Institute of Food, Nutrition and Human Health
AND
School of Engineering and Advanced Technology

280.371 PROCESS ENGINEERING OPERATIONS

Drying
Model Answers to Tutorial Problems

These notes are a culmination of the efforts of the following past and present staff of Massey University:

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DRYING EXAMPLE SOLUTIONS

Example 1

20ºC
60% RH

$$h = 20W/m^2K$$

$$\text{Purely convective system} \Rightarrow \theta_{surface} = \theta_{wb} = 15^oC$$

$$R_c = \frac{\phi}{Ah_{fg}}$$

$$h_{fg} @ 15^\circ C = 2465 kJ/kg \text{ and } \phi = hA(\theta_{air} - \theta_s)$$

$$R_c = \frac{w}{A} = \frac{\phi}{h_{fg}A} = \frac{k'_H A(H_s - H_{air})}{A} = \frac{hA(\theta_{air} - \theta_s)}{h_{fg}A}$$

$$\Rightarrow R_c = \frac{20 \times (20 - 15)}{2465000} = 4.057 \times 10^{-5} \frac{kg}{m^2 s}$$

I could also have calculated this from

$$Rc = k'_H(H_s - H_{air})$$

Assuming dries from both sides

$$\Rightarrow t = \frac{s\rho_s}{R_c}(X_1 - X_2)$$

$$= \frac{1 \times 10^{-3} \times 300}{4.057 x 10^{-5}}(2.5 - 1.0)$$

$$= 11093 secs \ (3.1 hours)$$

Or to solve with

$$R_c = k'_H(H_s - H_{air})$$

$$c_p = c_{pair} + H_{air}c_{psteam}$$

$$= 1004 + 0.0088 \times 1871$$

$$= 1020 kJ/kgK$$

$$k'_H = \frac{h}{c_p} = \frac{20}{1020} = 0.0196 \, kg/m^2 s$$

$$R_c = 0.0196(0.0108 - 0.0088)$$

$$= 3.92 \times 10^{-5} \, kg/m^2 s \quad close \ enough$$

Example 2

$$\theta = 100^{o}C$$
$$\theta_{wb} = 43^{o}C$$
$$\Rightarrow H_{air} = 0.0325\ kg/kg$$
$$\Rightarrow c_p = c_{pair} + H_{air}c_{psteam}$$
$$= 1010 + 0.0325 \times 1880$$
$$= 1071\ kJ/kgK$$

$$\theta_{element} = 500^{o}C$$
$$h = 8\ W/m^2K$$
$$\Rightarrow k_H^{'} = \frac{h}{c_p} = \frac{8}{1071} = 7.47 \times 10^{-3}\ kg/m^2s$$
$$\theta_s \neq \theta_{wb}\ as\ it\ is\ convection\ and\ radiation$$

Guess steak surface temp

$Assume\ \theta_s = 60^{o}C \Rightarrow$ $\quad H_s = 0.1524$ (given, as off psyschrometric chart)

Guess surface temp $\quad$ = outside range of charts

$$Mass\ Transfer\ \ R_c = 7.47 \times 10^{-3} = (\underset{H_s}{\overset{@60^{o}C}{0.1524}} - \underset{H_a}{\overset{air}{0.0325}}) = 8.96 \times 10^{-4}\ kg/m^2s$$

$$Heat\ Transfer\ R_c = \frac{\theta}{Ah_{fg}} = \frac{h_c(\theta_{air} - \theta_s) + h_{rad}(\theta_{rad} - \theta_s)}{h_{fg}}$$

$$h_{rad} = F_{12} \times 5.67 \times 10^{-8} \times (T_{rad} + T_s)(T_{rad}^{\ 2} - T_s^{\ 2})$$

$$R_c = \frac{8 \times (100 - 60) + 5.67 \times 10^{-8} \times (773 + 333)(773^2 + 333^2)(500 - 60)}{2358000}$$

@60ºC (333K)

$$= 8.43 \times 10^{-3}\ kg/m^2s$$

(factor 10 bigger than mass transfer)

$$\text{Assume}\,\theta_s = 88^oC \Rightarrow \qquad H_s = 1.112\frac{kg}{kg}$$

$$Mass\,Transfer\,R_c = 7.47\times10^{-3}\times(\underset{@88^oC}{1.112}-0.0325) = 8.064\times10^{-3}\,\frac{kg}{m^2s}$$

$$Heat\,Transfer\,R_c = \frac{8\times(100-88)+5.67\times10^{-8}\times(773+361)(773^2+361^2)\times(500-88)}{\underset{@88^oC}{2288000}}$$

$$=8.47x10^{-3}\,\frac{kg}{m^2s} \qquad \text{close enough}$$

$$t = \frac{s\rho_s}{R_c}(X_1 - X_2)$$

$$= \frac{5\times10^{-3}\times600}{8.4\times10^{-3}}\times(2-1.5)$$

$$= 177\,\text{secs} \quad (\approx 3\,\text{minutes})$$

Example 3

spherical corn kernels

$$r = 2.5\times10^{-3}\,m = \frac{D}{2} = \frac{5\times10^{-3}}{2}$$

$$\rho_s = 200\,kg/m^3$$

$$\left.\begin{aligned} X_1 &= 3\,kg/kg \\ X_2 &= 0.2\,kg/kg \\ X_c &= 0.5\,kg/kg \end{aligned}\right\} \Longrightarrow$$

drying in both constant rate and falling rate regimes

$$Rc = 5\times10^{-4}\,\frac{kg}{m^2 s}$$

$$X^* = 0.025\,kg/kg$$

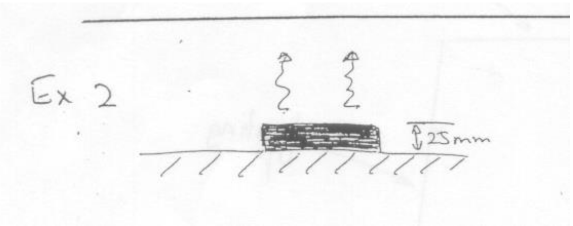
$$t_d = \frac{r\rho_s(X_1 - X_c)}{3Rc} + \frac{r\rho_s(X_c - X^*)}{3Rc}\,\ell n\left[\frac{X_c - X^*}{X_2 - X^*}\right]$$

$$t = \frac{r\rho_s}{3Rc}\left[(X_1 - X_c) + (X_c - X^*)\ln\left[\frac{X_c - X^*}{X_2 - X^*}\right]\right]$$

$$= \frac{2.5\times10^{-3}\times200}{3\times5\times10^{-4}}\left((3-0.5)+(0.5-0.025)\ln\left[\frac{0.5-0.025}{0.2-0.025}\right]\right)$$

$$= 991.4\,\text{secs}\ \ (16.5\,\text{minutes})$$

Example 4



$$X_1 = 2\,kg/kg$$

$$\theta_{air} = 90^oC$$

$$H_{air} = 0.03\,kg/kg$$

$$h = 50\,W/m^2K$$

$$\rho_s = 900\,kg/m^3$$

$$Xc = 0.5\,kg/kg$$

$$Rc = \frac{\phi}{Ah_{fg}} = \frac{h(\theta_{air} - \theta_s)}{h_{fg}} \qquad \theta_s = \theta_{wb}\,@\,41^oC$$

purely convective system $\Rightarrow \theta_s = \theta_{wb} = 41^oC$

$$\Rightarrow Rc = \frac{50(90-41)}{2404000_{@41^{o}C}} = 1.019x10^{-3}\ \frac{kg}{m^2 s}$$

$$(i)\, if\ X_2 = 0.6\, kg/kg (> X_c) \Rightarrow \text{drying in constant rate period only}$$

$$\Rightarrow td = \frac{s\rho_s}{R_c}(X_1 - X_2) = \frac{25\times10^{-3}\times900}{1.02\times10^{-3}}(2-0.6)$$

$$= 30882 \text{secs} \left(8.5\, hrs\right)$$

$$(ii)\, if\ X_2 = 0.1\,(< X_c) \Rightarrow \text{drying is in both constant rate and falling rate periods}$$

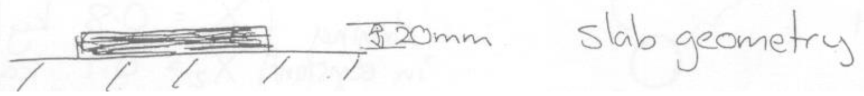
$$t_d = \frac{s\rho_s\,(X_1 - Xc)}{Rc} + \frac{s\rho_s\,(Xc - X^*)}{Rc}\ln\left[\frac{Xc - X^*}{X_2 - X^*}\right]$$

$$t_d = \frac{s\rho_s}{Rc}\left[(X_1 - Xc) + (Xc - X^*)\ln\left(\frac{Xc - X^*}{X_2 - X^*}\right)\right]$$

$$**\, free\ moisture contents \Rightarrow X^* = 0$$

$$\Rightarrow t_d = \frac{25\times10^{-3}\times900}{1.02\times10^{-3}}\left[(2-0.5) + (0.5-0)\ln\left(\frac{0.5-0}{0.1-0}\right)\right]$$

$$= 50840 \text{secs} \quad (14.12\, hrs)$$



Example 5

$X_1 = 0.51 kg/kg$ $\qquad$ $D\,at\,25^oC = 3.5\times10^{-9}\dfrac{m^2}{s}$

$X_2 = 0.033 kg/kg$

$X^* = 0.018 kg/kg$ $\qquad$ $X_c > 0.51 \Rightarrow \text{drying in falling rate regime only}$

$$t_d = \frac{4s^2}{\pi^2 D}\ \ln\left[\frac{8(X_1 - X^*)}{\pi^2(X_2 - X^*)}\right]$$

$$D\ at\ 45^oC = 3.5\times10^{-9}\times\left(\frac{273+45}{273+25}\right)^{1..5} = 3.86\times10^{-9}\frac{m^2}{s}$$

assuming exposed
on both surface only

$$t_d = \frac{4\times(10\times10^{-3})^2}{\pi^2\times3.86\times10^{-9}}\ln\left[\frac{8(0.51-0.018)}{\pi^2\times(0.033-0.018)}\right]$$

$= 34443\text{sec}\ (9.56\,hours)$ *This* is drying time to reach average moisture content of 0.033 kg/kg

$$Fo = \frac{Dt}{R^2} = \frac{3.86\times10^{-9}\times34443}{(10\times10^{-3})^2} = 1.33\ (greater\ than\ 0.1)$$

To determine the final moisture content at centre n=0

From Guerney Lurie chart (figure1)

For m=0 (no external mass transfer resistance)
n=0 (at the centre of the slab)

$$Y = 0.05 = \frac{X - X_{outside}}{X_{initial} - X_{outside}} \qquad Y = \frac{X - X^*}{X_i - X^*}$$

$$= \frac{X - 0.018}{0.51 - 0.018}$$

$$\Rightarrow X = 0.05\times(0.51-0.018)+0.018$$
$$= 0.0426 kg/kg$$

Example 6

drying in both constant rate and falling rate regions
$$\begin{cases} X_1 = 0.8\,kg/kg \\ X_2 = 0.1\,kg/kg \\ Xc = 0.6\,kg/kg \end{cases}$$

$$D = 50 \times 10^{-9}\, m^2/s$$

$X^* = 0$ because using free moisture contents

$$R_c = 8.5 \times 10^{-3}$$

$$\rho_s = 1100\,kg/m^3$$

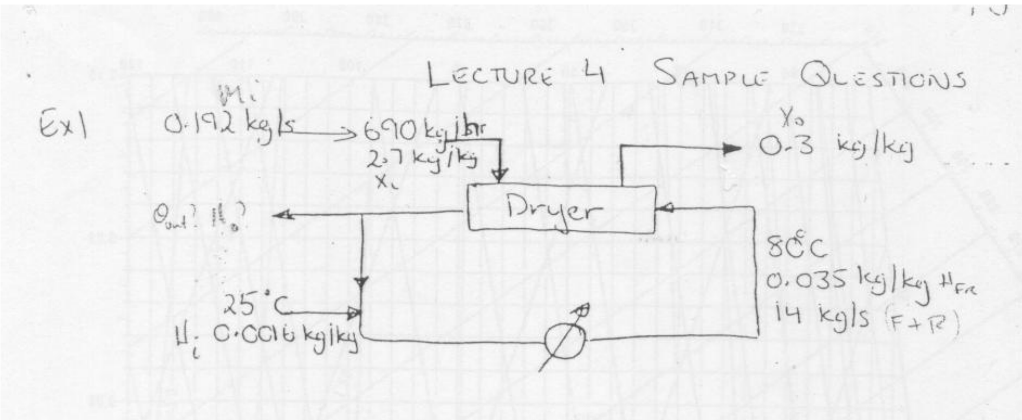
$$t_d = \frac{r\rho_s}{3Rc}(X_1 - Xc) + \frac{r^2}{\pi^2 D}\ln\left[\frac{6(Xc - X^*)}{\pi^2(X_2 - X^*)}\right]$$

$$= \frac{5 \times 10^{-3} \times 1100}{3 \times 8.5 \times 10^{-3}}(0.8 - 0.6) + \frac{(5 \times 10^{-3})^2}{\pi^2 \times 50 \times 10^{-9}}\ln\left[\frac{6(0.6 - 0)}{\pi^2(0.1 - 0)}\right]$$

$$= 43.14 \qquad + 65.56$$

$$= 108.7\,secs \qquad (1.81\,minutes)$$

Example 7



Solids balance

$$690\times\frac{1}{1+2.7}=M_o\times\frac{1}{1+0.3}$$

$$\Rightarrow M_o = 242 kg/hr \ \ (0.0673\ kg/s)$$

Water balance around the dryer (690 kg/hr = 0.192 kg/s)

$$M_i\frac{X_i}{1+X_i}+(F+R)H_{FR} = M_o\frac{X_o}{1+X_o}+(F+R)H_o$$

$$0.192\times\frac{2.7}{1+2.7}+14\times0.035 = 0.0673\times\frac{0.3}{1+0.3}+14\times \mathrm{H}_o$$

$$\Rightarrow Ho = 0.0439 kg/kg \Rightarrow outlet\ temp = 60^oC \text{ from psychrometric chart}$$

Water balance around the system

$$M_i\frac{X_i}{1+X_i}+F_{air}H_i = M_o\frac{X_o}{1+X_o}+F_{air}H_o$$

$$0.192\times\frac{2.7}{1+2.7}+F_{air}\times0.0016 = 0.0673\times\frac{0.3}{1+0.3}+F_{air}\times0.0439$$

$$\Rightarrow F_{air} = 2.95 kg/s$$

$2.95 kg/s$ of air is purged @ $0.0439 kg/kg$ and 60^oC

$11.05 kg/s$ of air is recycled

Enthalpy $h_O = h_{FR}$

$$\phi_{in} = F_{air}(h_o - h_i)$$

$h_o = 177 kJ/kg$ $h_i = 29.5 kJ/kg$ (from psychrometric charts)

$$\Rightarrow \phi_{in} = 2.95(177 - 29.5)$$

$\theta_{air} = 25°C$
$H_i = 0.0016$

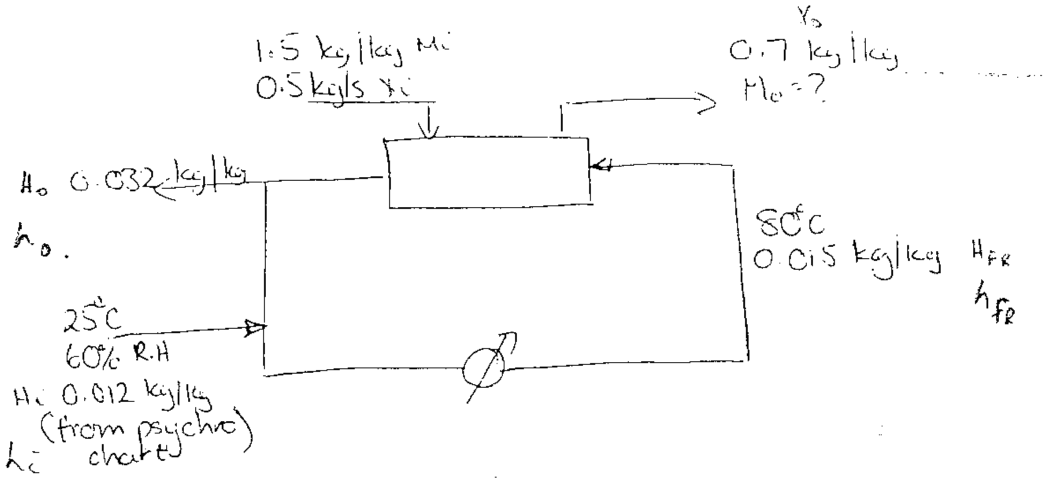
$$= 435.1 kJ/s$$

h_{fg}@41° C
θ_{wb}

$$\phi_{ideal} = \left(0.192 \times \frac{2.7}{1+2.7} - 0.0673 \times \frac{0.3}{1+0.3}\right) \times 2403$$

$$= 299.4\ kJ/s$$

$$\Rightarrow \eta = \frac{299.4}{435.1} = 68.8\%$$



Example 8

$$\text{Solids balance}$$

$$0.5\times\frac{1}{1+1.5}=Mo\times\frac{1}{1+0.7}\Rightarrow Mo=0.34\,kg/s$$

$$\text{Water balance around the system}$$

$$M_i\frac{X_i}{1+X_i}+F_{air}H_i=M_o\frac{X_o}{1+X_o}+F_{air}H_o$$

25°C,
RH = 60%

$$0.5\times\frac{1.5}{1+1.5}+F_{air}\times 0.012=0.34\times\frac{0.7}{1+0.7}+F_{air}\times 0.032$$

$$F_{air}=\frac{0.16}{(0.032-0.012)}=8\,kg/s$$

$$\text{Water balance around the dryer}$$

$$M_i\frac{X_i}{1+X_i}+(F+R)H_{FR}=M_o\frac{X_o}{1+X_o}+(F+R)H_o$$

$$0.5\times\frac{1.5}{1+1.5}+(F+R)\times 0.015=0.34\frac{0.7}{1+0.7}+(F+R)\times 0.032$$

$$\Rightarrow F+R=9.41\,kg/s$$

$8\,kg/s$ *of air is purged* @ 40^oC *and* $0.032\,kg/kg$ (From constant enthalpy line on psychrometric chart

1.41kg/s of air is recycled

$$\phi_{in}=F_{air}\left(h_o-h_i\right)\qquad \text{where } h_o=123\frac{kJ}{kg}\text{ and } h_i=56\frac{kJ}{kg}$$

$$=8(123-56)$$
$$=536\,kJ/s$$

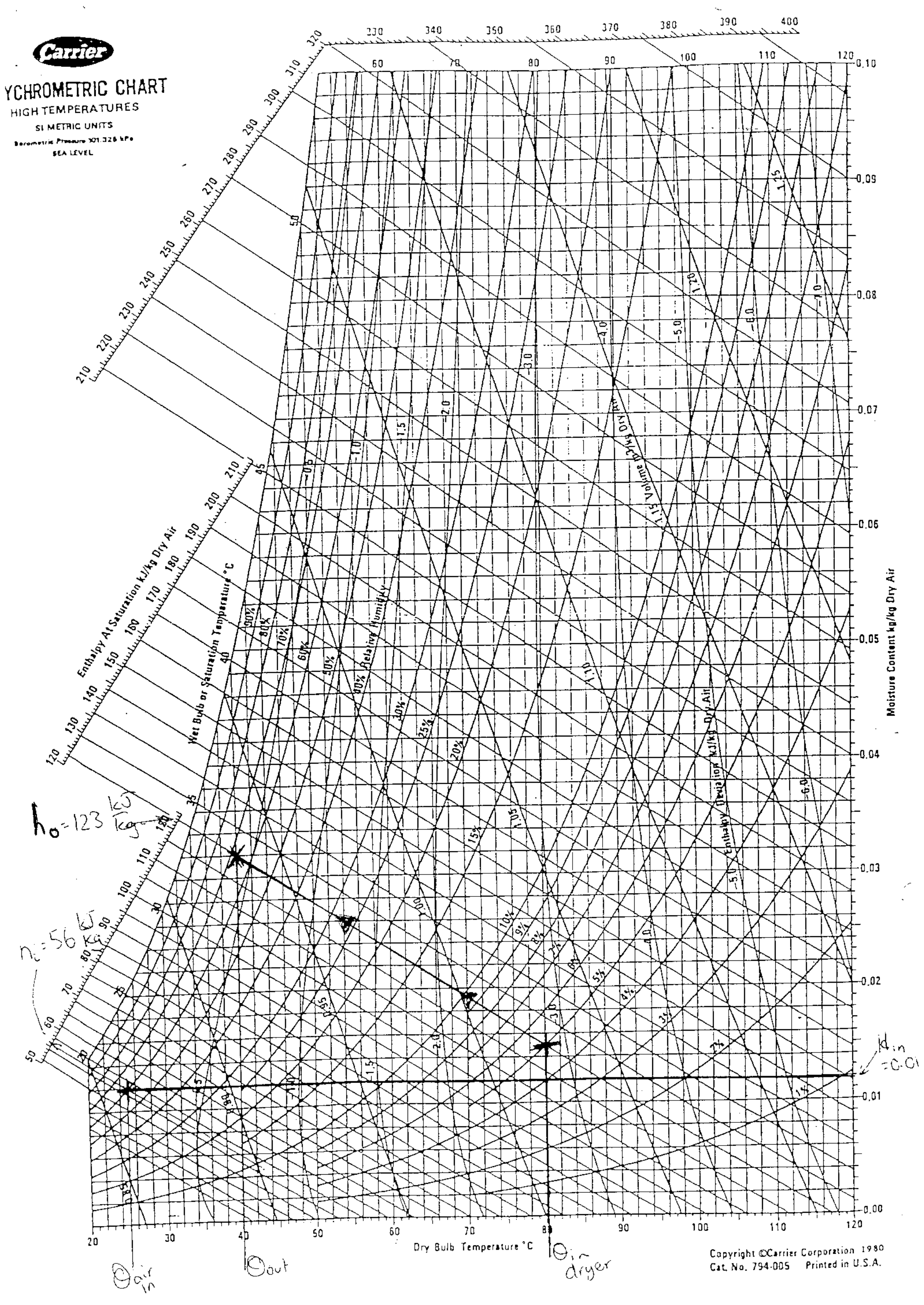
80°C,
ho = h_{FR}

25°C,
RH = 60%

$$\phi_{ideal}=\left(0.5\times\frac{1.5}{1+1.5}-0.34\times\frac{0.7}{1+0.7}\right)\times 2421$$

$$\eta=\frac{387.4}{536}=72.3\%$$

WB temp = 34°C,

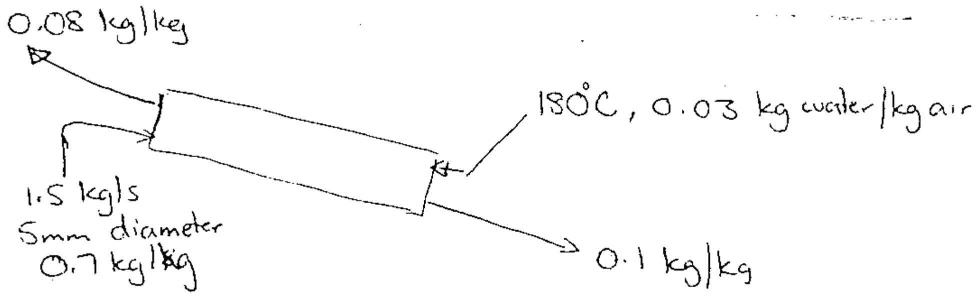

Carrier
YCHROMETRIC CHART
HIGH TEMPERATURES
SI METRIC UNITS
Barometric Pressure 101.325 kPa
SEA LEVEL
Enthalpy At Saturation kJ/kg Dry Air
Wet Bulb or Saturation Temperature °C
Relative Humidity
Volume m3/kg Dry Air
Enthalpy Deviation kJ/kg Dry Air
Moisture Content kg/kg Dry Air
Dry Bulb Temperature °C
Copyright ©Carrier Corporation 1980
Cat. No. 794-005 Printed in U.S.A.
h₀=123 kJ/kg
hᵢ=56 kJ/kg
H_in =0.01
θ air in
θ out
θ in dryer

LECTURE 4 DRYING

Example 9 Dryer selection

(a) Roller dryer
(b) Roller drum dryer
(c) Tray or tunnel dryer
(d) Fluidised bed dryer
(e) Spray dryer

Example 10



Solids balance

$$1.5\times\frac{1}{1+0.7}=M_o\times\frac{1}{1+0.1}$$
$$\Rightarrow M_o=0.971\,kg/s$$

Water balance

$$1.5\times\frac{0.7}{1+0.7}+F_{air}\times 0.03=0.971\times\frac{0.1}{1+0.1}+F_{air}\times 0.08$$
$$\Rightarrow F_{air}=\frac{0.529}{(0.08-0.03)}=10.59\,kg/s$$
$$Ideal\ heat\ required=\left[1.5\times\frac{0.7}{1+0.7}-0.971\times\frac{0.1}{1+0.1}\right]\times 2382$$
$$=1260\,kJ/s$$
$$Actual\ heat\ supplied=150\times 10.59=1589\,kJ/s$$
$$\Rightarrow efficiency=\frac{1260}{1589}=79\%$$

h_{fg}@
θ_{wb} = 50°C
psychro charts
180°C, H_{air} =0.03

Efficiency could be improved by supplying less air so that air exits at saturation temperature (= minimum air = minimum heat loss in exhaust air, assuming there’s spare heat transfer volume in dryer).

Or Efficiency could be improved by increasing the feed air temperature (This enables a greater fraction of the incoming heat to be utilised for evaporation).

$$G = 1kg/m^2s = \frac{4F_{air}}{\pi D^2}$$

$$\Rightarrow D = \sqrt{\frac{4 \times 10.59}{\pi \times 1}} = 3.67m$$

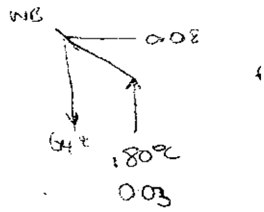
$$\Rightarrow Ua = \frac{200\,G^{0.67}}{D} = 54.5W/m^3K$$

Exit temperature of the air (from psychrometric chart) ≈ 64ºC (perhaps a bit less when I redraw the chart!).

Wet bulb temperature of the air (inlet and outlet)

= 50ºC from psychrometric chart

= surface temperature because it's a purely convective system



$$\Rightarrow \Delta\theta_{lm} = \frac{(180-50)-(64-50)}{\ln\left(\dfrac{180-50}{64-50}\right)}$$

$$= 52.05^oC$$

ϕ (the heat transfered to evaporate the water)

= ideal heat

$= 1260\,kJ/s$

$$1260 \times 10^3 = 54.5 \times V \times 52.05$$

$$\Rightarrow V = 444.2m^3$$

$$= \frac{\pi D^2 L}{4}$$

$$\Rightarrow L = 444.2 \times \frac{4}{\pi D^2} = 42m$$

$\frac{L}{D} = \frac{42}{3.67} = 11.4$ *is about right. (normally in range 4 ⇒ 10)*

But is the solids residence time sufficient.

$$t_d = \frac{\rho x h_{fg}(X_1 - X_2)}{Ua\Delta\theta_{lm}}$$

$$= \frac{1100 \times 0.1 \times 2382 \times 10^3 (X_1 - X_2)}{54.5 \times 52.05}$$

$$= 55420 \sec \quad (= 15.4 hrs)$$

For a 5mm diameter particle

$$t_d = \frac{r\rho_s}{3R_c}(X_1 - X_2)$$

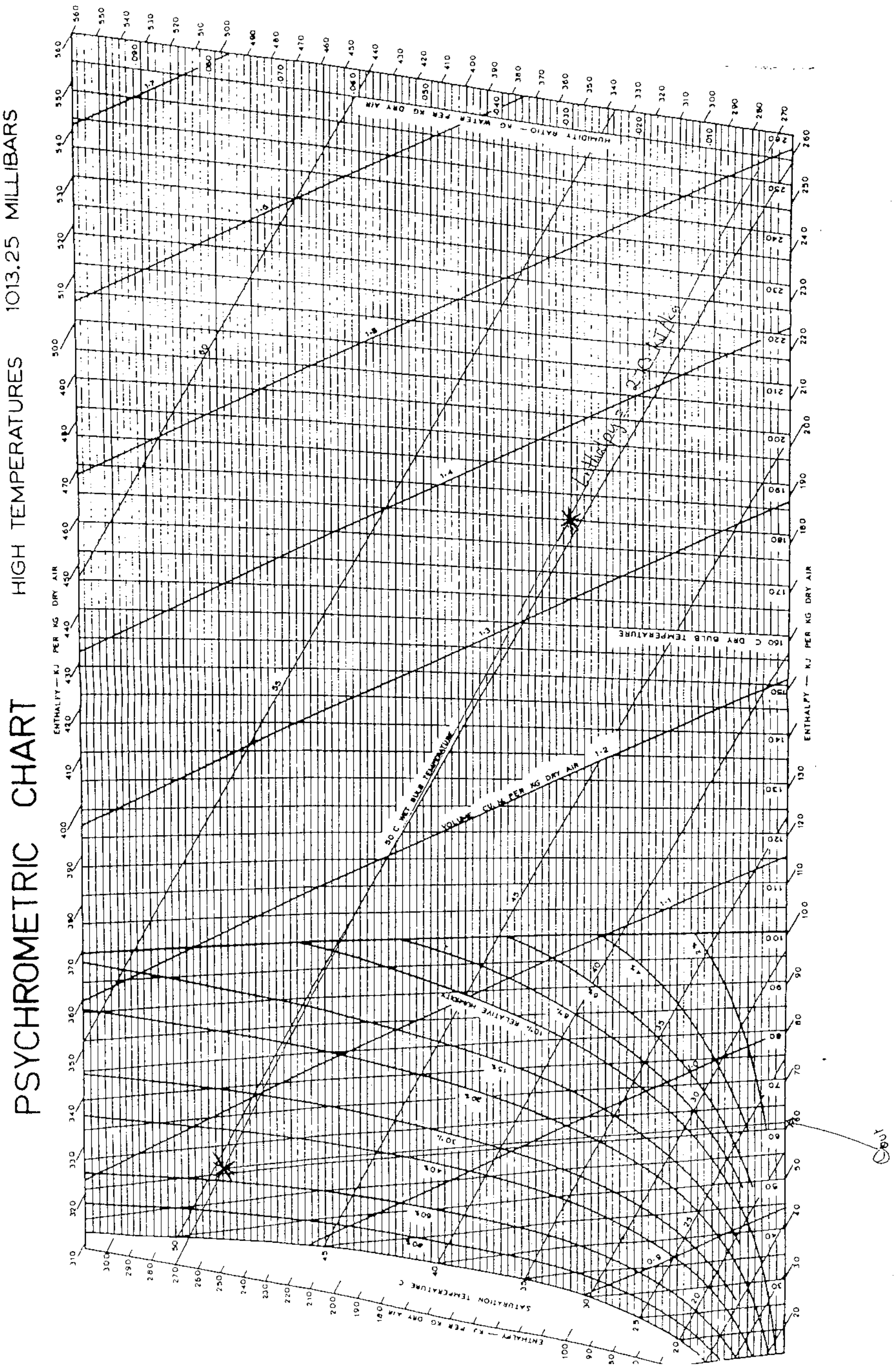
$$R_c = \frac{h\Delta\theta_{lm}}{h_{fg}} = \frac{50 \times 52.05}{2382000} = 1.093 \times 10^{-3} \frac{kg}{m^2 s}$$

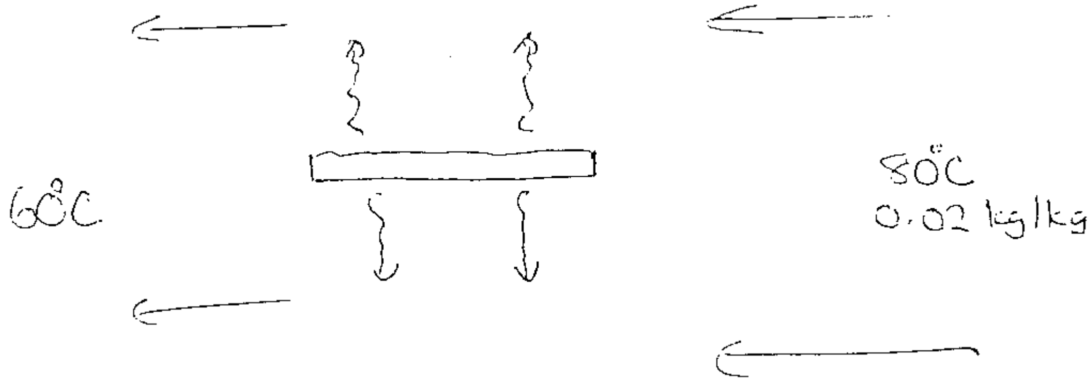
$$\Rightarrow t_d = \frac{\left(\frac{5 \times 10^{-3}}{2}\right) \times 1100}{3 \times 1.093 \times 10^{-3}} \times (0.7 - 0.1)$$

$$= 503.4 \sec$$

$$ratio = \frac{t_{dry,particle}}{t_{residencetime}} = \frac{503.4}{55420} = 0.009$$

Particle drying time is much shorter than the actual residence time, probably reflecting loss of surface area due to aggregation, conservative design etc. Could look to resize dryer but residence time should always be longer than actual drying time of dryer. Or could increase the solids volume fraction in the dryer.


PSYCHROMETRIC CHART
HIGH TEMPERATURES
1013.25 MILLIBARS
ENTHALPY — KJ PER KG DRY AIR
HUMIDITY RATIO — KG WATER PER KG DRY AIR
160 C DRY BULB TEMPERATURE
VOLUME CU M PER KG DRY AIR
50 C WET BULB TEMPERATURE
10% RELATIVE HUMIDITY
SATURATION TEMPERATURE C
ENTHALPY — KJ PER KG DRY AIR



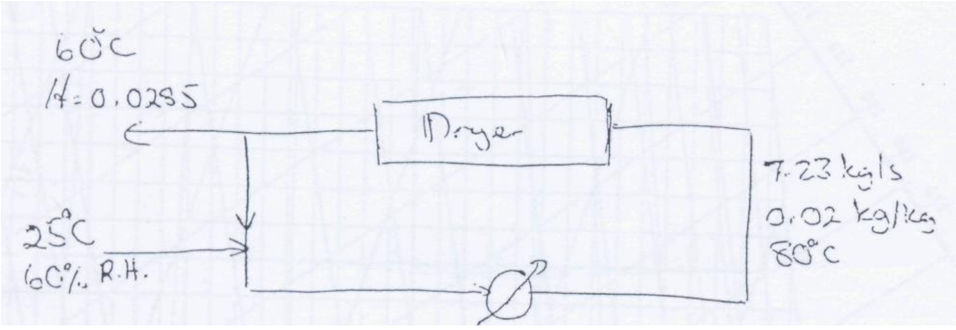
Example 11

$$X_1 = 0.8 kg/kg$$
$$X_2 = 0.3 kg/kg$$
$$A = 150 m^2$$
$$h = 30 W/m^2K$$
$$\phi = hA\Delta\theta_{lm}$$
$$\Delta\theta_{lm} = \frac{(80-36)-(60-36)}{\ln\left(\frac{80-36}{60-36}\right)} = 33^oC$$

$$\Rightarrow \phi = 30 \times 150 \times 33$$
$$= 148.5 kW$$
Evaporation rate:
$$= \left[M_1 \frac{X_1}{(1+X_1)} - M_o \frac{X_o}{(1+X_o)}\right] \Rightarrow evaporation\ rate = \frac{\phi}{h_{fg}} = \frac{148.5\times10^3}{2416000} = 0.061 kg/s$$
$$\text{and} \frac{evaporation\ rate}{Area} = \frac{w}{A} = R_c = \frac{0.061}{150} = 4.097\times10^{-4}\frac{kg}{m^2s}$$

Mass Balance around dryer
$$M_i \frac{X_i}{1+X_i} + (F+R)H_{FR} = M_o \frac{X_o}{1+X_o} + (F+R)H_o$$
$$\left[M_i \frac{X_i}{1+X_i} - M_o \frac{X_o}{1+X_o}\right] = (F+R)(H_o - H_{FR})$$
$$0.061 = (F+R)(H_o - H_{FR})$$

$$\theta_i = 80^oC,\ \mathrm{H_{FR}} = 0.02\ kg/kg$$
$$\theta_{out} = 60^oC$$
$$H_o = 0.0285 \text{ from psychrometric chart}$$
$$\Rightarrow (F+R) = 7.18 kg/s$$



$H_i = 0.012\, kg/kg$ $\qquad \left[M_i \frac{X_i}{1+X_i} - M_o \frac{X_o}{1+X_o} \right] = F_{air}(H_o - H_i)$

$F_{air}(H_o - H_i) = evaporation\ rate\ of\ water$

$$\Rightarrow F_{air} = \frac{0.061}{(0.0285 - 0.012)} = 3.72\, kg/s$$

$enthalpy\ air\ in = 57\, kJ/kg$ (psychrometric chart) $25^oC, RH = 60\%$
$enthalpy\ air\ out = 136\, kJ/kg$ $\qquad 60^oC, H_o = 0.0285$

$$\Rightarrow \phi_{supplied} = F_{air}(h_o - h_i) = 3.72 \times (136 - 57) = 293.88\, kJ/s$$
$$\phi_{ideal} = 0.061 \times 2416 = 148.5\, kJ/s$$
$$\Rightarrow \eta = 50.5\%$$
$$t_d = \frac{s\rho_s}{R_c}(X_1 - X_2)$$
$$= \frac{5 \times 10^{-3} \times 1000}{4.097 \times 10^{-4}}(0.8 - 0.2) = 7322\, secs$$
$$(2.034\, hours)$$
$time\ per\ batch = 2½\, hrs$

$\Rightarrow in\ eight\ hour\ day\ can\ do\ 3.2\ batches,\ round\ down\ to\ 3\ batches$

$Each\ batch\ volume = number\ of\ shelves \times width \times length \times thickness$
$$= 15 \times 5 \times 1 \times 10 \times 10^{-3} = 0.75 m^3$$

$\Rightarrow Total\ product\ volume\ in\ 8\ hours = 0.75 \times 3 = 2.25 m^3 \approx 2.25\ tonnes$

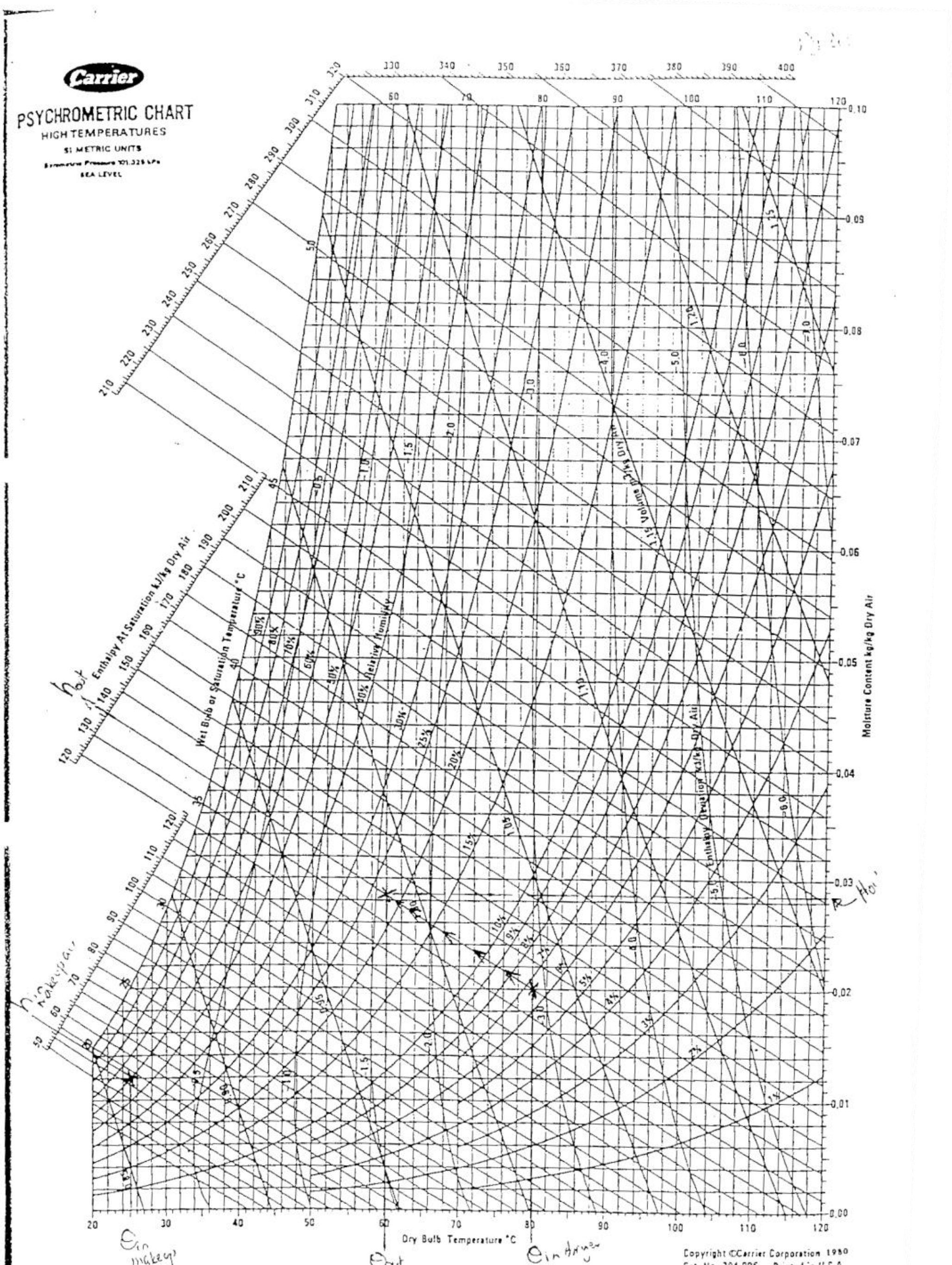
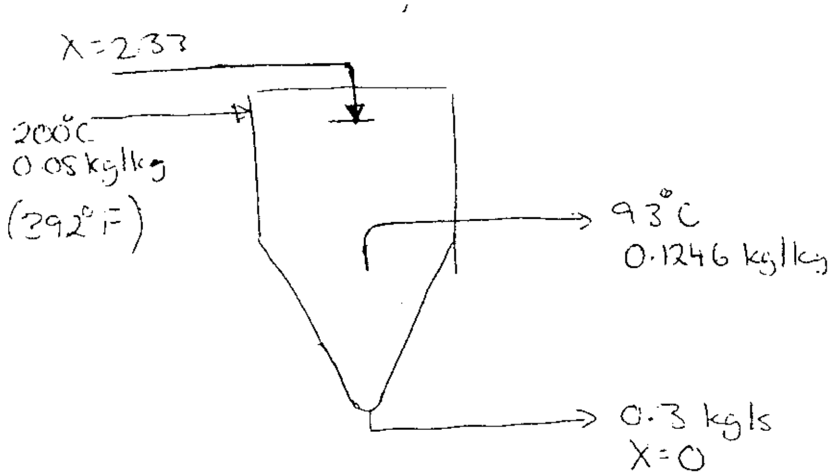


Figure 4.26b (Reprinted by permission of Carrier Corporation.)



Example 12

Solids Balance

$$M_i \frac{1}{1+2.33} = 0.3 \times \frac{1}{1+0}$$

$$\Rightarrow M_i = 0.999\, kg/s$$

$$\Rightarrow Water\ evaporated \quad = 0.999 - 0.3$$
$$= 0.699 kg/s$$

$$M_i \frac{X_i}{1+X_i} + F_{air} H_i = M_o \frac{X_o}{1+X_o} + F_{air} H_o$$

$$\left[\mathrm{M}_i \frac{\mathrm{X}_i}{1+\mathrm{X}_i} - \mathrm{M}_o \frac{\mathrm{X}_o}{1+\mathrm{X}_o} \right] = F_{air}(0.8 - 0.2)$$

$$0.699 = F_{air}(H_o - H_i)$$

$$\Rightarrow F_{air} = \frac{0.699}{(0.1246 - 0.08)} = 15.67 kg/s$$

$$\text{Using charts : evaporator capacity} = \frac{0.699}{0.4536} \times 3600 = 5550\,\underline{\underline{lb/h}}$$

$$\textit{from chart}\,(b)\ \textit{chamber volume} = 13{,}000 ft^3$$

$$\textit{chamber diameter} = 29ft\ (8.84m)$$

$$\textit{convective heat transfer}\ \text{in spray dryer}$$

$$R_c = \frac{w}{A} = \frac{\phi}{h_{fg}A} = \frac{hA(\Delta\theta_{lm})}{h_{fg}A}$$

$$\textit{spherical particle}$$

$$t_d = \frac{r\rho_s}{3R_c}(X_1 - X_2)\ \text{where}\ R_c = \frac{h\Delta\theta_{lm}}{h_{fg}}$$

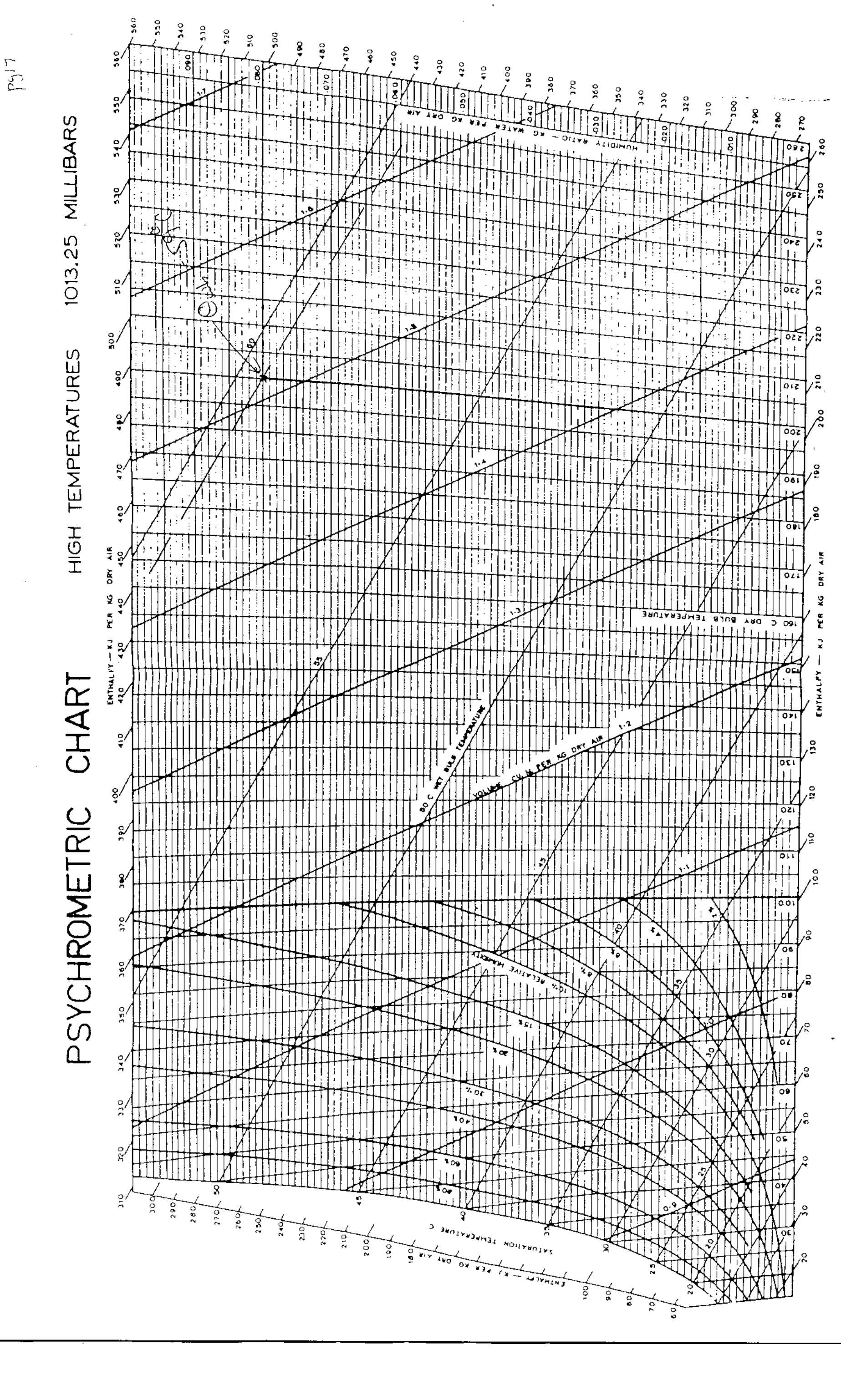
$$R_c = \frac{50\times}{2360000}\left[\frac{(200-59)-(93-59)}{\ln\left(\frac{200-59}{93-59}\right)}\right]$$

$$= 1.59\times10^{-3}\ kg/m^2s$$

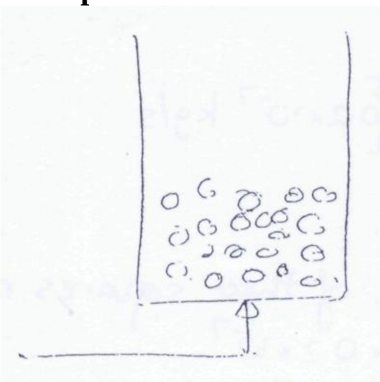
$$t_d = \frac{50\times10^{-6}\times300}{3\times1.59\times10^{-3}}(2.33-0)$$

$$= 7.31\sec$$

$$\Rightarrow 20\ \text{seconds is more than adequate}$$


PSYCHROMETRIC CHART
HIGH TEMPERATURES
1013.25 MILLIBARS

Example 13



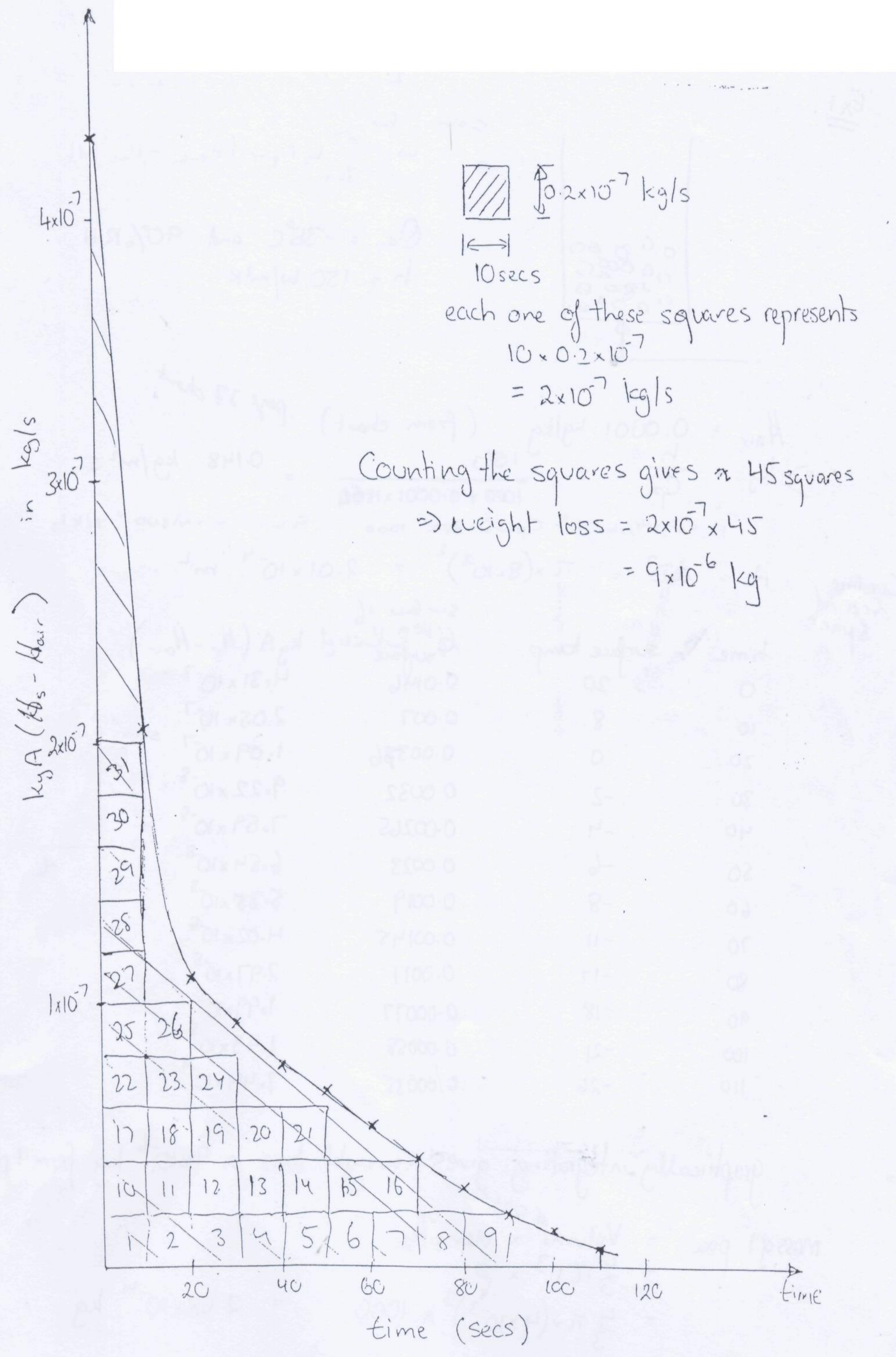
$$\text{Solve for}$$
$$w = \sum \int_{start}^{finish} k_y' A_{prod} (H_{surface} - H_{air}) dt$$
$$\theta_{air} = -38^oC \text{ and } 90\% R.H$$
$$h = 150\, W/m^2K$$

$$H_{air} = 0.0001\,kg/kg \text{(from chart – page 37)}$$
$$\Rightarrow k_y' = \frac{h}{c_p} = \frac{150}{1010 + 0.0001 \times 1800} = 0.148\,kg/m^2s$$
$$c_{pair} @ -38^oC \approx 1010\, J/kgK$$
$$c_{psteam} \approx 1800\, J/kgK$$
$$\text{surface area of a sphere} = A = \pi D^2 = \pi \times (8 \times 10^{-3})^2 = 2.01 \times 10^{-4}\, m^2$$

Surface of Pea is saturated

$time$	$surface\ temp$	$H_{surface}$	$k_y' A(H_s - H_{air})$
0	20	0.0116	4.31×10^{-7}
10	8	0.007	2.05×10^{-7}
20	0	0.00376	1.09×10^{-7}
30	–2	0.0032	9.22×10^{-8}
40	–4	0.00265	7.59×10^{-8}
50	–6	0.0023	6.54×10^{-8}
60	–8	0.0019	5.35×10^{-8}
70	–11	0.00145	4.02×10^{-8}
80	–14	0.0011	2.97×10^{-8}
90	–18	0.00077	1.99×10^{-8}
100	–21	0.00058	1.43×10^{-8}
110	–26	0.00035	7.44×10^{-9}

Graphically integrating gives weight loss $\approx 9 \times 10^{-6}$ kg from 1 pea
Mass of 1 pea = Volume of sphere × density

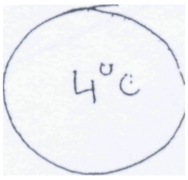


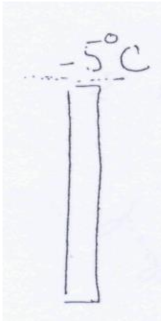
$$= \frac{4}{3}\pi r^3 \times \rho$$

$$= \frac{4}{3}\pi \times (4\times10^{-3})^3 \times 1000 = 2.68\times10^{-4}\, kg$$

$$\Rightarrow \%\, weight\ loss = \frac{9\times10^{-6}}{2.68\times10^{-4}}\times100 = 3.36\%$$

Example 14.





$$k_y = \frac{h}{c_p} \qquad h_{lettuce} = 15 \ \ W/m^2K \qquad h_{evap} = 20 \ \ W/m^2K$$

$$c_p = 1010 + H_{air} \times 1800 = 1010 + 0.005 \times 1800 = 1019 \ J/kgK$$

$$k_y = \frac{15}{1019} = 0.0147 \ kg/m^2s \qquad k_y = \frac{20}{1019} = 0.0196 \ kg/m^2s$$

$$\frac{1}{K_y' A} = \frac{1}{0.0147 \times \pi \times 0.2^2} + \frac{1}{0.0196 \times 0.2 \times 0.2}$$

$$= 1816.9$$

$$\Rightarrow K_y' \ A = 5.50 \times 10^{-4} \ kg/s$$

$$H_{lettuce} = 0.005 \ kg/kg \qquad 4^o\,C \, WB \, Temp \qquad i.e. 100\% \, sat$$

$$H_{evap} = 0.00245 \ kg/kg \quad -5^o\,C \quad (\,page\,55\,chart\,)$$

$$\Rightarrow w = K_y' A (H_{lettuce} - H_{evap})$$

$$= 5.50 \times 10^{-4} \times (0.005 - 0.00245)$$

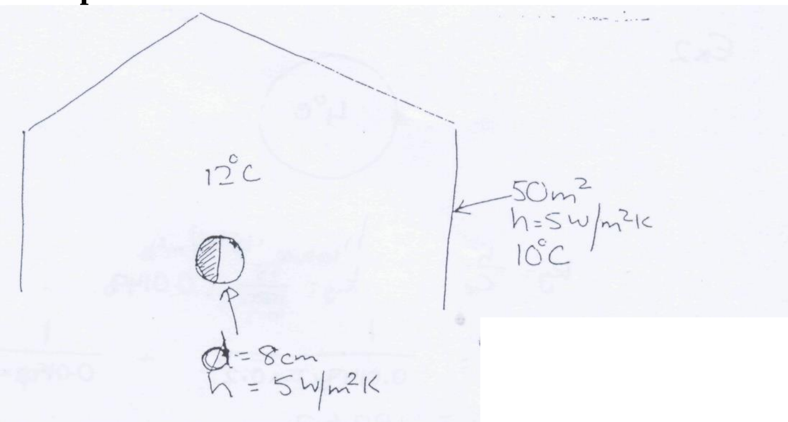
$$= 1.40 \times 10^{-6} \ kg/s$$

$$2\,days = 24 \times 2 \times 3600\,sec$$

$$\Rightarrow w = 0.242\,kg\,lost\,after\,2\,days$$

Air will be at lettuce temperature
Air will have humidity less than saturation (caused by moisture transport to the evaporator).

Example 15



Area exposed to atmosphere (that which doesn't have peel on it)

$$= ½(4\pi r^2) \qquad (= ½\pi D^2)$$

$$= ½\pi \times 0.08^2 = 0.0101 m^2$$

$$k_y' = \frac{h}{c_p} \approx \frac{5}{1010 + 0.0088 \times 1800} = 4.87 \times 10^{-3}\ kg/m^2 s \quad \text{(same for both surfaces)}$$

$$\Rightarrow \frac{1}{K_y' A} = \frac{1}{4.87 \times 10^{-3} \times 0.0101} + \frac{1}{4.87 \times 10^{-3} \times 50} = 20335$$

$$\Rightarrow K_y' A = 4.92 \times 10^{-5}\ kg/m^2 s$$

both at WB temp $\quad H_{apple} @ 12^oC = 0.0088 \frac{kg}{kg} \quad H_{shed} @ 10^oC = 0.0077 \frac{kg}{kg}$

assuming shed surface is wet and apple surface saturated

rate of weightloss

$$w = K_y' A (H_{apple} - H_{shed})$$
$$= 4.92 \times 10^{-5} \times (0.0088 - 0.0077)$$
$$= 5.412 \times 10^{-8}\ kg/s$$

Total weight loss is 0.2% *of the apple mass*

$$\textit{volume sphere} \qquad = 0.002 \times \left(\frac{1}{2} \times \frac{4}{3} \pi r^3 \rho \right)$$
$$= 0.002 \times \frac{1}{2} \times \frac{4}{3} \times \pi \times (0.04)^3 \times 1000$$
$$= 2.681 \times 10^{-4}\ kg$$

$$\Rightarrow \textit{time of exposure} = \frac{2.681 \times 10^{-4}}{5.412 \times 10^{-8}} = 4954\ \textit{seconds}$$

$$(1.38\ \textit{hours})$$

2006 Question 16:

(a) Refer to notes

(b) mass transfer (MT) $\frac{w}{A} = k_H(H_s - H_{air})$

heat transfer (HT) $\frac{w}{A} = \frac{\phi}{h_{fg}A}$

$$= \frac{h_c(\theta_{air} - \theta_s) + h_{rad}(\theta_{rad} - \theta_s)}{h_{fg}}$$

$\theta_{air} = 160^0 C$ $\theta_{WB} = 44^0 C$ $\theta_{rad} = 160^0 C$

$H_{air} = 0.011\, kg\, kg^{-1}$

$$k_H = \frac{h}{c_{pair} + Hc_{psteam}} = \frac{80}{1005 + 1880 \times 0.011} = 0.078\, kgm^{-2}s^{-1}$$

$guess \quad \theta_{surface} = 45^0 C$

$H_{sat} = 0.065\, kgkg^{-1}$ $h_{fg} = 2395\, kJ\, kg^{-1}$

$MT \qquad R_c = 0.078(0.065 - 0.011)$

$$= \underline{4.212 \times 10^{-3} \quad kgm^{-2}s^{-1}}$$

$$h_{rad} = 5.67 \times 10^{-8} \times 1 \times (433 + 318)(433^2 + 318^2)$$

$$= 12.3\, Wm^{-2}K^{-1}$$

$$HT \qquad R_c = \frac{hc(\theta_{air} - \theta_s) + h_{rad}(\theta_{rad} - \theta_s)}{h_{fg}}$$

$$= \frac{80(160 - 45) + 12.3(160 - 45)}{2395 \times 10^3}$$

$$= \underline{4.43 \times 10^{-3}\, kg\, m^{-2}s^{-1}}$$ close

guess again $\theta_s = 44.5^0 C$ $H_s = 0.0635\, kgkg^{-1}$ $h_{fg} = 2396\, kJkg^{-1}$

$$MT \quad R_c = 0.078(0.0635 - 0.011) = 4.1 \times 10^{-3}\ kgm^{-2}s^{-1}$$

$$HT \quad R_c = \frac{80(160 - 44.5) + 12.3(160 - 44.5)}{2396 \times 10^3} = 4.4 \times 10^{-3}$$

guess $\theta_s = 46^0 C$ $\quad H_s = 0.067\ kgkg^{-1}$ $\quad h_{fg} = 2390\ kJkg^{-1}$

$$MT \quad R_c = 0.078(0.067 - 0.011) = \underline{4.37 \times 10^{-3}\ kgm^{-2}s^{-1}}$$

$$HT \quad R_c = \frac{80(160 - 46) + 12.3(160 - 46)}{2390 \times 10^3}$$

$$= \underline{4.4 \times 10^{-3}\ kg\ m^{-2}s^{-1}}$$

<u>Closer</u>

$$\theta_{surface} = 46^0 C$$

$$\therefore \quad R_c = \left(\frac{4.4 + 4.37}{2}\right) \times 10^{-3} = 4.4 \times 10^{-3}\ kgm^{-2}s^{1}$$

$$t_d = \frac{s\rho_s (X_1 - X_c)}{R_c} + \frac{s\rho_s (X_c - X^*)}{R_c} \ln\left[\frac{(X_c - X^*)}{(X_2 - X^*)}\right]$$

$$= \frac{0.06 \times 150(1.2 - 0.8)}{4.4 \times 10^{-3}}$$

$$+ \frac{0.06 \times 150(0.8 - 0.15)}{4.4 \times 10^{-3}} \ln\left[\frac{(0.80 - 0.15)}{(0.4 - 0.15)}\right]$$

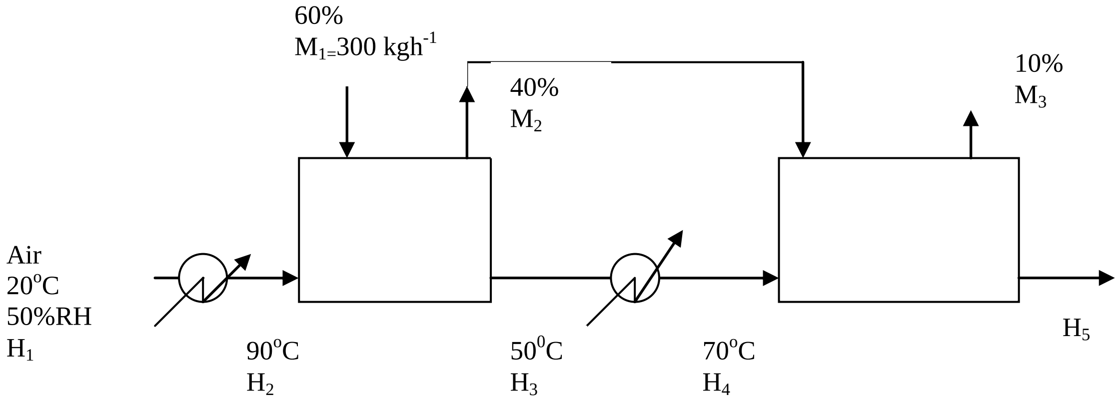
$$= 818 + 1330 \times 0.955$$

$$= 2088s$$

$$= 34 \min s$$

2006 Question 17

(a)



(i) $M_1 = 300\ kgh^{-1}$

$$X_1 = \frac{60}{40} = 1.5\ kg\ kg^{-1}$$

$$X_2 = \frac{40}{60} = 0.67\ kgkg^{-1}$$

$$X_3 = 0.11\ kg\,\mathrm{lg}^{-1}$$

$$M_1 \frac{1}{1+X_1} = 300 \frac{1}{1+1.5} = 120\ kgh^{-1}$$

$$M_1 \frac{1}{1+X_1} = M_2 \frac{1}{1+X_2} \quad \therefore M_2 = 200\ kgh^{-1}$$

$$M_2 \frac{1}{1+X_2} = M_3 \frac{1}{1+X_3} \quad \therefore M_3 = 133\ kgh^{-1}$$

(ii) From psychrometric charts

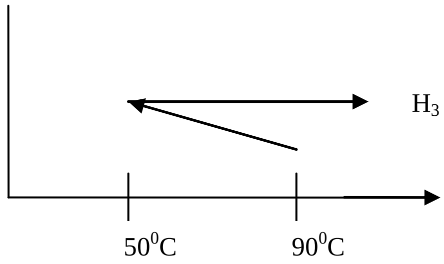
$\theta_{air} = 20^0 C \quad RH\ 50\% \qquad H_1 = 0.0073\ kgkg^{-1}$

$\theta_{air} = 90^0 c \qquad H_2 = 0.0073\ kgkg^{-1}$

$RH = 1.98\%$

$enthalpy \quad h = 115\ kJkg^{-1}$

$\theta_{WB} = 32.5^0 C$



$$H_3 = 0.024\ kgkg^{-1}$$

(iii)

(iv) $M_2 = 200\ kgh^{-1}$

$$M_1 \frac{X_1}{1+X_1} - M_2 \frac{X_2}{1+X_2} = F_{air}(H_3 - H_2)$$

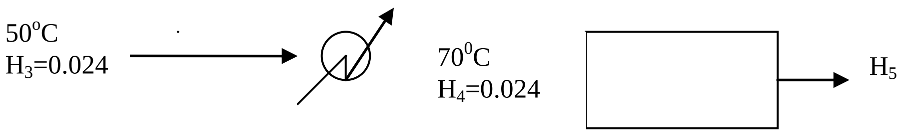
$$300 \times \frac{1.5}{1+1.5} - 200 \times \frac{0.667}{1+0.667} = F_{air}(0.024 - 0.0073)$$

$$100 = F_{air}(0.0167)$$

$$F_{air} = 5988\ kgh^{-1}$$

$$= 1.66\ kgs^{-1}$$

(v) $M_3 = 133\ kgh^{-1}$



$$M_2 \frac{X_2}{1+X_2} - M_3 \frac{X_3}{1+X_3} = F_{air}(H_5 - H_4)$$

$$200\frac{0.667}{1+0.667} - 133\frac{0.11}{1+0.11} = 5988(H_5 - 0.024)$$

$$50.24 = 5988\ H_5 - 143.7$$

$$\therefore\ H_5 = 0.32\ kg\,\mathrm{lg}^{-1}$$

From psychometric chart

$$\theta_{air\,out} = 52^0 C$$
$$RH = 35\%$$
$$h = 137\ kJ\,\mathrm{lg}^{-1}$$

(vi)

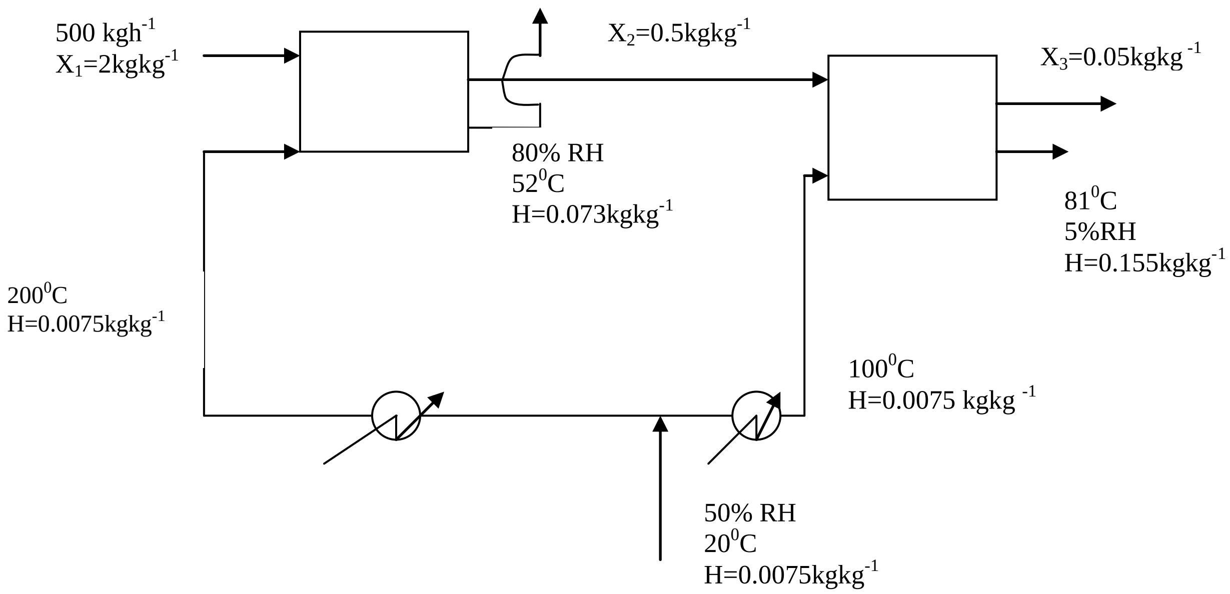
$$\phi_{in} = F_{air}(h_0 - h_i) \qquad @\,20^0C \quad 50\%RH \qquad h_i = 39\ kJ\ kg^{-1}$$
$$= 1.66(137 - 39)$$
$$= 162\ KW$$

(b) Finalised bed dryer

- refer to notes.

2007 Question 18

(a)



Stage 1 $\quad mass\ balance \quad M_i \dfrac{1}{1+X_i} = M_0 \dfrac{1}{1+X_0}$

$$0.14\frac{1}{1+2} = M_0 \frac{1}{1+0.5}$$

$$\therefore M_0 = 0.07\ kgs^{-1}$$

$$(= 252 kgh^{-1})$$

$$water\ balance \quad M_i \frac{X_i}{1+X_i} - M_o \frac{X_o}{1+X_o} = F_1(H_o - H_i)$$

$$0.14\frac{2}{1+2} - 0.07\frac{0.5}{1+0.5} = F_1(0.073 - 0.0075)$$

$$0.07 = F_1(0.0655)$$

$$F_1 = 1.07\ kgs^{-1}$$

$$energy\ balance \quad Fh_1 + \phi = Fh_2$$

$$\phi = F(h_2 - h_1)$$

$$\phi = 1.07(242 - 39)$$

$$\phi = 217.21\,kgs^{-1}$$

Stage 2 M_0 from stage 1 = M_I for stage 2

`

$$mass\,balance \quad 0.07\frac{1}{1+0.5} = M_0\frac{1}{1+0.05}$$

$$\therefore M_0 = 0.049\,kgs^{-1}$$

$$water\ balance \quad 0.07\ \frac{0.5}{1+0.5} - 0.049\ \frac{0.05}{1+0.05} = F_2(0.0155-0.0075)$$

$$0.021 = F_1(8\times10^{-3})$$

$$F_2 = 2.625\,kgs^{-1}$$

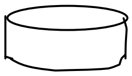
$$energy\ balance \quad \phi = 2.625(125-29)\ \text{energy balance}$$

$$= 225.75\ kJs^{-1}$$

$$\phi\ total = 217.21 + 225.75$$

$$444.96\ kJs^{-1}$$

2008 Question 19



8mm thickness

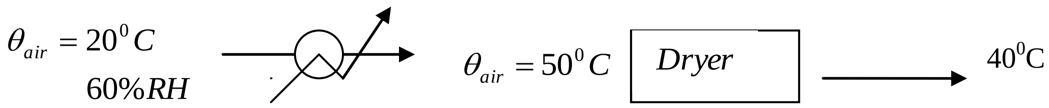
mesh trays
convective drying

$$Initial\ water\ content = 80\%$$

$$X_i = \frac{80}{20} = 4\ kg\, kg^{-1}$$

$$Final\ moisture\ content = 15\%$$

$$X_f = \frac{15}{85} = 0.176\ kg\ kg^{-1}$$



$Batch\ trial$

$m_i = 4kg$

$m_c = 80\%$

$R_c = 2.5 \times 10^{-3}\, kg\ m^{-2} s^{-1}$

$m_f = 2\ kg$

$$X^* = 5\%\ w/w = \frac{5}{95} = 0.053\, kg\, kg^{-1}$$

$\rho_s = 600\ kg\ m^{-3}$

(a)

$X_1 = 4\, kg\, kg^{-1}$

$X_2 = 0.176\, kg\, kg^{-1}$

$X = 0.053\, kg\, kg^{-1}$

$at\ end\ of\ constant\ rate\ period$

$$M_i \frac{1}{1 + X_i} = M_o \frac{1}{1 + X_0}$$

$$4\frac{1}{1+4} = 2\frac{1}{1+X_c}$$

$$0.8 = 2\frac{1}{1+X_c}$$

$$0.4 = \frac{1}{1+X_c}$$

$$1+X_c = 2.5$$

$$X_c = 1.5\ kg\,kg^{-1}$$

(b)

$$t_d = \frac{s\rho_s(X_1 - X_c)}{R_c} + \frac{s\rho_s(X_c - X^*)}{R_c}\ln\left[\frac{(X_c - X^*)}{(X_2 - X^*)}\right]$$

$$= \frac{4\times10^{-3}\times600(4-1.5)}{2.5\times10^{-3}}$$

$$+\frac{4\times10^{-3}\times600(1.5-0.053)}{2.5\times10^{-3}}\ln\left[\frac{1.5-0.053}{0.176-0.053}\right]$$

$$= 2400 + 3424$$

$$= 5824\,s$$

$$= 1.6\,hours$$

(c)

$\theta_{air}\ 20^0 C$

$60\%RH$ *find on psycometric chart*

$H = 0.0088\ kg\ kg^{-1}$

$\downarrow$

heat to $50^0 C$

$H = 0.0088\,kg\,kg^{-1}$

$RH = 11\%$

$$Fruit\ surface\ temperature = \theta_{WB} = 24.4^0 C$$

(d)

$$R_c = \frac{w}{A} = \frac{\phi}{h_{fg}\,A} = \frac{h_c \cancel{A}(\theta_{air} - \theta_s)}{h_{fg}\cancel{A}}$$

$$R_c = \frac{h_c(\theta_{air} - \theta_s)}{h_{fg}}$$

$$h_{fg}\,@\,\theta_{WB} = 2443.6\ kJ\ kg^{-1}$$

$$h_c = \frac{R_c\,h_{fg}}{(\theta_{air} - \theta_s)}$$

$$= \frac{2.5\times10^{-3}\times 2443.6\times10^{3}}{(50-24.4)}$$

$$= 238\,Wm^{-2}\,K^{-1}$$

(ii)

(a)

$$h_c = 20\,Wm^{-2}\ {}^0C^{-1}$$
$$\theta_{WB} = 10^0\,C$$

$$3\ days = 72\ hours$$

$$m_{fruit} = 110g$$

$$\theta_{air} = 14^0\,C$$
$$\theta_{WB} = 10^0\,C$$
$$H_{air} = 0.006\ kg\ kg^{-1} \qquad\qquad H_s = 0.0078\,kg\,kg^{-1}$$

$$c_{pH} = 1.005 + 1.88\times H_{air}$$
$$= 1.005 + 1.88\times 0.006$$
$$= 1.016 \quad kJ\ kg^{-1}\,K^{-1}$$

$$k_H{}^1 = \frac{h}{c_{pH}} = \frac{20}{1.016\times10^3} = 0.0197\,kg\,m^{-2}s^{-1}$$

$$A = 1.5 \times \pi r L$$
$$= 1.5 \times \pi \times 3 \times 10^{-2} \times 8 \times 10^{-2} = 0.0113 m^2$$

$$w = 0.0197 \times 0.0113 (0.0078 - 0.006)$$
$$= 4 \times 10^{-7}\ kg\ s^{-1}$$

$$H_2O\ loss\ over\ 3\ days = 4 \times 10^{-7} \times 72 \times 3600$$
$$= 0.103\ g$$
$$loss = \frac{103}{110g} = 94\%\ loss\ !!\ high\ ?$$

(b)

greater weight loss

higher temperature $\rightarrow$ *height* H_s $\rightarrow$ *increase in driving force for mass transfer*