2.1

1. $y'' = x + \cos x + \sin x \ /\int$

$y' = \frac{x^2}{2} + \sin x - \cos x + c_1 \ /\int$

$y = \frac{1}{2} \cdot \frac{x^3}{3} - \cos x - \sin x + C_1 x + C_2$

2. $y'' = \frac{1}{x} \ /\int$

$y' = \ln|x| + c_1 \ /\int$

$y = \int \underset{g'}{1} \cdot \underset{f}{\ln x} + C_1 x = x\ln x - \int \frac{1}{\cancel{x}} \cancel{x} \, dx = x\ln x - x + C_1 x + C_2$

3. $y'' = \ln x \ /\int$

$y' = x\ln x - x + c_1 \ /\int$

$y = \int (\underset{g'}{x} \cdot \underset{f}{\ln x}) - \frac{x^2}{2} + c_1 x = \frac{x^2}{2} \ln x - \frac{x^2}{4} - \frac{x^2}{2} + c_1 x + C_2$

$\int x\ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^{\cancel{2}}}{2} \cdot \frac{1}{\cancel{x}} = \frac{x^2}{2} \ln x - \frac{1}{2} \frac{x^2}{2} = \frac{x^2}{2} \ln x - \frac{x^2}{4}$

4. $y'' = 1 + tg^2 x \ /\int$

$y' = x + \int tg^2 x \, dx = x + \int \frac{\sin^2 x}{\cos^2 x} dx = x + \int \frac{1 - \cos^2 x}{\cos^2 x} dx = \cancel{x} + \int \frac{1}{\cos^2 x} dx - \cancel{x} = tgx + c_1$

$y = \int (tgx + c_1) dx = c_1 x + \int tgx \, dx = c_1 x + \int \frac{\sin x}{\cos x} = c_1 x - \ln|\cos x| + C_2$

5. $y'' = xe^x \ /\int$

$y' = \int \underset{f}{x} \underset{g'}{e^x} = xe^x - \int e^x dx = xe^x - e^x + c_1 \ /\int$

$y = \int xe^x - \int e^x = xe^x - e^x - e^x = xe^x - 2e^x + C_1 x + C_2$

6. $y'' = \dfrac{2x^2}{(1+x^2)^2} \quad / \int$

$y' = \int \dfrac{2x^2}{(1+x^2)^2}\,dx = \int \underset{f}{x} \cdot \underset{g'}{\dfrac{2x}{(1+x^2)^2}} = -x\,\dfrac{1}{x^2+1} + \int \dfrac{1}{x^2+1}\,dx = -\dfrac{x}{x^2+1} + \operatorname{arctg} x \quad / \int$

$\dfrac{2x}{(x^2+1)^2} = -\left(\dfrac{1}{x^2+1}\right)'$

$y = -\dfrac{1}{2}\int \dfrac{2x}{x^2+1} + \int \operatorname{arctg} x = -\dfrac{1}{2}\ln|x^2+1| + \int \underset{g'}{1} \cdot \underset{f}{\operatorname{arctg} x}\,dx =$

$= -\dfrac{1}{2}\ln|x^2+1| + x\operatorname{arctg} x - \dfrac{1}{2}\int 2x \cdot \dfrac{1}{x^2+1} = -\dfrac{1}{2}\ln|x^2+1| + x\operatorname{arctg} x - \dfrac{1}{2}\ln|x^2+1|$

$= -\ln|x^2+1| + x\operatorname{arctg} x + c_1 x + c_2$

2.2

1. $xy'' + y' + x = 0$

not $z = y' \Rightarrow xz' + z + x = 0 \quad /: x$

$z' + \dfrac{z}{x} = -1$

not $t = \dfrac{z}{x} \Rightarrow z = tx \Rightarrow z' = t'x + tx' = t'x + t$

$-1 - t = t'x + t$

$t'x = -1 - 2t$

$\left.\begin{aligned} t' &= -\dfrac{1}{x}(1+2t) \\ t' &= \dfrac{dt}{dx} \end{aligned}\right\} \Rightarrow \dfrac{dt}{1+2t} = -\dfrac{dx}{x} \quad / \int$

$\dfrac{1}{2}\int \dfrac{2}{1+2t} = -\ln|x| + c$

$\dfrac{1}{2}\ln|1+2t| = -\ln|x| + c$

$\Rightarrow \sqrt{|1+2t|} = |x|^{-1} c_1 \quad /()^2 \Rightarrow |1+2t| = |x|^{-2} c_1^2$

$1+2t = x^{-2}c_1^{2}$

$t = \dfrac{x^{-2}c_1^{2}-1}{2}$

$t = \dfrac{z}{x} \Rightarrow z = x\,\dfrac{x^{-2}c_1^{2}-1}{2} \Rightarrow y' = x\,\dfrac{x^{-2}c_1^{2}-1}{2} \;/\!\int$

$\Rightarrow y = \int \dfrac{x^{-1}c_1 - x}{2}$

$y = \dfrac{1}{2}\int \dfrac{c_1}{x} - \dfrac{x^2}{2} = \dfrac{c_1}{2}\ln|x| - \dfrac{x^2}{4} + c_2$

2. $xy'' = y'\ln\dfrac{y'}{x}$

not $z = y' \Rightarrow xz' = z\ln\dfrac{z}{x} \;/:x$

$z' = \dfrac{z}{x}\ln\dfrac{z}{x}$

not $\dfrac{z}{x} = t \Rightarrow z = xt \Rightarrow z' = xt' + t$

$xt' + t = t\ln t$

$$\left.\begin{array}{l} t' = (t\ln t - t)\dfrac{1}{x} \\ t' = \dfrac{dt}{dx} \end{array}\right\} \Rightarrow \frac{dt}{t\ln t - t} = \frac{1}{x}dx \;/\!\int$$

$\int \dfrac{1}{t\ln t - t} = \ln|x| + c$

$\int \dfrac{1}{t(\ln t - 1)} = \ln|x| + c$

not $\ln t = m$

$dm = \dfrac{1}{t}dt$

$\Rightarrow \int \dfrac{1}{m-1}dm = \ln|x| + c$

$\ln|\ln t - 1| = \ln|x| + c$

$\ln t - 1 = xc$

$\ln t = xc + 1$

$t = e^{xc+1}$

$g' = e^{xc+1} \Rightarrow g = \int e^{xc+1} = \dfrac{1}{c}\int e^{t}dt = \dfrac{1}{c}e^{t} = \dfrac{1}{c}e^{xc+1}$

not $t = xc+1$

$dt = c$

$t = \dfrac{z}{x} \Rightarrow z = e^{xc+1}\cdot x$

$z = y' \Rightarrow y' = xe^{xc+1} \;/\!\int$

$y = \int \underset{f}{x}\,\underset{g'}{e^{xc+1}} = x\cdot\dfrac{1}{c}e^{xc+1} - \int e^{xc+1} =$

$= x\,\dfrac{1}{c}e^{xc+1} - \dfrac{1}{c^2}e^{xc+1} + c_2$

3. $y'' - 2y' = -x^2$

not $z = y' \Rightarrow z' - 2z = -x^2$

$z' = -x^2 + 2z$

I omogenă

$z' - 2z = 0$

$$\left.\begin{array}{l} z' = 2z \\ z' = \dfrac{dz}{dx} \end{array}\right\} \Rightarrow \frac{dz}{2z} = dx \;/\int$$

$\frac{1}{2}\ln|z| = x + c$

$\sqrt{z} = e^{x+c} \;/()^2$

$z = e^{2(x+c)}$

$z = e^{2x+c}$

$g' = e^{2x} \Rightarrow g = \frac{1}{2}e^{2x}$

$\int e^{-2x} = \frac{1}{-2}e^{-2x}$

II particulară

$z_p = e^{2x+\varphi} \;/()'$

$z_p' = e^{2x+\varphi}\cdot(2+\varphi')$

$\Rightarrow e^{2x+\varphi}\cdot(2+\varphi') - 2\cdot e^{2x+\varphi} = -x^2$

$\varphi' e^{2x+\varphi} = -x^2$

$\varphi' = -\frac{x^2}{e^{2x+\varphi}} = -\frac{x^2}{e^{2x}\cdot e^{\varphi}}$

$f(x) = -\frac{x^2}{e^{2x}}$

$g(x) = \frac{1}{e^{\varphi}}$

$\varphi' = \frac{d\varphi}{dx} \Rightarrow d\varphi\cdot e^{\varphi} = dx\cdot\frac{-x^2}{e^{2x}} \;/\int$

$e^{\varphi} = -\int \frac{\overset{f}{x^2}}{\underset{g'}{e^{2x}}}dx \;/(-1)$

$\Leftrightarrow -e^{\varphi} = -\frac{1}{2}\cdot x^2\cdot e^{-2x} - \cancel{2}\cdot\frac{1}{-\cancel{2}}\int \overset{f}{x}\cdot \overset{g'}{e^{-2x}}dx$

$-e^{\varphi} = -\frac{x^2\cdot e^{-2x}}{2} + x\cdot\frac{1}{-2}e^{-2x} - \frac{1}{-2}\int e^{-2x}dx$

$-e^{\varphi} = \frac{-x^2\cdot e^{-2x}}{2} + \frac{xe^{-2x}}{-2} + \frac{1}{2}\cdot\frac{1}{-2}e^{-2x} \Rightarrow$

$\Rightarrow -e^{\varphi} = \frac{-x^2\cdot e^{-2x}}{2} - \frac{xe^{-2x}}{2} - \frac{1}{4}e^{-2x}$

$+e^{\varphi} = +\frac{1}{2}e^{-2x}(+x^2 + x + \frac{1}{2})$

$\varphi = \ln \frac{1}{2}e^{-2x}(x^2 + x + \frac{1}{2})$

$\varphi = \ln\left[\frac{1}{2}(x^2 + x + \frac{1}{2})\right] - 2x$

$\Rightarrow z_p = \frac{1}{2}(x^2 + x + \frac{1}{2})$

$z = z_0 + z_p = e^{2x+c} + \frac{1}{2}(x^2 + x + \frac{1}{2})$

$z = y' \Rightarrow \quad y' = e^{2x + c_1} + \frac{1}{2}(x^2 + x + \frac{1}{2})$

$\Rightarrow y = \int \underline{e^{2x + c_1}} + \int \frac{1}{2}(x^2 + x + \frac{1}{2})$

$y = \frac{1}{2} e^{2x + c_1} + \frac{1}{2}\left(\frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{2}\right) + c_2$

4. $(1+x^2)y'' + (y')^2 + 1 = 0$

not $z = y'$

$(1+x^2)z' + z^2 + 1 = 0$

$z' = \frac{-z^2 - 1}{1+x^2} = -\frac{z^2+1}{x^2+1}$ EVS

$z' = \frac{dz}{dx}$

$\Rightarrow \frac{dz}{-(z^2+1)} = \frac{dx}{x^2+1} \quad / \int$

$-\int \frac{1}{z^2+1} dz = \int \frac{1}{x^2+1} dx$

$-\operatorname{arctg} z = \operatorname{arctg} x + c$

$\operatorname{arctg}(-z) = \operatorname{arctg} x + c \quad / tg$

$-z = tg(\operatorname{arctg} x + c)$

$-z = \frac{x + tg c}{1 - x \cdot tg c}$

$-z = \frac{x + c_1}{1 - c_1 x} \rightarrow z = \frac{x + c_1}{c_1 x - 1}$

$\Rightarrow y = \int \frac{x + c}{cx - 1} dx = \int \frac{x}{cx-1} + \int \frac{c}{cx-1} =$

$= \ln|cx-1| + \int \frac{x}{cx-1} dx =$

$= \ln|cx-1| + \frac{1}{c}\int \frac{x}{x - \frac{1}{c}} dx =$

$= \ln|cx-1| + \frac{1}{c}\int \frac{x + \frac{1}{c} - \frac{1}{c}}{x - \frac{1}{c}} dx =$

$= \ln|cx-1| + \frac{1}{c}\left(x + \frac{1}{c}\int \frac{1}{x - \frac{1}{c}}\right) =$

$= \ln|cx-1| + \frac{x}{c} + \frac{1}{c^2}\ln|x - \frac{1}{c}| + c_1$

5. $(1+x^2)y'' = 2xy'$

not $z = y' \Rightarrow z'(1+x^2) = 2xz$

$$z' = \frac{2xz}{1+x^2} \quad \text{EVS}$$

$$z' = \frac{dz}{dx}$$

$$\Rightarrow \frac{dz}{z} = \frac{2x}{1+x^2}dx \; /\int$$

$$\Rightarrow \ln|z| = \ln|1+x^2| + c$$

$$\ln|z| = \ln c_1(1+x^2)$$

$$z = c_1(1+x^2)$$

$z = y' \to y = \int c_1(1+x^2)\,dx =$

$$= c_1\int(1+x^2)\,dx = c_1 x + c_1\frac{x^3}{3} + c_2$$

6. $y'(1+(y')^2) = ay''$, $a \in \mathbb{R}^*$

not $z = y' \Rightarrow z(1+z^2) = az' \Rightarrow z' = \frac{z(1+z^2)}{a}$ EVS

$$z' = \frac{dz}{dx}$$

$$\Rightarrow \frac{dz}{z(1+z^2)} = \frac{dx}{a} \; /\int$$

$$\Rightarrow \int \frac{1}{z(1+z^2)}\,dz = \frac{1}{a}x + c$$

$$\frac{a}{z} + \frac{bz+c}{1+z^2} = \frac{a + az^2 + bz^2 + cz}{(1+z^2)z} = \frac{z^2(a+b) + z\cdot c + a}{\text{--}} \Rightarrow \begin{array}{l} a = 1 \\ b = -1 \\ c = 0 \end{array}$$

$$\Rightarrow \int\frac{1}{z} - \frac{1}{2}\int\frac{2z}{1+z^2} = \frac{1}{a}x + c$$

$$\ln|z| - \frac{1}{2}\ln|1+z^2| = \frac{1}{a}x + c$$

$$\ln\frac{z}{\sqrt{1+z^2}} = \frac{1}{a}x + c \Rightarrow \frac{z}{\sqrt{1+z^2}} = e^{\frac{1}{a}x+c} \; /()^2 \Rightarrow \frac{z^2}{1+z^2} = e^{2(\frac{1}{a}x+c)}$$

$$\frac{z^2+1-1}{z^2+1} = e^{\frac{2x}{2}} \cdot c$$

$$1 - \frac{1}{z^2+1} = e^{\frac{2x}{2}} \cdot c$$

$$1 - e^{\frac{2x}{2}} \cdot c = \frac{1}{z^2+1} \Rightarrow z^2+1 = \frac{1}{1-c\cdot e^{\frac{2}{2}}} \Rightarrow z = \pm\sqrt{\frac{1}{1-ce^{\frac{x}{2}}} - 1}$$

$$\Rightarrow y = \int \sqrt{\frac{1}{1-ce^{\frac{2x}{2}}} - 1}$$

2.3

1. $y'' - y = 0$

$r^2 + 0 \cdot r + (-1) = 0 \Rightarrow r = \pm 1$

$y_1(x) = e^{1 \cdot x}$

$y_2(x) = e^{(-1)x}$

$\Rightarrow y_0 = c_1 \cdot e^x + c_2 e^{-x}$

2. $y'' + 2y' + y = 0$

$r^2 + 2r + 1 = 0$

$\Delta = 4 - 4 = 0 \Rightarrow r_{1,2} = \frac{-2}{2} = -1$

$y_1(x) = e^{-x}$

$y_2(x) = x e^{-x}$

$\Rightarrow y_0 = c_1 e^{-x} + c_2 x e^{-x}$

3. $y'' - 5y' + 6y = 6x^2 - 10x + 2$

$r^2 - 5r + 6 = 0$

$\Delta = 25 - 24 = 1 \Rightarrow r_{1,2} = \dfrac{5 \pm 1}{2} \Rightarrow \begin{array}{l} r_1 = 3 \\ r_2 = 2 \end{array}$

$y_1(x) = e^{2x}$

$y_2(x) = e^{3x}$

$\Rightarrow y_o = c_1 e^{2x} + c_2 e^{3x}$

$y_p = ax^2 + bx + c \,/()'$

$y_p' = 2ax + b \,/()'$

$y_p'' = 2a$

$\Rightarrow 2a - 5(2ax + b) + 6(ax^2 + bx + c) = 6x^2 - 10x + 2$

$x^2(6a) + x(-10a + 6b) + 2a - 5b + 6c = 6x^2 - 10x + 2$

$a = 1$

$-10 + 6b = -10 \Rightarrow b = 0$

$2 + 6c = 2 \Rightarrow c = 0$

$\Rightarrow y_p = ax^2 = x^2 \Rightarrow y = c_1 e^{2x} + c_2 e^{3x} + x^2$

4. $y'' + 3y' + 2y = e^x$

$r^2 + 3r + 2 = 0$

$\Delta = 9 - 8 = 1 \Rightarrow r_{1,2} = \dfrac{-3 \pm 1}{2} \Rightarrow \begin{array}{l} r_1 = -1 \\ r_2 = -2 \end{array}$

$y_o = c_1 e^{-x} + c_2 e^{-2x}$

1 nu e rădăcină a ec. $\Rightarrow y_p(x) = e^x \cdot c \;/ ()'$

$$y_p' = e^x c = y_p''$$

$\Rightarrow ce^x + 3ce^x + 2ce^x = e^x$

$e^x(c + 3c + 2c) = e^x$

$e^x \cdot 6c = e^x \Rightarrow c = \frac{1}{6} \Rightarrow y_p(x) = \frac{1}{6} e^x$

$\Rightarrow y = c_1 e^{-x} + c_2 e^{-2x} + \frac{1}{6} e^x$

5. $y'' + 2y' + 2y = xe^{-x} + 2x + 4$

$r^2 + 2r + 2 = 0$

$\Delta = 4 - 8 = -4 = (2i)^2 \Rightarrow r_{1,2} = \frac{-2 \pm 2i}{2} \Rightarrow \begin{matrix} r_1 = -1+i \\ r_2 = -1-i \end{matrix} \quad (\alpha + \beta i)$

$y_1(x) = e^{-x} \cdot \cos x$

$y_2(x) = e^{-x} \cdot \sin x$

$\Rightarrow y_0 = c_1 e^{-x} \cos x + c_2 e^{-x} \sin x$

$* \; y_p = y_{p_1} + y_{p_2} = e^{-x}(ax+b) + cx + d \;/ ()'$

$y_{p_1} = e^{-x} \cdot (ax+b)$

$y_{p_2} = cx + d$

$y_p' = -e^{-x}(ax+b) + e^{-x} \cdot a + c$

$= e^{-x}(-ax - b + a) + c \;/ ()''$

$y_p'' = -e^{-x}(-ax-b+a) + e^{-x}(-a) =$

$= e^{-x}(ax + b - a - a) = e^{-x}(ax + b - 2a)$

$\Rightarrow e^{-x}(ax+b-2a) + 2 \cdot e^{-x}(-ax-b+a) + 2c + 2e^{-x}(ax+b) + 2cx + 2d =$

$= xe^{-x} + 2x + 4$

$$e^{-x}(ax+b-\cancel{3a}-\cancel{2ax}-\cancel{2b}+\cancel{2a}+\cancel{2ax}+\cancel{2b})+x(2c)+2c+2d = xe^{-x}+2x+4$$

$$\Rightarrow ax+b = x \Rightarrow a=1, \quad b=0$$

$$2c = 2 \Rightarrow c=1$$

$$2c+2d=4 \Rightarrow 2+2d=4 \Rightarrow d=1$$

$$\Rightarrow y = c_1 e^{-x}\sin x + c_2 e^{-x}\cos x + e^{-x}\cdot x + x+1$$

6. $y''+y'-2y = 10\sin 2x$

$r^2+r-2=0$

$\Delta = 1+8=9 \Rightarrow r_{1,2} = \frac{-1\pm 3}{2} \Rightarrow r_1 = 1, \quad r_2=-2$

$y_0 = c_1 e^{x} + c_2 e^{-2x}$

$2i$ nu e răd. a ec. caract

$y_p = a\cos 2x + b\sin 2x \;/()'$

$y_p' = -2a\sin 2x + 2b\cos 2x \;/()'$

$y_p'' = -4a\cos 2x - 4b\sin 2x$

$-4a\cos 2x - 4b\sin 2x - 2a\sin 2x + 2b\cos 2x - 2(a\cos 2x + b\sin 2x) = 10\sin 2x$

$\cos 2x(-4a+2b-2a)+\sin 2x(-4b-2a-2b) = 10\sin 2x$

$$\Rightarrow \begin{cases} -6a+2b=0 \\ -6b-2a=10 \end{cases} \Rightarrow \begin{matrix} a=-\frac{1}{2} \\ b=-\frac{3}{2} \end{matrix} \Rightarrow y_p = -\frac{1}{2}\cos 2x - \frac{3}{2}\sin 2x$$

$\Downarrow$

$\Rightarrow 6a = 2b \Rightarrow b = \frac{6a}{2} = 3a$

$-6\cdot 3a - 2a = 10$

$-20a = 10 \Rightarrow a = -\frac{1}{2} \Rightarrow b = 3\cdot\frac{-1}{2} = b = -\frac{3}{2}$

$$\boxed{y = c_1 e^{x} + c_2 e^{-2x} - \frac{1}{2}\cos 2x - \frac{3}{2}\sin 2x}$$

$y'' + y = 4xe^{-x} + 2\cos x$

$r^2 + 1 = 0 \Rightarrow r_1 = i$

$r_2 = -i$

$y_0 = c_1 e^{0\cdot x} \cdot \cos x + c_2 e^{0x} \sin x = c_1 \cos x + c_2 \sin x$

$y_p = y_{p_1} + y_{p_2}$

$y_{p_1} = e^{-x} \cdot (ax+b) \; /()'$

$y_{p_1}' = -e^{-x}(ax+b) + e^{-x} \cdot a = e^{-x}(-ax-b+a) \; /()'$

$y_{p_1}'' = -e^{-x}(-ax-b+a) + e^{-x}(-a) = e^{-x}(ax+b-a-a) = e^{-x}(ax+b-2$

$\Rightarrow e^{-x}(ax+b-2a) + e^{-x}(ax+b) = 4xe^{-x}$

$\cancel{e^{x}}(2ax+2b-2a) = 4x\cancel{e^{-x}}$

$$\left.\begin{array}{l} 4x = 2ax \Rightarrow a = 2 \\ 2b-2a = 0 \Rightarrow b = 2 \end{array}\right\} \Rightarrow y_{p_1} = e^{-x}(2x+2)$$

$y_{p_2} =$ (i e răd) $x \cdot (a\cos x + b\sin x) \; /()'$

$y_{p_2}' = a\cos x + b\sin x + x(-a\sin x + b\cos x) = \sin x(b-ax) + \cos x(a+bx) /()$

$y_{p_2}'' = \cos x(b-ax) + \sin x \cdot (-a) + \cos x(b) - \sin x(bx+a) =$

$= \cos x(2b-ax) + \sin x(-bx-2a)$

$\Rightarrow \cos x(2b-ax) + \sin x(-bx-2a) + x(a\cos x + b\sin x) = 2\cos x$

$\cos x \cdot 2b + \sin x(-bx+b-2a) = 2\cos x$?? ceva eroare umană

7. $y'' - 3y' + 2y = 3e^x + 10\sin x$

$r^2 - 3r + 2 = 0$

$\Delta = 9 - 8 = 1 \Rightarrow r_{1,2} = \frac{3 \pm 1}{2} \Rightarrow r_1 = 2$

$r_2 = 1$

$y_0 = c_1 e^x + c_2 e^{2x}$

$y_p = y_{p_1} + y_{p_2}$

I $y_{p_1} = $ ($r=1$ este răd) $x^1 \cdot e^{1 \cdot x} \cdot a \ /()'$

$y_{p_1}' = ae^x + axe^x = ae^x(1+x) \ /()'$

$y_{p_1}'' = ae^x(1+x) + ae^x = ae^x(x+2)$

$\Rightarrow ae^x(x+2) - 3ae^x(x+1) + 2(axe^x) = 3e^x$

$ae^x(\cancel{x} + 2 - \cancel{3x} - 3 + \cancel{2x}) = 3e^x$

$-ae^x = 3e^x \Rightarrow \boxed{a = -3} \Rightarrow y_{p_1} = -3xe^x$

II $y_{p_2} = $ (nu e răd) $= a \cdot \cos x + b \sin x \ /()'$

$y_{p_2}' = -a\sin x + b\cos x \ /()'$

$y_{p_2}'' = -a\cos x - b\sin x$

$\Rightarrow -a\cos x - b\sin x + 3a\sin x - 3b\cos x + 2a\cos x + 2b\sin x = 10\sin x$

$\cos x(\underbrace{-a - 3b + 2a}_{a-3b}) + \sin x(\underbrace{-b + 3a + 2b}_{b+3a}) = 10\sin x$

$\Rightarrow \begin{cases} a - 3b = 0 \Rightarrow a = 3b \\ 3a + b = 10 \end{cases}$

$9b + b = 10 \Rightarrow b = 1 \Rightarrow a = 3 \Rightarrow y_{p_2} = 3\cos x + \sin x$

$\Rightarrow y_p = -3xe^x + 3\cos x + \sin x$

$\Rightarrow y = y_0 + y_p = c_1 e^x + c_2 e^{2x} + 3\cos x + \sin x - 3xe^x$