Accessing the nucleon transverse structure in inclusive deep inelastic scattering

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We revisit the standard analysis of inclusive Deep Inelastic Scattering off nucleons taking into account the fact that on-shell quarks cannot be present in the final state, but they rather decay into hadrons – a process that can be described in terms of suitable "jet" correlators. As a consequence, a spin-flip term associated with the invariant mass of the produced hadrons is generated non perturbatively and couples to the target's transversity distribution function. In inclusive cross sections, this provides an hitherto neglected and large contribution to the twist-3 part of the g_2 structure function, that can explain the discrepancy between recent calculations and fits of this quantity. It also provides an extension of the Burkhardt–Cottingham sum rule, putting constraints on the small-x behavior of the transversity function, as well as an extension of the Efremov–Teryaev–Leader sum rule, suggesting a novel way to measure the tensor charge of the proton.

I. INTRODUCTION

The tensor charge is a fundamental property of the nucleon that is at present poorly constrained but of fundamental importance, not the least because its knowledge can also be used to put constraints on searches for physics beyond the Standard Model [1–3]. The tensor charge has been estimated in lattice QCD (see, *e.g.*, [4–8]), but only limited information is available from direct measurements. Its experimental extraction requires first of all flavor-separated measurements of the so-called transversity parton distribution function, denoted by $h_1^q(x)$ (see Ref. [9] for a review and Refs. [10–12] for the most recent extractions). Secondly, one needs to perform flavor-by-flavor integrals of these, that correspond to the contribution of a parton flavor q to the tensor charge.

The transversity distribution is notoriously difficult to access because it is a chiral-odd function and needs to be combined with a spin-flip mechanism to appear in a scattering process [13]. Usually, this spin flip is provided by another nonperturbative distribution or fragmentation function, accessible in Drell-Yan or semi-inclusive Deep Inelastic Scattering (DIS) [14–17]. The only other known way to attain spin-flip terms in Quantum Electro-Dynamics and QCD is taking into account mass corrections. In fact, it is well known that the transversity distribution gives a contribution to the structure function g_2 in inclusive DIS (see, *e.g.*, [18] and references therein), and in particular to the violation of the so-called Wandzura–Wilczek relation for g_2 [19]. However, this contribution is proportional to the current quark mass and can be expected to be negligibly small.

In this paper, we discuss a novel way of accessing the transversity parton distribution function (PDF) and measuring the proton's tensor charge in totally inclusive Deep Inelastic Scattering. We revisit the standard analysis of the DIS handbag diagram, taking into account the fact that on-shell quarks cannot, in fact, be present in the final state, but they rather decay and form (mini)jets of hadrons. This is sufficient to modify the structure of the DIS cut diagram, even if none of those hadrons is detected in the final state. For a proper description of this effect, we include "jet correlators" into the analysis, and pay particular attention to ensuring that our results are gauge invariant.

The jet correlators describe interactions of a perturbative quark with vacuum fields, that break chiral symmetry and generate a non perturbative mass of order 100 MeV, much larger than the current quark mass for light flavors, as also heuristically advocated in Ref. [20] for a study of transverse target single-spin asymmetries in two-photon exchange processes. Here, we formalize this idea in the context of collinear factorization, and observe that jet correlators introduce a new contribution already in one-photon exchange processes, and more precisely to the inclusive g_2 structure function. The new term is proportional to the transversity distribution function multiplied by a new nonperturbative "jet mass", which will be precisely defined below, and has the interesting features that: (a) it violates the Wandzura–Wilczek relation; (b) it extends the Burkhardt–Cottingham sum rule, providing a useful bound on the small-x behavior of the transversity distribution; (c) it also extends the Efremov–Teryaev–Leader sum rule, providing a novel way to measure the proton's tensor charge. We estimate this new jet-mass-induced contribution based on a recent extraction of the transversity distribution, and show it can indeed be very large.

II. THE QUARK-QUARK JET CORRELATOR

Motivated by mass corrections to inclusive DIS structure functions at large values of the Bjorken invariant x_B, Accardi and Qiu [21] have introduced in the LO handbag diagram a "jet correlator", also called "jet factor" by

Collins and Rogers in Ref. [22], that accounts for invariant mass production in the current region and ensures that leading twist calculations in collinear factorization are consistent with the $x_B < 1$ requirement imposed by baryon number conservation. [21]. The jet correlator is depicted in Figure 1(a) and is defined as

$$\Xi_{ij}(l, n_+) = \int \frac{d^4\eta}{(2\pi)^4} \, e^{il\cdot\eta} \, \langle 0| \, \mathcal{U}^{n_+}_{(+\infty,\eta)} \, \psi_i(\eta)\bar{\psi}_j(0) \, \mathcal{U}^{n_+}_{(0,+\infty)} \, |0\rangle \,, \tag{1}$$

In this definition, l is the quark's four-momentum, Ψ the quark field operator (with quark flavor index omitted for simplicity), and $|0\rangle$ is the nonperturbative vacuum state. Furthermore, the correlator's gauge invariance is explicitly guaranteed the two Wilson line operators $\mathcal{U}^{n_+}$, that run to infinity first along along a light-cone plus direction determined by the vector n_+, then along the direction transverse to that vector, see [23] for details. This path choice for the Wilson line is required by QCD factorization theorems, and the vector n_+ is determined by the particular hard process to which the jet correlator contributes. For example, in the case of inclusive DIS discussed in this paper, this is determined by the four momentum transfer q and the proton's momentum p.

The correlator Ξ can be parametrized in terms of jet parton correlation functions A_i and B_i through a Lorentz covariant Dirac decomposition that utilizes the vectors l and n_+,

$$\Xi(l, n_+) = \Lambda A_1(l^2)\, \mathbf{1} + A_2(l^2)\slashed{l} + \frac{\Lambda^2}{l\cdot n_+}\slashed{n}_+ \, B_1(l^2) + \frac{i\Lambda}{2l\cdot n_+}[\slashed{l},\slashed{n}_+]\, B_2(l^2) \,, \tag{2}$$

where Λ is an arbitrary scale, introduced for power counting purposes. In this parametrization, no terms proportional to γ_5 enter because of parity invariance. Time reversal invariance in QCD requires $B_2 = 0$, while B_1 contributes only at twist-4 order and will not be considered further in this paper. We focus, instead, on the role of chiral odd terms in the g_2 structure function up to twist 3. At this order,

$$\Xi(l, n_+) = \Lambda A_1(l^2)\, \mathbf{1} + A_2(l^2)\slashed{l} + \mathcal{O}(\Lambda^2/Q^2) \tag{3}$$

is nothing else than the cut quark propagator; note however, that we consider here the full QCD vacuum rather than the perturbative one (or, in other words, the interacting rather than the free quark fields). The A_1 and A_2 terms can be interpreted in terms of the spectral representation of the cut quark propagator (see, e.g., Sec. 6.3 of [24] and Sec. 2.7.2 of [25]),

$$\Xi(l) = \int d\mu^2 \left[J_1(\mu^2)\, \mu\, \mathbf{1} + J_2(\mu^2)\slashed{l} \right] \delta(l^2 - \mu^2) \,, \tag{4}$$

where μ^2 can be interpreted as the invariant mass of the current jet, i.e., of the particles going through the cut in the top blob of Fig.1(a). The J_i are the spectral functions of the quark propagator, also called "jet functions" in [21], and can be interpreted as current-jet mass distributions. As a consequence of positivity constraints and CPT invariance, these satisfy [24–26]

$$J_2(\mu^2) \geq J_1(\mu^2) \geq 0 \quad \text{and} \quad \int d\mu^2 J_2(\mu^2) = 1 \,. \tag{5}$$

From a comparison of Eqs.(2) and (4), one can see that

$$A_1(l^2) = \frac{\sqrt{l^2}}{\Lambda} J_1(l^2) \qquad\qquad A_2(l^2) = J_2(l^2) \,. \tag{6}$$

When inserting the jet correlator in the handbag diagram for inclusive DIS, the integration over dl^+, or equivalently $dl^2/(2l^-)$, is kinematically coupled to the other integrations, and induces corrections of order $\mathcal{O}(1/Q^2)$ whose effect on the F_2 structure function has been studied in Ref. [21]. In this paper, where we limit our attention to effects of order $\mathcal{O}(1/Q)$, we can neglect k^- compared to q^-. As a consequence, we can extend the integration over dl^2 to infinity, with the consequence that the jet correlator decouples from the parton correlator Φ, and the inclusive structure functions only depend on the integrated jet correlator

$$\Xi(l^-, \boldsymbol{l}_T) \equiv \int \frac{dl^2}{2l^-} \Xi(l) = \frac{\Lambda}{2l^-} \xi_1 \mathbf{1} + \xi_2 \frac{\slashed{n}_-}{2} + \mathcal{O}(l_T/l^-) + \text{higher twists} \,. \tag{7}$$

The neglected $\boldsymbol{l}_T$-dependent and higher twist terms only contribute to $\mathcal{O}(1/Q^2)$ to the inclusive cross section. Note that thanks to Eq. (5) we obtain

$$\xi_1 = \int d\mu^2 \frac{\mu}{\Lambda} J_1(\mu^2) \equiv \frac{M_q}{\Lambda}, \qquad\qquad \xi_2 = \int d\mu^2 J_2(\mu^2) = 1 \,, \tag{8}$$
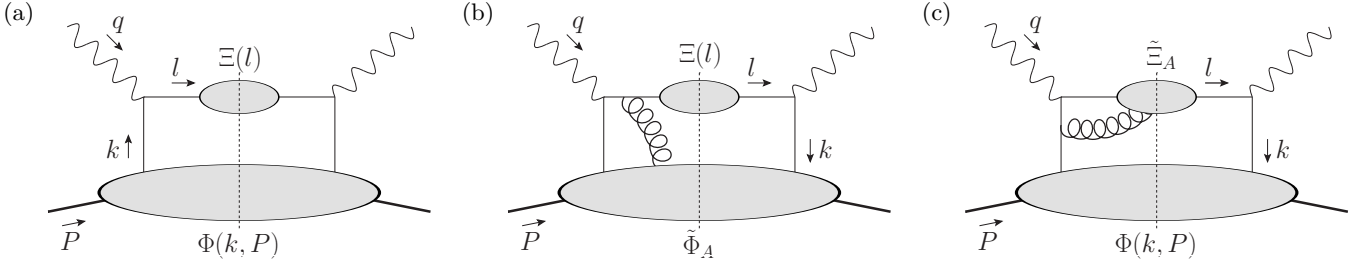
FIG. 1: Diagrams contributing to inclusive DIS scattering up to twist-3, including a jet correlator in the top part. Note the gluon attaches to both the nucleon and jet correlators. The Hermitian conjugates of diagrams (b) and (c), i.e., with gluons attaching to the right of the cut, are not shown.

where M_q can be interpreted as the average invariant mass produced in the spin-flip fragmentation processes of a quark of flavor q. It is important to notice that while $\xi_2 = 1$ exactly due to CPT invariance (see Sec. 10.7 of Ref. [26]), the jet mass $0 < M_q < \int d\mu^2 \mu J_2(\mu^2)$ is dynamically determined. From the analytic properties of the spectral functions we know that $J_2(\mu^2) = Z\delta(\mu^2 - m_q) + \bar{J}_2(\mu^2)\theta(\mu^2 - m_\pi^2)$ with $Z < 1$ and the continuum starting at m_π (the mass of the pion) due to color confinement effects. Taking into account that $J_1 < J_2$, we may therefore estimate

$$M_q = \mathcal{O}(100 \text{ MeV}) . \tag{9}$$

Although M_q is in general a nonperturbative quantity, it is interesting to notice that on the perturbative vacuum,

$$\Xi^{\text{pert}}(l) = (\slashed{l} + m_q \mathbf{1}) \, \delta(l^2 - m_q^2) + \mathcal{O}(\alpha_s) ,$$

where m_q is the current quark mass; therefore $M_q^{\text{pert}} = m_q$, and one recovers the result of the calculation with the conventional handbag diagram. However, we are here considering non perturbative effects in the quark propagation, and $M_q \gg m_q$. Therefore, differently from J_2, the J_1 function leaves an imprint on the inclusive DIS cross section even in the asymptotic $Q^2 \to \infty$ regime.

III. TWIST-3 ANALYSIS

Extending this analysis to the calculation of twist-3 structure functions requires not only to consider the ξ_1 term in the jet correlator, but also quark-gluon-quark correlators in both the proton and the vacuum as depicted in Figs.1(b) and (c), respectively.

In diagram (b), the ξ_1 term contributes to $\mathcal{O}(1/Q^2)$, so that up to $\mathcal{O}(1/Q)$ considered in this paper this give the same contribution as in the conventional handbag calculation. The novel element in our analysis, instead, is the jet's quark-gluon-quark correlator $\Xi_A^\mu(l, k)$ in diagram (c), defined as

$$\left(\tilde{\Xi}_A^\mu\right)_{ij} = \int \frac{d^4\eta}{(2\pi)^4} \, e^{ik\cdot\eta} \, \langle 0| \mathcal{U}_{(+\infty,\eta)}^{n_+} \, gA^\mu(\eta) \, \psi_i(\eta) \, \bar{\psi}_j(0) \mathcal{U}_{(0,+\infty)}^{n_+} |0 \rangle . \tag{10}$$

This diagram and its Hermitian conjugate are not only important to account for all contribution of order $\mathcal{O}(1/Q)$, but also to restore gauge invariance, which is broken in diagram 1(a) due to the different mass of the incoming and outgoing quark lines, namely, $m_q \neq M_q$.

Rather than directly using the definition (10), it is convenient and instructive to calculate the inclusive cross section as an integral of the semi-inclusive one summed over all produced hadron flavors, then utilize the QCD equations of motion, sum over all hadron flavors, and take advantage of

$$\sum_h \int d^2 p_{hT} \frac{dp_h^-}{2p_h^-} \, p_h^- \, \Delta^h(l, p_h) = l^- \, \Xi(l) , \tag{11}$$

where Δ^h is the quark fragmentation correlator for production of a hadron of flavor h and momentum p_h, discussed in detail in Ref. [23]. In terms of the TMD fragmentation functions we are interested in, the sum rule (11) reads

$$\sum_h \int dz d^2 p_{hT} z D_1^h(z, p_{hT}) = \xi_2 = 1 \tag{12}$$

$$\sum_h \int dz d^2 p_{hT} E^h(z, p_{hT}) = \xi_1 = M_q/\Lambda , \tag{13}$$

where $D_1^h(z, p_{hT})$ is the twist-2 quark fragmentation function, that depends on the hadron's collinear momentum fraction z and transverse momentum p_{hT}, and $E^h(z, p_{hT})$ is a chiral-odd twist-3 function defined in [23].

The role of the $\xi_1 = M_q/\Lambda$ term in inclusive DIS can be discussed by analyzing the following terms of the semi-inclusive hadronic tensor [27]:

$$2MW^{\mu\nu} = i\frac{2M}{Q}\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho}S_{\perp\rho} \tag{14}$$

$$\times \sum_q e_q^2 \left[2x_B g_T^q(x_B) \sum_h \int d^2p_{hT}dz\, z\, D_1^{q,h}(z, p_{hT}) + \frac{2\Lambda}{M}h_1^q(x_B)\sum_h \int d^2p_{hT}dz\, \tilde{E}^{q,h}(z, p_{hT}) \right] + \dots ,$$

where $g_T^q(z, p_{hT})$ and $\tilde{E}^q(z, p_{hT})$ are twist-3 TMDs originating, respectively, from the quark-quark and the quark-gluon-quark fragmentation correlators. Note that in Eq. (14) M is the proton's mass, and we identified the power counting scale M_h of Ref. [27] with our Λ. For clarity, we also reintroduced the quark flavors q, e_q being their respective electric charge. The first term can be easily integrated with the help of the sum rules (12) and (13). To integrate the second term, however, we first need make use of the relation $\tilde{E}(z) = E(z) - (m_q/\Lambda)zD_1(z)$, which is a consequence of the QCD equations of motion [23], then make again use of the sum rules (12)-(13) to obtain

$$\sum_h \int dz d^2p_{hT}\, \tilde{E}^{q,h}(z, p_{hT}) = \xi_1 - \frac{m_q}{\Lambda}\xi_2 = \frac{M_q - m_q}{\Lambda} . \tag{15}$$

This formula provides us with a non perturbative generalization of the commonly used $\int \tilde{E} = 0$ sum rule introduced in [13]. Indeed, calculating the jet correlator on the perturbative vacuum one would obtain, as already discussed, $M_q = m_q$ and the integral would vanish.

Finally, the contraction of the hadronic tensor with the leptonic tensor leads to the following result for the inclusive DIS cross section up to order M/Q [23]:

$$\frac{d\sigma}{dx_B\, dy\, d\phi_S} = \frac{2\alpha^2}{x_B yQ^2}\frac{y^2}{2\,(1-\varepsilon)}\left\{ F_T + \varepsilon F_L + S_\parallel \lambda_e\sqrt{1-\varepsilon^2}\, F_{LL} + |\boldsymbol{S}_\perp|\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\cos\phi_S\, F_{LT}^{\cos\phi_S} \right\} , \tag{16}$$

where ϕ_S is the angle between the transverse component of the proton spin vector and the lepton plane, ϵ is the ratio of the longitudinal and transverse photon fluxes, and λ_e is the electron's helicity. The structure functions on the right hand side read

$$F_T = x_B \sum_q e_q^2 f_1^q(x_B), \tag{17}$$

$$F_L = 0, \tag{18}$$

$$F_{LL} = x_B \sum_q e_q^2 g_1^q(x_B), \tag{19}$$

$$F_{UT}^{\sin\phi_S} = 0, \tag{20}$$

$$F_{LT}^{\cos\phi_S} = -x_B \sum_q e_q^2 \frac{2M}{Q}\left(x_B g_T^q(x_B) + \frac{M_q - m_q}{M}h_1^q(x_B)\right) , \tag{21}$$

where f_1^q, g_1^q and h_1^q are the unpolarized, polarized, and transversity PDFs, respectively. The second term in the last structure function is a new result from our analysis, it is proportional to the jet mass, and it is not suppressed as an inverse power of Q compared to the standard g_T term. Perturbatively, $M_q^{\text{pert}} = m_q$ and the new term vanishes. However, on the nonperturbative vacuum the jet mass M_q is much larger than the quark's current mass m_q, originating a nonnegligible term to the twist-3 part of the target's g_2 structure function, as we will discuss in the next section.

IV. THE g_2 STRUCTURE FUNCTION

The structure functions in Eqs. (19) – (21) can be related to the usual structure functions g_1 and g_2 defined from the following Lorentz decomposition of the antysymmetric part of the inclusive hadronic tensor

$$W_A^{\mu\nu}(P, q) = \frac{1}{P\cdot q}\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[S_\sigma g_1(x_B, Q^2) + \left(S_\sigma - \frac{S\cdot q}{P\cdot q}p_\sigma\right)g_2(x_B, Q^2)\right] . \tag{22}$$

Then, neglecting contrinutions of order $1/Q^2$, one obtains [23],

$$g_1 = \frac{1}{2x_B} F_{LL} \tag{23}$$

$$g_2 + g_1 = -\frac{Q}{4x_B^2 M} F_{LT}^{\cos \phi_S} \ . \tag{24}$$

Utilizing equations of motion and Lorentz invariance relations as discussed in Ref. [18] to decompose g_T into "pure twist-3" and twist-2 pieces, we arrive at

$$g_2(x_B) = g_2^{WW}(x_B) + \frac{1}{2}\sum_a e_a^2 \left(\widetilde{g}_T^{a\star}(x_B) + \int_{x_B}^1 \frac{dy}{y} \widehat{g}_T^q(y) + \frac{m_q}{M}\left(\frac{h_1^q}{x}\right)^\star(x_B) + \frac{M_q - m_q}{M}\frac{h_1^q(x_B)}{x_B} \right) , \tag{25}$$

where we used $f^*(x) = f(x) - \int_x^1 \frac{dy}{y} f(y)$, and $\tilde{g}_T$ and $\hat{g}_T$ are pure twist-3 functions that only depend on projections of quark-gluon-quark correlator, and are explicitly defined in that reference. The first four terms coincide with the result obtained in the conventional handbag approximation [18], while the last is new. Note that even if the relation is written for the sum over quark flavors weighted by their charge squared, it is also valid flavor by flavor; in fact, the steps leading to such a decomposition are formulated at the quark correlator level.

The first term is also known as the Wandzura-Wilczek function, $g_2^{WW} = -g_1^*$, with $g_1 = \frac{1}{2}\sum_q e_q^2 g_1^q$, and contains all the twist-2 chiral-even contributions to the g_2 structure coming from quark-quark correlators. The second and third terms contain all "pure twist-3" contributions, i.e., those coming from quark-gluon-quark correlators. The fourth and fifth terms contain chiral-odd twist-2 contributions and depend on the transversity distribution function, h_1. The fourth term is usually neglected for light quarks since it is proportional to $m_q = \mathcal{O}(1 \text{ MeV})$. The last term, new in our analysis, is again proportional to the transversity distribution but multiplied by the jet mass $M_q = \mathcal{O}(100 \text{ MeV})$, so that it cannot be a priori neglected.

It is important to estimate the size of the various contributions to the non Wandzura-Wilczek part of g_2. We define the shorthand notation

$$g_2^{\text{tw3}} = \frac{1}{2}\sum_q e_q^2 \left(\widetilde{g}_T^{q\star}(x_B) + \int_{x_B}^1 \frac{dy}{y}\widehat{g}_T^q(y) \right)$$

$$g_2^{\text{quark}} = \frac{1}{2}\sum_q e_q^2 \frac{m_q}{M}\left(\frac{h_1^q}{x}\right)^\star(x_B) , \tag{26}$$

$$g_2^{\text{jet}} = \frac{1}{2}\sum_q e_q^2 \frac{M_q - m_q}{M}\frac{h_1^q(x_B)}{x_B} \ .$$

These terms are compared in Figure 2 to the $g_2 - g_2^{WW}$ function obtained in the very recent JAM15 fit of polarized DIS asymmetries [28], that includes a large amount of precise data at large x_B and small Q^2 from Jefferson Lab, and simultaneously fits the higher-twist components of g_1 and g_2 to the data.[1] For the "pure twist-3" contribution, g_2^{tw3}, i.e., the contribution from quark-gluon-quark matrix elements, we show a recent light-front model calculation by Braun et al. [29] (for bag model calculations, see [30, 31]). To estimate the contributions from quark (g_2^{quark}) and jet mass (g_2^{jet}) effects, that depend on chiral-odd quark-quark matrix elements, we use the recent Pavia15 fit of the transversity distribution from Ref. [10], which is comparable also to other extractions [12, 32]. Furthermore, we choose the values of the mass parameters to be $m_q = 5$ MeV and $M_q = 100$ MeV.

As one can see, in the proton case the pure twist-3 contribution is quite smaller in magnitude than, and nearly opposite in sign compared to, the twist-3 term extracted in the JAM15 fit. The quark-mass contribution, as expected, is essentially negligible. For what concerns the jet-mass contribution, the uncertainties due to the h_1 extraction are very large, especially at low x_B. In addition, there is an overall normalization uncertainty due to the choice of M_q, not shown in the plot. In any case, the jet-mass contribution is strikingly large, and of the same order of magnitude as the chiral-even twist 3 term. If we assume the latter to be of the order of the model calculation by Braun et al., the breaking of the Wandzura-Wilczek relation can be used to constrain the extractions of the transversity distribution. This is in particular true at low x_B, where the pure twist-3 term is expected to vanish. Moreover, it is quite clear that

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[1] Notice, however, that the JAM15 fit imposes the $\int dx g_2(x) = 0$ Burkhardt–Cottingham sum rule, which, however, is broken by inclusion of jet correlators, as discussed in Section V.

the gap between the pure twist-3 g_2^{tw3} function and the JAM15 fit can be explained by the new jet-mass contribution we discuss in this paper.

In the neutron case, the jet contribution is very negative at intermediate to large values of x_B. If one trusts the order of magnitude of the g_2^{tw3} calculation by Braun et al., one would conclude that the jet contribution should not be that large. However, for a neutron target, g_2^{jet} depends strongly on the d quark's transversity, whose fit suffers from large systematic uncertainties and saturates the negative Soffer bound. Recent data in $p + p$ collisions indicate, in fact, that $h_1^{q=d}$ might be less negative than in the Pavia15 fits [33]. Correspondingly the jet contribution to the proton at $x_B \approx 0.1$ would become less positive, improving as well the agreement with the JAM15 fit.
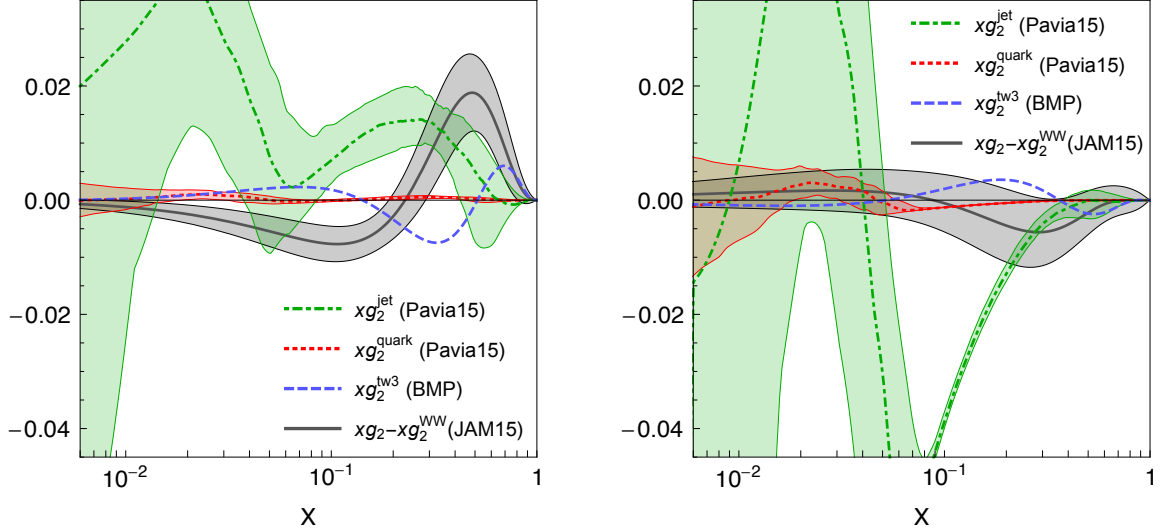

FIG. 2: Different contributions to the non Wandzura-Wilczek part of the proton (left) and neutron (right) g_2 structure functions compared to the JAM15 fit of the $g_2 - g_2^{\text{WW}}$ function (solid black) [28]. The quark and jet contributions are shown with a dotted red and a dot-dashed green line respectively, with uncertainty bands coming form the Pavia15 fit of the transversity function [10]. The uncertainty in the choice $m_q = 5$ MeV and $M_q = 100$ MeV is not shown. The pure twist-3 contribution calculated by Braun et al. [29] is shown as a dashed blue line (no uncertainty estimate was provided in the original reference).

V.   MOMENTS OF THE g_2 STRUCTURE FUNCTION

It is interesting to consider the moments of the non Wandzura-Wilczek contribution to g_2,

$$d_N \equiv (N + 1) \int_0^1 dx\, x^N \left(g_2(x) - g_2^{WW}(x) \right) . \tag{27}$$

For a generic function f, let us define it's N-th moment as $f[N] = \int_0^1 dx\, x^N f(x)$. It is then straightforward to verify that $f^*[N] = f[N]\, N/(N + 1)$ and

$$d_N = (N + 1)g_2[N] + N g_1[N] \tag{28}$$

$$= \frac{1}{2} \sum_q e_q^2 \left[N \tilde{g}_T^q[N] + \hat{g}_T^q[N] + \frac{(N + 1)M_q - m_q}{M} h_1^q[N - 1] \right] . \tag{29}$$

The zero-th moment, $d_0 = \int g_2$, provides an interesting relationship between transversity and the inclusive structure function g_2:

$$\int dx\, g_2(x) = \sum_q e_q^2 \frac{M_q - m_q}{M} \int dx\, \frac{1}{x} h_1^q(x) . \tag{30}$$

Assuming $M_{\text{light}} \equiv M_u - m_u \approx M_d - m_d$ and dominance of light quarks in the sum over flavors, we can also write

$$\int dx\, g_2(x) = \frac{M_{\text{light}}}{M} \int dx\, \frac{h_1(x)}{x} \,, \tag{31}$$

with h_1 the transversity structure function.

In Eq. (30), we used the fact that $\hat{g}_T^q[0]$ vanishes identically due to the symmetry properties of the quark-gluon-quark correlators [18]. Therefore all pure twist-3 terms have explicitly disappeared, and the only surviving term on the right-hand side is the new jet contribution. Thus, our new sum rule (30) generalizes the Burkhardt–Cottingham (BC) sum rule [34], which states that $\int_0^1 dx\, g_2(x) = 0$, while we have shown that jet-mass corrections, and in particular from invariant mass generation in spin-flip processes, can directly violate this. In fact, the possibility of a violation of the BC sum rule due to contributions from spin-flip processes was already mentioned in the original derivation, but these do not show up in treatments that only consider free-field quark propagators for the struck quark [13]. Although we formulated (30) in terms of a sum over quark flavors in order to display a clear connection to the structure function g_2, we stress that this is valid also flavor by flavor, i.e., for each single flavor the only measurable nonzero contribution to the zeroth moment of the structure function g_2^q comes from the coupling between its jet mass and transversity function[2].

One should notice that since h_1 is slowly driven to 0 by QCD evolution as $Q^2 \to \infty$, the BC sum rule is still satisfied at least asymptotically. At finite scales, however, the only way to preserve the validity of the BC sum rule is if $\int dx\, \frac{1}{x} h_1^q(x) = 0$. Interestingly, one can show that this constraint, if valid at any given scale Q_0 is conserved through QCD evolution. However, we think that it is unlikely to be satisfied in general, since it is explicitly broken in perturbative QCD [35], as well as in model calculations [36–41]. Then, if we assume that the BC sum rule is broken by a *finite* amount we obtain that $h_1(x)/x$ must be integrable, which implies the following bound on the small x behavior of the transversity:

$$h_1^q(x) \propto x^\epsilon \quad \epsilon > 0 \,. \tag{32}$$

This bound can be very useful, *e.g.*, in transversity fits, where the data at small x is, as yet, very limited, and in general for proper extrapolations to $x_B = 0$ when experimentally measuring moments. We also note that the small-x behavior of the longitudinal spin structure function g_1 has been recently investigated in Ref. [42]; it would be interesting to extend those techniques to the transverse spin structure functions $g_T = g_1 + g_2$ and h_1.

The first moment, d_1, is the first one to display a contribution from the pure twist-3 part of g_2:

$$d_1 = \frac{1}{2} \sum_q e_q^2 \left(2\tilde{g}_T^q[1] + \hat{g}_T^q[1] + \frac{2M_q - m_q}{M} h_1^q[0] \right) \tag{33}$$

where $h_1^q[0] = \int_0^1 dx\, h_1^q(x)$ is the contribution of a quark q to the target's tensor charge. The second moment,

$$d_2 = \frac{1}{2} \sum_q e_q^2 \left(3\tilde{g}_T^q[2] + \hat{g}_T^q[2] + \frac{3M_q - m_q}{M} h_1^q[1] \right) \,, \tag{34}$$

is also interesting because the pure twist-3 part can be related to quark-gluon-quark local matrix elements, see [13], and interpreted as the average color force experienced by the struck quark as it exits the nucleon [43]; for experimental measurements of d_2, see, e.g., Refs. [44–48].

For both the d_1 and d_2 moments, the transversity contribution is a background to the extraction of the pure twist-3 piece. Fortunately, it is a quantity that can be extracted from the lattice [4–8] or fitted [10–12]. Furthermore, the bound (32) promises to improve future transversity fits. Therefore the pure twist-3 part can, in principle, be properly isolated and measured.

We should also note that the M_q jet mass parameter can also be experimentally measured, *e.g.*, in electron-positron collisions. A promising avenue is through inclusive single hadron production, $e^+ e^- \to hX$, and inclusive dihadron production from the same hemisphere, $e^+ e^- \to hhX$, see Fig.3 In single-hadron production, the fragmentation functions play the role of PDFs in DIS and couple to the jet functions in an analogous way. To access the spin-flip J_1 function one needs to detect a polarized hadron, such as a Δ baryon. In double hadron production, the enlarged

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[2] This conclusion is true even if the BC sum rule is broken by a $J = 0$ fixed pole with non-polynomial residue [13], since this would appear as a $\delta(x_B)$ contribution and would not be measurable.
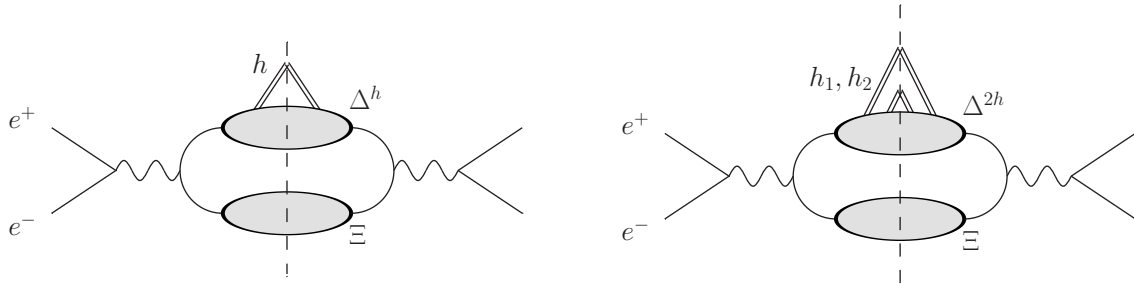
FIG. 3: Single hadron *(left)* and double hadron *(right)* production in e^+e^- collisions at LO with jet and fragmentation correlators.

number of Dirac structures of the dihadron fragmentation correlators related to the relative momentum of the two hadrons [49, 50] allows one to access the jet function in novel ways, and in particular to isolate the contribution from the helicity-flip J_1 term in combination with the chiral-odd fragmentation function $H_1^{\sphericalangle}$.

To conclude this section, we note that the jet contribution also leads to an explicit breaking of the Efremov–Teryaev–Leader (ETL) sum rule [51], in which the pure twist-3 contribution to the first moment of $g_2 - g_2^{WW}$ also disappears. To see this, let's define the valence contribution to a given structure function as $f^V = \frac{1}{2}\sum_q e_q^2(f^q - f^{\bar{q}})$. Then, as shown in [51], $\widehat{2\tilde{g}}_T^V[1] + g_T^V[1] = 0$, and from Eq. (33) we obtain

$$d_1^V = \frac{1}{2}\sum_q e_q^2 \frac{2M_q - m_q}{M}\left(h_1^q[0] - h_1^{\bar{q}}[0]\right) . \tag{35}$$

Assuming again dominance of light flavors, we can also see that

$$d_1^V = \frac{M_{\text{light}}}{M}\delta_T(p) , \tag{36}$$

This gives an alternative way to access the proton tensor charge, $\delta_T(p) = \sum_q e_q^2\left(h_1^q[0] - h_1^{\bar{q}}[0]\right)$, by measuring or fitting moments of the flavor separated g_2 structure function.

VI. CONCLUSIONS

In this paper, we revisited the inclusive DIS analysis, including the effects due to the production of a system of final state hadrons in the current direction, which we conveniently referred to as a "jet." We described this in terms of a jet correlator that corresponds, up to twist-4 contributions, to the nonperturbative quark cut propagator, or, equivalently, to the quark's spectral function, and of a quark-gluon-quark jet correlator needed to insure gauge invariance of the calculation. We then carried out the analysis of the DIS cross section up to contribution of order $1/Q$. The introduction of the jet correlators leads to a difference in the expression of the structure function g_2 in inclusive DIS with respect to the standard analysis: a new term appears, proportional to a jet mass parameter $M_q = \mathcal{O}(100 \text{ MeV})$ and to the transversity distribution function. This new term contributes to the violation of the Wandzura-Wilczek relation, in addition to the standard pure twist-3 terms and quark mass corrections. Contrary to these standard terms, however, the new jet mass correction does not necessarily integrate to zero and so violates also the Burkhardt–Cottingham and Efremov–Teryaev–Leader sum rules. This is yet another example of how surprising and rich the phenomenology of polarized inclusive DIS can be.

Detailed measurements of the g_2 structure function can be used to constrain the jet mass parameter M_q, the transversity distribution function and the nucleon tensor charge, helping their extraction from other observables, e.g., in electron-positron annihilation and semi-inclusive DIS. Knowledge of the jet mass parameter and of the transversity distribution will eventually be needed for a precise extraction of pure twist-3 terms from the g_2 structure function, or from transverse target single spin asymmetries [52].

Finally, studying and classifying all the contributions of jet correlators to single and double hadron production in electron-positron annihilation events will open up a rich phenomenology. Measurements in the asymptotically large Q^2 regime will provide access to the integral of the J_1 jet function, i.e., to the jet-mass parameter M_q, and therefore (in conjunction with precise measurements or lattice QCD calculations of the first h_1 moment) also of the target's tensor charge through the modified ETL sum rule. Equally interesting is the possibility to experimentally measure, at finite values of Q^2, the momentum dependence of the jet functions J_1 and J_2, that enter structure functions integrated

only up to $\mu^2 = Q^2(1/x_B - 1)$ [21]. In other words, it may become possible to experimentally access also the quark's spectral function itself.

Acknowledgments

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