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SIDIS SUM RULES

GOAL: • connect integrals of SIDIS correlators and FFs to jet correlators and jet functions

• Use these to calculate inclusive DIS structure functions up to $O(1/Q)$ with jet corrections

We start from: (differently from July 2016 version of the paper)

$$(1.1) \quad \boxed{\sum_h \int \frac{d^2 p_{hT}\, dp_h^-}{(2\pi)^3 2p_h^-}\, p_h^- \, \Delta^h(l, p_h) = l^- \, \Xi(l)}$$

NOTE: • This is akin to (2.16)$_{nt}$: $\int \sigma^{SIDIS} = \sigma^{DIS}$

• I used light-cone coordinate from the outset;
↳ while the measure is also equal to $\frac{d^3 p_h}{(2\pi)^3 2E_h}$,
$p_h^- = E_h - p_h^z = E_h \left(1 + O\left(\frac{1}{Q}\right)\right)$

so that $dp_h^- \, p_h^- = dE_h \, E_h$ only asymptotically

• For now the only justification is that it implies the known FF sum rules, e.g., $\int dz \, D(z)\, z =$

↳ BUT it should be valid on its own

→ NEED DIRECT PROOF and See 170501 NOTES

(I) INTEGRATED CORRELATORS:

- Integrate (1.1) over dl^+ $\Rightarrow$ obtain $\Delta(l^-, l_T; P_h)$ and $\Xi(l^-, l_T)$:

$$\sum_h \int \frac{d^2 P_{hT}\, dP_h^-}{(2\pi)^3\, 2P_h^-} P_h^- \underbrace{\int dl^+ \Delta^h(l; P_h)}_{\equiv\, \tilde{\Delta}^h(l^-, l_T; P_h)} = l^- \int dl^+ \Xi(l) \equiv \underbrace{\int \frac{dl^2}{2l^+} \Xi(l)}_{\equiv\, \tilde{\Xi}(l^-, l_T)} \qquad (2.1)$$

- Next we work on the l.h.s. for a while

  · Insert the definition of z:

$$\int dz\; 2\pi\, \delta\left(z - \frac{P_h^-}{l^-}\right) = 1$$

h is on shell: $P_h^+ = \frac{M_h^2 + P_T^2}{2P^-}$

$$\text{LHS} = \sum_h \int \frac{d^2 P_{hT}\, dP_h^-}{(2\pi)^{\cancel{3}\,2}\, 2P_h^-} \int dz\, \cancel{(2\pi)}\, \delta\left(z - \frac{P_h^-}{l^-}\right) \tilde{\Delta}(l^-, l_T; P_h^-, P_{hT})\, P_h^-$$

exchange dz & dP^-, and use $\frac{dP_h^-}{2P_h^-} = \frac{dz}{2z}$

$$\text{LHS} = \sum_h \int \frac{d^2 P_{hT}}{(2\pi)^2} \frac{dz}{2z} \underbrace{\int \frac{dP_h^-}{l^-}\, \delta\left(z - \frac{P_h^-}{l^-}\right)}_{=1} \tilde{\Delta}(l^-, l_T; z l^-, P_{hT})\, z l^-$$

$$(2.2) \qquad = \sum_h \int \frac{d^2 P_{hT}}{(2\pi)^2} \frac{dz}{2z}\, \tilde{\Delta}(l^-, l_T; z l^-, P_{hT})\, z l^-$$

Next, define the RELATIVE QUARK MOMENTUM

Notation as in [Signori] Ph.D.

$$\vec{P}_{hT} = z\left(\vec{l}_T + \vec{K}_T\right)$$

(3.1) $$K_T \equiv \frac{P_{hT}}{z} - l_T$$ ← RELATIVE QUARK'S VS HADRON'S TRANSVERSE MOMENTUM

and insert it as a δ-function in (2.2):

$$\mathrm{LHS} = \sum_h \int \frac{dz\, d^2P_{hT}}{2z\,\cancel{(2\pi)^2}} \int d^2K_T \cancel{(2\pi)^2}\, \delta^{(2)}\!\left(K_T - \frac{P_{hT}}{z} - l_T\right) \tilde{\Delta}\left(l^-, l_T; z l^-, P_{hT}\right) z l^-$$

from the $\delta^{(2)}$

$$= \sum_h \int dz\, dK_T \; \underbrace{\frac{z}{2}\, \tilde{\Delta}\left(l^-, l_T; z l^-, z(l_T + K_T)\right)}_{\equiv\, \Delta(z, K_T)} z l^-$$

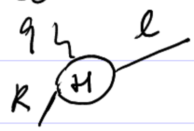
$\equiv \Delta(z, K_T)$ "TMD CORRELATOR" [Signori] (2.6.13)

So:

(3.2) $$\mathrm{LHS} = \int dz\, dK_T\; z l^-\, \Delta(z, K_T)$$

<u>NOTE</u>: even though the integration is written in terms of variables that have a partonic interpretation, the TMD correlator is in fact a function of the hadronic momenta, as is clear from the definition

(3.3) $$\Delta(z, K_T) \equiv \frac{1}{z}\, \tilde{\Delta}\left(l^-, l_T; P_h^-, \vec{P}_{hT}\right)\Big|_{\substack{P_h^- = z l^- \\ \vec{P}_{hT} = z(\vec{l}_T + \vec{K}_T)}}$$

l^- and l_T are here external parameters; in a DIS cross section they are determined by the hard scattering: 

, and the hadron comes out with its z and relative zK_T

NOTE 2: in usual SIDIS calculations, one uses a (P_h, P) collinear frame, so that $\vec{P}_{hT} = 0$, and the change of variables hides a bit the above remark:

$$\vec{P}_{hT} = 0 \Longrightarrow \vec{l}_T = -\vec{K}_T$$

- Finally, using (3.2) in (2.1) we obtain the sum rule at the level of TMD correlators:

(4.1) $$\sum_h \int dz\, d^2K_T\, z\, \Delta(z, \vec{K}_T; l^-, \vec{l}_T) = \Xi(l^-, \vec{l}_T)$$

where I explicitly wrote the dependence of Δ on the "external" quark momentum for clarity of notation

- It is also useful to write this in terms of the collinear FF correlator:

$$\begin{aligned}
\Delta(z) &= \int d^2K_T\, \Delta(z, K_T) \\
&= \int d^2K_T\, \frac{z}{z}\, \Delta(l^-, l_T; z l^-; \overbrace{z(l_T + K_T)}^{= P_{hT}}) \\
&= \int d^2P_{hT}\, \frac{1}{z z}\, \Delta(l^-, l_T; z l^-; \vec{P}_{hT}) \\
&= \int d^2P_{hT}\, \Delta(z, P_{hT})
\end{aligned}$$

this verifies the consistency of the definitions of $\Delta(z)$

- We then obtain:

(5.1) $$\boxed{\sum_h \int dz\, z\, \Delta(z; l^-, l_T) = \Xi(l^-, l_T)}$$

SUM RULE FOR INTEGRATED CORRELATORS

(II) SUM RULES FOR FFs:

- The final step is to sub in (5.1) the Dirac expansion of Δ and Ξ
- From Eq. (3.40)$_{nt}$ we have:

$$\Delta(z) = D_1(z)\frac{\not{n}_-}{2} + \frac{\Lambda}{2P_h^-} E_1(z) + \frac{i\Lambda}{2P_h^-} H(z) \frac{[\not{n}_-, \not{n}_+]}{2}$$

so that:

(5.2) $$z\Delta(z) = zD_1(z)\frac{\not{n}_-}{2} + \frac{\Lambda}{2l^-} E_1(z) + \frac{i\Lambda}{2l^-} H(z)\frac{[\not{n}_-, \not{n}_+]}{2}$$

- The jet correlator expansion reads:

(5.3) $$\Xi(l^-, l_T) = \xi_2 \frac{\not{n}_-}{2} + \frac{\Lambda}{2l^-}\xi_1 + \frac{i\Lambda}{2l^-}\beta_2 \frac{[\not{n}_-, \not{n}_+]}{2}$$

- and the sum rules in terms of FFs can be read off inserting (5.2) and (5.3) into (5.1)

$$\boxed{\begin{aligned} &(6.1) \quad \int dz\, z\, D(z) = \int dz\, d^2 p_{hT}\, z D(z, p_{hT}) = \xi_2 \\ &(6.2) \quad \int dz\; E(z) = \int dz\, d^2 p_{hT}\, E(z, p_{hT}) = \xi_1 \\ &(6.3) \quad \int dz\; H(z) = \int dz\, d^2 p_{hT}\, H(z, p_{hT}) = \beta_2 \end{aligned}}$$

NOTE: this is NEW!

where:

$$\xi_2 = \int dl^2\, J_2(l^2) = 1$$

$$\xi_1 = \int dl^2 \frac{\sqrt{l^2}}{\Lambda} J_1(l^2) = \frac{M_q}{\Lambda}$$

$$\beta_2 = \int dl^2\, B(l^2) \neq 0$$

← NO spectral interpretation known; keep β_2

<u>NOTE</u>: at $O(1/Q)$ $\xi_2 = 1$, $\xi_1 = M_q/\Lambda$;

→ it may seem an excess of notation to leave the ξ_i explicit above.

However, at order $1/Q^2$ the integration over $dl^+ = \frac{dl^2}{2l^-}$ does not decouple, and

$$\beta_i \longrightarrow \beta_i(l^2)$$

Therefore it is useful, in view of a possible twist-4 analysis to keep the β_i explicit in (6.1)-(6.3)

- Using the E.o.M. relations $(3.76)_{NT}$-$(3.79)_{NT}$,

(7.1) $$\tilde{E}(z) = E(z) - \frac{m_q}{\Lambda} z D_1(z)$$

(7.2) $$\tilde{H}(z) = H(z) - H_1^{\perp(1)}(z)$$

- Then:

GENERALIZES Eq. $(4.30b)_{NT}$

(7.3) $$\int dz\, \tilde{E}(z) = \frac{M_q - m_q}{\Lambda}$$

(7.4) $$\int dz\, \tilde{H}(z) = B_2 - \int dz\, H_1^{\perp(1)}(z)$$

= ?

- Eq (7.4) needs a more careful treatment:

↳ our sum rule (1.1) does not yield, at face value, information on integrals of p_{hT}^2 moments of TMDs

Indirectly we can say something if we trust $(2.22)_{NT}$:

(7.5) $$\sum_h \int dz\, z\, F_{UT}^{\sin\phi_S}(x,z) = 0$$

b/c of T-reversal invariance [Diehl - Sapeta]

NOTE: this is probably obtained directly at the s.fn. level without looking at factorization and quark FFs
⟹ likely to be really true in general

CHECK!

- Now, in collinear factorisation: [see $(4.24)_{NT}$]

$$F_{UT}^{\sin\phi_S}(x) = \frac{2\Lambda}{Q}\, x h_1(x) \sum_h \int dz\, \tilde{H}(z)$$

$\Rightarrow$ In order to satisfy (7.5) we need

$$\int dz\, H_1^{\perp(1)}(z) = \beta_2$$ $\leftarrow$ NEW FF SUM RULE !

NOTE: the proof is indirect, but it would be nice to find an argument directly at the correlator level. ‖

SUMMARY: NEW NON-PERT FF SUM RULES:

$$\boxed{\begin{aligned}
&\sum_h \int dz\, z D_1^h(z) = 1 && \leftarrow \text{confirms } (4.30a)_{NT}\\
&\sum_h \int dz\, \tilde{E}(z) = \frac{M_q - m_q}{\Lambda} && \leftarrow \text{NEW ONE! extends } (4.30b)_{NT}\\
&\sum_h \int dz\, \tilde{H}(z) = 0 && \leftarrow \text{confirms } (4.30c)_{NT}\\
&\left.\begin{aligned}&\sum_h \int dz\, H(z) = \beta_2\\ &\sum_h \int dz\, H_1^{\perp(1)}(z) = \beta_2\end{aligned}\right\} && \text{COMPLETELY NEW ONES}
\end{aligned}} \tag{8.1}$$

NOTE: "PERTURBATIVE LIMIT"

the "usual" sum rules can be recovered in a perturbative treatment of the jet correlator:

$$\Xi(\ell) = \text{F.T.} \langle 0 | \bar\psi(\eta)\, \psi(0)\, W[0,\eta; n_-] | 0 \rangle$$

$$\Rightarrow \Xi_{pert} = \text{F.T.}\ \overset{2\pi i\, \mathrm{Im}}{} \langle 0 | \bar\psi_{free}(\eta)\, \psi_{free}(0) | 0 \rangle$$

$$= \not\ell + m_q$$

$$\Rightarrow M_q^{pert} = \mathrm{Tr}\left[\Xi_{pert}\, \mathbb{1}\right] = m_q$$

$$\beta_2 = \mathrm{Tr}\left[\Xi_{pert}\, \not{\not{n}}\right] = 0$$

TO DO:

- Relate M_q to chiral condensate $\langle 0 | \bar\psi(0)\psi(0) | 0 \rangle$
  - ↳ How ??
- Evaluate β_2 (and M_q?) in a quark model
  - ↳ see e.g. calculations of B_* functions in appendix of [ABMS]