A novel chiral consedate contribution in inclusive deep inelastic scattering

Alberto Accardi[a], Alessandro Bacchetta[b]
[a] Jefferson Lab, Newport News, VA 23606, USA,
and Hampton University, Hampton, VA 23668, USA
[b] Dipartimento di Fisica, Università degli Studi di Pavia, and INFN, Sez. di Pavia, 27100 Pavia, Italy

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I. INTRODUCTION

II. JET CORRELATOR AND TWIST-2 STRUCTURE FUNCTIONS

Motivated by large-x mass corrections to inclusive DIS structure functions, Accardi and Qiu have introduced in the LO handbag diagram a "jet correlator" that accounts for invariant mass production in the current jet, and ensures that leading twist calculations in collinear factorization are consistent with the requirement imposed by baryon number conservation that $x_B < 1$ [**?**]. The jet correlator is depicted in Figure **??**(a) and is defined as

$$\Xi_{ij}(l, n_+) = \int \frac{d^4\eta}{(2\pi)^4}\, e^{il\cdot\eta}\, \langle 0|\, \mathcal{U}^{n_+}_{(+\infty,\eta)}\, \psi_i(\eta)\bar{\psi}_j(0)\, \mathcal{U}^{n_+}_{(0,+\infty)}\, |0\rangle \ , \tag{1}$$

In this definition, l is the quark four-momentum, Ψ the quark field operator, and $\|0\rangle$ is the nonperturbative vacuum. Furthermore, we explicitly guarantee the correlator's gauge invariance by introducing two Wilson line operators $\mathcal{U}^{n_+}$ along a light-cone plus direction determined by the vector n_+. This path choice for the Wilson line is required by QCD factorization theorems, and the vector will be determined by the particular hard process to which the jet correlator contributes. For example, in the case of inclusive DIS discussed in this paper, this is determined by the four momentum transfer q and the proton's momentum p.

The correlator Ξ can be parametrized in terms of scalar functions, using the vectors l and n_+

$$\Xi(l, n_+) = \Lambda A_1(l^2)\, \mathbf{1} + A_2(l^2)\not{l} + \frac{\Lambda^2}{l\cdot n_+}\not{n}_+\, B_1(l^2) + \frac{i\Lambda}{2P\cdot n_+}[\not{l}\, , \not{n}_+]\, B_2(l^2) \ . \tag{2}$$

Time reversal invariance in QCD requires $B_2 = 0$, while B_1 contributes only at twist-4 order, and will not be considered further in this paper. We focus on the role of chiral odd terms in the g_2 structure function up to twist 3. At this order,

$$\Xi(l, n_+) = \Lambda A_1(l^2)\, \mathbf{1} + A_2(l^2)\not{l} + O(\Lambda/Q^2) \tag{3}$$

is nothing else than the full quark propagator; note however, that we consider here the full QCD vacuum rather than the perturbative one.

The A_1 and A_2 terms can be nicely interpreted in terms of the spectral representation of the cut quark propagator,

$$\Xi(l) = \int d\mu^2 \left[J_1(\mu^2)\, \mu + J_2(\mu^2)\not{l}\,\right] \delta(l^2 - m_j^2) \ , \tag{4}$$

where μ^2 is interpreted as the invariant mass of the current jet, *i.e.*, of the particles going through the cut in the top blob of Fig.**??**(a), and the J_i are the spectral functions of the quark propagator, that have been also called "jet functions" in [**?**]. These satisfy [**? ?**]

$$J_2(\mu^2) \geq J_1(\mu^2) \geq 0 \quad \text{and} \quad \int d\mu^2 J_2(\mu^2) = 1 \ . \tag{5}$$

From a comparison of Eqns.(2) and (4), one can see that

$$A_1(l^2) = \frac{\sqrt{l^2}}{\Lambda} J_1(l^2) \qquad\qquad A_2(l^2) = J_2(l^2) \ . \tag{6}$$

When inserting the jet correlator in the handbag diagram for inclusive DIS, the invariant jet mass μ^2 is integrated from 0 to $Q^2(1/x_B - 1)$. This induces (kinematical) corrections of order $O(1/Q^2)$, whose effect on the F_2 structure function has been studied in Ref. [?]. In this paper we limit our attention to effects of order $O(1/Q)$ and therefore can extend the integration to $\mu^2 = \infty$. For example, the unpolarized DIS structure function F_2 reads

$$F_2(x_B) = \int_0^{Q^2(1/x_B-1)} d\mu^2\, J_2(\mu^2) F_2^{(0)}(x_B(1+\mu^2/Q^2), Q^2) \tag{7}$$

$$= \Big(\int_0^\infty d\mu^2\, J_2(\mu^2) \Big) F_2^{(0)}(x_B, Q^2) + O(\Lambda^2/Q^2) \tag{8}$$

$$= F_2^{(0)}(x_B, Q^2) + O(\Lambda^2/Q^2)\ , \tag{9}$$

where $F_2^{(0)}$ is the structure function calculated with the handbag diagram sporting a bare quark propagator instead of the jet correlator. At this order the jet function J_2 decouples and thanks to the sum rule (5) it integrates to 1, so that one recovers the conventional result. The same holds true also for the helicity structure function g_1. More in general, the jet correlator decouples from the parton correlator Φ in inclusive cross section calculation up to $O(1/Q)$, on which we are focusing the attention in this paper, and the inclusive structure functions depend on the integrated jet correlator

$$\Xi \equiv \int \frac{dl^2}{2l^-} d^2 l_T\, \Xi(l) = \frac{\Lambda}{2l^-}\, \xi_1 \mathbf{1} + \xi_2 \frac{\not{n}_-}{2} + \text{higher twists} \tag{10}$$

where

$$\xi_1 = \int d\mu^2 \frac{\mu}{\Lambda} J_1(\mu^2) \equiv \frac{M_q}{\Lambda}, \qquad\qquad \xi_2 = \int d\mu^2 J_2(\mu^2) = 1\ . \tag{11}$$

It is important to notice that $\xi_2 = 1$ exactly due to CPT invariance [?], while $0 < M_q < \int d\mu^2 \mu J_2(\mu^2)$ is dynamically determined. From the analytic properties of spectral functions we may expect [?] $J_2(\mu^2) = Z\delta(\mu^2 - m_q) + \bar{J}_2(\mu^2)\theta(\mu^2 - m_\pi)$ with the continuum starting at m_π, the mass of the pion, due to color confinement effects. Taking into account that $J_1 < J_2$, we may therefore expect

$$M_q = O(10 - 100\ \text{MeV})\ . \tag{12}$$

Although M_q is in general a nonperturbative quantity, it is interesting to notice that

$$M_q = \frac{\Lambda}{4} \int \text{Tr}\left[\Xi(l)\mathbf{1}\right] = \langle 0|\bar{\psi}_i(0)\psi_i(0)|0\rangle \tag{13}$$

Calculating this on the perturbative vacuum and limiting oneself to LO corresponds to taking the trace of the cut bare-quark propagator to obtain $M_q = {}_{\text{pert}}\langle 0|\bar{\psi}_i(0)\psi_i(0)|0\rangle_{\text{pert}} = m_q$, recovering the conventional result. However, we are here considering non perturbative effects on the quark fragmentation and $M_q \gtrsim m_q$.

III. TWIST-3 ANALYSIS

Extending the analysis of [?] to the calculation of twist-3 structure functions requires not only to consider the ξ_1 term in the jet correlator, but also quark-gluon-quark correlators in both the proton and the vacuum as depicted in Figs.**??**(b) and (c), respectively. In the former the ξ_1 terms contribute to $O(1/Q^2)$, so that up $O(1/Q)$ these give the same contribution as in the conventional handbag calculation.

The novel element in our analysis is the jet quark-gluon quark correlators $\Xi^\mu_A(l,k)$ in diagrams **??**(c),

$$\left(\Xi^\mu_A\right)_{ij} = \frac{1}{2} \sum_X \int \frac{d\eta^+\, d^2\boldsymbol{\eta}_T}{(2\pi)^3}\, e^{ik\cdot\eta}\, \langle 0|\, \mathcal{U}^{n_+}_{(+\infty,\eta)}\, gA^\mu(\eta)\ \psi_i(\eta)|X\rangle\langle X|\bar{\psi}_j(0)\, \mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\Big|_{\eta^-=0} . \tag{14}$$

These diagrams are not only important to account for all contribution of order $O(1/Q)$, but also in restoring up to twist-3 the gauge invariance broken in diagram **??**(a) by the different mass of the incoming and outgoing quark lines, namely, $m_q \neq M_q$.

Rather than directly using the definition (14), it is convenient to calculate the inclusive cross section as an integral of the semi-inclusive one, utilize the QCD equation of motions and furthermore summed over all hadron flavors, and take advantage of

$$\sum_h \int \frac{d^3 p_h}{(2\pi) 2E_h} \Delta^h(l, p_h) = \Xi(l)\ , \tag{15}$$

where Δ^h is the quark fragmentation correlator for production of a hadron of flavor h and momentum p_h [**?**]. The ξ_1 term is chiral-odd and therefore can appear in the inclusive cross section only coupled to the transversity function h_1. Therefore, for our analysis the relevant part of the semi-inclusive hadronic tensor is **[AA] I am using the notation in Piet Mulder's lecture notes - this will need to be checked. Decide also if we want this at parton or charge-weighted level.** •

$$2\Lambda \sum_h \int dz d^2 p_{hT} = i \frac{2\Lambda}{Q} \hat{t}^{[\mu} \epsilon_\perp^{\nu]\rho} S_{\perp\rho} \Big[2x_b g_T(x_B) \sum_h \int dz d^2 p_{hT} D_1^h(z) + 2h_1(x_B) \sum_h \int dz d^2 p_{hT} \tilde{E}^h(z) \Big] + \dots \tag{16}$$

where D_1^h is the twist-2 quark fragmentation function as a function of the hadron's collinear momentum fraction z, and $\tilde{E}^h$ is a twist-3 function defined in [**?**]. The former can be easily integrated with the help of the sum rule (15):

$$\sum_h \int dz d^2 p_{hT} z D_1^h(z) = \xi_2 = 1 \ . \tag{17}$$

To integrate the latter, we first make use of the relation [**?**] $\tilde{E}(z) = E(z) - m_q z D_1(z)$, which is a consequence of the QCD equations of motion, so that

$$\sum_h \int dz d^2 p_{hT} \tilde{E}(z) = \sum_h \int dz d^2 p_{hT} \Big[E(z) - \frac{m_q}{\Lambda} z D_1 z \Big] = \xi_1 - \frac{m_q}{\Lambda} \xi_2 = \frac{M_q - m_q}{\Lambda} \ . \tag{18}$$

This formula is the single most important result of this paper.

Finally, with suitable projections of the hadronic tensor, the inclusive cross section up to order M/Q turns out to be

$$\frac{d\sigma}{dx\, dy\, d\psi} = \frac{2\alpha^2}{xyQ^2} \frac{y^2}{2\,(1-\varepsilon)} \Big\{ F_{UU,T} + \varepsilon F_{UU,L} + S_\parallel \lambda_e \sqrt{1-\varepsilon^2}\; F_{LL} \tag{19}$$

$$+ |\boldsymbol{S}_\perp| \lambda_e \sqrt{2\,\varepsilon(1-\varepsilon)}\; \cos\phi_S\, F_{LT}^{\cos\phi_S} \Big\}, \tag{20}$$

where the structure functions on the right hand side are defined as

$$F_{UU,T} = x \sum_a e_a^2\, f_1^a(x), \tag{21}$$

$$F_{UU,L} = 0, \tag{22}$$

$$F_{LL} = x \sum_a e_a^2\, g_1^a(x), \tag{23}$$

$$F_{UT}^{\sin\phi_S} = 0, \tag{24}$$

$$F_{LT}^{\cos\phi_S} = -x \sum_a e_a^2 \frac{2\Lambda}{Q} \Big(x g_T^a(x) + \frac{M_q - m_q}{\Lambda}\, h_1^a(x) \Big). \tag{25}$$

The second term in the last struture function is a new result from our analysis; it is not suppressed as an inverse power of Q, and therefore survives even in the Bjorken limit. Note that calculating the jet correlator on the perturbative vacuum one would obtain, as already discussed, $M_q = m_q$ and the new term vanishes as it should. However, on the non-perturbative vacuum the jet mass is larger than the quark's, and this contributes a non-negligible term to the twist-3 part of the g_2 function, as we shall discuss in the next section.

IV. THE g_2 STRUCTURE FUNCTION

The new term in Eq.e:FLTint only appeears in the g_2 structure function. Following the derivation in Ref. [**?**], one finds

$$g_2(x_B) = g_2^{WW} + \frac{1}{2} \sum_a e_a^2 \Big(\widetilde{g}_T^{a\star}(x) + \int_x^1 \frac{dy}{y} \widetilde{g}_T^a(y) + \frac{m_q}{M} (h_1^a/x)^\star(x) + \frac{M_q - m_q}{M} h_1^a(x) \Big) \ , \tag{26}$$

where we defined $f^*(x) = -f(x) + \int_x^1 \frac{dy}{y} f(y)$. The first 4 terms coincide with the result obtained in the conventional handbag approximation [**?**], while the fifth is new.

The first term is also known as the Wandzura-Wilczeck function $g_2^{WW} = g_1^*(x)$, and contains all the "pure twist-2" chiral even contributions to the g_2 structure coming from quark-quark correlators. The second and third terms contain all "pure twist-3" contributions, i.e., those coming from quark-gluon-quark correlators. The fourth and fifth terms depend on the transversity parton distribution function, h_1. The former is usually neglected for light quarks since it is proportional to $m_q = O(1 \text{ eV})$. In the latter term, new in our analysis, the transversity distribution is multiplied by a constant of $O(100 \text{ GeV})$, and cannot be a priori neglected.

To estimate the size of the last two terms in Eq. (26), we use a recent parametrization of the transversity distribution from Ref. [1], which is comparable also to other extractions [2, 3]. We define the shorthand notation

$$g_2^{\text{quark}} = \frac{1}{2} \sum_a e_a^2 \frac{m_q}{M} (h_1^a/x)^\star(x), \qquad g_2^{\text{jet}} = \frac{1}{2} \sum_a e_a^2 \frac{M_q - m_q}{M} h_1^a(x). \tag{27}$$

We choose the values of the mass parameters to be $m_q = 5$ MeV and $M_q = 50$ MeV.

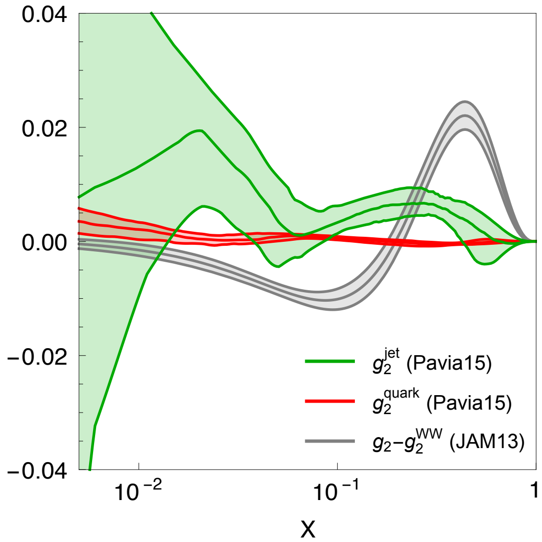

FIG. 1: Different contributions to the g_2 structure function.

- large-x: bridges Braun's pure twist-3 theory and the data
- samller-x: consttrains the small-x behavior of transversity.

Another consequence of the new chiral odd term induced by the non-vanishing of the chiral condensate on the non perturbative vacuum is that the Burkardt-Cottingham sum rule is broken:

$$\int dx\, g_2(x) = \frac{M_q - m_q}{\Lambda} \int dx\, h_1(x) \ , \tag{28}$$

where the integral over h_1 is related to target's tensor charge. Consequences:

- Inclusive DIS become sensitive to the tensor charge; furthermore, teh BC summ rule isolates the effects due to the chiral odd part of the jet correlator.
- Both the jet mass M_q and the tensor charge can in principle be calculated on the lattice
- Comparison to the Burkhardt-Cottingham sum rule can provide experimental verification of lattice calculation
- in turn these can be used to determine the size of the h_1 term in $g_2 - g_2^{WW}$ and allow an eperimental extraction of the pure twist-3 terms.

It is important to explore in which other process does M_q contribute, as to provide an experimental check of the formalism:

- inclusive Λ production in $e^+ + e^-$
- same-side dihadrons in $e^+ + e^-$

It would be cool to find a process where the M_q contribution is th eonly one (similar to the BC breaking) ...

V. CONCLUSIONS

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Acknowledgments

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