Practice Problems

1. Find the determinant of the following matrices

$$\mathbf{A} = \begin{pmatrix} 21 & -43 & 97 \\ 0.1 & -9.3 & 0.8 \\ 0.3 & -16 & 22.7 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -65 & 31.1 & -83 & -13.2 \\ -14.5 & 1.6 & -6.4 & 48.9 \\ -10.5 & 54.8 & -67.7 & 92.3 \\ 20.8 & -9.9 & -14.8 & 42.1 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 32.1 & -9.3 & -12.7 \\ 56.1 & -27.9 & 83.2 \\ -90.1 & 2 & 55 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} -11.4 & -17.2 & -48.9 & -12.6 \\ -32.5 & -90.9 & -15.8 & -43.3 \\ -12.9 & 6.8 & -41.7 & 83.9 \\ 0 & -40 & -20 & 46.6 \end{pmatrix}$$

2. Find the orthonormal basis of the following matrices

$$\mathbf{A} = \begin{pmatrix} 21 & -43 & 97 \\ 0.1 & -9.3 & 0.8 \\ 0.3 & -16 & 22.7 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -65 & 31.1 & -83 & -13.2 \\ -14.5 & 1.6 & -6.4 & 48.9 \\ -10.5 & 54.8 & -67.7 & 92.3 \\ 20.8 & -9.9 & -14.8 & 42.1 \end{pmatrix}$$

3. Diagonalize the following matrices

$$\mathbf{A} = \begin{pmatrix} 1 & 4 & -1 & 21 \\ 4 & 8 & 13 & -3 \\ -1 & 13 & 17 & -5 \\ 21 & -3 & -5 & 2 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} -2 & 91 & -1 & 2 \\ 9 & -3 & 3 & 31 \\ -1 & 3 & -4 & 54 \\ -2.1 & 51 & 5 & -5 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} -13.1 & 9.2 & -1.3 & 2.4 \\ 5.9 & -6.3 & 3.7 & 1.8 \\ 9.1 & 10.3 & 14.1 & 51.5 \\ 16.2 & -11.7 & -18.5 & 91.5 \end{pmatrix}$$

4. Convert this second order ODEa into a system of first order ODEs and use MATLAB to plot the solution.

$$3y'' + y' - y = 0,\ y(0) = 2.1,\ y'(0) = -4.4$$

5. Convert this second order ODEa into a system of first order ODEs and use MATLAB to plot the solution. What function is this?

$$y'' + y = 0,\ y(0) = 2,\ y'(0) = 3$$

6. Convert this second order ODEa into a system of first order ODEs and use MATLAB to plot the solution.

$$y'' - 2y' + = \frac{e^t}{t^2+1},\ y(0) = -4,\ y'(0) = -1$$

Solutions:

1. $\det(\mathbf{A})$ = -3961.790, $\det(\mathbf{B})$ = 10853722.549, $\det(\mathbf{C})$ = 74312.229, $\det(\mathbf{D})$ = 8499128.802

2.

$$\mathbf{Q}_A = \begin{pmatrix} -0.9696 & 0.18663 & -0.1583 \\ -0.0409 & -0.7616 & -0.6468 \\ -0.2412 & -0.6207 & 0.7461 \end{pmatrix} \quad \mathbf{Q}_B = \begin{pmatrix} -0.5165 & -0.8019 & -0.1584 & 0.2549 \\ -0.2426 & 0.2729 & -0.9096 & -0.1985 \\ -0.8055 & 0.3534 & 0.3825 & -0.2826 \\ -0.1596 & 0.3969 & -0.0355 & 0.9032 \end{pmatrix}$$

3.

$$\mathbf{D}_A = \begin{pmatrix} -20.3803 & 0 & 0 & 0 \\ 0 & -1.3522 & 0 & 0 \\ 0 & 0 & 21.2495 & 0 \\ 0 & 0 & 0 & 28.4829 \end{pmatrix}$$

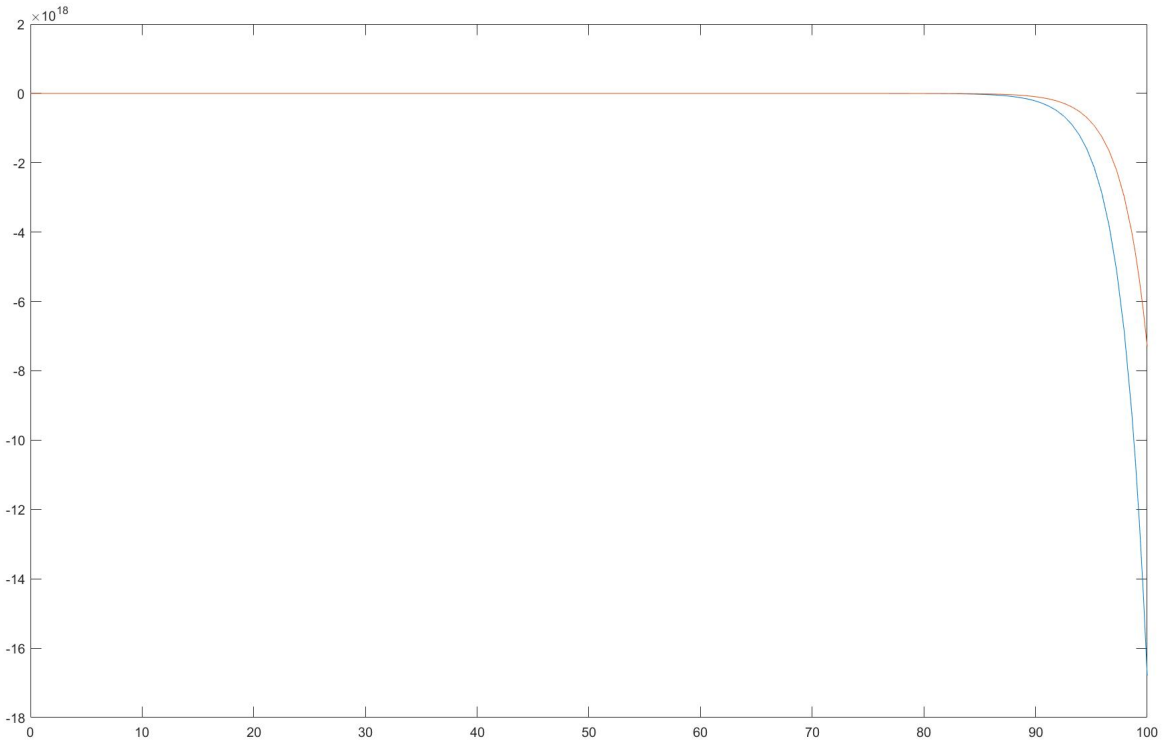
$$\mathbf{D}_B = \begin{pmatrix} 47.6845 & 0 & 0 & 0 \\ 0 & -53.4351 & 0 & 0 \\ 0 & 0 & 6.8856 & 0 \\ 0 & 0 & 0 & -15.1350 \end{pmatrix}$$

$$\mathbf{D}_C = \begin{pmatrix} 75.2886 & 0 & 0 & 0 \\ 0 & -18.4135 & 0 & 0 \\ 0 & 0 & -3.1731 & 0 \\ 0 & 0 & 0 & 32.4980 \end{pmatrix}$$

4. System:

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= -\frac{y_2}{3} + \frac{y_1}{3} \end{aligned}$$

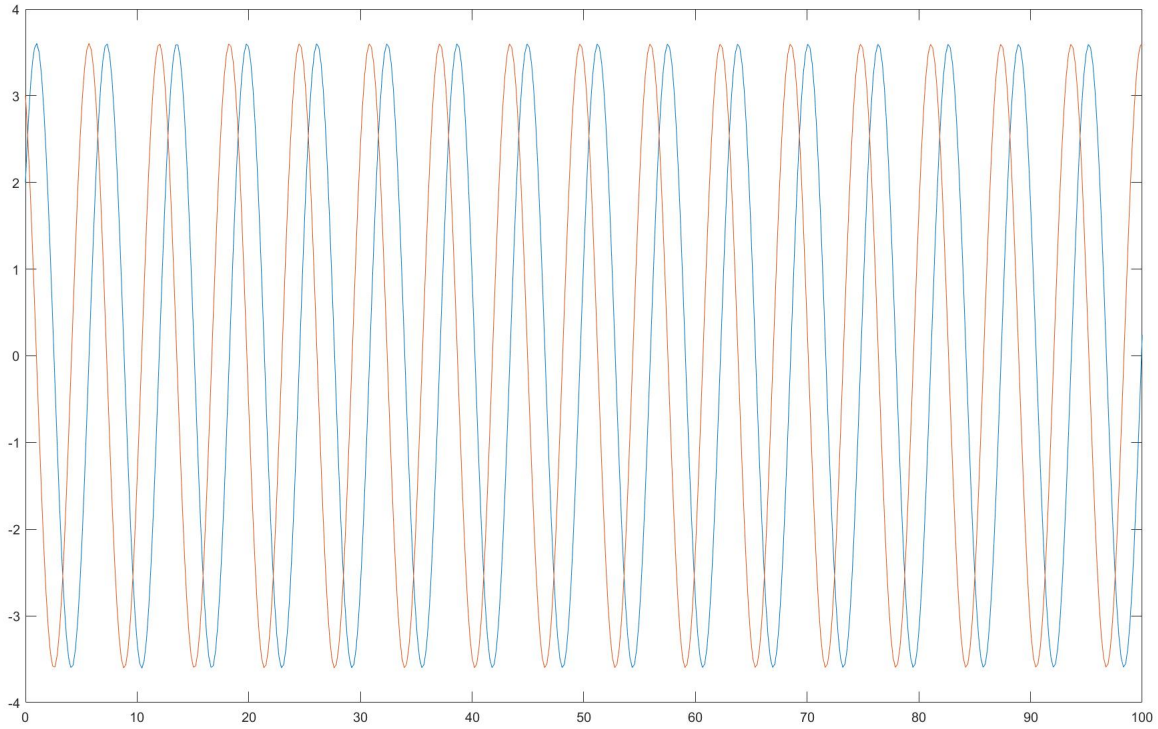
Plot:



5. System:

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= -y_1 \end{aligned}$$

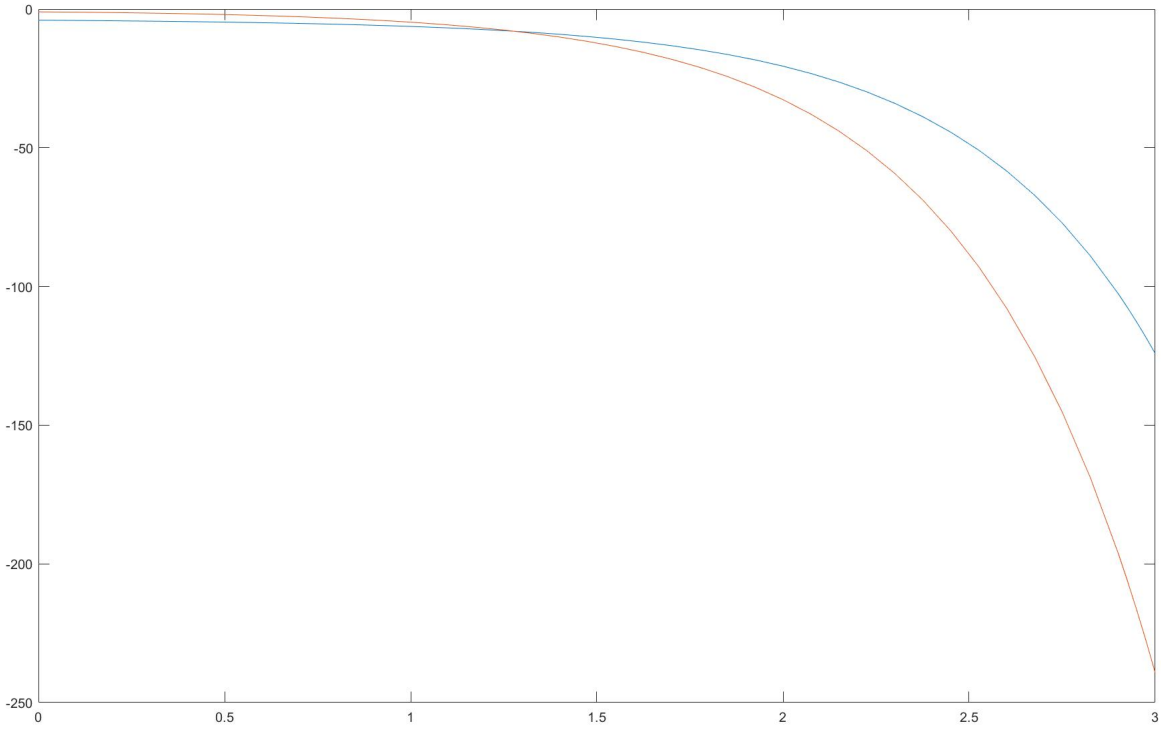
Plot:



6. System:

$$\begin{aligned} y_1' &= y_2 \\ y_2' = &= \frac{e^t}{t^2+1} + 2Y - 2 \end{aligned}$$

Plot:


0
-50
-100
-150
-200
-250
0
0.5
1
1.5
2
2.5
3