Final Project: Electric Function

Author: Aishazhan Abuova

PHYS 1211

Introduction

Flux is a concept that describes the interaction between a vector field and objects that are immersed in that field. More specifically, it identifies the quantity of field lines that are flowing through a given area. Maximum flux is achieved when an area plane is perpendiculat to the field lines. On the other hand, when the plane is parallel to the field, no flux is produced because the vectors pass over the area rather than through it. In Equation 1, a general formula for electric flux is defined as the scalar product of electric field and the area vector. Note that n is a unit vector normal to the surface area plane. Therefore, θ is referring to the angle between the electric field and the normal

$$\Phi = \vec{E} \cdot \hat{\mathrm{n}} A = \mathrm{EA}\cos(\theta) \text{ (1)}$$

Electric Flux Through 2-Dimensional Circular and Rectangular Planes

The Electric function calculates the flux through 2-dimensional circles and rectangles. When the 'circle' type is selected, Electric calculates the flux through the surface based on the input electric field, angle, and radius. When the 'rectangle' type is selected, Electric calculates flux base on the input electric field, width, length, and angle. The formulas for electric flux through a circle and a rectangle are given by Equations 2 and 3, respectively, and examples of the visualizations are shown below.

$$\Phi_{\mathrm{circle}} = \mathrm{EA}\cos(\theta) = E(\pi r^2)\cos(\theta) \text{ (2)}$$

$$\Phi_{\mathrm{rectangle}} = \mathrm{EA}\cos(\theta) = E(\mathrm{wl})\cos(\theta) \text{ (3)}$$

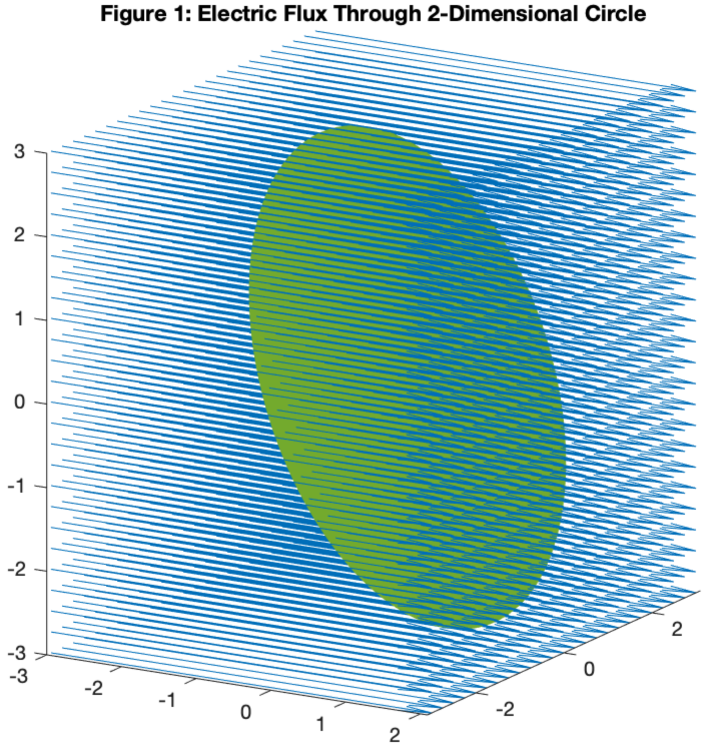
Example 1—Circle

For an input circle radius of 3 m, an electric field of 10 N/C, and an angle of 23 degrees, Electric will produce the followng outputs and visualization:

```
[flux] = Electric('circle', 10, 3, 23)
```

```
flux = 260.2666
```

```
hold off
```



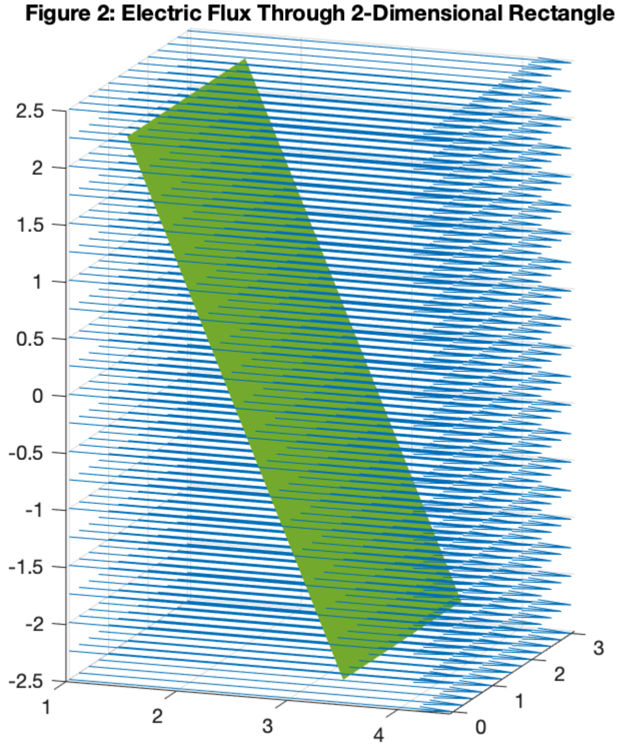
Example 2—Rectangle

For an input electric field of 10 N/C, width of 3 m, length of 5 m, and angle fo 23 degrees, Electric wil produce the following outputs and visualization:

```
[flux] = Electric('rectangle',10,3,5,23)
```

```
flux = 138.0757
```

```
hold off
```



Electric Flux and Field at Surface of Conducting Spheres

When charge is evenly distributed on the surface of a conducting sphere, the electric flux can be calculated using Gauss's Law, in which the enclosed charge is divided by the permitivity of free space as shown in Equation 4. The permitivity of free space (ε_0) is defined as 8.85×10^{-12}Farads/m in all calculations. The electric field at the surface can then be determined by dividing the electric flux by the surface area as shown in Equation 5. Note that the area vector is still normal to the plane at every point along the surface area and the electric field vectors point either radially outward or toward the surface depending on the sign of the charge. When moving far away from the conducting sphere, it essentially acts like a point charge. An example of a sphere visualization is given below.

$$\Phi_{\text{sphere}} = \text{EA} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \quad (4)$$

$$E = \frac{q_{\text{enclosed}}}{A\varepsilon_0} = \frac{q_{\text{enclosed}}}{(4\pi r^2)\varepsilon_0} \quad (5)$$

Example 3—Sphere

For an input of net charge 3 C and a sphere radius of 2 m, Electric produces the following outputs and visualization:

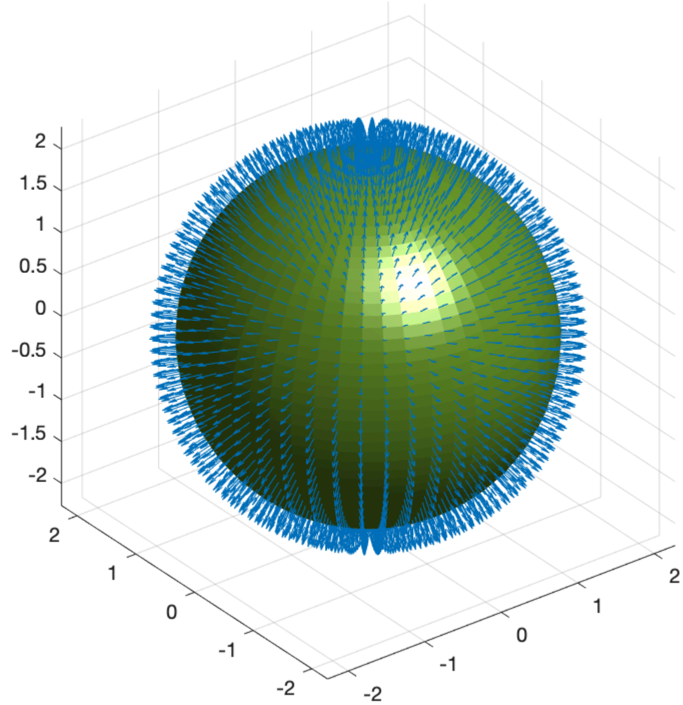
```
[flux,E] = Electric('sphere',3,2)
```

```
flux = 3.3898e+11
```

```
E = 6.7439e+09
```

```
hold off
```

Figure 3: Electric Flux and Field of Conducting Sphere



In the case of two concentric conducting spheres, the procedure is practically the same but with a few adjustments. In conducting materials, charge is entirely distributed on the surface. Electric requires that the net charge on each sphere is initially specified. All of the net charge on the inner sphere will rest on the surface, while the outer sphere net charge is equal to the sum of the charges on each side of outer the shell. Since only the surface charge is necessary for the calculation of electric flux and field, the outer sphere surface charge can be obtained by simply adding the two input net charges. This is valid because the inner surface charge of the outer shell has the same value as the inner sphere net charge but with the opposite sign. Equation 6 clarifies how the outer sphere surface charge was found.

$$q_{2\text{net}} = q_{2\text{inner}} + q_{2\text{outer}}$$

$$q_{2\text{inner}} = -q_{1\text{net}}$$

$$q_{2\text{outer}} = q_{2\text{net}} + q_{1\text{net}} \quad (6)$$

Gauss's Law was used to calculate the electric flux and field at each sphere surface as described for the single conducting sphere. The specific equations used in the Electric function are shown below. Positive surface charges should produce electric field lines that point radially outward while electric field lines due to negative charges should point radially inward.

inner sphere: $\Phi_{s1} = \frac{q_1}{\varepsilon_0}$ and $E = \frac{q_1}{\left(4\pi r_1^2\right)\varepsilon_0}$

outer sphere: $\Phi_{s1} = \frac{q_{2\text{outer}}}{\varepsilon_0}$ and $E = \frac{q_{2\text{outer}}}{\left(4\pi r_2^2\right)\varepsilon_0}$

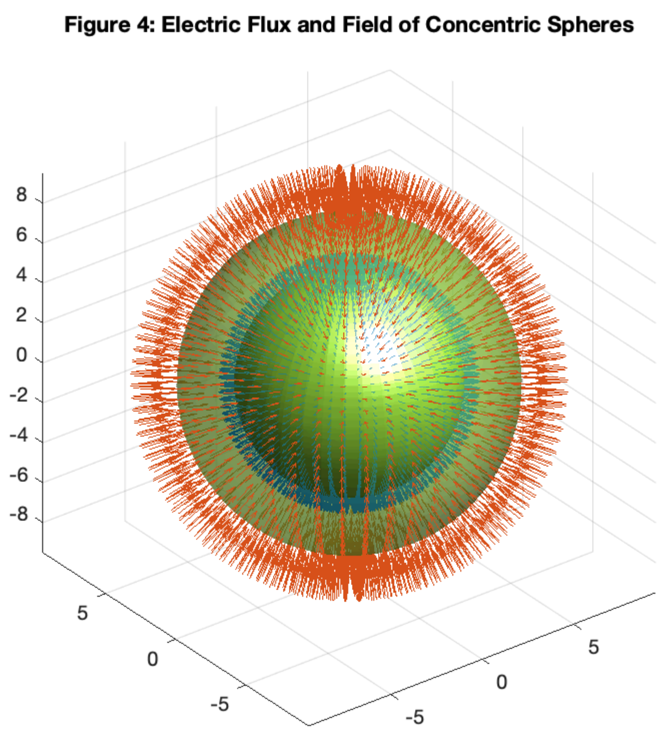
Example 4—Concentric Spheres

Electric outputs the following outputs and visualizations for inputs of q1 = 2 C, r1 = 5 m, q2 = -3 C, and r2 = 7.5 m:

```
[flux,E] = Electric('concentric spheres',2,5,-3,7.5)
```

```
flux = 2.2599e+11
E = 7.1934e+08
```

```
hold off
```



*Notice how in Figure 4 the electric field vectors on the outer surface are pointing inward due to the negative surface charge. The inner sphere electric field vectors are point radially outward due to the positive surface charge.

Function Summary and Parameters

Electric is a function that outputs various electric quantities based on the input object type. The four possible object types are circle, rectangle, sphere, and concentric spheres. When the 'circle' type is selected, three additional arguments—electric field, circle radius, and angle—are required in order to run the function. When the 'rectangle' type is selected, four additional arguments—electric field, width, length, and angle—are necessary. Once values have been input correctly, the function will output the electric flux through the selected surface and a visual of the object in a constant electric field. When the 'sphere' option is chosen, two addtional aruments, net charge and sphere radius, must be input. If the object type is 'concentric spheres,' four additional arguments must be input—inner net charge, inner radius, outer net charge, and outer radius. The 'sphere' case will yield outputs of surface electric flux and field as well as a visualization of the object with field vectors. In the 'concentric spheres' case, the outputs will be inner surface flux, inner surface field, outer surface flux, and outer surface field. All inputs are required for each case and must be input in the specified order.

<u>The input and output orders are clarified below:</u>

circle → [flux] = Electric('circle', electric field, radius, angle)

rectangle → [flux] = Electric('rectangle', electric field, width, length, angle)

sphere → [flux, E] = Electric('sphere', net charge, radius)

concentric spheres → [flux, E] = Electric('concentric spheres', inner net charge, inner radius, outer net charge, outer radius)

- Note that in this case the outer radius must be larger than the inner radius in order for the function to run.

<u>Electric inputs must be in the following units</u>:

1. electric field → Newtons/Coulomb (N/C)
2. radii, width, length → meters (m)
3. angle → degrees
4. charge → Coulombs (C)

<u>Electric outputs have the following units</u>:

1. electric flux → volt meters (Vm)
2. electric field → Newtons/Coulomb (N/C)

If the inputs are not in the required units, Electric will produce incorrect results.

Conclusion and Areas for Improvement

Electric functions smoothly for calculating electric flux and field. However, the visualizations, especially the electric field vectors, could be further fine tuned in order to achieve more accurate results. The vector fields generated above represent the direction of the respective electric fields rather than both direction and field intensity. The built-in function used to create the vector fields is called quiver3 and outputs an evenly spaced field based on the parameters of a given surface. For example, in the sphere visualizations, a field of vectors normal to the surface at each face of the sphere is produced. Therefore, the density of the field depends on the number of specified surface faces rather than the actual electric field. An attempt was made to solve this issue by setting the sphere face number to a factor of the calculated electric field. However, this solution was not effective since the calculated electric field is often very large and causes an interminable run cycle. Additionally, in order to produce electric field vectors that point toward the surface—as due to a negative surface charge—a second larger and invisible (no face color) sphere was created. This technique allowed for the vectors to still be seen even when inverted. A similar approach was taken in creating constant vector fields for the 2-dimensional visualizations. For both the circle and the rectangle, an invisible vertical meshgrid was created adjacent to each shape and was used to produce a constant horizontal electric field using surfnorm and quiver3. Although this solution is functional, there may be a more concise method in which to achieve the same result. Overall, the visualizations of Electric were successful in that they provide a clear image of each specific case. Of course, they could still be further refined in order to show greater complexity and vector field strength. Other aspects of the function that oculd be improved are error handling and unit conversion. Electric contains very minimal error handling and therefore will produce incorrect results if the inputs are improperly selected. A useful addtion to the sphere type could be checking which input radius is larger and using that value as the outer radius. As for unit conversion, the function could accept additional unit inputs for each quantity. However, this aspect was not included in Electric due to the predicted unnecessary complexity.

Sources

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