280371 Process Engineering Operations

Drying

Lecture 3

Diffusion mass transfer in falling rate regime

Gurney Lurie Charts

Diffusion Mass Transfer to the surface

- unsteady state condition
- variation in moisture content (X) in the solid
- X dependent on time and position.
- dependent rate of diffusion out of a solid
- Gurney Lurie Charts
- point moisture content (for slab, sphere, cylinder).

Diffusivity or Diffusion coefficient

D_v $Diffusivity$ $\left(\frac{m^2}{s}\right)$ T $temperature\ (K)$

- We require an effective diffusivity that accounts for the tortuous path through which water must diffuse in the solid. Experiments are generally required to determine this value.

- Temperature dependence:

- T_1 and T_2 in Kelvins

$$\frac{D_{v1}}{D_{v2}} = \left(\frac{T_1}{T_2}\right)^{1.5}$$

Gurney Lurie Charts

Gurney Lurie Charts had their origins in transient heat transfer. They offer a graphical method that enables rapid calculation of temperature as a function of position and time.

They consider three “ideal” geometries: slab, sphere and cylinder.

Gurney Lurie Charts can also be used in transient mass transfer problems, as is the case here.

Diffusion mass transfer

- When the Fourier number is greater than ~0.1, the first term of an infinite series equation can optionally be used to calculate drying time (instead of using the Gurney Lurie chart)

Fo	Fourier number (-)
t_d	time (s)
s	slab thickness (m)
r	radius of sphere (m)
D_V	Diffusivity (m^2/s)

$$\frac{\left(X_2 - X^*\right)}{\left(X_1 - X^*\right)} = \frac{8}{\pi^2}\left[e^{\frac{-\pi^2 Fo}{4}}\right] \quad \text{for} \quad Fo = \frac{D_v t_d}{s^2} > 0.1$$

$$t_d = \frac{4s^2}{\pi^2 D_v} \ln\left[\frac{8\left(X_1 - X^*\right)}{\pi^2\left(X_2 - X^*\right)}\right]$$

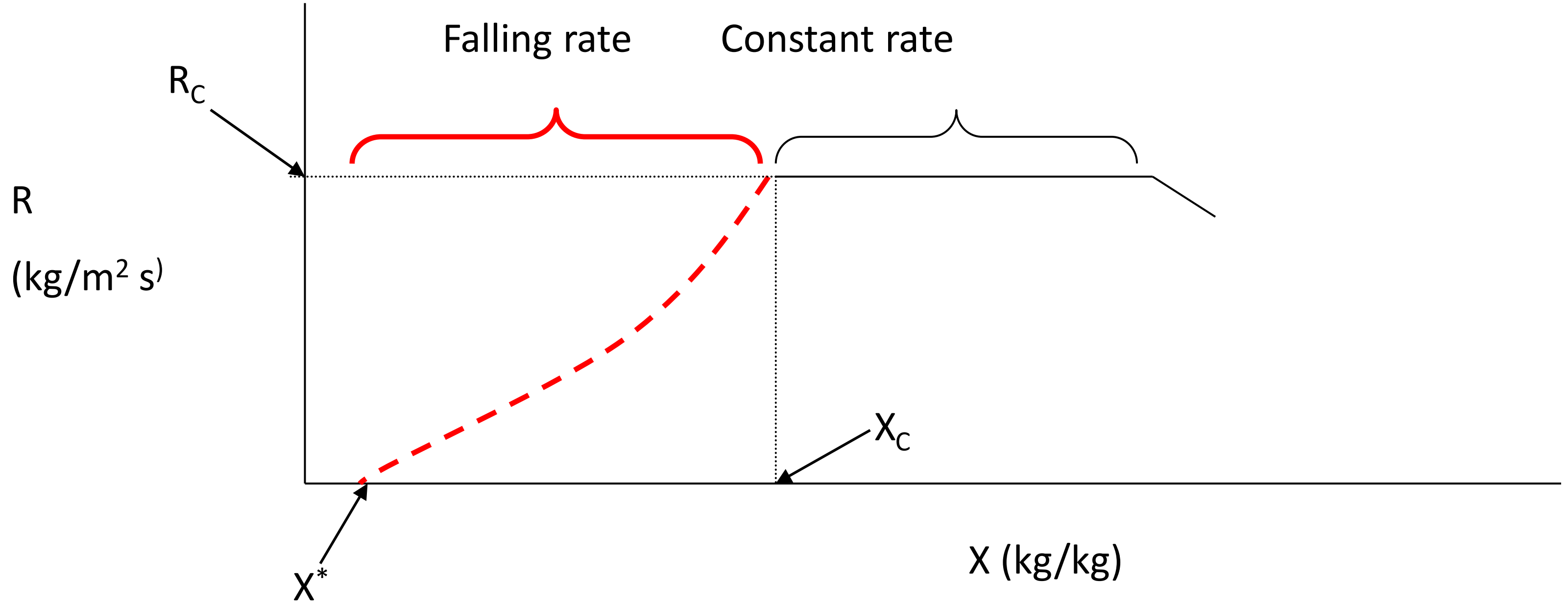
Drying time for diffusion mass transfer in falling rate period

slab

$$t_d = \frac{4s^2}{\pi^2 D_v} \ln\left[\frac{8\left(X_1 - X^*\right)}{\pi^2\left(X_2 - X^*\right)}\right]$$

sphere

$$t_d = \frac{r^2}{\pi^2 D_v} \ln\left[\frac{6\left(X_1 - X^*\right)}{\pi^2\left(X_2 - X^*\right)}\right]$$



- Dimension term n

- Biot number

$$Bi = \frac{internal\ resistances}{external\ resistances}$$

$$m = \frac{1}{Bi}$$

- Y = unaccomplished concentration change

$$Y = \frac{X - X^*}{X_i - X^*}$$

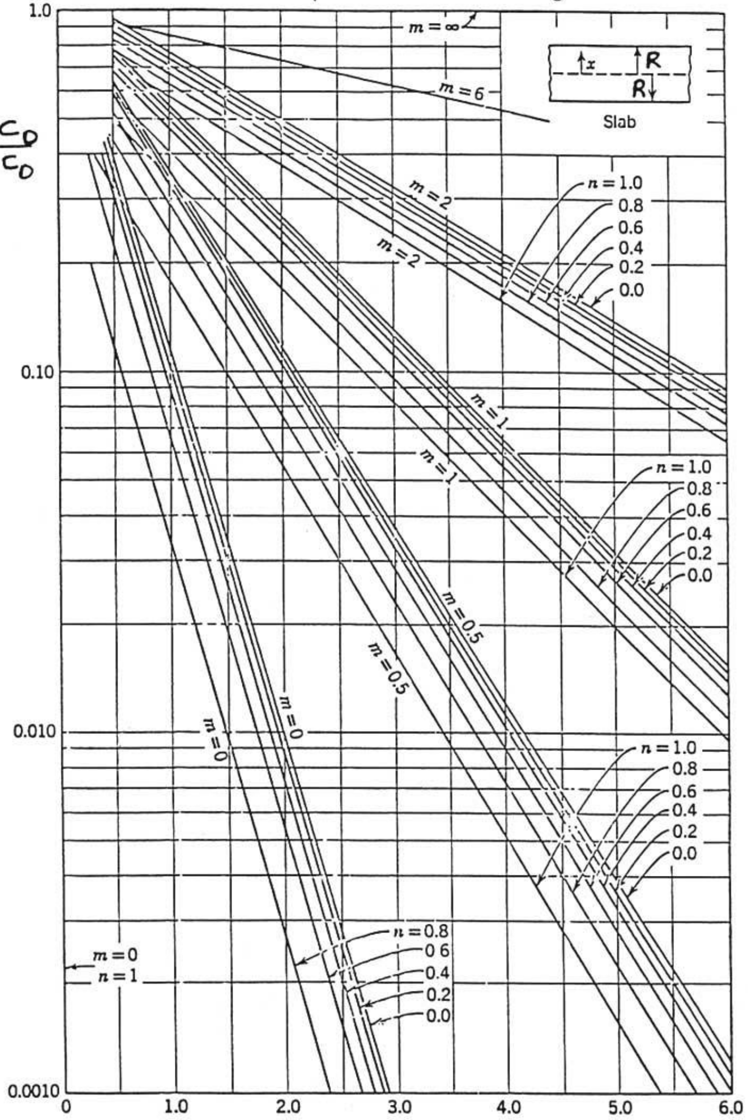
Slab

The Gurney Lurie charts use R as the half thickness of a slab. The equations in the course notes use s/2 as the half thickness.

$$Y = \frac{X - X^*}{X_i - X^*}$$

Concentration as a function of time and position - Slab geometry

$Y = \frac{c - c_0}{c_i - c_0}$



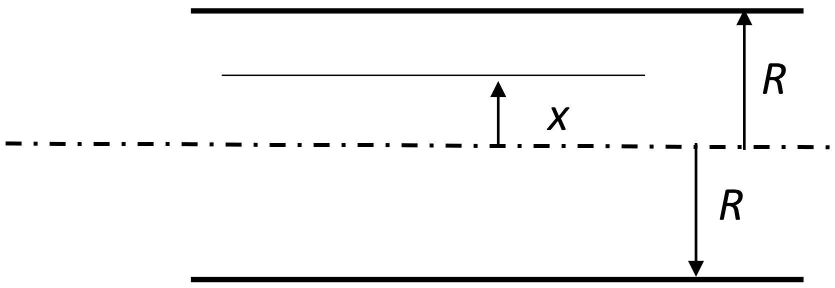
$\beta = Dt/R^2$

Unsteady-state transport in a large flat slab.

$n = \frac{x}{R}$

$m = 1/Bi$

slab



R Characteristic dimension

n x/R

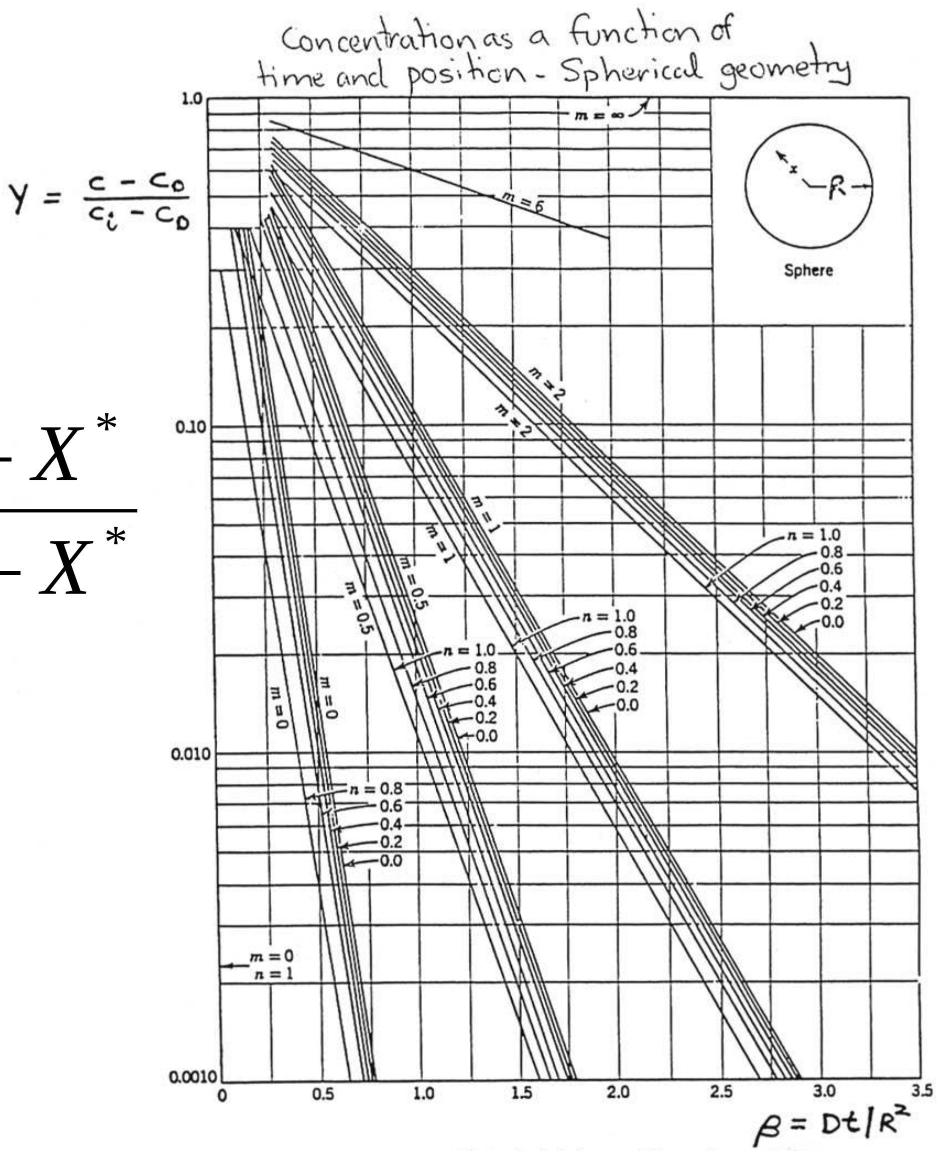
x distance from centre

m 1/Bi

$$Fo = \beta = \frac{Dt}{R^2}$$

Sphere

$$Y = \frac{X - X^*}{X_i - X^*}$$

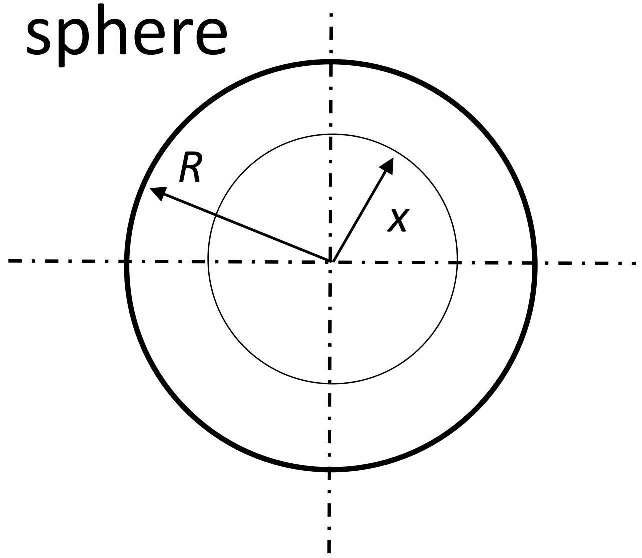


Unsteady-state transport in a sphere.

$n = \frac{x}{R}$

$m = 1/Bi$

sphere



R Characteristic dimension

n x/R

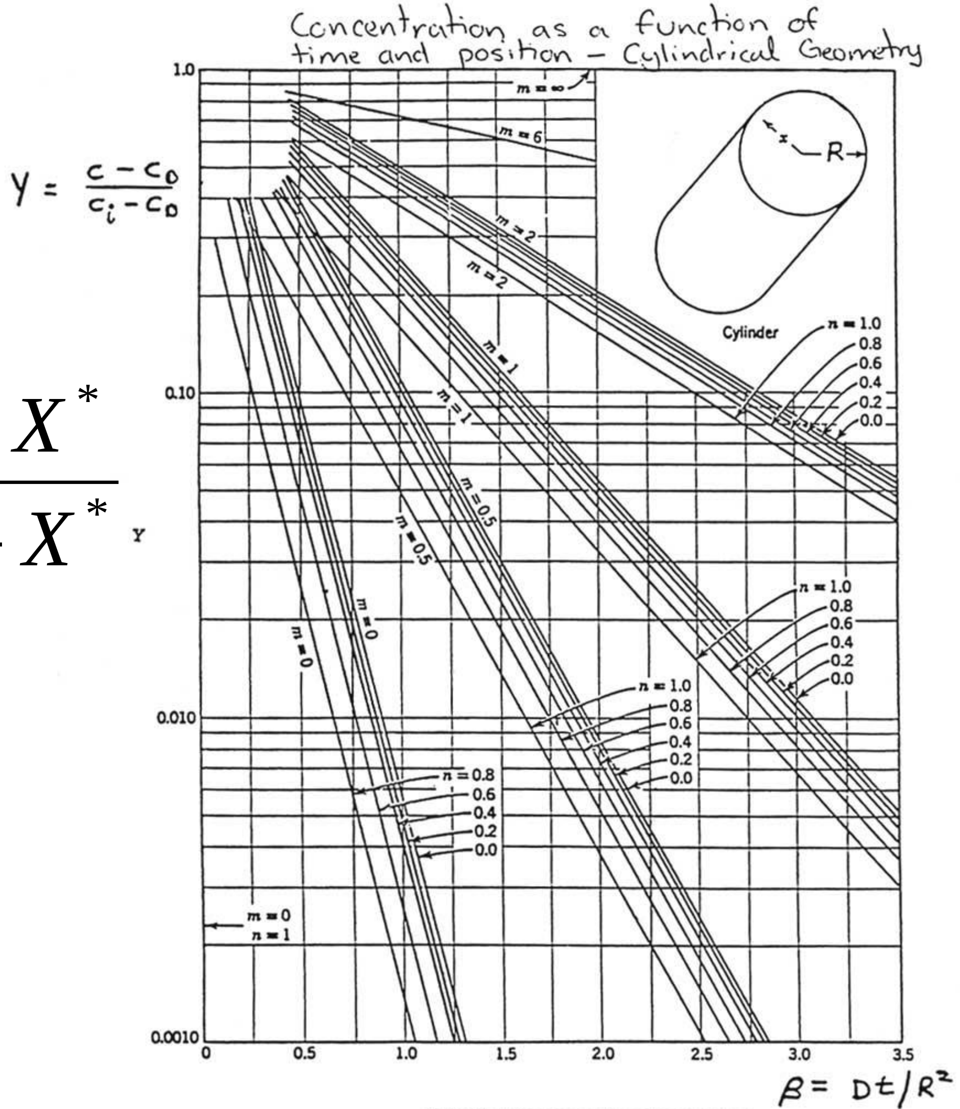
x distance from centre

m 1/Bi

$$Fo = \beta = \frac{Dt}{R^2}$$

Cylinder

$$Y = \frac{X - X^*}{X_i - X^*}$$



Unsteady-state transport in a long cylinder.

$n = \frac{x}{R}$

$m = 1/Bi$

R Characteristic dimension

n x/R

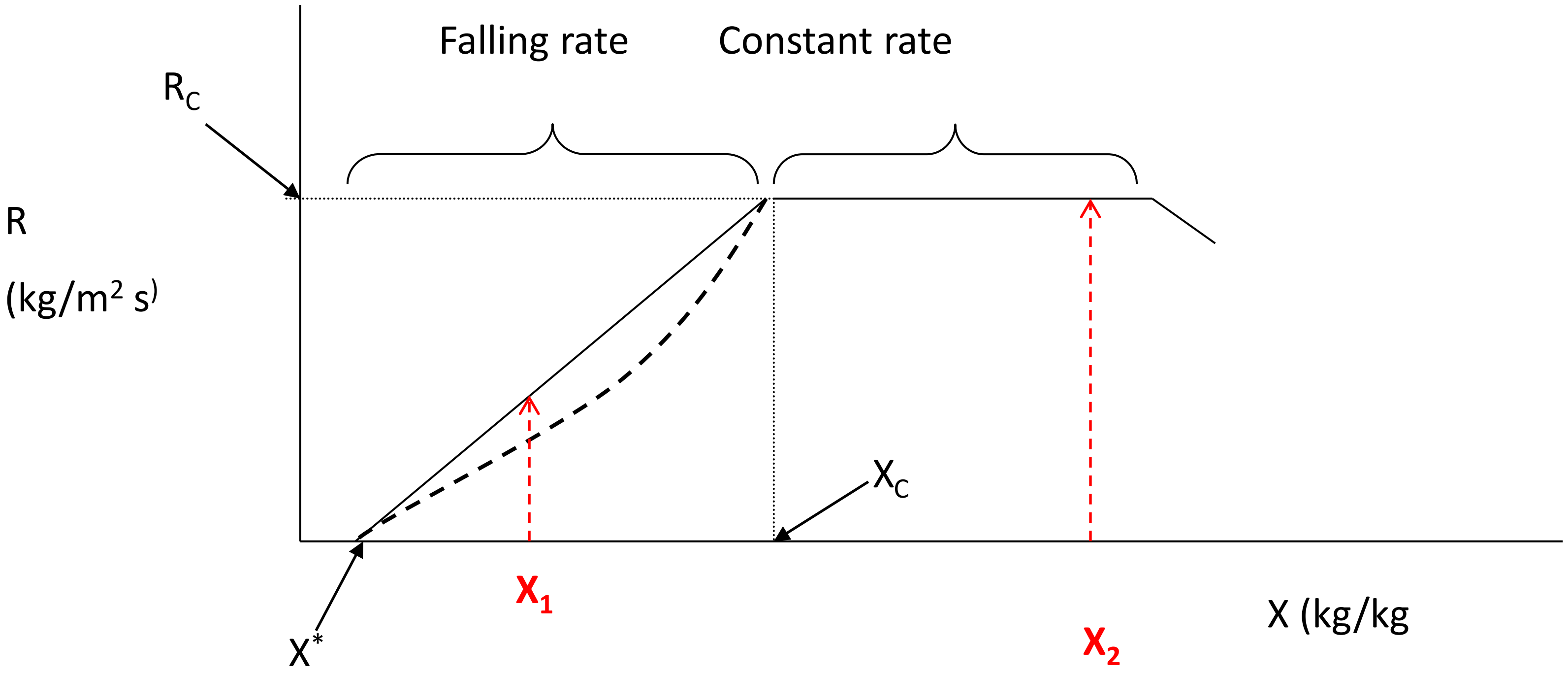
x distance from centre

m 1/Bi

$$Fo = \beta = \frac{Dt}{R^2}$$

Calculation of drying times

- The time required to dry solid from a moisture content above the critical point to a point below, can be obtained by summing the two drying periods



Solution procedure for drying time

1. Define geometry of material being dried
2. Identify X_C
3. From X_1 and X_2 determine what period drying is occurring over (constant or falling rate periods)
4. If drying occurs during the falling rate period, is the drying by capillary action or diffusion
5. Derive equation to determine drying time from X_1 to X_2
6. Note: if the falling rate period starts at the critical moisture content then $X_1 = X_c$ *for* $t_{d, falling\ rate\ period}$
7. Check if you using free moisture contents – impacts on value of X^*

Drying Time Equations

Capillary Action

	Constant rate only X_1 to X_2	Constant rate from X_1 to X_c and falling rate from X_c to X_2	Falling rate only X_1 to X_2
Slab geometry	$t_d = \frac{s\rho_s(X_1 - X_2)}{R_c}$	$t_d = \frac{s\rho_s(X_1 - X_c)}{R_c} + \frac{s\rho_s(X_c - X^*)}{R_c}\ln\left[\frac{(X_c - X^*)}{(X_2 - X^*)}\right]$	$t_d = \frac{s\rho_s(X_c - X^*)}{R_c}\ln\left[\frac{(X_1 - X^*)}{(X_2 - X^*)}\right]$
Spherical geometry	$t_d = \frac{r\rho_s(X_1 - X_2)}{3R_c}$	$t_d = \frac{r\rho_s(X_1 - X_c)}{3R_c} + \frac{r\rho_s(X_c - X^*)}{3R_c}\ln\left[\frac{(X_c - X^*)}{(X_2 - X^*)}\right]$	$t_d = \frac{r\rho_s(X_c - X^*)}{3R_c}\ln\left[\frac{(X_1 - X^*)}{(X_2 - X^*)}\right]$

Diffusion

	Constant rate only X_1 to X_2	Constant rate from X_1 to X_c and falling rate from X_c to X_2	Falling rate only X_1 to X_2
Slab geometry	$t_d = \frac{s\rho_s(X_1 - X_2)}{R_c}$	$t_d = \frac{s\rho_s(X_1 - X_c)}{R_c} + \frac{4s^2}{\pi^2 D_v}\ln\left[\frac{8(X_c - X^*)}{\pi^2(X_2 - X^*)}\right]$	$t_d = \frac{4s^2}{\pi^2 D_v}\ln\left[\frac{8(X_1 - X^*)}{\pi^2(X_2 - X^*)}\right]$
Spherical geometry	$t_d = \frac{r\rho_s(X_1 - X_2)}{3R_c}$	$t_d = \frac{r\rho_s(X_1 - X_c)}{3R_c} + \frac{r^2}{\pi^2 D_v}\ln\left[\frac{6(X_c - X^*)}{\pi^2(X_2 - X^*)}\right]$	$t_d = \frac{r^2}{\pi^2 D_v}\ln\left[\frac{6(X_1 - X^*)}{\pi^2(X_2 - X^*)}\right]$

If using free moisture contents set $X^* = 0$

Final points

- Use appropriate equations when the drying regime changes midway through the drying period
- Time required to dry from a moisture content above X_c to a point below obtained by summing the two drying periods (constant and falling rate)
- Use the half thickness if evaporation is taking place on both sides or full thickness if evaporation is only on one side
- If using free moisture contents then set $X^*=0$
- For diffusion mass transfer drying, unsteady state mass transfer is occurring.

Example 5: Drying time with slab geometry, in falling rate regimes under diffusion.

(i) Calculate the time required to dry a 20 mm thick slab of meat from an initial moisture content of 0.51 kg/kg (below critical) to a final average moisture content of 0.033 kg/kg.

The equilibrium moisture content of the meat in the air is 0.018 kg/kg. The meat is at 45^{o}C and the effective diffusivity of water through the meat is 3.5×10^{-9} m^2 /s at 25^{o}C.

(ii) Determine the final moisture content in the centre of the meat assuming the meat is exposed on both sides.

Drying Time Equations

Capillary Action

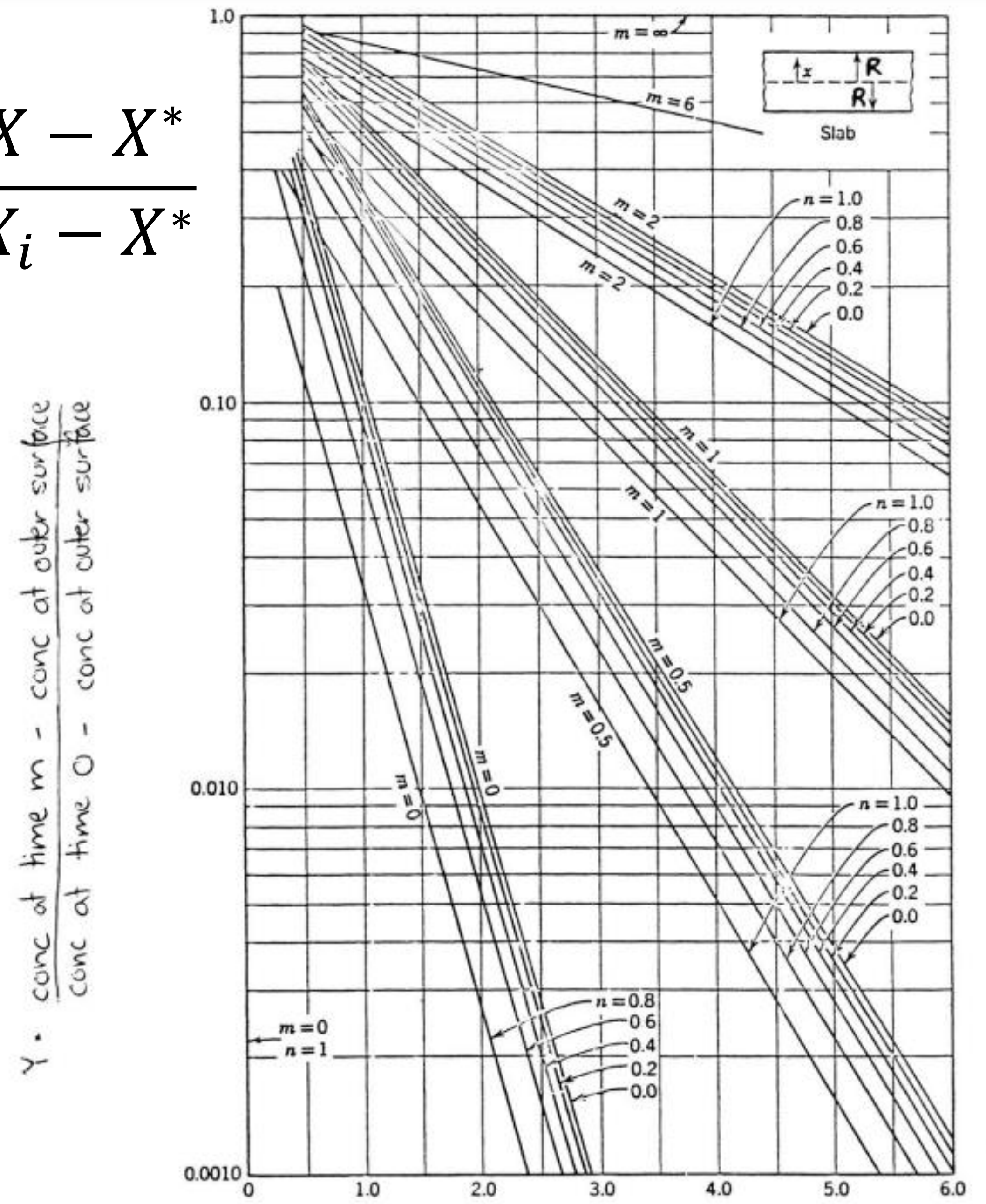
	Constant rate only X_1 to X_2	Constant rate from X_1 to X_c and falling rate from X_c to X_2	Falling rate only X_1 to X_2
Slab geometry	$t_d = \frac{s\rho_s(X_1 - X_2)}{R_c}$	$t_d = \frac{s\rho_s(X_1 - X_c)}{R_c} + \frac{s\rho_s(X_c - X^*)}{R_c}\ln\left[\frac{(X_c - X^*)}{(X_2 - X^*)}\right]$	$t_d = \frac{s\rho_s(X_c - X^*)}{R_c}\ln\left[\frac{(X_1 - X^*)}{(X_2 - X^*)}\right]$
Spherical geometry	$t_d = \frac{r\rho_s(X_1 - X_2)}{3R_c}$	$t_d = \frac{r\rho_s(X_1 - X_c)}{3R_c} + \frac{r\rho_s(X_c - X^*)}{3R_c}\ln\left[\frac{(X_c - X^*)}{(X_2 - X^*)}\right]$	$t_d = \frac{r\rho_s(X_c - X^*)}{3R_c}\ln\left[\frac{(X_1 - X^*)}{(X_2 - X^*)}\right]$

Diffusion

	Constant rate only X_1 to X_2	Constant rate from X_1 to X_c and falling rate from X_c to X_2	Falling rate only X_1 to X_2
Slab geometry	$t_d = \frac{s\rho_s(X_1 - X_2)}{R_c}$	$t_d = \frac{s\rho_s(X_1 - X_c)}{R_c} + \frac{4s^2}{\pi^2 D_v}\ln\left[\frac{8(X_c - X^*)}{\pi^2(X_2 - X^*)}\right]$	$t_d = \frac{4s^2}{\pi^2 D_v}\ln\left[\frac{8(X_1 - X^*)}{\pi^2(X_2 - X^*)}\right]$
Spherical geometry	$t_d = \frac{r\rho_s(X_1 - X_2)}{3R_c}$	$t_d = \frac{r\rho_s(X_1 - X_c)}{3R_c} + \frac{r^2}{\pi^2 D_v}\ln\left[\frac{6(X_c - X^*)}{\pi^2(X_2 - X^*)}\right]$	$t_d = \frac{r^2}{\pi^2 D_v}\ln\left[\frac{6(X_1 - X^*)}{\pi^2(X_2 - X^*)}\right]$

If using free moisture contents set $X^* = 0$

$$Y = \frac{X - X^*}{X_i - X^*}$$



$$Fo$$

$$n = \frac{x}{R}$$

$$m = \frac{1}{Bi}$$

$$Fo = \frac{Dt}{R^2}$$

To determine the final moisture content at centre n=0

From Guerney Lurie chart (figure1)

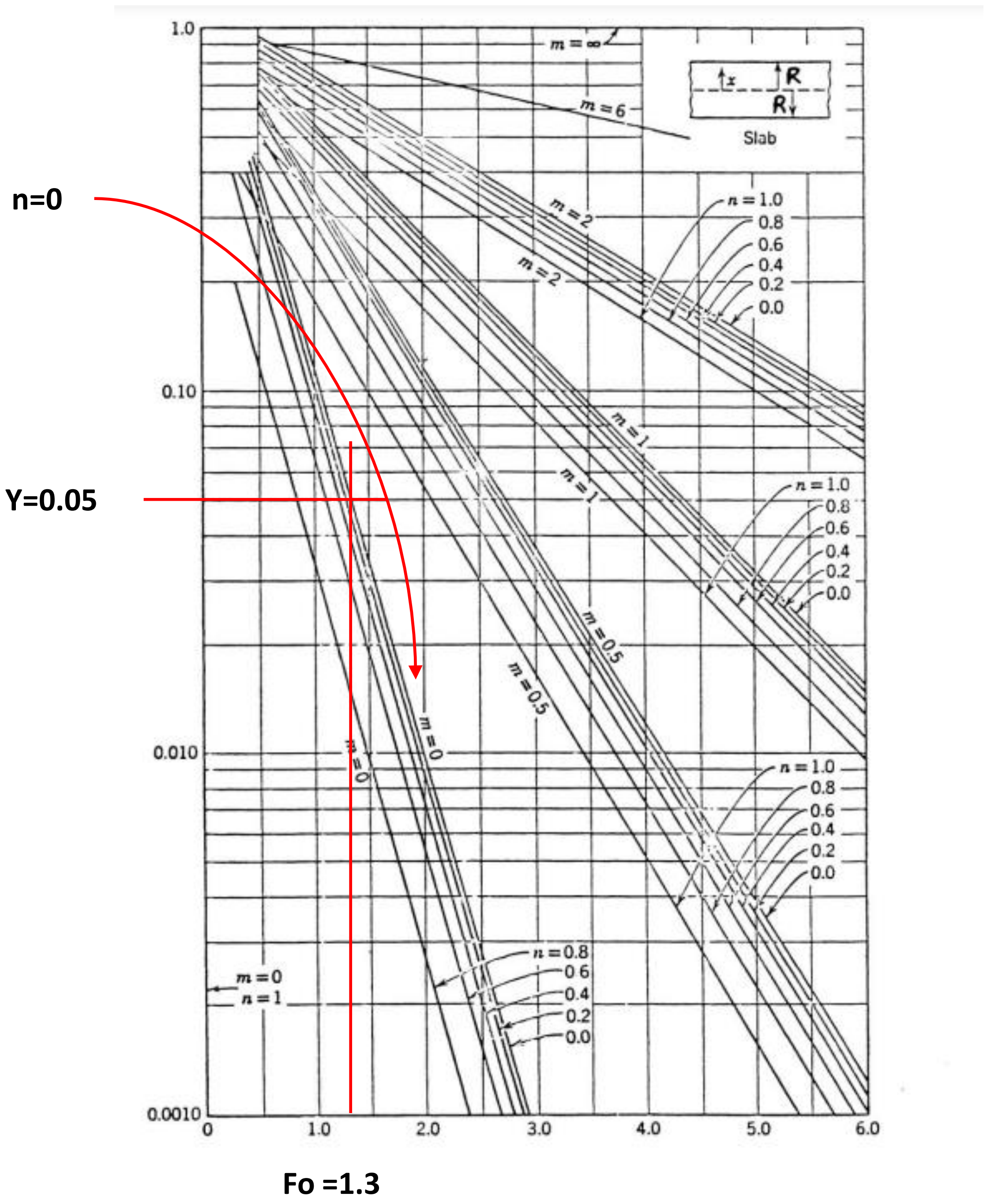
For m=0 (no external mass transfer resistance)
n=0 (at the centre of the slab)

$$Y = 0.05 = \frac{X - X_{outside}}{X_{initial} - X_{outside}}$$

$$Y = \frac{X - X^*}{X_i - X^*} = \frac{X - 0.018}{0.51 - 0.018}$$

$$\Rightarrow X = 0.05 \times (0.51 - 0.018) + 0.018$$

$$= 0.0426 kg/kg$$



Example 6: Drying time with particle geometry, in constant and falling rate regimes under diffusion.

10 mm diameter solid desiccant particles are to be dried in a hot air stream from an initial free moisture content of 0.8 kg/kg to 0.1 kg/kg.

The critical free moisture content of the desiccant is 0.6 kg/kg and the effective diffusivity of moisture in the particles is 50×10^{-9} m^2 /s.

The rate of drying in the constant rate period is 8.5×10^{-3} kg/m^2 s and the dry solid density is 1100 kg/m^3 .

Calculate the time required to dry the particles.

Drying Time Equations

Capillary Action

	Constant rate only X_1 to X_2	Constant rate from X_1 to X_c and falling rate from X_c to X_2	Falling rate only X_1 to X_2
Slab geometry	$t_d = \frac{s\rho_s(X_1 - X_2)}{R_c}$	$t_d = \frac{s\rho_s(X_1 - X_c)}{R_c} + \frac{s\rho_s(X_c - X^*)}{R_c}\ln\left[\frac{(X_c - X^*)}{(X_2 - X^*)}\right]$	$t_d = \frac{s\rho_s(X_c - X^*)}{R_c}\ln\left[\frac{(X_1 - X^*)}{(X_2 - X^*)}\right]$
Spherical geometry	$t_d = \frac{r\rho_s(X_1 - X_2)}{3R_c}$	$t_d = \frac{r\rho_s(X_1 - X_c)}{3R_c} + \frac{r\rho_s(X_c - X^*)}{3R_c}\ln\left[\frac{(X_c - X^*)}{(X_2 - X^*)}\right]$	$t_d = \frac{r\rho_s(X_c - X^*)}{3R_c}\ln\left[\frac{(X_1 - X^*)}{(X_2 - X^*)}\right]$

Diffusion

	Constant rate only X_1 to X_2	Constant rate from X_1 to X_c and falling rate from X_c to X_2	Falling rate only X_1 to X_2
Slab geometry	$t_d = \frac{s\rho_s(X_1 - X_2)}{R_c}$	$t_d = \frac{s\rho_s(X_1 - X_c)}{R_c} + \frac{4s^2}{\pi^2 D_v}\ln\left[\frac{8(X_c - X^*)}{\pi^2(X_2 - X^*)}\right]$	$t_d = \frac{4s^2}{\pi^2 D_v}\ln\left[\frac{8(X_1 - X^*)}{\pi^2(X_2 - X^*)}\right]$
Spherical geometry	$t_d = \frac{r\rho_s(X_1 - X_2)}{3R_c}$	$t_d = \frac{r\rho_s(X_1 - X_c)}{3R_c} + \frac{r^2}{\pi^2 D_v}\ln\left[\frac{6(X_c - X^*)}{\pi^2(X_2 - X^*)}\right]$	$t_d = \frac{r^2}{\pi^2 D_v}\ln\left[\frac{6(X_1 - X^*)}{\pi^2(X_2 - X^*)}\right]$

If using free moisture contents set $X^* = 0$