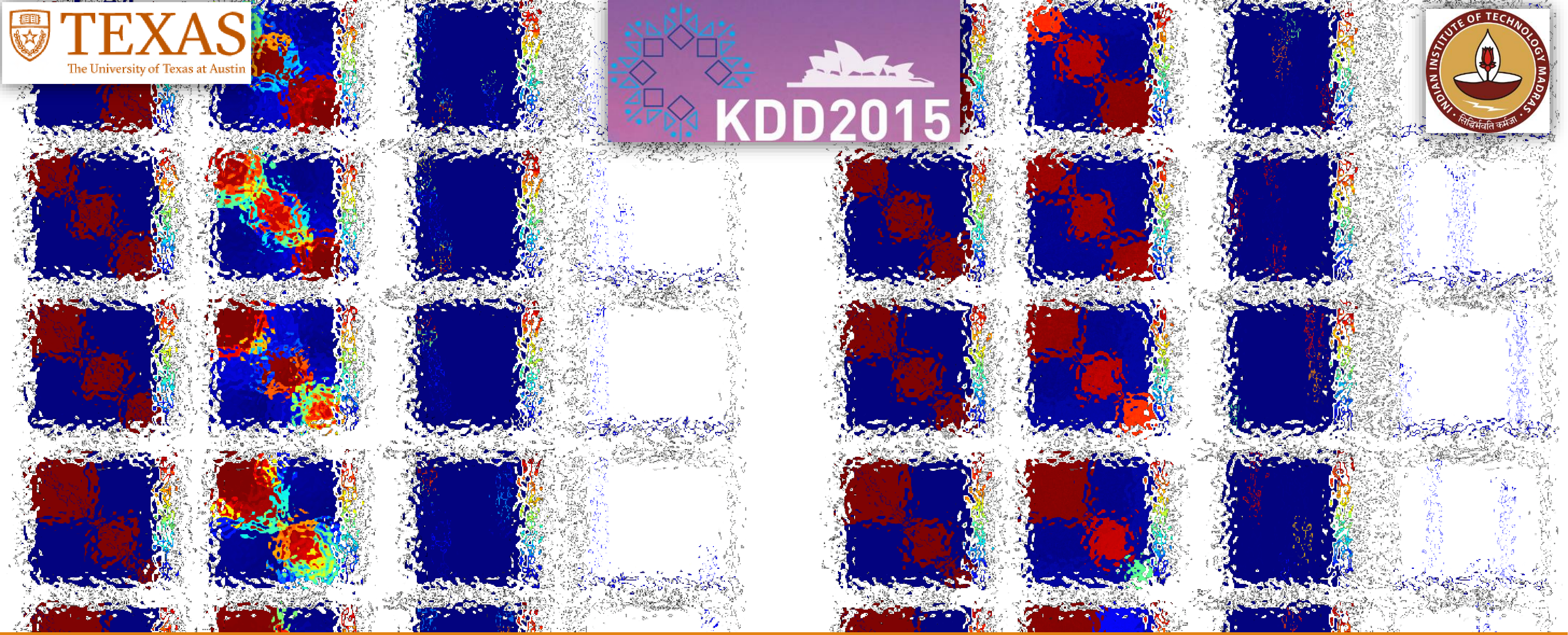

Nonparametric Dynamic Network Modeling

1st SIGKDD Workshop on Mining and Learning from Time Series 2015

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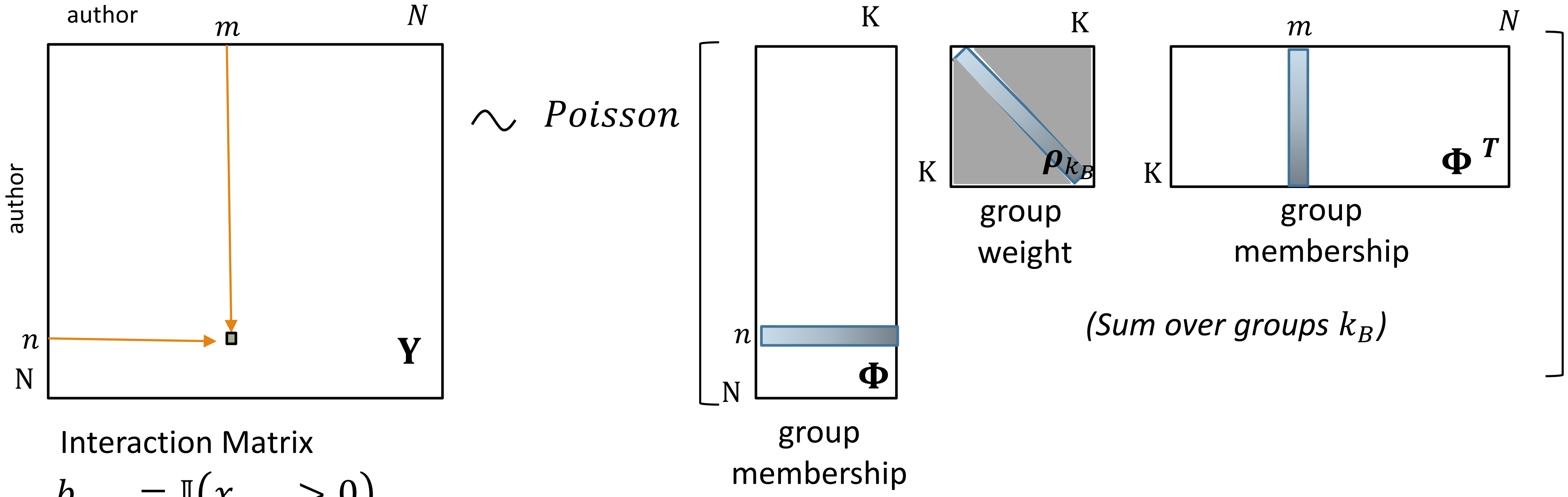
Poisson Factorization for Network Associations (N-GPPF)

Network association (blockmodel):



$$b_{n,m} = \mathbb{I}(x_{n,m} > 0)$$

$$x_{n,m} \sim Pois\left(\sum_{k_B} \rho_{k_B} \phi_{nk_B}\, \phi_{mk_B}\right)$$

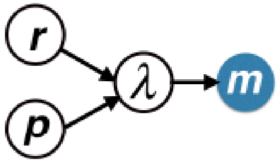
Priors: $\boldsymbol{\phi}_k \sim \sum_{n=1}^{N} Gamma(a_n, {}^1/_{c_d})$; $\boldsymbol{\rho}_k \sim Gamma({}^\nu/_k, {}^1/_c)$

Characteristics of N-GPPF

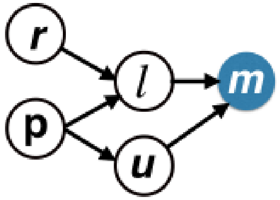
- Poisson factorization: $Y_{dw} \sim Pois(\langle \phi_n, \phi_{n'} \rangle)$, samples latent counts corresponding to non-zeros only
- Poisson factorization for imputing a graph
- Hierarchy of Gamma priors for less sensitivity towards initialization
- Non-parametric modeling with closed form inference updates

Negative Binomial Distribution (NB)

- Number of heads seen until r number of tails occurs while tossing a biased coin with probability of head p (or, number of successes before r failures in successive Bernoulli trials): $m \sim \text{NB}(r, p)$
- $m \sim \text{Poisson}(\lambda), \lambda \sim \text{Gam}(r, p)$ – Gamma-Poisson Construction
- $m \sim \sum_{t=1}^{\ell} u_t,\ u_t \sim \text{Log}(p),\ \ell \sim \text{Poisson}(-r \log(1-p))$ – Compound Poisson Construction



Gamma-Poisson Construction



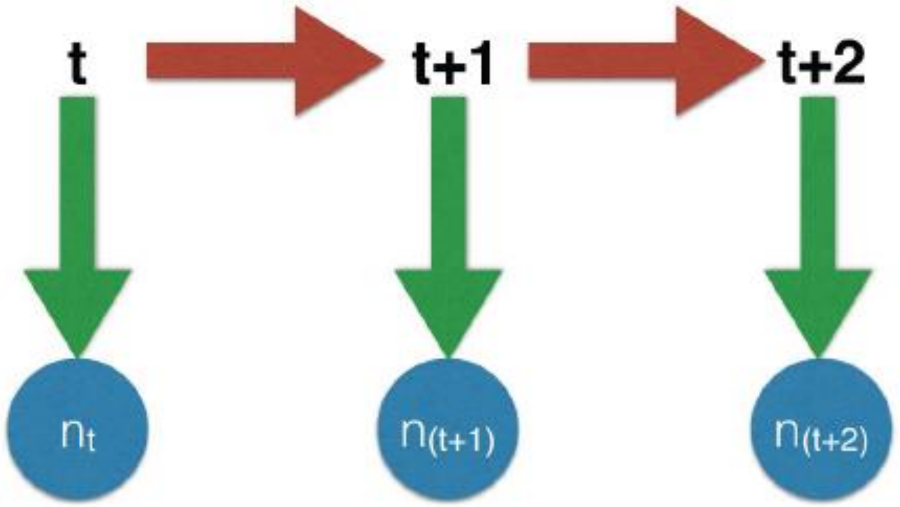
Compound Poisson Construction

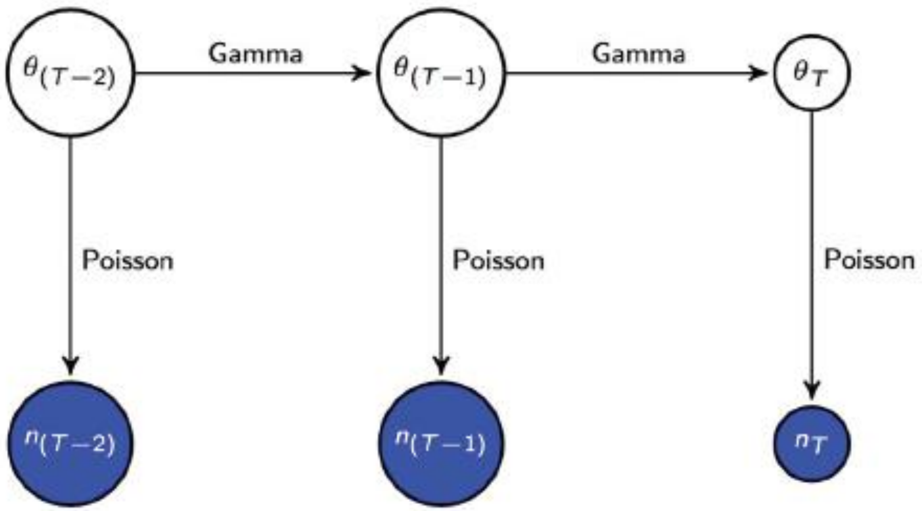
Figure: Constructions of Negative Binomial Distribution

Lemma

If $m \sim NB(r, p)$ is represented under its compound Poisson representation, then the conditional posterior of ℓ given m and r is given by $(\ell | m, r) \sim CRT(m, r)$, which can be generated via $\ell = \sum_{n=1}^{m} z_n, z_n \sim Bernoulli(r/(n-1+r))$.

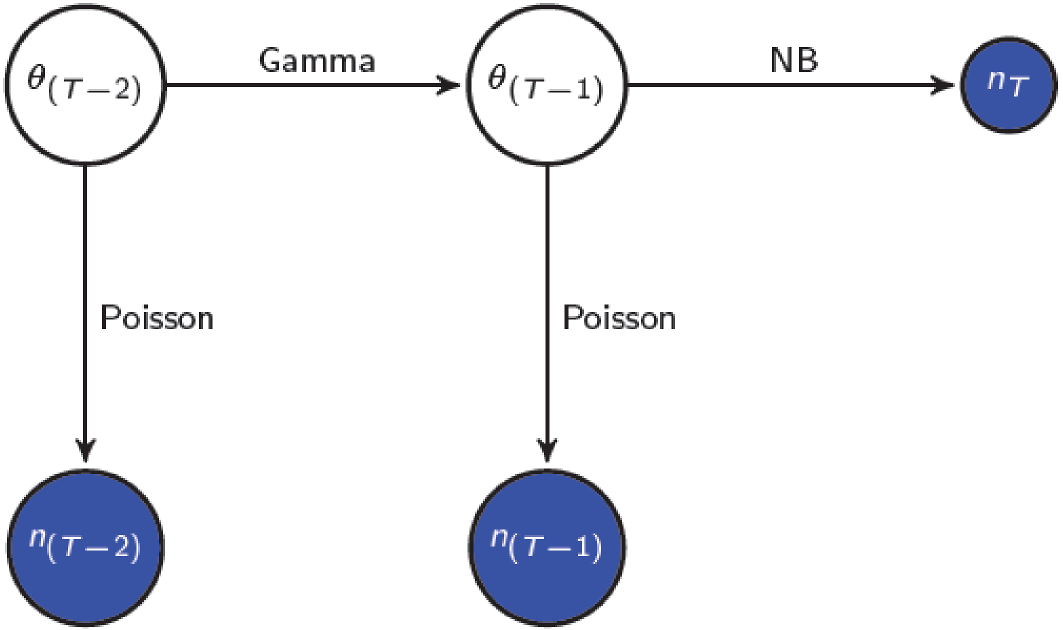
Gamma Poisson Autoregressive Model





- $\theta_t \sim \text{Gam}(\theta_{(t-1)}, 1/c)$, $n_t \sim \text{Pois}(\theta_t)$.
- Gamma-Gamma construction breaks conjugacy

Inference in Gamma Poisson Autoregressive Model



- use Gamma-Poisson construction of NB
- $n_T \sim \mathrm{NB}(\theta_{(T-1)}, 1/(c+1))$.

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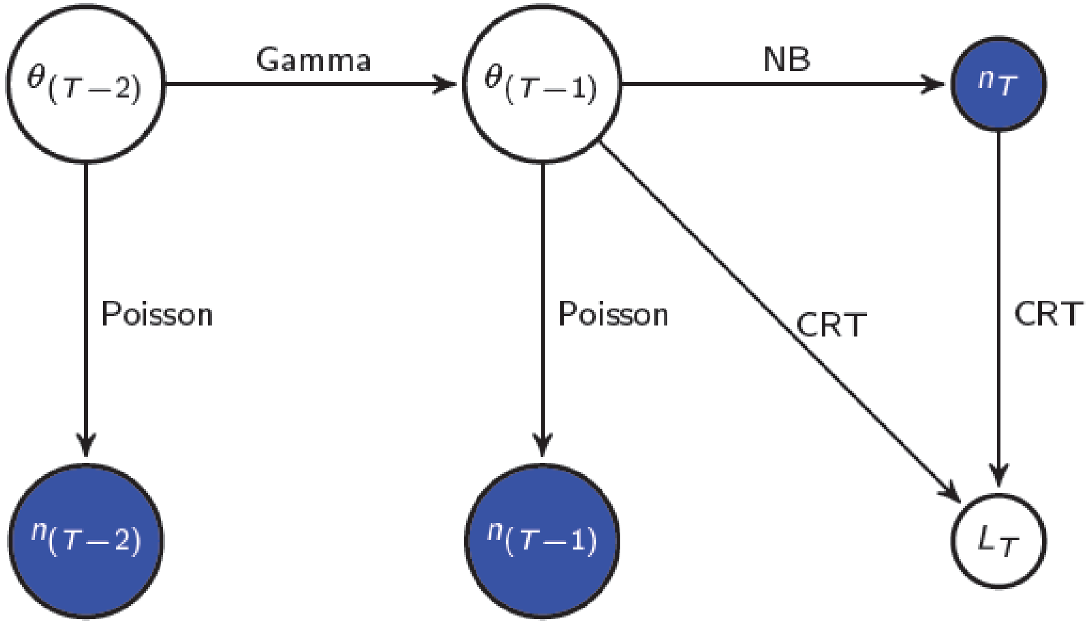
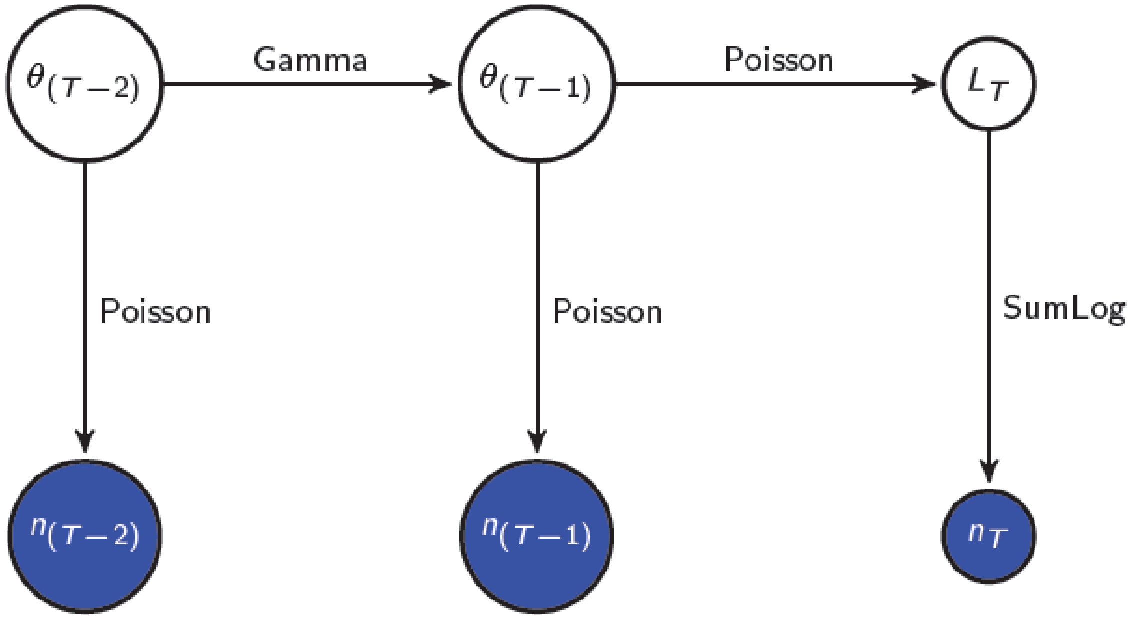
Inference in Gamma Poisson Autoregressive Model



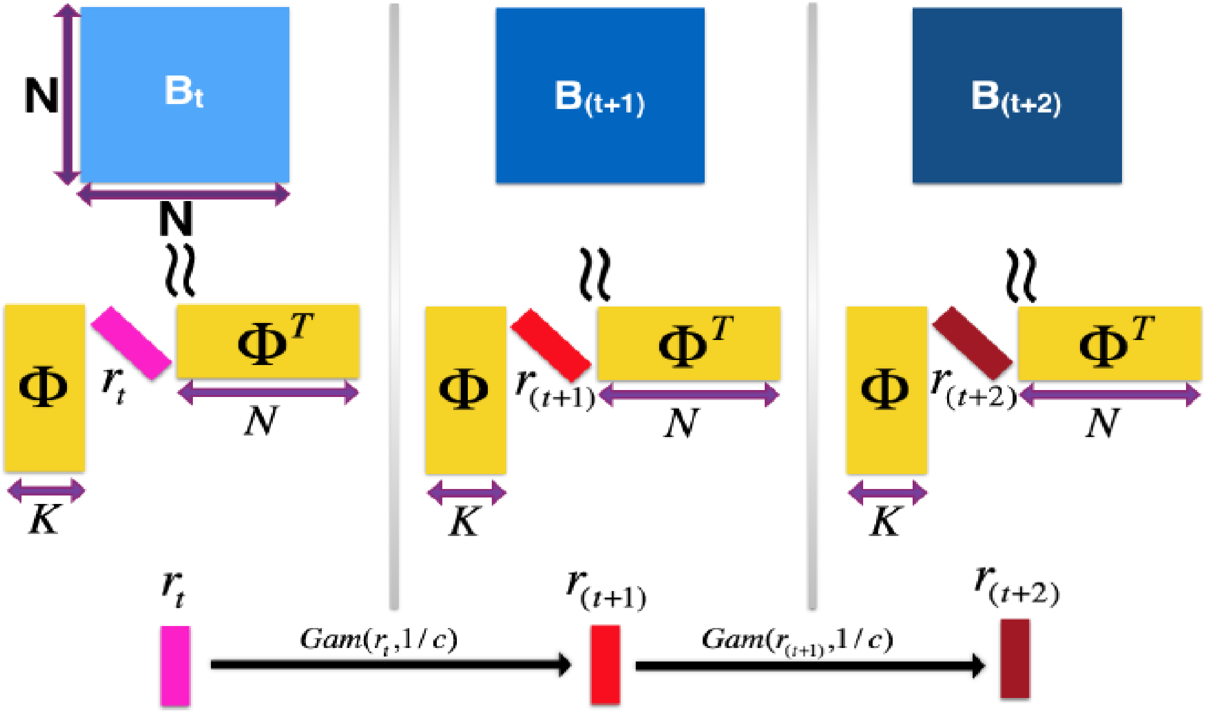
- $n_T \sim \mathrm{NB}(\theta_{(T-1)}, 1/(c+1))$. Augment $L_T \sim \mathrm{CRT}(n_T, \theta_{(T-1)})$.

Inference in Gamma Poisson Autoregressive Model



- use compound poisson construction of NB
- $n_T \sim \sum_{t=1}^{L_T} \mathrm{Log}(1/(c+1)), L_T \sim \mathrm{Poisson}(\theta_{(T-1)} \log((c+1)/c)).$
- Gamma-Poisson construction facilitates closed form Gibbs sampling.

Gamma Process Poisson Factorization for Dynamic Network Modeling (D-NGPPF)



- $b_{tnm} = I_{\{x_{tnm} \geq 1\}}$, $x_{tnm} = \sum_k x_{tnmk}$, $x_{tnmk} \sim \mathrm{Pois}(r_{tk}\phi_{nk}\phi_{mk})$.
- $r_{tk} \sim \mathrm{Gam}(r_{(t-1)k}/K, 1/c)$, $c \sim \mathrm{Gam}(g_0, 1/h_0)$, $r_{0k} \sim \mathrm{Gam}(\gamma_0, 1/f_0)$.
- $\phi_k \sim \prod_{n=1}^{N} \mathrm{Gam}(a_0, 1/c_n)$, $c_n \sim \mathrm{Gam}(c_0, 1/d_0)$.

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Results from Dynamic Network Modeling: Synthetic Data

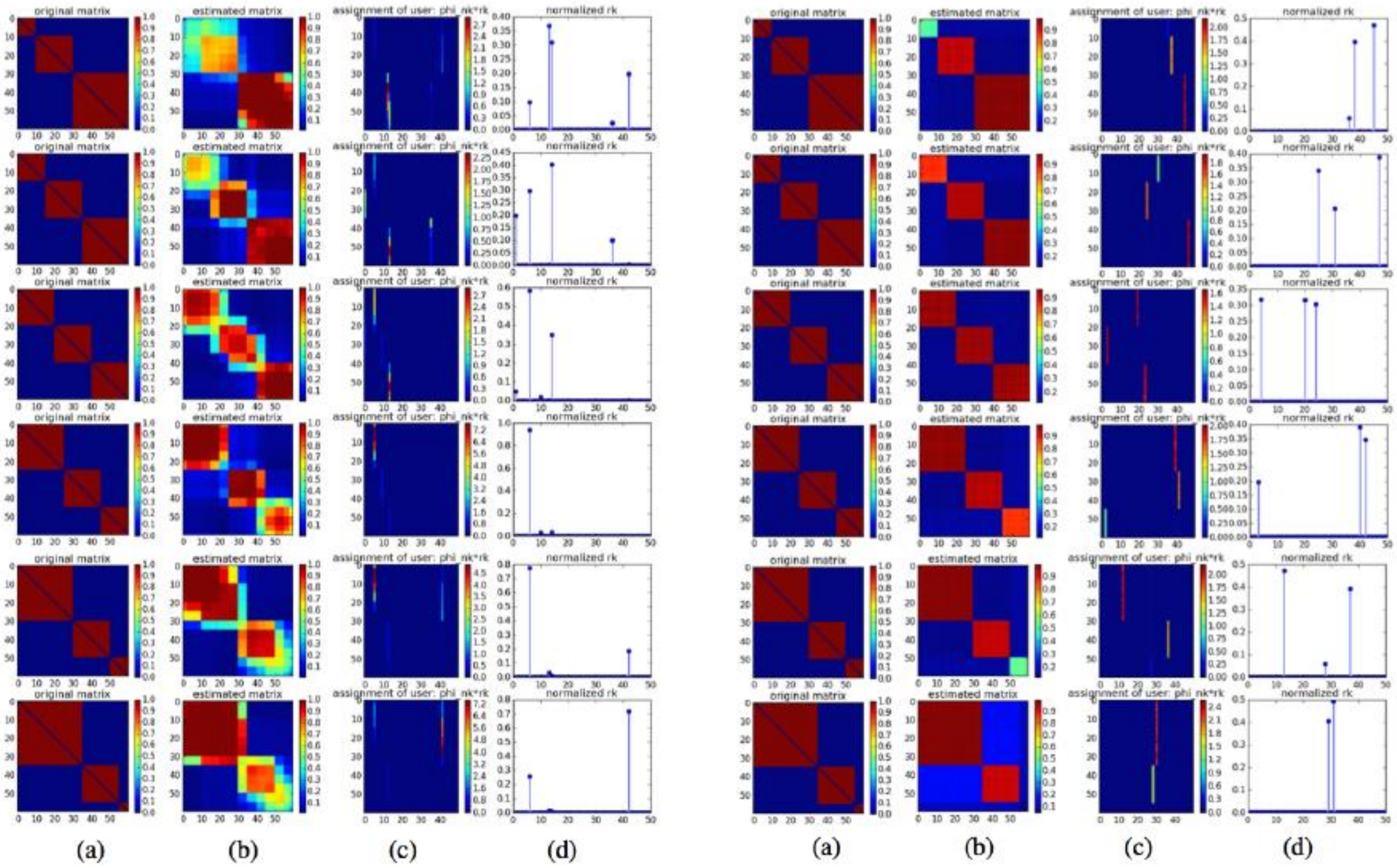

Figure: Results from dynamic model (left) and non-dynamic model (right)

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Results from Dynamic Network Modeling: Real-Wold Data

- DSBM: Dynamic stochastic block model
- N-GPPF: Gamma Process Poisson factorization for networks
- MMSB: Mixed membership stochastic block model

Dataset	D-NGPPF	DSBM	N-GPPF	MMSB
NIPS	**0.797** $\pm$ 0.016	0.780 $\pm$ 0.010	0.766 $\pm$ 0.012	0.740 $\pm$ 0.009
DBLP	**0.836** $\pm$ 0.013	0.810 $\pm$ 0.013	0.756 $\pm$ 0.020	0.749 $\pm$ 0.014
Infocom	**0.907** $\pm$ 0.008	0.901 $\pm$ 0.006	0.856 $\pm$ 0.011	0.831 $\pm$ 0.006

Figure: AUC Results

Method	D-NGPPF	DSBM	N-GPPF	MMSB
Complexity	$O((S+N+T)K)$	$O(N^2KT)$	$O((S+N)KT)$	$O(N^2KT)$

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