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Bag model predictions for polarized structure functions and their Q^2-evolutions

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Abstract. It is shown that modified versions of the MIT bag model can fairly describe non-singlet pieces of experimentally known structure functions if the Q^2-evolution equations are trusted down to the model scale. Further on we utilize these models to study the presently unmeasured transversely polarized structure functions g_2 and h_1. Using recently developed approximate evolution equations we focus on the relevance of the twist-3 contribution to g_2. Finally, a simple 'first guess'-model for the 'transversity' distribution h_1 is presented.

1 Introduction

QCD works perfectly well in relating structure functions at different scales Q^2 perturbatively but it fails in calculating some input distribution from first principles because of the poor understanding of nonperturbative QCD in the confinement domain. Special nonperturbative techniques must be invented to fill this gap. Apart from lattice QCD, regarded as the favorite technique but requiring immense numerical calculations and therefore has not been widely applied yet, are bag models [1] an interesting tool for calculating structure functions because they incorporate the confinement property in a simple manner [1]. Introducing phenomenological refinements into the basic model [1, 2] such as the 'one gluon exchange' [3] and the Peierls-Yoccoz projection [4], as recently done in [5], one can overcome, however, some of the major disadvantages of the original MIT model [6]. But, nevertheless, one is plagued with the problem of fixing the introduced model parameters which are not calculable yet, i.e. how to relate the model to experimental data. The structure functions obtained in the bag represent the leading twist non-singlet part of the physical structure function under consideration at a very low bound-state scale $Q^2=\mu^2_{\text{bag}}=\mathcal{O}(\Lambda^2_{\text{QCD}})$, which implies that they are not *directly* comparable, because the physical one may contain various higher twist contributions at such a low scale. Therefore, without making any ad hoc assumptions about the model parameters or μ^2_{bag}, one has to utilize the evolution equations to come into contact with data. In doing so one can determine the bag model parameters via a fit procedure.

Once the parameters are fixed another important application will be accessible: the investigation of experimentally unknown structure functions. Especially transversely polarized structure functions, the main issue of this work, attract a growing interest at present motivated by the surprising EMC-result for the longitudinally polarized structure function g_1 [7]. In particular, g_2 offers a deeper insight into the nucleon structure because of its directly extractable twist-3 contribution ($\bar{g}_2$) [8] and will be measurable by the SMC-[9] and the SLAC-E-142/143 [10] groups in the near future. The size of $\bar{g}_2$ at experimentally relevant scales Q^2 is a matter of a lively discussion which is rendered by the fact that the evolution equations for g_2 cannot be solved exactly [11]. We try to give an estimate of $\bar{g}_2$ at a relevant Q^2 using the bag model results as an input for the recently calculated approximate evolution kernels [12]. Finally, the chiral odd 'transversity' distribution h_1 is interesting in itself because it completes the twist-2 sector of parton distributions [13]. Theoretical studies are necessary in view of forthcoming experiments at RHIC [14].

This work is divided as follows: In Sect. 2 we briefly review the general framework for bag model calculations of structure functions and describe the models under consideration. In Sect. 3 we focus on the problem how to fix the model parameters, i.e. how to possibly relate the bag models to experimentally relevant Q^2-regions. Results for unpolarized (used for the fit) and longitudinally polarized (as a first consistency check) structure functions are given. Sections 4 and 5 cover the major issue of this work: to begin with, Sect. 4 deals with the transversely polarized structure function g_2, especially with the relevance of its twist-3 contribution and its approximate Q^2-evolution. Section 5 covers the 'transversity' structure function h_1 for which a 'first guess' model is developed. Finally we draw our conclusions in Sect. 6. The Appendix contains some functions/abbreviations employed in the present calculations.

2 Bag models – general framework

In order to introduce the notation adopted in Sect. 4 and 5 and to motivate our model prescription we have to review some of the bag model basics especially of the recent model of Schreiber, Signal and Thomas [5] (referred as SST model in the following).

Bag models can be considered as an extension of nonrelativistic quark models by introducing the important QCD property of confinement in a covariant manner [1], in general. Quark and gluon fields are restricted to a finite spacetime hypertube, the bag [1]. In order to achieve stable solutions one has to endow the bag with a constant universal energy density B [1], the main new ingredient of bag models. To admit exact solutions of the resulting equations of motion and boundary conditions [2], tractable in the calculation of structure functions, several far-reaching simplifications are necessary, namely neglecting any explicit gluonic degrees of freedom inside the bag and treating the bag as a sphere with fixed radius R [2, 6] (sometimes referred as the 'cavity approximation'). In doing so, one easily obtains the famous MIT bag wave functions [2] as solutions for the quarks inside the bag. Apart from studying any static properties of nucleons [2] one can utilize these wave functions to calculate structure functions from first principles [6]. Substituting the wave functions into the current correlation function

$$W_{\mu\nu}=\frac{1}{4\pi}\int d^4x\, e^{iqx}\langle ps|[J_\mu(x), J_\nu(0)]|ps\rangle \tag{2.1}$$

and using the well-known SU(6) nucleon wave functions $|ps\rangle$ in order to calculate the weights of quarks with definite spin and flavor inside the nucleon, one obtains [6]

$$u_v(x)=\frac{\omega^2}{\pi j_0^2(\omega)(\omega-1)}\int_{|y_{\min}|}^{\infty} dy\, y\left[t_0^2+2t_0t_1\frac{y_{\min}}{y}+t_1^2\right] +(x\to -x) \tag{2.2}$$

$$d_v(x)=\tfrac{1}{2}u_v(x) \tag{2.3}$$

as the unpolarized valence distributions of the proton within the MIT model (with $y_{\min}=\omega(4x-1)$ introducing the x-dependence of the parton distributions and $\omega=2.043$ as the lowest cavity quark mode [2]; for other abbreviations see Appendix). The $(x\to -x)$ prescription denotes how to treat the important contributions with $x<0$, namely as negative, and therefore unphysical, antiquark distributions [6, 15]. Although quark as well as antiquark distributions are unphysical, the valence distributions in (2.2) and (2.3) are physical, however, as strongly indicated by saturating the normalization constraint

$$\int_0^1 \begin{pmatrix} u_v(x) \\ d_v(x)\end{pmatrix}_{\text{proton}} dx=\begin{pmatrix}2\\1\end{pmatrix} \tag{2.4}$$

(otherwise (2.4) is spoiled!). The impossibility of getting physical antiquark distributions can be traced back to the ignorance of quantum fluctuations by fixing the bag shape [2, 6].

The MIT model contains some well-known disadvantages, e.g. the ratio $u_v(x)/d_v(x)$ is a constant ((2.2) and (2.3)) in striking disagreement with experiment (F_2^n/F_2^p!) or support problems for $x\to 1$. One can overcome most of the problems by introducing some phenomenological extensions such as the so called 'one-gluon-exchange' [3] which can be applied to the remaining diquark not participating in the hard process in order to obtain a different x-behavior for $u_v(x)$ and $d_v(x)$. To get rid of the support problems one has to restore the translational invariance in space and time, i.e. momentum and energy conservation must be retained [5]. Both improvements were recently adopted in the SST bag model [5]. Instead of (2.1) one already starts with the helicity dependent light-cone (LC) definition of quark distributions [16, 5]

$$q^{\uparrow\downarrow}(x)=p^+\sum_n \delta(p^+(1-x)-p_n^+)|\langle n|\Psi_+^{\uparrow\downarrow}(0)|ps\rangle|^2 \tag{2.5}$$

and analogously for $\bar{q}^{\uparrow\downarrow}$ (LC momenta p are defined by $p^+=p^0+p^z$). In order to proceed they [5] specified the initial ($|ps\rangle$) and intermediate ($|n\rangle$) states, with momentum p^+ (mass M) and p_n^+ ($M_{(n)}$), respectively, as composed of simple MIT bag wave functions but making them approximately translational invariant by using the Peierls-Yoccoz projection [4] (not the best but tractable technique [17]). Furthermore, it is argued [5] that in contrast to the MIT model the normalization constraint (2.4) can be fulfilled *without* taking any $(x\to -x)$ contributions into account. Apart from diquark states $|n=2\rangle$ they take $|n=4\rangle$ states into consideration too [5]. In doing so, one obtains several helicity dependent parton distributions belonging to both values of n, e.g. [5]

$$q_{(2)}^{\uparrow\downarrow}(x)=\frac{M}{(2\pi)^2}\sum_m\langle ps|\hat{P}_m^q|ps\rangle\int_{\left|\frac{M^2(1-x)^2-M_{(2)}^2}{2M(1-x)}\right|}^{\infty}\left[p_n\frac{|\phi_2(\mathbf{p}_n)|^2}{|\phi_3(\mathbf{0})|^2}\frac{1}{2}[f(\mathbf{p}_n)\pm(-1)^{m+\frac{3}{2}}g(\mathbf{p}_n)]\right]dp_n \tag{2.6}$$

(the operator $\hat{P}_m^q$ projects onto flavor q and quark spin projection m; for further abbreviations see Appendix). The x-dependence of the $u_{(2)}(x)$ and $d_{(2)}(x)$ distributions can be split up by the 'one-gluon exchange' [3, 5] simply by calculating the probability to find a quark of definite spin and flavor in the $SU(6)$ wave function $|ps\rangle$ leaving behind a diquark state $|n=2\rangle$ in a singlet (S) (triplet (T)) state of mass $M_{(2)}^{S(T)}$. Equipped with these formulas we have calculated the normalization constraint (2.4) by adding all valence contributions for a multitude of parameters (R, $M_{(2)}^{S,T}$, $M_{(4)}$) which seem to make physical sense (see Table 1), observing that (2.4) *cannot* be fulfilled ($\approx 80\%$ at

Table 1. The scanned parameter range for the modified bag models as well as a brief explanation for the choice

Parameter	Range	'Explanation'
R	0.6 ... 1.2 fm	$\mathcal{O}(R_{\text{nucleon}})$
M	938 MeV	nucleon mass (fixed)
$M_{(2)}^S$	$(\frac{1}{2}\cdots\frac{4}{5})M$	mass of the diquark $<M$
$M_{(2)}^T$	$M_{(2)}^T\approx M_{(2)}^S+200$ MeV	utilizing an estimate based on the Δ-N mass difference [3, 18]
$M_{(4)}$	$(\frac{5}{4}\cdots\frac{7}{4})M$	mass of 4 parton state $>M$

best). This leads to the disagreeable feature of introducing arbitrary normalization factors for each n ($N^q_{(2)}$, $N^q_{(4)}$) in further calculations to get rid of the sum rule constraint.*

We have figured out another way of 'repairing' the normalization defect resulting in a slightly modified model prescription (referred as MOD model further on). From a closer inspection of the diquark piece (2.6) one can easily obtain that the normalization condition (2.4) is perfectly well saturated if one adds the $(x \to -x)$ contributions to (2.6) leading to a model with only three parameters (R, $M^{S,T}_{(2)}$) whereas the arbitrary normalizations as well as the slightly suspicious $n=4$ contributions are not necessary any more. Despite of all arguments for neglecting the $(x \to -x)$ terms given in [5], these contributions seem to be urgently required in the sense of the original MIT attempt (2.2) [6] and the work of Jaffe [15] as strongly indicated by the normalization constraint (2.4). Furthermore, additional problems will arise in the SST model if one calculates polarized structure functions as discussed later on, all of them are solvable simply by adding the 'crossing' piece. But unfortunately there is no model without a weakness: contrary to the contributions for $0 \le x \le 1$ the $(x \to -x)$ terms violate energy-momentum conservation leading to the known support problem for $x \to 1$ (MIT model [6]) for the valence distributions but, fortunately, drastically reduced as compared to the MIT model because of utilizing projected wave functions.

3 Fixing the model parameters

In order to give predictions for presently unknown structure functions later on, it is first of all necessary to determine the model parameters as reliably as possible. For this purpose it is appropriate to utilize the copious data for unpolarized structure functions. As mentioned, bag models give predictions for valence distributions at a very low resolution scale $\mu^2_{\text{bag}} = \mathcal{O}(\Lambda^2_{\text{QCD}})$ as strongly indicated by the saturation of the momentum sum rule, far away from any data for the present. This situation is completely analogous to the attempt to generate purely dynamically all gluon and sea distributions merely from a nucleon composed of valence quarks exclusively [19]. The resulting momentum fraction carried by gluons at $Q^2 \approx 5-10\ \text{GeV}^2$ is in striking agreement with the experiment. Unfortunately the detailed x-dependence of the gluon and sea distributions turns out to be far to steep at small x. This problem can be solved by introducing a valence-like boundary condition for gluons and sea, sharing the momentum with valence quarks from the very beginning [20]. Of course, this is impossible in case of bag models and we therefore *have to* restrict ourselves to the non-singlet case, i.e. the evolution of the calculated valence distributions, concluding that gluon and sea distributions are *not* accessible in such models. For the sake of consistency we can only utilize the Altarelli-Parisi equations in leading order (LO) [21] because higher order evolution kernels for the various polarized structure functions to be studied afterwards are still missing. Nevertheless, this seems to be stable enough (we are not interested in a high precision analysis but in getting a first guess of unknown structure functions later on!) because on the one hand the relevant perturbative expansion parameter $\alpha_s/2\pi$ is still clearly less than one (0.5 . . .0.6) and on the other hand we can gather from [22] that higher order corrections to the original dynamical model [19] are rather moderate if one avoids the small x-range, a region we are not interested in anyway. In addition, by analyzing the convergence of various perturbative expansions for moments of singlet and non-singlet quantities it is argued in [23] that the non-singlet evolution converges very fast and may remain rather stable even for small values of $Q^2/\Lambda^2_{\text{QCD}}$. Therefore we hope that it will reliably interpolate between the model scale μ^2_{bag} and experimentally scales at $Q^2 \gg \mu^2_{\text{bag}}$. Note that we have *no* other choice anyway because introducing an ad hoc model scale $\mu^2 \gg \Lambda^2_{\text{QCD}}$ as done in [24] will result in valence distributions $(x q_v(x))$ roughly twice as large as experiment because of disregarding the saturated momentum sum rule.

To adjust the model parameters we utilize the LO parametrizations by Glück, Reya and Vogt (GRV(LO)) [20] as reference for the valence distributions (any other recent parametrization is useful as well because valence parts differ only slightly from set to set) together with experimental data for the valence sensitive structure functions $F_3^{\nu N}(x, Q^2) = u_v(x, Q^2) + d_v(x, Q^2)$ [25] and $F_2^p(x, Q^2) - F_2^n(x, Q^2) = \frac{1}{3} x (u_v(x, Q^2) - d_v(x, Q^2))$ [26]. Furthermore it might be convenient to fix the bag scale μ^2_{bag} based on the momentum carried by the valence quarks of the reference parametrization at some Q^2_{ref}. With the help of the renormalization group equations one easily obtains

$$\mu^2_{\text{bag}} = \Lambda^2_{\text{QCD}} e^L \quad \text{with}$$

$$L = \left(\frac{V_{\text{NS}}^{n=2}(Q^2_{\text{ref}})}{V_{\text{NS}}^{n=2}(\mu^2_{\text{bag}})} \right)^{1/a_{ns}^{n=2}} \ln\left(\frac{Q^2_{\text{ref}}}{\Lambda^2_{\text{QCD}}} \right) \tag{3.1}$$

where $V_{\text{NS}}^{n=2}$ denotes the momentum carried by the valence quarks, i.e. $V_{\text{NS}}^{n=2}(Q^2) = \int_0^1 dx\, x(u_v(x, Q^2) + d_v(x, Q^2))$, calculated in case of the bag model ($V_{\text{NS}}^{n=2}(\mu^2_{\text{bag}})$) or known from the reference parametrization ($V_{\text{NS}}^{n=2}(Q^2_{\text{ref}})$) and $a^n_{\text{NS}} = \gamma^n_{qq}/(11 - 2n_f/3)$. This offers the opportunity to use the sea quark distributions of the reference parametrizations (in our case [20]) together with the calculated bag model valence distributions when comparing with data for other not only valence sensitive structure functions such as F_2^p and F_2^n/F_2^p. By varying the model parameters within the 'physical range' (as specified in Table 1) we adjust the parameters as given in Table 2* (for the evolution we have used three active flavors, i.e. $n_f = 3$, and $\Lambda_{\text{QCD}}^{n_f=3} = 0.232$ GeV). Remember that there are no free parameters in the MIT model [6]. Our final results for the structure functions $F_3^{\nu N}$ and $F_2^p - F_2^n$ are shown in

*Allowing $N^q_{(2)} \neq 1$ deviates from the normalization procedure in [5] where only $N^q_{(4)} \neq 1$ was used but in our view the $n=2$ and $n=4$ contributions have to be treated on equal footing

*Our findings for the parameters as well as the model scale in case of the SST model are comparable to the one given in [5]

Table 2. Our findings for the bag model parameters and the underlying scales μ^2_{bag} for the SST, MOD and MIT models. Varying the parameters by the stated uncertainties leaves our results for the various structure functions almost unaltered

Parameter	SST model	MOD model	MIT model	Uncertainty
R in fm	0.6	0.7	—	0.05
$M^T_{(2)}$ in MeV	750	800	—	25
$M^S_{(2)}$ in MeV	550	560	—	25
$M_{(4)}$ in MeV	1500	—	—	50
$N^{u,d}_{(2)}$	1.1	—	—	0.01
$N^u_{(4)}$	fixed by	—	—	—
$N^d_{(4)}$	normalization	—	—	—
μ^2_{bag} in GeV^2	0.086	0.080	0.081	—

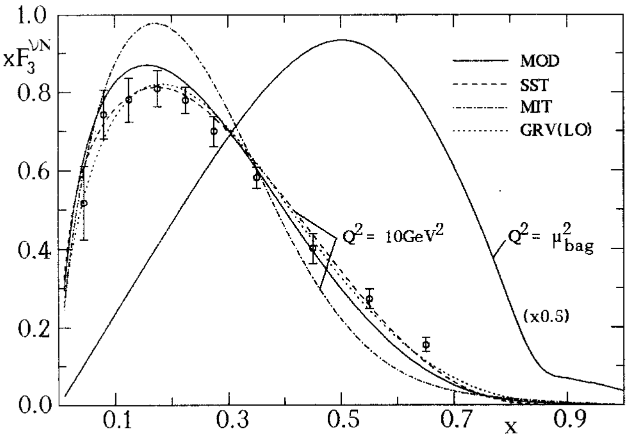

Fig. 1. Bag model results for the structure function $xF_3^{\nu N}$ at $Q^2=10\,\mathrm{GeV}^2$ in comparison with the corresponding result of the GRV parametrization (LO) [20]. In addition the MOD bag model result at the model scale μ^2_{bag} (multiplied by 1/2) is shown. Data are taken from CDHSW [25]

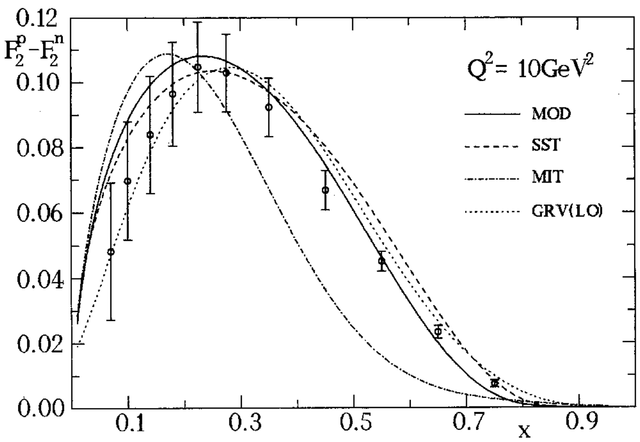

Fig. 2. Bag model results for the difference $F_2^p-F_2^n$ at $Q^2=10\,\mathrm{GeV}^2$ in comparison with the corresponding result of the GRV parametrization (LO) [20]. Data are taken from BCDMS [26]

Figs. 1 and 2, respectively. In order to demonstrate the necessity of the Q^2-evolution we present in addition the bag model (MOD) result for $xF_3^{\nu N}$ *at* the model scale in Fig. 1, which is in striking disagreement with experiment (even worse in case of the other models). If one presumes that perturbation theory makes sense down to the model scale $\mu^2_{\mathrm{bag}}\approx 0.08\,\mathrm{GeV}^2$, both modified models can fairly describe the data whereas the MIT model fails. The results for the SST model are slightly better but the arbitrary normalizations have to be taken into account (!) whereas the MOD model needs only three parameters.

As a first consistency check let us briefly focus on the bag model predictions for the longitudinally polarized structure function g_1. Figure 3 shows our results for the evolved bag model predictions for $g_1(x)=\frac{1}{2}(\frac{4}{9}\Delta u_v(x)+\frac{1}{9}\Delta d_v(x))$ at $Q^2=10\,\mathrm{GeV}^2$ in comparison with existing measurements of g_1 [7, 27] and a parametrization of the valence part [28] as well as an 'EMC' g_1 (by including a large polarized sea contribution) [28]. The underlying prescriptions for the calculations can be found in [29, 30] in case of the MIT model or simply being obtained from the helicity dependent distributions, like (2.6), by subtraction in case of the modified models, e.g. $\Delta q_v(x)=q^{\uparrow}_{(2)}(x)-q^{\downarrow}_{(2)}(x)$, according to the definition of longitudinally polarized distributions. Again, the two modified models give a fair description of the data, especially in the region where the valence is expected to be the dominant

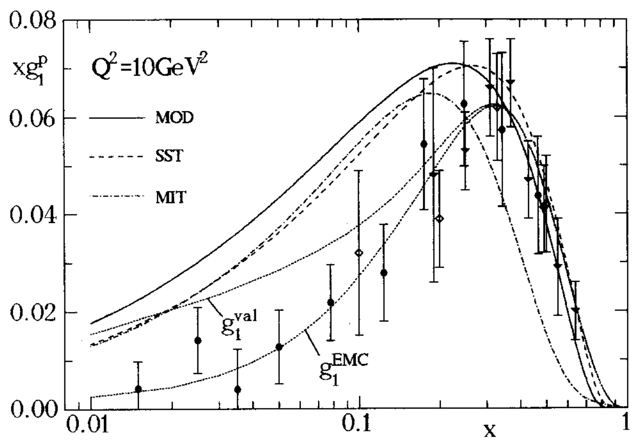

Fig. 3. Bag model predictions for g_1^p at $Q^2=10\,\mathrm{GeV}^2$ in comparison with the g_1-parametrizations of [28] (valence parts only (g_1^{val}) and large polarized sea contribution included to take the EMC result [7] into account (g_1^{EMC})). Data are taken from SLAC E-80 (rhombs), SLAC E-130 (triangles) and EMC (dots) [27, 7]

contribution to g_1 ($x>0.3$), whereas the MIT model fails. Naturally the bag model results are far to large for smaller values of x since a large polarized sea and/or gluon contribution is required to describe the data as we can gather from an analysis of the EMC measurement [7]. In

Table 3. Results for several quantities related with the topic of longitudinal polarization: first moments (Δu_v, Δd_v, g_1), axial coupling g_A and hyperon-β-decay parameters (F, D) for the three bag models in comparison with experiment [7, 31, 32]

Quantity	MIT	SST	MOD	Experiment
Δu_v	0.840	0.951	1.054	≈ 0.96
Δd_v	-0.210	-0.202	-0.264	≈ -0.29
g_A	1.050	1.153	1.318	1.254 ± 0.006
F	0.420	0.476	0.527	≈ 0.48
D	0.630	0.677	0.791	≈ 0.77
g_1^p	0.175	0.200	0.220	$0.126 \pm 0.010 \pm 0.015$

addition, we cover some related quantities such as first moments (Δu_v, Δd_v, g_1^p) in Table 3 demonstrating that all models can serve only as a qualitative description, e.g. in comparison with the experimentally extremely well known axial coupling g_A [31]. Finally it should be noted that in the SST model one has in addition to neglect the $n=4$ contributions to $g_1(x)$ in order to achieve a reasonable agreement with experiment as was already done in [5]. The $n=4$ contributions would decrease the SST predictions for the polarized valence distributions substantially, leading to an unacceptable $g_A \approx 1.0$ for example. Therefore we neglect this contribution in our further calculations of polarized structure functions.

4 The transversely polarized structure function $g_2(x, Q^2)$

Let us first briefly review some of the interesting properties of g_2 relevant for further discussions (more details can be found in [8, 30]). Unravelling the operator structure of g_2 one easily obtains that g_2 can be written as the sum of a twist-2 (g_2^{WW}) and a twist-3 contribution ($\bar{g}_2$):

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2). \tag{4.1}$$

Because of the Wandzura-Wilczek (WW) sum rule which expresses g_2^{WW} through the longitudinally polarized structure function g_1 [33]

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2) \tag{4.2}$$

a measurement of g_2 (and g_1) offers for the first time the possibility of extracting a higher twist contribution directly. The relevance, i.e. the size of $\bar{g}_2$ which, as shown by Shuryak and Vainshtein [34], is related to quark-gluon-correlations inside the nucleon, is subject of a lively discussion.

What do we learn about g_2, especially $\bar{g}_2$, from a bag model point of view? Polarizing the nucleon in some transverse direction one obtains for the experimentally accessible combination $g_T \equiv g_1 + g_2$ within the MIT bag model [35, 30]

$$g_1(x) + g_2(x) = \frac{5\omega^2}{9\pi j_0^2(\omega)(\omega-1)} \int_{|y_{\min}|}^{\infty} dy\, y \left[t_0^2 - t_1^2 \left[\frac{y_{\min}}{y}\right]^2 \right] + (x \to -x). \tag{4.3}$$

We have extended these calculations to the case of modified bag models, discussed so far, and obtain

$$g_T(x) = \tfrac{1}{6} N_{(2)}^q \left[2G_{(2)}^S + \tfrac{1}{3} G_{(2)}^T \right] \tag{4.4}$$

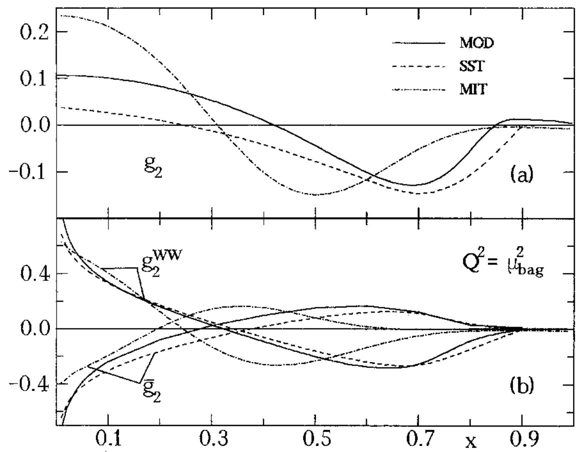

Fig. 4. The transversely polarized structure function g_2 (a) and its decomposition into the different twist contributions g_2^{WW} and $\bar{g}_2$ (b) in the three bag models at the model scale μ_{bag}^2 given in Table 2

with

$$G_{(2)}^{S,T} = \frac{M}{(2\pi)^2} \int_{\left|\frac{M^2(1-x)^2 - (M_{(2)}^{S,T})^2}{2M(1-x)}\right|}^{\infty} \left[p_n \frac{|\phi_2(\mathbf{p}_n)|^2}{|\phi_3(\mathbf{0})|^2} g(\mathbf{p}_n) \right] dp_n, \tag{4.5}$$

$$g(\mathbf{p}_n) = \frac{\pi R^3}{2} \frac{\omega^2}{\omega^2 - \sin^2\omega} \left[s_1^2(u) - \left[\frac{p_n^z}{|\mathbf{p}_n|}\right]^2 s_2^2(u) \right] \tag{4.6}$$

in the SST model (with obvious abbreviations, see Appendix). By setting $N_{(2)}^q = 1$ and adding the 'crossing' contribution ($x \to -x$) to (4.6) this is valid in the MOD model too. Figure 4 shows $g_2(x)$ and its decomposition into the two different twist pieces g_2^{WW} and $\bar{g}_2$ for the three distinct models at the model scale $\mu_{\mathrm{bag}}^2 \approx 0.08\,\mathrm{GeV}^2$ utilizing the prescriptions for $g_1(x)$, the WW sum rule (4.2) and the fixed parameters given in Table 2. Qualitatively all models show nearly the same results: g_2 starts positive for small x, crosses zero in the range between $x \approx 0.25$ and $x \approx 0.45$ and after passing through a minimum it tends to zero (the second crossing in the MOD model for large x is a relic of the lack of energy-momentum conservation for the $x \to -x$ contributions). The sign of g_2 is almost exclusively given by the sign of g_2^{WW} and, finally, the twist-3 part $\bar{g}_2$ is on equal footing with g_2^{WW} but with the sign reversed. Note that, as a remarkable distinction between the two modified models, the SST model again fails in fulfilling an important sum rule [36]:

$$\int_0^1 g_2(x, Q^2)\, dx = 0 \tag{4.7}$$

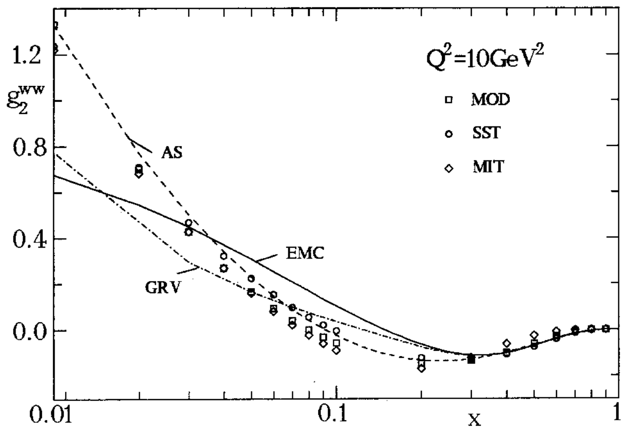

Fig. 5. The twist-2 contribution g_2^{WW} at $Q^2=10\,\mathrm{GeV}^2$ for the three distinct models (distinguished by symbols) in comparison with results for g_2^{WW} based on the g_1 valence parts of [38] (AS) and [28] (GRV) and on a g_1 which describes the EMC data [28] (EMC)

as one may easily check by inspecting the area under g_2 in Fig. 4 (this remains even worse if one adds the $n=4$-contributions). In particular, we obtain numerically for the first moment of g_2 in the various models: 0.0038 (MIT) and -0.0009 (MOD) in perfect agreement with (4.7) and -0.045 (SST) badly violating (4.7) indicating that something is missing in the SST model (in our view the crossing contribution $(x\to -x)$). The Burkhardt-Cottingham sum rule [36] (4.7) is of great interest since it cannot be proved within QCD [8] but it is considered as most probably true (especially in bag models [30] where an analytical proof exists in the case of the MIT model [29, 35]) because a violation would imply a rather non-conventional behaviour of $\bar{g}_2$.

One should bear in mind that the obtained large $\bar{g}_2$ says *nothing* about its relevance at $Q^2 \gg \mu_{\text{bag}}^2$! To relate the bag model results to scales experimentally relevant in the future, i.e. in order to give a prediction, one is plagued with the problem of evolving g_2. Before discussing this problem in some detail let us try to figure out where this large $\bar{g}_2$ comes from. At first sight one might attribute this to the virtuality $k^2 \neq 0$ of the quarks built up by confining them into a sphere of radius R, but such terms are simply neglected throughout the calculation. This can easily being justified, e.g. in the MIT model, by estimating the average k^2 from the maximum reached by the integrand in (4.3), i.e. $\langle k^2\rangle \approx 0.02\,\mathrm{GeV}^2$, which is sizeably smaller than the model scale μ_{bag}^2 given in Table 2. But virtuality is *not* the only origin of a nonvanishing $\bar{g}_2$, quark-gluon-correlations must be taken into account too [30]*. We take, however, the view that in the case of bag models the highly non-trivial qg-correlations are summed up by the bag-'background' field B. Therefore the resulting $\bar{g}_2$ can be traced back to the special noncovariant treatment of the static bag itself: the energy of a quark inside the bag is *fixed* by ω/R [2, 6] introducing the x-dependence of the distributions via the lower integration limit in (4.3). A complete treatment with explicit gluonic degrees of freedom and $k^2 \neq 0$-effects inside the bag is very complicated and has not been performed so far.

Let us now focus on the very important question of estimating g_2 at experimentally relevant scales Q^2 based on the model results presented so far. The evolutions of g_2^{WW} and $\bar{g}_2$ can be treated separately because contributions of different twist do not mix under renormalization. In particular, the evolution of g_2^{WW} contains nothing mysterious and is completely determined through the well-known evolution kernels of g_1 [21]. Figure 5 shows our results for g_2^{WW} $(x, Q^2=10\,\mathrm{GeV}^2)$ utilizing the evolved bag model results for the non-singlet pieces of g_1. Also shown are the results for g_2^{WW} where two distinct sets of g_1-valence distributions [28, 38] (derived from the EMC/SLAC data [7, 27] in each case) and an 'EMC' g_1 [28] are taken into consideration. Note that results for g_2^{WW} obtained by an 'EMC' g_1 represent our present knowledge about g_2 and remain exact if $\bar{g}_2$ will be negligible as argued e.g. in [37]. All three models, especially the modified ones, follow closely the shape obtained by the Altarelli-Stirling set [38]. Deviations from the 'EMC' result are only sizeable for $x<0.2$ where sea and/or gluon contributions to g_1 might be relevant. Again, starting the evolution of the bag model results at $\mu_{\text{bag}}^2 \approx 0.08\,\mathrm{GeV}^2$ is crucial, otherwise the discrepancy between the bag and the 'EMC' prediction for g_2^{WW} at $Q^2=10\,\mathrm{GeV}^2$ is very large as a comparison of Figs. 4 and 5 shows.

Let us now turn to the evolution of $\bar{g}_2$ which is in general an unsolved intricate problem because the number of independent twist-3 operators increases with the moment n under consideration [8, 11], impressively demonstrated by the 'tower-structure' of the anomalous dimensions belonging to them [12]. Already in the simplest case, namely the evolution of the non-singlet piece, important for our bag result, $(n-1)$ independent inputs, experimentally or theoretically, are necessary for solving the renormalization group equations for the nth moment of $\bar{g}_2$ [11]. Apart from $n=2$ and $n=4$ this is a completely impracticable task, in particular in the case of $\bar{g}_2$ for any given x and Q^2, one is actually interested in. Therefore practical approximations to the exact Q^2-evolution are urgently required. Recently, Ali, Braun and Hiller [12] have shown that for $N_C\to\infty$ (N_C denotes the number of colors) or $n\to\infty$ a simple evolution equation for the moments of $\bar{g}_2$ holds

$$\int_0^1 \mathrm{d}x\, x^n \bar{g}_2(x, Q^2) = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)}\right)^{-\gamma_n/b} \int_0^1 \mathrm{d}x\, x^n \bar{g}_2(x, \mu^2) \qquad (4.8)$$

with $b=11-\frac{2}{3}n_f$. They have calculated the required non-singlet anomalous dimensions γ_n in both cases. Only the N_C-limit might be useful for our purpose, because the $n\to\infty$ limit is equivalent with restricting oneself to the x-range very close to 1. For $N_C\to\infty$ the anomalous dimensions are given by [12]

$$\gamma_n^{NS} = 2N_C\left(\Psi(n+1)+\gamma_E-\frac{1}{4}+\frac{1}{2(n+1)}\right)+\mathcal{O}\left(\frac{1}{N_C^2}\right). \qquad (4.9)$$

Note that γ_n^{NS} differs substantially from the one naively obtained by ignoring the operator mixing as was done in

*Note that within covariant parton model approaches [37] such contributions cannot be taken into consideration, resulting in $g_2 \approx g_2^{WW}$

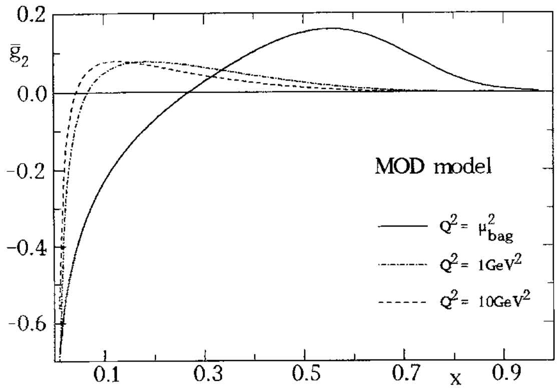

Fig. 6. The evolution of $\bar{g}_2$ in the MOD model

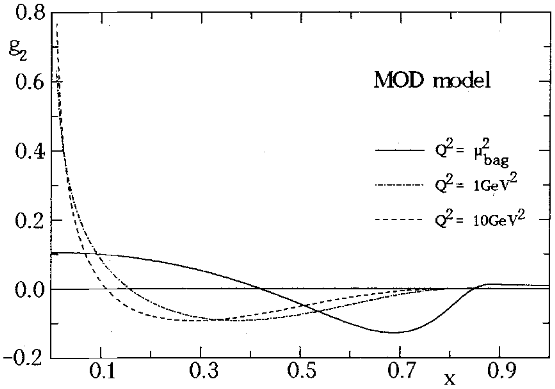

Fig. 7. Predictions for $g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2)$ with g_2^{WW} and $\bar{g}_2$ taken from Figs. 5 and 6, respectively

the early days of evolving g_2 [39]. As the only currently available evolution technique for $\bar{g}_2$, we optimistically apply (4.8) and (4.9) in the case $N_C = 3$ together with our bag model predictions. Figure 6 shows our results for $\bar{g}_2$ at various Q^2. In Fig. 7 we add the g_2^{WW} contribution in order to compare evolved and not evolved bag model predictions for g_2. Even after the evolution g_2^{WW} and $\bar{g}_2$ remain roughly on equal footing with their signs reversed. Like g_2^{WW} and $\bar{g}_2$, g_2 is shifted towards smaller values of x and becomes steep for $x < 0.1$, a behavior well-known from other more conventional structure functions. The question how far these results can be trusted is a rather difficult one because possibly important (especially for small values of x) singlet contributions to $\bar{g}_2$ are out of reach. Note that in a completely different nonperturbative approach using light-cone nucleon wave functions [40], an even more dominant $\bar{g}_2$ contribution shows up, resulting in a negative g_2 for small x. So, forthcoming experiments have to decide between parton model predictions (negligible $\bar{g}_2$) or nonperturbative models (large $\bar{g}_2$ is possible).

5 The 'transversity' structure function $h_1(x, Q^2)$

Recently Jaffe and Ji [13] have shown that for a complete theoretical description of parton distributions with twist-2 the inclusion of a *chiral odd* structure function, h_1, is necessary (the importance of these quantities was first recognized by Ralston and Soper [41] in a study of the transversely polarized Drell-Yan process). The chirality properties of h_1 lead to stringent experimental consequences in comparison with the structure functions discussed so far: a nonvanishing h_1 requires a helicity flip of the quarks in the hard process, i.e. h_1 is not measureable in fully inclusive deep inelastic scattering because of helicity conservation. Nevertheless, for comparisons with other structure functions it might be convenient to treat h_1 *formally* as a 'usual' structure function and write down in complete analogy to the longitudinal case

$$h_1(x, Q^2) = \frac{1}{2}\sum_i e_i^2(\Delta_T q_i(x, Q^2) + \Delta_T \bar{q}_i(x, Q^2)) \tag{5.1}$$

as the flavor decomposition in LO. The physical relevant and well-defined 'transversity' distributions $\Delta_T \overset{(-)}{q_i}$ contained in (5.1) measure the probability to find a (anti-) quark in an eigenstate of the transverse Pauli-Lubanski-operator $\gamma_5 \not{s}_\perp$ with eigenvalue $+1$ minus the probability to find it polarized oppositely in a transversely polarized nucleon with spin vector $s_\perp$ [13]. The 'transversity' distributions should become experimentally accessible to the forthcoming experiments at RHIC [14]. To decide if a specific process under consideration might be a useful tool for the extraction of the 'transversity' distributions, a model for the $\Delta_T \overset{(-)}{q_i}$ is absolutely necessary.

What can we gather from bag model considerations? In [13] the $\Delta_T q_i$ were calculated within the MIT model. One obtains for the non-singlet piece of the formally introduced structure function h_1 (5.1)

$$h_1(x) = \frac{5\omega^2}{9\pi j_0^2(\omega)(\omega-1)} \int_{|y_{\min}|}^{\infty} dy\, y \times \left[t_0^2 + 2t_0 t_1 \frac{y_{\min}}{y} + t_1^2 \left[\frac{y_{\min}}{y}\right]^2 \right] + (x \to -x). \tag{5.2}$$

Again, we have extended these calculations to the case of modified models. In the SST model h_1 is given by

$$h_1(x) = \tfrac{1}{6} N_{(2)}^q [2G_{(2)}^S + \tfrac{1}{3} G_{(2)}^T] \tag{5.3}$$

with $G_{(2)}^{S,T}$ as given in (4.5) and

$$g(\mathbf{p}_n) = \frac{\pi R^3}{2} \frac{\omega^2}{\omega^2 - \sin^2\omega} \left[s_1^2(u) + 2\frac{p_n^z}{|\mathbf{p}_n|} s_1(u) s_2(u) + \left[\frac{p_n^z}{|\mathbf{p}_n|}\right]^2 s_2^2(u) \right] \tag{5.4}$$

(with obvious abbreviations, see Appendix). By setting $N_{(2)}^q = 1$ and adding the 'crossing' terms $(x \to -x)$ to (5.4) this is also valid in the MOD model. We can establish the result of the MIT model [13] that $h_1(x)$ is only slightly larger than $g_1(x)$ at the model scale $\mu_{\text{bag}}^2 \approx 0.08\,\text{GeV}^2$, as shown in Fig. 8 in case of the MOD model (analogous in the SST model). But the observed inequality $h_1(x) > g_1(x)$ for their non-singlet pieces cannot hold at arbitrary high

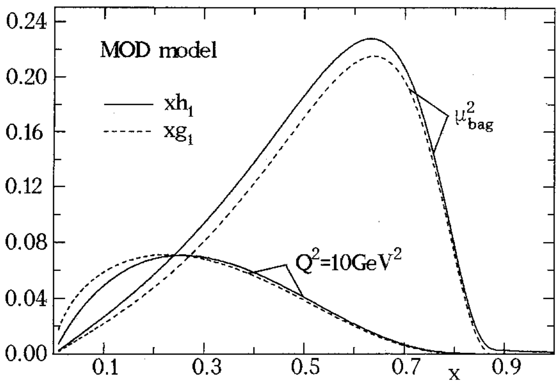

Fig. 8. The 'transversity' structure function $h_1(x)$ (multiplied by x) at $\mu^2_{\rm bag}$ and $Q^2 = 10\,\mathrm{GeV}^2$ in the MOD bag model in comparison with its longitudinally polarized counterpart $xg_1(x)$ in each case

scales $Q^2 > \mu^2_{\rm bag}$ because h_1 and g_1 obtain different evolution equations. Utilizing the well-known non-singlet evolution kernels for g_1 [21] and the one for h_1 [42]

$$\Delta_T \gamma_{qq}(n) = \frac{\alpha_s}{2\pi} C_2(R) \left[-3 + 4 \sum_{j=1}^{n} \frac{1}{j} \right] \tag{5.5}$$

we have evolved the bag model predictions to $Q^2 = 10\,\mathrm{GeV}^2$ in each case, shown in Fig. 8 too. It may not surprise that $g_1(x)$ starts to take over because, as one can easily gather from (5.5), the first moment of $h_1(x)$ slowly depolarizes with increasing Q^2 ($\Delta_T \gamma_{qq}(n=1) \neq 0$), whereas the longitudinally polarized quark distributions do not renormalize ($\Delta_L \gamma_{qq}(n=1) = 0$).

Arguing that the slight differences between the longitudinal $\Delta_L q(x)$ and the 'transversity' $\Delta_T q(x)$ distributions, induced by relativistic effects in case of bag models [13], might be not particularly relevant in a realistic hadron at some low scale μ^2, we prefer the following ansatz

$$\Delta_T q_v(x, \mu^2) = \Delta_L q_v(x, \mu^2) \tag{5.6}$$

as a 'first guess' for the 'transversity' valence distributions. The $\Delta_T q_v(x, Q^2 > \mu^2)$ are determined by (5.5), taking into account that the different evolution equations become more and more important with increasing Q^2. Such an ansatz seems to be even more reliable than utilizing the bag model predictions because on the one hand one can choose μ^2 safely in the perturbative region, e.g. $\mu^2 = 1\,\mathrm{GeV}^2$, and on the other hand one can use the experimentally more or less known distributions $\Delta_L q_v(x)$ as boundary condition (by scanning through the several distinct parametrizations of $\Delta_L q_v(x)$ one can figure out the ambiguities introduced by the choice of $\Delta_L q_v(x)$). Because of the well-known fact that no polarized gluon distribution can contribute to h_1 at leading twist [43, 42], only the 'transversity' sea distributions remain uncertain. Note that, as a consequence of the non-existing $g \to q$ splitting function in the 'transversity' case, no sea distribution can be generated dynamically or in other words if the sea vanishes at μ^2 it vanishes for all $Q^2 > \mu^2$. In order to give an upper bound for cross-sections involving 'transversity' distributions one can take the maximally allowed sea distributions (because of the positivity condition on probability) $|\Delta_T \bar{q}(x, \mu^2)| = \bar{q}(x, \mu^2)$ as an input for the evolution of the transversity' sea (together with (5.5)). The presented model scenario for the $\Delta_T \overset{(-)}{q_i}$ has been applied in some recent studies of processes involving 'transversity' distributions [44, 45].

6 Conclusions

First of all we have shown that, under the condition that the evolution equations are reliable at the model scale $\mu^2_{\rm bag} = \mathcal{O}(\Lambda^2_{\rm QCD})$, bag models can give a fair description of non-singlet pieces of unpolarized as well as longitudinally polarized structure functions if some modifications of the original MIT model are taken into account. We prefer the MOD model instead of the original SST model because it is free of normalization ambiguities and requires only three parameters. Further on we have calculated the transversely polarized structure function g_2 within these modified models and figured out that g_2^{WW} and $\bar{g}_2$ are on equal footing at the model scale $\mu^2_{\rm bag}$ but with sign reversed as already in the MIT model. Using recently developed evolution kernels for $\bar{g}_2$ we have obtained that at higher Q^2 this situation does not change and $\bar{g}_2$ remains an important ingredient of g_2. g_2 itself is shifted towards smaller values of x and becomes steep for $x < 0.1$. Finally we have studied the 'transversity' structure function h_1 and confirmed the MIT result that g_1 and h_1 are almost equal at $\mu^2_{\rm bag}$. Based on this result we present a 'first guess' model for the 'transversity' distributions which will be useful for estimating cross-sections. To proceed measurements of g_2 as well as the 'transversity' distributions $\Delta_T \overset{(-)}{q_i}$ are urgently required!

Appendix

For the sake of completeness we have collected here the explicit formulas for the shorthands used in the various presented bag model prescriptions for the calculation of structure functions. Let us start with the case of the MIT model: all formulas have the two functions [6, 30]

$$t_0 \equiv t_0(\omega, y) = \frac{\omega j_1(\omega) j_0(y) - y j_1(y) j_0(\omega)}{\omega^2 - y^2} \tag{A.1}$$

and

$$t_1 \equiv t_1(\omega, y) = \frac{y j_1(\omega) j_0(y) - \omega j_1(y) j_0(\omega)}{\omega^2 - y^2} \tag{A.2}$$

in common, both of them originating from the Fourier transform of the MIT wave functions [6] (j_0, j_1 are the well-known spherical Bessel functions). The dimensionless quantity y is related to the momentum $\mathbf{k}$ carried by the quarks inside the bag, $y = \frac{4\omega}{M}|\mathbf{k}|$, whereas the energy of a quark is *fixed* by ω/R, i.e. $k^\mu = (\frac{\omega}{R}, \mathbf{k})$ [2].

In case of the modified models we have adopted the notation of [5]. In the master formula (2.6) we have left out so far the functions [5]

$$f(\mathbf{p}_n)=\frac{\pi R^3}{2}\frac{\omega^2}{\omega^2-\sin^2\omega}\left[s_1^2(u)+2\frac{p_n^z}{|\mathbf{p}_n|}s_1(u)s_2(u)+s_2^2(u)\right] \tag{A.3}$$

and

$$g(\mathbf{p}_n)=\frac{\pi R^3}{2}\frac{\omega^2}{\omega^2-\sin^2\omega}\left[s_1^2(u)+2\frac{p_n^z}{|\mathbf{p}_n|}s_1(u)s_2(u)\right.$$
$$\left.+\left[1-2\left[\frac{p_n^\perp}{|\mathbf{p}_n|}\right]^2\right]s_2^2(u)\right] \tag{A.4}$$

relevant for the calculation of unpolarized $(q^\uparrow+q^\downarrow)$ and longitudinally polarized $(q^\uparrow-q^\downarrow)$ parton densities, respectively. Furthermore, the two functions $s_1(u)$ and $s_2(u)$ are completely analogous to the MIT functions t_0 (A.1) and t_1 (A.2), respectively, and are given by [5]

$$s_1(u)=\frac{1}{u}\left[\frac{\sin(u-\omega)}{u-\omega}-\frac{\sin(u+\omega)}{u+\omega}\right] \tag{A.5}$$

$$s_2(u)=2j_0(\omega)j_1(u)-\frac{u}{\omega}s_1(u). \tag{A.6}$$

Finally, the normalizations of the Peierls-Yoccoz projected [4] wave functions are given by [5]

$$|\phi_n(\mathbf{p}_n)|^2=\frac{4\pi\omega^n}{[2(\omega^2-\sin^2\omega)]^n}\left(\frac{2R}{\omega}\right)^3\int_0^\omega \mathrm{d}v$$
$$\times v^{2-n}T^n(v)j_0\left(\frac{2uv}{\omega}\right) \tag{A.7}$$

with

$$T(v)=\left(\omega-\frac{\sin^2\omega}{\omega}-v\right)\sin 2v-\left(\frac{1}{2}+\frac{\sin 2\omega}{2\omega}\right)\cos 2v$$
$$+\frac{1}{2}+\frac{\sin 2\omega}{2\omega}-\frac{\sin^2\omega}{\omega^2}v^2. \tag{A.9}$$

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