

Algorithms For Shaping a Particle Swarm With a Shared Control Input Using Boundary Interaction*

Shiva Shahrokhi, Arun Mahadev, and Aaron T. Becker

Abstract— Consider a swarm of particles controlled by global inputs. This paper presents algorithms for shaping such swarms in 2D using boundary walls. The range of configurations created by conforming a swarm to a boundary wall is limited. We describe the set of stable configurations of a swarm in two canonical workspaces, a circle and a square. To increase the diversity of configurations, we add boundary interaction to our model. We provide algorithms using friction with walls to place two robots at arbitrary locations in a rectangular workspace. Next, we extend this algorithm to place n agents at desired locations. We conclude with efficient techniques to control the covariance of a swarm not possible without wall-friction. Simulations and hardware implementations with 100 robots validate these results.

These methods may have particular relevance for micro- and nano-robots controlled by global inputs.

I. INTRODUCTION

Particle swarms propelled by a global force are common in applied mathematics, biology, and computer graphics. If the i th particle has position $[x_i, y_i]$ and velocity $[\dot{x}_i, \dot{y}_i]$, then

$$[\dot{x}_i, \dot{y}_i]^\top = \mathbf{u}(t), \quad i \in [1, n]. \quad (1)$$

These system dynamics represent particle swarms in low-Reynolds number environments, where viscosity dominates inertial forces and so velocity is proportional to input force [?]. In this regime, the input force command $\mathbf{u}(t)$ controls the velocity of the robots. The same model can be generalized to particles moved by fluid flow where the vector direction of fluid flow $\mathbf{u}(t)$ controls the velocity of particles, or for a swarm of robots that move at a constant speed in a direction specified by a global input $\mathbf{u}(t)$ [15]. Our control problem is to design the control inputs $\mathbf{u}(t)$ to make all n particles achieve a task. As a current example, micro- and nano-robots can be manufactured in large numbers, see [1], [2], [3], [4], [5], [6] or [7]. Someday large swarms of robots will be remotely guided to assemble structures in parallel and through the human body to cure disease, heal tissue, and prevent infection. For each application, large numbers of micro robots are required to deliver sufficient payloads, but the small size of these robots makes it difficult to perform onboard computation. Instead, these robots are often controlled by a global, broadcast signal. They require techniques to shape the swarm that can reliably exploit large populations despite severe underactuation ($2 \ll 2n$).

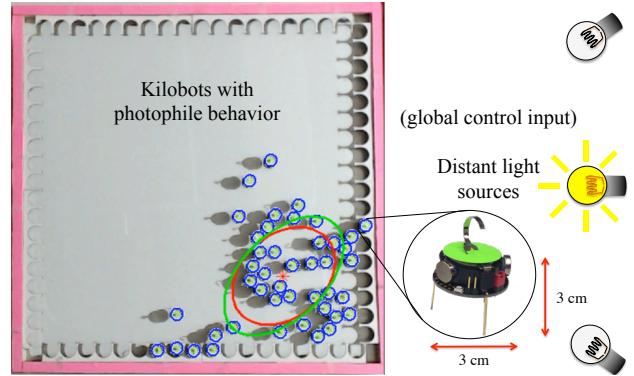


Fig. 1. Swarm of kilobots programmed to move toward the brightest light source as explained in Sec. VI. The current covariance ellipse and mean are shown in red, the desired covariance is shown in green. Navigating a swarm using global inputs is challenging because each member receives the same control inputs. This paper focuses on using boundary walls and wall friction to break the symmetry caused by the global input and control the shape of a swarm. See multimedia attachment for overview of the experiments.

Even without obstacles or boundaries, the mean position of the swarm in (1) is controllable. By adding rectangular boundary walls, some higher-order moments, such as the swarm's position variance orthogonal to the boundary walls (σ_x and σ_y for a workspace with axis-aligned walls), are also controllable [8]. These techniques could only compress a swarm orthogonal to obstacles. Swarm shape control is necessary for navigation through narrow passages, for self-assembly, and for manipulating objects.

The paper is arranged as follows. After a review of recent related work in Sec. II, Sec. III-A provides analytical position control results of stable configurations in two canonical workspaces with frictionless walls. These results are limited in the set of shapes that can be generated. To extend the range of possible shapes, Sec. III-B introduces general models for boundary interaction. We prove that two orthogonal boundaries with high friction are sufficient to arbitrarily position two robots in Sec. IV-A, and Sec. IV-B extends this to prove a rectangular workspace with high-friction boundaries can position a swarm of n robots arbitrarily within a subset of the workspace. Both algorithms require robots with low process noise and require time proportional to the number of robots, so Sec. IV-C introduces a robust and efficient technique to control swarm covariance. Sec. V describes implementations of the algorithms in simulation and Sec. VI describes hardware experiments with up to 100 robots, as shown in Fig. 1. We end with directions for future research in Sec. VII.

*This work was supported by the National Science Foundation under Grant No. [IIS-1553063].

Authors are with the Department of Electrical and Computer Engineering, University of Houston, Houston, TX 77204 USA {sshahrokhi2, aviswanathanmahadev, atbecker}@uh.edu

II. RELATED WORK

Controlling the *shape*, or relative positions, of a swarm of robots is a key ability for a range of applications. Correspondingly, it has been studied from a control-theoretic perspective in both centralized and decentralized approaches. For examples of each, see the centralized virtual leaders in [19], and the gradient-based decentralized controllers using control-Lyapunov functions in [20]. However, these approaches assume a level of intelligence and autonomy in individual robots that exceeds the capabilities of many systems, including current micro- and nano-robots. Current micro- and nano-robots, such as those in [1], [21], [22] lack onboard computation.

Instead, this paper focuses on centralized techniques that apply the same control input to each member of the swarm. Precision control requires breaking the symmetry caused by the global input. Symmetry can be broken using agents that respond differently to the global control, either through agent-agent reactions, see work modeling biological swarms [23], or engineered inhomogeneity [4], [24], [25]. This work assumes a uniform control (1) with homogenous agents, as in [26]. The techniques in this paper are inspired by fluid-flow techniques and artificial force-fields.

Fluid-flow: A shear force pushes different parts of a body in opposing directions. They are common in fluid flow along boundaries. Most introductory fluid dynamics textbooks provide models, e.g. [27]. Similarly, a swarm of robots under global control pushed along a boundary will experience shear forces. This is a position-dependent force, and so can be exploited to control the configuration or shape of the swarm. Physics-based swarm simulations used these forces to disperse a swarm's spatial position for coverage in [28].

Artificial Force-fields: Much research has focused on generating non-uniform artificial force-fields that can be used to rearrange passive components. Applications have included techniques to design shear forces for sensorless manipulation of a single object by [29]. [30], [31] demonstrated a collection of 2D force fields generated by 6DOF vibration inputs to a rigid plate. These force fields, including shear forces, could be used as a set of primitives for motion control to steer the formation of multiple objects. However unlike the uniform control model in this paper, theirs was multi-modal and position-dependent.

III. THEORY

A. Using Boundaries: stable configurations of a swarm

One method to control a swarm's shape in a bounded workspace is to simply push in a given direction until the swarm conforms to the boundary. Like fluid settling in a tank, the stable final configuration minimizes potential energy.

a) *Square workplace:* We first examine the mean (\bar{x}, \bar{y}) , covariance $(\sigma_x^2, \sigma_y^2, \sigma_{xy})$, and correlation ρ_{xy} of a very large swarm of robots as they move inside a square workplace under the influence of gravity pointing in the direction β . The swarm is large, but the robots are small in

comparison, and together occupy a constant area A ranging from $[0,1]$. Under a global input such as gravity, they flow like water, moving to a side of the workplace and forming a polygonal shape, as shown in Fig. 2, see also [9].

The range for the global input angle β is $[0, 2\pi)$. In this range, the swarm assumes eight different polygonal shapes. These shapes alternate between triangles and trapezoids when the area $A < 1/2$, and between squares with one corner removed and trapezoids when $A > 1/2$.

Computing means, variances, covariance, and correlation requires integrating over the region R , the region containing the swarm:

$$\bar{x} = \frac{\iint_R x \, dx \, dy}{A}, \quad \bar{y} = \frac{\iint_R y \, dx \, dy}{A} \quad (2)$$

$$\sigma_x^2 = \frac{\iint_R (x - \bar{x})^2 \, dx \, dy}{A}, \quad \sigma_y^2 = \frac{\iint_R (y - \bar{y})^2 \, dx \, dy}{A} \quad (3)$$

$$\sigma_{xy} = \frac{\iint_R (x - \bar{x})(y - \bar{y}) \, dx \, dy}{A}, \quad \rho_{xy} = \frac{\sigma_{xy}^2}{\sigma_x \sigma_y} \quad (4)$$

The region of integration R is the polygon containing the swarm. For example, if $A < 1/2$ and the force angle is β , the mean when R is a triangular region in the lower-left corner is:

$$\bar{x}(A, \beta) = \frac{\int_0^{\sqrt{2A \tan(\beta)}} \left(\int_0^{\cot(\beta)(\sqrt{2A \tan(\beta)} - x)} dy \right) x \, dx}{A} \\ = \frac{\sqrt{2}}{3} \sqrt{A \tan(\beta)} \quad (5)$$

$$\bar{y}(A, \beta) = \frac{\int_0^{\sqrt{2A \tan(\beta)}} \left(\int_0^{\cot(\beta)(\sqrt{2A \tan(\beta)} - x)} y \, dy \right) dx}{A} \\ = \frac{\sqrt{2}}{3} \sqrt{A \cot(\beta)} \quad (6)$$

The full equations are included in the appendix [11], and are summarized in Fig. 2. A few highlights are that the correlation is maximized $\pm 1/2$ when the swarm is in a triangular shape. The covariance of a triangle is always $\pm(A/18)$. Variance is minimized in the direction of β and maximized orthogonal to β when the swarm is in a rectangular shape. The range of mean positions are maximized when A is small.

b) *Circular workplace:* Though rectangular boundaries are common in artificial workspaces, biological workspaces are usually rounded. Similar calculations can be made for a circular workspace. The workspace is a circle centered at $(0,0)$ with radius 1 and thus area π . For notational simplicity, the swarm is parameterized by the global control input signal β and the fill-level h . Under a global input, the robot swarm fills the region under a chord with area

$$A(h) = \arccos(1 - h) - (1 - h)\sqrt{(2 - h)h}. \quad (7)$$

For a circular workspace, the locus of mean positions are aligned with β and the mean position is at radius $r(h)$ from



Fig. 2. Pushing the swarm against a square boundary wall allows limited control of the shape of the swarm, as a function of swarm area A and the commanded movement direction β . Left plot shows locus of possible mean positions for five values of A . The locus morphs from a square to a circle as A increases. The covariance ellipse for each A is shown with a dashed line. Center shows two corresponding arrangements of kilobots. At right is $\bar{x}(A)$, $\sigma_{xy}(A)$, $\sigma_x^2(A)$, and $\rho(A)$ for a range of β values. See online interactive demonstration at [9].



Fig. 3. Pushing the swarm against a circular boundary wall allows limited control of the shape of the swarm, as a function of the fill level h and the commanded movement direction β . Left plot shows locus of possible mean positions for four values of h . The locus of possible mean positions are concentric circles. See online interactive demonstration at [10].

the center:

$$r(h) = \frac{2(-(h-2)h)^{3/2}}{3(\sqrt{-(h-2)h}(h-1) + \arccos(1-h))} \quad (8)$$

Variance $\sigma_x^2(\beta, h)$ is maximized at $\beta = \pi/2 + n\pi$ and $h \approx 1.43$, while covariance is maximized at $\beta = \pi 3/4 + n\pi$ and $h \approx 0.92$. For small h values, correlation approaches ± 1 . Results are displayed in Fig. 3, see also [10].

B. Using Boundaries: Friction and Boundary Layers

Global inputs move a swarm uniformly. Shape control requires breaking this uniform symmetry. A swarm inside an axis-aligned rectangular workspace can reduce variance normal to a wall by simply pushing the swarm into the boundary. Controlling covariance by pushing the swarm into a boundary requires changes to the boundary. An obstacle

in the lower-right corner is enough to generate positive covariance, but generating both positive and negative covariance requires additional obstacles. Requiring a special obstacle configuration also makes shape control dependent on the local environment. Instead of pushing our robots directly into a wall, this paper examines an oblique approach, by using boundaries that generate friction with the robots. These frictional forces are sufficient to break the symmetry caused by uniform inputs. Robots touching a wall have a friction force that opposes movement along the boundary. This causes robots along the boundary to move more slowly than robots in free-space.

Let the control input be a vector force \vec{F} with magnitude F and orientation θ with respect to a line perpendicular to and into the nearest boundary. N is the normal or perpendicular force between the robot and the boundary. The force of friction F_f is nonzero if the robot is in contact with the

boundary and $\sin(\theta) < 0$. The resulting net force on the robot, $F_{forward}$, is aligned with the wall and given by

$$F_{forward} = F \sin(\theta) - F_f$$

$$\text{where } F_f = \begin{cases} \mu_f N, & \mu_f N < F \sin(\theta) \\ F \sin(\theta), & \text{else} \end{cases} \quad (9)$$

and $N = F \cos(\theta)$

Fig. 4 shows the resultant forces on two robots when one is touching a wall. Though each receives the same inputs, they experience different net forces. For ease of analysis, the following algorithms assume μ_f is infinite and robots touching the wall are prevented from sliding along the wall. This means that if one robot is touching the wall and another robot is free, the touching robot will not move when the control input is parallel or into the wall. There are many alternate models of friction that also break control symmetry. Fig. 4c shows fluid flow along a boundary. Fluid in the free-flow region moves uniformly, but flow decreases to zero in the boundary layer.

$$F_{forward}(y) = F - F_f \begin{cases} \left(\frac{y-h}{h}\right)^2, & y < h \\ 0, & \text{else} \end{cases} \quad (10)$$

The next section shows how a system with friction model (9) and two orthogonal walls can arbitrarily position two robots.

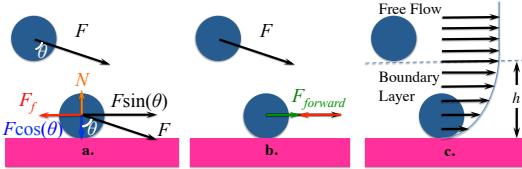


Fig. 4. (a,b) Wall friction reduces the force for going forward $F_{forward}$ on a robot near a wall, but not for a free robot. (c) velocity of a fluid reduces to zero at the boundary.

IV. ALGORITHMS

A. Position Control of 2 Robots Using Wall Friction

Alg. 1 uses wall-friction to arbitrarily position two robots in a rectangular workspace. This algorithm introduces concepts that will be used for multi-robot positioning. It only requires collisions with two orthogonal walls, in this case, the bottom and left walls. Fig. 5 shows a Mathematica implementation of the algorithm, and is useful as a visual reference for the following description.

Assume two robots are initialized at s_1 and s_2 with corresponding goal destinations e_1 and e_2 . Denote the current positions of the robots r_1 and r_2 . Subscripts x and y denote the x and y coordinates, i.e., s_{1x} and s_{1y} denote the x and y locations of s_1 . The algorithm assigns a global control input at every instance. The goal is to adjust $\Delta r_x = r_{2x} - r_{1x}$ from $\Delta s_x = s_{2x} - s_{1x}$ to $\Delta e_x = e_{2x} - e_{1x}$ and adjust $\Delta r_y = r_{2y} - r_{1y}$ from $\Delta s_y = s_{2y} - s_{1y}$ to $\Delta e_y = e_{2y} - e_{1y}$ using a shared global control input. This algorithm exploits the position-dependent friction model (9).

Our algorithm solves the positioning problem in three steps: First, $|\Delta r_x - \Delta e_x|$ is reduced to zero while Δr_y is kept constant in Alg. 2. Second, $|\Delta r_y - \Delta e_y|$ is reduced to zero while Δr_x is kept constant. Third, the robots, now correctly positioned relative to each other, are moved to their goal locations.

In the worse case, adjusting both Δr_x and Δr_y requires two iterations. Two iterations of Alg. 2 are only required if $|\Delta e_x - \Delta s_x| > L$. Similarly, two iterations are only required if $|\Delta e_y - \Delta s_y| > L$. The worst case path length is $6L$.

Algorithm 1 WallFrictionArrange2Robots(s_1, s_2, e_1, e_2, L)

Require: starting (s_1, s_2) and ending (e_1, e_2) positions of two robots. $(0, 0)$ is bottom corner, s_1 is rightmost robot, L is length of the walls. Current robot positions are (r_1, r_2) .

- 1: GenerateDesiredx-spacing(s_1, s_2, e_1, e_2, L)
- 2: GenerateDesiredy-spacing(r_1, r_2, e_1, e_2, L)
- 3: Move robots $e_1 - r_1$

Algorithm 2 GenerateDesired spacing(s_1, s_2, e_1, e_2, L)

Require: knowledge of starting (s_1, s_2) and ending (e_1, e_2) positions of two robots. $(0, 0)$ is bottom corner, s_1 is top-leftmost robot, L is length of the walls. Current robot positions are (r_1, r_2) .

Ensure: $(e_1, e_2), (s_1, s_2)$ all have ϵ distance with the walls.

Ensure: $s_{1.x} > s_{2.x}$ and $s_{1.y} \geq s_{2.y}$ if not, rotate the workspace 90° .

- 1: ϵ \leftarrow diameter of the robot
- 2: $\Delta e_y = e_2.y - e_1.y$
- 3: $\Delta e_x = e_2.x - e_1.x$
- 4: **if** $\Delta e_y \neq 0$ **then**
- 5: $sign \leftarrow \frac{|\Delta e_y|}{\Delta e_y}$
- 6: **else**
- 7: $sign \leftarrow 0$
- 8: **end if**
- 9: Move by $(-r_1.x, -\min(r_2.y, r_1.y + \Delta e_y) + \epsilon)$ \triangleright Touch left wall
- 10: Move by $(-\min(|-r_2.x + \Delta e_x|, r_2.x), sign \min(\max(r_1.y + \Delta e_y, r_2.y) - \epsilon, L - \epsilon) + \epsilon)$ \triangleright Adjust y
- 11: **if** $r_2.y \neq r_1.y$ **then**
- 12: Move by $(\max(\epsilon, -\Delta e_x + (r_2.x - r_1.x)), -\min(r_1.y, r_2.y))$ \triangleright Touch bottom wall
- 13: Move by $(\Delta e_x, (|sign|-1)\epsilon)$ \triangleright Adjust x
- 14: **end if**
- 15: **if** $\Delta e_y \neq r_2.y - r_1.y$ **then**
- 16: Move by $(L - r_1.x, \epsilon)$ \triangleright Touch Right wall
- 17: Move by $(0, \Delta e_y - r_2.y)$ \triangleright Adjust y
- 18: **end if**
- 19: Move by $(e_2.x - r_2.x, e_2.y - r_2.y)$ \triangleright Go to the goal position

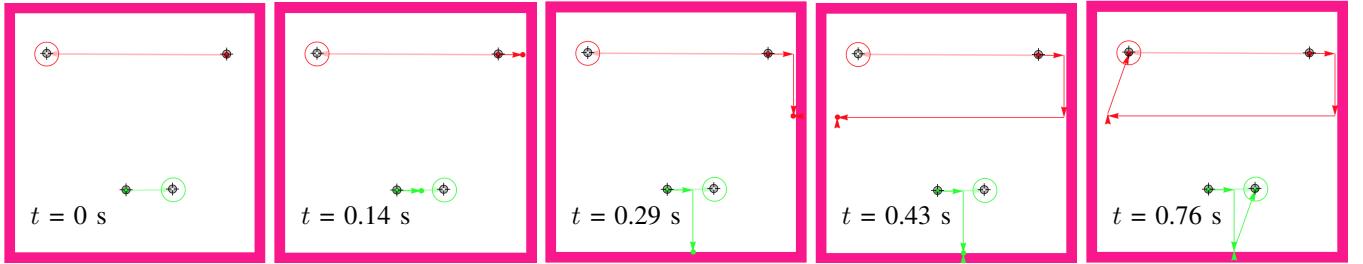


Fig. 5. Frames from an implementation of Alg. 1: two robot positioning using walls with infinite friction. The algorithm only requires friction along the bottom and right walls. Robot initial positions are shown by a crosshair, and final positions by a circled crosshair. Dashed lines show the shortest route if robots could be controlled independently. Solid arrows show path given by Alg. 1. Online demonstration and source code at [12].

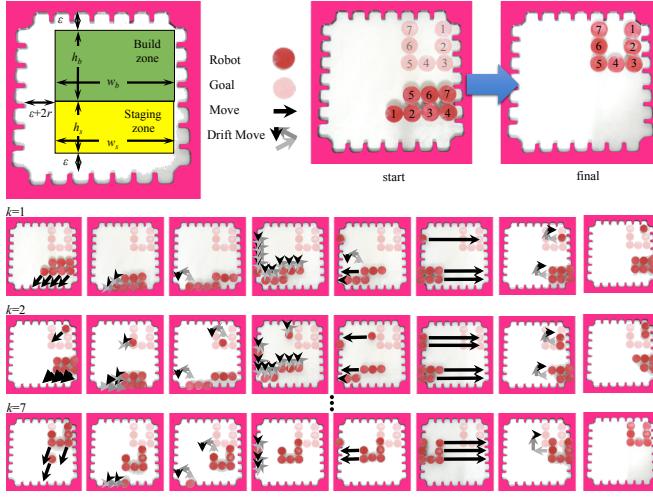


Fig. 6. Illustration of Alg. 3, n robot position control using wall friction.

B. Position Control of n Robots Using Wall Friction

Alg. 1 can be extended to control the position of n robots using wall friction under several constraints. The solution described here is an iterative procedure with n loops. The k th loop moves the k th robot from a *staging zone* to the desired position in a *build zone*. All robots move according to the global input, but due to wall friction, at the end the k th loop, robots 1 through k are in their desired final configuration in the build zone, and robots $k+1$ to n are in the staging zone. See Fig. 6 for a schematic of the build and staging zones.

Assume an open workspace with four axis-aligned walls with infinite friction. The axis-aligned build zone of dimension (w_b, h_b) containing the final configuration of n robots must be disjoint from the axis-aligned staging zone of dimension (w_s, h_s) containing the starting configuration of n robots. Without loss of generality, assume the build zone is above the staging zone. Furthermore, there must be at least ϵ space above the build zone, ϵ below the staging zone, and $\epsilon + 2r$ to the left of the build and staging zone, where r is the radius of a robot. The minimum workspace is then $(\epsilon + 2r + \max(w_b, w_s), 2\epsilon + h_s, h_b)$.

The n robot position control algorithm relies on a $\text{DriftMove}(\alpha, \beta, \epsilon)$ control input, shown in Fig. 7. A drift move consists of repeating a triangular movement sequence $\{(\beta/2, -\epsilon), (\beta/2, \epsilon), (-\alpha, 0)\}$. The robot touching a top wall

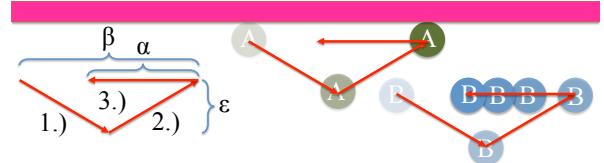


Fig. 7. A $\text{DriftMove}(\alpha, \beta, \epsilon)$ to the right repeats a triangular movement sequence $\{(\beta/2, -\epsilon), (\beta/2, \epsilon), (-\alpha, 0)\}$. Robot A touching a top wall moves right β units, while robots not touching the top move right $\beta - \alpha$.

moves right β units, while robots not touching the top right $\beta - \alpha$.

Let $(0, 0)$ be the lower left corner of the workspace, p_k the x, y position of the k th robot, and f_k the final x, y position of the k th robot. Label the robots in the staging zone from left-to-right and bottom-to-top, and the f_k configurations top-to-bottom and right-to-left as shown in Fig. 6.

Algorithm 3 PositionControlInRobotsUsingWallFriction(k)

```

1: move(  $-\epsilon, r - p_{ky}$ )
2: while  $p_{kx} > r$  do
3:   DriftMove( $\epsilon, \min(p_{kx} - r, \epsilon), \epsilon$ ) left
4: end while
5:  $m \leftarrow \text{ceil}(\frac{f_{ky}-r}{\epsilon})$ 
6:  $\beta \leftarrow \frac{f_{ky}-r}{m}$ 
7:  $\alpha \leftarrow \beta - \frac{r-p_{ky}-\epsilon}{m}$ 
8: for  $m$  iterations do
9:   DriftMove( $\alpha, \beta, \epsilon$ ) up
10: end for
11: move( $r + \epsilon - f_{kx}, 0$ )
12: move( $f_{kx} - r, 0$ )

```

Alg. 3 proceeds as follows: First, the robots are moved left away from the right wall, and down so robot k touches the bottom wall. Second, a set of DriftMoves are executed that move robot k to the left wall with no net movement of the other robots. Third, a set of DriftMoves are executed that move robot k to its target height and return the other robots to their initial heights. Fourth, all robots except robot k are pushed left until robot k is in the correct relative x position compared to robots 1 to $k-1$. Finally, all robots are moved right until robot k is in the desired target position. Running time is $O(n(w+h))$.

C. Controlling Covariance Using Wall Friction

Assume an obstacle-free, bounded workspace with infinite boundary friction. Goal variances and covariance are $(\sigma_{goalx}^2, \sigma_{goaly}^2, \sigma_{goalxy})$ and mean, variances and covariance of the swarm are $(\bar{x}, \bar{y}, \sigma_x^2, \sigma_y^2, \sigma_{xy})$.

- 1) swarm pushed into the left wall until $\sigma_x^2 \leq c_1 \sigma_{goalx}^2$.
- 2) swarm's mean position is moved to the center of the workspace
- 3) swarm pushed into the bottom wall until $\sigma_y^2 \leq \sigma_{goaly}^2$.
- 4) if $\sigma_{goalxy} > 0$ swarm slides right until $\sigma_{xy} \geq \sigma_{goalxy}$
else swarm slides left until $\sigma_{xy} \leq \sigma_{goalxy}$
- 5) swarm's mean position is moved to the center of the workspace

The constant c_1 in step 1 should be less than 1 to ensure σ_x^2 stays within tolerance during the subsequent moves.

V. SIMULATION

Two simulations were implemented using wall-friction for position control. The first controls the position of two robots, the second controls the position of n robots.

Two additional simulations were performed using wall-friction to control variance and covariance. The first is an open-loop algorithm that demonstrates the effect of varying friction levels. The second uses a closed-loop controller to achieve desired variance and covariance values.

A. Position Control of Two Robots

Algorithms 1 and 2, were implemented in Mathematica using point robots (radius = 0). Fig. 5 shows an implementation of this algorithm. Robot initial positions are shown by crosshairs, and final positions by circled crosshairs. Dashed lines show the shortest route if robots could be controlled independently. The path given by Alg. 1 is shown with solid arrows. For the sake of brevity axis-aligned moves were replaced with oblique moves that combine two moves simultaneously. An online interactive demonstration and source code of the algorithm are available at [12].

B. Position Control of n Robots

Alg. 3 was simulated in MATLAB using square block robots with unity width. Code is available at [13]. Simulation results are shown in Fig. 8 for arrangements with an increasing number of robots, $n = [8, 46, 130, 390, 862]$. The distance moved grows quadratically with the number of robots n . A best-fit line $210n^2 + 1200n - 10,000$ is overlaid by the data.

In Fig. 8, the amount of clearance is $\epsilon = 1$. Control performance is sensitive to the desired clearance. As ϵ increases, the total distance decreases asymptotically, as shown in Fig. 9, because the robots have more room to maneuver and fewer DriftMoves are required.

C. Efficient Control of Covariance

Random disturbances impair the performance of Alg. 1 and Alg. 3. However, due to the central limit theorem, we can control the covariance of large swarms. This section demonstrates controlling swarm covariance in simulation, using the 2D physics engine Box2D [14]. 144 disc-shaped

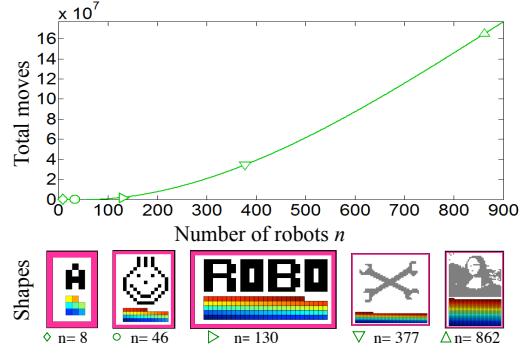


Fig. 8. The required number of moves under Alg. 3 using wall-friction to rearrange n square-shaped robots. See hardware implementation and simulation at [13].

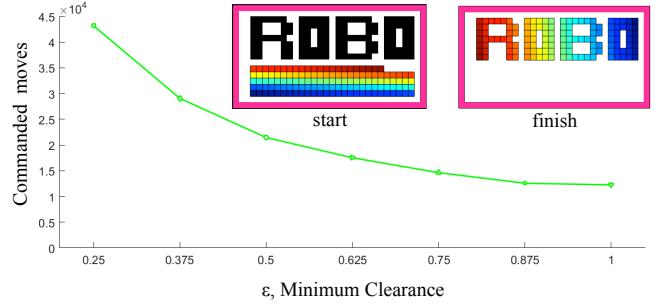


Fig. 9. Control performance is sensitive to the desired clearance ϵ . As ϵ increases, the total distance decreases asymptotically.

robots with boundary layer model (10) were controlled by an open-loop control input as illustrated in Fig. 10. All robots had the same initial conditions, but in four tests the boundary friction was $F_f = \{0, 1/3F, 2/3F, F\}$. Without friction, covariance has minimal variation. As friction increases, the covariance can be manipulated to greater degrees.

144 disc-shaped robots were also controlled by a closed-loop controller using the procedure in Sec. IV-C with $c_1 = 0.1$. Fig. 11 illustrates that covariance and variances in x and y directions were controlled from a set of initial conditions.

VI. EXPERIMENT

Our experiments are on centimeter-scale hardware systems called *kilobots*. These allows us to emulate a variety of dynamics, while enabling a high degree of control over robot function, the environment, and data collection. The kilobot, from [15, 16] is a low-cost robot designed for testing collective algorithms with large numbers of robots. It is available as an open source platform or commercially from [17]. Each robot is approximately 3 cm in diameter, 3 cm tall, and uses two vibration motors to move on a flat surface at speeds up to 1 cm/s. Each robot has one ambient light sensor that is used to implement *phototaxis*, moving towards a light source. In these experiments as shown in Fig. 13, we used $n=100$ kilobots, a 1 m \times 1 m whiteboard as the workspace, four 30W and four 50W LED floodlights arranged 1.5 m above the plane of the table at the midpoints and vertices of a 6 m square centered on the workspace. An

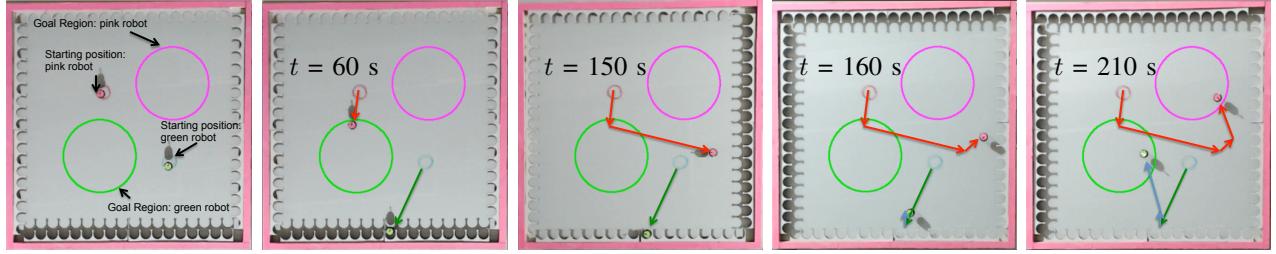


Fig. 12. Position control of two kilobots (Alg. 2) steered to corresponding colored circle. Boundary walls have nearly infinite friction, so the green robot is stopped by the wall from $t = 60\text{s}$ until the commanded input is directed away from the wall at $t = 150\text{s}$, while the pink robot in free-space is unhindered.

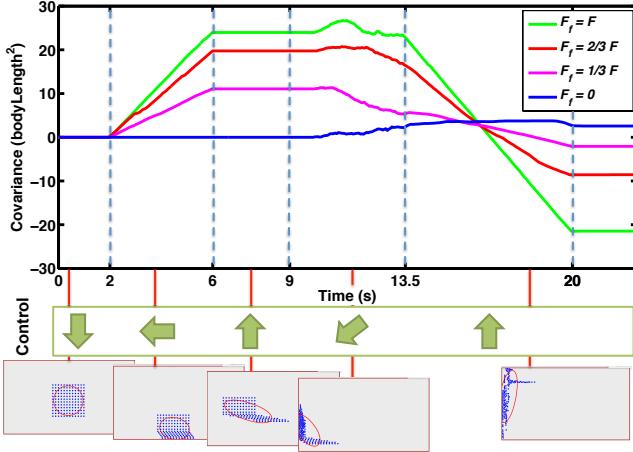


Fig. 10. Open-loop simulation with 144 disc robots and varying levels of boundary friction under the same initial conditions. Without friction, covariance is unchangeable. As friction increases, the covariance can be manipulated to greater degrees.

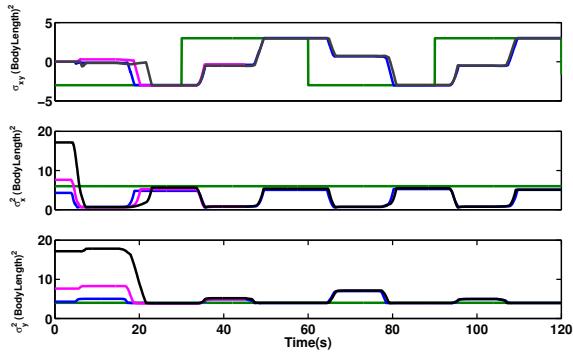


Fig. 11. Closed-loop simulation with 144 disc robots and three sets of initial conditions. The algorithm tracks goal variance and covariance values (green). The goal covariance switches sign every 30 s.

Arduino Uno connected to an 8-relay shield controlled the lights. Above the table, an overhead machine vision system tracked the position of the swarm.

A. Hardware Experiment: Position Control of Two Robots

The hardware platform walls have almost infinite friction, due to a laser-cut, scalloped border. When a kilobot is steered into the scalloped border, they are pinned to the wall unless the global input directs them away from the wall. Even though the kilobots are stochastic, this wall friction is

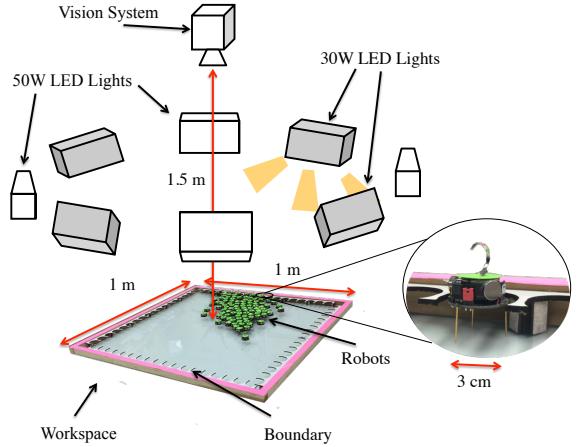


Fig. 13. Hardware platform: table with $1 \times 1\text{ m}$ workspace, surrounded by eight triggered LED floodlights, with an overhead machine vision system.

sufficient to enable independent control of two kilobots, as shown in Fig. 12.

B. Hardware Experiment: Position Control of n Robots

Kilobots have too much stochasticity to implement Alg. 3. Instead, a hardware setup with a bounded platform, magnetic sliders, and a magnetic guide board was used. Designs for each are available at [18]. The pink boundary is toothed with a white free space, as shown in Fig 6. Only discrete, 1 cm moves in the x and y directions are used. The goal configuration highlighted in the top right corner represents a ‘U’ made of seven sliders. The dark red configuration is the current position of the sliders. Due to the discretized movements allowed by the boundary, drift moves follow a 1 cm square. Free robots return to their start positions but robots on the boundary to move laterally, generating a net sliding motion.

Fig. 6 follows the motion of the sliders through iterations $k=1, 2$, and 7 . All robots receive the same control inputs, but boundary interactions break the control symmetry. Robots reach their respective goal position in a first-in, first-out arrangement beginning with the bottom-left robot from the staging zone occupying the top-right position of the build zone.

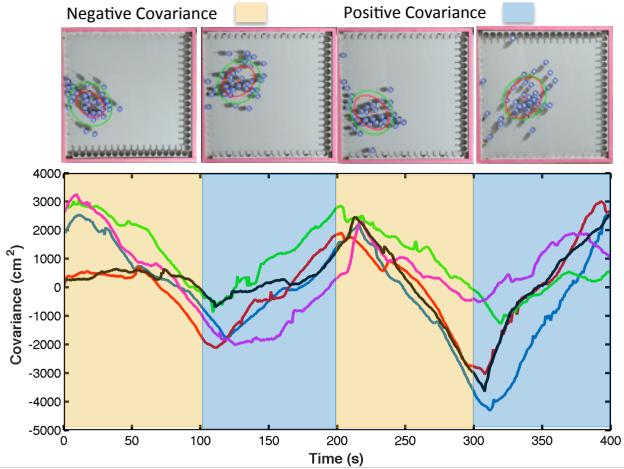


Fig. 14. Hardware demonstration steering ≈ 50 kilobot robots to desired covariance. The goal covariance is negative in first 100 seconds and is positive in the next 100 seconds. The actual covariance is shown in different trials. Frames above the plot show output from machine vision system and an overlaid covariance ellipse.

C. Hardware Experiment: Control of Covariance

To demonstrate covariance control, up to 100 kilobots were placed on the workspace and manually steered with lights, using friction with the boundary walls to vary the covariance from -4000 to 3000 cm^2 . The resulting covariance is plotted in Fig. 14, along with snapshots of the swarm.

VII. CONCLUSION AND FUTURE WORK

This paper presented techniques for controlling the shape of a swarm of robots using uniform global inputs and interaction with boundary friction forces. The paper provided algorithms for precise position control, as well as robust and efficient covariance control. Extending algorithms 1 and 2 to 3D is straightforward but increases the complexity. Future efforts should be directed toward improving the technology and tailoring it to specific robot applications.

With regard to technological advances, this includes designing controllers that efficiently regulate σ_{xy} , perhaps using Lyapunov-inspired controllers as in [3]. Additionally, this paper assumed nearly infinite wall friction. The algorithms require retooling to handle small μ_f friction coefficients. It may be possible to rank controllability as a function of friction.

ACKNOWLEDGMENTS

We thank Haoran Zhao, Jarret Lonsford, An Nguyen, and Lillian Lin for help in making structures for the experiments.

REFERENCES

- [1] S. Chowdhury, W. Jing, and D. J. Cappelleri, “Controlling multiple microrobots: recent progress and future challenges,” *Journal of Micro-Bio Robotics*, vol. 10, no. 1-4, pp. 1–11, 2015.
- [2] S. Martel, S. Taherkhani, M. Tabrizian, M. Mohammadi, D. de Lanauze, and O. Felfoul, “Computer 3d controlled bacterial transports and aggregations of microbial adhered nano-components,” *Journal of Micro-Bio Robotics*, vol. 9, no. 1-2, pp. 23–28, 2014.
- [3] P. S. S. Kim, A. Becker, Y. Ou, A. A. Julius, and M. J. Kim, “Imparting magnetic dipole heterogeneity to internalized iron oxide nanoparticles for microorganism swarm control,” *Journal of Nanoparticle Research*, vol. 17, no. 3, pp. 1–15, 2015.
- [4] B. R. Donald, C. G. Levey, I. Paprotny, and D. Rus, “Planning and control for microassembly of structures composed of stress-engineered mems microrobots,” *The International Journal of Robotics Research*, vol. 32, no. 2, pp. 218–246, 2013.
- [5] A. Ghosh and P. Fischer, “Controlled propulsion of artificial magnetic nanostructured propellers,” *Nano Letters*, vol. 9, no. 6, pp. 2243–2245, 2009.
- [6] Y. Ou, D. H. Kim, P. Kim, M. J. Kim, and A. A. Julius, “Motion control of magnetized *tetrahymena pyriformis* cells by magnetic field with model predictive control,” *Int. J. Rob. Res.*, vol. 32, no. 1, pp. 129–139, Jan. 2013.
- [7] F. Qiu and B. J. Nelson, “Magnetic helical micro-and nanorobots: Toward their biomedical applications,” *Engineering*, vol. 1, no. 1, pp. 21–26, 2015.
- [8] S. Shahrokh and A. T. Becker, “Stochastic swarm control with global inputs,” in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Sep. 2015, p. tbd.
- [9] H. Zhao and A. T. Becker, “Distribution of a robot swarm in a square under gravity, wolfram demonstrations project,” Jan. 2016. [Online]. Available: <http://demonstrations.wolfram.com/DistributionOfARobotSwarmInASquareUnderGravity/>
- [10] ———, “Distribution of a swarm of robots in a circular workplace under gravity, wolfram demonstrations project,” Feb. 2016. [Online]. Available: <http://demonstrations.wolfram.com/DistributionOfASwarmOfRobotsInACircularWorkplaceUnderGravity/>
- [11] S. Shahrokh, A. Mahadev, and A. T. Becker, “Algorithms for shaping a particle swarm with a shared control input using boundary interaction,” *ArXiv e-prints*, Sep. 2016.
- [12] S. Shahrokh and A. T. Becker, “Moving Two Particles with Shared Control Inputs Using Wall Friction, Wolfram Demonstrations Project,” Nov. 2015. [Online]. Available: <http://demonstrations.wolfram.com/MovingTwoParticlesWithSharedControlInputsUsingWallFriction/>
- [13] A. V. Mahadev and A. T. Becker, “Arranging a robot swarm with global inputs and wall friction [discrete],” MATLAB Central File Exchange,” Dec. 2015. [Online]. Available: <https://www.mathworks.com/matlabcentral/fileexchange/54526>
- [14] E. Catto, “User manual, Box2D: A 2D physics engine for games, <http://www.box2d.org/>,” 2010.
- [15] M. Rubenstein, C. Ahler, and R. Nagpal, “Kilobot: A low cost scalable robot system for collective behaviors,” in *IEEE Int. Conf. Rob. Aut.*, May 2012, pp. 3293–3298.
- [16] M. Rubenstein, A. Cornejo, and R. Nagpal, “Programmable self-assembly in a thousand-robot swarm,” *Science*, vol. 345, no. 6198, pp. 795–799, 2014.
- [17] K-Team, “Kilobot, www.k-team.com,” 2015.
- [18] A. Mahadev, A. Nguyen, and A. T. Becker, “Position control using boundary interaction,” Sep. 2016. [Online]. Available: <http://www.thingiverse.com/thing:1761909>
- [19] M. Egerstedt and X. Hu, “Formation constrained multi-agent control,” *IEEE Trans. Robotics Automat.*, vol. 17, pp. 947–951, 2001.
- [20] M. A. Hsieh, V. Kumar, and L. Chaimowicz, “Decentralized controllers for shape generation with robotic swarms,” *Robotica*, vol. 26, no. 05, pp. 691–701, 2008.
- [21] S. Martel, “Magnetotactic bacteria for the manipulation and transport of micro- and nanometer-sized objects,” *Micro-and Nanomanipulation Tools*, 2015.
- [22] X. Yan, Q. Zhou, J. Yu, T. Xu, Y. Deng, T. Tang, Q. Feng, L. Bian, Y. Zhang, A. Ferreira, and L. Zhang, “Magnetite nanostructured porous hollow helical microwimmers for targeted delivery,” *Advanced Functional Materials*, vol. 25, no. 33, pp. 5333–5342, 2015.
- [23] A. L. Bertozi, T. Kolokolnikov, H. Sun, D. Uminsky, and J. Von Brecht, “Ring patterns and their bifurcations in a nonlocal model of biological swarms,” *Communications in Mathematical Sciences*, vol. 13, no. 4, 2015.
- [24] T. Bretl, “Control of many agents using few instructions,” in *Proceedings of Robotics: Science and Systems*, Atlanta, GA, USA, June 2007.
- [25] A. Becker, C. Onyukse, T. Bretl, and J. McLurkin, “Controlling many differential-drive robots with uniform control inputs,” *Int. J. Robot. Res.*, vol. 33, no. 13, pp. 1626–1644, 2014.
- [26] A. Becker, G. Habibi, J. Werfel, M. Rubenstein, and J. McLurkin, “Massive uniform manipulation: Controlling large populations of simple robots with a common input signal,” in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Nov. 2013, pp. 520–527.

- [27] B. R. Munson, A. P. Rothmayer, T. H. Okiishi, and W. W. Huebsch, *Fundamentals of Fluid Mechanics*, 7th ed. Wiley, 2012.
- [28] D. Spears, W. Kerr, and W. Spears, “Physics-based robot swarms for coverage problems,” *The international journal of intelligent control and systems*, vol. 11, no. 3, 2006.
- [29] F. Lamiraux and L. E. Kavraki, “Positioning of symmetric and non-symmetric parts using radial and constant fields: Computation of all equilibrium configurations,” *International Journal of Robotics Research*, vol. 20, no. 8, pp. 635–659, 2001.
- [30] T. Vose, P. Umbarhowar, and K. Lynch, “Friction-induced velocity fields for point parts sliding on a rigid oscillated plate,” *The International Journal of Robotics Research*, vol. 28, no. 8, pp. 1020–1039, 2009.
- [31] T. H. Vose, P. Umbarhowar, and K. M. Lynch, “Sliding manipulation of rigid bodies on a controlled 6-dof plate,” *The International Journal of Robotics Research*, vol. 31, no. 7, pp. 819–838, 2012.