Propositional

DNF → models
CNF → anti-models

$\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

- a formula has 2^n INTERPRETATIONS

MODEL = an INTERPRETATION which evaluates the formula as TRUE

ANTI-MODEL = an INTERPRETATION which evaluates the formula as FALSE

INTERPRETATION = an assignment of meaning to the symbols of a formal language

FORMULA
- consistent : it has a MODEL
- valid (TAUTOLOGY) : all its INTERPRETATIONS are TRUE
- inconsistent : all its INTERPRETATIONS are FALSE
- contingent : consistent $\wedge$ $\neg$ valid

THEOREM = a formula derivable only from the axioms and using modus ponens as inference rule

* formula V is a ~~semantic~~ LOGICAL CONSEQUENCE of U : $U \models V$

$U_1, \dots, U_m \models V$

if $\left.\begin{array}{l} \forall i : F_P \rightarrow \{T, F\} \\ i(U) = T \end{array}\right\} \Rightarrow i(V) = T$

$i(U_1, \dots, i(U_m)) = T \rightarrow i(V) = T$

* formulas V and U are LOGICAL EQUIVALENT : $U \equiv V$

if they have identical truth tables

DEDUCTION

* formula V is a SYNTACTIC CONSEQUENCE of $U_1, \dots, U_m$: $U_1, \dots, U_m \vdash V$

if $\exists (f_1 \dots f_m)$ s.t. $\begin{cases} f_m = V \\ \forall i \in \{1, \dots, m\} \end{cases}$
- $f_i \in A_P$ (axiom)
- $f_i \in \{U_1, \dots, U_m\}$ (hypotheses)
- $f_{i1}, f_{i2} \vdash_{mp} f_i$ $\quad i_1 < i, \; i_2 < i$

THEOREM OF DEDUCTION

if $U_1, \dots, U_{n-1}, U_n \vdash V$, then $U_1, \dots, U_{n-1} \vdash U_n \to V$

REVERSE OF THE THEOREM OF DEDUCTION

if $U_1, \dots, U_{n-1} \vdash U_n \to V$, then $U_1, \dots, U_{n-1}, U_n \vdash V$

By applying n times T.D and R.T.D we obtain:

$U_1, \dots, U_n \vdash V$ iff

$U_1, \dots, U_{n-1} \vdash U_n \to V$ iff

$\vdots$

$\vdash U_1 \to (U_2 \to (\dots \to (U_n \to V)) \dots)$

...ate logic

a : b 4 se divide cu 2
b | a 2 [illegible]

Resolution in PROPOSITIONAL LOGIC

soundness : if $S \vdash_{res} \square \Rightarrow S$ is INCONSISTENT

completeness : S is INCONSISTENT $\Rightarrow S \vdash_{res} \square$

s + c : $S \vdash_{res} \square$ iff S INCONSISTENT

texis)

GENERAL RESOLUTION

U is a theorem (TAUTOLOGY) iff $CNF(\neg U) \vdash_{res} \square$

- $U_1, \dots, U_n \vdash / \vDash V$ iff $\underbrace{CNF(U_1 \wedge \dots \wedge U_n \wedge \neg V)}_{S} \vdash_{res} \square$

LEVEL SATURATION STRATEGY

→ generates levels of resolvants corresponding to the exploration of the whole search space which contains all the possible resolvents

$S^{k} = \emptyset \rightarrow S$ consistent

$\square \in S^{k} \rightarrow S$ INCONSISTENT

DELETION STRATEGY

→ the resolvents that are TAUTOLOGYES or are SUBSUMED by other clauses in the set S of clauses are eliminated and they will not be used further in the resolution process because they produce redundant clauses

SET OF SUPPORT STRATEGY

→ avoids resolving two clauses belonging to a consistent subset of the initial set of clauses, because the resolvents derived from a consistent set are irrelevant in the process of deriving $\square$

LOCK RESOLUTION

→ the set of clauses has each literal arbitrarly indexed with an integer

→ 2 clauses resolve upon a literal x if x and ¬x have the lowest index in their clause

LINEAR RESOLUTION

→ based on backtracking algorithm

$S \vdash_{res} \square$ → S is INCONSISTENT

all backtracking ⇒ S CONSISTENT

⊛ we choose a top clause

- UNIT - the central clauses have at least a unit clause as a parent clause
- INPUT - all side clauses are initial clauses

DAVIS PUTMAN STRATEGY

- DELETE the clauses that are tautologyes
- DELETE the clauses subsumed by other clauses of the set
- DELETE every clause that contains a pure literal
- let C = l be an UNIT CLAUSE of the set
  - → DELETE every clause that contains l
  - → DELETE ¬l from every remaining clause

Predicate logic

$a \vdots b$ 4 se divide cu 2
$b \mid a$ 2 divizibil cu 4

DEDUCTION

$U_1, \dots, U_n \vdash V$

if $\exists \underbrace{(f_1, \dots, f_m)}_{\text{deduction of } V}$ s.t. $\forall i \in \{1, \dots, m\}$
- $\to f_i \in A_{P_r}$ (axiom)
- $\to f_i \in \{U_1, \dots, U_n\}$ (hypotesis)
- $\to f_{i1}, f_{i2} \vdash_{mp} f_i$
- $\to f_j \vdash_{\text{univ. general.}} f_i$

TH. OF DEDUCTION

If $U_1, \dots, U_{n-1}, U_n \vdash V \Rightarrow U_1, \dots, U_{n-1} \vdash U_n \to V$

REVERSE OF TH. OF DEDUCTION

If $U_1, \dots, U_{n-1} \vdash U_n \to V \Rightarrow U_1, \dots, U_n \vdash V$

REFUTATION THEOREM

If $U_1, \dots, U_n \cup \{\neg V\}$ - INCONSISTENT $\Rightarrow U_1, \dots, U_n \vdash V$

Inference rules

- universal INSTANTIATION : $(\forall x) U(x) \vdash_{\text{univ. i.}} U(t)$ t - constant
- universal GENERALIZATION : $U(x) \vdash_{\text{univ. g}} (\forall x) U(x)$
- existential INSTANTIATION : $(\exists x) U(x) \vdash_{\text{exist. i}} U(c)$ c - new constant
- existential GENERALIZATION : $U(t) \vdash_{\text{exist. g}} (\exists x) U(x)$

Predicate logic

$a \vdots b$ 4 se divide cu 2
$b \mid a$ 2 divizibil cu 4

DEDUCTION

$U_1, \ldots, U_n \vdash V$

if $\exists \underbrace{(f_1, \ldots, f_m)}_{\text{deduction of } V}$ s.t. $\forall i \in \{1, \ldots, m\}$

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- $f_{i1}, f_{i2} \vdash_{mp} f_i$
- $f_j \vdash_{\substack{univ.\\general.}} f_i$

TH. OF DEDUCTION

If $U_1, \ldots, U_{m-1}, U_m \vdash V \Rightarrow U_1, \ldots, U_{m-1} \vdash U_m \to V$

REVERSE OF TH. OF DEDUCTION

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- existential GENERALIZATION : $U(t) \vdash_{exist.\,g} (\exists x)\, U(x)$

Logical equivalences

- expansion laws
  - $(\forall x) A(x) \equiv (\forall x) A(x) \wedge A(t)$
  - $(\exists x) A(x) \equiv (\exists x) A(x) \vee A(t)$
- DeMorgan laws

  $\neg(\exists x) \equiv (\forall x)$

  $\neg(\forall x) \equiv (\exists x)$
- same quantifiers can interchange

$v_a^I(\neg A) = \neg v_a^I(A)$

$v_a^I(A \to B) = v_a^I(A) \to v_a^I(B)$

$v_a^I(A \wedge B) \equiv v_a^I(A) \wedge v_a^I(B)$

$v_a^I(A \vee B) \equiv v_a^I(A) \vee v_a^I(B)$

$v_a^I((\exists x) A(x)) = T$
$\iff v_{a'}^I(A(x)) = T$ for a function a'

$v_a^I((\forall x) A(x)) = T$
$\iff v_{a'}^I(A(x)) = T$ for any function a'

Extraction of QUANTIFIERS

$A \vee (\exists x) B(x) \equiv (\exists x)(A \vee B(x))$

$A \wedge (\exists x) B(x) \equiv (\exists x)(A \wedge B(x))$

$A \vee (\forall x) B(x) \equiv (\forall x)(A \vee B(x))$

$A \wedge (\forall x) B(x) = (\forall x)(A \wedge B(x))$

$(\exists x)(A(x) \vee B(x)) \equiv (\exists x) A(x) \vee (\exists x) B(x)$

$(\forall x)(A(x) \wedge B(x)) \equiv (\forall x)(A(x) \wedge (\forall x) B(x)$

semidistributivity

$\exists$ over $\left< \begin{matrix} \wedge \\ \to \end{matrix} \right.$

$\forall$ over $\left< \begin{matrix} \vee \\ \to \end{matrix} \right.$

$(\exists x) A(x) \; \underline{\vee / \wedge} \; B \equiv (\exists x)(A(x) \; \underline{\vee / \wedge} \; B)$

$(\forall x) A(x) \; \underline{\vee / \wedge} \; B \equiv (\forall x)(A(x) \; \underline{\vee / \wedge} \; B)$

When is a formula $\forall$

Semantic tableaux method - PROPOSITIONAL LOGIC

* conjunctive formulas : **α rules**

$$A \wedge B \quad / \quad A \quad / \quad B$$

$$\neg(A \vee B) \equiv \neg A \wedge \neg B \quad / \quad \neg A \quad / \quad \neg B$$

$$\neg(A \rightarrow B) \equiv A \wedge \neg B \quad / \quad A \quad / \quad \neg B$$

* disjunctive formulas : **β rules**

$$A \vee B \quad / \quad A \;\;\mid\;\; B$$

$$\neg(A \wedge B) \equiv \neg A \vee \neg B \quad / \quad \neg A \;\;\mid\;\; \neg B$$

$$A \rightarrow B \equiv \neg A \vee B \quad / \quad \neg A \;\;\mid\;\; B$$

$\neg U$ INCONSISTENT $\Rightarrow$ U TAUTOLOGY → TH. OF SOUNDNESS AND COMPLETENESS

$$\left.\begin{array}{l} U_1, \dots, U_m \models V \\ U = U_1 \wedge \dots \wedge U_m \wedge \neg V \end{array}\right\} \text{HOLDS?} \begin{cases} \text{YES} : & U \text{ closed} \\ \text{NO} : & U \text{ open} \end{cases}$$

THE $\odot$ branches provide → partial models

models for U ≡ anti-models for $\neg U$

BRANCH
- → $\otimes$: $l + \neg l$
- → $\odot$: $\neg \otimes$
- → complete
  - → $\otimes$
  - or
  - → all formulas are decomposed

$\odot$ → provides MODELS for the formula

TABLEAU
- → $\otimes$: all branches are $\otimes$
- → $\odot$: at least 1 branch $\odot$
- → complete : all branches are complete

$\otimes$ → INCONSISTENT formula

$\odot$ → CONSISTENT formula

Semantic tableaux method - PREDICATE LOGIC

SEMANTIC TABLEAU
- $\neg U \to$ finite
  - U = tautology ($\otimes$ for $\neg U$)
  - U = NOT tautology ($\odot$ for $\neg U$)
- $\neg U \to$ infinite : we don't know

Church THEOREM

* the problem of validity is SEMI-DECIDABLE

U
- $\to$ valid : PROC ends with the corresponding answer
- $\to \neg$valid
  - $\to$ PROC ends with the corresponding answer
  - $\to$ PROC may never stop

*** γ rules** gama

$$\begin{array}{c}
(\forall x)\, A(x) \\
| \\
A(c_1) \\
\vdots \\
| \\
A(c_n) \\
| \\
(\forall x)\, A(x)
\end{array}
\qquad\qquad
\begin{array}{c}
\neg(\exists x)\, A(x) \\
| \\
\neg A(c_1) \\
\vdots \\
| \\
\neg A(c_n) \\
| \\
\neg(\exists x)\, A(x)
\end{array}$$

$c_1, \dots, c_n$ - CONSTANTS

*** ∂ rules** δ - delta

$$\begin{array}{c}
(\exists x)\, A(x) \\
| \\
A(c)
\end{array}
\qquad\qquad
\begin{array}{c}
\neg(\forall x)\, A(x) \\
| \\
\neg A(c)
\end{array}$$

c - new CONSTANT on the branch

BOOLEAN

- MONOM = conjunction of variables

* MINTERM = $\underset{all}{n}$ variables conjunction $\rightarrow m_1$
* MAXTERM = all n variables disjunction $\rightarrow M_2$

$$\begin{cases} m_i \wedge m_j = 0 \\ M_i \vee M_j = 1 \end{cases}$$

$M_i = \overline{m_i}$ and $m_i = \overline{M_i}$

CCF $\rightarrow$ values 0 of the functions $M_1 \wedge M_2 \wedge \ldots$
DCF $\rightarrow$ values 1 of the functions $m_3 \vee m_4 \ldots$

set MAXIMAL MONOMS = $M(f)$
- minterms obtained by using FACTORIZATION

set CENTRAL MONOMS = $C(f)$
- maximal monom which has at least one minterm circled only once

LOGIC CIRCUITS

VEITCH

	x_1		$\bar{x}_1$		
x_2	15	13	5	7	x_4
	14	12	4	6	$\bar{x}_4$
$\bar{x}_2$	10	8	0	2	
	11	9	1	3	x_4
	x_3	$\bar{x}_3$		x_3	

	x_1		$\bar{x}_1$	
x_2	7	6	2	3
$\bar{x}_2$	5	4	0	1
	x_3	$\bar{x}_3$		x_3

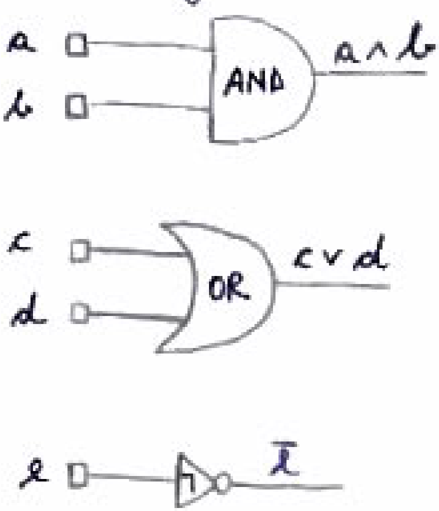
KARNAUGH

ab\cd	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

		$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
	x\yz	00	01	11	10
$\bar{x}$	0	0	1	3	2
x	1	4	5	7	6

LOGIC CIRCUITS

Basic gates



Derived gates