MODEL

1. Considerăm ca bază de coordonate $(\vec{a}, \vec{b}, \vec{c}) \Rightarrow \vec{OP}(1,1,-1), \vec{OQ}(2,3,0), \vec{OR}(0,-1,-t), t \in \mathbb{R}$

$$\left.\begin{array}{l} \vec{PR}(-1,-2,1-t) \\ \vec{PQ}(1,2,1) \end{array}\right\} \text{ coliniari} \Rightarrow \text{au coordonate proporționale} \Rightarrow \frac{-1}{1} = \frac{-2}{2} = \frac{1-t}{1} \Rightarrow t = 2$$

2. ABCD - tetraedru

$A(1,-6,10)$, $B(-1,-3,7)$, $C(5,-1,a)$, $D(7,-4,7)$, $a \in \mathbb{R}$

$V_{ABCD} = 11 = \frac{1}{6}|\Delta|$

$$\Delta = \begin{vmatrix} -2 & 3 & -3 \\ 4 & 5 & a-10 \\ 6 & 2 & -3 \end{vmatrix} = 30 - 24 + 18(a-10) + 90 + 4(a-10) + 36 = 22a - 88$$

$$\begin{matrix} -2 & 3 & -3 \\ 4 & 5 & a-10 \end{matrix}$$

$\Rightarrow 11 = |22a - 88| \cdot \frac{1}{6} \quad /:11$

$1 = \frac{1}{6}|2a-8|$

$|2a-8| = 6 \quad /:2$

$|a-4| = 3 \Rightarrow$ I $a-4 = 3 \Rightarrow a = 7$

II $a - 4 = -3 \Rightarrow a = 1$

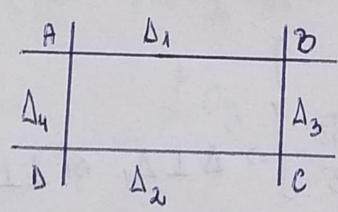
3. $\vec{a} \times (\vec{b} \times \vec{a}) = \vec{a} \times \vec{m} \Rightarrow$ perpendicular pe $\vec{a}$

4. $\Delta_1: x + 2y + 3 = 0 \Rightarrow \vec{a_1}(-2,1)$

$\Delta_2: x + 2y - 7 = 0 \Rightarrow \vec{a_2}(-2,1)$

$\Delta_3: 2x - y - 4 = 0 \Rightarrow \vec{a_3}(1,+2)$

$\Delta_4: ?$



aflăm pct. $B = \Delta_1 \cap \Delta_2$ $\begin{cases} x+2y+3=0 \\ x+2y-7=0 \end{cases} \Rightarrow \begin{matrix} x = 1 \\ y = -2 \end{matrix} \Rightarrow B(1,-2)$

$\Rightarrow$ vectorul director al lui $\vec{BD} = \frac{\vec{a_1} + \vec{a_3}}{2} = \frac{(-1,3)}{2} \Rightarrow$ putem lua $(-1,3)$

↳ formulă, dar puteam să adun cu paralelogramul

$\Rightarrow$ vect. director $\vec{BD}$

$\Rightarrow$ ec. vectorială $\vec{r}(x,y) = (1,-2) + t(-1,3)$

↳ punct de pe BD

$$\begin{cases} x = 1 - t \\ y = -2 + 3t \end{cases} \Rightarrow \begin{array}{l} t = 1 - x \\ t = \frac{y+2}{3} \end{array} \Rightarrow 1 - x = \frac{y+2}{3} \Rightarrow BD: 3x + y - 1 = 0$$

aflăm $D = BD \cap CD \Rightarrow \begin{cases} 3x + y - 1 = 0 \\ x + 2y - 7 = 0 \end{cases} \Rightarrow \begin{array}{l} x = -1 \\ y = 4 \end{array} \Rightarrow D(-1, 4)$

$\Rightarrow$ avem un pct. di pe dreaptă, ne mai trebuie vectorul director

$\Delta_4 \parallel \Delta_3 \Rightarrow$ vect. director Δ_4 este $(-b, a) = (1, 2)$

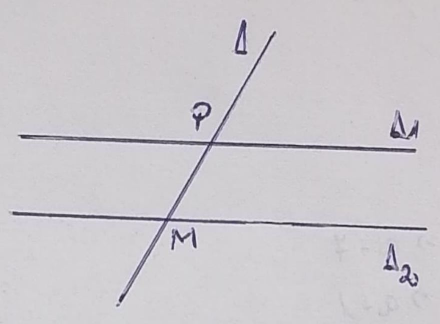
$\Rightarrow$ ec. vectorială : $r(x,y) = (-1,4) + t(1,2) \Rightarrow \begin{cases} x = -1 + t \\ y = 4 + 2t \end{cases} \Rightarrow \begin{cases} t = x + 1 \\ t = \frac{y-4}{2} \end{cases} \Rightarrow x + 1 = \frac{y-4}{2}$

$\Updownarrow$

$2x + 2 = y - 4$

$\Rightarrow AD : 2x - y + 6 = 0$

5. $A(-5,4) \in \Delta$



$\Delta_1 : x + 2y + 1 = 0$

$\Delta_2 : x + 2y - 1 = 0$ $\quad \Rightarrow \vec{a}_{1,2}(-2, 1)$

$\Delta_1 \parallel \Delta_2$

$d = \frac{2\sqrt{5}}{5}$

not. $\vec{a}(x_a, y_a)$ vect director al lui Δ

$$\left.\begin{array}{l} \text{pt. } y = 0 \Rightarrow B(-1, 0) \in \Delta_1 \\ d(B, \Delta_2) = \frac{|1\cdot(-1) + 2\cdot 0 + 1|}{\sqrt{1^2 + 2^2}} = \frac{2\sqrt{5}}{5} \end{array}\right\} \Rightarrow \Delta \perp \Delta_1 \text{ și } \Delta \perp \Delta_2$$

$\Delta \perp \Delta_1 \Rightarrow \vec{a} \cdot \vec{a}_{1,2} = 0 \Rightarrow (x_a, y_a)(-2, 1) = 0 \Rightarrow -2x_a + y_a = 0 \Rightarrow y_a = 2x_a$

$\Rightarrow \Delta : 2x - y + \text{constanta} = 0$

$A \in \Delta \Rightarrow 2(-5) - 1\cdot 4 + \text{constanta} = 0 \Rightarrow \text{constanta} = 14$

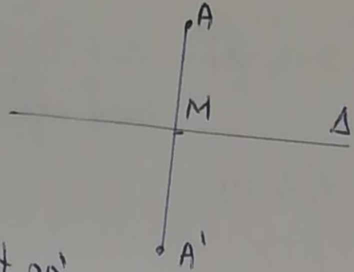
$\Rightarrow \Delta : 2x - y + 14 = 0$

6. $A(4, 13)$

$\Delta : 5x+y+6=0 \rightarrow \vec{n}(5,1)$

$\Downarrow$

vect. director pt. AA'



$\Rightarrow AA' : \vec{r}(x,y) = (4,13) + t(5,1) \Rightarrow \begin{cases} x = 4+5t \\ y = 13+t \end{cases} \Rightarrow \begin{cases} t = \frac{x-4}{5} \\ t = y+13 \end{cases} \Rightarrow \frac{x-4}{5} = y+13 \Rightarrow x-5y-69=0$

$M = \Delta \cap AA' \Rightarrow \begin{cases} 5x+y+6=0 \\ x-5y-69=0 \end{cases} \Rightarrow \begin{matrix} x = \frac{3}{2} \\ y = -\frac{27}{2} \end{matrix} \Rightarrow M\left(\frac{3}{2}, \frac{-27}{2}\right)$

$$\left.\begin{matrix} \frac{4+x_{A'}}{2} = \frac{3}{2} \Rightarrow 4+x_{A'} = 3 \Rightarrow x_{A'} = -1 \\ \frac{13+y_{A'}}{2} = -\frac{27}{2} \Rightarrow y_{A'} = 14 \end{matrix}\right\} \Rightarrow A'(-1, 14)$$

7. $A(-2,3,1)$

$P(-3,5,2) \in \Delta$

face unghiuri egale cu axele

$\vec{a}(x_0, y_0, z_0)$

$$\cos\varphi = \frac{\vec{a}\cdot\vec{i}}{\|\vec{a}\|\cdot\|\vec{i}\|} = \underbrace{\frac{x_0}{\sqrt{x_0^2+y_0^2+z_0^2}}}_{\text{pt. } Ox} = \underbrace{\frac{y_0}{\sqrt{x_0^2+y_0^2+z_0^2}}}_{\text{pt. } Oy} = \underbrace{\frac{z_0}{\sqrt{x_0^2+y_0^2+z_0^2}}}_{\text{pt. } Oz} \Rightarrow x_0 = y_0 = z_0 = \text{oricât}$$

$\downarrow$

putem lua ca vector director $\vec{a}(1,1,1)$ pentru Δ

$$\left.\begin{matrix} \Rightarrow \text{ec. vectorială } \vec{r} = \vec{r}_0 + t\vec{a} \\ \vec{r}_0(-3,5,2) \in \Delta \\ \vec{a}(1,1,1) \end{matrix}\right\} \Rightarrow d = \frac{\|[(-2,3,1)-(-3,5,2)] \times (1,1,1)\|}{\|(1,1,1)\|} =$$

→ cu det cu $\vec{i}, \vec{j}, \vec{k}$ dă $-\vec{i}-2\vec{j}+3\vec{k}$

$$= \frac{\|(1,-2,-1)\times(1,1,1)\|}{\sqrt{3}} = \frac{\sqrt{(-1)^2+(-2)^2+3^2}}{\sqrt{3}} = \sqrt{\frac{14}{3}}$$

8. $\Delta_1: \frac{x-5}{0} = \frac{y}{3-a} = \frac{z}{-2} \Rightarrow \vec{a}_1(0, 3-a, -2)$

sunt coplanare

$\Delta_2: \frac{x-a}{0} = \frac{y}{-1} = \frac{z}{2-a} \rightarrow \vec{a}_2(0, -1, 2-a)$

$\Delta_1: \begin{cases} x-5 = 0t \\ y = (3-a)t \\ z = -2\cdot t \end{cases} \Rightarrow \begin{cases} x = 5 \\ y = (3-a)t \\ z = -2t \end{cases}$ pt. $t=0 \Rightarrow A(5,0,0) \in \Delta_1$

$\Delta_2: \begin{cases} x-a = 0\cdot t \\ y = -1\cdot t \\ z = (2-a)t \end{cases} \Rightarrow \begin{cases} x = a \\ y = -t \\ z = (2-a)t \end{cases}$ pt. $t=1 \Rightarrow B(a,-1,2-a) \in \Delta_2$ (dacă am fi luat tot 0 urm. determinant ar fi fost inutil)

$\vec{AB} = (a-5, -1, 2-a)$

dacă produsul mixt $=0 \Rightarrow$ vectorii sunt coplanari

$$\begin{vmatrix} a-5 & -1 & 2-a \\ 0 & 3-a & -2 \\ 0 & -1 & 2-a \end{vmatrix} = 0 \Leftrightarrow (-1)^2(a-5)\cdot\begin{vmatrix} 3-a & -2 \\ -1 & 2-a \end{vmatrix} = 0$$

$\Leftrightarrow (a-5)(a^2-5a+4) = 0 \Rightarrow$ I $a-5=0 \Rightarrow a=5$

II $a^2-5a+4=0 \Rightarrow$ II.1 $a=4$

II.2 $a=1$

9. $A(1,6,3)$

$\Delta: \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} \rightarrow \vec{v}(1,2,3)$

P - va fi piciorul perpendicularei $P(x_p, y_p, z_p)$

$\vec{AP}\cdot\vec{v} = 0$ (sunt $\perp$) $\qquad \vec{AP}(x_p-1, y_p-6, z_p-3)$

$\Downarrow$

$x_p - 1 + 2(y_p-6) + 3(z_p-3) = 0$

$x_p + 2y_p + 3z_p = 22$

$P \in \Delta \Rightarrow \frac{x_p}{1} = \frac{y_p-1}{2} = \frac{z_p-2}{3} \Rightarrow \begin{cases} x_p = \frac{y_p-1}{2} \\ x_p = \frac{z_p-2}{3} \end{cases} \Rightarrow \frac{z_p-2}{3} = \frac{y_p-1}{2} \Rightarrow y_p = \frac{2z_p-1}{3}$

înlocuim în prima $\Rightarrow \frac{z_p - 2}{3} + 2 \cdot \frac{2z_p - 1}{3} + 3 \cdot z_p = 22 \Rightarrow z_p = 5$

$x_p = 1 \Rightarrow P(1, 3, 5)$

$y_p = 3$