830-ph2-report

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PHASE II - Method of Steepest Descent

Objective:

In phase II, we hope to locate the optimal settings of the factors that were identified as significant in the screening phase, that is, to reach the vicinity of optimum. This can make sure our second-order model in phase 3 is capable of modeling concavity/convexity.

There are 3 factors we need to explore. Among them, Preview Type is a categorical factor with 2 levels (TT and AC). We first explored the vicinity optimum of Preview Length (x1) and Match Score (x2) with Preview Type = TT, then we take a similar process with Preview Type = AC.

Experiment

First We begin with a 2^2 factorial experiment with a center point condition using the default setting (see the left table on page 2) :

The linear regression model with linear predictor

$$\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{PQ} x_{PQ}$$

was fit (the pure quadratic effect $\beta_{PQ} = \sum_{j=1}^{K'} \beta_{jj}$). The result shows that the p-value of the test for overall curvature$H_0 : \beta_{PQ} = 0$ is 1.65e-05, compared to other p-values, it is not high enough to reject the null hypothesis.

So we need to begin the method of steepest descent procedure. After fitting the first order regression model with linear predictor $\widehat{\eta} = \widehat{\beta}_0 + \widehat{\beta}_1 x_1 + \widehat{\beta}_2 x_2$, we can obtain the betas, thus the gradients

$$\mathrm{g} = \nabla \widehat{\eta} = \begin{bmatrix} \frac{\partial \hat{\eta}}{\partial x_1} & \frac{\partial \hat{\eta}}{\partial x_2} \end{bmatrix}^T = [0.9595, 0.8679]^T$$

We defined the step size as

$$\lambda = \frac{\Delta x_j}{\left|\widehat{\beta}_j\right|} == \frac{\Delta x_1}{\left|\widehat{\beta}_1\right|}$$

where Δx_j is the step size of Prev.Length (which is 5 in natural unit) in the coded unit, which is 0.5.

Then we begin the steps of steepest descend. To find the minimum of response, we descend the surface by moving in the direction of -g. we can obtain new data using the x1 and x2 values shown in each step.

To save the resources, we generated new data using the x1 and x2 values shown in each step. Then record the mean value of the average browsing time. When we observe an increase of average browsing time, then stop this iteration.

Step 2 (x1=100, x2=81) corresponded to the lowest observed average browsing time. The average browsing time in each condition is visualized in the plot below. So we should perform another test of curvature in this region (x1=(105,95), x2=(86,76)) to determine whether we've reached the vicinity of the optimum. (see the middle table)

So we generated new data using the above setting, and fit another model. The p-value of β_{PQ} is 0.00466, which has surprisingly been higher. *(Note that in the hindsight, this large p-value may be caused by the reason that the region we chose is so small. It is hard to determine the curvature in such a small region. But this does not influence the final output.)*

We restart the whole steepest descend process by putting the center point in x1=100, x2=81, and reorienting toward the optimum. This time $g = [1.2998, 0.7397]$, $\Delta x_1 = 1$.

Similarly, we can observe the lowest average browse time at step 3, which Prev.Length = 85, Match.Score = 72.

Regenerate the data with the below setting, we can see the average browse time of the center point is obviously smaller than most factorial points.

initial

Prev.Length	Match. Score	average browse time
100	80	16.13501
120	80	19.35158
100	100	19.16823
120	100	19.78986
110	90	19.06782

intermediate

Prev.Length	Match. Score	average browse time
95	76	14.20650
105	76	16.64484
95	86	15.52470
105	86	18.28539
100	81	16.49285

final

Prev.Length	Match. Score	average browse time
75	62	12.12609
95	62	13.59431
75	82	10.92799
95	82	14.91981
85	72	11.40038

vicinity of optimum

* final row is the center point

pvalue of xPQ: 1.65e-05 0.00466 -2e-16

Figure 1: setting

P-value of xPQ $= -2e - 16$ shows that we are confident that here is the vicinity of optimum.

For type = "AC", we also use intermediate two-level designs to reorient toward the optimum. First, we move to the region around x1=85, x2=71, but the p-value is quite large as 0.005007. Then we restart the process to move to the region around x1=80, x2=68. Then we finally find the evidence of curvature.

Conclusion

In phase II, we found the vicinity of optimum in both levels of preview type. For preview type is TT, the region is $x_1 \in (75, 95), x_2 \in (62, 82)$. For preview type is AC, the region is $x_1 \in (70, 90), x_2 \in (58, 78)$.

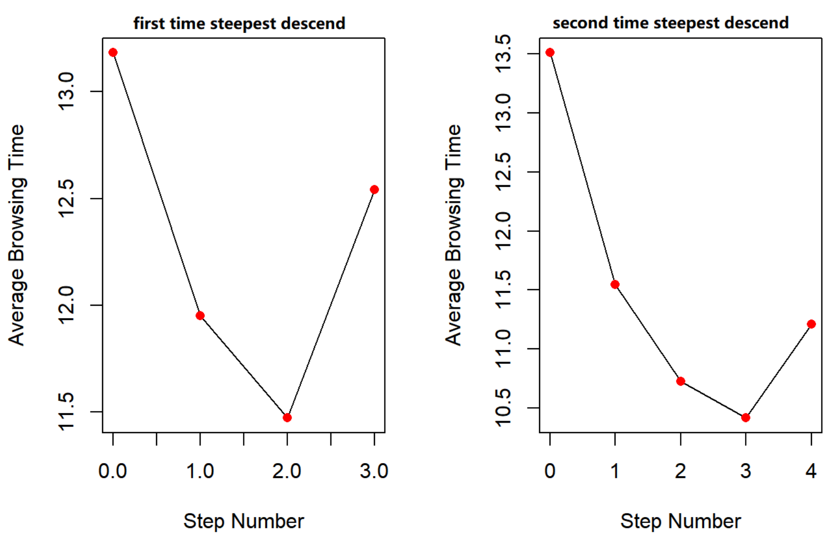

Figure 2: curve