Practice Problems

1. Practice turning the following systems in augmented matrices, and use MATLAB to solve for their solution, if they have one.

$$\begin{aligned} x + 11y + z &= -15 \\ x + 2z - 3y &= 33 \\ 5y + 12z - x + 2 &= 5 \end{aligned}$$

$$\begin{aligned} 3x - 8x - 1.3y - 2z &= 71 + z \\ x - y + z &= 13 + y + 2.2z \\ -9.1x + 2.8y + 82.3z &= 6.5 + 1.9x - 9.4y - 3.3z \end{aligned}$$

2. Find the magnitude of the following vectors: $\overrightarrow{a} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, $\overrightarrow{b} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$, $\overrightarrow{c} = \begin{bmatrix} 0.5 \\ 2.4 \\ 10.2 \\ 8.7 \end{bmatrix}$

3. Let $\overrightarrow{a} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $\overrightarrow{b} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$, $\overrightarrow{c} = \begin{bmatrix} 1 \\ -8 \end{bmatrix}$. Find $\overrightarrow{v}$ if $\overrightarrow{v} = 2\overrightarrow{a} - 3\overrightarrow{b} + 4\overrightarrow{c}$

4. Find the unit vector of the following vectors: $\overrightarrow{x} = \begin{bmatrix} 1 \\ -8 \end{bmatrix}$, $\overrightarrow{y} = \begin{bmatrix} -3 \\ 6 \\ 7 \end{bmatrix}$, $\overrightarrow{z} = \begin{bmatrix} 10 \\ -2 \\ -8 \\ 2 \end{bmatrix}$

5. Check whether the following pairs of vectors are orthogonal:

$$\overrightarrow{a} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \text{ and } \overrightarrow{b} = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$$

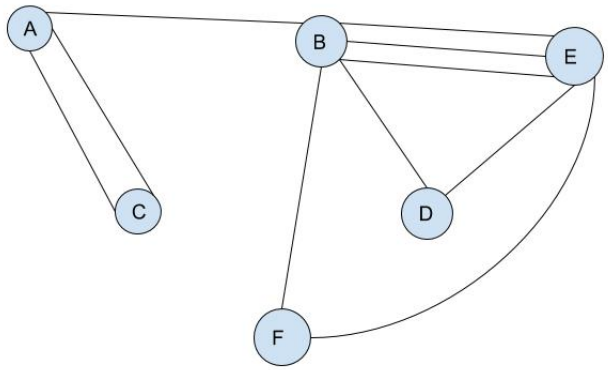
$$\vec{c} = \begin{bmatrix} 12 \\ 4 \\ -2 \end{bmatrix} \text{ and } \vec{d} = \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix}$$

$$\vec{e} = \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix} \text{ and } \vec{f} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

6. Find the transpose and inverse of the following matrix:

$$\mathbf{P} = \begin{pmatrix} 21 & -1 & 43 \\ 91 & -12 & 41 \\ 17 & -26 & -65 \end{pmatrix}$$

7. 6 towns, named A through F, have a series of roads connecting them. If you look at the picture, you can see that there are two roads connecting A and C, for example.



Create a matrix that displays how many roads connect each of the towns. Your matrix should look like

$$\left(\begin{array}{c|cccccc} & A & B & C & D & E & F \\ \hline A & 0 & 1 & 2 & 0 & 0 & 0 \\ B & & & & & & \\ C & & & & & & \\ D & & & & & & \\ E & & & & & & \\ F & & & & & & \end{array}\right)$$

The first row is filled out to demonstrate the solution. Fill in the rest of the spaces.

Solutions

1. $\left(\begin{array}{ccc|c} 1 & 11 & 1 & -15 \\ 1 & 2 & -3 & 33 \\ -1 & 5 & 12 & 3 \end{array}\right)$

   Solution: $\begin{bmatrix} 92.094 \\ -10.868 \\ 12.453 \end{bmatrix}$

   $\left(\begin{array}{ccc|c} -5 & -1.3 & -3 & 71 \\ 1 & -2 & -1.2 & 13 \\ -11 & 12.2 & 85.6 & 6.5 \end{array}\right)$

   Solution: $\begin{bmatrix} -11.225 \\ -12.349 \\ 0.393 \end{bmatrix}$

2. $|\overrightarrow{a}| = \sqrt{10}$, $|\overrightarrow{a}| = \sqrt{21}$, $|\overrightarrow{a}| = \sqrt{185.74}$

3. $\begin{bmatrix} 26 \\ 26 \end{bmatrix}$

4. $\overrightarrow{x}_u = \begin{bmatrix} \frac{1}{\sqrt{65}} \\ \frac{-8}{\sqrt{65}} \end{bmatrix}$, $\overrightarrow{y}_u = \begin{bmatrix} \frac{-3}{\sqrt{94}} \\ \frac{6}{\sqrt{94}} \\ \frac{7}{\sqrt{94}} \end{bmatrix}$, $\overrightarrow{z}_u = \begin{bmatrix} \frac{10}{\sqrt{172}} \\ \frac{-2}{\sqrt{172}} \\ \frac{-8}{\sqrt{172}} \\ \frac{2}{\sqrt{172}} \end{bmatrix}$

5. $\overrightarrow{a} \cdot \overrightarrow{b} = -22$, $\overrightarrow{c} \cdot \overrightarrow{d} = 0$, $\overrightarrow{e} \cdot \overrightarrow{f} = 0$

6. $\mathbf{P}^T = \begin{pmatrix} 21 & 91 & 17 \\ -1 & -12 & -26 \\ 43 & -41 & -65 \end{pmatrix}$, $\mathbf{P}^{-1} = \begin{pmatrix} -0.030 & 0.019 & -0.008 \\ -0.109 & 0.034 & -0.050 \\ 0.036 & -0.009 & 0.003 \end{pmatrix}$

7.

	A	B	C	D	E	F
A	0	1	2	0	0	0
B	1	0	0	1	3	1
C	2	0	0	0	0	0
D	0	1	0	0	1	0
E	0	3	0	1	0	1
F	0	1	0	0	1	0