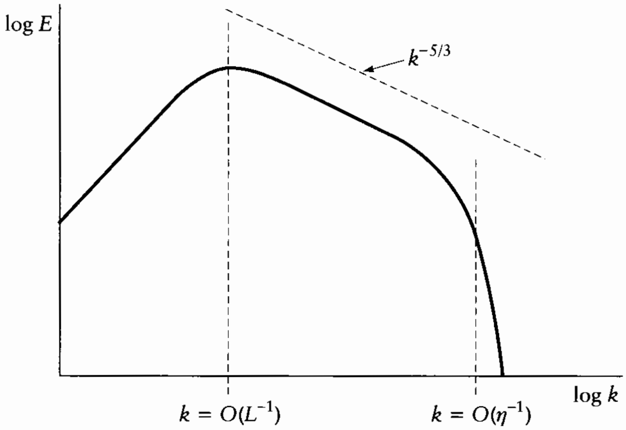

Figure 6.1. Sketch of the energy spectrum of turbulence in traditional log–log format, which brings out power laws as straight lines. The figure is drawn for a sufficiently high Reynolds number that there is a $k^{-5/3}$ inertial range, a feature we will address later in this chapter and in more detail in the next.

power laws, often found in turbulent spectra, appear as straight lines. Notice that the relationship, $\ell \leftrightarrow k^{-1}$, between spatial length scale and wavenumber is reciprocal: small wavenumbers correspond to large spatial scales and vice versa.

Although the identification of wavenumber k with spatial scales of order k^{-1} can be very useful in qualitative interpretation of spectral properties, both theoretically and experimentally, it should not be pushed too far. The relationship $\ell = O(k^{-1})$ represents an order of magnitude and one cannot, for instance, meaningfully distinguish between length scales k^{-1} and, say, $2\pi/k$ (which is the wavelength of the Fourier component $e^{i\mathbf{k}.\mathbf{x}}$). The Fourier transform of some spatially localized function of spatial width $O(\ell)$ extends over all k, even if its largest values occur at $k = O(\ell^{-1})$. In the same way, turbulent spatial scales of $O(\ell)$ will mostly contribute to the spectrum for k of $O(\ell^{-1})$, but will have effects at all k. In short, the correspondence $\ell \leftrightarrow k^{-1}$ is often used, but should be treated as an order-of-magnitude, interpretative relationship.

In a similar vein, it should be borne in mind that spectra do not, in general, contain full statistical information about turbulence. This is perhaps obvious, after all second-order moments like R_{ij} are usually insufficient to fully define a random process such as $u_i(\mathbf{x}, t)$. However, it is easy to forget this fact when faced with a spectrum. It will be recalled from Chapter 2 that the full statistics of Gaussian variables are determined by their mean (which is zero here) and second-order moments, but the turbulent fluctuating velocities are not usually Gaussian, although they can be approximately so.

In Chapter 3, we observed that turbulence contains a wide range of spatial scales. The largest scales are of size $O(L)$, where L is a correlation scale and is determined by the physical processes that created the turbulence and does not depend on the fluid viscosity. The dynamics of the large scales are essentially independent of viscosity at high Reynolds numbers. These large scales are intrinsically unstable and tend to progressively give up their energy to smaller scales by a process which is inviscid and depends on the nonlinear convective terms of the Navier–Stokes equation (see Chapter 3 and Section 4.4). Eventually, as the spatial scale decreases, a point is reached at which the increasing importance of viscosity stops the cascade of energy to smaller scales. Viscous energy dissipation acts to destroy the kinetic energy that the small structures inherited from their larger parents. This intervention of viscosity takes place around the Kolmogorov scale, η, which therefore gives the order of size of the smallest scales of turbulence. Thus, we have spatial scales ranging all the way from $O(\eta)$ up to $O(L)$. Since η decreases with the viscosity, the ratio, L/η, grows