

Problem set 3:

Exercise 1)

a) User - User CF

$$\begin{aligned} \bar{r}_A &= (4+5+5+1+3+2)/6 = \frac{10}{3} \\ \bar{r}_B &= (3+4+3+1+2+1)/6 = \frac{7}{3} \\ \bar{r}_C &= (2+1+3+4+5+3)/6 = 3 \\ \bar{r}_D &= (1+1+4+4+2)/5 = \frac{12}{5} \end{aligned}$$

$\bar{r}_{AS} - \bar{r}_A$
 Row A: $\left[\frac{2}{3}, \frac{5}{3}, 0, \frac{5}{3}, -\frac{1}{3}, 0, -\frac{1}{3}, -\frac{4}{3} \right]$
 Row B: $\left[0, \frac{2}{3}, \frac{5}{3}, \frac{2}{3}, -\frac{4}{3}, -\frac{1}{3}, -\frac{4}{3}, 0 \right]$
 Row C: $\left[-1, 0, -2, 0, 0, 1, 2, 0 \right]$
 Row D: $\left[-\frac{7}{5}, 0, \frac{2}{5}, 0, \frac{8}{5}, 0, \frac{8}{5}, -\frac{2}{5} \right]$

From CF Similarity:

$$\text{sim}(c, A) = \frac{\left(\frac{2}{3} \cdot (-1) \right) + \left(\left(-\frac{1}{3} \right) \cdot 2 \right)}{\sqrt{1^2 + 2^2 + 1^2 + 2^2} \cdot \sqrt{\left(\frac{2}{3} \right)^2 + \left(\frac{5}{3} \right)^2 + \left(\frac{5}{3} \right)^2 + \left(\frac{7}{3} \right)^2 + \left(\frac{1}{3} \right)^2 + \left(\frac{4}{3} \right)^2}} = \frac{-\frac{4}{3}}{\frac{20\sqrt{3}}{3}} = -\frac{\sqrt{3}}{15} = -0,11547$$

$$\text{sim}(c, A) = -\frac{\sqrt{3}}{15} = -0,11547$$

$$\text{sim}(c, B) = \frac{\left(\frac{5}{3} \cdot (-2) \right) + \left(\left(-\frac{1}{3} \right) \cdot 1 \right) + \left(\left(-\frac{4}{3} \right) \cdot 2 \right)}{\sqrt{10} \cdot \sqrt{\left(\frac{2}{3} \right)^2 + \left(\frac{5}{3} \right)^2 + \left(\frac{2}{3} \right)^2 + \left(\frac{4}{3} \right)^2 + \left(\frac{1}{3} \right)^2 + \left(\frac{5}{3} \right)^2}} = \frac{-\frac{19}{3}}{\frac{2\sqrt{165}}{3}} = -0,73957$$

$$\text{sim}(c, B) = -0,73957$$

$$\text{sim}(c, D) = \frac{\left(-1 \cdot \left(-\frac{7}{5} \right) \right) + \left(\left(-2 \right) \cdot \left(-\frac{7}{5} \right) \right) + \left(2 \cdot \frac{8}{5} \right)}{\sqrt{10} \cdot \sqrt{\left(\frac{2}{3} \right)^2 + \left(\frac{5}{3} \right)^2 + \left(\frac{2}{3} \right)^2 + \left(\frac{4}{3} \right)^2 + \left(\frac{1}{3} \right)^2 + \left(\frac{5}{3} \right)^2}} = \frac{\frac{37}{5}}{\sqrt{230}} = 0,77150$$

$$\text{sim}(c, D) = 0,77150$$

=> highest 2 neighbours: D, A

$$\begin{aligned} r_{ce} &= \bar{r}_c + \frac{\sum_{y \in N(c)} \text{sim}(c, y) \cdot (\bar{r}_{ye} - \bar{r}_c)}{\sum_{y \in N(c)} |\text{sim}(c, y)|} \\ &= 3 + \frac{0,77150 \cdot (4-3) + (-0,11547) \cdot (1-3)}{0,77150 + |(-0,11547)|} = \frac{1,00244}{0,88697} + 3 = 3 + 1,13018 \\ &= 4,13018 \end{aligned}$$

r_{ce} is 4,13018

b) Item-Item CF

$$\begin{aligned}
 m_a &= (4+2+1)/3 = \frac{7}{3} & \text{column}_a &= \left[\frac{5}{3}, 0, -\frac{1}{3}, -\frac{4}{3} \right] \\
 m_b &= (5+3)/2 = 4 & \text{column}_b &= [1, -1, 0, 0] \\
 m_c &= (4+1+1)/3 = 2 & \text{column}_c &= [0, 2, -1, -1] \\
 m_d &= (5+3+3)/3 = \frac{11}{3} & \text{column}_d &= \left[\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}, 0 \right] \\
 m_e &= (1+1+4)/3 = 2 & \text{column}_e &= [-1, -1, 0, 2] \\
 m_f &= (2+4)/2 = 3 & \text{column}_f &= [0, -1, 1, 0] \\
 m_g &= (3+1+5+4)/4 = \frac{13}{4} & \text{column}_g &= \left[-\frac{1}{4}, -\frac{3}{4}, \frac{7}{4}, \frac{3}{4} \right] \\
 m_h &= (2+3+2)/3 = \frac{7}{3} & \text{column}_h &= \left[-\frac{1}{3}, 0, \frac{2}{3}, -\frac{1}{3} \right]
 \end{aligned}$$

Similaritys

$$\text{sim}(e, a) = \frac{(\frac{5}{3} \cdot (-1)) + ((-\frac{4}{3}) \cdot 2)}{\sqrt{1^2 + 1^2 + 2^2} \cdot \sqrt{\left(\frac{5}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{4}{3}\right)^2}} = \frac{-\frac{13}{3}}{2\sqrt{7}} = -\frac{13\sqrt{7}}{42} = -0,81892$$

$$\text{sim}(e, b) = \frac{(1 \cdot (-1)) + ((-1) \cdot (-1))}{\sqrt{6} \cdot \dots} = \frac{0}{\dots} = 0$$

$$\text{sim}(e, c) = \frac{(2 \cdot (-1)) + ((-1) \cdot 2)}{\sqrt{6} \cdot \sqrt{2^2 + 1^2 + 1^2}} = \frac{-4}{6} = -\frac{2}{3} = -0,6$$

$$\text{sim}(e, d) = \frac{\left(\frac{4}{3} \cdot (-1) \right) + \left(-\frac{2}{3} \cdot (-1) \right)}{\sqrt{6} \cdot \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2}} = \frac{-\frac{2}{3}}{\frac{2\sqrt{6}}{3}} = -\frac{1}{6} = -0,16$$

$$\text{sim}(e, f) = \frac{(-1) \cdot (-1)}{\sqrt{6} \cdot \sqrt{1^2 + 1^2}} = \frac{1}{2\sqrt{3}} = 0,28868$$

$$\text{sim}(e, g) = \frac{(-1) \cdot (-\frac{1}{4}) + (-1) \cdot (-\frac{3}{4}) + (2 \cdot \frac{3}{4})}{\sqrt{6} \cdot \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^2}} = \frac{4}{2\sqrt{10}} = 0,55205$$

$$\text{sim}(e, h) = \frac{\left(-1 \cdot -\frac{1}{3} \right) + \left(2 \cdot \frac{1}{3} \right)^2}{\sqrt{6} \cdot \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2}} = -\frac{1}{2} = -0,5$$

→ highest 2 neighbours: g, f

$$\begin{aligned}r_{ce} &= \frac{\sum_{j \in N(e, c)} \text{sim}(e, j) \cdot r_{cj}}{\sum_{j \in N(e, c)} \text{sim}(e, j)} \\&= \frac{(0,28868 \cdot 4) + (0,55205 \cdot 5)}{0,28868 + 0,55205} = 4,65663\end{aligned}$$