5.6 WORKED EXAMPLE

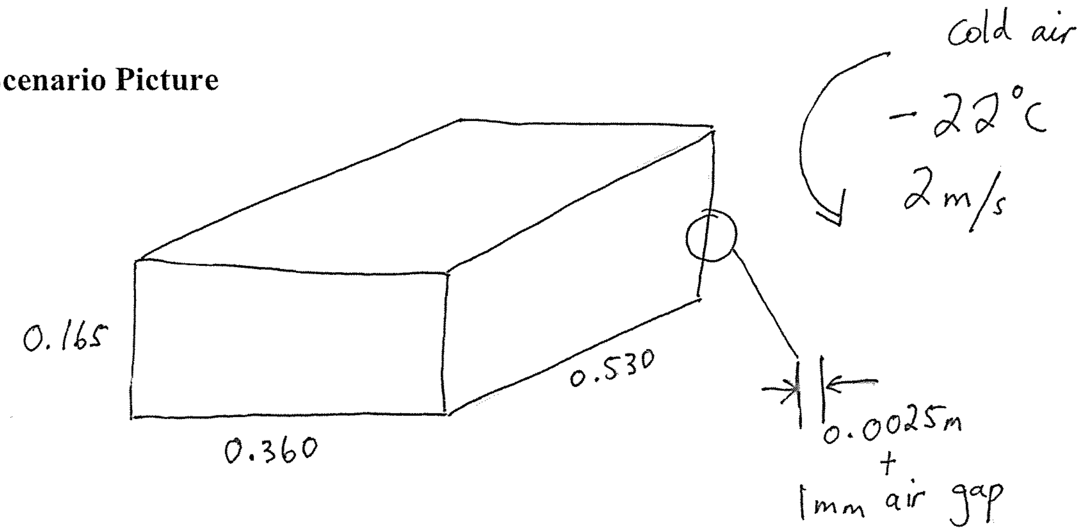
Cartons of boneless beef containing 27 kg have dimensions of about 165mm x 360 mm x 530 mm. The product starts at 10^0C and is frozen to -18^0C. A 48 hours cycle carton freezer using air at -22^0C and an average air velocity of 2 m/s is used. The cartons presently used are 2.5 mm thick solid walls. As the meat will not contact the sides, top and bottom of the carton perfectly, assume that there is a 1 mm air gap as well as the packaging that is a plastic liner as well as the carton. Will the carton freezer work?

In the calculation first we need thermal properties of the boneless beef. These were looked up in data tables: Product freezing temp $-1\,^\circ c$

c_l = 3600 J/kgK
c_s = 1900 J/kgK
k_s = 1.50 W/mK
ρ = 1060 kg/m^3
Δh_f = 215,000 J/kg

$10\,^\circ C \rightarrow -18\,^\circ C$

5.6.1 Scenario Picture



5.6.2 Planks Equation

$$R = \frac{0.165}{2} = 0.0825\,m$$

- cooling fluid heat transfer co-efficient

$$h_a = 7.3\,V^{0.8}$$
$$= 7.3\,(2)^{0.8}$$
$$= 12.71\,\frac{W}{m^2 K}$$

- heat transfer co-efficient

$$\frac{1}{h_e} = \frac{1}{h_a} + \frac{x_c}{k_c} + \frac{x_{air}}{k_{air}}$$
$$= \frac{1}{12.71} + \frac{0.0025}{0.08} + \frac{0.001}{0.025}$$

$$h_e = 6.67\,\frac{W}{m^2 K}$$

Sphere	$E = 3$
Infinitely long cylinders	$E = 2$
Infinite slabs	$E = 1$
Lamb leg	$E = 2.1$
Lamb shoulder	$E = 1.4$
Ewe leg	$E = 2.0$
Beef leg	$E = 1.3$
Tuna	$E = 1.4$

For 2-dimensional shapes β_2 is assumed to be infinitely large.

- **Planks Equation**

$$t_f = \frac{\rho \Delta h_f}{(\theta_f - \theta_a)} \left(\frac{R}{h_e} + \frac{R^2}{2 k_s} \right)$$

$$= \frac{1060 \times 215000}{(-1 - {}^{-}22)} \left(\frac{0.0825}{6.67} + \frac{0.0825^2}{2 \times 1.5} \right)$$

$$= 158852.4\ s$$

$$= 44.12\ hrs$$

5.6.3 Phams Modification to Planks Equation

- Shape factor E $\quad 1 + \dfrac{1 + \frac{2}{Bi}}{\beta_1^2 + \frac{2\beta_1}{Bi}} + \dfrac{1 + \frac{2}{Bi}}{\beta_2^2 + \frac{2\beta_2}{Bi}}$

- Smallest cross-sectional area of product through the thermal centre

- $\beta_1 = \dfrac{A}{\pi R^2} = \dfrac{0.165 \times 0.360}{\pi \times 0.0825^2} = 2.78$

- Volume $\quad 0.165 \times 0.360 \times 0.530$

  $= 0.0306\ m^3$

- $\beta_2 \quad \dfrac{3V}{4\pi R^3 \beta_1} = \dfrac{3 \times 0.0306}{4\pi (0.0825^3 \times 2.78)} = 4.68$

- $Bi \quad \dfrac{h_e R}{k_s} = \dfrac{6.67 \times 0.0825}{1.50} = 0.37$

- **Shape Factor** $\quad 1 + \dfrac{1 + \frac{2}{0.37}}{2.78^2 + \frac{2 \times 2.78}{0.37}} + \dfrac{1 + \frac{2}{0.37}}{4.68^2 + \frac{2 \times 4.68}{0.37}}$

$$= 1 + 0.281 + 0.136$$

$$E = 1.42$$

- $\theta_{fm} = 1.8 + 0.263 \times \theta_{out} + 0.105\,\theta_a$
  $= 1.8 + 0.263 \times -18 + 0.105 \times -22$
  $= -5.244$

Heat release in pre cooling
- $\Delta H_1 \; \rho C_l(\theta_{in} - \theta_{fm})$
  $= 1060 \times 3600(10 - {}^{-}5.244) = 5.82 \times 10^7 \; J/m^3$

Heat release in freezing
- $\Delta H_2 = \rho C_s(\theta_{fm} - \theta_{out}) + \rho \Delta h_f$
  $= 1060 \times 1900(-5.244 - {}^{-}18) + 1060 \times 215000 = 2.53 \times 10^8 \; J/m^3$

- $\Delta\theta_1 \; \dfrac{\theta_{in} - \theta_{fm}}{2} - \theta_a = \dfrac{10 - {}^{-}5.244}{2} - {}^{-}22 = 29.62$

- $\Delta\theta_2 = \theta_{fm} - \theta_a$
  $= -5.244 - {}^{-}22 = 16.756$

- **Phams Modification to Planks Equation**

$$t_f = \frac{1}{E}\left(\frac{\Delta H_1}{\Delta\theta_1} + \frac{\Delta H_2}{\Delta\theta_2}\right)\left(\frac{R}{h_e} + \frac{R^2}{2k_s}\right)$$

$$= \frac{1}{1.42}\left(\frac{5.82\times10^7}{29.62} + \frac{2.53\times10^8}{16.756}\right)\left(\frac{0.0825}{6.67} + \frac{0.0825^2}{2\times1.5}\right)$$

$$= 12016871\,(0.014637)$$

$$= 175898\,s$$

$$= 48.86\,hr$$