MNTC P01 - Week #8 - Second Order Differential Equations

1. Use `ode45` to generate a graph of the solution to the following DEs, over the specified interval, given the initial condition.

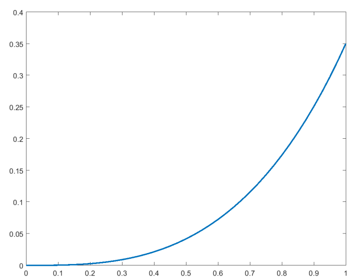
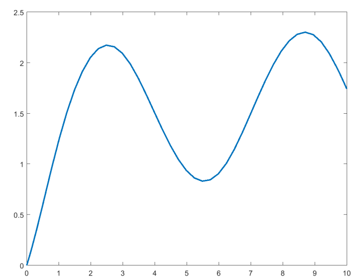
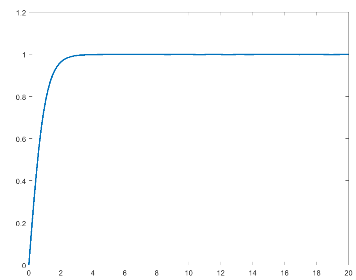
   (a) $\dfrac{dy}{dt} = t^2 + y^2$, $y(0) = 0$, and $0 \le t \le 1$.

   (b) $\dfrac{dy}{dt} = \sin(t) + \cos(y)$, $y(0) = 0$, and $0 \le t \le 10$.

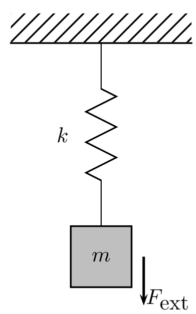
   (c) $\dfrac{dy}{dt} = (1 - y^2) + 0.2\sin(t)$, $y(0) = 0$, and $0 \le t \le 20$.

Link to the MATLAB code:
W08DE01.m

Here are the graphs of the solutions.

2. Consider the single spring/mass system shown below, with no damper:



where F_{ext} is an external applied force.

Newton's second law gives us the relationship:

$$ma = \sum F = F_{\text{spring}} + F_{\text{ext}}$$
$$mx'' = -kx + F_{\text{ext}}$$

where k is the spring constant.

   (a) By hand, write this second order DE as a system of 1st order DEs, using the new variables $w_1 = x$ and $w_2 = x'$

   (b) Set $m = 0.5$ kg, $k = 10$ N/m, and F_{ext} to zero (no external force). Define the differential equations in MATLAB in `springDE1.m`. Use `ode45` to simulate the motion of the spring, given an initial displacement of $x(0) = 0.2$ m, and initial velocity of zero: $x'(0) = 0$. Generate a plot with

   - position against time (do *not* show the velocity), and
   - either choosing the time interval used for the `ode45` simulation, or setting the graph's display limits on the graph with `xlim`, to show the first 3 to 4 cycles only.

   (c) With the same initial conditions and constants as in (b), simulate the motion of the spring if we now apply an external force of $F_{\text{ext}} = \sin(t)$. To do this, you will need to have to add both t and F_{ext} as arguments to the DE. e.g.

```
function dw_dt = springDE2(t, w, m, k, F_ext)
```

Generate a simulation over the time span $t = [0, 40]$ seconds, and plot the position against time.

Explain why the motion looks so disorganized.

(d) Repeat part (c), but with an external force of $F_{\text{ext}} = \sin(4t)$. Explain why the motion has cyclic waves in its amplitude.

(a) The first-order system would be:

$$\frac{d}{dt}w_1 = \dot{x} = w_2$$
$$\frac{d}{dt}w_2 = \ddot{x} = \frac{1}{m}\left(-kx + F_{\text{ext}}\right) = \frac{1}{m}\left(-kw_1 + F_{\text{ext}}\right)$$

(b) The files
springDE1.m
and
W08SpringSimulation01.m
have the code that will run this simulation.

In the resulting plot, we see a very nice example of simple harmonic motion.