Accessing the nucleon tensor structure in inclusive deep inelastic scattering

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We revisit the standard analysis of inclusive DIS on protons taking into account the fact that on-shell quarks cannot be present in the final state, but they rather decay into hadrons. As a consequence, a spin-flip term associated with the invariant mass of this (mini)jet of hadrons is generated non perturbatively, and couples to the target's transversity distribution function. In inclusive cross sections, this provides an hitherto neglected and large contribution to the twist-3 part of the g_2 structure function, that can explain the discrepancy between recent calculations and fits of this quantity. It also provides an extension of the Burkardt-Cottingham sum rule, that puts stringent constraints on the small-x behavior of the transversity function. Perspectives to measure the new spin flip term, and applications to spin-1 targets will be briefly discussed.

I. INTRODUCTION

The tensor charge is a fundamental property of the nucleon, at present poorly constrained. It has been estimated in lattice QCD (see, *e.g.*, [1–5]), but only limited information is available from direct measurements. The way to extract the tensor charge from experimental measurements requires first of all the extraction the so-called transversity parton distribution function, denoted by $h_1^q(x)$ (see Ref. [6] for a review on transversity and Refs. [7–9] for the most recent extractions). The integral of the transversity distribution corresponds to the contribution of flavor q to the tensor charge. The knowledge of the tensor charge can be used also to put constraints on the search of physics beyond the Standard Model [10–12].

In this paper, we discuss the possibility of observing the effect of the transversity parton distribution function (PDF) in totally inclusive Deep Inelastic Scattering.

The transversity distribution is notoriously difficult to access because it is a chiral-odd function and needs to be combined with a spin-flip mechanism to appear in a scattering process [13]. Usually, this spin flip is provided by another nonperturbative distribution or fragmentation function, implying that transversity cannot be accessed in inclusive DIS, but only in more complex processes such as semi-inclusive DIS or Drell-Yan [14**?** −16].

The only other way to attain spin-flip terms in QED and QCD is taking into account mass corrections. In fact, it is well known that transversity gives a contribution to the structure function g_2 in inclusive DIS (see, *e.g.*, [17] and references therein), and in particular to the violation of the so-called Wandzura–Wilczek relation for g_2 [18]. However, this contribution is proportional to the current quark mass and can be expected to be very small.

We revisit the standard analysis of inclusive DIS taking into account the fact that on-shell quarks cannot be present in the final state, but they rather decay into hadrons (ideally, forming jets of hadrons). This is sufficient to modify the structure of the DIS cut-diagram, even if none of the hadrons is detected in the final state. For a proper description of this effect, we include proper "jet correlators" into the analysis, and pay particular attention to ensuring that our results are gauge invariant. We observe that the inclusion of jet correlators introduces a new contribution to the inclusive g_2 structure function. This term has the interesting features that: a) violates the Wandzura–Wilczek relation, b) potentially violates the Burkhardt–Cottingham sum rule, c) is proportional to the transversity distribution function multiplied by a nonperturbative "jet mass" parameter, likely much larger than the mass of light quarks. We provide estimates of this contribution based on a recent extraction of transversity and show that it could be very large.

II. JET CORRELATOR AND TWIST-2 STRUCTURE FUNCTIONS

Motivated by large-x mass corrections to inclusive DIS structure functions, Accardi and Qiu have introduced in the LO handbag diagram a "jet correlator" (also called "jet factor" by Collins and Rogers in Ref. [19]) that accounts for invariant mass production in the current jet, and ensures that leading twist calculations in collinear factorization are consistent with the requirement imposed by baryon number conservation that $x_B < 1$ [20]. The jet correlator is depicted in Figure 1(a) and is defined as

$$\Xi_{ij}(l, n_+) = \int \frac{d^4\eta}{(2\pi)^4}\, e^{il\cdot\eta}\, \langle 0|\, \mathcal{U}^{n_+}_{(+\infty,\eta)}\, \psi_i(\eta)\bar{\psi}_j(0)\, \mathcal{U}^{n_+}_{(0,+\infty)}\, |0\rangle\ , \tag{1}$$

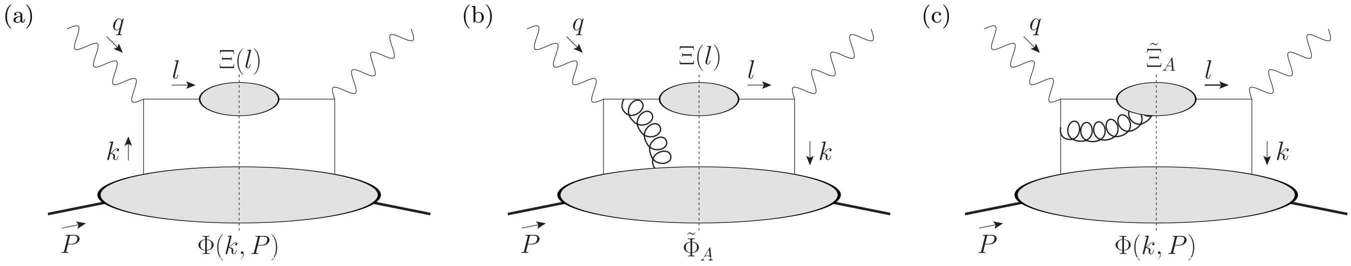

FIG. 1: Diagrams contributing to DIS scattering up to twist-3 expansion, including a jet correlator in the top part. Note the gluon attaches to both the nucleon and jet correlators. The Hermitian conjugates of diagrams (b) and (c), i.e., with gluons attaching to the right of the cut, are not shown.

In this definition, l is the quark four-momentum, Ψ the quark field operator (with quark flavor index omitted for simplicity), and $|0\rangle$ is the nonperturbative vacuum state. Furthermore, we explicitly guarantee the correlator's gauge invariance by introducing two Wilson line operators $\mathcal{U}^{n_+}$ along a light-cone plus direction determined by the vector n_+. This path choice for the Wilson line is required by QCD factorization theorems, and the vector is determined by the particular hard process to which the jet correlator contributes. For example, in the case of inclusive DIS discussed in this paper, this is determined by the four momentum transfer q and the proton's momentum p.

The correlator Ξ can be parametrized in terms of jet parton correlation functions (PCFs), using the vectors l and n_+:

$$\Xi(l, n_+) = \Lambda A_1(l^2)\,\mathbf{1} + A_2(l^2)\not{l} + \frac{\Lambda^2}{l\cdot n_+}\not{n}_+\, B_1(l^2) + \frac{i\Lambda}{2l\cdot n_+}[\not{l}, \not{n}_+]\, B_2(l^2)\ . \tag{2}$$

Time reversal invariance in QCD requires $B_2 = 0$, while B_1 contributes only at twist-4 order, and will not be considered further in this paper. We focus, instead, on the role of chiral odd terms in the g_2 structure function up to twist 3. At this order,

$$\Xi(l, n_+) = \Lambda A_1(l^2)\,\mathbf{1} + A_2(l^2)\not{l} + O(\Lambda^2/Q^2) \tag{3}$$

is nothing else than the full quark propagator; note however, that we consider here the full QCD vacuum rather than the perturbative one. The A_1 and A_2 terms can be nicely interpreted in terms of the spectral representation of the cut quark propagator (see, e.g., Sec. 6.3 of [**?**] and Sec. 2.7.2 of [**?**]),

$$\Xi(l) = \int d\mu^2 \left[J_1(\mu^2)\,\mu + J_2(\mu^2)\not{l}\,\right] \delta(l^2 - \mu_j^2)\ , \tag{4}$$

where μ^2 is interpreted as the invariant mass of the current jet, *i.e.*, of the particles going through the cut in the top blob of Fig.1(a), and the J_i are the spectral functions of the quark propagator, that have been also called "jet functions" in [20]. These satisfy

$$J_2(\mu^2) \geq J_1(\mu^2) \geq 0 \qquad \text{and} \qquad \int d\mu^2 J_2(\mu^2) = 1\ . \tag{5}$$

From a comparison of Eqns.(2) and (4), one can see that

$$A_1(l^2) = \frac{\sqrt{l^2}}{\Lambda} J_1(l^2) \qquad\qquad A_2(l^2) = J_2(l^2)\ . \tag{6}$$

When inserting the jet correlator in the handbag diagram for inclusive DIS, the invariant jet mass μ^2 is integrated from 0 to $Q^2(1/x_B - 1)$. This induces (kinematical) corrections of order $O(1/Q^2)$, whose effect on the F_2 structure function has been studied in Ref. [20]:

$$F_2(x_B) = \int_0^{Q^2(1/x_B - 1)} d\mu^2\, J_2(\mu^2) F_2^{(0)}(x_B(1 + \mu^2/Q^2))\ , \tag{7}$$

where $F_2^{(0)}$ is the structure function calculated with the handbag diagram sporting a bare quark propagator instead of the jet correlator, and $\xi = 2x_B/(1 + \sqrt{1 + 4x_B^2 M^2/Q^2})$ with M the nucleon's mass is the Nachtmann scaling variable.

(We also omitted the dependence of the structure function on Q^2 for clarity of notation). In this paper we limit our attention to effects of order $O(1/Q)$ and therefore can extend the integration to $\mu^2 = \infty$. Therefore, the jet function J_2 decouples and, thanks to the sum rule (5), integrates to 1. One then recovers the conventional result,

$$F_2(x_B) = \Big[\int_0^\infty d\mu^2 \, J_2(\mu^2) \Big] F_2^{(0)}(x_B) + O(\Lambda^2/Q^2) = F_2^{(0)}(x_B) + O(\Lambda^2/Q^2) \ . \tag{8}$$

More in general, the jet correlator decouples from the parton correlator Φ in any inclusive cross section calculation up to $O(1/Q)$, and the inclusive structure functions only depend on the integrated jet correlator

$$\Xi(l^-) \equiv \int \frac{dl^2}{2l^-} d^2 l_T \, \Xi(l) = \frac{\Lambda}{2l^-} \, \xi_1 \mathbf{1} + \xi_2 \frac{\not{n}_-}{2} + \text{higher twists} \tag{9}$$

where

$$\xi_1 = \int d\mu^2 \frac{\mu}{\Lambda} J_1(\mu^2) \equiv \frac{M_q}{\Lambda}, \qquad\qquad \xi_2 = \int d\mu^2 J_2(\mu^2) = 1 \ . \tag{10}$$

where M_q can be interpreted as the average invariant mass produced in the spin-flip fragmentation processes of a quark of flavor q. It is important to notice that $\xi_2 = 1$ exactly due to CPT invariance (see Sec. 10.7 of Ref. [21]), while $0 < M_q < \int d\mu^2 \mu J_2(\mu^2)$ is dynamically determined. From the analytic properties of spectral functions we may expect [20] $J_2(\mu^2) = Z\delta(\mu^2 - m_q) + \bar{J}_2(\mu^2)\theta(\mu^2 - m_\pi^2)$ with the continuum starting at m_π, the mass of the pion, due to color confinement effects. Taking into account that $J_1 < J_2$, we may therefore expect

$$M_q = O(10^2 \text{ MeV}) \ . \tag{11}$$

Although M_q is in general a nonperturbative quantity, it is interesting to notice that

$$M_q = \frac{\Lambda}{4} \int \text{Tr} \left[\Xi(l) \mathbf{1} \right] = \langle 0 | \bar{\psi}_i(0) \psi_i(0) | 0 \rangle \tag{12}$$

Calculating this on the perturbative vacuum and to leading order corresponds to taking the trace of the cut bare-quark propagator to obtain $M_q = {}_{\text{pert}}\langle 0 | \bar{\psi}_i(0) \psi_i(0) | 0 \rangle_{\text{pert}} = m_q$, with m_q the quark mass, recovering the conventional result. However, we are here considering non perturbative effects on the quark fragmentation and $M_q \gtrsim m_q$.

III. TWIST-3 ANALYSIS

Extending the analysis of [20] to the calculation of twist-3 structure functions requires not only to consider the ξ_1 term in the jet correlator, but also quark-gluon-quark correlators in both the proton and the vacuum as depicted in Figs.1(b) and (c), respectively. In the former the ξ_1 terms contribute to $O(1/Q^2)$, so that up $O(1/Q)$ these give the same contribution as in the conventional handbag calculation.

The novel element in our analysis is the jet's quark-gluon-quark correlator $\Xi_A^\mu(l,k)$ in diagrams 1(c),

$$(\Xi_A^\mu)_{ij} = \frac{1}{2} \sum_X \int \frac{d\eta^+ \, d^2\boldsymbol{\eta}_T}{(2\pi)^3} \, e^{ik\cdot\eta} \, \langle 0 | \mathcal{U}^{n_+}_{(+\infty,\eta)} \, gA^\mu(\eta) \ \psi_i(\eta) | X \rangle \langle X | \bar{\psi}_j(0) \, \mathcal{U}^{n_+}_{(0,+\infty)} | 0 \rangle \Big|_{\eta^-=0} . \tag{13}$$

This diagram and its Hermitian conjugate are not only important to account for all contribution of order $O(1/Q)$, but also to restore the gauge invariance, which is broken in diagram 1(a) due to the different mass of the incoming and outgoing quark lines, namely, $m_q \neq M_q$.

Rather than directly using the definition (13), it is convenient to calculate the inclusive cross section as an integral of the semi-inclusive one, utilize the QCD equation of motions and furthermore summed over all hadron flavors, and take advantage of

$$\sum_h \int \frac{d^3 p_h}{(2\pi) 2E_h} \Delta^h(l, p_h) = \Xi(l) \ , \tag{14}$$

where Δ^h is the quark fragmentation correlator for production of a hadron of flavor h and momentum p_h [22]. In terms of the TMD fragmentation functions we are interested in, this reads

$$\sum_h \int dz d^2 p_{hT} z D_1^h(z, p_{hT}) = \xi_2 = 1 \tag{15}$$

$$\sum_h \int dz d^2 p_{hT} E(z, p_{hT}) = \xi_1 \ , \tag{16}$$

where $D_1^h(z, p_{hT})$ is the twist-2 quark fragmentation function as a function of the hadron's collinear momentum fraction z and transverse momentum p_{hT}, and $\tilde{E}^h(z, p_{hT})$ is a chiral-odd twist-3 function defined in [22].

The relevant part of the semi-inclusive hadronic tensor for our analysis is

$$2\Lambda W^{\mu\nu} = i\frac{2\Lambda}{Q}\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}S_{\perp\rho}\sum_q e_q^2\Big[2x_b g_T(x_B)\sum_h\int dz d^2p_{hT}D_1^{q,h}(z,p_{hT}) + 2h_1(x_B)\sum_h\int dzd^2p_{hT}\tilde{E}^{q,h}(z,p_{hT})\Big] + \dots \tag{17}$$

For clarity, here we reintroduced the quark flavor q, e_q being its electric charge. The first term can be easily integrated with the help of the sum rule (15). To integrate the latter, we first need make use of the relation $\tilde{E}(z) = E(z) - (m_q/\Lambda)zD_1(z)$, which is a consequence of the QCD equations of motion [22], then make use of the sum rule (14):

$$\sum_h\int dzd^2p_{hT}\tilde{E}^{q,h}(z,p_{hT}) = \sum_h\int dzd^2p_{hT}\Big[E^{q,h}(z,p_{hT}) - \frac{m_q}{\Lambda}zD_1^{q,h}(z,p_{hT})\Big] = \xi_1 - \frac{m_q}{\Lambda}\xi_2 = \frac{M_q - m_q}{\Lambda}\ . \tag{18}$$

This formula is the single most important result of this paper, and provides a non perturbative generalization of the commonly used $\int\tilde{E} = 0$ sum rule introduced in [13]. Indeed, calculating the jet correlator on the perturbative vacuum one would obtain, as already discussed, $M_q = m_q$ and the new term would vanish.

Finally, the contraction of the hadronic tensor with the leptonic tensor leads to the following well known result for the inclusive DIS cross section up to order Λ/Q [22]

$$\frac{d\sigma}{dx\,dy\,d\psi} = \frac{2\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\Big\{F_{UU,T} + \varepsilon F_{UU,L} + S_\parallel\lambda_e\sqrt{1-\varepsilon^2}\,F_{LL} + |\boldsymbol{S}_\perp|\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}\Big\}\ , \tag{19}$$

where the structure functions on the right hand side correspond to

$$F_{UU,T} = x\sum_q e_q^2\, f_1^q(x), \tag{20}$$

$$F_{UU,L} = 0, \tag{21}$$

$$F_{LL} = x\sum_q e_q^2\, g_1^q(x), \tag{22}$$

$$F_{UT}^{\sin\phi_S} = 0, \tag{23}$$

$$F_{LT}^{\cos\phi_S} = -x\sum_q e_q^2\frac{2\Lambda}{Q}\Big(xg_T^q(x) + \frac{M_q - m_q}{\Lambda}\,h_1^q(x)\Big). \tag{24}$$

The second term in the last structure function is a new result from our analysis; it is not suppressed as an inverse power of Q compared to the standard term. On the nonperturbative vacuum the jet mass is larger than the quark's, and this contributes a nonnegligible term to the twist-3 part of the g_2 function, as we will discuss in the next section.

IV. THE g_2 STRUCTURE FUNCTION

The new term in Eq.(24) only appears in the g_2 structure function. Following the derivation in Ref. [17], one finds

$$g_2(x_B) = g_2^{WW} + \frac{1}{2}\sum_a e_a^2\Big(\tilde{g}_T^{a\star}(x) + \int_x^1\frac{dy}{y}\hat{g}_T^q(y) + \frac{m_q}{\Lambda}\Big(\frac{h_1^q}{x}\Big)^\star(x) + \frac{M_q - m_q}{\Lambda}\frac{h_1^q(x)}{x}\Big)\ , \tag{25}$$

where we defined $f^*(x) = -f(x) + \int_x^1\frac{dy}{y}f(y)$. The first four terms coincide with the result obtained in the conventional handbag approximation [17], while the fifth is new. Note that even if the relation is written for the sum of the quark weighted by their charge squared, it can be considered valid also flavor by flavor. In fact, the steps leading to such a decomposition are formulated at the correlator level.

The first term is also known as the Wandzura-Wilczeck function $g_2^{WW} = -g_1^*(x)$, and contains all the twist-2 chiral-even contributions to the g_2 structure coming from quark-quark correlators. The second and third terms contain all "pure twist-3" contributions, i.e., those coming from quark-gluon-quark correlators. The fourth and fifth terms depend on the transversity parton distribution function, h_1. The fourth term is usually neglected for light quarks since it is proportional to $m_q = O(1\text{ MeV})$. In the last term, new in our analysis, the transversity distribution is multiplied by a constant of $O(100\text{ MeV})$, and cannot be a priori neglected.

It is important to estimate the size of the various contributions to the non Wandzura-Wilczek part of g_2. We define the shorthand notation

$$g_2^{\rm tw3} = \frac{1}{2}\sum_q e_q^2\left(\widetilde{g}_T^{q\star}(x) + \int_x^1 \frac{dy}{y}\widetilde{g}_T^q(y)\right) \quad g_2^{\rm quark} = \frac{1}{2}\sum_q e_q^2 \frac{m_q}{\Lambda}(h_1^q/x)^\star(x), \quad g_2^{\rm jet} = \frac{1}{2}\sum_q e_q^2 \frac{M_q - m_q}{\Lambda}\frac{h_1^q(x)}{x}. \tag{26}$$

These are compared in Figure 2 to the $g_2 - g_2^{WW}$ function obtained in the very recent JAM15 fit of polarized DIS asymmetries [23], that includes a large amount of precise data at large x from Jefferson Lab, and simultaneously fits the higher-twist components of in g_1 and g_2 to the data. For the "pure twist-3" contribution, $g_2^{\rm tw3}$, *i.e.*, the contribution from quark-gluon-quark matrix elements, we show a model calculation by Braun et al. [24]; for (modified) bag model calculations, see [25, 26]. To estimate the contributions from quark (g_2^q) and jet mass ($g_2^{\rm jet}$) effects, that depend on chiral odd quark-quark matrix elements, we use the recent Pavia15 fit of the transversity distribution from Ref. [7], which is comparable also to other extractions [9, 27]. Furthermore, we choose the values of the mass parameters to be $m_q = 5$ MeV and $M_q = 100$ MeV.

As one can see, in the proton case the pure twist-3 contribution is quite smaller in magnitude, and opposite in sign, compared to the JAM15 fit. As expected, the quark-mass contribution is essentially negligible. For what concerns the jet-mass contribution, the uncertainties due to the h_1 extraction are very large, especially at low x. In addition, there is an overall normalization uncertainty due to the choice of M_q, not shown in the plot. In any case, the jet-mass contribution is strikingly large. If we assume that the pure twist-3 contributions are of the order of the model calculation by Braun et al., the breaking of the Wandura-Wilczek relation can be used to constrain the extractions of the transversity distribution, in particular at low x. Moreover, it is quite clear that the gap between the pure twist-3 $g_2^{\rm tw3}$ function and the JAM15 fit can be explained by the new jet-mass contribution we discussed in this paper.

In the neutron case, the jet contribution is very negative at intermediate to large values of x. If one trusts the order of magnitude of the $g_2^{\rm tw3}$ calculation by Braun et al., one would conclude that the jet contribution should not be that large. However, the jet contribution is strongly influenced by the d quark's transversity, whose fit suffers from large systematic uncertainties and saturates the negative Soffer bound. Recent deata in $p+p$ collisions indicate, however, that h_1^d might be less negative than in the Pavia15 fits [28]. Correspondingly the jet contribution to the proton at $x \approx 0.1$ would become less positive, inproving as well the agreement with the JAM15 fit.

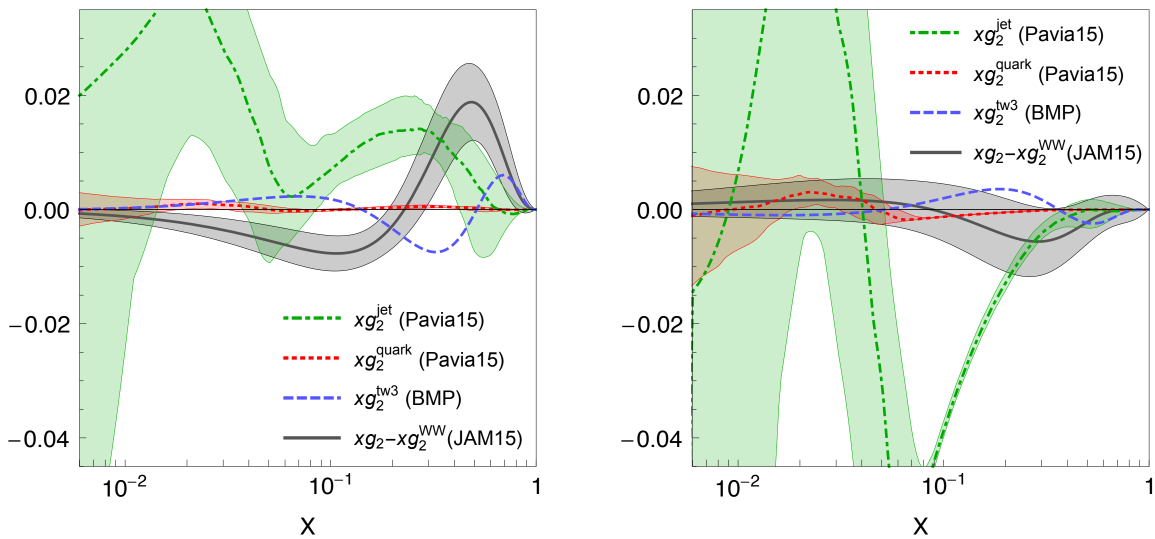

FIG. 2: Different contributions to the non Wandzura-Wilczek part of the proton (left) and neutron (right) g_2 structure function compared to the JAM15 fit of the $g_2 - g_2^{\rm WW}$ (solid black) [23]. The quark and jet contributions are shown with a dotted red and a dot-dashed green line respectively, with uncertainty bands coming form the Pavia15 fit of the transversity function [7]. Theunceratainty in the choice $m_q = 5$ GeV and $M_q = 100$ GeV is not shown. The pure twist-3 contribution calculated by Braun et al. [24] is shown as a dashed blue line (no uncertainty estimate was provided in the original reference).

It is interesting to consider the moments of the non Wandzura-Wilczek contribution to g_2,

$$d_N \equiv (N+1)\int_0^1 x^N\Big(g_2(x) - g_2^{WW}(x)\Big) . \tag{27}$$

For a generic function f, let us define it's N-th moment as $f[N] = \int_0^1 dx\, x^N f(x)$. It is then straightforward to verify that $f^*[N] = N/(N+1) \times f[N]$ and

$$\begin{aligned} d_N &= (N+1)g_2[N] + N g_1[N] & (28)\\ &= \frac{1}{2}\sum_q e_q^2\Big(N\tilde{g}_T^q[N] + \hat{g}_T^q[N] + \frac{(N+1)M_q - m_q}{\Lambda}h_1^q[N]\Big) . & (29)\end{aligned}$$

The zero-th moment provides an interesting relationship between transversity and the inclusive structure function g_2:

$$\int dx\, g_2(x) = \sum_q e_q^2 \frac{M_q - m_q}{\Lambda}\int dx\, \frac{1}{x}h_1^q(x) , \tag{30}$$

where we used the fact that $\hat{g}_T^q[0] = 0$ identically due to the symmetry properties of the quark-gluon-quark correlators. The sum rule (30) generalizes the Burkhardt-Cottingham (BC) sum rule [29], which states that $\int_0^1 dx\, g_2(x) = 0$. However, we show that jet-mass corrections violate the BC sum rule. The possibility of a violation of the sum rule due to contributions from spin-flip processes was already mentioned in the original derivation [29], but do not show up in treatments that only consider free field quark propagators for the struck quark [13]. Since h_1 is driven to 0 by QCD evolution as $Q^2 \to \infty$, the BC sum rule $\int_0^1 dx\, g_2(x) = 0$ is satisfied at least asymptotically. Although we formulated (30) in terms of sum over quark flavors in order to display a clear connection to the structure function g_2, we stress that it is valid also flavor by flavor, i.e., for each single flavor the only measurable nonzero contribution to the zeroth moment of the structure function g_2 can come from the jet-mass corrections and transversity.[1] The fact that $g_2^{\rm quark}$ disappears in the zeroth moment of g_2 is due to the definition of the $*$ functions. The fact that $g_2^{\rm tw3}$ disappears is due to the symmetry properties of $\widehat{g}_T^q$, or equivalently is a consequence of the Lorentz invariance of QCD interactions, that entails $\int_0^1 dx g_1^q(x) = \int_0^1 g_T^q(x)$.

At finite scales, the only way to preserve the validity of the Burkhardt-Cottingham sum rule is if

$$\int dx\, \frac{1}{x}h_1^q(x) = 0 . \tag{31}$$

Interestingly, one can show that this constraint, if valid at any given scale Q_0 is conserved through QCD evolution. However, we think that this constrain cannot be satisfied in general, since it is broken in perturbative QCD [30] and models (see, e.g., []).

E or explicitly as, *e.g.*, in the quark target model [30] – i.e., if g_2 integrates to a finite but nonzero number – If we assume that the BC sum rule is broken by a *finite* amount, we obtain that $h_1(x)/x$ must be integrable, implying a a bound on the small x behavior of the transversity,

$$h_1^q(x) \propto x^\epsilon \quad \epsilon > 0 . \tag{32}$$

This bound will be very useful, *e.g.*, in transversity fits, where the data at small x is, as yet, very limited, and in general for proper extrapolations when calculating moments.

The first moment is the first in which a contribution from the pure twist-3 part of g_2 appears:

$$d_1 = \frac{1}{2}\sum_q e_q^2\Big(2\tilde{g}_T^q[1] + \hat{g}_T^q[1] + \frac{2M_q - m_q}{\Lambda}h_1^q[1]\Big) \tag{33}$$

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[1] This conclusion is true even if the BC sum rule is broken by a $J = 0$ fixed pole with non-polynomial residue [13], since this would appear as a $\delta(x)$ contribution and would not be measurable.

where $h_1^q[1] = \int_0^1 dx h_1^q(x)$ is the contribution of a quark q to the tensor charge. The third moment is also interesting because the pure twist-3 part can be related to quark-gluon-quark correlators, see [13], and interpreted as as the average color force experienced by the struck quark as it exits the nucleon [**?**]:

$$d_2 = \frac{1}{2}\sum_q e_q^2 \left(3\tilde{g}_T^q[2] + \hat{g}_T^q[2] + \frac{3M_q - m_q}{\Lambda} h_1^q[2] \right) \tag{34}$$

In both cases, the transversity contribution is a background to the extraction of the pure twist-3 piece. Fortunately, it is a quantity that can be extracted from the lattice [1–5] or fitted [7–9]. Furthermore, the new sum rule (31) and the bound (32) promise to improve future transversity fits. What is less obvious is how to calculate $M_q - m_q$ from first principles, since this is related to the spectral function of the quark propagator. We will, however, briefly discuss perspectives on how to measure it. Therefore the pure twist-3 part can, in principle, be properly isolated.

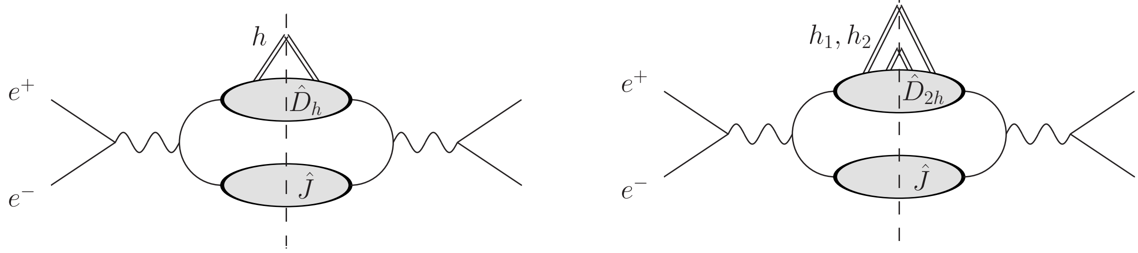

FIG. 3: Single hadron *(left)* and double hadron *(right)* production in e^+e^- collisions at LO with jet and fragmentation correlators.

A promising avenue to experimentally access jet functions is, however, through inclusive single hadron production, $e^+e^- \to hX$, and inclusive dihadron production from the same hemisphere, $e^+e^- \to hhX$, see Fig. 3. In single-hadron production, the fragmentation functions D_h play the role of PDFs in DIS, and couple to the jet functions in an analogous way. In double hadron production the enlarged number of Dirac structures of the dihadron fragmentation correlator D_{2h} allows one to access the jet function in novel ways, and in particular to isolate the contribution from the helicity-flip J_0 term. Studying and classifying all the possibilities offered by single and double hadron production will open up a rich phenom, which will in turn be needed to extract pure twist-3 matrix elements from the g_2 structure function, and more in general to perform precise jet mass corrections in DIS.

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It is important to explore in which other process does M_q contribute, as to provide an experimental check of the formalism:

- inclusive Λ production in $e^+ + e^-$
- same-side dihadrons in $e^+ + e^-$

It would be cool to find a process where the M_q contribution is the only one (similar to the BC breaking) ...

V. CONCLUSIONS

Acknowledgments

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