**Adam Smith Business School**
**Subject of Economics**
**Degree of MSc**
**Degree Exam**
**Mathematical Finance, ECON5020**
Thursday, 06 December 2018, 09:30-11:30

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**Please ensure that you write the course code (as above), your student ID, date of birth and the number of the question that you have attempted on each answer sheet.**

**How to complete this exam:**

- Students should answer ONE question from Section A and ONE question from Section B.

**Materials allowed:**

- Calculators:
  - You may use Business School approved models only: Casio FX-83GT/Casio-83GT+, Casio FX-85GT/Casio FX-85GT+, Sharp EL531WH, Aurora AX-582BL, Sharp EL-233SBBK

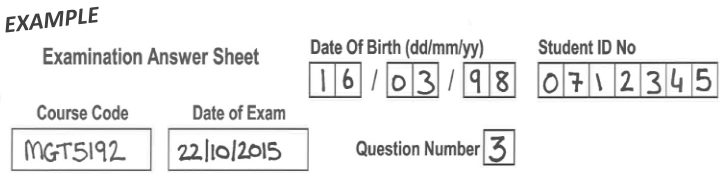
**Instructions to students:**

- Both entry and exit to the examination hall will be at the absolute discretion of the invigilator.
- No candidate will be permitted to leave within the first hour or the last half hour of this exam.

0 1 2 3 4 5 6 7 8 9

A B C D E F G H I J K L M N O

P Q R S T U V W X Y Z




**Using exam answer sheets**

- Always use a **black pen**.
- Complete personal information on all white sheets supplied before the exam begins.
- Use the standard character set, printed above, when hand writing in data boxes. E.g. Student ID, Date of Birth, Question Number. Keep your characters inside the boxes.
- Use **one** white written answer sheet per question, using BOTH sides if required.
- For this exam, the required number of white answer sheets is **2**
- Request yellow continuation sheets to continue writing answer if there is not enough space on the white sheet.
- You must return all answer sheets to the invigilator even if you have not attempted all questions.

This page has been left blank for student notes – anything written here will not be marked.

## Section A

**You must answer ONE question from this section.**

**Please use ONE WHITE answer sheet per question. If there is not enough space on a white sheet, please raise your hand to request a YELLOW answer sheet in order to continue your answer.**

1. Consider the discrete-time binomial tree model with three periods of length 1, i.e. $T = 2$ and $t = 0, 1, 2, 3$. Assume that the factor for moving up is $u = 2$, the factor for moving down is $d = 1/2$, and that the interest rate is $r = \frac{1}{5}$. The probability for moving up is $q = 3/4$, the probability for moving down $(1 - q) = 1/4$, and the initial stock price is $S_0 = 1$.

   1.1. Compute the price process (i.e. prices at all times and states) for a *American put option* on the stock with strike price $K = 1$ and maturity $T = 3$. In which periods should the option be exercised?

   25%

   1.2. Compute the price at time $t = 0$ of the following option $(\frac{S_{\max}}{S_{\min}} - K)^+$ with $K = 3$. Here $S_{min}$ denotes the minimum stock price along the path, i.e. $S_{min}(\omega) = \min\{S_t(\omega), t = 0 \dots 3\}$ and $S_{\max}$ denotes the maximum stock price along the path, $S_{\max}(\omega) = \max\{S_t(\omega), t = 0 \dots 3\}$ Note: As this option is path dependent, you will not be able to use the recursive method, nor will you be able to use the CRR formula.

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2. Consider the stock price under the Black-Scholes assumption, i.e.

$$S_t = S_0 \exp\left(\left(r - \frac{1}{2}\sigma^2\right) t + \sigma W_t\right)$$

   where $r$ denotes the interest rate. Consider an option with payoff

$$h\left(S_T\right) = S_T \left(\log \frac{S_T}{K}\right)$$

   where $T$ is the time of maturity and $K$ is a constant. Decide whether

$$V\left(t, x\right) = x\left[\log\left(\frac{x}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)\left(T - t\right)\right]$$

   is the Black-Scholes price of the option at time $t$ assuming that $S_t = x$. Present your arguments. Compute the Greeks of this option.

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**Section B**

**You must answer one question from this section. Please use a new WHITE answer sheet. If there is not enough space on a white sheet, please raise your hand to request a YELLOW answer sheet in order to continue your answer.**

3. Consider a single period model with a finite number of states of nature and a finite number of tradable assets. State a condition of no arbitrage opportunity. Define a risk-neutral probability measure. State and explain the Fundamental Theorem of Asset Pricing for this model. Define a complete market. Under what conditions a market is complete?

50%

4. What is hedging and how is it applied in the context of option pricing? In your discussion use examples of discrete and continuous time models.

50%

***Before your exam answers are collected:***

**Please ensure that you have written the course code (on the front of this exam paper), your student ID, date of birth and the question number on <u>each answer sheet.</u>**

**Put your exam answers together, keeping white and yellow answer sheets together and in the correct order.**

**Do not place any other exam materials, including the exam paper, beside the answer sheets.**