Exercise 5

Given: x = amount Pagerank supplied from outside to t

n = # pages in Web

$y = r_t$ = Pagerank of page t

k = # second-tier nodes

m = # supporting pages

Pagerank of 2^{nd} tier page:

$$r_{2nd tier} = \frac{\beta \cdot y}{k} + \frac{1-\beta}{n}$$

Pagerank of supporting page:

$$r_{supp} = \frac{\beta \cdot r_{2nd tier}}{\frac{m}{k}} + \frac{1-\beta}{n}$$

$$= \beta \cdot r_{2nd tier} \cdot \frac{k}{m} + \frac{1-\beta}{n}$$

$$= \frac{\beta k \cdot r_{2nd tier}}{m} + \frac{1-\beta}{n}$$

$$= \frac{\beta k \cdot \left[\frac{\beta y}{k} + \frac{1-\beta}{n}\right]}{m} + \frac{1-\beta}{n}$$

$$= \frac{\frac{\beta^2 \cancel{k} y}{\cancel{k}} + \frac{\beta k (1-\beta)}{n}}{m} + \frac{(1-\beta)}{n}$$

$$= \frac{1}{m}\left[\beta^2 y + \frac{\beta k - \beta^2 k}{n}\right] + \frac{1-\beta}{n}$$

Pagerank of page t:

$$y = x + \beta m \cdot r_{supp} + \cancel{\frac{1-\beta}{n}}$$

$$= x + \beta m \cdot \left[\frac{1}{m}\left[\beta^2 y + \frac{\beta k - \beta^2 k}{n}\right] + \frac{1-\beta}{n}\right]$$

$$= x + \frac{\beta \cancel{m}}{\cancel{m}}\left[\beta^2 y + \frac{\beta k - \beta^2 k}{n}\right] + \frac{1-\beta(\beta m)}{n}$$

$$y = x + \beta^3 y + \frac{\beta^2 k - \beta^3 k}{n} + \frac{\beta m - \beta^2 m}{n}$$

$$y - \beta^3 y = x + \frac{\beta^2 k - \beta^3 k}{n} + \frac{\beta m - \beta^2 m}{n}$$

$$y(1-\beta^3) = x + \frac{k(\beta^2 - \beta^3)}{n} + \frac{m(\beta - \beta^2)}{n}$$

$$y = \frac{1}{1-\beta^3} \cdot x + \frac{k}{n} \cdot \frac{\beta^2 - \beta^3}{1-\beta^3} + \frac{m}{n} \cdot \frac{\beta - \beta^2}{1-\beta^3}$$

$$y = \underbrace{\frac{1}{1-\beta^3}}_{a} x + \underbrace{\frac{\beta-\beta^2}{1-\beta^3}}_{b} \cdot \frac{m}{h} + \underbrace{\frac{\beta^2-\beta^3}{1-\beta^3}}_{c} \cdot \frac{k}{n}$$

Given $\beta = 0{,}85$:

$$a = \frac{1}{1-\beta^3} = \frac{1}{1-0{,}85^3} = \underline{\underline{2{,}592}}$$

$$b = \frac{\beta-\beta^2}{1-\beta^3} = \frac{0{,}85-0{,}85^2}{1-0{,}85^3} = \underline{\underline{0{,}330}}$$

$$c = \frac{\beta^2-\beta^3}{1-\beta^3} = \frac{0.85^2-0{,}85^3}{1-0.85^3} = \underline{\underline{0{,}281}}$$