

MNTC P01 - Week #2 - Derivatives - Linearization and Applications

Linear Approximations and Tangent Lines

- Find the equation of the tangent line to the graph of f at $(1,1)$, where f is given by $f(x) = 2x^3 - 2x^2 + 1$.
- (a) Find the equation of the tangent line to $f(x) = x^3$ at $x = 2$.
(b) Sketch the curve and the tangent line on the same axes, and decide whether using the tangent line to approximate $f(x) = x^3$ would produce *over-* or *under-*estimates of $f(x)$ near $x = 2$.
- Find the equation of the line tangent to the graph of f at $(3,57)$, where f is given by $f(x) = 4x^3 - 7x^2 + 12$.
- Given a power function of the form $f(x) = ax^n$, with $f'(3) = 16$ and $f'(6) = 128$, find n and a .
- Find the equation of the line tangent to the graph of f at $(2,1)$, where f is given by $f(x) = 2x^3 - 5x^2 + 5$.
- Find all values of x where the tangent lines to $y = x^8$ and $y = x^9$ are parallel.
- Consider the function $f(x) = 9 - e^x$.
 - Find the slope of the graph of $f(x)$ at the point where the graph crosses the x -axis.
 - Find the equation of the tangent line to the curve at this point.
 - Find the equation of the line perpendicular to the tangent line at this point. (This is the *normal* line.)
- Consider the function $y = 2^x$.
 - Find the tangent line based at $x = 1$, and find where the tangent line will intersect the x axis.
 - Find the point on the graph $x = a$ where the tangent line will pass through the origin.
- Find the tangent line approximation to $f(x) = e^x$ at $x = 0$.
 - Use a sketch of $f(x)$ and the tangent line to determine whether the tangent line produces over- or under-estimates of $f(x)$.
 - Use your answer from part (b) to decide whether the statement $e^x \geq 1 + x$ is always true or not.
- The speed of sound in dry air is
$$f(T) = 331.3\sqrt{1 + \frac{T}{273.15}} \text{ m/s}$$
where T is the temperature in degrees Celsius. Find a linear function that approximates the speed of sound for temperatures near 0° C .
- Find the equations of the tangent lines to the graph of $y = \sin(x)$ at $x = 0$, and at $x = \pi/3$.
 - Use each tangent line to approximate $\sin(\pi/6)$.
 - Would you expect these results to be equally accurate, given that they are taken at equal distances on either side of $\pi/6$? If there is a difference in accuracy, can you explain it?
- Consider the graphs of $y = \sin(x)$ (regular sine graph), and $y = ke^{-x}$ (exponential decay, but scaled vertically by k).
If $k \geq 1$, the two graphs will intersect. What is the smallest value of k for which two graphs will be *tangent* at that intersection point?
- Show that $1+kx$ is the local linearization of $(1+x)^k$ near $x = 0$.
 - Someone claims that the square root of 1.1 is about 1.05. Without using a calculator, is this estimate about right, and how can you decide using part (a)?
- Find the local linearization of e^x near $x = 0$.
 - Square your answer to part (a) to find an approximation to e^{2x} .
 - Compare your answer in part (b) to the actual linearization to e^{2x} near $x = 0$, and discuss which is more accurate.
- Show that $1 - x$ is the local linearization of $\frac{1}{1+x}$ near $x = 0$.
 - From your answer to part (a), show that near $x = 0$,
$$\frac{1}{1+x^2} \approx 1 - x^2.$$
 - Without differentiating, what do you think the derivative of $\frac{1}{1+x^2}$ is at $x = 0$?

MATLAB Graphing

16. Create a smooth-looking graph of the function $y = \cos(x)$, x in radians, on the interval $[-\pi, 5\pi]$.
17. Create a smooth-looking graph of the function $y = e^{-x^2}$, over the interval $[-3, 3]$.
18. It is common in scientific plots to draw functions as lines, and plot data as distinct points.

The following points mark the distance between Saturn and several of its moons:

Planet/Object	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Mean Distance (AU), d	0.39	0.72	1	1.52	5.20	9.54	19.18	30.06	39.44
Period (Earth years), T	0.24	0.62	1	1.88	11.86	29.46	84.01	164.8	247.7

The best-fit curve to this data is given by the formula $T = d^{3/2}$.

Plot both the raw data (as points) and best fit curve (as a line) on a single graph. The best-fit graph should look smooth.

Newton's Method

19. Consider the equation $e^x + x = 2$. This equation has a solution near $x = 0$. By replacing the left side of the equation by its linearization near $x = 0$, find an approximate value for the solution.

(In other words, perform one step of Newton's method, starting at $x = 0$, by hand.)

20. Use Newton's Method with the equation $x^2 = 2$ and initial value $x_0 = 3$ to calculate x_1, x_2, x_3 (the next three solution estimates generated by Newton's method). Do the calculations by hand.
21. Using MATLAB, write a script that applies Newton's Method to solve the equation $x^3 = 5$. Use 10 iterations of Newton's method.
Compute the values of x^3 when you are done to confirm that it is close to 5.
22. Use Newton's Method to approximate $4^{1/3}$ and compare

with the value obtained from a calculator.

(Hint: write out a simple equation that $4^{1/3}$ would satisfy, and use Newton's method, with MATLAB, to solve that.)

23. Consider the equation $10xe^{-2x} = 0.4$.

- (a) On a single set of axes, draw both the graphs $y = 10xe^{-2x}$ and $y = 0.4$. The x locations of the intersections between these two graphs are the solutions.
- (b) Continue your MATLAB script so that you use Newton's Method to find **both** solutions to $10xe^{-2x} = 0.4$.
- (c) Confirm both solutions by subbing them into the original equation and verifying that the left and right hand sides of the equation are equal.