Production of two hadrons in semi-inclusive deep inelastic scattering

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We present the general expression, in terms of structure functions, of the cross section for the production of two hadrons in semi-inclusive deep inelastic scattering. We analyze this process including full transverse-momentum dependence up to subleading twist and check, where possible, the consistency with existing literature.

I. INTRODUCTION

Deep-inelastic lepton-nucleon scattering (DIS) is one of the key experimental tools for studying the structure of nucleons, including their spin. Particularly insightful can be semi-inclusive DIS (SIDIS), where one or more final-state hadrons are detected. In this article, we take into consideration two-particle-inclusive DIS, i.e., DIS with two detected hadrons in the final state. And, in particular, those hadron pairs with small invariant mass.

One-particle-inclusive DIS has been studied in depth, including transverse-momentum dependence (see, e.g., [1]). The analysis usually consists of two parts: (1) a study of the general form of the cross section in terms of structure functions, which applies to any lepton-hadron scattering process with at least one hadron in the final state and relies on the single-photon-exchange approximation, (2) a study of the specific form of the structure functions in terms of parton distribution and fragmentation functions. The second step relies on the existence of factorization theorems for the process under consideration. Such factorization theorems have been established at the leading-twist level [2–10], i.e., up to corrections of order M/Q, where M is the mass of the nucleon and Q is the modulus of the transferred four-momentum. When trying to extend these theorems to the subleading-twist level, i.e., including corrections of order M/Q, technical complications arise from the contribution of soft gluon radiation [11,12]. The extension of factorization theorems to subleading twist is still an important open problem, and it is the main focus of several active investigations [12–18]. In spite of this problem, the analysis of structure functions in terms of partonic densities has been carried out including subleading twist [1,19].

In this article, we follow the same approach but for the two-particle inclusive case. At leading twist, if the invariant mass of the pair is much smaller than Q, the underpinnings

of factorization theorems for the single-hadron case remain valid also for the two-hadron case (the situation may be different for a large invariant mass [20]). Then, we apply a parton-model scheme to the analysis of structure functions assuming the validity of factorization theorems, including subleading twist.

The production of two hadrons in SIDIS has been studied in several papers. The first comprehensive study has been presented in Ref. [21] up to leading twist, where the relevant dihadron fragmentation functions have been defined. Reference [22] introduced the method of partial-wave analysis, which is important for our present discussion and clarifies the connection between two-hadron production and spin-one production [23]. Reference [24] extended the analysis up to subleading twist, but only integrated over transverse momentum. Recent work has considered the problem of two-hadron production where one hadron is in the current region and one in the target region [25–27]. The analysis has been carried out at leading twist and is somewhat complementary to our present work.

This paper presents a slight modification to the definition of the fragmentation functions compared to, e.g., Ref. [24]. This not only may help in the interpretation and presentation of cross section moments, but it also has the practical advantage that the two-hadron SIDIS cross sections, at any twist, can be derived from single-hadron SIDIS. Using this method, in this paper we present for the first time the expression of the transverse-momentum dependent (TMD) two-hadron SIDIS cross section at subleading twist. We cross-checked our result with existing literature for specific cases.

The paper is organized as follows. In Sec. II, we describe our notation, the kinematics, and we list the general expression for cross section in terms of structure functions. In Sec. III, we describe our new more general definition of the transverse-momentum dependent two-hadron fragmentation functions, including their new partial-wave expansion. In Sec. IV, the structure functions are mapped onto specific convolutions involving the TMD distributions and two-hadron fragmentation functions. In Sec. V, we compare

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with known results for specific cases in order to clarify our nomenclature. Finally, some conclusions are drawn in Sec. VI.

II. CROSS SECTION IN TERMS OF STRUCTURE FUNCTIONS

A. Definitions

We consider the process

$$\ell(l) + N(P) \to \ell(l') + h_1(P_1) + h_2(P_2) + X, \quad (1)$$

where ℓ denotes the beam lepton, N the nucleon target, and h the produced hadron, and where four-momenta are given in parentheses. We work in the one-photon exchange approximation and neglect the lepton mass. We denote by M the mass of the nucleon and by S its polarization. The final hadrons have masses M_1, M_2 and momenta P_1, P_2. We introduce the pair total momentum $P_h = P_1 + P_2$ and relative momentum $R = (P_1 - P_2)/2$. The invariant mass of the pair is $P_h^2 = M_h^2$.

As usual we define $q = l - l'$, where $Q^2 = -q^2$ is the hard scale of the process. We introduce the variables

$$x_B = \frac{Q^2}{2P \cdot q}, \qquad y = \frac{P \cdot q}{P \cdot l},$$

$$z_h = \frac{P \cdot P_h}{P \cdot q}, \qquad \gamma = \frac{2Mx_B}{Q}. \quad (2)$$

The longitudinal polarization factor for the beam will be denoted λ_e and α is the fine structure constant.

Of particular relevance for our discussions are the angles involved in the process. Two different sets of transverse projections are usually taken into consideration. In fact, we can define two different transverse planes: the first is perpendicular to (P, q), and the projection of a generic 4-vector V onto it will be denoted by $V_\perp$; the second one is perpendicular to (P, P_h) and the projection is indicated by V_T. The corresponding projection operators, up to terms of order M^4/Q^4, turn out to be[1]

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu P^\nu + P^\mu q^\nu}{P \cdot q(1 + \gamma^2)} + \frac{\gamma^2}{1 + \gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right), \quad (3)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{P_\rho q_\sigma}{P \cdot q\sqrt{1 + \gamma^2}} \quad (4)$$

and

$$g_T^{\mu\nu} = g^{\mu\nu} - \frac{2x_B}{Q^2 z_h}(P^\mu P_h^\nu + P_h^\mu P^\nu) + \frac{M_h^2\gamma^2}{Q^2 z_h^2}\left(\frac{P^\mu P^\nu}{M^2} + \frac{P_h^\mu P_h^\nu}{M_h^2}\right), \quad (5)$$

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[1]We use the convention $\epsilon^{0123} = 1$.

$$\epsilon_T^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{P_\rho P_{h\sigma}}{P \cdot P_h}. \quad (6)$$

We define the azimuthal angles [1,28]

$$\cos\phi_h = -\frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \sin\phi_h = -\frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad (7)$$

where $l_\perp^\mu = g_\perp^{\mu\nu} l_\nu$ and $P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$. The azimuthal angle of the spin vector, ϕ_S, is defined in analogy to ϕ_h, with P_h replaced by S.

For dihadron fragmentation functions, we need to introduce one more azimuthal angle. We first introduce the vector R_T, i.e., the component of R perpendicular to P and P_h. Defining the invariant

$$\zeta_h = \frac{2R \cdot P}{P_h \cdot P}, \quad (8)$$

neglecting terms of order M^4/Q^4 we can write

$$R_T^\mu = g_T^{\mu\nu} R_\nu = R^\mu - \frac{\zeta_h}{2}P_h^\mu + x_B\frac{\zeta_h M_h^2 - (M_1^2 - M_2^2)}{Q^2 z_h}P^\mu. \quad (9)$$

However, the cross section will depend on the azimuthal angle of R_T measured in the plane perpendicular to (P, q). Therefore, we need to use Eq. (7) replacing P_h with R_T. We will denote the azimuthal angle of R_T in this frame by $\phi_{R_\perp}$. This choice is similar to what has been done in Ref. [29], but here it has been realized in a covariant way. In Appendix A, we compare our definition with other noncovariant ones available in the literature, pointing out the potential differences depending on the choice of the reference frame.

It is anyway convenient to give the expression of the involved angles in specific frames of reference. The azimuthal angles are usually written in the target rest frame (or in any frame reached from the target rest frame by a boost along q)

$$\phi_h = \frac{(q \times l) \cdot P_h}{|(q \times l) \cdot P_h|}\arccos\frac{(q \times l) \cdot (q \times P_h)}{|q \times l||q \times P_h|}, \quad (10)$$

$$\phi_{R_\perp} = \frac{(q \times l) \cdot R_T}{|(q \times l) \cdot R_T|}\arccos\frac{(q \times l) \cdot (q \times R_T)}{|q \times l||q \times R_T|}. \quad (11)$$

In the center-of-mass (cm) frame of the two hadrons, the emission occurs back to back and the key variable is the polar angle ϑ between the directions of the emission and of P_h [22]. The variable ζ_h can be written in terms of the ϑ as follows:

$$|\boldsymbol{R}| = \frac{1}{2}\sqrt{M_h^2 - 2(M_1^2 + M_2^2) + (M_1^2 - M_2^2)^2/M_h^2},$$

$$\zeta_h = \frac{1}{M_h}\left(\sqrt{M_1^2 - |\boldsymbol{R}|^2} - \sqrt{M_2^2 - |\boldsymbol{R}|^2} - 2|\boldsymbol{R}|\cos\vartheta\right).$$

$$(12)$$

At this point, we remark also that the analysis of two-hadron production can be done in terms of the variables $(z_{h1}, z_{h2}, P_{h1T}, P_{h2T})$, of the two individual hadrons, instead of introducing the sum and difference of their momenta. This choice is more reasonable if one hadron is in the current region and one in the target region [25–27].

B. Complete dependence of the cross section

The cross section is split in parts denoted by σ_{XY}, based on the target (X) and beam (Y) polarization, X and Y taking values U (unpolarized), L (longitudinally polarized), and T (transversely polarized). The structure functions will likewise have subscripts XY, with the same meaning. In a few cases the structure functions have an additional subscript, indicating a longitudinal (L) or transverse (T) virtual photon polarization.

We introduce the depolarization factors [1]

$$A(x,y) = \frac{y^2}{2(1-\epsilon)} = \frac{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}{1 + \gamma^2}$$

$$\approx \left(1 - y + \frac{1}{2}y^2\right), \qquad (13)$$

$$B(x,y) = \frac{y^2}{2(1-\epsilon)}\epsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 + \gamma^2}$$

$$\approx (1 - y), \qquad (14)$$

$$C(x,y) = \frac{y^2}{2(1-\epsilon)}\sqrt{1-\epsilon^2} = \frac{y(1 - \frac{1}{2}y)}{\sqrt{1+\gamma^2}}$$

$$\approx y\left(1 - \frac{1}{2}y\right), \qquad (15)$$

$$V(x,y) = \frac{y^2}{2(1-\epsilon)}\sqrt{2\epsilon(1+\epsilon)}$$

$$= \frac{2-y}{1+\gamma^2}\sqrt{1 - y - \frac{1}{4}\gamma^2 y^2}$$

$$\approx (2-y)\sqrt{1-y}, \qquad (16)$$

$$W(x,y) = \frac{y^2}{2(1-\epsilon)}\sqrt{2\epsilon(1-\epsilon)}$$

$$= \frac{y}{\sqrt{1+\gamma^2}}\sqrt{1 - y - \frac{1}{4}\gamma^2 y^2}$$

$$\approx y\sqrt{1-y}. \qquad (17)$$

The approximations, which no longer depend on x, are valid up to corrections of order M^2/Q^2.

The cross section will be differential in the following variables:

instead of ζ_h

$$\frac{d\sigma}{dx_B\,dy\,d\psi\,dz_h\,d\phi_h\,dP_{h\perp}^2\,d\phi_{R_\perp}\,dM_h\,d\cos\theta}. \qquad (18)$$

The angle ψ is the azimuthal angle of ℓ' around the lepton beam axis with respect to an arbitrary fixed direction, which in case of a transversely polarized target we choose to be the direction of S. The corresponding relation between ψ and ϕ_S is given in Ref. [30]; neglecting corrections of order M^2/Q^2, one has $d\psi \approx d\phi_S$.

The dependence of the cross section on the polar angle $\cos\vartheta$ and on the azimuthal angles $\phi_h, \phi_{R_\perp}$ is transformed by expanding it on a basis of spherical harmonics. In particular, for the $\cos\vartheta$ dependence we adopt the basis of Legendre polynomials, the first few of which read

$$P_{0,0} = 1, \qquad P_{2,0} = \frac{1}{2}(3\cos^2\vartheta - 1), \qquad (19)$$

$$P_{1,0} = \cos\vartheta, \qquad P_{2,1} = \sin 2\vartheta, \qquad (20)$$

$$P_{1,1} = \sin\vartheta, \qquad P_{2,2} = \sin^2\vartheta\ldots \qquad (21)$$

with $P_{\ell,-m} = P_{\ell,m}$.

For the one-particle-inclusive SIDIS case, the hadronic tensor is built by using 3 four-vectors, q, P, P_h, and 1 pseudo four-vector, S. Since the target is a spin-1/2 particle, the hadronic tensor can be at most linear in S. By imposing the invariance under the usual transformations (parity, time-reversal, gauge), the hadronic tensor can be parametrized in terms of 18 structure functions [1,19,31]. In the two-particle-inclusive SIDIS, even in the simplest case when the target and the two final hadrons are unpolarized, the pseudovector S is replaced by R and the hadronic tensor does not necessarily need to be linear in R. Actually, the number of partial waves depending on $\phi_{R_\perp}$ is in principle not limited, and so the number of structure functions is also not limited.

The structure of the cross section for unpolarized beam and unpolarized target is similar to the one for the one-particle-inclusive SIDIS, because it is dictated by the helicity density matrix of the virtual photon: there are two diagonal elements related to its transverse (T) and longitudinal (L) polarization, and there are two interference terms. Then, in this cross section four different parts can be identified, each one displaying an infinite number of structure functions:

$$d\sigma_{UU} = \frac{\alpha^2}{4\pi xyQ^2}\left(1 + \frac{\gamma^2}{2x}\right)\sum_{\ell=0}^{\ell_{max}}\left\{A(x,y)\sum_{m=0}^{\ell}\left[P_{\ell,m}\cos(m(\phi_h - \phi_{R_\perp}))\left(F_{UU,T}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))} + \epsilon F_{UU,L}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))}\right)\right]\right.$$

$$+ B(x,y)\sum_{m=-\ell}^{\ell}P_{\ell,m}\cos((2-m)\phi_h + m\phi_{R_\perp})F_{UU}^{P_{\ell,m}\cos((2-m)\phi_h+\phi_{R_\perp})}$$

$$\left.+ V(x,y)\sum_{m=-\ell}^{\ell}P_{\ell,m}\cos((1-m)\phi_h + m\phi_{R_\perp})F_{UU}^{P_{\ell,m}\cos((1-m)\phi_h+m\phi_{R_\perp})}\right\}. \tag{22}$$

As explained above, there is no upper limit to ℓ_{max}: there are infinitely many azimuthal modulations, in contrast to the 18 structure functions of single-hadron production. In this case, what limits the number of structure functions is conservation of angular momentum, as discussed for instance in Ref. [30]. Here, however, the presence of two sources of angular momentum (the total and relative angular momenta of the pair) allows for infinitely many combinations. The only constraint is that the sum of the coefficients of ϕ_h and $\phi_{R_\perp}$ should be limited to at most 3: this is the maximum mismatch of angular momentum projections in the γP system.

For practical purposes, in certain situations it may be possible to restrict the value of ℓ_{max}. For instance,

when considering two hadrons emitted through a vector meson resonance, $\ell_{max} = 2$. However, it is not possible in general to distinguish between resonant and non-resonant dihadron production, e.g., between nonresonant $\pi^+\pi^-$ and resonant ρ^0. The nonresonant dihadrons are not necessarily restricted to any finite ℓ value, although at small invariant mass higher ℓ should be suppressed.

The structure functions on the right-hand side of Eq. (22) depend on x_B, Q^2, z_h, $P_{h\perp}^2$, M_h^2.

Along the same lines, the cross section for longitudinally polarized beam and unpolarized target reads

$$d\sigma_{LU} = \frac{\alpha^2}{4\pi xyQ^2}\left(1 + \frac{\gamma^2}{2x}\right)\lambda_e\sum_{\ell=0}^{\ell_{max}}\left\{C(x,y)\sum_{m=1}^{\ell}\left[P_{\ell,m}\sin(m(\phi_h - \phi_{R_\perp}))2\left(F_{LU,T}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))} + \epsilon F_{LU,L}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))}\right)\right]\right.$$

$$\left.+ W(x,y)\sum_{m=-\ell}^{\ell}P_{\ell,m}\sin((1-m)\phi_h + m\phi_{R_\perp})F_{LU}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp})}\right\}. \tag{23}$$

The cross section for an unpolarized beam and longitudinally polarized target reads

$$d\sigma_{UL} = \frac{\alpha^2}{4\pi xyQ^2}\left(1 + \frac{\gamma^2}{2x}\right)S_L\left\{A(x,y)\sum_{\ell=1}^{\ell_{max}}\sum_{m=1}^{\ell}P_{\ell,m}\sin(-m\phi_h + m\phi_{R_\perp})F_{UL}^{P_{\ell,m}\sin(-m\phi_h+m\phi_{R_\perp})}\right.$$

$$+ B(x,y)\sum_{\ell=0}^{\ell_{max}}\sum_{m=-\ell}^{\ell}P_{\ell,m}\sin((2-m)\phi_h + m\phi_{R_\perp})F_{UL}^{P_{\ell,m}\sin((2-m)\phi_h+m\phi_{R_\perp})}$$

$$\left.+ V(x,y)\sum_{\ell=0}^{\ell_{max}}\sum_{m=-\ell}^{\ell}P_{\ell,m}\sin((1-m)\phi_h + m\phi_{R_\perp})F_{UL}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp})}\right\}. \tag{24}$$

The cross section for a longitudinally polarized beam and longitudinally polarized target reads

$$d\sigma_{LL} = \frac{\alpha^2}{4\pi xyQ^2}\left(1 + \frac{\gamma^2}{2x}\right)\lambda_e S_L$$

$$\times\sum_{\ell=0}^{\ell_{max}}\left\{C(x,y)\sum_{m=0}^{\ell}2^{2-\delta_{m0}}P_{\ell,m}\cos(m(\phi_h - \phi_{R_\perp}))F_{LL}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))}\right.$$

$$\left.+ W(x,y)\sum_{m=-\ell}^{\ell}P_{\ell,m}\cos((1-m)\phi_h + m\phi_{R_\perp})F_{LL}^{P_{\ell,m}\cos((1-m)\phi_h+m\phi_{R_\perp})}\right\}. \tag{25}$$

The cross section for an unpolarized beam and transversely polarized target reads

$$d\sigma_{UT} = \frac{\alpha^2}{4\pi x y Q^2}\left(1+\frac{\gamma^2}{2x}\right)|\boldsymbol{S}_\perp|\sum_{\ell=0}^{\ell_{max}}\sum_{m=-\ell}^{\ell}\left\{A(x,y)\left[P_{\ell,m}\sin((m+1)\phi_h - m\phi_{R_\perp}-\phi_S))\right.\right.$$
$$\left.\times\left(F_{UT,T}^{P_{\ell,m}\sin((m+1)\phi_h - m\phi_{R_\perp}-\phi_S)} + \epsilon F_{UT,L}^{P_{\ell,m}\sin((m+1)\phi_h - m\phi_{R_\perp}-\phi_S)}\right)\right]$$
$$+ B(x,y)\left[P_{\ell,m}\sin((1-m)\phi_h + m\phi_{R_\perp}+\phi_S)F_{UT}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp}+\phi_S)}\right.$$
$$\left.+ P_{\ell,m}\sin((3-m)\phi_h + m\phi_{R_\perp}-\phi_S)F_{UT}^{P_{\ell,m}\sin((3-m)\phi_h+m\phi_{R_\perp}-\phi_S)}\right]$$
$$+ V(x,y)\left[P_{\ell,m}\sin(-m\phi_h + m\phi_{R_\perp}+\phi_S)F_{UT}^{P_{\ell,m}\sin(-m\phi_h+m\phi_{R_\perp}+\phi_S)}\right.$$
$$\left.\left.+ P_{\ell,m}\sin((2-m)\phi_h + m\phi_{R_\perp}-\phi_S)F_{UT}^{P_{\ell,m}\sin((2-m)\phi_h+m\phi_{R_\perp}-\phi_S)}\right]\right\}. \tag{26}$$

Lastly, the cross section for a longitudinally polarized beam and transversely polarized target reads

$$d\sigma_{LT} = \frac{\alpha^2}{4\pi x y Q^2}\left(1+\frac{\gamma^2}{2x}\right)\lambda_e|\boldsymbol{S}_\perp|\sum_{\ell=0}^{\ell_{max}}\sum_{m=-\ell}^{\ell}\left\{C(x,y)2P_{\ell,m}\cos((1-m)\phi_h + m\phi_{R_\perp}-\phi_S))F_{LT}^{P_{\ell,m}\cos((1-m)\phi_h+m\phi_{R_\perp}-\phi_S))}\right.$$
$$+ W(x,y)\left[P_{\ell,m}\cos(-m\phi_h + m\phi_{R_\perp}+\phi_S)F_{LT}^{P_{\ell,m}\cos(-m\phi_h+m\phi_{R_\perp}+\phi_S)}\right.$$
$$\left.\left.+ P_{\ell,m}\cos((2-m)\phi_h + m\phi_{R_\perp}-\phi_S)F_{LT}^{P_{\ell,m}\cos((2-m)\phi_h+m\phi_{R_\perp}-\phi_S)}\right]\right\}. \tag{27}$$

In the above equations, we can identify 21 different classes $F_{XY,Z}^{f(\phi_h,\phi_{R_\perp},\phi_S)}$ of structure functions, for a total of

$$21\sum_{\ell=0}^{\ell_{max}}(2\ell+1) = 21(\ell_{max}+1)^2 \tag{28}$$

structure functions. A common choice for dihadrons with invariant mass $M_h \lesssim 1$ GeV is to stop the sum at $\ell_{max} = 2$. The structure functions are then 189.

C. Integrated cross section

As already observed in Ref. [30], the cross section for two-hadron production integrated over the pair's transverse momentum has a similar form to the cross section of single-hadron production [1], with ϕ_h replaced by $\phi_{R_\perp}$, i.e.,

$$\frac{d\sigma}{dx_B dy d\psi dz_h d\phi_{R_\perp} dM_h d\cos\theta} = \frac{\alpha^2}{x_B y Q^2}\left(1+\frac{\gamma^2}{2x_B}\right)$$
$$\times\left\{A(x,y)F_{UU,T} + B(x,y)F_{UU,L} + \frac{1}{2}V(x,y)\cos\phi_{R_\perp}F_{UU}^{\cos\phi_{R_\perp}} + B(x,y)\cos(2\phi_{R_\perp})F_{UU}^{\cos 2\phi_{R_\perp}}\right.$$
$$+ \lambda_e\frac{1}{2}W(x,y)\sin\phi_{R_\perp}F_{LU}^{\sin\phi_{R_\perp}} + S_L\left[\frac{1}{2}V(x,y)\sin\phi_{R_\perp}F_{UL}^{\sin\phi_{R_\perp}} + B(x,y)\sin(2\phi_{R_\perp})F_{UL}^{\sin 2\phi_{R_\perp}}\right]$$
$$+ S_L\lambda_e\left[C(x,y)2F_{LL} + \frac{1}{2}V(x,y)\cos\phi_{R_\perp}F_{LL}^{\cos\phi_{R_\perp}}\right]$$
$$+ |\boldsymbol{S}_\perp|\left[\sin(\phi_{R_\perp}-\phi_S)(A(x,y)F_{UT,T}^{\sin(\phi_{R_\perp}-\phi_S)} + B(x,y)F_{UT,L}^{\sin(\phi_{R_\perp}-\phi_S)})\right.$$
$$+ B(x,y)\sin(\phi_{R_\perp}+\phi_S)F_{UT}^{\sin(\phi_{R_\perp}+\phi_S)} + B(x,y)\sin(3\phi_{R_\perp}-\phi_S)F_{UT}^{\sin(3\phi_{R_\perp}-\phi_S)}$$
$$\left.+ \frac{1}{2}V(x,y)\sin\phi_S F_{UT}^{\sin\phi_S} + \frac{1}{2}V(x,y)\sin(2\phi_{R_\perp}-\phi_S)F_{UT}^{\sin(2\phi_{R_\perp}-\phi_S)}\right]$$
$$\left.+ |\boldsymbol{S}_\perp|\lambda_e\left[C(x,y)\cos(\phi_{R_\perp}-\phi_S)2F_{LT}^{\cos(\phi_{R_\perp}-\phi_S)} + \frac{1}{2}V(x,y)\cos\phi_S F_{LT}^{\cos\phi_S} + \frac{1}{2}V(x,y)\cos(2\phi_{R_\perp}-\phi_S)F_{LT}^{\cos(2\phi_{R_\perp}-\phi_S)}\right]\right\}, \tag{29}$$

where the structure functions on the right-hand side depend on x_B, Q^2, z_h, M_h. The above formula can be obtained from the sum of the formulas in the previous section, observing that the only surviving contributions are the ones with values of m that cancel the coefficients of the ϕ_h angle. Each of the 18 structure functions in Eq. (29) corresponds to a specific class $F_{XY,Z}^{f(\phi_h,\phi_{R_\perp},\phi_S)}$ in Eqs. (22)–(27). Only three classes do not survive the integration upon $P_{h\perp}$: the ones containing the fragmentation functions $F_{LU,T}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))}$ and $F_{LU,L}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))}$ in Eq. (23), and the one containing the fragmentation functions $F_{UL}^{P_{\ell,m}\sin(-m\phi_h+m\phi_{R_\perp})}$ in Eq. (24).

III. NEW DEFINITION FOR TWO-HADRON FRAGMENTATION FUNCTIONS

Throughout the paper, we will adopt the following notation for intrinsic momenta: k denotes the parton momentum in the distribution functions, p the parton momentum occurring in the fragmentation functions, while P refers to the final hadron momentum.

The general fragmentation process is rigorously defined starting from the correlation matrix Δ [19]. Using the common shorthand notation for the trace of projections of the correlation matrix [19,21],

$$\Delta^{[\Gamma]}(z, \cos\vartheta, |\boldsymbol{R}_T|, |\boldsymbol{p}_T|, \boldsymbol{R}_T \cdot \boldsymbol{p}_T)$$
$$= 4\pi \frac{z|\boldsymbol{R}|}{16M_h} \int dk^+ \text{Tr}[\Gamma\Delta(p, P_h, R)]|_{p^-=P_h^-/z}, \quad (30)$$

with $p = \{p^-, p^+ \equiv (p^2+\boldsymbol{p}_T^2)/(2p^-), \boldsymbol{p}_T\}$, for each choice of the operator Γ we can define a specific class of fragmentation functions. At leading twist, we have the three classes

$$D_1 = \Delta^{[\gamma^-]}, \quad (31)$$

$$G_1 = \Delta^{[\gamma^-\gamma_5]}, \quad (32)$$

$$i\frac{|\boldsymbol{p}_T|}{M_h}e^{i\phi_p}H_1^\perp = \Delta^{[-i(\sigma^{1-}+i\sigma^{2-})\gamma^5]}$$
$$= \Delta^{[(\gamma^2-i\gamma^1)\gamma^-\gamma^5]}, \quad (33)$$

where ϕ_p is the azimuthal angle referred to $\boldsymbol{p}_T$ [32]. Their probabilistic interpretation is depicted in Fig. 1. With respect to the helicity density matrix of the fragmenting quark, the unpolarized D_1 corresponds to the sum of diagrams with no quark spin flip [those with $(\chi = \chi' = 1/2)$ and $(\chi = \chi' = -1/2)$ in Fig. 1]; the polarized G_1 corresponds to their difference; and the chiral-odd $H_1^\perp$, the generalized Collins fragmentation function, corresponds to the sum of diagrams with quark spin flip $(\chi \neq \chi')$.

One advantage of this new convention is that at leading twist it associates the name and symbol of the fragmentation
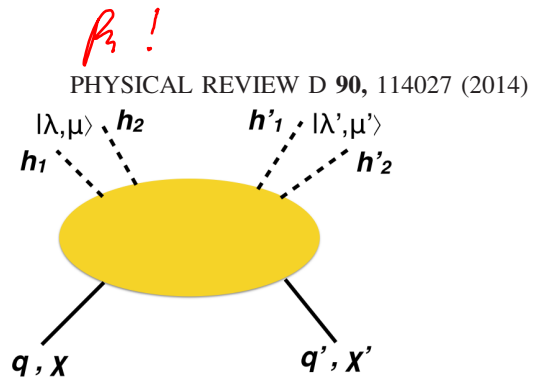

FIG. 1 (color online). The generic diagram for leading-twist fragmentation functions. The quarks are indicated as q, q', with their helicities χ, χ', respectively. The hadron pairs (h_1, h_2) and (h_1', h_2') are, respectively, in the partial waves $|\lambda, \mu\rangle$ and $|\lambda', \mu'\rangle$.

functions with the quark spin states (χ, χ'), while the various polarization states of the produced hadron system $(|\lambda, \mu\rangle$ and $|\lambda', \mu'\rangle)$ are associated with partial waves of the fragmentation functions. At subleading twist, the projection operators Γ select the so-called "bad" light-cone components which are associated to quark-gluon combinations; hence, a clear identification of such components in terms of good quark helicity states is not applicable.

A. Partial wave expansion

The fragmentation functions can be expanded in partial waves in the direct product basis $|\lambda, \mu\rangle|\lambda', \mu'\rangle$, with $\lambda\lambda'$, the relative partial waves of each hadron pair in Fig. 1. However, the structure functions in the cross section of Eqs. (22)–(27) are related to specific partial waves in the direct sum basis. Then, it is more convenient to reexpress the expansion in the basis $|\ell, m\rangle$, where $\ell = \lambda \oplus \lambda'$ is the total partial wave of the hadronic system.

For each class of fragmentation functions in Eqs. (31)–(33), the partial wave expansion is accomplished by expanding the dependence on the polar angle $\cos\vartheta$ and on the azimuthal angle $\phi_{R_\perp} - \phi_p$ on a basis of spherical harmonics, in the same way as was done for the cross sections (22)–(27). Then, we have

$$D_1 = \sum_{\ell=0}^{\ell_{max}} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos\vartheta)\cos\left(m(\phi_{R_\perp} - \phi_p)\right)$$
$$\times D_1^{|\ell,m\rangle}(z, M_h, |\boldsymbol{p}_T|), \quad (34)$$

$$G_1 = \sum_{\ell=1}^{\ell_{max}} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos\vartheta)\sin\left(m(\phi_{R_\perp} - \phi_p)\right)$$
$$\times G_1^{|\ell,m\rangle}(z, M_h, |\boldsymbol{p}_T|), \quad (35)$$

$$H_1^\perp = \sum_{\ell=0}^{\ell_{max}} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos\vartheta)e^{im(\phi_{R_\perp} - \phi_p)}$$
$$\times H_1^{\perp|\ell,m\rangle}(z, M_h, |\boldsymbol{p}_T|), \quad (36)$$

and likewise for the higher twist fragmentation functions. All nonexpanded classes explicitly depend on the variables

z, M_h, $|\boldsymbol{p}_T|$, $\cos\vartheta$, $\phi_{R_\perp} - \phi_p$, and implicitly depend on Q^2.

If the correlator Δ of Eq. (30) is considered as a Hermitian 2×2 matrix in the quark helicity basis, each diagonal element is complex conjugate of the other. Their sum gives twice their real part and is proportional to the class D_1; their difference gives twice their imaginary part and is proportional to the class G_1. Hence, when D_1 and G_1 are expanded onto the basis of spherical harmonics, the former contains only cosine components of the azimuthal angle $\phi_{R_\perp} - \phi_p$ while the latter only sine components. The class $H_1^\perp$, being related to helicity-flip matrix elements, contains both components.

Each expanded fragmentation function appears in specific structure functions in Eqs. (22)–(27) for a given partial wave (ℓ, m).

Note that the cross section for any final state polarization, when written in terms of nonexpanded fragmentation functions, is identical to that for a single pseudoscalar meson production, which is simply the $|0, 0\rangle$ component of the final state polarization. This allows one to compute the cross section for any final state polarization, at any twist level, given the cross section for pseudoscalar meson production at the corresponding twist level.

IV. STRUCTURE FUNCTIONS IN TERMS OF DISTRIBUTION AND FRAGMENTATION FUNCTIONS

In the following, the results for the structure functions are obtained in a parton-model framework, with no QCD corrections. Factorization theorems exist for the leading-twist structure functions in single-hadron production. Such theorems should be applicable also to two-hadron production if $M_h \ll Q$. Then, the resulting structure functions can be written in the same form as the parton-model results. At subleading twist, the proof of factorization is still pending. Nevertheless, the parton-model expressions are assumed to be valid also in this case.

Therefore, for a specific partial wave (ℓ, m) each structure function occurring in Eqs. (22)–(27) can be written as a sum of convolutions with the form

$$\mathcal{I}[wfD] = \sum_q e_q^2 \int d^2\boldsymbol{k}_T d^2\boldsymbol{p}_T \delta^2\left(\boldsymbol{k}_T - \boldsymbol{p}_T - \frac{\boldsymbol{P}_{h\perp}}{z}\right)$$
$$\times w_m(x, z_h, M_h, \phi_h, \boldsymbol{k}_T, \boldsymbol{p}_T)$$
$$\times x f^q(x, \boldsymbol{k}_T) D^{q|\ell,m\rangle}(z_h, M_h, |\boldsymbol{p}_T|), \quad (37)$$

where the fragmentation functions $D^{q|\ell,m\rangle}$ have been defined in Eqs. (34)–(36) for the leading twist case (and similarly for higher twists).

The leading twist structure functions for unpolarized beam and unpolarized target are

$$F_{UU,L}^{P_{\ell,m}\cos(m(\phi_h - \phi_{R_\perp}))} = 0, \quad (38)$$

$$F_{UU,T}^{P_{\ell,m}\cos(m(\phi_h - \phi_{R_\perp}))} = \mathcal{I}\left[\cos(m(\phi_h - \phi_p)) f_1 D_1^{|\ell,m\rangle}\right], \quad (39)$$

$$F_{UU}^{P_{\ell,m}\cos((2-m)\phi_h + m\phi_{R_\perp})}$$
$$= -\mathcal{I}\left[\frac{|\boldsymbol{k}_T||\boldsymbol{p}_T|}{MM_h}\cos((m-2)\phi_h + \phi_k + (1-m)\phi_p) h_1^\perp H_1^{\perp|\ell,m\rangle}\right], \quad (40)$$

while the twist-3 structure functions are

$$F_{UU}^{P_{\ell,m}\cos((1-m)\phi_h + m\phi_{R_\perp})}$$
$$= -\frac{2M}{Q}\mathcal{I}\left[\frac{|\boldsymbol{p}_T|}{M_h}\cos((m-1)\phi_h + (1-m)\phi_p)\right.$$
$$\times \left(xhH_1^{\perp|\ell,m\rangle} + \frac{M_h}{M}f_1\frac{\tilde{D}^{\perp|\ell,m\rangle}}{z}\right)$$
$$+ \frac{|\boldsymbol{k}_T|}{M}\cos((m-1)\phi_h + \phi_k - m\phi_p)$$
$$\left.\times \left(xf^\perp D_1^{|\ell,m\rangle} + \frac{M_h}{M}h_1^\perp\frac{\tilde{H}^{|\ell,m\rangle}}{z}\right)\right]. \quad (41)$$

The leading twist structure functions for longitudinally polarized beam and unpolarized target are

$$F_{LU,L}^{P_{\ell,m}\sin(m(\phi_h - \phi_{R_\perp}))} = 0, \quad (42)$$

$$F_{LU,T}^{P_{\ell,m}\sin(m(\phi_h - \phi_{R_\perp}))} = -\mathcal{I}\left[2\cos(m(\phi_h - \phi_p)) f_1 G_1^{|\ell,m\rangle}\right], \quad (43)$$

while the twist-3 structure functions are

$$F_{LU}^{P_{\ell,m}\sin((1-m)\phi_h + m\phi_{R_\perp})}$$
$$= \frac{2M}{Q}\mathcal{I}\left[-\frac{|\boldsymbol{p}_T|}{M_h}\cos((1-m)(\phi_p - \phi_h))\right.$$
$$\times \left(xeH_1^{\perp|\ell,m\rangle} + \frac{M_h}{M}f_1\frac{\tilde{G}^{\perp|\ell,m\rangle}}{z}\right)$$
$$+ \frac{|\boldsymbol{k}_T|}{M}\cos((m-1)\phi_h + \phi_k - m\phi_p)$$
$$\left.\times \left(xg^\perp D_1^{|\ell,m\rangle} + \frac{M_h}{M}h_1^\perp\frac{\tilde{E}^{|\ell,m\rangle}}{z}\right)\right]. \quad (44)$$

The leading twist structure functions for unpolarized beam and longitudinally polarized target are

$$F_{UL}^{P_{\ell,m}\sin((2-m)\phi_h + m\phi_{R_\perp})}$$
$$= -\mathcal{I}\left[\frac{|\boldsymbol{k}_T||\boldsymbol{p}_T|}{MM_h}\cos((m-2)\phi_h + \phi_k + (1-m)\phi_p) h_{1L}^\perp H_1^{\perp|\ell,m\rangle}\right], \quad (45)$$

$$F_{UL}^{P_{\ell,m}\sin(m(\phi_h-\phi_{R_\perp}))} = -\mathcal{I}\left[2\cos(m(\phi_h-\phi_p))g_{1L}G_1^{|\ell,m\rangle}\right],\tag{46}$$

while the twist-3 structure functions are

$$F_{UL}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp})} = \frac{2M}{Q}\mathcal{I}\left[-\frac{|\boldsymbol{p}_T|}{M_h}\cos((1-m)(\phi_p-\phi_h))\left(xh_LH_1^{\perp|\ell,m\rangle}+\frac{M_h}{M}g_{1L}\frac{\tilde{G}^{\perp|\ell,m\rangle}}{z}\right)\right.$$
$$\left.+\frac{|\boldsymbol{k}_T|}{M}\cos((m-1)\phi_h+\phi_k-m\phi_p)\left(xf_L^\perp D_1^{|\ell,m\rangle}-\frac{M_h}{M}h_{1L}^\perp\frac{\tilde{H}^{|\ell,m\rangle}}{z}\right)\right].\tag{47}$$

The leading twist structure functions for longitudinally polarized beam and longitudinally polarized target are

$$F_{LL}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))} = \mathcal{I}[\cos(m(\phi_h-\phi_p))g_{1L}D_1^{|\ell,m\rangle}],\tag{48}$$

while the twist-3 structure functions are

$$F_{LL}^{P_{\ell,m}\cos((1-m)\phi_h+m\phi_{R_\perp})} = \frac{2M}{Q}\mathcal{I}\left[\frac{|\boldsymbol{p}_T|}{M_h}\cos((1-m)(\phi_p-\phi_h))\left(xe_LH_1^{\perp|\ell,m\rangle}-\frac{M_h}{M}g_{1L}\frac{\tilde{D}^{\perp|\ell,m\rangle}}{z}\right)\right.$$
$$\left.-\frac{|\boldsymbol{k}_T|}{M}\cos((m-1)\phi_h+\phi_k-m\phi_p)\left(xg_L^\perp D_1^{|\ell,m\rangle}+\frac{M_h}{M}h_{1L}^\perp\frac{\tilde{E}^{|\ell,m\rangle}}{z}\right)\right].\tag{49}$$

The leading twist structure functions for unpolarized beam and transversely polarized target are

$$F_{UT,L}^{P_{\ell,m}\sin((1+m)\phi_h-m\phi_{R_\perp}-\phi_S)} = 0,\tag{50}$$

$$F_{UT,T}^{P_{\ell,m}\sin((1+m)\phi_h-m\phi_{R_\perp}-\phi_S)} = -\mathcal{I}\left[\frac{|\boldsymbol{k}_T|}{M}\cos(\phi_k+m\phi_p-(1+m)\phi_h)(f_{1T}^\perp D_1^{|\ell,m\rangle}+\text{sign}[m]g_{1T}G_1^{|\ell,m\rangle})\right],\tag{51}$$

$$F_{UT}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp}+\phi_S)} = -\mathcal{I}\left[\frac{|\boldsymbol{p}_T|}{M_h}\cos((1-m)(\phi_p-\phi_h))h_1H_1^{\perp|\ell,m\rangle}\right],\tag{52}$$

$$F_{UT}^{P_{\ell,m}\sin((3-m)\phi_h+m\phi_{R_\perp}-\phi_S)} = -\mathcal{I}\left[\frac{|\boldsymbol{k}_T|^2|\boldsymbol{p}_T|}{2M^2M_h}\cos((m-3)\phi_h+2\phi_k+(1-m)\phi_p)h_{1T}^\perp H_1^{\perp|\ell,m\rangle}\right].\tag{53}$$

and at twist-3 the structure functions are

$$F_{UT}^{P_{\ell,m}\sin(\phi_S)} = \frac{2M}{Q}\mathcal{I}\left\{\cos(m(\phi_h-\phi_p))\left(xf_TD_1^{|\ell,m\rangle}-\frac{M_h}{M}h_1\frac{\tilde{H}}{z}\right)-\frac{1}{2}\frac{|\boldsymbol{k}_T||\boldsymbol{p}_T|}{MM_h}\cos(m\phi_h+\phi_k-(m+1)\phi_p)\right.$$
$$\left.\times\left[\left(xh_TH_1^{\perp|\ell,m\rangle}+\frac{M_h}{M}g_{1T}\frac{\tilde{G}^{\perp|\ell,m\rangle}}{z}\right)-\left(xh_T^\perp H_1^{\perp|\ell,m\rangle}-\frac{M_h}{M}f_{1T}^\perp\frac{\tilde{D}^{\perp|\ell,m\rangle}}{z}\right)\right]\right\},\tag{54}$$

$$F_{UT}^{P_{\ell,m}\sin((2-m)\phi_h+m\phi_{R_\perp}-\phi_S)} = \frac{2M}{Q}\mathcal{I}\left\{\frac{|\boldsymbol{k}_T|^2}{2M^2}\cos((m-2)\phi_h+2\phi_k-m\phi_p)\left(xf_T^\perp D_1^{|\ell,m\rangle}-\frac{M_h}{M}h_{1T}^\perp\frac{\tilde{H}}{z}\right)\right.$$
$$-\frac{1}{2}\frac{|\boldsymbol{k}_T||\boldsymbol{p}_T|}{MM_h}\cos((m-2)\phi_h+\phi_k+(1-m)\phi_p)\left[\left(xh_TH_1^{\perp|\ell,m\rangle}+\frac{M_h}{M}g_{1T}\frac{\tilde{G}^{\perp|\ell,m\rangle}}{z}\right)\right.$$
$$\left.\left.+\left(xh_T^\perp H_1^{\perp|\ell,m\rangle}-\frac{M_h}{M}f_{1T}^\perp\frac{\tilde{D}^{\perp|\ell,m\rangle}}{z}\right)\right]\right\}.\tag{55}$$

The leading twist structure functions for longitudinally polarized beam and transversely polarized target are

$$F_{LT}^{P_{\ell,m}\cos((1-m)\phi_h+m\phi_{R_\perp}-\phi_S)} = \mathcal{I}\left[\frac{|\boldsymbol{k}_T|}{M}\cos((m-1)\phi_h+\phi_k-m\phi_p)(g_{1T}D_1^{|\ell,m\rangle}+\mathrm{sign}[m]f_{1T}^\perp G_1^{|\ell,m\rangle})\right], \qquad (56)$$

while the twist-3 structure functions are

$$F_{LT}^{P_{\ell,m}\cos(\phi_S)} = \frac{2M}{Q}\mathcal{I}\left\{-\cos(m(\phi_h-\phi_p))\left(xg_TD_1^{|\ell,m\rangle}+\frac{M_h}{M}h_1\frac{\tilde{E}}{z}\right)+\frac{1}{2}\frac{|\boldsymbol{k}_T||\boldsymbol{p}_T|}{MM_h}\cos(m\phi_h+\phi_k-(m+1)\phi_p)\right.$$
$$\left.\times\left[\left(xe_TH_1^{\perp|\ell,m\rangle}-\frac{M_h}{M}g_{1T}\frac{\tilde{D}^{\perp|\ell,m\rangle}}{z}\right)+\left(xe_T^\perp H_1^{\perp|\ell,m\rangle}+\frac{M_h}{M}f_{1T}^\perp\frac{\tilde{G}^{\perp|\ell,m\rangle}}{z}\right)\right]\right\}, \qquad (57)$$

$$F_{LT}^{P_{\ell,m}\cos((2-m)\phi_h+m\phi_{R_\perp}-\phi_S)} = \frac{2M}{Q}\mathcal{I}\left\{-\frac{|\boldsymbol{k}_T|^2}{2M^2}\cos((m-2)\phi_h+2\phi_k-m\phi_p)\left(xg_T^\perp D_1^{|\ell,m\rangle}+\frac{M_h}{M}h_{1T}^\perp\frac{\tilde{E}}{z}\right)\right.$$
$$+\frac{1}{2}\frac{|\boldsymbol{k}_T||\boldsymbol{p}_T|}{MM_h}\cos((m-2)\phi_h+\phi_k+(1-m)\phi_p)\left[\left(xe_TH_1^{\perp|\ell,m\rangle}-\frac{M_h}{M}g_{1T}\frac{\tilde{D}^{\perp|\ell,m\rangle}}{z}\right)\right.$$
$$\left.\left.-\left(xe_T^\perp H_1^{\perp|\ell,m\rangle}+\frac{M_h}{M}f_{1T}^\perp\frac{\tilde{G}^{\perp|\ell,m\rangle}}{z}\right)\right]\right\}. \qquad (58)$$

V. RELATION WITH EXISTING NOMENCLATURE

If the final hadronic system is made of pairs of mesons, then in Fig. 1 we can have $\lambda, \lambda' = 0, 1$. Then, in the direct sum basis 16 states can be formed:

$$(1\oplus 0)\otimes(1\oplus 0) = 2\oplus 1\oplus 1\oplus 1\oplus 0\oplus 0. \qquad (59)$$

In reality, the cross section appears as if there were 9 states with ℓ taking the values $\ell = 0, 1, 2$, where the $\ell = 1$ and $\ell = 0$ states are three times and two times degenerate, respectively. The $\ell = 0$ states can be distinguished, because one of them is the cross section for a single pseudoscalar meson production, while the other one is the angular integrated dihadron cross section. The $\ell = 1$ states are not experimentally distinguishable. They contain a contribution from the interference of relative partial waves s and p ($[\lambda = 0]\otimes[\lambda' = 1]$ and vice versa), and from the interference of two p waves ($[\lambda = 1]\otimes[\lambda' = 1]$), in agreement with Ref. [22].

We now clarify our notation by recovering known results in the literature for specific final hadronic systems.

A. Single-hadron SIDIS

For the production of a pseudoscalar meson, only the $|0,0\rangle$ final state polarization is possible. Hence, for the case $\ell = 0$, $m = 0$, the cross sections (22)–(27) with the structure functions (38)–(58) reduce to the ones in Ref. [1] with the following obvious identifications:

$$D_1^{|0,0\rangle} = D_1, \qquad H_1^{\perp|0,0\rangle} = H_1^\perp, \qquad \tilde{H}^{|0,0\rangle} = \tilde{H}, \qquad (60)$$

$$\tilde{D}^{\perp|0,0\rangle} = \tilde{D}^\perp, \qquad \tilde{G}^{\perp|0,0\rangle} = \tilde{G}^\perp, \qquad \tilde{E}^{|0,0\rangle} = \tilde{E}. \qquad (61)$$

The D_1 and $H_1^\perp$ are the usual unpolarized fragmentation function and the Collins function, respectively, and correspond to the reduction of Eqs. (34)–(36) to the case $\ell = 0$, $m = 0$. In this limit, no contribution emerges from the class of fragmentation functions G_1.

B. Two-hadron SIDIS

If the final state is represented by two mesons, the cross-check with existing literature can be made by comparing Eqs. (22)–(27), including the leading twist contributions to the structure functions of Eqs. (38)–(58), with Eqs. (C4)–(C10) of Ref. [22]. For the chiral-even fragmentation functions we have

$$D_1^{|0,0\rangle} = \frac{1}{4}D_{1,OO}^s+\frac{3}{4}D_{1,OO}^p, \qquad (62)$$

$$D_1^{|1,0\rangle} = D_{1,OL}, \qquad D_1^{|1,1\rangle} = D_1^{|1,-1\rangle} = \frac{1}{2}D_{1,OT}, \qquad (63)$$

$$D_1^{|2,0\rangle} = \frac{1}{2}D_{1,LL}, \qquad D_1^{|2,1\rangle} = D_1^{|2,-1\rangle} = \frac{1}{4}D_{1,LT}, \qquad (64)$$

$$D_1^{|2,2\rangle} = D_1^{|2,-2\rangle} = \frac{1}{2}D_{1,TT}, \qquad (65)$$

$$G_1^{|0,0\rangle} = G_1^{|1,0\rangle} = G_1^{|2,0\rangle} = 0, \qquad (66)$$

$$G_1^{|1,1\rangle} = G_1^{|1,-1\rangle} = -\frac{|\boldsymbol{p}_T||\boldsymbol{R}|}{2M_h^2}G_{1,OT}^\perp, \qquad (67)$$

$$G_1^{|2,1\rangle} = G_1^{|2,-1\rangle} = -\frac{|\boldsymbol{p}_T||\boldsymbol{R}|}{4M_h^2} G_{1,LT}^\perp, \qquad (68)$$

$$G_1^{|2,2\rangle} = G_1^{|2,-2\rangle} = -\frac{|\boldsymbol{p}_T||\boldsymbol{R}|}{4M_h^2} G_{1,TT}^\perp, \qquad (69)$$

while for the chiral-odd function,

$$H_1^{\perp|0,0\rangle} = \frac{1}{4} H_{1,OO}^{\perp s} + \frac{3}{4} H_{1,OO}^{\perp p}, \qquad (70)$$

$$H_1^{\perp|1,0\rangle} = H_{1,OL}^\perp, \qquad H_1^{\perp|2,0\rangle} = \frac{1}{2} H_{1,LL}^\perp, \qquad (71)$$

$$H_1^{\perp|1,1\rangle} = \frac{|\boldsymbol{R}|}{|\boldsymbol{p}_T|} H_{1,OT}^\sphericalangle, \qquad H_1^{\perp|1,-1\rangle} = H_{1,OT}^\perp, \qquad (72)$$

$$H_1^{\perp|2,1\rangle} = \frac{|\boldsymbol{R}|}{2|\boldsymbol{p}_T|} H_{1,LT}^\sphericalangle, \qquad H_1^{\perp|2,-1\rangle} = \frac{1}{2} H_{1,LT}^\perp, \qquad (73)$$

$$H_1^{\perp|2,2\rangle} = \frac{|\boldsymbol{R}|}{|\boldsymbol{p}_T|} H_{1,TT}^\sphericalangle, \qquad H_1^{\perp|2,-2\rangle} = H_{1,TT}^\perp. \qquad (74)$$

Using the above relations, one can then cross-check the formulas listed in Secs. II and IV. There is consistency between the published literature and the present work, although in some case there are typographical errors (for a detailed list, see Appendix B).

VI. CONCLUSION

In this paper, we have presented a slightly modified definition of the fragmentation functions compared to, e.g., Ref. [24]. We have proposed a new partial wave expansion for fragmentation functions, which allows a consistent framework for fragmentation into final states of any polarization.

This not only helps in the interpretation of cross section moments, but also has the advantage that the two-hadron SIDIS cross sections, at any twist, can be derived from single-hadron SIDIS. Using this method, in this paper we present for the first time the expression of the two-hadron SIDIS cross section up to subleading twist, including the dependence upon the transverse momentum of involved particles.

The cross section has also been given in terms of structure functions, and the resulting expressions have been cross-checked with existing literature for specific cases.

ACKNOWLEDGMENTS

APPENDIX A: DEFINITION OF AZIMUTHAL ANGLES

As explained in Sec. II A, the SIDIS cross section for dihadron production depends also on the azimuthal angle $\phi_{R_\perp}$ of the vector R_T measured in the plane perpendicular to (P, q), where R_T is given by Eq. (9) and $\phi_{R_\perp}$ is defined by

$$\cos\phi_{R_\perp} = -\frac{l_\mu R_{T\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 R_{T\perp}^2}},$$

$$\sin\phi_{R_\perp} = -\frac{l_\mu R_{T\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 R_{T\perp}^2}}, \qquad (A1)$$

with $l_\perp^\mu = g_\perp^{\mu\nu} l_\nu$ and $R_{T\perp}^\mu = g_\perp^{\mu\nu} R_{T_\nu}$.

Depending on the reference frame, the vector R_T can have a nonvanishing component along $\boldsymbol{q}$, but $g_\perp^{\mu\nu}$ projects out only its spatial components transverse to $\boldsymbol{q}$. Hence, in order to compare with other noncovariant definitions we inspect in the following the expressions of only $\boldsymbol{R}_{T_\perp} = \{R_{T_x}, R_{T_y}\}$.

The most natural choice of frame is the target rest frame (TRF). There, from Eq. (9) we have

$$\boldsymbol{R}_{T\perp}|_{\text{TRF}} = \frac{z_2 \boldsymbol{P}_{1T} - z_1 \boldsymbol{P}_{2T}}{z} + \mathcal{O}\left(\frac{1}{Q^3}\right). \qquad (A2)$$

The above result coincides (up to corrections of order $1/Q^3$) with the transverse spatial components of $\boldsymbol{R} - \boldsymbol{P}_h \boldsymbol{R} \cdot \boldsymbol{P}_h / \boldsymbol{P}_h^2$, which is the definition of $\boldsymbol{R}_T$ used in the analysis of dihadron production from SIDIS data by the HERMES Collaboration [29]. It is also equal, in the same limit, to the definition used in Ref. [33], that has been adopted in the analyses of dihadron production from SIDIS data by the COMPASS Collaboration [34] and from e^+e^- annihilation data by the Belle Collaboration [35].

If we boost all four-vectors to the so-called infinite momentum frame (IMF), where the momentum of the virtual photon is purely spacelike, our definition reduces to

$$\boldsymbol{R}_{T\perp}|_{\text{IMF}} = \frac{z_2 \boldsymbol{P}_{1T} - z_1 \boldsymbol{P}_{2T}}{z} + \mathcal{O}\left(\frac{1}{Q^2}\right), \qquad (A3)$$

which again coincides with all other noncovariant definitions, but now up to corrections of order $1/Q^2$. We find the same result if we boost the four-vectors to the Breit frame

of the virtual photon-proton system, i.e. where the vector $q + P$ is purely timelike.

In conclusion, we find that our definition of the azimuthal angle $\phi_{R_\perp}$ of the vector $R_{T\perp}^\mu = g_\perp^{\mu\nu} R_{T\nu}$, with R_T given in Eq. (9), is equivalent to all other definitions found in the literature up to corrections of order $1/Q^2$. In the target rest frame, the equivalence with the definition of Ref. [29] holds including $1/Q^2$ corrections, i.e. up to correction of order $1/Q^3$. Of course, our definition is preferable because it is covariant, hence valid in any frame. Recently, a new definition appeared [36] which in the notation of this paper reads $\boldsymbol{R}_\perp = (\boldsymbol{P}_{1\perp} - \boldsymbol{P}_{2\perp})/2$; this definition is different from all the other ones (see also the discussion in Ref. [37]).

APPENDIX B: CROSS-CHECK OF STRUCTURE FUNCTIONS

As explained in Sec. V B, the formulas for the cross sections and structure functions listed in Secs. II and IV, respectively, can be cross-checked at the leading twist level with Eqs. (C4)–(C10) of Ref. [22]. In the cross-check, the different convention in the definition of azimuthal angles must be accounted for, because Ref. [22] was published before the release of the so-called Trento conventions [28]. There is a general consistency between the two groups of equations, but the latter one displays some typographical errors that are listed here below.

For unpolarized beam and target, the cross section $d\sigma_{UU}$ of Eq. (C4) can be divided in two groups enclosed in braces, the former multiplied by $A(y)$ and the latter by $B(y)$. The third term of the former group, involving the function $D_{1,OT}$ and corresponding to the component $\ell = 1, m = \pm 1$ in Eq. (22), should change sign. In the latter group, a term proportional to

$$\cos\vartheta\cos 2\phi_h \mathcal{I}\left[\frac{2\boldsymbol{k}_T\cdot\hat{\boldsymbol{P}}_{h\perp}\boldsymbol{p}_T\cdot\hat{\boldsymbol{P}}_{h\perp} - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}h_1^\perp H_{1,OL}^\perp\right]$$

is missing, that corresponds to the component $\ell = 1$, $m = 0$ in Eq. (22).

For unpolarized beam and longitudinally polarized target, the cross section $d\sigma_{UL}$ of Eq. (C6) can also be divided in two groups enclosed in braces, the former multiplied by $A(y)$ and the latter by $B(y)$. The overall $(-)$ sign in front of the former group should be dropped. The first term of the latter group, involving $\frac{1}{4}H_{1,OO}^{\perp s} + \frac{3}{4}H_{1,OO}^{\perp p}$ and corresponding to the component $\ell = 0$, $m = 0$ in Eq. (24), should change sign. Finally, a term proportional to

$$\cos\vartheta\sin 2\phi_h \mathcal{I}\left[\frac{2\boldsymbol{k}_T\cdot\hat{\boldsymbol{P}}_{h\perp}\boldsymbol{p}_T\cdot\hat{\boldsymbol{P}}_{h\perp} - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}h_{1L}^\perp H_{1,OL}^\perp\right]$$

is missing, that corresponds to the component $\ell = 1$, $m = 0$ in Eq. (24).

For longitudinally polarized beam and target, in the cross section $d\sigma_{LL}$ of Eq. (C7) the fourth term proportional to $\sin\vartheta\cos(\phi_h - \phi_{R_\perp})$ should involve the function $D_{1,OT}$. It corresponds to the component $\ell = 1$, $m = 1$ in Eq. (25).

For unpolarized beam and transversely polarized target, the cross section $d\sigma_{UT}$ of Eq. (C8) can also be divided in two groups enclosed in braces, the former multiplied by $A(y)$ and the latter by $B(y)$. The eighth term of the former group, involving the function $D_{1,OL}$ and corresponding to the component $\ell = 1$, $m = 0$ in Eq. (26), should read

$$\cos\vartheta\sin(\phi_h - \phi_S)\mathcal{I}\left[\frac{\boldsymbol{k}_T\cdot\hat{\boldsymbol{P}}_{h\perp}}{M}f_{1T}^\perp D_{1,OL}\right].$$

In the latter group, the first and ninth terms, corresponding to the component $\ell = 0$, $m = 0$ in Eq. (26), should read

$$\sin(\phi_h + \phi_S)\mathcal{I}\left[\frac{\boldsymbol{p}_T\cdot\hat{\boldsymbol{P}}_{h\perp}}{M_h}h_1\left(\frac{1}{4}H_{1,OO}^{\perp s} + \frac{3}{4}H_{1,OO}^{\perp p}\right)\right],$$

$$-\sin(3\phi_h - \phi_S)\mathcal{I}\left[\frac{4(\boldsymbol{k}_T\cdot\hat{\boldsymbol{P}}_{h\perp})^2\boldsymbol{p}_T\cdot\hat{\boldsymbol{P}}_{h\perp} - 2\boldsymbol{k}_T\cdot\hat{\boldsymbol{P}}_{h\perp}\boldsymbol{k}_T\cdot\boldsymbol{p}_T - \boldsymbol{k}_T^2\boldsymbol{p}_T\cdot\hat{\boldsymbol{P}}_{h\perp}}{M^2 M_h}h_{1T}^\perp\left(\frac{1}{4}H_{1,OO}^{\perp s} + \frac{3}{4}H_{1,OO}^{\perp p}\right)\right].$$

Finally, the terms

$$-\cos\vartheta\sin(\phi_h + \phi_S)\mathcal{I}\left[\frac{\boldsymbol{p}_T\cdot\hat{\boldsymbol{P}}_{h\perp}}{M_h}h_1 H_{1,OL}^\perp\right],$$

$$\cos\vartheta\sin(3\phi_h - \phi_S)\mathcal{I}\left[\frac{4(\boldsymbol{k}_T\cdot\hat{\boldsymbol{P}}_{h\perp})^2\boldsymbol{p}_T\cdot\hat{\boldsymbol{P}}_{h\perp} - 2\boldsymbol{k}_T\cdot\hat{\boldsymbol{P}}_{h\perp}\boldsymbol{k}_T\cdot\boldsymbol{p}_T - \boldsymbol{k}_T^2\boldsymbol{p}_T\cdot\hat{\boldsymbol{P}}_{h\perp}}{M^2 M_h}h_{1T}^\perp H_{1,OL}^\perp\right]$$

are missing, that correspond to the component $\ell = 1$, $m = 0$ in Eq. (26).

For longitudinally polarized beam and transversely polarized target, in the cross section $d\sigma_{LT}$ of Eq. (C9) the terms

$$-\cos\vartheta\cos(\phi_h-\phi_S)\mathcal{I}\left[\frac{\boldsymbol{k}_T\cdot\hat{\boldsymbol{P}}_{h\perp}}{M}g_{1T}D_{1,OL}\right],$$

$$-\sin\vartheta\cos(\phi_{R_\perp}-\phi_S)\mathcal{I}\left[\frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M}\left(g_{1T}\frac{1}{2|\boldsymbol{p}_T|}D_{1,OT}-f_{1T}^\perp\frac{|\boldsymbol{R}|}{2M_h^2}G_{1,OT}^\perp\right)\right],$$

$$-\sin\vartheta\cos(2\phi_h-\phi_{R_\perp}-\phi_S)\mathcal{I}\left[\frac{2\boldsymbol{k}_T\cdot\hat{\boldsymbol{P}}_{h\perp}\boldsymbol{p}_T\cdot\hat{\boldsymbol{P}}_{h\perp}-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M}\left(g_{1T}\frac{1}{2|\boldsymbol{p}_T|}D_{1,OT}+f_{1T}^\perp\frac{|\boldsymbol{R}|}{2M_h^2}G_{1,OT}^\perp\right)\right]$$

are missing, that correspond to the components $\ell=1$, $m=0$, and $\ell=1$, $m=1$, and $\ell=1$, $m=-1$, respectively, in Eq. (27).

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