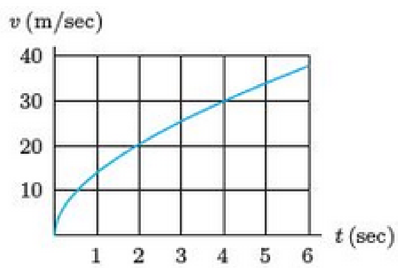
MNTC P01 - Week #4 - Integrals - Foundations

Distance And Velocity

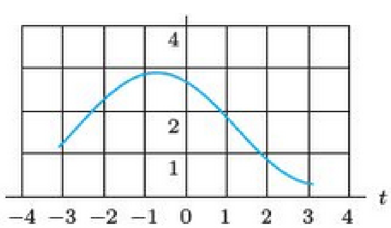
1. The graph below shows the velocity, v, of an object (in meters/sec). Estimate the total distance the object traveled between $t = 0$ and $t = 6$.



2. The figure below shows the velocity of a particle, in cm/sec, along the t-axis for $-3 \leq t \leq 3$ (t in seconds).

   (a) Describe the motion in words. Is the particle changing direction or always moving in the same direction? Is the particle speeding up or slowing down?

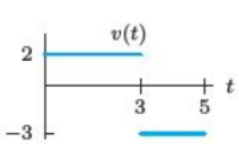
   (b) Make over- and underestimates of the distance traveled for $-3 \leq t \leq 3$.



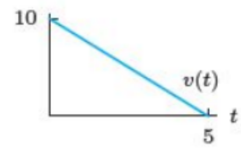
For questions 3 to 6, the graph shows the velocity, in cm/sec, of a particle moving along the x-axis. Compute the particle's change in position, left (negative) or right (positive), between times $t = 0$ and $t = 5$ seconds.

3.

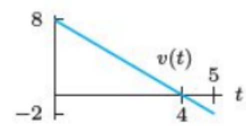
4.



5.



6.



7. A car going 80 ft/s (about 90 km/h) brakes to a stop in five seconds. Assume the deceleration is constant.

   (a) Graph the velocity against time, t, for $0 \leq t \leq 5$ seconds.

   (b) Represent, as an area on the graph, the total distance traveled from the time the brakes are applied until the car comes to a stop.

   (c) Find this area and hence the distance traveled.

8. A baseball thrown directly upward at 96 ft/sec has velocity $v(t) = 96 - 32t$ ft/ sec at time t seconds.

   (a) Graph the velocity from $t = 0$ to $t = 6$.

   (b) When does the baseball reach the peak of its flight? How high does it go?

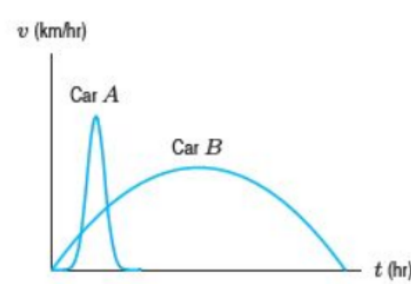
   (c) How high is the baseball at time $t = 5$?

9. Two cars start at the same time and travel in the same direction along a straight road. The graph below gives the velocity, v, of each car as a function of time, t. Which car:

   (a) Attains the larger maximum velocity?

   (b) Stops first?

   (c) Travels farther?

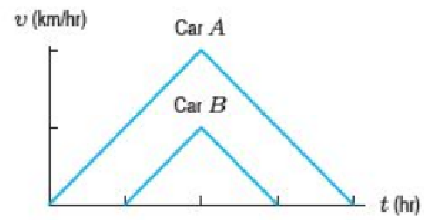


10. Two cars travel in the same direction along a straight road. The graph below shows the velocity, v, of each car at time t. Car B starts 2 hours after car A and car B reaches a maximum velocity of 50 km/ hr.

(a) For approximately how long does each car travel?

(b) Estimate car A's maximum velocity.

(c) Approximately how far does each car travel?

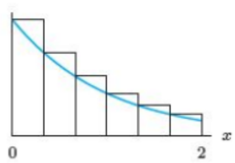


The Definite Integral

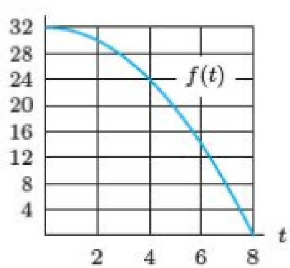
11. The figure below shows a Riemann sum approximation with n subdivisions to $\int_a^b f(x)dx$.

   (a) Is it a left- or right-hand approximation? Would the other one be larger or smaller?

   (b) What are a, b, n and Δx?



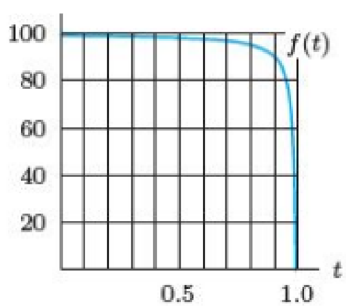
12. Using the figure below, draw rectangles representing each of the following Riemann sums for the function f on the interval $0 \le t \le 8$. Calculate the value of each sum.



   (a) Left-hand sum with $\Delta t = 4$.

   (b) Left-hand sum with $\Delta t = 2$.

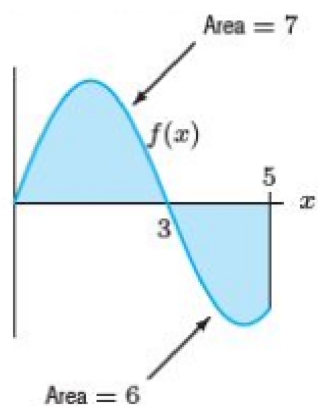
13. The graph of a function $f(t)$ is given in the figure below.



Which of the following four numbers could be an estimate of $\int_0^1 f(t)\, dt$, accurate to two decimal places? Explain how you chose your answer.
(a) -98.35 (b) 71.84 (c) 100.12 (d) 93.47

14. (a) What is the area between the graph of $f(x)$ shown below and the x-axis, between $x = 0$ and $x = 5$?

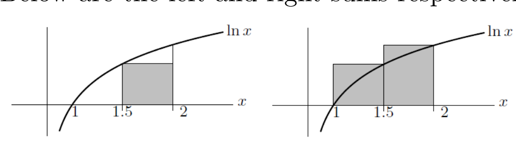
   (b) What is $\int_0^5 f(x)\, dx$?



15. (a) On a sketch of $y = \ln(x)$, represent the left Riemann sum with $n = 2$ approximating $\int_1^2 \ln(x)\, dx$. Write out the terms in the sum, but do not evaluate it.

   (b) Will this LEFT(4) estimate be an overestimate or an underestimate of $\int_1^2 \ln(x)\, dx$?

   (a) Below are the left and right sums respectively.



$$\begin{aligned}\text{Left sum} &= f(1)\Delta x + f(1.5)\Delta x \\ &= \underbrace{(\ln 1)}_{=0} 0.5 + \ln(1.5)0.5 = (\ln 1.5)0.5\end{aligned}$$

(b) Because the y values on the left end of each interval are the smallest on that interval, the sum built on them will be an underestimate of the exact integral value.

16. Estimate $\int_1^2 x^2\ dx$ using left-hand sum with four subdivisions.

17. Without computation, decide if $\int_0^{2\pi} e^{-x} \sin x\ dx$ is positive or negative. [Hint: use MATLAB to draw $e^{-x} \sin(x)$ over the given interval.]

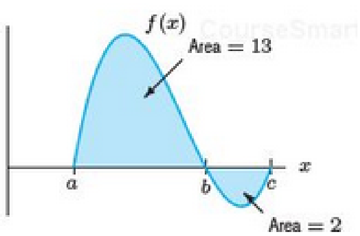
18. (a) Graph $f(x) = \begin{cases} 1-x & \text{if } 0 \le x \le 1 \\ x-1 & \text{if } 1 < x \le 2 \end{cases}$

(b) Find the *exact* value of $\int_0^2 f(x)\ dx$ (hint: sketch and see what shapes you get).

(c) Calculate the 4-term left Riemann sum approximation to the definite integral. How does the approximation compare to the exact value?

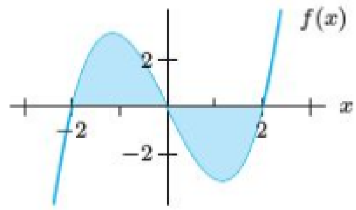
19. Using the figure below, find the values of

(a) $\int_a^b f(x)\ dx$ (b) $\int_b^c f(x)\ dx$
(c) $\int_a^c f(x)\ dx$ (d) $\int_a^c |f(x)|\ dx$



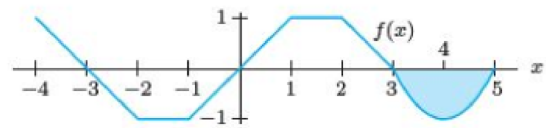
20. Given the figure below, and the statement that $\int_{-2}^0 f(x)\ dx = 4$, estimate

(a) $\int_0^2 f(x)\ dx$ (b) $\int_{-2}^2 f(x)\ dx$
(c) The total shaded area.



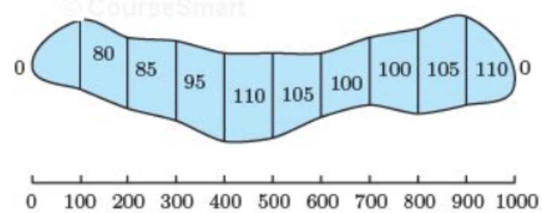
21. (a) Using the graph below, find $\int_{-3}^0 f(x)\ dx$.

(b) If the area of the shaded region is A, estimate $\int_{-3}^4 f(x)\ dx$.



22. Find the exact value of $\int_0^{2\pi} \sin\theta d\theta$ without calculation (i.e. from a sketch).

23. The width, in feet, at various points along the fairway of a hole on a golf course is given in the figure below. If one pound of fertilizer covers 200 square feet, use a LEFT estimate the amount of fertilizer needed to fertilize the fairway.



Definite Integrals in Modeling

24. The rate at which the world's oil is being consumed is continuously increasing. Suppose the rate of oil consumption (in billions of barrels per year) is given by the function $r = f(t)$, where t is measured in years and $t = 0$ is the start of 2004.

(a) Write a definite integral which represents the total quantity of oil used between the start of 2004 and the start of 2009.

(b) Suppose $r = 32e^{0.05t}$. Using a left-hand sum with five subdivisions, find an approximate value for the total quantity of oil used between the start of 2004 and the start of 2009.

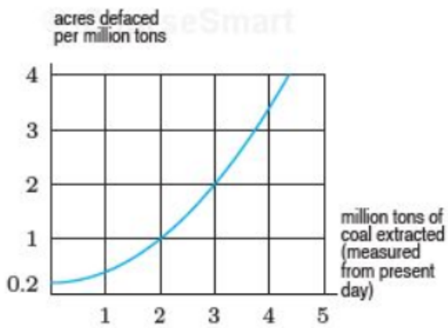
(c) Interpret each of the five terms in the sum from part (b) in terms of oil consumption.

25. As coal deposits are depleted, it becomes necessary to strip-mine larger areas for each ton of coal. The graph below shows the number of acres of land per million tons of coal that will be defaced during strip-mining as a function of the number of million tons removed, starting from the present day.

(a) Estimate the total number of acres defaced in extracting the next 4 million tons of coal (measured from the present day). Draw four rectangles under the curve, and compute their area.

(b) Re-estimate the number of acres defaced using rectangles above the curve.

(c) Combine your answers to parts (a) and (b) to get a better estimate of the actual number of acres defaced.



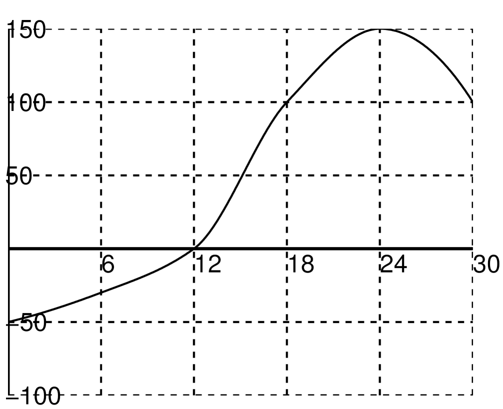
26. Coal gas is produced at a gasworks. Pollutants in the gas are removed by scrubbers, which become less and less efficient as time goes on. The following measurements, made at the start of each month, show the rate at which pollutants are escaping (in tons/ month) in the gas:

Time (months)	0	1	2	3	4	5	6
Rate pollutants escape (tons/month)	5	7	8	10	13	16	20

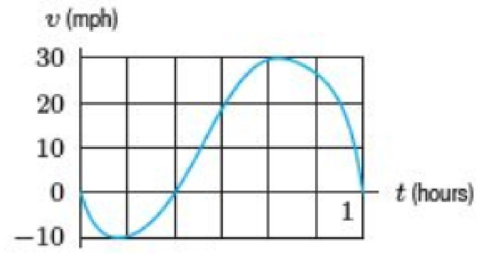
(a) Make an overestimate and an underestimate of the total quantity of pollutants that escape during the **first month**.

(b) Make an overestimate and an underestimate of the total quantity of pollutants that escape during the **six months** shown in the table.

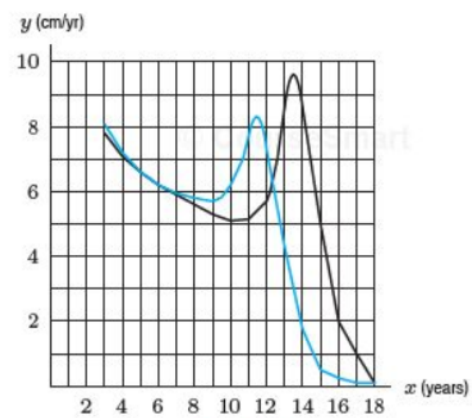
27. The graph below shows the rate of change of the quantity of water in a water tower, in liters per day, during the month of April. If the tower had 12,000 liters of water in it on April 1, estimate the quantity of water in the tower on April 30.



28. A bicyclist pedals along a straight road with velocity v given in the graph below. She starts 5 miles from a lake; positive velocities take her away from the lake and negative velocities take her toward the lake. When is the cyclist farthest from the lake, and how far away is she then?



29. Height velocity graphs are used by endocrinologists (doctors specializing in the study of hormones) to follow the progress of children with growth deficiencies. The graph below shows the height velocity curves of an average boy and an average girl between ages 3 and 18.



(a) Which curve is for girls and which is for boys? Explain how you can tell.

(b) About how much does the average boy grow between ages 3 and 10?

(c) The growth spurt associated with adolescence and the onset of puberty occurs between ages 12 and 15 for the average boy and between ages 10 and 12.5 for the average girl. Estimate the height gained by each average child during this growth spurt.

(d) When fully grown, about how much taller is the average man than the average woman? (The average boy and girl are about the same height at age 3.)

Properties of Definite Integrals

30. Without integrating, show that

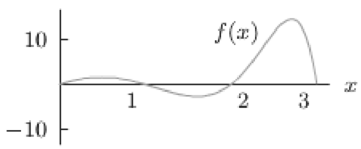
$$2 \leq \int_0^2 \sqrt{1+x^3}\, dx \leq 6.$$

31. Without calculating the integral, explain why the following statements are false.

(a) $\int_{-2}^{-1} e^{x^2}\, dx = -3$

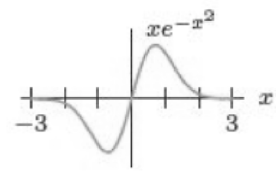
(b) $\int_{-1}^{1} \left| \frac{\cos(x+2)}{1+\tan^2 x} \right| dx = 0.$

32. Using the graph of $f(x)$ shown below, arrange the following quantities in increasing order, from least to greatest.



(i) $\int_0^1 f(x)\, dx$ (ii) $\int_1^2 f(x)\, dx$
(iii) $\int_0^2 f(x)\, dx$ (iv) $\int_2^3 f(x)\, dx$
(v) $-\int_1^2 f(x)\, dx$ (vi) The number 0
(vii) The number 20 (viii) The number -10

33. (a) Use the graph of $y = xe^{-x^2}$ shown below to explain why $\int_{-3}^{3} xe^{-x^2}\, dx = 0$.



(b) Find the left-hand sum approximation with $n = 3$ to $\int_0^3 xe^{-x^2}\, dx$. Give your answer to four decimal places.

(c) Repeat part (b) for $\int_{-3}^{0} xe^{-x^2}\, dx$.

(d) Do your answers to parts (b) and (c) add to 0? Explain.

Integration By Anti-Derivatives

To practice computing anti-derivatives, do as many of the problems from the following section as you need.

These problems are a good start for practicing anti-derivatives or integration. If you want more though, consult any calculus textbook.

Evaluate the following integrals.

34. $\int_{-1}^{2} (x^3 - 2x)dx$

35. $\int_{-1}^{1} x^{100} dx$

36. $\int_1^4 (5 - 2t + 3t^2)dt$

37. $\int_0^1 (1 + \frac{1}{2}u^4 - \frac{2}{5}u^9)du$

38. $\int_1^9 \sqrt{x} dx$

39. $\int_{1}^{8} x^{-2/3} dx$

40. $\int_{\pi/6}^{\pi} \sin\theta d\theta$

41. $\int_{-5}^{5} e \; dx$

42. $\int_{0}^{1} (u+2)(u-3) du$

43. $\int_{0}^{4} (4-t)\sqrt{t} \; dt$

44. $\int_{1}^{9} \frac{x-1}{\sqrt{x}} dx$

45. $\int_{0}^{2} (y-1)(2y+1) dy$

46. $\int_{0}^{\pi/4} \sec^2 t \; dt$

47. $\int_{0}^{\pi/4} \sec\theta \tan\theta d\theta$

48. $\int_{1}^{2} (1+2y)^2 dy$

49. $\int_{0}^{3} (2\sin x - e^x) dx$

50. $\int_{1}^{2} \frac{v^3+3v^6}{v^4} dv$

51. $\int_{1}^{18} \sqrt{\frac{3}{z}} dz$

52. $\int_{0}^{1} (x^e + e^x) dx$

53. $\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2} dx$

54. $\int_{1}^{2} \frac{4+u^2}{u^3} du$

55. $\int_{-1}^{1} e^{u+1} du$

56. $\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}} dx$

57. $\int_{0}^{\pi} f(x) dx$ where $f(x) = \begin{cases} \sin x & \text{if } 0 \leq x < \pi/2 \\ \cos x & \text{if } \pi/2 \leq x \leq \pi \end{cases}$

58. $\int_{-2}^{2} f(x) dx$ where $f(x) = \begin{cases} 2 & \text{if } -2 \leq x \leq 0 \\ 4 - x^2 & \text{if } 0 < x \leq 2 \end{cases}$

59. $\int (x^2 + x^{-2}) dx$

60. $\int (\sqrt{x^3} + \sqrt[3]{x^2}) dx$

61. $\int (x^4 - \frac{1}{2}x^3 + \frac{1}{4}x - 2) dx$

62. $\int (u + 4)(2u + 1) du$

63. $\int v(v^2 + 2)^2 dv$

64. $\int \frac{x^3 - 2\sqrt{x}}{x} dx$

65. $\int \left(x^2 + 1 + \frac{1}{x^2+1} \right) dx$

Numerical Integration with MATLAB

In Questions 66-69, consider the integrals which were evaluated earlier in the practice problems using anti-derivatives and the Fundamental Theorem of Calculus.

For each integral, in MATLAB, now use the LEFT(n) rule with $n = 1000$ to approximate the value of the integral; this does **not** require using anti-derivatives.

66. $\displaystyle\int_{-1}^{2} (x^3 - 2x) dx$, which equals $\dfrac{3}{4}$.

67. $\displaystyle\int_{-1}^{1} x^{100} dx$, which equals $\dfrac{2}{101}$.

68. $\displaystyle\int_{1}^{4} (5 - 2t + 3t^2) dt$, which equals 63.

69. $\displaystyle\int_{0}^{1} (1 + \frac{1}{2}u^4 - \frac{2}{5}u^9) du$, which equals $\dfrac{53}{50}$.