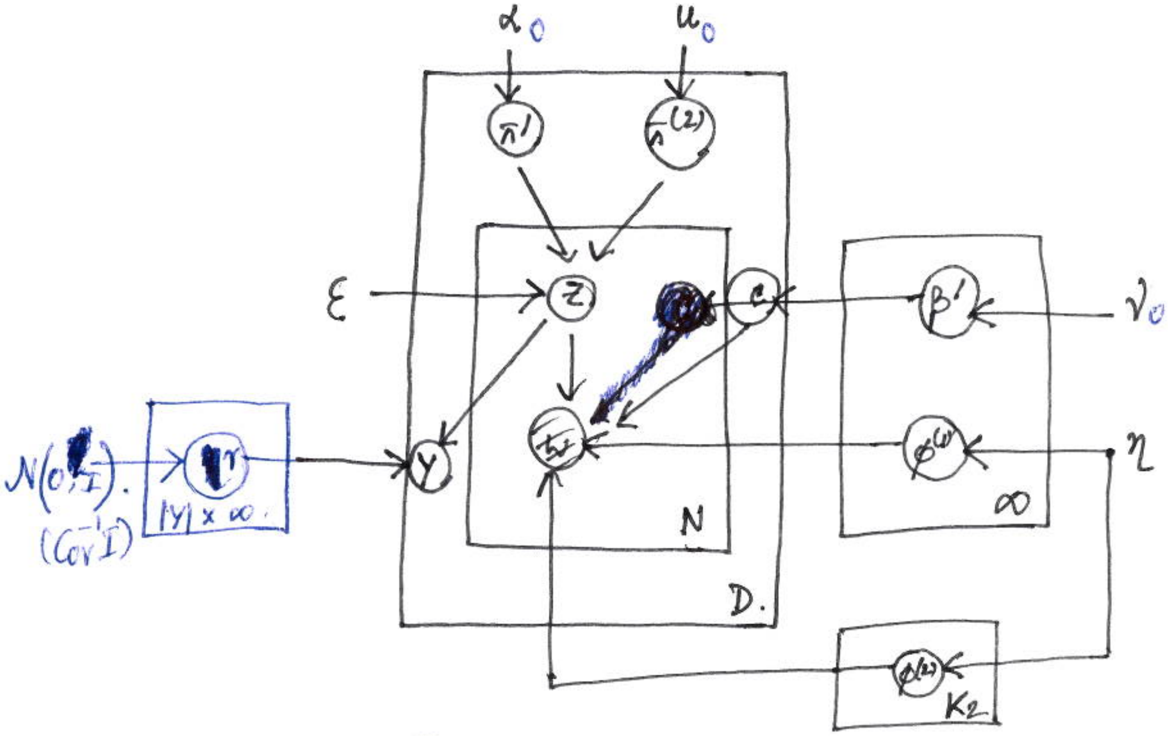
$$p(w_{jn} \mid c_j, z_{jn}, \phi) = \prod_{k_1=1}^{\infty} \left(\prod_{v=1}^{V} \left(\phi^{(1)}_{k_1 v}\right)^{\mathbb{I}\{w_{jn}=v\}} \right)^{\mathbb{I}\{c_j z_{jn} = k_1\}} \prod_{k_2=1}^{K_2} \left(\prod_{v=1}^{V} \left(\phi^{(2)}_{k_2 v}\right)^{\mathbb{I}\{w_{jn}=v\}} \right)^{\mathbb{I}\{z_{jn}=k_2\}}.$$

$\therefore \log p(w_{jn} \mid c_j, z_{jn}, \phi)$

$$= \sum_{k_1=1}^{\infty} \left[\mathbb{I}\{c_j z_{jn} = k_1\} \left[\sum_{v=1}^{V} \mathbb{I}\{w_{jn}=v\} \log\left(\phi^{(1)}_{k_1 v}\right) \right] \right] + \sum_{k_2=1}^{K_2} \left[\mathbb{I}\{z_{jn} = k_2\} \left[\sum_{v=1}^{V} \mathbb{I}\{w_{jn}=v\} \log\left(\phi^{(2)}_{k_2 v}\right) \right] \right]$$



$\therefore \mathbb{E}_q\left[\log p(w_{jn} \mid c_j, z_{jn}, \phi)\right]$

$$= \sum_{k_1=1}^{K_1} \left[\sum_{z_{jn}} q(z_{jn}) \prod_{t=1}^{T} \Phi(c_{jt})^{c_{jt}} \, \mathbb{I}\{c_j z_{jn} = k_1\} \sum_{v=1}^{V} \mathbb{I}\{w_{jn}=v\} \int \log\left(\phi^{(1)}_{k_1 v}\right) q\left(\phi^{(1)}_{k_1} \mid \lambda^{(1)}_{k_1}\right) d\phi^{(1)}_{k_1} \right] + \cdots$$

$$= \sum_{k_1=1}^{K_1} \left[\left(\sum_{t=1}^{T} \varsigma_{jnt} \, \varphi_{jtk_1} \right) \sum_{v=1}^{V} \mathbb{I}\{w_{jn}=v\} \left[\Psi\left(\lambda^{(1)}_{k_1 v}\right) - \Psi\left(\sum_{v=1}^{V} \lambda^{(1)}_{k_1 v}\right) \right] \right] + \cdots$$

$$\therefore \text{term 7:} = \sum_{j=1}^{D} \sum_{n=1}^{N_j} \left[\sum_{k_1=1}^{K_1} \left(\sum_{t=1}^{T} \varsigma_{jnt} \varphi_{jtk_1} \right) \sum_{v=1}^{V} \mathbb{I}\{w_{jn}=v\} \left[\Psi\left(\lambda^{(1)}_{k_1 v}\right) - \Psi\left(\sum_{v=1}^{V} \lambda^{(1)}_{k_1 v}\right) \right] + \sum_{k_2=1}^{K_2} \varsigma_{jnk_2} \sum_{v=1}^{V} \mathbb{I}\{w_{jn}=v\} \left[\Psi\left(\lambda^{(2)}_{k_2 v}\right) - \Psi\left(\sum_{v=1}^{V} \lambda^{(2)}_{k_2 v}\right) \right] \right]$$

Cover Page

Hidden Vars: ~~Variational params~~: $\beta', c, \pi', \pi^{(2)}, z, \phi$.

Model param: $\eta, \nu, u, \alpha, \mathcal{E}, \gamma$.

Variational Param: ~~Hidden Vars~~: (u, v); (a, b), $\lambda^{(1)}, \lambda^{(2)}$,

φ, ζ, u

Joint distribution:

$p(D, \beta', \pi', \pi^{(2)}, c, z, \phi)$

term1. dimension V. term 8. dimension V. term 3.

$$= \underbrace{\prod_{k_1=1}^{\infty} p(\phi^{(1)}_{k_1} | \eta) \prod_{k_2=1}^{K_2} p(\phi^{(2)}_{k_2} | \eta)}_{\text{Dirichlet}(\eta)} \cdot \underbrace{\prod_{k_1=1}^{\infty} p(\beta'_{k_1} | \nu_0)}_{\text{Beta}(1, \nu)}$$

term2. make it μ. term4

$$\underbrace{\prod_{j=1}^{D} p(\pi^{(2)}_j | u)}_{\text{Dirichlet}(u)} \cdot \underbrace{\prod_{j=1}^{D} \prod_{t=1}^{\infty} p(\pi'_{jt} | \alpha_0)}_{\text{Beta}(1,\alpha)} \cdot \underbrace{\prod_{j=1}^{D} \prod_{t=1}^{\infty} p(c_{jt} | \pi(\beta'))}_{\text{multinomial}} \rightarrow \text{term 5.}$$

dimension K_2

$\pi(\beta') \equiv \beta$

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$$\prod_{j=1}^{D} \prod_{n=1}^{N_j} p(z_{jn} | \mathcal{E}, \pi^{(1)}_j, \pi^{(2)}_j) \rightarrow \text{term 6.}$$

$$\prod_{j=1}^{D} \prod_{n=1}^{N_j} p(w_{jn} | \phi, c_{j z_{jn}}, z_{jn}) \rightarrow \text{term 7}$$

$\rightarrow$ multinomial.

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$$\boxed{\begin{aligned} \pi^{(1)}_{jt} &= \pi'_{jt} \prod_{l=1}^{(t-1)} (1 - \pi'_{jt}). \\ \beta_t &= \beta'_t \prod_{l=1}^{(t-1)} (1 - \beta'_l). \end{aligned}}$$

~~$T \geq K_1$~~

$\boxed{K_1 > T}$

Factorised Approx:

$$q(\beta', c, \pi', \pi^{(2)}, z, \phi) = \prod_{k_1=1}^{K_1} q(\beta'_{k_1} | u_{k_1}, v_{k_1}). \rightarrow \text{Beta distr.}$$

term 9.

lower ~~higher~~ level truncation

$$\prod_{j=1}^{D} \prod_{t=1}^{T} q(\pi'_{jt} | a_{jt}, b_{jt}) \rightarrow \text{Beta distr.} \rightarrow \text{term10}$$

higher ~~lower~~ level truncation

term 11. term 12.

$$\prod_{k_1=1}^{(K_1)} q(\phi^{(1)}_{k_1} | \lambda^{(1)}_{k_1}) \prod_{k_2=1}^{K_2} q(\phi^{(2)}_{k_2} | \lambda^{(2)}_{k_2}). \rightarrow \text{Dirichlet distr.}$$

term13.

$$\prod_{j=1}^{D} \prod_{t=1}^{T} q(c_{jt} | \varphi_{jt}) \rightarrow \text{multinomial. distr.}$$

dimension K_1.

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Cover Page

$$\prod_{j=1}^{D}\prod_{n=1}^{N_j} \overset{\nearrow \text{term } 14}{q(z_{jn} \,|\, \varsigma_{jn})}. \rightarrow \boxed{\text{dimension } (T+K_2)} \rightarrow \text{multinomial distr.}$$

$$\prod_{j=1}^{D} \underbrace{\overset{\nearrow \text{term } 15}{q(\pi_j^{(2)} \,|\, \mu_j)}}_{\boxed{\text{dimension } K_2}}. \rightarrow \text{dirichlet distr.}$$

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term 7: $E_q\left[\log p(w \,|\, c, z, \phi)\right]$

see 2nd page

$$= \sum_{j=1}^{D}\sum_{n=1}^{N_j}\Bigg[\cancel{\sum_{t=1}^{T} \varsigma_{jn(K_2+t)} \sum_{v=1}^{V} \mathbb{1}_{\{w_{jn}=v\}} \left[\psi(\lambda^{(1)}_{tv}) - \psi\Big(\sum_{v=1}^{V} \lambda^{(1)}_{tv}\Big)\right]}$$

$$+ \sum_{k_2=1}^{K_2} \varsigma_{jnk_2} \sum_{v=1}^{V} \mathbb{1}_{\{w_{jn}=v\}} \left[\psi(\lambda^{(2)}_{k_2 v}) - \psi\Big(\sum_{v=1}^{V} \lambda^{(2)}_{k_2 v}\Big)\right]\Bigg]$$

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Cover Page

term1: $$E_q\left[\log p(\phi^{(1)}|\eta)\right] = \sum_{k_1=1}^{K_1}\left[\left[\log\Gamma\left(\sum_{v=1}^{V}\eta_v\right) - \sum_{v=1}^{V}\log\Gamma(\eta_v)\right] + \sum_{v=1}^{V}(\eta_v-1)\left[\Psi\left(\lambda^{(1)}_{k_1 v}\right) - \Psi\left(\sum_{v=1}^{V}\lambda^{(1)}_{k_1 v}\right)\right]\right]$$

term8: $$E_q\left[\log p(\phi^{(2)}|\eta)\right] = \sum_{k_2=1}^{K_2}\left[\left[\log\Gamma\left(\sum_{v=1}^{V}\eta_v\right) - \sum_{v=1}^{V}\log\Gamma(\eta_v)\right] + \sum_{v=1}^{V}(\eta_v-1)\left[\Psi\left(\lambda^{(2)}_{k_2 v}\right) - \Psi\left(\sum_{v=1}^{V}\lambda^{(2)}_{k_2 v}\right)\right]\right]$$

term 2: $$E_q\left[\log p(\pi^{(2)}|\mu)\right] = \sum_{j=1}^{D}\left[\left[\log\Gamma\left(\sum_{k_2=1}^{K_2}\mu_{k_2}\right) - \sum_{k_2=1}^{K_2}\log\Gamma(\mu_{k_2})\right] + \sum_{k_2=1}^{K_2}(\mu_{k_2}-1)\left[\Psi(\mu_{jk_2}) - \Psi\left(\sum_{k_2=1}^{K_2}\mu_{jk_2}\right)\right]\right]$$

term3: $$E_q\left[\log p(\beta'|\nu)\right] = \sum_{k_1=1}^{K_1}\left[\left[\log\Gamma(1+\nu) - \log\Gamma(\nu)\right] + (\nu-1)\left[\Psi(v_{k_1}) - \Psi(u_{k_1}+v_{k_1})\right]\right]$$

$$= \sum_{k_1=1}^{K_1}\left[\left[\log\Gamma(\nu_{01}+\nu_{02}) - \log\Gamma(\nu_{01}) - \log\Gamma(\nu_{02})\right] + (\nu_{01}-1)\left(\Psi(u_{k_1}) - \Psi(u_{k_1}+v_{k_1})\right) + (\nu_{02}-1)\left(\Psi(v_{k_1}) - \Psi(u_{k_1}+v_{k_1})\right)\right]$$

Cover Page

term 4: $\quad E_q\left[\log p(\pi' \mid \alpha_0)\right]$

$$= \sum_{j=1}^{D}\sum_{t=1}^{T}\left[\left[\log\Gamma(1+\alpha) - \log\Gamma(\alpha)\right] + (\alpha-1)\left[\psi(b_{jt}) - \psi(a_{jt}+b_{jt})\right]\right].$$

$$= \sum_{j=1}^{D}\sum_{t=1}^{T}\left[\left[\log\Gamma(\alpha_{01}+\alpha_{02}) - \log\Gamma(\alpha_{01}) - \log\Gamma(\alpha_{02})\right] + (\alpha_{01}-1)\left(\psi(a_{jt}) - \psi(a_{jt}+b_{jt})\right) + (\alpha_{02}-1)\left(\psi(b_{jt}) - \psi(a_{jt}+b_{jt})\right)\right].$$

term 5: $\quad E_q\left[\log p(c \mid \beta')\right].$

$$= \sum_{j=1}^{D}\sum_{t=1}^{T}\sum_{k_1=1}^{K_1}\left[\left(\sum_{k=k_1+1}^{K_1}\varphi_{jtk}\right)\left[\psi(v_{k_1}) - \psi(u_{k_1}+v_{k_1})\right] + \varphi_{jtk_1}\left[\psi(u_{k_1}) - \psi(u_{k_1}+v_{k_1})\right]\right].$$

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(−).
term 9: $\quad \sum_{k_1=1}^{K_1} E_q\left[\log q(\beta'_{k_1} \mid u_{k_1}, v_{k_1})\right]$

$$= \sum_{k_1=1}^{K_1}\left[\left[\log\Gamma(u_{k_1}+v_{k_1}) - \log\Gamma(u_{k_1}) - \log\Gamma(v_{k_1})\right] + (u_{k_1}-1)\left[\psi(u_{k_1}) - \psi(u_{k_1}+v_{k_1})\right] + (v_{k_1}-1)\left[\psi(v_{k_1}) - \psi(u_{k_1}+v_{k_1})\right]\right].$$

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(-). term 10: $\sum_{j=1}^{D}\sum_{t=1}^{T} E_q\left[\log q(\pi_{jt}' \mid a_{jt}, b_{jt})\right]$.

$$= \sum_{j=1}^{D}\sum_{t=1}^{T}\left[\left[\log\Gamma(a_{jt}+b_{jt}) - \log\Gamma(a_{jt}) - \log\Gamma(b_{jt})\right] + (a_{jt}-1)\left[\psi(a_{jt}) - \psi(a_{jt}+b_{jt})\right] + (b_{jt}-1)\left[\psi(b_{jt}) - \psi(a_{jt}+b_{jt})\right]\right].$$

(-) term 11: $\sum_{k_1=1}^{(K_1)} E_q\left[\log q(\phi^{(1)}_{k_1} \mid \lambda_{k_1})\right]$.

$$= \sum_{k_1=1}^{(K_1)}\left[\left[\log\Gamma\left(\sum_{v=1}^{V}\lambda^{(1)}_{k_1 v}\right) - \sum_{v=1}^{V}\log\Gamma(\lambda^{(1)}_{k_1 v})\right] + \sum_{v=1}^{V}(\lambda^{(1)}_{k_1 v}-1)\left[\psi(\lambda^{(1)}_{k_1 v}) - \psi\left(\sum_{v=1}^{V}\lambda^{(1)}_{k_1 v}\right)\right]\right].$$

(-) term 12: $\sum_{k_2=1}^{K_2} E_q\left[\log q(\phi^{(2)}_{k_2} \mid \lambda_{k_2})\right]$.

$$= \sum_{k_2=1}^{K_2}\left[\left[\log\Gamma\left(\sum_{v=1}^{V}\lambda^{(2)}_{k_2 v}\right) - \sum_{v=1}^{V}\log\Gamma(\lambda^{(2)}_{k_2 v})\right] + \sum_{v=1}^{V}(\lambda^{(2)}_{k_2 v}-1)\left[\psi(\lambda^{(2)}_{k_2 v}) - \psi\left(\sum_{v=1}^{V}\lambda^{(2)}_{k_2 v}\right)\right]\right].$$

(-). term 14: $\sum_{j=1}^{D}\sum_{n=1}^{N_j} E_q\left[\log q(z_{jn} \mid \xi_{jn})\right]$

$$= \sum_{j=1}^{D}\sum_{n=1}^{N_j}\sum_{t=1}^{(K_1+K_2)} \xi_{jnt}\log\xi_{jnt}$$

term 15: (−). $\sum_{j=1}^{D} E_q\left[\log q\left(\pi_j^{(2)} \middle| \mu_j\right)\right]$

$$= \sum_{j=1}^{D}\left[\left[\log\Gamma\left(\sum_{k_2=1}^{K_2}\mu_{jk_2}\right) - \sum_{k_2=1}^{K_2}\log\Gamma\left(\mu_{jk_2}\right)\right] + \sum_{k_2=1}^{K_2}\left(\mu_{jk_2}-1\right)\cdot\left[\Psi\left(\mu_{jk_2}\right) - \Psi\left(\sum_{k_2=1}^{K_2}\mu_{jk_2}\right)\right]\right]$$

term 13: (−). $\cancel{\sum_{j=1}^{D}\sum_{t=1}^{T-1} E_q\left[\log q\left(c_{jt} \middle| \varphi_{jt}\right)\right] = \sum_{j=1}^{D}\sum_{t=1}^{T-1} E_q.} \log.$

$$q(c_{jt}|\varphi_{jt}) = \prod_{k_1=1}^{K_1}\left(\varphi_{jtk_1}\right)^{\mathbb{1}\{c_{jt}=k_1\}}.$$

$$\log q\left(c_{jt}\middle|\varphi_{jt}\right) = \sum_{k_1=1}^{K_1}\mathbb{1}\{c_{jt}=k_1\}\log\left(\varphi_{jtk_1}\right).$$

$$\therefore E_q\left[\log q\left(c_{jt}\middle|\varphi_{jt}\right)\right] = \sum_{k_1=1}^{K_1}\varphi_{jtk_1}\log\varphi_{jtk_1}$$

$\therefore$ term13 : (−). $\sum_{j=1}^{D}\sum_{t=1}^{T}\sum_{k_1=1}^{K_1}\varphi_{jtk_1}\log\varphi_{jtk_1}$.

term 6: $\sum_{j=1}^{D} \sum_{n=1}^{N_j} E_q \left[\log p(z_{jn} \mid \varepsilon, \pi^{(1)}, \pi^{(2)}) \right]$

$$p(z_{jn} \mid \varepsilon, \pi^{(1)}, \pi^{(2)}) = \prod_{k_2=1}^{K_2} \left(\varepsilon \, \pi_{jk_2}^{(2)} \right)^{\mathbb{I}\{z_{jn}=k_2\}} \cdot \prod_{t=1}^{\infty} (1-\pi'_{jt})^{\mathbb{I}\{z_{jn}>t\}} \, (\pi'_{jt}) \cdot (1-\varepsilon)^{\mathbb{I}\{z_{jn}=t\}}$$

$$= \prod_{k_2=1}^{K_2} \{\varepsilon \, \pi_{jk_2}^{(2)}\}^{\mathbb{I}\{z_{jn}=k_2\}} \cdot (1-\varepsilon) \cdot \prod_{k_1=1}^{\infty} \left[\prod_{t=1}^{\infty} (1-\pi'_{jt})^{\mathbb{I}\{\pi'_{jt} > k_1\}} \, \pi'_{j}{}^{\mathbb{I}\{\pi'_j = k_1\}} \right]^{\mathbb{I}\{z_{jn}=\}}$$

$\therefore \log p(z_{jn} \mid \varepsilon, \pi^{(1)}, \pi^{(2)})$:

$$\therefore \text{term 6} := \sum_{j=1}^{D} \sum_{n=1}^{N_j} \left[\sum_{k_2=1} \delta_{jnk_2} \left[\log \varepsilon + \Psi(\mu_{jk_2}) - \Psi\left(\sum_{k_2=1}^{K_2} \mu_{jk_2}\right) \right] + \sum_{t=1}^{T} \left[\delta_{jnt} \left[\log(1-\varepsilon) + \Psi(a_{jt}) - \Psi(a_{jt}+b_{jt}) + \left(\sum_{t'=t+1}^{T} \delta_{jnt'}\right) \left[\Psi(b_{jt}) - \Psi(a_{jt}+b_{jt}) \right] \right] \right] \right]$$

$$= \sum_{k_2=1}^{K_2} \mathbb{I}\{z_{jn}=k_2\} \cdot \log\left(\varepsilon \, \pi_{jk_2}^{(2)}\right)$$

$$+ \sum_{k_1=1}^{\infty} \mathbb{I}\{z_{jn}=k_1\} \log\left[(1-\varepsilon) \cdot \prod_{t=1}^{\infty} (1-\pi'_{jt})^{\mathbb{I}\{z_{jn} > k_1\}} \cdot \pi'_{j}{}^{\mathbb{I}\{z_{jn} = k_1\}} \right]$$