



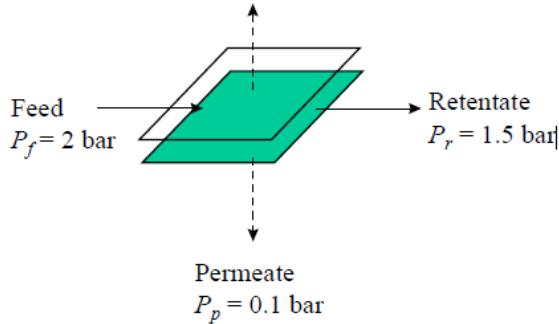
## **280.371 Process Engineering Operations**

### **Membrane Separation Processes – Tutorial Problems**

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- 1.** For the filter module shown below, calculate the pressure drop on the feed side and the transmembrane pressure.



**2.** The permeate flux through a membrane under optimal processing conditions is  $6 \times 10^{-4} \text{ m}^3 \text{m}^{-2} \text{s}^{-1}$ . How much membrane area would you need to produce  $1440 \text{ m}^3$  of filtered broth within an 8-h shift, if the flux does not change?

**3.** An ultrafiltration membrane has a pure water flux of  $5.6 \times 10^{-5} \text{ m}^3 \text{m}^{-2} \text{s}^{-1}$  at a transmembrane pressure of  $3 \times 10^5 \text{ Pa}$ . When the membrane is used to concentrate an aqueous slurry of yeast cells at a transmembrane pressure of  $4 \times 10^5 \text{ Pa}$ , the steady-state flux is reduced to  $8.3 \times 10^{-6} \text{ m}^3 \text{m}^{-2} \text{s}^{-1}$  because of the buildup of a cake layer of cells. The specific resistance of the cake is  $1.5 \times 10^{18} \text{ m}^{-2}$ . Calculate the thickness of the yeast cake. How would you attempt to reduce the resistance offered by the cake? What is the likely result of increasing the processing temperature by 5 °C? (The viscosity of water at the operating conditions was  $1.2 \times 10^{-3} \text{ Pa.s}$ ).

**4.** During steady-state concentration of albumin by ultrafiltration, the permeate stream contained albumin at a concentration of  $0.002 \text{ kg m}^{-3}$ . The concentration of albumin in the bulk feed was  $2 \text{ kg m}^{-3}$ . Calculate the solute rejection coefficient, SRC.

## Problems for Reverse Osmosis

### 5. Osmotic pressure / RO membrane performance

Estimate the osmotic pressure of a 3 wt % (w/v) solution of the following compounds at 30°C:

- (a) NaCl (MW = 58.5)
- (b) glucose (MW = 180)
- (c) albumin protein (MW ≈ 65,000)

An RO membrane is characterised by :

$$A = 5 \times 10^{-6} \text{ m s}^{-1} \text{ bar}^{-1}$$

$$B = 5 \times 10^{-7} \text{ m s}^{-1}$$

Estimate the flux and SRC for a feed concentration of 5,000 ppm NaCl, operating temperature of 35°C, and  $\Delta P$  of  $2.5 \times 10^6$  Pa.

### 6. Osmotic pressure and Reverse osmosis

An RO system is used to concentrate an electroplating waste stream available at 25°C. Given:

- the retentate stream contains 2 kg m<sup>-3</sup> ZnCl<sub>2</sub> (MW: Zn = 65.4, Cl = 35.5)
- the SRC is 0.95
- the feed pressure is 20 bar (with negligible permeate pressure)
- the flux is 25 LMH

Calculate the values of the membrane constants A and B.

### 7. Osmotic pressure and Reverse osmosis

It is proposed to concentrate a salt brine (NaCl, MW = 58.5) at 30°C using an RO membrane with membrane constants of  $A = 9 \times 10^{-7} \text{ m.s}^{-1}.\text{bar}^{-1}$  and  $B = 7 \times 10^{-7} \text{ m.s}^{-1}$ . The retentate brine concentration is 10,000 mg.l<sup>-1</sup> and the permeate salt concentration is not to exceed 1,000 mg. l<sup>-1</sup>.

Estimate the flux and minimum required operating pressure. The osmotic pressure is given by:

$$\pi = \beta nRT$$

where R = 8,314, for  $\pi$  in Pa and n in kg-mole.m<sup>-3</sup>.

### 8. Osmotic pressure and Reverse osmosis

Tests have been performed to characterise the performance of a reverse osmosis membrane. In these tests, a retentate NaCl brine concentration of 1.5% (w/v)

produced a flux of 40 LMH and SRC of 0.99 when the operating pressure and temperature were 60 bar and 40°C respectively.

The RO membrane is now to be used to concentrate a  $\text{CaCl}_2$  brine to a final concentration of 2.5% (w/v). The molecular weight of the dissolved salt is 111 and the temperature is 40°C.

Estimate the operating pressure required if the flux and solute rejection coefficient (SRC) are to have a minimum of 35 LMH and 0.98, respectively. State any assumptions made.

### 9. Reverse osmosis design problem

A reverse osmosis membrane plant is used to concentrate a  $\text{NaCl}$  brine solution. At an operating pressure of 20 bar and temperature of 25°C, a flux of 32 LMH was measured when the brine concentration in the retentate was 2% w/v. The membrane's solute rejection coefficient is 0.98 ( $\text{NaCl}$ , MW = 58.5).

- (a) Determine the membrane constants A and B.
- (b) If the brine concentration in the retentate is increased to 3% w/v and the operating pressure is increased to 40 bar, determine the flux that should be expected and the new solute rejection coefficient. The operating temperature remains the same. State any assumptions made.

## Problems for Ultrafiltration

### 10. Worked Example

#### DESIGN OF UF PLANTS FOR WHEY CONCENTRATION

The flux obtained during UF concentration of acid casein whey is described in Table 1:

Table 1: Flux data for UF whey	
VCF	Flux (LMH)
1.0	45.0
1.19	43.0
1.45	41.0
1.83	38.5
2.42	35.5
3.41	32.5
5.29	28.2
9.42	22.5
20.0	15.0

Calculate the area required to process 7,500 l/hr whey to VCF = 20 for:

- A batch plant, i.e. to process 7,500 litres in 1 hour (Note: in a preliminary batch experiment 100 litres whey was concentrated to VCF = 20 in 1.36 h with 2 m<sup>2</sup> plant)
- A single-stage continuous plant
- A two-stage plant if the membrane areas in each stage are to be approximately equal

#### (a) Batch operation

##### (1) Estimate the average flux

(2) Calculate the required area:

**(b) Single-stage Continuous System**

(1) Determine flux that the plant will operate at:

(2) Calculate the required area:

**(c) Two-stage Continuous System**

As required here, in multi-stage systems the VCF for each stage is usually chosen so as to give an equal area for each stage. This is convenient and also gives a total area very close to the minimum area. The VCF for all but the last stage must be selected on a trial and error basis.

Before starting the flux data must be plotted (preferably on semi-log paper)

(1) Select VCF values and determine the corresponding flux values for each stage:

(2) Calculate the flow rates at each stage and the area required:

Table 2: Flow rates and module areas for the two-stage system, Trial 1

Stage	VCF	Feed flow (l/h)	Retentate flow (l/h)	Permeate flow (l/h)	Flux (l/m <sup>2</sup> h)	Area (m <sup>2</sup> )
1						
2						

(3) Adjust the estimates and recalculate:

Table 3: Flow rates and module areas for the two-stage system, Trial 2

Stage	VCF	Feed flow (l/h)	Retentate flow (l/h)	Permeate flow (l/h)	Flux (l/m <sup>2</sup> h)	Area (m <sup>2</sup> )
1						
2						

## 11. Ultrafiltration – VCF

A continuous flow ultrafiltration plant is required to concentrate  $7.5 \text{ m}^{-3} \text{ h}^{-1}$  of whey to a volume concentration factor (VCF) of 20. Calculate the membrane area needed.  
(Note: A batch process development run showed that  $0.1 \text{ m}^3$  of whey could be concentrated to a VCF of 20 in 1.36 h with a  $2 \text{ m}^2$  membrane module.)

## 12. Ultrafiltration – gel layer model

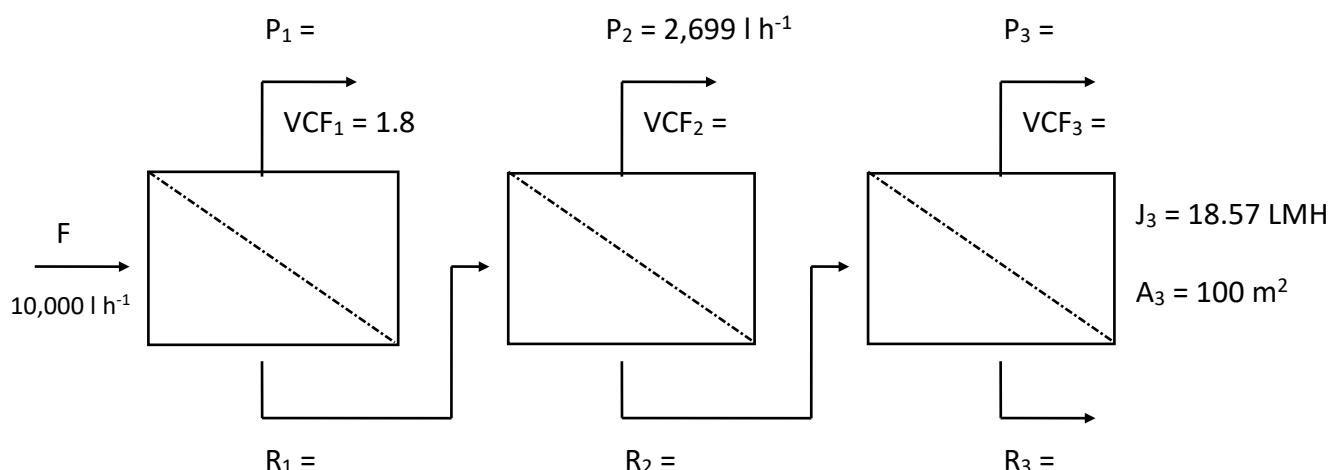
The gel-layer model of concentration polarisation gives

$$J = k \ln\left(\frac{c_g}{c_b}\right)$$

Estimate the values of  $k$  (in  $\text{m s}^{-1}$ ) and  $c_g$  for the whey data given in the Worked Example using the flux value at VCF = 20. Assume the initial protein concentration is 0.6 wt % and that SRC = 1.0. Confirm your answer by calculating the flux at VCF = 10.

## 13. Ultrafiltration – multistage process

Calculate the missing values for a three-stage plant as shown:



## 14. Ultrafiltration

Determine the gel concentration (i.e., concentration at membrane) of human serum albumin (hSA) in an ultrafiltration module operated using feeds with various concentrations of hSA. The hSA concentration in the feed and the corresponding steady state flux of permeate are given below:

hSA concentration (wt. %)	Flux ( $\text{L m}^{-2} \text{ h}^{-1}$ )
2	196
3	165
4	146
5	125

Assume that the properties of the feed solution were unaffected by the amount of dissolved hSA present. The cross flow rate and the transmembrane pressures were identical in all runs.

### **15. Ultrafiltration – mass transfer coefficients**

A UF process is designed to recover a sizing agent from a textile plant effluent. The sizing effluent is collected in batches of 2,000 l and must be processed in 5h to yield a retentate stream of 250 l, which is then recycled. The UF plant has a membrane area of 10 m<sup>2</sup>. The initial flux of the sizing agent (at a concentration of 3 kg.m<sup>-3</sup>) is 40 LMH. Essentially all the sizing agent is retained by the membrane.

- (i) Calculate the average and final fluxes.
- (ii) Estimate the mass transfer coefficient ( $k$ ) for the process, and  $c_m$ .
- (iii) What processing time would be required if the temperature was raised to increase the value of the mass transfer coefficient ( $k$ ) by 30%?

### **16. Ultrafiltration – batch process**

A protein solution is concentrated by batch ultrafiltration (UF) in a laboratory trial. The membrane area was 0.1 m<sup>2</sup> and 12 litres of permeate was collected in 4 hours with the plant operating at 20°C. The initial feed volume was 15 litres and the initial protein concentration was 1.5% (w/v). The flux at 4 hours was measured as 18 LMH.

- (i) Calculate the average flux, and estimate the gel layer concentration ( $c_g$ ) and mass transfer coefficient ( $k$ ).
- (ii) If a large batch plant is to process 4,000 litres of feed material in an eight hour period with the same concentration factor, what area would be required? This plant will run at 10°C, with the consequence that  $k$  will reduce by one third from the value at 20°C.

### **17. Ultrafiltration – batch process**

A batch ultrafiltration (UF) process is used to pre-concentrate skim milk prior to cheese making. During UF the proteins are completely retained by the membrane. After 5 hours operation the retentate volume is 500 litres and the retentate is four times more concentrated than the feed solution, which was initially 34 g/l. The initial and final fluxes were 42 LMH and 6 LMH respectively. The membrane area is 10 m<sup>2</sup>.

- (a) Calculate the average flux.
- (b) Estimate the mass transfer coefficient ( $k$ ) and the solute concentration at the membrane ( $c_m$ ).

### **18. Ultrafiltration**

The total feed volume flow rate through a flat plate ultrafiltration module (10 channels of 2 mm height and 15 cm width) is  $4 \times 10^{-3} \text{ m}^3\text{s}^{-1}$ . Calculate the mass transfer

coefficient  $k$  of the solute (albumin) in the filter module and the thickness of the concentration polarization layer (or gel layer). The density and the viscosity of the bulk feed are  $1010 \text{ kg m}^{-3}$  and  $1.2 \times 10^{-3} \text{ Pa s}$ , respectively. The diffusivity of the solute is  $6.81 \times 10^{-11} \text{ m}^2\text{s}^{-1}$ .

### 19. Microfiltration

Microfiltration is used to concentrate a bacterial fermentation broth from 1% to 10% cells. Use of an effective backflush ensures that the flux remains at  $0.1 \text{ m}^3 \text{ m}^2\text{h}^{-1}$  throughout the process. The fermentor has a working volume of  $10 \text{ m}^3$  and the available area of the membrane is  $12 \text{ m}^2$ . Calculate the batch filtration time if the membrane fully retains the cells.