- Group Project

Question 1: $f(x) = \dfrac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}}$

$$F(x) = \int_0^x \frac{\alpha\beta^\alpha}{(t+\beta)^{\alpha+1}}\,dt = -\beta^\alpha\cdot(t+\beta)^{-\alpha}\Big|_0^x = -\beta^\alpha(x+\beta)^{-\alpha}+1$$

$$E(X) = \int_0^\infty x\cdot\alpha\beta^\alpha\cdot(x+\beta)^{-\alpha-1}\,dx = \int_0^\infty -x\cdot\beta^\alpha\cdot d(x+\beta)^{-\alpha}$$

$$= -x\cdot(x+\beta)^{-\alpha}\cdot\beta^\alpha\Big|_0^\infty + \int_0^\infty \beta^\alpha\cdot(x+\beta)^{-\alpha}\,dx$$

$$= \frac{\beta^\alpha}{1-\alpha}\cdot(x+\beta)^{1-\alpha}\Big|_0^\infty = \frac{\beta}{\alpha-1} \quad \text{if } \alpha>1$$

$$E(X^2) = \int_0^\infty x^2\cdot\alpha\beta^\alpha\cdot(x+\beta)^{-\alpha-1}\,dx = \int_0^\infty -x^2\cdot\beta^\alpha\,d(x+\beta)^{-\alpha}$$

$$= -x^2\cdot\beta^\alpha\cdot(x+\beta)^{-\alpha}\Big|_0^\infty + 2\beta^\alpha\int_0^\infty x\cdot(x+\beta)^{-\alpha}\,dx$$

$$= \frac{2\beta^\alpha}{1-\alpha}\int_0^\infty x\,d(x+\beta)^{1-\alpha} = \frac{2\beta^\alpha}{1-\alpha}\,x(x+\beta)^{1-\alpha}\Big|_0^\infty - \int_0^\infty (x+\beta)^{1-\alpha}\,dx\cdot\frac{2\beta^\alpha}{1-\alpha}$$

$$= -(x+\beta)^{2-\alpha}\cdot\frac{1}{2-\alpha}\Big|_0^\infty\cdot\frac{2\beta^\alpha}{1-\alpha}$$

if $\alpha > \cancel{\beta}\,2$ when $x\to\infty$ $-(x+\beta)^{2-\alpha}\to 0$

$$\therefore E(X^2) = \frac{\cancel{\beta}^2}{2-\cancel{\alpha}} \qquad \cancel{var(X) = E(X^2)-(E(X))^2 = \frac{\beta^{2+}}{2-\alpha} - \frac{\beta^2}{(\alpha-1)^2} = \frac{\beta^2\cdot(\alpha^2-2\alpha+1+\alpha-2)}{(2-\alpha)(\alpha-1)^2}}$$

$$= \frac{2\cdot\beta^{2+\cancel{\alpha}}}{(\alpha-2)(\alpha-1)}$$

$$Var(X) = E(X^2) - (E(X))^2 = \frac{2\cdot\beta^{\cancel{\alpha+2}\,2}}{(\alpha-2)(\alpha-1)} - \frac{\beta^2}{(\alpha-1)^2}$$

$$= \frac{2\cdot\beta^2\cdot(\alpha-1)-\beta^2\cdot(\alpha-2)}{(\alpha-2)(\alpha-1)^2}$$

$$= \frac{\beta^2\cdot(2\alpha-2-\alpha+2)}{(\alpha-2)(\alpha-1)^2}$$

$$= \frac{\beta^2\alpha}{(\alpha-2)(\alpha-1)^2}$$

median of X: $\frac{1}{2} = \int_{-\infty}^{m} \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}} dx = -\beta^\alpha (m+\beta)^{-\alpha} + 1 = \frac{1}{2}$

$(m+\beta)^{-\alpha} = \frac{1}{2}\beta^{-\alpha} \Rightarrow 2\beta^\alpha = (m+\beta)^\alpha \quad m = -\beta + \left(\frac{1}{2}\right)^{\frac{1}{\alpha}} \cdot \beta$