Transformări geometrice

Transformări plane

Translaţia în plan

$$\text{Trans}(\mathbf{w}) = \begin{bmatrix} 1 & 0 & w_1 \\ 0 & 1 & w_2 \\ 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

Rotaţia

$$\text{Rot}(Q, \theta) = \begin{bmatrix} \cos\theta & -\sin\theta & q_1(1-\cos\theta) + q_2\sin\theta \\ \sin\theta & \cos\theta & -q_1\sin\theta + q_2(1-\cos\theta) \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

Scalarea uniformă

$$\text{Scale}(Q, s) = \begin{pmatrix} s & 0 & (1-s)q_1 \\ 0 & s & (1-s)q_2 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

Scalarea neuniformă

$$\text{Scale}(Q, s_x, s_y) = \begin{pmatrix} s_x & 0 & (1-s_x)q_1 \\ 0 & s_y & (1-s_y)q_2 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4}$$

Reflexia

$$\text{Mirror}(Q, \mathbf{w}) = \begin{pmatrix} I_2 - 2\left(\mathbf{w}^\perp \otimes \mathbf{w}^\perp\right) & 2\left(\mathbf{w}^\perp \otimes \mathbf{w}^\perp\right) \cdot Q \\ 0 & 1 \end{pmatrix}. \tag{5}$$

Forfecarea

$$\text{Shear}(Q, \mathbf{w}, \theta) = \begin{pmatrix} I_2 + \text{tg}\,\theta\left(\mathbf{w}^\perp \otimes \mathbf{w}\right) & -\text{tg}\,\theta\left(\mathbf{w}^\perp \otimes \mathbf{w}\right) \cdot Q \\ 0 & 1 \end{pmatrix}. \tag{6}$$

Transformări în spaţiu

Translaţia

$$\text{Trans}(\mathbf{w}) = \begin{pmatrix} 1 & 0 & 0 & w_1 \\ 0 & 1 & 0 & w_2 \\ 0 & 0 & 1 & w_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{7}$$

Rotaţia

$$\text{Rot}(Q, \mathbf{u}, \theta) = \begin{pmatrix} \text{Rot}(\mathbf{u}, \theta) & (I_3 - \text{Rot}(\mathbf{u}, \theta)) \cdot \mathbf{Q} \\ 0 & 1 \end{pmatrix}, \tag{8}$$

unde

$$\text{Rot}(\mathbf{u}, \theta) = \cos\theta \cdot I_3 + (1 - \cos\theta)(\mathbf{u} \otimes \mathbf{u}) + \sin\theta \cdot (\mathbf{u} \times -) \quad \text{şi} \quad \mathbf{u} \times - = \begin{pmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{pmatrix}.$$

Scalarea uniformă

$$\text{Scale}(Q, s) = \begin{pmatrix} s & 0 & 0 & (1-s) \cdot q_x \\ 0 & s & 0 & (1-s) \cdot q_y \\ 0 & 0 & s & (1-s) \cdot q_z \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{9}$$

Scalarea neuniformă

$$\text{Scale}(Q, s_x, s_y, s_z) = \begin{pmatrix} s_x & 0 & 0 & (1-s_x) \cdot q_x \\ 0 & s_y & 0 & (1-s_y) \cdot q_y \\ 0 & 0 & s_z & (1-s_z) \cdot q_z \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{10}$$

Reflexia

$$\text{Mirror}(Q, \mathbf{n}) = \begin{pmatrix} I_3 - 2(\mathbf{n} \otimes \mathbf{n}) & 2(\mathbf{n} \otimes \mathbf{n}) \cdot Q \\ 0 & 1 \end{pmatrix} \tag{11}$$

Forfecarea

$$\text{Shear}(Q, \mathbf{n}, \mathbf{u}, \theta) = \begin{pmatrix} I_3 + \operatorname{tg}\theta \cdot (\mathbf{n} \otimes \mathbf{u}) & -\operatorname{tg}\theta \cdot (\mathbf{n} \otimes \mathbf{u}) \cdot Q \\ 0 & 1 \end{pmatrix}. \tag{12}$$