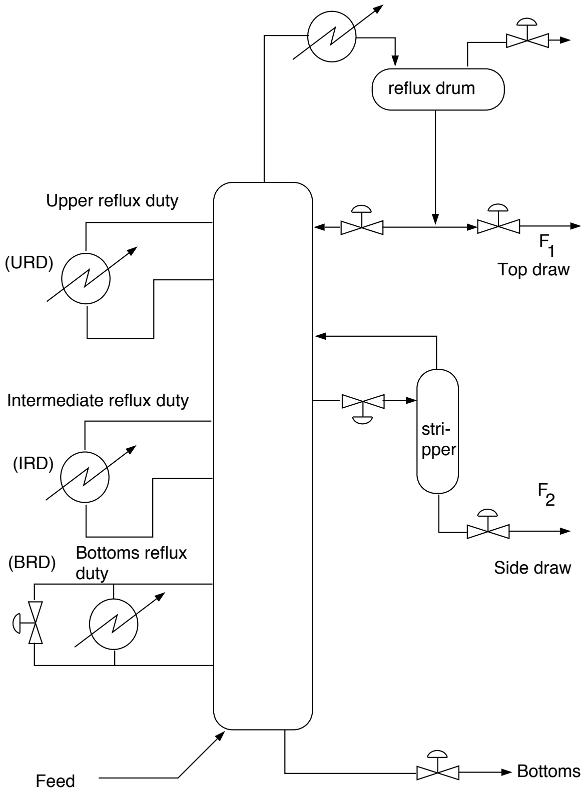
Model Predictive Control: Theory and Applications

Assignment 12

Background

Shell Oil Fractionator Challenge Problem (SCP) was a multivariate control problem presented by engineers at the Shell Oil Company. It was one of the standard problems in the literature, which highlighted some of the key issues in control of large process systems, viz., possibly a non-square MIMO system, time delays, inferential measurements and constraints.

Schematic of the Fractionator is shown in the adjoining figure. The feed enters the bottom, and products are taken from top-draw, side-draw and bottom. Let's do a role-play. All of us are engineers in NPTEL Control Systems Ltd., who are tasked with designing the MPC.



The two primary outputs are compositions of the top and side streams (y_1, y_2 respectively). Additionally, temperatures of all the outlet streams and the reflux streams are also measured. The inputs to the system include the top-draw and side-draw (u_1, u_2, respectively) flowrates. The reflux duties form auxiliary or disturbance variables. Engineers at the Shell Oil Company performed multiple step tests to develop the first order plus time delay (FOPTD) models for the system.

Problem Definition

You can refer to the paper [Yu et al., *Chem. Eng. Sci.*, 1994, **49**: 285–301] for more details of the exact system. In this assignment, we will consider a simplified problem statement to solve using MPC toolbox as an assignment problem.

The system (i.e., "plant") is defined as:

$$\mathbf{Y}(s) = G_p\mathbf{U}(s) + G_d\mathbf{D}(s)$$

$$G_p^{\text{plant}} = \begin{bmatrix} \frac{4.05e^{-27s}}{50s+1} & \frac{1.77e^{-28s}}{60s+1} & \frac{5.88e^{-27s}}{50s+1} \\ \frac{5.39e^{-18s}}{50s+1} & \frac{5.72e^{-14s}}{60s+1} & \frac{6.9e^{-15s}}{40s+1} \\ \frac{4.38e^{-20s}}{33s+1} & \frac{4.42e^{-22s}}{44s+1} & \frac{7.2}{19s+1} \end{bmatrix}, \quad G_d^{\text{plant}} = \begin{bmatrix} \frac{1.2e^{-27s}}{45s+1} & \frac{1.44e^{-27s}}{40s+1} \\ \frac{1.52e^{-15s}}{25s+1} & \frac{1.83e^{-15s}}{20s+1} \\ \frac{1.14}{27s+1} & \frac{1.26}{32s+1} \end{bmatrix}$$

In the problem definition, Shell had defined $\mathbf{D}(s)$ as *unmeasured disturbances*. This means that we cannot know the values of $d(k)$. At most, we can estimate them through the available measurements. Yu et al. (1994) developed state space MPC for the system, where different options for disturbance modeling were evaluated. We will consider two of those options.

Control Objectives

- Sampling time, $\Delta t = 2$
- Unmeasured disturbances occur in reflux duties: $d_1(k) = -0.5,\ d_2(k) = -0.5$.
- Maintain set-points: $y_1^{\text{SP}} = 0,\ y_2^{\text{SP}} = 0$.
- Maintain bottom reflux above a minimum value, $-0.5 \le y_7$ (output constraint)
- Maintain output constraints: $-0.5 \le y_1 \le 0.5$ and $-0.5 \le y_7$ as mentioned above
- Maintain input constraints: $-0.5 \le u_1, u_2, u_3 \le 0.5$
- Maintain input rate constraints: $-0.05 \le \Delta u_1, \Delta u_2, \Delta u_3 \le 0.05$

Solving the Control Problem

Some students had asked in the live session to go over steps taken in industry to solve a problem such as the above. I have modified the text below to demonstrate the same.

Step-1: Generating Data for Model Identification

The most expensive step in industrial implementation of MPC is to generate data for obtaining the model. Typically, this is through input step or pseudo-random tests. As NCSL engineers, we performed plant tests and generated the data for the system.

Step-2: Model-Plant Mismatch (MPM)

The data generated in the above tests is then used to develop the model for MPC. Following a model identification process, we as NCSL engineers obtained the following model.

$$G_p^{\text{model}} = \begin{bmatrix} \frac{6e^{-27s}}{50s+1} & \frac{1.4e^{-28s}}{60s+1} & \frac{6.2e^{-27s}}{50s+1} \\ \frac{8e^{-18s}}{50s+1} & \frac{5.2e^{-14s}}{60s+1} & \frac{7.6e^{-15s}}{40s+1} \\ \frac{7.5e^{-20s}}{33s+1} & \frac{3.7e^{-22s}}{44s+1} & \frac{8.5}{19s+1} \end{bmatrix}, \quad G_w^{\text{model}} = \begin{bmatrix} \frac{1.3e^{-27s}}{45s+1} & \frac{1.6e^{-27s}}{40s+1} \\ \frac{1.6e^{-15s}}{25s+1} & \frac{2e^{-15s}}{20s+1} \\ \frac{1.3}{27s+1} & \frac{1.4}{32s+1} \end{bmatrix}$$

Step-3: Tuning Parameters

Please note the controller tuning parameters used by Yu et al. (1994). Let us assume Dr. Yu is our team leader at NCSL. His research gave us the following MPC tuning parameters.

- Control and prediction horizons: $m = 10,\ p = 40$
- Output weights: $\text{diag}\{1,\ 1,\ 0\}$
- Input rate weights: $\text{diag}\{1.5,\ 0.15,\ 1.5\}$
- Use output constraint softening, if required
- Estimator parameters: Based on the choice of disturbance model, use $R_\varepsilon = I,\ R_\nu = 0$

Step-4: Implementation of MPC

Please set up MPC algorithm and implement it for the following two cases:

Option 4.1: Output Load Disturbance Model

This was Case-1 in the paper. The output load disturbance model is of the form:

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + \eta(k) + \nu(k)$$

where, $\eta(k)$ is integrating white noise. Note that in this case, the model G_w^{model} is ignored. Please implement MPC with this option

Option 4.2: Unmeasured Disturbance Model

This was Case-2 in the paper. The disturbance model is of the form

$$x(k+1) = Ax(k) + Bu(k) + B_e w(k)$$
$$y(k) = Cx(k) + \nu(k)$$

where, $w(k)$ is an integrating white noise sequence. Please implement MPC with this option.

Submission of Assignment: Please include the MATLAB code as well as input and output profiles (*u* vs. *t* and *y* vs. *t* plots) for all the three problems in this assignment.