

More on the Burkhardt-Cottingham sum rule in QCD

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Received 25 August 1994; revised manuscript received 8 November 1994

Editor: M. Dine

Abstract

The QCD higher order effects to the polarized structure function $g_2(x, Q^2)$ are reanalyzed for massive quarks in the context of the operator product expansion. We confirm that the lowest moment of $g_2(x, Q^2)$ which corresponds to the Burkhardt-Cottingham sum rule does not suffer from radiative corrections in perturbative QCD.

In the polarized lepton production, we have two structure functions g_1 and g_2 . The QCD effects on g_1 have been extensively studied mainly in connection with the problem of so-called "spin crisis". On the other hand, a new measurement of g_2 is expected to be performed at CERN, SLAC and DESY in the near future. Such an experiment is very important since it has been known that the twist-3 operators as well as the twist-2 operators contribute to g_2 [1] and furthermore we can check the well-known sum rule

$$\int_0^1 dx g_2(x, Q^2) = 0 \quad (1)$$

called the Burkhardt-Cottingham (BC) sum rule.

The fact that the twist-3 operators also contribute to the moment of g_2 in the leading order of $1/Q^2$ produces new features which do not appear in the analyses of other structure functions. At the higher-twist level, in general, the appearance of the composite operators in the operator product expansion (OPE) which are proportional to the equation of motion makes the operator mixing problems complicated [2]. This problem has been discussed by many authors [3-5] after the old papers [6-8] on the polarized process. However, as far as the first moment ($n = 1$) of g_2 is concerned, the above complexity is irrelevant since there is no operators corresponding to $n = 1$. Therefore it is naively expected that the BC sum rule is exact at least perturbatively.

Recently two groups [9,10] have discussed the QCD effects at order α_s to BC sum rule for massive quarks and reached different conclusions on the validity of BC sum rule Eq. (1). This controversial situation must be

¹ Supported in part by the Monbusho Grant-in-Aid for Scientific Research No. C-05640351.

² Supported in part by the Monbusho Grant-in-Aid for Scientific Research No. C-06640392 and No.06221239.

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resolved especially for the experimentalists who plan the measurement of g_2 . The purpose of this note is to present an independent calculation of the QCD corrections to BC sum rule in the framework of OPE and try to settle the above issue.

Spin-dependent structure functions are defined by the antisymmetric (A) part of the Fourier transform of the commutator of two electromagnetic currents sandwiched between polarized nucleon states.

$$W_{\mu\nu} = \frac{1}{2\pi} \int d^4x e^{iq \cdot x} \langle p, s | [J_\mu(x), J_\nu(0)] | p, s \rangle \equiv W_{\mu\nu}^S + iW_{\mu\nu}^A,$$

where p (s) is the nucleon's momentum (covariant spin) and q is the virtual photon momentum. We also introduce the current correlation function

$$T_{\mu\nu} = i \int d^4x e^{iq \cdot x} \langle p, s | T(J_\mu(x) J_\nu(0)) | p, s \rangle \equiv T_{\mu\nu}^S + iT_{\mu\nu}^A, \quad (2)$$

such that

$$W_{\mu\nu} = \frac{1}{\pi} \text{Im} T_{\mu\nu}.$$

The antisymmetric part $W_{\mu\nu}^A$ is expressed in terms of two structure functions g_1 and g_2 .

$$W_{\mu\nu}^A = \varepsilon_{\mu\nu\lambda\sigma} q^\lambda \left\{ s^\sigma \frac{1}{p \cdot q} g_1(x, Q^2) + (p \cdot q s^\sigma - q \cdot s p^\sigma) \frac{1}{(p \cdot q)^2} g_2(x, Q^2) \right\},$$

where x is the Bjorken variable $x = Q^2/2p \cdot q$ and $q^2 = -Q^2$. We will make the same tensor decomposition for also the current correlation function.

$$T_{\mu\nu}^A = \varepsilon_{\mu\nu\lambda\sigma} q^\lambda \left\{ s^\sigma \frac{1}{p \cdot q} t_1(\omega, Q^2) + (p \cdot q s^\sigma - q \cdot s p^\sigma) \frac{1}{(p \cdot q)^2} t_2(\omega, Q^2) \right\}, \quad (3)$$

where $\omega = 1/x$.

According to OPE, the current correlation function Eq. (2) is written as follows in the Bjorken limit [5] where we keep only the leading twist terms,

$$T_{\mu\nu}^A = -\varepsilon_{\mu\nu\lambda\sigma} q^\lambda \sum_{n:\text{odd}} \left(\frac{2}{Q^2} \right)^n q_{\mu_1} \cdots q_{\mu_{n-1}} \left\{ E_q^n \langle p, s | R_q^{\sigma\mu_1 \cdots \mu_{n-1}} | p, s \rangle + \sum_j E_j^n \langle p, s | R_j^{\sigma\mu_1 \cdots \mu_{n-1}} | p, s \rangle \right\}. \quad (4)$$

R_i 's are the composite operators and E_i 's the corresponding coefficient functions. For simplicity, here we consider the flavor non-singlet case. In Eq. (4), $R_q^{\sigma\mu_1 \cdots \mu_{n-1}}$, which will be abbreviated as R_q^n , is the twist-2 operator of the form bilinear in the fermion field ψ . The remaining R_j^n 's consist of three types of twist-3 operators: R_F^n which is bilinear in fermion field, R_m^n which is proportional to the quark mass m and R_k^n ($k = 1, \dots, n-2$) which depends on the gluon field strength and is bilinear in fermion field. We refer to Ref. [5] for the details including explicit expressions of these operators.

Denoting the nucleon matrix elements of R_q^n , R_F^n , R_m^n and R_k^n by a_n , d_n , e_n and f_n^k , respectively, the moment sum rule for g_1 and g_2 becomes

$$\int_0^1 dx x^{n-1} g_1(x, Q^2) = \frac{1}{2} a_n E_q^n(Q^2). \quad (5)$$

$$\int_0^1 dx x^{n-1} g_2(x, Q^2) = -\frac{n-1}{2n} a_n E_q^n(Q^2) + \frac{1}{2} \left[\frac{n-1}{n} d_n E_F^n(Q^2) + e_n E_m^n(Q^2) + \sum_k f_n^k E_k^n(Q^2) \right]. \quad (6)$$

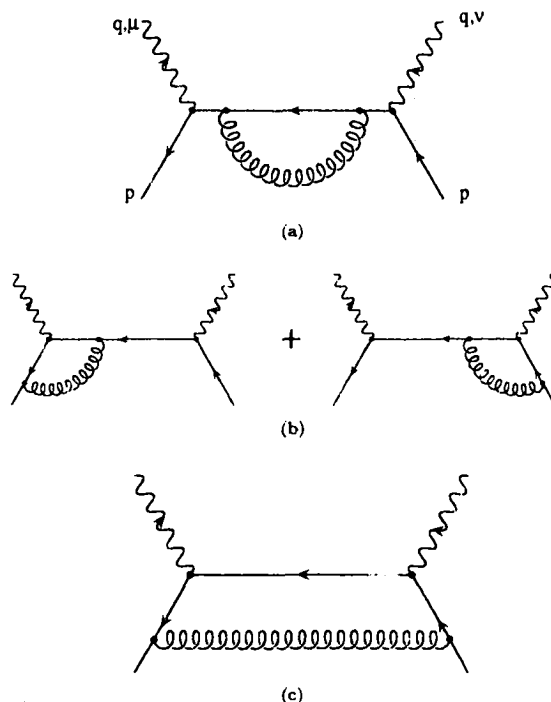


Fig. 1. Diagrams (a) self-energy, (b) vertex and (c) box, contributing to the current correlation function at order g^2 .

where we have assumed that the unsubtracted dispersion relations hold for t_1 and t_2 . As mentioned before, the twist 3 operators can *not* be defined for $n = 1$. Namely, d_n , e_n , f_n^k 's are identically zero for $n = 1$ in (6). Also we have a “kinematical” factor $n - 1$ in front of the contribution from the twist 2 operators in the moment sum rule for g_2 Eq. (6). Therefore the OPE analysis implies that the BC sum rule does not suffer from the radiative corrections at all.

Here note that when expanding $t_1(\omega, Q^2)$ and $t_2(\omega, Q^2)$ in Eq. (3) in powers of ω , their n -th coefficients are just equal to the moment sum rules Eq. (5) and Eq. (6) respectively [7]. This is the standard procedure to obtain the higher order corrections for the coefficient functions. Therefore if we know $t_2(\omega, Q^2)$, we can check whether the above *naïve* argument for $n = 1$ holds or not. The order of α_s diagrams contributing to $T_{\mu\nu}^A$ are shown in Fig. 1. The calculation is performed with massive quarks. The ultraviolet divergences are regularized with momentum cut off Λ . To regularize the infrared singularities, we give a mass λ to the gluon. Note that the collinear singularities are already regulated by quark mass. We choose the on-shell renormalization scheme. Although we have calculated the diagrams without making any approximations for m^2/Q^2 , we present only the results after taking the limit $m^2/Q^2 \rightarrow 0$ in accordance with the fact that we kept only the leading twist terms in the OPE. The contributions from each diagrams contain current non-conserved terms which cancel out after adding all contributions. So we will neglect such terms in the following results.

The results for each diagrams read ($C_F (= 4/3)$ is the Casimir operator for quarks):

(a) Born + self-energy contribution of Fig. 1a :

$$t_1^{(a)} = \sum_n \omega^n \left[1 + \frac{g^2}{16\pi^2} C_F \left\{ \ln \frac{Q^2}{m^2} + 3 + \frac{1}{n} - \sum_{r=1}^n \frac{1}{r} + 2 \ln \frac{\lambda^2}{m^2} \right\} \right],$$

$$t_2^{(a)} = 0.$$

(b) vertex contribution of Fig. 1b:

$$t_1^{(b)} = \frac{g^2}{8\pi^2} C_F \sum_n \omega^n \left[\left(-1 - 2 \sum_{r=2}^n \frac{1}{r} \right) \ln \frac{Q^2}{m^2} - 4 + \frac{1}{n} + 2 \sum_{r=1}^n \frac{1}{r} - 2 \sum_{r=1}^n \frac{1}{r^2} - 2 \sum_{s=1}^n \frac{1}{s} \sum_{r=1}^s \frac{1}{r} - 2 \ln \frac{\lambda^2}{n^2} \right],$$

$$t_2^{(b)} = \frac{g^2}{8\pi^2} C_F \sum_n \omega^n \left[\left(-1 + \frac{1}{n} \right) \ln \frac{Q^2}{m^2} + \frac{1}{2} - \frac{3}{n} + \frac{1}{n^2} + \left(\frac{1}{2} + \frac{1}{n} \right) \sum_{r=1}^n \frac{1}{r} \right].$$

(c) box contribution of Fig. 1c:

$$t_1^{(c)} = \frac{g^2}{8\pi^2} C_F \sum_n \omega^n \left[\frac{1}{n(n+1)} \ln \frac{Q^2}{m^2} - \frac{4}{n} + \frac{4}{n+1} + \frac{1}{n^2} - \frac{2}{(n+1)^2} + \left(2 + \frac{1}{n(n+1)} \right) \sum_{r=1}^n \frac{1}{r} + \ln \frac{\lambda^2}{m^2} \right],$$

$$t_2^{(c)} = \frac{g^2}{8\pi^2} C_F \sum_n \omega^n \left[\left(\frac{2}{n+1} - \frac{1}{n} \right) \ln \frac{Q^2}{m^2} + \frac{3}{n} - \frac{6}{n+1} + \frac{1}{n^2} + \frac{4}{(n+1)^2} + \left(\frac{2}{n+1} - \frac{1}{n} \right) \sum_{r=1}^n \frac{1}{r} \right].$$

Now the entire expressions for t_1 and t_2 in which the above three contributions are added together are:

$$t_1 = \sum_n \omega^n \left[1 + \frac{g^2}{8\pi^2} C_F \left\{ -\frac{1}{2} \left(1 - \frac{2}{n(n+1)} + 4 \sum_{r=2}^n \frac{1}{r} \right) \ln \frac{Q^2}{m^2} - \frac{5}{2} - \frac{5}{2n} + \frac{4}{n+1} + \frac{1}{n^2} - \frac{2}{(n+1)^2} + \left(\frac{7}{2} + \frac{1}{n(n+1)} \right) \sum_{r=1}^n \frac{1}{r} - 2 \sum_{r=1}^n \frac{1}{r^2} - 2 \sum_{s=1}^n \frac{1}{s} \sum_{r=1}^s \frac{1}{r} \right\} \right], \quad (7)$$

$$t_2 = \frac{g^2}{8\pi^2} C_F \sum_n \omega^n \left[-\frac{1}{2} \frac{2(n-1)}{n+1} \ln \frac{Q^2}{m^2} + \frac{1}{2} - \frac{6}{n+1} + \frac{4}{(n+1)^2} + \frac{n+5}{2(n+1)} \sum_{r=1}^n \frac{1}{r} \right]. \quad (8)$$

Eq. (7) and its interpretation is in agreement with the result of Ref. [9]. However the result Eq. (8) is in agreement with that of Ref. [10]. The first moment ($n=1$) which corresponds to the BC sum rule vanishes. So our calculation reconfirms the validity of the BC sum rule and shows that the OPE analysis is consistent with the QCD perturbation theory. Here we note that for general n , by subtracting the matrix elements of the composite operators from the above expressions, we get the coefficient functions which are scheme-dependent but free from mass-singularities. But for the $n=1$ moment of g_2 , there exists no such complication.

In conclusion we have calculated the virtual Compton scattering amplitude at order α_s and shown that the BC sum rule does not receive any corrections in perturbative QCD based on the OPE. In this respect, here we note that the BC sum rule is not only protected from QCD radiative corrections but also free from target mass effects [11].

Finally, we expect that future experiments on g_2 will confirm the BC sum rule in its original form.

We would like to thank Yoshiaki Yasui for discussions.

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