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Phenomenological constraints on A_N in $p^\uparrow p \rightarrow \pi X$ from Lorentz invariance relations

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ABSTRACT

We present a new analysis of A_N in $p^\uparrow p \rightarrow \pi X$ within the collinear twist-3 factorization formalism. We incorporate recently derived Lorentz invariance relations into our calculation and focus on input from the kinematical twist-3 functions, which are weighted integrals of transverse momentum dependent (TMD) functions. In particular, we use the latest extractions of the Sivers and Collins functions with TMD evolution to compute certain terms in A_N . Consequently, we are able to constrain the remaining contributions from the lesser known dynamical twist-3 correlators.

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1. Introduction

The endeavor to probe the spin structure of the proton through transverse single-spin asymmetries (TSSAs), denoted A_N , in high-energy single inclusive lepton-hadron and hadron-hadron scattering processes, i.e., $A^\uparrow + B \rightarrow C + X$,¹ has received considerable attention from both the experimental and theoretical communities [1,2]. For the case where the produced particle C 's transverse momentum $P_{CT} \gg \Lambda_{QCD}$, TSSAs manifest themselves as sub-leading twist (twist-3) effects calculable within perturbative QCD (pQCD). The computational techniques and methodology of this collinear twist-3 factorization framework were developed rigorously in Refs. [3–27]. Over the last 40 years, there have been many measurements of large TSSAs [28–44], whose description, therefore, has become a fundamental test of this pQCD formalism.

Schematically, one writes the (polarized) differential cross section for $A^\uparrow + B \rightarrow C + X$ as

$$d\sigma(S_T) = H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{C/c(2)} + H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{C/c(2)} + H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)}, \quad (1)$$

where S_T is the transverse spin vector of hadron A , $f_{a/A(t)}$ is the twist- t parton distribution function (PDF) associated with parton a in hadron A (similarly for $f_{b/B(t)}$), while $D_{C/c(t)}$ is the twist- t fragmentation function (FF) associated with hadron C in parton c . The twist-3 correlators can either be of the 2-parton or 3-parton type and are categorized into intrinsic, kinematical, and dynamical functions [1,26]. The intrinsic functions are twist-3 Dirac projections of collinear 2-parton correlators, while the kinematical functions are first transverse momentum moments of transverse momentum dependent (TMD) 2-parton functions. The dynamical functions are 3-parton correlators. The factors H , H' , and H'' are the hard parts for each term, and the symbol \otimes denotes convolutions in the appropriate parton momentum fractions. One then can calculate the TSSA A_N as

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¹ One could also have C transversely polarized instead of A .

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$$A_N \equiv \frac{d\Delta\sigma(S_T)}{d\sigma}, \text{ where } d\Delta\sigma(S_T) \equiv \frac{1}{2} [d\sigma(S_T) - d\sigma(-S_T)], \text{ and } d\sigma \equiv \frac{1}{2} [d\sigma(S_T) + d\sigma(-S_T)]. \quad (2)$$

In this Letter, we will focus on A_N in the process $p^\uparrow p \rightarrow \pi X$, i.e., single inclusive pion production in collisions between transversely polarized protons and unpolarized protons.

Let us first review the current situation for this observable, which receives contributions from all three terms in Eq. (1). That is, the full result involves twist-3 functions from (i) the transversely polarized proton, (ii) the unpolarized proton, and (iii) the (unpolarized) final-state pion. For (i) there are two types of terms that arise: a soft-gluon pole (SGP) term and a soft-fermion pole (SFP) term. The former was calculated in Refs. [7,9] for qgq correlators involving the Qiu–Sterman (QS) function $F_{FT}(x, x)$, as well as in Ref. [24] for tri-gluon (ggg) ones. The latter (SFP) was computed in Ref. [15]. We note that $F_{FT}(x, x)$ has an important, model-independent relation [45] to the first k_T -moment of the TMD Sivers function $f_{1T}^\perp(x, k_T^2)$ [46].

The initial attempt to describe A_N in $p^\uparrow p \rightarrow \pi X$ did so under the assumption that the QS function was the dominant source of the asymmetry [7,9]. However, a series of studies [47–49] have demonstrated that $F_{FT}(x, x)$ cannot be the main cause of A_N in this reaction. Also, the tri-gluon term has been shown to give small effects in the forward region [24] where A_N is most significant. While the SFP piece might play some role, it cannot account for all of the asymmetry [50,51]. In addition, term (ii) couples two chiral-odd functions: the quark transversity PDF from the transversely polarized proton and the twist-3 function $H_{FU}(x, x)$ [8,52–54] from the unpolarized proton, which is related [45] to the first k_T -moment of the Boer–Mulders function $h_1^\perp(x, k_T^2)$ [55]. It was shown in [56] that such a contribution to A_N is negligible due to the small size of the corresponding hard partonic cross section.

Based on these observations, attention has recently been given to term (iii) involving twist-3 effects from partons fragmenting into the final-state pion. A few years ago, the complete calculation of this contribution was carried out in Ref. [22] and involves three parts: the kinematical function $H_1^{\perp(1)}(z)$ (i.e., the first p_\perp -moment of Collins function $H_1^\perp(z, z^2 \vec{p}_\perp^2)$ [57]²) the intrinsic function $H(z)$, and the dynamical function³ $\hat{H}_{FU}^\mathfrak{Z}(z, z_1)$. Later, a phenomenological analysis of A_N incorporated this twist-3 fragmentation piece along with the QS term [59]. The procedure employed by the authors of Ref. [59] was (1) to use a QCD equation of motion relation (EOMR) to write $H(z)$ in terms of $H_1^{\perp(1)}(z)$ and an integral of $\hat{H}_{FU}^\mathfrak{Z}(z, z_1)$; (2) then fix $H_1^{\perp(1)}(z)$ in terms of the Collins function and $F_{FT}(x, x)$ in terms of the Sivers function, both of which had been previously extracted from TMD processes using a simple Gaussian ansatz for the respective transverse momentum dependence [60,61]; (3) finally, fit $\hat{H}_{FU}^\mathfrak{Z}(z, z_1)$ to the RHIC data. The result was a successful description of A_N data from RHIC for both neutral and charged pion production, where the twist-3 fragmentation piece dominated the asymmetry [59]. This was a crucial step towards clarifying the mechanism behind TSSAs in hadronic collisions.

Since that study, there has been a recent development, due to the work in Ref. [26], that has provided an additional operator constraint, a Lorentz invariance relation (LIR), on the twist-3 FFs $H_1^{\perp(1)}(z)$, $H(z)$, and an integral of $\hat{H}_{FU}^\mathfrak{Z}(z, z_1)$ that enter A_N , which was not known at the time of the computation in Ref. [59]. From a purely theoretical standpoint, the work in Ref. [26] has elucidated that multi-parton (dynamical) correlators are the fundamental objects that cause transverse spin observables. For example, one can solve the relevant LIR and EOMR and write $H_1^{\perp(1)}(z)$ and $H(z)$ in terms of integrals of $\hat{H}_{FU}^\mathfrak{Z}(z, z_1)$. That is, $\hat{H}_{FU}^\mathfrak{Z}(z, z_1)$ is the “base” function from which $H_1^{\perp(1)}(z)$ and $H(z)$ are derived. However, from a practical phenomenological standpoint, one can use LIRs to eliminate “unknown” functions in terms of “known” functions. To be more specific, the only twist-3 functions that enter A_N in $p^\uparrow p \rightarrow \pi X$ that we *a priori* have any information on are the kinematical correlators because they are connected to TMD functions.⁴ Therefore, we can use these relations to re-express A_N in terms of the maximum number of kinematical correlators.⁵

To this end, we present a new analysis of the TSSA in $p^\uparrow p \rightarrow \pi X$, where we compute the QS term and the twist-3 fragmentation piece as was done in [59]. However, we do not aim at a fit of the 3-parton FF $\hat{H}_{FU}^\mathfrak{Z}(z, z_1)$. Rather, we first employ the LIR and EOMR to rewrite the cross section, eliminating unknown functions where possible. We then determine the contribution only from the kinematical correlators, where we use the latest TMD evolved extractions of those functions. Consequently, we are able to provide a constraint on the parts that remain from the lesser known dynamical functions. The Letter is organized as follows: in Sec. 2 we review the relevant analytical formulae needed in our analysis, and, in particular, rewrite the fragmentation term using the aforementioned EOMR and LIR. Next, in Sec. 3 we conduct our numerical study of A_N and compare our results to experimental data. Finally, in Sec. 4 we summarize our work.

2. The Qiu–Sterman and fragmentation contributions to $p^\uparrow p \rightarrow \pi X$

We consider TSSAs in the single-inclusive production of pions from proton–proton collisions,

$$p(P, S_T) + p(P') \rightarrow \pi(P_h) + X, \quad (3)$$

where we have indicated the momenta and polarizations of the particles. The spin-averaged differential cross section $d\sigma$ in Eq. (2) at leading-order can be written as

$$E_h \frac{d\sigma}{d^3\vec{p}_h} = \frac{\alpha_s^2}{S} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^2} \int_0^1 \frac{dx'}{x'} \int_0^1 \frac{dx}{x} \delta(\hat{s} + \hat{t} + \hat{u}) f_1^a(x) f_1^b(x') D_1^{\pi/c}(z) S_U^i, \quad (4)$$

² Studies of the contribution to A_N from the convolution of the transversity and Collins TMD functions in the framework of the so-called Generalized Parton Model are presented in Ref. [58].

³ The dynamical FFs are complex, and we indicate their real and imaginary parts by \Re and \Im superscripts, respectively.

⁴ Of course there are still uncertainties in these TMD inputs because they are not well-constrained in the forward x_F region where A_N is large.

⁵ Note from our discussion before that we can never write the entire cross section only in terms of kinematical functions.

where \sum_i is a sum over all partonic interaction channels, a can be a quark, anti-quark, or gluon and likewise for b, c , α_s is the strong coupling constant, and $f_1(x)$ ($D_1(z)$) is the standard twist-2 unpolarized PDF (FF). We have made explicit that parton c fragments into a pion. The well-known hard factors for the unpolarized cross section are denoted by S_U^i [62,63] and can be found in, e.g., Appendix A of Ref. [9]. They are functions of the partonic Mandelstam variables $\hat{s} = xx'S$, $\hat{t} = xT/Z$, and $\hat{u} = x'U/Z$, where $S = (P + P')^2$, $T = (P - P_h)^2$, and $U = (P' - P_h)^2$.

Let us now turn to the spin-dependent differential cross section $d\Delta\sigma(S_T)$ in Eq. (2). All three terms in Eq. (1) enter into the analysis. However, as stated in Section 1, we will focus on the qgq SGP (QS) piece of the first term and the third (fragmentation) term:

$$E_h \frac{d\Delta\sigma(S_T)}{d^3\vec{P}_h} = E_h \frac{d\Delta\sigma^{QS}(S_T)}{d^3\vec{P}_h} + E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h}. \quad (5)$$

The definitions of the relevant functions can be found in Refs. [1,26]. First, we give the expression for the QS term, which reads [7,9]

$$E_h \frac{d\Delta\sigma^{QS}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M}{S} \epsilon^{P'P_h S_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx \delta(\hat{s} + \hat{t} + \hat{u}) \frac{\pi}{\hat{s}\hat{u}} \times f_1^b(x') D_1^{\pi/c}(z) \left[F_{FT}^a(x, x) - x \frac{dF_{FT}^a(x, x)}{dx} \right] S_{FT}^i, \quad (6)$$

where the Levi-Civita tensor is defined with $\epsilon^{0123} = +1$, and the hard factors are denoted by S_{FT}^i and can be found in Appendix A of Ref. [9]. There is an operator identity that relates the QS function $F_{FT}^q(x, x)$ to the first k_T -moment of the Siverson function [45],

$$\pi F_{FT}^q(x, x) = f_{1T}^{\perp(1),q}(x)|_{\text{SIDIS}} = -f_{1T}^{\perp(1),q}(x)|_{\text{DY}}, \quad (7)$$

where

$$f_{1T}^{\perp(1),q}(x) \equiv \int d^2\vec{k}_T \frac{\vec{k}_T^2}{2M^2} f_{1T}^{\perp,q}(x, \vec{k}_T^2). \quad (8)$$

In Eq. (7) we have indicated that the Siverson function is either the one extracted from semi-inclusive deep-inelastic scattering (SIDIS) or the Drell-Yan (DY) process [64,65].

Next, we look at the fragmentation term, which was first fully calculated in Ref. [22] and reads

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S} \epsilon^{P'P_h S_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx \delta(\hat{s} + \hat{t} + \hat{u}) \frac{1}{\hat{s}(-x'\hat{t} - x\hat{u})} \times h_1^a(x) f_1^b(x') \left\{ \left[H_1^{\perp(1),\pi/c}(z) - z \frac{dH_1^{\perp(1),\pi/c}(z)}{dz} \right] S_{H_1^+}^i + \frac{1}{z} H^{\pi/c}(z) S_H^i + \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\left(\frac{1}{z} - \frac{1}{z_1}\right)^2} \hat{H}_{FU}^{\pi/c,\mathfrak{N}}(z, z_1) S_{\hat{H}_{FU}}^i \right\}, \quad (9)$$

where M_h is the pion mass, and $h_1(x)$ is the standard twist-2 transversity PDF. The functions $H_1^{\perp(1)}(z)$, $H(z)$, and $\hat{H}_{FU}^{\mathfrak{N}}(z, z_1)$ are, respectively, the kinematical, intrinsic, and dynamical unpolarized twist-3 FFs discussed in Section 1. The hard factors associated with them are represented by S^i with the corresponding subscript, and they can be found in Appendix A of Ref. [22].⁶ Note that $H_1^{\perp(1)}(z)$ is the first p_\perp -moment of the Collins function,

$$H_1^{\perp(1),q}(z) \equiv z^2 \int d^2\vec{p}_\perp \frac{\vec{p}_\perp^2}{2M_h^2} H_1^{\perp,q}(z, z^2\vec{p}_\perp^2). \quad (10)$$

These collinear twist-3 FFs are related to each other through an EOMR [17,22,23,66–68],

$$H^q(z) = -2z H_1^{\perp(1),q}(z) + 2z \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^q(z, z_1). \quad (11)$$

It is important to mention that there is a similar EOMR involving TMD FFs [68,69]

$$H^q(z, z^2\vec{p}_\perp^2) = \frac{\vec{p}_\perp^2}{M_h^2} H_1^{\perp,q}(z, z^2\vec{p}_\perp^2) + \tilde{H}^q(z, z^2\vec{p}_\perp^2), \quad (12)$$

where $H(z, z^2\vec{p}_\perp^2)$ is the TMD version of $H(z)$, $H_1^{\perp}(z, z^2\vec{p}_\perp^2)$ is the usual TMD Collins function, and $\tilde{H}(z, z^2\vec{p}_\perp^2)$ is a twist-3 qgq TMD FF. Upon integration over \vec{p}_\perp , we find⁷

$$H^q(z) = -2z H_1^{\perp(1),q}(z) + \tilde{H}^q(z). \quad (13)$$

⁶ Note that in Ref. [22], $\hat{H}(z) \equiv H_1^{\perp(1)}(z)$.

⁷ Note that $D(z) = z^2 \int d^2\vec{p}_\perp D(z, z^2\vec{p}_\perp^2)$ for a generic FF.

Therefore, comparing Eqs. (11) and (13) one can make the identification⁸ (using the variable substitution $y = z/z_1$)

$$\tilde{H}^q(z) = 2z \int_0^1 dy \frac{1}{1-y} \hat{H}_{FU}^{q,\tilde{N}}(z, z/y). \quad (14)$$

That is, the r.h.s. of Eq. (14) is actually a TMD FF integrated over \vec{p}_\perp . We will revisit this observation below.

Another set of formulae relating the intrinsic, kinematical, and dynamical functions are LIRs [10,26,72–74]. For twist-3 FFs, these expressions were derived for the first time in Ref. [26] using identities among non-local operators and taking into account constraints from Lorentz invariance. For our purposes, we focus on the one relating $H_1^{\perp(1)}(z)$, $H(z)$, and $\hat{H}_{FU}^{\tilde{N}}(z, z_1)$ [26]:

$$\frac{H^q(z)}{z} = -\left(1 - z \frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\tilde{N}}(z, z_1)}{(1/z - 1/z_1)^2}. \quad (15)$$

We now employ both the EOMR (11) and LIR (15) to re-express the fragmentation contribution to A_N in Eq. (9) in terms of the kinematical function $H_1^{\perp(1)}(z)$ and the dynamical correlator $\hat{H}_{FU}^{\tilde{N}}(z, z_1)$ (via $\tilde{H}(z)$),

$$E_h \frac{d\Delta\sigma^{\text{Frag}}(S_T)}{d^3\vec{p}_h} = -\frac{4\alpha_s^2 M_h}{S} \epsilon^{P'P P_h S_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx \delta(\hat{s} + \hat{t} + \hat{u}) \frac{1}{\hat{s}} \\ \times h_1^q(x) f_1^b(x') \left\{ \left[H_1^{\perp(1),\pi/c}(z) - z \frac{dH_1^{\perp(1),\pi/c}(z)}{dz} \right] \tilde{S}_{H_1^\perp}^i + \left[-2H_1^{\perp(1),\pi/c}(z) + \frac{1}{z} \tilde{H}^{\pi/c}(z) \right] \tilde{S}_H^i \right\}, \quad (16)$$

where we have used the identity (14) for the last term in brackets. The hard factors are given by

$$\tilde{S}_{H_1^\perp}^i \equiv \frac{S_{H_1^\perp}^i - S_{H_{FU}}^i}{-x'\hat{t} - x\hat{u}} \quad \text{and} \quad \tilde{S}_H^i \equiv \frac{S_H^i - S_{H_{FU}}^i}{-x'\hat{t} - x\hat{u}}, \quad (17)$$

which explicitly read

$$\tilde{S}_{H_1^\perp}^{qg \rightarrow qg} = -\frac{1}{N_c^2} \frac{1}{\hat{t}} + \frac{1}{N_c^2 - 1} \frac{\hat{s}(\hat{u} - \hat{s})}{\hat{t}^3} - \frac{\hat{s}^2}{\hat{t}^2 \hat{u}}, \quad \tilde{S}_H^{qg \rightarrow qg} = \frac{1}{N_c^2 - 1} \frac{\hat{s}(\hat{u} - \hat{s})}{\hat{t}^3} + \frac{1}{N_c^2} \frac{\hat{s} - \hat{u}}{2\hat{t}\hat{u}} + \frac{(\hat{s} - \hat{u})(\hat{t}^2 - 2\hat{t}\hat{u} - 2\hat{u}^2)}{2\hat{t}^3 \hat{u}}, \quad (18)$$

$$\tilde{S}_{H_1^\perp}^{qq' \rightarrow qq'} = \frac{1}{N_c^2} \frac{\hat{s}(\hat{u} - 2\hat{t})}{\hat{t}^3} + \frac{\hat{s}}{\hat{t}^2}, \quad \tilde{S}_H^{qq' \rightarrow qq'} = \frac{1}{N_c^2} \frac{\hat{s}(2\hat{u} - \hat{t})}{\hat{t}^3} - \frac{\hat{s}\hat{u}}{\hat{t}^3}, \quad (19)$$

$$\tilde{S}_{H_1^\perp}^{qq \rightarrow qq} = \frac{1}{N_c^3} \frac{\hat{s}(\hat{t} - \hat{u})}{\hat{t}^2 \hat{u}} + \frac{1}{N_c^2} \frac{\hat{s}(\hat{u} - 2\hat{t})}{\hat{t}^3} + \frac{\hat{s}}{\hat{t}^2}, \quad \tilde{S}_H^{qq \rightarrow qq} = \frac{1}{N_c^3} \frac{\hat{s}(\hat{t} - 3\hat{u})}{2\hat{t}^2 \hat{u}} + \frac{1}{N_c^2} \frac{\hat{s}(2\hat{u} - \hat{t})}{\hat{t}^3} - \frac{1}{N_c} \frac{\hat{s}^2}{2\hat{t}^2 \hat{u}} - \frac{\hat{s}\hat{u}}{\hat{t}^3}, \quad (20)$$

$$\tilde{S}_{H_1^\perp}^{\tilde{q}\tilde{q} \rightarrow \tilde{q}\tilde{q}} = \frac{1}{N_c^3} \frac{\hat{s}}{\hat{t}^2} + \frac{1}{N_c^2} \frac{\hat{s}(\hat{t} - \hat{s})}{\hat{t}^3} - \frac{1}{N_c} \frac{1}{\hat{t}}, \quad \tilde{S}_H^{\tilde{q}\tilde{q} \rightarrow \tilde{q}\tilde{q}} = \frac{1}{N_c^3} \frac{3\hat{s} - \hat{t}}{2\hat{t}^2} + \frac{1}{N_c^2} \frac{\hat{s}(\hat{t} - 2\hat{s})}{\hat{t}^3} + \frac{1}{N_c} \frac{\hat{u}}{2\hat{t}^2} + \frac{\hat{s}^2}{\hat{t}^3}, \quad (21)$$

$$\tilde{S}_{H_1^\perp}^{\tilde{q}\tilde{q} \rightarrow q\tilde{q}} = \frac{1}{N_c^3} \frac{\hat{s}}{\hat{t}\hat{u}} - \frac{1}{N_c} \frac{1}{\hat{t}}, \quad \tilde{S}_H^{\tilde{q}\tilde{q} \rightarrow q\tilde{q}} = \frac{N_c^2 + 1}{N_c^3} \frac{\hat{s} - \hat{u}}{2\hat{t}\hat{u}}, \quad (22)$$

$$\tilde{S}_{H_1^\perp}^{\tilde{q}\tilde{q}' \rightarrow q\tilde{q}'} = \frac{1}{N_c^2} \frac{\hat{s}(\hat{t} - \hat{s})}{\hat{t}^3}, \quad \tilde{S}_H^{\tilde{q}\tilde{q}' \rightarrow q\tilde{q}'} = \frac{1}{N_c^2} \frac{\hat{s}(\hat{t} - 2\hat{s})}{\hat{t}^3} + \frac{\hat{s}^2}{\hat{t}^3}. \quad (23)$$

Note that the factor $1/(-x'\hat{t} - x\hat{u})$ in Eq. (17) is canceled in each of the above expressions (18)–(23). Such a simplification also occurred for A_N in $\ell p^\dagger \rightarrow \pi X$ after using the LIR (15) [26] and appears to be a non-trivial cross-check of the result. We also mention that one can obtain the channels involving anti-quark fragmentation by charge conjugating the above partonic processes $ab \rightarrow cd$. The hard parts are the same as the ones above, i.e., $\tilde{S}^{ab \rightarrow cd} = \tilde{S}^{ab \rightarrow cd}$.

At this point, we emphasize again that $\tilde{H}(z)$ in Eq. (16) is the twist-3 qqq TMD FF $\tilde{H}(z, z^2 \vec{p}_\perp^2)$ integrated over \vec{p}_\perp . Thus, we stress that one could obtain some information on $\hat{H}_{FU}^{\tilde{N}}(z, z_1)$ (via $\tilde{H}(z)$ or $\tilde{H}(z, z^2 \vec{p}_\perp^2)$) through transverse spin observables within the collinear twist-3 and/or TMD factorization formalisms. For example, the processes $p^\dagger p \rightarrow \pi X$ and $\ell p^\dagger \rightarrow \pi X$ depend on $\tilde{H}(z)$. On the other hand, at small- q_T , where q is the momentum of the virtual photon, the unintegrated correlator $\tilde{H}(z, z^2 \vec{p}_\perp^2)$ enters (along with several other terms) the asymmetries $A_{UT}^{\sin \phi_S}$, $A_{UU}^{\cos \phi_h}$, $A_{UL}^{\sin \phi_h}$, and $A_{UT}^{\sin(2\phi_h - \phi_S)}$ in SIDIS [68] as well as the analogous ones in $e^+e^- \rightarrow h_a h_b X$ [69]. At the same time, a measurement of $A_{UT}^{\sin \phi_S}$ in SIDIS integrated over P_{hT} is sensitive to a single term⁹ that couples $\tilde{H}(z)$ to the transversity PDF

⁸ This connection was made previously in Ref. [70], where $H(z, z^2 \vec{p}_\perp^2)$ and $\tilde{H}(z, z^2 \vec{p}_\perp^2)$ were calculated in a spectator model, and in Ref. [71] in the context of the asymmetry $A_{UT}^{\sin \phi_S}$ in SIDIS.

⁹ In $e^+e^- \rightarrow h_a h_b X$ there is an additional term coupling $D_T(z_a)$ to $D_1(z_b)$, whereas in SIDIS the analogous term coupling $f_T(x)$ to $D_1(z)$ vanishes due to time-reversal invariance.

$h_1(x)$ [68].¹⁰ We want to reiterate once more that $\hat{H}_{FU}^3(z, z_1)$ (via $\tilde{H}(z)$ or $\tilde{H}(z, z^2 \vec{p}_\perp^2)$) does show up in several different observables that can provide more information on this function in the future. In the next section, we perform an analysis of the TSSA for $p^\uparrow p \rightarrow \pi X$ and comment on what constraints on $\tilde{H}(z)$ it can provide.

3. Phenomenology

We now give our numerical estimate for A_N . Using Eqs. (4), (6), (16), we are able to express, respectively, the numerator and denominator of A_N in Eq. (2) as

$$d\Delta\sigma(S_T) = \frac{2P_{hT}\alpha_S^2}{S} \sum_i \sum_{a,b,c,z_{min}} \int \frac{dz}{z^3} \int \frac{dx}{x} \frac{1}{x'} \frac{1}{xS + U/z} f_1^b(x') \left[M_h h_1^a(x) \mathcal{H}^{\pi/c,i}(x, x', z) + \frac{M}{\hat{u}} \mathcal{F}^{a,i}(x, x', z) D_1^{\pi/c}(z) \right], \quad (24)$$

$$d\sigma = \frac{\alpha_S^2}{S} \sum_i \sum_{a,b,c,z_{min}} \int \frac{dz}{z^2} \int \frac{dx}{x} \frac{1}{xS + U/z} f_1^a(x) f_1^b(x') D_1^{\pi/c}(z) S_U^i, \quad (25)$$

where $z_{min} = -(T + U)/S$, $x_{min} = -(U/z)/(T/z + S)$, and $x' = -(xT/z)/(xS + U/z)$. In Eq. (24) the quantities $\mathcal{H}^{\pi/c,i}(x, x', z)$ and $\mathcal{F}^{a,i}(x, x', z)$ are

$$\mathcal{H}^{\pi/c,i}(x, x', z) = \left[H_1^{\perp(1),\pi/c}(z) - z \frac{dH_1^{\perp(1),\pi/c}(z)}{dz} \right] \tilde{S}_{H_1^1}^i + \left[-2H_1^{\perp(1),\pi/c}(z) + \frac{1}{z} \tilde{H}^{\pi/c}(z) \right] \tilde{S}_H^i, \quad (26)$$

$$\mathcal{F}^{a,i}(x, x', z) = \pi \left[F_{FT}^a(x, x) - x \frac{dF_{FT}^a(x, x)}{dx} \right] S_{FT}^i. \quad (27)$$

We focus on the contributions in Eqs. (26), (27) from the functions $F_{FT}(x, x)$ and $H_1^{\perp(1)}(z)$, for which we have information on from the Siverson function and Collins function, respectively, that have been extracted from TMD processes [60,61,76–82]. In our analysis we will ignore $\tilde{H}(z)$ in Eq. (26), which is equivalent to approximating Eq. (13) as

$$H^q(z) = -2z H_1^{\perp(1),q}(z) + \tilde{H}^q(z) \approx -2z H_1^{\perp(1),q}(z). \quad (28)$$

This is known as a Wandzura–Wilczek-type approximation [83], which for TMDs was explored in Refs. [84,85]. We emphasize that Eq. (28) is a statement that $\tilde{H}(z)/z$ is parametrically smaller than $H_1^{\perp(1)}(z)$, not that $\hat{H}^3(z, z_1)$ is zero. In fact, $\hat{H}_{FU}^3(z, z_1)$ must be nonzero because it was shown in Ref. [26] that $H_1^{\perp(1)}(z)$ is an integral of $\hat{H}_{FU}^3(z, z_1)$. That is, $\hat{H}_{FU}^3(z, z_1) = 0$ implies $H_1^{\perp(1)}(z) = 0$, and consequently, Eq. (26) would vanish identically. Moreover, we know from current extractions of the Collins function that $H_1^{\perp(1)}(z) \neq 0$. Here, the purpose of our computation is not to offer a complete analysis of A_N ; it is to use recent TMD evolved extractions of known (i.e., kinematical) inputs to the observable, along with a new constraint from the LIR (15), to assess how well we are currently able to describe the data and ascertain what contributions remain from the dynamical functions. This will help guide a future fit of these correlators, in particular $\hat{H}_{FU}^3(z, z_1)$ (or $\tilde{H}(z)$), where one would be able to confirm or refute the approximation (28). The function $\hat{H}_{FU}^3(z, z_1)$ was originally extracted in Ref. [59] before the LIR (15) was derived, and, therefore, that work must be updated to include this constraint.

As mentioned above, we will ignore $\tilde{H}(z)$ and compute the terms in Eqs. (26), (27) that involve $F_{FT}(x, x)$ and $H_1^{\perp(1)}(z)$, using the latest fits of the Siverson and Collins functions that incorporate TMD evolution. We elaborate a bit here. TMD evolution is conventionally carried out in coordinate space, and thus one usually defines the Fourier transform of the TMD in a two-dimensional \vec{b} -space [86]. Take the quark Siverson function as an example [80,87],

$$f_{1T}^{\perp,q(\alpha)}(x, \vec{b}; Q) = \frac{1}{M} \int d^2\vec{k}_T e^{-i\vec{k}_T \cdot \vec{b}} \vec{k}_T^\alpha f_{1T}^{\perp,q}(x, \vec{k}_T^2; Q), \quad (29)$$

which is probed at the momentum scale Q . TMD evolution evolves the above Siverson function from an initial low scale Q_0 to a high Q : $f_{1T}^{\perp,q(\alpha)}(x, \vec{b}; Q_0) \rightarrow f_{1T}^{\perp,q(\alpha)}(x, \vec{b}; Q)$. At the same time, one can further perform an operator product expansion to relate the TMD at the initial scale Q_0 to a corresponding collinear function [87]. For the Siverson function at small b , one has a connection to the QS function:

$$f_{1T}^{\perp,q(\alpha)}(x, \vec{b}; Q_0) \propto \frac{i\vec{b}^\alpha}{2} \int \frac{dx'}{x'} C(x/x', b, Q_0) \otimes F_{FT}(x', x', Q_0), \quad (30)$$

where the coefficient C was derived in [88,89]. Thus, in the global analysis of the Siverson asymmetry that incorporates TMD evolution, one can directly extract the QS function $F_{FT}(x, x)$. Similarly we can directly extract $H_1^{\perp(1)}(z)$ through an analysis of the Collins asymmetry with TMD evolution. For details, see Ref. [82].

In the following numerical calculations, we use $H_1^{\perp(1)}(z)$ extracted in Ref. [82], evolved through the “diagonal” piece of its evolution equations [16,90]. We found that the QS function extracted in Ref. [80] leads to a negligible contribution from Eq. (27) to A_N , so we will ignore it in the plots below. Instead, we focus on the fragmentation term in Eq. (24) driven by (26), which gives the dominant contribution

¹⁰ This corresponds to a leading-order (LO) computation for this asymmetry within the usual collinear twist-3 factorization formalism [75]. At next-to-leading order (NLO), typically a more complicated structure will appear that likely contains more terms [23].

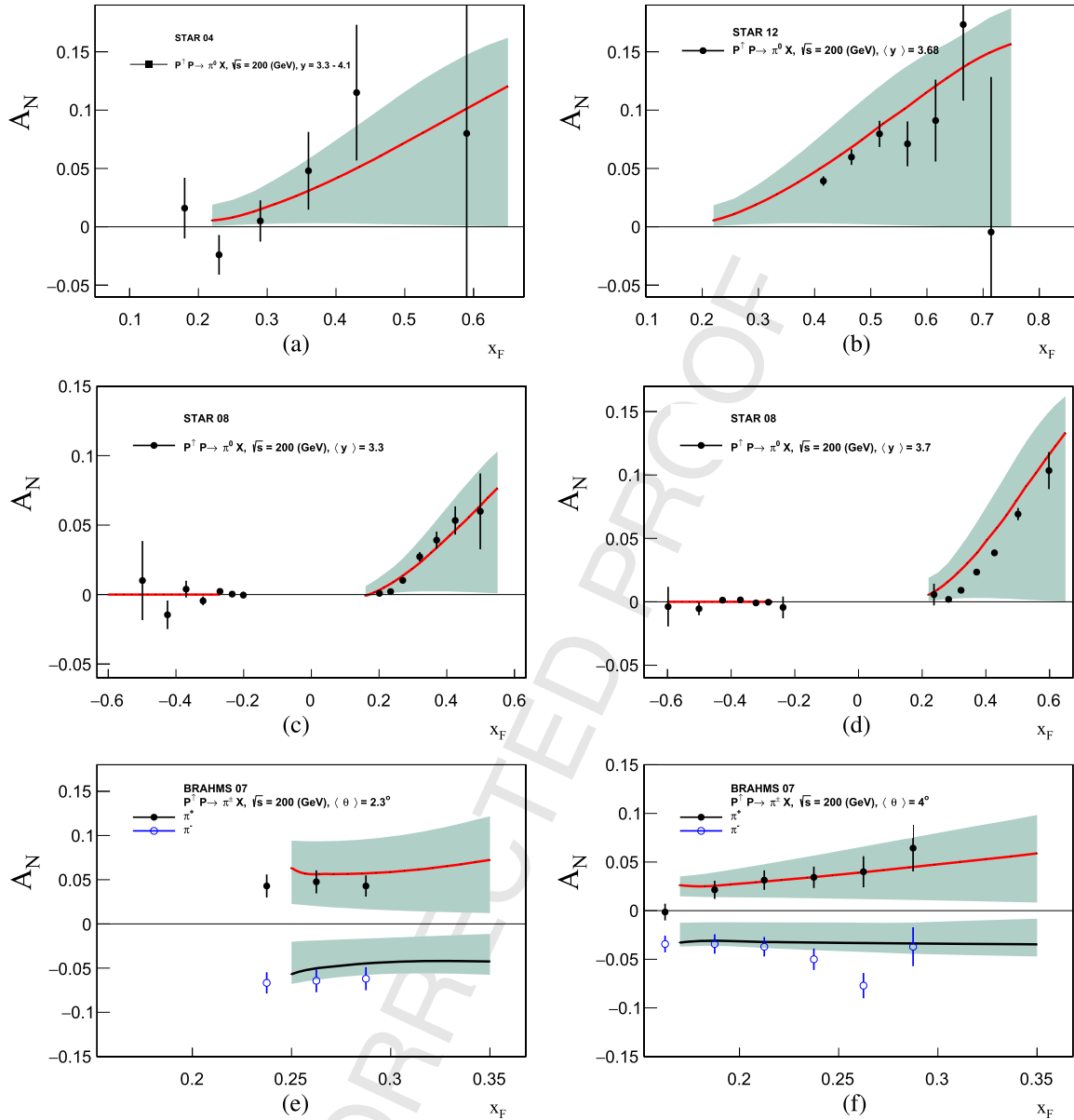


Fig. 1. Calculations of the contribution to A_N from Eq. (16) (using the approximation (28)) compared to (a) STAR Collaboration 2004 data [33] on A_N for π^0 , (b) STAR Collaboration 2012 data [36] on A_N for π^0 at $\langle y \rangle = 3.68$, (c) STAR Collaboration 2008 data [39] on A_N for π^0 at $\langle y \rangle = 3.3$, (d) STAR Collaboration 2008 data [39] on A_N for π^0 at $\langle y \rangle = 3.7$, (e) BRAHMS Collaboration 2007 data [35] on A_N for π^\pm (black closed data π^+ , blue open data π^-) at $\langle \theta \rangle = 2.3^\circ$, (f) BRAHMS Collaboration 2007 data [35] on A_N for π^\pm (black closed data π^+ , blue open data π^-) at $\langle \theta \rangle = 4^\circ$. The solid lines correspond to the results using the central parameters from Ref. [82] for $h_1(x)$ and $H_1^{\perp(1)}(z)$. The shaded regions correspond to an estimate of 90% C.L. error band from Ref. [82] due to uncertainties in $h_1(x)$ and $H_1^{\perp(1)}(z)$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

to the asymmetry. This confirms the original findings in Ref. [59]. A few additional comments are in order about the size of the QS term. The smallness of this term is because of the large- x behavior of the Siverson function [60,61,91], which can be explored and constrained in future experiments, like those at the 12 GeV upgrade of Jefferson Lab (JLab 12) [92,93]. More specifically, one can only generate a non-negligible QS term if the β parameters (that control the large- x behavior of the Siverson/QS function) are different for up and down quarks [59,91], whereas in Ref. [80] one has $\beta_u = \beta_d$. While there does exist an extraction of the Siverson function where $\beta_u \neq \beta_d$ [61], this extraction is not at the NLL' accuracy that we use for other functions in our analysis. Moreover, if such a Siverson function is used, as explored in Ref. [59], it most likely will not overwhelm the large effect from the fragmentation term. Nevertheless, it will be interesting to include both the QS functions and the fragmentation terms in the global analysis in order to understand A_N fully. Such a study will be explored in the future.

In Fig. 1, we show the result of our calculation compared with the BRAHMS charged pion and STAR neutral pion data for A_N as a function of x_F [33,35,36,39]. The error bands for A_N in those plots are based on the uncertainties in $h_1(x)$ and $H_1^{\perp(1)}(z)$ from the fit in Ref. [82]. In the large- x_F (i.e., large- x and large- z) region these functions are not well-constrained, and consequently, one obtains large errors due to these inputs. We see that, although the theoretical calculations undershoot or overshoot A_N in some places, the central curves do a reasonable job in describing the data. We are especially encouraged by these plots given that the contribution from $\tilde{H}(z)$ still needs to be included. This clearly demonstrates that this function must be nonzero. Through this computation, we now have a constraint

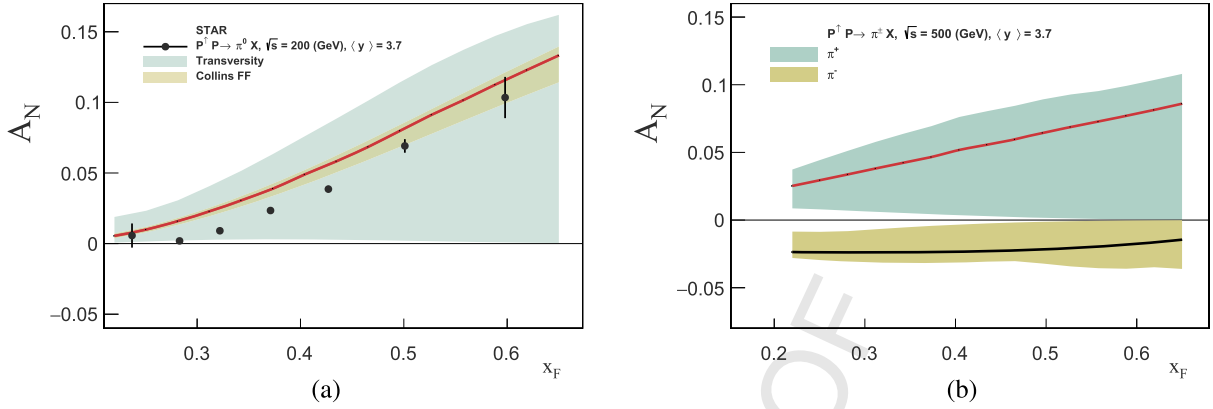


Fig. 2. (a) STAR Collaboration data on A_N for π^0 as function of x_F at $\langle y \rangle = 3.7$ for positive x_F . The two error bands correspond to the theoretical uncertainty from $H_1^{\perp(1)}(z)$ (the narrow band) and $h_1(x)$ (the wide band). (b) Predictions for A_N in π^\pm production as function of x_F at $\langle y \rangle = 3.7$ for positive x_F using Eq. (16) along with the approximation (28). The error bands correspond to the theoretical uncertainty from both $h_1(x)$ and $H_1^{\perp(1)}(z)$.

on $\tilde{H}(z)$ and leave a fit of this function to A_N data for future work. We emphasize again that the unintegrated version of this correlator also enters multiple asymmetries in SIDIS and $e^+e^- \rightarrow h_a h_b X$, while $\tilde{H}(z)$ itself can be directly measured in $A_{UT}^{\sin\phi_S}$ in SIDIS integrated over P_{hT} .

Moreover, since $F_{FT}(x, x)$ and $H_1^{\perp(1)}(z)$, $h_1(x)$ enter the TMD evolution equations for the Sivvers and Collins asymmetries, respectively, in SIDIS and $e^+e^- \rightarrow h_a h_b X$, one can eventually perform a global analysis that includes all these observables along with A_N in proton–proton and lepton–nucleon collisions (where $F_{FT}(x, x)$, $\tilde{H}(z)$, $H_1^{\perp(1)}(z)$, $h_1(x)$ all enter). This would better constrain the large- x_F behavior of these functions and greatly reduce the error bands in our plots since we have data from RHIC in this region. We found that the uncertainty in $h_1(x)$ in this regime is what dominates the error over that from $H_1^{\perp(1)}(z)$, see Fig. 2(a). Thus, it is evident that the A_N data would allow us to drastically improve the extraction of transversity. Also, future measurements at JLab12 can improve the situation in the large- x region [93]. In order to demonstrate the powerful capability of RHIC future measurements [2], we present our predictions for A_N in π^\pm production at 500 GeV in Fig. 2(b). One can clearly see that large- x_F measurements of A_N will reduce the uncertainty of the large- x behavior of transversity and, together with other data sets, allow us to explore the missing contribution from $\tilde{H}(z)$. In addition, we also give our result for A_N as a function of P_{hT} in Fig. 3 compared with the STAR data from Ref. [94]. One can see that our calculations exhibit a flat behavior, similar to that shown in Ref. [59]. The reason is that in the forward region, where \hat{t} becomes very small, the $qg \rightarrow qg$ channel dominates, and the hard function $S_{H_1^{\perp}}^{qg \rightarrow qg} \propto 1/\hat{t}^3$ compensates the twist-3 $(P_{hT})^{-1}$ fall off of the asymmetry. Again, one has to

keep in mind that there is still a term missing, $\tilde{H}(z)$ from our analysis, which needs to be fit to data as has been emphasized above. It is also important to emphasize that the experimental data has a very large uncertainty which prevents an unambiguous identification of the P_{hT} -dependence. These open issues can only be addressed by future experimental measurements and theoretical work.

We end this section with a brief comment about the fragmentation contribution to A_N in $p^\dagger A \rightarrow \pi^0 X$. Recently, a calculation of this term was carried out in Ref. [95] that included gluon saturation effects in the unpolarized nucleus. The authors found that the first two terms in braces in Eq. (9) are proportional to $A^{-1/3}$ (see also [96]), while the third term is proportional to A^0 . Since in Ref. [59] one finds that this third term is negligible (see Fig. 3 of Ref. [59]), the authors of Ref. [95] concluded that the fragmentation piece to A_N in $p^\dagger A$ collisions is proportional to $A^{-1/3}$, which is in contradiction to recent STAR measurements [97] that find no suppression with A . However, as we have mentioned, the fit in Ref. [59] was performed before the LIR (15) was derived. Using both the EOMR (11) and LIR (15) we can write

$$\frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\left(\frac{1}{z} - \frac{1}{z_1}\right)^2} \hat{H}_{FU}^{\pi/c, \mathcal{N}}(z, z_1) = H_1^{\perp(1), \pi/c}(z) + z \frac{dH_1^{\perp(1), \pi/c}(z)}{dz} - \frac{1}{z} \tilde{H}^{\pi/c}(z). \quad (31)$$

With Eq. (31) in hand, along with using the known input from the Collins function for $H_1^{\perp(1), q}(z)$, we can obtain a new estimate for the contribution of the third term in Eq. (9) to A_N . To be specific, we replace $\mathcal{H}^{\pi/c, i}(x, x', z)$ in Eq. (24) with

$$\mathcal{H}^{\pi/c, i}(x, x', z) \Big|_{3^{rd} \text{ term in (9)}} = \left[H_1^{\perp(1), \pi/c}(z) + z \frac{dH_1^{\perp(1), \pi/c}(z)}{dz} - \frac{1}{z} \tilde{H}^{\pi/c}(z) \right] \frac{S_{\hat{H}_{FU}}^i}{-x'\hat{t} - x\hat{u}}, \quad (32)$$

where, as before, $S_{\hat{H}_{FU}}^i$ can be found in Appendix A of Ref. [22], and we include only terms involving $H_1^{\perp(1), \pi/c}(z)$.¹¹ Our estimate is shown in Fig. 4. We see that the contribution to A_N from the third term in Eq. (9) is actually moderate in size and certainly not negligible. Since this part of A_N in $p^\dagger A$ collisions is proportional to A^0 [95], the fragmentation term for A_N is not inconsistent with the STAR data [97] on the asymmetry in $p^\dagger A \rightarrow \pi^0 X$. Recently, the author of Ref. [98] has found that, in contrast to Ref. [99], the QS piece contribution to A_N in pA collisions is small (and may very well vanish), leaving the fragmentation term as the only source of A_N in pA collisions.¹²

¹¹ This of course is not a complete calculation because we still must include/fit $\tilde{H}(z)$.

¹² This conclusion is reached within the so-called hybrid approach of twist-3 and color glass condensate [98,99].

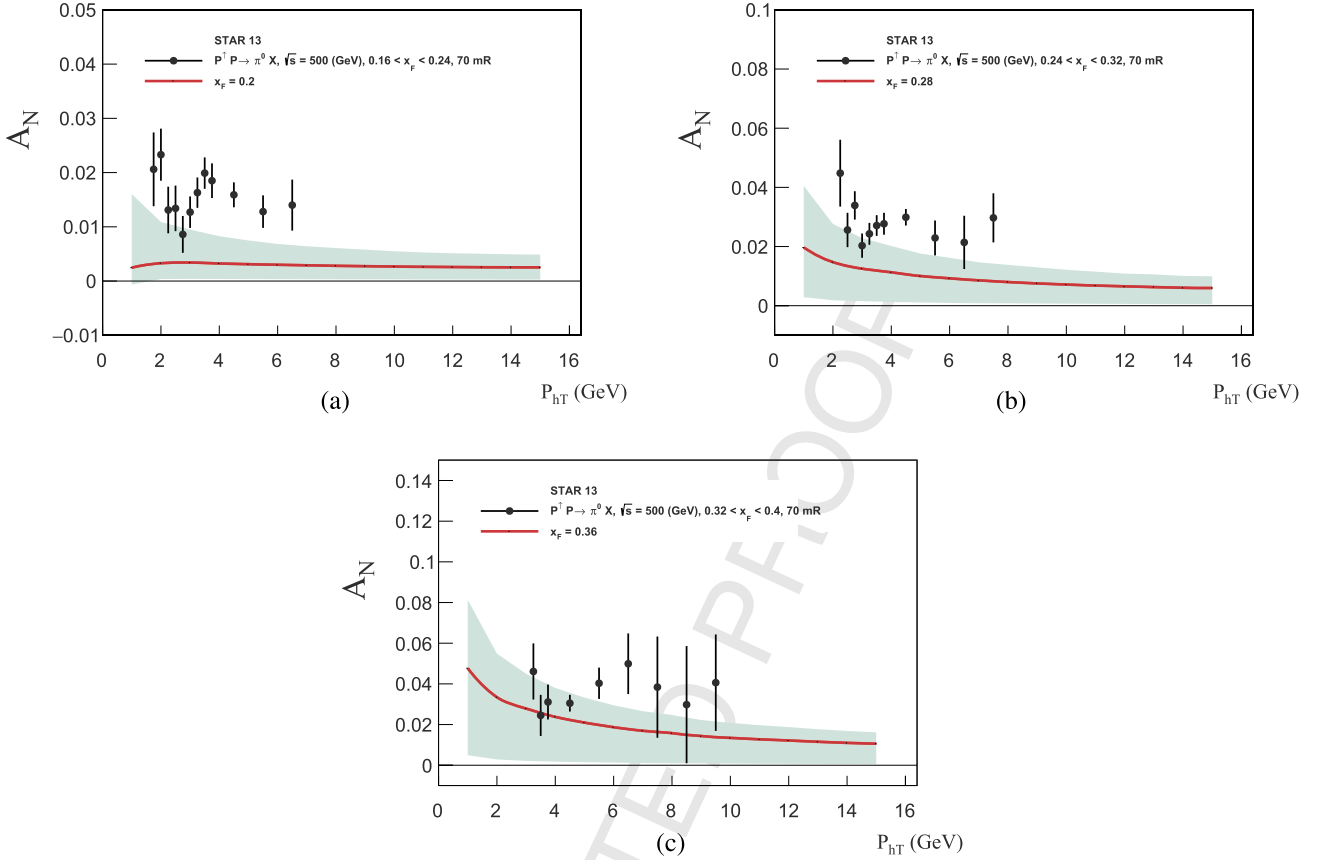


Fig. 3. Calculations of the contribution to A_N from Eq. (16) (using the approximation (28)) compared to STAR Collaboration 2013 data [94] on A_N for π^0 at $\sqrt{s} = 500$ GeV as a function of P_{hT} (a) $0.16 < x_F < 0.24$, (b) $0.24 < x_F < 0.32$, and (c) $0.32 < x_F < 0.4$. The solid lines correspond to results using the central parameters from Ref. [82] for $h_1(x)$ and $H_1^{\perp(1)}(z)$. The shaded regions correspond to an estimate of 90% C.L. error band from Ref. [82] due to uncertainties in $h_1(x)$ and $H_1^{\perp(1)}(z)$.

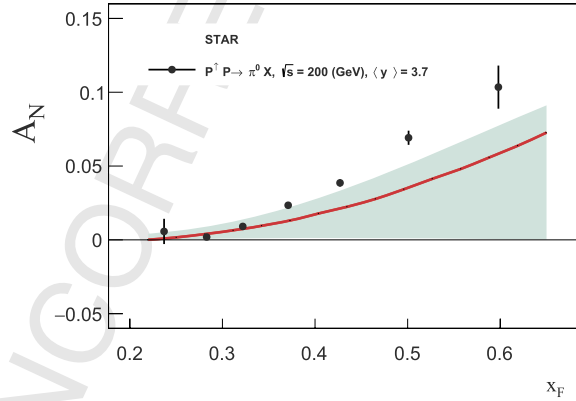


Fig. 4. STAR Collaboration data on A_N for π^0 as function of x_F at $\langle y \rangle = 3.7$ for positive x_F . The solid line and the shaded area correspond to calculations using Eq. (32).

4. Summary

We have given an estimate of A_N in $p^\uparrow p \rightarrow \pi X$ due to the kinematical twist-3 functions, which are first (k_T or p_\perp) moments of TMD functions. Using the newly derived LIR (15) from Ref. [26], we have written A_N in terms of a maximum number of these correlators, which helps optimize the phenomenology. In particular, we computed the terms in A_N that involve $H_1^{\perp(1)}(z)$, using the most recent TMD evolved extraction of that function [82] and confirmed the conclusion of Ref. [59] that the fragmentation piece can be the dominant source of the asymmetry. Note that the dynamical twist-3 FF $\hat{H}_{FU}^3(z, z_1)$, which played a key role in the analysis of Ref. [59], is also crucial to our work here since $H_1^{\perp(1)}(z)$ (and $\tilde{H}(z)$) is an integral of this function. We have simply “re-shuffled” the pieces by incorporating LIRs (and EOMRs) in order to write the cross section in terms of a maximal number of known functions. We also found that the existing extractions of the QS function lead to a negligible contribution to A_N , although one must be mindful that this function is not well-constrained at large x . Our results give a reasonable description of the data, although the central curves cannot fully account for all of the asymmetry. However, we know the pieces that remain from the dynamical twist-3 functions, in particular $\hat{H}_{FU}^3(z, z_1)$ (via $\tilde{H}(z)$), must be nonzero.

Therefore, we must fit this function to A_N in order to obtain a complete result for this observable. In addition, we can reduce the rather large theoretical uncertainties by including both the transversity and Sivers functions simultaneously in such a fit. We leave both of these matters for future work, which are needed to definitively resolve the puzzle of what causes A_N .

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References

- [1] D. Pitonyak, Int. J. Mod. Phys. A 31 (2016) 1630049, arXiv:1608.05353.
- [2] E.-C. Aschenauer, et al., arXiv:1602.03922.
- [3] A.V. Efremov, O.V. Teryaev, Sov. J. Nucl. Phys. 36 (1982) 140.
- [4] A.V. Efremov, O.V. Teryaev, Phys. Lett. B 150 (1985) 383.
- [5] J.-W. Qiu, G. Sterman, Phys. Rev. Lett. 67 (1991) 2264.
- [6] J.-W. Qiu, G. Sterman, Nucl. Phys. B 378 (1992) 52.
- [7] J.-W. Qiu, G. Sterman, Phys. Rev. D 59 (1998) 014004, arXiv:hep-ph/9806356.
- [8] Y. Kanazawa, Y. Koike, Phys. Lett. B 478 (2000) 121, arXiv:hep-ph/0001021.
- [9] C. Kouvaris, J.-W. Qiu, W. Vogelsang, F. Yuan, Phys. Rev. D 74 (2006) 114013, arXiv:hep-ph/0609238.
- [10] H. Eguchi, Y. Koike, K. Tanaka, Nucl. Phys. B 752 (2006) 1, arXiv:hep-ph/0604003.
- [11] H. Eguchi, Y. Koike, K. Tanaka, Nucl. Phys. B 763 (2007) 198, arXiv:hep-ph/0610314.
- [12] Y. Koike, K. Tanaka, Phys. Lett. B 646 (2007) 232, arXiv:hep-ph/0612117.
- [13] Y. Koike, K. Tanaka, Phys. Rev. D 76 (2007) 011502, arXiv:hep-ph/0703169.
- [14] J. Zhou, F. Yuan, Z.-T. Liang, Phys. Rev. D 78 (2008) 114008, arXiv:0808.3629.
- [15] Y. Koike, T. Tomita, Phys. Lett. B 675 (2009) 181, arXiv:0903.1923.
- [16] F. Yuan, J. Zhou, Phys. Rev. Lett. 103 (2009) 052001, arXiv:0903.4680.
- [17] Z.-B. Kang, F. Yuan, J. Zhou, Phys. Lett. B 691 (2010) 243, arXiv:1002.0399.
- [18] H. Beppu, Y. Koike, K. Tanaka, S. Yoshida, Phys. Rev. D 82 (2010) 054005, arXiv:1007.2034.
- [19] A. Metz, J. Zhou, Phys. Lett. B 700 (2011) 11, arXiv:1006.3097.
- [20] Y. Koike, S. Yoshida, Phys. Rev. D 84 (2011) 014026, arXiv:1104.3943.
- [21] Y. Koike, S. Yoshida, Phys. Rev. D 85 (2012) 034030, arXiv:1112.1161.
- [22] A. Metz, D. Pitonyak, Phys. Lett. B 723 (2013) 365, arXiv:1212.5037.
- [23] K. Kanazawa, Y. Koike, Phys. Rev. D 88 (2013) 074022, arXiv:1309.1215.
- [24] H. Beppu, K. Kanazawa, Y. Koike, S. Yoshida, Phys. Rev. D 89 (2014) 034029, arXiv:1312.6862.
- [25] K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak, Phys. Rev. D 91 (2015) 014013, arXiv:1410.3448.
- [26] K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak, M. Schlegel, Phys. Rev. D 93 (2016) 054024, arXiv:1512.07233.
- [27] Y. Koike, K. Yabe, S. Yoshida, Phys. Rev. D 92 (2015) 094011, arXiv:1509.06830.
- [28] R.D. Klem, et al., Phys. Rev. Lett. 36 (1976) 929.
- [29] G. Bunce, et al., Phys. Rev. Lett. 36 (1976) 1113.
- [30] D.L. Adams, et al., E581, Phys. Lett. B 261 (1991) 201.
- [31] K. Krueger, et al., Phys. Lett. B 459 (1999) 412.
- [32] C.E. Allgower, et al., Phys. Rev. D 65 (2002) 092008.
- [33] J. Adams, et al., STAR, Phys. Rev. Lett. 92 (2004) 171801, arXiv:hep-ex/0310058.
- [34] S.S. Adler, et al., PHENIX, Phys. Rev. Lett. 95 (2005) 202001, arXiv:hep-ex/0507073.
- [35] J.H. Lee, F. Videbaek, BRAHMS, AIP Conf. Proc. 915 (2007) 533.
- [36] B.I. Abelev, et al., STAR, Phys. Rev. Lett. 101 (2008) 222001, arXiv:0801.2990.
- [37] I. Arsene, et al., BRAHMS, Phys. Rev. Lett. 101 (2008) 042001, arXiv:0801.1078.
- [38] L. Adamczyk, et al., STAR, Phys. Rev. D 86 (2012) 032006, arXiv:1205.2735.
- [39] L. Adamczyk, et al., STAR Collaboration, Phys. Rev. D 86 (2012) 051101, arXiv:1205.6826.
- [40] L. Bland, et al., AnDY Collaboration, arXiv:1304.1454, 2013.
- [41] A. Adare, et al., PHENIX Collaboration, arXiv:1312.1995.
- [42] A. Adare, et al., PHENIX Collaboration, arXiv:1406.3541.
- [43] A. Airapetian, et al., HERMES Collaboration, Phys. Lett. B 728 (2014) 183, arXiv:1310.5070.
- [44] K. Allada, et al., Jefferson Lab Hall A Collaboration, Phys. Rev. C 89 (2014) 042201, arXiv:1311.1866.
- [45] D. Boer, P.J. Mulders, F. Pijlman, Nucl. Phys. B 667 (2003) 201, arXiv:hep-ph/0303034.
- [46] D.W. Sivers, Phys. Rev. D 41 (1990) 83.
- [47] Z.-B. Kang, J.-W. Qiu, W. Vogelsang, F. Yuan, Phys. Rev. D 83 (2011) 094001, arXiv:1103.1591.
- [48] Z.-B. Kang, A. Prokudin, Phys. Rev. D 85 (2012) 074008, arXiv:1201.5427.
- [49] A. Metz, et al., Phys. Rev. D 86 (2012) 094039, arXiv:1209.3138.
- [50] K. Kanazawa, Y. Koike, Phys. Rev. D 82 (2010) 034009, arXiv:1005.1468.
- [51] K. Kanazawa, Y. Koike, Phys. Rev. D 83 (2011) 114024, arXiv:1104.0117.
- [52] J. Zhou, F. Yuan, Z.-T. Liang, Phys. Rev. D 79 (2009) 114022, arXiv:0812.4484.
- [53] Z.-B. Kang, J.-W. Qiu, Phys. Rev. D 79 (2009) 016003, arXiv:0811.3101.
- [54] Z.-B. Kang, J.-W. Qiu, Phys. Lett. B 713 (2012) 273, arXiv:1205.1019.
- [55] D. Boer, P.J. Mulders, Phys. Rev. D 57 (1998) 5780, arXiv:hep-ph/9711485.
- [56] Y. Kanazawa, Y. Koike, Phys. Lett. B 490 (2000) 99, arXiv:hep-ph/0007272.
- [57] J.C. Collins, Nucl. Phys. B 396 (1993) 161, arXiv:hep-ph/9208213.
- [58] M. Anselmino, et al., Phys. Rev. D 86 (2012) 074032, arXiv:1207.6529.
- [59] K. Kanazawa, Y. Koike, A. Metz, D. Pitonyak, Phys. Rev. D 89 (2014) 111501(R), arXiv:1404.1033.
- [60] M. Anselmino, et al., Eur. Phys. J. A 39 (2009) 89, arXiv:0805.2677.
- [61] M. Anselmino, et al., Phys. Rev. D 87 (2013) 094019, arXiv:1303.3822.
- [62] J.F. Owens, Rev. Mod. Phys. 59 (1987) 465.
- [63] Z.-B. Kang, I. Vitev, H. Xing, Phys. Rev. D 88 (2013) 054010, arXiv:1307.3557.
- [64] S.J. Brodsky, D.S. Hwang, I. Schmidt, Phys. Lett. B 530 (2002) 99, arXiv:hep-ph/0201296.
- [65] J.C. Collins, Phys. Lett. B 536 (2002) 43, arXiv:hep-ph/0204004.

- [66] H.D. Politzer, Nucl. Phys. B 172 (1980) 349.
- [67] D. Boer, P.J. Mulders, O.V. Teryaev, Phys. Rev. D 57 (1998) 3057, arXiv:hep-ph/9710223.
- [68] A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P.J. Mulders, M. Schlegel, J. High Energy Phys. 0702 (2007) 093, arXiv:hep-ph/0611265.
- [69] D. Boer, R. Jakob, P.J. Mulders, Nucl. Phys. B 504 (1997) 345, arXiv:hep-ph/9702281.
- [70] Z. Lu, I. Schmidt, Phys. Lett. B 747 (2015) 357, arXiv:1501.04379.
- [71] X. Wang, Z. Lu, Phys. Rev. D 93 (2016) 074009, arXiv:1601.01574.
- [72] J. Kodaira, K. Tanaka, Prog. Theor. Phys. 101 (1999) 191, arXiv:hep-ph/9812449.
- [73] A.V. Belitsky, Leading order analysis of the twist-three space-like and time-like cut vertices in QCD, in: Annual 31st PNPI Winter School on Nuclear and Particle Physics, St. Petersburg, Russia, February 24–March 2, 1997, arXiv:hep-ph/9703432.
- [74] A. Accardi, A. Bacchetta, W. Melnitchouk, M. Schlegel, J. High Energy Phys. 11 (2009) 093, arXiv:0907.2942.
- [75] Z.-B. Kang, I. Vitev, H. Xing, Phys. Rev. D 87 (2013) 034024, arXiv:1212.1221.
- [76] M. Anselmino, et al., Phys. Rev. D 75 (2007) 054032, arXiv:hep-ph/0701006.
- [77] M. Anselmino, et al., Nucl. Phys. B, Proc. Suppl. 191 (2009) 98, arXiv:0812.4366.
- [78] M. Anselmino, M. Boglione, S. Melis, arXiv:1204.1239, 2012.
- [79] M. Anselmino, et al., Phys. Rev. D 88 (2013) 054023, arXiv:1304.7691.
- [80] M.G. Echevarria, A. Idilbi, Z.-B. Kang, I. Vitev, Phys. Rev. D 89 (2014) 074013, arXiv:1401.5078.
- [81] M. Anselmino, et al., Phys. Rev. D 92 (2015) 114023, arXiv:1510.05389.
- [82] Z.-B. Kang, A. Prokudin, P. Sun, F. Yuan, Phys. Rev. D 93 (2016) 014009, arXiv:1505.05589.
- [83] S. Wandzura, F. Wilczek, Phys. Lett. B 72 (1977) 195.
- [84] A. Metz, P. Schweitzer, T. Teckentrup, Phys. Lett. B 680 (2009) 141, arXiv:0810.5212.
- [85] T. Teckentrup, A. Metz, P. Schweitzer, Mod. Phys. Lett. A 24 (2009) 2950, arXiv:0910.2567.
- [86] J. Collins, Foundations of Perturbative QCD, Cambridge University Press, Cambridge, England, 2011.
- [87] S.M. Aybat, J.C. Collins, J.-W. Qiu, T.C. Rogers, Phys. Rev. D 85 (2012) 034043, arXiv:1110.6428.
- [88] Z.-B. Kang, B.-W. Xiao, F. Yuan, Phys. Rev. Lett. 107 (2011) 152002, arXiv:1106.0266.
- [89] P. Sun, F. Yuan, Phys. Rev. D 88 (2013) 114012, arXiv:1308.5003.
- [90] Z.-B. Kang, Phys. Rev. D 83 (2011) 036006, arXiv:1012.3419.
- [91] L. Gamberg, Z.-B. Kang, A. Prokudin, Phys. Rev. Lett. 110 (2013) 232301, arXiv:1302.3218.
- [92] J. Dudek, et al., Eur. Phys. J. A 48 (2012) 187, arXiv:1208.1244.
- [93] Z. Ye, et al., arXiv:1609.02449.
- [94] S. Heppelmann, STAR, PoS D IS2013 (2013) 240.
- [95] Y. Hatta, B.-W. Xiao, S. Yoshida, F. Yuan, Phys. Rev. D 95 (2017) 014008, arXiv:1611.04746.
- [96] Z.-B. Kang, F. Yuan, Phys. Rev. D 84 (2011) 034019, arXiv:1106.1375.
- [97] S. Heppelmann, STAR, talk at SPIN 2016.
- [98] J. Zhou, arXiv:1704.04901.
- [99] Y. Hatta, B.W. Xiao, S. Yoshida, F. Yuan, Phys. Rev. D 94 (2016) 054013, arXiv:1606.08640.