Question 4 Drying **(From 2017 Exam)**

After graduating from a well-known University, you immediately find employment with a company manufacturing food products.

One of your first tasks is to estimate some of the drying parameters for a new product being developed for the novelty snack-food market.

You are given the following information and guidelines:

- Each product unit is spherical, and has a diameter of 40 mm.
- The effective product density, for all calculations, is 1150 kg/m^3.
- In drying trials in the constant rate regime, a drying time of 4 minutes and 50 seconds was required to reduce moisture content from an initial value of 0.84 kg water/ kg dry solid to the critical moisture content at the end of constant rate period; the drying rate was 9 x 10^{-3} kg/m^2 s.
- In the falling rate regime, drying for this material is diffusion controlled.
- In exploratory drying trials in the falling rate regime, a drying time of 5 minutes was required to reduce the moisture content from the critical moisture content at the end of the constant rate period to 0.3 kg water/ kg dry solid.
- The effective diffusivity of water through the product is 3.315 x 10^{-9} m^2 /s at 20^{0}C.
- To meet specification, the final product must be dried to a condition where the moisture content at a radius of 12 mm is 0.0138 kg water/ kg dry solid; drying is carried out at 42°C.
- You can assume that there is no external resistance to mass transfer (Bi is very high).
- Gurney-Lurie charts are provided, and equations you will need for your calculations are in the list below.

Question four continued over …….

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(a) Calculate the equilibrium value of the moisture content at the end of the constant rate period, of the new product.

[3 marks]

(b) Calculate the residual moisture at equilibrium, X^*, for the new product.

[7 marks]

(c) Calculate the *total* time required to dry the new product from its initial moisture content (0.84 kg water/ kg dry solid) to the moisture content that meets the final product specification.

(If you do not have a value X^* from Part (b) you can use a value of 0.008 kg water/ kg dry solid)

[9 marks]

(d) Do you think the total drying time you calculated in (c) is correct? Explain your answer.

[1 marks]

[Total 20 marks]

You must show all your calculations; failure to do so will be penalised. All lines you draw on a Gurney-Lurie chart to extract information must be clearly visible, and the charts provided must be attached to your answer book.

Useful equations:

$$t_d = \frac{r\rho_s}{3R_c}(X_1 - X_2)$$

$$t_d = \frac{s\rho_s}{R_c}(X_1 - X_2)$$

$$t_d = \frac{r^2}{\pi^2 D_v}\ln\left[\frac{6(X_1\text{-}X^*)}{\pi^2(X_2\text{-}X^*)}\right]$$

$$t_d = \frac{4s^2}{\pi^2 D_v}\ln\left[\frac{8(X_1\text{-}X^*)}{\pi^2(X_2\text{-}X^*)}\right]$$

$$\frac{D_{v1}}{D_{v2}} = \left(\frac{T_1}{T_2}\right)^{1.5}$$

Drying 2017 SOLUTION.

(a) Choose equation: $t_D = \frac{r\,\rho_s}{3R_c}(X_1 - X_2)$
- CONSTANT RATE t_D -

here $X_1 = 0.84$ kg/kg (given)

here, $X_2 = X_c$

$t_D = 4\,\text{min}\,50\,\text{s} = (4 \times 60 + 50)\,\text{s} = 290\,\text{s}$ (given)

$\rho_s = 1150\ \text{kg/m}^3$ (given)

$R_c = 9 \times 10^{-3}\ \text{kg/m}^2\text{s}$ (given)

$r = 20\,\text{mm} = 0.02\,\text{m}$ (given)

substitute into equation for CONSTANT RATE t_D.

$$290 = \frac{0.02 \cdot 1150}{3 \cdot 9 \times 10^{-3}}(0.84 - X_c)$$

$$= 851.85\,(0.84 - X_c)$$

rearrange; $X_c = 0.4995 \simeq \underline{0.5}$ kg/kg

ANSWER (a) $X_c = 0.5$ kg/kg.

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(b) Choose equation: $t_D = \frac{r^2}{\pi^2 D_v} \ln\left[\frac{6}{\pi^2}\frac{(X_1 - X^*)}{(X_2 - X^*)}\right]$

here, $t_D = 5\ min = 5 \times 60s = 300s$ (given)

$r = 0.02m$ (given)

D_v – Given $D_v = 3.315 \times 10^{-9} \frac{m^2}{s}$

at 20°C

must calculate D_v for 42°C
(42°C is the drying temperature – given)

$$\frac{D_{v1}}{D_{v2}} = \left(\frac{T_1}{T_2}\right)^{1.5}$$

Temperatures in this equation are K.

$20°C = 273 + 20 = 293K$
$42°C = 273 + 42 = 315K$

rearrange: $D_{v315} = D_{v293}\left(\frac{T_2}{T_1}\right)^{1.5}$

$$= 3.315 \times 10^{-9} \underbrace{\left(\frac{315}{293}\right)^{1.5}}_{1.1147} = 3.695 \times 10^{-9}\ m^2/s$$

here, $X_1 = X_c$ (from (a)) $= 0.5\ \frac{kg}{kg}$.
$X_2 = 0.3$ (given)

Substitute, $300 = \frac{0.02^2}{\pi^2 3.695 \times 10^{-9}} \ln\left[\frac{6}{\pi^2}\frac{(0.5 - X^*)}{(0.3 - X^*)}\right]$

$$0.02735 = \ln\left[0.60793 \frac{(0.5 - X^*)}{(0.3 - X^*)}\right]$$

$$\exp[0.02735] = 0.60793 \frac{(0.5 - X^*)}{(0.3 - X^*)}$$

cont^{d} ⇒

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(3)

(2) continued.

$$1.02773 = 0.60793 \frac{(0.5 - x^*)}{(0.3 - x^*)}$$

$$1.6905(0.3 - x^*) = (0.5 - x^*)$$

$$.50715 - 1.6905x^* = 0.5 - x^*$$

$$0.00715 = 0.6905x^*$$

ANSWER (b) $\underline{\underline{x^*}} = 0.01035 \simeq 0.0104$ kg/kg.

(c) Calculate drying time in falling rat regim

Need to use Gurney-Lurie chart.

$$Y = \frac{X - x^*}{X_i - x^*}$$

$X = 0.0138$ (given)

$X_i = 0.5$ (given)

$X^* = 0.0104$ (from (b))

$$Y = \frac{0.0138 - 0.0104}{0.5 - 0.0104} = 0.00694 \simeq 0.007$$

Use the Gurney-Lurie Chart

$m = \frac{1}{Bi} \simeq 0$; $n = \frac{12}{20} = 0.6$

For $Y = 0.007$, $m = 0$, $n = 0.6$, $\beta = 0.5$

continued ⇒

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$$\beta = \mathcal{D}_v t_D / R^2 = 0.5$$

$$0.5 = \frac{3.695 \times 10^{-9} \; t_D}{(0.02)^2}$$

$$t_{D_{\text{FALLING RATE}}} = 54127 \text{ s} \quad (= 15.035 \text{ hours})$$

$$\text{TOTAL DRYING TIME} = t_{D_{\text{CONST RATE}}} + t_{D_{\text{FALLING RATE}}}$$

$$= 290 \text{ s} + 54127$$

$$\text{ANSWER (c)} = \underline{\underline{54417 \text{ s}}}$$

15 h
6 min
57 s

(d) The answer is correct.

The target moisture is close to the residual moisture, and the approach to this value is slow,