Oct 28, 2019, Intro to Robotics
Name: ______________________________

PeopleSoft ID: ______________________________

Problem	**Score**	**Possible**
1		15
2		25
3		15
4		25
5		15
6		10
Totals		105

You may have on your desk:

- Your student ID card
- 1 handwritten 8.5"x11" double-sided crib sheet
- This exam (provided by Professor)

Grading: $\text{(problem difficulty)} \times \begin{cases} \text{2 for trying} \\ \text{3 if partially correct} \\ \text{5 if correct} \end{cases}$

As in Appendix A, $\theta = \text{Atan2}(\cos(\theta), \sin(\theta)) = \text{Atan2}(x, y)$

Concepts: Covers chapters 1—4, 11.1—11.2

Rotations & transformations

- Composition of rotations about world or current frame
- Construct a homogenous transform

Kinematics

- Assign DH parameters
- Given DH parameters, construct A matrix
- Given two A matrices, construct T matrix

Inverse Kinematics

- Two-argument arc tangent function
- Solve inverse position kinematics for a 3-link arm

Jacobian

- Construct Jacobian given sketch and T matrices

Computer Vision

- Move from camera frame to world frame, reason from image coordinates

Problem 1: _____/15

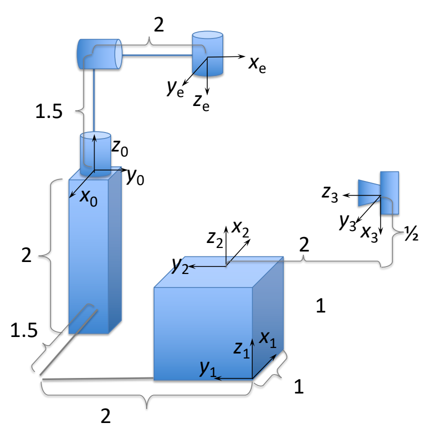
I. **(5 pt)** Write the matrix product that will give the resulting rotation matrix (*DO NOT perform the matrix multiplications, DO simplify*):

a. Rotate by α about the current z-axis
b. Rotate by β about the world y-axis
c. Rotate by γ about the world z-axis
d. Rotate by Φ about the current z-axis
e. Rotate by θ about the current x-axis
f. Rotate by ψ about the world y-axis

II. **(5 pt)** Suppose the three coordinate frames $o_1x_1y_1z_1$, $o_2x_2y_2z_2$, and $o_3x_3y_3z_3$ are given, and suppose

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, R_1^3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$ Find the matrix R_3^2

III. **(5 pt)** Consider diagram at right. The cube front right bottom corner is (1.5,2,-2) meters from the robot base. The cube has 1-meter sides. A frame $o_1x_1y_1z_1$ is fixed to the side of the cube as shown. A second coordinate frame $o_2x_2y_2z_2$ is centered on the top of the cube as shown. A camera is situated 2 meters right and ½ meter above frame 2 with frame $o_3x_3y_3z_3$ attached as shown. Find the **homogenous transform** relating the camera frame to the frame $o_ex_ey_ez_e$, that is, H_3^e:



Problem 2: _____/25 Rotation matrices

Let $R_{YZX}=\begin{bmatrix} c_\alpha c_\theta & -c_\beta s_\alpha + c_\alpha s_\beta s_\theta & s_\alpha s_\beta + c_\alpha c_\beta s_\theta \\ s_\alpha c_\theta & c_\alpha c_\beta + s_\alpha s_\beta s_\theta & -c_\alpha s_\beta + c_\beta s_\alpha s_\theta \\ -s_\theta & s_\beta c_\theta & c_\beta c_\theta \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

If r_{11} and r_{21} are not both zero, then $r_{31} \neq \pm 1$, $-s_\theta = r_{31}$, and $c_\theta = \pm\sqrt{1 - r_{31}^2}$.

If c_θ>0 (c_θ positive), then

a. **(5 pt)** $\alpha =$

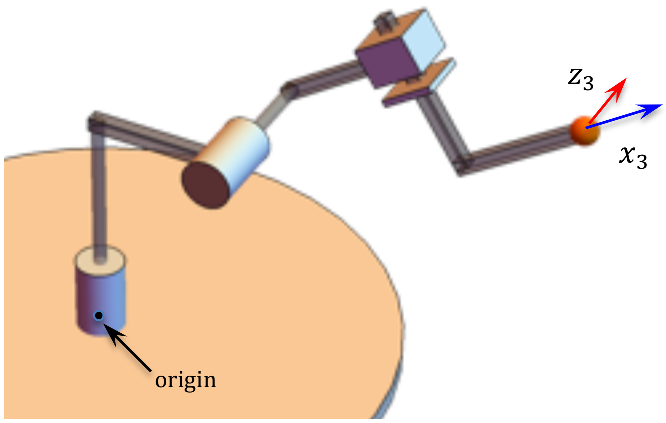
b. **(5pt)** $\beta =$

c. **(15 pts)** Matrix Identification, state **Yes** or **No**. +1 for each correctly listed

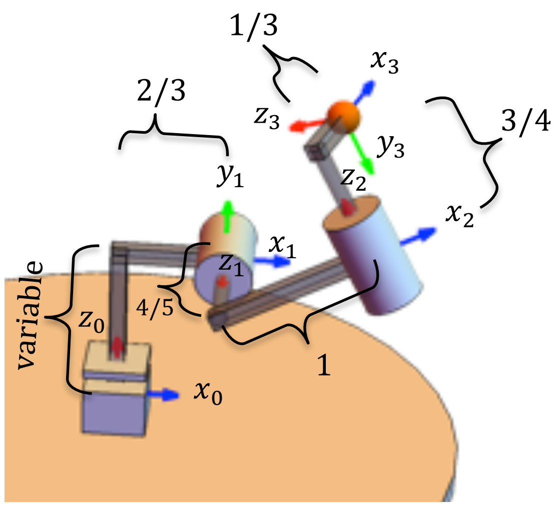
	valid so(k)	valid SE(n)	valid SO(n)
$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$			
$\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$			
$\begin{bmatrix} 0 & -1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \end{bmatrix}$			
$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$			
$\begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$			

Problem 3: _____/15, Forward Kinematics

a.) **(5 pt)** For the 3-link robot below, **draw** and **label** the *z* and *x*-axes according to the DH convention



b.) **(5pt)** Give the DH parameters for this PRR robot.



* indicates variable

Link	r_i	α_i	d_i	θ_i
1				
2				
3				

c.) **(5pt)** Compute the transformation matrix A_2 using the DH parameters:

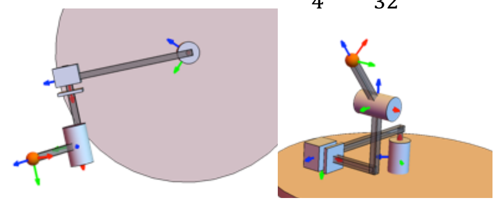
* indicates variable

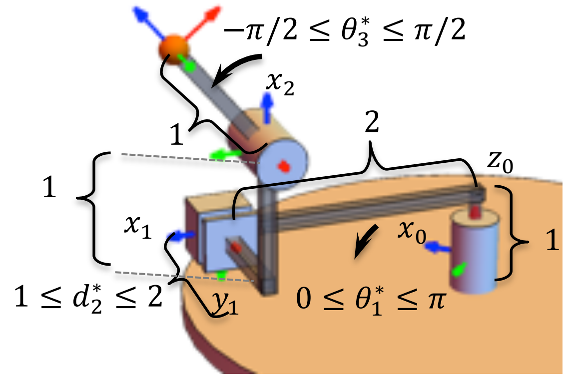
Link	r_i	α_i	d_i	θ_i
1	3	45°	5	θ_1^*
2	5	90°	2	θ_2^*
3	7	90°	4	θ_3^*
4	9	−90°	d_4^*	−90°

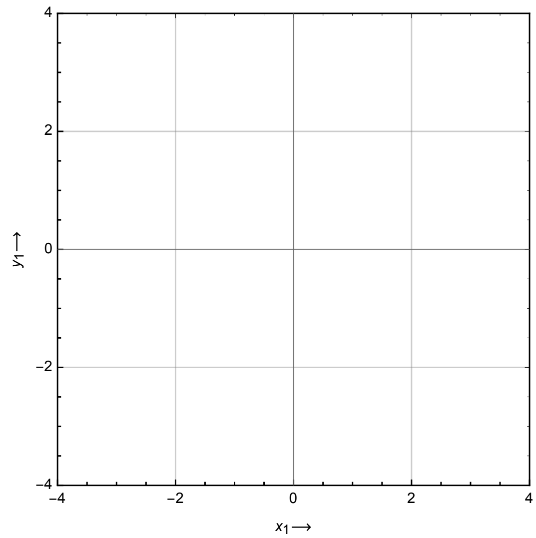
Problem 4: _____/25

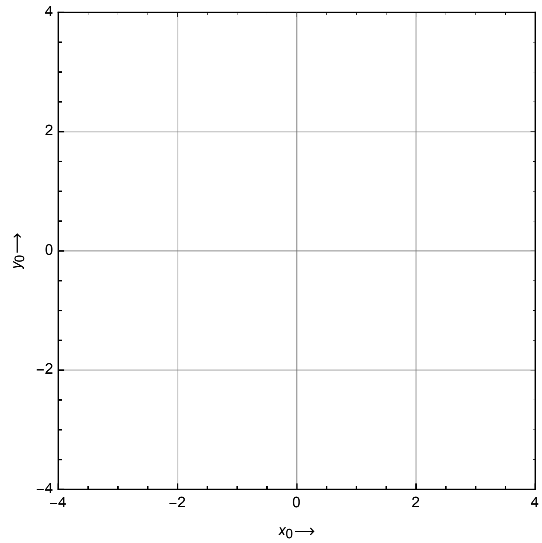
RPR robot for inverse kinematics.

Shown: $(\theta_1, d_2, \theta_3) = (\frac{\pi}{4}, 1, \frac{7\pi}{32})$









a.) **(2pt)** draw & shade x_1, y_1 cross-section of manipulator's *workspace* at $z_1 = 1$. Label all radii.

b.) **(8pt)** draw & shade x_0, y_0 cross-section of manipulator's *workspace* at $z_0 = 2$. Label all radii.

What joint variables place the end-effector at the point $[x_c, y_c, z_c]$ specified in the frame $o_0 x_0 y_0 z_0$? Assume the point is reachable and that $0 \leq \theta_3^* \leq \pi/2$.

c.) **(5pt)** $\theta_3^* =$

d.) **(5pt)** $d_2^* =$

e.) **(5pt)** $\theta_1^* =$

Problem 5: _____/15

Calculate the manipulator Jacobian of the 2-link RR arm at the position $o_2 = o_c$.

$$T_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 & 4c_1 \\ s_1 & c_1 & 0 & 4s_1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_2^0 = \begin{bmatrix} c_{1+2} & 0 & -s_{1+2} & 4c_1 + 2\,c_{1+2} \\ s_{1+2} & 0 & c_{1+2} & 4s_1 + 2\,s_{1+2} \\ s_2 & 0 & 0 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a. **(3 pts)** Write out the J matrix in terms of z_i and o_i.

b. **(2 pt)** Write out the z_i and o_i values needed for part a.

c. **(10 pts)** Write out the J matrix. Calculate the cross products.

Problem 6: _____/10 Computer Vision

a. **(5 pt)** frames $o_1x_1y_1z_1$ and $o_0x_0y_0z_0$ are related by homogenous transformation H_0^1. A particle has position $[4,2,-6]^T$ relative to frame $o_0x_0y_0z_0$. What is the position of the particle in frame $o_1x_1y_1z_1$?

$$H_0^1 = \begin{bmatrix} 0 & 0 & 1 & -8 \\ -1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b. **(5 pt)** For a camera with focal length $\lambda = 4$, suppose the optical axis is parallel with the world y-axis, the camera x-axis is parallel to the negative world z-axis, (and thus camera y-axis parallel to negative world x-axis) and the center of projection has world coordinates $[3,-9,6]^T$.

Compute $[18,3,18]^W$→ (, ,)^C

And convert these to image plane coordinates → (u,v)=