Numerical Methods

Lab 9 - Numerical Integration

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Submitted by

Aabhusan Aryal

076BCT001

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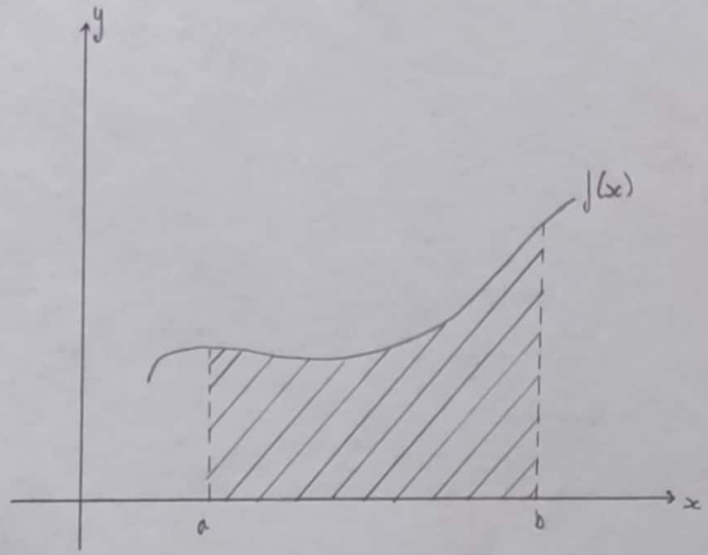
Department of Applied Sciences and
Chemical Engineering, Pulchowk Campus

Numerical Integration :-

Numerical Integration methods allow us to calculate ~~the~~ any definite integral without having to go through the tedious process of finding its primitive (antiderivative).

These methods define a set of steps that can be followed for any function $f(x)$ to calculate the integral.

$$I = \int_a^b f(x)\,dx$$



We know,

$$\int_a^b f(x)\,dx = \text{area under the curve } f(x) \text{ bet}^n\ a \ \& \ b.$$

i) Trapezoid rule :-

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_n) + 2\left[f(x_1) + f(x_2) + \dots + f(x_{n-1}) \right] \right]$$

ii) Simpson's 1/3 rule :-

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0) + f(x_n) + 4\left[f(x_1) + f(x_3) + \dots + f(x_{n-1}) \right] + 2\left[f(x_2) + f(x_4) + \dots + f(x_{n-2}) \right] \right]$$

iii) Simpson's 3/8 rule :-

$$\int_a^b f(x)\,dx = \frac{3}{8}h\left[f(x_0) + f(x_n) + 3\left[f(x_1) + f(x_2) + \dots + f(x_{n-1}) \right] + 2\left[f(x_3) + f(x_6) + \dots + f(x_{n-3}) \right] \right]$$

:- Algorithm

1) Start

2) Declare two arrays x[] and y[], which will hold the points of the curve

3) ~~Declare~~ Input limits of integration - x0, xn ; and the number of points - n.

4) If the user wants to use predefined function :-

    4.1) for (int i=0; i<n+1 ; i++) :-

        4.1.a) x[i] = x0 + h * i ;

        4.1.b) y[i] = f(x[i])

5) else, the user wants to input points :-

    5.1) for (int i=0; i<n+1 ; i++) :-

        5.1.a) x[i] = x0 + h * i

        5.1.b) y[i] = input (" Enter f(x[i]) ")

6) print " Trapezoid : "

7) set sum = 0

8) for (int i=0; i<n+1; i++):

    8.1) set scale = 1 if i=0 or i=n, else scale=2

    8.2) set sum = sum + scale * y[i]

9) print " Result = sum * h/2 "

10) print " Simpson 1/3 : "

11) set sum = 0

12) for (int i=0; i<n+1 ; i++)

    12.1) if i=0 or i=n, set scale = 1

    12.2) if i % 2 = 0, set scale = 2 ; else set scale = 4

    12.3) sum = sum + scale * y[i]

13) print " Result = sum * h/3 "

14) print " Simpson 3/8: "

15) set Sum = 0

16) for (int i = 0 ; i < n+1 ; i++):

   16.1) ~~set~~ if i = 0 or i == n , set scale = 1

   16.2) if i % 3 = 0 , set scale = 2 else set scale = 3

   16.3) set sum = sum + scale * y [i]

17) print . "Result = sum * 3 * h / 8;

18) Stop

+ Source Code:-

```
#include <iostream>
using namespace std;
float f( float x) {
    return x
}
int main () {
    bool use Function;
    float x0, xn, h, *x, *y ;
    int n ;

    cout << " Enter the limits of integration, x0 and xn : ";
    cin >> x0>>xn ;

    cout << " Enter the number of intervals, n : ";
    cin >> n;

    cout << " Press 1 to use predefined function and
     press 0 to give input points : " ;
    cin >> use Function ;

    x = new float [n+1];
```

```
y= new float [n+1];
h= (xn-x0)/n;

// Populating x[] and y[]
for (int i=0; i<n+1; i++){
    x[i] = x0 +h*i;
    if (useFunction) y[i] = f(x[i]);
    else {
        cout<<"Enter y(x"<<i<<") = y("<<x[i]<<")=";
        cin >> y[i];
    }
}
Cout<<"Trapezoid:\n";
//Main Trapezoid logic
float sum =0;
for (int i=0; i<n+1; i++){
    int scale = (i==0 || i==n)? 1:2;
    sum += scale * y[i];
}
float soln = sum * h/2;
cout<<"The area under the given curve is:<<
        soln << endl;

Cout<<"Simpson 1/3:\n";
// Main Simpson 1/3 logic
sum =0;
for ( int i=0; i<n+1; i++){
    int scale = (i==0 || i==n)? 1:(i%2==0? 2: 4);
    sum += Scale * y[i];
}
Soln = sum * h/3;
Cout<<"The area under curve is: "<< soln << endl;
```

```
Cout<< " Simpson 3/8 : \n ";

// Main simpson 3/8 logic

sum = 0;
for (int i=0; i<n+1; i++) {
    int scale = (i==0 || i==n)? 1 : (i%3 == 0? 2 : 3);
    sum += scale * y[i];
}
soln = sum * 3 * h /8;
Cout<< "The area under the given curve is: << soln << endl;
```

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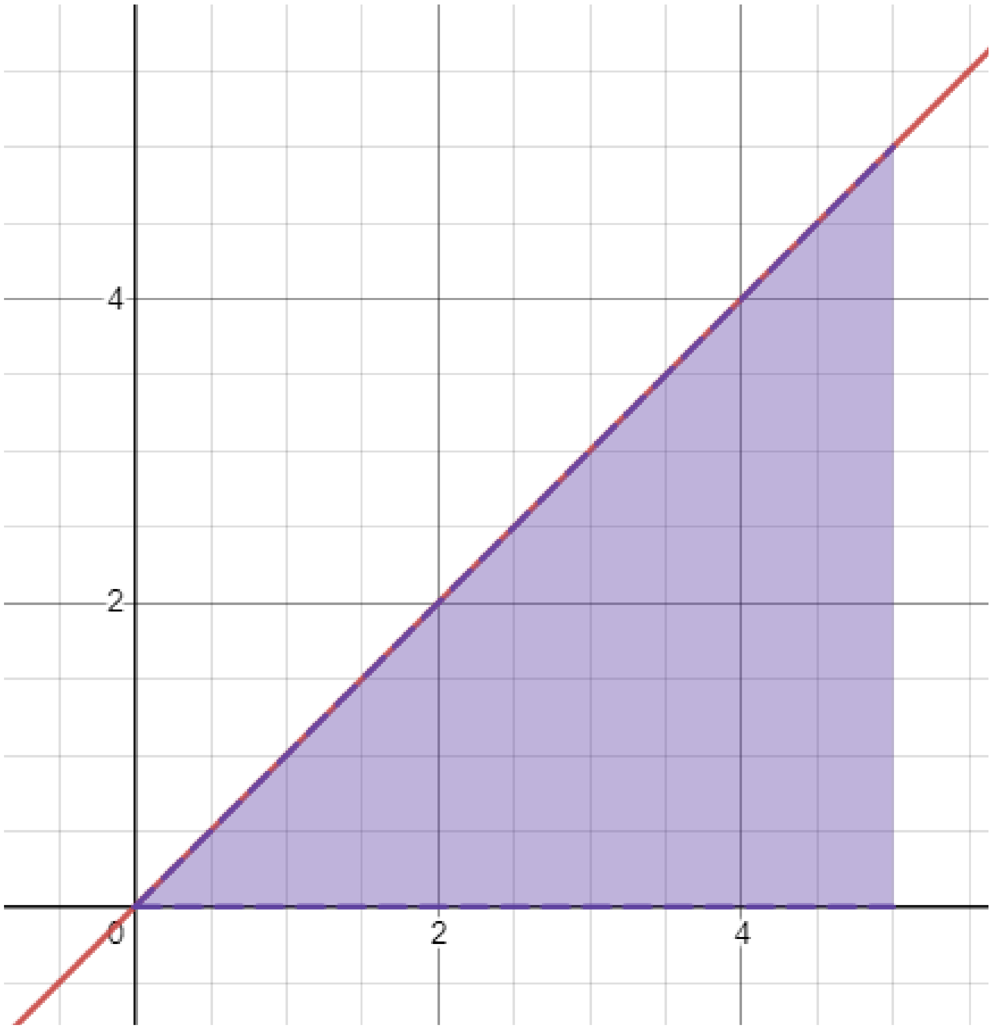
Outputs

1. $I = \int_{a}^{b} f(x) = \int_{0}^{5} x$



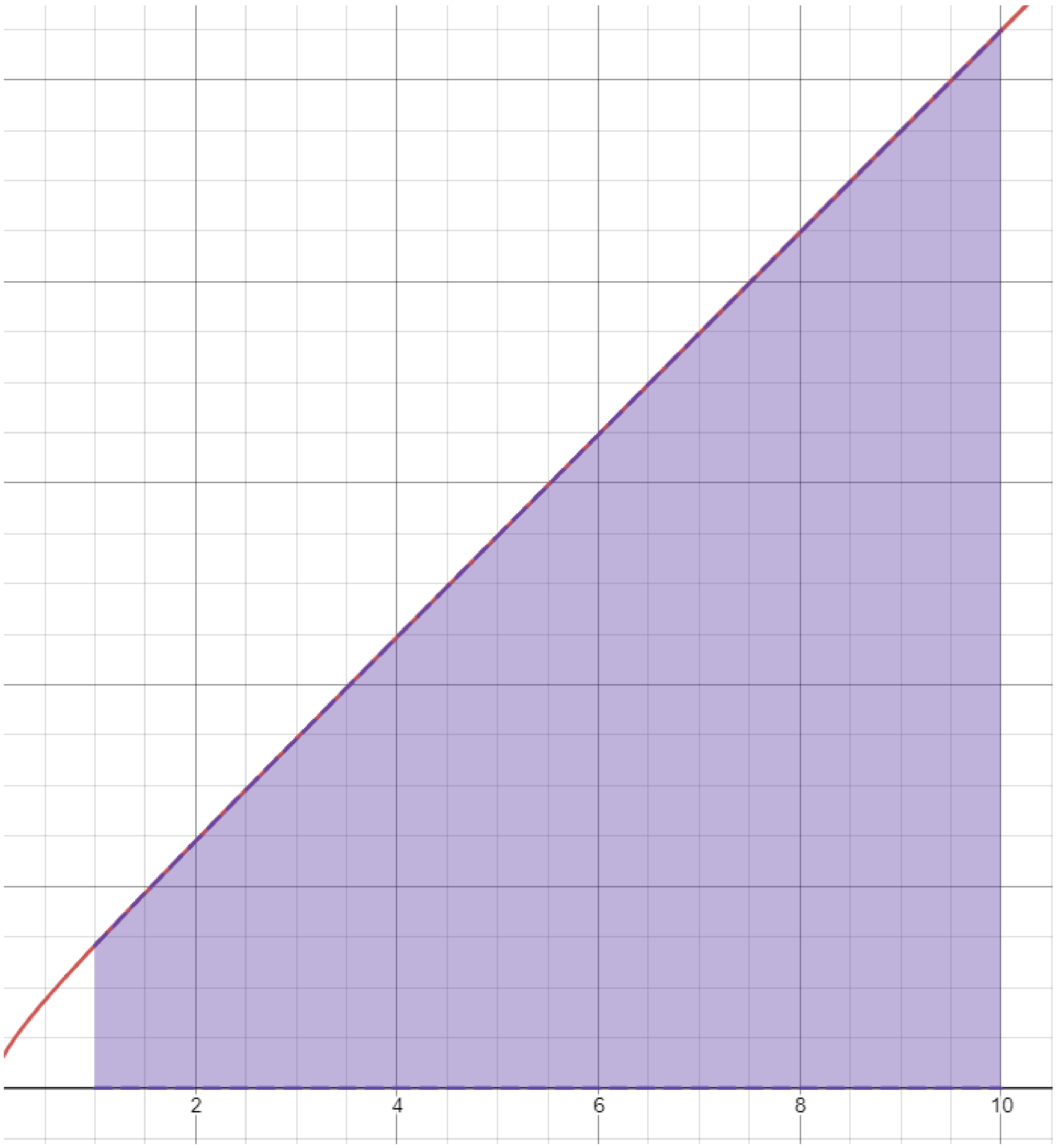
Using non-numerical method, I = 12.5

Using numerical methods:

```
Enter the limits of integration, x0 and xn: 0 5
Enter the number of intervals, n: 10
Press 1 to use predefined function and press 0 to give input points: 1
Trapezoid:
The area under the given curve is: 12.5
Simpson 1/3:
The area under the given curve is: 12.5
Simpson 3/8:
The area under the given curve is: 11.9062
```

Here, the Simpson's ⅜ rule is giving an incorrect result. This is because of the fact that the number of intervals (10) is not a multiple of 3.

2. $I = \int_{a}^{b} f(x) = \int_{1}^{10} \sqrt{x^2 + x}$

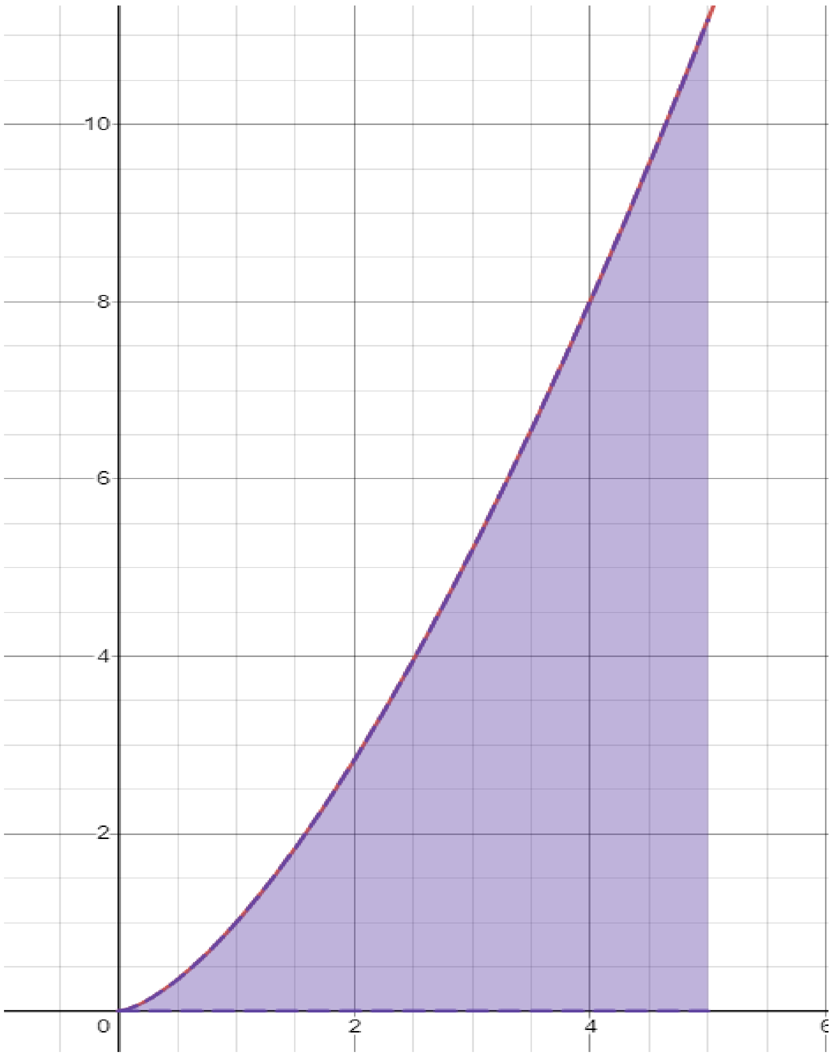


Using non-numerical method, I = 53.755

Using numerical methods:

```
Enter the limits of integration, x0 and xn: 1 10
Enter the number of intervals, n: 12
Press 1 to use predefined function and press 0 to give input points: 1
Trapezoid:
The area under the given curve is: 53.7523
Simpson 1/3:
The area under the given curve is: 53.7548
Simpson 3/8:
The area under the given curve is: 53.7546
```

3. $I = \int_{a}^{b} f(x) = \int_{0}^{5} \sqrt{x^{3}}$



Using non-numerical method, I = 22.36

Using numerical methods:

```
Enter the limits of integration, x0 and xn: 0 5
Enter the number of intervals, n: 12
Press 1 to use predefined function and press 0 to give input points: 1
Trapezoid:
The area under the given curve is: 22.4063
Simpson 1/3:
The area under the given curve is: 22.3623
Simpson 3/8:
The area under the given curve is: 22.363
```

4.

x	$f(x) = x$
0.000	0.000
0.417	0.417
0.833	0.833
1.250	1.250
1.667	1.667
2.083	2.083
2.500	2.500
2.917	2.917
3.333	3.333
3.750	3.750
4.167	4.167
4.583	4.583
5.000	5.000

```
Enter the limits of integration, x0 and xn: 0 5
Enter the number of intervals, n: 12
Press 1 to use predefined function and press 0 to give input points: 0
Enter y(x0) = y(0) =
0.000
Enter y(x1) = y(0.416667) = 0.417
Enter y(x2) = y(0.833333) = 0.833
Enter y(x3) = y(1.25) = 1.250
Enter y(x4) = y(1.66667) = 1.667
Enter y(x5) = y(2.08333) = 2.083
Enter y(x6) = y(2.5) = 2.500
Enter y(x7) = y(2.91667) = 2.917
Enter y(x8) = y(3.33333) = 3.333
Enter y(x9) = y(3.75) = 3.750
Enter y(x10) = y(4.16667) = 4.167
Enter y(x11) = y(4.58333) = 4.583
Enter y(x12) = y(5) = 5.000
Trapezoid:
The area under the given curve is: 12.5
Simpson 1/3:
The area under the given curve is: 12.5
Simpson 3/8:
The area under the given curve is: 12.5
```

5.

x	$f(x) = \sqrt{x^2 + x}$
1.000	1.414
1.750	2.194
2.500	2.958
3.250	3.717
4.000	4.472
4.750	5.226
5.500	5.979
6.250	6.731
7.000	7.483
7.750	8.235
8.500	8.986
9.250	9.737
10.000	10.488

```
Enter the limits of integration, x0 and xn: 1 10
Enter the number of intervals, n: 12
Press 1 to use predefined function and press 0 to give input points: 0
Enter y(x0) = y(1) =
1.414
Enter y(x1) = y(1.75) = 2.194
Enter y(x2) = y(2.5) = 2.958
Enter y(x3) = y(3.25) = 3.717
Enter y(x4) = y(4) = 4.472
Enter y(x5) = y(4.75) = 5.226
Enter y(x6) = y(5.5) = 5.979
Enter y(x7) = y(6.25) = 6.731
Enter y(x8) = y(7) = 7.483
Enter y(x9) = y(7.75) = 8.235
Enter y(x10) = y(8.5) = 8.986
Enter y(x11) = y(9.25) = 9.737
Enter y(x12) = y(10) = 10.488
Trapezoid:
The area under the given curve is: 53.7518
Simpson 1/3:
The area under the given curve is: 53.7545
Simpson 3/8:
The area under the given curve is: 53.7539
```

6.

x	$f(x) = \sqrt{x^3}$
0.000	0.000
0.417	0.269
0.833	0.761
1.250	1.398
1.667	2.152
2.083	3.007
2.500	3.953
2.917	4.981
3.333	6.086
3.750	7.262
4.167	8.505
4.583	9.812
5.000	11.180

```
Enter the limits of integration, x0 and xn: 0 5
Enter the number of intervals, n: 12
Press 1 to use predefined function and press 0 to give input points: 0
Enter y(x0) = y(0) = 0.000
Enter y(x1) = y(0.416667) = 0.269
Enter y(x2) = y(0.833333) = 0.761
Enter y(x3) = y(1.25) = 1.398
Enter y(x4) = y(1.66667) = 2.152
Enter y(x5) = y(2.08333) = 3.007
Enter y(x6) = y(2.5) = 3.953
Enter y(x7) = y(2.91667) = 4.981
Enter y(x8) = y(3.33333) = 6.086
Enter y(x9) = y(3.75) = 7.262
Enter y(x10) = y(4.16667) = 8.505
Enter y(x11) = y(4.58333) = 9.812
Enter y(x12) = y(5) = 11.180
Trapezoid:
The area under the given curve is: 22.4067
Simpson 1/3:
The area under the given curve is: 22.3625
Simpson 3/8:
The area under the given curve is: 22.3633
```

Discussion and Conclusion

Hence various numerical methods were employed to solve different definite integrals. The advantage of using numerical methods over non-numerical methods is that we need not know different formulas for integrating different functions. One method can be applied on any function to obtain the result. Another major advantage of using numerical methods is that it may not always be possible to find the primitive of a certain function. For eg- there exists no function whose derivative is equal to $sin(x)/x$. Which means that we can't simply integrate the function $sin(x)/x$ to find its primitive. But numerical methods can be employed to calculate the integral $\int_a^b \frac{sin(x)}{x}$ over the range a to b.

It was also observed that, at times, the Simpson's ⅓ method was producing more accurate results as compared to Simpson's ⅜ method. Also, the Trapezoid method produced a more accurate result than the Simpson's ⅜ method when the number of intervals were not divisible by 3 (in observation 1). This reminds us that we may choose any number of intervals in Trapezoid method, even number of intervals in the Simpson's ⅓ method and a number of intervals divisible by 3 in Simpson's ⅜ result. Higher number of intervals satisfying the above constraints produced the most accurate results. Hence, 12 intervals were taken in most of our calculations.