a) $F(x) = \begin{cases} 0 & x \le 0 \\ \int_0^x \frac{\alpha\beta^\alpha}{(t+\beta)^{\alpha+1}}\,dt & x > 0 \end{cases}$

$$\int_0^x \frac{\alpha\beta^\alpha}{(t+\beta)^{\alpha+1}}\,dt = -\beta^\alpha(t+\beta)^{-\alpha}\Big|_0^x = 1-\beta^\alpha(x+\beta)^{-\alpha}$$

$$E(X) = \int_{-\infty}^{+\infty} xf(x)\,dx = \int_0^\infty x\cdot\frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}}\,dx$$

$$= \int_0^\infty (x+\beta)\cdot\frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}}\,dx - \int_0^\infty \frac{\alpha\beta^{\alpha+1}}{(x+\beta)^{\alpha+1}}\,dx$$

$$= \int_0^\infty \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha}}\,dx + \beta^{\alpha+1}(x+\beta)^{-\alpha}\Big|_0^\infty$$

$$= \frac{\alpha\beta^\alpha}{1-\alpha}(x+\beta)^{1-\alpha}\Big|_0^\infty - \beta = \frac{\beta}{\alpha-1} \qquad \alpha > 1, \beta > 0$$

Median:

Let $F(x) = 1-\beta^\alpha(x+\beta)^{-\alpha} = \frac{1}{2} \quad \Rightarrow \quad x = \beta(2^{1/\alpha}-1)$

$$Var(X) = E(X^2) - E(X)^2$$

$$= \int_0^\infty x^2\cdot\frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}}\,dx - (\frac{\beta}{\alpha-1})^2$$

$$= \int_0^\infty (x+\beta)^2\cdot\frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}}\,dx - 2\beta\int_0^\infty x\cdot\frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}}\,dx$$

$$-\beta^2\int_0^\infty \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}}\,dx - (\frac{\beta}{\alpha-1})^2$$

$$= \int_0^\infty \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha-1}}\,dx - \frac{2\beta^2}{\alpha-1} - \beta^2 - (\frac{\beta}{\alpha-1})^2$$

$$= \frac{\alpha\beta^\alpha}{2-\alpha}(x+\beta)^{2-\alpha}\Big|_0^\infty - \frac{2\beta^2}{\alpha-1} - \beta^2 - (\frac{\beta}{\alpha-1})^2$$

$$= \frac{\alpha\beta^2}{2-\alpha} - \frac{2\beta^2}{\alpha-1} - \beta^2 - (\frac{\beta}{\alpha-1})^2 = \frac{\beta^2\alpha}{(\alpha-1)^2(\alpha-2)} \qquad \alpha > 2, \beta > 0$$

b) $F\big(F^{-1}(u)\big) = u$

$1-\beta^\alpha(F^{-1}(u)+\beta)^{-\alpha} = u$

$(\frac{\beta}{(F^{-1}(u)+\beta))^\alpha} = 1-u$

$F^{-1}(u) = \beta\cdot(1-u)^{-1/\alpha} - \beta$