DATA 303 Assignment 1 (2021)

Due: Tuesday 24 August 2021, 12 noon.

1. Find all stationary points of $f(x,y) = x^2 - y^2$ subject to the constraint $g(x,y) = 4 - x^2 - (y-1)^2 \geq 0$.

   Also find all critical points of f subject to $g \geq 0$, where a critical point is a point satisfying $g = 0$ and $\nabla f = \lambda \nabla g$ for some value of λ.

2. A timber company's client has ordered a_1 lengths of 1.8 metre long timber, a_2 lengths of 2.4 metre long timber, and a_3, lengths of 3 metre long timber. The uncut timber comes in 6 metre lengths, and the timber company wishes to cut as few 6 metre lengths as possible to fill the client's order.

   (a) List all the ways (or cutting patterns) a 6 metre length of timber can be cut into 1.8, 2.4, and 3 metre lengths, such that the remaining part of the 6 metre length is less than 1.8 metres long. (Cutting patterns which leave lengths of 1.8 metres or more are clearly not optimal, and so are not considered). Several different lengths may appear in a cutting pattern.

   (b) Formulate an LP to find the smallest number of 6 metre lengths which must be used to fill the clients order. Each cutting pattern should have a decision variable which gives the number of 6 metre lengths cut according to that pattern.

   Remark: *In practice the solution to the LP might contain some non-integer variables. For this type of problem, a very good, but not necessarily optimal, solution can be found by rounding each such non-integer variable up to the nearest integer. For other types of problems, rounding off non-integer answers is not advised, and might not produce a useful answer.*

3. Apply one iteration of the Newton-Raphson method to the system of equations

   $$f(x,y) = x^2 - x - y = 0 \qquad \text{and} \qquad g(x,y) = -x + y^2 = 0$$

   Start from the initial point $(x,y) = (1,0)$.

4. The goal of this question is to fit a parabola to 11 data points. The data points are of the form (t_i, y_i) where

   $$t_i = \frac{i\,\pi}{10} \qquad \text{and} \qquad y_i = \sin(t_i) \qquad i = 0, \ldots, 10$$

   and the quadratic model has the form

   $$y = F(t,x) = x_1 + x_2 t + x_3 t^2$$

   with x_1, x_2, and x_3 to be determined by the fitting process.

(a) Using `fitmydata303` plot the 1, 2, and max norm fits of a quadratic model to the data points (t_i, y_i), $i = 0.\ldots, 10$.

Use `fitmydata303(A,t,y,TypeOfFit)` to fit the data. Here `A` is the design matrix, and `t` and `y` are the vectors of t_i and y_i values. `TypeOfFit` is one of `'one'` `'two'` `'inf'` `'max'` `'all'` depending on the norm used for the fit. `fitmydata303` returns the x and the vector of residuals for each type of fit. If `'all'` is used it returns the infinity norm x and residuals only.

NB: `max` and `inf` do the same thing. Make sure `t` and `y` are entered in the correct positions.

`fitmydata303` calls `simplex303`. Both of these codes must be in the same directory so that `fitmydata303` can find and use `simplex303`.

(b) Change the middle data point value from $y_5 = 1$ to $y_5 = 0$ to create an outlier. (NB This middle data point will be the sixth data point in your `y` vector, and have the t value $t = \pi/2$.) Redo the plot of the three fits.

(c) Comment on the different responses of the three types of fit to the altered point.

For parts (a) and (b), just print off the design matrix A, the t and the y you used, the instruction you used to generate the fits, and the two plots that matlab generates.