

$$d(\gamma) = (\text{term1} + \text{term8})$$

$$\text{term1: } \sum_{k_1=1}^{K_1} \left[\log \tau \left(\sum_{v=1}^V n_v \right) - \sum_{v=1}^V \log \tau(n_v) \right] + \sum_{v=1}^V (n_v - 1) \left(\psi(\gamma_{k_1 v}^{(1)}) - \psi \left(\sum_{v=1}^V \gamma_{k_1 v}^{(1)} \right) \right)$$

$$\text{term2: } \sum_{j=1}^D \left[\log \tau \left(\sum_{k_2=1}^{K_2} u_{jk_2} \right) - \sum_{k_2=1}^{K_2} \log \tau(u_{jk_2}) \right] + \sum_{k_2=1}^{K_2} (u_{jk_2} - 1) \left(\psi(u_{jk_2}) - \psi \left(\sum_{k_2=1}^{K_2} u_{jk_2} \right) \right)$$

$$\text{term3: } \sum_{k_1=1}^{K_1} \left[\left(\log \tau(v_{01} + v_{02}) - \log \tau(v_{01}) - \log \tau(v_{02}) \right) + (v_{01} - 1) \left(\psi(u_{k_1}) - \psi(u_{k_1} + v_{k_1}) \right) \right. \\ \left. + (v_{02} - 1) \left(\psi(u_{k_1}) - \psi(u_{k_1} + v_{k_1}) \right) \right]$$

$$\text{term4: } \sum_{j=1}^D \sum_{t=1}^T \left[\left(\log \tau(a_{01} + a_{02}) - \log \tau(a_{01}) - \log \tau(a_{02}) \right) + (a_{01} - 1) \left(\psi(a_{jt}) - \psi(a_{jt} + b_{jt}) \right) \right. \\ \left. + (a_{02} - 1) \left(\psi(b_{jt}) - \psi(a_{jt} + b_{jt}) \right) \right]$$

$$\text{term5: } \sum_{j=1}^D \sum_{t=1}^T \sum_{k_1=1}^{K_1} \left[\left(\sum_{k=1+t+k_1}^{K_1} \gamma_{jtk} \right) \left(\psi(v_{k_1}) - \psi(u_{k_1} + v_{k_1}) \right) + \gamma_{jtk_1} \left(\psi(u_{k_1}) - \psi(u_{k_1} + v_{k_1}) \right) \right]$$

$$\text{term6: } \sum_{j=1}^D \sum_{n=1}^{N_j} \left[\sum_{t=1}^T \left[\delta_{jn}(t+k_2) \left[\log(1-\epsilon) + \left(\psi(a_{jt}) - \psi(a_{jt} + b_{jt}) \right) \right] + \sum_{t=1+t}^T \delta_{jn}(t+k_2) \cdot \left(\psi(b_{jt}) - \psi(a_{jt} + b_{jt}) \right) \right] \right. \\ \left. + \sum_{k_2=1}^{K_2} \delta_{jn k_2} \left[\log \epsilon + \psi(u_{jk_2}) - \psi \left(\sum_{k_2=1}^{K_2} u_{jk_2} \right) \right] \right]$$

$$\text{term7: } \sum_{j=1}^D \sum_{n=1}^{N_j} \left[\sum_{k_1=1}^{K_1} \left(\sum_{t=1}^T \delta_{jn t} \gamma_{jtk_1} \right) \sum_{v=1}^V \mathbb{I}_{\{w_{jn}=v\}} \left(\psi(\gamma_{k_1 v}^{(1)}) - \psi \left(\sum_{v=1}^V \gamma_{k_1 v}^{(1)} \right) \right) \right. \\ \left. + \sum_{k_2=1}^{K_2} \delta_{jn k_2} \sum_{v=1}^V \mathbb{I}_{\{w_{jn}=v\}} \left(\psi(\gamma_{k_2 v}^{(2)}) - \psi \left(\sum_{v=1}^V \gamma_{k_2 v}^{(2)} \right) \right) \right]$$

$$\text{term 8:} \sum_{k_2=1}^{K_2} \left[\log \tau \left(\sum_{v=1}^V n_{kv} \right) - \sum_{v=1}^V \log \tau(n_{kv}) + \sum_{v=1}^V (n_{kv}-1) \left(\psi(\lambda_{kv}^{(2)}) - \psi \left(\sum_{v=1}^V \lambda_{kv}^{(2)} \right) \right) \right]$$

$$\begin{aligned} \text{term 9:} & \sum_{k_1=1}^{K_1} \left[\log \tau(u_{k_1} + v_{k_1}) - \log \tau(u_{k_1}) - \log \tau(v_{k_1}) \right. \\ & \left. + (u_{k_1}-1) (\psi(u_{k_1}) - \psi(u_{k_1} + v_{k_1})) + (v_{k_1}-1) (\psi(v_{k_1}) - \psi(u_{k_1} + v_{k_1})) \right] \end{aligned}$$

$$\begin{aligned} \text{term 10:} & \sum_{j=1}^D \sum_{t=1}^T \left[\log \tau(a_{jt} + b_{jt}) - \log \tau(a_{jt}) - \log \tau(b_{jt}) \right. \\ & \left. + (a_{jt}-1) (\psi(a_{jt}) - \psi(a_{jt} + b_{jt})) + (b_{jt}-1) (\psi(b_{jt}) - \psi(a_{jt} + b_{jt})) \right] \end{aligned}$$

$$\text{term 11:} \sum_{k_1=1}^{K_1} \left[\log \tau \left(\sum_{v=1}^V \lambda_{k_1 v}^{(1)} \right) - \sum_{v=1}^V \log \tau(\lambda_{k_1 v}^{(1)}) + \sum_{v=1}^V (\lambda_{k_1 v}^{(1)}-1) \left(\psi(\lambda_{k_1 v}^{(1)}) - \psi \left(\sum_{v=1}^V \lambda_{k_1 v}^{(1)} \right) \right) \right]$$

$$\text{term 12:} \sum_{k_2=1}^{K_2} \left[\log \tau \left(\sum_{v=1}^V \lambda_{k_2 v}^{(2)} \right) - \sum_{v=1}^V \log \tau(\lambda_{k_2 v}^{(2)}) + \sum_{v=1}^V (\lambda_{k_2 v}^{(2)}-1) \left(\psi(\lambda_{k_2 v}^{(2)}) - \psi \left(\sum_{v=1}^V \lambda_{k_2 v}^{(2)} \right) \right) \right]$$

$$\text{term 13:} \sum_{j=1}^D \sum_{t=1}^T \sum_{k_1=1}^{K_1} \varphi_{j,t,k_1} \log \varphi_{j,t,k_1}$$

$$\text{term 14:} \sum_{j=1}^D \sum_{n=1}^{N_j} \sum_{t=1}^{(K_2+T)} s_{jnt} \log s_{jnt}$$

$$\text{term 15:} \sum_{j=1}^D \left[\log \tau \left(\sum_{k_2=1}^{K_2} \mu_{jk_2} \right) - \sum_{k_2=1}^{K_2} \log \tau(\mu_{jk_2}) + \sum_{k_2=1}^{K_2} (\mu_{jk_2}-1) \left(\psi(\mu_{jk_2}) - \psi \left(\sum_{k_2=1}^{K_2} \mu_{jk_2} \right) \right) \right]$$

model parameters: $\eta, d_0, v_0, \mu_0, \gamma$