

# Parameterization of the quark–quark correlator of a spin- $\frac{1}{2}$ hadron

K. Goeke, A. Metz, M. Schlegel

*Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

Received 25 April 2005; accepted 17 May 2005

Available online 23 May 2005

Editor: N. Glover

## Abstract

The general parameterization of the quark–quark correlation function for a spin- $\frac{1}{2}$  hadron is considered. The presence of the Wilson line ensuring color gauge invariance of the correlator induces structures that were not given explicitly in the existing literature. In particular, the general form of the transverse momentum dependent correlator entering various hard scattering processes is derived. In this case two new time-reversal odd parton distributions appear at the twist-3 level.

© 2005 Elsevier B.V. Open access under [CC BY license](http://creativecommons.org/licenses/by/4.0/).

**1.** The purpose of this Letter is to provide the general structure of the quark–quark correlation function of a spin- $\frac{1}{2}$  hadron,

$$\Phi_{ij}(P, k, S|n_-) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik \cdot \xi} \langle P, S | \bar{\psi}_j(0) \mathcal{W}(0, \xi | n_-) \psi_i(\xi) | P, S \rangle. \quad (1)$$

The target state is characterized by its four-momentum  $P$  and the covariant spin vector  $S$  ( $P^2 = M^2$ ,  $S^2 = -1$ ,  $P \cdot S = 0$ ), while  $k$  denotes the momentum of the quark. The Wilson line  $\mathcal{W}(0, \xi | n_-)$  guarantees color gauge invariance of the correlator, where the specific path of the gauge link will be given below. Several articles in the literature [1–4] are already dealing with the general parameterization of  $\Phi$ , but none of them contains explicitly the complete decomposition.

The knowledge of the correlator in Eq. (1) is particularly useful in order to obtain the general form of the transverse momentum dependent ( $k_T$ -dependent) correlator  $\Phi(x, \vec{k}_T, S)$ , which enters the description of hard scattering processes like transverse momentum dependent semi-inclusive DIS and the unintegrated Drell–Yan reaction. The connection between both objects is given by the relation

$$\Phi(x, \vec{k}_T, S) = \int dk^- \Phi(P, k, S|n_-), \quad (2)$$

*E-mail address:* [metza@tp2.ruhr-uni-bochum.de](mailto:metza@tp2.ruhr-uni-bochum.de) (A. Metz).

with  $x$  defining the plus-momentum of the quark via  $k^+ = xP^+$ . Recently, a lot of work has been devoted to the experimental investigation of  $k_T$ -dependent parton distributions—determined through the correlator in (2)—and fragmentation functions [5–10]. Most of these studies have focused on so-called time-reversal odd (T-odd) correlation functions which typically give rise to single spin asymmetries. Also on the theoretical side there has been a tremendous activity in this field of research during the past years comprising conceptual (see, e.g., Refs. [11–24]) and phenomenological work (see, e.g., Refs. [25–34]). Because many of the mentioned studies are dealing with subleading twist (twist-3) effects it is important to have a complete description of the correlator (2) including the twist-3 level. In the present work we intend to present such a description for the first time. We also would like to emphasize that the totally unintegrated correlator in Eq. (1) should not merely be considered as a mathematical object, but may in fact be used in the description of hard processes, in which it is appropriate to not integrate upon the minus-momentum of the quark [35].

Our work is mainly based on the crucial observation made in Ref. [3] according to which the direction of the Wilson line in (1), specified by the light-cone vector  $n_-$ , leads to more terms in the decomposition than the ones considered in [1,2]. However, Ref. [3] contains only the spin-independent part of the correlator (1) explicitly, even though certain spin-dependent terms were used in order to derive the violation of three specific relations (so-called Lorentz invariance relations) between forward twist-3 parton distributions and moments of  $k_T$ -dependent parton distributions (see also Refs. [36,37]). In fact, also the spin-independent part given in [3] was not entirely complete which has subsequently been corrected in Ref. [4]. It is quite interesting that the one additional structure advocated in [4] implies also a new structure (associated with a new twist-3 parton distribution, called  $g^\perp$  in Ref. [4]) on the level of the  $k_T$ -dependent correlator in Eq. (2). In Ref. [38] the existence of  $g^\perp$  was already anticipated based on a calculation of the single spin asymmetry  $A_{LU}$  (longitudinally polarized lepton beam and unpolarized target) for semi-inclusive DIS in the framework of a spectator model (see also Ref. [39]).

In the present work we want to give the complete structure of the correlator in Eq. (1) for a spin- $\frac{1}{2}$  hadron including all terms generated by the presence of the Wilson line. We find as a particular consequence two new T-odd parton distributions that appear at twist-3 level in the correlator (2). Altogether the twist-3 part of (2) contains 16 parton distributions and shows a high degree of symmetry.

## 2. We start by specifying the Wilson line that appears in Eq. (1),

$$\mathcal{W}(0, \xi | n_-) = [0, 0, \vec{0}_T; 0, \infty, \vec{0}_T] \times [0, \infty, \vec{0}_T; \xi^+, \infty, \vec{\xi}_T] \times [\xi^+, \infty, \vec{\xi}_T; \xi^+, \xi^-, \vec{\xi}_T], \quad (3)$$

where  $[a^+, a^-, \vec{a}_T; b^+, b^-, \vec{b}_T]$  denotes a gauge link connecting the points  $a^\mu = (a^+, a^-, \vec{a}_T)$  and  $b^\mu = (b^+, b^-, \vec{b}_T)$  along a straight line. It is important to note that the contour in Eq. (3) not only depends on the coordinates of the initial and final points but also on the light-cone direction  $n_-$ , which is opposite to the direction of the target momentum [3]. The path is chosen such that, upon integration over the minus-momentum of the quark, it leads to a proper definition of the correlator in (2) as given in Refs. [12–14,18,40]. The choice of the contour depends on the process under consideration [12]. Here we restrict ourselves to the case of semi-inclusive DIS, but all our arguments hold as well for other processes like Drell–Yan. It has been pointed out [19,40] that in general light-like Wilson lines as used in (3) can lead to divergences, which can be avoided, however, by adopting a near light-cone direction. Again, our general reasoning remains valid if we use such a direction instead of  $n_-$ .

To write down the most general expression of the correlator in (1), we impose the following constraints due to hermiticity and parity,

$$\Phi^\dagger(P, k, S | n_-) = \gamma_0 \Phi(P, k, S | n_-) \gamma_0, \quad (4)$$

$$\Phi(P, k, S | n_-) = \gamma_0 \Phi(\bar{P}, \bar{k}, -\bar{S} | \bar{n}_-) \gamma_0, \quad (5)$$

where  $\bar{P}^\mu = (P^0, -\vec{P})$ , etc. In the case of the correlators (1), (2) time-reversal does not give an additional constraint [12]. To avoid redundant terms in the decomposition we make use of the identity

$$g^{\alpha\beta}\varepsilon^{\mu\nu\rho\sigma} = g^{\mu\beta}\varepsilon^{\alpha\nu\rho\sigma} + g^{\nu\beta}\varepsilon^{\mu\alpha\rho\sigma} + g^{\rho\beta}\varepsilon^{\mu\nu\alpha\sigma} + g^{\sigma\beta}\varepsilon^{\mu\nu\rho\alpha}. \quad (6)$$

With these ingredients it is possible to obtain the general form of the correlator in Eq. (1). One ends up with 32 matrix structures multiplied by scalar functions ( $A_i$ ,  $B_i$ ),

$$\begin{aligned} \Phi(P, k, S|n) = & M A_1 + \not{P} A_2 + \not{k} A_3 + \frac{i}{2M} [\not{P}, \not{k}] A_4 + i(k \cdot S) \gamma_5 A_5 + M \not{S} \gamma_5 A_6 + \frac{k \cdot S}{M} \not{P} \gamma_5 A_7 \\ & + \frac{k \cdot S}{M} \not{k} \gamma_5 A_8 + \frac{[\not{P}, \not{S}]}{2} \gamma_5 A_9 + \frac{[\not{k}, \not{S}]}{2} \gamma_5 A_{10} + \frac{(k \cdot S)}{2M^2} [\not{P}, \not{k}] \gamma_5 A_{11} \\ & + \frac{1}{M} \varepsilon^{\mu\nu\rho\sigma} \gamma_\mu P_\nu k_\rho S_\sigma A_{12} + \frac{M^2}{P \cdot n_-} \not{n}_- B_1 + \frac{iM}{2P \cdot n_-} [\not{P}, \not{n}_-] B_2 + \frac{iM}{2P \cdot n_-} [\not{k}, \not{n}_-] B_3 \\ & + \frac{1}{P \cdot n_-} \varepsilon^{\mu\nu\rho\sigma} \gamma_\mu \gamma_5 P_\nu k_\rho n_{-\sigma} B_4 + \frac{1}{P \cdot n_-} \varepsilon^{\mu\nu\rho\sigma} P_\mu k_\nu n_{-\rho} S_\sigma B_5 \\ & + \frac{iM^2}{P \cdot n_-} (n_- \cdot S) \gamma_5 B_6 + \frac{M}{P \cdot n_-} \varepsilon^{\mu\nu\rho\sigma} \gamma_\mu P_\nu n_{-\rho} S_\sigma B_7 + \frac{M}{P \cdot n_-} \varepsilon^{\mu\nu\rho\sigma} \gamma_\mu k_\nu n_{-\rho} S_\sigma B_8 \\ & + \frac{(k \cdot S)}{M(P \cdot n_-)} \varepsilon^{\mu\nu\rho\sigma} \gamma_\mu P_\nu k_\rho n_{-\sigma} B_9 + \frac{M(n_- \cdot S)}{(P \cdot n_-)^2} \varepsilon^{\mu\nu\rho\sigma} \gamma_\mu P_\nu k_\rho n_{-\sigma} B_{10} \\ & + \frac{M}{P \cdot n_-} (n_- \cdot S) \not{P} \gamma_5 B_{11} + \frac{M}{P \cdot n_-} (n_- \cdot S) \not{k} \gamma_5 B_{12} + \frac{M}{P \cdot n_-} (k \cdot S) \not{n}_- \gamma_5 B_{13} \\ & + \frac{M^3}{(P \cdot n_-)^2} (n_- \cdot S) \not{n}_- \gamma_5 B_{14} + \frac{M^2}{2P \cdot n_-} [\not{n}_-, \not{S}] \gamma_5 B_{15} + \frac{(k \cdot S)}{2P \cdot n_-} [\not{P}, \not{n}_-] \gamma_5 B_{16} \\ & + \frac{(k \cdot S)}{2P \cdot n_-} [\not{k}, \not{n}_-] \gamma_5 B_{17} + \frac{(n_- \cdot S)}{2P \cdot n_-} [\not{P}, \not{k}] \gamma_5 B_{18} + \frac{M^2(n_- \cdot S)}{2(P \cdot n_-)^2} [\not{P}, \not{n}_-] \gamma_5 B_{19} \\ & + \frac{M^2(n_- \cdot S)}{2(P \cdot n_-)^2} [\not{k}, \not{n}_-] \gamma_5 B_{20}. \end{aligned} \quad (7)$$

The first twelve structures that are multiplied by the amplitudes  $A_i$  were already written down for the corresponding fragmentation correlator in Ref. [2]. (See Ref. [41] in the case of parton distributions.) These terms constitute a complete decomposition as long as the Wilson line is neglected. They give a sufficient parameterization if the correlator is evaluated in some model of non-perturbative QCD which does not contain gluonic degrees of freedom.

The spin-independent terms associated with the  $n_-$ -dependence and the amplitudes  $B_{1,2,3}$  were given in [3], while the  $B_4$ -term can be found for the first time in [4]. The remaining 16  $B$ -terms are relevant once the target spin is involved. Note that in order to specify the Wilson line in Eq. (3) a rescaled vector  $\lambda n_-$  with some parameter  $\lambda$  could be used instead of  $n_-$ . By construction, the terms in (7) are not affected by such a rescaling. The various factors of the target mass  $M$  are introduced in order to assign the same mass dimension to all scalar amplitudes. Finally, we mention that the following twelve amplitudes are associated with T-odd matrix structures:  $A_4, A_5, A_{12}, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9, B_{10}$ .

### 3. We now focus our attention on the $k_T$ -dependent correlator in Eq. (2),

$$\Phi_{ij}(x, \vec{k}_T, S) = \int \frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3} e^{i(k^+\xi^- - \vec{k}_T \cdot \vec{\xi}_T)} \langle P, S | \bar{\psi}_j(0) \mathcal{W}_1(0, \xi) \psi_i(\xi) | P, S \rangle \Big|_{\xi^+ = 0}, \quad (8)$$

which can be derived from the general result (7) in a straightforward manner. The Wilson line in this correlator is connected to the one in (3) through

$$\mathcal{W}_1(0, \xi) = \mathcal{W}(0, \xi | n_-)|_{\xi^+=0}. \quad (9)$$

We will specify the  $k_T$ -dependent correlator in (8) in terms of all possible Dirac traces given by

$$\begin{aligned} \Phi^{[\Gamma]}(x, \vec{k}_T, S) &\equiv \frac{1}{2} \text{Tr}(\Phi(x, \vec{k}_T, S) \Gamma) \\ &= \int \frac{d\xi^- d^2 \vec{\xi}_T}{2(2\pi)^3} e^{i(k^+ \xi^- - \vec{k}_T \cdot \vec{\xi}_T)} \langle P, S | \bar{\psi}_j(0) \Gamma \mathcal{W}_1(0, \xi) \psi_i(\xi) | P, S \rangle \Big|_{\xi^+=0}. \end{aligned} \quad (10)$$

These traces immediately provide the definition of the various  $k_T$ -dependent parton distributions. In order to have a twist-classification it is convenient to use the Sudakov decomposition of the four-vectors in (7),

$$P^\mu = P^+ n_+^\mu + \frac{M^2}{2P^+} n_-^\mu, \quad (11)$$

$$k^\mu = x P^+ n_+^\mu + k^- n_-^\mu + k_T^\mu, \quad (12)$$

$$S^\mu = \lambda \frac{P^+}{M} n_+^\mu - \lambda \frac{M}{2P^+} n_-^\mu + S_T^\mu, \quad (13)$$

with  $k_T^\mu = (0, 0, \vec{k}_T)$  and  $S_T^\mu = (0, 0, \vec{S}_T)$ . The two light-like vectors  $n_-$ ,  $n_+$  satisfy the usual conditions  $n_-^2 = n_+^2 = 0$  and  $n_- \cdot n_+ = 1$ . We consider  $P^+$  as the large component of the target momentum. This input, together with the relation (2), is sufficient to obtain the final result for the  $k_T$ -dependent correlator.

For the sake of completeness and of later comparison we start with the result for the twist-2 case, which has already been given in the literature [2,41],

$$\Phi^{[\gamma^+]} = f_1(x, \vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{1T}^\perp(x, \vec{k}_T^2), \quad (14)$$

$$\Phi^{[\gamma^+ \gamma_S]} = \lambda g_{1L}(x, \vec{k}_T^2) + \frac{\vec{k}_T \cdot \vec{S}_T}{M} g_{1T}(x, \vec{k}_T^2), \quad (15)$$

$$\Phi^{[i\sigma^{+i} \gamma_S]} = S_T^i h_{1T}(x, \vec{k}_T^2) + \frac{k_T^i}{M} \left( \lambda h_{1L}^\perp(x, \vec{k}_T^2) + \frac{\vec{k}_T \cdot \vec{S}_T}{M} h_{1T}^\perp(x, \vec{k}_T^2) \right) - \frac{\varepsilon_T^{ij} k_{Tj}}{M} h_1^\perp(x, \vec{k}_T^2). \quad (16)$$

Here we use the definition  $\varepsilon_T^{ij} = \varepsilon^{-+ij}$  and the standard notation  $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$ . All eight twist-2 parton distributions are given by  $k^-$ -integrals of certain linear combinations of the scalar amplitudes in (7). For brevity we refrain from listing these relations here. The functions  $f_{1T}^\perp$  (Sivers function [42]) and  $h_1^\perp$  [41] are T-odd and have recently attracted an enormous interest because they are considered to be at the origin of the observed interesting single spin phenomena in certain hard processes. If the correlator is integrated upon  $k_T$  only three functions (the forward unpolarized, helicity and transversity distribution of a quark) survive.

In the twist-3 case, characterized through a suppression by one power in  $P^+$ , we find

$$\Phi^{[1]} = \frac{M}{P^+} \left[ e(x, \vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} e_T^\perp(x, \vec{k}_T^2) \right], \quad (17)$$

$$\Phi^{[i \gamma_S]} = \frac{M}{P^+} \left[ \lambda e_L(x, \vec{k}_T^2) + \frac{\vec{k}_T \cdot \vec{S}_T}{M} e_T(x, \vec{k}_T^2) \right], \quad (18)$$

$$\Phi^{[\gamma^i]} = \frac{M}{P^+} \left[ \frac{k_T^i}{M} \left( f^\perp(x, \vec{k}_T^2) - \frac{\varepsilon_T^{jk} k_{Tj} S_{Tk}}{M} f_{T'}^\perp(x, \vec{k}_T^2) \right) + \frac{\varepsilon_T^{ij} k_{Tj}}{M} \left( \lambda f_L^\perp(x, \vec{k}_T^2) + \frac{\vec{k}_T \cdot \vec{S}_T}{M} f_T^\perp(x, \vec{k}_T^2) \right) \right], \quad (19)$$

$$\Phi^{[\gamma^i \gamma_5]} = \frac{M}{P^+} \left[ S_T^i g'_T(x, \vec{k}_T^2) + \frac{k_T^i}{M} \left( \lambda g_L^\perp(x, \vec{k}_T^2) + \frac{\vec{k}_T \cdot \vec{S}_T}{M} g_T^\perp(x, \vec{k}_T^2) \right) - \frac{\varepsilon_T^{ij} k_{Tj}}{M} g^\perp(x, \vec{k}_T^2) \right], \quad (20)$$

$$\Phi^{[i\sigma^{ij} \gamma_5]} = \frac{M}{P^+} \left[ \frac{S_T^i k_T^j - k_T^i S_T^j}{M} h_T^\perp(x, \vec{k}_T^2) - \varepsilon_T^{ij} h(x, \vec{k}_T^2) \right], \quad (21)$$

$$\Phi^{[i\sigma^{+-} \gamma_5]} = \frac{M}{P^+} \left[ \lambda h_L(x, \vec{k}_T^2) + \frac{\vec{k}_T \cdot \vec{S}_T}{M} h_T(x, \vec{k}_T^2) \right]. \quad (22)$$

The twist-4 result, which is basically a copy of the twist-2 case, reads

$$\Phi^{[\gamma^-]} = \frac{M^2}{(P^+)^2} \left[ f_3(x, \vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{3T}^\perp(x, \vec{k}_T^2) \right], \quad (23)$$

$$\Phi^{[\gamma^- \gamma_5]} = \frac{M^2}{(P^+)^2} \left[ \lambda g_{3L}(x, \vec{k}_T^2) + \frac{\vec{k}_T \cdot \vec{S}_T}{M} g_{3T}(x, \vec{k}_T^2) \right], \quad (24)$$

$$\Phi^{[i\sigma^{-i} \gamma_5]} = \frac{M^2}{(P^+)^2} \left[ S_T^i h_{3T}(x, \vec{k}_T^2) + \frac{k_T^i}{M} \left( \lambda h_{3L}^\perp(x, \vec{k}_T^2) + \frac{\vec{k}_T \cdot \vec{S}_T}{M} h_{3T}^\perp(x, \vec{k}_T^2) \right) - \frac{\varepsilon_T^{ij} k_{Tj}}{M} h_3^\perp(x, \vec{k}_T^2) \right]. \quad (25)$$

The twist-4 case is of course only of academic interest but is included for completeness. We would like to add several points:

(1) In total there are 32  $k_T$ -dependent parton distributions which exactly agrees with the number of the independent amplitudes in Eq. (7). This result seems non-trivial to us for the following reason: if the same calculation is performed neglecting the  $n_-$ -dependent terms in (7) then the number of structures/functions on the level of the  $k_T$ -dependent correlator is larger than the number of the amplitudes  $A_i$ . This feature gives rise to the Lorentz invariance relations between certain parton distributions [2,41]. In a gauge theory, however, these relations no longer hold.

(2) At twist-3 there appear 16 functions, where 8 of them ( $e_T^\perp$ ,  $e_L$ ,  $e_T$ ,  $f_L^\perp$ ,  $f_T^\perp$ ,  $f_T^{\perp'}$ ,  $g^\perp$ ,  $h$ ) are T-odd.

(3) The structure of the  $k_T$ -dependent fragmentation correlator is completely analogous to the case of parton distributions considered here. For fragmentation we refer the reader in particular to [2].

(4) With the exception of  $e_T^\perp$ ,  $f_T^\perp$ ,  $f_T^{\perp'}$ ,  $g^\perp$  all other twist-3 functions were already given in Ref. [2] (for the fragmentation case). As mentioned above, the function  $g^\perp$  was introduced in [4]. The remaining three parton distributions are discussed here for the first time. Actually  $\Phi^{[\gamma^i]}$  in [2] contains a term of the type  $\varepsilon_T^{ij} S_{Tj} f_T(x, \vec{k}_T^2)$ , which is not present in our result (19). To get maximal symmetry of the final result we have eliminated such a contribution by means of the identity

$$\vec{k}_T^2 \varepsilon_T^{ij} S_{Tj} = -k_T^i \varepsilon_T^{jk} k_{Tj} S_{Tk} + \varepsilon_T^{ij} k_{Tj} \vec{k}_T \cdot \vec{S}_T, \quad (26)$$

which immediately follows from Eq. (6). The terms associated with the functions  $f_T^\perp$  and  $f_T^{\perp'}$  are absent in [2], which means that  $\Phi^{[\gamma^i]}$  in that reference contains only three instead of four independent functions.

(5) In our work the function  $g^\perp$  in (20) has the opposite sign as compared to Ref. [4]. We propose this sign reversal because in that case the structure of  $\Phi^{[\gamma^i \gamma_5]}$  completely coincides with the twist-2 structure  $\Phi^{[i\sigma^{+i} \gamma_5]}$  in (16).

(6) The parton distributions  $e_T^\perp$ ,  $g^\perp$  and the independence of the functions  $f_T^\perp$  and  $f_T^{\perp'}$  only appear if the gauge link is taken into account in the unintegrated correlator in Eq. (7). All these functions are T-odd, which is consistent with the fact that they vanish once the gauge link is neglected [12,43].

(7) If the correlation functions in Eqs. (14)–(25) are integrated upon  $k_T$  one obtains the light-cone correlators  $\Phi^{[\Gamma]}(x)$ . In these objects all T-odd functions have to vanish due to time-reversal invariance of QCD [43], which

implies the following constraints:

$$\int d^2\vec{k}_T e_L(x, \vec{k}_T^2) = 0, \quad (27)$$

$$\int d^2\vec{k}_T \vec{k}_T^2 (f_T^\perp(x, \vec{k}_T^2) + f_T^{\perp'}(x, \vec{k}_T^2)) = 0, \quad (28)$$

$$\int d^2\vec{k}_T h(x, \vec{k}_T^2) = 0. \quad (29)$$

Such relations do not hold in the case of the corresponding fragmentation functions.

(8) The new functions appear in transverse momentum dependent semi-inclusive DIS and in the unintegrated Drell–Yan process at subleading twist. To be specific, in semi-inclusive DIS  $e_T^\perp$  enters the double polarized cross section  $\sigma_{LT}$  (multiplied with the Collins function), while  $f_T^\perp$  and  $f_T^{\perp'}$  enter  $\sigma_{UT}$  (multiplied with the unpolarized fragmentation function  $D_1$ ). It is beyond the scope of this Letter to give a complete (parton model) description of these observables up to twist-3, because one has to deal also with quark–gluon–quark matrix elements. (In this context see, e.g., Refs. [2,18].)

**4.** In summary, we have derived the general structure of the quark–quark correlation function for a spin- $\frac{1}{2}$  hadron. In order to obtain a full parameterization of the correlator in QCD it is crucial to consider also the dependence on an additional light-like vector specifying the direction of the Wilson line, which ensures color gauge invariance of the correlator. We have used the result to write down the most general form of the  $k_T$ -dependent quark–quark correlator  $\Phi(x, \vec{k}_T, S)$  appearing in the description of various hard scattering processes. Our final result for this correlator shows a high degree of symmetry. In particular, we have found two new  $k_T$ -dependent T-odd parton distributions at subleading twist.

## Acknowledgements

We are grateful to P.V. Pobylitsa and M.V. Polyakov for discussions. The work of M.S. has been supported by the Graduiertenkolleg “Physik der Elementarteilchen an Beschleunigern und im Universum”. The work has also been partially supported by the Verbundforschung (BMBF) and the Transregio/SFB Bochum–Bonn–Giessen. This research is part of the EU Integrated Infrastructure Initiative Hadronphysics Project under contract number RII3-CT-2004-506078.

## References

- [1] J.P. Ralston, D.E. Soper, Nucl. Phys. B 152 (1979) 109.
- [2] P.J. Mulders, R.D. Tangerman, Nucl. Phys. B 461 (1996) 197;  
P.J. Mulders, R.D. Tangerman, Nucl. Phys. B 484 (1997) 538, Erratum.
- [3] K. Goeke, A. Metz, P.V. Pobylitsa, M.V. Polyakov, Phys. Lett. B 567 (2003) 27.
- [4] A. Bacchetta, P.J. Mulders, F. Pijlman, Phys. Lett. B 595 (2004) 309.
- [5] HERMES Collaboration, A. Airapetian, et al., Phys. Rev. Lett. 84 (2000) 4047.
- [6] HERMES Collaboration, A. Airapetian, et al., Phys. Rev. D 64 (2001) 097101.
- [7] CLAS Collaboration, H. Avakian, et al., Phys. Rev. D 69 (2004) 112004.
- [8] STAR Collaboration, J. Adams, et al., Phys. Rev. Lett. 92 (2004) 171801.
- [9] HERMES Collaboration, A. Airapetian, et al., Phys. Rev. Lett. 94 (2005) 012002.
- [10] COMPASS Collaboration, V.Y. Alexakhin, et al., hep-ex/0503002.
- [11] S.J. Brodsky, D.S. Hwang, I. Schmidt, Phys. Lett. B 530 (2002) 99.
- [12] J.C. Collins, Phys. Lett. B 536 (2002) 43.

- [13] X. Ji, F. Yuan, Phys. Lett. B 543 (2002) 66.
- [14] A.V. Belitsky, X. Ji, F. Yuan, Nucl. Phys. B 656 (2003) 165.
- [15] A. Metz, Phys. Lett. B 549 (2002) 139.
- [16] M. Burkardt, Phys. Rev. D 66 (2002) 114005.
- [17] P.V. Pobylitsa, hep-ph/0301236.
- [18] D. Boer, P.J. Mulders, F. Pijlman, Nucl. Phys. B 667 (2003) 201.
- [19] J.C. Collins, Acta Phys. Pol. B 34 (2003) 3103.
- [20] M. Burkardt, Phys. Rev. D 69 (2004) 091501.
- [21] X. Ji, J.P. Ma, F. Yuan, Phys. Rev. D 71 (2005) 034005.
- [22] C.J. Bomhof, P.J. Mulders, F. Pijlman, Phys. Lett. B 596 (2004) 277.
- [23] J.C. Collins, A. Metz, Phys. Rev. Lett. 93 (2004) 252001.
- [24] X. Ji, J.P. Ma, F. Yuan, hep-ph/0503015.
- [25] D. Boer, S.J. Brodsky, D.S. Hwang, Phys. Rev. D 67 (2003) 054003.
- [26] A.V. Efremov, K. Goeke, P. Schweitzer, Eur. Phys. J. C 32 (2003) 337.
- [27] A. Bacchetta, A. Schaefer, J.J. Yang, Phys. Lett. B 578 (2004) 109.
- [28] L.P. Gamberg, D.S. Hwang, K.A. Oganessyan, Phys. Lett. B 584 (2004) 276.
- [29] D. Boer, W. Vogelsang, Phys. Rev. D 69 (2004) 094025.
- [30] U. D'Alesio, F. Murgia, Phys. Rev. D 70 (2004) 074009.
- [31] A.V. Efremov, K. Goeke, S. Menzel, A. Metz, P. Schweitzer, Phys. Lett. B 612 (2005) 233.
- [32] A.V. Efremov, K. Goeke, P. Schweitzer, hep-ph/0412420.
- [33] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin, hep-ph/0501196.
- [34] I. Schmidt, J. Soffer, J.J. Yang, hep-ph/0503127.
- [35] J.C. Collins, X. Zu, hep-ph/0411332.
- [36] R. Kundu, A. Metz, Phys. Rev. D 65 (2002) 014009.
- [37] M. Schlegel, A. Metz, hep-ph/0406289.
- [38] A. Metz, M. Schlegel, Eur. Phys. J. A 22 (2004) 489.
- [39] A. Afanasev, C.E. Carlson, hep-ph/0308163.
- [40] J.C. Collins, D.E. Soper, Nucl. Phys. B 194 (1982) 445.
- [41] D. Boer, P.J. Mulders, Phys. Rev. D 57 (1998) 5780.
- [42] D.W. Sivers, Phys. Rev. D 41 (1990) 83;  
D.W. Sivers, Phys. Rev. D 43 (1991) 261.
- [43] J.C. Collins, Nucl. Phys. B 396 (1993) 161.