Question2

Build up the model:

$$Z = a + cn - S$$

Z: the assets of the company at the end of the year.

$a = 250000$ represents the current assets of the company.

$c = 6000$ represents the annual premium.

$n = 1000$ represents the number of the customers.

Total claim:

$$S = \sum_{i=1}^{N} X_i$$

X_i~Pareto distribution i.i.d.

$N = \sum_{j=1}^{1000} Y_j$: the number of clients making a claim this year.

For Y_j $\mathrm{i.i.d.}$:

$$\begin{cases} \mathrm{P}(Y_j = 1) = 0.1 \\ \mathrm{P}(Y_j = 0) = 0.9 \end{cases}$$

We should calculate the bankruptcy probability:

$$\mathrm{P}(Z < 0)$$

Question4 d)

We can extend the model from following aspects:

1. Consider 10 years' time period
2. The number of customers is not fixed every year and the premium is not necessarily paid at the start of a year. The total number of paying for annual premium has a Poisson distribution.
3. The probability of customers making a claim is not identical for everyone. Assume the number of times making a claim also has a Poisson distribution.

We have a new model:

$$Z(t) = a + cM(t) - S(t)$$

$Z(t)$: the assets of the company at the end of the year t.

$a = 250000$ represents the current assets of the company.

$c = 6000$ represents the annual premium.

$M(t)$: the number of times for paying for premium until year t. It has a Poisson distribution of parameter $\lambda_1 t$. The pdf of $M(t)$ is:

$$\mathrm{P}(M(t) = k) = \frac{(\lambda_1 t)^k}{k!} e^{-\lambda_1 t}, \quad k = 0,1,2 \ldots$$

Total claim till the end of year t:

$$S(t) = \sum_{i=1}^{N(t)} X_i$$

X_i~Pareto distribution i.i.d.

$N(t)$: the number of claims made until year t. It has a Poisson distribution of parameter$\lambda_2 t$. The pdf of $N(t)$ is:

$$\mathrm{P}(N(t)=k)=\frac{(\lambda_2 t)^k}{k!}e^{-\lambda_2 t},\quad k=0,1,2\ldots$$

Assume $\lambda_1 = 1000$, $\lambda_2 = 100$, we can simulate the assets of the company in the end of year t. Compare the probability of bankruptcy at the end of each year.