Accessing the nucleon tensor structure in inclusive deep inelastic scattering

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We revisit the standard analysis of inclusive Deep Inelastic Scattering (DIS) off nucleons taking into account the fact that on-shell quarks cannot be present in the final state, but they rather decay into hadrons. As a consequence, a spin-flip term associated with the invariant mass of the produced hadrons is generated non perturbatively, and couples to the target's transversity distribution function. In inclusive cross sections, this provides an hitherto neglected and large contribution to the twist-3 part of the g_2 structure function, that can explain the discrepancy between recent calculations and fits of this quantity. It also provides an extension of both the Burkardt-Cottingham sum rule, putting constraints on the small-x behavior of the transversity function and can help constraining the nucleon tensor charge, and of the Efremov-Teryaev-Leader sum rule, suggesting a novel way to mwasure the tensor charge of the proton.

I. INTRODUCTION

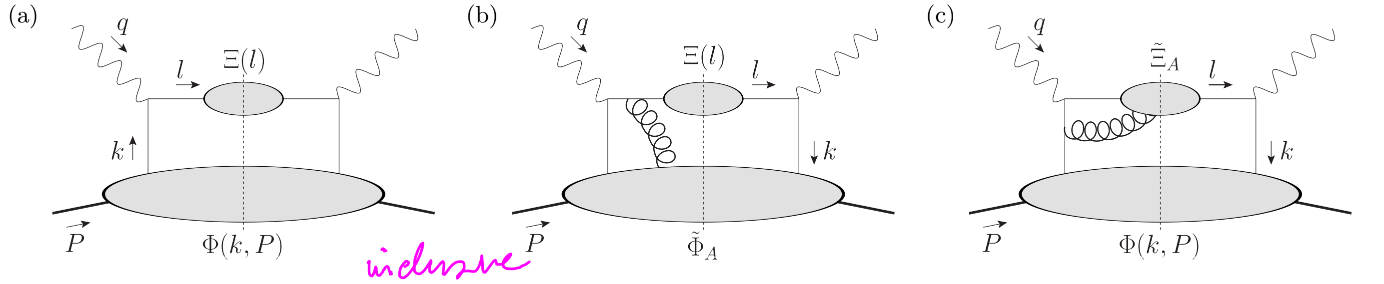
The tensor charge is a fundamental property of the nucleon, that is at present poorly constrained but of fundamental importance, not the least because its knowledge can be used also to put constraints on the search of physics beyond the Standard Model [1–3]. The tensor charge has been estimated in lattice QCD (see, *e.g.*, [4–8]), but only limited information is available from direct measurements. Its experimental extraction requires first of all the measurement of the so-called transversity parton distribution function, denoted by $h_1^q(x)$ (see Ref. [9] for a review on transversity and Refs. [10–12] for the most recent extractions). Secondly, one needs to perform a flavor decomposition and evaluate its integral, that corresponds to the contribution of flavor q to the tensor charge.

The transversity distribution is notoriously difficult to access because it is a chiral-odd function and needs to be combined with a spin-flip mechanism to appear in a scattering process [13]. Usually, this spin flip is provided by another nonperturbative distribution or fragmentation function, accessible in Drell-Yan or semi-inclusive DIS [14–17]. The only other known way to attain spin-flip terms in QED and QCD is taking into account mass corrections. In fact, it is well known that the transversity distribution gives a contribution to the structure function g_2 in inclusive DIS (see, *e.g.*, [18] and references therein), and in particular to the violation of the so-called Wandzura–Wilczek relation for g_2 [19]. However, this contribution is proportional distribution to the current quark mass and can be expected to be negligibly small. This implies that the transversity cannot be accessed in inclusive DIS, but only in more complex processes such as semi-inclusive DIS or dilepton production in hadron-hadron collisions.

In this paper, we discuss a novel way of accessing the transversity parton distribution function (PDF) and measuring the proton's tensor charge in totally inclusive Deep Inelastic Scattering. We revisit the standard analysis of inclusive DIS taking into account the fact that on-shell quarks cannot be present in the final state, but they rather decay into hadrons (ideally, forming (mini) jets of hadrons). This is sufficient to modify the structure of the DIS cut-diagram, even if none of the hadrons is detected in the final state. For a proper description of this effect, we include "jet correlators" into the analysis, and pay particular attention to ensuring that our results are gauge invariant. We observe that ~~the inclusion of~~ jet correlators introduces a new contribution to the inclusive g_2 structure function. This term is proportional to the transversity distribution function multiplied by a nonperturbative "jet mass" parameter, likely much larger than the mass of light quarks, and has the interesting features that: (a) it violates the Wandzura–Wilczek relation; (b) it extends the Burkhardt–Cottingham sum rule, providing a useful bound on the small-x behavior of the transversity distribution; (c) it also extends the Efremov-Teryaev-Leader sum rule, and provides a novel way to measure the proton's tensor charge. We provide estimates of this new jet-mass induced contribution based on a recent extraction of transversity, and show that it can indeed be very large.

II. THE JET CORRELATOR

Motivated by large-x mass corrections to inclusive DIS structure functions, Accardi and Qiu [21] have introduced in the LO handbag diagram a "jet correlator" (also called "jet factor" by Collins and Rogers in Ref. [20]) that accounts for invariant mass production in the current jet, and ensures that leading twist calculations in collinear factorization are consistent with the requirement imposed by baryon number conservation that $x_B < 1$ [~~21~~]. The jet correlator is



inclusive

FIG. 1: Diagrams contributing to DIS scattering up to twist-3, including a jet correlator in the top part. Note the gluon attaches to both the nucleon and jet correlators. The Hermitian conjugates of diagrams (b) and (c), i.e., with gluons attaching to the right of the cut, are not shown.

and .... perpendicular... see [25]

depicted in Figure 1(a) and is defined as

$$\Xi_{ij}(l,n_+) = \int \frac{d^4\eta}{(2\pi)^4}\, e^{il\cdot\eta}\, \langle 0|\, \mathcal{U}^{n_+}_{(+\infty,\eta)}\, \psi_i(\eta)\bar\psi_j(0)\, \mathcal{U}^{n_+}_{(0,+\infty)}\, |0\rangle \ , \tag{1}$$

In this definition, l is the quark's four-momentum, Ψ the quark field operator (with quark flavor index omitted for simplicity), and $|0\rangle$ is the nonperturbative vacuum state. Furthermore, we explicitly guarantee the correlator's gauge invariance by introducing two Wilson line operators $\mathcal{U}^{n_+}$ along a light-cone plus direction determined by the vector n_+. This path choice for the Wilson line is required by QCD factorization theorems, and the vector is determined by the particular hard process to which the jet correlator contributes. For example, in the case of inclusive DIS discussed in this paper, this is determined by the four momentum transfer q and the proton's momentum p.

The correlator Ξ can be parametrized in terms of jet parton correlation functions (PCFs), using the vectors l and n_+:

$$\Xi(l,n_+) = \Lambda A_1(l^2)\,\mathbf{1} + A_2(l^2)\,\not{l} + \frac{\Lambda^2}{l\cdot n_+}\not{n}_+\, B_1(l^2) + \frac{i\Lambda}{2l\cdot n_+}[\not{l},\not{n}_+]\, B_2(l^2) \ , \tag{2}$$

where Λ is an arbitrary scale, introduced for power counting purposes. In this parametrization, no terms proportional to γ_5 enter~~s~~ because of parity invariance. Time reversal invariance in QCD requires $B_2 = 0$, while B_1 contributes only at twist-4 order, and will not be considered further in this paper. We focus, instead, on the role of chiral odd terms in the g_2 structure function up to twist 3. At this order,

$$\Xi(l,n_+) = \Lambda A_1(l^2)\,\mathbf{1} + A_2(l^2)\,\not{l} + O(\Lambda^2/Q^2) \tag{3}$$

is nothing else than the cut quark propagator; note however, that we consider here the full QCD vacuum rather than the perturbative one (or, in other words, the interacting rather than the free quark fields). The A_1 and A_2 terms can be nicely interpreted in terms of the spectral representation of the cut quark propagator (see, e.g., Sec. 6.3 of [22] and Sec. 2.7.2 of [23]),

Reihenfolge Vorzeichen

$$\Xi(l) = \int d\mu^2\, [J_1(\mu^2)\, \mu\, \mathbb{1} + J_2(\mu^2)\,\not{l}\,]\, \delta(l^2 - \mu^2) \ , \tag{4}$$

where μ^2 is interpreted as the invariant mass of the current jet, *i.e.*, of the particles going through the cut in the top blob of Fig.1(a), and ~~the~~ J_i are the spectral functions of the quark propagator, also called "jet functions" in [21]. These satisfy [..., ...]

$$J_2(\mu^2) \geq J_1(\mu^2) \geq 0 \quad \text{and} \quad \int d\mu^2 J_2(\mu^2) = 1 \ . \tag{5}$$

From a comparison of Eqns.(2) and (4), one can see that

$$A_1(l^2) = \frac{\sqrt{l^2}}{\Lambda} J_1(l^2) \qquad\qquad A_2(l^2) = J_2(l^2) \ . \tag{6}$$

When inserting the jet correlator in the handbag diagram for inclusive DIS, the invariant jet mass μ^2 is integrated from 0 to $Q^2(1/x_B - 1)$, where x_B is the Bjorken invariant. This induces (kinematical) corrections of order $O(1/Q^2)$, whose effect on the F_2 structure function has been studied in Ref. [21]:

$$F_2(x_B) = \int_0^{Q^2(1/x_B-1)} d\mu^2\, J_2(\mu^2) F_2^{(0)}(\xi(1+\mu^2/Q^2)) \ , \tag{7}$$

and approximately $\xi \simeq x_B$

where $F_2^{(0)}$ is the structure function calculated with the handbag diagram sporting a bare quark propagator instead of the jet correlator, and $\xi = 2x_B/(1+\sqrt{1+4x_B^2M^2/Q^2})$ with M the nucleon's mass is the Nachtmann scaling variable. (We omitted the dependence of the structure function on Q^2 for clarity of notation). In this paper we limit our attention to effects of order $O(1/Q)$ and therefore can extend the integration to $\mu^2 = \infty$. As a consequence, the jet function J_2 decouples and, thanks to the sum rule (5), integrates to 1. One then recovers the conventional result,

$$F_2(x_B) = \Big[\int_0^\infty d\mu^2\, J_2(\mu^2)\Big] F_2^{(0)}(x_B) + O(\Lambda^2/Q^2) = F_2^{(0)}(x_B) + O(\Lambda^2/Q^2)\ . \tag{8}$$

More in general, the jet correlator decouples from the parton correlator Φ in any inclusive cross section calculation up to $O(1/Q)$, and the inclusive structure functions only depend on the integrated jet correlator

$$\Xi(l^-) \equiv \int \frac{dl^2}{2l^-}\, d^2l_T\, \Xi(l) = \frac{\Lambda}{2l^-}\,\xi_1 \mathbf{1} + \xi_2 \frac{\not{n}_-}{2} + \text{higher twists} \tag{9}$$

the leading-twist terms are independent of l_T, from Eq.(4),

where

$$\xi_1 = \int d\mu^2 \frac{\mu}{\Lambda} J_1(\mu^2) \equiv \frac{M_q}{\Lambda}, \qquad \xi_2 = \int d\mu^2 J_2(\mu^2) = 1\ . \tag{10}$$

and the higher-twist terms only contribute to $O(1/Q^2)$ to the cross section,

Here, M_q can be interpreted as the average invariant mass produced in the spin-flip fragmentation processes of a quark of flavor q. It is important to notice that $\xi_2 = 1$ exactly due to CPT invariance (see Sec. 10.7 of Ref. [24]), while $0 < M_q < \int d\mu^2 \mu J_2(\mu^2)$ is dynamically determined. From the analytic properties of spectral functions we may expect [21] $J_2(\mu^2) = Z\delta(\mu^2 - m_q) + \bar{J}_2(\mu^2)\theta(\mu^2 - m_\pi^2)$ with the continuum starting at m_π, the mass of the pion, due to color confinement effects. ~~Taking into account that $J_1 < J_2$,~~ We may therefore ~~expect~~ estimate

$$M_q = O(100\ \text{MeV})\ . \tag{11}$$

~~Although M_q is in general a nonperturbative quantity, it is interesting to notice that~~

$$\cancel{M_q = \frac{\Lambda}{4}\int \mathrm{Tr}\,[\Xi(l)\mathbf{1}] = \langle 0|\bar\psi_i(0)\psi_i(0)|0\rangle} \tag{12}$$

~~On the non-perturbative vacuum, this corresponds the contribution of a quark of flavor q to the chiral condensate, and is calculable in principle in lattice QCD. Considering the perturbative vacuum and calculating the same to leading order in the strong coupling constant, corresponds, instead, to taking the trace of the cut bare-quark propagator to obtain $M_q = {}_{\text{pert}}\langle 0|\bar\psi_i(0)\psi_i(0)|0\rangle_{\text{pert}} = m_q$, with m_q the quark mass.~~ One recovers, therefore, the result of the calculation with the conventional handbag diagram. However, we are here considering non perturbative effects on the quark fragmentation and $M_q \gtrsim m_q$. Therefore, differently from J_2, the J_1 function leaves an imprint on the inclusive DIS cross section even in the asymptotic $Q^2 \to \infty$ regime.

III. TWIST-3 ANALYSIS

Extending the analysis of [21] to the calculation of twist-3 structure functions requires not only to consider the ξ_1 term in the jet correlator, but also quark-gluon-quark correlators in both the proton and the vacuum as depicted in Figs.1(b) and (c), respectively. In the former the ξ_1 terms contribute to $O(1/Q^2)$, so that up to $O(1/Q)$ these give the same contribution as in the conventional handbag calculation.

The novel element in our analysis is the jet's quark-gluon-quark correlator $\Xi_A^\mu(l,k)$ in diagram 1(c),

$$(\Xi_A^\mu)_{ij} = \frac{1}{2}\sum_X \int \frac{d\eta^+\, d^2\boldsymbol{\eta}_T}{(2\pi)^3}\, e^{ik\cdot\eta}\, \langle 0|\mathcal{U}^{n_+}_{(+\infty,\eta)}\, gA^\mu(\eta)\ \psi_i(\eta)|X\rangle\langle X|\bar\psi_j(0)\,\mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\Big|_{\eta^-=0}. \tag{13}$$

This diagram and its Hermitian conjugate are not only important to account for all contribution of order $O(1/Q)$, but also to restore gauge invariance, which is broken in diagram 1(a) due to the different mass of the incoming and outgoing quark lines, namely, $m_q \neq M_q$.

Rather than directly using the definition (13), it is convenient to calculate the inclusive cross section as an integral of the semi-inclusive one, utilize the QCD equation of motions, sum over all hadron flavors, and take advantage of

$$\sum_h \int \cancel{\frac{d^3p_h}{(2\pi)^3 2E_h}}\, \Delta^h(l,p_h) = \Xi(l)\ , \tag{14}$$

$d^2p_{hT}\dfrac{dp_h^-}{2p_h^-}$

~~Now quantitatively, one can calculate the functions $A(l^2)$ and $A_2(l^2)$ have been calculated non perturbatively, e.g., in Schwinger-Dyson approaches such as in Ref [...], where they are called m, M and can be identified with $M(...)$ and Z, if one uses $\Lambda = 1$ GeV in Eq. (...). Utilizing the results plotted in Fig. (...) of the reference, one obtains~~

~~$$M_q \approx \ldots\ldots \text{ MeV}$$~~

Although M_q is in general a non pert. quantity, it is interesting to notice that on the perturbative vacuum,

$$\Xi^{pert}(l) = (\not{l} + m_q \mathbb{I})\,\delta(l^2 - m_q^2) + O(\alpha_s)$$

Therefore, $M_q = m_q + O(\alpha_s)$ with m_q the current quark mass, and one recovers the result of the calculation with the conventional handbag diagram.

However ....

$$\int dx \int dl^2$$

$$\cancel{\int dl^+} \quad \equiv$$

$$\int (d^4 k)(d^4 l)\, \phi(k)\, \Delta(l)\, \delta^{(4)}(k + q - l)$$

$$\cancel{\delta^{(-)}(}$$

$$\delta(q^- + k^- - l^-)\, \delta(q^+ + k^+ - l^+)$$

$$l^+ = \frac{l^2}{2l^-}$$

$$\int dl^+ \delta(q^+ + k^+ - l^+)$$

$$"=" \int \frac{dl^2}{2l^-} \delta\left(q^+ + k^+ - \frac{l^2}{2l^-}\right) \qquad q^+ = -\xi p^+$$

$$= \int dl^2\, \delta(l^2 - q^+ l^- + k^+ l^-) \qquad l^- = q^- + k^-$$

$$l^2 - (q^+ + k^+)(q^- + k^-) \to$$

$$q^+ q^- + k^+ q^- + q^+ k^- + k^+ k^-$$

$$\frac{Q^2}{2\xi} + \frac{x}{\xi}\frac{Q^2}{2} - \frac{\xi}{x}\frac{k^2 + k_T^2}{2} + \frac{k^2}{2}$$

$$k^2, k_T^2 \to 0 \implies \boxed{l^2 - \frac{Q^2}{2\xi}\,\cancel{(x+\xi)}\,(x - \xi)}$$

where Δ^h is the quark fragmentation correlator for production of a hadron of flavor h and momentum p_h [25]. In terms of the TMD fragmentation functions we are interested in, this reads

$$\sum_h \int dz d^2 p_{hT} z D_1^h(z, p_{hT}) = \xi_2 = 1 \tag{15}$$

$$\sum_h \int dz d^2 p_{hT} E^h(z, p_{hT}) = \xi_1 = M_q/\Lambda \ , \tag{16}$$

and depends on

where $D_1^h(z, p_{hT})$ is the twist-2 quark fragmentation function ~~as a function of~~ the hadron's collinear momentum fraction z and transverse momentum p_{hT}, and $E^h(z, p_{hT})$ is a chiral-odd twist-3 function defined in [25].

The relevant part of the semi-inclusive hadronic tensor for our analysis is

$$2\Lambda W^{\mu\nu} = i\frac{2\Lambda}{Q}\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho} S_{\perp\rho} \sum_q e_q^2 \Big[2x_B g_T^q(x_B) \sum_h \int dz d^2 p_{hT} D_1^{q,h}(z, p_{hT}) + 2h_1^q(x_B) \sum_h \int dz d^2 p_{hT} \tilde{E}^{q,h}(z, p_{hT}) \Big] + \dots \tag{17}$$

For clarity, here we reintroduced the quark flavor q, e_q being its electric charge. The first term can be easily integrated with the help of the sum rule (15). To integrate the latter, we first need make use of the relation $\tilde{E}(z) = E(z) - (m_q/\Lambda) z D_1(z)$, which is a consequence of the QCD equations of motion [25], then make use of the sum rules (15 - 16):

$$\sum_h \int dz d^2 p_{hT} \tilde{E}^{q,h}(z, p_{hT}) = \sum_h \int dz d^2 p_{hT} \Big[E^{q,h}(z, p_{hT}) - \frac{m_q}{\Lambda} z D_1^{q,h}(z, p_{hT}) \Big] = \xi_1^q - \frac{m_q}{\Lambda}\xi_2 = \frac{M_q - m_q}{\Lambda} \ . \tag{18}$$

This formula is the single most important result of this paper, and provides a non perturbative generalization of the commonly used $\int \tilde{E} = 0$ sum rule introduced in [13]. Indeed, calculating the jet correlator on the perturbative vacuum one would obtain, as already discussed, $M_q = m_q$ (19) and the integral would vanish.

Finally, the contraction of the hadronic tensor with the leptonic tensor leads to the following ~~well known~~ result for the inclusive DIS cross section up to order Λ/Q [25]:

$$\frac{d\sigma}{dx_B\, dy\, d\phi_S} = \frac{2\alpha^2}{x_B y Q^2} \frac{y^2}{2(1-\varepsilon)} \Big\{ F_{UU,T} + \varepsilon F_{UU,L} + S_\parallel \lambda_e \sqrt{1-\varepsilon^2}\, F_{LL} + |\boldsymbol{S}_\perp| \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S\, F_{LT}^{\cos\phi_S} \Big\} . \tag{19}$$

%

~~where~~ The structure functions on the right hand side correspond to

$$F_{UU,T} = x_B \sum_q e_q^2 f_1^q(x_B), \tag{20}$$

$$F_{UU,L} = 0, \tag{21}$$

$$F_{LL} = x_B \sum_q e_q^2 g_1^q(x_B), \tag{22}$$

$$\cancel{F_{UT}^{\sin\phi_S} = 0,} \tag{23}$$

$$F_{LT}^{\cos\phi_S} = -x_B \sum_q e_q^2 \frac{2\Lambda}{Q} \Big(x_B g_T^q(x_B) + \frac{M_q - m_q}{\Lambda} h_1^q(x_B) \Big) \tag{24}$$

%

~~**Alessandro, please double check that h_1 appears naked in Eq. (24), and that this implies dividing it by x in Eq. (25). It is fundamental for the conclusion about the ETL sum rule later on.**~~

The second term in the last structure function is a new result from our analysis; it is not suppressed as an inverse power of Q compared to the standard term. On the nonperturbative vacuum, the jet mass is larger than the quark's, and this contributes a nonnegligible ~~term~~ to the twist-3 part of the g_2 function, as we will discuss in the next section.

e new term

anomalous

IV. THE g_2 STRUCTURE FUNCTION

of the pdfs

The new term in Eq.(24) only appears in the g_2 structure function. ~~[AA: But in the past we mentioned that it could also appear in some fancier structure function – the one that also obays a WW-type relation.~~

On the perturbative vacuum it vanishes.

(*) where ϕ_S is the angle between the transverse component of the proton's spin vector and the lepton plane, at asymptotic Q^2 values, ε is the ratio of the longitudinal and transverse photon flux, and λ_e the electron's helicity [illegible].

(**) where f_1^q, g_1^q, and h_1^q are the unpolarized, polarized and transversity PDFs, and g_T^q is a twist 3 quark distribution that couples directly to the proton's transverse spin [illegible]

~~Or maybe, we were saying that ξ_1 might appear there, and the sentence as written here is correct.]~~
Following the derivation in Ref. [18], one finds

$$g_2(x_B) = g_2^{WW}(x_B) + \frac{1}{2}\sum_a e_a^2 \left(\widetilde{g}_T^{a\star}(x_B) + \int_{x_B}^1 \frac{dy}{y}\widehat{g}_T^q(y) + \frac{m_q}{\Lambda}\left(\frac{h_1^q}{x}\right)^\star (x_B) + \frac{M_q - m_q}{\Lambda}\frac{h_1^q(x_B)}{x_B} \right) , \tag{25}$$

where we defined $f^*(x) = -f(x) + \int_x^1 \frac{dy}{y} f(y)$. The first four terms coincide with the result obtained in the conventional handbag approximation [18], while the fifth is new. Note that even if the relation is written for the sum of the quark flavors weighted by their charge squared, it ~~can be considered valid~~ valid also flavor by flavor. In fact, the steps leading to such a decomposition are formulated at the correlator level.

The first term is also known as the Wandzura-Wilczeck function $g_2^{WW} = -g_1^*$, and contains all the twist-2 chiral-even contributions to the g_2 structure coming from quark-quark correlators. The second and third terms contain all "pure twist-3" contributions, i.e., those coming from quark-gluon-quark correlators. The fourth and fifth terms contain chiral-odd twist-2 contributions and depend on the transversity parton distribution function, h_1. The fourth term is usually neglected for light quarks since it is proportional to $m_q = O(1 \text{ MeV})$. The last term, new in our analysis, is again proportional to the transversity distribution but multiplied by a constant of $O(100 \text{ MeV})$, and cannot be a priori neglected.

It is important to estimate the size of the various contributions to the non Wandzura-Wilczek part of g_2. We define the shorthand notation

$$\begin{aligned} g_2^{\rm tw3} &= \frac{1}{2}\sum_q e_q^2 \left(\widetilde{g}_T^{q\star}(x_B) + \int_{x_B}^1 \frac{dy}{y}\widehat{g}_T^q(y) \right) \\ g_2^{\rm quark} &= \frac{1}{2}\sum_q e_q^2 \frac{m_q}{\Lambda}(h_1^q/x)^\star(x_B) \ , \\ g_2^{\rm jet} &= \frac{1}{2}\sum_q e_q^2 \frac{M_q - m_q}{\Lambda}\frac{h_1^q(x_B)}{x_B} \ . \end{aligned} \tag{26}$$

These terms are compared in Figure 2 to the $g_2 - g_2^{WW}$ function obtained in the very recent JAM15 fit of polarized DIS asymmetries [26], that includes a large amount of precise data at large x_B and small Q^2 from Jefferson Lab, and simultaneously fits the higher-twist components of g_1 and g_2 to the data. For the "pure twist-3" contribution, $g_2^{\rm tw3}$, *i.e.*, the contribution from quark-gluon-quark matrix elements, we show a recent light-front model calculation by Braun et al. [27]; for (modified) bag model calculations, see [28, 29]. To estimate the contributions from quark ($g_2^{\rm quark}$) and jet mass ($g_2^{\rm jet}$) effects, that depend on chiral-odd quark-quark matrix elements, we use the recent Pavia15 fit of the transversity distribution from Ref. [10], which is comparable also to other extractions [12, 30]. Furthermore, we choose the values of the mass parameters to be $m_q = 5$ MeV and $M_q = 100$ MeV. from Eq (....)

As one can see, in the proton case the pure twist-3 contribution is quite smaller in magnitude, and opposite in sign, ~~compared to~~ the JAM15 fit. The quark-mass contribution, as expected, is essentially negligible. For what concerns the jet-mass contribution, the uncertainties due to the h_1 extraction are very large, especially at low x_B. In addition, there is an overall normalization uncertainty due to the choice of M_q, not shown in the plot. In any case, the jet-mass contribution is strikingly large. If we assume that the pure twist-3 contributions are of the order of the model calculation by Braun et al., the breaking of the Wandura-Wilczek relation can be used to constrain the extractions of the transversity distribution. This is inparticular true at low x_B, where the pure twist-3 term is expected to vanish. Moreover, it is quite clear that the gap between the pure twist-3 $g_2^{\rm tw3}$ function and the JAM15 fit can be explained by the new jet-mass contribution we discuss in this paper.

In the neutron case, the jet contribution is very negative at intermediate to large values of x_B. If one trusts the order of magnitude of the $g_2^{\rm tw3}$ calculation by Braun et al., one would conclude that the jet contribution should not be that large. However, this contribution depends strongly on the d quark's transversity, whose fit suffers from large systematic uncertainties and saturates the negative Soffer bound. Recent deata in $p + p$ collisions indicate, in fact, that h_1^d might be less negative than in the Pavia15 fits [31]. Correspondingly the jet contribution to the proton at $x_B \approx 0.1$ would become less positive, inproving as well the agreement with the JAM15 fit.

V. MOMENTS OF THE g_2 STRCUTURE FUNCTION

It is interesting to consider the moments of the non Wandzura-Wilczek contribution to g_2,

$$d_N \equiv (N+1)\int_0^1 x^N \left(g_2(x) - g_2^{WW}(x) \right) . \tag{27}$$

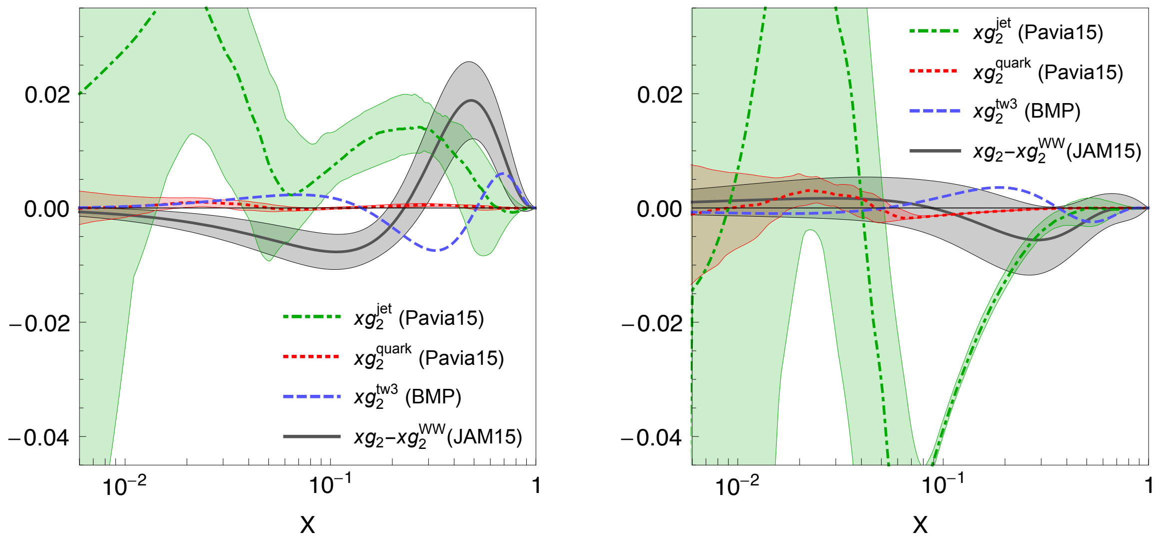


FIG. 2: Different contributions to the non Wandzura-Wilczek part of the proton (left) and neutron (right) g_2 structure function compared to the JAM15 fit of the $g_2 - g_2^{\rm WW}$ (solid black) [26]. The quark and jet contributions are shown with a dotted red and a dot-dashed green line respectively, with uncertainty bands coming form the Pavia15 fit of the transversity function [10]. The unceratainty in the choice $m_q = 5$ GeV and $M_q = 100$ GeV is not shown. The pure twist-3 contribution calculated by Braun et al. [27] is shown as a dashed blue line (no uncertainty estimate was provided in the original reference).

For a generic function f, let us define it's N-th moment as $f[N] = \int_0^1 dx\, x^N f(x)$. It is then straightforward to verify that $f^*[N] = N/(N+1)\, f[N]$ and

$$d_N = (N+1) g_2[N] + N g_1[N] \tag{28}$$

$N-1$

$$= \frac{1}{2} \sum_q e_q^2 \left(N \tilde{g}_T^q[N] + \hat{g}_T^q[N] + \frac{(N+1)M_q - m_q}{\Lambda} h_1^q[N] \right) . \tag{29}$$

do, $\quad$ hm $\quad$ h1

The zero-th moment provides an interesting relationship between transversity and the inclusive structure function g_2:

[ABMS]

$$\int dx\, g_2(x) = \sum_q e_q^2 \frac{M_q - m_q}{\Lambda} \int dx\, \frac{1}{x} h_1^q(x) . \tag{30}$$

Here we used the fact $f^*[0] = 0$ for any function f, which eliminates the contribution from the quark-mass and the twist-3 $\tilde{g}_T$ terms; instead, $\hat{g}_T^q[0]$ vanishes identically due to the symmetry properties of the quark-gluon-quark correlators, or equivalently as a consequence of the Lorentz invariance of QCD interactions, that entails $\int_0^1 dx g_1^a(x) = \int_0^1 g_T^q(x)$. The only surviving term on the right-hand side is the new jet contribution. The sum rule (30) generalizes the Burkhardt-Cottingham (BC) sum rule [32], which states that $\int_0^1 dx\, g_2(x) = 0$. Here, we have shown that jet-mass corrections can directly violate the BC sum rule. In fact, the possibility of a violation of the BC sum rule due to contributions from spin-flip processes was already mentioned in the original derivation, but these do not show up in treatments that only consider free-field quark propagators for the struck quark [13]. Although we formulated (30) in terms of a sum over quark flavors in order to display a clear connection to the structure function g_2, we stress that this is valid also flavor by flavor, i.e., for each single flavor the only measurable nonzero contribution to the zeroth moment of the structure function g_2 can come from the coupling between the jet mass and the transversity function[1].

One should notice that since h_1 is slowly driven to 0 by QCD evolution as $Q^2 \to \infty$, the BC sum rule is still satisfied at least asymptotically. At finite scales, however, the only way to preserve the validity of the Burkhardt-Cottingham

---

[1] This conclusion is true even if the BC sum rule is broken by a $J = 0$ fixed pole with non-polynomial residue [13], since this would appear as a $\delta(x_B)$ contribution and would not be measurable.

this is a bit cryptic!

is unlikely to

sum rule is if

$$\int dx\, \frac{1}{x} h_1^q(x) = 0\ . \tag{31}$$

Interestingly, one can show that this constraint, if valid at any given scale Q_0 is conserved through QCD evolution. However, we think that this constrain cannot be satisfied in general, since it is explicitly broken in perturbative QCD [33] as well as in model calculations (see, e.g., [34–39]). If, nonetheless, we assume that the BC sum rule is broken by a *finite* amount, we obtain that $h_1(x)/x$ must be integrable, implying a a bound on the small x behavior of the transversity,

to zero like x

$$h_1^q(x) \propto x^\epsilon \quad \epsilon > 0\ . \tag{32}$$

This bound can be very useful, *e.g.*, in transversity fits, where the data at small x is, as yet, very limited, and in general for proper extrapolations when measuring moments.

The first moment is the first one to display a contribution from the pure twist-3 part of g_2:

$$d_1 = \frac{1}{2}\sum_q e_q^2 \Big(2\tilde{g}_T^q[1] + \hat{g}_T^q[1] + \frac{2M_q - m_q}{\Lambda} h_1^q[0] \Big) \tag{33}$$

check W/ (31)

where $h_1^q[1] = \int_0^1 dx h_1^q(x)$ is the contribution of a quark q to the target's tensor charge. The second moment is also interesting because the pure twist-3 part can be related to quark-gluon-quark correlators, see [13], and interpreted as as the average color force experienced by the struck quark as it exits the nucleon [40]:

check if $q+\bar{q}$ or $q-\bar{q}$!?

$$d_2 = \frac{1}{2}\sum_q e_q^2 \Big(3\tilde{g}_T^q[2] + \hat{g}_T^q[2] + \frac{3M_q - m_q}{\Lambda} h_1^q[1] \Big) \tag{34}$$

For experimental measurements of d_2, see, e.g., Refs. [41–45]. For both of these moments, the transversity contribution is a background to the extraction of the pure twist-3 piece. Fortunately, it is a quantity that can be extracted from the lattice [4–8] or fitted [10–12]. Furthermore, the new sum rule (31) and the bound (32) promise to improve future transversity fits. Finally, the M_q mass parameter can as well be evaluated on the lattice, or also measured, *e.g.*, in electron-positron collisions, as we will briefly discuss later. Therefore the pure twist-3 part can, in principle, be properly isolated and measured.

Finally, we note that the jet contribution also leads to an explicit breaking of the Efremov-Teriayev-Leader (ETL) sum rule [46], in which the pure twist-3 contribution to the first moment of $g_2 - g_2^{WW}$ also disappears. To see this, let's define the valence contribution to a given structure function as $f^V = \frac{1}{2}\sum_q e_q^2 (f^q - f^{\bar{q}})$. Then, as shown in [46], $\tilde{g}_T^V[1] + \hat{g}_T^V[1] = 0$, and from Eq. (33) we obtain

$$d_1^V = \frac{1}{2}\sum_q e_q^2 \frac{2M_q - m_q}{\Lambda} \big(h_1^q[0] - h_1^{\bar{q}}[0] \big)\ . \tag{35}$$

lattice? check $p-\bar{p}$ vs $p+\bar{p}$?

$M_{light} \equiv$

Assuming $M_u \approx M_d \gg m_u, m_d$ ~~and isospin symmetry of the proton and neutron~~, we can also see that

$$d_1^V = \frac{M_{\rm light}}{\Lambda} \delta_T(p)\ , \tag{36}$$

with for proton.

This gives an alternative way to ~~experimentally~~ access the proton tensor charge, $\delta_T(p) = \sum_q e_q^2 (h_1^q[0] - h_1^{\bar{q}}[0])$, by measusing moments of the flavor separated g_2 structure function. ~~**[AA: I tried to build an observable for d_1^V such as $p - n$ or $p - \bar{p}$, but I am messing up a bit. We may try and discuss this at the Gordon.]**~~

VI. MEASURING THE JET PCFS

To experimentally measure jet functions, and in particular the jet mass parameter M_q, a promising avenue is through inclusive single hadron production, $e^+e^- \to hX$, and inclusive dihadron production from the same hemisphere, $e^+e^- \to hhX$, see Fig. 3. In single-hadron production, the fragmentation functions D^h play the role of PDFs in DIS, and couple to the jet functions in an analogous way. To access the spin-flip J_1 function one needs to detected a polarized hadron, such as a Λ baryon. In double hadron production, the enlarged number of Dirac structures of the dihadron fragmentation correlator allows one to access the jet function in novel ways, and in particular to isolate

cite Bacchetta Radici

but one integrates only up to $k^2 = Q^2(\frac{1}{x_B} - \dots)$

ally

comment on virtuality PDFs by Radyushkin

the contribution from the helicity-flip J_1 term in combination with the chiral-odd fragmentation function $H_1^{\sphericalangle}$. While measurements in the asymptotic large Q^2 regime will provide access to the integral of the J_1 jet function, i.e., to the jet-mass parameter M_q, perhaps even more interesting is the possibility to experimentally measure at finite values of Q^2 the momentum dependence of the jet functions J_1 and J_2, that enter structure functions in a way analogous to Eq. (7). In other words, it becomes possible to experimentally access the quark's spectral function.

Studying and classifying all the possibilities offered by single and double hadron production in electron-positron annihilation events will open up a rich phenomenology, which will in turn be needed to extract pure twist-3 matrix elements from the g_2 structure function, and more in general to perform precise jet mass corrections in DIS.

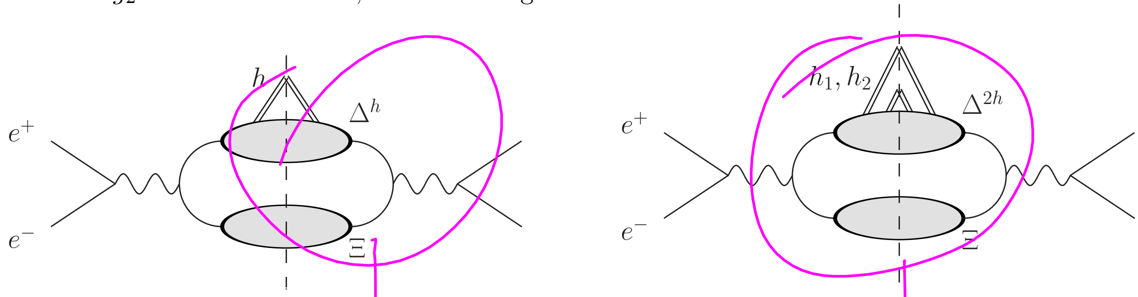


FIG. 3: Single hadron *(left)* and double hadron *(right)* production in e^+e^- collisions at LO with jet and fragmentation correlators.

pull down the figs (overlay the text)

VII. CONCLUSIONS

perturbative

In this paper, we revisited the inclusive DIS analysis, including the effects due to the production of a system of final state hadrons in the current direction, which we conveniently referred to as a "jet." We described this in terms of a jet correlator that corresponds, up to twist-4 contributions, to the nonperturbative quark cut propagator, or, equivalently, to the quark's spectral function. We then carried out the analysis of the DIS cross section up to twist-3 level. The introduction of this jet correlator leads to a difference in the expression of the structure function g_2 in inclusive DIS with respect to the standard analysis: a new term appears, proportional to a jet mass parameter and to the transversity distribution function. This new term contributes to the violation of the Wandzura-Wilczek relation, in addition to the standard pure twist-3 terms and quark mass corrections. Contrary to these standard terms, however, the new jet mass correction does not necessarily integrate to zero and so violates also the Burkhardt-Cottingham sum rule. This is yet another example of how surprising and rich the phenomenology of polarized inclusive DIS can be. Detailed measurements of the g_2 structure function can be used to constrain the jet mass parameter, the transversity distribution function and the nucleon tensor charge, helping their extraction from other observables in, e.g., electron-positron annihilation and semi-inclusive DIS, respectively. The knowledge of jet mass parameters and transversity distribution will be eventually needed for a precise extraction of pure twist-3 terms from the structure function g_2, for precise mass corrections of inclusive structure functions at small Q^2, and perhaps to experimentally access the recently proposed VPDFs!

Acknowledgments

~~whatever~~

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[AA: NOTE: the umlauts do not show up in the bibliography!! See, e.g., reference [5]. Need to fix this.]

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