6 CHILLNG TIME PREDICTION

As was the case with freezers, the factor most limiting the throughput of chillers is the chilling time. In second year you covered methods to predict chilling and heating times of regular shapes (slabs, cylinders, spheres, rectangular rods, finite cylinders, bricks). In this section an approximate method for predicting the chilling times of irregular shapes is presented so that you have methodology that can be applied to all shapes. Many of the data required are the same as used for freezing:

- product size, measured as the half-thickness of product (use "radius" from "thermal centre" to surface on shortest route), R (m)
- thermal properties of the unfrozen product:
  - Thermal conductivity k_l (W/mK)
  - Specific heat capacity c_l (J/kgK)
  - Density ρ (kg/m3)
- cooling medium temperature, θ_a (°C)
- product initial temperature, θ_{in} (°C)
- required final product centre temperature, θ_c (°C)
- surface heat transfer coefficient, h_e (W/m2K)
- equivalent heat transfer dimensionality E. This has different numerical values to freezing.

6.1 MODEL PHYSICAL INTERPRETATION

The model can be conceptualised as shown in Figure 47.

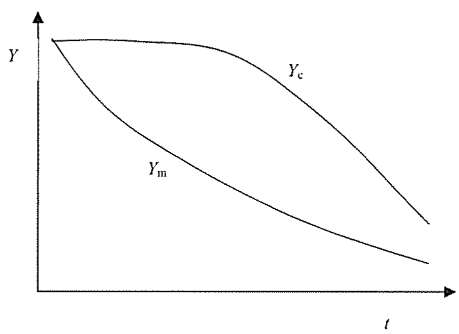


Figure 47, Conceptual Model of Chilling Time Prediction Technique

After an initial period there is a linear relationship between *ln Y* and time as shown in Figure 48.

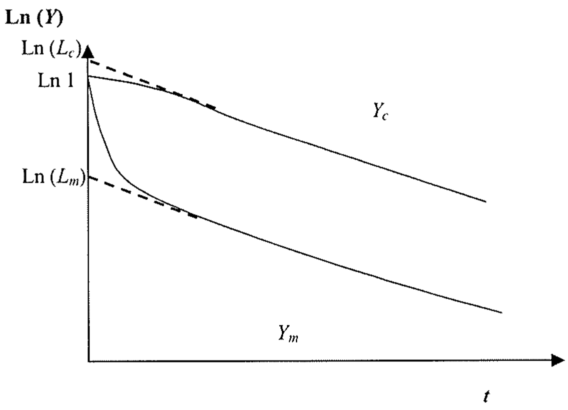


Figure 48, Conceptual Model of Chilling Time Prediction Technique

6.2 THE METHOD

The various calculation steps are given below. As expressed, the method applies to all shapes. The source is:

Lin, Z., Cleland, A.C., Cleland, D. J. and Serrallach, G. F. (1996). A simple method for prediction of chilling times: extension to three-dimensional shapes. *International Journal Refrigeration*: **19**; 107-114.

1. Measurement of the object dimensions D_1, D_2 and D_3:
   D_1 = shortest dimension through the geometric centre of the object (first dimension):
   D_2 = shortest dimension through geometric centre of the object taken at right angles to the first dimension (second dimension);
   D_3 = longest dimension through geometric centre of the object taken as close as possible to right angles to both the first and second dimensions (third dimension).

   For irregular shapes this implies approximation of the real shape to an equivalent infinite ellipse or ellipsoid. In using this measurement technique any obvious protrusions are ignored.

2. Calculation of characteristic dimension R and two dimension R and two dimensional ratios β_1 and β_2:

$$R = \frac{D_1}{2} \qquad \beta_1 = \frac{D_2}{D_1} \qquad \beta_2 = \frac{D_3}{D_1}$$

3. Calculation of the Biot number (Bi).

$$Bi = \frac{h_e R}{k_L}$$

4. Calculation of the equivalent heat transfer dimensionality at $Bi = 0$ (E_o) using:
   (i) Ellipsoid or three-dimensional irregular shapes:

$$E_o = \frac{3\left[\beta_1 + \beta_2 + \beta_1^2(1+\beta_2) + \beta_2^2(1+\beta_1)\right]}{2\beta_1\beta_2(1+\beta_1+\beta_2)} - \frac{\left[(\beta_1 - \beta_2)^2\right]^{0.4}}{15}$$

(ii) Ellipse or two-dimensional irregular shapes:

$$E_o = \left(1 + \frac{1}{\beta_1}\right)\left(1 + \left(\frac{\beta_1 - 1}{2\beta_1 + 1}\right)^2\right)$$

(iii) Finite cylinders, bricks, infinite rectangular rods:

$$E_o = 1 + \frac{1}{\beta_1} + \frac{1}{\beta_2}$$

(iv) sphere ($E_0 = 3$), infinite cylinder ($E_0 = 2$), infinite slab ($E_0 = 1$);

5. Calculation of the equivalent heat transfer dimensionality at $Bi = \infty$ (E_∞) using

$$E_\infty = 0.75 + P_1 f(\beta_1) + P_2 f(\beta_2)$$

where

$$f(\beta) = \frac{1}{\beta^2} + 0.01 P_3 \exp\left(\beta - \frac{\beta^2}{6}\right)$$

values of P_1 and P_2 are found in Table 3.

6. Calculation of E:

$$E = \frac{Bi^{\frac{4}{3}} + 1.85}{\left(\frac{Bi^{\frac{4}{3}}}{E_\infty} + \frac{1.85}{E_o}\right)}$$

7. Calculation of a so-called lag factor L at $Bi = \infty$ (designated L_∞):

$$L_\infty = 1.271 + 0.305\exp\left(0.172\gamma_1 - 0.115\gamma_1^2\right) + 0.425\exp\left(0.09\gamma_2 - 0.128\gamma_2^2\right)$$

where values of λ are given in Table 3.

8. Calculation of the lag factor for the object thermal centre position (L_c):

$$L_c = \frac{Bi^{1.35} + \frac{1}{\lambda}}{\left(\frac{Bi^{1.35}}{L_\infty} + \frac{1}{\lambda}\right)}$$

where values of λ are given in Table 3.

Shape	N	P_1	P_2	P_3	γ_1	γ_2	λ
Infinite slab ($\beta_1 = \beta_2 = \infty$)	1	0	0	0	∞	∞	1
Inf, rectangular rod ($\beta_1 \geq 1$, $\beta_2 = \infty$	2	0.75	0	-1	$4\beta_1/\pi$	∞	γ_1
Brick ($\beta_1 \geq 1$, $\beta_2 \geq \beta_1$)	3	0.75	0.75	-1	$4\,\beta_1/\pi$	$1.5\beta_2$	γ_1
Infinite cylinder ($\beta_1 = \beta_2 = \infty$)	2	1.01	0	0	1	∞	1
Infinite ellipse ($\beta_1 > 1$, $\beta_2 = \infty$)	2	1.01	0	1	β_1	∞	γ_1
Squat cylinder ($\beta_1 = \beta_2$, $\beta_1 \geq 1$)	3	1.01	0.75	-1	$1.225\beta_1$	$1.225\beta_1$	γ_1
Short cylinder ($\beta_1 = 1$, $\beta_2 \geq 1$)	3	1.01	0.75	-1	β_1	$1.5\beta_2$	γ_1
Sphere ($\beta_1 = \beta_2 = 1$	3	1.01	1.24	0	1	1	1
Ellipsoid ($\beta_1 \geq 1$, $\beta_2 \geq \beta_1$)	3	1.01	1.24	1	β_1	β_2	γ_1

Table 3, Values of geometric parameters N, P_1, P_2, P_3, γ_1, γ_2 and λ for a variety of shapes

9. Calculation of the lag factor for the mass-average temperature of the object (L_m)

$$L_m = \mu L_c$$

where

$$\mu = \left(\frac{1.5 + 0.69Bi}{1.5 + Bi}\right)^N$$

N is the number of dimensions of an object in which heat is significant; values are stated in Table 3.

10. Calculation of the first root of the transcendental equation for a sphere (α)

$$\alpha \cot\alpha + Bi - 1 = 0$$

Note that $0 \leq \alpha \geq 3.14159$. Table 4 gives some numerical values.

Bi	α	*Bi*	α	*Bi*	α	*Bi*	α
0.01	0.173	0.60	1.264	1.80	1.959	8.50	2.786
0.02	0.244	0.65	1.310	2.00	2.029	9.00	2.804
0.03	0.299	0.70	1.353	2.20	2.092	9.50	2.821
0.04	0.345	0.75	1.393	2.40	2.148	10.00	2.836
0.05	0.385	0.80	1.432	2.60	2.200	11.00	2.863
0.06	0.422	0.85	1.469	2.80	2.246	12.00	2.885
0.08	0.486	0.90	1.504	3.00	2.289	13.00	2.904
0.10	0.542	0.95	1.538	3.20	2.328	14.00	2.921
0.12	0.593	1.00	1.571	3.40	2.364	15.00	2.935
0.14	0.639	1.05	1.602	3.60	2.397	16.00	2.948
0.16	0.682	1.10	1.632	3.80	2.427	18.00	2.969
0.18	0.722	1.15	1.661	4.00	2.456	20.00	2.986
0.20	0.759	1.20	1.689	4.50	2.518	25.00	3.017
0.25	0.845	1.25	1.716	5.00	2.570	30.00	3.037
0.30	0.921	1.30	1.741	5.50	2.615	35.00	3.052
0.35	0.990	1.35	1.766	6.00	2.654	40.00	3.063
0.40	1.053	1.40	1.791	6.50	2.687	50.00	3.079
0.45	1.111	1.45	1.814	7.00	2.717	60.00	3.089
0.50	1.16	1.50	1.837	7.50	2.742	80.00	3.102
0.55	1.27	1.60	1.880	8.00	2.765	100.00	3.110

Table 4, Values of the first root of the transcendental equation for a sphere (α)

11. Calculation of the chilling time to reach achieve a desired temperature (t_c or t_m) or temperature reached after a defined chilling time (θ_c or θ_m):

For thermal centre:

$$t_c = \frac{3\rho c_L R^2}{\alpha^2 k_L E} \ln\left(\frac{\theta_{in} - \theta_a}{\theta_c - \theta_a} L_c\right)$$

or

$$\theta_c = L_c \exp\left(\frac{-k_L t_c E \alpha^2}{3\rho c_L R^2}\right)(\theta_{in} - \theta_a) + \theta_a$$

For mass-average:

$$t_m = \frac{3\rho c_L R^2}{\alpha^2 k_L E} \ln\left(\frac{\theta_{in} - \theta_a}{\theta_m - \theta_a} L_m\right)$$

or

$$\theta_m = L_m \exp\left(\frac{-k_L t_m E \alpha^2}{3\rho c_L R^2}\right)(\theta_{in} - \theta_a) + \theta_a$$

where θ_m = mass-average temperature (^{0}C)

12. Checking of the range of fractional unaccomplished temperature change (Y). If $Y_c > 0.7$ or $Y_m > 0.55$, results calculated using the prediction method may be unreliable.

$$Y_c = \frac{\theta_c - \theta_a}{\theta_{in} - \theta_a}$$

$$Y_m = \frac{\theta_m - \theta_a}{\theta_{in} - \theta_a}$$

This method is approximate only but generally gives chilling time estimates within ±10% provided accurate data are used – any data error, particularly the need to average time-variable conditions, or uncertainties in heat transfer coefficients increases the error significantly.

As a general principle for regular shapes (slabs, cylinders, spheres, finite cylinders, rectangular rods, bricks) you should use the Schack and Gurney – Luries charts covered in second year for greatest accuracy and restrict the above approximate method to irregular shapes.

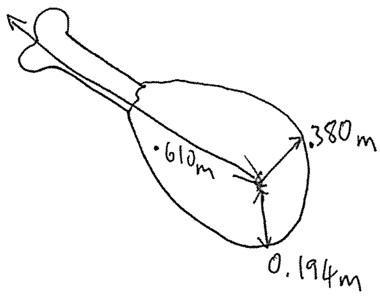
6.3 WORKED EXAMPLE

A 125 kg side of lean beef is being chilled to a mass-average temperature of 8°C from an initial temperature of 40°C using air at 4°C and a velocity of 2 m/s. The critical area is the deep leg position (hip bone). After excursion of the leg and the body through the rib cage the measured dimensions are 194 mm x 380 mm x 610 mm.

Thermal properties of beef are:

$$c_L = 3400 \text{J/kgK}$$
$$\rho = 1030 \text{ kg/m}^3$$
$$k_L = 0.46 \text{ W/mK}$$

We can follow the calculations routinely:
Picture:



Assume ellipsoid

(1) Evaluate D_1, D_2 and D_3:

$$D_1 = 0.194 \quad D_2 = 0.380 \quad D_3 = 0.610$$

(2) Evaluate R, β_1 and β_2

$$R = \frac{0.194}{2} = 0.097\text{m} \quad \beta_1 = \frac{0.380}{0.194} = 1.96 \quad \beta_2 = \frac{0.610}{0.194} = 3.14$$

(3) Evaluate $Bi = h_e R/k_L$. First find h_e for large oval shapes.

from page 59

$$h_e = 12.5 V^{0.6}$$
$$= 12.5(2)^{0.6}$$
$$= 18.95 \frac{W}{m^2K}$$

$$Bi = \frac{18.95 \times 0.097}{0.46} = 4$$

(4) Calculation of E_0.

$$E_0 \approx \frac{3[\beta_1 + \beta_2 + \beta_1^2(1+\beta_2) + \beta_2^2(1+\beta_1)]}{2\beta_1\beta_2(1+\beta_1+\beta_2)} - \frac{[(\beta_1-\beta_2)^2]^{0.4}}{15}$$

$$\approx \frac{3[1.96 + 3.14 + 1.96^2(1+3.14) + 3.14^2(1+1.96)]}{2(1.96)(3.14)(1+1.96+3.14)} - \frac{[(1.96-3.14)^2]^{0.4}}{15}$$

$$\approx$$

$$\simeq 1.93$$

(5) Calculation of E_∞

From table 3

$$P_1 = 1.01$$

$$P_2 = 1.24$$

$P_3 = 1$

$$f(\beta) = \frac{1}{\beta^2} + 0.01 P_3 \exp\left[\beta - \frac{\beta^2}{6}\right]$$

$$\begin{aligned}
f(\beta_1) &= \frac{1}{\beta_1^2} + 0.01(1)\exp\left[\beta_1 - \frac{\beta_1^2}{6}\right] \\
&= \frac{1}{1.96^2} + 0.01\exp\left[1.96 - \frac{1.96^2}{6}\right] \\
&= 0.298 \\
f(\beta_2) &= \frac{1}{\beta_2^2} + 0.01 P_3 \exp\left[\beta_2 - \frac{\beta_2^2}{6}\right] \\
&= \frac{1}{3.14^2} + 0.01\exp\left[3.14 - \frac{3.14^2}{6}\right] \\
&= 0.146 \\
E_\infty &= 0.75 + P_1 f(\beta_1) + P_2 f(\beta_2) \\
&= 0.75 + 1.01(0.298) + 1.24(0.146) = 1.23
\end{aligned}$$

(6) Calculation of E

$$E = \frac{Bi^{4/3} + 1.85}{\frac{Bi^{4/3}}{E_\infty} + \frac{1.85}{E_0}} = \frac{4^{4/3} + 1.85}{\frac{4^{4/3}}{1.23} + \frac{1.85}{1.93}} = 1.34$$

(7) Calculation of L_∞

$$\begin{aligned}
\gamma_1 &= \beta_1 = 1.96 \\
\gamma_2 &= \beta_2 = 3.14 \\
L_\infty &= 1.271 + 0.305\exp(0.172\gamma_1 - 0.115\gamma_1^2) + 0.425\exp(0.09\gamma_2 - 0.128\gamma_2^2) \\
&= 1.271 + 0.305\exp(0.172(1.96) - 0.115(1.96)^2) + 0.425\exp(0.09(3.14) - 0.128(3.14)^2) \\
&= 1.70
\end{aligned}$$

(8) Calculation of lag factor for thermal centre position

$\lambda = 1.96$ (Table 3)

$$L_c = \frac{Bi^{1.35} + \frac{1}{\lambda}}{\frac{Bi^{1.35}}{L_\infty} + \frac{1}{\lambda}} = \frac{4^{1.35} + \frac{1}{1.96}}{\frac{4^{1.35}}{1.70} + \frac{1}{1.96}} = 1.62$$

(9) Calculation of L_m

$N = 3$ (Table 3)

$$\mu = \left(\frac{1.5 \times 0.69\,Bi}{1.5 + Bi}\right)^N = \left[\frac{1.5 \times 0.69(4)}{1.5 + 4}\right]^3 = 0.465$$

and $L_m = \mu L_c$

$= 0.465(1.62) = 0.75$

(10) Determination of α. Table 4

$\alpha = 2.456$

You might check this by substitution into the equation.

(11) Calculation of the chilling time to be found:

Noting temperatures: $\theta_\alpha = 4$

$\theta_{in} = 40$

$\theta_m = 8$

$$t_m = \frac{3\rho C_L R^2}{\alpha^2 k_L E} \ln\left(\frac{\theta_{in} - \theta_a}{\theta_m - \theta_a} L_m\right)$$

$=$

$$= \frac{3 \times 1030 \times 3400 \times 0.097^2}{2.456^2 \times 0.46 \times 1.34} \times \ln\left(\frac{40-4}{8-4} \times 0.75\right) = 50768\,s$$

$= 14.10\,hr$

(12) Check Y_m

$$Y_m = \frac{\theta_m - \theta_a}{\theta_{in} - \theta_a} = \frac{8-4}{40-4} = 0.111 < 0.55 \text{ OK!}$$

Lastly, we might want to know the value of θ_c at the completion of chilling.

$$\theta_c = L_c \exp\left(\frac{-k_L t_c E\alpha^2}{3\rho C_L R^2}\right)\left(\theta_{in} - \theta_a\right) + \theta_a$$

Assume $t_m = t_c$ for now

$$= 1.62 \exp\left(\frac{-0.46 \times 50768 \times 1.34 \times 2.456^2}{3(1030)(3400)(0.097^2)}\right)(40-4) + 4$$

$= 12.6\,°C$