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# Dynamic portfolio allocation with time-varying jump risk ☆

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ABSTRACT

This paper solves the dynamic portfolio allocation problem with account of time-varying jump risk. We find that both the initial jump intensity as a state variable and the jump dynamics including the average jump intensity and jump persistence are important for the investor's optimal portfolio decision. The risk-averse investor can benefit from the optimal dynamic strategy instead of the myopic strategy. The out-of-sample results show that compared with the no-jump model, constant-jump model or the equal weighted portfolio, the dynamic portfolio with account of time-varying jump risk can produce better performance, and is more preferred by the risk-averse investor.

## 1. Introduction

It has been well documented that the risky assets prices exhibit large jumps due to unexpected major events, and the jump risk is an important factor that can affect the investors' portfolio allocation. For instance, Liu et al. (2003) showed that investors faced with event risk are unwilling to take leveraged positions. Das and Uppal (2004) documented that the market-wide systematic jumps reduce the gains from the portfolio diversification, and make the investors unwilling to take leveraged positions of risky assets. Kole et al. (2006) extended the study of Das and Uppal (2004) by building a regime-switching model which can allow for crisis persistence. They showed that the investors would allocate less to the risky assets, especially in the crisis-prone emerging markets. Ang and Bekaert (2002) and Guidolin and Timmermann (2008) highlighted the important role of regime switching in the dynamic asset allocation. They showed that ignorance of regime switching can result in significant economic loss and lower out-of-sample performance. Aït-Sahalia et al. (2009) provided a close-form solution to the consumption–portfolio selection problem with account of both Brownian risk and jump risk. Li and Zhou (2018) solved the optimal dynamic portfolio problem under the double exponential jump diffusion distribution and examined how the asymmetric jump distribution affects the optimal dynamic asset allocation.

Most of the previous studies assume that the jump intensity is constant when examining the dynamic asset allocation problem. Given the stylized fact that the jump intensity is time-varying,[1] in this paper we solve the dynamic asset allocation problem with account of time-varying jump risk.

Following Chan and Maheu (2002) and Maheu and McCurdy (2004), we build a generalized autoregressive conditional heteroskedasticity with autoregressive jump intensity (GARCH-ARJI) model to capture the time-varying jumps in the risky asset returns. The returns innovations are decomposed into two parts: a diffusion component and a jump component. The diffusion part is normally distributed, where the conditional variance follows a GARCH(1, 1) process. The jump part follows a compound Poisson distribution, where the jump intensity is time-varying and follows an AR(1) process. Chan and Maheu (2002) and

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[1] See e.g. Chan and Maheu (2002), Eraker (2004), Barndorff-Nielsen and Shephard (2006), Lee and Mykland (2008), Christoffersen et al. (2012), Aït-Sahalia et al. (2015) among others.

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Maheu and McCurdy (2004) found that by allowing the jump intensity to be time-varying, the GARCH-ARJI model could improve the forecasts of volatilities, particularly after large shocks in stock returns. Nyberg and Wilhelmsson (2009), Chang et al. (2011), Su and Hung (2011) and Su (2014) also built GARCH-ARJI models to describe the dynamics of risky asset returns, and found that the jump dynamics are important for tail risk measurements. Based on the GARCH-ARJI model, Maheu et al. (2013) found that the jump and jump dynamics are important factors to determine the market equity risk premium.

We estimate the GARCH-ARJI model using daily returns on S&P 500 index future. The empirical results show that the jump intensity is time-varying and reaches a relatively high level during US early 2000s economic recession, the US 2008 subprime crisis and the 2010–2011 European debt crisis. The average jump intensity from the model is 0.4109, indicating that the probability that at least a jump occurs on a trading day is 33.7%. Moreover, compared with the GARCH model which does not take account of jump risk, and GARCH-CJI model which assumes the jumps arrive at a constant rate, we find the GARCH-ARJI model can better fit the data.

It should be noted that there are two other approaches to detect jumps in the asset returns. The first method is parametric and based on the option data, see e.g. Pan (2002), Yan (2011), Christoffersen et al. (2012) and Ornthanalai (2014) among others. The empirical results based on the S&P 500 index option show that on average the index jumps once every year, and the mean and standard deviation of jump size are around $-3\%$ and 4% respectively. The other method to detect jumps is nonparametric and based on the high-frequency data, see e.g. Barndorff-Nielsen and Shephard (2006), Lee and Mykland (2008) and Aït-Sahalia and Jacod (2009) among others. Using 5-minute S&P 500 index future data, Miao et al. (2014) found that during the period from January of 2001 to December of 2010, the number of days that at least one significant jump occurred account for 20% of the total trading days.

The ARJI model used in this paper is a parametric method to model the time-varying jump intensity in the asset return. We can see that the average jump probability on a trading day from the ARJI model is close to that from the high-frequency data, but is much larger than that from option data. Compared with the model which use the option data to detect jumps, the ARJI model produces higher jump intensity, but lower mean and standard deviation of jump size. Therefore the method using option data tends to detect very large and rare jumps, while the ARJI model and high-frequency data method detect relatively small and frequent jumps.

Given the return dynamics described by the GARCH-ARJI model, we study a dynamic portfolio allocation problem with a CRRA (Constant Relative Risk Aversion) preference. As the problem has no close-form solution, we rely on the numerical algorithm to solve it. There are mainly two algorithms that are commonly used to calculate the value function of the dynamic allocation problem. The first one is to discretize the state variables and approximate the value function among the grids of the state variables, see e.g. Barberis (2000) and Guidolin and Timmermann (2008). The second approach simulates a large number of sample paths and obtain the value function by across-path regression. Brandt et al. (2005) noted that compared with the first method, the simulation-based method can handle a large number of state variables with path-dependent dynamics. Under the GARCH-ARJI model, the dynamics of the three state variables, including the one-period ahead forecasts of return, conditional variance and jump intensity, are nonlinear and path-dependent. In this paper, we rely on the simulation-based method to solve the dynamic asset allocation problem.

The empirical results show that the jump intensity as a state variable is an important factor to affect the investor's portfolio decision. The investor will decrease the proportion of her wealth invested in the risky asset when the initial jump intensity increases, since more frequent negative jumps make the risky asset less attractive to the risk-averse investor, especially to the more risk-averse investor. As a result, the less risk averse investor can benefit more from the optimal dynamic strategy, and has a larger certainty equivalent rate from the dynamic strategy.

We also examine how the jump dynamics affect the dynamic portfolio allocation. Under the GARCH-ARJI model, the jump dynamics are described by three parameters: $\theta$, $\rho$ and $\phi$. The first parameter $\theta$ measures the long-term average jump intensity. We find that the risky asset is negatively dependent on $\theta$. The second parameter $\rho$ measures the jump persistence, and its impact on the risky asset weight depends on the initial jump intensity. On one hand, if the initial jump intensity is low, a high $\rho$ indicates a persistence of low jump intensity in the future, which leads to a high risky asset weight. On the other hand, if the initial jump intensity is high, a high $\rho$ means a possible high jump intensity in the future. In this case, the risky asset weight will be small. Finally, $\phi$ is a coefficient of the jump innovation term, and measures the impact of past shocks on the jump intensity. As the expected value of the jump innovation is zero, we find that $\phi$ has no monotonic relationship with the risky asset weight.

The out-of-sample results show that compared with the GARCH and GARCH-CJI models, the GARCH-ARJI model with account of time-varying jump takes lower risk and yields realized returns with lower standard deviation and lower average return. Using the Sharpe ratio as the performance measure, the GARCH-ARJI model cannot beat the GARCH-CJI model, but can beat the GARCH model and the equal weighted portfolio. If using the utility-based performance measure, the GARCH-ARJI model can produce higher certainty equivalent rates than the GARCH model, GARCH-CJI model and equal weighted portfolio. This indicates that the GARCH-ARJI model is more preferred by the risk-averse investor. Moreover, the results are robust when we take the transaction costs into account.

The remainder of the paper is organized as follows. Section 2 firstly introduces the ARJI model to characterize the dynamics of S&P 500 index future returns. Then, we build a dynamic portfolio allocation model for the investor with a CRRA preference. Section 3 presents the empirical results, and finally Section 4 concludes the paper.

## 2. The model

### 2.1. Daily returns with jump dynamics

In this section we use a AR(1)-GARCH(1, 1)-ARJI model to describe the daily asset returns, which enables us to capture the dynamic process of both volatility and jump intensity. Let $r_t$ be the logarithmic risky asset return from $t-1$ to $t$, and its dynamic is specified as

$$r_t = \alpha + \beta r_{t-1} + \varepsilon_t, \tag{1}$$

where the conditional mean follows the AR(1) process to account for potential serial correlation in the return series.

Following Chan and Maheu (2002) and Maheu and McCurdy (2004), the return innovation $\varepsilon_t$ has two contemporaneously independent components:

$$\varepsilon_t = \varepsilon_{1,t} + \varepsilon_{2,t}, \tag{2}$$

where $\varepsilon_{1,t}$ represents the smoothly changing component and $\varepsilon_{2,t}$ captures the discrete jump driven by some major event. Given $I_{t-1}$, the information set available at time $t-1$, the term $\varepsilon_{1,t}$ follows the normal distribution $\varepsilon_{1,t}|I_{t-1} \sim N\left(0, h_t\right)$, where the time-varying conditional variance $h_t$ is modeled as a GARCH(1, 1) process[2]

$$h_t = \kappa + \eta h_{t-1} + \psi \varepsilon_{t-1}^2. \tag{3}$$

The jump innovation term $\varepsilon_{2,t}$ is a compensated compound Poisson random variable

$$\varepsilon_{2,t} = \sum_{k=1}^{N_t} J_k - \lambda_t \mu, \tag{4}$$

where the jump size $J_k$ follows the normal distribution $J_k \sim N\left(\mu, \delta^2\right)$, and $\mu$ and $\delta$ are the mean and standard deviation of jump size, respectively. The random variable $N_t$ counts the number of jumps during the interval $(t-1, t)$, and it follows the Poisson distribution given $I_{t-1}$. Therefore the conditional density function of $N_t$ can be written as

$$\Pr(N_t = i|I_{t-1}) = \frac{\lambda_t^i}{i!}\exp\left(-\lambda_t\right), i = 0, 1, \ldots . \tag{5}$$

where the jump intensity $\lambda_t = E_{t-1}(N_t)$ is time-varying, and is specified as

$$\lambda_t = \theta + \rho \lambda_{t-1} + \phi \xi_{t-1}. \tag{6}$$

In Eq. (6), the autoregressive coefficient $\rho$ is a measure of the persistence of jump intensities, and can capture the lasting impacts of major events on financial market. The innovation term $\xi_{t-1}$ is given by

$$\xi_{t-1} = E_{t-1}(N_{t-1}) - \lambda_{t-1}, \tag{7}$$

where $E_{t-1}(N_{t-1}) = E\left(N_{t-1}|I_{t-1}\right)$ is the ex post filter on $N_{t-1}$ based on information set $I_{t-1}$ and $\lambda_{t-1} = E_{t-2}(N_{t-1})$ is the conditional forecast of $N_{t-1}$ based on information $I_{t-2}$. So $\xi_{t-1}$ represents the update of the conditional forecast of $N_{t-1}$ when information at time $t-1$ becomes available. Therefore if an unexpected event occurred during the time interval $(t-2, t-1)$, the innovation term $\xi_{t-1}$ could capture the impact of the event on the market. Moreover, as Maheu and McCurdy (2004) pointed out, $\xi_t$ is a martingale with respect to $I_{t-1}$, and satisfies $cov(\xi_s, \xi_t) = 0$ if $s \neq t$, which is a desirable property for a well-specified innovation term.

To assure the positivity of $\lambda_t$, following Chan and Maheu (2002) and Maheu and McCurdy (2004), in Eq. (6) we impose the constraint that $\theta, \phi \geq 0$ and $\phi \leq \rho < 1$. The parameters are estimated using the maximum likelihood estimation method. More details about the estimation procedure can be found in Appendix A.

### 2.2. Dynamic portfolio allocation with time-varying jump risk

Consider an investor who maximizes her expected utility of terminal wealth by rebalancing her portfolio of one risky asset and one risk-free asset at the beginning of each period (i.e, each day) over a $T$-period horizon. The investor's asset allocation problem can be written as

$$\max_{\{\omega_\tau\}_{\tau=t}^{T-1}} E_t\left[U(W_T)\right] \tag{8}$$
$$\text{s.t. } W_{t+1} = W_t\left(\omega_t R_{t+1}^e + R_f\right), t = 0, \ldots, T-1,$$

where $W_t$ is the wealth, $\omega_t$ is the risky asset weight, $R_{t+1}^e = \exp\left(r_{t+1}\right) - R_f$ is the excess return on the risky asset, and $R_f$ is the gross return on the risk-free asset. The investor's preference over terminal wealth is described by the following CRRA utility function:

$$U(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma}, \tag{9}$$

where $\gamma$ is the relative risk aversion coefficient.

For the CRRA utility function, the wealth is not a state variable. Let $Z_t = \left(\bar{r}_{t+1|t}, \bar{h}_{t+1|t}, \bar{\lambda}_{t+1|t}\right)$ denote the vector of state variables, where $\bar{r}_{t+1|t}$, $\bar{h}_{t+1|t}$ and $\bar{\lambda}_{t+1|t}$ denote respectively the one-period ahead forecast of return, conditional variance and jump intensity

[2] As Maheu and McCurdy (2004) pointed out, it is difficult to perfectly separate the diffusion and jump innovations based on the observed return series. In other words, it is hard to estimate the model parameters if $\varepsilon_t$ is replaced by $\varepsilon_{1,t}$ in Eq. (3). Therefore following Maheu and McCurdy (2004) and Chan (2004), we specify total innovation $\varepsilon_{t-1}$ as the innovation term in Eq. (3).

**Table 1**
Summary statistics.

| Mean | Std Dev | Skewness | Excess Kurtosis | LBQ Test | ARCH Test |
|---|---|---|---|---|---|
| 0.0128 | 1.2032 | −0.0784 | 11.413 | 76.934*** | 1473.8*** |

The table reports summary statistics of S&P 500 index future returns (in percent). The Ljung–Box Q test of Ljung and Box (1978) and the ARCH LM test of Engle (1982) is conducted with 20 lags. The asterisks *** represent 1% significance level. The sample period covers from January 3, 2001 to December 29, 2017.

given information available at time $t$. The optimal risky asset weight can be obtained by recursively calculating $Q_t\left(Z_t\right)$ from $T-1$ to 0 as

$$Q_t\left(Z_t\right)=\min_{\omega_t} E_t\left[\left(\omega_t R^e_{t+1}+R_f\right)^{1-\gamma} Q_{t+1}\left(Z_{t+1}\right)\right], \tag{10}$$

with the terminal condition given by

$$Q_T\left(Z_T\right)=1.$$

When the risky asset return follows the GARCH-ARJI process, there is no close-form solution to the problem. We rely on the simulation-based method proposed by Brandt et al. (2005) to solve the problem. We provide more details of the algorithm in Appendix B.

## 3. Empirical results

### 3.1. Data and preliminary analysis

To minimize the transaction costs and avoid the short-selling constraints, investors are assumed to trade the S&P 500 index future in our empirical investigation. The daily closing prices of the continuous index future are collected from Datastream. The risk-free rate is assumed to be zero for ease of exploration. The sample covers a period from January 3, 2000 to December 29, 2017 with a total of 4695 observations. The future return is calculated as the log-difference between the current price and the previous price multiplied by 100, and the summary statistics of the logarithmic future returns are presented in Table 1. The Ljung–Box Q tests using 20 lags show significant autocorrelations in the returns. The ARCH tests conducted by regressing the squared returns on their own 20 lags indicate the existence of significant ARCH effect in future returns series.

### 3.2. Parameters estimates

To examine whether it is important to incorporate the time-varying jump risk when making dynamic portfolio decision, the GARCH model without jump component (GARCH model) and GARCH model with constant jump intensity (GARCH-CJI) are used as two natural benchmarks to the GARCH-ARJI model. Table 2 displays the parameters estimates and standard errors under the GARCH, GARCH-CJI, and GARCH-ARJI models using the maximum likelihood estimation method.

Columns 2 and 3 present the estimation results of the GARCH model, where $\alpha$ and $\beta$ are the parameters describing the conditional mean of daily returns given in Eq. (1). The slope coefficient $\beta$ in the mean equation is negative and significantly different from zero at the 1% level, capturing the negative serial correlation in the asset returns. The parameters $\kappa$, $\eta$ and $\psi$ describe the conditional variance of daily returns given in Eq. (3). The estimate of $\eta$ is 0.8865, indicating that strong GARCH effect prevails.

Columns 4 and 5 report the results of the GARCH-CJI model, where the jump component with constant jump intensity is included in the return dynamics. In particular, the constant jump intensity $\theta$ is specified and $\mu$ and $\delta$ are the mean and standard deviation of the jump size respectively. The estimate of the constant jump intensity is 0.0477 and statistically significant. The estimate of average jump size $\mu$ is −0.4069, capturing the negative skewness of the daily returns distribution.

Column 6 and 7 report the results of the GARCH-ARJI model, where dynamic jump component is specified and the arrival rate of jumps is assumed to follow an AR process. Specifically, $\theta$, $\rho$, $\phi$ denote the constant, autoregressive and past innovation coefficients of the conditional jump intensity, while $\mu$ and $\delta$ are the mean and standard deviation of the jump size, respectively. With the estimate of $\rho$ being 0.9614 and significantly different from zero at the 1% level, the time-varying jump intensity is persistent. The coefficient of past shock $\phi$ is estimated as 0.3627, capturing the impact of past shocks on the jump intensity. The estimates of the mean and standard deviation of the jump size are similar to those from the GARCH-CJI model.

Finally, compared with the GARCH and GARCH-CJI models, the GARCH-ARJI model has a larger log-likelihood, a lower AIC and a lower BIC, suggesting that the GARCH-ARJI model can better fit the data than the other two models. Meanwhile, the likelihood ratio test statistics are all significant at 1% level, suggesting that we can reject the null in favor of GARCH or GARCH-CJI model against GARCH-ARJI model.

Fig. 1 plots the time-varying jump intensities from the GARCH-ARJI model. As we can see, the S&P 500 index future experienced a great turmoil and had high jump intensities during US early 2000s economic recession, the US 2008 subprime crisis and the 2010–2011 European debt crisis. Based on the time-series of jump intensities, we can calculate the average jump intensity as 0.4109, meaning that the probability that at least a jump occurs on a trading day is

$$1-\exp(-0.4109)=0.3370.$$

**Table 2**
Parameters estimates.

| | GARCH | | GARCH-CJI | | GARCH-ARJI | |
|---|---|---|---|---|---|---|
| | Estimates | Std Err | Estimates | Std Err | Estimates | Std Err |
| $\alpha$ | 0.0545*** | 0.0116 | 0.0446*** | 0.0106 | 0.0151 | 0.0112 |
| $\beta$ | −0.0488*** | 0.0165 | −0.0565*** | 0.0158 | −0.0412*** | 0.0158 |
| $\kappa$ | 0.0160*** | 0.0015 | 0.0037*** | 0.0012 | 0.0021*** | 0.0006 |
| $\eta$ | 0.8865*** | 0.0064 | 0.8998*** | 0.0064 | 0.9544*** | 0.0041 |
| $\psi$ | 0.1016*** | 0.0060 | 0.0834*** | 0.0059 | 0.0235*** | 0.0025 |
| $\theta$ | | | 0.0477*** | 0.0154 | 0.0184*** | 0.0032 |
| $\rho$ | | | | | 0.9614*** | 0.0063 |
| $\phi$ | | | | | 0.3627*** | 0.0539 |
| $\mu$ | | | −1.5401*** | 0.2855 | −1.0204*** | 0.1159 |
| $\delta$ | | | 0.4173* | 0.2463 | 0.4345*** | 0.0972 |
| $L$ | −6,351 | | −6,229 | | −6,138 | |
| $AIC$ | 12,712 | | 12,474 | | 12,295 | |
| $BIC$ | 12,744 | | 12,526 | | 12,360 | |
| $LR$ | 426.53*** | | 182.60*** | | – | |

1 This table presents the parameter estimates and standard errors of GARCH, GARCH-CJI and GARCH-ARJI models. Compared with GARCH model, GARCH-CJI model includes a jump component with constant jump intensity, while GARCH-ARJI introduces a jump component with time varying jump intensity. $\alpha$ and $\beta$ are the parameters describing the conditional mean of daily returns given in Eq. (1) in the text, $\kappa$, $\eta$ and $\psi$ are the parameters describing the conditional variance of daily returns given in Eq. (3) in the text, $\theta$, $\rho$, $\phi$ are the parameters describing the time-varying jump intensity given in Eq. (5), and finally $\mu$ and $\delta$ are the mean and variance of the jump size, respectively.
2 $L$ is the log likelihood function, and a model with higher LLF better fits the data. The Akaike information criterion (AIC) and Bayesian information criterion (BIC) are two widely-used model selection methods. The model with a lower AIC or BIC is more preferred.
3 $LR$ is the likelihood ratio test statistic for the null hypothesis in favor of alternative model against GARCH-ARJI model. $LR$ follows the $\chi^2$ distribution with degree of freedom equal to the difference in dimensionality of unknown parameters between GARCH-ARJI model and the alternative model. Therefore the $LR$ statistic for GARCH model against GARCH-ARJI model follows $\chi^2(5)$ distribution, and $LR$ statistic for GARCH-CJI model against GARCH-ARJI model follows $\chi^2(2)$ distribution.
4 The asterisks *, **, *** represent 10%, 5%, 1% significance levels respectively.

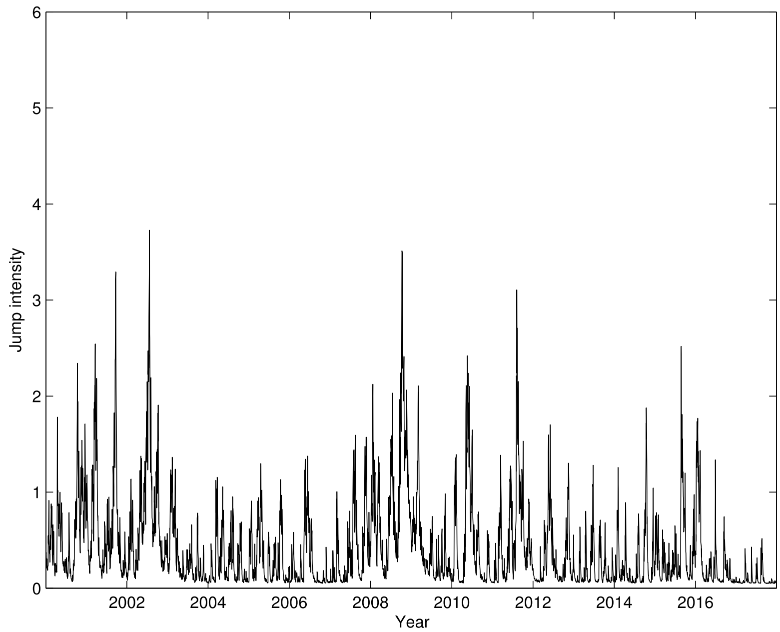


**Fig. 1.** Jump intensities of ARJI model.

The result is close to the findings by Miao et al. (2014), who used the high-frequency data and showed that the number of days that at least one significant jump occurs account for 20% of the total trading days during the period from January of 2001 to December of 2010.

**Table 3**
Dynamic asset weights and certainty equivalent rates.

| | $\gamma = 2$ | | | $\gamma = 10$ | | |
|---|---|---|---|---|---|---|
| | $\lambda = 0.05$ | $\lambda = 0.5$ | $\lambda = 1$ | $\lambda = 0.05$ | $\lambda = 0.5$ | $\lambda = 1$ |
| Panel A: Risky asset weights | | | | | | |
| $T = 1$ | 1.0313 | 0.8379 | 0.6184 | 0.1992 | 0.1333 | 0.0743 |
| $T = 5$ | 1.0863 | 0.9029 | 0.6630 | 0.2198 | 0.1576 | 0.0916 |
| $T = 20$ | 1.1051 | 0.9179 | 0.6674 | 0.2273 | 0.1636 | 0.0923 |
| Panel B: Certainty equivalent rates | | | | | | |
| $T = 1$ | 2.2812 | 2.4931 | 1.9787 | 0.4558 | 0.4775 | 0.3305 |
| $T = 5$ | 13.2180 | 16.0370 | 15.7810 | 2.6839 | 3.2555 | 3.1809 |
| $T = 20$ | 19.4470 | 18.8260 | 17.7440 | 3.9649 | 3.8085 | 3.5838 |
| Panel C: Gains from optimal dynamic trading | | | | | | |
| $T = 5$ | 10.5330 | 13.8760 | 14.0820 | 2.1404 | 2.8259 | 2.8839 |
| $T = 20$ | 16.9720 | 16.8310 | 15.8450 | 3.4574 | 3.4044 | 3.2655 |

1 In Panel A of this table, we report the optimal fraction of wealth invested in S&P 500 index future based on the GARCH-ARJI model. The initial jump intensity as a state variable can be 0.05, 0.5 or 1, meaning the risky asset can jump once every 20 days, every 2 days, or every day. The investment horizon can be 1 day, 5 days or 20 days, and the investor's risk aversion parameter can be 2 or 10.
2 Panel B reports the annualized certainty equivalent rates of returns for the optimal dynamic strategy. The rates are expressed in percent.
3 Panel C reports the increase in annualized certainty equivalent rates from investing optimally instead of myopically.

### *3.3. Dynamic portfolio weights*

In this section, we solve the dynamic asset allocation problem to examine how jump intensity and its dynamics affect the investor's dynamic portfolio decision.

Firstly, we focus on the role of jump intensity as a state variable in determining the optimal risky asset weight. The initial jump intensity can be 0.05, 0.5 or 1, and corresponds respectively to that on average the risky asset returns can jump once every 20 days, 2 days, or 1 day. The other two state variables, including the forecasts of asset return and conditional variance, are set to be equal to their sample averages.

Using the simulation-based method, we calculate the optimal risky asset weight where the investment horizon can be 1 day, 5 days or 20 days, and the risk aversion coefficient can be 2 or 10. The results are tabulated in Panel A of Table 3. We find that the risk averse investor will decrease the proportion of her wealth invested in the risky asset when the jump intensity increases. The results are intuitive since more frequent negative jumps make the risky asset less attractive to the risk averse investors, especially to the more risk-averse investor.

In Panel B of Table 3, we report the annualized certainty equivalent rates of returns (CER)[3] from the dynamic strategy. As a more risk-averse investor has less investment in the risky asset and has a lower expected return, we can see from the table that the CER decreases with the risk aversion, consistent with the results of Brandt (2010). Meanwhile, CER increases with the horizon, indicating that the investor can benefit more from dynamic trading as the investment horizon extends.

Finally, in Panel C we report the increases in annualized CER if using optimal dynamic strategy instead of using myopic strategy. We can find that the gains from optimal dynamic strategy are more prominent for the less risk-averse investor with $\gamma = 2$, and the annualized gains can be over 10%. For the more risk-averse investor with $\gamma = 10$, the annualized gains are around 3%.

In Figs. 2–4 we investigate how the parameters of the jump dynamics, including $\theta$, $\rho$ and $\phi$ as shown in Eq. (6), affect the dynamic portfolio decision. We consider three initial jump intensities, which are 0.05, 0.5 and 1. The investment horizon ranges from 1 day to 20 days.

As $\frac{\theta}{1-\rho}$ represents the long-term average jump intensity, we would expect that an increase in $\theta$ leads to a less fraction of wealth invested in the risky asset. The observations shown in Fig. 2 are consistent with our expectation. We can see that for different initial jump intensities, the risky asset weight decreases when $\theta$ increases from 0.01 to 0.03.

From Fig. 3, we can see that the impact of jump persistence parameter $\rho$ on the optimal weight is dependent on the initial jump intensity. On one hand, if the initial jump intensity is relatively low, a large $\rho$ means a large persistence of low jump intensity in the future, and as a result the investor will allocate more in the risky asset. On the other hand, if the initial jump intensity is high, a large $\rho$ means that the jump intensity will remain relatively large in the future. In this case the investor will allocate less of her wealth in the risky asset.

Finally, the observations in Fig. 4 show that the jump innovation coefficient $\phi$ has no monotonic impact on the risky asset. The reason can be seen from Eq. (6), which describes the dynamics of the jump intensity. From the equation we can see that when we use the simulation method to generate the sample paths, the expect jump intensity does not depend on $\phi$, since we always have $E(\xi_t) = 0$.

3 For the CRRA utility function, the CER can be calculated as

$$\frac{(1+CER)^{1-\gamma}}{1-\gamma} = Q(Z_t),$$

where $Q(Z_t)$ is the indirect value function defined in (10).

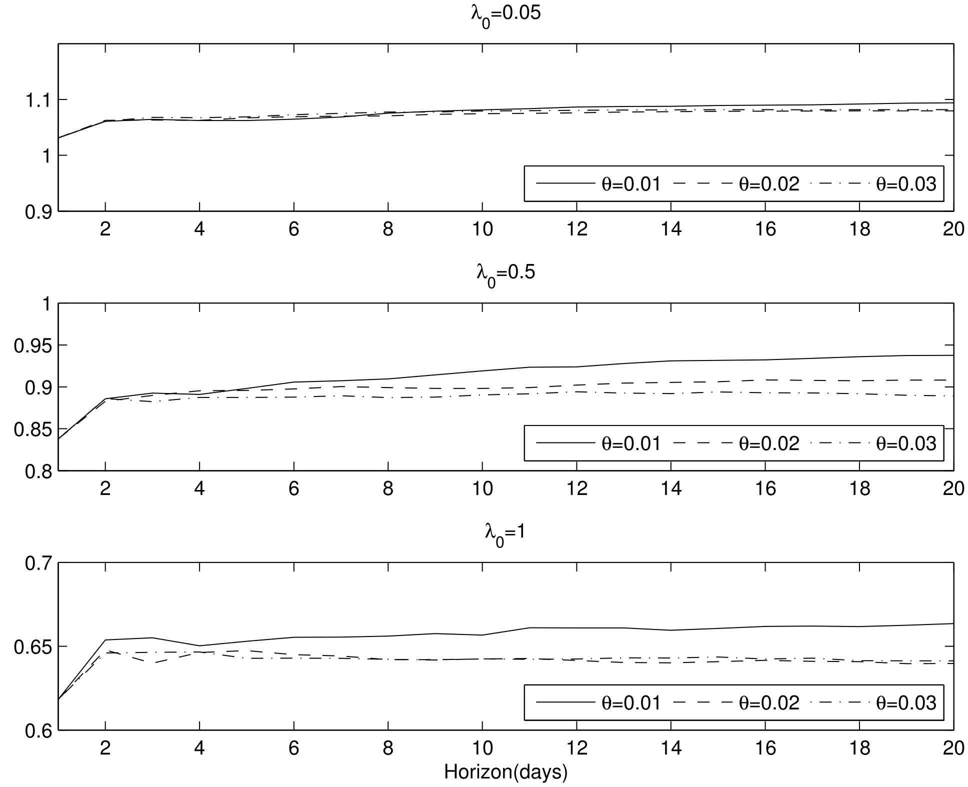


**Fig. 2.** Impacts of average jump intensity on risky asset weight.

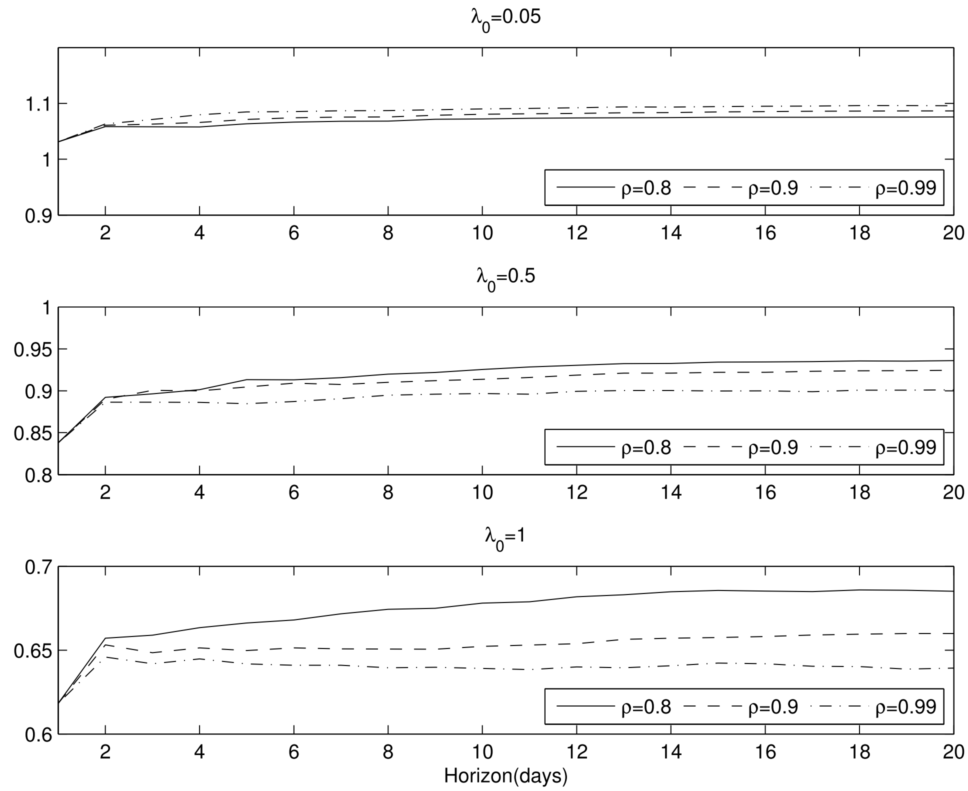


**Fig. 3.** Impacts of jump persistence on risky asset weight.

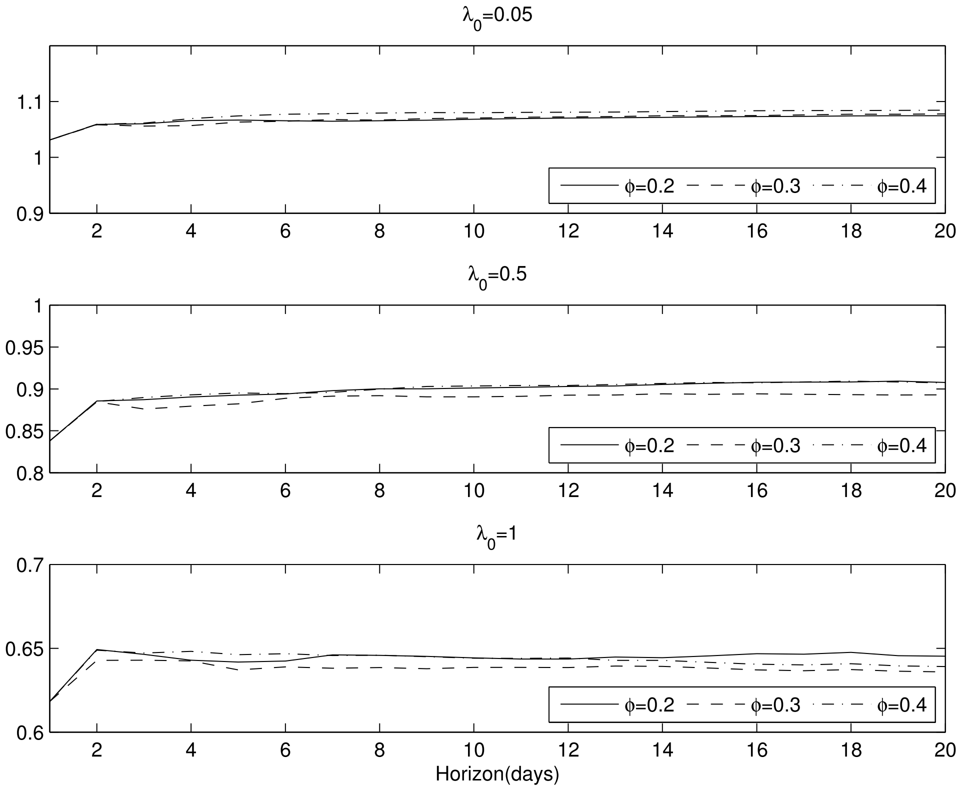


**Fig. 4.** Impacts of jump innovation on risky asset weight.

### 3.4. Out-of-sample performance

In this section, we explore the out-of-sample performances of different models. We firstly use the data before December 27, 2006 to estimate the model parameters. We use these estimates to obtain the one-day forecast of the return, conditional variance and jump intensity.[4] Using the three state variables, we rely on the simulation-based method to solve the dynamic portfolio allocation problem. When the time rolls over to the next day and a new observation becomes available, we estimate the model parameters based on the rolling-window method.

In the empirical analysis, we report the annualized mean, standard deviation (SD), Sharpe ratio (SR) of the realized portfolio returns, and the annualized certainty equivalent rate of return (CER) for the average realized utility. Moreover, to evaluate whether the GARCH-ARJI model can yield statistically significant better performance than the alternative model, we calculate the p-values for the differences using the bootstrapping method described in DeMiguel et al. (2013). Specifically, we generate $N-T$ pairs of $T$−period portfolio returns, where $N$ is the size of the out-of-sample data and $T$ is the investment horizon, by drawing randomly with replacement from the realized $T$−period returns of the GARCH-ARJI model and the alternative model. The performance measure for both strategies then can be calculated for this trial. We obtain the distribution of the difference of performance measure between the GARCH-ARJI model and the alternative model across $B = 10{,}000$ trials. We calculate the one-side $p$-value for the null hypothesis that the difference of the performance measure between the GARCH-ARJI model and the alternative strategy is lower than zero, so a small $p$-value would indicate that we can reject the null. Table 4 presents the out-of-sample performances of the GARCH, GARCH-CJI and GARCH-ARJI models. The 50/50 portfolio strategy is also included for comparison.

Compared with the GARCH model which does not take account of jump risk, the GARCH-ARJI model is more conservative, producing realized returns with significant lower means and significant lower standard deviations for different risk aversions and different investment horizons. When the risk aversion parameter $\gamma$ is 2 and the investment horizon $T$ is 1 day, the annualized Sharpe ratio of the GARCH-ARJI model is 0.7608, higher than that of the GARCH model, which is 0.7341. However, given that the $p$-value is 0.4292, the difference is not statistically significant. The Sharpe ratio of GARCH-ARJI model is significant higher than that of the GARCH model when the horizon extends to be 20 days. Regarding the utility-based performance measure, the CER of the GARCH-ARJI model is 13.9903, much higher than that of the GARCH model, which is only −0.3328. As a result, the $p$-value of the difference is 0.1057. When the horizon is 5 days or 20 days, the CER of GARCH-ARJI model is significant higher than that of the GARCH model.

[4] For the GARCH and GARCH-CJI models, the dynamic asset weight can also be obtained by simulation-based method. Under the GARCH and GARCH-CJI models, the return and conditional variance are the state variables, and the jump intensity is not a state variable.

**Table 4**
Out-of-sample performance.

| | $\gamma = 2$ | | | | $\gamma = 10$ | | | |
|---|---|---|---|---|---|---|---|---|
| | Mean | SD | SR | CER | Mean | SD | SR | CER |
| Panel A: $T = 1$ day | | | | | | | | |
| GARCH-ARJI | 31.9072 | 42.0175 | 0.7608 | 13.9903 | 6.4032 | 8.4393 | 0.7602 | 2.8089 |
| GARCH | 53.2658 | 72.8430 | 0.7341 | −3.2228 | 10.7066 | 14.5631 | 0.7380 | −0.2946 |
| | (0.9594) | (1.0000) | (0.4292) | (0.1057) | (0.9584) | (1.0000) | (0.4414) | (0.1185) |
| GARCH-CJI | 54.2100 | 65.7874 | 0.8268 | 8.7661 | 10.9195 | 13.2331 | 0.8278 | 1.9002 |
| | (0.9848) | (1.0000) | (0.6801) | (0.3286) | (0.9843) | (1.0000) | (0.6878) | (0.3468) |
| 50/50 | 3.7154 | 9.8879 | 0.3757 | 2.7368 | 3.7154 | 9.8879 | 0.3757 | −1.1817 |
| | (0.0060) | (0.0000) | (0.1048) | (0.1649) | (0.1627) | (1.0000) | (0.0990) | (0.0768) |
| Panel B: $T = 5$ days | | | | | | | | |
| GARCH-ARJI | 31.5876 | 39.6234 | 0.7984 | 15.0921 | 6.5718 | 8.1087 | 0.8120 | 3.1388 |
| GARCH | 50.4126 | 67.2791 | 0.7508 | −1.5488 | 10.7392 | 13.7630 | 0.7823 | 0.1580 |
| | (0.9999) | (1.0000) | (0.2425) | (0.0038) | (0.9999) | (1.0000) | (0.3332) | (0.0103) |
| GARCH-CJI | 52.7881 | 60.3974 | 0.8760 | 9.5421 | 11.1448 | 12.5604 | 0.8903 | 1.9636 |
| | (1.0000) | (1.0000) | (0.8700) | (0.1816) | (1.0000) | (1.0000) | (0.8736) | (0.2050) |
| 50/50 | 3.6668 | 8.7569 | 0.4202 | 2.8906 | 3.6668 | 8.7569 | 0.4202 | −0.4242 |
| | (0.0000) | (0.0000) | (0.0040) | (0.0082) | (0.0051) | (0.9926) | (0.0025) | (0.0020) |
| Panel C: $T = 20$ days | | | | | | | | |
| GARCH-ARJI | 30.3235 | 37.2668 | 0.8139 | 16.3562 | 6.5313 | 7.4419 | 0.8780 | 3.7027 |
| GARCH | 46.7912 | 64.6359 | 0.7242 | 1.7500 | 10.5357 | 12.7872 | 0.8244 | 1.6540 |
| | (1.0000) | (1.0000) | (0.0032) | (0.0000) | (1.0000) | (1.0000) | (0.0515) | (0.0002) |
| GARCH-CJI | 50.0796 | 56.7987 | 0.8822 | 12.3948 | 11.0494 | 11.5227 | 0.9600 | 3.2524 |
| | (1.0000) | (1.0000) | (0.9749) | (0.0913) | (1.0000) | (1.0000) | (0.9858) | (0.2602) |
| 50/50 | 3.5512 | 8.0035 | 0.4448 | 2.8906 | 3.5512 | 8.0035 | 0.4448 | −0.1382 |
| | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.9903) | (0.0000) | (0.0000) |

1 The table presents out-of-sample performances of different models. The investment horizon can be 1 day, 5 days or 20 days, and the risk aversion can be 2 or 10. The annualized mean, standard deviation (SD), Sharpe ratio (SR), and certainty equivalent rate of return (CER) are reported.
2 In the parentheses we report the one-side p-values for the null hypothesis that the performance measure of the GARCH-ARJI model is lower than that of the alternative model, so a small $p$-value would indicate that we can reject the null.

For the GARCH-CJI model which assumes that the jump intensity is constant, the means and standard deviations of its realized returns are significant higher than those of the GARCH-ARJI model for different investment horizons and different risk aversion parameters. The annualized Sharpe ratios of the GARCH-ARJI model are lower than those of the GARCH-CJI model, but the differences are not significant when the horizon is 1 day or 5 days, but become significant when the investment horizon is 20 days. When using the utility-based performance measure, the GARCH-ARJI model can produce higher CERs than the GARCH-CJI model, and the difference is significant when the horizon is 20 days and risk aversion parameter is 2. This indicates that compared with the GARCH-CJI model, the GARCH-ARJI model is more preferred by the risk-averse investor.

Finally, the 50/50 portfolio yields returns with lower mean and lower standard deviation than the GARCH-ARJI model when the risk aversion parameter is 2. When the risk aversion parameter is 10, the GARCH-ARJI model can yield higher average return and lower standard deviation than the 50/50 portfolio. As a result, when using the Sharpe ratio and CER as the performance measures, the GARCH-ARJI model can produce higher performance than the 50/50 portfolio, and the superiority of GARCH-ARJI model over the 50/50 portfolio is significant when the horizon is 5 or 20 days.

In Table 5, we further calculate the realized returns net of transaction costs, and provide the performance measures of different models. Let $W_t$, $P_t$ and $R_{p,t}$ denote respectively the wealth, the price of the S&P 500 index future and the gross portfolio return without transaction cost at time $t$. Suppose the transaction cost for the S&P 500 index future is \$$c$ per index unit, then the wealth at time $t+1$ net of transaction costs can be written as

$$W_{t+1} = W_t R_{p,t+1} - c \left| \frac{W_t R_{p,t+1} \omega^*_{t+1}}{P_{t+1}} - \frac{W_t \omega^*_t}{P_t} \right|,$$

where $\omega^*_t$ is the optimal risky asset weight at time $t$. As a result, the gross return net of transaction costs is given by

$$\tilde{R}_{p,t+1} = \frac{W_{t+1}}{W_t} = R_{p,t+1} - c \left| \frac{R_{p,t+1} \omega^*_{t+1}}{P_{t+1}} - \frac{\omega^*_t}{P_t} \right|.$$

Following Fleming et al. (2001), suppose the bid/ask spread and round-trip commission cost for S&P 500 index future is \$0.10 per index unit. As we can see, the results in Table 5 are quite consistent with those tabulated in Table 4, indicating that when considering the transaction costs, the GARCH-ARJI model can still produce better performance than the alternative models.

**Table 5**
Out-of-sample performance net of transaction costs.

| | $\gamma = 2$ | | | | $\gamma = 10$ | | | |
|---|---|---|---|---|---|---|---|---|
| | Mean | SD | SR | CER | Mean | SD | SR | CER |
| Panel A: $T = 1$ day | | | | | | | | |
| GARCH-ARJI | 27.8190 | 42.0000 | 0.6637 | 9.9059 | 5.5835 | 8.4380 | 0.6632 | 1.9890 |
| GARCH | 45.6620 | 72.8010 | 0.6300 | −10.8140 | 9.1887 | 14.5610 | 0.6338 | −1.8145 |
| | (0.9305) | (1.0000) | (0.4122) | (0.0636) | (0.9301) | (1.0000) | (0.4208) | (0.0749) |
| GARCH-CJI | 47.6360 | 65.7500 | 0.7272 | 2.2076 | 9.5999 | 13.2300 | 0.7282 | 0.5807 |
| | (0.9734) | (1.0000) | (0.6768) | (0.2506) | (0.9725) | (1.0000) | (0.6808) | (0.2666) |
| 50/50 | 2.8618 | 9.8871 | 0.2893 | 1.8832 | 2.8618 | 9.8871 | 0.2893 | −2.0354 |
| | (0.0139) | (0.0000) | (0.1107) | (0.2442) | (0.1595) | (1.0000) | (0.1060) | (0.0750) |
| Panel B: $T = 5$ days | | | | | | | | |
| GARCH-ARJI | 27.5230 | 39.5260 | 0.6975 | 11.0810 | 5.7579 | 8.0933 | 0.7129 | 2.3354 |
| GARCH | 45.0640 | 67.1770 | 0.6723 | −6.8525 | 9.6794 | 13.7500 | 0.7059 | −0.8905 |
| | (0.9997) | (1.0000) | (0.3565) | (0.0018) | (0.9999) | (1.0000) | (0.4607) | (0.0066) |
| GARCH-CJI | 47.5190 | 60.2900 | 0.7901 | 4.3284 | 10.0910 | 12.5460 | 0.8072 | 0.9229 |
| | (1.0000) | (1.0000) | (0.9130) | (0.1321) | (1.0000) | (1.0000) | (0.9122) | (0.1560) |
| 50/50 | 3.4900 | 8.7564 | 0.4000 | 2.7138 | 3.4900 | 8.7564 | 0.4000 | −0.6013 |
| | (0.0000) | (0.0000) | (0.0186) | (0.0532) | (0.0231) | (0.9937) | (0.0109) | (0.0080) |
| Panel C: $T = 20$ days | | | | | | | | |
| GARCH-ARJI | 26.2220 | 37.0910 | 0.7072 | 12.3230 | 5.7226 | 7.4222 | 0.7714 | 2.9047 |
| GARCH | 41.6780 | 64.4100 | 0.6473 | −3.2630 | 9.5623 | 12.7730 | 0.7491 | 0.6934 |
| | (1.0000) | (1.0000) | (0.0327) | (0.0000) | (1.0000) | (1.0000) | (0.2578) | (0.0002) |
| GARCH-CJI | 44.9270 | 56.6080 | 0.7942 | 7.3088 | 10.0490 | 11.5100 | 0.8740 | 2.2620 |
| | (1.0000) | (1.0000) | (0.9947) | (0.0460) | (1.0000) | (1.0000) | (0.9974) | (0.1775) |
| 50/50 | 3.4971 | 8.0036 | 0.4381 | 2.8364 | 3.4971 | 8.0036 | 0.4381 | −0.1937 |
| | (0.0000) | (0.0000) | (0.0004) | (0.0000) | (0.0003) | (0.9925) | (0.0001) | (0.0001) |

1 The table presents out-of-sample performances of different models net of transaction costs. The investment horizon can be 1 day, 5 days or 20 days, and the risk aversion can be 2 or 10. The annualized mean, standard deviation (SD), Sharpe ratio (SR), and certainty equivalent rate of return (CER) are reported.
2 In the parentheses we report the one-side p-values for the null hypothesis that the performance measure of the GARCH-ARJI model is lower than that of the alternative model, so a small $p$-value would indicate that we can reject the null.

## 4. Conclusion

This paper solves the dynamic portfolio allocation problem with account of time-varying jump risk. We capture the jump dynamics of the returns on the S&P 500 index future using a GARCH-ARJI model, and study a dynamic portfolio allocation problem with a CRRA preference. The three state variables of the problem, including the one-period ahead forecasts of return, conditional variance and jump intensity, are quite nonlinear and path-dependent, and so we solve the problem using the simulation-based method.

The empirical results show that the jump intensity as a state variable is an important factor to affect the investor's portfolio decision. The investor will decrease the weight in the risky asset when the initial jump intensity increases, since more frequent negative jumps make the risky asset less attractive to the risk-averse investor. The risk-averse investor can benefit from the optimal dynamic strategy instead of myopic strategy.

Under GARCH-ARJI model, the risky asset weight depends on the jump dynamics, which are described by three parameters: $\theta$, $\rho$ and $\phi$. The first parameter $\theta$ measures the average jump intensity, so a low $\theta$ leads to a high risky asset weight. The second parameter $\rho$ measures the jump persistence, and its impact on the risky asset weight depends on the initial jump intensity. On one hand, a low initial jump intensity and a high $\rho$ indicate a relatively low jump intensity in the future, which leads to a high risky asset weight. On the other hand, a high initial jump intensity and a high $\rho$ indicate a high jump intensity in the future. In this case the investor would invest less in the risky asset. Finally, we find that the jump innovation coefficient $\phi$ has no monotonic relationship with the risky asset weight.

In the out-of-sample analysis, we find that compared with the GARCH model which does not consider jump risk, the GARCH-CJI model which assumes the jump intensity is constant, and the equal weighted portfolio, the GARCH-ARJI model has better performance and can yield higher certainty equivalent rates of returns. This is also true if we consider the transaction costs. The results indicate that the dynamic strategy based on the GARCH-ARJI model is more preferred by the risk-averse investors than the alternative strategies.

## Appendix A. Estimation of ARJI model

Let $\Theta = \{\alpha, \beta, \kappa, \eta, \psi, \theta, \rho, \phi, \mu, \delta\}$ be the set of unknown parameters of the ARJI model. The parameters can be estimated using maximal likelihood estimation method. More specifically, the likelihood function can be written as

$$L = \sum_{t=1}^{T} \log\left[q\left(r_t | I_{t-1}\right)\right],$$

where $T$ is the number of observations, $q\left(r_t|I_{t-1}\right)$ is the density function and can be calculated as

$$q\left(r_t|I_{t-1}\right)=\sum_{i=0}^{+\infty}q\left(r_t|N_t=i,I_{t-1}\right)\Pr\left(N_t=i|I_{t-1}\right). \tag{11}$$

$\Pr\left(N_t=i|I_{t-1}\right)$ can be obtained from (5). Moreover, conditional on $N_t=i$ and information set $I_{t-1}$, from Eqs. (1) and (2), $r_t$ is given by

$$r_t=\alpha+\beta r_{t-1}+\varepsilon_{1,t}+\sum_{k=1}^{i}J_k-\lambda_t\mu.$$

Therefore $q\left(r_t|N_t=i,I_{t-1}\right)$ can be calculated as

$$q\left(r_t|N_t=i,I_{t-1}\right)=\frac{1}{\sqrt{2\pi\sigma_r^2}}\exp\left[-\frac{\left(r_t-\mu_r\right)^2}{2\sigma_r^2}\right], \tag{12}$$

where $\mu_r=\alpha+\beta r_{t-1}+\left(i-\lambda_t\right)\mu$ and $\sigma_r^2=h_t+i\delta^2$.

It remains to determine the ex post filter $E\left(N_t|I_t\right)$ in Eq. (7). By definition $E\left(N_t|I_t\right)$ is given by

$$E\left(N_t|I_t\right)=\sum_{i=0}^{+\infty}i\Pr(N_t=i|I_t).$$

where, based on Bayes' rule, $\Pr(N_t=i|I_t)$ can be calculated as

$$\Pr(N_t=i|I_t)=\frac{q(r_t|N_t=i,I_{t-1})\Pr(N_t=i|I_{t-1})}{q(r_t|I_{t-1})}.$$

## Appendix B. Procedure for solving the dynamic asset allocation problem

From Eq. (10), the first order condition can be written as

$$E_t\left[\left(\omega_t R_{t+1}^e+R_f\right)^{-\gamma}Q_{t+1}\left(Z_{t+1}\right)R_{t+1}^e\right]=0.$$

Using the fourth-order Taylor expansion the equation can be written as

$$E_t\left(A_{t+1}\right)+E_t\left(B_{t+1}\right)\omega_t+E_t\left(C_{t+1}\right)\omega_t^2+E_t\left(D_{t+1}\right)\omega_t^3=0, \tag{13}$$

where

$$\begin{aligned}
A_{t+1}&=R_f^{-\gamma}Q_{t+1}\left(Z_{t+1}\right)R_{t+1}^e,\\
B_{t+1}&=-\gamma R_f^{-\gamma-1}Q_{t+1}\left(Z_{t+1}\right)\left(R_{t+1}^e\right)^2,\\
C_{t+1}&=\frac{1}{2}\gamma(\gamma+1)R_f^{-\gamma-2}Q_{t+1}\left(Z_{t+1}\right)\left(R_{t+1}^e\right)^3,\\
D_{t+1}&=-\frac{1}{6}\gamma(\gamma+1)(\gamma+2)R_f^{-\gamma-3}Q_{t+1}\left(Z_{t+1}\right)\left(R_{t+1}^e\right)^4.
\end{aligned}$$

Therefore the optimal risky asset weight $\omega_t^*$ can be obtained if we can calculate the conditional expectation of $A_{t+1},B_{t+1},C_{t+1},D_{t+1}$. We take the following steps to calculate them.

Step 1: We run Monte Carlo simulations to generate $M$ sample paths, and each path has $T$ days of observations. For each sample path $m$, and given the state variables including the next-period return forecast $\bar{r}_{t+1|t}^{(m)}$, the conditional variance forecast $\bar{h}_{t+1|t}^{(m)}$, and the jump intensity forecast $\bar{\lambda}_{t+1|t}^{(m)}$, we can calculate the return in the next period as

$$r_{t+1}^{(m)}=\bar{r}_{t+1|t}^{(m)}+\varepsilon_{1,t+1}^{(m)}+\varepsilon_{2,t+1}^{(m)},$$

where $\varepsilon_{1,t+1}^{(m)}$ and $\varepsilon_{2,t+1}^{(m)}$ are two random variables to be simulated. We sample $\varepsilon_{1,t+1}^{(m)}$ from the normal distribution $N\left(0,\bar{h}_{t+1|t}^{(m)}\right)$. As $\varepsilon_{2,t+1}^{(m)}=\sum_{k=1}^{N_{t+1}^{(m)}}J_k^{(m)}-\bar{\lambda}_{t+1|t}^{(m)}\mu$, we sample $N_{t+1}^{(m)}$ from the Poisson distribution with arrival rate $\bar{\lambda}_{t+1|t}^{(m)}$ and sample $J_k^{(m)}$ from the normal distribution $N\left(\mu,\delta^2\right)$.

Given the return sample $r_{t+1}^{(m)}$, we can update the state variables as

$$\begin{aligned}
\bar{r}_{t+2|t+1}^{(m)}&=\alpha+\beta r_{t+1}^{(m)},\\
\bar{h}_{t+2|t+1}^{(m)}&=\kappa+\eta\bar{h}_{t+1|t}^{(m)}+\psi\left(\epsilon_{t+1}^{(m)}\right)^2,\\
\bar{\lambda}_{t+2|t+1}^{(m)}&=\theta+\rho\bar{\lambda}_{t+1|t}^{(m)}+\phi\xi_{t+1}^{(m)},
\end{aligned}$$

where $\epsilon_{t+1}^{(m)}=r_{t+1}^{(m)}-\bar{r}_{t+1|t}^{(m)}$ is the return innovation, and $\xi_{t+1}^{(m)}=E(N_{t+1}|r_{t+1}^{(m)})-\bar{\lambda}_{t+1|t}^{(m)}$ is the jump intensity innovation.

Step 2: We calculate recursively the optimal risky asset weight at each date $t$ and for each sample path $m$, where $t = 0, \ldots, T-1$, and $m = 1, \ldots, M$. Suppose that we are at time $t$, for each sample path $m$ the optimal weights from $t+1$ to $T-1$ have been calculated and denoted as $\omega_s^{(m)*}$, where $s = t+1, \ldots, T-1$. Using these optimal weights, we can calculate $A_{t+1}^{(m)}, B_{t+1}^{(m)}, C_{t+1}^{(m)}, D_{t+1}^{(m)}$ for each $m$.

Let $y_{t+1}$ be a generic element of $A_{t+1}, B_{t+1}, C_{t+1}$ or $D_{t+1}$. We follow Brandt et al. (2005) and parameterize its conditional expectation $E_t\left(y_{t+1}\right)$ as a linear function of the state variables:

$$E_t\left(y_{t+1}\right) = a + b' Z_t.$$

Therefore $a$ and $b$ can be estimated using the following across-path regression:

$$y_{t+1}^{(m)} = a + b' Z_t^{(m)} + u_{t+1}^{(m)},$$

where $u_{t+1}^{(m)}$ is the error term. We can substitute the fitted value of the regressions $\hat{A}_{t+1}^{(m)}, \hat{B}_{t+1}^{(m)}, \hat{C}_{t+1}^{(m)}$ or $\hat{D}_{t+1}^{(m)}$ into Eq. (13) and get the optimal weight $\omega_t^{(m)*}$.

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