Exercise 1

Prove that $\sum_{k=0}^{N} \binom{N}{k} = 2^N$ for $N \geq 1$

- $N = 1 \Rightarrow \binom{1}{0} + \binom{1}{1} = 1 + 1 = 2 = 2^1 = 2^N$

- $N = 7 \Rightarrow \binom{7}{0} + \binom{7}{1} + \binom{7}{2} + \binom{7}{3} + \binom{7}{4} + \binom{7}{5} + \binom{7}{6} + \binom{7}{7}$

$= 1 + 7 + 21 + 35 + 35 + 21 + 7 + 1 = 128 = 2^7 = 2^N$

- You can prove it with the binomial theorem of natural exponents.

$$\boxed{\text{Binomial theorem:} \quad (a+b)^n = \sum_{k=0}^{n} \binom{n}{k} \cdot a^{n-k} \cdot b^k}$$

$$2^N = (1+1)^N = \sum_{k=0}^{n} \binom{N}{k} \cdot 1^{n-k} \cdot 1^k = \sum_{k=0}^{n} \binom{N}{k}$$

- You can prove it with Pascal's triangle:

The values of the binomial coefficient can be read from Pascal's triangle, where the value of N^k is in the row n and column k.

Therefore the binomial coefficient can be intepreted as a pascal triangle.

If the values of a row are added in the Pascal triangle, then it can be seen that the sum of the values is exactly twice the sum of the values of the row above.

$\Rightarrow$ In general it can be written that the summed values of the row n of the Pascal triangle result in the value 2^N