MODEL 1.

8. Determinați imag. pătratului ABCD, cu $A(0,0)$, $B(2,0)$, $C(2,2)$, $D(0,2)$ printr-o translație de vector $w(1,1)$, urmată de o rotație de unghi $30°$ în jurul punctului $Q(1,1)$. Desen.

$$trans(w) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$rot(Q,\theta) = \begin{pmatrix} \cos 30° & -\sin 30° & (1-\cos 30°)+\sin 30° \\ \sin 30° & \cos 30° & -\sin 30° + (1-\cos 30°) \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{3-\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1-\sqrt{3}}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^{2)}1-\frac{\sqrt{3}}{2}+\frac{1}{2} = \frac{3-\sqrt{3}}{2} \qquad -\frac{1}{2}+{}^{2)}1-\frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$$

$$rot \cdot trans = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{3-\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1-\sqrt{3}}{2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(A', B', C', D') = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \sqrt{3}+1 & \sqrt{3} & 0 \\ 1 & 2 & \sqrt{3}+2 & \sqrt{3}+1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$\Rightarrow A'(1,1)$

$B'(\sqrt{3}+1, 2)$

$C'(\sqrt{3}, \sqrt{3}+2)$

$D'(0, \sqrt{3}+1)$

2. Ec. planului tangent la elipsoidul $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1=0$ în pct. (x_0, y_0, z_0)

$$\frac{xx_0}{a^2}+\frac{yy_0}{b^2}+\frac{zz_0}{c^2}-1=0$$

3. Dreapta $\frac{x}{2}=\frac{y}{3}=\frac{z}{1}$ intersectează paraboloidul $\frac{x^2}{4}-\frac{y^2}{9}=2z \,/\cdot 36$ în:

$$\begin{cases} 3x=2y \\ y=3z \end{cases}$$

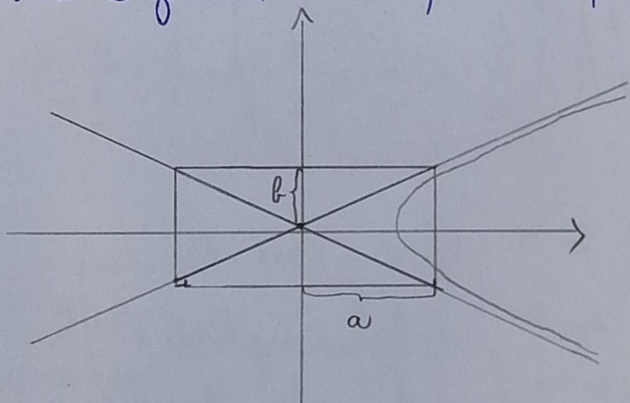
$$\frac{\cancel{36}^{9}x^2}{\cancel{4}}-\frac{\cancel{36}^{4}y^2}{\cancel{9}}=72z \iff 9x^2-4y^2=72z$$

$$\Rightarrow \begin{cases} 3x=2y \Rightarrow 3x=6z \Rightarrow x=2z \\ y=3z \\ 9x^2-4y^2=72z \Rightarrow 9(2z)^2-4(3z)^2=72z \end{cases}$$

$$\cancel{36z^2}-\cancel{36z^2}=72z \Rightarrow z=0 \Rightarrow x=0 \Rightarrow y=0$$

4. ✓

5. Aria Δ format de asimptotele hiperbolei $\frac{x^2}{4}-\frac{y^2}{9}=1$ şi $d: 2x+3y-7=0$.



asimptote $y=\pm\frac{b}{a}x=\pm\frac{3}{2}x$

$$\underline{I} \begin{cases} y=\frac{7-2x}{3} \\ y=\frac{3}{2}x \end{cases} \qquad \underline{II} \begin{cases} y=\frac{7-2x}{3} \\ y=-\frac{3}{2}x \end{cases}$$

$\underline{I}$ $\frac{7-2x}{3}=\frac{3x}{2} \iff 14-4x=9x \Rightarrow x=\frac{14}{13} \Rightarrow y=\frac{3}{\cancel{2}}\cdot\frac{\cancel{14}^{7}}{13}=\frac{21}{13}$

$\underline{II}$ $\frac{7-2x}{3}=\frac{-3x}{2} \Rightarrow 14-4x=-9x \Rightarrow x=\frac{14}{-5} \Rightarrow y=-\frac{3}{\cancel{2}}\cdot\frac{\cancel{14}^{7}}{-5}=\frac{21}{5}$

$\underline{III}$ $\begin{cases} \frac{3}{2}x=y \\ -\frac{3}{2}x=y \end{cases} \Rightarrow x=0 \quad y=0$

$$\Rightarrow (0,0),\left(\frac{14}{13},\frac{21}{13}\right),\left(-\frac{14}{5},\frac{21}{5}\right) \Rightarrow \frac{1}{2}\left|\begin{array}{ccc} 0 & \frac{14}{13} & -\frac{14}{5} \\ 0 & \frac{21}{13} & \frac{21}{5} \\ 1 & 1 & 1 \\ 0 & \frac{14}{13} & -\frac{14}{5} \\ 0 & \frac{21}{13} & \frac{21}{5} \end{array}\right| = \frac{1}{2}\left(\frac{14}{13}\cdot\frac{21}{5}+\frac{14}{5}\cdot\frac{21}{13}\right) = \frac{294}{65}$$

6. Ec. tangentei la parabola $y^2 = 4x$, perpendiculară pe $d: x+y-6=0$ este:

$y^2 = 2px \Rightarrow p = 2$

vect. normal (a, b)

vect. director $(-b, a)$

ec. tangentei: $yy_0 = p(x+x_0)$

$$\begin{cases} yy_0 = 2(x-x_0) \Leftrightarrow \underline{yy_0} \underline{-2x} + 2x_0 = 0 \Rightarrow \vec{n_1}(-2, y_0) \\ \vec{n_2}(1,1) \end{cases}$$

$\vec{n_1} \cdot \vec{n_2} = -2 + y_0 = 0 \Rightarrow y_0 = 2 \Rightarrow y_0^2 = 4x_0 \Rightarrow 4x_0 = 4 \Rightarrow x_0 = 1$

$\Rightarrow$ ec. tangentei: $2y = 2(x+1) \Leftrightarrow 2x - 2y + 2 = 0 \Leftrightarrow x - y + 1 = 0$

7. Det. ec. suprafeței cilindrice generate de o familie de drepte de vector director $u(1,1,1)$ și care intersectează suprafața $y^2 + 2x^2 = 4$, $z = 2$.

$\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ are vect. dir $(1,1,1) \Rightarrow x = y = z$

$$\begin{cases} x = y \\ y = z \end{cases} \Leftrightarrow \begin{cases} x - y = 0 \\ y - z = 0 \end{cases}$$

$$\begin{cases} \begin{cases} x - y = \lambda \\ y - z = \mu \end{cases} \text{ ec. generatoarelor} \\ z = 2 \\ y^2 + 2x^2 = 4 \end{cases}$$

luăm 3 cele mai simple $\Rightarrow y = \mu + 2$

$x - \mu - 2 = \lambda \Rightarrow x = \lambda + \mu + 2$

înlocuim în ultima

$(\mu+2)^2 + 2(\lambda+\mu+2)^2 = 4 \Leftrightarrow \mu^2 + 4\mu + 4 + 2(\lambda^2 + \mu^2 + 4 + 2\lambda\mu + 4\mu + 4\lambda) = 4$

$\mu^2 + 4\mu + 4 + 2\lambda^2 + 2\mu^2 + 8 + 4\lambda\mu + 8\mu + 8\lambda = 4$

$3\mu^2 + 2\lambda^2 + 12\mu + 8\lambda + 4\mu\lambda + 8 = 0$

le rescriem în fct. de sistem la loc.

$$3(y-z)^2 + 2(x-y)^2 + 12(y-z) + 8(x-y) + 4(y-z)(x-y) + 8 = 0$$

$$3(y^2+z^2-2yz) + 2(x^2-2xy+y^2) + 12y - 12z + 8x - 8y + 4(xy - y^2 - zx + zy) + 8 = 0$$

$$\underline{3y^2} + \underline{3z^2} - \underline{6yz} + \underline{2x^2} - 4xy + \underline{2y^2} + 12y - 12z + 8x - 8y + 4xy - \underline{4y^2} - \underline{4zx} + \underline{4zy} + 8 = 0$$

$$2x^2 + y^2 + 3z^2 - 4xz - 2yz + 8x + 4y - 12z + \underline{\underline{8}} = 0$$