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How much can illiquidity affect corporate debt yield spread?☆

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ABSTRACT

We present a structural method for measuring the upper bound for the illiquidity risk of liabilities issued by a levered firm. The method calculates the upper bound of illiquidity spread of a corporate bond given its duration and the issuing firm's asset risk and leverage ratio. Consistent with the empirical literature the illiquidity spread is positively related to the issuing firm's asset risk and leverage ratio and the illiquidity component increases with a bond's credit quality. The term structure of illiquidity spread has a humped shape, where its maximum level depends on the firm's leverage ratio. Finally, we demonstrate how the method's implied restricted trading period can be used as a measure for illiquidity in the bonds' market.

1. Introduction

The effect of illiquidity on the valuation of corporate bonds has been extensively studied (Bao et al., 2011; Chen et al., 2007; Huang and Huang, 2012; Helwege et al., 2014; Longstaff et al., 2005). In particular, financial research has focused on the relative effects of illiquidity and credit risk in determining a corporate bond yield spread, and how illiquidity varies with a bond's credit quality and a debt's duration.[1] The 2007–2009 crisis, when market liquidity dried up (White, 2008; Cukierman, 2011, 2013), highlights the importance of understanding this relationship, since both illiquidity and credit risk intensified at the same time, and the relative contribution of each component was not clear (Bao et al., 2011; Friewald et al., 2012; Goodhart, 2008; Pelizzon et al., 2015; Schwarz, 2014). The Basel Committee on Banking Supervision (2013) recently recognized that liquidity and solvency risks are often interlinked, but are frequently treated separately in (supervisory) stress tests, whereby this separate treatment of capital and liquidity understates bank risk.

In this paper we present a simple theoretical method that calculates an upper bound for the illiquidity discount of corporate liabilities. In the financial literature there are several models for finding the upper bound for the illiquidity discount of securities issued by an unlevered firm (Longstaff, 1995; Finnerty, 2012). However, none of these models suggests a general solution that accounts for the case of corporate liabilities issued by a levered firm. The suggested generalization is implemented by integrating structural models for pricing corporate liabilities (Longstaff and Schwartz, 1995; Merton, 1974) with a model that measures illiquidity discount. While previous works only consider a firm's asset risk and the length of the restricted trading period as the determining factors of illiquidity discount, the generalized approach, suggested in this paper, also considers the firm's capital structure and the duration of its debt. Further, by using a conventional analogy between structural models and reduced-form models, the illiquidity discount can be analyzed using a bond's recovery rate and probability of default.

The upper bound for the illiquidity discount is calculated in two stages. In the first stage, the value of the corporate bond is unbundled into a long position in a risk-free asset and a short position in the potential loss of the bondholder in the event of default by using

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[1] A corporate bond's yield spread is the difference between the bond's yield to maturity and a given default-free interest rate of the same maturity.

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a structural model. In the second stage, we value the corporate liability under the assumption of no trading restrictions and calculate the illiquidity discount as the difference between this value and the bond's price in the existence of such restrictions.[2] The value of the liability with no trading restrictions is calculated by assuming, as in Longstaff (1995), a hypothetical investor with perfect market-timing ability. In the absence of trading restrictions, this investor would sell at the maximum price that the security reaches during this period. The value under the trading restriction is the value of the corporate liability at the end of the trading restriction period under the structural model, which accounts for credit risk only. Thus, in this method, we can unbundle credit risk from liquidity risk.[3]

The method provides a number of important new insights into the potential effects of illiquidity on the pricing of corporate bonds. First, for a given restricted trading period and a firm's asset risk, we find that the illiquidity discount of a corporate bond is smaller than of a stock for an identical unlevered firm.[4] This result implies that using models such as Longstaff (1995) and Finnerty (2012) that ignore leverage and duration and consequently treat a bond's illiquidity in the same way as stocks would overprice the illiquidity discount of a corporate debt.

Second, while the relationship between asset risk and illiquidity discount has been explored in the theoretical literature (Longstaff, 1995), our paper is the first to establish a link between leverage and illiquidity. We find that illiquidity spread (the part of a corporate bond's yield spread that is due to liquidity risk) increases with a firm's leverage and asset risk. Our findings are consistent with the empirical literature showing that illiquidity spread decreases with a bond's credit quality (Bao et al., 2011; Chen et al., 2007; Dick-Nielsen et al., 2012; Friewald et al., 2012; Huang and Huang, 2012; Longstaff et al., 2005). We show that a firm's leverage has a major effect on the size of the illiquidity spread of a corporate bond. For example, the upper bound for the illiquidity spread of a zero-coupon corporate bond with 4 years to maturity of a firm with an asset risk of 30% and a leverage ratio of 30% equals 3 (6) basis points when the length of the restricted trading period equals 10 (30) days. Ceteris paribus, a corporate bond of a firm with a leverage ratio of 70% has an illiquidity spread of 29 (50) basis points for a restricted trading period of 10 (30) days.

Further, the method yields an illiquidity component (i.e., the illiquidity spread out of a bond's total yield spread) that increases with a corporate bond's credit quality. Again, this result is consistent with the empirical literature that shows that the portion of yield spread explained by credit risk increases as a bond's credit quality decreases (Friewald et al., 2012; Longstaff et al., 2005; Huang and Huang, 2012).

Third, the model's term structure of illiquidity spread, which describes the relation between illiquidity spread and a bond's duration, has a humped shape. This term structure is consistent with the shape predicted by Koziol and Sauerbier (2007). However, our model provides a new testable hypothesis by showing that the duration at which the curve reaches its maximum depends on a firm's leverage ratio. As leverage increases, the curve reaches its maximum at a shorter duration, where the curve becomes downward-sloping. The results are also consistent with the empirical downward-sloping shape of the term structure found by Ericsson and Renault (2006), since bonds with less than one year to maturity are excluded. In addition, we find that the term structure of illiquidity spread reaches its maximum at the same duration, irrespective of the choice of the restricted trading period (duration of one year). This means that in periods of financial distress as well as in periods of financial stability the illiquidity spread reaches its peak in bonds with the same duration.

The implications of our method for corporate bond pricing are derived from the interpretation that is given to the restricted trading period. If we simply understand it as the period of time in which an investor is forbidden from trading a security, the method can be used to estimate the required illiquidity spread of private placement under the SEC Rule 144A, which allows the trading of privately placed securities among qualified institutional buyers. These debt instruments are traded at a discount with respect to publicly offered debt (Chaplinsky and Ramchand, 2004; Livingston and Zhou, 2002). Moreover, the method can be used for the valuation of syndicated loans, where the syndication process is timely and usually an immediate transaction is almost impossible.

A broader interpretation of the restricted trading period is an expected future period of market dry-out in which trading in a security is limited. The model's implied restricted trading period can be extracted given the illiquidity spread of a bond and be used by investors, risk managers, and regulators as a novel measure of the expected period of market dry-out. Applying this approach, we refer to Dick-Nielsen et al. (2012) who estimate that during the subprime crisis the illiquidity component accounted for approximately 29% of the yield spread of corporate bonds that mature in 3–5 years (relative to 13% in the post-crisis period).[5] We show that for corporate bonds with features that are typical to a rating category of A and BB, an illiquidity component of 29% implies minimum restricted trading periods of 82 and 160 days, respectively. This is consistent with the observed period of market dry-out in the financial markets, which implies a restricted trading period between 3 and 5 months.

Finally, our work contributes to the understanding of the "credit spread puzzle," that is, the claim that yield spreads of corporate bonds are larger than what can be explained by the default risk implied by structural models (Collin-Dufresne et al., 2001; Elton et al., 2001; Huang and Huang, 2012). According to Longstaff et al. (2005), the non-default yield spread is strongly related to measures of corporate bond illiquidity and does not relate to the differential state tax treatment given to Treasury and corporate bonds. Our work can be used as a yardstick for testing equilibrium models for the valuation of both the predicted credit spread and illiquidity spread. To this end we show how our method can be expressed in terms of reduced-form models that are based on a firm's default probability and a bond's recovery rate.

The rest of the paper is organized as follows. Section 2 describes the related literature. Section 3 presents the framework of analysis for calculating the upper bound for illiquidity of corporate liabilities, namely, stocks and bonds. Section 4 discusses the implications for the analysis of corporate liabilities. Section 5 concludes the paper.

2. Related literature

The effect of a restricted trading period on the pricing of securities is modeled first on Mayers (1973, 1976), Brito (1977), Stapleton and Subrahmanyam (1979), and Boudouch and Whitelaw (1993) by

[2] Restricted trading is an extreme case of illiquidity and therefore serves as an upper bound for illiquidity.

[3] Finnerty (2012) suggests an alternative trading rule for calculating an illiquidity premium by assuming that investors cannot perfectly time the market and instead compare between the average price of the asset during the restricted trading period and its value at the end of the period.

[4] Usually, the observed restricted trading period of stocks is much shorter than the observed restricted period of corporate bonds, as they are traded in a centralized clearing.

[5] The view that illiquidity discount increases sharply when the general state of the economy is bad is supported also by Schuster and Uhrig-Homburg (2015) and Schwarz (2014).

presenting equilibrium models of illiquid asset returns. Longstaff (2009) finds that trading constraints decrease the price of restricted shares and might increase the price of their freely traded counterparts.

It is well known that a significant part of a corporate bond yield spread cannot be attributed solely to default risk, and non-default components are strongly relate to measures of illiquidity (Chen et al., 2007; Longstaff et al., 2005). This finding is consistent with evidence on illiquidity in secondary corporate bond markets (e.g., Bao et al., 2011; Edwards et al., 2007). Nonetheless, structural credit-risk modeling almost exclusively focuses on explaining the "default component" and only several equilibrium models focus on illiquidity spread. Ericsson and Renault (2006) are an exception, applying a binomial model for defaultable illiquid bonds while assuming that spread is decreasing with maturity, as are He and Xiong (2012), who develop a theoretical model that considers the interaction between debt market liquidity and credit risk. The model shows that when debt market liquidity decreases, equity holders face increasing rollover losses in issuing a new bond to replace the maturing bond. He and Milbradt (2014) endogenize market liquidity to feature liquidity-driven default and default-driven liquidity components in addition to pure liquidity and default components in credit spread. Chen et al. (2014) propose a decomposition scheme to capture both credit and illiquidity risk premium by calibrating historical moments of default probabilities and measures of illiquidity, and embed countercyclical macroeconomic fundamentals as well as procyclical liquidity frictions.

Similarly to Longstaff (1995), our method has an advantage over full equilibrium models because it is free of any assumptions of an investor's preference and the existence of asymmetric information. The disadvantage of this is that we only obtain bounds, while equilibrium models aim to extend theoretical credit-risk models in a way that fits actual market yield spreads.

3. The framework of analysis

The upper bound for the illiquidity discount of corporate liabilities issued by a levered firm is derived in two stages. First, we apply a structural model for pricing corporate liabilities (Merton, 1974), where a firm's asset risk, leverage, and a bond's maturity are necessary inputs. Second, to calculate the upper bound for illiquidity we adopt the theoretical framework developed by Longstaff (1995) that assumes that an investor is restricted from trading in a security during a predetermined period. If the investor can perfectly time the market, the best she can do is to sell the security at its maximum price during the period. Integrating the models results in an upper bound for the illiquidity discount of a corporate liability, which is the present value of the difference between the maximum value of the liability during the restricted trading period and its value at the end of this period, where the stochastic variable in our method is the firm's asset value.

While we price corporate liabilities using Merton's (1974) model and illiquidity using Longstaff's (1995) model, it should be noted that our method is not limited to a specific pricing model. The pricing model for corporate liabilities can be replaced by any structural model for pricing corporate liabilities or by a reduced-form model.[6] We choose Merton's (1974) model because of the tractability of relating it to the two main factors that affect a bond's credit quality: leverage ratio and asset risk. Further, the risk-neutral default probability and the recovery rate of a bond can be extracted from Merton (1974), which enables us to create an analogy with reduced-form models such as those of Jarrow and Turnbull (1995) and Duffie and Singleton (1999). In addition, the assumption of Longstaff (1995) that with no restricted trading period an investor can perfectly time the market can be replaced as well. Recently, Finnerty (2012) assumed that an investor can only achieve the average price of a security during the restricted trading period.[7]

Our method is based on several assumptions that simplify the analysis framework. These assumptions, which are common for structural models and for methods of calculating the upper bound for illiquidity, may affect the calculated results. The first assumption is that the corporate liabilities are fully liquid outside the restricted trading period. In case the assumption is relaxed (for example, trades can be executed, but not continuously), the value of the bond will further decline because of illiquidity. A second assumption is that the investor is a perfect-timer, thus yielding an upper bound of illiquidity discount (rather than the illiquidity discount itself). Naturally, if the investor has no special information, the illiquidity discount decreases. Third, at the event of default the debtholder receives the entire residual asset of the firm. The relaxation of the assumption causes violation of the absolute priority rule at bankruptcy, where stockholders can extract some value in a firm's default. In such a case the value of the stock will increase over the value of the bond and consequently the yield spread due to illiquidity will increase while the illiquidity component will decrease. An additional assumption is that the firm has a simple capital structure that includes only two types of securities: a senior bond and a stock. Adding a mezzanine tranche (sometimes called subordinated debt) will create a bond with an illiquidity discount higher than the senior bond and lower than the stock (the residual). Finally, the method assumes that there are no bankruptcy costs and the event of bankruptcy can occur only at debt maturity. In the case that bankruptcy costs exist, the bond holder will have a lower recovery rate and the value of the bond will decrease and, consequently, the yield spread will increase due to illiquidity. In case bankruptcy occurs before the debt matures, both the debtholder power and their ability to extract value at the event of default will increase. In such a case, the value of the stock will decrease over the value of the bond and consequently the yield spread will decrease due to illiquidity.

3.1. A structural model for pricing corporate liabilities

In this subsection we briefly describe the pricing model for corporate liabilities (Merton, 1974) and how to calculate its implied credit spread. We assume a firm with an asset value of V, financed by equity with a market value of S, and debt maturing at time T with face value F and market value B. Default may occur only at debt's maturity, T. In the case of default, the debtholder takes over the firm without incurring any distress costs and realizes an amount of V_T. Otherwise, the debt is fully paid and the debtholder receives the face value of debt, F. The payoff to the debtholder at maturity is expressed as:

$$B_T = \min(F, V_T) = F - \max(F - V_T, 0) \tag{1}$$

Under the Black and Scholes (1973) and Merton (1974) assumptions, the value of the firm's asset is governed by a Geometric Brownian Motion process under a risk-neutral measure, where

$$dV_t = rV_t dt + \sigma V_t dW_t, \tag{2}$$

[6] Examples of structural models include Black and Cox (1976), Leland (1994), and Longstaff and Schwartz (1995). Examples of reduced-form models include Jarrow and Turnbull (1995), Duffie and Singleton (1999), and Hull and White (2000).

[7] Appendix A briefly describes our method for pricing illiquidity of corporate liabilities using Finnerty's (2012) model.

where r is the risk-free rate, σ is the volatility of the firm's assets, and W_t is a standard Brownian motion. Hence, the current value of debt is expressed as:

$$B = F \cdot e^{-rT} - Put(V, F, T, \sigma, r) \tag{3}$$

We define the firm's leverage ratio as $LR = F \cdot e^{-rT}/V$. Under this formulation, the value of the corporate bond can be rewritten as a function of its leverage ratio and asset risk, the two main factors that affect a bond's credit quality. Thus, the credit spread of the risky debt over the risk-free rate, *cs,* is entirely determined by the value of the put option and can be expressed as:

$$cs = -\frac{1}{T} \cdot \ln\left(\frac{B}{F}\right) - r - \frac{1}{T} \cdot \ln\left(\frac{F \cdot e^{-rT} - Put\left(1, LR \cdot e^{rT}, T, \sigma, r\right)}{F}\right) - r \tag{4}$$

At debt maturity the stockholder receives the difference between the value of asset and the face value of debt if the value of asset is greater than the face value of debt and, otherwise, nothing. This payoff is expressed as:

$$S_T = \max(V_T - F, 0) \tag{5}$$

The present value of this payoff can be replicated by a call option on the value of the firm's asset with a strike price equal to the face value of debt:

$$S = Call(V, F, T, \sigma, r) \tag{6}$$

3.2. The upper bound of illiquidity of security issued by an unlevered firm

To price the upper bound for illiquidity of corporate liabilities, we generalize Longstaff's (1995) model, which derives the upper bound for illiquidity of securities issued by an unlevered firm. The model assumes a hypothetical investor with a perfect market-timing ability who is restricted from trading in a specific security during a predetermined period t. With no trading restriction, the perfect timer can sell the security at time τ, and reinvest the proceeds at the risk-free rate till the end of the period, such that the payoff at the end of the restricted trading period will be maximal. Hence, the payoff of the unrestricted investor with perfect timing ability, at the end of the period, denoted by M_t, can be expressed as:

$$M_t = Max_{0 \le \tau \le t}\left(e^{r(t-\tau)} \cdot V_\tau\right) \tag{7}$$

where r denotes the risk-free interest rate and V_τ denotes the security value at time τ during the restricted trading period t (i.e., $0 \le \tau \le t$).

If the "perfect timer" investor is restricted from trading the security during the restricted trading period, the result is a loss that is equal to the incremental value $Max(0, M_t - V_t) \equiv M_t - V_t$ at time t. The present value of this incremental cash flow represents the upper bound for the value of illiquidity, and is written as $e^{-rt} \cdot (E[M_t] - E[V_t])$. The expression has a closed-form solution and can be replicated by a Lookback option, as developed by Goldman et al. (1979).

3.3. An upper bound for illiquidity of corporate liabilities

Our method generalizes Longstaff's (1995) model for pricing the upper bound for illiquidity of corporate liabilities, consisting of a bond and a stock. The generalization is conducted by replacing the value of the unlevered asset with the value of the firm's liabilities. The stochastic variable in our model is the value of the firm's asset, V_S, which follows a Geometric Brownian Motion (GBM) process as in Eq. (2) at any time s, where $0 \le s \le T$.[8]

The maximum value of a corporate bond during the restricted trading period can be expressed by integrating Eq. (3), for the value of the bond, and Eq. (7), which describes the maximum value of a security during a restricted trading period, t: [9]

$$M_t = \max_{0 \le \tau \le t}\left[e^{r(t-\tau)} \cdot \left(F \cdot e^{-r\cdot(T-\tau)} - Put\left(\frac{F \cdot e^{-r\cdot(T-\tau)}}{LR_\tau}, F, T-\tau, \sigma, r\right)\right)\right] \tag{8}$$

where the corporate bond value, at time τ, is expressed as:

$$B_\tau = F \cdot e^{-r\cdot(T-\tau)} - Put\left(\frac{F \cdot e^{-r\cdot(T-\tau)}}{LR_\tau}, F, T-\tau, \sigma, r\right)$$

Since the investor is restricted from selling the security at its maximum price, at time τ, her maximum loss due to illiquidity at the end of the restricted trading period, t, is the incremental value $M_t - B_t$. This value can be simplified to the difference between the value of a put option on the firm's asset at the end of the restricted trading period and its minimum value during this period:

$$\begin{aligned} M_t - B_t &= Put\left(\frac{F \cdot e^{-r\cdot(T-t)}}{LR_t}, F, T-t, \sigma, r\right) \\ &-\min_{0 \le \tau \le t}\left[e^{r(t-\tau)} \cdot Put\left(\frac{F \cdot e^{-r\cdot(T-\tau)}}{LR_\tau}, F, T-\tau, \sigma, r\right)\right] \end{aligned} \tag{9}$$

By using the standard risk-neutral valuation framework from option-pricing theory, the present value of Eq. (9) is the corporate bond's illiquidity discount $P(F, LR, \tau, t, T, \sigma, r)$ equals:

$$P(F, LR, \tau, t, T, \sigma, r) = e^{-rt} \cdot \left(\begin{aligned} &E^Q\left[Put\left(\frac{F \cdot e^{-r\cdot(T-t)}}{LR_t}, F, T-t, \sigma, r\right)\right] \\ &-E^Q\left[\min_{0 \le \tau \le t}\left[e^{r(t-\tau)} \cdot Put\left(\frac{F \cdot e^{-r\cdot(T-\tau)}}{LR_\tau}, F, T-\tau, \sigma, r\right)\right]\right] \end{aligned}\right) \tag{10}$$

Since there is no closed-form solution for Eq. (10) we present a semi-analytical solution that combines the Black and Scholes (1973) and Merton (1974) closed-form solutions for pricing the put option with a Monte Carlo simulation, creating simulation paths for finding its minimum price and its final price during the end of the restricted trading period.

It should be noted that according to Eq. (10), the maximum illiquidity discount of corporate bonds is a function of the length of the restricted trading period and the firm's credit risk. The latter is quantified by the value of the put option in the Merton model, and is a function of the firm's asset risk and its leverage ratio, and the bond's time to maturity.

Since $P(F, LR, \tau, t, T, \sigma, r)$ represents the upper bound for illiquidity discount, it follows that $B^{MinLiq} = B - P(F, LR, \tau, t, T, \sigma, r)$ is the minimum value of the corporate bond while accounting for illiquidity risk. Applying Merton (1974) and using Eq. (4) for the yield spread, we can now separate a bond's total yield spread into its credit-risk and illiquidity-risk components. The upper bound for illiquidity spread is calculated as the difference between the yield spread of B^{MinLiq} and the yield spread of the corporate bond B

[8] Another potential stochastic factor that may affect the value of the corporate bond is the risk-free rate, r. However, as observed in the 2007–2009 crisis, liquidity in the Treasury market was relatively high, and interest-rate exposure can be hedged by a "perfect timer" by shorting an all else identical bond issued by the Treasury. Hence, we remain with the assumption of a constant risk-free interest rate throughout the paper.

[9] We assume without loss of generality that the illiquidity period begins today and ends at time t. The framework can easily be extended to the case where the illiquidity period occurs at an arbitrary period until the bond's maturity, from time t_1 to time t_2.

according to Merton's (1974) model, which does not account for illiquidity risk. Thus, the maximum yield spread, ys^{max}, and the upper bound for illiquidity spread, ls^{max} (in basis points), are:

$$ys^{\max} = -\frac{1}{T}\cdot \ln\left(\frac{B^{MinLiq}}{F}\right) - r \tag{11}$$

$$ls^{\max} = -\frac{1}{T}\cdot \left[\ln\left(\frac{B^{MinLiq}}{F}\right) - \ln\left(\frac{B}{F}\right)\right] \tag{12}$$

Special attention is given in the financial literature to the illiquidity component, which is defined as the percentage of illiquidity spread out of the total yield spread of a corporate bond. The literature finds that the illiquidity component usually increases with credit quality (Huang and Huang, 2012; Longstaff et al., 2005). To align our work with the existing empirical literature, we define this component as $illiqc^{max}$, where its value is calculated as follows:

$$illiqc^{\max} = \frac{\ln\left(\frac{B^{MinLiq}}{F}\right) - \ln\left(\frac{B}{F}\right)}{\ln\left(\frac{B^{MinLiq}}{F}\right) - r} \tag{13}$$

Finally, for a comprehensive framework for corporate liability we derive the upper bound for illiquidity of a stock that is issued by a levered firm. In this setting, the stock of the levered firm exhibits a period of illiquidity and during this period it cannot be traded until time t. Since the stock is replicated by a call option on the value of the firm's asset, as in Eq. (6), the maximum value of the option during the illiquidity period at time t is:

$$M_t = max_{0\le\tau\le t}\left(e^{r(t-\tau)}\cdot S_\tau\right) = \max_{0\le\tau\le t}\left[e^{r(t-\tau)} \cdot \left(Call\left(\frac{F\cdot e^{-r\cdot(T-\tau)}}{LR_\tau}, F, T-\tau, \sigma, r\right)\right)\right] \tag{14}$$

and the maximum discount due to illiquidity at time t, $M_t - S_t$, is the difference between the value of the option during the illiquidity period and its value at the end of the period. Following Eq. (14) and applying the standard assumptions of the risk-neutral valuation framework, the upper bound for illiquidity discount, $LSP(F, LR, \tau, t, T, \sigma, r)$, equals:

$$LSP(F, LR, \tau, t, T, \sigma, r) = e^{-rt}\cdot\left(\begin{array}{l} E^Q\left[\max_{0\le\tau\le t}\left[e^{r(t-\tau)}\cdot Call\left(\frac{F\cdot e^{-r\cdot(T-\tau)}}{LR_\tau}, F, T-\tau, \sigma, r\right)\right]\right] \\ -E^Q\left[Call\left(\frac{F\cdot e^{-r\cdot(T-t)}}{LR_t}, F, T-t, \sigma, r\right)\right]\end{array}\right) \tag{15}$$

According to Eq. (15), the illiquidity discount of a levered stock is a function of the restriction period and the firm's credit quality. The latter is quantified by the value of the call option in Merton's structural model, and is a function of the firm's asset risk, its leverage ratio, and the bond's time to maturity. The illiquidity discount (in percent) of the levered stock is $(S - LSP(F, LR, \tau, t, T, \sigma, r))/S$.[10]

4. Implications for the analysis of corporate liabilities

4.1. Illiquidity price discount for liabilities of a levered firm

The empirical literature shows that, *ceteris paribus*, the illiquidity discount of corporate bonds usually increases as bond credit quality declines (Chen et al., 2010; Longstaff et al., 2005). Our method is consistent with these findings. In Table 1 we report the upper bound for illiquidity discount (in percent) of corporate liabilities. Where the base case parameters are asset volatility of 0.3, a risk-free rate of 2.75%, and time to maturity of 4 years, the upper

Table 1
The illiquidity discounts (in percent) for stocks and bonds.

			Asset risk			
			20%	**30%**	**40%**	**50%**
Panel A: 10 days						
Stock of unlevered firm			2.67%	4.02%	5.39%	6.78%
Stock and bond of a levered firm						
Leverage ratio	**0.20**	**Stock**	3.24%	4.79%	6.20%	7.53%
		Bond	0.00%	0.03%	0.20%	0.57%
	0.30	**Stock**	3.65%	5.21%	6.57%	7.83%
		Bond	0.01%	0.13%	0.49%	1.02%
	0.40	**Stock**	4.09%	5.60%	6.89%	8.09%
		Bond	0.04%	0.32%	0.83%	1.44%
	0.50	**Stock**	4.53%	5.95%	7.17%	8.31%
		Bond	0.13%	0.57%	1.17%	1.82%
	0.60	**Stock**	4.97%	6.29%	7.42%	8.51%
		Bond	0.30%	0.85%	1.49%	2.16%
	0.70	**Stock**	5.41%	6.59%	7.65%	8.69%
		Bond	0.52%	1.14%	1.80%	2.47%
Panel B: 30 days						
Stock of unlevered firm			4.66%	7.05%	9.48%	11.96%
Stock and bond of a levered firm						
Leverage ratio	**0.20**	**Stock**	5.76%	8.53%	11.08%	13.49%
		Bond	0.00%	0.04%	0.34%	0.98%
	0.30	**Stock**	6.48%	9.28%	11.74%	14.05%
		Bond	0.01%	0.22%	0.85%	1.77%
	0.40	**Stock**	7.26%	9.98%	12.32%	14.51%
		Bond	0.07%	0.56%	1.43%	2.51%
	0.50	**Stock**	8.06%	10.62%	12.83%	14.91%
		Bond	0.23%	0.99%	2.03%	3.18%
	0.60	**Stock**	8.85%	11.22%	13.29%	15.27%
		Bond	0.51%	1.48%	2.61%	3.79%
	0.70	**Stock**	9.63%	11.77%	13.70%	15.60%
		Bond	0.89%	1.99%	3.14%	4.33%
Panel C: 60 days						
Stock of unlevered firm			6.64%	10.08%	13.61%	17.23%
Stock and bond of a levered firm						
Leverage ratio	**0.20**	**Stock**	6.56%	9.96%	13.44%	16.99%
		Bond	0.06%	0.71%	2.01%	3.63%
	0.30	**Stock**	7.29%	11.04%	14.76%	18.40%
		Bond	0.27%	1.36%	2.91%	4.64%
	0.40	**Stock**	8.19%	12.17%	15.89%	19.42%
		Bond	0.61%	2.00%	3.67%	5.43%
	0.50	**Stock**	9.22%	13.25%	16.85%	20.22%
		Bond	1.04%	2.60%	4.30%	6.06%
	0.60	**Stock**	10.33%	14.26%	17.68%	20.91%
		Bond	1.50%	3.13%	4.85%	6.60%
	0.70	**Stock**	11.48%	15.20%	18.42%	21.50%
		Bond	1.95%	3.62%	5.33%	7.06%

The table reports the illiquidity discount (in percent) as a function of the firm's asset risk and the leverage ratio. For a stock of an unlevered firm, the illiquidity discount is calculated using Longstaff's (1995) model. For a levered firm the illiquidity discount of stocks and corporate bonds is calculated using the method for illiquidity of corporate liabilities, as a function of different leverage ratios and asset risks. Leverage ratio is defined as the ratio of debt to total assets. The values in the table refer to a 4-year zero-coupon corporate bond for a firm with an asset value of 100 and a risk-free interest rate of 2.75%. The corporate bond's face value is set according to its leverage ratio. Panels A, B, and C in the table present results for a restricted trading period of 10 days, 30 days, and 60 days, respectively.

[10] Our method rely on Longstaff (1995) framework that assume that after the restriction trading period the bond is perfectly liquid. In case this assumption does not holds (for example during periods of financial distress), and the underlying bond will trade infrequently after the restriction trading period, the illiquidity discount and the illiquidity spread will be smaller since the effect of illiquidity is less severe.

bound for illiquidity discount equals 0.85% (1.48%) of the bond price for restricted trading period of 10 (30) days.

The analysis shows a significant effect of leverage on the illiquidity discount of corporate liabilities. As leverage decreases, the illiquidity discount of both stocks and corporate bonds decreases, where the illiquidity discount of a stock converges to the level calculated by Longstaff (1995) for a security of an unlevered firm. In Table 1, using the base case parameters, for a restricted trading period of 10 days and an asset risk of 30%, the illiquidity discount of a stock of an unlevered firm equals 4.02%. The illiquidity discount of a stock issued by an all else equal firm, except for issuing a debt with a maturity of 4 years, equals 5.21% (6.29%) for a leverage ratio of 0.3 (0.6). Thus, the illiquidity discount of a stock of a firm with a leverage of 0.6 is 56.5% greater than that of the unlevered firm. The results are qualitatively similar for longer restricted trading periods, such as 30 and 60 days, as demonstrated in Panels B and C of Table 1.

Comparing our method and the models that ignore the firm's capital structure (Longstaff, 1995; Finnerty, 2012), one can claim that using the stock volatility in the latter models will yield similar results to our method. We examine this claim by inferring the stock volatility from the firm's leverage and asset risk, as suggested by Jones et al. (1984) who calculates the stock volatility within the Merton (1974) framework.[11] While this procedure seems promising it has the drawback of using a constant asset risk and leverage over time. To demonstrate this, for a given leverage ratio and asset volatility we compare the illiquidity discount using our method (as appears in Table 1) to find what is the implied stock volatility which will yield the same illiquidity discount as obtained when using the Longstaff (1995) model. Then, we compare it to the stock volatility as obtained by Jones et al. (1984). We find that the levels of stock volatility according to Longstaff (1995) differ from the stock volatility obtained by Jones et al. (1984) for a levered firm, and this difference increases with leverage and asset risk. For example, when leverage is equal to 0.6 and asset risk is 30% the stock volatility implied by Longstaff's (1995) model is 47%, while the Jones et al. (1984) leads to a stock volatility of 68%. Therefore, we cannot use a simple transformation to infer the illiquidity discount of levered securities.

4.2. *The effect of credit quality on illiquidity yield spread of corporate bonds*

The method for calculating the upper bound for illiquidity of corporate liabilities is a function of the firm's capital structure and its asset's volatility. Therefore, we focus on quantitatively analyzing the effect of a firm's leverage on the corporate debt's illiquidity spread.

We find that the upper bound for illiquidity spread decreases as a bond's credit quality increases, while the percent of the illiquidity spread out of the total yield spread (the illiquidity component) increases with a bond's credit quality, independent of the length of the restricted trading period. These findings are demonstrated in Table 2, which reports the effect of asset risk and leverage ratio on a corporate bond's illiquidity. Panels A, B, and C relate to restricted trading periods of 1, 10, and 30 days, respectively. In each panel,

Table 2
The effect of asset risk and leverage ratio on illiquidity of corporate bonds.

Panel A: 1 day

		Asset risk					
		25%	**30%**	**35%**	**40%**	**45%**	**50%**
Leverage ratio	**0.2**	0	0	1	1	3	4
		(5.89)	(5.07)	(4.48)	(4.03)	(3.68)	(3.39)
	0.3	0	1	2	4	5	7
		(4.78)	(4.17)	(3.73)	(3.40)	(3.15)	(2.94)
	0.4	1	2	4	6	8	10
		(4.02)	(3.57)	(3.24)	(3.00)	(2.80)	(2.65)
	0.5	2	4	6	8	11	13
		(3.47)	(3.13)	(2.89)	(2.70)	(2.56)	(2.44)
	0.6	4	6	8	11	13	16
		(3.04)	(2.80)	(2.62)	(2.48)	(2.37)	(2.28)
	0.7	6	8	11	13	15	18
		(2.71)	(2.54)	(2.41)	(2.31)	(2.22)	(2.15)

Panel B: 10 days

		Asset risk					
		25%	**30%**	**35%**	**40%**	**45%**	**50%**
Leverage ratio	**0.2**	0	1	2	5	9	14
		(17.11)	(15.13)	(13.62)	(12.44)	(11.49)	(10.72)
	0.3	1	3	7	12	19	26
		(14.38)	(12.81)	(11.64)	(10.74)	(10.03)	(9.45)
	0.4	4	8	14	21	28	36
		(12.41)	(11.18)	(10.29)	(9.60)	(9.05)	(8.61)
	0.5	8	14	22	29	38	46
		(10.90)	(9.97)	(9.29)	(8.76)	(8.34)	(8.00)
	0.6	14	21	29	38	46	55
		(9.72)	(9.03)	(8.52)	(8.12)	(7.80)	(7.52)
	0.7	21	29	37	45	54	62
		(8.77)	(8.28)	(7.90)	(7.61)	(7.36)	(7.14)

Panel C: 30 days

		Asset risk					
		25%	**30%**	**35%**	**40%**	**45%**	**50%**
Leverage ratio	**0.2**	0	1	4	8	16	25
		(25.81)	(23.24)	(21.23)	(19.61)	(18.30)	(17.21)
	0.3	2	6	12	21	32	45
		(22.25)	(20.12)	(18.50)	(17.23)	(16.22)	(15.39)
	0.4	6	14	24	36	49	64
		(19.57)	(17.86)	(16.58)	(15.60)	(14.81)	(14.17)
	0.5	14	25	38	51	66	81
		(17.46)	(16.13)	(15.15)	(14.38)	(13.77)	(13.27)
	0.6	24	37	51	66	81	97
		(15.76)	(14.76)	(14.02)	(13.43)	(12.96)	(12.56)
	0.7	36	50	65	80	95	111
		(14.37)	(13.65)	(13.10)	(12.67)	(12.30)	(11.99)

The table reports the upper bound for illiquidity of corporate bonds as a function of the firm's asset risk and leverage ratio.The first row in the table shows the upper bound for illiquidity spread (in basis points) of the corporate bond due to the restricted trading period. The second row, presented in parentheses, is the illiquidity component, which is the percent of the illiquidity spread out of the total yield spread of the corporate bond. The values in the table refer to a 4-year zero-coupon bond for a firm with an asset value of 100 and a risk-free interest rate of 2.75%. Leverage ratio is defined as the ratio of debt to total assets. The corporate bond's face value is set according to its leverage ratio. Panels A, B, and C in the table present results for a restricted trading period of 1 day, 10 days, and 30 days, respectively.

the first row presents the illiquidity spread and the second row (in parentheses) presents the illiquidity component, which is calculated by dividing the illiquidity spread by the sum of the credit spread, as calculated by the Merton (1974) model, and the illiquidity spread. All the other parameters are the same as in the baseline case. When leverage equals 30% and asset risk equals 30%, which are typical levels for investment grade bonds (Huang and Huang, 2012), the illiquidity spread equals 1, 3, and 6 bps for restricted trading periods of 1, 10, and 30 days, respectively. The illiquidity component in this case equals 4.17%, 12.81%, and 20.12% out of

[11] Jones et al. (1984) shows that the relation between the asset risk σ_A and the equity risk σ_E within the Merton (1974) framework is $\sigma_E = \left(A/E\right) \cdot N(d_1) \cdot \sigma_A$. In this equation, A denotes the firm's assets, E denotes the firm's equity and $N(.)$ is the cumulative normal distribution function. The equation for the term d_1 is $d1 = \left[\ln\left(A/D\right) + \left[r + 0.5\sigma_A^2\right] \cdot T\right] / \left[\sigma_A \cdot \sqrt{T}\right]$, where D denotes the firm's debt, r is the risk-free rate in the market, and T is the debt's time to maturity. To the best of our knowledge, a transformation from asset volatility to debt volatility does not exist in the financial literature. Hence, such a comparison cannot be conduct.

the total yield spread. As the asset risk increases to a level of 45%, which is typical of high yield bonds, the illiquidity spread shifts to levels of 5, 19, and 32 bps for restricted trading periods of 1, 10, and 30 days, respectively. While the total spread due to illiquidity increases with the asset risk, the illiquidity component (out of the total yield spread) behaves in the reverse way; that is, when asset risk equals 45%, the illiquidity component decreases to 3.15%, 10.03%, and 16.22% for restricted trading periods of 1, 10, and 30 days, respectively.

A firm's leverage has a similar effect to asset risk on illiquidity spread and on the illiquidity component: as a firm's leverage ratio increases, the illiquidity spread increases, while the illiquidity component decreases. For the base case parameters, when the leverage ratio equals 0.6, a typical ratio for a corporation that issues a high yield bond in the non-financial sectors (Huang and Huang, 2012), illiquidity spread equals 6, 21, and 37 bps and the illiquidity component equals 2.8%, 9.03%, and 14.76% for restricted trading periods of 1, 10, and 30 days, respectively. Thus, the illiquidity component is highly sensitive to a firm's leverage ratio.

The relationships between a bond's credit quality and its illiquidity spread and illiquidity component are consistent with the ample empirical literature that finds similar evidence in terms of both size and direction. Longstaff et al. (2005) investigate the components of the corporate yield spread and conclude that the non-default component (out of the total yield spread) is strongly related to measures of corporate bond illiquidity and does not relate to the differential state tax treatment given to Treasury and corporate bonds. Moreover, consistent with our method, the non-default component is higher for a bond with better credit quality, as measured by their credit rating. Further, Longstaff et al. (2005) estimate that the non-default component ranges between 20 and 100 basis points. This level is found in our analysis for all bonds that mature in 4 years with a leverage ratio greater than or equal to 0.5 and asset risk greater than 30% and for bonds with leverage ratio above 0.3 and asset risk of 40% and above. Our findings are also consistent with studies that examine measures of corporate bond illiquidity directly and not via the non-default component (see Chen et al., 2007; Chen et al., 2010; Ericsson and Renault, 2006; Friewald et al., 2012; Han and Zhou, 2008).

4.3. The term structure of illiquidity yield spread

The term structure of illiquidity spread describes the relation between illiquidity spread and a bond's duration. Existing models for the term structure of illiquidity spread predict either a humped curve (Koziol and Sauerbier, 2007) or a downward-sloping curve where bonds with short-term maturity are excluded (Ericsson and Renault, 2006).[12] Our method is consistent with these models. It predicts a humped-shape term structure and the maximum level of the curve is received at a relative short maturity. The humped-shape term structure first increases with duration since the illiquidity spread of a bond with maturity of zero is equal to zero (Koziol and Sauerbier, 2007). The value of the Lookback option increases with the time to maturity t, but the marginal increase in value declines, yielding a downward-sloping term structure for longer maturities. In addition, our pricing method is based on Merton (1974) model which yields a hump-shaped term structure of credit spread (Sundaram and Das, 2010).

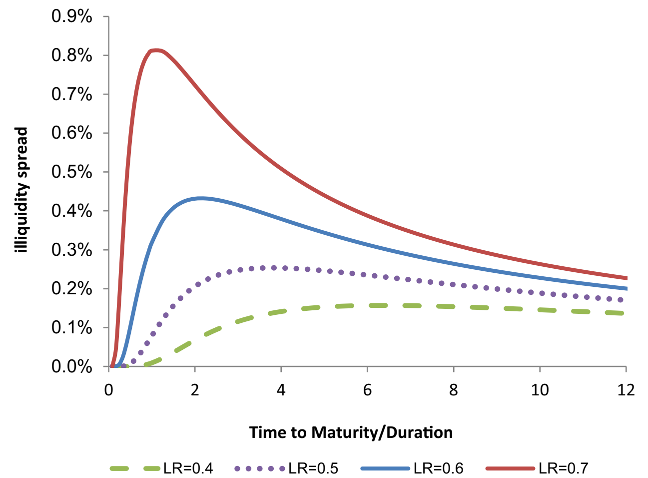

Fig. 1. The term structure of illiquidity spread as a function of the firm's leverage ratio.
The figure plots the term structure of illiquidity spread for corporate bonds with different leverage ratios. The bond's restricted trading period is 10 days. The values in the figure refer to a zero-coupon bond for a firm with an asset value of 100, asset risk of 30%, and a risk-free interest rate of 2.75%. The corporate bond's face value is set according to Merton (1974). Leverage is defined as the ratio of debt to total assets.

More importantly, our method provides a new testable hypothesis by showing that the duration in which the illiquidity curve reaches its maximum depends on a firm's leverage ratio. All else equal, as leverage increases, the maximum level of illiquidity spread is reached at a shorter duration and the relationship becomes downward sloping at a shorter maturity. Several empirical papers study the term structure of illiquidity (Amihud and Mendelson, 1991; Kempf et al., 2012; Longstaff, 2004). However, none of them relate to the effect of leverage.

The effect of leverage on the relationship between a bond's duration and its illiquidity spread is expressed in Fig. 1 and Table 3. When leverage is relatively high and equals 0.7, the upper bound for illiquidity spread reaches its maximum value at a maturity of 12 months and equals 81 bps. The illiquidity spread decreases with maturity and equals 22.7 bps at a maturity of 12 years. When leverage is relatively low, the effect of a given restricted period on a bond yield spread is not meaningful. Thus, the upper bound for illiquidity reaches its maximum at a longer duration. For example, when leverage is equal to 0.4, the upper bound for illiquidity spread reaches its maximum value at maturity a of 42 months and equals 25 bps.

An additional prediction of our method that relates to the term structure of illiquidity spread is to examine the term structure for different restricted trading periods (denoted by τ). The results are presented in Fig. 2, which plots the term structure of illiquidity spread with different restricted trading periods: 1 day, 5 days, 10 days, and 30 days. According to Fig. 2, the illiquidity curve reaches its maximum at duration of 1 year, irrespective of the choice of the restricted trading period, τ. This result is interpreted to show that even in periods of crisis, when the restricted trading period increases, the illiquidity spread (in bps) still reaches its maximum in bonds with the same duration (1 year); that is, the relative sensitivity of bonds to illiquidity conditions is free from the length of the restricted trading period.

[12] Both our method and Koziol and Sauerbier (2007) are based to some extend on similar methodology, i.e. – a Lookback option approach. However, Koziol and Sauerbier (2007) differ from our notion of illiquidity (and from the original setting of Longstaff, (1995), since they assume an illiquid bond that can be traded on any desired set of dates during its lifetime rather than a specific period of illiquidity. In addition, Koziol and Sauerbier (2007) analysis is focused on interest rate risk (market risk) as the stochastic factor that can affect the value of the bond price, while in our setting the stochastic variable is the value of the firm's assets (credit risk). Therefore, the illiquidity spread of Koziol and Sauerbier (2007) is mainly appropriate for government bonds with thin trading, while our is more appropriate for corporate bonds, where credit is the major source of risk.

Table 3
The effect of duration on illiquidity of corporate bonds.

Panel A: LR of 30%

		Duration (years)					
		2	**4**	**6**	**8**	**10**	**12**
Restricted trading period (days)	**1**	1.9	3.6	3.8	3.7	3.4	3.2
		(6.06)	(3.43)	(2.47)	(1.96)	(1.64)	(1.42)
	5	4.6	8.7	9.3	8.9	8.3	7.7
		(13.18)	(7.83)	(5.74)	(4.60)	(3.88)	(3.37)
	10	6.3	12.2	13.1	12.6	11.8	10.9
		(17.39)	(10.68)	(7.93)	(6.40)	(5.42)	(4.72)
	15	7.6	14.8	16.0	15.4	14.4	13.4
		(20.31)	(12.72)	(9.52)	(7.72)	(6.55)	(5.72)
	20	8.9	17.5	18.9	18.2	17.1	15.9
		(22.94)	(14.63)	(11.04)	(9.00)	(7.66)	(6.71)
	30	10.7	21.1	22.9	22.2	20.8	19.4
		(26.27)	(17.18)	(13.10)	(10.74)	(9.19)	(8.07)

Panel B: LR of 70%

		Duration (years)					
		2	**4**	**6**	**8**	**10**	**12**
Restricted trading period (days)	**1**	21.3	13.1	9.5	7.5	6.2	5.3
		(3.65)	(2.32)	(1.79)	(1.48)	(1.28)	(1.13)
	5	51.1	31.7	23.1	18.2	15.0	12.8
		(8.34)	(5.45)	(4.23)	(3.52)	(3.05)	(2.71)
	10	72.1	45.0	32.8	25.9	21.4	18.3
		(11.39)	(7.56)	(5.91)	(4.94)	(4.29)	(3.82)
	15	88.0	55.2	40.3	31.8	26.3	22.4
		(13.56)	(9.11)	(7.15)	(6.00)	(5.22)	(4.65)
	20	103.8	65.3	47.7	37.7	31.2	26.6
		(15.61)	(10.60)	(8.36)	(7.03)	(6.13)	(5.47)
	30	126.0	79.6	58.3	46.1	38.2	32.6
		(18.34)	(12.63)	(10.03)	(8.47)	(7.41)	(6.62)

The table reports the upper bound for illiquidity of corporate bonds as a function of the corporate bond's time to maturity and the length of the restricted trading period (in days). The first row in the table is the illiquidity spread (in basis points). The second row, presented in parentheses, is the illiquidity component, (i.e., the percent of the illiquidity spread out of the total yield spread of the corporate bond). The values in the table refer to a 4-year zero-coupon bond for a firm with an asset value of 100 and a risk-free interest rate of 2.75%. Leverage ratio is defined as the ratio of debt to total assets. The corporate bond's face value is set according its leverage ratio. Panel A (Panel B) in the table presents results for a firm with a leverage ratio of 30% (70%).

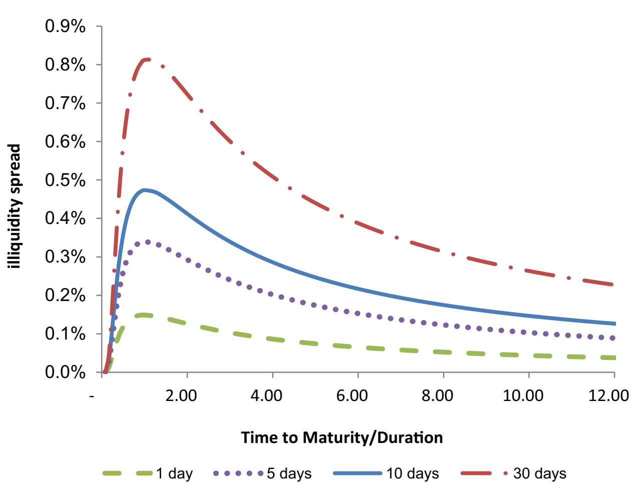

Fig. 2. The term structure of illiquidity spread as a function of the restricted trading period.
The figure plots the term structure of illiquidity spread for corporate bonds with different restricted trading periods. The values in the figure refer to a zero-coupon bond for a firm with an asset value of 100, asset risk of 30%, leverage ratio of 70%, and a risk-free interest rate of 2.75%. The corporate bond's face value is set according to Merton (1974). Leverage is defined as the ratio of debt to total assets. The figure plots graphs for restricted trading periods of 1 day, 5 days, 10 days, and 30 days.

4.4. *Implied restricted trading period as a measure of market dry-out*

The 2007-2009 financial crisis, when market liquidity and funding liquidity dried up (White, 2008), has underscored the need to explicitly take into account liquidity risk in stress-testing frameworks. One major element of a reliable stress test is a scenario that defines the (exogenous) shocks that stress the set of risks subject to stress (Borio et al., 2014). However, assuming a sudden arbitrary restricted trading period (for example 30 days) is not empirically helpful. After all, if the projected restricted trading period is too long all financial institutions would become insolvent. Therefore, it is important to consider experiments that economic agents believe are possible (Hilscher et al., 2014). Hence, we calculate the implied restricted trading period out of the actual illiquidity spread (or the illiquidity component) that is observed in the market.

The restricted trading period implied by the model is of special interest for two main reasons. First, in the case of newly issued debt there is uncertainty about the length of a future market freeze. This information can be extracted from the value of existing traded bonds with similar features.[13] Second, the average implied restricted trading period of bonds with similar credit quality and duration can be used by market participants as a novel measure of consensus among investors in the market regarding the expected time of a market dry-out.

The bond's implied restricted trading period is extracted in two steps. First, the total yield spread of a bond is calculated as the difference between its total yield and the yield on a similar government bond. Second, Merton's (1974) model is used to calculate the bond's credit spread. The difference between the total yield spread and the credit spread can serve as a proxy for a bond's illiquidity spread (based on Longstaff et al., 2005). Finally, using a numerical search, the implied restricted trading period, τ, that matches this difference is extracted from Eqs. (10) and (12).[14]

Fig. 3 presents the illiquidity component, which is expressed in Eq. (13), as a function of the length of the restricted trading period. The figure presents graphs of corporate bonds with similar attributes to four rating categories. The rating categories represent different leverage ratios and asset risks for corporate bonds that mature in 4 years, according to the analysis made by Huang and Huang (2012). Leverage ratio and asset risk equal 13.1% and 36.2%, respectively, for rating category of "AAA," where their levels for rating categories of "A," "BB," and "B" are 32.0% and 29.8%, 53.5% and 34.3%, and 65.7% and 39.5% respectively. The figure shows that as the restricted trading period increases so does the illiquidity component. Consistent with Section 4.2, as a bond's credit quality increases, so does the illiquidity component.

In seeking a proxy for the expected market dry-out in the corporate bond market, we relate to Dick-Nielsen et al. (2012), who find that post the subprime crisis the illiquidity component explained 13% of the yield spread of corporate bonds with 3–5 years to maturity, and increased to 29% during the subprime crisis (see Table 5 in Dick-Nielsen et al., 2012).

The horizontal graph (that marks the 29%) is a representation of this finding. For non-investment grade bonds, with ratings of "BB", 29% of the total yield spread represents restricted trading periods of

[13] Obviously, such an analysis is limited to the extent to which illiquidity risk is systemic.

[14] In the calculation of the implied restricted trading period, both the yield spread and the estimated volatility (which is used as an input in the Merton (1974) model) are obtained from market data and affected from illiquidity. Hence, a potential identification problem exists. In this respect, Campbell et al. (1997) show that non-synchronous trading has a substantial effect on the covariance between the asset returns and the market returns, however its impact on the security's volatility is marginal. It is shown (see chapter 3.1, page 89).

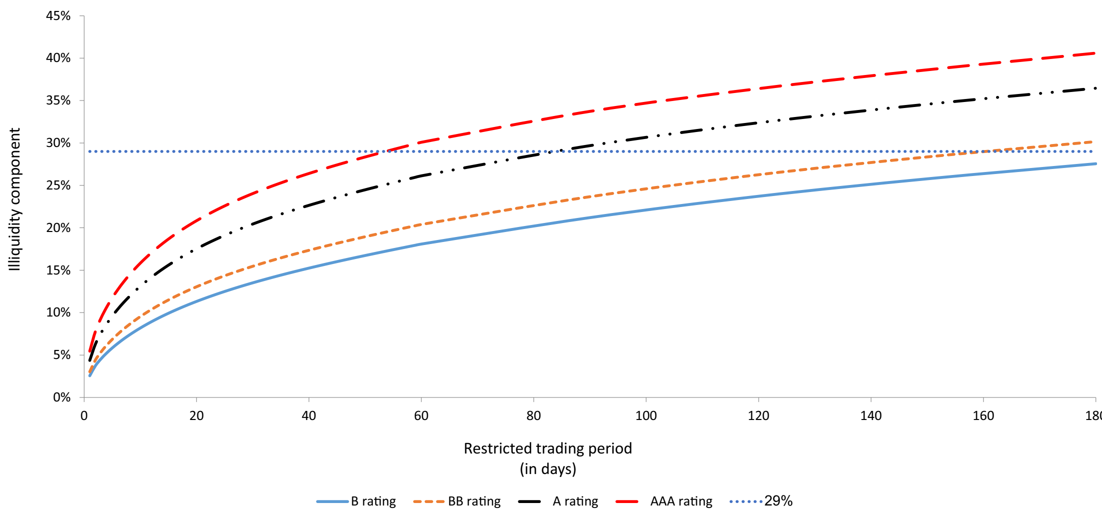

Fig. 3. Implied restricted trading periods.
The figure plots the illiquidity component, which is the percent of the illiquidity spread out of the total yield spread of the corporate bond, against the length of the restricted trading period for different credit rating categories AAA, A BB, and B. Leverage ratio and firm's asset risk are set according to Huang and Huang (2012), who report this data according to rating categories for corporate bonds that mature in 4 years (see Panel B of Table 2 in Huang and Huang, 2012) and a risk-free interest rate of 2.75%. Leverage ratio is defined as the ratio of debt to total assets. The corporate bond's face value is set according to its leverage ratio. The dashed line is set to 29%, which is the illiquidity component of corporate bonds with 3–5 years of maturity during the subprime crisis, as measured by Dick-Nielsen et al. (2012) (see Panel B of Table 5).

160. The expected restricted trading period of bonds with ratings of "AAA" and "A," namely, investment grade bonds is lower and equals only 55 and 82 days, respectively. These results are consistent with market events observed in the 2007–2009 financial crisis, where the dry-out in the non-investment grade category was longer and more severe than for investment grade bonds.

An interesting question is how the length of an implied restricted trading period is affected in times of financial crisis. The model's implied restricted trading period can be used as a yardstick for estimating the projected effect of a severe market dry-out. We find that a restricted trading period of 4 months has the effect of 1 basis point on the illiquidity spread of a typical AAA bond, which is 36.99% of the bond's total spread. This spread increases to 129 basis points for a BB bond, while the percentage of illiquidity out of the total spread decreases to 23.94%. While these levels are relatively high, they are consistent with the empirical finding that shows that in times of market crisis the illiquidity measures are more than doubled (Friewald et al., 2012; Dick-Nielsen et al., 2012; Schwarz, 2014; Schuster and Uhrig-Homburg, 2015).

4.5. *The effect of a bond's recovery rate and probability of default on illiquidity spread*

A common practice by credit analysts is to use reduced-form models that ignore a firm's capital structure for the valuation of corporate securities. These models are usually based on two main components, namely, the probability of default of the issuing firm and the expected recovery rate in the event of default (Jarrow and Turnbull, 1995; Duffie and Singleton, 1999; Hull and White, 2000). In reduced-form models, these two inputs are estimated from historical data by credit-rating agencies or can be extracted from the price of traded corporate bonds. In this subsection we demonstrate how our method is aligned with reduced-form models. This is meaningful since, as expressed by Altman et al. (2004), exogenously specifying the cash flows to risky debt in the event of bankruptcy and simplifying the bankruptcy process improves the first class of models that price corporate bonds. Moreover, reduced form models can capture different correlation structure across the business cycles between a bond's probability of default and its recovery rate (Jokivuolle and Viren, 2013).

To align our analysis with the reduced-form framework, we assume a constant recovery rate at the event of default *à la* Longstaff and Schwartz (1995) and compute the implied default probability using the Merton (1974) model. In the Merton (1974) model, the risk-neutral probability of default is equivalent to the probability that the value of a firm's asset is below the face value of debt at maturity and is expressed as:

$$\text{Probability of default} \equiv PD \equiv P(V_T < F) = N(-d_2),$$

$$\text{where}: \quad d_2 = \frac{\ln\left(V/F\right) + \left(r + \frac{\sigma^2}{2}\right)\cdot T}{\sigma\cdot\sqrt{T}} - \sigma\cdot\sqrt{T}.$$

Denoting RR as the percent of the recovery rate out of the firm's asset, the discount due to the corporate bond's credit risk relative to the value of a risk-less bond is:

$$F\cdot e^{-rT} - F\cdot e^{-rT}\cdot[RR\cdot N(-d_2) + (1 - N(-d_2))] = F\cdot e^{-rT}\cdot N(-d_2)\cdot(1 - RR)$$

While in the Merton (1974) model a bond's credit risk is equivalent to a put option on the value of a firm's asset, in the case of an exogenous recovery rate we can no longer base our analysis on this assumption. Since the upper bound for loss due to illiquidity at time t is $Max(M_t - B_t, 0) \equiv M_t - B_t$, we incorporate the expression in $e^{-rt}\cdot(E[M_t] - E[V_t])$ and receive the illiquidity premium as a function of the firm's recovery rate in case of default.

Table 4 shows the illiquidity spread for corporate bonds with different credit-rating categories. The credit categories are derived from the analysis of Huang and Huang (2012). We use Merton (1974) to calculate the risk-neutral probability of default of each bond in each rating category and assign an implied recovery rate as reported by Elkamhi et al. (2014). Table 3 in Elkamhi et al. (2014) presents firm-by-firm recovery rate estimations based on several loss functions. For each rating category we calculate the average recovery rate (for all the estimation types). We unite the rating categories AAA and AA and the rating categories BB and B since their number of observations is small (we obtain 5 [50] corporate bonds for AAA-AA [A] rating categories and 75 [22] corporate bonds for BBB [BB-B] rating categories).

Table 4
The effect of credit rating and recovery rate on illiquidity spread of corporate bonds.

Credit rating	Leverage ratio (%)	Assets vol (%)	Recovery rate	Prob. of default	Credit spread	Illiquidity spread 1 day	1 Week	2 Weeks	4 Weeks	2 Months
AAA	0.131	36.2%	62.5%	0.7%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.212	34.4%	62.5%	2.8%	0.06%	0.00%	0.01%	0.01%	0.01%	0.02%
A	0.32	29.8%	59.8%	5.3%	0.19%	0.01%	0.02%	0.03%	0.04%	0.06%
BBB	0.433	28.9%	52.6%	12.3%	0.72%	0.02%	0.06%	0.09%	0.12%	0.19%
BB	0.535	34.3%	44.3%	28.5%	2.49%	0.06%	0.16%	0.24%	0.33%	0.50%
B	0.657	39.6%	44.3%	44.7%	5.04%	0.09%	0.26%	0.38%	0.54%	0.82%

The table reports the illiquidity spread (in basis points) of corporate bonds as a function of the corporate bond's credit rating for different restricted trading periods. The corporate bond is a 4-year zero-coupon bond for a firm with an asset value of 100 and a risk-free interest rate of 2.75%. Leverage ratio is defined as the ratio of debt to total assets. The corporate bond's face value is set according to its leverage ratio. The corporate bond's leverage ratio and asset risk are determined according to its credit categories, as in Huang and Huang (2012). The corporate bond's recovery rate is determined according to the implied recovery rate, as in Elkamhi et al. (2014). Leverage ratio is defined as the ratio of debt to total assets. Credit spread and probability of default are calculated using Merton's (1974) model.

For example, a bond with a rating of "A" has a default probability of 5.3% and a recovery rate of 59.8%. Using these parameters leads to a credit spread of 19 bps for a 4 year bond and an upper bound for illiquidity of 4 (6) bps for a restricted trading period of 30 (60) days. When we analyze a bond with features that are similar to a non-investment "BB" grade bond, the bond's probability of default equals 28.5% and its recovery rate equals only 44.3%. Using the pricing method that adjusts for an exogenous recovery rate yields a credit spread of 249 bps and an upper bound for illiquidity spread of 33 (50) bps for a restricted trading period of 30 (60) days.

5. Conclusion

In this paper we present a simple theoretical method that calculates an upper bound for the illiquidity discount of corporate liabilities. The suggested generalization is conducted by integrating structural models for pricing corporate liabilities (Merton, 1974; Longstaff and Schwartz, 1995) with a model that measures illiquidity discount (Longstaff, 1995; Finnerty, 2012). By doing so, we are the first to suggest a solution that accounts for the upper bound of illiquidity of corporate liabilities issued by levered firms. While previous work for upper bound of illiquidity considers only a firm's asset risk and the length of the restricted trading period as the determining factors of illiquidity discount, our generalized approach considers the firm's capital structure and a bond's duration. Further, by using the conventional analogy between structural models and reduced-form models, the illiquidity discount can be analyzed in terms of a bond's recovery rate and probability of default.

The method provides a number of important insights into the potential effects of illiquidity on the pricing of corporate bonds. First, for a given restricted trading period and a firm's asset risk we find that the illiquidity discount of corporate liabilities increases with leverage. This result implies that using models such as those of Longstaff (1995) and Finnerty (2012), which ignore leverage and duration, would misprice the cost of illiquidity of a corporate debt.

Second, while the relationship between asset risk and illiquidity discount has already been explored in the theoretical literature (Longstaff, 1995), this paper establishes a link between leverage and illiquidity. We find that illiquidity spread increases with a firm's leverage and asset risk, while the illiquidity component decreases with these factors. These results are consistent with the empirical literature.

Third, the effect of leverage on the term structure of illiquidity spread is not explored in the literature. We provide a new testable hypothesis by showing that the duration in which the term structure of illiquidity spread reaches its maximum depends on a firm's leverage ratio. As leverage increases, the curve reaches its maximum at a shorter duration, where the curve becomes downwards sloping.

The implications of our method are derived from the interpretation that is given to the restricted trading period. While a simple interpretation is a time period in which an investor is forbidden from trading a security, a broader interpretation is the expected future period of market dry-out in which trading in a security is limited due to systemic factors. The latter was demonstrated in the recent financial crisis, which highlighted the effect of political uncertainty and macroeconomic factors on corporate debt pricing (Waisman et al., 2015; Bruneau et al., 2012). Thus, the method can be used by investors, risk managers, and regulators as a novel measure for the effect of a severe market dry-out as observed in the 2007–2009 financial crisis.

Appendix A. : Illiquidity spread for an investor with no timing ability

A.1 Step 1: Describing Finnerty's (2012) average-strike put option model

Finnerty (2012) derives the value of illiquidity using the value of an average-price Asian put. The model assumes that in the absence of any restrictions and inside information, the investor is equally likely to sell the shares anytime during the restricted trading period. The other assumptions of Finnerty (2012) are similar to Longstaff's (1995) model. In this case, the investor bears an opportunity cost if the arithmetic average price of the security during the restricted trading period is higher than the price at the end of the illiquidity period t. Formally, the illiquidity opportunity cost is:

$$\frac{1}{t+1}\cdot\sum_{\tau=0}^{t}e^{r(t-\tau)}\cdot B_\tau > B_t$$

and the upper bound of the investor's opportunity cost is:

$$\max\left(\frac{1}{t+1}\cdot\sum_{\tau=0}^{t}\left[e^{r(t-\tau)}\cdot B_\tau\right]-B_t, 0\right).$$

The upper bound represents the payoff of an average-strike put option.

A.2 Step 2: Integrating Finnerty's and Merton's frameworks

Assume that the firm's corporate bond exhibits a restricted trading period and that during this period the bond cannot be traded. As in Finnerty (2012), and without loss of generality, we assume that this period begins at the current time and lasts till time t. The corporate bond holder is equally likely to sell the corporate bond at any time during the restricted trading period t. Applying Merton's

(1974) framework (Section 3.1), the value of the corporate bond holder at the end of the restricted trading period t is expressed as:

$$\max\left(\frac{1}{t+1}\cdot\sum_{\tau=0}^{t}\left[e^{r(t-\tau)}\cdot B_\tau\right]-B_t,0\right)$$
$$=\max\left(\begin{array}{l}\frac{1}{t+1}\cdot\sum_{\tau=0}^{t}\left[e^{r(t-\tau)}\cdot\left(F\cdot e^{-r\cdot(T-\tau)}-Put\left(\frac{F\cdot e^{-r\cdot(T-\tau)}}{LR_\tau},F,T-\tau,\sigma,r\right)\right)\right]\\ -\left(F\cdot e^{-r\cdot(T-t)}-Put\left(\frac{F\cdot e^{-r\cdot(T-t)}}{LR_t},F,T-t,\sigma,r\right)\right),0\end{array}\right)$$
$$=\max\left(Put\left(\frac{F\cdot e^{-r\cdot(T-t)}}{LR_t},F,T-t,\sigma,r\right)-\frac{1}{t+1}\cdot\sum_{\tau=0}^{t}\left[e^{r(t-\tau)}\cdot Put\left(\frac{F\cdot e^{-r\cdot(T-\tau)}}{LR_\tau},F,T-\tau,\sigma,r\right)\right],0\right)$$

Again, the present value of the bond's illiquidity premium in this case is determined using the standard risk-neutral valuation. The present value of the illiquidity premium using Finnerty's (2012) model equals:

$$P(F,LR,\tau,t,T,\sigma,r)=$$
$$e^{-rt}\cdot\left(E^Q\left[\max\left(\begin{array}{l}Put\left(\frac{F\cdot e^{-r\cdot(T-t)}}{LR_t},F,T-t,\sigma,r\right)\\ -\frac{1}{t+1}\cdot\sum_{\tau=0}^{t}\left[e^{r(t-\tau)}\cdot Put\left(\frac{F\cdot e^{-r\cdot(T-\tau)}}{LR_\tau},F,T-\tau,\sigma,r\right)\right],0\end{array}\right)\right]\right)$$

The use of the average-strike put option model is appropriate for corporate bonds that are restricted from trading due to contractual or legal restrictions and their price may also rise during the restricted trading period (and not due to market dry-outs, which usually occur during financial crises).

Appendix B. : A Brief Review of the Monte Carlo Simulation

The following is a brief review of the basics of our numerical procedure for approximating the upper bound for the illiquidity discount.

The objective of the Monte Carlo simulation is to estimate $P(F,LR,\tau,t,T,\sigma,r)$, which is presented in Eq. (10) for the case of corporate bonds, and to estimate $LSP(F,LR,\tau,t,T,\sigma,r)$, which is presented in Eq. (15) for the case of stocks. The simulation is intended to calculate the firm's asset value V_t under a GBM as presented at Eq. (2): $dV_t=rV_tdt+\sigma dW_t$. Hereafter, we describe the Monte-Carlo simulation and then the valuation of the firm's claims (the stock and the bond).

B.1 Monte Carlo estimation for corporate bonds

Eq. (10) calculates the corporate bond's illiquidity discount $P(F,LR,\tau,t,T,\sigma,r)$, which equals:

$$e^{-rt}\cdot\left(\begin{array}{l}E^Q\left[Put\left(\frac{F\cdot e^{-r\cdot(T-t)}}{LR_t},F,T-t,\sigma,r\right)\right]\\ -E^Q\left[\min_{0\le\tau\le t}\left[e^{r(t-\tau)}\cdot Put\left(\frac{F\cdot e^{-r\cdot(T-\tau)}}{LR_\tau},F,T-\tau,\sigma,r\right)\right]\right]\end{array}\right)$$

The simulation generates sample paths out of the GBM process for the value of the firm's asset V_t. At each simulation path we calculate for each sample point the value of a put option on the firm's assets, with a strike price that equals the face value of the corporation debt, and compound its future value to the end of the restricted trading period using the risk-free rate. The outcome of each simulation path is the minimum of the compounded put option value. The discounted average value of the difference between the value of the put option at the end of the restricted trading period and the minimum compounded option value at each path is the illiquidity discount (in present value terms) of a corporate bond.

B.2 Monte Carlo estimation for stocks

Eq. (15) calculates the upper bound of the stock's illiquidity discount$LSP(F,LR,\tau,t,T,\sigma,r)$, which equals

$$e^{-rt}\cdot\left(\begin{array}{l}E^Q[\max_{0\le\tau\le t}[e^{r(t-\tau)}\cdot Call(\frac{F\cdot e^{-r\cdot(T-\tau)}}{LR_\tau},F,T-\tau,\sigma,r)]]\\ -E^Q[Call(\frac{F\cdot e^{-r\cdot(T-t)}}{LR_t},F,T-t,\sigma,r)]\end{array}\right)$$

The simulation generates sample paths out of the GBM process for the value of the firm's asset V_t. At each simulation path we calculate for each sample point the value of a call option on the firm's assets, with a strike price that equals the face value of the corporation debt, and compound its future value to the end of the restricted trading period using the risk-free rate. The outcome of each simulation path is the maximum of the compounded call option value. The discounted average value of the difference between the maximum compounded call option value at each path and its value at the end of the restricted trading period is the illiquidity discount (in present value terms) of the corporation's stock.

For each calculation we apply 30,000 simulation paths. In each simulation, the asset value V_t is calculated in a discrete way. Hence, we control in the number of daily sample of the asset value. According to our calculations, we reach a precision of a hundredth by employing 96 daily samples (i.e., for a continuously traded asset it means that the asset value is calculated every 15 minutes). The convention in the market is to quote yield spread in basis points (1 bps = 0.01%) and consequently robust results should be accurate to the first four digits. Increasing the number of the daily sample does not change the first four figures of our results. To avoid simulation bias, we apply one set of paths to determine the asset value and calculate the illiquidity value for different values of asset risk, leverage, restricted trading periods and duration (in the case of corporate bonds).

To examine the robustness of the Monte Carlo simulation results, we compare the discount obtained using$(S-LSP(F,LR,\tau,t,T,\sigma,r))/S$ for a leverage ratio LR of zero with Longstaff's (1995) close-form solution (the comparison is for the same asset risk and restricted trading period). The results of this comparison indicate that with 96 daily samples and 10,000 simulations we can accurately replicate the results of the closed form solution. Nonetheless, we use 30,000 simulation paths for each option, as stated above. Finally, since both the models we apply (i.e., Merton, 1974; Longstaff, 1995) assume continuous trading, we express the number of days over a calendar year (365 days) and not the number of trading days (252).

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