MNTC P01 - Week #7 - Differential Equations - Introduction

Verifying Solutions

1. Show that $y = \dfrac{2}{3}e^x + e^{-2x}$ is a solution of the differential equation $y' + 2y = 2e^x$.

2. (a) For what values of r does the function $y = e^{rx}$ satisfy the differential equation $2y'' + y' - y = 0$?

   (b) If r_1 and r_2 are the values of r that you found in part (a), show that every member of the family of functions $y = ae^{r_1 x} + be^{r_2 x}$ is also a solution.

3. (a) For what values of k does the function $y = \cos(kt)$ satisfy the differential equation $4y'' = -25y$?

   (b) For those values of k, verify that every member of the vamily of functions $y = A\sin kt + B\cos kt$ is also a solution.

4. Consider the differential equation $\dfrac{dy}{dx} = -y^2$.

   (a) If you were asked whether the solutions to this equation would *increase* or *decrease* as x increased, what could you say based on only the equation itself?

   (b) Verify that all members of the family $y = 1/(x + C)$ are solutions of the equation in part (a).

   (c) Can you think of a (very simple) solution of the differential equation $y' = -y^2$ that is *not* a member of the family in part (b)?

   (d) Find the solution to the initial-value problem

$$y' = -y^2 \qquad y(0) = 0.5$$

Numerical ODE Solutions With MATLAB

5. Create a plot for the solution to the differential equation $y' - \frac{y^2}{x^3} = 0$ if y(2) = 1. Include a large enough `xspan` to see the long-term behaviour.

6. Create a plot for the solution to the differential equation $(2y - 4)y' - 3x^2 = 4x - 4$, if y(1) = 3.

7. Create a plot for the solution to the differential equation $y' = e^{-y}(2t - 4)$ if $y(0) = 5$

8. Create a plot for the solution to the differential equation $ty' - 2y = t^5 \sin(2t) - t^3 + 4t^4$, if $y(\pi) = \frac{3}{2}\pi^4$

9. Create a plot for the solution to the differential equation $ty' + 2y = t^2 - t + 1$, if $y(1) = 0.5$.

10. Create a plot for the solution to the differential equation $2xy^2 + 4 = 2(3 - x^2 y)y'$ if $y(5) = 8$.