MODEL PROF

1. $\begin{cases} x^3+y^3-3xy=0 \\ z=0 \end{cases}$ curbă directoare

generatoarele au v. dir. $v(2,-2,1)$

$\frac{x}{2}=\frac{y}{-2}=\frac{z}{1}$ (dreaptă cu v. dir v)

$\begin{cases} -2x=2y \Leftrightarrow x=-y \\ y=-2z \end{cases} \Leftrightarrow \begin{cases} x+y=\lambda \\ y+2z=\mu \end{cases}$

luăm 3 cele mai simple și rezolvăm

$\begin{cases} x+y=\lambda \Rightarrow x=\lambda-\mu \\ y+2z=\mu \Rightarrow y=\mu \\ z=0 \end{cases}$

înlocuim în ultima

$(\lambda-\mu)^3+\mu^3-3(\lambda-\mu)\mu=0$ apoi înlocuim înapoi din sistem GATA

2. $x^2+y^2-z^2=1$

$x^2-z^2=1-y^2 \Leftrightarrow (x-z)(x+z)=(1-y)(1+y)$

I $\Delta_1: \begin{cases} \lambda(x+z)=\mu(1+y) \\ \mu(x-z)=\lambda(1-y) \end{cases}$

înlocuim $(1,1,1)=(x,y,z) \Rightarrow 2\lambda=2\mu \Rightarrow \lambda=\mu$ alegem $=1$

$\Rightarrow \Delta_1: \begin{cases} x+z=1+y \\ x-z=1-y \end{cases}$

II $\begin{cases} \alpha(x-z)=\beta(1+y) \\ \beta(x+z)=\alpha(1-y) \end{cases} \Leftrightarrow 2\beta=0 \Rightarrow \beta=0 \Rightarrow$ alegem $\alpha=1$

$\Rightarrow \Delta_2: \begin{cases} x-z=0 \\ 1-y=0 \end{cases}$

4. $7x^2 - 2y^2 = 14$ tg la hiperbola $\quad$ v. dir$(-b, a)$

$x + 2y - 3 = 0$ $\perp$ pe dreapta d

ec. tg: $7x x_0 - 2y y_0 = 14$ are vect. director $(2y_0, -7x_0)$

$\perp$ pe d: $\Rightarrow$ v dir $\cdot$ v dir d $= 0$

$(2y_0, +7x_0)(-2, 1) = 0 \Rightarrow -4y_0 - 7x_0 = 0$

$\Rightarrow x_0 = \frac{4y_0}{+7}$

punctul verifică ec. tangentei: $\left(\frac{4y_0}{+7}\right)^2 \cdot 7 - 2y_0^2 = 14$

$\frac{16y_0^2}{7} - 2y_0^2 = 14 \quad / \cdot 7$

$16y_0^2 - 14y_0^2 = 98$

$2y_0^2 = 98 \Rightarrow y_0^2 = 49 \Rightarrow y_0 = \pm 7$

$\Rightarrow x_0 = \frac{4(\pm 7)}{-7}$

$x_0 = \mp 4$

$\Rightarrow$ I $7x \cdot 4 - 2y \cdot (+7) = 14 \Rightarrow 2x - y = 1. \Rightarrow 2x - y - 1 = 0$

II $7x(-4) - 2y(-7) = 14 \Rightarrow -2x + y = 1 \Rightarrow -2x + y - 1 = 0$

$\Rightarrow$ a, c

5. $\frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{8}=1$ tg la elipsoidul

$3x-2y+5z=0$ $\parallel$ cu planul

ec. tg: $\frac{xx_0}{9}+\frac{yy_0}{4}+\frac{zz_0}{8}=1$ are vect. normal $\left(\frac{x_0}{9}, \frac{y_0}{4}, \frac{z_0}{8}\right)$

au coordonate proporționale $\Rightarrow$ $\frac{\frac{x}{9}}{3}=\frac{\frac{y}{4}}{-2}=\frac{\frac{z}{8}}{5} \Leftrightarrow \frac{x}{27}=\frac{y}{-8}=\frac{z}{40}$

$\Rightarrow \begin{cases} -8x=27y \\ 40y=-8z \end{cases} \Leftrightarrow \begin{cases} -8x=27y \\ 5y=-z \end{cases}$

atașăm prima ecuație:

$\begin{cases} -8x_0=27y_0 \Rightarrow -8x_0=27\cdot\frac{-z_0}{5} \Rightarrow x_0=\frac{27}{5}(-z)\cdot\frac{1}{-8}=\frac{27}{40}z_0 \\ 5y_0=-z_0 \Rightarrow y_0=-\frac{z}{5} \\ {}^{8/}\frac{x_0^2}{9}+{}^{18/}\frac{y_0^2}{4}+{}^{9/}\frac{z_0^2}{8}=1^{72} \end{cases}$

$8x^2+18y^2+9z^2=72$ apoi înlocuiesc, aflu z_0, x_0, y_0 și înlocuiesc în ec. tg.

3. Se consideră ΔABC cu vârfurile $A(1,1)$, $B(4,1)$, $C(2,3)$. Determinați imaginea triunghiului printr-o translație de vector $v(-2,-1)$, urmată de o reflexie față de dreapta $3x+y+2=0$. Desen.

v_1 și v_2

$$trans(-2,-1) = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$mirror(Q, \omega) = \begin{pmatrix} I_2 - 2(\omega^{\perp} \otimes \omega^{\perp}) & 2(\omega^{\perp} \otimes \omega^{\perp}) \cdot Q \\ 0 & 1 \end{pmatrix}$$

$$\omega^{\perp}\otimes\omega^{\perp}=\begin{pmatrix}-\omega_2\\ \omega_1\end{pmatrix}\begin{pmatrix}-\omega_2 & \omega_1\end{pmatrix}=\begin{pmatrix}\omega_1\omega_1 & \omega_1\omega_2\\ \omega_1\omega_2 & \omega_2\omega_2\end{pmatrix}=\begin{pmatrix}\frac{1}{10} & +\frac{3}{10}\\ +\frac{3}{10} & \frac{9}{10}\end{pmatrix}$$

! vectorul director al $3x+y+2=0$ este $(1,-3)$ => un versor director este $\omega=\left(\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right)$

calculăm elementele matricei:

$$I_2-2(\omega^{\perp}\otimes\omega^{\perp})=\begin{pmatrix}1&0\\0&1\end{pmatrix}-\begin{pmatrix}\frac{1}{5} & +\frac{3}{5}\\ +\frac{3}{5} & \frac{9}{5}\end{pmatrix}=\begin{pmatrix}1-\frac{1}{5} & -\frac{3}{5}\\ -\frac{3}{5} & 1-\frac{9}{5}\end{pmatrix}=\begin{pmatrix}\frac{4}{5} & -\frac{3}{5}\\ -\frac{3}{5} & -\frac{4}{5}\end{pmatrix}$$

avem nevoie de un punct de pe dreaptă: alegem $Q(-1,1)$

$$2(\omega^{\perp}\otimes\omega^{\perp})\cdot Q=\begin{pmatrix}\frac{1}{5} & \frac{3}{5}\\ \frac{3}{5} & \frac{9}{5}\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}-\frac{1}{5}+\frac{3}{5}\\ -\frac{3}{5}+\frac{9}{5}\end{pmatrix}=\begin{pmatrix}\frac{2}{5}\\ \frac{6}{5}\end{pmatrix}$$

=> matricea reflexiei: $\begin{pmatrix}\frac{4}{5} & -\frac{3}{5} & \frac{2}{5}\\ -\frac{3}{5} & -\frac{4}{5} & \frac{6}{5}\\ 0 & 0 & 1\end{pmatrix}$

$(A',B',C')=$ trans·reflexie$\cdot(A,B,C)$

$$\text{trans}\cdot\text{reflexie}=\begin{pmatrix}1&0&-2\\0&1&-1\\0&0&1\end{pmatrix}\begin{pmatrix}\frac{4}{5} & -\frac{3}{5} & \frac{2}{5}\\ -\frac{3}{5} & -\frac{4}{5} & \frac{6}{5}\\ 0&0&1\end{pmatrix}=\begin{pmatrix}\frac{4}{5} & -\frac{3}{5} & \frac{2}{5}-\frac{5}{5}2\\ -\frac{3}{5} & -\frac{4}{5} & \frac{6}{5}-\frac{5}{5}1\\ 0&0&1\end{pmatrix}=\begin{pmatrix}\frac{4}{5} & -\frac{3}{5} & -\frac{8}{5}\\ -\frac{3}{5} & -\frac{4}{5} & \frac{1}{5}\\ 0&0&1\end{pmatrix}$$

$$(A',B',C')=\begin{pmatrix}\frac{4}{5} & -\frac{3}{5} & -\frac{8}{5}\\ -\frac{3}{5} & -\frac{4}{5} & \frac{1}{5}\\ 0&0&1\end{pmatrix}\begin{pmatrix}1&4&2\\1&1&3\\1&1&1\end{pmatrix}=\begin{pmatrix}\frac{4}{5}-\frac{3}{5}-\frac{8}{5} & \frac{16}{5}-\frac{3}{5}-\frac{8}{5} & \frac{8}{5}-\frac{9}{5}-\frac{8}{5}\\ -\frac{3}{5}-\frac{4}{5}+\frac{1}{5} & -\frac{12}{5}-\frac{4}{5}+\frac{1}{5} & -\frac{6}{5}-\frac{12}{5}+\frac{1}{5}\\ 1&1&1\end{pmatrix}$$

=> $A'\left(-\frac{7}{5}, -\frac{6}{5}\right)$, $B'\left(\frac{4}{5}, -\frac{15}{5}\right)$, $C'\left(-\frac{9}{5}, -\frac{17}{5}\right)$ apoi desen