

$$x > 0 : \\ 7. \text{ a.s } F(x) = \int_0^x \frac{\beta^x}{(x+\beta)^{x+1}} dx$$

$$1. \text{ a.s } F(x) = \begin{cases} 0 & x \leq 0 \\ \int_0^x \frac{\beta^x}{(x+\beta)^{x+1}} dx & x > 0 \end{cases}$$

$$\int_0^x \frac{\beta^x}{(x+\beta)^{x+1}} dx = -\beta^x (x+\beta)^{-x} \Big|_0^x = -\beta^x (x+\beta)^{-x}$$

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_0^{\infty} x \cdot \frac{\beta^x}{(x+\beta)^{x+1}} dx$$

$$= \int_0^{\infty} (x+\beta) \cdot \frac{\beta^x}{(x+\beta)^{x+1}} dx - \int_0^{\infty} \frac{\beta^{x+1}}{(x+\beta)^{x+1}} dx$$

$$= \int_0^{\infty} \frac{\beta^x}{(x+\beta)^x} dx + \beta^{x+1} (x+\beta)^{-x} \Big|_0^{\infty}$$

$$= \frac{\beta^x}{x-1} (x+\beta)^{1-x} \Big|_0^{\infty} - \beta = \frac{\beta(2-\beta)}{\beta-1} \frac{\beta}{\beta-1} \quad \beta > 1$$

median: Let $F(x) = 1 - \beta^x (x+\beta)^{-x} = \frac{1}{2}$

$$x = \beta(2^{\frac{1}{2}} - 1)$$

$$\text{Var}(x) = \int_0^{\infty} \left(x - \frac{\beta(2^{\frac{1}{2}} - 1)}{\beta-1} \right)^2 \cdot \frac{\beta^x}{(x+\beta)^{x+1}} dx$$

$$\text{Var}(x) = E(x^2) - E(x)^2 = \int_0^{\infty} x^2 \frac{\beta^x}{(x+\beta)^{x+1}} dx - \left(\frac{\beta}{\beta-1} \right)^2$$

$$= \int_0^{\infty} (x+\beta)^2 \frac{\beta^x}{(x+\beta)^{x+1}} dx - 2\beta \int_0^{\infty} x \frac{\beta^x}{(x+\beta)^{x+1}} dx - \beta^2 \int_0^{\infty} \frac{\beta^x}{(x+\beta)^{x+1}} dx$$

$$- \left(\frac{\beta}{\beta-1} \right)^2 = \int_0^{\infty} \frac{\beta^x}{(x+\beta)^{x+1}} dx - \frac{2\beta^2}{\beta-1} - \beta^2 - \left(\frac{\beta}{\beta-1} \right)^2$$

$$= \frac{\frac{d\beta^2}{dz}}{z-2} (x+\beta)^{2-z} \Big|_0^\infty - \frac{2\beta^2}{z-1} - \beta^2 - \left(\frac{\beta}{z-1}\right)^2$$

$$= \frac{\frac{d\beta^2}{dz}}{z-2} - \frac{2\beta^2}{z-1} - \beta^2 - \left(\frac{\beta}{z-1}\right)^2 = \frac{\beta^2 z}{(z-1)(z-2)} \quad z > 2 \\ \beta > 0$$