1 Tutorial Week 2: Elementary Market Model

1. What is the price at time 0 of a contingent claim represented by the payoff $h(S_1) = S_1$?

2. Assume the parameters in the two-state market model are given by: $r = \frac{1}{3}$, $S_0 = 1$, $u = 2$, $d = .5$ and $p = \frac{3}{5}$. Suppose the strike price of the European call option $K = 1$ and maturity $T = 1$. Suppose the strike price of the European put option $K = 1$ and maturity $T = 1$.

   For these Call and Put options compute the hedging strategies.

2 Tutorial Week 2: Solutions

1. Let (x, φ) be the replicating strategy for contingent claim $X = h(S_1) = S_1$, that is

$$V_1(x, \varphi) = (x - \varphi S_0)(1 + r) + \varphi S_1 = S_1$$

   More explicitly, for every $\omega_i \in \Omega = \{\omega_1, \omega_2\}$

$$V_1(x, \varphi)(\omega_i) = (x - \varphi S_0)(1 + r) + \varphi S_1(\omega_i) = S_1(\omega_i)$$

   If a contingent claim X is offered at a price different from S_0 then an arbitrage opportunity arises. Hence the unique fair value of he claim $X = S_1$ at time 0 equals S_0.

   Note also that with $\varphi = 1$

$$x = \frac{(1 - \varphi)}{(1 + r)} S_1(\omega_i) + \varphi S_0 \overset{\text{arbitrage price}}{=} S_0$$

2. For the call option we need to solve

$$V_1(x, \varphi) = (x - \varphi S_0)(1 + r) + \varphi S_1 = C_1 = (S_1 - K)^+$$

   or, more explicitly

$$\begin{aligned}
(x - \varphi)\frac{4}{3} + 2\varphi &= (2 - 1)^+ = 1 \\
(x - \varphi)\frac{4}{3} + 0.5\varphi &= (0.5 - 1)^+ = 0
\end{aligned}$$

   so we get $(x, \varphi) = \left(\frac{15}{36}, \frac{2}{3}\right)$. Note that

$$x - \varphi S_0 = \frac{15}{36} - \frac{2}{3} \times 1 = -\frac{9}{36}$$

   meaning that after selling the call option at the price $\frac{15}{36}$ in order to buy $\frac{2}{3}$ of the stock we need to borrow $\frac{9}{36}$ units of cash.

   For the put option we solve

$$V_1(x, \varphi) = (x - \varphi S_0)(1 + r) + \varphi S_1 = P_1 = (K - S_1)^+$$

that is

$$\begin{aligned}(x-\varphi)\frac{4}{3}+2\varphi &= (1-2)^{+}=0\\(x-\varphi)\frac{4}{3}+0.5\varphi &= (1-0.5)^{+}=\frac{1}{2}\end{aligned}$$

We obtain $(x,\varphi)=\left(\frac{1}{6},-\frac{1}{3}\right)$. Note that

$$x-\varphi S_0=\frac{1}{6}+\frac{1}{3}\times 1=\frac{1}{2}$$

meaning that after short-selling of $\frac{1}{3}$ of the stock and selling the put option at the price $\frac{1}{6}$ we invest $\frac{1}{2}$ units of cash in the savings account.