The role of jet mass corrections in polarized deep inelastic scattering

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I. INTRODUCTION

II. THE BODY

We start from the case of semi-inclusive deep inelastic scattering integrated over transverse momentum. The general formula of the cross section is (see, e.g., [**?**])

$$\begin{aligned}\frac{d\sigma}{dx\,dy\,d\psi\,dz} &= \frac{\alpha^2\,y}{8z\,Q^4}\,2MW^{\mu\nu}\,L_{\mu\nu}\\ &= \frac{2\alpha^2}{xy\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon\,F_{UU,L}+S_\parallel\lambda_e\,\sqrt{1-\varepsilon^2}\,F_{LL}\\ &\quad+|\boldsymbol{S}_\perp|\,\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+|\boldsymbol{S}_\perp|\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}\Big\}. \end{aligned} \tag{1}$$

Limiting ourselves to the leading and first subleading term in the $1/Q$ expansion of the cross section and to graphs with the hard scattering at tree level, we can express the hadronic tensor as

$$\begin{aligned}2MW^{\mu\nu} &= 2z\sum_a e_a^2\,\mathrm{Tr}\Big\{\Phi^a(x)\gamma^\mu\Delta^a(z)\gamma^\nu\\ &\quad-\frac{1}{Q\sqrt{2}}\Big[\gamma^\alpha\not{n}_+\gamma^\nu\,\tilde{\Phi}^a_{A\,\alpha}(x)\,\gamma^\mu\Delta^a(z)+\gamma^\alpha\not{n}_-\gamma^\mu\,\tilde{\Delta}^a_{A\,\alpha}(z)\,\gamma^\nu\Phi^a(x)+\text{h.c.}\Big]\Big\},\end{aligned} \tag{2}$$

with corrections of order $1/Q^2$, where the sum runs over the quark and antiquark flavors a, and e_a denotes the fractional charge of the struck quark or antiquark.

Instead of working with the fragmentation correlation function, Δ, we consider here the case where we insert a *jet correlation function* **AB: I don't know if we should include the gauge links explicitly** •

$$\Xi_{ij}(l) = \int\frac{d^4\eta}{(2\pi)^4}\,e^{ik\cdot\eta}\,\langle 0|\,\mathcal{U}^{n_+}_{(+\infty,\eta)}\,\psi_i(\eta)\bar{\psi}_j(0)\,\mathcal{U}^{n_+}_{(0,+\infty)}\,|0\rangle \tag{3}$$

The correlator can be parametrized in terms of scalar functions, using the vectors l and n_+ **AB: I don't know if we should include the last term or not. It has to do with whether or not we want/need to address the complications related to the gauge link**. •

$$\Xi(l) = \Lambda A_1(l^2)\,\mathbf{1}+A_2(l^2)\not{l}+\frac{\Lambda^2}{l\cdot n_+}\not{n}_-\,B_1(l^2)+\frac{i\Lambda}{2P\cdot n_-}[\not{l},\not{n}_+]\,B_2(l^2) \tag{4}$$

After the necessary integrations, we obtain a general form of the jet correlation function in terms of jet components

$$\Xi\equiv\int dl^+d^2l_T\,\Xi(l)\equiv\xi_1\frac{\not{n}_-}{2}+\frac{\Lambda}{2l^-}\,\xi_2\mathbf{1}+\frac{\Lambda^2}{4(l^-)^2}\,\xi_3\not{n}_++i\frac{\Lambda}{2l^-}\xi_4\frac{[\not{n}_-,\not{n}_+]}{2}. \tag{5}$$

To understand the nature of the ξs, we consider the spectral representation of the jet correlator

$$\Xi_{ij}(l) = \int\frac{dm_j^2}{2\pi}J(m_j^2)\,i(\not{l}+m_j)(-2\pi i)\delta(l^2-m_j^2)\,\delta^2(l_T) \tag{6}$$

where the jet function J has the property $\int dm_j^2 J(m_j^2)=1$ and where the last δ comes from a choice of frame where the l momentum has no transverse components. We obtain then

$$\begin{aligned}\int dl^+d^2l_T\,\Xi(l) &= \int\frac{dl^2}{2l^-}d^2l_T\int\frac{dm_j^2}{2\pi}J(m_j^2)\,i(\not{l}+m_j)(-2\pi i)\delta(l^2-m_j^2)\,\delta^2(l_T)\\ &= \int dm_j^2 J(m_j^2)\Big(\frac{\gamma^+}{2}+\frac{m_j}{2l^-}\mathbf{1}+\frac{m_j^2}{4(l^-)^2}\gamma^-\Big)\end{aligned} \tag{7}$$

The result is in agreement with the general decomposition once we identify

$$\xi_1 = \int dm_j^2 J(m_j^2) = 1, \qquad \xi_2 = \int dm_j^2 \frac{m_j}{\Lambda} J(m_j^2) = \frac{\langle m_j \rangle}{\Lambda}, \tag{8}$$

$$\xi_4 = 0, \qquad \xi_4 = \int dm_j^2 \frac{m_j^2}{\Lambda^2} J(m_j^2) = \frac{\langle m_j^2 \rangle}{\Lambda^2}. \tag{9}$$

We need to consider also the quark-gluon correlator $\tilde{\Xi}_{D\,\alpha}$. Due to the absence of transverse-momentum effects, the correlator corresponds to

$$\Xi_A^\alpha(l^-) = \Xi_D^\alpha(l^-) \tag{10}$$

where

$$(\Xi_D^\mu)_{ij} = \frac{1}{2} \sum_X \int \frac{d\eta^+ \, d^2\boldsymbol{\eta}_T}{(2\pi)^3} \, e^{ik\cdot\eta} \, \langle 0| \mathcal{U}^{n_+}_{(+\infty,\eta)} \, iD^\mu(\eta) \; \psi_i(\eta)|X\rangle\langle X|\bar{\psi}_j(0)\, \mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle \bigg|_{\eta^-=0}. \tag{11}$$

with the covariant derivative being $iD^\mu(\eta) = i\partial^\mu + gA^\mu$.

By analogy with the standard fragmentation correlator, we can decompose it in the following way

$$\tilde{\Xi}_A^\alpha(l^-) = \frac{\Lambda}{4} \left(\tilde{\xi}_4 + i\,\tilde{\xi}_2\right) i\gamma_T^\alpha \not{n}_-. \tag{12}$$

Relations between correlation functions of different twist are provided by the equation of motion for the quark field

$$\left[i\not{D}(\eta) - m_q\right]\psi(\eta) = \left[\gamma^+ iD^-(\eta) + \gamma^- iD^+(\eta) + \gamma_T^\alpha \, iD_\alpha(\eta) - m_q\right]\psi(\eta) = 0, \tag{13}$$

where m_q is the quark mass. Applying the equations of motion, we obtain the following relations

$$\tilde{\xi}_2 = \xi_2 - \frac{m_q}{\Lambda}\,\xi_1, \tag{14}$$

$$\tilde{\xi}_4 = \xi_4. \tag{15}$$

The calculation of the hadronic tensor leads to the following result

$$2MW^{\mu\nu} = 2\sum_a e_a^2 \bigg[-g_\perp^{\mu\nu} \; f_1^a(x) + i\epsilon_\perp^{\mu\nu} \; g_1^a(x) + i\frac{2M}{Q}\,\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho} S_{\perp\rho}\Big(xg_T(x) + \frac{M_j - m_q}{M} h_1(x)\Big)\bigg] \tag{16}$$

The cross section up to order M/Q turns out to be

$$F_{UU,T} = x \sum_a e_a^2 \, f_1^a(x), \tag{17}$$

$$F_{UU,L} = 0, \tag{18}$$

$$F_{LL} = x \sum_a e_a^2 \, g_1^a(x), \tag{19}$$

$$F_{UT}^{\sin\phi_S} = 0, \tag{20}$$

$$F_{LT}^{\cos\phi_S} = -x \sum_a e_a^2 \, \frac{2M}{Q} \left(xg_T^a(x) + \frac{M_j - m_q}{M} \, h_1^a(x)\right). \tag{21}$$

III. CONCLUSIONS

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APPENDIX A: SOME DETAILS

The calculation of the hadronic tensor can be done starting from the following expression in TMD formalism

$$\begin{aligned} 2MW^{\mu\nu} &= 2z \sum_a e_a^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^2\Big(\boldsymbol{p}_T - \frac{\boldsymbol{P}_{h\perp}}{z} - \boldsymbol{k}_T\Big) \operatorname{Tr}\Big\{\Phi^a(x,\boldsymbol{p}_T)\gamma^\mu \Delta^a(z,\boldsymbol{k}_T)\gamma^\nu \\ &\quad - \frac{1}{Q\sqrt{2}}\Big[\gamma^\alpha \not{n}_+ \gamma^\nu\, \tilde{\Phi}^a_{A\,\alpha}(x,\boldsymbol{p}_T)\, \gamma^\mu \Delta^a(z,\boldsymbol{k}_T) + \gamma^\alpha \not{n}_- \gamma^\mu\, \tilde{\Delta}^a_{A\,\alpha}(z,\boldsymbol{k}_T)\, \gamma^\nu \Phi^a(x,\boldsymbol{p}_T) + \text{h.c.}\Big]\Big\}, \end{aligned} \tag{A1}$$

The correlators Φ and Δ are usually written in terms of light cone vectors n_-, n_+, which are collinear to P and P_h. When including $1/Q$ corrections, we have to take into consideration the difference between those vectors and the vectors n'_- and n'_+, which are collinear to P and q. The difference is (see, e.g., Eq. (20) and (21) of [**?**])

$$n_+^\mu = n_+'^\mu \qquad\qquad n_-^\mu = n_-'^\mu + \frac{2P_{h\perp}^\mu}{zQ\sqrt{2}}. \tag{A2}$$

In the jet case under consideration here, we are integrating over $dz\, d^2\boldsymbol{P}_{h\perp}$ because we are not observing the kinematics of the final state. Moreover, we have $k_T = 0$ and $z = 1$. The expression for the hadronic tensor becomes

$$\begin{aligned} 2MW^{\mu\nu} &= 2\sum_a e_a^2 \int dz z \int d^2\boldsymbol{P}_{h\perp} d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^2\Big(\boldsymbol{p}_T - \frac{\boldsymbol{P}_{h\perp}}{z} - \boldsymbol{k}_T\Big) \operatorname{Tr}\Big\{\Phi^a(x,\boldsymbol{p}_T)\gamma^\mu \Xi^a \delta(z-1)\delta^2(\boldsymbol{k}_T)\gamma^\nu \\ &\quad - \frac{1}{Q\sqrt{2}}\Big[\gamma^\alpha \not{n}_+ \gamma^\nu\, \tilde{\Phi}^a_{A\,\alpha}(x,\boldsymbol{p}_T)\, \gamma^\mu \Xi^a \delta(z-1)\delta^2(\boldsymbol{k}_T) + \gamma^\alpha \not{n}_- \gamma^\mu \tilde{\Xi}^a_{A\,\alpha} \delta(z-1)\delta^2(\boldsymbol{k}_T)\, \gamma^\nu \Phi^a(x,\boldsymbol{p}_T) + \text{h.c.}\Big]\Big\} \\ &= 2\sum_a e_a^2 \int d^2\boldsymbol{p}_T \operatorname{Tr}\Big\{\Phi^a(x,\boldsymbol{p}_T)\gamma^\mu \Xi^a \gamma^\nu \\ &\quad - \frac{1}{Q\sqrt{2}}\Big[\gamma^\alpha \not{n}_+ \gamma^\nu\, \tilde{\Phi}^a_{A\,\alpha}(x,\boldsymbol{p}_T)\, \gamma^\mu \Xi^a + \gamma^\alpha \not{n}_- \gamma^\mu \tilde{\Xi}^a_{A\,\alpha}\, \gamma^\nu \Phi^a(x,\boldsymbol{p}_T) + \text{h.c.}\Big]\Big\}. \end{aligned} \tag{A3}$$

In the last line, we can neglect the component of vector n_- perpendicular to $\hat{t}$ and $\hat{z}$, but we have to take it into account in the preceding line:

$$\begin{aligned} 2MW^{\mu\nu} &= 2\sum_a e_a^2 \operatorname{Tr}\Big\{\int d^2\boldsymbol{p}_T \Big(\Phi^a(x,\boldsymbol{p}_T)\gamma^\mu \Xi^a \gamma^\nu\Big) \\ &\quad - \frac{1}{Q\sqrt{2}}\Big[\gamma^\alpha \not{n}_+ \gamma^\nu\, \tilde{\Phi}^a_{A\,\alpha}(x)\, \gamma^\mu \Xi^a + \gamma^\alpha \not{n}_- \gamma^\mu \tilde{\Xi}^a_{A\,\alpha}\, \gamma^\nu \Phi^a(x) + \text{h.c.}\Big]\Big\} \\ &= 2\sum_a e_a^2 \operatorname{Tr}\Big\{\Phi^a(x)\gamma^\mu \Xi^a \gamma^\nu + \frac{1}{Q\sqrt{2}} \Phi^a_{\partial\,\alpha}(x)\gamma^\mu \Xi^a\Big|_{\not{n}_-\to 2\gamma^\alpha} \gamma^\nu \\ &\quad - \frac{1}{Q\sqrt{2}}\Big[\gamma^\alpha \not{n}_+ \gamma^\nu\, \tilde{\Phi}^a_{A\,\alpha}(x)\, \gamma^\mu \Xi^a + \gamma^\alpha \not{n}_- \gamma^\mu \tilde{\Xi}^a_{A\,\alpha}\, \gamma^\nu \Phi^a(x) + \text{h.c.}\Big]\Big\}\Big|_{\not{n}_-\to\not{n}'_-}. \end{aligned} \tag{A4}$$

The previous steps are pretty safe. The following are still to be checked. If they are correct, the final result is simple and appealing:

$$\begin{aligned} 2MW^{\mu\nu} &= 2\sum_a e_a^2 \operatorname{Tr}\Big\{\Phi^a(x)\gamma^\mu \Xi^a \gamma^\nu - \frac{1}{Q\sqrt{2}}\gamma^\alpha \not{n}_+ \gamma^\nu\, \Phi^a_{\partial\,\alpha}(x)\gamma^\mu \Xi^a \\ &\quad - \frac{1}{Q\sqrt{2}}\Big[\gamma^\alpha \not{n}_+ \gamma^\nu\, \tilde{\Phi}^a_{A\,\alpha}(x)\, \gamma^\mu \Xi^a + \gamma^\alpha \not{n}_- \gamma^\mu \tilde{\Xi}^a_{A\,\alpha}\, \gamma^\nu \Phi^a(x) + \text{h.c.}\Big]\Big\}\Big|_{\not{n}_-\to\not{n}'_-} \\ &= 2\sum_a e_a^2 \operatorname{Tr}\Big\{\Phi^a(x)\gamma^\mu \Xi^a \gamma^\nu \\ &\quad - \frac{1}{Q\sqrt{2}}\Big[\gamma^\alpha \not{n}_+ \gamma^\nu\, \Phi^a_{D\,\alpha}(x)\, \gamma^\mu \Xi^a + \gamma^\alpha \not{n}_- \gamma^\mu \Xi^a_{D\,\alpha}\, \gamma^\nu \Phi^a(x) + \text{h.c.}\Big]\Big\}\Big|_{\not{n}_-\to\not{n}'_-}. \end{aligned} \tag{A5}$$

FIG. 1:

where

$$\Phi_\partial^\alpha(x) = \int d^2\boldsymbol{p}_T p_T^\alpha \Phi(x,\boldsymbol{p}_T) = \frac{M}{2}\Big\{ S_T^\alpha g_{1T}^{(1)}(x)\gamma_5 - S_L h_{1L}^{\perp(1)}\gamma^\alpha + f_{1T}^{\perp(1)}\epsilon_T^{\beta\alpha} S_{T\beta} - i h_1^{\perp(1)}\gamma^\alpha \Big\} \not{n}_+ \tag{A6}$$

APPENDIX B: EXPLICIT CALCULATIONS OF JET PROPAGATOR

To check some of the general statements about the jet propagator, we can compute it at first oder in the coupling constant of some theory. Let us start from a pseudoscalar Yukawa interaction. The corresponding Feynman diagram is shown in Fig. 1 on the left. The formula is

$$\begin{aligned}\Xi(l) &= \frac{i(\not{l}+m)}{l^2-m^2}\bigg(\int \frac{d^4p}{(2\pi)^4} ig\gamma_5(\not{l}-\not{p}+m)ig\gamma_5(2\pi)\delta\big[p^2-m_\pi^2\big](2\pi)\delta\big[(l-p)^2-m^2\big]\bigg)\frac{(-i)(\not{l}+m)}{l^2-m^2} \\ &\equiv \frac{i(\not{l}+m)}{l^2-m^2}(A\not{l}+Bm)\frac{(-i)(\not{l}+m)}{l^2-m^2} = \frac{1}{\big(l^2-m^2\big)^2}\Big[\Big(m^2(A-2B)+A\,l^2\Big)\not{l} + \Big(l^2(B-2A)+B\,m^2\Big)m\Big]\end{aligned} \tag{B1}$$

The first step is justified by the fact that we have only one external vector l, therefore in general the result of the calculation of the integral in the first line can be writte in terms of $\not{l}$ and m. Already at this level, it is possible to say that the function multiplying $\not{l}$ is different from that multiplying m.

Let us focus first on the integral. The formula is

$$\begin{aligned}A\not{l}+Bm &= -\frac{g^2}{(2\pi)^2}\int d^4p\,\gamma_5(\not{l}-\not{p}+m)\gamma_5\ \delta\big[p^2-m_\pi^2\big]\delta\big[(l-p)^2-m^2\big] \\ &= \frac{g^2}{(2\pi)^2}\int d^4p\,(\not{l}-\not{p}-m)\ \delta\big[p^2-m_\pi^2\big]\delta\big[(l-p)^2-m^2\big] \\ &= \frac{g^2}{(2\pi)^2}(\not{l}-m)\int d^4p\,\delta\big[p^2-m_\pi^2\big]\delta\big[(l-p)^2-m^2\big] - \frac{g^2}{(2\pi)^2}\int d^4p\,\not{p}\ \delta\big[p^2-m_\pi^2\big]\delta\big[(l-p)^2-m^2\big].\end{aligned} \tag{B2}$$

The last term must be proportional to $\not{l}$. We can therefore write

$$\int d^4p\ \not{p}\ \delta\big[p^2-m_\pi^2\big]\delta\big[(l-p)^2-m^2\big] = \not{l}\ F_1 \tag{B3}$$

where

$$\begin{aligned}F_1 &= \frac{1}{l^2}\int d^4p\ l\cdot p\ \delta\big[p^2-m_\pi^2\big]\delta\big[(l-p)^2-m^2\big] \\ &= \frac{1}{2}\bigg(1-\frac{m^2-m_\pi^2}{l^2}\bigg)\int d^4p\ \delta\big[p^2-m_\pi^2\big]\delta\big[(l-p)^2-m^2\big].\end{aligned} \tag{B4}$$

In summary, we obtain

$$\begin{aligned}A\not{l}+Bm &= \frac{g^2}{(2\pi)^2}\bigg[(\not{l}-m)-\not{l}\frac{1}{2}\bigg(1-\frac{m^2-m_\pi^2}{l^2}\bigg)\bigg]\int d^4p\,\delta\big[p^2-m_\pi^2\big]\delta\big[(l-p)^2-m^2\big] \\ &= \frac{g^2}{(2\pi)^2}\bigg[\frac{1}{2}\bigg(1+\frac{m^2-m_\pi^2}{l^2}\bigg)\not{l}-m\bigg]\int d^4p\,\delta\big[p^2-m_\pi^2\big]\delta\big[(l-p)^2-m^2\big]\end{aligned} \tag{B5}$$

We can further write the explicit result for the last integral

$$I_1 = \int d^4p\, \delta\big[p^2 - m_\pi^2\big]\, \delta\big[(l-p)^2 - m^2\big] = \frac{\pi}{2l^2}\sqrt{\lambda(l^2, m^2, m_\pi^2)}\; \theta(l^2 - (m+m_\pi)^2) \tag{B6}$$

where we have introduced the so-called Källen function, $\lambda(l^2, m^2, m_\pi^2) = [l^2 - (m+m_\pi)^2][l^2 - (m-m_\pi)^2]$.

To simplify the discussion, let us take the limit $m \to 0$. We obtain

$$\Xi(l^2) = \frac{1}{l^4} A\, l^2 \not{l} = \frac{g^2}{16\pi\, l^6}\big(l^2 - m_\pi^2\big)^2\, \theta\big(l^2 - m_\pi^2\big) \not{l} \tag{B7}$$