JMC @ Twist-3

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<u>GOAL</u>: analyse JMC at order M/Q, determine effects of mass term
⟹ the only effect will be on g_2

<u>IDEA</u>: Consider

current correlator

χ

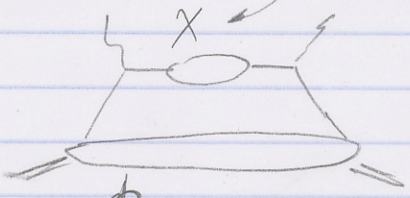


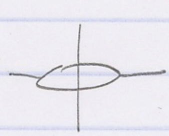
FIG. (1.1.)

ϕ ↖ target correlator

Gehrmann QFT lectures 2010

$$\chi(\ell) = \int dm^2\, J(m^2)\, \frac{i}{\not{\ell} - m} =$$ 

$$\mathrm{Im}\, \chi(\ell) = \int dm^2\, J(m^2)\,(\not{\ell} + m)\,\delta(\ell^2 - m^2)$$

$$\stackrel{!}{=}$$ 

|| What are the consequences of this "jet propagator" $\not{q}$, in an expansion where we retain terms up to $\frac{1}{Q}$?

NOTE THAT:

$$\left.\begin{array}{l} \int_0^\infty dm^2\, J(m^2) = 1 \\ J(m^2) > 0 \end{array}\right\} \Rightarrow \int_0^\infty dm\, m\, J(m^2) = \overline{m}_j \gneq 0$$

At $O(M/Q)$, $\xi \approx x$ ⟹ PDFs decouple from dm^2

we expect no changes, but for $m_q \to \overline{m}_j$

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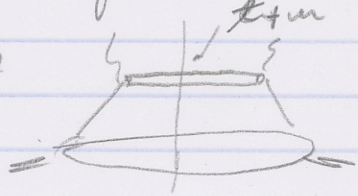
- Using the spectral representation it looks like we are dealing with a massive quark of mass m

$\int dm^2$ 

However this interpretation can be misleading, since due care is needed to distinguish:

m = mass of the final state
(what goes through the cut in Fig(1.1))

m_q = current mass: the mass of the field in the current correlator χ:

$$\chi = \langle 0 | \bar{\psi} \quad \psi | 0 \rangle = {}_{m_q}\!\!-\!\!\phi\!\!-$$

The difference is:

m_q: enters the e.o.m.

m: matters for external kinematics.

$$x \to \xi \left(1 + \frac{m^2}{Q^2} \right) = x + O\left(\frac{M^2}{Q^2}\right) + O\left(\frac{m^2}{Q^2}\right)$$

and the mass term $m J_2(m)\, \mathbb{I}$

This distinction is very important to demonstrate the gauge invariance of

$$\mathrm{Tr}\left[\phi\, \gamma^\mu \chi\, \gamma^\nu \right]$$

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@ $O(M/Q^2)$

– FINAL RESULT (we believe)

$$g_1(x) \longrightarrow \int dm^2 J(m^2)\, g_1(x) \equiv g_1(x)$$

$$h_1(x) \longrightarrow \int dm^2 J(m^2)\, h_1(x) = h_1(x)$$

$$\frac{m_q}{M}\left(\frac{h_1}{x}\right)^* \longrightarrow \frac{1}{M}\int dm^2\, m\, J(m^2)\left(\frac{h_1(x)}{x}\right)^* \equiv \frac{\overline{m}_j}{m_q}\left(\frac{h_1}{x}\right)^*$$

this is the term in $g_{2T} - g_2^{WW}$

• PHENOM. CONSEQUENCES:

$\overline{m}_j^* = O(200\ \text{MeV}) \gg m_q$

$\Rightarrow$ mass term in $g_2 - g_2^{WW}$ NOT negligible

In fact, from quick numerics with ~~Ale~~ Alessandro's h_1 fit,

$\frac{\overline{m}_j}{M}\left(\frac{h_1}{x}\right)^* \approx \Delta_{WW}$ from our paper

Since this is positive, the extracted HT matrix of elements

$$Tw3 = \Delta_{WW} - \frac{\overline{m}_j}{M}\left(\frac{h_1}{x}\right)^*$$

become much more similar to the calculation by Braun et al.

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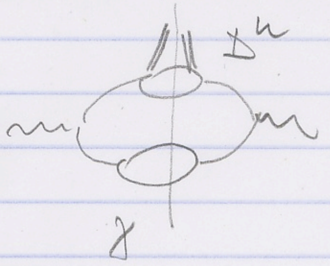
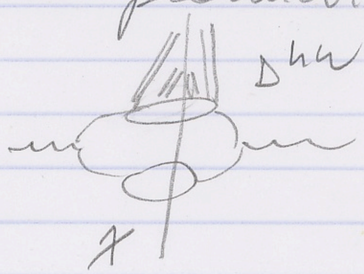
- How to measure $\bar{m}_j$?

$\bar{m}_j$ can be also accessed in single, and same side dihadron production in e^+e^-:





↓ convoluted with the equivalent of h_1

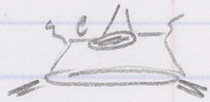
$\bar{m}_j\, H_1^h$

↓ likely in a www like breaking term, as well

$\bar{m}_j\, H_1^{\sphericalangle}$

- These expectations have yet to be verified
- likely facilities.
  - ↳ BES (low energy, higher sensitivity to this physics than LEP, say)

- General idea of the computation:

• Follow the same as for SIDIS 

• Differences, only a small subset of PDFs survives:

$$\Delta(\ell, P_h, S_h) \quad \text{vs} \quad \chi(\ell)$$

$$\chi(\ell) = \chi_h(\ell^2)\,\mathbb{I} + \chi_\ell(\ell^2)\,\not{\ell} + \text{tw.-3}$$

these I don't remember, but need to be included

↳ In particular, apply eq. of motion to extract tw-2 from "$\tilde{E}$"

• Then:

↳ take all the traces
↳ verify gauge invariance
↳ finally: use spectral representat. for tw-2 pieces.

• NOTE: $\chi_h(\ell^2) = \int dm^2\, m\, J_2(m^2)\, \delta(m^2 - \ell^2)$

understood as operator

→ the mass term is known if $J(m^2)$ is known

• Jump

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* PROOF (VERY ROUGH)   and cavalier!

Identify: (here I am very schematic, only to try and not forget the idea)

$D_1 \longrightarrow \cancel{x}\, J_2$

$E \longrightarrow \cancel{x}\, J_0$

$H \longrightarrow\ ?$

↳ Ale needs to make the real calculation.

possible? survives inclusive DIS?

From Bacchetta & Co.:

twist 2

$$\Delta(z) = z^2 \int d^2k_T\, D(z,k_T) = D_1 \frac{\not{n}_-}{2} + \frac{M_h}{2P_h^-} E + \frac{M_h}{2P_h^-}\, iH \frac{[\not{n}_-, \not{n}_+]}{2}$$

$$\Big\downarrow z=1$$

$$X = \qquad = J_2 \frac{\not{n}_-}{2} + \Lambda_q J_0 + \Lambda_q\, i J_H \frac{[\not{n}_-, \not{n}_+]}{2}$$

?

twist 3

$\Delta_D \longrightarrow$ only the $\tilde{E}$ term survives

e.o.m:

$$\tilde{E} = E - 2\frac{m}{M_h} D_1$$

$$\downarrow \qquad \downarrow \qquad \downarrow z\to 1,\ P_h^- = \ell^-$$

$$\tilde{J} = J_0 - \frac{m_q}{\Lambda_q} J_2$$

(6b)

because of e.o.m

Finally, from Eq (4.25)

(6b.1) $$F_{LT}^{\cos\phi_S} = -x\sum e_a^2 \frac{2M}{Q}\left(x g_T(x) D_1(z) + \frac{M_h}{M} h_1(x) \frac{\tilde{E}(z)}{z}\right)$$

$M_h \to m_q$

Integrating over dz, they use $\int \tilde{E}(z)\,dz = 0$.
[neglecting χSC]

For us, $\int dz \to \int dm^2$

$$\Longrightarrow \int dz\, D_1(z) \to \int dm^2\, J(m^2) = 1$$

$$\int dz\, \tilde{E}(z) \to \int dm^2\, J(m^2) - \frac{1}{\Lambda_q}\int dm^2\, m\, J(m^2)$$

Choose $\Lambda_q = m_q$

(6b.2) $$= \cancel{1} - \frac{1}{m_q}\Big[\cancel{m_q} + \underbrace{\int_{m_\pi}^{\infty} dm^2\, m\, J(m^2)}_{\overline{m}_c}\Big)$$

$$= -\frac{\overline{m}_c}{m_q}$$

should be +

Now $x g_T \overset{\text{e.o.m}}{=} x\tilde{g}_T + x\hat{g}_T + \frac{m_q}{M} h_1 +$

So that:

$$F_{LT}^{\cos\phi_S} \propto \frac{2M}{Q}\Big[\text{pure tw-3} + \frac{m_q}{M}h_1 + \cancel{\frac{m_q}{M}h_1}\frac{\overline{m}}{\cancel{m_q}}h_1\Big)$$

$$= \frac{2M}{Q}\Big[\text{pure tw-3} + \frac{\overline{m}_j}{M}h_1\Big]$$

end

→ (6c)

The cancellation in (6b.2), and the m_q gained back putting that into (6b.1) look like black magic,

However, they are not a formal exercise: they come from:

e.o.m. for g_T $\rightsquigarrow m_q$

e.o.m for $\tilde{J}$ $\rightsquigarrow m_q$

δ-fn in $J(m^2)$ $\rightsquigarrow m\,\delta(m^2 - m_q^2) \equiv m_q$

One may wonder if all this gimmick is necessary, or one could find an easier way

Probably it is necessary, because e.o.m are one of the ingredients in the proof of the WW relations that isolates ~~twist~~ pure tw-3 terms.

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OPEN QUESTIONS

* Does $\bar{m}_j$ enter some other structure function?
  ↳ at twist 3 probably not
  ↳ at twist-4 ?

Note: generally the substitution $m_q \to \bar{m}_j$ is not necessarily valid: for g_2 it looks like an accident: m_q is cancelled between Δg_2^{ww} (obtained through e.o.m) and the ~~m_q~~ δ-function in J_2:

$$\bar{m}_j = m_q + \int_{m_\pi}^{\infty} J_2\Big|_{cont}(m^2)\, dm^2$$

So, there may be some m_q left in the general analysis (say in the WW for h_L?, somewhere else?)

* What is the role of the gauge link in χ?
  So far we sort of neglected it, and in fact it may not enter much the DIS cross section, except for $g_2 - g_2^{ww}$

$$\chi = \chi(\ell, n^+) = J_0(\ell^2)\,\mathbb{1} + J_2(\ell^2)\,\ell\!\!/ + \not{n}^- \quad ??$$

⇝ But $\ell^+\ell^- \to h\,(?h)$??

* How are $\tilde{g}_2, \hat{g}_1$ jet-mass corrected? I would say with $\int J_2 \Longrightarrow$ no corrections @ tw-3, but corrections of $O(\frac{M}{Q}, \frac{m_j^2}{Q^2})$ may be there.

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* Does the WW relation survive JMK?

At order $1/Q$ yes, as g_1 doesn't receive corrections.

But what about all-orders à la Accardi-Qiu?

Probably yes:

$$g_{2T}\Big|_{LT} = \left[J * g_1 \right]^{WW}$$

* FOR 2ND PAPER

include kinematics @ all twists:

Argue: • "exact" kinematics ⇒ J, MJ don't decouple
→ large corrections numerically if x large, Q^2 small

• But pure twist 4 may really be small (smaller than pure tw 3)

⇒ It makes sense to have up to pure tw. 3, but all orders kinematics

⇝ interesting to work it out and show numerically how large these corrections may be!

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* Transversity with JLC ???

From [Bacchetta & C.], eq. (4.24)

$$F_{UT}^{\sin\phi_S} = -x \sum_a e_a^2 \frac{2M_h}{Q} h_1 \frac{\tilde{H}(z)}{z}$$

$$\tilde{H} = H - z\frac{k_T^2}{M_h^2} H_1^\perp$$

integrated becomes

$$\tilde{H} = H - \frac{z}{M_h^2} \hat{H}_1^\perp$$

relevant for JLC !!

Following the heuristic before and $D \leftrightarrow X$..

$$F_{UT}^{\sin\phi_S} \propto \frac{\overline{m}_J}{Q} h_1 + \ldots ??$$

IT looks like we can measure transversity in INCLUSIVE DIS ??? Against general theorem...

But maybe I am missing something and

(9.1) $\tilde{H} \longleftrightarrow 0$

Fragm. Jet fn

It would be nice to prove explicitly (9.1) following the same identification process as for g_2.