

Chapter 2

Governing Equations

The ARW dynamics solver integrates the compressible, nonhydrostatic Euler equations. The equations are cast in flux form using variables that have conservation properties, following the philosophy of [Ooyama \(1990\)](#). The equations are formulated using a terrain-following mass vertical coordinate ([Laprise, 1992](#)). In this chapter we define the vertical coordinate and present the flux form equations in Cartesian space, we extend the equations to include the effects of moisture in the atmosphere, and we further augment the equations to include projections to the sphere.

2.1 Vertical Coordinate and Variables

The ARW equations are formulated using a terrain-following hydrostatic-pressure vertical coordinate denoted by η and defined as

$$\eta = (p_h - p_{ht})/\mu \quad \text{where} \quad \mu = p_{hs} - p_{ht}. \quad (2.1)$$

p_h is the hydrostatic component of the pressure, and p_{hs} and p_{ht} refer to values along the surface and top boundaries, respectively. The coordinate definition (2.1), proposed by [Laprise \(1992\)](#), is the traditional σ coordinate used in many hydrostatic atmospheric models. η varies from a value of 1 at the surface to 0 at the upper boundary of the model domain (Fig. 2.1). This vertical coordinate is also called a mass vertical coordinate.

Since $\mu(x, y)$ represents the mass per unit area within the column in the model domain at (x, y) , the appropriate flux form variables are

$$\mathbf{V} = \mu \mathbf{v} = (U, V, W), \quad \Omega = \mu \dot{\eta}, \quad \Theta = \mu \theta. \quad (2.2)$$

$\mathbf{v} = (u, v, w)$ are the covariant velocities in the two horizontal and vertical directions, respectively, while $\omega = \dot{\eta}$ is the contravariant ‘vertical’

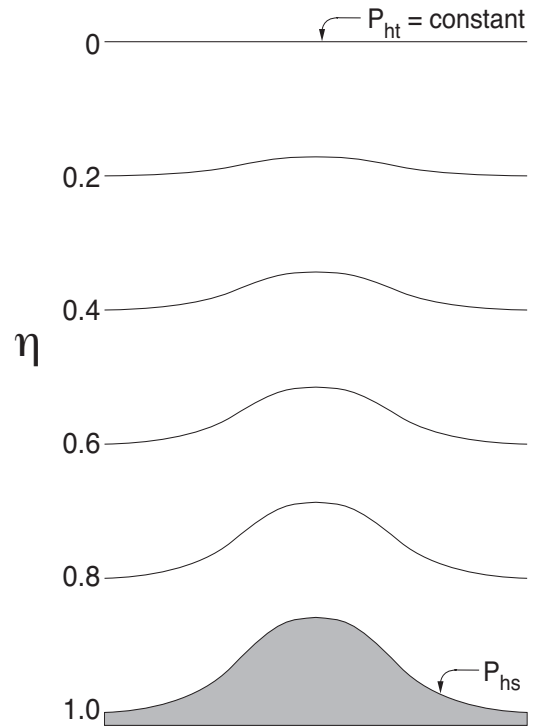


Figure 2.1: ARW η coordinate.