Liquidity and Liquidity Risk

Professor Alexander Barinov

School of Business Administration
University of California Riverside

MGT 295F Empirical Methods

Outline

Two Facets of Liquidity

- The measure of liquidity is the costs of instantaneously trading in the asset you desire to buy/sell
- The fundamental idea of asset pricing is that risk is the covariance of returns with something we care about
- As for firm characteristics, we only care about them because they are correlated with covariances
- Liquidity is special: the liquidity of an individual stock (characteristic) can impact the expected returns to this stock

Liquidity and Expected Returns

- Assume that, given the level of systematic risk, the expected return of asset A is 12%
- Assume also that the round-trip cost of trading A is 1%
- If we are to hold A for a year, its return before trading cost should be 13%, so that we get the fair compensation for risk (12%) net of the trading costs
- One of the first papers to point it out is Amihud and Mendelson (JFE 1986)

Different Holding Periods

- Now assume we will hold A for 5 years, not 1
- Return before trading costs of 12.2% should be enough
- The extra return we ask from illiquid assets then depends on the time period we intend to hold them for, that is, it depends on the investor, not only the asset
- This quite unusual, since we usually assume that investors only choose their individual level of risk, but tend to view all assets similarly (e.g., in the CAPM the market portfolio aka MVE is everyone's favorite)

Clientele Effects

- Short-term investors have strong preferences for liquid stocks (low trading costs)
- Long-term investors can outbid short-term investors on all stocks, but they cannot buy the whole country and have to choose
- Long-term investors choose illiquid stocks, since for these stocks they do not have to outbid anyone by accepting lower return

Clientele Effects

- Long-term investors earn rents (positive abnormal returns) by holding illiquid stock with high expected returns
- Short-term investors just get the fair compensation for risk
- **Prediction (Amihud and Mendelson, JFE 1986): illiquid firms have higher expected returns and positive alphas**

Example 1: 2 Stocks, 2 Investors

- Consider two stocks: A has the bid-ask spread of 2%, B has the bid-ask spread of 0.25%
- There are two investors in the market: X holds his stocks for 1 year, Y holds her stocks for 5 years
- Assume that if the bid-ask spreads were zero, the fair return to both A and B would be 12% per year
- What are the returns to A and B in the world with the bid-ask spread?

Example 1: 2 Stocks, 2 Investors

- We know that X will hold Stock B - it has to yield at least 12.25% for him to do that
- So, Stock B has the expected return of 12.25% and Y would make 0.2% holding it, because her costs of doing so are only 0.25%/5=0.05% per year
- But we need Y to choose Stock A, for which the holding costs are 2%/5=0.4%, and she has to make at least 0.2% on top of that, because otherwise she will switch to B
- Hence, the minimum return to Stock A is 12.61%, but it can be as high as 13.99% (if it is 14%, X will start buying A too)
- So, the return to A is 13.99%, and Y earns the abnormal return of 13.99%-12%-0.4%=1.59% holding it

Example 2: 3 Stocks, 3 Investors

- Consider stocks A, B, C with expected return of 12% and with the bid-ask spreads of 3%, 1%, and 0.5%, respectively
- Consider also investors X, Y, Z with the investment horizons of 1, 5, 10 years, respectively

Costs	**Stock A**	**Stock B**	**Stock C**
X, 1yr	3	1	0.5
Y, 5 yrs	0.6	0.2	0.1
Z, 10 yrs	0.3	0.1	0.05

Example 2: 3 Stocks, 3 Investors

- X holds Stock C, so Stock C has to make 12.5%, and Y and Z would get the abnormal return of 0.4% and 0.45% holding it
- Y has to hold Stock B, so Y has to make the abnormal return of 0.41% holding it, and his costs are 0.2% - Stock B has to make 12.61%
- Z holds Stock A at the cost of 0.3%, and if the return to A exceeds 13%, Y will switch to A
- So, Stock A makes 13%, and Z makes 0.7% - Z would make 0.51% holding B, so Z does not want to switch

Bid-Ask Spread and Returns

- Amihud and Mendelson (1986) is one of the first papers to document the impact of liquidity on expected returns and develop the model with clientele effects
- They claim that the spread-return relation should be convex: high and very high spread stocks command similar return, low spread stock command much smaller return
- They divide all stock in seven groups based on past bid-ask spread and find that the least liquid stocks earn by 86 bp per month (10.5% per year) more than the most liquid stocks

Amihud and Mendelson: Concerns

- Most importantly, the effect seems unbelievably strong
- If the extra return to the stocks with the highest bid-ask spread is 10.5%, just imagine how large the bid-ask spread has to be
- Take the middle spread group: still 3.5% on top of the return to the most liquid group
- In most theoretical models, trading costs cannot have that huge effect on expected returns

Amihud (JFM (2002) Measure

- The Amihud measure is the ratio of daily return to daily volume averaged for each firm-month or firm-year
- It intends to measure price impact: by how much you would move prices if you make a $1 million trade within a day
- Further studies with high-frequency data confirm that the Amihud measure has it right
- Amihud (JFM 2002): the Amihud illiquidity measure is positively related to returns
- If the price impact goes up by 1%, expected returns go up by about 15 bp per month

Example 3: 2 Stocks, 2 Investors

- Consider two stocks: A has the Amihud measure of 0.5%, B has the Amihud measure of 0.2%
- There are two investors in the market: X usually trades $10 million, Y usually trades $2 million
- Assume that if the price impact was zero, the fair return to both A and B would be 12% per year
- What are the returns to A and B in the world with price impact?

Example 3: 2 Stocks, 2 Investors

- When you trade two-way, you lose one bid-ask spread and two price impacts, that is, you lose what you trade times the Amihud measure
- X will hold B, the stock with low price impact, and will lose 2%, so Stock B has to make 14%
- Y will hold Stock A - if X held A, he would require at least 17% from it, because his costs are 5%
- So, if Stock A returns 16.99%, it is enough to keep X out and Y will make the abnormal return of 3.99%, because his costs of holding A are 1%
- Y can also hold B and make 1.6%, hence he is better off with A

Example 4: 3 Stocks, 2 Investors

- Suppose there is also Stock C with the Amihud measure of 0.1%
- If X is rich and wants both Stock B and Stock C, then C makes 13% and the returns to A and B are as above
- If Y is rich and wants both Stock A and Stock B, C makes 13%, B makes 13.99% to keep X out, C still makes 16.99%
- Y makes 1.59% abnormal return on Stock B and would make only 0.8% on C, so Y leaves Stock C to X

Example 5: 2 Stocks, 3 Investors

- Suppose that we only have Stock A and Stock B, but we also have investor Z who usually trades $1 million
- Z can share Stock A with Y, and then the returns to the stocks are the same as in Example 3
- Z earns 4.49% abnormal return on A and would earn 1.8% on B, so he is not tempted to switch
- Alternatively, Z can try to push Y to holding B - Y makes 1.6% holding B, and the cost of holding A to Y is 1%
- If A returns 14.59%, Y switches to B, Z makes 2.09% on A and would make 1.8% on B - another equilibrium
- What will happen depends on whether Z is rich enough to hold A all by himself

Liquidity Risk and Liquidity CAPM: Acharya and Pedersen (JFE 2005)

- Assume that each stock has its trading cost of c_i and the market portfolio has the trading cost of c_m (can be average trading cost, but can be the cost of buying the index futures)
- Standard CAPM: $Ret_i = RF + \lambda \cdot \beta_i$, where λ is the market risk premium, and $\beta = \dfrac{Cov(R_i, R_m)}{Var(R_m)}$
- If trading is not free, we can get only $R_i - C_i$ instead of R_i and $R_m - C_m$ instead of R_m
- Liquidity CAPM: $Ret_i = RF + \lambda \cdot \beta_i$, where $\beta = \dfrac{Cov(R_i - C_i, R_m - C_m)}{Var(R_m - C_m)}$

Liquidity CAPM

$$\beta = \frac{Cov(R_i - C_i, R_m - C_m)}{Var(R_m - C_m)} = \beta + \beta_{L1} + \beta_{L2} + \beta_{L3}$$

- $\beta \sim Cov(R_i, R_m)$ - the covariance with the market factor is still there, as always
- $\beta_{L1} \sim -Cov(C_i, R_m)$ - high trading costs during market declines is the first form of liquidity risk
- $\beta_{L2} \sim -Cov(R_i, C_m)$ - losses when the whole market is illiquid is the second form of liquidity risk
- $\beta_{L3} \sim Cov(C_i, C_m)$ - high trading costs when the market liquidity drops is the third form of liquidity risk

Liquidity CAPM Attempt

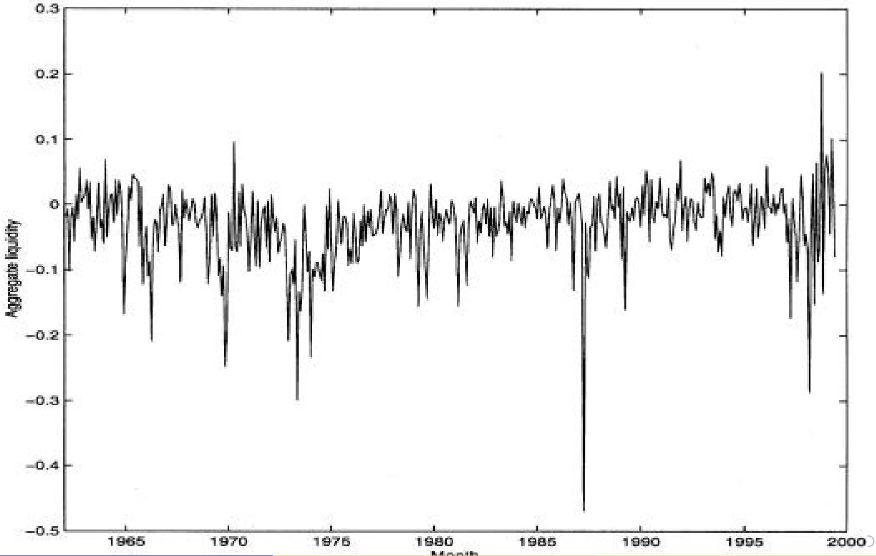
- Averaging the Amihud measure across all firms yields market illiquidity (average price impact in the market)
- Amihud (JFM 2002) tries using it as an ICAPM state variable
- Higher average price impact means negative returns today and higher risk going forward
- Both effects are stronger for small stocks ("flight-to-quality")
- The results survive the control for other predictors of the market risk premium

Pastor-Stambaugh Gamma

$$R_{t+1} = \theta + \phi R_t + \gamma \cdot \mathrm{sign}(R_t) \cdot \mathrm{Vol}_t$$

- γ is a measure of price impact: if prices were pushed too high because everyone was buying, tomorrow we will see a bounce-back
- Hence, more negative gamma (more bounce-back) means higher price impact
- Pastor and Stambaugh (JPE 2003) take the average gamma in each month and construct the market-wide illiquidity measure
- Subsequent studies raise doubts if this measure is a good measure of price impact

Market-Wide Illiquidity Through Time



Innovations to Market Illiquidity

Specifically, to construct innovations in liquidity, we first scale the monthly difference in liquidity measures, averaged across the N_t stocks with available data in both the current and previous month,

$$\Delta\hat{\gamma}_t = \left(\frac{m_t}{m_1}\right)\frac{1}{N_t}\sum_{i=1}^{N_t}(\hat{\gamma}_{i,t} - \hat{\gamma}_{i,t-1}). \tag{6}$$

We then regress $\Delta\hat{\gamma}_t$ on its lag as well as the lagged value of the scaled level series:

$$\Delta\hat{\gamma}_t = a + b\Delta\hat{\gamma}_{t-1} + c\left(\frac{m_{t-1}}{m_1}\right)\hat{\gamma}_{t-1} + u_t. \tag{7}$$

Liquidity CAPM Attempt

- Pastor and Stambaugh define liquidity risk as the risk of losses during the periods of market illiquidity
- This is not the only way to define liquidity risk: they could have also used the firm-level gammas to look at the risk of illiquidity during market declines or periods of market illiquidity
- Liquidity betas are the stock return exposure to innovation in the market-wide gamma
- The exposures are estimated from the four-factor model with the Fama-French factors and the innovation

Sorting on Liquidity Betas

- Pastor and Stambaugh sort firms on liquidity risk this way:
  - First, for each firm estimate the loading on the non-traded factor using data from past 5 years
  - Then, sort firms into ten equal bins (deciles) on the loading you just estimated
  - Observe each decile for a year, then repeat the estimation
- Both low liquidity risk firms and high liquidity risk firms have significant alphas
- The return spread between the firms with the lowest and the highest liquidity risk is between 4-8% per year
- The negative alphas are more important

Traded Liquidity Factor

- The innovations to the market-wide gamma are often referred to as the non-traded Pastor-Stambaugh factor
- Why positive loading on the non-traded factor is risk: negative gamma is illiquidity, negative shock to the market-wide average gamma is more illiquidity, bad returns in response to this shock are risk
- Traded Pastor-Stambaugh factor is the 10-1 return spread from the sorts in the previous slide
- Why positive liquidity beta wrt the traded factor is risk: positive liquidity beta means that the firm comoves with other firms with positive loadings on the non-traded factor

Liquidity Factor and Momentum

- Liquidity factor explains momentum, especially in the later sample
- Equal-weighted factor works better - it better captures the liquidity of small firms that populate winners and losers quintiles)
- Looks like liquidity can explain 25-50% of momentum
- Winners will be hit more by an increase in market illiquidity than losers?

Acharya and Pedersen (JFE 2005)

- Acharya and Pedersen (JFE 2005) complete the ICAPM-style analysis with the Amihud liquidity measure
- They trim it a little before averaging across firms in order to remove outliers
- They compute innovations to the market-wide average price impact
- Then they do the real liquidity CAPM: they look at all three covariances that measure liquidity risk

Liquidity CAPM

$$\beta = \frac{Cov(R_i - C_i, R_m - C_m)}{Var(R_m - C_m)} = \beta + \beta_{L1} + \beta_{L2} + \beta_{L3}$$

- $\beta \sim Cov(R_i, R_m)$ - the covariance with the market factor is still there, as always
- $\beta_{L1} \sim -Cov(C_i, R_m)$ - high trading costs during market declines is the first form of liquidity risk
- $\beta_{L2} \sim -Cov(R_i, C_m)$ - losses when the whole market is illiquid is the second form of liquidity risk
- $\beta_{L3} \sim Cov(C_i, C_m)$ - high trading costs when the market liquidity drops is the third form of liquidity risk

Acharya and Pedersen: Recap

- The model suggests that all four betas should have the same price of risk, which probably means the model is too simplistic
- Acharya and Pedersen single out the market beta and lump all liquidity betas together
- The joint liquidity beta is significant in liquidity sorts, a bit shaky in size/BM sorts
- Splitting it up in four separate betas kills the significance and makes all signs but one wrong
- This is not supposed to happen, it raises questions if we were lumping together wrong things to start with

Momentum and Liquidity Risk

- We will look at the exposure of the MOM factor to liquidity risk
- We will use the Amihud non-traded factor (changes in the average price impact)
- And we will use the Pastor-Stambaugh traded factor (return differential between high and low liquidity risk firms)
- We will be adding these factors to the CAPM

Momentum and Amihud Factor

$$MOM_t = \underset{(0.20)}{0.75} - \underset{(0.04)}{0.17} \cdot (MKT_t - RF_t) - \underset{(0.04)}{2.83} \cdot DIlliq_t$$

- Can we interpret the intercept as abnormal return?
- High average price impact is a bad thing, and negative loading on DIlliq means bad returns to the momentum strategy if average price impact increases
- Hence, the negative loading we see is liquidity risk
- Magnitude issue: if price impact increases by 1%, MOM trails the CAPM by 2.83%, but keep in mind that the standard deviation of DIlliq is only 0.18%

Momentum and PS Factor

$$MOM_t = \underset{(0.20)}{0.77} - \underset{(0.04)}{0.14} \cdot (MKT_t - RF_t) - \underset{(0.06)}{0.05} \cdot PS_t$$

- Can we interpret the intercept as abnormal return?
- The negative Pastor-Stambaugh beta suggests that MOM comoves more with low-risk firms, the ones that respond positively to the increase in the market illiquidity
- In the modern sample, the Pastor-Stambaugh liquidity factor does not explain momentum