

$$X_t = \exp\left(\frac{1}{2}t\right) \cos(W_t)$$

Ito Formula as in lectures - most general one:

For the process

$$dx(t) = \phi(t, x(t)) dt + \sigma(t, x(t)) dW(t)$$

take composite function

$$g(t) = G(t, x(t))$$

then its process is

$$\begin{aligned} dg(t) = & \left(G_t(t, x(t)) + G_x(t, x(t)) \phi(t, x(t)) + \frac{1}{2} G_{xx}(t, x(t)) \sigma^2(t, x(t)) \right) dt \\ & + G_x(t, x(t)) \sigma(t, x(t)) dW(t) \end{aligned}$$

You need to understand what is what in $X_t = \exp\left(\frac{1}{2}t\right) \cos(W_t)$

An obvious way:

$$\begin{aligned} x(t) &= W(t) \\ \sigma(t, x(t)) &= 1 \\ \phi(t, x(t)) &= 0 \\ g(t) &= G(t, x(t)) = \exp\left(\frac{1}{2}t\right) \cos(x(t)) \end{aligned}$$

Then only three terms remain:

$$dg(t) = \left(G_t(t, x(t)) + \frac{1}{2} G_{xx}(t, x(t)) \right) dt + G_x(t, x(t)) \sigma(t, x(t)) dW(t)$$