Model Predictive Control: Theory and Applications

Assignment 10

Problem Description[1]

Consider a distillation column which has state $x \in \mathcal{R}^{20}$. Consider the following definitions:

$$u = \begin{bmatrix} L \\ V \end{bmatrix}, \qquad w = \begin{bmatrix} F \\ x_F \end{bmatrix}, \qquad y_c = \begin{bmatrix} x_D \\ x_B \end{bmatrix}, \qquad y_m = \begin{bmatrix} T_D \\ T_B \end{bmatrix}$$

- Manipulated inputs (u): Reflux and reboiler flowrates
- Unmeasured disturbances (w): Feed flowrate and composition
- Measured outputs (y_m): Temperatures
- Unmeasured outputs (y_c): Distillate and bottoms compositions

A MATLAB file is uploaded, which provides system matrices for the model:

$$x(k+1) = Ax(k) + Bu(k) + B_w w(k)$$
$$y_m(k) = Cx(k) + v(k)$$
$$y_c(k) = Hx(k)$$

The *unmeasured* disturbances, $w(k)$, are non-stationary in nature. In Problem-1, we will setup state estimator using Kalman Filter and in Problem-2, we will set up LQR control. Problem-3 is non-graded problem for practice only.

The following files are uploaded:

`sysMat.mat`: Uploaded file contains the system matrices: `A`, `B`, `Bw`, `C`, `H`, `R1` and `R2`.

`kfExample.mat`: Uploaded file contains input and measured data: `L`, `V`, `yc` and `ym`.

Problem 1: State Estimation

Problem 1a: Kalman filter with $w(k)$ assumed to be white noise sequences **(30% Grade)**

Assume $w(k)$ is a white noise sequence with the covariances R_1 and R_2 as given in the uploaded file. With this, use an unsteady-state Kalman filter:

$$\hat{x}(k+1|k) = Ax(k|k) + Bu(k)$$
$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K_k\big(y_m(k) - C\hat{x}(k+1|k)\big)$$

Assume $\hat{x}(0|0) = \mathbf{0}$ and $P(0|0) = \alpha I$ (where α is a small number).

Please perform the following steps.

[1] This problem is courtesy of Prof. Jay H. Lee, Korea Advanced Institute of Science and Technology (KAIST). This problem and accompanying data is shared for *educational purposes only*.

Start at $k = 0$, and at each time k:

- Compute $\hat{x}(k+1|k)$ and $\bar{P}(k+1|k)$ using previous $\hat{x}(k|k),\ P(k|k)$
- Compute Kalman gain, K_{k+1}. Also compute covariance $P(k+1|k+1)$
- Use K_{k+1} to compute $\hat{x}(k+1|k+1)$
- Use $\hat{x}(k+1|k+1)$ to compute $\hat{y}_c(k+1|k+1)$

1. Make a plot of $y_c(k)$ and $\hat{y}_c(k+1|k+1)$ vs. time.

2. Compute and return the `SSE` = $\sum_{k=1}^{200}(y_c - \hat{y}_c)^T(y_c - \hat{y}_c)$

Problem 1b: Kalman Filter with $w(k)$ assumed to be integrating white noise **(40% Grade)**

With $w(k)$ as integrating white noise (IWN) sequence, you will have to use one of the approaches discussed in class to handle non-white noise sequences. Note that for IWN,

$$w(k) = w(k-1) + \varepsilon(k) \quad \text{or} \quad \Delta w(k) = \varepsilon(k)$$

where, $\text{cov}\{\varepsilon\} = R_1$.

With $\hat{x}(0|0) = \mathbf{0}$ and $P(0|0) = \alpha I$ (α is a small number), repeat all the steps in Part-1.

3. Make a plot of $y_c(k)$ and $\hat{y}_c(k+1|k+1)$ vs. time.

4. Compute and return the `SSE` = $\sum_{k=1}^{200}(y_c - \hat{y}_c)^T(y_c - \hat{y}_c)$ for this case as well

Problem 2: Output Feedback LQG Control

The control objective is to reject the unmeasured disturbances, $w(k)$, and control y_c at the origin. The control objective is given as:

$$\min_{u(\cdot)} \sum_{i=0}^{\infty} y_c^T \bar{Q}_y y_c + u^T R u$$

where, $Q = I$ and $R = I$. Note that from the output model, the above objective is written in standard LQR form as:

$$\min_{u(\cdot)} \sum_{i=0}^{\infty} x^T \left(H^T \bar{Q}_y H\right) x + u^T R u$$

The rate form of the equation is given by the following augmentation

$$z(k) = \begin{bmatrix} \Delta x(k) \\ y_m(k) \\ y_c(k) \end{bmatrix}$$

Since the equations are

$$\Delta x(k+1) = A\Delta x(k) + B\Delta u(k) + B_w \varepsilon(k)$$
$$y(k) = C\Delta x(k) + y(k-1) + \nu(k)$$

we will be able to write the equations in the following form:

$$z(k+1) = \Phi z(k) + \Gamma u(k) + \Psi \varepsilon(k)$$
$$y_m(k) = \Xi z(k) + \nu(k)$$

Part-1: Provide the matrices **(20% Grade)**

Provide the values of `phi`, `gamma`, `psi` and `xi` matrices. Ensure that

$$Q = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \bar{Q}_y \begin{bmatrix} 0 & 0 & I \end{bmatrix}, \qquad R = I$$

Part-2: Design Infinite-Horizon LQR Gain **(10% Grade)**

Start with $S = I$

Use the Ricatti difference equation using $\Phi, \Gamma, \Psi, \Xi, Q, R$ from Part-1. Iterate for 50 iterations.

Report the value of S after 50^{th} iteration.

Report the corresponding value of L_∞ if S has converged

Note: We are not asking you to solve the LQG problem yet. State Augmentation in appropriate form is asked in Problem 2.1, and obtaining LQR gain for the system is asked in Problem 2.2. The next problem (not graded) is where you can run LQG simulations.

Problem-3 for Practice (not graded / do not submit): LQG Simulations

Assume a unit step-change in F and step-change of magnitude 0.5 in x_F. Since these are unmeasured disturbances, you will also need to design Kalman Filter. Note that only the temperatures (i.e., y_m) are measured. The augmented state $\hat{z}(k|k)$ must be estimated from these measurements.

Perform simulations of the LQG to ensure regulation of the controlled outputs.