Optimal Population Group Targeting and Poverty Reduction

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Abstract - *In this note, a new anti-poverty group targeting algorithm is developed for the case of a fixed budget of transfers. The suggested algorithm to optimize the reduction in poverty is assumed to be valid for all of the additive poverty indices. Also, this note recalls the limits of the precedent developed algorithms, which only cover a subset of poverty indices.*

1- INTRODUCTION

The main aim of this technical note is to introduce a new anti-poverty group-targeting algorithm in the case of a fixed budget of transfers. Precisely, it is assumed that the planner agent do not dispose a perfect information on the level of wellbeing of each individual of the population. The available information on well-being only concerns the sampled households. At population level, the agent disposes the information on the group of each individual. Also, it is assumed that the population is divided into mutually exclusive population groups, for instance, the living area of the individual. The problem that the agent is faced is to find the optimal groups' transfers in order to the reduction the most the aggregate poverty. Thus, the aggregate poverty index becomes the objective function to be minimized.

The analytical optimization requires a set of basic conditions. Mainly, for the minimization, the objective function must be strictly quasi-convex in the arguments of interest. Unfortunately, it is not the all of the popular poverty indices, as the headcount or the poverty gap, that obey to this condition. A lot of theoretical and empirical works have focused on the case of the squared poverty gap index that satisfy the set of the optimization conditions. In a chronic way, one can recall the works of Kanbur (1987), Ravallion and Chao (1989) and Elbers et al. (2004). While the first work has focused on the theoretical rules of optimization, the two last works have in addition proposed numerical algorithms to solve the problem of maximizing the reduction in squared poverty gap index with a fixed budget of transfers.

In this note, a new data-graph algorithm is developed. This algorithm is conceived to find the optimal groups' transfers to reduce optimally any of the additive poverty additives like the FGT index. The rest of this note is organized as follow. In section 2, we introduce the poverty reduction function and its normalized form by the cost of transfers. In section 3, we discuss the data-graph algorithm to find the optimal transfers. Also, this note, two Stata modules are introduced, and which can be used for the optimization of the poverty reduction.

2- THE GROUP POVERTY REDUCTION CURVES

The poverty reduction indicator

Assume that the lump-sum transfer is attributed only to the population group g. The per capita lump-sum transfer is denoted by τ_g. The change in the contribution the individual i within the targeted group g is denoted by $d\pi_{g,i}$. For the headcount, we have that:

$$d\pi_{g,i}(\tau_g;\alpha=0) = \mathrm{I}[y_{g,i}<z]\ \mathrm{I}\left[\left(y_{g,i}+\frac{\tau_g}{\varphi_g}\right)>z\right].$$

Where φ_g is the population share of the targeted group g and the indicator: $\mathrm{I}[condition]$ is equal to one if the condition is satisfied and zero otherwise. As it can be observed, there are two conditions to escape poverty. The first is that the individual must be poor. The second, is that the transferred amount must be enough to escape poverty. Similarly, for the poverty gap, one can write:

$$d\pi_{g,i}(\tau_g;\alpha=1) = \mathrm{I}[y_{g,i}<z]\ \min\left(\frac{\tau_g}{\varphi_g},\left(z-y_{g,i}\right)_+\right),$$

and for the squared poverty gap we have that:

$$d\pi_{g,i}\ (\tau_g;\alpha=2) = \mathrm{I}[y_{g,i}<z] * \left(\left(z-y_{g,i}\right)_+^2-\left(z-\min\left(y_{g,i}+\frac{\tau_g}{\varphi_g},z\right)\right)_+^2\right).$$

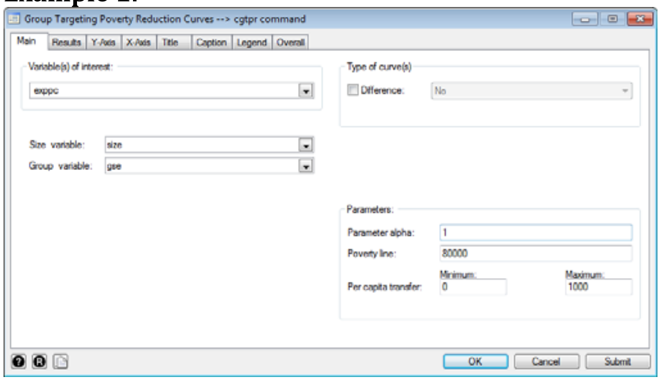
Based on this and for the case of additive poverty indices, it is easy to define the reduction in the aggregate poverty as follows:

$$PR(\tau_g;\alpha)=\varphi_g\frac{1}{n_g}\sum_{i=1}^{n_g}d\pi_{g,i}(\tau_g;\alpha).$$ [1]

In what follow the function $PR_g(\tau_g;\alpha)$ is called the *Group Targeting Poverty Reduction* (GTPR). Thus, this function is equal to the reduction in the aggregate poverty according to the per capita transfer τ_g. The Stata module ***cgtpr.ado*** can be used to draw the GTPR curves.

[1] For simplicity, we omit the common denominator $1/z^\alpha$ for the class of the FGT poverty indices.

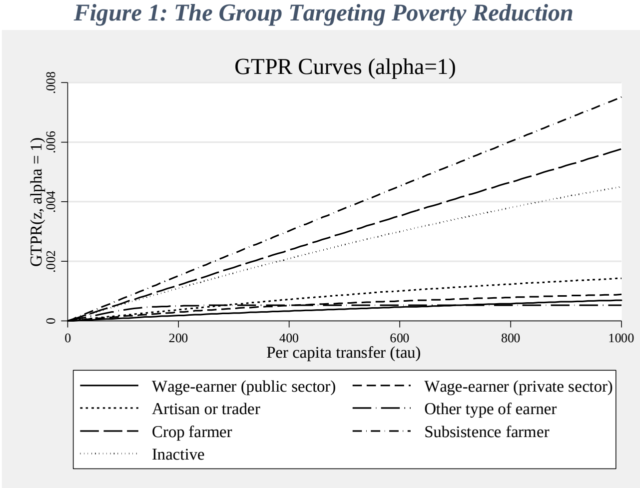
Example 1:



The syntax:

cgtpr exppc, alpha(1) hsize(size) hgroup(gse) min(0) max(1000) \\\ pline(80000) xline(600)

Figure 1: The Group Targeting Poverty Reduction



3- THE OPTIMAL TRANSFERS

Case 1: Single group targeting

Proposal 1:

For a given per capita transfer $\bar{\tau}$ and in case of a single group targeting constraint, the socially efficient group to be targeted is that with the highest GTPR curve at $\bar{\tau}$.

For instance, based on the results of the Figure 1, the socially efficient transfer is that where we target the *subsistence farmer group*.

Case 2: Multiple group targeting

Now, assume that the planner can target more than one group, and this by different levels of the lump-sum transfers. The task becomes to find the optimal transfers to reduce the aggregate poverty, and this, under the transfer constraint: $\tau = \sum_{g=1}^{G} \tau_g$), as well as, the G borne constraints: $0 \le \tau_g \le z \ \forall\, g$. Formally, let the reduction in aggregate poverty $PR(\boldsymbol{\tau};\alpha) = \sum_{g=1}^{G} PR(\tau_g;\alpha)$, the optimization problem can be written as follows:

$$\max_{args:\,\tau_g} \{PR(\boldsymbol{\tau};\alpha)\} \quad s.t. \quad \tau = \sum_{g=1}^{G} \tau_g \text{ and } 0 \le \tau_g \le z \ \forall g$$

where the vector $\boldsymbol{\tau} = \{\tau_1, \tau_2, \cdots, \tau_G\}$. Without focusing on the optimization conditions and based on the analytical approach, the FOC condition of maximization is:

$$\frac{\partial PR_g(\tau_g;\alpha)}{\partial \tau_g} - \lambda = 0 \quad \forall g$$

The SOC condition of optimization requires that: $\frac{\partial\partial PR_g(\tau_g;\alpha)}{\partial {\tau_g}^2} \le 0$. However, for the class of the FGT indices, this condition is only satisfied with $\alpha > 1$.

This result corroborates with that of Kanbur (1987) to minimize in the aggregate poverty. Kanbur has described a potential algorithm to minimize the FGT(α>1). As he reports, basically to minimize the FGT poverty class for α>1, the group with the higher FGT(α−1) should be targeted on the margin. For instance, to minimize the squared poverty gap, target groups should be ranked by the poverty gap (FGT with α=1) and lump-sum transfers made until the poverty gap of the poorest group becomes equal to that in the next poorest group, and so on, until using all of the budget of transfers. Unfortunately, this Kanbur's rule is only valid for the case of α>1 and it fails to cover other popular indices like the headcount or the poverty gap.

Thus, the simple algebraically optimization rules are not valid for the cases of α=0, 1. Indeed, the poverty reduction is not always a decreasing function with the marginal increase of transfer. This is explained by the different levels of concentration of individuals around the different levels of income. The new suggested algorithm tries to overcome this difficulty. Also, it takes into account the importance of the group population sizes, and where the optimization may require to target in priority the groups with small population shares, and this, even if they are less poor.

The data-graph algorithm

In what follow, the three main steps of the new developed algorithm are introduced.

STEP I: Estimating the normalized poverty reduction

The first stage starts with the computation of the reduction in aggregate poverty (at population level), and this, for different levels of the per capita transfer. This is similar to estimate the function $PR_g(\tau_g;\alpha)$ for different levels of τ_g. For instance, if the fixed budget of the per capita transfer is equal to $\bar{\tau}$, for each group, it can be estimated the reduction in aggregate poverty for the transfers: $\frac{\bar{\tau}}{1000}, \frac{2\bar{\tau}}{1000,}, \frac{3\bar{\tau}}{1000,} \cdots, \frac{1000\bar{\tau}}{1000,}$. After that, we normalize the estimated $PR_g(\tau_g;\alpha)$ by the corresponding per capita transfer (τ_g). For simplicity, we denote the ratio between the reduction in aggregate poverty and the per capita transfer for the group g by:

$$\theta_g(\tau_g;\alpha) = PR_g(\tau_g;\alpha) / \tau_g.$$

Note that, by covering the whole potential levels of $\tau_g \in [0, \bar{\tau}]\ \forall g$, the algorithm seeks for a global maximum of poverty reduction.

STEP II: ranking the normalized aggregate poverty reduction

For each group, we classify the θ_g results in a descending order. Note that, for our problem of maximization, this form of ranking enables to converge quickly to the global optimum. This is because, with a highest θ_g and its corresponding transfer τ_g we cannot do better with lowest transfers for the group g. This result will be more discussed latter. After that step, we will have the basic data information that be can be used to seek the optimal transfers. In clear, the results can be organized as follows:

	Group 1		**Group 2**		**... Group G**	
Position	θ_1	τ_1	θ_2	τ_2	θ_G	τ_G
1						
2						
.						
.						
1000						

In what follow, the combination ($\theta_{g.p}$; $\tau_{g.p}$) will refer to the two elements: 1- the normalized poverty reduction and 2- the per capita transfer for the group g at the position p in the table above.

STEP III: seeking for the optimal transfers

- Starting from the first position of the table of results, we seek for the group with the highest $\theta_{g,1}$, and then we attribute the corresponding transfer to that group (for example: g). Obviously, the transferred amount must satisfy the different constraints. Otherwise, we omit this highest $\theta_{g,1}$ for the group g and we perform the comparison across groups again (we compare between $\theta_{g,2}$ and the rest of $\theta_{k,1}$ such that $k \neq g$).
- We seek for the group with the second highest θ_g, and then we attribute the corresponding transfer to that group. Remark that the incrementing in position of comparison of the group is only performed when the transfer is attributed to that group. Also, the different constraints must be satisfied, this requires for instance a group transfer that will not exceed the poverty line, or in more refined form, the maximum individual poverty gap within the targeted group.
- We continue the process until using all of the fixed budget.

Proposal 2:

The data-graph algorithm solution will converge to the global optimum.

Proof

Since, for each of the incrementing steps, we attribute in priority the transfer $\tau_{g,l}$ to the group g with the highest reduction in poverty ($\theta_{g.l} > \theta_{k.m}$), or equivalently, when:

$$PR_g(\tau_{g,l};\alpha) / \tau_{g,l} > PR_k(\tau_{k,m};\alpha) / \tau_{k,m}, \text{ then}$$

this progressive form of attribution of transfers is optimal. Starting from the highest $\theta_{g.1}$ (first position of the table), with the attributed first $\tau_{g,1}$ dollars to the group g, we cannot do better if we attribute this transfer to another group. Also, even if we decide to share the $\tau_{g,1}$ between different groups, the total reduction in poverty will be lower. Why? This is because, by per dollar of transfers, the poverty reduction with $\tau_{g,1}$ is higher than that of any other combination of transfers $\left(\tau_{g,1} = \sum_{l=1}^{G} \tau_{l,p}\right)$[2]. The same discussion can be made f succeeding incrementing.

The following example details again the STEP III. Assume that we only have two population groups (1, 2) with population shares of 75% and of 25% respectively. Also, assume that the total fixed per capita transfer is 50$. Also, assume that the step II gives the following table.

	Group 1		Group 2	
Position (p)	θ_1	τ_1	θ_1	τ_2
1	0.009	10	0.010	20
2	0.004	40	0.006	10
3	0.003	20	0.008	40
4	0.002	30	0.007	30
5	0.001	50	0.006	50

[2] Remember that if : $\frac{C}{A+B} > \frac{D}{A}$ and $\frac{C}{A+B} > \frac{E}{B}$, then $C > D + E$.

Step III.1

Starting from the first position, the highest $\theta_{g,1}$ is that with a per capita transfer of 20$ to the group 2, or equivalently, a transfer of 80$ to each individual of that group. The new position of comparison of group 2 is now the third. This is because we have that $\theta_{2,2} < \theta_{2,1}$ and also $\tau_{2,2} < \tau_{2,1}$. The total per capita cost of transfers until this step is: 0.25*80 = 20$, and it is lower than the planned 50$.

Step III.2

We set the transfer of group 1 to 10$ ($\theta_{1,1} > \theta_{2,3}$). The new position of comparison of group 1 is now the second. The total per capita transfer is now: 20 + 10 = 30$, which it is lower than the planned 50$.

Step III.3

We update the level of transfer of the group 2 to be 40$ ($\theta_{2,3} > \theta_{1,2}$). The new position of comparison of group 2 is now the fourth. The total per capita cost at this step is: 10+ 40 = 50$. Since we reach the budget limit, the algorithm ends at this step.

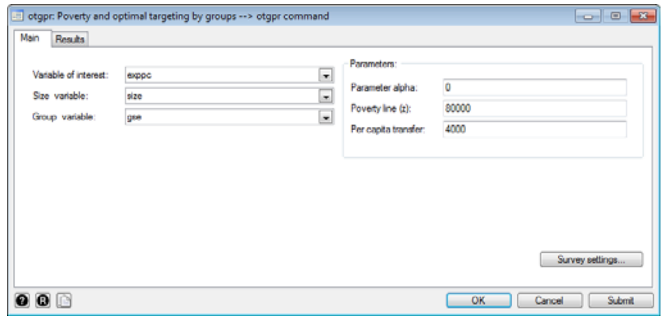
Thus, the optimal per capita transfers are 10$ for the first group and 40$ for the second group. Equivalently, the optimal group-per capita transfers are 13.33 $ (10/0.75) for the first group and 160$ (40/0.25) for the second group.

The virtues of this data-graph algorithm:

- It is assumed to give the highest reduction in poverty for a fixed budget of transfers;
- It can be applied to any additive poverty index;
- It considers the corner solution for which some groups can have a lower or upper limits of the transfer in the optimal solution.

The Stata routine **otgpr** can be used to automatically estimate the optimal transfers for the reduction of the FGT indices, including the headcount and the poverty gap. The estimation takes in general few seconds using usual household surveys:

Example:




```
. otgpr exppc, hgroup(gse) hsize(size) alpha(0) pline(80000) trans(4000) ered(1)

    Optimal Targeting of Groups for Poverty Reduction
    Number of observations      :8478
    Time of computation       : 2.69      second(s)
    Per capita transfer  :  4000
    Household size  :  size
    Sampling weight :  weight
    Group variable  :  gse
    Parameter alpha :  0.00
```

Group	Fgt Index	Population Share	Optimal Transfer
Wage-earner (public sector)	0.075	4.140	0.000
Wage-earner (private sector)	0.139	2.904	1440.000
Artisan or trader	0.155	5.580	0.000
Other type of earner	0.355	0.569	27440.000
Crop farmer	0.492	16.781	20630.682
Subsistence farmer	0.606	65.355	0.000
Inactive	0.463	4.672	7280.000

Total Poverty Reduction

Variable	Estimate	Std. Err.	t	P>|t|	[95% Conf.	interval]	Pov. line
exppc	.5180611	.0109517	47.3042	0.0000	.4965334	.5395888	80000
exppc_tr	.4771605	.0111802	42.6791	0.0000	.4551836	.4991374	80000
diff.	-.0409006	.0042044	-9.72805	0.0000	-.0491652	-.032636	---

In this example, the Burkina Faso household survey of 1998 was used. For the computation of the optimal transfers for seven socio-economic groups and 8478 observations, the execution time takes less than three seconds. As it can be seen based on the results above, for the reduction of the headcount index, the optimization requires to target the groups with lowest population shares. Obviously, this can help to reduce a significant proportion of poor with a low cost of transfers, but also, this form of targeting can be socially undesirable.

BASIC REFERENCES

- **Elbers, Chris & Tomoki Fujii & Lanjouw, Peter & Ozler, Berk & Yin**, Wesley, 2004. "*Poverty alleviation through geographic targeting: how much does disaggregation help?*," Policy Research Working Paper Series 3419, The World Bank.
- **Kanbur, R.**, 1987. *Measurement and Alleviation of Poverty*. IMF Staff Papers, 34:60-85.
- **Ravallion, M., and K. Chao**. 1989. "*Targeted Policies for Poverty Alleviation under Imperfect Information: Algorithms and Applications*." Journal of Policy Modeling 11: 213–24.

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