Accessing the nucleon tensor structure in inclusive deep inelastic scattering

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I. INTRODUCTION

The nucleon tensor charge is a fundamental property of the nucleon and is at present poorly constrained. Several estimates based on lattice QCD are available, but only limited information from direct measurements. The way to extract the tensor charge from experimental measurements requires first the extraction the so-called transversity parton distribution function (PDF), denoted by $h_1^q(x)$. The transversity distribution is then integrated over x to yield the contribution of flavor q to tensor charge.

In this paper, we discuss the possibility of observing the effect of the transversity distribution in totally inclusive Deep Inelastic Scattering.

The transversity distribution is notoriously difficult to access because it is a chiral-odd function and needs to be combined with a spin-flip mechanism to appear in a scattering process. Usually, this spin flip is provided by another nonperturbative distribution or fragmentation function, implying that transversity cannot be accessed in inclusive DIS, but only in more complex processes such as semi-inclusive DIS or Drell-Yan.

The only other way to attain spin-flip terms in QED and QCD is taking into account mass corrections. In fact, it is well known that transversity gives a contribution to the structure function g_2 in inclusive DIS, and in particular to the violation of the so-called Wandzura–Wilczek relation for g_2. However, this contribution is proportional to the current quark mass and can be expected to be very small.

We revisit the standard analysis of inclusive DIS taking into account the fact that on-shell quarks cannot be present in the final state, but they rather decay into hadrons (ideally, the form jets of hadrons). This is sufficient to modify the structure of the DIS cut-diagram, even if none of the hadrons is detected in the final state. For a proper description of this effect, we include "jet correlators" into the analysis. We pay particular attention to insuring that our results are gauge invariant. We observe that the inclusion of jet correlators introduces a new contribution to the inclusive g_2 structure function. This term has the interesting features that: a) violates the Wandzura–Wilczek relation, b) violates the Burkhardt–Cottingham sum rule, c) is proportional to the transversity distribution function multiplied by a nonperturbative "jet mass" parameter. We provide estimates of this contribution based on a recent extraction of transversity and show that it could be very large.

II. JET CORRELATOR AND TWIST-2 STRUCTURE FUNCTIONS

Motivated by large-x mass corrections to inclusive DIS structure functions, Accardi and Qiu have introduced in the LO handbag diagram a "jet correlator" that accounts for invariant mass production in the current jet, and ensures that leading twist calculations in collinear factorization are consistent with the requirement imposed by baryon number conservation that $x_B < 1$ [**?**]. The jet correlator is depicted in Figure 1(a) and is defined as

$$\Xi_{ij}(l, n_+) = \int \frac{d^4\eta}{(2\pi)^4}\, e^{il\cdot\eta}\, \langle 0|\, \mathcal{U}^{n_+}_{(+\infty,\eta)}\, \psi_i(\eta)\bar{\psi}_j(0)\, \mathcal{U}^{n_+}_{(0,+\infty)}\, |0\rangle \;, \tag{1}$$

In this definition, l is the quark four-momentum, Ψ the quark field operator (with quark flavor index omitted for simplicity), and $|0\rangle$ is the nonperturbative vacuum state. Furthermore, we explicitly guarantee the correlator's gauge invariance by introducing two Wilson line operators $\mathcal{U}^{n_+}$ along a light-cone plus direction determined by the vector n_+. This path choice for the Wilson line is required by QCD factorization theorems, and the vector is determined by the particular hard process to which the jet correlator contributes. For example, in the case of inclusive DIS discussed in this paper, this is determined by the four momentum transfer q and the proton's momentum p.

The correlator Ξ can be parametrized in terms of scalar functions, using the vectors l and n_+:

$$\Xi(l, n_+) = \Lambda A_1(l^2)\,\mathbf{1} + A_2(l^2)\not{l} + \frac{\Lambda^2}{l\cdot n_+}\not{n}_+\, B_1(l^2) + \frac{i\Lambda}{2l\cdot n_+}[\not{l}\,,\not{n}_+]\, B_2(l^2) \;. \tag{2}$$

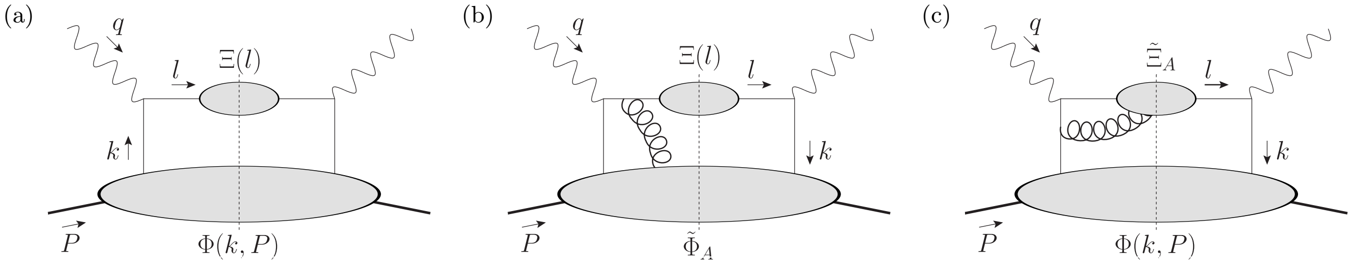

FIG. 1: Diagrams contributing to DIS scattering up to twist-3 expansion, including a jet correlator in the top part. Note the gluon attaches to both the nucleon and jet correlators. The Hermitian conjugates of diagrams (b) and (c), i.e., with gluons attaching to the right of the cut, are not shown.

Time reversal invariance in QCD requires $B_2 = 0$, while B_1 contributes only at twist-4 order, and will not be considered further in this paper. We focus, instead, on the role of chiral odd terms in the g_2 structure function up to twist 3. At this order,

$$\Xi(l, n_+) = \Lambda A_1(l^2)\,\mathbf{1} + A_2(l^2)\not{l} + O(\Lambda^2/Q^2) \tag{3}$$

is nothing else than the full quark propagator; note however, that we consider here the full QCD vacuum rather than the perturbative one. The A_1 and A_2 terms can be nicely interpreted in terms of the spectral representation of the cut quark propagator [**?** **?**],

$$\Xi(l) = \int d\mu^2 \big[J_1(\mu^2)\,\mu + J_2(\mu^2)\not{l}\,\big]\,\delta(l^2 - \mu_j^2)\ , \tag{4}$$

where μ^2 is interpreted as the invariant mass of the current jet, *i.e.*, of the particles going through the cut in the top blob of Fig.1(a), and the J_i are the spectral functions of the quark propagator, that have been also called "jet functions" in [**?** **?** **?**]. These satisfy

$$J_2(\mu^2) \geq J_1(\mu^2) \geq 0 \qquad \text{and} \qquad \int d\mu^2 J_2(\mu^2) = 1\ . \tag{5}$$

From a comparison of Eqns.(2) and (4), one can see that

$$A_1(l^2) = \frac{\sqrt{l^2}}{\Lambda} J_1(l^2) \qquad\qquad A_2(l^2) = J_2(l^2)\ . \tag{6}$$

When inserting the jet correlator in the handbag diagram for inclusive DIS, the invariant jet mass μ^2 is integrated from 0 to $Q^2(1/x_B - 1)$. This induces (kinematical) corrections of order $O(1/Q^2)$, whose effect on the F_2 structure function has been studied in Ref. [**?**]:

$$F_2(x_B) = \int_0^{Q^2(1/x_B-1)} d\mu^2\, J_2(\mu^2) F_2^{(0)}(x_B(1+\mu^2/Q^2))\ , \tag{7}$$

where $F_2^{(0)}$ is the structure function calculated with the handbag diagram sporting a bare quark propagator instead of the jet correlator, and $\xi = 2x_B/(1+\sqrt{1+4x_B^2M^2/Q^2})$ with M the nucleon's mass is the Nachtmann scaling variable. (We also omitted the dependence of the structure function on Q^2 for clarity of notation). In this paper we limit our attention to effects of order $O(1/Q)$ and therefore can extend the integration to $\mu^2 = \infty$. Therefore, the jet function J_2 decouples and, thanks to the sum rule (5), integrates to 1. One then recovers the conventional result,

$$F_2(x_B) = \Big[\int_0^\infty d\mu^2\, J_2(\mu^2)\Big] F_2^{(0)}(x_B) + O(\Lambda^2/Q^2) = F_2^{(0)}(x_B) + O(\Lambda^2/Q^2)\ . \tag{8}$$

More in general, the jet correlator decouples from the parton correlator Φ in any inclusive cross section calculation up to $O(1/Q)$, and the inclusive structure functions only depend on the integrated jet correlator

$$\Xi(l^-) \equiv \int \frac{dl^2}{2l^-} d^2l_T\, \Xi(l) = \frac{\Lambda}{2l^-}\,\xi_1 \mathbf{1} + \xi_2 \frac{\not{n}_-}{2} + \text{higher twists} \tag{9}$$

where

$$\xi_1 = \int d\mu^2 \frac{\mu}{\Lambda} J_1(\mu^2) \equiv \frac{M_q}{\Lambda}, \qquad\qquad \xi_2 = \int d\mu^2 J_2(\mu^2) = 1 \ . \tag{10}$$

where M_q can be interpreted as the average invariant mass produced in the spin-flip fragmentation processes of a quark of flavor q. It is important to notice that $\xi_2 = 1$ exactly due to CPT invariance [?], while $0 < M_q < \int d\mu^2 \mu J_2(\mu^2)$ is dynamically determined. From the analytic properties of spectral functions we may expect [?] $J_2(\mu^2) = Z\delta(\mu^2 - m_q) + \bar{J}_2(\mu^2)\theta(\mu^2 - m_\pi^2)$ with the continuum starting at m_π, the mass of the pion, due to color confinement effects. Taking into account that $J_1 < J_2$, we may therefore expect

$$M_q = O(10 - 100 \text{ MeV}) \ . \tag{11}$$

Although M_q is in general a nonperturbative quantity, it is interesting to notice that

$$M_q = \frac{\Lambda}{4} \int \mathrm{Tr}\left[\Xi(l)\mathbf{1}\right] = \langle 0|\bar{\psi}_i(0)\psi_i(0)|0\rangle \tag{12}$$

Calculating this on the perturbative vacuum and limiting oneself to LO corresponds to taking the trace of the cut bare-quark propagator to obtain $M_q = {}_{\text{pert}}\langle 0|\bar{\psi}_i(0)\psi_i(0)|0\rangle_{\text{pert}} = m_q$, with m_q the quark mass, recovering the conventional result. However, we are here considering non perturbative effects on the quark fragmentation and $M_q \gtrsim m_q$.

III. TWIST-3 ANALYSIS

Extending the analysis of [?] to the calculation of twist-3 structure functions requires not only to consider the ξ_1 term in the jet correlator, but also quark-gluon-quark correlators in both the proton and the vacuum as depicted in Figs.1(b) and (c), respectively. In the former the ξ_1 terms contribute to $O(1/Q^2)$, so that up $O(1/Q)$ these give the same contribution as in the conventional handbag calculation.

The novel element in our analysis is the jet's quark-gluon-quark correlator $\Xi_A^\mu(l,k)$ in diagrams 1(c),

$$(\Xi_A^\mu)_{ij} = \frac{1}{2} \sum_X \int \frac{d\eta^+ \, d^2\boldsymbol{\eta}_T}{(2\pi)^3} \ e^{ik\cdot\eta} \ \langle 0|\,\mathcal{U}^{n_+}_{(+\infty,\eta)}\, gA^\mu(\eta) \ \psi_i(\eta)|X\rangle\langle X|\bar{\psi}_j(0)\,\mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\bigg|_{\eta^-=0} . \tag{13}$$

This diagram and its hermitian conugate are not only important to account for all contribution of order $O(1/Q)$, but also in restoring up to twist-3 the gauge invariance broken in diagram 1(a) by the different mass of the incoming and outgoing quark lines, namely, $m_q \neq M_q$.

Rather than directly using the definition (13), it is convenient to calculate the inclusive cross section as an integral of the semi-inclusive one, utilize the QCD equation of motions and furthermore summed over all hadron flavors, and take advantage of

$$\sum_h \int \frac{d^3p_h}{(2\pi)2E_h} \Delta^h(l,p_h) = \Xi(l) \ , \tag{14}$$

where Δ^h is the quark fragmentation correlator for production of a hadron of flavor h and momentum p_h [?]. In terms of the TMD fragmentation functions we are interested in, this reads

$$\sum_h \int dz d^2p_{hT} z D_1^h(z,p_{hT}) = \xi_2 = 1 \tag{15}$$

$$\sum_h \int dz d^2p_{hT} E(z,p_{hT}) = \xi_1 \ , \tag{16}$$

where $D_1^h(z,p_{hT})$ is the twist-2 quark fragmentation function as a function of the hadron's collinear momentum fraction z and transverse momentum p_{hT}, and $\tilde{E}^h(z,p_{hT})$ is a chiral-odd twist-3 function defined in [?].

The relevant part of the semi-inclusive hadronic tensor for our analysis is **[AA] I am using the notation in Piet Mulder's lecture notes - this will need to be checked.** •

$$2\Lambda W^{\mu\nu} = i\frac{2\Lambda}{Q}\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho} S_{\perp\rho} \sum_q e_q^2 \Big[2x_b g_T(x_B) \sum_h \int dz d^2p_{hT} D_1^h(z,p_{hT}) + 2h_1(x_B) \sum_h \int dz d^2p_{hT} \tilde{E}^h(z,p_{hT})\Big] + \ldots \tag{17}$$

Here and we reintroduced the quark flavor q for maximum clarity, and e_q its electric charge. The first term can be easily integrated with the help of the sum rule (15). To integrate the latter, we first need make use of the relation $\tilde{E}(z) = E(z) - (m_q/\Lambda) z D_1(z)$, which is a consequence of the QCD equations of motion [?], then utilize the sum rule (14):

$$\sum_h \int dz d^2 p_{hT} \tilde{E}^{q,h}(z, p_{hT}) = \sum_h \int dz d^2 p_{hT} \Big[E^{q,h}(z, p_{hT}) - \frac{m_q}{\Lambda} z D_1^{q,h}(z, p_{hT}) \Big] = \xi_1 - \frac{m_q}{\Lambda} \xi_2 = \frac{M_q - m_q}{\Lambda} \ . \tag{18}$$

This formula is the single most important result of this paper, and provides a non perturbative generalization of the commonly used $\int \tilde{E} = 0$ sum rule introduced in [?].

Finally, with suitable projections of the hadronic tensor, the inclusive cross section up to order Λ/Q can be written as

$$\frac{d\sigma}{dx\, dy\, d\psi} = \frac{2\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \Big\{ F_{UU,T} + \varepsilon F_{UU,L} + S_\parallel \lambda_e \sqrt{1-\varepsilon^2}\, F_{LL} + |\boldsymbol{S}_\perp| \lambda_e \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, F_{LT}^{\cos\phi_S} \Big\} \ , \tag{19}$$

where the structure functions on the right hand side are defined as

$$F_{UU,T} = x \sum_a e_a^2\, f_1^a(x), \tag{20}$$

$$F_{UU,L} = 0, \tag{21}$$

$$F_{LL} = x \sum_a e_a^2\, g_1^a(x), \tag{22}$$

$$F_{UT}^{\sin\phi_S} = 0, \tag{23}$$

$$F_{LT}^{\cos\phi_S} = -x \sum_a e_a^2 \frac{2\Lambda}{Q} \left(x g_T^a(x) + \frac{M_q - m_q}{\Lambda}\, h_1^a(x) \right). \tag{24}$$

The second term in the last structure function is a new result from our analysis; it is not suppressed as an inverse power of Q, and therefore survives even in the Bjorken limit. Note that calculating the jet correlator on the perturbative vacuum one would obtain, as already discussed, $M_q = m_q$ and the new term wouls vanishe, as it should. However, on the non-perturbative vacuum the jet mass is larger than the quark's, and this contributes a non-negligible term to the twist-3 part of the g_2 function, as we shall discuss in the next section.

IV. THE g_2 STRUCTURE FUNCTION

The new term in Eq.(24) only appears in the g_2 structure function. Following the derivation in Ref. [?], one finds

$$g_2(x_B) = g_2^{WW} + \frac{1}{2} \sum_a e_a^2 \left(\widetilde{g}_T^{a\star}(x) + \int_x^1 \frac{dy}{y} \widehat{g}_T^a(y) + \frac{m_q}{M} (h_1^a/x)^\star(x) + \frac{M_q - m_q}{M} h_1^a(x) \right) \ , \tag{25}$$

where we defined $f^*(x) = -f(x) + \int_x^1 \frac{dy}{y} f(y)$. The first 4 terms coincide with the result obtained in the conventional handbag approximation [?], while the fifth is new.

The first term is also known as the Wandzura-Wilczeck function $g_2^{WW} = g_1^*(x)$, and contains all the "pure twist-2" chiral even contributions to the g_2 structure coming from quark-quark correlators. The second and third terms contain all "pure twist-3" contributions, i.e., those coming from quark-gluon-quark correlators. The fourth and fifth terms depend on the transversity parton distribution function, h_1. The former is usually neglected for light quarks since it is proportional to $m_q = O(1 \text{ MeV})$. In the latter term, new in our analysis, the transversity distribution is multiplied by a constant of $O(100 \text{ MeV})$, and cannot be a priori neglected.

To estimate the size of the last two terms in Eq. (25), we use a recent parametrization of the transversity distribution from Ref. [?], which is comparable also to other extractions [? ?]. We define the shorthand notation

$$g_2^{\text{quark}} = \frac{1}{2} \sum_a e_a^2 \frac{m_q}{M} (h_1^a/x)^\star(x), \qquad g_2^{\text{jet}} = \frac{1}{2} \sum_a e_a^2 \frac{M_q - m_q}{M} h_1^a(x). \tag{26}$$

We choose the values of the mass parameters to be $m_q = 5$ MeV and $M_q = 50$ MeV.

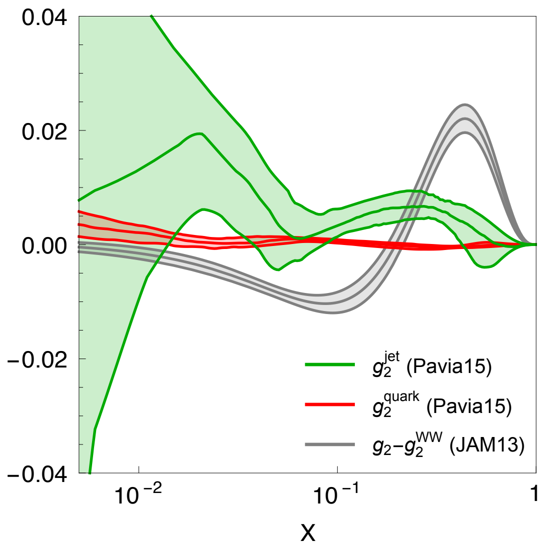

FIG. 2: Different contributions to the non Wandzura-Wilczek part of the g_2 structure function compared to the JAM15 fit of $g_2 - g_2^{\text{WW}}$ [**?**].

- large-x: bridges Braun's pure twist-3 theory and the data
- smaller-x: constrains the small-x behavior of transversity.
- in general, g_2^{quark} is quite smaller than the other contribution, as customarily assumed.

Another consequence of the new chiral odd term induced by the non-vanishing of the chiral condensate on the non perturbative vacuum is that the Burkardt-Cottingham sum rule is broken:

$$\int dx\, g_2(x) = \frac{M_q - m_q}{\Lambda} \int dx\, h_1(x) \ , \tag{27}$$

where the integral over h_1 is related to target's tensor charge. Consequences:

- Inclusive DIS become sensitive to the tensor charge; furthermore, teh BC summ rule isolates the effects due to the chiral odd part of the jet correlator.
- Both the jet mass M_q and the tensor charge can in principle be calculated on the lattice
- Comparison to the Burkhardt-Cottingham sum rule can provide experimental verification of lattice calculation
- in turn these can be used to determine the size of the h_1 term in $g_2 - g_2^{WW}$ and allow an eperimental extraction of the pure twist-3 terms.

It is important to explore in which other process does M_q contribute, as to provide an experimental check of the formalism:

- inclusive Λ production in $e^+ + e^-$
- same-side dihadrons in $e^+ + e^-$

It would be cool to find a process where the M_q contribution is th eonly one (similar to the BC breaking) ...

V. CONCLUSIONS

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Acknowledgments

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