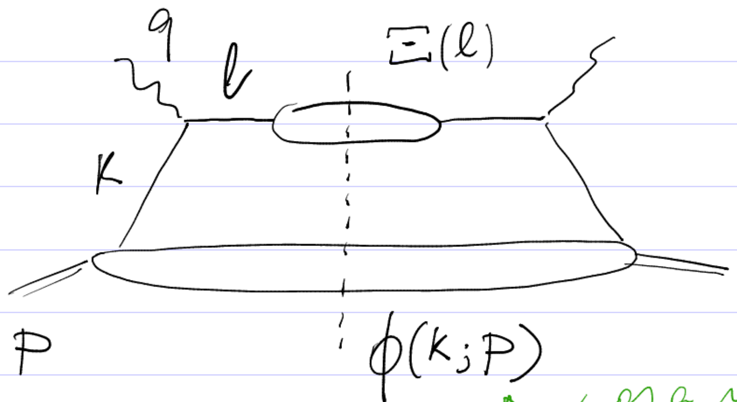
22 Apr 2017

COLLINEAR FACTORIZATION WITH A JET FUNCTION

— NOTES FOR SECTION II OF AB PAPER —

GOAL: establish formulas for collinear factorization with jet functions at LO
→ similar to notes from July 2015
but aligned with paper from July 2016



as a matter of notation, I keep "fixed" momenta to the right of the semicolon (it will be important when connecting Ξ with TMD correlators)

• Hadronic tensor:

means "trace of"

$$W^{\mu\nu} \sim \int d^4k \int d^4l \left[\phi(k)\gamma^\mu \Xi(l)\right] \delta^{(4)}(k+q-l)$$

$$= \int dk^+ dk^- d^2k_T \int dl^- \frac{dl^2}{2l^-} d^2l_T \left[\phi(k)\gamma^\mu \Xi(l)\gamma^\nu\right]$$

$$\times\, \delta(k^+ + q^+ - l^+)\, \delta(k^- + q^- - l^-)\, \delta^{(2)}(k_\perp + q_\perp - l_\perp)$$

same as dl^+, but establishes parallel with spectral function repr.

in fact, $q_\perp = 0$ in (q,P) frame

⓪ JET CORRELATOR: DEFINITION AND EXPANSION

- From our draft, we define:

$$(1.1)\quad \Xi_{ij}(\ell, n_+) = \int \frac{d^4\eta}{(2\pi)^4} e^{i\ell\cdot\eta} \langle 0| U^{n_+}_{(+\infty,\eta)} \psi_i(\eta) \bar{\psi}_j(0) U_{(0,+\infty)} |0\rangle$$

The notation is consistent with the FF correlator defined in $(3.35)_{NT}$, and includes a gauge link at infinity:

$$U^{n_+}_{(+\infty,\eta)} \equiv U^T(\infty_T, \eta_T; +\infty^+) U^{n_+}(+\infty^+, \xi^+; \xi_T)$$

$$U^{n_+}_{(0,\infty)} \equiv U^{n_+}(0^+, +\infty^+; 0_T) U^T(0_T, \infty_T; +\infty^+)$$

- This can be expanded as

$$\Xi(\ell, n_+) = \Lambda A_1(\ell^2)\mathbb{1} + A_2(\ell^2)\not{\ell} + \frac{\Lambda^2}{\ell\cdot n_+}\not{n}_+ B_1(\ell^2) + \frac{i\Lambda}{2\ell\cdot n_+}[\not{\ell}, \not{n}_+] B_2(\ell^2)$$

NOTE:

* Andrea says that at the UNINTEGRATED LEVEL, B_2 is not T-odd, due to the staple shape of the Wilson lines

↳ The unintegrated $\boxed{B_2(\ell^2) \neq 0 \text{ in general}}$

* even at the ℓ^+-integrated level, which is what we need in COL. FACT INCLUSIVE DIS (see below) this remains true b/c $\eta_T \neq 0$ in (1.1)

(I) KINEMATIC APPROXIMATION ON THE LIGHT CONE

(a) $k^- = \dfrac{k^2 + k_T^2}{2k^+} \ll k^+$ e.g. Breit frame $k^+ \sim Q$ $\Rightarrow k^- = O\left(\dfrac{k^2+k_T^2}{Q^2}\right) k^+$ with $k^2 + k_T^2 \sim \Lambda^2_{had}$

then, can neglect it in the δ functions:

$l^- \sim O(Q) - O(k^+)$ hence "kinematic approx": neither the scattering nor the correlators are approximated

$$(2.1) \quad \delta(k^- + q^- - l^-) \approx \delta(q^- - l^-)$$

· Thus the dk^- integration acts only on Φ:

$$W^{\mu\nu} \sim \int dk^+ d^2k_\perp \int dl^- \frac{dl^2}{2l^-} d^2l_T \left[\Phi(k^+, k_T) \gamma^\mu \Xi(l) \gamma^\nu \right]$$
$$\times \delta(k^+ + q^+ - l^+)\, \delta(q^- - l^-)\, \delta^{(2)}(k_\perp - l_\perp)$$

where:

$$\Phi(k^+, k_\perp) = \int dk^- \phi(k) \quad \text{INTEGRATED TMD CORRELATOR}$$

(b) Likewise we approximate the scattered quark momentum:

$$l^+ = \frac{l^2 + l_T^2}{2l^-} \approx \frac{\Lambda^2}{2l^-} = \frac{\Lambda^2}{2\, Q^2/2k^+} = \frac{\Lambda^2}{Q^2} k^+ \ll k^+$$

$l^- = q^-$ from (2.1)

$$\Longrightarrow \delta(k^+ + q^+ - l^+) \approx \delta(k^+ + q^+)$$

and the integral over dl^+ acts only on Ξ:

$$W^{\mu\nu} \sim \int dk^+ d^2k_T \int dl^- d^2l_T \left[\Phi(k^+, k_T)\,\gamma^\mu\, \Xi(l^-, l_T) \right] \delta(k^+ + q^+)\,\delta(q^- - l^-)\,\delta^{(2)}(k_T - l_T) \tag{3.1}$$

$q_T \Rightarrow$ due to choice of (P, q) frame

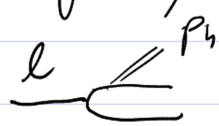
where:

$$\Xi(l^-, l_T) \equiv \int \frac{dl^+}{2l^-}\, \Xi(l) \qquad \text{INTEGRATED JET CORRELATOR}$$

NOTE: this is NOT a TMD correlator, even if it explicitly depends on the transverse l_T

- Rather, l_T is determined by the hard scattering kinematics – in this LO calc, $l_T = k_T$ from the $\delta^{(2)}$ function in Eq (3.1)

- It is best to think about l^-, l_T as "fixed" momenta; a bit like in a fragmentation correlator one thinks as the probability of a hadron to be produced with a variable momentum P_h^-, P_{hT} out of a quark with a given l^-, l_T:



- This way of thinking will be useful when we will look at the "FRAGMENTATION SUM RULES" later on, connecting $\Delta(P_h; l)$ to $\Xi(l)$

NOTE: l is to the right of the semicolon

(II) COLLINEAR EXPANSION OF THE CORRELATORS

<u>No other kinematic approximations are necessary</u> to obtain "collinear PDFs", i.e., k_T-integrated (traces of) parton correlators

→ what we can do, instead, is an expansion in inverse powers of the large momenta k^+ and l^- of the correlators, that will also separate out the k_T^μ, l_T^μ dependence of the correlators.

- this is akin to the Taylor expansion in Qiu's factorization method

- it is also referred to as a "TWIST-EXPANSION" (a bit loosely, in the sense that terms of order $\frac{1}{k^{+\,n}}$ contribute to order $\frac{1}{Q}$ in the DIS cross section)

- The way we achieve this is to perform a decomposition in Dirac space of the correlators ϕ and Ξ:

- To simplify the discussion, we first expand Ξ and express the hadronic tensor in terms of k_T moments of the correlator Φ

use $(9)_{AB}$

$$(4.1)\quad \Xi(l^-, l_T) = \int \frac{dl^2}{2l^-}\,\Xi(l, n_+) = \frac{\Lambda}{2l^-}\,\xi_1 \mathbb{I} + \xi_2 \frac{\not{n}_-}{2} + \frac{i\Lambda}{2}[\not{n}_-, \not{n}_+]\,\beta_2 + \{tw\text{-}4\}$$

$$\equiv \Xi^{(2+3)}(l^-)$$

twist up to $\tau = 3$

where: $\xi_1 = \int dl^2 \frac{\sqrt{l^2}}{\Lambda} J_1(l^2) = \frac{m_q}{\Lambda}$

$\xi_2 = \int dl^2\, J_2(l^2) = 1$

b/c of properties of spectral functions

$\beta_2 = \int dl^2\, B_2(l^2)$ ← I am not aware of a spectral function interpretation

NOTE: according to Andrea's argument, β_2 can in general be $\neq 0$.

NOTE:

- In principle, $\xi_2 \frac{k_T}{2l^-}$ is also a twist-3 term;

however, after k_T integration, it gives zero unless it couples with another term linear in k_T coming from the decomposition of $\phi(x, k_T)$, see $(3.15)_{nt}$ and $(3.45)_{nt}$.

↳ then, this term would start contributing only at order $1/Q^2$ in the cross section, and has been included in the $\{tw\text{-}4\}$ term of Eq. (4.1) ← an explicit calculation would be nice and was started in the July 2015 notes.

- In sum, <u>NO k_T-DEPENDENT TERM APPEARS IN (4.1)</u> up to order $1/Q$, and the d^2k_T integration hits $\phi(x, k_T)$

Then we can define:

$$x = k^+/p^+ \qquad \phi(x) = \int d^2k_T\, \phi(xp^+, k_T)$$

(5.2)

• We can then decompose the integrated ϕ according to (3.31)nt:

$$\phi(x) = \frac{1}{2}\left\{ f_1(x)\not{n}_+ + S_L\, g_1(x)\gamma_5\not{n}_- + h_1(x)\frac{[\not{S}_T,\not{n}_+]}{2}\gamma_5 \right\}$$

$$+ \frac{\Lambda}{2p^+}\left\{ g_T \gamma_5 \not{S}_T \right\} + \dots + \text{tw-4} \tag{6.1}$$

where I kept only the terms that survive the Dirac trace in the hadronic tensor, up to twist-3.

↳ This claim should be verified explicitly, but the FF sum rule argument to be developed in the next set of notes, there should be no surprise

↳ Nonetheless, we could do this up to twist-4 here, since the twist-4 SIDIS is not available

$$\Rightarrow \begin{cases} \text{check the role of } \beta_1 \\ \text{look where else } \beta_2 \text{ appears} \end{cases}$$

③ HADRONIC TENSOR AND DIS CROSS SECTION

- Using (4.1) and (5.2) in the had. tensor (3.1) we get:

$$W^{\mu\nu} \sim \int dk^+ dl^- \left[\Phi(k^+) \gamma^\mu \Xi^{(2+3)}(l^-) \gamma^\nu \right] \delta(k^+ + q^+) \delta(l^- - q^-)$$

$$\boxed{W^{\mu\nu} \sim \int dx\, dl^- \left[\Phi(x) \gamma^\mu \Xi^{(2+3)}(l^-) \gamma^\nu \right] \delta(x - x_B) \delta(l^- - q^-) + \{\text{twist-4}\}} \quad (7.1)$$

This is the basic formula for collinear DIS w/ jet function.

- The next step would be to use the expansions (4.1) and (6.1), contract with the leptonic tensor $L^{\mu\nu}$ and calculate σ; then extract the inclusive structure functions.

- In the AB paper, instead, we take a short cut to be developed in the next "SIDIS SUM RULES" set of notes

↳ as noted above, it is still interesting to carry out the purely inclusive calculation:

→ verify that no other PDF needed in ϕ expansion (6.1)

→ extend to twist-4 ⇒ {role of β_1; where else does β_2 appear?}

(and show explictly that $O(k_T)$ terms in ϕ, Ξ have zero contribution)