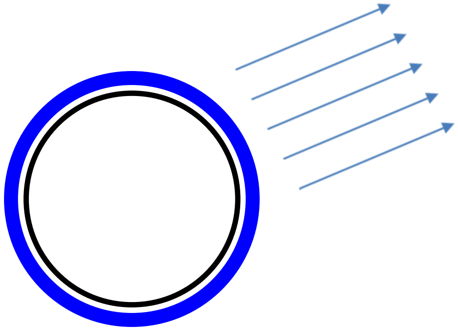
fish (cod)

Milk

Dairy factory

Moisture is lost from the surface

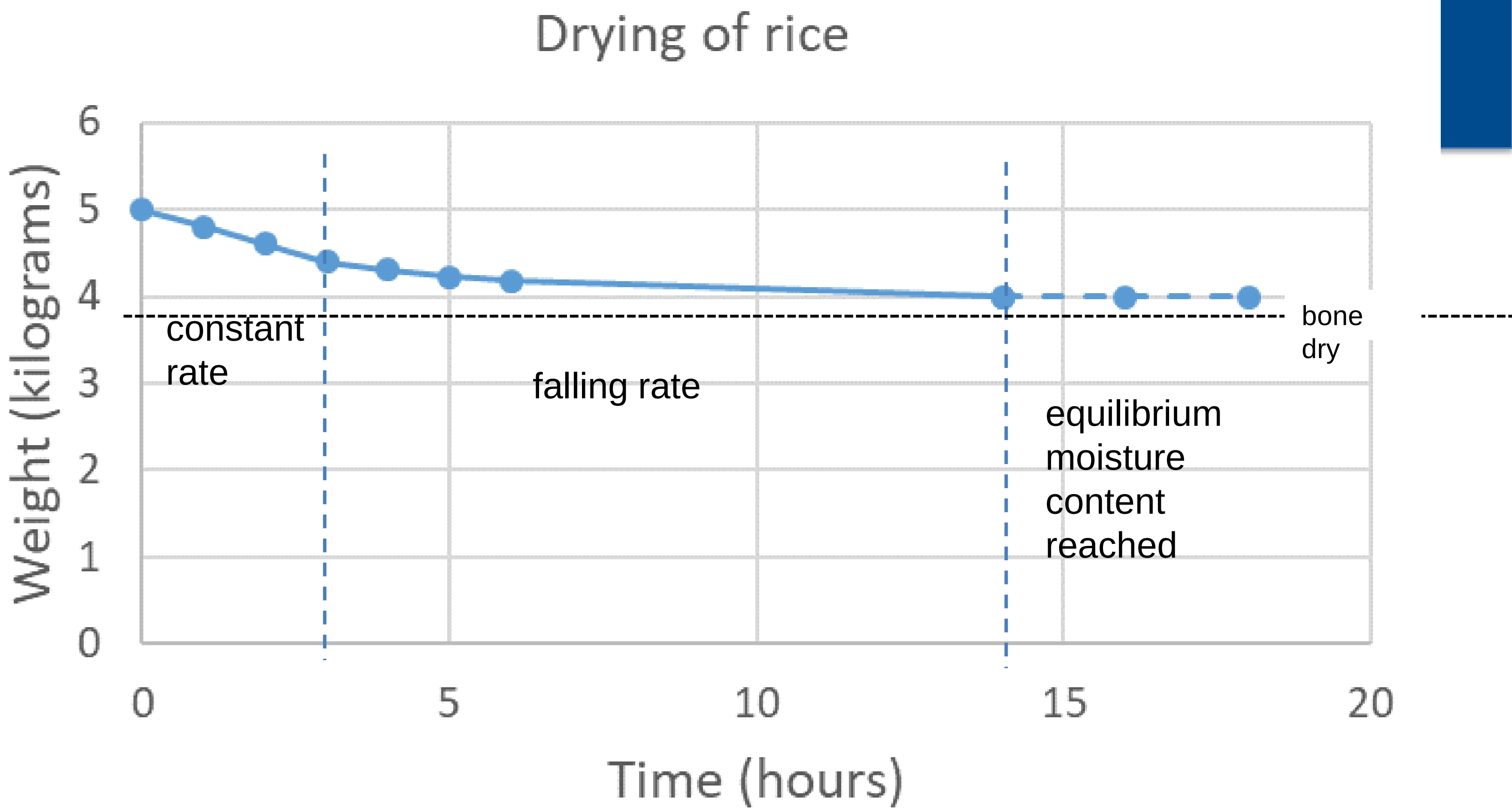




5 kilograms of rice dried in air

Weight of rice grains during drying

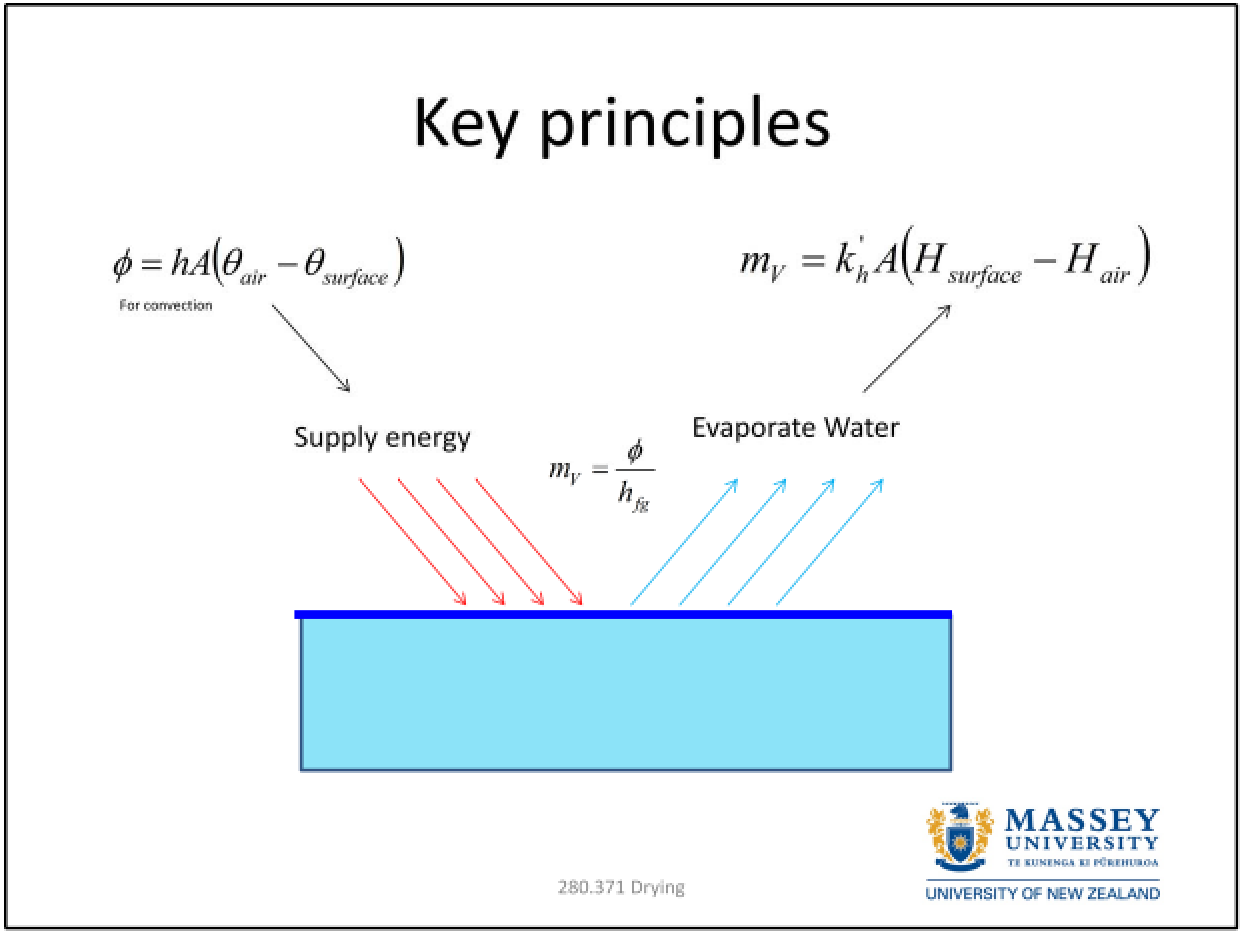
Time (h)	Weight (kg)
0	5.0
1	4.8
2	4.6
3	4.4
4	4.3
5	4.22
6	4.18
14	4.0



- Drying can take a long time
- After a heating up period, the rate of weight loss is constant - Constant rate period
- Then the rate of weight loss decreases - Falling rate period (Two regimes: capillary and diffusion)
- Finally, there is no further weight loss - Equilibrium moisture content reached

We are interested in being able to measure and calculate:

- Water content of material being dried
- Rate of drying
- Time of drying (the time it takes to dry something)
- How moisture content varies inside something being dried
- Solids and Air flowrates in drying equipment
- Temperatures and Humidities



θ Temperature ºC

H Humidity (kg water/kg dry air)

Heat transfer and mass transfer equations can be related to the same process – the evaporation of water from the surface. In heat transfer the driving force is temperature difference, in mass transfer it is concentration difference.

Difference in Humidity

In drying, the concentration is equivalent to the moisture content in the air more commonly known as the humidity. For part of the drying process (constant rate), these two process are equivalent; that is the drying rate defined by heat transfer is equal to the drying rate defined by mass transfer.

Nomenclature and Units

- Moisture content of a solid is expressed on a dry basis. Here, dry solid means solid with all the water removed and is sometimes termed bone-dry solid this can be done by drying with air of zero humidity for a long time.

variable	symbol	units
Moisture content of a solid	X	kg/kg
Equilibrium moisture content	X^*	kg/kg
Free moisture content	$X_{free} = (X-X^*)$	kg/kg
Critical moisture content	X_c	kg/kg
Humidity of air (kg of water/ kg of dry air)	H	kg/kg

Key principles

- Drying is thermal evaporation of water to produce a solid product
- Heat of vaporisation usually provided by contact with hot air → air is cooled in process
- For drying of liquid foods, evaporation often precedes drying

Nomenclature

- Water evaporated
  - m_v kg.s^{-1}
- Drying rate:
  - R kg.m^{-2}.s^{-1}
  - R_C constant rate

$$R = \frac{m_v}{A} = \frac{L_s}{A}\frac{dX}{dt}$$

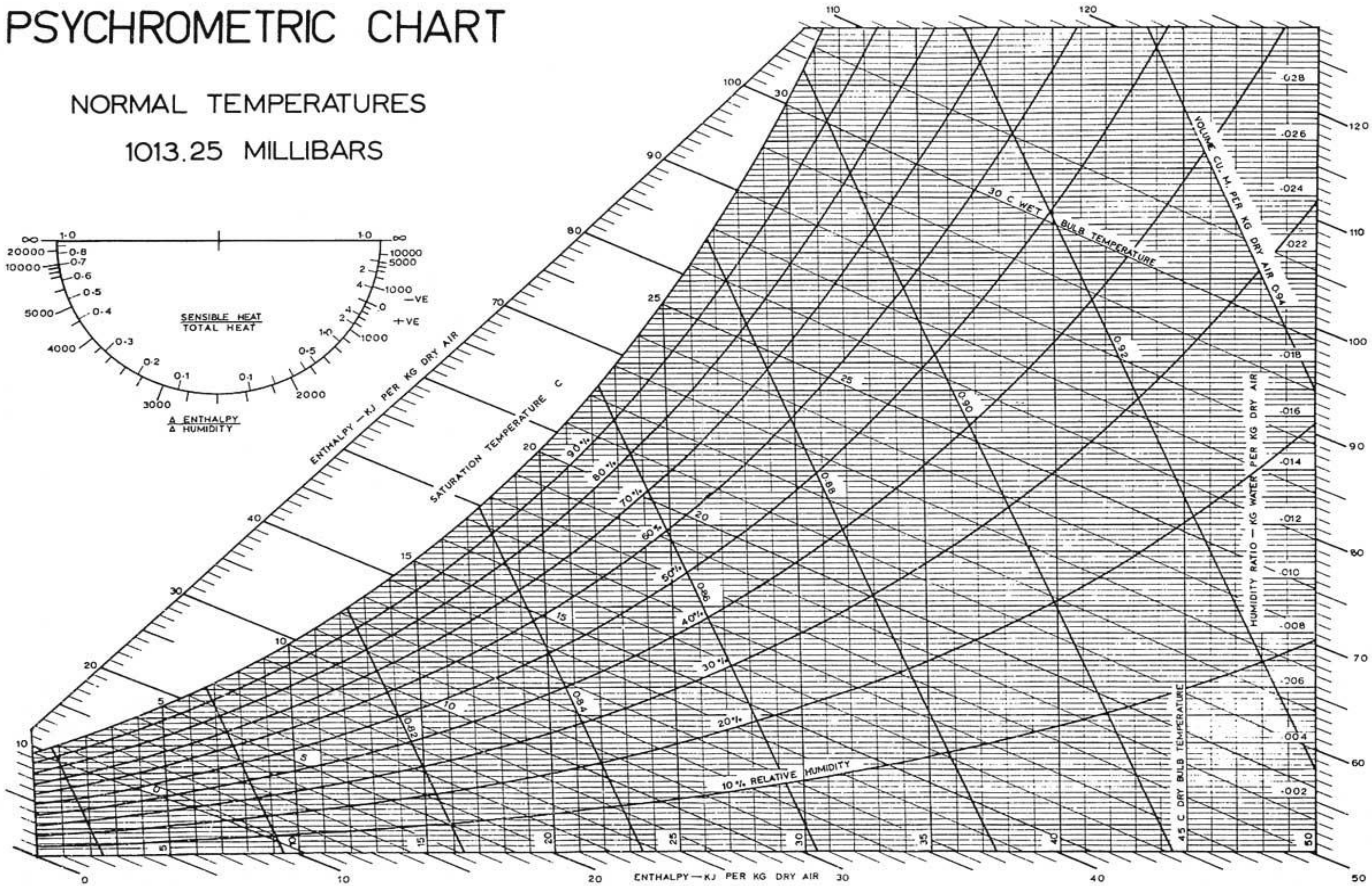
- Surface area:
  - A m^2
- Moisture content:
  - X kg.kg^{-1} (kg water. kg dry solid^{-1})
  - X_c critical moisture content (end of constant rate period)
- Humidity:
  - H kg.kg^{-1} (kg water vapour.kg dry air)

Psychrometry Revision

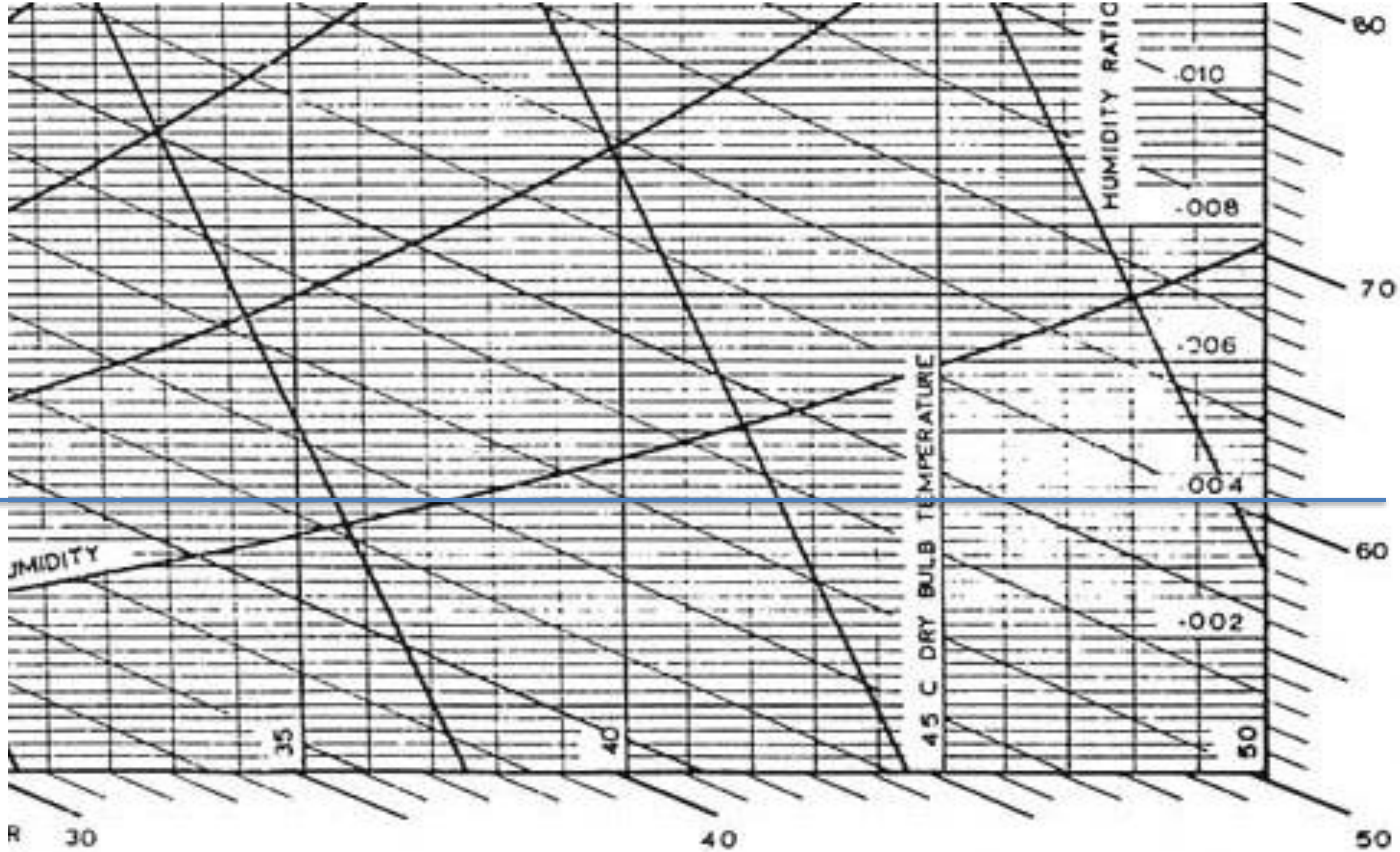
- Physical properties of a given air stream $\rightarrow$ **only** need two pieces of information on the air.
- Dry bulb (θ_{air}) temperature of the air plus:
  - the wet bulb temperature (θ_{WB})
  - the relative humidity (RH)
  - the absolute humidity (H)
- Line of constant enthalpy.

Humidity

Enthalpy



Relative Humidity: The ratio of the partial pressure of the vapour to the vapour pressure of the liquid at the same temperature.
The amount of water vapour present in air expressed as a percentage of the amount needed for saturation at the same temperature.



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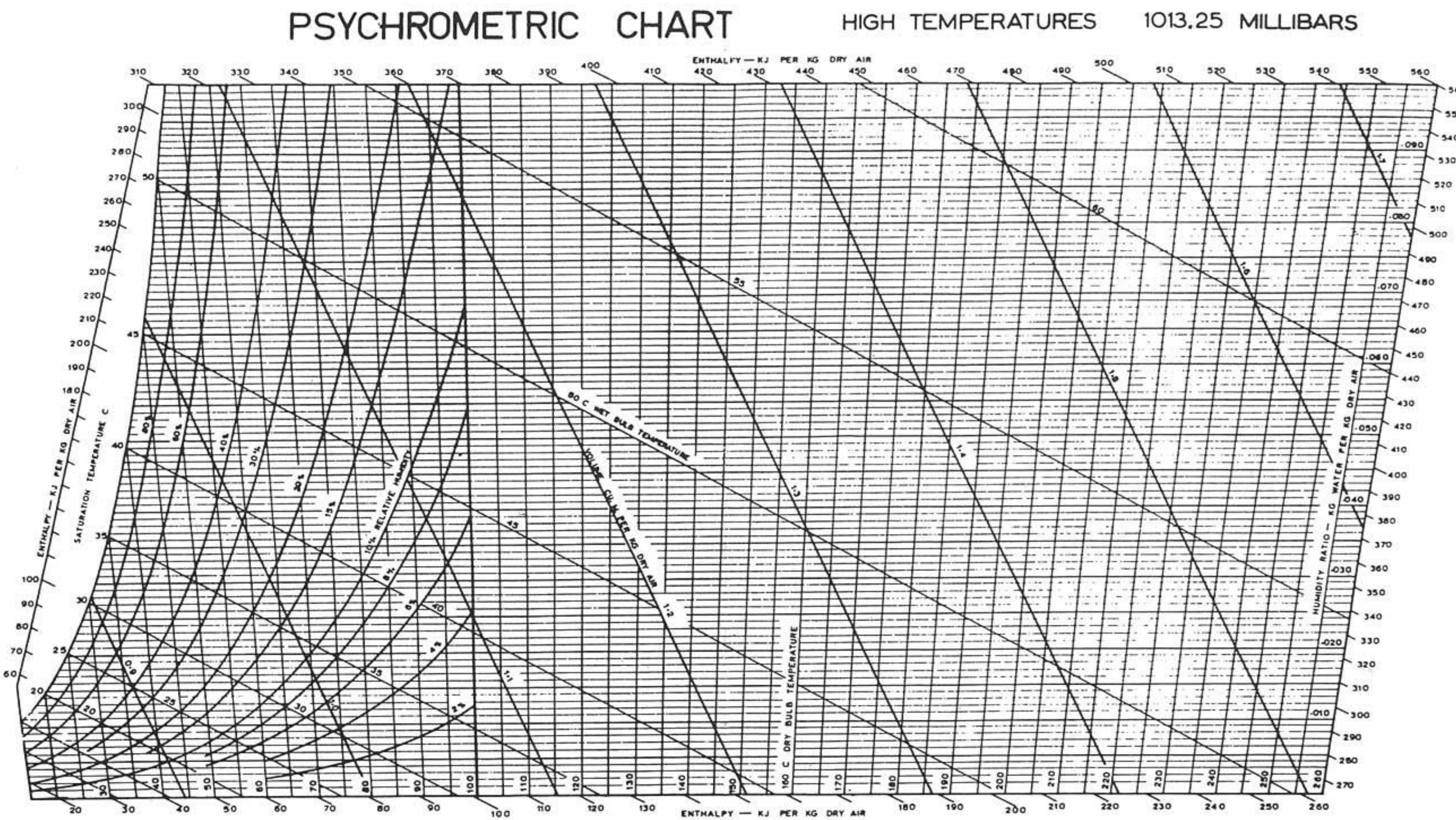


PSYCHROMETRIC CHART
HIGH TEMPERATURES
1013.25 MILLIBARS
ENTHALPY — KJ PER KG DRY AIR
SATURATION TEMPERATURE C
80 C WET BULB TEMPERATURE
VOLUME CU M PER KG DRY AIR
10% RELATIVE HUMIDITY
160 C DRY BULB TEMPERATURE
HUMIDITY RATIO — KG WATER PER KG DRY AIR
ENTHALPY — KJ PER KG DRY AIR

Constant rate drying

- A fully-wetted surface exposed to air will achieve a dynamic equilibrium:
  - Rate of heat transfer from the air = rate of heat loss from surface by evaporation of water
  - Water transfers from the interior to the drying surface as fast as it evaporates
  - Surface temperature is constant and governed by heat inputs

Evaporation rate

The rate of evaporation (i.e. the drying rate) can be calculated by simultaneously solving:

- the heat and mass transfer equations
- the heat and mass transfer analogy equation (Lewis)
- for a given surface equilibrium condition, depending on the nature of heat transfer

- For convection heat transfer

$$\theta_{surface} = \theta_{WB}$$

- If other heat transfer mechanisms are involved, combine all resistances to heat transfer

$$\theta_{surface} > \theta_{WB}$$

Heat & mass transfer equations

- The rate of mass transfer = drying rate

$$m_v = k'_H A(H_{surface} - H_{air})$$

$$R = k'_H (H_{surface} - H_{air})$$

- The rate of heat transfer to provide drying rate, m_v

$$\phi = m_v h_{fg} \qquad R = \frac{\phi}{h_{fg} A} \qquad R = \frac{kg\ water}{m^2 s}$$

- h_{fg} is the latent heat of evaporation evaluated at the surface temperature
- $\phi = q =$ rate of heat transfer (W)

Heat & Mass transfer analogy

- The mass transfer coefficient analogy with the heat transfer coefficient is:

$$k'_H \approx \frac{h}{c_{pH}}$$

Lewis relationship

$$c_{pH} = c_{p,air} + H_{air}\, c_{p,steam}$$

$C_{p,air}$ – specific heat of dry air at θ_{air}

$C_{p,stream}$ – specific heat at θ_{WB}

Equilibrium condition

- $H_{surface}$ is a function of $\theta_{surface}$
- Given by psychrometric chart

For Convection only – on all sides

$$\phi = h_c A\left(\theta_{air} - \theta_{surface}\right)$$

$$\theta_{surface} = \theta_{WB}$$

- *Remember for Mass transfer* $R = k_H^{'}(H_{surface} - H_{air})$

And for Heat transfer $R = \dfrac{\phi}{h_{fg} A}$

For Convection & conduction

- *Convection from above combined with conduction from below:*

$$\phi = \left(h_c + \frac{1}{\frac{\Delta x}{\lambda}} \right) A\left(\theta_{air} - \theta_{surface}\right)$$

$$\theta_{surface} > \theta_{WB}$$

For Convection & conduction

- *Convection from above combined with convection and conduction from below:*

$$\phi = \left(h_c + \frac{1}{\frac{1}{h_c} + \frac{\Delta x}{\lambda}} \right) A \left(\theta_{\mathrm{air}} - \theta_{surface} \right)$$

$$\theta_{surface} > \theta_{WB}$$

Convection, conduction and radiation

- *Convection and radiation from above combined with convection and conduction from below*

$$\phi = \left(h_c + \frac{1}{\frac{1}{h_c} + \frac{\Delta x}{\lambda}} \right) A\left(\theta_{air} - \theta_{surface}\right) + h_{rad} A\left(\theta_{radr} - \theta_{surface}\right)$$

Convection & radiation

- *Convection and radiation from above*

$$\phi = h_c A\left(\theta_{air} - \theta_{surface}\right) + h_{rad} A\left(\theta_{radr} - \theta_{surface}\right)$$

Generic solution procedure

1. Calculate the mass transfer coefficient from the analogy
2. Assume a surface temperature
3. Determine the surface saturation humidity from the psychrometric chart
4. Calculate the rate of evaporation using the mass transfer equation
5. Calculate the rate of evaporation using the heat transfer equation
6. If the rates from steps 4 and 5 are the same, you guessed correctly. Otherwise go back to step 2

Moisture content

- *X (kg water/kg dry solid)*

Take note of the units.

This is not equal to % moisture.

e.g. 80% moisture

For 1 kg of wet material (0.8 kg water 0.2 kg dry solid)

$$X = \frac{0.8}{0.2} = 4 \qquad kg\ water / kg\ dry\ solid$$

Drying rate per unit surface area

$$R_c = \frac{m_v}{A}$$

$$R_c = s\rho_s \frac{dX}{dt} \qquad (for\ a\ slab)$$

$$R_c = \frac{r}{3}\rho_s \frac{dX}{dt} \qquad (for\ a\ sphere)$$

R_c = rate of drying per unit area (kg/m^{2}s)
s = characteristic dimension of slab (m)
r = radius of sphere (m)
ρ_s = density of dry solid (kg/m^3)
X = moisture content (kg water/kg bone dry solid)

Drying time – constant rate

$$t_d = \frac{L_s}{AR_c}(X_{initial} - X_{final})$$

- *Drying time for slab geometry*

$$t_d = \frac{s\rho_s}{R_c}(X_{initial} - X_{final})$$

<u>s</u> is ½ thickness (2-side drying) or full thickness (1-side drying)

- *Drying time for particle (spherical) geometry*

$$t_d = \frac{r\rho_s}{3R_c}(X_{initial} - X_{final})$$

Drying time – falling rate

- *Drying time for slab geometry*

$$t_d = \frac{s\rho_s}{R_c}\left[(X_1 - X_C) + X_c \ln\left(\frac{X_C}{X_2}\right)\right]$$

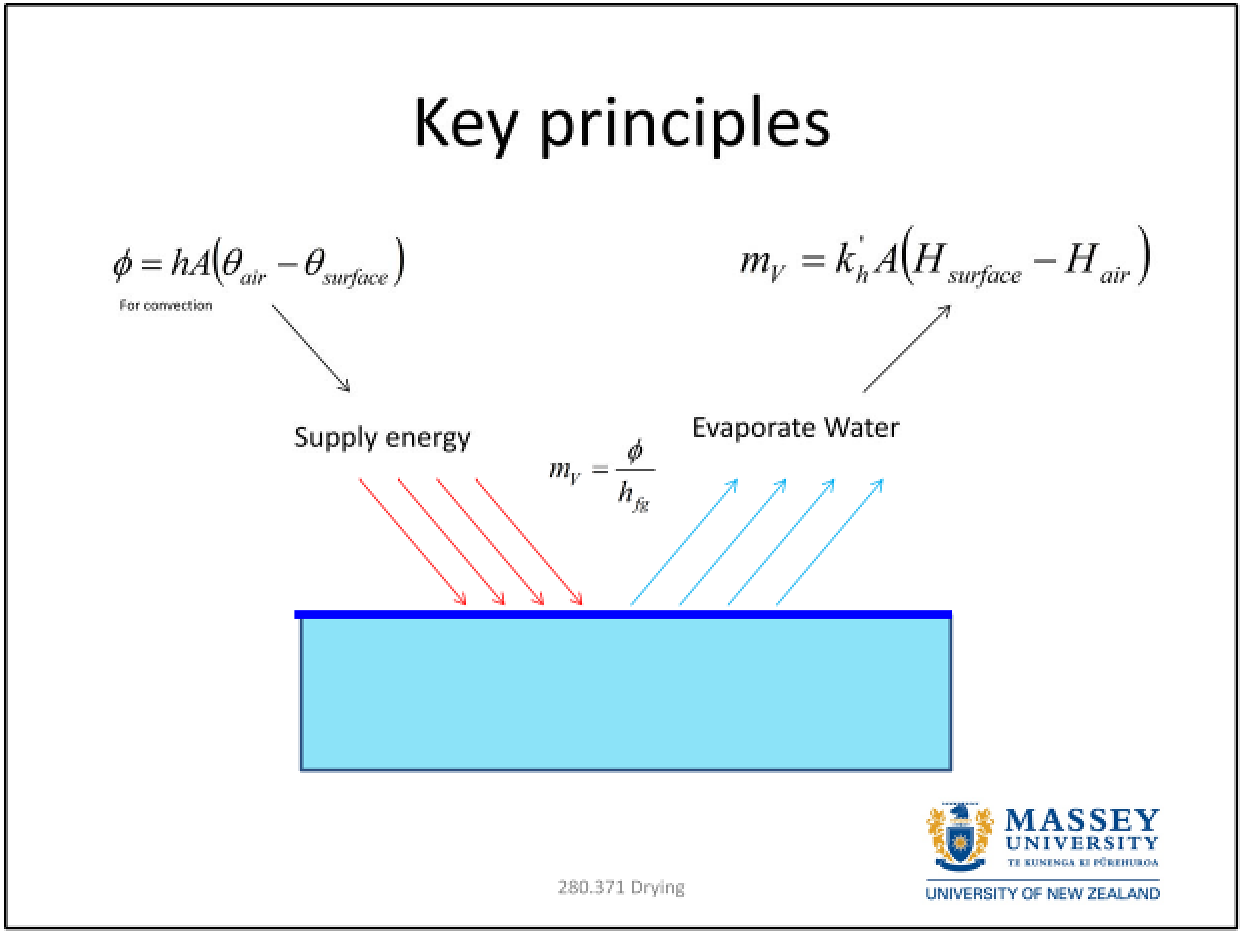
Drying Problems

Example 1: Evaporation from a wet surface (convection heat transfer only)

Clothes drying on a line are exposed to air at 20ºC and 60% relative humidity. The convective heat transfer coefficient from the air to the clothes is 20 W/m^2 K and radiation can be ignored.

(a) Determine the rate of water removal (assuming the surface to be fully wet) from one m^2 of surface area of clothes. What will the surface temperature of the clothes be?

(b) How long will it take to dry the clothes from a moisture content of 2.5 kg/kg to a moisture content of 1.0 kg/kg assuming that the material is 2mm thick with a dry density of 300 kg/ m^3 ?



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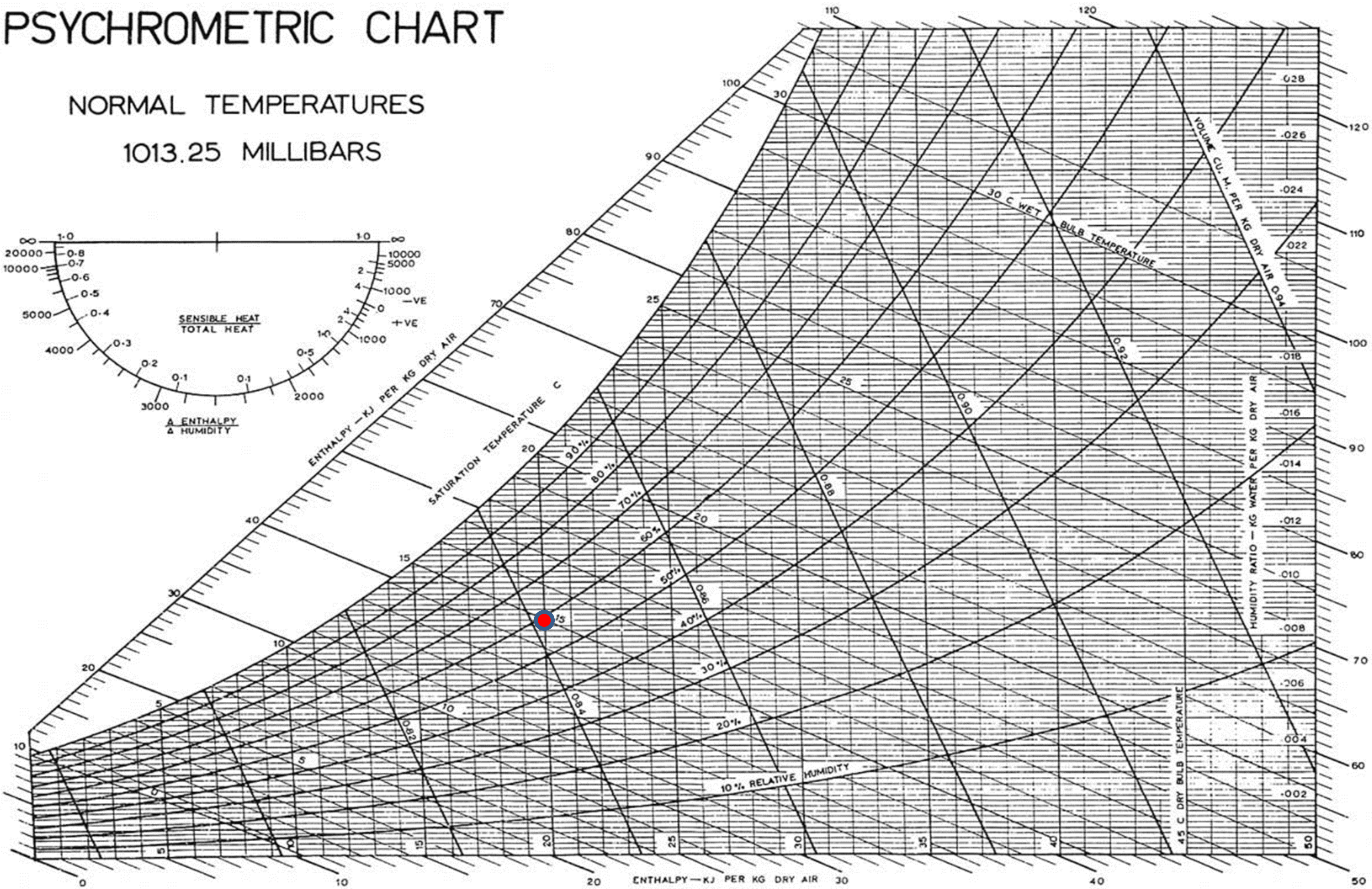
PSYCHROMETRIC CHART
NORMAL TEMPERATURES
1013.25 MILLIBARS
SENSIBLE HEAT / TOTAL HEAT
Δ ENTHALPY / Δ HUMIDITY
ENTHALPY — KJ PER KG DRY AIR
SATURATION TEMPERATURE C
30 C WET BULB TEMPERATURE
VOLUME CU. M. PER KG DRY AIR 0.94
HUMIDITY RATIO — KG WATER PER KG DRY AIR
45 C DRY BULB TEMPERATURE
10 % RELATIVE HUMIDITY
20 %
30 %
40 %
50 %
60 %
70 %
80 %
90 %
0.82
0.84
0.86
0.88
0.90
0.92
ENTHALPY — KJ PER KG DRY AIR

3 Saturated Data

P	θ_{sat}	v_l	v_v	u_l	u_v	h_l	$\Delta(h_{vap})$	h_v	s_l	s_v
MPa	°C	m^3/Mg		kJ/kg		kJ/kg			kJ/(kgK)	
0.0010	6.970	1.000	129178	29.30	2384	29.30	2484	2514	0.106	8.975
0.0015	13.02	1.001	87959	54.68	2393	54.68	2470	2525	0.196	8.827
0.0020	17.49	1.001	66987	73.43	2399	73.43	2459	2533	0.261	8.723
0.0025	21.08	1.002	54240	88.42	2404	88.42	2451	2539	0.312	8.642
0.0030	24.08	1.003	45653	101.0	2408	101.0	2444	2545	0.354	8.576