

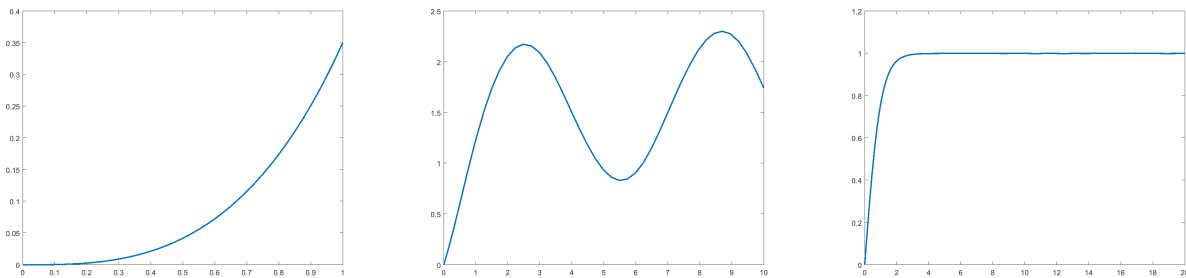
## MNTC P01 - Week #8 - Second Order Differential Equations

1. Use `ode45` to generate a graph of the solution to the following DEs, over the specified interval, given the initial condition.

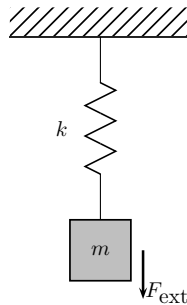
- (a)  $\frac{dy}{dt} = t^2 + y^2$ ,  $y(0) = 0$ , and  $0 \leq t \leq 1$ .
- (b)  $\frac{dy}{dt} = \sin(t) + \cos(y)$ ,  $y(0) = 0$ , and  $0 \leq t \leq 10$ .
- (c)  $\frac{dy}{dt} = (1 - y^2) + 0.2 \sin(t)$ ,  $y(0) = 0$ , and  $0 \leq t \leq 20$ .

Link to the MATLAB code:  
W08DE01.m

Here are the graphs of the solutions.



2. Consider the single spring/mass system shown below, with no damper:



where  $F_{\text{ext}}$  is an external applied force.

Newton's second law gives us the relationship:

$$ma = \sum F = F_{\text{spring}} + F_{\text{ext}}$$

$$mx'' = -kx + F_{\text{ext}}$$

where  $k$  is the spring constant.

- (a) By hand, write this second order DE as a system of 1st order DEs, using the new variables  $w_1 = x$  and  $w_2 = x'$
- (b) Set  $m = 0.5$  kg,  $k = 10$  N/m, and  $F_{\text{ext}}$  to zero (no external force). Define the differential equations in MATLAB in `springDE1.m`. Use `ode45` to simulate the motion of the spring, given an initial displacement of  $x(0) = 0.2$  m, and initial velocity of zero:  $x'(0) = 0$ . Generate a plot with
  - position against time (do *not* show the velocity), and
  - either choosing the time interval used for the `ode45` simulation, or setting the graph's display limits on the graph with `xlim`, to show the first 3 to 4 cycles only.
- (c) With the same initial conditions and constants as in (b), simulate the motion of the spring if we now apply an external force of  $F_{\text{ext}} = \sin(t)$ . To do this, you will need to have to add both  $t$  and  $F_{\text{ext}}$  as arguments to the DE. e.g.

```
function dw_dt = springDE2(t, w, m, k, F_ext)
```

Generate a simulation over the time span  $t = [0, 40]$  seconds, and plot the position against time.

Explain why the motion looks so disorganized.

- (d) Repeat part (c), but with an external force of  $F_{\text{ext}} = \sin(4t)$ . Explain why the motion has cyclic waves in its amplitude.

- (a) The first-order system would be:

$$\begin{aligned}\frac{d}{dt}w_1 &= \dot{x} = w_2 \\ \frac{d}{dt}w_2 &= \ddot{x} = \frac{1}{m}(-kx + F_{\text{ext}}) = \frac{1}{m}(-kw_1 + F_{\text{ext}})\end{aligned}$$

- (b) The files  
springDE1.m  
and  
W08SpringSimulation01.m  
have the code that will run this simulation.

In the plot, we see very nice example of harmonic motion.

