ST425 Group Project

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Background and Assumption

An insurance company with fixed commercial customers.

BASIC ASSUMPTIONS

- Current asset: £250000
- Number of commercial customers: 1000
- Annual premium: £6000
- Probability of making a claim: 0.1
- The size of claim variables: Pareto distribution
- $F(x) = \begin{cases} 0 & x \le 0 \\ \int_0^x \frac{\alpha\beta^\alpha}{(t+\beta)^{\alpha+1}} dt & x > 0 \end{cases}$

OTHER ASSUMPTIONS

- One customer can only **make one claim each year.**
- The probability of making a claim is **fixed and equal for each customer**.
- Claims are made **independently of each other** and no customer is influenced by the other.

OTHER ASSUMPTIONS

- **The value of the premium is fixed** irrespective of the size of the claim.
- All customers are loyal and will **not drop out at any circumstance**(eg:- When premium increases)
- **No deductibles** are paid by any customers.
- The company will **retain all premiums charged** with itself and not invest on anything else.

2

Simulation the Size of Claims

Let X be the size of a typical claim.

a) Calculate the cumulative distribution function F(x) of X, the expectation E(X), the median of X, and the variance of X. State any conditions on α and/or β that need to be satisfied.

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EXPECTATION OF X

$$F(x) = \begin{cases} 0 & x \leq 0 \\ \int_0^x \frac{\alpha\beta^\alpha}{(t+\beta)^{\alpha+1}}\,dt & x > 0 \end{cases}$$

$$\int_0^x \frac{\alpha\beta^\alpha}{(t+\beta)^{\alpha+1}}\,dt = -\beta^\alpha(t+\beta)^{-\alpha}\Big|_0^x = 1 - \beta^\alpha(x+\beta)^{-\alpha}$$

$$E(X) = \int_{-\infty}^{+\infty} xf(x)\,dx = \int_0^\infty x \cdot \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}}\,dx$$

$$= \int_0^\infty (x+\beta) \cdot \frac{\alpha\beta^\alpha}{(x+\beta)^{\alpha+1}}\,dx - \int_0^\infty \frac{\alpha\beta^{\alpha+1}}{(x+\beta)^{\alpha+1}}\,dx$$

$$= \int_0^\infty \frac{\alpha\beta^\alpha}{(x+\beta)^\alpha}\,dx + \beta^{\alpha+1}(x+\beta)^{-\alpha}\Big|_0^\infty$$

$$= \frac{\alpha\beta^\alpha}{1-\alpha}(x+\beta)^{1-\alpha}\Big|_0^\infty - \beta = \frac{\beta}{\alpha-1} \qquad \alpha > 1, \beta > 0$$

MEDIAN AND VARIANCE OF X

Let $F(x) = 1 - \beta^{\alpha}(x+\beta)^{-\alpha} = \frac{1}{2} \quad \Rightarrow \quad x = \beta(2^{1/\alpha} - 1)$

$Var(X) = E(X^2) - E(X)^2$

$= \int_0^{\infty} x^2 \cdot \frac{\alpha\beta^{\alpha}}{(x+\beta)^{\alpha+1}} dx - (\frac{\beta}{\alpha-1})^2$

$= \int_0^{\infty} (x+\beta)^2 \cdot \frac{\alpha\beta^{\alpha}}{(x+\beta)^{\alpha+1}} dx - 2\beta \int_0^{\infty} x \cdot \frac{\alpha\beta^{\alpha}}{(x+\beta)^{\alpha+1}} dx$

$-\beta^2 \int_0^{\infty} \frac{\alpha\beta^{\alpha}}{(x+\beta)^{\alpha+1}} dx - (\frac{\beta}{\alpha-1})^2$

$= \int_0^{\infty} \frac{\alpha\beta^{\alpha}}{(x+\beta)^{\alpha-1}} dx - \frac{2\beta^2}{\alpha-1} - \beta^2 - (\frac{\beta}{\alpha-1})^2$

$= \frac{\alpha\beta^{\alpha}}{2-\alpha}(x+\beta)^{2-\alpha}\Big|_0^{\infty} - \frac{2\beta^2}{\alpha-1} - \beta^2 - (\frac{\beta}{\alpha-1})^2$

$= \frac{\alpha\beta^2}{2-\alpha} - \frac{2\beta^2}{\alpha-1} - \beta^2 - (\frac{\beta}{\alpha-1})^2 = \frac{\beta^2\alpha}{(\alpha-1)^2(\alpha-2)} \qquad \alpha > 2, \beta > 0$

b) Describe how the inversion method may be used to simulate from X.

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INVERSION METHOD

$$F\left(F^{-1}(u)\right) = u$$

$$1 - \beta^{\alpha}(F^{-1}(u) + \beta)^{-\alpha} = u$$

$$(\frac{\beta}{(F^{-1}(u)+\beta))^{\alpha}} = 1 - u$$

$$F^{-1}(u) = \beta \cdot (1 - u)^{-1/\alpha} - \beta$$

Inversion method

1. Generate ui ∈ (0, 1).

2. Set ui = F(xi) and solve for xi, which is the desired random observation from the distribution with cdf F.

c) Simulation of 10000 values drawn from X. Comparison with the true density function superimposed.

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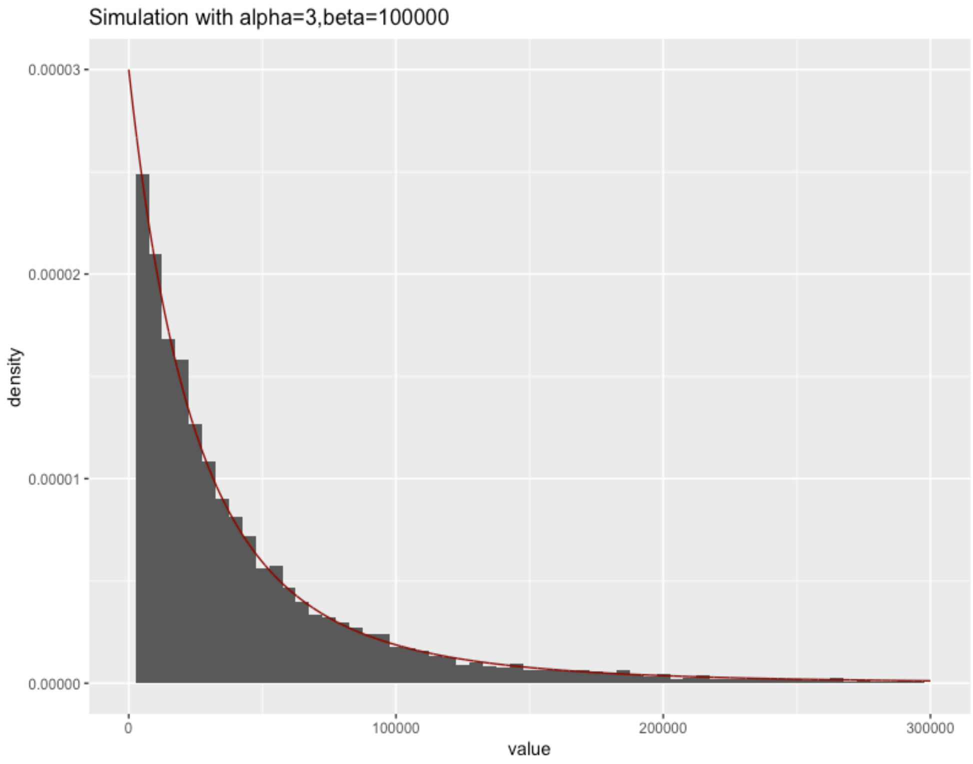
SIMULATION FOR SIZE OF CLAIMS



d) Reasons for use of a Pareto distribution to describe insurance claims.

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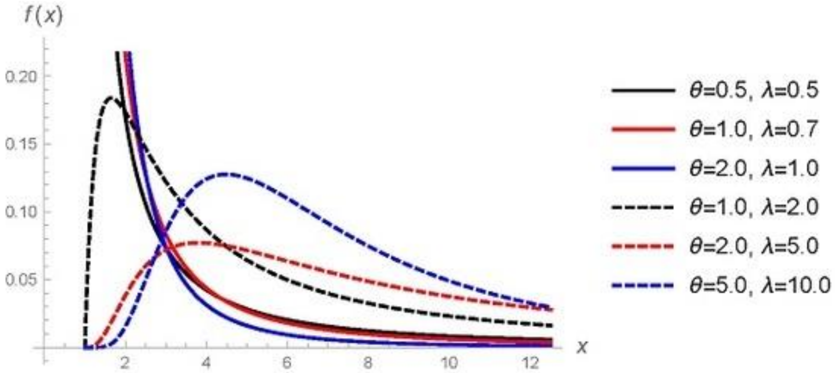
PARETO DISTRIBUTION

- The Pareto distribution **is positively skewed** and has a **heavy tail on the right**. Insurance applications, Pareto for this reason can be used for **modelling extreme loss**, especially for more **risky types of insurance.**



Resource: MDPI

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Model of Year End Assets

To find a simplified model of insurance company

BUILDING UP THE MODEL

$$Z = a + cn - S$$

$Z:$ the assets of the company at the end of the year.

$a = 250000$ represents the current assets of the company.

$c = 6000$ represents the annual premium.

$n = 1000$ represents the number of the customers.

BUILDING UP THE MODEL

Total claim:

$$S = \sum_{i=1}^{N} X_i$$

$X_i \sim$Pareto distribution i.i.d.

$N = \sum_{j=1}^{1000} Y_j$: the number of clients making a claim this year.

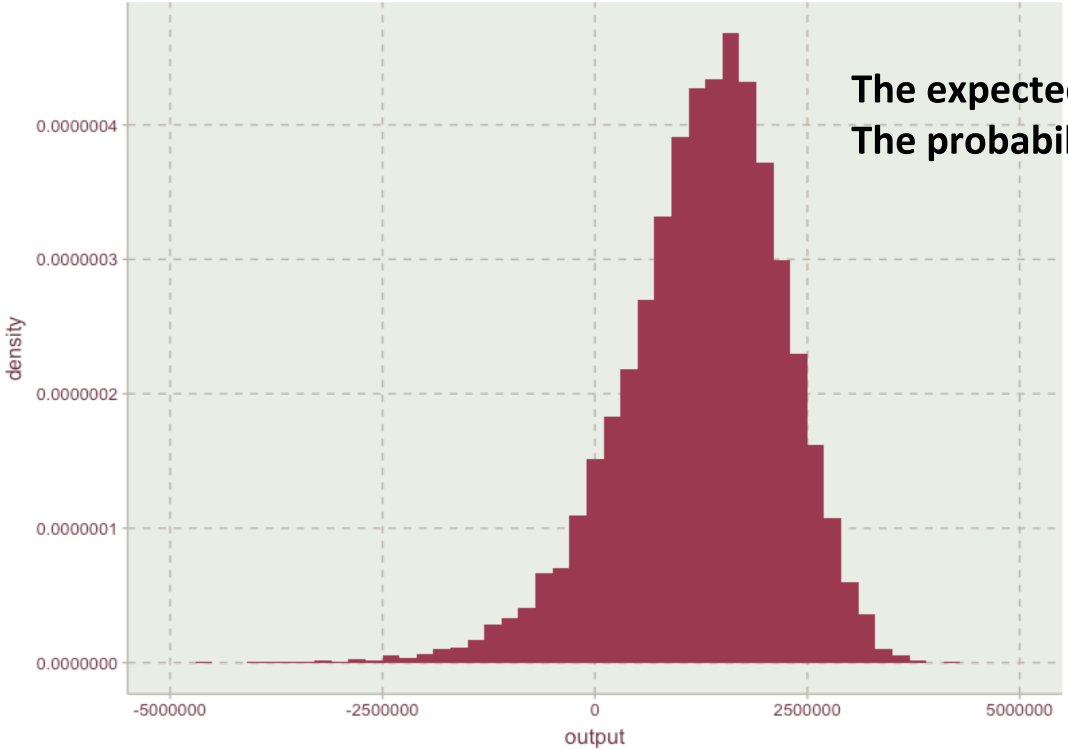
For Y_j i. i. d.:

$$\begin{cases} \mathrm{P}(Y_j = 1) = 0.1 \\ \mathrm{P}(Y_j = 0) = 0.9 \end{cases}$$

We should calculate the bankruptcy probability:

$$\mathrm{P}(Z < 0)$$

YEAR END TOTAL ASSET



The expected asset: 1249603.2379

The probability of bankruptcy: 0.0977

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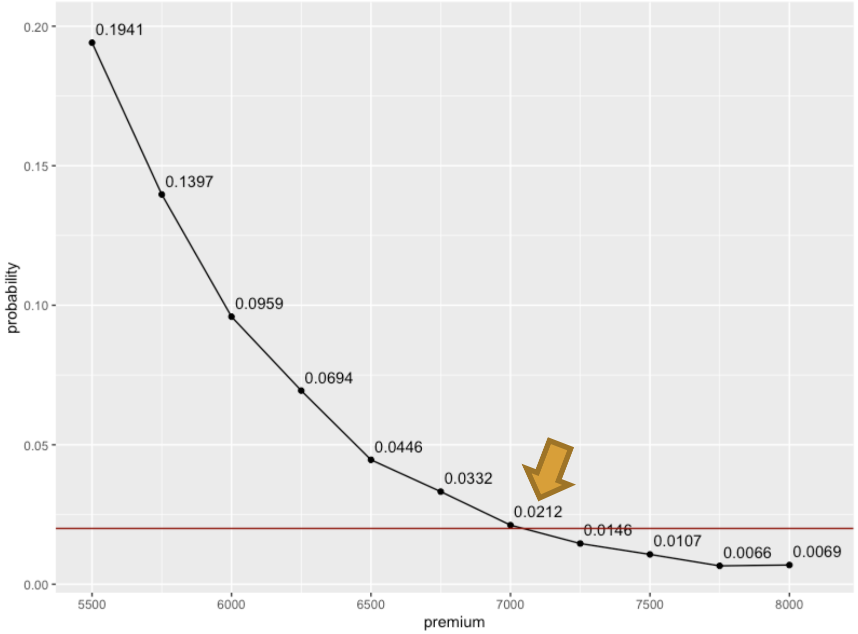
Changing Inputs of the Model

Changing the annual premiums and probability of making a claim.

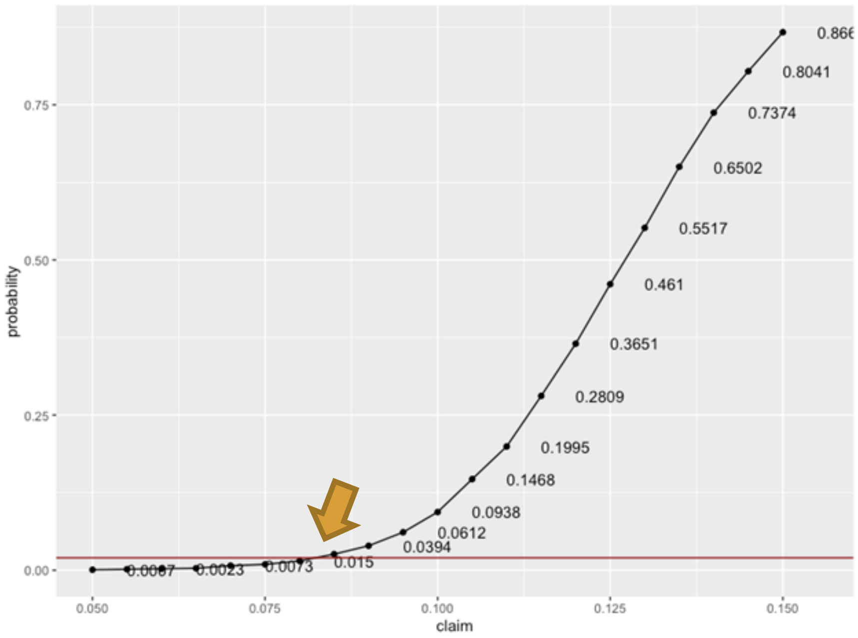
CHANGE OF PREMIUM



Minimal premium level: 7250

PREMIUM	BALANCE	PROBABILITY
5500	741136.6002	0.1941
5750	1003137.7332	0.1397
6000	1255157.6144	0.0959
6250	1498083.9745	0.0694
6500	1759673.6819	0.0446
6750	2002743.6275	0.0332
7000	2251394.9805	0.0212
7250	2490678.6568	0.0146
7500	2740845.1092	0.0107
7750	2994727.1776	0.0066
8000	3255043.8666	0.0069

CHANGE OF PROBABILITY



Minimal premium level: 7250

CLAIM	BALANCE	PROBABILITY
0.05	3759849.6438	0.0007
0.055	3497623.5856	0.0016
0.06	3260109.4721	0.0023
0.065	3009525.6470	0.0032
0.07	2746500.6304	0.0073
0.075	2490617.9996	0.0094
0.08	2243506.9708	0.015
0.085	1998117.9679	0.0259
0.09	1740876.9193	0.0394
0.095	1499039.7931	0.0612
0.1	1251768.8584	0.0938
0.105	993418.8423	0.1468
0.11	754042.4828	0.1995
0.115	502591.5330	0.2809
0.12	252371.5275	0.3651
0.125	-1004.4900	0.461
0.13	-242388.0170	0.5517
0.135	-497185.3079	0.6502
0.14	-746170.9921	0.7374
0.145	-991378.6177	0.8041
0.15	-1247132.7022	0.8669

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Discussions on the Model

Advices, reservations and extending models.

Advices on factors that the company may be able to control and reservations of the model

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FACTORS THAT THE COMPANY CAN CONTROL

Premium

Try to maintain the premium at **£7250** as this is the minimal premium level to ensure that the probability of loss is no more than 2%.

Probability

The maximum estimated probability of a customer making a claim is **0.08** for the risk of bankruptcy to be **less than 2%.** Forecast the probability of a customer making a claim before accepting customers and accept those who are **at most 8% likely** to make a claim.

FACTORS THAT THE COMPANY CAN CONTROL

Updated Evaluations

Do not wait till the **net balance has reached zero** to calculate the risk of bankruptcy. Always keep updated with the evaluations.

Number of Claims

Find the relationship between number of claims made per year and risk of bankruptcy and limit to the **maximum number of claims with minimal risk.**

Other Advices

Conduct frequent seminars, providing advice to minimize avoidable risks and network with the customers.

Work together and maintain dialogues with customers throughout the year and not limit to the time of bankruptcy.

FACTORS THAT THE COMPANY CAN CONTROL

```
> prob<-as_tibble(prob) %>%mutate(premium=0)
> for (i in 1:length(prob$value)){
+   j=1
+   while (j <=length(premium)) {
+     prob0<-mean(AssetSim(1000,premium[j],prob$value[i])<0)
+     if(prob0<0.02){
+       prob$premium[i]<-premium[j]
+       break}
+     j=j+1
+   }
+ }
```

Finding Optimal Portfolios

PROBABILITY	PREMIUM
0.05	5500
0.055	5500
0.06	5500
0.065	5500
0.07	5500
0.075	5750
0.08	5750
0.085	6250
0.09	6500
0.095	7250
0.1	7250
0.105	7750
0.11	7750

- This outcome is from a given probability, to calculate the **minimal premium needed to avoid bankruptcy**.
- For probability (of making a claim) more than 0.11, the premium will be beyond the upper limit (which is 8000) of given premium intervals, we consider those as **abnormal situations** that will not be accepted. (it means that even when the premium approaches 8000, you can't reduce the probability of bankruptcy to 0.02)

RESERVATIONS OF THE MODEL

- **The probability of a customer making a claim** is practically not fixed. It varies with each customer and is dependent on age, annual income, etc.
- Throughout our analysis**, the values for α and β were fixed**. The risk of bankruptcy may vary with different values of α and β. But our calculation is restricted to $\alpha=3$ and $\beta=1000$.
- Claims made by customers are generally **not independent of each other.**
- In reality, one customer can **make more than one claim per year.**
- There is a **time constraint** in the above analysis, which restricts the company to come to conclusions due to insufficient information.

EXTENDING MODEL

We can extend the model from following aspects:

Consider **10 years' time period**

The number of customers **is not fixed** every year and the premium is not necessarily paid **at the start of a year**. The total number of pays of annual premium has a **Poisson distribution.**

The probability of customers making a claim is **not identical** for everyone. Assume the number of times making a claim also has a Poisson distribution.

EXTENDING MODEL

We have a new model:

$$Z(t) = a + cM(t) - S(t)$$

$Z(t)$: the assets of the company at the end of the year t.

$a = 250000$ represents the current assets of the company.

$c = 6000$ represents the annual premium.

$M(t)$: the number of times for paying for premium until year t. It has a Poisson distribution of parameter $\lambda_1 t$. The pdf of $M(t)$ is:

$$P(M(t) = k) = \frac{(\lambda_1 t)^k}{k!} e^{-\lambda_1 t}, \quad k = 0,1,2 \ldots$$

EXTENDING MODEL

Total claim till the end of year t:

$$S(t) = \sum_{i=1}^{N(t)} X_i$$

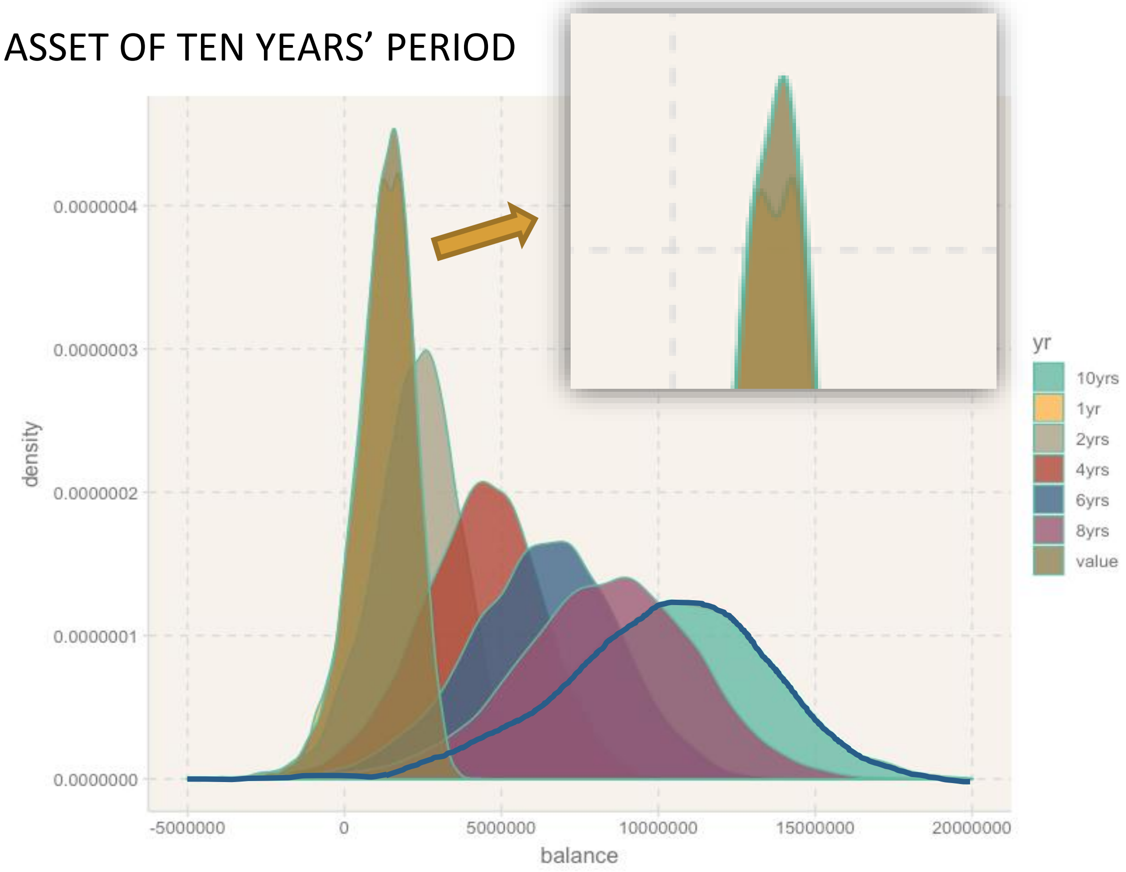
X_i~Pareto distribution i.i.d.

$N(t)$: the number of claims made until year t. It has a Poisson distribution of parameter$\lambda_2 t$. The pdf of $N(t)$ is:

$$P(N(t) = k) = \frac{(\lambda_2 t)^k}{k!} e^{-\lambda_2 t}, \quad k = 0,1,2 \ldots$$

Assume $\lambda_1 = 1000$, $\lambda_2 = 100$, we can simulate the assets of the company in the end of year t. Compare the probability of bankruptcy at the end of each year.

ASSET OF TEN YEARS' PERIOD



- The probability of bankruptcy in ten years: **0.1456**
- The mean balance at the end of each year:

1255702 2254113 3253047 4244971 5253847 6267491 7286082 8271802 9267766 10265101

- Minimal premium level: **7500**
- To reduce bankruptcy down to 0.02, at any year of the 10 years, the minimum premium should be 7500.

Thanks!