SUBIECT IULIAN

1. $\sqrt{x^2+1}\, y' - y = 0$

$$\left.\begin{array}{l} y' = y \cdot \frac{1}{\sqrt{x^2+1}} \quad \text{EVS} \\ y' = \frac{dy}{dx} \end{array}\right\} \Rightarrow \frac{dy}{y} = \frac{dx}{\sqrt{x^2+1}} \ /\int \Rightarrow \ln|y| = \ln(x+\sqrt{x^2+1}) + c$$

$$y = (x+\sqrt{x^2+1}) \cdot c_1$$

2. $\begin{cases} y' + 2xy = x \Leftrightarrow y' = x(1-2y) \\ y(0) = -\frac{1}{2} \end{cases}$

$$\left.\begin{array}{l} y' = x(1-2y) \quad \text{EVS} \\ y' = \frac{dy}{dx} \end{array}\right\} \Rightarrow \frac{dy}{1-2y} = x\,dx \ /\int$$

$$-\frac{1}{2}\int \frac{2}{2y-1}\, dy = \frac{x^2}{2} + c$$

$$-\frac{1}{2} \ln|2y-1| = \frac{x^2}{2} + c$$

$$(2y-1)^{-\frac{1}{2}} = e^{\frac{x^2}{2}+c} \ /()^{-2}$$

$$2y-1 = \frac{1}{e^{x^2+c_1}} \Leftrightarrow y = \frac{1}{2}\left(\frac{1}{e^{x^2+c_1}} + 1\right)$$

$$y = \frac{1}{2} \quad \frac{1}{ce^{x^2}} + \frac{1}{2}$$

$$y = \frac{1}{ce^{x^2}} + \frac{1}{2}$$

$$y(0) = -\frac{1}{2} \Rightarrow \frac{1}{c} + \frac{1}{2} = -\frac{1}{2} \Leftrightarrow \frac{1}{c} = -\frac{2}{2} \Rightarrow \boxed{c=-1}$$

$$\Downarrow$$

$$y = \frac{1}{-e^{x^2}} + \frac{1}{2}$$

3. $y'' = 1 - (y')^2$

not $z = y' \Rightarrow$

$$\left.\begin{array}{l} z' = 1 - z^2 \quad \text{EVS} \\ z' = \frac{dz}{dx} \end{array}\right\} \Rightarrow -\int \frac{1}{z^2-1}\, dz = x + c$$

$$-\frac{1}{2} \ln\left|\frac{z-1}{z+1}\right| = x + c \ /\cdot(-2)$$

$$\ln \frac{z-1}{z+1} = -2x + c$$

$$1 - \frac{2}{z+1} = ce^{-2x} \Leftrightarrow \frac{2}{z+1} = 1 - ce^{-2x} \Leftrightarrow z+1 = \frac{2}{1-ce^{-2x}}$$

$$z = \frac{2}{1-ce^{-2x}} - 1$$

$$y' = z \Rightarrow y = \int \left(\frac{2}{1-ce^{-2x}} - 1 \right) dx = \int \frac{1+ce^{-2x}}{1-ce^{-2x}} dx = \int \frac{1-ce^{-2x}}{1-ce^{-2x}} dx + \int \frac{2ce^{-2x}}{1-ce^{-2x}} =$$

$$= \underline{\underline{x + \ln|1-ce^{-2x}| + C_1}}$$

4. $\begin{cases} y'' - 2y' + 5y = 5x^2 - 4x + 2 \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$

I rez. ec. omogenă

$r^2 - 2r + 5 = 0$

$\Delta = 4 - 20 = -16 = (4i)^2$

$r_{1,2} = \frac{2 \pm 4i}{2} \Rightarrow r_1 = 1+2i$
$\quad r_2 = 1-2i$

$\Rightarrow y_0 = C_1 e^x \cos 2x + C_2 e^x \sin 2x$

II $y_p = ax^2 + bx + c \;/()'$

$y_p' = 2ax + b \;/()'$

$y_p'' = 2a$

$\Rightarrow 2a - 2(2ax+b) + 5(ax^2+bx+c) = 5x^2 - 4x + 2$

$x^2(5a) + x(-4a+5b) + 2a - 2b + 5c = 5x^2 - 4x + 2$

$\left.\begin{array}{l} \Rightarrow a = 1 \\ -4 + 5b = -4 \Rightarrow b = 0 \\ 2 + 5c = 2 \Rightarrow c = 0 \end{array}\right\} \Rightarrow y_p = x^2$

$\Rightarrow y = y_0 + y_p = C_1 e^x \cos 2x + C_2 e^x \overset{0}{\sin 2x} + \overset{0}{x^2}$

$y' = C_1(e^x \cos 2x - 2\overset{0}{\sin(2x)} e^x) + C_2(e^x \overset{0}{\sin 2x} + 2\underset{1}{\cos(2x)} e^x) + 2x$

$y'(0) = C_1 + 2C_2 \Leftrightarrow C_1 + 2C_2 = 1 \Rightarrow C_1 = 1 - 2C_2$

$y(0) = C_1 \Rightarrow C_1 = 1 \Rightarrow C_2 = 0$

$\Rightarrow y = e^x \cos 2x + x^2$

$$\begin{cases} y_1' = 2y_1 + y_2 \Rightarrow y_2 = y_1' - 2y_1 \\ y_2' = y_1 + 2y_2 \end{cases}$$

$$y_1'' = 2y_1' + y_2' = 2(2y_1 + y_2) + y_1 + 2y_2 = 5y_1 + 4y_2 = 5y_1 + 4(y_1' - 2y_1) = -3y_1 + 4y_1'$$

$$\Rightarrow y_1'' - 4y_1' + 3y_1 = 0$$

$$r^2 - 4r + 3 = 0$$

$$\Delta = 16 - 12 = 4 \Rightarrow r_{1,2} = \frac{4 \pm 2}{2} \Rightarrow r_1 = 3$$
$$r_2 = 1$$

$$\left.\begin{aligned} \Rightarrow y_1 &= c_1 e^{x} + c_2 e^{3x} \\ y_1' &= c_1 e^{x} + 3c_2 e^{3x} \end{aligned}\right\} \Rightarrow \begin{aligned} y_2 &= c_1 e^{x} + 3c_2 e^{3x} - 2(c_1 e^{x} + c_2 e^{3x}) \\ y_2 &= -c_1 e^{x} + c_2 e^{3x} \end{aligned}$$