280.371 Process Engineering Operations

Membrane Separation Processes
Model Answers to Tutorial Problems

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MEMBRANE SEPARATION PROCESSES

Model Answers

Problem 1.

$$\text{Feed side presure drop} = P_f - P_r = 0.5 \text{ bar}$$

Transmembrane pressure = 1.65 bar

Problem 2

$$J = \frac{P}{A \times t}$$

Permeate flow rate needed = 0.05 m^3/s $\left(\frac{1440}{8 \times 3600}\right)$

Area = 83.333 m^2

Problem 3

$$J = \frac{\Delta P}{\mu(R_m + R_c)}$$

When R_c=0

R_m =4.46 x 10^{12} m^{-1}

For the yeast slurry
J = 8.3x10^{-6} $m^3m^{-2}s^{-1}$

Then
R_c= 3.57x10^{13} m^{-1}

Thickness of cake = $\frac{R_c}{specific\ cake\ resis\tan ce} = 23.8\mu m$

Problem 4.

SRC = $1 - \frac{c_p}{c_f} = 0.999$

Problem 5.
3 wt % soln

$$\frac{3g}{100ml} = \frac{30g}{l} = \frac{30{,}000g}{m^3}$$

(a) NaCl MW = 58.5 g/mol

$$n = \frac{30{,}000}{58.5} = 512.8 \quad molm^{-3} \qquad\qquad \beta = 2$$

$$\pi = \beta nRT = 2 \times 512.8 \times 8.314 \times (273 + 30)$$

$$\pi = 2.58 MPa$$

check units

(b) glucose MW = 180 g/mol

$$n = \frac{30{,}000}{180} = 166.7 molm^{-3} \qquad\qquad \beta = 1$$

$$\pi = 1 \times 166.7 \times 8.314 \times 303 = 0.42 MPa$$

(c) albumin protein MW = 65,000 g/mol

$$n = 0.46 molm^{-3} \qquad\qquad \beta = 1$$

$$\pi = 1 \times 0.46 \times 8.314 \times 303 = 1158 Pa$$

Estimate $\Delta\pi$ across the membrane

Assume $c_p = 0 \quad \Rightarrow \quad \Delta\pi = \pi_{feed}$

5000 ppm = 5000 gm^{-3}

$$n = \frac{5000}{58.5} = 85.5 molm^{-3}$$

$$\pi = \beta nRT \qquad = 2 \times 85.5 \times 8.314 \times (273 + 35) = 4.38 \times 10^5 Pa$$

Estimate flux

$A = 5 \times 10^{-6}$ m s^{-1} bar^{-1} = 5×10^{-11} m s^{-1} Pa^{-1}

$$J = A(\Delta P - \Delta\pi) = 5 \times 10^{-11}(2.5 \times 10^6 - 4.38 \times 10^5)$$

$$J = 1.03 \times 10^{-4} m^3\, m^{-2}\, s^{-1}$$

$$SRC = \frac{J}{J + B} = \frac{1.03 \times 10^{-4}}{1.03 \times 10^{-4} + 5 \times 10^{-7}} = 0.995$$

Problem 6.

$$J = 25\ LMH = 25\frac{l}{m^2 h} \times \frac{m^3}{1000l} \times \frac{1h}{3600s}$$

$$= 6.94 \times 10^{-6}\, m^3 m^{-2} s^{-1}$$

$$\Delta P = 20bar = 20 \times 10^5\, Pa$$

$$\pi_{feed} = \beta nRT \qquad \beta = 3$$

$$n = 2 \times 10^3\, \frac{g}{m^3} \times \frac{1}{136.4 gmol^{-1}}$$

$$= 14.66\ mol\, m^{-3}$$

$$\pi_{feed} = 3 \times 14.66 \times 8.314 \times 298 = 1.09 \times 10^5\, Pa$$

SRC = 0.95

$$SRC = 1 - \frac{c_p}{c_b} \qquad \Rightarrow \qquad c_p = c_b(1 - SRC)$$

$$c_p = 2\ kgm^{-3} \times (1 - 0.95) = 0.1\, kgm^{-3}$$

$$\pi_{permeate} = 5.46 \times 10^3\, Pa$$

$$\therefore\ \Delta\pi = 109 - 5.46 = 103.54 kPa = 1.035 \times 10^5\, Pa$$

$$J = A(\Delta P - \Delta\pi)$$

$$A = \frac{J}{(\Delta P - \Delta\pi)} = \frac{6.94 \times 10^{-6}}{(20 \times 10^5 - 1.035 \times 10^5)}$$

$$= 3.66 \times 10^{-12}\, ms^{-1} Pa^{-1} = 3.66 \times 10^{-7}\, ms^{-1} bar^{-1}$$

$$SRC = \frac{J}{J + B}$$

$$B = \frac{J}{SRC} - J$$

$$= \frac{6.94 \times 10^{-6}}{0.95} - 6.94 \times 10^{-6}$$

$$= 3.65 \times 10^{-7}\, ms^{-1}$$

Problem 7.

$$SRC = 1 - \frac{c_p}{c_b}$$

$$= 1 - \frac{1{,}000}{10{,}000} = 0.9$$

$$\Delta c = 10{,}000 - 1{,}000 mgl^{-1} \times \frac{g}{1000mg} \times \frac{1000l}{m^3} = 9000\ gm^{-3}$$

$$\Delta n = 9000\ gm^{-3} \times \frac{mol}{58.5g} \quad = 153.8\ mol\, m^{-3}$$

$$\therefore \begin{aligned} \Delta\pi &= 2 \times 153.8 \times 8.314 \times 303 \\ &= 7.75 bar \end{aligned}$$

$$SRC = \frac{J}{J+B} \qquad SRC \cdot J + SRC \cdot B = J$$

$$SRC \cdot B = J \cdot (1 - SRC)$$

$$J = \frac{SRC \cdot B}{(1 - SRC)}$$

$$J = \frac{0.9 \times (7 \times 10^{-7})}{(1 - 0.9)}$$

$$J = 6.3 \times 10^{-6}\, ms^{-1}$$

$$\Delta P = \frac{6.3 \times 10^{-6}}{9 \times 10^{-7}} + 7.75 = 14.75\ bar$$

Problem 8.

NaCl = 1.5%, MW = 58.5 g/mol
$CaCl_2$ = 2.5%, MW = 111 g/mol

NaCl
1.5% w/v = 1.5×10^4 g/m^3
÷ 58.5 = 256 mol/m^3

$$\pi_{feed} = 2 \times 256 \times 8.314 \times 313 = 13.3\ bar = 13.3 \times 10^5\ Pa$$

SRC = 0.99

$$SRC = 1 - \frac{c_p}{c_b} \qquad \Rightarrow c_p = 0.015\%$$

$$\pi_p = 2 \times 2.56 \times 8.316 \times 313 = 0.133\ bar = 13.3 \times 10^3\ Pa$$

$$\Delta\pi = 13.17 bar$$

$$J = A(\Delta P - \Delta\pi) \qquad \Rightarrow \qquad A = \frac{J}{(\Delta P - \Delta\pi)} = \frac{40}{(60 - 13.17)} = 0.854\ l\, m^{-2} h^{-1} bar^{-1}$$

$$= 2.37 \times 10^{-12}\ m^3 m^{-2} s^{-1} Pa^{-1}$$

$CaCl_2$
2.5% w/v = 225 mol/m^3

$$c_p = (1 - 0.98)2.5 = 0.05\%$$

$$\pi_{feed} = 3 \times 225 \times 8.314 \times 313 = 17.56\ bar$$

$$\pi_p = 3 \times 4.5 \times 8.314 \times 313 = 0.35\ bar$$

$$\Delta\pi = 17.56 - 0.35 = 17.21\ bar$$

$$J = A(\Delta P - \Delta\pi) \qquad \Rightarrow \quad \Delta P = \frac{J}{A} + \Delta\pi = \frac{35}{0.854} + 17.21 = 58.19\ bar$$

$$= 5.819 MPa$$

Problem 9.

NaCl 2%w/v = $20 \times 10^3\ gm^{-3}$, MW = 58.5

For NaCl feed, $n = 20 \times 10^3 \div 58.5 = 342\ molm^{-3}$

$$\pi_{feed} = \beta nRT = 2 \cdot 342 \cdot 8.314 \cdot 298$$
$$\pi_{feed} = 16.9 \quad bar$$

SRC = 0.98

$$c_p = (1 - SRC)c_b = 400\ gm^{-3}$$
$$\therefore \quad n = 6.8\ molm^{-3}$$
$$\pi_{permeate} = 2 \cdot 6.8 \cdot 8.314 \cdot 298$$
$$= 0.34\ bar$$

$$\Delta\pi = 16.9 - 0.34$$
$$= 16.6\ bar$$

$$J = 32LMH = 8.9 \times 10^{-6}\ m^3 m^{-2} s^{-1}$$

$$J = A(\Delta P - \Delta\pi)$$

$$A = \frac{8.9 \times 10^{-6}}{(20 - 16.6)} = 2.6 \times 10^{-6}\ m^3 m^{-2} s^{-1} bar^{-1} = 2.6 \times 10^{-11} m^3 m^{-2} s^{-1} Pa^{-1}$$
$$= 9.41\ LMHbar^{-1}$$

$$SRC = \frac{J}{J + B}$$
$$SRC \cdot J + SRC \cdot B = J$$
$$SRC \cdot B = J(1 - SRC)$$
$$B = \frac{J(1 - SRC)}{SRC}$$
$$B = \frac{8.9 \times 10^{-6}(1 - 0.98)}{0.98} = 1.8 \times 10^{-7}\ m^3 m^2 s^{-1}$$
$$= 0.65\ LMH$$

For a 3%w/v NaCl solution, and $\Delta P = 40\ bar$

$$\pi_{feed} = \frac{30 \times 10^3}{58.5} \times 2 \times 8.314 \times 298 = 25.4\ bar$$

As $\Delta P \uparrow \quad then \quad SRC \uparrow$

Assume SRC = 0.98 (at least)

$$\pi_{permeate} = \frac{600}{58.5} \times 2 \times 8.314 \times 298 = 0.05\ bar$$

As $\pi_{permeate} << \pi_{feed}$ *then* $\Delta\pi \approx \pi_{feed} = 25.4\ bar$
We will also assume SRC ≈ 1

$$J = A(\Delta P - \Delta\pi) = 2.6 \times 10^{-6}(40 - 25.4) = 3.76 \times 10^{-5}\ m^3 m^{-2} s^{-1}$$
$$= 136\ LMH$$

$$SRC = \frac{J}{J+B} = \frac{3.76 \times 10^{-5}}{3.76 \times 10^{-5}} + 1.8 \times 10^{-7} = 0.995$$

close to SRC = 1, hence initial assumption OK

Ultrafiltration

10.

Design of UF Plants for Whey Concentration

The flux obta ined during UF concentration of acid casein whey is described in Table l :

Table 1: **Flux data for UF of whey (from Matthews 1980)**	
VCF	**Flux(LMH)**
1.0	45.0
1.19	43.0
1.45	41.0
1.83	38.5
2.42	35.5
3.41	32.5
5.29	28.2
9.42	22.5
20.0	15.0

Calculate the area required to process 7,500 I/hr whey to VCF=20 for:

- A batch plant, i.e. to process 7,500 l in 1 hour (Note: in a preliminary batch experiment 100 1 whey was concentrated to VCF=20 in 1.36 h with 2m^2 plant)
- A single-stage continuous plant
- A two-stage plant if the membrane areas in each stage are to be approximately equal

1. Batch Operation:

(1) Estimate the average flux

From the data, for whey concentration to VCF=20, i.e., the retentate has a concentration which is 20 times that of the initial solution. For 100 litres of the process fluid, we would have 5 litres of retentate, or 95 litres of penneate.

$$J_{avg} = \frac{Permeate\ removed}{Area\ \times time} = \frac{95\ litres}{1.36\ h \times 2m^2} = 34.9\ LMH$$

(2) Calculate the required area:

For VCF=20, the final retentate volume= 7500/20 = 375 litres

The permeate volume removed= 7500 - 375 = 7125 litres

The required area = (permeate removed) / (time) (J_{avg})

= 7125 / (1) (35)
= 203.6 m^2

2. Single-stage Continuous System

(1) Determine flux that plant will operate at:
For VCF=20, the required flux, J = 15 LMH (from Table 1)

(2) Calculate the required area:
For processing 7,500 litres per hour, the required permeate removal = 7125 l/h

$$\therefore \textit{The area required} = \frac{7125\ l/h}{15\ LMH} = 475\ m^2$$

3. Two-stage Continuous System

As required here, in multi-stage systems the VCF for each stage is usually chosen so as to give an equal area for each stage. This is convenient and also gives a total area very close to the minimum area. The VCF for all but the last stage must be selected on a trial and error basis.

Before starting the flux data must be plotted (preferably on semi-log paper).

(1) Select VCF values and determine the corresponding flux values for each stage. VCF of the final stage must be 20,
i.e. VCF_2 = 20 and J_2 = 15LMH

Select $VCF_1 = 2.42$, then $J_1 = 35.5$LMH

(2) Calculate the flowrates at each stage and the area required:

Table 2: Flowrates and module areas for two-sta2e system, Trial 1

Stage	VCF	Feed flow I/hr	Retentate tlow,1/hr	Permeate flow: I/hr	Flux Um²hr	Area m2
1	2.42	7500	3099	4401	35.5	124
2	20	3099	375	2724	15	182

(3) Adjust the estimates and recalculate:
with the semi-log plot, take $VCF_1 = 3.0$, $J_1 = 3\ 3.8$ LMH
$VCF_2 = 20$, $J_2 = 15$LMH

Table 3: Flowrates and module areas for two-stage system, Trial 2

Stage	VCF	Feed flow I/hr	Retentate flow, I/hr	Permeate flow, I/hr	Flux l/m²hr	Area m2
1	3.0	7500	2500	5000	33.8	148
2	20	2500	375	2125	15	142

Aim to have membrane areas for both stages ± $5m^2$

Total membrane area is 148 + 142 = 290 m²

Problem 11.

$$F = 7.5\ \mathrm{m^3 \cdot h^{-1}}$$
$$VCF = \frac{F}{R} = 20$$
$$\therefore\ R = \frac{F}{VCF} = \frac{7.5}{20} = 0.375\mathrm{m^3/h}$$
$$F = P + R$$
$$\therefore\ P = F - R = 7.5 - \left(\frac{7.5}{20}\right) = 7.125\mathrm{m^3/h}$$

From the batch run data

$$P_{\mathrm{batch}} = F_{\mathrm{batch}} - R_{\mathrm{batch}} = 0.1 - \frac{0.1}{VCF} = 0.095\mathrm{m^3}$$

$$J_{\mathrm{av\,batch}} = \frac{P_{\mathrm{batch}}}{A_{\mathrm{batch}} \times t_{\mathrm{batch}}} = \frac{0.095}{2 \times 1.36} = 0.035\mathrm{m^3 m^{-2} h^{-1}}$$

$$J_{\mathrm{av\,continuous}} = J_{\mathrm{av\,batch}} = \frac{P}{A} = \frac{7.125}{A} = 0.035\mathrm{m^3 m^{-2} h^{-1}}$$

$$\therefore\ A = \frac{7.125}{0.035} = \underline{\underline{203.6\mathrm{m^2}}}$$

Problem 12.

Using data from Worked Example

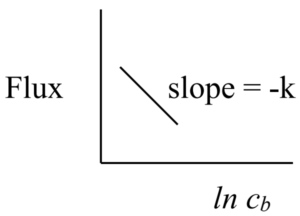
$$VCF = \frac{V_{feed}}{V_{retentate}} \qquad \text{also} \qquad V_{feed} \cdot c_{feed} = V_{retentate} \cdot c_{retentate}$$

$$\Rightarrow \qquad VCF = \frac{V_{feed}}{V_{retentate}} = \frac{c_{retentate}}{c_{feed}}$$

If SRC =1 then $c_p = 0$ all solutes are retained in retentate

Let $c_{retentate} = c_b$

We know $c_{feed} = 6\ kgm^{-3}$ $\left[0.6wt\% = \frac{0.6g}{100\,ml} = \frac{6kg}{m^3}\right]$



VCF = 3.41 J = 32.5 LMH $c_b = 3.41 \times 6 = 20.5$ kg m^{-3}
VCF = 9.42 J = 22.5 LMH $c_b = 9.42 \times 6 = 56.5$ kg m^{-3}

$$slope = \frac{22.5 - 32.5}{[(\ln 56.5) - (\ln 20.5)]} = -\frac{10}{1.014} = -9.86 LMH$$

$$\Rightarrow \quad k = 9.86 \text{ LMH}$$

$$J = k \ln\left(\frac{c_g}{c_b}\right)$$

@ VCF = 20 $\quad c_b = 20 \times 6\ kgm^{-3} = 120\ kgm^{-3}$

$$\Rightarrow \quad 15 = 9.86 \ln\left(\frac{c_g}{120}\right)$$

$$c_g = 120 \times \exp^{15/9.86} = 549\ kg\,m^{-3}$$

Check. @ VCF = 10 $\quad c_b = 60\ kgm^{-3} \Rightarrow \quad J = 9.86 \ln\left(\frac{549}{60}\right) = 21.8 LMH$

cf 21.7 LMH from graph, OK.

Problem 13.

Stage	VCF	Feed flow *(lh⁻¹)*	Retentate flow *(lh⁻¹)*	Permeate flow *(lh⁻¹)*	Flux (LMH)	Area (m^2)
1	1.8	10,000	5,556	4,444		
2	3.5	5,556	2,857	2,699		
3	10.0	2,857	1,000	1,857	18.57	100

Problem 14

$$\frac{C_m}{C_b} = \exp\left(\frac{J}{k}\right)$$

$$\therefore \; J = k \ln C_m - k \ln C_b$$

Plot J (flux) versus ln C_b.

Slope of plot = $-k$; y-intercept of plot = $k \ln C_m$.

Need to calculate C_m.

From the regression line,
y-intercept = 249.1 $L \cdot m^{-2}h^{-1}$ and slope = –76.1

$\therefore$ k = 76.1 $L \cdot m^{-2}h^{-1}$ and

$$C_m = \exp\left(\frac{249.1}{76.1}\right) = \underline{\underline{26.4 \text{ wt.\%}}}.$$

Problem 15.

(i) $J_i = 40 LMH$

$$J_{ave} = \frac{Permeate\ volume}{area \times time} = \frac{2000 - 250}{10 \times 5} = 35 LMH$$

$$J_{ave} = J_f + \tfrac{2}{3}(J_i - J_f)$$

$\Rightarrow$ $J_{ave} = J_f + \tfrac{2}{3} J_i - \tfrac{2}{3} J_f$

$\Rightarrow$ $J_{ave} - \tfrac{2}{3} J_i = J_f (1 - \tfrac{2}{3})$

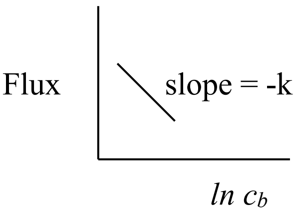
$\Rightarrow$ $J_f = 3(J_{ave} - \tfrac{2}{3} J_i)$

$J_f = 3(35 - \tfrac{2}{3} \times 40) = \quad 25\ LMH$

(ii)

$$VCF = \frac{2000}{250} = 8 \qquad\qquad c_{final} = 8 \times c_{feed} = 24\ kgm^{-3}$$

Estimate k



$$slope = \frac{25 - 40}{[\ln 24 - \ln 3]} = -7.2 LMH \qquad \therefore \qquad k = 7.2 \text{ LMH}$$

$$c_m = c_b \exp^{(J/k)} = 3 \exp^{(40/7.2)} = 776\ kgm^{-3}$$

(iii)

k increased by 30% $\quad\therefore\quad$ k = 9.36 LMH

$c_m = 776\ kgm^{-3}$ (constant)

$$J_i = k\ln\left(\frac{c_m}{c_b}\right) = 9.36\ln\left(\frac{776}{3}\right) = 52\ LMH$$

$$J_f = 9.36\ln\left(\frac{776}{24}\right) = 32.5\ LMH$$

$$J_{ave} = J_f + {}^2\!/_3(J_i - J_f) = 32.5 + {}^2\!/_3(52 - 32.5) = 45.5\ LMH$$

$$Processing\ time = \frac{Permeate\ volume}{J \times A} = \frac{1750}{45.5 \times 10} = \quad 3.85\ hours$$

Problem 16.

(i)

$$J_{ave} = \frac{Permeate\ volume}{area \times time} = \frac{12}{4 \times 0.1} = 30LMH$$

$$J_{final} = 18\ LMH$$

$$J_{ave} = J_f + {}^2\!/_3(J_i - J_f)$$

$$\Rightarrow \qquad J_{ave} = J_f + {}^2\!/_3 J_i - {}^2\!/_3 J_f$$

$$\Rightarrow \qquad J_{ave} - {}^1\!/_3 J_f = {}^2\!/_3 J_i$$

$$\Rightarrow \qquad J_i = {}^3\!/_2(J_{ave} - {}^1\!/_3 J_f)$$

$$J_i = {}^2\!/_3(30 - {}^1\!/_3 \times 18) = 36\ LMH$$

$$VCF = \frac{15}{3} = 5$$

J_i = 36 LMH; $c_{b,\ initial}$ = 1.5 %
J_f = 18 LMH; $c_{b,\ final}$ = 1.5 × 5 = 7.5 %

$$-\mathrm{k} = slope = \frac{36 - 18}{[\ln 1.5 - \ln 7.5]} = -11.2LMH \ \therefore \qquad k = 11.2\ LMH$$

$$c_g = c_b \exp^{(J/k)} = 1.5 \exp^{(36/11.2)} = 37.3\ \%$$

(ii)

VCF = 5

Feed = 4,000 *litres* $\quad \therefore Retentate = \frac{4{,}000}{5} = 800\ litres$

Processing time = 8 hours

$$k = \frac{2}{3} \times 11.2 = 7.46\ LMH$$

$$J_i = 7.46\, ln\left(\frac{37.3}{1.5}\right) = 23.97\ LMH \qquad J_f = 7.46\, ln\left(\frac{37.3}{7.5}\right) = 11.97\ LMH$$

$$J_{ave} = J_f + \tfrac{2}{3}(J_i - J_f)$$

$$J_{ave} = 11.97 + \tfrac{2}{3}(23.97 - 11.97) = 19.97\ LMH$$

$$Membrane\ area = \frac{4000 - 800}{19.97 \times 8} = \quad 20\ m^2$$

Problem 17.

VCF = 4
$V_{retentate}$ = 500 litres
$V_{feed} = 4\times 500$ = 2000 litres
$V_{permeate}$ = 1500 litres

c_f =34 g l^{-1}
$c_{retentate}$ = 136 g l^{-1}

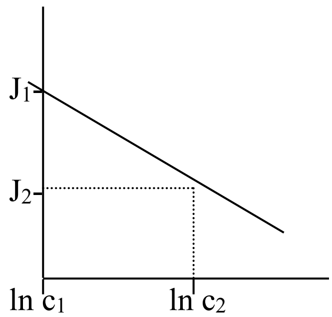
$$J_{avg} = \frac{1500}{5\times 10} = 30\ LMH$$

$$J_{avg} = J_f + \frac{2}{3}(J_i - J_f) \quad \rightarrow \quad J_f = 3\left(J_{avg} - \frac{2}{3}J_i\right)$$

$$J_f = 3\left(30 - \frac{2}{3}\times 42\right) = 6\ LMH$$

$$J_1 = k\ ln\left(\frac{c_m}{c_1}\right) = k\ ln\, c_m - k\ ln\, c_1$$

$$J_2 = k\ ln\left(\frac{c_m}{c_2}\right) = k\ ln\, c_m - k\ ln\, c_2$$



slope = -k

$$J_1 - J_2 = k(\ln c_2 - \ln c_1)$$

$$k = \frac{J_1 - J_2}{(\ln c_2 - \ln c_1)} = \frac{42-6}{(\ln 136 - \ln 34)} = 26\ LMH$$

For J_1

$$J_1 = k\ ln\left(\frac{c_m}{c_1}\right)$$

$$c_m = exp\left(\frac{J_1}{k}\right)c_1$$

$$= exp\left(\frac{42}{26}\right)\times 34 = 171\ gl^{-1}$$

Problem 18

$$d_h = \frac{4 \times \text{cross sectional area for flow}}{\text{wetted perimeter}}$$

$$= \frac{4(0.002 \times 0.15)}{2(0.002 + 0.15)} = 3.95 \times 10^{-3}\,\text{m}$$

$$u = \frac{\text{volume flow rate}}{10 \times \text{cross sectional area of channel}}$$

$$= \frac{4 \times 10^{-3}}{10(0.002 \times 0.15)} = 1.33\,\text{m} \cdot \text{s}^{-1}$$

Is flow turbulent? (i.e. is Reynolds number >2400)

$$\text{Re} = \frac{\rho d_h u}{\mu} = \frac{1010 \times 3.95 \times 10^{-3} \times 1.33}{1.2 \times 10^{-3}} = 4421.7$$

$\alpha = 0.023$ (turbulent flow)

$b = 1.0$ (turbulent flow)

$$k = 0.023 \times 4421.7 \times \left(\frac{1.2 \times 10^{-3}}{1010 \times 6.81 \times 10^{-11}}\right)^{1/3} \left(\frac{6.81 \times 10^{-11}}{3.95 \times 10^{-3}}\right)$$

$$= \underline{\underline{4.55 \times 10^{-5}\,\text{m} \cdot \text{s}^{-1}}}$$

Thickness of gel layer

$$k = \frac{D}{\delta}$$

$$\therefore\ \delta = \frac{D}{k} = \frac{6.81 \times 10^{-11}}{4.55 \times 10^{-5}} = \underline{\underline{14.96 \times 10^{-7}\,\text{m or } 1.5\,\mu\text{m}}}$$

Problem 19

$J = 0.1\ \mathrm{m^3 m^{-2} h^{-1}}$

$A = 12\ \mathrm{m^2}$

$$-\frac{dV_f}{dt} = JA$$

$$-\int_{V_{f0}}^{V_f} dV_f = JA\int_0^t dt$$

$$V_{f0} - V_f = JAt$$

Concentration from 10 m^3 to 1 m^3

$\therefore\ V_{fo} = 10\ \mathrm{m^3}$ and $V_f = 1\ \mathrm{m^3}$

$$t = \frac{(V_{f0} - V_f)}{JA} = \frac{(10-1)}{0.1\times 12} = \underline{\underline{7.5\mathrm{h}}}$$