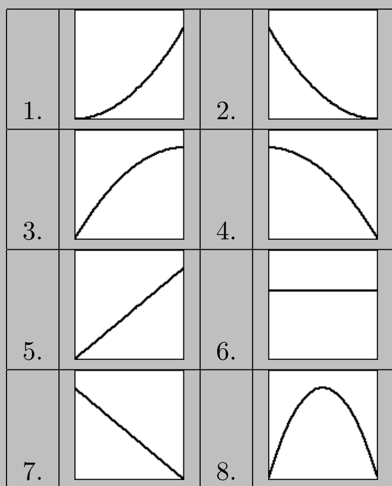
Week #1 - Derivatives - Foundations

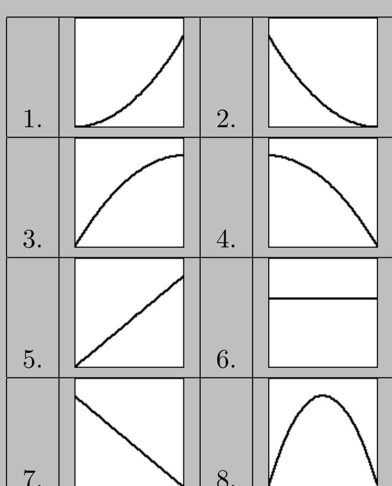
Derivative Concept and Definition

1. A car is driven at a speed that is initially high and then decreases, starting at noon. Which of the following could be a graph of the distance the car has traveled as a function of time past noon?



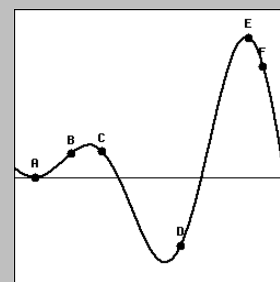
Because the car is driven at a decreasing speed, the distance traveled for different time intervals of the same length must decrease as time goes on. Therefore the slope of the graph of distance traveled must decrease with increasing time, and must be 3.

2. A car is driven at a constant speed, starting at noon. Which of the following could be a graph of the distance the car has traveled as a function of time past noon?



Because the car is driven at a constant speed, the change in the distance traveled is the same for different time intervals of the same length. Thus the graph of the distance traveled must have a constant positive slope, and must be graph 5.

3. Match the points labeled on the curve below with the given slopes in the following table.

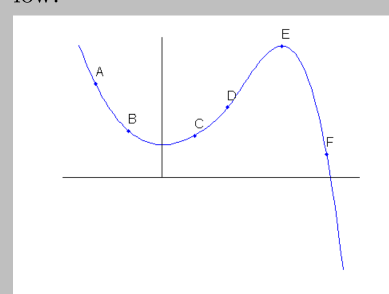


slope	-3	-1	-1/2	0	1	2
label						

We match the points and slopes by noting that the slope is zero where the tangent to the curve is horizontal, negative where the function is decreasing, and positive where the function is increasing. The magnitude of the slope gives the rate at which the function is increasing or decreasing, so that we have

slope	-3	-1	-1/2	0	1	2
label	F	C	E	A	B	D

4. Consider the function shown in the graph below.



(a)For each labelled point, is the slope of the graph positive, negative or zero?
(b) At which labeled point does the graph have the greatest (i. e., most positive) slope?
(c) At which labeled point does the graph have the **largest negative** slope?

(a)

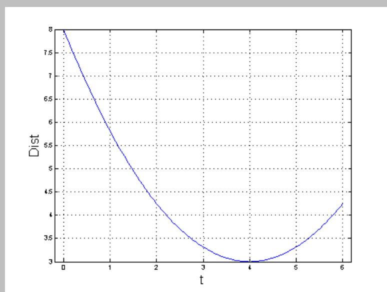
A. Negative
B. Negative
C. Positive

D. Positive
E. Zero
F. Negative

(b) D

(c) F

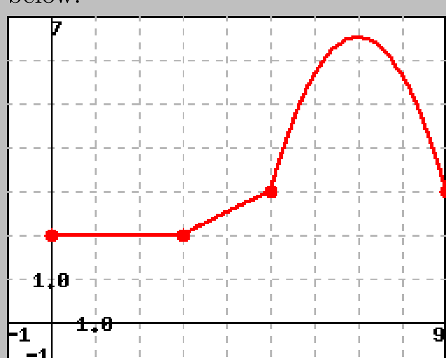
5. Consider the **distance vs time** graph shown below.



Rank the following quantities as if along the real line (i.e. largest negative, through zero, up to largest positive).
A - Instantaneous velocity at $t = 1$.
B - Instantaneous velocity at $t = 3$.
C - Instantaneous velocity at $t = 4$.
D - Instantaneous velocity at $t = 5$.
E - Average velocity over $t = 1 \ldots 3$.
F - Average velocity over $t = 4 \ldots 5$.

The order, from largest negative through to largest positive, is:
$A \to E \to B \to C \to F \to D$

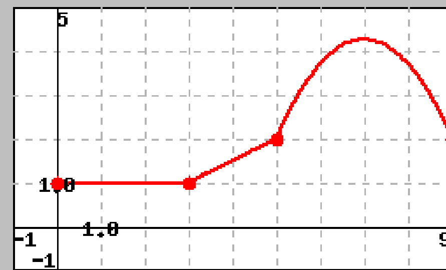
6. Let $f(x)$ be the function whose graph is shown below.



Determine the derivative of $f(a)$ at the points $a = 1, 2, 4, 7$.

Remember that the value of the derivative of f at $x = a$ can be interpreted as the slope of the line tangent to the graph of $y = f(x)$ at $x = a$. From the figure, we see that the graph of $y = f(x)$ is a horizontal line (that is, a line with zero slope) on the interval $0 \le x \le 3$. Accordingly, $f'(1) = f'(2) = 0$. On the interval $3 \le x \le 5$, the graph of $y = f(x)$ is a line of slope 0.5; thus, $f'(4) = 0.5$. Finally, the line tangent to the graph of $y = f(x)$ at $x = 7$ is horizontal, so $f'(7) = 0$.

7. Let $f(x)$ be the function whose graph is shown below.



Which is larger?

- A. $f'(6.5)$
- B. $f'(5.5)$

The line tangent to the graph of $y = f(x)$ at $x = 5.5$ has a larger slope than the line tangent to the graph of $y = f(x)$ at $x = 6.5$. Therefore, $f'(5.5)$ is larger than $f'(6.5)$.

8. Use algebra to evaluate the following limit.

$$\lim_{h \to 0} \frac{(3+h)^2 - 9}{h}$$

$\frac{(3+h)^2-9}{h} = \frac{3^2+6h+h^2-9}{h} = \frac{6h+h^2}{h} = 6 + h$

Thus, as $h \to 0$ we have $\frac{(3+h)^2-9}{h} \to 6$.

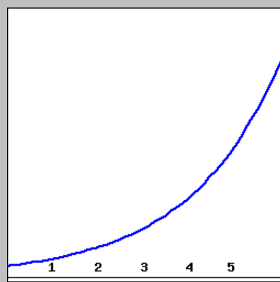
9. Estimate the following limit by substituting smaller and smaller values of h, and by using algebra (the two answers should be very similar!).

$$\lim_{h \to 0} \frac{(4+h)^3 - 64}{h}$$

We try successively smaller values of h: $\frac{(4+0.1)^3-64}{0.1} = 49.2100$ $\frac{(4+0.01)^3-64}{0.01} = 48.1201$ $\frac{(4+0.001)^3-64}{0.001} = 48.0120$ $\frac{(4+0.0001)^3-64}{0.0001} = 48.0012$ $\frac{(4+0.00001)^3-64}{0.00001} = 48.0001$

These values suggest that the limit is 48.000.

10. Suppose $y = f(x)$ graphed in the figure below represents the cost of manufacturing x kilograms of a chemical.

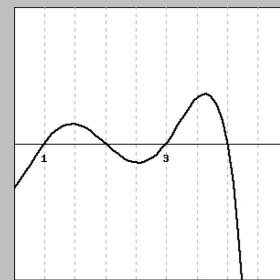


$f(x)/x$ represents the average cost of producing 1 kilogram of the chemical when x kilograms are made. This problem asks you to visualize these averages graphically.

(a) We can represent $f(4)/4$ as the slope of a line. Through what points does this line extend?

(b) Which is larger, $f(3)/3$ or $f(4)/4$?

11. Consider the function $y = f(x)$ graphed below.



Give the x-coordinate of a point where:

(a) The derivative of the function is positive.

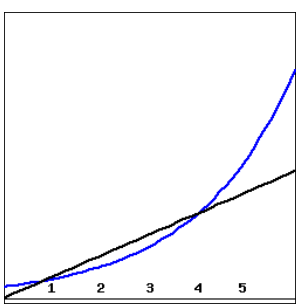
(b) The value of the function is positive.

(c) The derivative of the function is largest.

(d) The derivative of the function is zero.

(e) The derivative of the function is approximately the same as the derivative at $x = 3.25$.

(a) $f(4)/4$ is the slope of the line connecting $(0,0)$ to $(4, f(4))$, as shown below.



(b) Extending the line described in part (a) for both the points $x = 3$ and $x = 4$, we will get a higher slope in the second case, so

$$\frac{f(3)}{3} < \frac{f(4)}{4}$$

(a) Recall that the derivative is positive when the function is increasing. So we're looking for a point on the graph where the function is increasing; one such point is $x = 1$.

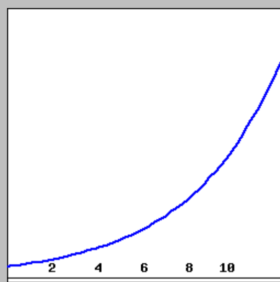
(b) The function is positive when the function is above the x-axis. One such point is $x = 1.5$.

(c) The derivative of the function is largest when the function is increasing the fastest. This occurs at $x \approx 3.25$.

(d) The derivative of the function is zero when the function has a horizontal tangent. One such point is $x \approx 1.45$.

(e) Finally, we're looking for a point where the derivative is the same as the derivative at $x = 3.25$. This means that we want the slope of a tangent to the curve to be approximately the same as it is at $x = 3.25$. Such a point is $x = 0.75$.

12. Consider the graph of the function $f(x)$ shown below.



Using this graph, for each of the following pairs of numbers decide which is larger. *Be sure that you can explain your answer.*

(a) $f(8)$ vs. $f(10)$

(b) $f(8) - f(6)$ vs. $f(6) - f(4)$

(c) $\dfrac{f(6) - f(4)}{6 - 4}$ vs. $\dfrac{f(8) - f(4)}{8 - 4}$

(d) $f'(4)$ vs. $f'(10)$

(a) Since f is increasing, $f(8) < f(10)$.

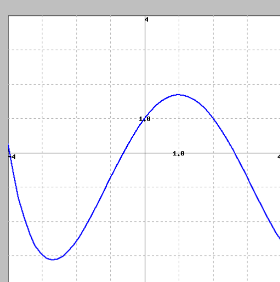
(b) From the figure, it appears that $f(8) - f(6) > f(6) - f(4)$.

(c) The quantity $\frac{f(6)-f(4)}{6-4}$ represents the slope of the secant line connecting the points on the graph at $x = 4$ and $x = 6$. This is less than the slope of the secant line connecting the points at $x = 4$ and $x = 8$, which is $\frac{f(8)-f(4)}{8-4}$

(d) The function is less steep at $x = 4$ than at $x = 10$, so $f'(4) < f'(10)$.

The Derivative as a Function

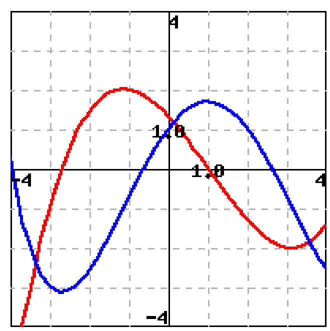
13. Consider the function $f(x)$ shown in the graph below.



Carefully sketch the derivative of the given function (you will want to estimate values on the derivative function at different x values as you do this). Use your derivative function graph to estimate the following values on the derivative function.

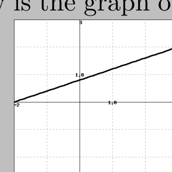
at $x =$	-3	-1	1	3
the derivative is	___	___	___	___

At each value of x, we estimate the derivative by estimating the slope (=rise/run) from the graph. This allows us to both sketch the graph of the derivative and estimate the derivatives that are requested. The derivative (in red) is shown with the original function (in blue) below.

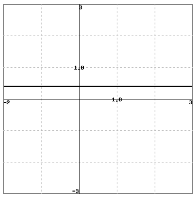


From the graph and our calculations, it appears that
$f'(-3) \approx -1$,
$f'(-1) \approx 2$,
$f'(1) \approx 0$, and
$f'(3) \approx -2$.

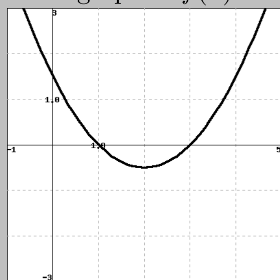
14. Below is the graph of $f(x)$. Sketch the graph of $f'(x)$.



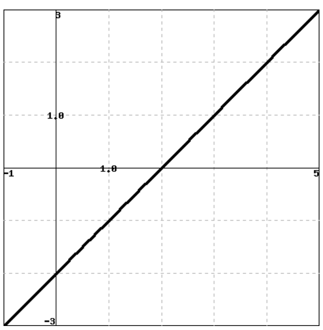
The **slope** on the original function is constant, so the **derivative** value is constant.



15. Below is the graph of $f(x)$. Sketch the graph of $f'(x)$.

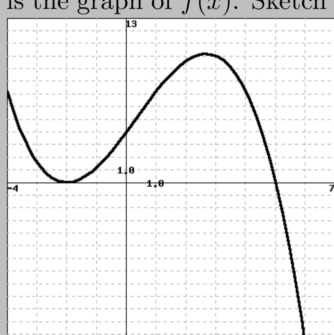


There is a point with **slope** zero at $x = 2$, so there must be a **derivative** value of zero at $x = 2$.
For smaller values of x, the **slopes** are negative, so the **derivative** values will be negative.
For larger values of x, the **slopes** are positive, so the **derivative** values will be positive.



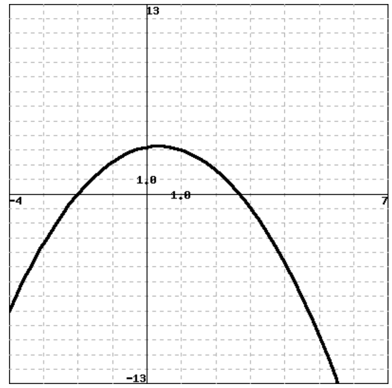
Note that the graph of $f'(x)$ shown here has an accurate vertical scale; however, that would be difficult for you to estimate by eye based simply on the graph of $f(x)$, so we do not expect that level of precision on a test or exam.

16. Below is the graph of $f(x)$. Sketch the graph of $f'(x)$.



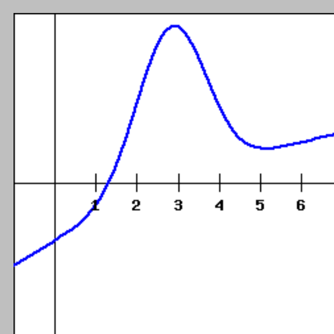
There are two points with **slope** zero, at $x = 2$ and $x \approx 2.5$, so there must be a **derivative** value of zero at those two x locations.

In between those landmark points, the slopes of f go: negative, positive, negative.

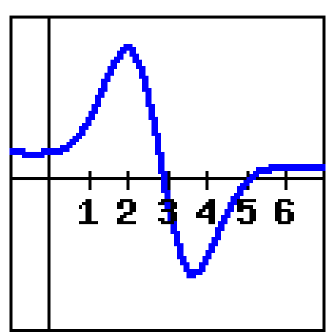


Note that the graph of $f'(x)$ shown here has an accurate vertical scale; however, that would be difficult for you to estimate by eye based simply on the graph of $f(x)$, so we do not expect that level of precision on a test or exam.

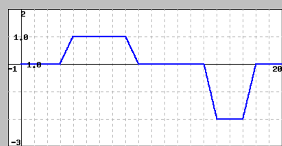
17. For the function $f(x)$ shown in the graph below, sketch a graph of the derivative.



Because the derivative gives the slope of the original function at each point x, we know that the derivative is negative where $f(x)$ is decreasing and positive where it is increasing. Applying this to $f(x)$, a sketch of the derivative would resemble the graph below:



18. A child fills a pail by using a water hose. After finishing, the child plays in a sandbox for a while before tipping the pail over to empty it. If $V(t)$ gives the volume of the water in the pail at time t, then the figure below shows $V'(t)$ as a function of t.



At what time does the child:

(a) Begin to fill the pail?

(b) Finish filling the pail?

(c) Tip the pail over?

The child begins filling the pail when $V'(t)$ becomes positive, which is when the volume of water in the pail starts increasing. This is at $t = 3$. The child finishes filling the pail when the volume stops increasing, which is at $t = 9$. The child tips the pail over when the volume of water in the pail start decreasing, which is where $V'(t)$ first becomes negative, which is at $t = 14$.

Computing Derivatives

Below are a **small sample of problems** involving the computation of derivatives. They are **not enough** to properly learn and memorize how to apply all the derivative rules. You should practice with as many problems as you need to become proficient at computing derivatives.

Further practice problems can be found in any calculus textbook. For example,

From Hughes-Hallett 5th edition,
Section 3.1 - 7-47 (odd)†
Section 3.2 - 1-33 (odd)
Section 3.3 - 3-29 (odd)
Section 3.4 - 1-49 (odd)
Section 3.5 - 3-39 (odd)
Section 3.6 - 1-33 (odd)

From Hughes-Hallett 6th edition,
Section 3.1 - 7-49 (odd)†
Section 3.2 - 1-25 (odd)
Section 3.3 - 3-29 (odd)
Section 3.4 - 1-55 (odd)
Section 3.5 - 3-47 (odd)
Section 3.6 - 1-41 (odd)

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19. Let $f(x) = 4e^x - 9x^2 + 5$. Compute $f'(x)$.

$f'(x) = 4e^x - 18x$

20. Let $f(x) = 2x^6\sqrt{x} + \dfrac{-5}{x^3\sqrt{x}}$. Compute $f'(x)$.

You can use the product rule here if you like, but it is far easier to rewrite the function before you start, since all the terms are just powers of x.

$$f(x) = 2x^6\sqrt{x} + \frac{-5}{x^3\sqrt{x}}$$

$$\begin{aligned} f(x) &= 2x^6 \cdot x^{1/2} + \frac{-5}{x^3 \cdot x^{1/2}} \\ &= 2x^{\frac{13}{2}} - 5x^{\frac{-7}{2}} \end{aligned}$$

Differentiating,

$$\begin{aligned} f'(x) &= 2\frac{13}{2}x^{\frac{11}{2}} - 5\left(\frac{-7}{2}\right)x^{\frac{-9}{2}} \\ &= 13x^5\sqrt{x} + \frac{35}{2x^4\sqrt{x}} \end{aligned}$$

†For this section, simplify products and fractions *before* you differentiate, rather than using the product rule and quotient rule.
†For this section, simplify products and fractions *before* you differentiate, rather than using the product rule and quotient rule.

21. Let $f(x) = \dfrac{7x^2 + 7x + 5}{\sqrt{x}}$.

(a) Compute $f'(x)$. (b) Find $f'(3)$.

You can use the quotient rule here if you like, but it is far easier to rewrite the function before you start, since all the terms are just powers of x.

$$\begin{aligned} f(x) &= \frac{7x^2 + 7x + 5}{\sqrt{x}} \\ &= 7x^{3/2} + 7x^{1/2} + 5x^{-1/2} \end{aligned}$$

(a) $f'(x) = \dfrac{21}{2}x^{1/2} + \dfrac{7}{2}x^{-1/2} - \dfrac{5}{2}x^{-3/2}$

(b) Evaluating $f'(x)$ at $x = 3$, we obtain ≈ 19.726.

22. Let $f(t) = 7t^{-7}$.

(a) Compute $f'(t)$. (b) Find $f'(3)$.

(a) $f'(x) = -49\ t^{-8}$

(b) $f'(3) = -49\ (3^{-8})$

23. Let $f(x) = 4e^x + e^1$. Compute $f'(x)$.

Don't be thrown off by the e^1: that's a constant (equal to e or ≈ 2.7), so the derivative of that term is zero.

$f'(x) = 4e^x$

24. Let $f(x) = 4e^x + 4x$. Compute $f'(x)$.

$f'(x) = 4e^x + 4$

25. $f(x) = (3x^2 - 2)(6x + 3)$.

(a) Compute $f'(x)$. (b) Find $f'(4)$.

You can either expand the product before differentiating (and obtain $f(x) = 18x^3 + 9x^2 - 12x - 6$), or use the product rule. Both give the same answer.

(a) $f'(x) = 54x^2 + 18x - 12$.

(b) $f'(4) = 924$.

26. Let $f(x) = \dfrac{\sqrt{x} - 4}{\sqrt{x} + 4}$. Compute $f'(9)$.

We need to use the quotient rule for this function.

$$\begin{aligned} f'(x) &= \frac{\frac{1}{2}x^{-1/2}(x^{1/2} + 4) - (x^{1/2} - 4)\left(\frac{1}{2}\right)x^{-1/2}}{(\sqrt{x} + 4)^2} \\ &= \frac{1 + 4/\sqrt{x} - 1 + 4/\sqrt{x}}{2(\sqrt{x} + 4)^2} \\ &= \frac{4}{\sqrt{x}(\sqrt{x} + 4)^2} \\ f'(9) &= \frac{4}{3(3 + 4)^2} \\ &= \frac{4}{147} \end{aligned}$$

27. Consider $f(x) = \dfrac{4x + 3}{3x + 2}$.

(a) Compute $f'(x)$. (b) Find $f'(5)$.

(a) $f'(x) = \dfrac{4(3x + 2) - (4x + 3)(3)}{(3x + 2)^2}$

(b) $f'(5) = \dfrac{-1}{17^2}$

28. Consider $f(x) = \dfrac{7 - x^2}{7 + x^2}$

(a) Compute $f'(x)$. (b) Find $f'(1)$.

(a) $f'(x) = \dfrac{-28x}{(x^2 + 7)^2}$

(b) $f'(1) = \dfrac{-28}{64}$

29. Let $f(x) = -2x(x - 3)$.

(a) Compute $f'(x)$. (b) Find $f'(-5)$.

(a) $f'(x) = -4x + 6$

(b) $f'(-5) = 26$

30. $f(x) = \dfrac{4x^3 - 3}{x^4}$

(a) Compute $f'(x)$. (b) Find $f'(2)$.

(a) $-4x^{-2} + 12x^{-5}$

(b) -0.625

31. $g(x) = \dfrac{e^x}{5 + 4x}$. Compute $g'(x)$.

$$g'(x) = \frac{(1 + 4x)e^x}{(5 + 4x)^2}$$

32. $f(x) = \dfrac{4x^2 \tan x}{\sec x}$.

(a) Find $f'(x)$. (b) Find $f'(3)$.

This should definitely be simplified before you differentiate!

Recall: $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$, and $\sec(x) = \dfrac{1}{\cos(x)}$, so $\dfrac{\tan(x)}{\sec(x)} = \dfrac{\sin(x)}{\cos(x)}\cos(x) = \sin(x)$.

(a) $f'(x) = 8x\sin(x) + 4x^2\cos(x)$

(b) $f'(3) = 24\sin(3) + 36\cos(3) \approx -32.253$

33. $f(x) = 7\sin x + 12\cos x$

(a)Compute $f'(x)$. (b) Find $f'(1)$.

(a) $f'(x) = 7\cos(x) - 12\sin(x)$

(b) $f'(1) = 7\cos(1) - 12\sin(1) \approx -6.3155$

34. Let $f(x) = \cos x - 2\tan x$. Compute $f'(x)$.

$f'(x) = -\sin(x) - 2\sec^2(x)$

35. $f(x) = \dfrac{5\sin x}{3+\cos x}$

(a) Compute $f'(x)$. (b) Find $f'(2)$.

Don't forget the identity $\sin^2(x) + \cos^2(x) = 1$.

(a) $f'(x) = (15\cos(x) + 5)/(3 + \cos(x))^2$

(b) $f'(2) = (15\cos(2) + 5)/(3 + \cos(2))^2 \approx -0.18606$

36. $f(x) = 7x(\sin x + \cos x)$

(a) Compute $f'(x)$. (a) Find $f'(3)$.

(a) $f'(x) = 7(\sin(x) + \cos(x)) + 7x(\cos(x) - \sin(x))$

(b) $f'(3) \approx -29.695$

37. Let $f(x) = \cos(\sin(x^2))$. Compute $f'(x)$.

$f'(x) = -\sin(\sin(x^2))\cos(x^2)\cdot(2x)$

38. Let $f(x) = 2\sin^3 x$. Compute $f'(x)$.

$f'(x) = 6\sin^2(x)\cos(x)$

39. Let $y = (8 + \cos^2 x)^6$. Compute $\dfrac{dy}{dx}$.

$\dfrac{dy}{dx} = 6(8 + cos^2(x))^5(2\cos(x)(-\sin(x)))$

40. Let $f(x) = -3\ln[\sin(x)]$. Compute $f'(x)$.

$f'(x) = \dfrac{-3}{\sin(x)}\cos(x)$ or, using trig identities $= -3\cot(x)$

41. Let $f(x) = 2\ln(4 + x)$. Compute $f'(x)$.

$f'(x) = \dfrac{2}{4+x}$

42. Consider the function $\cos(\arccos(x))$.

(a) Simplify the expression to get a simple equation.

(b) Differentiate both sides of the equation, and then solve for $\dfrac{d}{dx}\arccos(x)$.

(c) Use the trig identity $\sin(\theta) = \sqrt{1-\cos^2(\theta)}$ to simplify your expression for $\dfrac{d\arccos}{dx}(x)$.

(a) Since cos and arccos are inverse functions, if we start with x then apply both functions, we get back to x:

$$\cos(\arccos(x)) = x$$

.

(b) Differentiating both sides,

$$\begin{aligned}
\frac{d}{dx}\cos(\arccos(x)) &= \frac{d}{dx}x \\
-\sin(\arccos(x))\cdot\frac{d}{dx}\arccos(x) &= 1 \\
\text{so } \frac{d}{dx}\arccos(x) &= \frac{1}{-\sin(\arccos(x))}
\end{aligned}$$

(c) To simplify the denominator, we note that

$$\begin{aligned}
\sin(\arccos(x)) &= \sqrt{1-\cos(\arccos(x))} \\
&= \sqrt{1-x^2}
\end{aligned}$$

so

$$\begin{aligned}
\frac{d}{dx}\arccos(x) &= \frac{1}{-\sqrt{1-x^2}} \\
&= \frac{-1}{\sqrt{1-x^2}}
\end{aligned}$$

43. Consider the function $\tan(\arctan(x))$.

(a) Simplify the expression to get a simple equation.

(b) Differentiate both sides of the equation, and then solve for $\dfrac{d}{dx}\arctan(x)$.

(c) Use the trig identity $\sec^2(\theta) = 1 + tan^2(\theta)$ to simplify your expression for $\dfrac{d\arctan}{dx}(x)$.

(a) Since tan and arctan are inverse functions, if we start with x then apply both functions, we get back to x:

$$\tan(\arctan(x)) = x$$

.

(b) Differentiating both sides,

$$\begin{aligned}
\frac{d}{dx}\tan(\arctan(x)) &= \frac{d}{dx}x \\
\sec^2(\arctan(x))\cdot\frac{d}{dx}\arctan(x) &= 1 \\
\text{so } \frac{d}{dx}\arctan(x) &= \frac{1}{\sec^2(\arctan(x))}
\end{aligned}$$

(c) To simplify the denominator, we note that

$$\sec^2(\theta) = 1 + tan^2(\theta)$$

so

$$\begin{aligned}\sec^2(\arctan(x)) &= 1 + \tan^2(\arctan(x)) \\ &= 1 + x^2\end{aligned}$$

meaning the derivative can be rewritten as

$$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$$

44. Take the derivative of $y = \arcsin(4x)$.

$$\frac{d}{dx}\arcsin(4x) = \frac{1}{\sqrt{1-(4x)^2}}(4)$$

45. If $f(x) = x\arctan(x)$, find $f'(x)$.

$$f'(x) = (1)\arctan(x) + x\left(\frac{1}{1+x^2}\right)$$

46. Find the slope of $y = \dfrac{3x}{\arccos(x)}$ at the point $x = 0$.

$$y' = \frac{(3)\arccos(x) - (3x)\frac{1}{\sqrt{1-x^2}}}{(\arccos(x))^2}$$

Evaluating this at $x = 0$,

$$\begin{aligned}y'(0) &= \frac{(3)\arccos(0) - (0)\frac{1}{\sqrt{1-0^2}}}{(\arccos(0))^2} \\ &= \frac{(3)\arccos(0)}{(\arccos(0))^2}\end{aligned}$$

Remembering that since $\cos(\pi/2) = 0$, then $\arccos(0) = \pi/2$,

$$\begin{aligned}y'(0) &= \frac{(3)\frac{\pi}{2}}{(\frac{\pi}{2})^2} \\ &= \frac{6}{\pi}\end{aligned}$$

47. Let $f(x) = \arcsin(x)$.

(a) Compute $f'(x)$ (b) Find $f'(0.4)$.

Recall that the derivative of $\arcsin(x)$ is:
$f'(x) = \frac{1}{\sqrt{1-x^2}}$
Evaluating this at $x = 0.4$, we get:
$f'(0.4) \approx 1.0911$

48. Let $f(x) = \dfrac{\arccos(14x)}{\arcsin(14x)}$. Compute $f'(x)$.

Applying the quotient rule:

$$\begin{aligned}f'(x) &= \frac{\arcsin(14x)\,(\arccos(14x))' - \arccos(14x)\,(\arcsin(1}{(\arcsin(14x))^2} \\ &= \frac{\frac{-14\arcsin(14x)}{\sqrt{1-(14x)^2}} - \frac{14\arccos(14x)}{\sqrt{1-(14x)^2}}}{(\arcsin(14x))^2} \\ &= \frac{-14\,(\arcsin(14x) + \arccos(14x))}{(\arcsin(14x))^2\sqrt{1-(14x)^2}}\end{aligned}$$

49. Let

$$P = \frac{V^2R}{(R+r)^2}.$$

Calculate $\dfrac{dP}{dr}$, assuming that r is variable and R and V are constant.

Note that V is also constant. Let

$$f(r) = \frac{V^2R}{(R+r)^2} = \frac{V^2R}{R^2+2Rr+r^2}.$$

Using the quotient rule:

$$f'(r) = \frac{(R^2+2Rr+r^2)(0) - (V^2R)(2R+2r)}{(R+r)^4}$$

$$= -\frac{2V^2R(R+r)}{(R+r)^4} = -\frac{2V^2R}{(R+r)^3}.$$

Inverse Trig Functions

50. (a) Sketch the graph of $y = \arccos(x)$.

(b) Find the exact values of

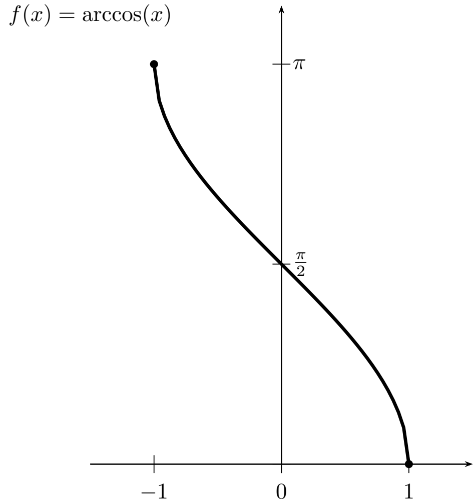
(i) $\arccos(0)$

(ii) $\arccos(0.5)$

(iii) $\arccos(1)$

(iv) $\arccos(-1)$

(a) The graph of $y = arccos(x)$ can be created by reflecting the $y = \cos(x)$ graph across the $y = x$ line.



(b) These calculations depend on the inverse relationship between cos and arccos, and your background knowledge of the $\cos(x)$ graph/function.

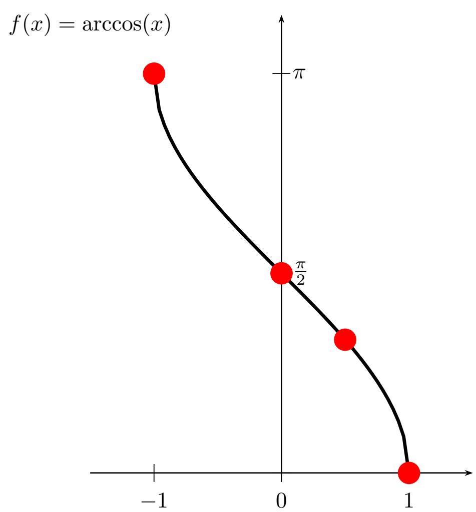
(i) Since $\cos(\pi/2) = 0$, then $\arccos(0) = \pi/2$.

(ii) Since $\cos(\pi/3) = 0.5$ (try drawing a 30/60 triangle to confirm), then $\arccos(0.5) = \pi/3$.

(iii) Since $\cos(0) = 1$, then $\arccos(1) = 0$.

(iv) Since $\cos(\pi) = -1$, then $\arccos(-1) = \pi$.

All the points are indicated on the $y = \arccos(x)$ graph below.



51. (a) Sketch the graph of $y = \arctan(x)$.

(b) Find the exact values of

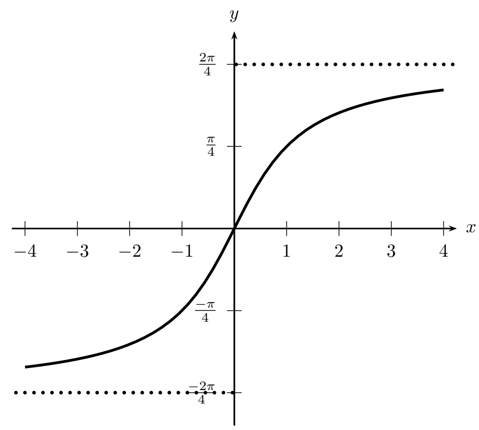
(i) $\arctan(0)$

(ii) $\arctan(1)$

(iii) $\arctan(-1)$

(iv) $\arctan(x)$ as x gets very large.

(a) The graph of $y = \arctan(x)$ can be created by reflecting the $y = \tan(x)$ graph across the $y = x$ line.



(b) These calculations depend on the inverse relationship between tan and arctan, and your background knowledge of the $\tan(x)$ graph/function.

(i) Since $\tan(0) = 0$, then $\arctan(0) = 0$.

(ii) Since $\tan(\pi/4) = 1$ (try drawing a 45/45 triangle to confirm), then $\arctan(1) = \pi/4$.

(iii) Similarly $\tan(-\pi/4) = -1$, so $\arctan(-1) = \pi/4$.

(iv) Looking at the horizontal asymptote of the $y = \arctan(x)$ graph, the limiting value as x increases is $y \to \dfrac{\pi}{2}$.