(1). $F(x)=\int_0^x \frac{\alpha\beta^\alpha}{(u+\beta)^{\alpha+1}}du = \alpha\beta^\alpha\int_0^x \frac{1}{(u+\beta)^{\alpha+1}}d(u+\beta) = -\beta^\alpha \frac{1}{(u+\beta)^\alpha}\Big|_0^x = 1-(\frac{\beta}{x+\beta})^\alpha$.

Let $Y=X+\beta$. then $E(X)=E(Y)-\beta$. $Var(X)=Var(Y)$.

$E(Y^r)=\int_0^\infty (x+\beta)^r \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}}dx = \alpha\beta^\alpha\cdot\frac{1}{r-\alpha}(x+\beta)^{r-\alpha}\Big|_0^\infty$ (1).

(1) $=\frac{\alpha\beta^r}{\alpha-r}$ provided $\alpha>r$.

so when $\alpha>1$. EY exists. $EY=\frac{\alpha\beta}{\alpha-1}$. $EX=\frac{\beta}{\alpha-1}$.

when $\alpha>2$. EY^2 exists., $VarY$ exsits. $VarY=EY^2-(EY)^2$

$=\frac{\alpha\beta^2}{\alpha-2}\ \frac{\alpha^2\beta^2}{(\alpha-1)^2}=\frac{\alpha\beta^2}{(\alpha-2)(\alpha-1)^2}=VarX$

Median of $X \Rightarrow F^{-1}(0.5)=u_{0.5}$. then $F(u_{0.5})=0.5$.

$1-(\frac{\beta}{u_{0.5}+\beta})^\alpha=\frac{1}{2}$. $u_{0.5}=(2^{\frac{1}{\alpha}}-1)\beta$. $\alpha\neq0$.

Summary: is a density function. ~~$\alpha\beta\neq0$~~. $f(x)>0$. for any $x\geq0$.

so $\alpha.\beta>0$. If $VarX$ exist, $\alpha>2$. $\begin{cases}\alpha>2.\\ \beta>0.\end{cases}$

(2) Set $u=F(x)=1-(\frac{\beta}{x+\beta})^\alpha$. to get $x=\beta(\frac{1}{(1-u)^{\frac{1}{\alpha}}}-1)$.

so the procedure is:

(1) generate $u_i\in(0,1)$.

(2) ~~set~~ draw values from $x_i=\beta(\frac{1}{(1-u_i)^{\frac{1}{\alpha}}}-1)$.