DATA 303 Assignment two : 2021
Due: Monday 18 October 2021.

Please Note: **Briefly discuss** means one or two sentences.

1. Put the following matrix A in row echelon form using Givens rotations via the MATLAB routine `GivensRollup`. Then do it a second time using Householder reflections via the MATLAB routine `House303`. You should need six Givens rotations, and three Householder reflections.

$$A = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 3 & 4 \\ 1 & 4 & 5 & 1 \end{pmatrix}$$

Your answer should include your matlab instructions, and the final row echelon matrices.

Note the two row echelon forms for A for this question have some rows scaled by -1. Their Q matrices are also different. Scaling row j of R and column j of Q by -1 leaves QR unchanged.

2. Using least squares ($\rho = 0$) fit polynomial models $\sum_{j=0}^{n} x_j t^j$ with $n = 2, 4, 6$ to the data (t_i, y_i) where t and y are given by the row vectors $t = \begin{pmatrix} -3 & -2 & -1 & 0 & 1 & 2 & 3 \end{pmatrix}$ and $y = (0\ 0\ 0\ 1\ 0\ 0\ 0)$. Use the `ridgemodel303` code to fit the models and draw the plots. Comment briefly on the behaviour of the fits as the size of the polynomial increases.

3. Using the data in the previous question, fit a sixth order polynomial to the data using least squares ($\rho = 0$) and ridge regression with $\rho = 0.25, 1$, and 4. Centre the data. Briefly comment on the how the model behaves as ρ is increased.

4. This question looks at fitting the function $y = 2^{t+1}$ at $t = -1$, 0, and 1.001 using the polynomial model $F(x, t) = x_1 + x_2 t + x_3 t^3$. Note this model has a cubic term but no quadratic term.

   (a) Write down the design matrix for this problem.

   (b) Use `ridgemodel303` to find the least squares ($\rho = 0$) fit to the data, along with the ridge regression fits with $\rho = 10^s$ for integer values of s from $s = -5$ to $s = 1$. Do not centre the data. Briefly comment on the how the model behaves as ρ is increased. NB: Do the least squares plot separately, or the other curves will be too small to see.

   (c) Repeat part (b) of this question, but this time centre the data. Briefly comment on the effect of centering the data on the model's behaviour as ρ varies.

   (d) Using `ridgexr303` plot $\|x\|_2$, $\|r\|_2$ and x against ρ for ρ in the range $10^{-5} \le \rho \le 10$. Use about 70 different values of ρ, and do not centre the data. The plot of x is a 3D plot, and by holding the left mouse button down with the cursor on it, you should be able to rotate it to get a good view from several angles.

      (i) Briefly discuss the behaviour of $\|x\|_2$ and $\|r\|_2$.

      (ii) There is a 'knee' in the path of x on the x path plot. Briefly discuss the 'knee' in the x path in light of the fact that the design matrix has two nearly parallel columns.

      (iii) The log-log plot of $\|x\|_2$ against ρ also shows three distinct parts. Identify these parts with parts of the graph showing the x path.

      Hint: the crosses on the graph show the locations of each plotted point. They give an indication of how quickly x is changing with changing ρ.

   For questions 2 to 4, you might find it helpful to write your answers on the plots, and point to the parts of the plots you are talking about.