

## PRML functions documentation:

- Kernel matrix(K) formula:  $(\text{gamma}(X_1 X_2^T) + \text{coeff})^{\text{degree}}$  where  $X_1$  and  $X_2$  are feature matrix. For degree = 1, this kernel will behave as a linear kernel.  
Default values: gamma = 1, coeff = 0, degree = 1 (Unless otherwise stated explicitly.)
- add\_intercept: It will add one's column in feature matrix X whose shape is:  
(no of samples x no of features)
- $y_{\text{predicted}} = K(\text{diag}(y_{\text{train}})\alpha)$  for classification (additional parameters and functions(like round ,sign or sigmoid) will be applied according to algorithm) and  $y_{\text{predicted}} = K\alpha$  for regression where  $\alpha$  is a dual coefficient of shape (no of training samples,) and  $K$  is a kernel matrix of shape (no of test samples x no of training samples) while prediction and (no of training samples x no of training samples) while training.
- Log loss :  $J = -[y_{\text{true}}^T \log(y_h) + (1 - y_{\text{true}}^T) \log(1 - y_h)] / m$
- Log loss gradient:  $\frac{\partial J}{\partial \alpha} = \frac{\text{diag}(y_{\text{train}})K(y_h - y_{\text{train}})}{m}$
- Log loss Hessian:  $\frac{\partial^2 J}{\partial \alpha^2} = \frac{\text{diag}(y_{\text{train}})K \text{diag}(y_h)(1 - \text{diag}(y_h))K \text{diag}(y_{\text{train}})}{m}$   
Where  $y_h = \sigma(K(\text{diag}(y_{\text{train}})\alpha))$

- Theoretical Perceptron Bound used in the report:

$$r = \max(\{|x_i| : i = 1, \dots, m\}) = \max(\text{diag}(K)) \dots (1)$$

$$\rho = \min\left(\frac{\text{diag}(y)X\omega}{\|\omega\|}\right) = \min\left(\frac{\text{diag}(y)XX^T(\text{diag}(y)\alpha)}{(\alpha^T \text{diag}(y))XX^T(\text{diag}(y)\alpha)}\right)$$

$$= \min\left(\frac{\text{diag}(y)K(\text{diag}(y)\alpha)}{(\alpha^T \text{diag}(y))K(\text{diag}(y)\alpha)}\right) \dots (2)$$

$$\delta = \sqrt{\sum_t d_t^2} \text{ where } d_t = \max\{0, \rho - y_t(v \cdot x_t)\} \dots (3)$$

$$v \text{ can be any unitvector, for simplicity } v = \left[\frac{1}{\sqrt{m}}, \frac{1}{\sqrt{m}}, \dots, \frac{1}{\sqrt{m}}\right]_{m \times 1}$$

$$\text{Bound} = \left(\frac{r + \delta}{\rho}\right)^2 \dots (4)$$

**\*Note:** To keep  $\rho > 0$  all such  $x_i$  are not considered in equation 2 which gives non positive values due to non-separability. And as  $r$  is norm, and there exists at least one  $x$  in the dataset, which is not a zero vector, so  $r > 0$  condition is automatically satisfied. Also, equation 4 gives general bound for non-linearly separable sample, so kernel modifications are implemented in the above equations like replacing  $K$  for  $XX^T$  or  $\text{diag}(K)$  for  $\{|x_i| : i = 1, \dots, m\}$ .

- Equation for kernel ridge regression:  $(K + \lambda I)K\alpha = K^T y$
- Other default values: (Unless otherwise stated explicitly.)

$T = 100$  (perceptron epochs),  $b = 0$  (bias for perceptron), learning rate = 1 (for perceptron and newton method), learning rate = 0.01 for gradient descent, iterations = 1000 (for gradient descent),  $C = 1.0$  (for soft margin SVM),  $C = 1e10$  (for hard margin SVM), initial dual coefficients = zero vector of shape: (no of samples,) (For perceptron algorithm too.)