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Higher Twist contributions to the Structure Functions $F_2(x, Q^2)$ and $g_2(x, Q^2)$

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We report on recent results on higher twist contributions to the unpolarized structure functions $F_2^{p,d}(x, Q^2)$ at N^{3}LO in the large x region and constraints on the twist–3 contribution to polarized structure function $g_2(x, Q^2)$.

1 Introduction

Higher twist terms contribute to the nucleon structure functions at lower scales Q^2. The range in which these terms may be safely neglected against the leading twist contributions, partly depends on the size of the experimental errors in the respective measurement. Highly precise data at low values of Q^2 allow to access these contributions, the detailed physical understanding of which is presently still in an early stage. It has been outlined in Refs. [1, 2] how the higher twist contributions can be extracted in a phenomenological way in case of the structure functions $F_2(x, Q^2)$ and $F_L(x, Q^2)$ in the valence quark region. In this note we report on recent results of an improved analysis. Another interesting question concerns the structure function $g_2(x, Q^2)$ in the polarized case, which has been measured to a higher precision during the last years [3]. Here we try to extract first information on the twist-3 contributions to $g_2(x, Q^2)$.

2 Higher Twist Contributions to $F_2^{p,d}(x, Q^2)$

We have carried out a QCD analysis in the valence region including more recent data from JLAB following earlier work [1]. In the present analysis tails from sea-quarks and the gluon in the valence region were dealt with based on the ABKM distributions [4]. Both the valence quark distributions $xu_v(x, Q_0^2)$ and $xd_v(x, Q_0^2)$ at $Q_0^2 = 4$ GeV^2 are effected only very little. The values of $\alpha_s(M_Z^2)$ change marginally w.r.t. the earlier analysis [1]. We obtain : $\alpha_s(M_Z^2) = 0.1148 \pm 0.0019$ NLO, $= 0.1134 \pm 0.0020$ NNLO; 0.1141 ± 0.0021 N^{3}LO*. Here, the N^{3}LO*-analysis accounts for the three-loop Wilson coefficients and a Padé-model for the non-singlet four-loop anomalous dimension, to which we attached a $\pm 100\%$ uncertainty, cf. [1] for details. Furthermore, we found that the response of the individual deep-inelastic data sets in the valence region respond stable values, which are in accordance with the central value obtained moving from NLO to N^{3}LO*. The present result agrees very well with determinations of $\alpha_s(M_Z^2)$

in Refs. [4–6], see also [7]. A survey on the current status of $\alpha_s(M_Z^2)$ based on precision measurements in different reactions has been given in [8]. In the present analysis we obtain a lower value of α_s than the world average, cf. [8], and values being obtained in [9, 10] at NNLO. Reasons for the difference to the values given in [9, 10] have been discussed in Refs. [6, 7] in detail. In particular, the partial response of α_s in case of the BCDMS and SLAC data in [9, 10] turns out to be partly different comparing to the results in [4–6]. There are also differences between the analyses [9] and [10] w.r.t. several data sets contributing.

The higher twist contributions can be determined by extrapolating the fit-results at leading twist for $W^2 > 12.5$ GeV^2 to the region $4 < W^2 < 12.5$ GeV^2, $Q^2 \geq 4$ GeV^2, cf. [2, 11].[1] The results for the coefficients $C_{\rm HT}^{p,d}(x)$

$$F_2(x,Q^2) = F_2(x,Q^2)\left[\frac{O^{\rm TM}[F_2(x,Q^2)]}{F_2(x,Q^2)} + \frac{C_{\rm HT}(x)}{Q^2[{\rm GeV}^2]}\right] \qquad (1)$$

are shown in Figure 1, where we averaged over the respective range in Q^2. We applied the target mass corrections [12] to the leading twist contributions.[2] The result for the higher twist coefficients for proton and deuteron targets depends on the order to which the leading twist distribution is described. The higher twist terms become smaller moving from NLO to N^{3}LO*. Within the present theoretical and experimental accuracy the curves stabilize for $x < 0.65$, while at larger values there are still differences.

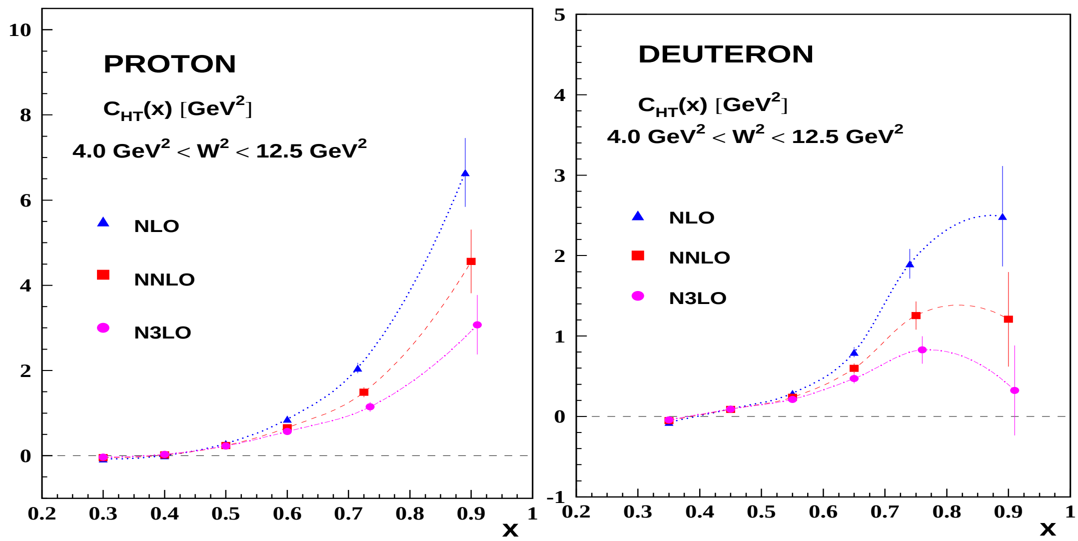

Figure 1: The empiric higher twist contributions to $F_2^{p,d}(x,Q^2)$ in the valence region, Eq. (1), extracted by calculating the leading twist part at NLO, NNLO, and N^{3}LO*, [11].

[1] In Ref. [6] also higher twist contributions for x below the valence region have been determined.
[2] An unfolding of the target mass corrections of the DIS world data for F_2 and F_L including the JLAB data, has been performed in [13] recently.

3 $g_2^{\rm tw3}(x, Q^2)$

Higher twist contributions to the polarized structure function $g_1(x,Q^2)$ have been studied in Refs. [14, 15] in phenomenological approaches aiming on the twist-4 contributions. However, the structure function $g_2(x,Q^2)$, together with other polarized electro-weak structure functions [16–18], receives also twist-3 contributions. $g_2(x,Q^2)$ obeys the Burkhardt-Cottingham relation [19]

$$\int_0^1 dx g_2(x,Q^2) = 0 \; . \tag{2}$$

Since the Wandzura-Wilczek relation [20] implies, that the first moment of the twist-2 part vanishes separately also

$$\int_0^1 dx g_2^{\rm tw3}(x,Q^2) = 0 \tag{3}$$

holds. The errors on the present world data from E143, E155, HERMES and NMC [3] on $g_2(x,Q^2)$ are still large but yet one may try the fit of a profile in x.

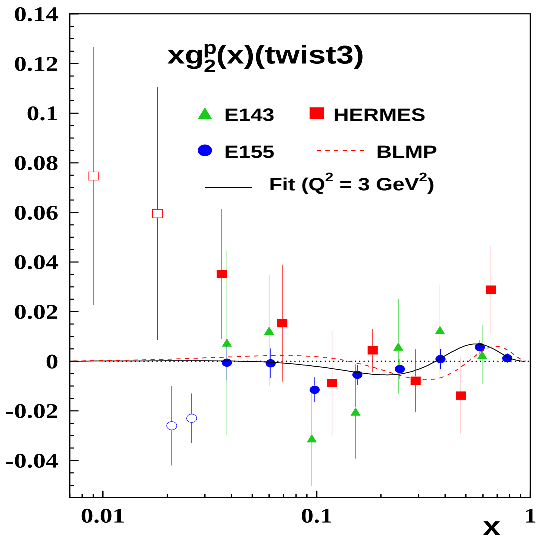

Figure 2: The twist-3 contributions to $g_2(x,Q^2)$ subtracting the twist-2 part according to the Wandzura-Wilczek relation [20] using the result of [15] for the twist-2 contribution to $g_1(x,Q^2)$ on experimental data from E143, E155, and HERMES [3] fitting the shape (4) (full line). Open symbols refer to data in the region $Q^2 < 1$ GeV^2. The dashed line shows the result of a calculation at $Q^2 = 1$ GeV^2 given in [21].

In Ref. [21] the parameterization

$$g_2^{\rm tw3}(x) = A\left[\ln(x) + (1-x) + \frac{1}{2}(1-x)^2\right] + (1-x)^3\left[B - C(1-x) + D(1-x)^2\right] \tag{4}$$

has been proposed. Since the data points are measured at different values of Q^2 an evolution has to be performed to a common scale. Furthermore, the target mass corrections [18] have to be taken into account. In Figure 2 the results of the fit to $g_2^{\rm tw3}(x,Q^2)$ are presented for $Q^2 = 3$ GeV^2. We limited the analysis to the region $Q^2 > 1$ GeV^2. The present errors are still large and the data of E155 dominate in the fit. We may compare with a theoretical prediction given in [21]. Indeed both results are quite similar. The twist-3 contribution to the structure function $g_1(x,Q^2)$ can be obtained from that to $g_2(x,Q^2)$ by the integral-relation [18]

$$g_1^{\rm tw3}(x,Q^2) = \frac{4x^2M^2}{Q^2}\left[g_2^{\rm tw3}(x,Q^2) - 2\int_x^1 \frac{dy}{y} g_2^{\rm tw3}(y,Q^2)\right] , \quad (5)$$

cf. [22]. Due to the large errors of the data the present results are of more qualitative character. To study the twist-3 contributions both to the structure functions $g_2(x,Q^2)$ and $g_1(x,Q^2)$ in detail, a high luminosity machine, like the planned EIC [23], is needed.

4 Conclusions

We performed a re-analysis of the present deep-inelastic world data on proton and deuteron targets for the structure function $F_2(x,Q^2)$ in the valence region $x > 0.3$ accounting for remaining non-valence tails, which were calculated using the ABKM09 distributions [4]. We obtain a slightly lower value of $\alpha_s(M_Z^2)$ than in our previous analysis [1] at N^{3}LO*, however, far within the 1σ error range. Very stable predictions are obtained going from NLO to N^{3}LO*, both for the valence distribution functions and $\alpha_s(M_Z^2)$. The values being obtained for the different subsets of experimental data in the present fit are well in accordance with our global result. We do not confirm the significant differences reported by MSTW between the SLAC *ep* and *ed* data at NNLO [9]. We also disagree with the large value of NNPDF [10] for the BCDMS data at NLO, which also contradicts the corresponding result by MSTW [9]. Our results are in agreement with those of the GJR collaboration [5] and the singlet analyses [4,6]. We obtained an update of the dynamical higher twist contributions to $F_2^{p,d}(x,Q^2)$ in the valence region, which depends on the order to which the leading twist contributions were calculated. The effect stabilizes including corrections up to N^{3}LO* in the range $0.3 < x \lesssim 0.65$. At larger values of x still higher order corrections may be needed. A first estimate on the quarkonic twist-3 contributions to the polarized structure function $g_2(x,Q^2)$ is given in a fit to the available world data on $g_2(x,Q^2)$. The contributions to $g_1(x,Q^2)$ are obtained by an integral relation, cf. Ref. [18].

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