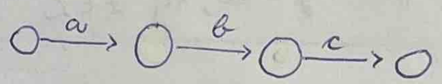
Pumping Lema for Regular Language

→ L - limbaj regular ⇒ este generat de un AFD (automat finit determinist)
→ then there exists a constant n (nb. of states of AFD) such that for every string w in L, $|w| \geq n$
→ we can break w into 3 strings, $w = xyz$ such that:

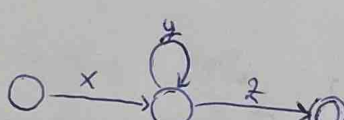
1. $y \neq \varepsilon$ (or) $|y| > 0$
2. $|xy| \leq n$
3. for all $k \geq 0$, the string xy^kz is also in L

$n = 3$
$w = abc$ ⇒ ne trebuie minim 4 stări în AFD



nu e bine! noi am zis că ne trebuie maxim 3 stări, deci unele simb. vor fi „pompate"

⇒



y poate fi „pompat" de oricâte ori

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exemplu: Prove that $L = \{0^n 1^n \mid n \geq 1\}$ is not regular.

* assume L is regular
* let n be a constant (n of states of AFD)
* let $w = 0^n 1^n$, $|w| \geq n$
* split $w = xyz$ such that
  1. $y \neq \varepsilon$ (or) $|y| > 0$
  2. $|xy| \leq n$
  3. for all $k \geq 0$, $xy^kz \in L$

Luăm o încercare

$w = 0^n 1^n$

$n = 2 \Rightarrow w = \underline{00}11$

$xy = 00$ from 2 ⇒ $y = 0$ ⇒ $z = 11$ (from 1)
$x = 0$

$k = 2 \Rightarrow xy^2z = 00011 \notin L$ contradiction ⇒ L nu e regular exemplu Prove that $L=\{a^i b^j \mid i<j\}$ is not regular

$\{aabb, aabbb, aabbbb, abb, \dots\}$

* assume that L is regular
* let n be the number of states of AFD
* alegem un string din L: $w=a^n b^{n+1}$, $|w|\geq n$
* split $w=xyz$ such that

  1. $y\neq\varepsilon$
  2. $|xy|\leq n$
  3. for any $k\geq 0$, $xy^kz\in L$

$n=2$

$w=aabbb$

$xy=aa$ (from 2)

$x=a$

$y=a$ (from 1)

$\Rightarrow z=bbb$

for all $k\geq 0$, $xy^kz\in L$

pick $k=2 \Rightarrow aaabbb\in L$

$k=3 \Rightarrow aaaabbb\notin L \Rightarrow L$ nu e regular (contradicție)

exemplu $L=\{0^{j^2} \mid j\geq 1\}$ is not regular. $\Rightarrow L=\{0, 0000, 0^9, \dots\}$

$w=0^{p^2}=0^n$ where $n=p^2$

split $w=xyz$

$w=0^n$

$xy=0^m$ where $m\leq n$

$y=0^r$ where $r<m$ și $r>0$

$z=0^{n-m}$

$xy^kz=xyy^{k-1}z=$

$=0^m(0^r)^{k-1}0^{n-m}$

$=0^{\cancel{m}+rk-r+n-\cancel{m}}=0^{r(k-1)+n}$

pick $k=1\Rightarrow xy^1z=0^n=0^{p^2}\in L$

$k=2\Rightarrow xy^2z=0^{r+n}=0^{r+p^2}\notin L$

$\Rightarrow L$ nu e regulară (contradicție)

Pumping lema for context free languages

* Let L be a context free language (CFL - independent de context)
* Let n be a constant
* any string z in L, $|z| \geq n$
* split $z = uvwxy$ such that

  $|vwx| \leq n$

  $vx \neq \varepsilon$ or $|vx| \geq 1$

  for all $i \geq 0$, $uv^i w x^i y \in L$

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exemplu $L = \{ a^n b^n c^n \mid n \geq 1 \}$ is not CFL

* let L be a CFL
* let n be a constant
* let $z = a^n b^n c^n$

  $|z| \geq n$

* split $z = uvwxy$

  $u = a^n$

  $vwx = b^n$, $|vwx| \leq n$

  $vx = b^{n-m}$, $|vx| \geq 1$, $m < n$

  $y = c^n$

* $uv^i w x^i y = uvv^{i-1} w x x^{i-1} y =$

  $= uvwx\, (vx)^{i-1}\, y =$

  $= a^n b^n (b^{n-m})^{i-1} c^n =$

  $= a^n b^n b^{ni-n-mi+m} c^n =$

  $= a^n b^{\cancel{n}+ni-\cancel{n}-mi+m} c^n =$

  pick $i = 0 \Rightarrow uv^i w x^i y = a^n b^m c^n \notin L \Rightarrow$ nu este CFL