Studies of the transverse spin-dependent structure function $g_2(x,Q^2)$

R. L. Jaffe and Xiangdong Ji
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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We study the transverse spin-dependent nucleon structure function $g_2(x,Q^2)$ in leading order at large Q^2. After reviewing the operator-product-expansion analysis of $g_2(x,Q^2)$ we calculate the twist-3 contributions which arise from quark-gluon interactions and quark masses in the bag model. The result confirms the presence of significant twist-3 effects in $g_2(x,Q^2)$. We discuss two questions raised by our analysis: the validity of the Burkhardt-Cottingham sum rule and the relation to previous calculations of $g_2(x,Q^2)$ in versions of the parton model.

I. INTRODUCTION

Spin-averaged deep-inelastic lepton-nucleon scattering has greatly contributed to our knowledge about the quark-gluon structure of the nucleon. Further understanding of nucleon structure can be obtained from scattering by a polarized nucleon.[1] The proton's dominant spin-structure function $g_1(x,Q^2)$ has been measured at SLAC (Ref. 2) and CERN (Ref. 3) with increasing accuracy. The data has stimulated many theoretical papers on the spin structure of the proton.[4] Recently, the HERMES and SMC Collaborations have proposed to measure the spin structure functions of both the proton and neutron with unprecedented accuracy.[5] Undoubtedly, future data will continue to stimulate interest in the spin physics of the nucleon.

The nucleon's second spin-dependent structure function,[6] $g_2(x,Q^2)$ has never been measured, and there have been few theoretical studies of it. This paper seeks a qualitative understanding of $g_2(x,Q^2)$, in particular, its physical significance and origin. In Sec. II we review the standard operator-product-expansion analysis which shows that $g_2(x,Q^2)$ is related to the matrix elements of both twist-2 and 3 operators. The twist-3 operators arise from quark-gluon interactions and quark masses. In Sec. III we develop further the space-time interpretation of $g_2(x,Q^2)$. Ignoring radiative corrections, we sum the operator-product expansion and obtain an "impulse approximation" to $g_2(x,Q^2)$. This approach is easily applied to the bag model. Also, it provides a formalism for analyzing the Burkhardt-Cottingham sum rule,[7] $\int_0^1 g_2(x,Q^2)dx=0$, showing explicitly how it may be violated. In Sec. IV, we calculate the twist-3 contribution using the bag model, with the bag boundary simulating the soft gluons responsible for quark confinement. The result shows that the twist-3 contribution is as important as twist 2, the latter being related to $g_1(x,Q^2)$ by Wandzura-Wilczek sum rules.[8] Therefore the future measurement of $g_2(x,Q^2)$ will provide a unique opportunity to study quark-gluon interactions through high-twist effects. In Sec. V we explain the transverse spin-structure function in a general parton picture and show explicitly that the transverse momentum and off-shell partons are responsible for the large effects associated with the transverse nucleon polarization. Finally, we summarize our discussions in Sec. VI.

II. OPERATOR-PRODUCT EXPANSION AND KINEMATICS

We start our discussions on the spin-dependent structure functions with the operator-product-expansion (OPE) method, which is the most reliable for analyzing the deep-inelastic processes. Unfortunately, the early OPE analysis of $g_2(x,Q^2)$ in Refs. 9–14 is partly wrong because of incorrect identification of twist-3 operators. The correct analysis was done later in Refs. 15–18. We largely follow and slightly expand their discussions in this section.

We begin with the hadron tensor

$$W_{\mu\nu}(q,P,S)=\frac{1}{4\pi}\int d^4\xi\, e^{iq\cdot\xi}\langle PS|[J_\mu(\xi),J_\nu(0)]|PS\rangle\ , \tag{1}$$

where q is the virtual-photon four-momentum and P and S are the target four-momentum and spin, respectively ($S^2=-M^2$, $P^2=M^2$, $S\cdot P=0$). We normalize the nucleon state $\langle PS|P'S'\rangle=2P^0(2\pi^2)\delta^3(\mathbf{P}-\mathbf{P}')\delta_{SS'}$.

In polarized electron and nucleon scattering, spin-dependent effects are related to the antisymmetric part of the hadron tensor $W^A_{\mu\nu}$. By Lorentz invariance and gauge invariance $W^A_{\mu\nu}$ can be constructed from two scalar functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$ for a spin-$\frac{1}{2}$ target:

$$W^A_{\mu\nu}=i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda}{\nu}\left[g_1(x,Q^2)S^\sigma+g_2(x,Q^2)\left[S^\sigma-P^\sigma\frac{q\cdot S}{\nu}\right]\right]\ , \tag{2}$$

where $Q^2=-q^2$ and $\nu=P\cdot q$. In the Bjorken limit in QCD, both $g_1(x,Q^2)$ and $g_2(x,Q^2)$ scale to $g_1(x)$ and $g_2(x)$ modulo logarithms.

The spirit of the operator-product expansion is to expand the product of currents at light-cone separation in terms of the renormalized local operators. In the Bjorken limit the importance of an operator is determined by its twist or light-cone singularity of its coefficient function. Keeping twist-2 and -3 operators in the spin-dependent part of the expansion we have

$$J_\mu(\xi)J_\nu(0)|_A=i\epsilon_{\mu\nu\lambda\sigma}\partial^\lambda \sum_{n=0,2,\ldots}^{\infty}\left[\sum_i F_{2,i}^n(\xi^2-i\epsilon\xi_0,\mu^2)O_{2,i}^{\sigma\mu_1\cdots\mu_n}(\mu^2)+\sum_i F_{3,i}^n(\xi^2-i\epsilon\xi_0,\mu^2)O_{3,i}^{\sigma\mu_1\cdots\mu_n}(\mu^2)\right]\xi_{\mu_1}\xi_{\mu_2}\cdots\xi_{\mu_n}\ , \tag{3}$$

where $O_{2,i}^{\sigma\mu_1\cdots\mu_n}$ and $O_{3,i}^{\sigma\mu_1\cdots\mu_n}$ denote twist-2 and -3 operators, respectively, and $F_{2,i}^n$ and $F_{3,i}^n$ are corresponding coefficient functions. The index i runs over operators with the same Lorentz structure and twist.

Using the expansion (3), we calculate the virtual-photon forward Compton amplitude on the nucleon:

$$T_{\mu\nu}(\nu,q^2)=i\int d^4\xi\, e^{iq\cdot\xi}\langle PS|T(J_\mu(\xi)J_\nu(0))|PS\rangle\ , \tag{4}$$

for $|\nu|<-q^2/2$ in the complex ν plane (otherwise the expansion is not convergent). The result is

$$T_{\mu\nu}(\nu,q^2)=i\epsilon_{\mu\nu\lambda\sigma}q^\lambda \sum_{n=0,2,\ldots}\left[\frac{2}{Q^2}\right]^{n+1} q_{\mu_1}q_{\mu_2}\cdots q_{\mu_n}\times\left[\sum_i F_{2,i}^n(Q^2,\mu^2)\langle PS|O_{2,i}^{\sigma\mu_1\cdots\mu_n}|PS\rangle+\frac{2n}{n+1}F_{3,i}^n(Q^2,\mu^2)\langle PS|O_{3,i}^{\sigma\mu_1\cdots\mu_n}|PS\rangle\right]\ , \tag{5}$$

where the $F(Q^2)$'s are Fourier transformations of the corresponding ξ^2 functions:

$$F_{2,i}^n(Q^2,\mu^2)=\tfrac{1}{2}(-i)^n(-q^2)^{n+1}\left[\frac{\partial}{\partial q^2}\right]^n\int d^4\xi\, e^{iq\cdot\xi}F_{2,i}^n(\xi^2-i\epsilon,\mu^2)\ , \tag{6}$$

$$F_{3,i}^n(Q^2,\mu^2)=\tfrac{1}{4}(-i)^n(-q^2)^{n+1}\frac{n+1}{n}\left[\frac{\partial}{\partial q^2}\right]^n\int d^4\xi\, e^{iq\cdot\xi}F_{3,i}^n(\xi^2-i\epsilon,\mu^2)\ , \tag{7}$$

where we choose the appropriate factors to simplify the expressions under the impulse approximation.

To proceed further, we introduce the nucleon forward matrix elements of the local operators:

$$\langle PS|O_{2,i}^{\sigma\mu_1\mu_2\cdots\mu_n}|PS\rangle=2a_i^n\mathcal{S}_1\{S^\sigma P^{\mu_1}P^{\mu_2}\cdots P^{\mu_n}\}-(\text{traces})\ , \tag{8}$$

where $\mathcal{S}_1$ symmetrizes all indices, and

$$\langle PS|O_{3,i}^{\sigma\mu_1\mu_2\cdots\mu_n}|PS\rangle=2d_i^n\mathcal{S}\mathcal{A}\{S^\sigma P^{\mu_1}P^{\mu_2}\cdots P^{\mu_n}\}-(\text{traces})\ , \tag{9}$$

where $\mathcal{A}$ antisymmetrizes σ and μ_1 and $\mathcal{S}$ symmetrizes $\mu_1,\mu_2,\ldots,\mu_n$. The scalar matrix elements a_i^n and d_i^n depend on the nucleon structure and the renormalization scale μ^2 at which the operators are defined.

To connect the Compton amplitude in the region where the expansion (5) holds with the physical hadron tensor in Eq. (1), we use the dispersion relations. The analysis is standard, and the result is an infinite set of sum rules for the moments of the structure functions:

$$\int_0^1 x^n g_1(x,Q^2)dx=\tfrac{1}{2}\sum_i a_i^n(\mu^2)F_{2,i}^n(Q^2,\mu^2),\quad n=0,2,4,\ldots\ , \tag{10}$$

$$\int_0^1 x^n g_2(x,Q^2)dx=-\frac{n}{2(n+1)}\left[\sum_i a_i^n(\mu^2)F_{2,i}^n(Q^2,\mu^2)-\sum_i d_i^n(\mu^2)F_{3,i}^n(Q^2,\mu^2)\right],\quad n=2,4,\ldots\ . \tag{11}$$

We emphasize again that all operators with a twist higher than 3 are negligible at high Q^2 in the sum rules. Equations (10) and (11) show that $g_1(x,Q^2)$ receives contribution from twist-2 operators alone, whereas $g_2(x,Q^2)$ receives contributions from both twist-2 and -3 operators. The twist-2 part of the $g_2(x,Q^2)$ can be constructed from $g_1(x,Q^2)$:[8]

$$g_2(x,Q^2)^{\mathrm{WW}}=-g_1(x,Q^2)+\int_x^1\frac{g_1(y,Q^2)}{y}dy\ . \tag{12}$$

In general, we write

$$g_2(x,Q^2)=g_2(x,Q^2)^{\mathrm{WW}}+\bar{g}_2(x,Q^2)\ , \tag{13}$$

where $\bar{g}_2(x,Q^2)$ represents the twist-3 contributions only:

$$\int_0^1 x^n \bar{g}_2(x,Q^2)dx=\frac{n}{2(n+1)}\sum_i d_i^n(\mu^2)F_{3,i}^n(Q^2,\mu^2),\quad n=2,4,\ldots\ . \tag{14}$$

Now we consider candidates for twist-2 operators in QCD. For definiteness we assume three flavor quarks: u, d, and s. For fixed n there are eight flavor-octet twist-2 operators involving quark fields only:

$$O_{2,k}^{\sigma\mu_1\mu_2\cdots\mu_n}=i^n\mathcal{S}_1\{\bar{\psi}(0)\gamma^\sigma\gamma^5 D^{\mu_1}D^{\mu_2}\cdots D^{\mu_n}\lambda_k\psi(0)\}-(\text{traces})\ , \tag{15}$$

where λ_k $(k=1,\ldots,8)$ are SU(3) Gell-Mann matrices and $D^\mu=\partial^\mu+igA^\mu$ is the covariant derivative. There are two flavor-singlet twist-2 operators. One is the same as Eq. (15) with λ_k replaced by the unit matrix. The other involves gluon fields:

$$O_{2,g}^{\sigma\mu_1\mu_2\cdots\mu_n}=i^{n-1}\mathcal{S}_1\{\tilde{G}^{\sigma\lambda}D^{\mu_1}D^{\mu_2}\cdots D^{\mu_{n-1}}G_\lambda^{\mu_n}\}\ . \tag{16}$$

The number of twist-3 operators is an increasing function of n. For simplicity we neglect the flavor structure of operators in the following discussion since it is the same as for the twist-2 operators. The following two sets of operators are all twist 3:

$$R_l^{\sigma\mu_1\cdots\mu_n}=\frac{i^{n-3}}{4}g\mathcal{S}\{\bar{\psi}(0)D^{\mu_1}\cdots D^{\mu_{l-1}}G^{\sigma\mu_l}D^{\mu_{l+1}}\cdots D^{\mu_{n-1}}\gamma^{\mu_n}\gamma^5\psi(0)\}-(\text{traces})\ , \tag{17}$$

$$S_l^{\sigma\mu_1\cdots\mu_n}=\frac{i^{n-2}}{4}g\mathcal{S}\{\bar{\psi}(0)D^{\mu_1}\cdots D^{\mu_{l-1}}\tilde{G}^{\sigma\mu_l}D^{\mu_{l+1}}\cdots D^{\mu_{n-1}}\gamma^{\mu_n}\psi(0)\}-(\text{traces})\ . \tag{18}$$

However, not all of them enter the operator-product expansion [Eq. (3)] independently. The current product is even under charge conjugation, whereas operators R_l and S_l are not charge-conjugation eigenstates. It is easy to show that the combinations R_l-R_{n-l} and S_l+S_{n-l} are charge-conjugation even. The former are $n/2-1$ in number, and the latter are $n/2$, giving a total of $n-1$ operators. In addition, there is a twist-3 explicitly quark-mass-dependent operator

$$O_m^{\sigma\mu_1\mu_2\cdots\mu_n}=\frac{i^{n+1}}{4}\mathcal{S}\{\bar{\psi}(0)m_q[\gamma^\sigma,\gamma^{\mu_1}]\gamma^5 D^{\mu_2}\cdots D^{\mu_n}\psi(0)\}\ , \tag{19}$$

where m_q is the quark-mass matrix. Therefore, for the flavor nonsinglet there are a total of n linearly independent operators which enter the operator-product expansion. For the flavor singlet there are additional operators constructed from the gluon fields alone.

The twist-3 operators are a direct manifestation of quark-gluon interaction and quark masses. The main effect of the gluons inside of the hadron is, of course, confining quarks. Therefore, the twist-3 contributions should be large in any realistic nucleon model with confinement. Their effect on $g_2(x,Q^2)$ cannot *a priori* be neglected in comparison to the twist-2 operators. Our bag-model calculation in Sec. IV will be an explicit example of a large twist-3 contribution.

III. $g_1(x)$ AND $g_2(x)$ IN THE IMPULSE APPROXIMATION: MODELS AND SUM RULES

The operator-product description outlined in the previous section is rather cumbersome for the treatment of simple models. In particular, if one is not interested in QCD radiative corrections, it is convenient to sum the OPE to obtain a light-cone description of deep-inelastic structure functions. Quark and gluon distribution functions are then related to the Fourier transforms of matrix elements of bilocal operators along the light cone. At any stage radiative corrections can be restored by taking moments of the distributions, which project out local operators or, equivalently, by using the distributions as initial data in Altarelli-Parisi evolution equations.[19]

In this section we present the light-cone treatment of the structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$. We have two objectives in mind: First, we will use this formalism to calculate g_1 and g_2 in the bag model; second, this is a particularly easy way to see what might go wrong with the derivation of the Burkhardt-Cottingham sum rule.

To calculate $g_1(x,Q^2)$ and $g_2(x,Q^2)$ we take the Bjorken limit of Eq. (1) and use the light-cone expansion

$$[J_\mu(\xi),J_\nu(0)]=\bar{\psi}(\xi)\mathcal{Q}^2\gamma_\mu S(\xi)\gamma_\nu\psi(0)-\bar{\psi}(0)\mathcal{Q}^2\gamma_\nu S(-\xi)\gamma_\mu\psi(\xi)+\cdots\ , \tag{20}$$

where $\mathcal{Q}$ is the quark charge matrix and the ellipses denote terms less singular on the light cone. $S(\xi)$ is the free-field causal function

$$\boxed{S(\xi)=\{\psi(\xi),\bar{\psi}(0)\}=(i\partial\!\!\!/+m)\Delta(\xi^2,m)=\frac{\partial\!\!\!/}{2\pi}\delta(\xi^2)\epsilon(\xi^0)+\cdots\ .} \tag{21}$$

NOTE only 1 functi $\Delta(\xi)$

Substituting for S, performing some γ-matrix algebra, and isolating terms antisymmetric in $\mu\leftrightarrow\nu$, we find

$$W^A_{\mu\nu}=i\epsilon_{\mu\nu\lambda\sigma}q^\lambda\left[-\frac{i}{8\pi}\right]\int d^4\xi\, e^{iq\cdot\xi}\delta(\xi^2)\epsilon(\xi^0)\langle PS|\bar\psi(\xi)Q^2\gamma^\sigma\gamma_5\psi(0)+\bar\psi(0)Q^2\gamma^\sigma\gamma_5\psi(\xi)|PS\rangle\ . \tag{22}$$

In Eq. (22) we have ignored terms which vanish as $Q^2\to\infty$. This result could equally well be derived from the OPE since all the coefficient functions are simple if radiative corrections are ignored. In particular, all twist-3 operators can be summed to give a simple operator

$$O_3^{\sigma\mu_1\mu_2\cdots\mu_n}=i^n\mathcal{S}\mathcal{A}\{\bar\psi(0)Q^2\gamma^\sigma\gamma^5D^{\mu_1}D^{\mu_2}\cdots D^{\mu_n}\psi(0)\}-(\text{traces})\ . \tag{23}$$

Combining it with the twist-2 operator, we immediately arrive at Eq. (22).

As written, Eq. (22) is not gauge invariant. This is because we have suppressed factors of the form

$$I(\Gamma)\equiv\mathcal{P}\exp\left[ig\int_\Gamma A_\mu(\lambda)d\lambda^\mu\right]\ , \tag{24}$$

where Γ is a path connecting 0 to ξ^μ and $\mathcal{P}$ denotes path ordering of the exponential. This term is important in the calculation of the twist-3 contribution to $g_2(x,Q^2)$, but not in the largely kinematic calculation of this section, and so we will not write it explicitly.

The matrix elment in Eq. (22) may be parametrized in terms of three invariant functions of ξ^2 and $\xi\cdot P$:

$$\begin{aligned}\langle PS|\bar\psi(\xi)Q^2\gamma^\sigma\gamma_5\psi(0)+(0\leftrightarrow\xi^\mu)|PS\rangle\\ =S^\sigma f(\xi^2,\xi\cdot P)-iP^\sigma\xi\cdot S\, g(\xi^2,\xi\cdot p)\\ +\xi^\sigma\xi\cdot S\, h(\xi^2,\xi\cdot p)\ .\end{aligned} \tag{25}$$

Comparison with the OPE analysis of the previous section shows that f, g, and h are finite at $\xi^2=0$ (modulo logarithms generated by radiative corrections). In light of the $\delta(\xi^2)$ in Eq. (22), only $f(0,\xi\cdot P)\equiv f(\xi\cdot P)$, $g(0,\xi\cdot P)\equiv g(\xi\cdot P)$, and $h(0,\xi\cdot P)\equiv h(\xi\cdot P)$ will contribute to $W^A_{\mu\nu}$. A simple dimensional analysis shows that $h(\xi\cdot P)$ does not contribute to leading order in Q^2 to either $g_1(x,Q^2)$ or $g_2(x,Q^2)$, and so we ignore it henceforth.

To proceed, we introduce Fourier transforms for $f(\xi\cdot P)$ and $g(\xi\cdot P)$:

$$\begin{aligned}f(\xi\cdot P)&=\int d\alpha\, e^{i\alpha\xi\cdot P}\bar f(\alpha)\ ,\\ g(\xi\cdot P)&=\int d\alpha\, e^{i\alpha\xi\cdot P}\bar g(\alpha)\ ,\end{aligned} \tag{26}$$

with the proviso that $\bar f(\alpha)$ and $\bar g(\alpha)$ are to be understood as distributions at $\alpha=0$. Substituting Eqs. (25) and (26) into $W^A_{\mu\nu}$, we find

$$W^A_{\mu\nu}=i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda}{\nu}\left[S^\sigma\frac{\bar f(x)}{4}+P^\sigma\frac{q\cdot S}{\nu}\frac{\bar g'(x)}{2}\right]\ , \tag{27}$$

and comparing with Eq. (2), we conclude

$$g_1(x)+g_2(x)=\tfrac14\bar f(x)\ , \tag{28}$$

$$g_2(x)=\tfrac12\bar g'(x)\ . \tag{29}$$

With these results in hand we investigate the Burkhardt-Cottingham sum rule[7]

$$\int_0^1 g_2(x)dx=0\ . \tag{30}$$

Inverting Eq. (26) and differentiating,

$$\bar g'(x)=-\frac{i}{2\pi}\int_{-\infty}^{\infty}d\lambda\, e^{-ix\lambda}\lambda g(\lambda)\ , \tag{31}$$

where $\lambda\equiv\xi\cdot P$. We can calculate $\int dx\, g_2(x)$ from Eqs. (29) and (31) [remembering $g_2(x)=g_2(-x)$], and *provided we can exchange the x integration with the* λ *integration,* we obtain

$$\int_0^1 dx\, g_2(x)=\int_{-\infty}^{\infty}d\lambda\,\lambda\delta(\lambda)g(\lambda)=0\ , \tag{32}$$

since $g(\lambda)$ is expected to be regular at $\lambda=0$ (on the basis of the OPE or the structure of equal-time commutators). The only way the sum rule can fail is if the x and λ integrals cannot be exchanged. This will happen in the following circumstances

(1) If $\bar g'(x)$ is so singular that $\int_0^1 dx\, g_2(x)$ does not exist. This occurs if $g(\lambda)\sim 1/\lambda^p$ with $p<1$ as $\lambda\to\infty$.

(2) If $g(\lambda)\sim 1/\lambda$ as $\lambda\to\infty$, then $g_2(x)$ has a δ function at $x=0$, and while Eq. (31) still holds formally, the δ function is not detectable experimentally, and so the measured $g_2(x)$ would appear to violate the sum rule.

To verify that this may indeed occur, suppose

$$g(\lambda)=\frac{c\lambda}{\lambda^2+a^2}\ . \tag{33}$$

[The $g_2(x)$ arising from this $g(\lambda)$ does not vanish for $x>1$. This shortcoming is inessential: $g(\lambda)$ can be altered so that the support $(x\le 1)$ restriction can be satisfied, leaving the long-range ($\lambda\to\infty$) behavior intact.] Then an easy calculation gives

$$g_2(x)=\tfrac14 cae^{-a|x|}-\tfrac12 c\delta(x)\ , \tag{34}$$

which formally satisfies the sum rule. Note that the success or failure of the Burkhardt-Cottingham sum rule depends on the *long-range* behavior of $g(\xi\cdot P)$ and cannot be determined by examining short-distance properties of the current product [Eq. (3)] alone. Also note that such singular long-range behavior of light-cone correlation functions has been known for many years.[20]

Note finally that subtleties in the g_2 sum rule do not spoil the important sum rule obeyed by $g_1(x)$: Integrating Eq. (28),

$$\begin{aligned}\int_{-1}^1 dx\,[g_1(x)+g_2(x)]&=\int_{-\infty}^{\infty}d\lambda\,\delta(\lambda)f(\lambda)\\ &=f(0)\ ,\end{aligned} \tag{35}$$

and since $\int dx\, g_2(x)=0$ regardless of subtleties such as δ functions, we have

$$\int_0^1 dx\, g_1(x)=f(0)\ , \tag{36}$$

which is equivalent to the Bjorken or Ellis-Jaffe sum rule, depending on the target.

The long-range behavior which invalidates the Burkhardt-Cottingham sum rule does not occur in simple models. Equation (30) is known to be satisfied in QCD perturbation theory through order g^2.[14] It is also manifestly valid in models such as the bag model in which correlation functions have finite support in space-time. In these models Fourier transforms such as $\bar{f}(\alpha)$ and $\bar{g}(\alpha)$ are *analytic* functions of α and cannot behave like Eq. (15). Of course, such models also fail to produce ordinary Regge behavior, which requires nonanalytic behavior in structure functions as $x\to 0$.

IV. BAG-MODEL CALCULATION

As we discussed in the previous sections, the structure function $g_2(x)$ receives contributions from both twist-2 and -3 operators, and the latter are related to the quark-gluon interactions and quark masses. In order to estimate the importance of the twist-3 operator on $g_2(x)$, which determines how interesting $g_2(x)$ is, we evaluate $g_2(x)$ in the bag model. The bag boundary simulates the confinement effects which arise from quark-gluon interactions. The twist-3 effects are therefore due to the bag boundary. As a result, the measurement of $g_2(x)$ will provide nontrivial constraints on the nucleon models. We do not expect this calculation to provide a quantitatively accurate prediction for $\bar{g}_2(x)$. Instead, we view this as a toy model or cartoon, lacking Regge behavior and the proper $x\to 1$ limit, but giving a rough estimate of the size of $\bar{g}_2(x)$.

The impulse-approximation formulas for $g_1(x)$ and $g_2(x)$ in Eqs. (28) and (29) are determined covariantly. However, for model studies it is easier to calculate them in a particular frame. For simplicity, let us choose the nucleon rest frame where $P_\mu=(M,0,0,0)$. To calculate $g_1(x)$ we polarize the nucleon in the z direction (defined to be the opposite direction of the virtual-photon three-momentum); then $S^z_\mu(0,0,0,M)$. Substituting S and P into the general decomposition for $W^A_{\mu\nu}$ in Eq. (2) and the impulse-approximation limit [Eq. (22)], we identify

$$g_1(x)=-\frac{i\sqrt{2}q_3}{8\pi^2}\int d^4\xi\, e^{iq\cdot\xi}\delta(\xi^2)\epsilon(\xi^0) \times\langle PS^z|\bar{\psi}(\xi)Q^2\gamma^+\gamma^5\psi(0) +\bar{\psi}(0)Q^2\gamma^+\gamma^5\psi(\xi)|PS^z\rangle\ , \tag{37}$$

where q_3 is the photon three-momentum. After integrating over ξ_+ and $\xi_\perp$ in Eq. (37), we have

$$g_1(x)=\frac{1}{8\pi}\int d\xi_-e^{iq_+\xi_-}\langle PS^z|\bar{\psi}(\xi_-)Q^2\gamma^+\gamma^5\psi(0) +\bar{\psi}(0)Q^2\gamma^+\gamma^5\psi(\xi_-)|PS^z\rangle\ , \tag{38}$$

where $q_+=-Mx/\sqrt{2}$. Substituting in the bag wave function from Ref. 21, we have

$$g_1(x)=\frac{5\omega_n RM}{36\pi j_0^2(\omega_n)(\omega_n-1)} \times\int_{|y_{\rm min}|}^\infty y\,dy\left\{t_0^2+2t_0t_1\frac{y_{\rm min}}{y} +t_1^2\left[2\left[\frac{y_{\rm min}}{y}\right]^2-1\right]\right\} +(x\to -x)\ , \tag{39}$$

where ω_n is the nth root of the bag eigenequation,

$$\tan\omega_n=-\frac{\omega_n}{\omega_n-1}\ , \tag{40}$$

and $y_{\rm min}=|xRM-\omega_n|$. R and M are bag radius and nucleon mass, respectively. In our calculation we fixed the dimensionless parameter $RM=4\omega_n$. The functions t_0 and t_1 are defined through

$$t_l(\omega_n,y)=\int_0^1 du\, u^2 j_l(u\omega_n)j_l(uy)\ . \tag{41}$$

We used the SU(6) wave function for the proton. The result of $g_1(x)$ is plotted as solid curve in Fig. 1.

Next, consider the nucleon to be polarized perpendicular to the photon momentum, say, in the x direction; then $S^x_\mu=(0,M,0,0)$. In this case we find the transverse structure function

$$g_1(x)+g_2(x)=-\frac{iq_3}{8\pi^2}\int d^4\xi\, e^{iq\cdot\xi}\delta(\xi^2)\epsilon(\xi^0) \times\langle PS^x|\bar{\psi}(\xi)Q^2\gamma^1\gamma^5\psi(0) +\bar{\psi}(0)Q^2\gamma^1\gamma^5\psi(\xi)|PS^x\rangle\ . \tag{42}$$

After integrating over ξ_+ and $\xi_\perp$, we have

$$g_1(x)+g_2(x) =\frac{1}{8\pi\sqrt{2}}\int d\xi_-e^{iq_+\xi_-} \times\langle PS^x|\bar{\psi}(\xi_-)Q^2\gamma^1\gamma^5\psi(0) +\bar{\psi}(0)Q^2\gamma^1\gamma^5\psi(\xi_-)|PS^x\rangle\ . \tag{43}$$

Again, after substituting in the bag wave function, it becomes

$$g_1(x)+g_2(x)=\frac{5\omega_n MR}{36\pi j_0(\omega_n)^2(\omega_n-1)} \times\int_{|y_{\rm min}|}^\infty y\,dy\left[t_0^2-t_1^2\left[\frac{y_{\rm min}}{y}\right]^2\right] +(x\to -x)\ . \tag{44}$$

Now the structure function $g_2(x)$ can be obtained by subtracting out the known $g_1(x)$. It is shown in Fig. 2 as a

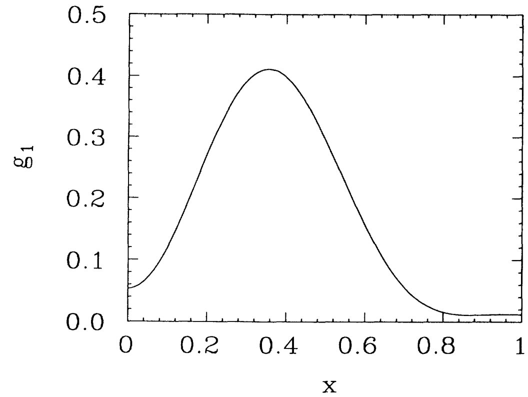

FIG. 1. Proton's spin-dependent structure function $g_1(x)$ in the bag model.

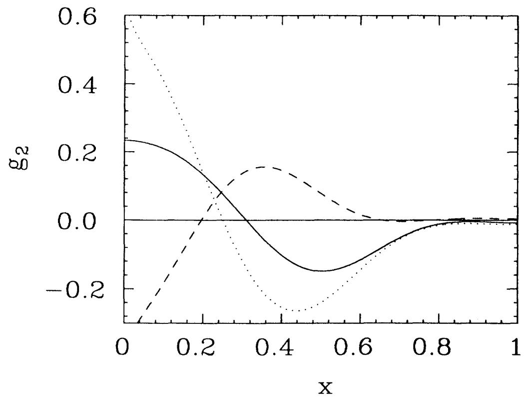

FIG. 2. Proton's spin-dependent structure function $g_2(x)$ in the bag model. The dotted line is the twist-2 contribution, the dashed line is twist 3, and the solid line is the sum of the two.

solid curve.

The $g_2(x)$ calculated from Eq. (44) contains contributions from both twist-2 and -3 operators. The twist-2 part can be calculated from Eq. (12) once $g_1(x)$ is known and is shown as a dotted curve in Fig. 2. The twist-3 part is, therefore, just the difference between the solid and dotted curves. We show it as a dashed curve in the same figure. The result clearly shows that, at least in the bag model, the twist-3 contribution to $g_2(x)$ is not small. It is interesting, however, that the twist-3 and -2 contributions are of opposite sign, and the net $g_2(x)$ bears the sign of a twist-2 part.

Following Ref. 15, the twist-3 contribution can also be calculated from a set of operators which explicitly depends on the interactions. Using the equation of motion for quarks in the bag,

$$i\not{\partial}\psi(\xi)=\delta(\xi-R)\psi(\xi)\ , \tag{45}$$

we transform the twist-3 operator in Eq. (23) into

$$O_3^{\sigma\mu_1\mu_2\cdots\mu_n}=\frac{i^n}{2}\mathcal{S}[\bar{\psi}(0)\sigma^{\sigma\mu_1}\gamma^5\partial^{\mu_2}\cdots\partial^{\mu_n}\delta_S\psi(0)+\bar{\psi}(0)\delta_S\sigma^{\sigma\mu_1}\gamma^5\partial^{\mu_2}\cdots\partial^{\mu_n}\psi(0)]\ , \tag{46}$$

where we used the shorthand $\delta_S=\delta(\xi-R)$ and $\sigma_{\sigma\mu_1}=(i/2)[\gamma_\sigma,\gamma_{\mu_1}]$. If there is no bag boundary, this operator is identically zero. We define a function $U(x)$ through

$$\begin{aligned}(S_\perp P_+ - P_\perp S_+)xU(x)&=\frac{i}{8\pi}P^+\int d\xi_- e^{iq_+\xi_-}\\&\quad\times\langle PS|\bar{\psi}(\xi_-)\sigma_{\perp+}\gamma^5\delta_S\psi(0)\\&\qquad-\bar{\psi}(0)\delta_S\sigma_{\perp+}\gamma^5\psi(\xi_-)|PS\rangle\ .\end{aligned} \tag{47}$$

Then it is easy to show

$$\int_0^1 U(x)x^n dx=\tfrac{1}{2}d_n\ . \tag{48}$$

Therefore, the twist-3 contribution to $g_2(x)$ is

$$\bar{g}_2(x)=U(x)-\int_x^1\frac{U(y)}{y}dy\ . \tag{49}$$

In the bag model we obtain

$$U(x)=\frac{5\omega_n}{36\pi j_0^2(\omega_n)(\omega_n-1)x}\int_{y_{\min}}^{\infty}y\,dy\left[t_0j_0(\omega_n)j_0(y)+\frac{y_{\min}}{y}[t_0j_1(\omega_n)j_1(y)+t_1j_0(\omega_n)j_0(y)]+\left[\frac{y_{\min}}{y}\right]^2 t_1j_1(\omega_n)j_1(y)\right]+(x\to -x)\ . \tag{50}$$

The result of Eq. (50) agrees with the dashed curve in Fig. 2. This confirms the physical interpretation of $\bar{g}_2(x)$ and is a check on the algebra.

There is another twist-3 operator [Eq. (19)] if the quarks are massive. The contribution from the mass operator is expected to be small for the valence quarks in the nucleon because the up and down quarks are nearly massless. However, the strange-quark mass is of the same order as $\Lambda_{\rm QCD}$ and cannot be neglected. To appreciate the effect of quark mass, we calculated $g_2(x)$ for a fictitious proton with mass 1650 MeV and up- and down-quark mass 150 MeV in the bag model. The formula for the mass contribution can be obtained from $U(x)$ by replacing δ_S by m_q. The result is shown in Fig. 3. As before, the dashed curve shows the total twist-3 contributions. The bag-surface contribution is shown by the heavy-dotted curve, whereas the mass contribution is shown by the dot-dashed curve. Both contributions add constructively in most of the x range.

Actually, the spin-dependent structure functions $g_1(x)$ and $g_2(x)$ in the bag model were calculated sometime ago in Ref. 22. The result for $g_2(x)$, however, is somewhat different from ours. To understand what the difference is, we rewrite Eq. (22) as

$$W^A_{\mu\nu}=i\epsilon_{\mu\nu\lambda\sigma}q^\lambda A^\sigma \ , \tag{51}$$

where A^σ is a four-vector:

$$A^\sigma=-\frac{i}{8\pi^2}\int d^4\xi\, e^{iq\cdot\xi}\delta(\xi^2)\epsilon(\xi^0) \times\langle PS|\bar\psi(\xi)Q^2\gamma^\sigma\gamma^5\psi(0) +\bar\psi(0)Q^2\gamma^\sigma\gamma^5\psi(\xi)|PS\rangle \ . \tag{52}$$

In general, A^σ can be decomposed as

$$A^\sigma=\frac{g_1+g_2}{\nu}S^\sigma-\frac{g_2}{\nu^2}(q\cdot S)P^\sigma-\frac{g_3}{\nu^3}M^2(q\cdot S)q^\sigma \ . \tag{53}$$

The term we have labeled g_3 does not contribute to either measurable structure function. If g_3 were omitted and one took a positive or negative component of A^σ and identified it with the standard decomposition in Eq. (2), one would possibly pick up an extra term for $g_1(x)$ and $g_2(x)$. Hughes[22] chose the $\sigma=$negative component with the proton spin in the z direction. Without subtracting $g_3(x)$, he arrived at

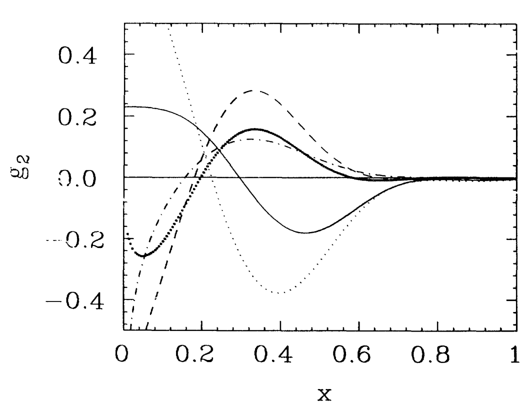

FIG. 3. $g_2(x)$ for a fictitious proton with mass 1650 MeV and the up- and down-quark masses 150 MeV. The heavy-dotted line is the twist-3 bag-surface contribution, and the dot-dashed line is the twist-3 quark-mass contribution. The meanings of the other curves are the same as in Fig. 2.

$$g_2^H(x)=-\frac{1}{8\pi\sqrt{2}}\int d\xi_- e^{i\xi_-q_+} \times\langle PS|\bar\psi(\xi_-)Q^2\gamma^0\gamma^5\psi(0) +\bar\psi(0)Q^2\gamma^0\gamma^5\psi(\xi_-)|PS\rangle \ , \tag{54}$$

which is wrong. In our calculation we have deliberately chosen the nucleon polarization to avoid the unphysical $g_3(x)$ contribution.

As a final check on the calculation of $g_1(x)$ and $g_2(x)$, we start with the sum rules (10) and (11). The simplest expressions are obtained by choosing the tensor indices in Eqs. (8) and (9) and the nucleon polarization so that the trace terms are minimized or vanish. The result coincides with our expressions for $g_1(x)$ and $g_2(x)$. Other index choices require trace terms which show up in the final expressions. If the choice is made as in Ref. 22, a trace term is needed in Eq. (51) to cancel out the $g_3(x)$ contribution. We followed that path and obtained the same result as with Eq. (44).

V. REMARKS ABOUT THE RELATION TO PARTON MODELS

The nucleon's deep-inelastic structure functions can usually be interpreted in terms of Feynman's parton model.[23] For the unpolarized structure functions $F_1(x)$ and $F_2(x)$, the naive parton picture, in which the on-shell partons move collinearly with the nucleon in the infinite-momentum frame, was shown to be equivalent to the impulse approximation or OPE in the simplest version. We will argue in this section that such a *simple* parton picture breaks down for the transverse spin phenomena. Our argument is based on reviewing various parton models for $g_2(x)$ and comparing them to the OPE analysis. In the end we come to the conclusion that the parton-model realization of the impulse approximation is complicated; in particular, the parton's transverse momentum, off-shell partons, or alternatively the quark-gluon-coupled densities must be introduced. Therefore, the parton-model language loses its advantage of simplicity in this case.

The OPE analysis in Sec. II shows that there are three sources of contributions to $g_2(x)$:

$$g_2(x)=g_2^{\rm WW}(x)+g_2^{\rm int}(x)+g_2^{m}(x) \ . \tag{55}$$

The first term in the right-hand side of Eq. (55) comes from the twist-2 operators in Eq. (15), the second from the twist-3 operators responsible for the quark-gluon interactions, and the third from the twist-3 mass operator in Eq. (19). Now we turn to examine how each term is contributed by parton models.

Let us first consider deep-inelastic scattering from a *free* quark with momentum p^μ, spin s^μ, and mass m_q. Denote $W_{\mu\nu}$ in this simple case as $w_{\mu\nu}$. An elementary calculation yields

$$w^A_{\mu\nu}=\tfrac{1}{2}\bar{u}(p,s)[\gamma_\mu(\not{p}+\not{q}+m_q)\gamma_\nu-(\mu\leftrightarrow\nu)]$$
$$\times u(p,s)\delta((p+q)^2+m_q^2)$$
$$=i\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma\frac{1}{p\cdot q}\delta\left[1-\frac{Q^2}{2p\cdot q}\right], \tag{56}$$

from which we read off

$$g_1^0(x)=\delta(x-1),$$
$$g_2^0(x)=0. \tag{57}$$

In deriving Eqs. (57) we used $(\not{p}-m_q)u(p,s)=0$ and $u(p,s)\gamma^\sigma\gamma^5u(p,s)=2s^\sigma$. This is a very surprising result for a free particle for which one would think the Wandzura-Wilczek relation [Eq. (12)] holds. If Eqs. (57) were recast in the operator language of Sec. II, it would be found that the twist-2 contribution to $g_2(x)$ given by Eq. (12) is exactly canceled by the contribution of the mass operator $O_m^{\sigma\mu_1\mu_2\cdots\mu_n}$ [Eq. (19)], yielding $g_2^0=0$. The explicit factor of m_q in $O_m^{\sigma\mu_1\mu_2\cdots\mu_n}$ is canceled by a factor $1/m_q$ in the free-quark matrix element of the operator.

Now let us construct a parton model for the nucleon. The simplest picture would be that the three massive quarks, each of which with one-third mass of the nucleon, rest inside of it with no mutual interactions. Then $g_1(x)$ and $g_2(x)$ of the nucleon would be just the sum of these of free quarks:

$$g_1(x)=c\delta(x-\tfrac{1}{3}),$$
$$g_2(x)=0, \tag{58}$$

where c is just a constant. The result for $g_1(x)$ is completely reasonable, but for $g_2(x)$ is somewhat suspicious. For a confined quark the $1/m_q$ infrared singularity appearing in the free-quark matrix element of the quark-mass operator will be replaced by $1/\Lambda_{\rm QCD}$. The matrix element of the mass operator between the nucleon ground state will vanish smoothly as $m_q\to 0$, as it should. Therefore, we conclude that $g_2(x)$ in the parton model is plaqued by an anomalous contribution from the quark-mass operator, which persists in the massless limit.

A more realistic parton picture for the nucleon would be to add a nontrivial quark momentum distribution. Then the δ-function distribution for $g_1(x)$ would be smoothed out, and $g_2(x)$ might be nonzero. However, we suspect that the result for $g_2(x)$ would still be unreliable on account of the mass-operator contribution. If it were possible to eliminate the quark-mass-operator contribution while keeping quarks on shell, then one would expect to obtain the Wandzura-Wilczek result for $g_2(x)$. A covariant parton model was studied in Ref. 24 in which the Wandzura-Wilczek relation between $g_1(x)$ and $g_2(x)$ was derived. However, it is not clear in their formalism how the quark-mass-operator contribution was discarded.

At this point we would like to comment on the claim that $g_T=g_1(x)+g_2(x)$ is small in the parton model.[25,26] This claim was generated from Feynman's derivation of the parton model for transverse spin, in which he arrived at

$$g_T(x)=\sum_i e_i^2\frac{m_q}{2xM}[k_i^+(x)-k_i^-(x)], \tag{59}$$

with the parton distribution function $k_i^+(x)$ [$k_i^-(x)$] representing the number of flavor-i quarks with polarization in the same [opposite] direction of the nucleon spin and with x fraction of the nucleon momentum. If the massless limit is taken, one indeed obtains $g_T=0$. However, in this model the zero-mass limit is inconsistent with the nucleon's rest mass. The consistent way is to set $m_q=xM$, as Feynman did in deriving Eq. (33.15) in his book. If we apply Eq. (59) to the nucleon's rest frame, then rotational invariance implies $g_2(x)=0$, consistent with the naive result in Eq. (58).

Therefore, parton models with on-shell partons at best reproduce the correct twist-2 contribution if one knows how to get rid of the singular contribution of the quark-mass operator. All the twist-3 effects associated with the gluon-quark mixing operators (17) and (18) are missing from the parton picture. This is because the matrix elements of these operators between the free-quark states is proportional to the antisymmetric tensor $s_\mu k_\nu - s_\nu k_\mu$, which vanishes for massless on-shell quarks. Indeed, this was the argument used by Wandzura and Wilczek to conclude that the twist-3 contributions to g_2 are small.[8]

Apparently, the essential ingredient missing from various parton models is their off-shell nature. The quarks inside of the nucleon are off shell because of confinement. For an off-shell quark, s_μ does not have to be parallel to k_μ even if the mass of the quark is zero. The twist-3 contributions to $g_2(x)$ are directly proportional to the off-shell quark k^2. The off-shell quarks are also welcome because the contribution of the mass operator is naturally suppressed by introducing the confinement scale. The large off-shell nature of the quark in the bag is directly responsible for the large twist-3 effect shown in Figs. 2 and 3. An alternative way is to introduce gluon partons. Then the matrix elements of the twist-3 operators are related to the coupled quark-gluon distributions.[17,18] However, both pictures are complicated to develop.

Therefore, a complete account of the transverse spin-structure function requires studies of off-shell partons as well as the transverse momentum. Both, of course, are related to quark-gluon interactions. Simple as the impulse approximation [Eq. (42)] looks, the physics in terms of the parton picture is much more complicated than that of the spin-averaged structure functions.

VI. SUMMARY

We have discussed the structure function $g_2(x,Q^2)$ in the leading order in the Bjorken limit. We calculated the twist-3 contributions in the bag model and found they are non-negligible compared to twist 2, which can be determined from the measurement of $g_1(x)$. Therefore, the future measurement of $g_2(x)$ can be used to study twist-3 operators which are intimately related to the quark-gluon

$(p+k)^2=\ell^2 \quad k^2+Q^2\left(\frac{1}{x}-1\right)=\ell^2$

interactions.

We did not discuss the Q^2 evolution of the $g_2(x,Q^2)$ when higher-order QCD corrections are included. If we accept the point of view that model calculations for the structure functions $g_1(x,\mu^2)$ and $g_2(x,\mu^2)$ are defined for some μ^2 characteristic of hadrons, we must calculate all the twist-3 quark-gluon matrix elements at this scale in order to calculate the full $g_2(x,Q^2)$ at experimentally interesting scales. Therefore, we need a nucleon model with explicit gluonic degrees of freedom which is not presently available.

Finally, the study of transverse spin physics in the parton model is quite complicated. It requires the consistent inclusion of the parton transverse-momentum distribution as well as the off-shell partons. Both effects are the same order of magnitude as the confinement scale. Furthermore, the singular contribution from the mass operator must be carefully treated.

Note added in proof. After finishing this work, we learned that the error in Ref. 22 was also pointed out by J. Bartelski, Phys. Rev. D **20**, 1229 (1979).

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[1]V. W. Hughes and J. Kuti, Annu. Rev. Nucl. Part. Sci. **33**, 611 (1983).

[2]M. J. Alguard *et al.*, Phys. Rev. Lett. **37**, 1261 (1978); **41**, 70 (1978); G. Baum *et al.*, *ibid.* **51**, 1135 (1981).

[3]J. Ashman *et al.*, Phys. Lett. B **206**, 364 (1988).

[4]For a review, see R. L. Jaffe and A. Manohar, Nucl. Phys. **B337**, 509 (1990).

[5]The HERMES and SMC Collaborations.

[6]J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

[7]H. Burkhardt and W. N. Cottingham, Ann. Phys. (N.Y.) **56**, 453 (1970).

[8]W. Wandzura and F. Wilczek, Phys. Lett. B **172**, 195 (1977).

[9]A. J. G. Hey and J. E. Mandula, Phys. Rev. D **5**, 2610 (1972).

[10]K. Sasaki, Prog. Theor. Phys. **54**, 1816 (1975).

[11]M. A. Ahmed and G. G. Ross, Phys. Lett. **56B**, 385 (1975).

[12]M. A. Ahmed and G. G. Ross, Nucl. Phys. **B111**, 441 (1976).

[13]J. Kodaira, S. Matsuda, K. Sasaki, and T. Uematsu, Nucl. Phys. **B159**, 99 (1979).

[14]I. Antoniadis and C. Kounnas, Phys. Rev. D **24**, 505 (1981).

[15]E. V. Shuryak and A. I. Vainshtein, Nucl. Phys. **B201**, 141 (1982).

[16]A. P. Bukhvostov, E. A. Kuraev, and L. N. Lipatov, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 406 (1983) [JETP Lett. **37**, 482 (1983)].

[17]A. V. Efremov and O. V. Teryaev, Yad. Fiz. **39**, 1517 (1984) [Sov. J. Nucl. Phys. **39**, 962 (1984)].

[18]P. G. Ratcliffe, Nucl. Phys. **B264**, 493 (1986).

[19]G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).

[20]D. Corrigan, Phys. Rev. D **4**, 1053 (1971); A. Zee, *ibid.* **3**, 2432 (1971); D. J. Broadhurst, J. Gunion, and R. L. Jaffe, Ann. Phys. (N.Y.) **81**, 88 (1973).

[21]R. L. Jaffe, Phys. Rev. D **11**, 1953 (1975); T. H. Hansson and R. L. Jaffe, *ibid.* **28**, 882 (1983).

[22]R. J. Hughes, Phys. Rev. D **16**, 622 (1977).

[23]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, New York, 1972).

[24]J. D. Jackson, G. G. Ross, and R. G. Roberts, Phys. Lett. B **226**, 159 (1989).

[25]G. L. Kane, J. Pumplin, and W. Repko, Phys. Rev. Lett. **41**, 1689 (1978).

[26]B. L. Ioffe, V. A. Khoze, and L. N. Lipatov, *Hard Processes* (North-Holland, Amsterdam, 1984), Vol. 1.