SUBIECT (A)

100.

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- subalg cmp(n)
    - dacă n=1 atunci
        @stop
    - sf dacă
    cmp(n/2)
- sf subalg
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justificare:

$$T(n) = \begin{cases} 1, \text{dacă } n=1 \\ 1+T(n/2), \text{ altfel} \end{cases}$$

$T(n) = 1 + T(n/2)$

$T(n/2) = 1 + T(n/4)$

$\vdots$

$T(2) = 1 + T(1)$

$T(1) = 1$

considerăm $n = 2^k \Rightarrow T(2^k) = 1 + T(2^{k-1})$

$T(2^k) = 1+1+T(2^{k-2})$

$T(2^k) = 3 + T(2^{k-3})$

$T(2^k) = k-1+T(2^{k-(k-1)}) = k-1+T(2)$

$= k-1+2 = k+1$

$\Rightarrow T(n) = 1+\log_2 n \in \theta(\log_2 n)$

$$\begin{cases} 2^k = n \mid \log_2 \\ k \cdot 1 = \log_2 n \end{cases}$$

102.

$$T(n) = \begin{cases} 1 \text{ dacă } n=1 \\ 1+2T(n-1) \text{ altfel} \end{cases}$$

$T(n) = 1+2T(n-1) = 1+2+4T(n-2) = 2^0+2^1+2^2 \cdot T(n-2)$

$T(n) = 2^0+2^1+2^2+\ldots+2^n = 2^{n+1}-1 \in \theta(2^n)$

03. - subalg cmp (n)

- pentru $i \leftarrow 1, n*n$

@ op elementară

└ sf pentru

- dacă $n=0$

@STOP

└ sf dacă

cmp (n-1)

└─ sf subalg

justificare:

$$T(n) = \begin{cases} 1 & \text{dacă } n=0 \\ n^2 + T(n-1) & \text{altfel} \end{cases}$$

$$T(n) = n^2 + T(n-1) = n^2 + (n-1)^2 + T(n-2)$$

$$\Rightarrow T(n) = 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+6)}{6} \in \Theta(n^3)$$

cum dem.?

$$\lim_{n\to\infty} \frac{n(n+1)(2n+6)}{6} \cdot \frac{1}{n^3} = \frac{1}{3} \text{ const} \neq 0, \infty$$

$$\Rightarrow \in \Theta(n^3)$$

104. $T(n) = \begin{cases} 1 & \text{dacă } n=1 \\ 1+T(n/2) & \text{altfel} \end{cases}$

justificare:

$$T(n) = 1 + T(n/2) = 1+1+T(n/4) = 2 + T(n/4)$$

considerăm $n = 2^k$

$$T(2^k) = k + T(2^{k-k}) = k + T(1) = k+1$$

$$2^k = n \Rightarrow k = \log_2 n \Rightarrow T(n) = \log_2 n + 1 \in \Theta \log_2 n$$

105. un for $1, n^3$

106. $T(n) = \begin{cases} 1 & \text{dacă } n \leq 1 \\ 2 + T(n/2) & \text{altfel} \end{cases}$ $\Rightarrow T(n) = 2 \cdot \log_2 n + 1 \in \Theta(\log_2 n)$

107. complexitate rec: $\log_2 i$ (ca sus)

complexitate operatie: $T(n) = 2 + (1 + \log_2(1)) + (1 + \log_2 2) + (1 + \log_2 4) + \dots + (1 + \log_2 n) =$

$= \log_2 n + 2 + \log_2(1 \cdot 2 \cdot 4 \cdot \dots \cdot n) = \log_2 n + 2 + \log_2(2^0 \cdot 2^1 \cdot \dots \cdot 2^{\log_2 n}) = \cancel{\log_2 n + 2 + \log_2}$

$= \log_2 n + 2 + \log_2 2^{0+1+2+3+\dots+\log_2 n} = \log_2 n + 2 + \frac{\log_2 n(\log_2 n + 1)}{2} \cdot 1 =$

$\Theta(\log_2^2 n)$

108. $T(n) = \begin{cases} 1 & \text{dacă } n <= 1 \\ (n-1) + 2 \cdot T(n/2) \end{cases}$

$T(n) = (n-1) + 2 \cdot T(n/2) = (n-1) + 2(\frac{n}{2} - 1) + 4T(n/4) = (n-1) + (n-2) + 4T(n/4)$

$= (n-1) + (n-2) + (n-4) + 8T(n/8) = (n-1) + (n-2) + (n-4) + (n-8) + 16T(n/16)$

$\cancel{\frac{n}{2} T(2) = \frac{n}{2}(1 + 2T(1)) = \frac{n}{2} \cdot 3 = n \frac{3}{2} =}$

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$2^k \cdot T(\frac{n}{2^k}) = 2^k(\frac{n}{2^k} - 1) + 2^{k+1} T(\frac{n}{2^{k+1}}) =$

$= (n - 2^k) + 2^{k+1} T(\frac{n}{2^{k+1}})$

$k \leftarrow \log_2(n-1)$

$T(2) = 3 \Rightarrow 2^{k-1} \cdot T(\frac{2^k}{2^{k-1}}) = 2^{k-1} \cdot T(2) = 2^{k-1} \cdot 3 = 2^{\log_2 n - 1} \cdot 3 =$

$n \leftarrow 2^k$

$= n \cdot \frac{3}{2}$

109. - subalg cb(v, el, st, dr) {inițial st = 0, dr = n}

dacă st > dr atunci @ el nu există sf dacă

mij ← (st + dr)/2

- dacă v[mij] = el atunci

@stop

- sf dacă

- dacă v[mij] > el atunci

cb(v, el, st, mij-1)

altfel

cb(v, el, mij+1, dr)

- sf dacă

- sf subalg cb

$T(n) = \begin{cases} 1 & \text{dacă } n = 0 \\ 1 + T(n/2) & \text{altfel} \end{cases}$

$n = abs(st - dr)$

$\Rightarrow O(\log_2 n)$

112. $T(n) = \begin{cases} 1 & \text{dacă } n = 1 \\ n + T(n/2) & \text{altfel} \end{cases}$

$$T(n) = n + T(n/2) = n + \frac{n}{2} + T(n/4) = n + \frac{n}{2} + \frac{n}{4} + \dots + 1 = \frac{n}{2^0} + \frac{n}{2^1} + \frac{n}{2^2} + \dots + \frac{n}{2^{\log_2 n}} =$$

$$= n\left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 + \dots + \left(\frac{1}{2}\right)^{\log_2 n}\right] = n \cdot \frac{\left(\frac{1}{2}\right)^{\log_2 n \to 0} - 1}{\frac{1}{2} - 1} = n \cdot 2 \in \Theta(n)$$

114. complexitate f: $\log_{10} n$

complexitate A: $1 + f(3) + f(5) + \dots + f(2n+1) =$

$$f(3) \downarrow \log_{10} 3 \qquad f(5) \downarrow \log_{10} 5 \qquad f(2n+1) \downarrow \log_{10}(2n+1)$$

$$= 1 + \log_{10} 3 + \log_{10} 5 + \dots + \log_{10}(2n+1) =$$

$$\lim_{n\to\infty} \frac{1 + \log_{10} 3 + \dots + \log_{10}(2n+1)}{f(n) \to \infty} = \text{const}$$

$$\overset{S-C}{=} \lim_{n\to\infty} \frac{\log_{10}(2n+3)}{f(n+1) - f(n)} \Rightarrow \lim \text{ pt } f(n) = n \log_{10} n \text{ e const} \neq \{0, \infty\} \Rightarrow \text{alg e } \Theta(n \log n)$$

$$\cancel{f(n) = n \cdot \log_{10} n \Rightarrow f(n+1) - f(n) = (n+1)\log_{10}(n+1) - n \log_{10}(n)}$$

$$\cancel{= n\left(\log_{10} \frac{n}{n+1}\right) + \log_{10} n =}$$

$$\cancel{= \log_{10} \left(1 + \frac{1}{n+1}\right)^n + \log_{10} n}$$

$$\downarrow e \dots$$

$$f(n) = n \cdot \log_{10} n \Rightarrow f(n+1) - f(n) = (n+1)\log_{10}(n+1) - n \log_{10} n =$$

$$= n\left(\log_{10}(n+1) - \log_{10} n\right) + \log_{10} (n+1)$$

$$= n \log_{10} \frac{n+1}{n} + \log_{10}(n+1) = \log_{10} e + \log_{10}(n+1)$$

$$\downarrow \log_{10} e$$