# ASSIGNMENT 8

Problem 1 :-

$$x(K+1) = \begin{bmatrix} 0.3 & -0.4 \\ 0.4 & 0.25 \end{bmatrix} x(K) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(K)$$

$$A = \begin{bmatrix} 0.3 & -0.4 \\ 0.4 & 0.25 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad L = \begin{bmatrix} a & b \end{bmatrix}$$

For dead-beat controller, place the poles of $(A-BL)$ @ origin.
eigenvalues

$$A - BL = \begin{bmatrix} 0.3 & -0.4 \\ 0.4 & 0.25 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} a & b \end{bmatrix}$$

$$= \begin{pmatrix} 0.3 & -0.4 \\ 0.4 & 0.25 \end{pmatrix} - \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0.3 - a & -0.4 - b \\ 0.4 & 0.25 \end{pmatrix}$$

$|A - \lambda I| = 0$

$$\begin{vmatrix} 0.3 - a - \lambda & -0.4 - b \\ 0.4 & 0.25 - \lambda \end{vmatrix} = 0.$$

$$0.25(0.3 - a - \lambda) - \lambda(0.3 - a - \lambda) + (0.4b + 0.16) = 0$$

or $\lambda^2 + \underbrace{(-0.25 - 0.3 + a)}_{\to 0}\lambda + \underbrace{(0.4b + 0.16 + 0.075 - 0.25a)}_{\to 0} = 0$

or $\lambda^2 + (a - 0.55)\lambda + (0.235 + 0.4b - 0.25a) = 0.$

$a = 0.55, \quad b = -0.24375.$

1. $L_{db} = \begin{bmatrix} 0.55 & -0.24375 \end{bmatrix}$

2. The poles of $(A-BL)$ have to be placed at $\lambda = 0.25, 0.4$

From the quadratic equation,

$$\lambda^2 + (a - 0.55)\lambda + (0.235 + 0.4b - 0.25a) = 0.$$

(i) First place $\lambda = 0.25$

(ii) Place $\lambda = 0.4$

(iii) Simultaneously solve both relations in a and b.

$$L_{pp} = \begin{bmatrix} -0.1 & -0.4 \end{bmatrix}$$

Problem 3:

8. Statements that are true:-

(i) The magnitude of LQR gain decreases as R is increased

(ii) The LQR poles go closer to open loop poles when R increases.

Problem 4:

$L\alpha = [0.3 \quad -0.35]$

$x(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$A = \begin{bmatrix} 0.3 & -0.4 \\ 0.4 & 0.25 \end{bmatrix}$ $\qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$x(1) = (A - BL\alpha)\, x(0)$

$x(1) = \left( \begin{bmatrix} 0.3 & -0.4 \\ 0.4 & 0.25 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0.3 & -0.35 \end{bmatrix} \right) \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

9. $x(1) = \begin{bmatrix} -0.1 \\ 0.9 \end{bmatrix}$

$x(2) = (A - BL\alpha)\, x(1)$

10. $x(2) = \begin{bmatrix} -0.045 \\ 0.185 \end{bmatrix}$

$x(3) = (A - BL\alpha)\, x(2)$

11. $x(3) = \begin{bmatrix} -0.0093 \\ 0.0282 \end{bmatrix}$

12. $x(4) = (A - BL\alpha)\, x(3)$

$x(4) = \begin{bmatrix} -0.0014 \\ 0.0034 \end{bmatrix}$

**PROBLEM 1:**

**Q2.**

```
COMMAND WINDOW
A =

    0.3000   -0.4000
    0.4000    0.2500

>> B=[1;0]

B =

     1
     0

>> eigen_X=[0.25;0.4]

eigen_X =

    0.2500
    0.4000

>> L_pp=place(A,B,eigen_X)

L_pp =

   -0.1000   -0.4000
```

**2.** P_pp= [**-0.1  -0.4**]

**PROBLEM 2:**

COMMAND WINDOW

```
>> A=[0.3 -0.4;0.4 0.25];
>> B=[1;0];
>> Q=eye(2);
>> R=1;
>> [X,K,P]=idare(A,B,Q,R)

X =

    1.2398    0.0473
    0.0473    1.1566


K =

    0.1745   -0.2161


P =

   0.1877 + 0.2640i
   0.1877 - 0.2640i
```

**3.** K_gain= [**0.1745    -0.2161**]

**4.** Closed loop poles of the system is= **0.1877** ± **0.2640** j

**PROBLEM 3:**

**5.** LQR_gain=[ **0.3329 -0.3745** ]

COMMAND WINDOW

```
>> A=[0.3 -0.4;0.4 0.25];
>> B=[1;0];
>> Q=100*eye(2);
>> R=1;
>> [X,K,L]=idare(A,B,Q,R)

X =

  117.0431   10.4564
   10.4564  106.7552


K =

    0.3329   -0.3745


L =

    0.2075
    0.0096
```

**PROBLEM 3:**

**6.** LQR_gain=[**0.3329   -0.3745**]

COMMAND WINDOW

```
>> A=[0.3 -0.4;0.4 0.25];
>> B=[1;0];
>> Q=eye(2);
>> R=0.01;
>> [X,L,Q]=idare(A,B,Q,R)

X =

    1.1704    0.1046
    0.1046    1.0676


L =

    0.3329   -0.3745


Q =

    0.2075
    0.0096
```