Exercise 1

a)

$$\mathbf{M} = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{pmatrix} \frac{1}{3} & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \end{array}$$

Power Iteration:

$$r^{(0)} = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$$

$$r^{(1)} = \begin{pmatrix} 5/18 \\ 2/9 \\ 4/9 \end{pmatrix}, \epsilon = \begin{pmatrix} 1/18 \\ 1/9 \\ 1/9 \end{pmatrix} \nless \frac{1}{12}$$

$$r^{(2)} = \begin{pmatrix} 11/54 \\ 13/54 \\ 23/54 \end{pmatrix}, \epsilon = \begin{pmatrix} 2/27 \\ 1/54 \\ 1/54 \end{pmatrix} < \frac{1}{12}$$

b)

$$formula: Ax = \lambda x \equiv Mr = \lambda r$$

assuming eigenvalue $\lambda = 1$:

$$formula: Mr = r$$

Compute eigenvector r of matrix M:

$$|M - \lambda I| = 0 \Leftrightarrow$$

$$|\begin{bmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}| = 0$$

$$|\begin{bmatrix} 1/3 - \lambda & 1/2 & 0 \\ 1/3 & -\lambda & 1/2 \\ 1/3 & 1/2 & 1/2 - \lambda \end{bmatrix}| = 0 \Leftrightarrow$$

$$\frac{1-\lambda}{3} * det \begin{bmatrix} -\lambda & 1/2 \\ 1/2 & 1/2 - \lambda \end{bmatrix} - \frac{1}{2} * det \begin{bmatrix} 1/3 & 1/2 \\ 1/3 & 1/2 - \lambda \end{bmatrix} = 0$$

$$\frac{1-\lambda}{3} * ((-\lambda * 1/2 - \lambda) - (1/2 * 1/2)) - \frac{1}{2} * (1/3 * 1/2 - \lambda) - (1/3 * 1/2) = 0$$

$$\frac{1-\lambda}{3} * ((-\lambda * 1/2 - \lambda) - (1/4)) - \frac{1}{2} * ((1/3 * 1/2 - \lambda) - (1/6)) = 0$$

$$(\lambda^2 - \frac{1}{2}\lambda + \frac{1}{18}) + \frac{1}{6}\lambda = 0$$

$$\lambda^2 - \frac{1}{3}\lambda + \frac{1}{18} = 0$$

No solution in reel set.

c)

$$A = \beta M + (1-\beta)\left[1/N\right] \Leftrightarrow$$

$$A = 0.8M + 0.2 \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \Leftrightarrow$$

$$A = \begin{bmatrix} 4/15 & 6/15 & 0 \\ 4/15 & 0 & 6/15 \\ 4/15 & 6/15 & 6/15 \end{bmatrix} + \begin{bmatrix} 1/15 & 1/15 & 1/15 \\ 1/15 & 1/15 & 1/15 \\ 1/15 & 1/15 & 1/15 \end{bmatrix} \Leftrightarrow$$

$$A = \begin{bmatrix} 5/15 & 7/15 & 1/15 \\ 5/15 & 1/15 & 7/15 \\ 5/15 & 7/15 & 7/15 \end{bmatrix}$$

Initilization:

$$r = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Iteration 1:

Please note: in_a means node a links to the node that we currently pagerank

$$r_a = in_b + in_a = (0.8\frac{1}{2} + 0.2\frac{1}{3}) + (0.8\frac{1}{3} + 0.2\frac{1}{3}) = 0.8$$

$$r_b = in_a + in_c = (0.8\frac{1}{3} + 0.2\frac{1}{3}) + (0.8\frac{1}{2} + 0.2\frac{1}{3}) = 0.8$$

$$r_c = in_a + in_b + in_c = (0.8\frac{1}{3} + 0.2\frac{1}{3}) + (0.8\frac{1}{2} + 0.2\frac{1}{3}) + (0.8\frac{1}{2} + 0.2\frac{1}{3}) = 1.266$$

$$r = Ar \Leftrightarrow r = \begin{bmatrix} 0.8 \\ 0.8 \\ 1.266 \end{bmatrix}, \epsilon = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.266 \end{bmatrix} \not< \frac{1}{12}$$

Iteration 2:

$$r_a = in_b + in_a = (0.8\frac{0.8}{2} + 0.2\frac{1}{3}) + (0.8\frac{0.8}{3} + 0.2\frac{1}{3}) = 2/3$$

$$r_b = in_a + in_c = (0.8\frac{0.8}{3} + 0.2\frac{1}{3}) + (0.8\frac{1.266}{2} + 0.2\frac{1}{3}) = 0.853$$

$$r_c = in_a + in_b + in_c = (0.8\frac{0.8}{3} + 0.2\frac{1}{3}) + (0.8\frac{0.8}{2} + 0.2\frac{1}{3}) + (0.8\frac{1.266}{2} + 0.2\frac{1}{3}) = 1.24$$

$$r = Ar \Leftrightarrow r = \begin{bmatrix} 2/3 \\ 0.853 \\ 1.24 \end{bmatrix}, \epsilon = \begin{bmatrix} 0.13 \\ 0.053 \\ 0.026 \end{bmatrix} \not< \frac{1}{12}$$

Iteration 3:

$$r_a = in_b + in_a = (0.8\frac{0.853}{2} + 0.2\frac{1}{3}) + (0.8\frac{2/3}{3} + 0.2\frac{1}{3}) = 0.652$$

$$r_b = in_a + in_c = (0.8\frac{2/3}{3} + 0.2\frac{1}{3}) + (0.8\frac{1.24}{2} + 0.2\frac{1}{3}) = 0.807$$

$$r_c = in_a + in_b + in_c = (0.8\frac{2/3}{3} + 0.2\frac{1}{3}) + (0.8\frac{0.853}{2} + 0.2\frac{1}{3}) + (0.8\frac{1.24}{2} + 0.2\frac{1}{3}) = 1.215$$

$$r = Ar \Leftrightarrow r = \begin{bmatrix} 0.652 \\ 0.807 \\ 1.215 \end{bmatrix}, \epsilon = \begin{bmatrix} 0.014 \\ 0.046 \\ 0.024 \end{bmatrix} < \frac{1}{12}$$

d)

$$A = \begin{bmatrix} 5/15 & 7/15 & 1/15 \\ 5/15 & 1/15 & 7/15 \\ 5/15 & 7/15 & 7/15 \end{bmatrix}$$

$$formula: Ax = \lambda x \equiv Ar = \lambda r$$

assuming eigenvalue $\lambda = 1$:

$$formula: Ar = r$$

Compute eigenvector r of matrix A:

$$|A - \lambda I| = 0 \Leftrightarrow$$

$$|\begin{bmatrix} 5/15 & 7/15 & 1/15 \\ 5/15 & 1/15 & 7/15 \\ 5/15 & 7/15 & 7/15 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}| = 0$$

$$|\begin{bmatrix} 5/15 - \lambda & 7/15 & 1/15 \\ 5/15 & 1/15 - \lambda & 7/15 \\ 5/15 & 7/15 & 7/15 - \lambda \end{bmatrix}| = 0$$

$$\frac{5}{15} - \lambda * det \begin{bmatrix} 1/15 - \lambda & 7/15 \\ 7/15 & 7/15 - \lambda \end{bmatrix} - \frac{7}{15} * det \begin{bmatrix} 5/15 & 7/15 \\ 5/15 & 7/15 - \lambda \end{bmatrix} + \frac{1}{15} * det \begin{bmatrix} 5/15 & 1/15 - \lambda \\ 5/15 & 7/15 \end{bmatrix} = 0$$

$$\frac{5}{15} - \lambda * ((1/15 - \lambda * 7/15 - \lambda) - 7/15 * 7/15) - \frac{7}{15} * (5/15 * 7/15 - \lambda - 7/15 * 5/15) \\ + \frac{1}{15} * (5/15 * 7/15 - 1/15 - \lambda * 5/15) = 0 \quad (1)$$

$$\frac{5}{15} - \lambda * ((\lambda^2 - \frac{8}{15}\lambda + \frac{7}{225}) - \frac{49}{225}) - (\frac{35}{225}\lambda) +$$

$$\lambda^3 - \frac{13}{15}\lambda^2 - \frac{2}{225}\lambda + \frac{210}{3375}$$