

Adam Smith Business School

Subject of Economics

Degree of MSc

Degree Exam

Mathematical Finance, ECON5020

Tuesday, 17 December 2019, 09:30-11:30

Please ensure that you write the course code (as above), your student ID, date of birth and the number of the question that you have attempted on each answer sheet.

How to complete this exam:

- Students should answer ONE question from Section A and ONE question from Section B.

Materials allowed:

- Calculators: Casio FX-83GT/Casio-83GT+, Casio FX-85GT/Casio FX-85GT+, Sharp EL531WH, Aurora AX-582BL, Sharp EL-233SBBK/Sharp ELSI MATE EL-233S

Instructions to students:

- Both entry and exit to the examination hall will be at the absolute discretion of the invigilator.
- No candidate will be permitted to leave within the first hour or the last half hour of this exam.

Using exam answer sheets

0 1 2 3 4 5 6 7 8 9

A B C D E F G H I J K L M N O

P Q R S T U V W X Y Z

EXAMPLE

Examination Answer Sheet

Date Of Birth (dd/mm/yy)

Student ID No

16 / 03 / 98

0712345

Course Code

Date of Exam

MG-T5192

22/10/2015

Question Number 3

- Always use a **black pen**.
- Complete personal information on all white sheets supplied before the exam begins.
- Use the standard character set, printed above, when hand writing in data boxes. E.g. Student ID, Date of Birth, Question Number. Keep your characters inside the boxes.
- Use **one** white written answer sheet per question, using BOTH sides if required.
- For this exam, the required number of white answer sheets is **TWO**.
- Request yellow continuation sheets to continue writing answer if there is not enough space on the white sheet.
- You must return all answer sheets to the invigilator even if you have not attempted all questions.

This page has been left blank for student notes – anything written here will not be marked.

You must use ONE WHITE answer sheet per question as per the number supplied. To provide an answer that exceeds the space on the answer sheet, please raise your hand to request a YELLOW answer sheet.

Answer ONE question from Section A and ONE question from Section B

Section A

1. Consider the discrete-time binomial tree model with three periods of length 1, i.e. $T = 3$ and $t = 0, 1, 2, 3$. In each period the price can move up or down, S_{t+1} is either uS_t or dS_t . Assume that the factor for moving up is $u = 4/3$, the factor for moving down is $d = 3/4$, and that the interest rate is $r = 0.0$. The initial stock price is $S_0 = 1$.
 - (a) Compute the price process (i.e. prices at all times and states) for a European *Put option* on the stock with strike price $K = 1$ and maturity $T = 3$.
 - (b) Compute the price at time $t = 0$ of the Australian option $\left(K - \frac{\frac{1}{T+1} \sum_{t=0}^T S_t}{S_T}\right)^+$ with $K = 1$. Note: As this option is path dependent, you will not be able to use the recursive method, nor will you be able to use the CRR formula.

[50%]

2. Consider the stock price under the Black-Scholes assumption, i.e.

$$S_t = S_0 \exp \left(\left(r - \frac{1}{2} \sigma^2 \right) t + \sigma W_t \right)$$

where r denotes the interest rate. Consider an option with payoff

$$h(S_T) = \left(\log \frac{S_T}{K} \right)^2$$

where T is the time of maturity and K is a constant. Compute the Greeks of this option. Decide whether

$$V(t, x) = \exp(-r(T-t)) \left[\left(\log \left(\frac{x}{K} \right) + \left(r - \frac{1}{2} \sigma^2 \right) (T-t) \right)^2 + \sigma^2 (T-t) \right]$$

is the Black-Scholes price of the option at time t assuming that $S_t = x$. Present your arguments.

[50%]

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Section B

3. Critically discuss similarities and differences of American and a European *call* and *put* options.

[50%]

4. State and explain the Ito formula. You do not need to prove it, but you have to illustrate the application of the formula by applying it to the geometric Brownian motion. Comment on the role of geometric Brownian motion in the modelling of stocks.

[50%]

Before your exam answers are collected:

- Please ensure that you have written the course code (on the front of this exam paper), your student ID, date of birth and the number of the question that you have attempted on each answer sheet.
- Put your exam answers in the order of the question number, ensuring that yellow answer sheets follow the appropriate white answer sheet.
- Do not place any other exam materials, including the exam paper, beside the A3 answer sheets.

END OF QUESTION PAPER