Bisection :

Q.1 Find the root of $e^{-x} - x$. Correct upto 3 decimal place.

Soln : Here,

$f(x) = e^{-x} - x$

$f(0) = 1$ (+ve)

$f(1) = -0.63$ (−ve)

∴ root lies betn 0 and 1

$C = \frac{A+B}{2}$: $e^{-c} - c$

a +ve	b −ve	c = a+b/2	f(c)
0	1	0.5	+ve
0.5	1	0.75	−ve
0.5	0.75	0.625	−ve
0.5	0.625	0.56250	+ve
0.56250	0.625	0.59375	−ve
0.56250	0.59375	0.57813	−ve
0.56250	0.57813	0.57032	∓ve
0.56250	0.57032	0.56641	+ve
0.56641	0.57032	0.56837	−ve
0.56641	0.56837	0.56739	−ve
0.56641	0.56739	0.56690	+ve
0.56690	0.56739	0.56715	+ve
0.56715	0.56739	0.56727	−ve

∴ The root is $\boxed{0.56727}$

False Position

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Algorithm

1. Define function $f(x)$ and error (e)
2. Take two initial guesses (a, b), such that $f(a)$ & $f(b) < 0$
3. Find next point by $c = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$
4. If $f(a)$ is negative.
   $f(a) * f(c) > 0$, $a = c$, else, $b = c$;
5. Repeat 3 & 4 until $f(c)$ < error.
6. Stop & print 'c' as root.

Q(1) $f(x) = e^{x} - x$ upto 3 d.p

Soln: Here,

$$f(x) = e^{x} - x$$
$$f(0) = +ve\ (1)$$
$$f(1) = -ve\ (-0.63212)$$

The root lies in 0 to 1.

$$C = \frac{a * (e^{-b} - b) - b(e^{-a} - a)}{(e^{-b} - b) - (e^{-a} - a)} \quad : e^{-c} - c$$

Interation	a	b	c	f(c)
1	0	1	0.61270	-ve
2	0	0.61270	0.57218	-ve
3	0	0.57218	0.56770	-ve
4	0	0.56770	0.56721	-ve

Q.2. $x \log_{10} x - 1.2$

$$\frac{a(b\log_{10} b - 1.2) - b(a\log_{10} a - 1.2)}{(b\log_{10} b - 1.2) - (a\log_{10} a - 1.2)}$$

$f(x) = x\log_{10} x - 1.2$

$f(1) = -ve$

$f(2) = -ve$

$f(3) = +ve$

Root lies in Beth 2 and 3

Using,

$$C = \frac{a\,[b\log_{10} b - 1.2] - b\,[a\log_{10} a - 1.2]}{[b\log_{10} b - 1.2] - [a\log_{10} a - 1.2]} \;:\; C\log_{10} c - 1.2$$

~~Intera~~ Iteration	b	a	c	f(c)
3	2	3	2.72101	−ve
3	2.72101	3	2.74021	−ve
3	2.74021	3	2.74064	−ve

Root = 2.74064

Secant

$\cos x = xe^{x}$

$f(x) = \cos x - xe^{x}$

let $x_1 = 5$ and $x_2 = 6$

Formula for second method is

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$= b - \frac{(\cos b - be^{b})(b-a)}{(\cos b - be^{b}) - (\cos a - ae^{a})}$$

Iteration	x_1 (a)	x_2 (b)	x_3
1	5	6	4.55789
2	6	4.55789	4.24189
3	4.55789	4.24189	3.57250
4	3.57250	3.57250	3.05998
5	3.57250	3.05998	2.51096
6	3.05998	2.51096	2.00628
7	2.51096	2.00628	1.55968
8	2.00628	1.55968	1.192449
9	1.55968	1.192449	0.83120
10	1.192449	0.83120	0.683223
11	0.83120	0.63223	0.54273
12	0.63223	0.54273	0.52000
13	0.54273	0.52000	0.51780
14	0.52000	0.51780	0.51776

Newton Raphson

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Q2. $x^3 - 4x - 9$

$f(x) = x^3 - 4x - 9$
$f'(x) = 3x^2 - 4$
let $x_0 = 5$

From Formula,

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

$$= x - \frac{x^3 - 4x - 9}{3x^2 - 4}$$

Iteration	x_{n-1}	x_n
1	5	3.64789
2	3.64789	2.95328
3	2.95328	2.73019
4	2.73019	2.70678
5	2.70678	$\boxed{2.70653}$

Root is 2.70653

Fixed Point Iteration

$\sin x + 3x - 2$

$f(x) = \sin x + 3x - 2 = 0$

$3x = 2 - \sin x$

$x = \left(\frac{2 - \sin x}{3}\right) \rightarrow \phi(x)$

$\phi(x) = \frac{2 - \sin x}{3} = \frac{2}{3} - \frac{\sin x}{3}$

Working Formula is

$x_n = \frac{2 - \sin x_{n-1}}{3}$

let

$x_{n-1} = 2$

Iteration	x_{n-1}	x_n
1	2	0.36357
2	0.36357	0.54813
3	0.54813	0.49297
4	0.49297	0.50892
5	0.50892	0.50426
6	0.50426	0.50561
7	0.50561	0.50522

II

Q. 2 y at $x = 2.5$

x	1	2	4	5	7
y	1	1.414	1.732	2	2.6
	y_0	y_1	y_2	y_3	y_4

$x = 2.5$ & $y = ?$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	1				
		$\frac{1.414 - 1}{2 - 1} = 0.41400$			
2	1.414		# $\frac{0.15900 - \cancel{0.41400} - 0.41400}{4-1}$ $\cancel{0.26800 - 0.41400}$ $= -0.08500$ $\cancel{-0.71582}$		
		$= 0.15900$		# $\frac{0.03633 - (-0.08500)}{5-1} = 0.03033$	
4	1.732		# ~~2~~ # $\frac{0.26800 - 0.15900}{5-2} = 0.03633$		# $\frac{-0.00513 - 0.03033}{7-1} = -0.00591$
		$= 0.26800$		# $\frac{0.01067 - 0.03633}{7-2} = -0.00513$	
5	2		# $\frac{0.3000 - 0.26800}{7-4} = 0.01067$		
		$= 0.3000$			
7	2.6				

Note: The value of independent x must be ascending.
~~o~~ Chaina vane banaune.

Now,

$$y(2.5) = y_0 + \left[(x - x_0)\Delta y_0\right] + \left[(x - x_0)(x - x_1)\Delta^2 y_0\right]$$
$$+ \left[(x - x_0)(x - x_1)(x - x_2)\Delta^3 y_0\right] +$$
$$\left[(x - x_0)(x - x_1)(x - x_2)(x - x_3)\Delta^4 y_0\right]$$

$$= \overset{1}{\cancel{0.41406}} + \left[(2.5 - 1)\,0.41400\right] + \left[(2.5 - 1)(2.5 - 2)\,0.08\ldots\right]$$
$$+ \left[(2.5 - 1)(2.5 - 2)(2.5 - 4)\,0.03033\right] + \left[(2.5 - 1)\right.$$
$$\left.(2.5 - 2)(2.5 - 4)(2.5 - 5)\,0.00591\right]$$

$$= 1 + 0.62100 + 0.06375 + (-0.03412) + 0.016\ldots$$

$$= \cancel{4.66725}\ \boxed{1.5065}$$ //Ans.

$$y(2.5) = y_0 + \left[(x - x_0)\Delta y_0\right] + \left((x - x_0)(x - x_1)\Delta^2 y_0\right] +$$
$$\left[(x - x_0)(x - x_1)(x - x_2)\Delta^3 y_0\right] +$$
$$\left[(x - x_0)(x - x_1)(x - x_2)\Delta^4 y_0\right)$$

Lag Range Interpolation

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It is also used for unequal interval in independent variable. It is normally used for higher order ~~polin~~ polynomial

Formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} \times y_0 +$$

$$\frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} \times y_1 + \ldots$$

Q.1. Find y at $x = 2.5$

	0	1	2	3	4
x	1	2	4	5	7
y	1	1.414	1.732	2	2.6

$$y = f(x) = \frac{(\overset{x}{\cancel{2.5}}-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \times y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \times y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \times y_2 +$$

$$= \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \times y_3 +$$

$$= \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} \times y_4$$

$$\frac{(2.5-2)(2.5-4)(2.5-5)(2.5-7)}{(1-2)(1-4)(1-5)(1-7)} \times 1 = -0.11719$$

$$\frac{(2.5-1)(2.5-4)(2.5-5)(2.5-7)}{(2-1)(2-4)(2-5)(2-7)} \times 1.414 = 1.19306$$

$$\frac{(2.5-1)(2.5-2)(2.5-5)(2.5-7)}{(4-1)(4-2)(4-5)(4-7)} \times 1.732 = \cancel{0.8188}\ 0.8118$$

$$\frac{(2.5-1)(2.5-2)(2.5-4)(2.5-7)}{(5-1)(5-2)(5-4)(5-7)} \times 2 = \cancel{1.6406}\ -0.4219$$

$$\frac{(2.5-1)(2.5-2)(2.5-4)(2.5-5)}{(7-1)(7-2)(7-4)(7-5)} \times 2.6 = 0.0406$$

NOW,

$$-0.11719 + 1.19306 + 0.8188 + \cancel{1.6406}\ {-0.4219} + 0.04063$$

$\cancel{3.568}$ 1.5065

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$x = 2.7$

$y(x)$

$RE = \frac{X_E - X_C}{X_E}$

Q.2 Find $y(2.7)$ & also calculate Relative error.

	0		1	2
x	3.2	2.7	1	4.8
y	22	17.8	14.2	38.3

$x = 2.7$; $y = ?$

As $x=2.7$ is x_1 cannot be 2.7 so,

	0	1	2
x	3.2	1	4.8
y	22	14.2	38.3

Formula Use :

$$y = f(x) = \frac{(2.7 - 1)(2.7 - 4.8)}{(3.2 - 1)(3.2 - 4.8)} \times 22 = 22.312$$

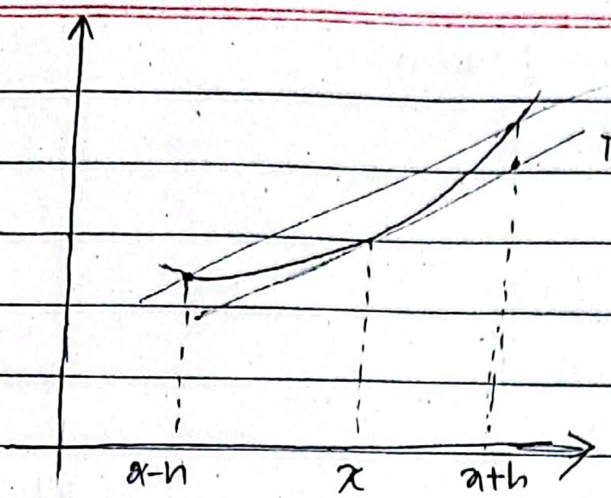
$$+$$

$$= \frac{(2.7 - 3.2)(2.7 - 4.8)}{(1 - 3.2)(1 - 4.8)} \times 14.2 = 1.783$$

$$+$$

$$= \frac{(2.7 - \overset{3.2}{\cancel{4.8}})(2.7 - \cancel{3}1)}{(4.8 - 3.2)(4.8 - 1)} \times 38.3 = -5.3544$$

$= 22.312 + 1.783 - 5.3544 = 18.74$ //



TS → True slope

ie. 3-point approximation.

∴ 3-point ~~gives~~ has better approximation than 2-point as 3-point gives parallel and slope is equal.

Numerical Differentiation & Integration

Q.1.) Find 1^{st} derivative of $f(x) = x^2$ at $x = 1$ for $h = 0.2$, 0.1, 0.05, 0.01.

Soln: Here,

$f(x) = x^2$

Direct method.

$f'(x) = 2x$

$f'(1) = 2 \times 1 = 2$

$\left\{ f'(1) = \frac{f(x+h) - f(x)}{h} \right\}$

h	f'(1)	
0.2	2.2	
0.1	2.1	
0.05	2.05	$\rightarrow \frac{1.1025 - 1}{0.05}$
0.01	2.01	

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$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

h	$f'(1)$
0.2	2
0.1	2
0.05	2
0.01	2

$\frac{f(1+0.2) - f(1-0.2)}{2 \times 0.2}$

$\frac{f(1.2) - f(0.8)}{0.4}$

$\frac{1.44 - 0.64}{0.4}$

we need to take value of h as small as possible.
usually we take $\boxed{h = 0.0001}$ //

2nd order derivative

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

find 2nd derivative of $\cos x$ at $x = 0.75$

$f(x) = \cos x$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$= \cancel{-0.999}\ -0.7316$

Now,

$$f''(x) = f(x+h) - 2f(x) + f(x-h) / h^2$$

h	$f''(x)$
0.0001	$= 0.7317$

$$= \frac{f(0.7501) - 2 \times f(0.75) f(0.7499}{\cancel{2\times 0.5625} \; 1\times 10^{-8}}$$

$$= \frac{0.7316 - 1.4633 + 0.7317}{1 \times 10^{-8}}$$

Differentiating Tabulated Function.

Values maybe given at the ~~starting~~ edge, right or ce of a table.

Forward Formula.

$$f'(x) = \frac{-3f(x) + 4(x+h) - f(x+2h)}{2h}$$

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Q.4. Evaluate $\int_0^6 \frac{1 \cdot dx}{1+x^2}$ by Trapezoidal and 1/3 Rule. Also find percentage error

→ $a = 0 \quad b = 6 \quad f(x) = \frac{1}{1+x^2} dx$

$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6
y	1	0.5	0.2	0.1	0.0588	0.0385	0.0270

By Trapezoidal rule,

$$\int_0^6 \frac{dx}{1+x^2} f(x) = \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \dots + y_{n-1})\right]$$

$$= \frac{1}{2}\left[1 + 0.0270 + 2(0.5 + 0.2 + 0.1 + 0.0588 + 0.0385)\right]$$

$$= 1.4108$$

By simpson's 1/3 Rule

$$\int_0^6 \frac{dx}{1+x^2} f(x) = \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3 + \dots) + 2(y_2 + y_4 + \dots)\right]$$

$$= \frac{1}{3}\left[1 + 0.0270 + 4(0.5 + 0.1 + 0.0385) + 2(0.2 + 0.0588)\right]$$

$$= 1.3662$$

from trapezoidal

Percentage Error = $\left| \frac{\text{Exact value} - \text{Approximate Value}}{\text{Exact value}} \right| \times 100$

$= \frac{1.4056 - 1.4108}{1.4056} \times 100$

$= |0.0037 \times 100|$

$= 0.37\%$

from Simpson's 1/3

Percentage Error = $\frac{1.4056 - 1.3662}{1.4056} \times 100$

$= 2.8031$

5) Evaluate $\int_0^2 ex^2 \, dx$ by suitable method.

→ $a=0$ ~~b=0~~ $b=2$ $f(x) = ex^2 \, dx$

$h = \frac{b-a}{n} = \frac{2-0}{6} = \frac{2}{6} = \frac{1}{3}$

x	0	1/3	2/3	1	4/3	5/3
y	0	0.3020	1.2081	2.7183	4.8325	7.550