~~$x>0$:~~

~~1. a> $F(x) = \int_0^x \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}} dx$~~

1. a> $F(x) = \begin{cases} 0 & x \le 0 \\ \int_0^x \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}} dx & x > 0 \end{cases}$

$$\int_0^x \frac{\alpha\beta^\alpha}{(t+\beta)^{\alpha+1}} dt = -\beta^\alpha (t+\beta)^{-\alpha} \Big|_0^x = 1 - \beta^\alpha (x+\beta)^{-\alpha}$$

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_0^\infty x \cdot \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}} dx$$

$$= \int_0^\infty (x+\beta) \cdot \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}} dx - \int_0^\infty \frac{\alpha\beta^{\alpha+1}}{(x+\beta)^{\alpha+1}} dx$$

$$= \int_0^\infty \frac{\alpha\beta^\alpha}{(x+\beta)^\alpha} dx + \beta^{\alpha+1}(x+\beta)^{-\alpha} \Big|_0^\infty$$

$$= \frac{\alpha\beta^\alpha}{1-\alpha}(x+\beta)^{1-\alpha} \Big|_0^\infty - \beta = \cancel{\frac{\beta(2\alpha-1)}{1-\alpha}} \quad \frac{\beta}{\alpha-1} \qquad \alpha > 1, \ \beta > 0$$

median: Let $F(x) = 1 - \beta^\alpha (x+\beta)^{-\alpha} = \frac{1}{2}$

$x = \beta(2^{\frac{1}{\alpha}} - 1)$

~~$Var(X) = \int_0^\infty \left(x - \frac{\beta}{\alpha-1}\right)^2 \cdot \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}} dx$~~

$$Var(X) = E(X^2) - E(X)^2 = \int_0^\infty x^2 \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}} dx - \left(\frac{\beta}{\alpha-1}\right)^2$$

$$= \int_0^\infty (x+\beta)^2 \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}} dx - 2\beta \int_0^\infty x \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}} dx - \beta^2 \int_0^\infty \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}} dx$$

$$- \left(\frac{\beta}{\alpha-1}\right)^2 = \int_0^\infty \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha-1}} dx - \frac{2\beta^2}{\alpha-1} - \beta^2 - \left(\frac{\beta}{\alpha-1}\right)^2$$

$$= \frac{\alpha\beta^{\alpha}}{2-\alpha}(x+\beta)^{2-\alpha}\Big|_0^{\infty} - \frac{2\beta^2}{\alpha-1} - \beta^2 - \left(\frac{\beta}{\alpha-1}\right)^2$$

$$= \frac{\alpha\beta^2}{\alpha-2} - \frac{2\beta^2}{\alpha-1} - \beta^2 - \left(\frac{\beta}{\alpha-1}\right)^2 = \frac{\beta^2\alpha}{(\alpha-1)^2(\alpha-2)}$$

$\alpha > 2$

$\beta > 0$