The price elasticity and the monopolistic behavior

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In this brief note, we introduce briefly the monopolistic behavior based on the market reaction through the demand function. For simplicity, we assume the case of a constant cost per produced unit. We start with the classical case of the linear demand function where the monopole must produce a quantity within the elastic part of the demand curve. After that, we show some other cases where the monopole can produce even within the inelastic parts of the demand curve.

Case 1: Single monopole & the linear demand curve

Let the classical linear demand curve*:*

- $p(q) = a - \delta q.$
- $q(p) = \frac{a}{\delta} - \frac{1}{\delta}p$

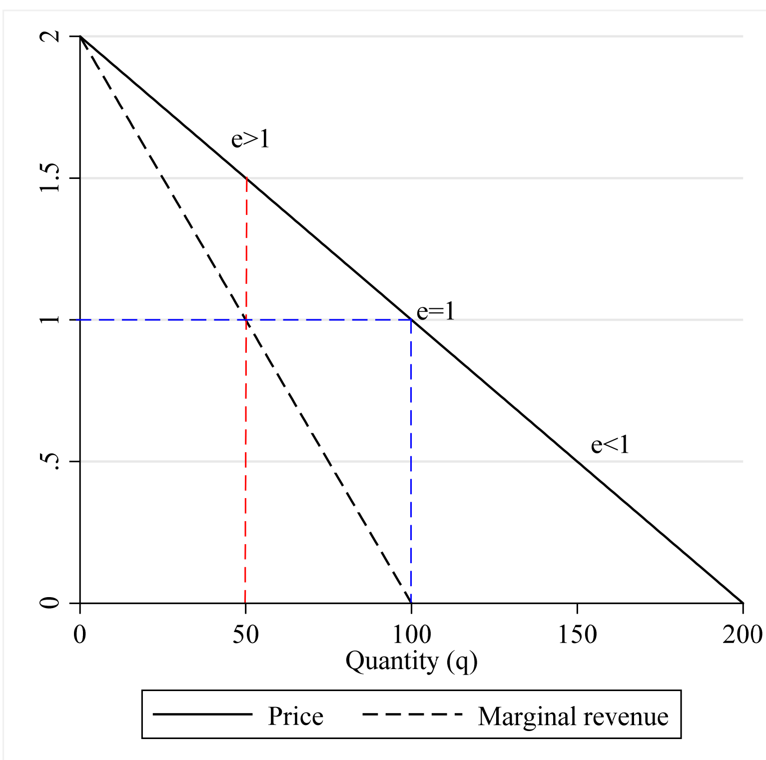
The price-elasticity is equal to $\varepsilon = \frac{\partial q}{\partial p}\frac{p(q)}{q} = -\frac{p}{\delta q}$.

The marginal revenue is: $MR(q) = a - 2\delta q = p - \delta q = p\left(1 + \frac{1}{\varepsilon}\right)$.

Then, $MR(q) > 0 \Rightarrow \varepsilon < -1$. This is the classical conclusion, and where the supplied quantity by the monopole must be situated within the elastic part of the demand curve.

The following graph is an example with: $p(q) = 2 - 0.01q$.

Figure 1: Marginal profit and price-elasticity



Case 2: Partial collusive oligopoly & the linear demand curve

In this case, we assume that the collusive oligopoly firms can only control a residual share of the supply and where the rest a constant and supplied a large number of the small non-collusive firms, which cannot adjust their supply in the shorter and medium terms. For simplicity, we assume that the marginal cost is constant, and it is equal to one. The small firms supplies the constant quantity q^S.

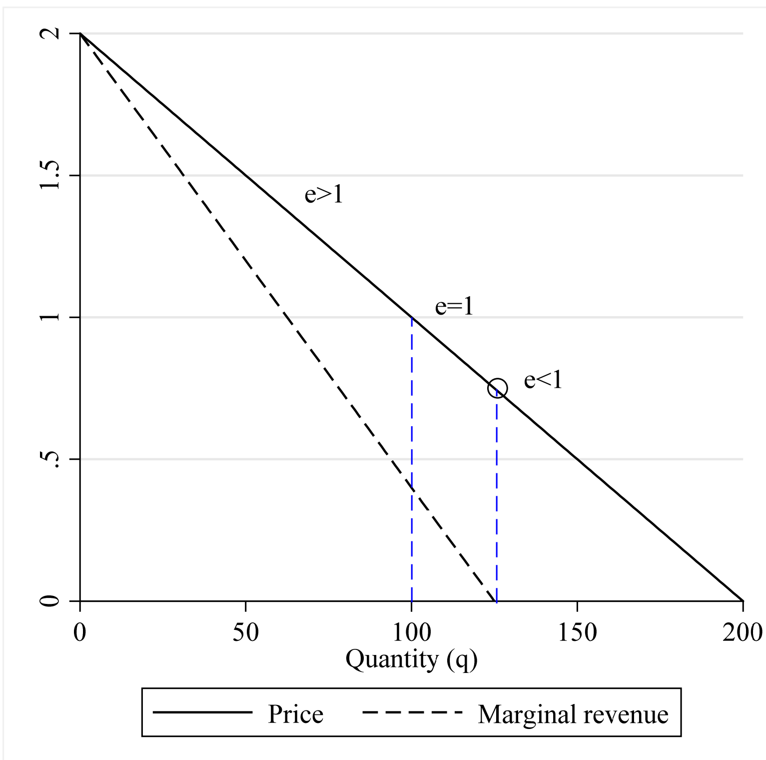
- $p(q) = a - \delta q.$
- $q(p) = \frac{a}{\delta} - \frac{1}{\delta}p$

The total revenue is: $\mathrm{TR}(q^{PCO}) = (a - \delta(q^S + q^{PCO}))q^{PCO}$

The marginal revenue is: $MR(q^{PCO}) = -\delta q^{PCO} + p = -\phi\delta\ q + p = p\left(1 + \frac{\phi}{\varepsilon}\right)$

The following graph is an example with: $p(q) = 2 - 0.01q$.

Figure 2: Marginal profit of the PCO group and price-elasticity



Case 2: Constant elasticity demand curves

The general form of the CES demand function is:

- $q(p) = ap^{-\delta}$
- $p(q) = \left(\frac{a}{q}\right)^{-\frac{1}{\delta}}$.

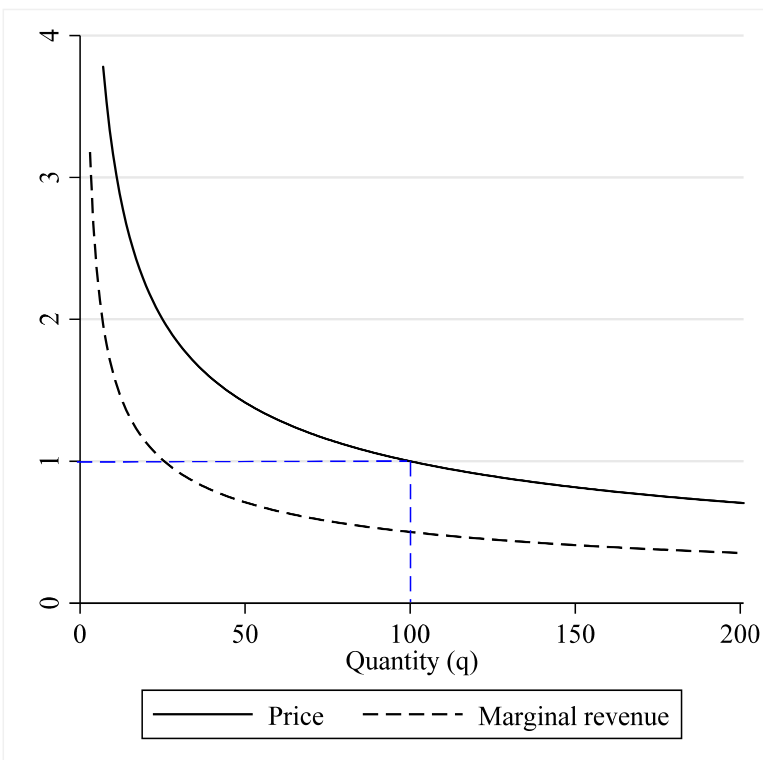
100=100*1

The price-elasticity is $\varepsilon = \frac{\partial q}{\partial p}\frac{p(q)}{q} = -\delta a p^{-\delta-1}\frac{p}{ap^{-\delta}} = -\delta < 0$.

The MR is: $(1-\delta)a^{-\frac{1}{\delta}}q^{-\delta}$. Obviously, this case is unrealistic, especially when $\delta < 1$, and where the monopole must indefinitely increase the produced quantity!

Example: $q(p) = 100p^{-2}$

Figure 3: The CES demand function and the marginal revenue



Case 3: The segmented-linear demand curve

Now, we suggest introducing some less familiar demand curves. For instance, assume the following demand segmented-linear curve:

- $p(q) = \begin{cases} a_1 - \delta_1 q & \text{if} \quad q \in [0, \quad q^*] \\ a_2 - \delta_2 q & \text{if} \quad q \in]q^*, +\infty] \end{cases};$

- $q(p) = \begin{cases} \frac{a_1}{\delta_1} - \frac{1}{\delta_1} p & \text{if} \quad p \in [0, \quad p^*] \\ \frac{a_2}{\delta_2} - \frac{1}{\delta_2} p & \text{if} \quad p \in]p^*, +\infty] \end{cases};$

and $q^* = \frac{a_2 - a_1}{\delta_2 - \delta_1}$ and $p^* = a_1 - \delta_1 \left(\frac{a_2 - a_1}{\delta_2 - \delta_1}\right)$.

Example:

$p(q) = \begin{cases} 150 - 1.5q & \text{if} \quad q \in [0, \quad 50] \\ 100 - 0.5q & \text{if} \quad q \in]50, +\infty] \end{cases}$, *and* the constant cost per unit is 30.

Figure 3: The segmented linear demand function and the marginal revenue

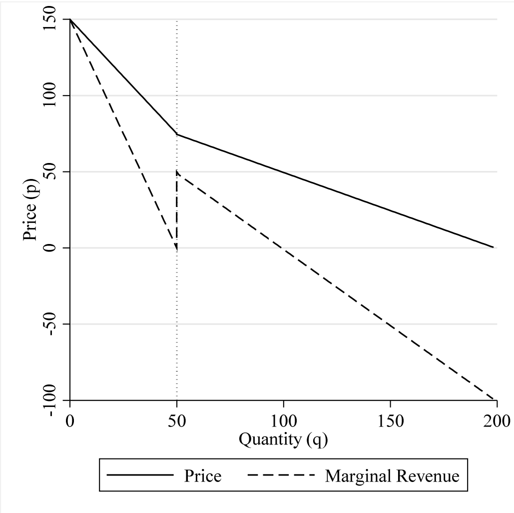
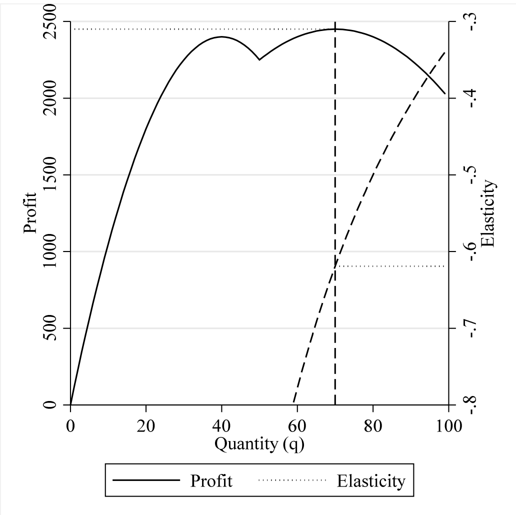


Figure 4: The segmented linear demand function and the profit



As we can observe string from the Figure 03, the form of this demand curve looks like a quasi-convex curve, which it is close to the CES form. This functional form of demand models the increasing decrease in the demand with the increase in price. In Figure 4, we show that the monopole maximises its profit even by supplying quantity within the inelastic part of the demand function.

Note that the CES form represents a special case of modeling the accelerating decrease in demand with the increase in price. In figure 05, we show how the segmented form is simply an intermediate case between the two familiar forms (i.e. the linear and the CES).

Figure 5: Demand function forms