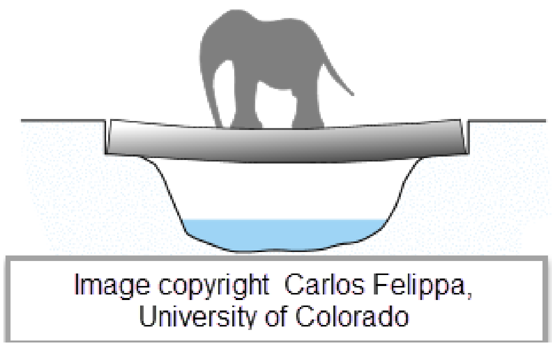
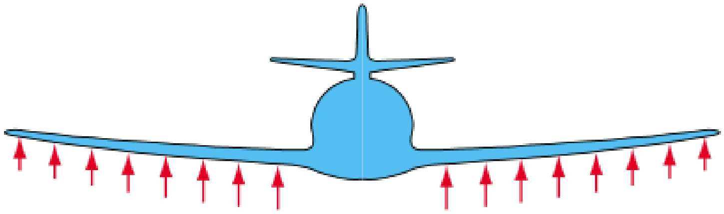
Week #9 : Differential Equations and Engineering

Goals:

- Take problems that can be modeled by differential equations, both first and second order, and give solutions both by hand and MATLAB (CLO5, CLO8)
- Examine case studies of differential equations applied to engineering problems and reproduce those solutions

Deformation of a Loaded Beam





The physics of how beams deform isn't central to this course, and engineering students in particular will take a course on structures that goes into deeper detail. For our purposes, it is enough to be able to use the resulting differential equations.

$$EIy^{(4)} = p(x)$$

where
$y(x)$ is the deflection (distance away from a straight line),
$p(x)$ is the loading in N/m at point x along the beam,
E is a value related to the material of the beam, and
I is a value derived from the cross-sectional size and shape of the beam.

Problem. Find the form of y_c for the beam deflection DE, assuming E and I are constant.

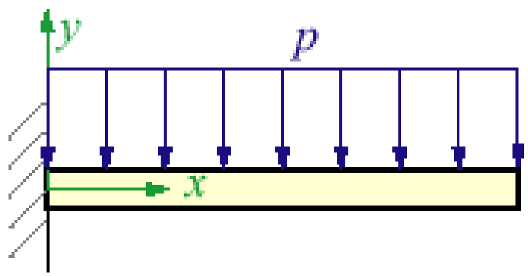
$$EIy^{(4)} = p(x)$$

Cantilevered Beam Under Uniform Load

Under a uniform loading (constant force per unit length), a *cantilevered beam* which is $L = 2$ m long, made out of a pine "2 by 4" satisfies

$p(x) = 100$ N/m, (or roughly 10 kg applied to each meter)

$I = 2.23 \times 10^{-6}\ \mathrm{m}^4, \qquad E = 9.1 \times 10^9\ \mathrm{N/m}^2,$



and the *boundary* conditions $y(0) = 0$, $y'(0) = 0$, $y''(2) = 0$, and $y'''(2) = 0$.

Problem. Find the amount of deflection of the beam at the tip under this load.

$$EIy^{(4)} = 100$$
$$y(0) = 0,\ y'(0) = 0,$$
$$y''(2) = 0, \text{ and } y'''(2) = 0.$$

Problem. If the maximum allowable deflection in such a beam is only 0.2 cm (say in a building code), what would the maximum uniform load be?