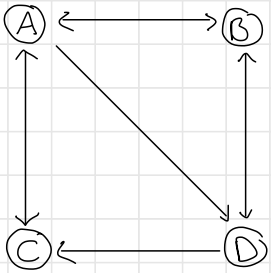
Problem Set 5

Exercise 4



$\beta = 0{,}8 \quad \Rightarrow 1-\beta = 0{,}2 = \frac{1}{5}$

$v' = \beta M v + \frac{(1-\beta)\, e_S}{|S|}$ with $|S|$ = size of set S

$$M = \begin{bmatrix} 0 & \frac{1}{2} & 1 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

a) Teleport set is A only

$S = \{A\}$ => size of $S = 1$

e_S has 1 in the components for A and 0 in the components for B, C, D

$$v' = 0{,}8 \cdot \begin{bmatrix} 0 & \frac{1}{2} & 1 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix} v + \begin{bmatrix} \frac{\frac{1}{5} \cdot 1}{1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$v' = \begin{bmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{bmatrix} v + \begin{bmatrix} \frac{1}{5} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Here are the first few iterations of this equation

- 0. iteration $= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ => surfers only on the pages in the teleport set

- 1. iteration => $v' = \begin{bmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{5} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} \\ \frac{4}{15} \\ \frac{4}{15} \\ \frac{4}{15} \end{bmatrix}$

$v' = \beta M \cdot v + \frac{(1-\beta)\, e_S}{|S|}$

- 2. iteration => $v' = \begin{bmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{3} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{5} \\ \frac{4}{15} \\ \frac{4}{15} \\ \frac{4}{15} \end{bmatrix} + \begin{bmatrix} \frac{1}{5} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{13}{25} \\ \frac{4}{25} \\ \frac{4}{25} \\ \frac{4}{25} \end{bmatrix}$

• 3. iteration $\Rightarrow$ $v' = \begin{bmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{3} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{13}{25} \\ \frac{4}{25} \\ \frac{4}{25} \\ \frac{4}{25} \end{bmatrix} + \begin{bmatrix} \frac{1}{5} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{49}{125} \\ \frac{76}{375} \\ \frac{76}{375} \\ \frac{76}{375} \end{bmatrix}$

b) Teleport set is A and C

$S = \{A, C\}$ => size of $S = 2$

e_S has 1 in the components for A, C and 0 in the components for B, D

$$v' = 0{,}8 \cdot \begin{bmatrix} 0 & \frac{1}{2} & 1 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix} v + \begin{bmatrix} \frac{\frac{1}{2} \cdot 1}{2} \\ 0 \\ \frac{\frac{1}{2} \cdot 1}{2} \\ 0 \end{bmatrix}$$

$$v' = \begin{bmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{bmatrix} v + \begin{bmatrix} \frac{1}{10} \\ 0 \\ \frac{1}{10} \\ 0 \end{bmatrix}$$

Here are the first few iterations of this equation

- 0. Iteration = $\begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \\ 0 \end{bmatrix}$ => surfers only on the pages in the teleport set

- 1. Iteration => $v' = \begin{bmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{10} \\ 0 \\ \frac{1}{10} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{2}{15} \\ \frac{1}{2} \\ \frac{2}{15} \end{bmatrix}$

$v' = \beta M \cdot v + \frac{(1-\beta) e_S}{|S|}$

- 2. Iteration => $v' = \begin{bmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} \\ \frac{2}{15} \\ \frac{1}{2} \\ \frac{2}{15} \end{bmatrix} + \begin{bmatrix} \frac{1}{10} \\ 0 \\ \frac{1}{10} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{83}{150} \\ \frac{28}{150} \\ \frac{43}{150} \\ \frac{28}{150} \end{bmatrix}$

- 3. Iteration $\Rightarrow$ $v' = \begin{bmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{3} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{83}{150} \\ \frac{28}{150} \\ \frac{43}{150} \\ \frac{28}{150} \end{bmatrix} + \begin{bmatrix} \frac{1}{10} \\ 0 \\ \frac{1}{10} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{101}{250} \\ \frac{2}{9} \\ \frac{29}{90} \\ \frac{2}{9} \end{bmatrix}$

- 4. Iteration $\Rightarrow$ $v' = \begin{bmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{3} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{101}{250} \\ \frac{2}{9} \\ \frac{29}{90} \\ \frac{2}{9} \end{bmatrix} + \begin{bmatrix} \frac{1}{10} \\ 0 \\ \frac{1}{10} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{5025}{11250} \\ \frac{2212}{11250} \\ \frac{3337}{11250} \\ \frac{2212}{11250} \end{bmatrix}$

- 5. Iteration $\Rightarrow$ $v' = \begin{bmatrix} 0 & \frac{2}{5} & \frac{4}{5} & 0 \\ \frac{4}{15} & 0 & 0 & \frac{2}{5} \\ \frac{4}{15} & 0 & 0 & \frac{2}{3} \\ \frac{4}{15} & \frac{2}{5} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{5025}{11250} \\ \frac{2212}{11250} \\ \frac{3337}{11250} \\ \frac{2212}{11250} \end{bmatrix} + \begin{bmatrix} \frac{1}{10} \\ 0 \\ \frac{1}{10} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{7799}{18750} \\ \frac{3708}{18750} \\ \frac{5583}{18750} \\ \frac{3708}{18750} \end{bmatrix}$