Demand Systems, Prices and Income Elasticities

Araar, A. Carlos, R.C., Sergio, O. and Eduardo, A.M.C.

Technical Note[1]

World Bank

April 2018

[1] This is a part of the WELCOM project research, which was sponsored by The World Bank. The authors gratefully acknowledges Tara Vishwanath, and members of the Global Solutions Group of Markets and Institutions for Poverty Reduction at the Poverty and Equity Global Practice for helpful comments.

Contents

1. Demands system models, elasticities and WELCOM

As reported before, the consumer reaction to the change in prices is among the main determinant of the market power. Disposing the information on the price elasticities is thus primordial to study the impacts of the market power on well-being. Depending on, on the specific context of each study, the practitioner can decide to use the available information on the elasticity in the other empirical studied or even estimate the required price elasticities. As an important contribution of the WELCOM package, is to introduce the popular and most updated demand system models and to provide the user-friendly tools for the estimations.

Because of the complex-specific details of each demand system model, it will be helpful to start by introducing in a more concise form each of the models and thus the WELCOM Stata module to estimate it. Thus, our aim is to remember briefly the most popular models and especially to focus on the computational framework, as well as to document or to develop the Stata code for the estimations of price elasticities. During the last decades, there is a significant progress in developing the flexible demand system models, as the AIDS/QUIDS or the EASI models. However, handling the data and estimating the models require some expertise. The models that we cover are:

- the Deaton (1988) Unit Value Model (DUVM);
- the AIDS/QUAIDS model(s);
- and he EASI model.

2. Deaton's (1988) Unit Value Model

2.1 The Deaton's theoretical model

Estimating a demand system requires information on the household expenditures at different price levels. The price can vary spatially or temporally. However, accurate price-data are rare. This scarcity of data justifies switching to what we call the *unit value model*. In particular, for the case of large consumption groups with weak separability, Deaton (1988) develops a model to estimate the price and income elasticities, and this, using unit values, and where the model enables to compensate for the quality effect.

As reported by Deaton (1988), we assume that we have a given main set of expenditures, where each of them is composed of a predetermined group of items. Let E_c denotes the household expenditures on the group of items c.

$$E_c = p_c q_c \qquad (01)$$

where p_c and q_c are vectors denoting the prices and the quantities of the group c. Within the group of commodities c, let λ_c denotes the proxy unique price of the commodities c and v_c the indicator of quality such that:

$$E_c = \lambda_c v_c Q_c \qquad (02)$$

For the same level of expenditures E_c the aggregated quantity within the group c (Q_c) will depend on the quality of bought goods, v_c. More important, the estimated unit value $V_c = \frac{E_c}{Q_c} = \lambda_c v_c$ will depend on the product of proxy unique price and quality. Based on this model, the price elasticity is defined as follows:

$$e_c = \frac{\partial Q_c}{\partial \lambda_c} = \frac{\partial \ln Q_c}{\partial \ln V_c}\left(1 + \frac{\partial \ln v_c}{\partial \ln \lambda_c}\right) \qquad (03)$$

Deaton (1988) proposes the adoption of the weak separable utility function with demands:

$$q_c = f_c\left(\frac{E_c}{\lambda_c}, p_c^*\right) \qquad (04)$$

The consumer selects in the first stage the –normalized- budget $\frac{E_c}{\lambda_c}$ that is allocated to the group *c*. After that, and depending on the vector of the prevailed prices according to the quality (p_c^*), the consumer will select the levels of expenditures on the different items of the group *c*. In addition, the assumption of weak separability leads to:

$$\frac{\partial \ln v_c}{\partial \ln \lambda_c} = \frac{\frac{\partial \ln v_c}{\partial \ln x}\frac{\partial \ln Q_c}{\partial \ln \lambda_c}}{\frac{\partial \ln Q_c}{\partial \ln x}} \qquad (05)$$

2.2 The empirical implementation and estimation

We start with the case of the standard unit value model, and where we assume that the quality effect is nil or can be neglected. As suggested by Deaton (1988), the simple model with the weak separability assumption is as follows:

$$w_{c,i} = \alpha_0 + \beta_0 x_i + \gamma_0 z_i + \phi_0 \ln\left(V_{c,i}\right) + u_{0,i} \tag{06}$$

where

- $w_{c,i}$: The budget share of group c.
- x_i : The household income.
- z_i : Household characteristics
- $V_{c,i}$: Unit value of group c.

We have that:

$$\frac{\partial \ln Q_c}{\partial \ln V_c} = \frac{\frac{\partial \ln w_c}{\partial \ln V_c}}{w_c} - 1$$

$$e_c = \frac{\partial Q_c}{\partial \lambda_c} = \left(\frac{\frac{\partial \ln w_c}{\partial \ln V_c}}{w_c} - 1\right)\left(1 + \frac{\partial \ln v_c}{\partial \ln \lambda_c}\right) \tag{07}$$

For the standard unit value model and by assuming that the quality is uncorrelated with the price changes (i.e. $\frac{\partial \ln v_c}{\partial \ln \lambda_c} = 0$), we have that:

$$\hat{e}_c = \left(\frac{\hat{\phi}_0}{\overline{w}_c} - 1\right) \tag{08}$$

The case where $\frac{\partial \ln v_c}{\partial \ln \lambda_c}$ is not nil and with the assumption of weak separability, we have that:

$$e_c = \frac{\boldsymbol{\partial Q_c}}{\boldsymbol{\partial \lambda_c}} = \left(\frac{\frac{\partial \ln w_c}{\partial \ln V_c}}{w_c} - 1\right)\left(1 + \frac{\frac{\partial \ln v_c}{\partial \ln x}\frac{\boldsymbol{\partial \ln Q_c}}{\boldsymbol{\partial \ln \lambda_c}}}{\frac{\partial \ln Q_c}{\partial \ln x}}\right) \tag{09}$$

Deaton recommends three steps to estimate the parameters and the elasticities:

Step 1: using the simultaneous equations, we estimate the components $\frac{\partial \ln v_c}{\partial \ln x}$ and $\frac{\partial \ln Q_c}{\partial \ln x}$,

$$w_{c,i} \quad = \alpha_1 + \beta_1 x_i + \gamma_1 z_i + f_c + u_{1,i} \tag{10}$$

$$\ln(V_{c,i}) = \alpha_2 + \beta_2 x_i + \gamma_2 z_i + f_c + u_{2,i} \tag{11}$$

Where f_c is the fixed effect of the group c.

Step 2: estimating the component $\frac{\partial \ln w_c}{\partial \ln V_c}$ using the average predicted values of $w_{c,i}$ and $V_{c,i}$ at the cluster level and this using the following model.

$$\widetilde{w}_c = \alpha_3 + \phi_3 \ln(\tilde{V}_{c,i}) + u_3 \tag{12}$$

It can be noted here that the estimation at this second stage is not ordinary least squares. Indeed, Deaton (1988) uses the correlation between the first stage residuals to estimate the severity of the measurement error. This is used to adjust the estimates and to correct for the structural correlation between quantity and unit value.

Step 3: Solving the equation (09) for the argument $\frac{\partial Q_c}{\partial \lambda_c}$.

References

- ***Angus Deaton***. Quality, quantity, and spatial variation of price. American Economic Review, 78(3), Jun 1988.
- ***Angus Deaton***. Price elasticities from survey data: Extensions and Indonesian results. Journal of Econometrics, 44:281–309, 1990.
- ***Angus Deaton***. The Analysis of Household Surveys: A Microeconometric Approach to Development Policy. Johns Hopkins University Press, 1997.
- ***McKelvey, Christopher***, 2011. "Price, unit value, and quality demanded," Journal of Development Economics, Elsevier, vol. 95(2), pages 157-169, July.

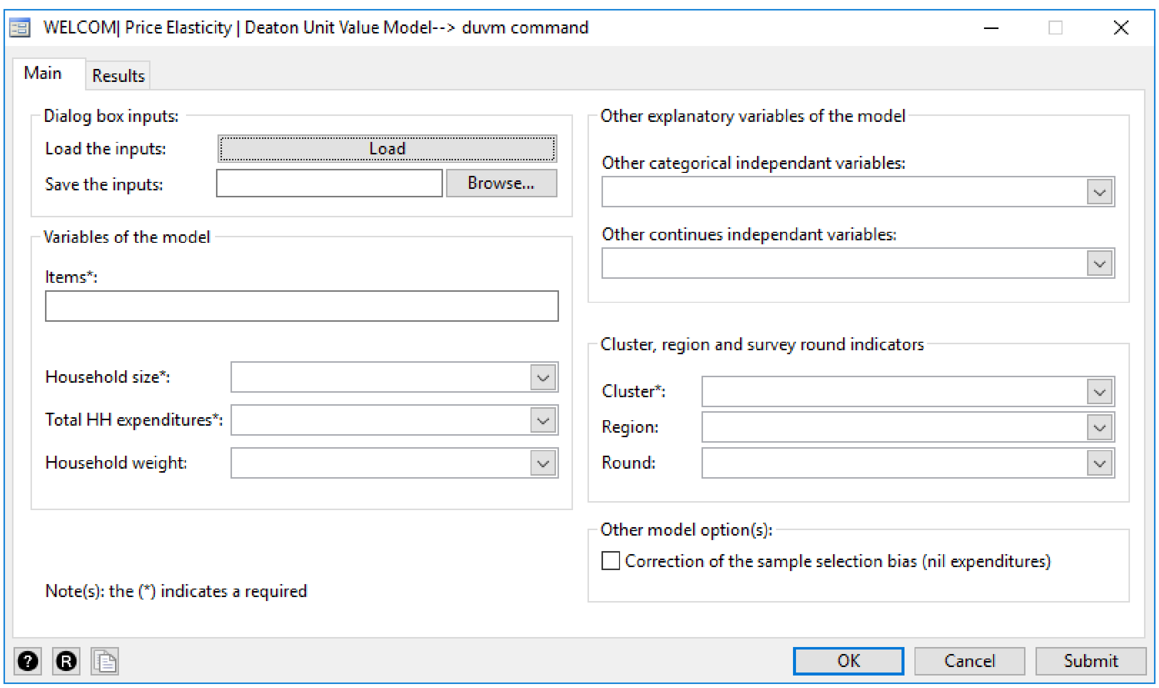
2.3 The DUVM Stata module

The Deaton (1997) book's includes a set of the Stata do files, which were updated in the WELCOM package to perform the estimation of unit value models. Our main contribution was to update the Stata code to produce a concise Stata ado file, which will be easier for the practitioner. Among the improvements, we find:

1. The possibility of using the sampling weight to consider the level of representativeness of each observation;
2. The possibility of tacking into account for the nil expenditures using the Heckman approach, or in short, the IMR component in the estimation;
3. Producing the elasticities different population groups;
4. The use of dialog box, saving the information of the dialog box;
5. Saving results in the Excel format;
6. Etc.

As we can show with some examples, these aspects can largely influence the estimated elasticities. The name of the new Stata command is **duvm**.

Figure 1 : The DUVM dialog box



The basic syntax of the DUVM module is as follows:

duvm **namelist , [options]**

The **namelist** should contain the names of items of expenditures. Note that the data must contain two variables for each item in the **namelist**. The name of the first variable is composed of the letter **w** followed by the name of the item. The name of second variable is composed of the word **luv** and the name of the item. For instance, if the name is of the item is *flour*, we must have the two variables: *wflour* and *luvflour*, which refers to the expenditure share and the log of the unit value of the flour item respectively. The options of the **duvm** command are (required in bold):

inisave	To save the duvm dialog box information. Mainly, all-important information in the dialog box will be saved in this file. In another session, the user can open the project using the command duvm_db_ini followed by the name.
hhsize	The household size.
expend	The household expenditures.
hgroup	Variable that captures a socio-demographic group. For example, this variable could equal 1 for rural households and 2 for urban ones.
indcat	The list of the independent variables that are categorical.
indcon	The list of the independent variables in continues form.
cluster	The cluster is required in an intermediate step to estimate the derivative of the log(exp_share) with regards to the log(unit value).
region	The region area.
subround	The round of the surveyed household.
csb	Correction of Selection Bias. In the case of csb(1), for each item, the routine estimates the beforehand IML ratio based on the binary model (Consumption is not nil), and then, it uses the IMR variable in models of the first stage.
hweight	The sampling weight of the variable.
boot	boot(number of replications) : to estimate the standard errors of the price elasticities.
xfil	To indicate the name of Excel file, which will be used to save the results (*.xml format).
dec	To indicate number of decimals of the displayed results.
sname	To indicate the short names of items.
dregres	To display the regression results.

2.4 Examples of DUVM module

The data

For the ENIGH Mexican data of 2014, we have four groups of expenditures on the ***cereal products***: *corn, wheat, rice and other_cereals*. The size of the full dataset is 19477 observations/households. By using the extended data on expenditures and quantities, we can remark that:

- The household may or may not consume a part of the sub items, or even the item/group c;
- Each group (or item) c can be composed of a set of sub items (c, j);

CORN
A001 Grain corn
A002 Maize flour
A003 Mass of corn
A004 Corn tortilla
A005 Toast
A006 Other maize products

WHEAT
A007 Wheat flour
A008 Flour tortilla
A009 Pasta for soup
A010 Sweet cookies
A011 Crackers
A012 White bread: bolillo, telera, baguette, etc.
A013 Sweet bread in pieces
A014 Sweet bread packaged
A015 Bread for sandwich, hamburger, hot-dog and toasted
A016 Cakes and pastries in pieces or in bulk
A017 Packaged cakes and pastries
A018 Other Wheat Products

RICE
A019 Rice grain
A020 Other rice products

OTHER CEREALS
A021 Corn, wheat, rice, oat, granola, etc. cereal
A022 Botanas: fritters, popcorn, Cheetos, Doritos et cetera (except potatoes)
A023 Instant soups
A024 other cereals

For each group c, we need two information (for each household), which are the unit value and the expenditure share. The household expenditure share is easy to be computed, and it ranges between 0 and 1. For the unit value, the Laspyers index is usually used.

$$V_{c,i} = \sum_{j=1}^{J_c} w_{c,j,i} V_{c,j,i}$$

In the case where the household does not consume any item of the group, we can attribute to this household the average unit value of the group *c* at the level of his cluster. This can be justified by the fact that the household will have the same consumption habits and prices of the other households living in the same cluster. As another step in this example is that we exclude some observations wit where the expenditure share (relative to the current income) exceeds the 95%. After finishing the treatment, the data file (15866 observations) will contain among others the following variables:

The variable	**The definition**
cluster	Primary sampling unit
hh_current_inc	Current income of the household
hhsize	Household size
sweight	sampling weight
sex	Sex of household head
age	Age of household head
psu	Primary sampling unit
education	Diploma of the household head
wcorn	expenditures share of corn
wwheat	expenditures share of wheat
wrice	expenditures share of rice
wother	expenditures share of the rest of cereals
luvcorn	log of the unit val of flour
luvwheat	log of the unit val of semolina
luvrice	log of the unit val of couscous
luvother	log of the unit val of pasta

For more information on the treatment of data, type the following Stata command:

view http://dasp.ecn.ulaval.ca/welcom/examples/ds/example_cereals_data.do

Example I-1

#delimit ;
use http://dasp.ecn.ulaval.ca/welcom/examples/ds/Mexico_2014_Cereals.dta , replace;
duvm corn wheat rice other, hhsize(hhsize) expend(hh_current_inc) hweight(sweight) cluster(psu) region(rururb)
 inisave(ex1_duvm_db) indcat(sex educ) indcon(age)   xfil(myfile) dregres(1) hgroup(decile) boot(50) ;

After executing the line command, a series of results are displayed.

Part_1: Estimations of the first stage.

```
. #delimit ;
delimiter now ;
. use http://dasp.ecn.ulaval.ca/welcom/examples/ds/Mexico_2014_Cereals.dta , replace;

.  duvm corn wheat rice other,  hhsize(hhsize) expend(hh_current_inc) hweight(sweight) cluster(psu) region(rururt
>  inisave(ex1_duvm_db)  indcat(sex educ)  indcon(age)   xfil(myfile) dregres(1) hgroup(decile) boot(50) ;
(note: file ex1_duvm_db.duvm not found)
The cluster fixed effect regression(s)
```

	(1) luvcorn	(2) luvwheat	(3) luvrice	(4) luvother
Log of expenditures	0.0138***	0.00629**	-0.000234	0.0108***
Log of hhsize	-0.0637***	-0.0122***	-0.00508	-0.00581*
Age of head of hou~d	-0.000402***	-0.000169	-0.000333**	-0.0000884
sex==1	0.00646	-0.00431	0.00826*	-0.00239
educ==2	-0.0632	-0.000972	-0.0269	0.00988
educ==3	0.00283	0.00512	0.00733	-0.00158
educ==4	0.00459	-0.00343	0.00848	-0.00106
educ==5	0.000119	-0.0112	-0.0182	-0.00520
educ==6	0.0105	0.00276	-0.00335	-0.00702
educ==7	0.0208**	0.0113*	0.00496	-0.00373
Constant	2.725***	3.792***	2.725***	3.897***
Observations	15375	15340	13172	12166
R-squared	0.911	0.928	0.797	0.958

* p<0.05, ** p<0.01, *** p<0.001

The budget shares regression(s)

	(1) wcorn	(2) wwheat	(3) wrice	(4) wother
Log of expenditures	-0.0171***	-0.00797***	-0.000674***	0.000189***
Log of hhsize	0.0126***	0.00526***	0.000866***	0.000145**
Age of head of hou~d	0.0000642***	-0.0000181	0.00000121	-0.00000966***
sex==1	-0.000223	-0.000653*	-0.0000224	-0.0000792
educ==2	0.0149***	0.00905**	0.00106	-0.000679
educ==3	0.00259***	0.00246***	0.000190	0.0000395
educ==4	0.00203**	0.00379***	0.000228*	0.000242*
educ==5	0.00111	0.00443***	0.000561***	0.000227
educ==6	0.00239**	0.00480***	0.000144	0.000195
educ==7	-0.0000400	0.00407***	0.0000706	0.000262*
Constant	0.162***	0.0800***	0.00654***	-0.000619
Observations	15866	15866	15866	15866
R-squared	0.503	0.336	0.274	0.224

* p<0.05, ** p<0.01, *** p<0.001

Part_2: Average expenditure shares, income and quality elasticities.

Table 01: Average expenditures shares (in %)

	corn	wheat	rice	other
AV_share	2.317	1.497	0.153	0.100

Table 02: Expenditure elasticities

	corn	wheat	rice	other
Elasti~y	0.250	0.461	0.560	1.178

Table 03: Quality elasticities

	corn	wheat	rice	other
Elasti~y	0.014	0.006	-0.000	0.011

In this first example, the elasticity of quality is very low. This is explained by the nature of goods. For instance, the quality of the flour group cannot vary largely.

Part_3: Price elasticities.

Table 04: Price elasticities: without quality correction | without symmetry restricted estimators

	corn	wheat	rice	other
corn	-0.877	0.059	-0.019	-0.010
wheat	0.075	-0.813	0.178	0.016
rice	-0.111	0.089	-1.108	-0.095
other	0.110	0.126	-0.039	-0.720

Table 05: Price elasticities: without quality correction | with symmetry restricted estimators

	corn	wheat	rice	other
corn	-0.877	0.056	-0.013	-0.004
wheat	0.081	-0.809	0.069	0.014
rice	-0.203	0.676	-1.152	-0.101
other	-0.107	0.192	-0.156	-0.710

Table 06: Price elasticities: with quality correction | without symmetry restricted estimators

	corn	wheat	rice	other
corn	-0.869	0.059	-0.019	-0.010
wheat	0.073	-0.800	0.175	0.016
rice	-0.111	0.089	-1.108	-0.095
other	0.110	0.126	-0.039	-0.720

Table 07: Price elasticities: with quality correction | with symmetry restricted estimators

	corn	wheat	rice	other
corn	-0.869	0.055	-0.013	-0.004
wheat	0.080	-0.796	0.068	0.013
rice	-0.203	0.676	-1.152	-0.101
other	-0.107	0.192	-0.156	-0.710

Table 08: Standard errors: with the bootstrap method

	corn	wheat	rice	other
corn	0.003	0.003	0.020	0.048
wheat	0.002	0.004	0.021	0.034
rice	0.001	0.002	0.008	0.007
other	0.002	0.002	0.005	0.010

Treatment of hgroups : 1:2:3:4:5:6:7:8:9:10:

Table 09: Own Price Elasiticies by hgroups

	corn	STE	wheat	STE	rice	STE	other	STE
Decile_1	-0.708	0.012	-1.060	0.014	-1.830	0.054	-1.261	0.049
Decile_2	-0.953	0.007	-0.982	0.009	-1.351	0.029	-1.512	0.064
Decile_3	-0.882	0.007	-1.087	0.010	-1.131	0.026	-0.633	0.049
Decile_4	-0.987	0.007	-0.890	0.012	-1.076	0.034	-0.925	0.047
Decile_5	-1.040	0.006	-0.887	0.012	-1.317	0.024	-0.863	0.034
Decile_6	-0.936	0.006	-0.937	0.010	-1.384	0.027	-0.780	0.031
Decile_7	-0.984	0.006	-0.803	0.009	-1.163	0.029	-0.841	0.024
Decile_8	-0.964	0.006	-0.794	0.013	-0.866	0.029	-0.623	0.029
Decile_9	-1.016	0.006	-0.671	0.012	-1.409	0.032	-0.553	0.015
Decil~10	-0.751	0.009	-0.407	0.011	-0.756	0.040	-0.231	0.021

The price elasticity may vary largely across deciles. It is important to consider this aspect to assess accurately the impact of price changes on the poor group.

3. AIDS & QUAIDS Demand System Models

3.1 The Almost Ideal Demand System theoretical model (AIDS)

To present the AIDS model, we start by defining the PIGLOG model. This model can be represented via an expenditure function $c(\mathbf{P}, U)$ that defines the minimum level of expenditure to reach a predetermined level of utility given prevailing prices:

$$\log(c(p, U)) = (1 - U)\log(a(p)) + U\log(b(p)) \tag{01}$$

where U is the utility located between 0, the level of subsistence, and 1, the level of beatitude. The function $a(p)$ is found through the TRANSLOG form:

$$\log(a(p)) = \alpha_0 + \sum_{i=1}^{K} \alpha_i \log(p_i) + \sum_{i=1}^{K} \sum_{j=1}^{K} \gamma_{i,j}^{*} \log(p_i)\log(p_j) \tag{02}$$

The component $b(p)$ is defined as :

$$\log(b(p)) = \log(a(p)) + \beta_0 \prod_{i=1}^{K} {p_i}^{\beta_i} \tag{03}$$

Thus, we find that:

$$\log(c(p, u)) = \alpha_0 + \sum_{i=1}^{K} \alpha_i \log(p_i) + \sum_{i=1}^{K} \sum_{j=1}^{K} \gamma_{i,j}^{*} \log(p_i)\log\left(p_j\right) + u\beta_0 \prod_{i=1}^{K} {p_i}^{\beta_i}. \tag{04}$$

Using Shephard's Lemma $(\partial c(p, u)/ \partial p_k = x_k)$, the expenditure share on good i becomes:

$$w_i = \alpha_i + \sum_{i=1}^{K} \sum_{j=1}^{K} \gamma_{i,j} \log(p_j) + \beta_i u \beta_0 \prod_{i=1}^{K} {p_i}^{\beta_i} \tag{05}$$

and

$$\gamma_{i,j} = \frac{1}{2}\left(\gamma_{i,j}^{*} + \gamma_{j,i}^{*}\right) \tag{06}$$

Expenditure shares also simplify as:

$$w_i = \alpha_i + \sum_{j=1}^{K} \gamma_{i,j} \log(p_j) + \beta_i \log(m/a(p)) \tag{07}$$

where $a(p)$ can be perceived as a price index equalling:

$$\log(a(p)) = \alpha_0 + \sum_{i=1}^{K} \alpha_i \log(p_i) + \sum_{i=1}^{K} \sum_{j=1}^{K} \gamma_{i,j}^{*} \log(p_i)\log(p_j) \tag{08}$$

The additional conditions of the model are:

$$\begin{array}{lll} I: & \sum_{i=1}^{K} \alpha_k = 1 & \mathit{Sum\ of\ expenditures\ shares\ is\ 1} \\ II: & \sum_{i=1}^{K} \gamma_{i,j} = 0\ \forall j\ \ \mathit{and}\ \ \sum_{i=1}^{K} \beta_i = 0 & \mathit{Homogeneity\ of\ degree\ 0\ of\ demand\ functions} \\ III: & \gamma_{i,j} = \gamma_{j,i} & \mathit{Symmetry\ of\ the\ Slutsky\ matrix} \end{array} \quad (09$$

3.2 The Quadratic Almost Ideal System theoretical model (QUAIDS)

Banks Blundell and Lewbel (1997) have proposed the Quadratic Almost Ideal System (QUAIDS) model that adds the quadratic logarithmic income term to the AIDS specification of

Deaton and Muellbauer (1980). This was proposed in order to take into account the potential quadratic form of the Engel curve behaviour for some durable and luxury goods. The specification is as follows:

$$w_i = \alpha_i + \sum_{j=1}^{K} \gamma_{i,j} \log(p_j) + \beta_i \log(m/a(p)) + \frac{\lambda_i}{b(p)} \log(m/a(p))^2 \qquad (09)$$

The price index is given by:

$$\log(a(p)) = \alpha_0 + \sum_{i=1}^{K} \alpha_i \log(p_i) + \sum_{i=1}^{K} \sum_{j=1}^{K} \gamma_{i,j}^{*} \log(p_i) \log(p_j) \qquad (10)$$

The price aggregator is given by:

$$b(p) = \prod_{i=1}^{K} p_i^{\beta_i} \qquad (11)$$

The income and demand elasticities are defined as follows:

$$\begin{array}{lll} I: & e_i = \mu_i / w_i - 1 & \textit{Income elasticity} \\ II: & e_{i,j}^{nc} = \mu_{i,j} / w_i - \delta_{i,j} & \textit{Non compensated elasticity} \\ III: & e_{i,j}^{c} = \mu_{i,j} / w_i - e_i w_i & \textit{Compensated elasticity} \end{array} \qquad (12)$$

where

$$\mu_i = \frac{\partial w_i}{\partial \log(m)} = \beta_i + 2\lambda_i \log\left(\frac{m}{a(p)}\right)$$

$$\mu_{i,j} = \frac{\partial w_i}{\partial \log(p_j)} = \gamma_{i,j} + \mu_i\left(\alpha_j + \sum_{k=1}^{K} \gamma_{k,j} \log(p_k)\right) - \frac{\lambda_i \beta_j}{b(p)} \left\{\log\left(\frac{m}{a(p)}\right)\right\}^2$$

References

- Banks, J., R. Blundell, and A. Lewbel. 1997. Quadratic Engel curves and consumer demand. *Review of Economics and Statistics* 79: 527–539.
- Deaton, A. S., and J. Muellbauer. 1980a. *Economics and Consumer Behaviour*. Cambridge: Cambridge University Press.
- ———. 1980b. An almost ideal demand system. *American Economic Review* 70: 312–326.

Remark:

The Stata module for the estimation of the AIDS and the QAIDS models is already available and well developed by Brian Poi (2012).

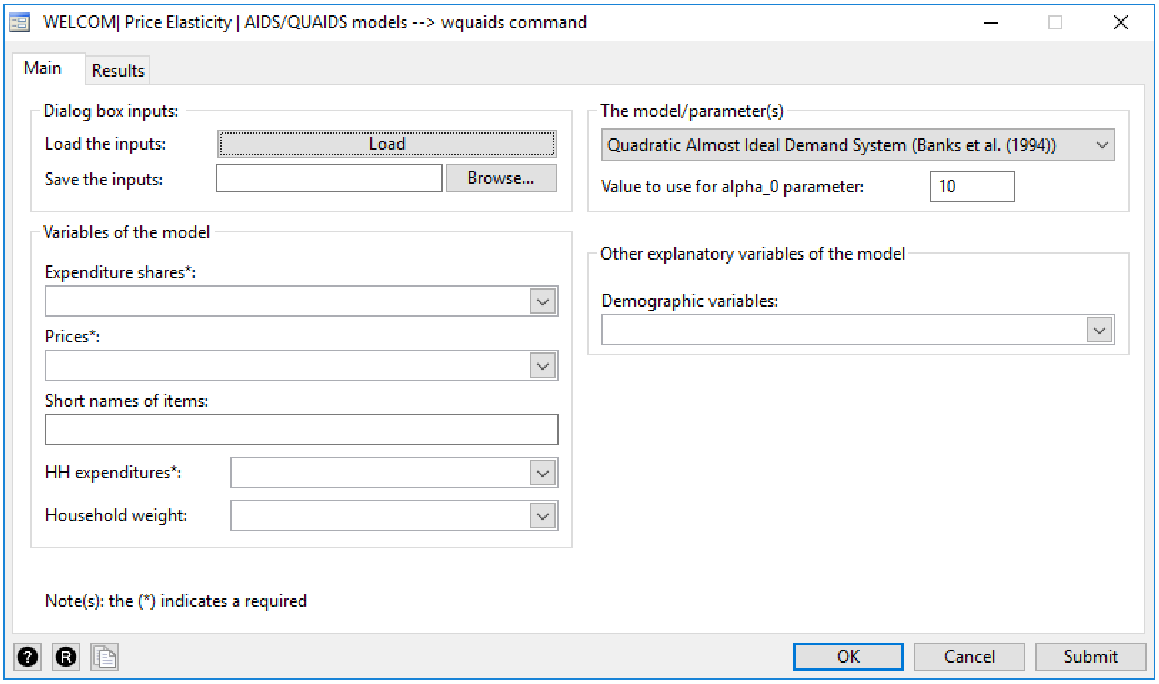
- *Poi, Brian P., (2012), Easy demand-system estimation with quaids, Stata Journal, 12, issue 3, p. 433-446*

The module **wquaids** uses implicitly the Brian Poi quaids Stata command (or its updated version wquaids). The improvements are:

1. The possibility of using the sampling weight to consider the level of representativeness of each observation;
2. The use of dialog box, saving the information of the dialog box;

3. Saving results in the Excel format;
4. Etc.

Figure 2: the dialog box of the AIDS/QUAIDS models



3.3 Practical exercise: The QUAIDS model

The data

We use the same data of the DUVM example (see the subsection 2.4). For simplicity, we consider unit values as the observed prices.

```
/* The Stata code  */

#delimit ;
use http://dasp.ecn.ulaval.ca/welcom/examples/ds/Mexico_2014_Cereals.dta , replace;
 wquaids wcorn wwheat wrice wother wcomp,
anot(9.5)
prices(pcorn pwheat price pother pcomp)
snames(Corn Wheat Rice Other_cereal Rest)
expend(hh_current_inc)
hweight(sweight)
model(1)
 inisave(ex1_quaids_db)
demographics(sex educa2- educa7 age)
dregres(1) dislas(0) xfil(myfil);
```

Remarks:

- The command *wquaids* is followed by the expenditure share variables. The variables *wcomp* is the expenditure share on the rest of items (outside the cereal products). The sum of the expenditure shares must be equal to one.
- The option anot() is the initial level of the parameter alpha_0 of the AIDS/QUAIDS model. It can be approximated to the log of the min(household income).
- The option model(1) is for the QUAIDS model. (2) is for the AIDS model.
- The option dislas(0) indicates that the results of the last item (complement or rest of items) will not be displayed.

The results:

QUAIDS MODEL	Standard DUV MODEL

Income elasticities

QUAIDS MODEL:

	Corn	Wheat	Rice	Other_ce~l
Elasticity	0.4455	0.6109	0.7575	1.4826

Standard DUV MODEL:

	corn	wheat	rice	other
Elasti~y	0.2405	0.4569	0.5480	1.1811

Uncompensated price elasticities

QUAIDS MODEL:

	Corn	Wheat	Rice	Other_ce~l
Corn	-0.9771	0.0451	-0.0136	-0.0022
Wheat	0.0682	-0.8793	0.0119	0.0010
Rice	-0.2254	0.1139	-1.2268	-0.1680
Other_cereal	-0.0803	-0.0023	-0.2364	-0.7818

Standard DUV MODEL:

	corn	wheat	rice	other
corn	-0.8699	0.0558	-0.0114	-0.0007
wheat	0.0810	-0.7955	0.0682	0.0123
rice	-0.1788	0.6667	-1.1300	-0.1058
other	-0.0390	0.1756	-0.1648	-0.7132

As we can remark, the two-demand system models give close estimates of the price elasticities.

4. Exact Affine Stone Index (EASI)

4.1 The EASI theoretical model?

To deal with the empirical nonlinear form of the Engel curve and to propose a more flexible model, Lewbel and Pendakur (2009) use the Shephard's lemma to approximate real income. This linear approximation implies the use of the Stone price index (SPI), as in the case of the Linear Approximate Almost Ideal Demand System (LA/AIDS). Even with this restriction, among the advantages of the EASI model is the possibility of using a higher order of the polynomial real income, which enables to better fit the Engel function. Formally, the approximated EASI model can be defined through the implicit marshallian budget share as follow:

$$w_i = \sum_{r=1}^{o} b_r \tilde{y}^r + \sum_{j=1}^{K} a_k \log(p_k) + \sum_{k=1}^{K} b_k \log(p_k)\tilde{y} + \tilde{\varepsilon}$$

$$= \sum_{r=1}^{o} b_r \tilde{y}^r + Ap + Bp\tilde{y} + \tilde{\varepsilon} \qquad (01)$$

where $\tilde{y}$ denotes here the log of the approximated real income: $y \approx \tilde{y} = log(m) - \sum_{k=1}^{K} w_k \log(p_k)$. The parameter o is the polynomial order of the real income and the parameter $\tilde{\varepsilon}$ is simply the error term of the estimation. For simplicity and compared to the Lewbel and Pendakur (2009) presentation of the model, we omit the household characteristic determinants. Based on the Shephard's lemma and the cost function, Arthur and Pendakur (2009) show that the exact real income is equal to:

$$y = \frac{m - p\prime w + p\prime Ap}{1 - 0.5 * p\prime Bp} \qquad (02)$$

Thus, the exact EASI model can be defined as follows:

$$w_i = \sum_{r=1}^{o} b_r \left(\frac{m - p\prime w + p\prime Ap}{1 - 0.5 * p\prime Bp}\right)^r + Ap + Bp\left(\frac{m - p\prime w + p\prime Ap}{1 - 0.5 * p\prime Bp}\right) + \varepsilon \qquad (03)$$

As was the case for the AIDS or the QAIDS models, additional conditions are imposed:

$$\begin{array}{lll} I: & \sum_{i=1}^{K} a_{i,j} = 0 \ \forall j \ \ and \ \ \sum_{i=1}^{K} b_i = 0 & Homogeneity \ of \ degree \ 0 \ of \ demand \ functions \\ II: & a_{i,j} = a_{j,i} & Symmetry \ of \ the \ Slutsky \ matrix \end{array} \qquad (04)$$

Among the recommended econometric methods to estimate the model is the nonlinear three stages least squares (3SLS). Let p_c denotes the vector of the log of prices after the change.

4.2 Computational tools to estimate the EASI model?

In what follow, we introduce briefly the different tools, which can be used to estimate the EASI model and the elasticities. Mainly, we suggest three tools:

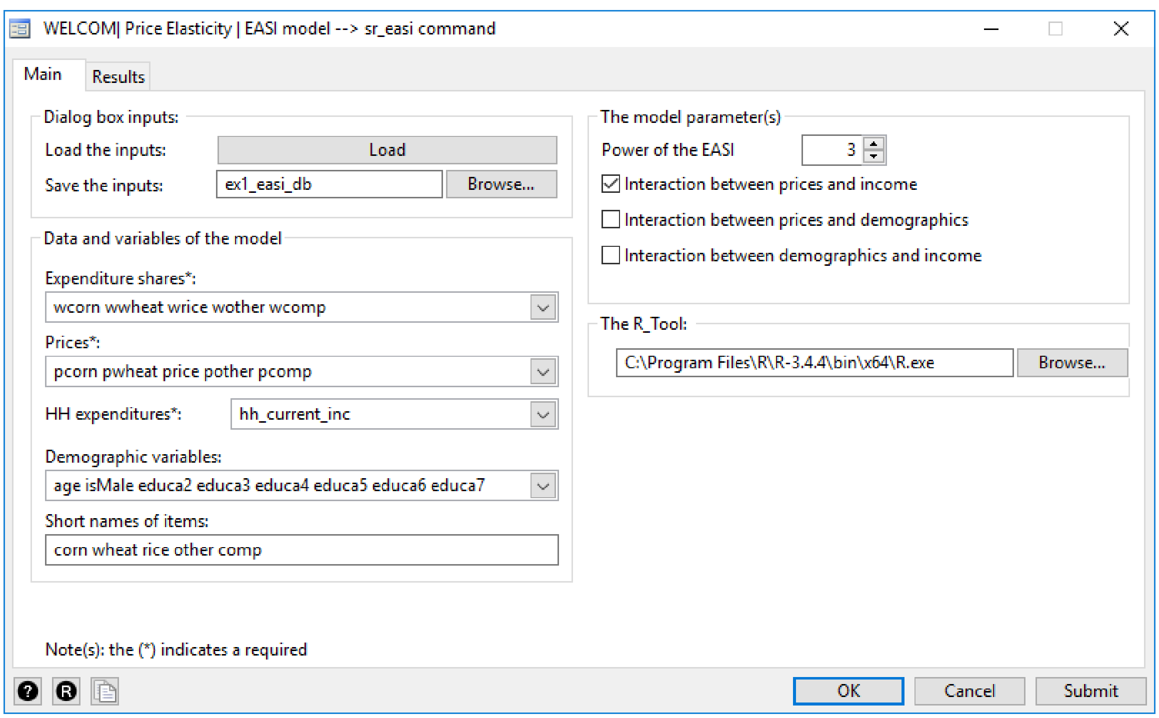
1- *A set of Stata do files*: EASI demand system with no interactions and EASI demand system with interactions (Lewbel, A. (2009)).
2- *Estimating EASI in R* (by Stephane Hoareau, Guy Lacroix, Mirella Hoareau, Luca Tiberti (2012)) **easi-r**.
3- *Estimating EASI in R with Stata*: the ***sr_easi*** Stata module (Araar, A. (2018)).

The first two tools may require more skill in Stata and R. The third, enables to estimate the EASI model in an easy way, but it uses intermediately the easi R package.

Reference(s)

St_ephane Hoareau, Guy Lacroix, Mirella Hoareau. Luca Tiberti (2012) , Exact Affine Stone Index Demand System in R: The easi Package ***http://www2.uaem.mx/r-mirror/web/packages/easi/vignettes/easi.pdf***

Figure 3: The dialog box of the sr_easi module



4.3 Practical exercise: The EASI model

The data

We use the same data of the DUVM example (see the subsection 2.4). For simplicity, we consider unit values as the observed prices. For this practical example, I simply use a random subsample of 8000 observations of *the Mexican national household survey of 2014* (INEGI-2014) to estimate the elasticities of the he cereal products (corn, wheat, rice and other cereals). Thus, we have four cereals variable shares in addition to the complement share of expenditures on the rest of goods. Further, for the comparison purpose, we will estimate the income and price elasticities using the DUVM and the QUAIDS models.

The Stata code

```
/****** THE DATA******************/
#delimit ;
use http://dasp.ecn.ulaval.ca/welcom/examples/ds/Mexico_2014_Cereals.dta , replace;
set seed 4321;
bsample 8000;

/****** THE DUVM MODEL******************/
duvm corn wheat rice other , hhsize(hhsize) expend(hh_current_inc)
cluster(psu) indcon(age isMale educa1-educa10 nocup1-nocup4)
csb(0) boot(0)  xfil(myres1);

/****** THE QUAIDS MODEL******************/
wquaids wcorn wwheat wrice wother wcomp, anot(6)
prices(pcorn pwheat price pother pcomp)
expenditure(hh_current_inc)
demographics(hhsize age isMale educa1-educa10 nocup1-nocup4)
snames(corn wheat rice other complement) dislas(0)  xfil(myres2) ;


/****** THE EASI MODEL******************/
sr_easi wcorn wwheat wrice wother wcomp,  prices(pcorn pwheat price pother pcomp)
snames(corn wheat rice other comp)
expenditure(hh_current_inc)
demographics(hhsize age isMale educa1-educa10 nocup1-nocup4)   dec(4) dregres(1) dislas(0)
inpy(0) inpz(0)   inzy(0)  power(5) xfil(myres3);
```

The results:

Income elasticities

	corn	wheat	rice	other
EASI	0.4375	0.5654	0.6920	1.2910
QUAIDS	0.3702	0.5042	0.6769	1.3472
DUVM	0.2763	0.4789	0.5607	1.2162

Price elasticities

	EASI			
	corn	**wheat**	**rice**	**other**
corn	-0.8993	0.0910	-0.1513	0.0199
wheat	0.0611	-0.9290	0.1263	0.0249
rice	-0.0041	0.0176	-1.2443	-0.1754
other	0.0078	0.0065	-0.1332	-0.7849
	QUAIDS			
	corn	**wheat**	**rice**	**other**
corn	-0.923	0.105	-0.009	0.029
wheat	0.166	-0.982	0.047	-0.028
rice	-0.155	0.464	-1.310	-0.152
other	0.606	-0.395	-0.213	-0.865
	DUVM			
	corn	**wheat**	**rice**	**other**
corn	-0.781	0.068	0.004	-0.001
wheat	0.106	-0.819	0.085	0.033
rice	0.055	0.844	-1.223	-0.147
other	-0.054	0.450	-0.206	-0.647