Conditional CAPM

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MGT 295F Empirical Methods

Outline

Changing Risk Argument

- CAPM: risk premium for a stock is its beta times the market risk premium
- Your investment can become more risky because
  - The market as a whole becomes more risky (expected market return increases)
  - The risk exposure (aka market beta) of your investment increases
- Higher risk during risky (bad) times is undesirable, hence investors would appreciate stocks with procyclical betas (low in recessions, high in booms)
- Such stocks will have high prices and low expected returns

Market Timing Argument (ICAPM)

- Increase in risk means higher expected returns and decline in price
- Stocks with larger increases in risk during recessions suffer larger losses, and losses during bad times are undesirable
- Stocks with procyclical betas witness smaller increase in risk and smaller losses during recessions, which makes investors want them
- These stocks are natural market timers: they load less on the market factor when the market goes down and load more on it when the market goes up
- Stocks with procyclical betas will have high prices and low expected returns

Static CAPM vs Conditional CAPM

- CAPM is a one-period model, but the world is multi-period
- When we estimate CAPM, we assume that all periods are about the same and the CAPM holds on average
- Conditional CAPM assumes that CAPM holds period-by-period (but may not hold on average)
- Conditional CAPM lets betas and risk premiums vary across time
- Empirically, we use time-varying betas, because the market risk premium will vary anyway

Stock A: Countercyclical Beta

	Market risk premium	Stock beta	Stock risk premium
Recession (p=1/4)	12%	2	24%
Expansion (p=3/4)	4%	2/3	8/3%
Average	6%	1	8%
Static CAPM	6%	1	6%

- Stock A has countercyclical beta: $\beta_A = 2$ in recessions, $\beta_A = 2/3$ in expansions
- Under Conditional CAPM, its risk premium is

$$\frac{3}{4} \cdot \left(\frac{2}{3} \cdot 4\% \right) + \frac{1}{4} \cdot \left(2 \cdot 12\% \right) = 8\%$$

- Static CAPM would just multiply the average market risk premium by the average beta, estimating the risk premium of stock A at 6%

Stock B: Procyclical Beta

	Market risk premium	Stock beta	Stock risk premium
Recession (p=1/4)	12%	1/2	6%
Expansion (p=3/4)	4%	7/6	14/3%
Average	6%	1	5%
Static CAPM	6%	1	6%

- Stock B has procyclical beta: $\beta_A = 1/2$ in recessions, $\beta_A = 7/6$ in expansions
- Under Conditional CAPM, its risk premium is

$$\frac{3}{4} \cdot \left(\frac{7}{6} \cdot 4\%\right) + \frac{1}{4} \cdot \left(\frac{1}{2} \cdot 12\%\right) = 5\%$$

- Static CAPM would just multiply the average market risk premium by the average beta, estimating the risk premium of stock B at 6%

How Static CAPM Failed

- Static CAPM averages, then multiplies
- Conditional CAPM multiplies, then averages
- We needed to calculate the expected excess return to the stock, $E(R_S^e)$

$$E[R_S^e] = E[\beta_S \cdot R_{MKT}^e] = E(\beta_S) \cdot E(R_{MKT}^e) + Cov(\beta_S, R_{MKT}^e)$$

- Static CAPM missed the covariance between the beta and the market risk premium
- In order for the covariance to be non-zero, both the beta and the market risk premium have to change!
- Preferably they both should depend on the same variables)

How Static CAPM Failed

- For Stock A (countercyclical beta), the covariance was positive
- Static CAPM underestimated stock A's expected return (said 6% when it was in fact 8%)
- Stock A will have positive $\alpha = 2\%$ in static CAPM and $\alpha = 0$ in conditional CAPM
- For Stock B (procyclical beta), the covariance was negative
- Static CAPM overestimated stock B's expected return (said 6% when it was in fact 5%)
- Stock B will have negative $\alpha = -1\%$ in static CAPM and $\alpha = 0$ in conditional CAPM

Playing with Covariance

- By definition
  $Cov(R_i, R_j) = E[(R_i - E(R_i)) \cdot (R_j - E(R_j)] =$
- Open the brackets inside of the expectation:

$$= E[R_i \cdot R_j - R_i \cdot E(R_j) - R_j \cdot E(R_i) + E(R_i) \cdot E(R_j)] =$$

- Average sum is the sum of the averages:

$$= E[R_i \cdot R_j] - E[R_i \cdot E(R_j)] - E[R_j \cdot E(R_i)] + E(R_i) \cdot E(R_j) =$$

Playing with Covariance

- $E(R_i)$ is just a number, hence

$$E[R_j \cdot E(R_i)] = E(R_i) \cdot E(R_j)$$

$$= E[R_i \cdot R_j] - E(R_i) \cdot E(R_j) - E(R_i) \cdot E(R_j) + E(R_i) \cdot E(R_j) \Rightarrow$$

- Covariance is average product minus the product of the averages:

$$\Rightarrow Cov(R_i, R_j) = E[R_i \cdot R_j] - E(R_i) \cdot E(R_j) \Rightarrow$$

- Average product differs from the product of the averages by the covariance:

$$\Rightarrow E[R_i \cdot R_j] = E(R_i) \cdot E(R_j) + Cov(R_i, R_j)$$

What We Learned about Risk

- For static CAPM, risk is the covariance with the market
- **CAPM: Risk is losses when the market goes down**
- Conditional CAPM introduces another dimension of risk: covariance of the beta with the expected market risk premium
- **Conditional CAPM: Risk is also higher beta (and greater losses) during risky times**

Magnitude Issue

- In the numerical example above, we assumed that the expected market risk premium changes from 4% per year in expansion to 12% per year in recession
- 8% per year change in the market risk premium is close to the largest change we can predict from predictive regressions
- We also assumed that the beta can double or even triple from its high to its low - can be true for a few stocks, but such stocks are rare
- Even with these extreme assumptions, the deviations from the static CAPM were at most 2% each way
- Economically, conditional CAPM produces small effects
- This is called **Lewellen-Nagel** (JFE 2006) **critique**

Market-to-Book

- Definition: market capitalization divided by book value of equity
- Firms with high market-to-book are *growth firms*
- Firms with low market-to-book are *value firms*
- Behavioral finance: high market-to-book means the firm is overvalued, so value firms should beat growth firms
- Asset pricing initial belief: growth options are riskier because they are options, so growth firms should beat value firms

Value Effect

$$HML_t = \underset{(0.12)}{0.58} - \underset{(0.03)}{0.27} \cdot (MKT_t - RF_t)$$

- HML buys value and short sells growth
- Regression above shows that growth firms indeed have higher beta than value firms
- However, value firms beat growth firms by 58 bp per month (7% per year) on risk-adjusted basis
- **"Value beats growth" is the value effect**

Why Value is Riskier than Growth?

- Value firms are mature and capital-intensive, growth firms are young and only plan to invest
- Zhang (JF 2005) argues that value is riskier in recessions, because value firms are locked in with useless capital
- Growth is riskier in expansions, when growth firms lack capital
- Main prediction: value is riskier in recessions and therefore riskier overall, but static CAPM does not see it

Petkova and Zhang (JFE 2005) Estimate Conditional CAPM

- We estimated $HML_t = \alpha + \beta \cdot (MKT_t - RF_t)$ and found the value effect - static CAPM is not valid
- Probably, the problem is the correlation between the market beta of HML and the market risk premium
- Then $HML_t = \alpha + \beta_t \cdot (MKT_t - RF_t)$ should work, i.e. produce $\alpha = 0$
- It was first estimated in Petkova and Zhang (JFE 2005)

Empirical Strategy

- Assume that the beta of HML is a linear function of the variables that predict market return:
  - Default spread (DEF)
  - Dividend yield (DIV)
  - Treasury bill rate (TB)
  - Term premium (TERM)

$$\beta_t = \gamma_0 + \gamma_1 \cdot DEF_{t-1} + \gamma_2 \cdot DIV_{t-1} + \gamma_3 \cdot TB_{t-1} + \gamma_4 \cdot TERM_{t-1}$$

- If this is the case, then the beta of HML and the market risk premium are surely correlated

Empirical Strategy

- We estimate $HML_t = \alpha + \beta_t \cdot (MKT_t - RF_t)$ with β_t as specified above

$$HML_t = \alpha + (\gamma_0 + \gamma_1 \cdot DEF_{t-1} + \gamma_2 \cdot DIV_{t-1}$$

$$+\gamma_3 \cdot TB_{t-1} + \gamma_4 \cdot TERM_{t-1}) \cdot (MKT_t - RF_t)$$

- Or, rearranging, we just regress HML on the market return and its products with the macro variables:

$$HML_t = \alpha + \gamma_0(MKT_t - RF_t) + \gamma_1 DEF_{t-1} \cdot (MKT_t - RF_t) +$$

$$+\gamma_2 DIV_{t-1} \cdot (MKT_t - RF_t) + \gamma_3 TB_{t-1} \cdot (MKT_t - RF_t) +$$

$$+\gamma_4 TERM_{t-1} \cdot (MKT_t - RF_t)$$

Petkova and Zhang: Main Result

$$HML_t = \underset{(0.115)}{0.45} + (-\underset{(0.09)}{0.36} - \underset{(0.08)}{0.11} DEF_{t-1} + \underset{(0.03)}{0.21} DIV_{t-1} -$$

$$- \underset{(0.02)}{0.07} TB_{t-1} - \underset{(0.03)}{0.01} TERM_{t-1}) \cdot (MKT_t - RF_t)$$

- As expected (Lewellen-Nagel critique), we go in the right direction, but the distance covered is too small
- The alpha declines by 14 bp, but still remains large and significant

Do the Signs Make Sense?

$$\beta_t = -\underset{(0.09)}{0.36} - \underset{(0.08)}{0.11}\ DEF_{t-1} + \underset{(0.03)}{0.21}\ DIV_{t-1} -$$
$$- \underset{(0.02)}{0.07}\ TB_{t-1} - \underset{(0.03)}{0.01}\ TERM_{t-1})$$

- Since the alpha improves when we use conditional CAPM instead of static CAPM, the market beta of HML has to be countercyclical, as we want it to be
- The beta of HML is high when:
  - Default spread is low - wrong, but insignificant
  - Dividend yield is high - right and significant
  - Treasury bill rate (aka expected inflation) is low - right and significant
  - Term premium is low - wrong, but insignificant

Are Magnitudes of the Coefficients above Realistic?

- If dividend yield increases by 1% per year, the market beta of HML increases by 0.2
- Dividend yield changes by about 2% peak to trough - the market beta of HML changes by 0.4 peak to trough. Realistic
- If expected inflation increases by 1% per year, the market beta of HML decreases by 0.07
- Expected inflation changes by about 4% peak to trough - the market beta of HML should change by 0.28 peak to trough. Realistic

Market Betas of HML in Expansions and Recessions

- I use the estimated equation for β_t to estimate the time-varying market beta of HML
- I split the sample into recessions and expansions based on the expected market return from the same regression (above median - recession, below median - expansion)
- Market beta of HML is -0.41 in expansion, -0.135 in recession
- Even with quite large variation in risk premium of 1% per month between expansions and recessions, the 0.275 difference in beta will explain less than half of the HML alpha
- Also, growth firms are still always riskier than value firms

Why Changing Beta Matters

- Market beta of HML is -0.41 in expansion, -0.135 in recession, -0.27 on average
- Suppose you invest $100 in HML and buy the market for $27 to hedge out the market movements and earn the (relatively) constant alpha
- Then the expansion happens, the beta of HML becomes -0.41
- The beta of your position "invest $100 in HML and buy the market for $27" becomes -0.14
- In the expansion, the market goes up, and you lose, since you have a negative beta (you under-bought the market)

Why Changing Beta Matters

- Then the recession happens, the beta of HML becomes -0.135
- The beta of your position "invest $100 in HML and buy the market for $27" becomes 0.135
- In the recession, the market goes down, and you lose, since you have a positive beta (you over-bought the market)
- The countercyclicality of the beta means that if you hedge your position based on average beta, you lose no matter where the market goes
- This is what is wrong with countercyclical beta

Hedging Changing Beta

$$\beta_t = -0.36 - 0.11 \cdot DEF_{t-1} + 0.21 \cdot DIV_{t-1} - \\ -0.07 \cdot TB_{t-1} - 0.01 \cdot TERM_{t-1})$$

- The equation below (borrowed from the estimation of the Conditional CAPM) can be used to predict beta in each period of time
- Example: in October 2001, DEF=0.86%, DIV=1.23%, TB=2.38%, TERM=1.91%
- Substituting into the beta equation above, one can calculate that the expected beta of HML in November 2001 is -0.384

Dynamic Hedging

- Hence, in October 2001 the trader that turns around \$100 in HML should have bought the market for \$38.4 to hedge out the market exposure
- One can redo the calculation using November 2001 values of DEF, DIV, TB, TERM and figure out that the beta was expected to be at -0.36 in December 2001
- Thus, going into December 2001, the trader will have to adjust his position in the market in the end of November 2001
- He wants to make sure he holds \$36 of the market for each \$100 in HML, not \$38.4, as in the end of October 2001

Recap

- Conditional CAPM introduces a new dimension of risk (in addition to static market beta)
- **Risk is higher beta (and greater losses) during risky times**
- However, we have to assume unrealistically large changes in market risk premium and betas to produce economically meaningful effects
- Conditional CAPM can point correctly to where the risk is coming from, but cannot measure its magnitude