MODEL 1

1. $X = [0, 1, 2, 3, 4, 5]$

$Y = [0, 0, 1, 2, 3]$

a - perm. aleatoare a lui X

b, c - perm. aleatoare indep. a lui Y

a) $P(\{a = (5, 1, 2, 3, 4, 0)\})$

caz. fav. = 1

caz. totale = 5! $\Rightarrow \frac{1}{5!}$

b) P(„c(1) - nr. nenul")

↳ primul elem din c

perm. cu repetiție: $\frac{n!}{n_1! \cdot \ldots \cdot n_k!} = \frac{4!}{1! \cdot 1! \cdot 2!} = 12$ cazuri fav (pt. 1 pe prima poz)

$\Rightarrow 3 \cdot 12$ cazuri fav — cazuri posibile: $\frac{5!}{2!} = 60$

$\Rightarrow$ P(„c(1) - nenul") $= \frac{36}{60}$

c)

Z va fi definit astfel P(Z = -1) = P(„c(1) - nr. par"), P(Z = 1) = P(„c(1) - nr impar").

$E(Z) = ?$

pt. 1 pe prima poz = 12
pt. 3 pe prima poz = 12 (de sus)

$\Rightarrow$ P(„c(1) - impar") $= \frac{24}{60} \Rightarrow P(Z=1) = \frac{24}{60}$

$P(Z=-1) = \frac{36}{60}$

$\Rightarrow E(Z) = -1 \cdot \frac{36}{60} + 1 \cdot \frac{24}{60} = -\frac{12}{60} = -\frac{1}{5}$

d) $P(\{b=[3,1,2,0,0]\} \cup \{c=[3,0,1,2,0]\})$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$ și sunt independente

$\Rightarrow P(E)=\frac{1}{60}+\frac{1}{60}-(P(A)\cdot P(B))=\frac{2}{60}-\frac{1}{60^2}$

e) au aceeași prob $\frac{1}{60}$

② $y_1,\dots,y_5$ date statistice pt Y, $p\in(0,0,5)$

$P(Y=-1)=0.5-p$

$P(Y=0)=p$

$P(Y=2)=0.5$

aproximare pt. $\hat{p}$ cu met. momentelor, știind $\overline{y_5}=0.9$

$$Y\sim\begin{pmatrix}-1 & 0 & 2\\ 0.5-p & p & 0.5\end{pmatrix}$$

$E(Y)=-1\cdot(0,5-p)+0+2\cdot 0,5=-\frac{1}{2}+p+1=\frac{1}{2}+p$

$E(Y)=\overline{y_5}\Rightarrow \frac{1}{2}+\hat{p}=0,9\Rightarrow \hat{p}=0,9-0,5=0,4$

③ U,T v.a. indep. cu aceeași $f:\mathbb{R}\to[0,\infty)$ $f(x)=\begin{cases}3x^2, & x\in[0,1]\\ 0, & \text{altfel}\end{cases}$

a) $\overset{=x^3}{F_U(x)}$, $x\in[0,1]$ și $\overset{=0}{F_T(x)}$, $x\in[1,3]$

$$F(x)=\begin{cases}3\int_0^x t^2\,dt=3\frac{t^3}{3}\Big|_0^x=x^3 & x\in[0,1]\\ 1, & x>1\\ 0, & x<0\end{cases}$$

b) $P(U \leq \frac{1}{2}, T \leq \frac{3}{2})$ -> sunt independente

$$F(\tfrac{1}{2}) = \frac{1}{8}$$

$$F(\tfrac{3}{2}) = 1$$

$$\Rightarrow P = \frac{1}{8} \cdot 1 = \frac{1}{8}$$

c) $E(5U^2 - 4T) = 5E(U^2) - 4E(T) = \cancel{5} \cdot \frac{3}{\cancel{5}} - \cancel{4} \cdot \frac{3}{\cancel{4}} = 0$

$$E(U^2) = \int_{\mathbb{R}} x^2 \cdot f(x)\,dx = \int_0^1 x^2 3x^2\,dx = 3\frac{x^5}{5}\Big|_0^1 = \frac{3}{5}$$

$$E(T) = \int_0^1 x \cdot 3x^2\,dx = 3 \cdot \frac{x^4}{4} = \frac{3}{4}$$