Percentile Classes

Sequences and Series

Arithmetic progression (AP)

A sequence is called an arithmetic progression if the difference of a term and the previous term is always same, i.e.,

$a_{n+1} - a_n$ = constant (= d) for all $n \in N$

= independent of n

The constant difference generally denoted by d is called common difference.

n th TERM OF A.P

If a is the first term and d the common difference of an AP, then its nth term a_n is given by

$a_n = a + (n - 1)d$

Properties of AP

* $T_p = q$, $T_q = P$ then $T_r = p + q - r$
* $T_p = \frac{1}{q}$ $T_q = \frac{1}{p}$ Then $T_{pq} = 1$
* $P\ Tp = q$, $q.Tq = p$ Then $Tp + q = 0$
* If $T_p = a$, $T_q = b$, $T_r = c$, Then

For AP → $(q - r)\, a + (r - p)\, b + (p - q)\, c = 0$

For GP → $a^{q-r} . b^{r-p} . c^{p-q} = 1$

For HP → $(q - r)\, \frac{1}{a} + (r - p)\, \frac{1}{b} + (p - q)\, \frac{1}{c} = 0$

Selection of term in A.P and G.P

	AP	GP
3 terms →	$a - d, a, a + d$	$\frac{a}{r}, r, ar$
4 terms →	$a-3d, a-d, a+d, a+3d$	$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$
5 terms →	$a-2d, a-d, a, a+d, a+2d$	$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

SUM OF n TERM

$$S_n = \frac{n}{2}[2a + (n-1)\, d]$$

$$S_n = (a + l)n/2$$

Geometric Progression

A sequence is called a geometric progression if the ratio of a next term to the previous term is always constant $a_1, a_2, a_3, \ldots\ldots, a_n$ is in GP. If

$\frac{a_{n+1}}{a_n}$ = a constant [common ratio r]

= independent of n.

If a is first term and r common ratio then GP will be $a, ar, ar^2, ar^3, \ldots\ldots\ldots\ldots\ldots ar^{n-1}$

n^{th} TERM OF GP

* $a, ar, ar^2, ar^3, \ldots\ldots\ldots\ldots\ldots ar^{n-1}$

$T_n = ar^{n-1}$

Properties of GP

* If T_p, T_q, T_r of an GP is in G.P. then p, q, r are in A.P.
* If T_p, T_q, T_r of an AP is in GP then $r = \frac{q-r}{p-q}$
* If T_p, T_q, T_r, T_s of an AP is in GP

then $p - q$, $q - r$, $r - s$ is in GP

Sum of n terms of a GP and Infinite GP.

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right),\ r \neq 1.$$

$S_n = a/r\text{-}1$, $|r| < 1$.

Geometric means

GM of *a* and *b* is G = $\pm\sqrt{ab}$

* If *a*, *b*, *c* are in GP then $b = \pm\sqrt{ac}$

* If a_1 , a_2 , a_3 .....a_n are *n* non-zero numbers then their GM will be $G = (a_1 , a_2 , a_3 .....a_n)^{1/n}$

Harmonic Progression

If a,b,c are in HP then 1/a, 1/b, 1/c are in AP

n th TERM OF HP

A sequence a_1, a_2, …., a_n,…. of non-zero numbers is called Harmonic progression, if the sequence $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots, \frac{1}{a_n}, \ldots$ is an Arithmetic progression.

T_n of HP = $\dfrac{1}{\frac{1}{a_1}+(n-1)d}$

Harmonic Mean (s)

If *a* and *b* are two non-zero numbers, then the harmonic mean of *a* and *b* is a number *H* such that the numbers *a*, *H*, *b* are in HP. We have

$$\frac{1}{H} = \frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right) \Rightarrow H = \frac{2ab}{a+b} = \frac{G^2}{A},$$

If a_1, a_3,…..,a_n are *n* non-zero numbers, then the harmonic mean *H* of these number is

dk given by

$$\frac{1}{H} = \frac{1}{n}\left(\frac{1}{a_1}+\frac{1}{a_2}+\ldots..+\frac{1}{a_n}\right).$$

$$\Rightarrow \quad d = \frac{a-b}{(n+1)ab}$$

AM ≥ GM ≥ HM

Exercise – 01

Arithmetic Progression

1. Find the sum of the first 15 terms of the series whose nth term is (4n +1)
   (a) 485 (b) 495 (c) 505 (d) 630

2. If the sum of AP is the same for p terms as for the q terms, find the sum for (p+q) terms.
   (a) 2 (b) 0 (c) 4 (d) none of these

3. Let t_r denote the rth term of an AP. If $t_m = \frac{1}{n}$ and $t_n = \frac{1}{m}$, then t_{mn} equals:
   (a) $\frac{1}{mn}$ (b) $\frac{1}{m} + \frac{1}{n}$ (c) 1 (d) 0

4. The 10^{th} common term between the series 3 + 7 + 11 + … and 1 + 6+ 11 + … is;
   (a) 191 (b) 193 (c) 211 (d) none of these

5. If a, b and c are in an AP, then a + $\frac{1}{bc}$, b + $\frac{1}{ca}$, c +$\frac{1}{ab}$
   (a) AP (b) GP (c) HP (d) none of these

6. If t_n denotes the nth term of the series 2 + 3 + 6 + 11 + 18 + ……then t_{50} is:
   (a) $49^2 - 1$ (b) 49^2 (c) $50^2 + 1$ (d) $49^2 + 2$

7. The 5^{th} term of an AP is 15 and the 9^{th} term is 23, find the 14^{th} term.
   (a) 31 (b) 33 (c) 35 (d) 37

8. If 9 times the 9^{th} term in an AP is equal to 15 times the 15^{th} term in the AP what is the 24^{th} term?
   (a) -1 (b) -3 (c) 0 (d) 1

9. Find the sum of all the two digit number that leave remainder 1 when divided by 5.
   (a) 963 (b) 968 (c) 967 (d) 960

10. The number of terms common between the series 1 + 2 + 4 + 8 + …to 100 terms and 1 + 4 + 7 + 10 + … to 100 terms is:
    (a) 6 (b) 4 (c) 5 (d) none of these

11. Find the 41st term of the progression 3, 8, 13. 15.
    (a) 102 (b) 203 (c) 304 (d) none of these

12. What term of the A.P, 2. 5. 8, ... is 56 ?
    (a) 20 (b) 21 (c) 19 (d) 15

13. If the 3rd and 7th terms of an A P. are 17 and 27 respectively. Find the first term of the A.P.:
    (a) 9 (b) 12 (c) 14 (d) none of these

14. If 7 times the seventh term of an A.P. is equal to 11 limes its eleventh term, the value of eighteenth term of the A P is:
    (a) 0 (b) -8 (d) 18 (d) 77

15. Find the sum of me A.P. 11,13,15….99:
    (a) 2475 (b) 2500 (c) 1122 (d) 1580

16. Find the sum of 222,224, 226, ..,888:
(a) 185370 (b) 195300 (c) 183000 (d) 899000

17. How many terms of the A.P. 1,4, 7,... are needed to give the sum 925 ?
(a) 20 (b) 22 (c) 24 (d) 25

18. p, q. r, s, t are first five terms of an A.P. such that p + r + t = - 12 and p.q.r = 8. Find the first term of the above A.P.:
(a) 3 (b) 2 (c) 4 (d) – 4

19. How many terms are there in A.P. whose first and fifth terms are -14 and 2 respectively and the sum is 40:
(a) 12 (b) 10 (c) 16 (d) 8

20. If you save Rs. 1 today, Rs. 2 the next day, Rs, 3 the succeeding day and so on, what will be your total savings in 365 days?
(a) 66579 (b) 66795 (c) 56795 (d) none of these

21. If m times the mth term of an A.P. is equal to n times its nth term, find (m+n) th term of the A.P.
(a) -1 (b) 0 (c) ,m (d) 4

22. If the first term of an AP is 2 and the first five terms is equal to one fourth of the next five terms. Then find the 20th term.
(a) -114 (b) -82 (c) -112 (d) none of these

23. Find the number of terms of the AP 98, 91, 84, … must be taken to give a sum of zero:
(a) 14 (b) 15 (c) 31 (d) 29

24. What is the least possible sum of the AP , -23, -19, -15….:
(a) -3 (b)-78 (c) -87 (d) none of these

25. There is an AP 11, 13 15... Which term of this AP is 65?
(a) 25th (b) 26th (c) 27th (d) 28th

26. Find the 25th term of the sequence 50, 45, 40, …
(a) -55 (b) -65 (c) -70 (d) -75

27. The 6th and 20th terms of an AP are 8 and -20 respectively. Find the 30th term.
(a) -34 (b) -40 (c) -32 (d) -30

28. How many terms are there in the AP 10, 15, 30,…120?
(a) 21 (b) 22 (c) 23 (d) 24

29. How many natural numbers between 100 to 500 are multiples of 9?
(a) 44 (b) 48 (c) 47 (d) 50

30. A number 39 is divided into three parts which are in A.P. and the sum of their squares is 515. Find the largest number.
(a) 17 (b) 15 (c) 13 (d) 11

31. The number of terms of the series 26 + 24 + 22 +... such that the sum is 182 is

(a) 13 (b) 14 (c) Both a and b (d) 15

32. Find the 1st term of an AP whose 8th and 12th terms are respectively 60 and 80.
(a) 15 (b) 20 (c) 25 (d) 30

33. The least value of n for which the sum of the series 5 + 10 + 15 … n terms is not less than 765 is
(a) 17 (b) 18 (c) 19 (d) 20

34. How many terms are identical in the two A.P.s 21, 23, 25, …up to 120 terms and 23, 26, 29, …up to 80 terms
(a) 39 (b) 40 (c) 41 (d) none of these

35. The sum of the first four terms of an A.P. is 56 and sum of the first eight terms of the same A.P. is 176, Find the sum of the first 16 terms of the A.P.?
(a) 646 (b) 640 (c) 608 (d) 536

Exercise – 01 (AP)

Solutions

1. Ans. (b)
Solution: Method I using summation formula
Sum of 15 terms = $\sum_{n-1}^{15} 4n + \sum_{n-1}^{15} 1$ =
4[1+2+3+4+5+….15] + 15 = 4 x 120 + 15 = 495
Method 2 using summation of AP formula
Terms are: 5, 9, 13, …., 61
Hence sum of AP = $\frac{first\ term+Last\ term}{2}$ x Number of terms
= $\frac{5+61}{2}$ x 15 = 33 x 15 = 495

2. Ans. (b)
Solution: If S_p = S_q then [S_{p+q} = 0]
You my use the result of this question as a standard result.

. If x = 1 + a + a^2 + a^3 + …. to ∞
(|a|<1),
y = 1 + b + b^2 + b^3 + ….. to ∞
(|b|<1),

3. Ans. (c)
Solution: Given that $t_m = \frac{1}{n}, t_n = \frac{1}{m}$
$t_m = a + (m-1)$d $= \frac{1}{n}$ ….(i)

t_n = a + (n-1)d = $\frac{1}{m}$ …(ii)

Solving (i) and (ii) we get (m-n) d = $\frac{1}{n}$ - $\frac{1}{m}$
d = $\frac{1}{mn}, a = \frac{1}{mn}$
$t_{mn} = a + (mn-1)d = \frac{1}{mn} + \frac{(mn-1)}{mn}$ = 1

4. Ans. (a)
Solution: 3 + 7 + 11 + …
1 + 6 + 11 + …
Common series will be
11, 31, 51
A = 11, d = 20
T_{10} = 11 + 19 x 20 = 191

5. Ans. (a)
Solution: Let a, b, and c be 1, 2, 3
a + $\frac{1}{bc}$ = 1 + $\frac{1}{2\,x\,3}$ = $\frac{7}{6}$
b+ $\frac{1}{ca}$ = 2 + $\frac{1}{3}$ = $\frac{7}{3}$
c + $\frac{1}{ab}$ = 3 + $\frac{1}{2}$ = $\frac{7}{2}$

6. Ans. (d)
Solution: $T_n = (n-1)^2 + 2$
For n = 1, $T_1 = 2$
N = 2, $T_2 = 3$
Therefore, $T_{50} = 49^2 + 2$

7. Ans. (b)
Solution: T_5 = 15, T_9 = 23
a+4d = 15, a+8d = 23
d = 2, a=7
$T_{14} = 7 + 13\,x\,2 = T_{14} = 33$

8. Ans. (c)
Solution: 9th term = a + 8d and 15th term = a+ 14d
Given that, 9(a+8d) = 15(a+14d)
→ 90 + 72d = 15a + 210d
→ 6a + 138d = 0 → a + 23d = 0
This is nothing but t_{24} (24th term) of the same AP. Hence, option (c) is the answer.

9. Ans. (a)

Solution: Series will be
11, 16, 21, 26,…,96
n = 18
S = $\frac{18}{2}$ (11 + 96) = 963

10. Ans. (c)
Solution: 1 + 2 + 4 + 8 + 16 +32 +64 +128 + 256 + ……
1 + 4 + 7 + 10+ …
The second series is in AP. T_{100} = 1 + 99 x 3 = 298
Therefore, common terms will be 1, 4, 16, 64, 256,
i.e., 5 terms.

11. Ans. (b)
Solution: T_{41} = 3 + 40 x 5 = 203
($\therefore T_n = a + (n-1)d$)

12. Ans. (c)
Solution: $T_n = l = a + (n-1)d$
= 2 + (n-1)3 = 56
= 3 (n-1) = 56 – 2
n = 19

13. Ans. (b)
Solution: T_3 = a + 2d = 17 …(i)
$T_7 = a + 6d = 27$ …(ii)
∴ Subtracting equation (1) from equation (2) we get
4d = 10 ➔ d = 2.5
a + 2 x 2.5 = 17
a = 12

14. Ans. (a)
Solution: $T_7 = a + 6d$
$T_{11} = a$ + 10d
∴ 7. T_7 = 11. T_{11}
⇨ 7 (a+6d) = 11 (a+10d)
⇨ 4a = -68d
⇨ A = -17d
Now, $T_{18} = a + 17d$
T_{18} = -17d + 17d
T_{18} = 0

15. Ans. (a)
Solution: 11 + 13 + 15 + … +99
Here number of terms = 50 – 5 = 45
Or number of terms = $\left(\frac{99-11}{2}\right)$ +1 = 45
Note: When both the extremes of a sequence are counted then number of terms = $\left(\frac{l-a}{d}\right)$ + 1
Now the sum of the progression = $\left(\frac{a+l}{2}\right)$n
= $\left(\frac{11+99}{2}\right)$ x 45= 2475
Alternatively: 11 + 13 + …99
= (1 + 3 + 5 + …99) – (1+3+…+9)
= $(50)^2$ - $(5)^2$= 2475

16. Ans. (a)
Solution: Number of terms = $\left(\frac{888-222}{2}\right)$ + 1 = 334
$\therefore S_{334}$ = $\left(\frac{222+888}{2}\right)$ x 334
= 555 x 334 = 185370

Alternatively: 222 + 224 +….+ 888
=(2 + 4 + 6 + …+888) – (2+4+6+….+220)
= (444 x 445) – (110 x 111)
= 111 (4 x 445 - 110) = 111 (1780-110)
= 111 x 1670 = 185370

17. Ans. (d)
Solution: Let n = 24, then
T_{24} = 1 + 23 x 3 = 70
S_{24} = $\left(\frac{1+70}{2}\right)$ x 24 = 71 x 12 = 852
S_{24} = 852 < 925, hence wrong
Thus it is obvious that n must be greater than 24, which gives us n = 24, per the choices given.
∴ S_{25} = $\left(\frac{a+T_{25}}{2}\right)$ 25 ($\therefore T_{25} = T_{24} + d =$ 73)

= $\left(\frac{1+73}{2}\right)$ x 25 = 925

Hence (d) is correct.
Alternatively: S_n = 925 = $\frac{n}{2}$ [2x1+(n-1)3]

⇨ 1850 = 2n + 3n^2 – 3n
⇨ 3n^2 – 75n + 74n - 1850 = 0
⇨ 3n (n-25) + 74 (n-25) = 0
⇨ n = 25 or n = $-\frac{74}{3}$
the only admissible value of n = 25
Since number of terms cannot be negative and as a fraction too.
Hence (d).

18. Ans. (b)
Solution: p= (r-2d), q = (r-d), r = r
S = (r+d), t = (r+2d)
P + q + t = 3r = -12
➔r = -4
Again p.q.r = 8
⇨ pq = -2 (∴ r = -4)
⇨ (r-2d) (r-d) = -2
⇨ (-4-2d) (-d-d) = -2
⇨ (2+d) (4+d) = -1

⇨ $d^2 + 6d + 9 = 0$
⇨ $(d+3)^2 = 0$
⇨ d = -3
∴ p= (r-2d) = -4 -2 x (-3) = 2

19. Ans. (b)
Solution: $T_1 = a = -14$
T_5 = a + 4d = 2
d = 4

⇨ $S_n = \frac{n}{2}[2a + (n-1)d]$

40 = $\frac{n}{2}[-28 + (n-1)4]$
⇨ n^2-8n -20 = 0
⇨ $n = 10 \; or \; n = -2$
Since number of terms can not be negative, hence n = 10

20. Ans. (b)
Solution: $S_{365} = \frac{365 \text{ x } 366}{2}$ $\left(S_n = \frac{n(n+1)}{2}\right)$

S_{365} = 66795

21. Ans. (b)
Solution: m. T_m = n. T_n
m [a + (m-1)d] = n [a+nd-d]
⇨ m[a+md-d] = n[a+nd-d]
⇨ ma+$m^2 - dm$ = na + $n^2 d - nd$
⇨ (m-n)a = $(m^2 - n^2)\ d$ + (m-n)d
⇨ (m-n)a = (m-n)d [1-(m-n)]
⇨ a = [1-(m+n)]d
Now $T_{(m+n)} = a + [(m+n) - 1]$d
= [1-(m-n)]d + [(m+n)-1]d = 0

22. Ans. (c)
Solution: a → 2
And $S_5 = \frac{1}{4}\ S(_{10} - S_5)$
5 $(S_5) = S_{10}$

⇨ $5\left[\frac{5}{2}(4+4d)\right] = \frac{10}{2}[4+9d]$
⇨ $d = -6$
⇨ T_{20} = a + 19d
⇨ = 2 + 19 x (-6) = -112

23. Ans. (d)
Solution: $S_n = \frac{n}{2}$[2a+ (n-1)d]
0 = $\frac{n}{2}$[196+(n-1)x (-7)]
0 = n (203-7n)
7n = 203 or no = 0
n = 29
Since n = 0 is not acceptable.
The AP is 98, 91, 84,….0,….,-91, -98
(Try to understand it as it is a problem of common sense,
Therefore is no need to calculate with the help of formula.)

24. Ans. (b)
Solution: The least possible sum can be obtained only when all the terms of AP be non positive.
Let the last term be T_n which must be non positive
$T_n \leq 0$
A+ (n-1)d ≤ 0
-23 + (n-1) 4 ≤ 0
4n ≤ 27
n ≤ 6
n = 6
$T_n = T_6 = -23 + 5 \text{ x } 4 = -3$
$S_6 = \left[\frac{-23 \text{ x} -3}{2}\right]$ x 6 = -78
(The required A.P. is -23, -19, -15, -11, -7, -3)

25. Ans. (d)
Solution: the number of terms in a series are found by:
$\frac{\text{Difference between first and last terms}}{\text{Common Difference}}$ + 1 = $\frac{65-11}{2}$ + 1 = 27 + 1 = 28[TH] term option (d) is correct.

26. Ans. (c)
Solution: The first term is 50 and the common difference is -5, thus the 25th term is: 50 + 24 x (-5) = -70. Option (c) is correct.

27. Ans. (b)
Solution: a + 5d = 8 and a + 19d = -20. Solving we get 14d = -28 -> d = -2. 30th term = 20th term + 10d =
-20 + 10 x (-2) = -40. Option (b) is correct.

28. Ans. (c)
Solution: in order to count the number of terms in the AP, use the shortcut;
[(last term - first term}/ common difference] + 1.
In this case it would become: [(120 – I0/5] +1- 23. Option (c) is correct.

29. Ans. (a)
Solution: the series will be 108, 117, 126, …495.
Hence answer = $\frac{495-108}{9}$ + 1 = 44. Option (a) is correct

30. Ans. (b)
Solution: the three parts are 11, 13, and 15 since $11^2 + 13^2 + 15^2 = 515$.
Since, we want the largest number, the answer would be 15

31. Ans. (c)

Solution: Use trial and error by using various values from the options.

If you find the sum of the series till 13 terms the value is 182. The 14^{th} term of the given series is 0, so also for 14 terms the value of the sum would be 182. Option (c) is correct.

32. Ans. (c)

Solution: Since the 8th and the 12th terms of the AP are given as 60 and 80, respectively, the difference between the two terms would equal 4 times the common difference. Thus we get 4d = 80 - 60 = 20. This gives us d = 5, Also, the 8th term in the AP is represented by a + 7d, we get: a + 7d = 60 → a + 7 x 5 = 60 → a = 25. Option (c) is correct.

33. Ans. (a)

Solution: Solve this question through trial and error by using values of n from the options; For 16 terms, the series would be 5 + 10 + 15 + ... + 80 which would give us a sum for the series as 8 x 85 = 680. The next term (17th term of the series) would be 85. Thus, 680 + 85 = 765 would be the sum to 17 terms. It can thus be concluded that for 17 term the value of the sum of the series is not less than 765. Option (a) is correct

34. Ans. (b)

Solution: The first common term is 23, the next will be 29 (Notice that the second common term is exactly 6 away from the first common term. 6 is also the LCM of 2 and 3 which are the respective common differences of the two series.)

Thus, the common terms will be given by the A.P 23, 29, 35 ...... last term. To find the answer you need to find the last term that will be common to the two series.

The first series is 23, 25, 27 ... 259

While the second series is 23, 26, 29 ..... 260.

Hence, the last common term is 257,

Thus our answer becomes $\frac{257-23}{6}$ + 1 = 40.

Option (b) is correct.

35. Ans. (c)

Solution: Think like this:

The average of the first 4 terms is 14, while the average of the first 8 terms must be 22.

Now visualise this:

1st $\underbrace{2nd \quad 3rd}_{average=14}$ $\underbrace{4th \quad 5th}_{average=22}$ 6th 7lh 8lh

Hence, d = 8/2 = 4 [Note: understand this as a property of an A.P.]

Hence, the average of the 8th and 9th term = 22 + 4.4 = 38 Rut this 38 also represents the average of the 16 term A.P.

Hence, required answer = 16 x 38 = 608. Option (c) is correct.

Exercise – 02

Geometric Progression

1. if a, b and c are in GP, then a+b, 2b, b+c are in

(a) AP (b) GP (c) HP (d) none of these

2. In the sequence 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8, …., where n consecutive terms have the value of n, then 1025^{th} term is:

(a) 2^9 (b) 2^{10} (c) 2^{11} (d) 2^8

3. The sum of an infinite geometric series is 4 and the sum of the cubes of the terms of the same GP is 192. The common ratio of the original geometric series is:

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

4. In an infinite GP, every term is equal to the sum of all the terms that follow. Find the common ratio.

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/6

5. In a GP, the product of the first four terms is 4 and the second term is the reciprocal of the fourth term. The sum of the GP upto infinite terms is:
   (i) 8 (ii) -8 (iii) $\frac{8}{3}$ (iv) $-\frac{8}{3}$
   (a) Only i and ii (b) only ii and iii (c) only I, ii, and iii (d) I, ii, iii and iv

6. In an infinite GP, each term is equal to four times the sum of all the terms that follow, find the common ratio.
   (a) ¼ (b) 1/5 (c) 1/8 (d) 1/7

7. Find the 22^{nd} term of the G.P. 2, 2, -2, ….;
   (a) -22 (b) -2 (c) 2 (d) none of these

8. The 5^{th}, 8^{th}, and 11^{th} terms of a G.P. are a,b,c respectively, then which one of the following is true?
   (a) 2b = ac (b) $b^2 = ac$ (c) a +b +c = 0 (d) none of these

9. What is the least number of terms of the G.P. 5 + 10 + 20 + ...whose sum would surely exceed 10^6?
   (a) 17 (b) 18 (c) 19 (d) 21

10. The sum of 3 numbers in G.P. is 38 and their product is 1728. Find the greatest number :
    (a) 24 (b) 18 (c) 16 (d)8

11. The third term of a G.P. is 4. The product of first five terms is:
    (a) 4^3 (b) 4^4 (c) 4^5 (d) none of these

12. Sum of three consecutive terms in a G.P. is 42 and their product is 512. Find the largest of these numbers :
    (a) 28 (b) 16 (c) 32 (d) none of these

13. The sum of four terms in G.P. is 312. The sum of first and fourth term is 252. Find the product of second and third term
    (a) 500 (b)150 (c) 60 (d) none of these

14. A bouncing tennis ball rebounds each time to a height equal to one half the height of the previous bounce. If it is dropped from a height of 16 m. find the total distance it has travelled when it hits the ground for the 10th time:
    (a) $47\frac{15}{16}$ (b) $37\frac{5}{16}$ (c) $67\frac{11}{16}$ (d) none of these

15. Find the common ratio of the GP whose first and last terms are 5 and $\frac{32}{625}$ respectively and the sum of the GP in $\frac{5187}{625}$.
    (a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{5}{3}$ (d) $\frac{4}{5}$

16. Find the sum of the series $2\sqrt{3}, 2\sqrt{2}, \frac{4}{\sqrt{3}}, \ldots$
    (a) $6(\sqrt{3} + \sqrt{2})$ (b) $6(\sqrt{3} - \sqrt{2})$
    (c) $\frac{6}{(\sqrt{3}+\sqrt{2})}$ (d) none of these

17. Find the sum of the three numbers in G.P. whose product is 216 and the sum of the products of them taken in pairs is 126:
    (a) 28 (b)21 (c) 35/4 (d) none of these

18. The sum of an infinite G.P. is 4 and the sum of their cubes is 192. Find the first term:

(a) 4 (b) 8 (c) 6 (d) 2

19. If x, y , z are in GP and $a^x = b^y = c^z$, then;
(a) $log_b a = log_c b$ (b) $log_b a = log_a c$
(c) $log_c b = log_a c$ (d) none of these

20. A square is drawn by joining the mid points of the sides of a given square. A third square is drawn inside the second square in the same way and this process continues indefinitely. If a side of the first square is 4 cm determine the sum of the areas of all the squares:
(a) 32 cm^2 (b) 16 cm^2 (c) 20 (d) none of these

21. If pth, qth rth and sth terms of an A.P. are in G.P. then p-q, q-r, r-s are in:
(a) A.P. (b) G.P. (c) H.P. (d) none of these

22. If x, y , z are in G.P. and $a^x = b^y = c^z$, then $log_b a.\ log_b c$ is equal to :
(a) 0 (b) 1 (b) ac (d) none of these

23. The 3rd and 8th term of a GP are 1/3 and 81, respectively. Find the 2nd term.
(a) 3 (b) 1 (c) 1/27 (d) 1/9

24. Find the general term of the GP with the third term 1 and the seventh term 8.
(a) $(2^{3/4})^{n-3}$ (b) $(2^{3/2})^{n-3}$ (c) $(2^{3/4})^{3-n}$ (d) $(2^{3/4})^{2-n}$

25. How many terms are there in the G.P. 5, 10, 20, 40, …1280?
(a) 6 (b) 8 (c) 9 (d) 10

26. What will be the value of $2^{1/3}.2^{1/6}.2^{1/12}$… to infinity.
(a) 2^2 (b) $2^{2/3}$ (c) $2^{3/2}$ (d) 8

27. The middle points of the sides of a triangle are joined forming a second triangle. Again a third triangle is formed by joining the middle points of this second triangle and this process is repealed infinitely. If the perimetre and area of the outer triangle are P and A respectively, what will be the sum of perimetres of triangles thus formed?
(a) 2P (b) P^2 (c) 3P (d) $P^2/2$

Exercise – 02 (GP)

Solutions

1. Ans. (c)
Solution: Since a, b, c are in GP, assume that a, b, c are 2, 4, 8, respectively.
Then a +b = 6, 2b = 8, b+c=12
Thus, 6, 8 and 12 are in HP.

2. Ans. (b)
Solution: 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8
First term = 1, Second term = 2, fourth term = 4, eigth term = 8
Therefore, terms are in GP.
$\therefore S_n = \frac{a(r^n-1)}{r-1} = \frac{1(2^n-1)}{2-1}$
For n = 10 $S_{10} = 1023$
Therefore, 1025th term will be 2^{10}.

3. Ans. (b)
Solution: $s_\infty = \frac{a}{1-r}$
$\frac{a}{1-r} = 4, \frac{a^3}{1-r^3} = 192$
$\frac{a^3}{(1-r)^3} = 64, \frac{a^3}{1-r^3} = 192$
$\frac{(1-r)^3}{(1-r^3} = 3$
On saving, we get = r= $-\frac{-1}{2}$

4. Ans. (a)
Solution: Since a = ar + $ar^2 + \cdots \infty$, [r<1]
$S_\infty = \frac{a}{1-r}$
a = $\frac{ar}{1-r}$, Hence r = $\frac{1}{2}$

5. Ans. (d)
Solution: Let us consider a GP
a, ar, ar^2, ar^3
a x ar x ar^2 x ar^3 = 4
or $a^4, r^6 = 4, hence\ a^2\ r^3$= ±2
Also, ar = $\frac{1}{ar^3}$
$a^2\ r^4$ = 1, hence, ar^2 = ±1
On solving, we get, a = 4, -4, r = $\frac{1}{2}$, - $\frac{1}{2}$
Now, when a = 4, r = 1/2 when a = 4, r = -$\frac{1}{2}$
S_∞ = 8 $\qquad$ $S_\infty = \frac{8}{3}$
When a = -4, r = 1/2 when a = -4, r = $-\frac{1}{2}$
S_∞ = -8 $\qquad$ $S_\infty = -\frac{8}{3}$
Hence, option (d) is the answer.

6. Ans. (b)
Solution: $S_\infty = \frac{a}{1-r}$
Since a = 4 (ar + ar^2 + ar^3 ... ∞)
i.e., a = 4 $\frac{ar}{1-r}$
So, r = $\frac{1}{5}$
Hence, option (b) is the answer.

7. Ans. (c)
Solution: Every even term is equal to 2
Common ratio of the GP is -1

8. Ans. (b)
Solution: $T_5 = a_1.r^4 = a$
$T_8 = a_1.r^7 = b$
$T_{11} = a_1.r^{10} = c$

Let b^2 = ac
Then, $(a_1r^7)^2 = a_1r^4.\ a_1r^{10}$
⇨ $a_1^2r^{14} = a_1^2r^{14}$
Hence (b) is correct.

9. Ans. (b)
Solution: $S_n > 10^6, for\ least\ possible\ n\ \in I^+$
Then, $\frac{5(2^n-1)}{(2-1)} > 10^6$
⇨ $2^n - 1$> 2 x 10^5
⇨ 2^n= 200001
At n = 17, 2^n= 2^{17}= 131072
And at n = 18, $2^n = 2^{18}$ = 262144
Hence the least possible value of n = 18

10. Ans. (b)
Solution: let a ,b , c be the three numbers in GP then
a+ b+ c = 38
And abc = 1728
⇨ b = $(abc)^{\frac{1}{3}}$ = $(1728)^{\frac{1}{3}}$ = 12
⇨ a x c = 144
and a + c = 26
⇨ Now, go through option
⇨ Let c = 18, then a = 8
a. c = 8 x 18 = 144

Alternatively : let the three numbers in GP be
$\frac{a}{r}, a, ar$
$\frac{a}{r} + a + ar = 38$
...(1)
And
$\frac{a}{r}a.ar = 1728$
...(2)

a = 12
∴ $\frac{12}{r}$ = + 12 + 12r = 38
Or
$\frac{6}{r} + 6 + 6r = 19$

⇨ $6r^2 - 13r + 6 = 0$
⇨ $6r^2 - 9r - 4r + 6 = 0$

⇨ (2r-3) (3r-2) = 0
⇨ r = $\frac{3}{2}$ or r = $\frac{2}{3}$
∴ Three number is GP are 8, 12, 18, or 18, 12, 8.
Hence (b) is correct.

11. Ans. (c)
Solution: Let a, b,c, d, e, be give consecutive terms in GP them
A = $\frac{c}{r^2}, b = \frac{c}{r}$, d = cr, e= cr^2

∴ a.b.c.d.e = $\frac{c}{r^2}.\frac{c}{r}.c.cr.cr^2 = c^5$
(∴ c = 4)
∴ a.b.c.d.e = 4^5

12. Ans. (c)
Solution:
a+ b+ c+ = 42.
⇨ abc = 512
⇨ b = 8 $\qquad$ $(\therefore (abc)^{1/3} = a = 8)$
ac = 64
and a + c = 34
Now, go through options (or by hit and trial)
∴ either a = 2, then c = 32
Or a = 32 then c = 2
Hence (c) is correct.

13. Ans. (a)

Solution: a + b + c+ d = 312

A + ar + $ar^2 + ar^3 = 312$

And a + d = 252

b + c = 312 – 252 = 60

And a + ar^3 = 60

⇨ a(1 + r^3) = 252

and a (r+r^2) = 60

⇨ $\frac{a(1+r^3)}{a\,(r+r^2)} = \frac{252}{60} = \frac{21}{5}$

➔ $\frac{1+r^3}{r+r^2} = \frac{21}{5}$

⇨ $\frac{(1+r)(1+r^2-r)}{r\,(1+r)} = \frac{21}{5}$

(∴ $a^3 + b^3$ = (a+b) ($a^2 + b^2$ - ab))

⇨ $\frac{1+r^2-r}{r} = \frac{21}{5}$

⇨ 5r^2 + 26r + 5 = 10

⇨ r = 5 or ar x ar^2 = 500 (when r = 5)

and a = 250 and ar * ar^2 = 500 (when r = $\frac{1}{5}$)

Go through options.

14. Ans. (a)

Solution: Distance travelled before the first hit = 16m

Distance travelled before the second hit = 16 + $\frac{1}{2}$ x 16 x 2

= (16 + 16)m

Distance travelled before the third hit

= 16 + 16 + ½ x 8 x 2

= (16 + 16 + 8 + 4)m

And so on.

Hence the require distance

= 16 + 16 + 8+ 4+ 2+ 1+ $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

= 16 + $\left(16 + 8 + 4 + \cdots . + \frac{1}{16}\right)$

= 16 + $\frac{16\left(1-\left(\frac{1}{2}\right)^9\right)}{1-\frac{1}{2}}$ = 16 + 16 x $\left(\frac{511}{512}\right)$ x $\frac{2}{1}$

= 16 + $\frac{511}{16}$ = 47$\frac{15}{16}$

15. Ans. (b)

Solution: Since a > l, therefore r < 1

Now, since $s_n = \frac{a-lr}{1-r}$ if r < 1

$\frac{5187}{625} = \frac{5-\frac{32}{625}r}{1-r}$

(1-r)$\frac{5187}{625}$ = 5 - $\frac{32r}{625} = r = \frac{2}{5}$

16. Ans. (a)

Solution: r = $\frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\frac{4}{\sqrt{3}}}{2\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$

$S_\infty = \frac{a}{1-r|}; |r < 1$

= $\frac{2\sqrt{3}}{1-\sqrt{\frac{2}{3}}} = \frac{2\sqrt{3} \text{ x } \sqrt{3}}{\sqrt{3}-\sqrt{2}}$

= $\frac{6}{\sqrt{3}-\sqrt{2}}$

6($\sqrt{3}$ + $\sqrt{2}$)

17. Ans. (b)

Solution: a. b. c = a.ar. $ar^2 = 216$

b = ar = 6 ...(1)

and ab + bc + ac = a. ar+ ar. A$r^2 + a.ar^2 = 126$

⇨ $a^2r + a^2r^3 + a^2\,r^2 = 126$

⇨ $a^2r + a^2r^3 = 90$ (∴ ar = 6)

⇨ $6a + 36r = 90$

⇨ a + 6r = 15 ...(2)

from equ. (1) and (2) we get

$\frac{6}{r}$ + 6r = 15

⇨ 6r^2 – 15 + 6= 0

⇨ r = 2 or $\frac{1}{2}$

∴ a, ar, ar^2 = 3, 6, 12, or 12, 6, 3

a+ b+ c = 21

Alternatively: go through options

18. Ans. (c)

Solution: a + ar + ar^2 + ar^3...+ ∞ = 4

⇨ $\frac{a}{1-r} = 4$

or

⇨ $\frac{a^3}{(1-r)^3} = 64$ ....(1)

⇨ And a^3 + $(ar)^3$ + $(ar^3)^3$ + ... ∞ = 192

= $\frac{a^3}{1-r^3}$ = 192 ...(2)

From equation (1) and (2)

$64\,(1-r)^3 = (1-r^3)192$

⇨ $2r^2 + 3r^2 - 3r - 2 = 0$
r = $-\frac{1}{2}, 1, -1$

but |r| < 1
∴ r = $-\frac{1}{2}$ is the only admissible value

$\frac{a}{1-r} = 4$

⇨ $\frac{a}{1-(-\frac{1}{2})} = 4 ==> a = 6$

19. Ans. (a)
Solution: Let x , y , z be 1 , 2, 4 respectively
∴ $a^1 = b^2 = c^4$

⇨ $16^1 = 4^2 = 2^4$
⇨ $a = 16, b = 4$ *and* $c = 2$
⇨ $log_4 16 = log_2 4$
⇨ 2 = 2
Hence choice (a is correct.)

20. Ans. (a)
Solution: side of the first square is 4 cm
Side of the second square is $2\sqrt{2}\ cm$
And side of the third square is 2 cm
And so on.
i.e. each side of first, second, third square….
= $4, 2\sqrt{2}, 2, \sqrt{2}, 1, \frac{1}{\sqrt{2}}, \ldots$

∴ Area of each of first, second , third square….
= 16, 8, 4, 2, 1, $\frac{1}{2}$, …
∴ Sum of the area of first, second, third square …
= 16 + 8 + 4 + 2 + … = $\frac{16}{1-\frac{1}{2}} = 32\ cm^2$

21. Ans. (b)
Solution: Let the A.P. be 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, …
Where p = 1, q = 2 r = 4, s = 8
∴ p – q = - 1 and q – r = -2 and r – s = -4
Hence (p-q), (q-r) and (r-s) are in G.P. with common ratio 2 and first term – 1.

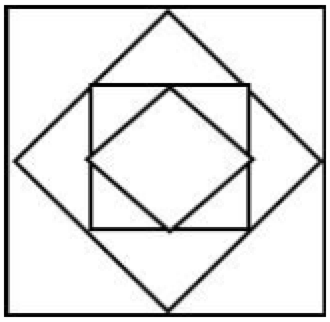
22. Ans. (b)
Solution: Let x = 1, y = 2, z = 4

∴ a = 16, b = 4, c = 2 $(\because a^x = b^x = c^z)$

$log_b a.\ log_b c = log_4 16.\ log_4 2$

= $(2log_4 4)\left(\frac{log 2}{2log 2}\right)$
= $(2.1)\left(\frac{1}{2}\right) = 1$

23. Ans. (d)
Solution: 3rd term ar^2 = 1/3, 8th term ar^7=81 , ar^5 = 243 Gives us: r = 3.
Hence, the second term will be given by (3rd term/r) = 1/3. 1/3 = 1/9. Option (d) is correct.
[Note: To go forward in a G.P. you multiply by the common ratio, to go backward in a G.P. you divide by the common ratio.]



24. Ans. (a)
Solution: Go through the options. The correct option should give value as 1, when n = 3 and as 8 when n = 7. Only option (a) satisfies both conditions.

25. Ans. (c)
Solution: 1280 = 5. 2^{n-1} or n - 1 = 8 or n = 9. Thus, there are total of 9 terms in the series. Option (c) is correct.

26. Ans. (b)
Solution: The expression can be written as $2^{\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\cdots} = 2^{INFINTE\ SUM\ OF\ THE\ GP} = 2^{2/3}$. option (b) is correct.

27. Ans. (a)
Solution: Each subsequent triangle would have the sum of sides halved from the previous triangle . thus, the sum of the perimetres would be given by P + P/2 + P/4 + P/8 + … till infinite terms. Hence the sum of all the perimetres of the infinite triangles would be 2P.

Exercise – 03

Special Series

1. Find the sum of the 37th bracket of the following series,
(1) + $(7+7^2+7^3)$ + $(7^4+7^5+7^6+7^7+7^8)$ + $(7^9+7^{10}+7^5+......+7^{15})$...
(a) $\frac{7^{37}}{6}(7^{37}-1)$ (b) $\frac{(7^{73}-1)}{6}$
(c) $\frac{7^{71}}{6}(7^{73}-1)$ (d) none of these

2. Find the sum of the series:
$1.2^0 + 2.2^1 + 3.2^2 + 4.2^3 + ...+ 100.2^{99}$
(a) $99 \times 2^{100} - 1$ (b) $99 \times 2^{101}+1$
(c) $99 \times 2^{101} - 1$ (d) $99 \times 2^{100} + 1$

3. What is the sum of the following series?
(a) 7 + 26 + 63 + 124 + ...999
(a) 3014 (b) 3013 (c) 3015 (d) none of these

4. Find the sum of the series : $-1 + 1^2 - 2 + 2^3 - 3 + 3^2 + ... n + n^2$.
(a) $\frac{-n(n+1)}{3}$ (b) $\frac{n(n+1)(n-1)}{3}$ (c) $\frac{n(n-1)}{3}$ (d) none of these

5. Find the sum to n terms of the series.
7 + 77 + 777 + 7777+ ....
(a) $\frac{7}{9}\{10(10^n-1)-n\}$ (b) $\frac{7}{9}\left\{\frac{10}{9}(10^n-1)-n\right\}$
(c) $7\left\{\frac{10}{9}(10^n-1)-n\right\}$ (d) none of these

6. Find the sum to n terms of 0.8 + 0.88+ 0.888 + ...
(a) $\frac{8}{9}[\frac{1}{9}(1-(0.1)^n]$ (b) $\frac{8}{9}\left[\frac{1}{10}(1-(0.1)^n)\right]$
(c) $\frac{8}{9}\left[n-\frac{1}{9}(1-(0.1)^n)\right]$ (d) none of these

7. Find the sum of $\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\cdots.+\frac{2n-1}{2^n}$.
(a) $\left(\frac{2n-3}{2^n}\right)$ (b) $\left(3-\frac{2n+3}{2^n}\right)$
(c) $\left(\frac{2n+3}{2^n}\right)$ (d) none of these

8. Find the sum to n terms of the series
11 + 102 + 1003+ 10004+ ....;
(a) $(10^n-1)+\frac{n(n+1)}{2}$ (b) $\frac{10}{9}(10^n-1)+\frac{n(n+1)}{2}$
(c) 10^n+n^2-1 (d) none of these

9. Find the sum to first n groups of
(1) + (1+3) + (1+3+9) + (1+3+9+27) + ...
(a) $\frac{1}{2}(3^n-1)$ (b) $\frac{3}{4}(3^n-1)-\frac{n}{2}$
(c) n^2+1 (d) none of these

10. Find the sum to first n terms;

$1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+\cdots$

(a) $\frac{3}{4}-\frac{(3+2n)}{4(3^{n-1})}$ (b) $\frac{9}{4}-\frac{3}{4}\left[\frac{3+2n}{3^n}\right]$

(c) $2n-\left(\frac{1}{n}\right)^2$ (d) none of these

11. Find the sum of the series;

$1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\cdots to\ n\ terms;$

(a) $\frac{35}{16}+\frac{7n}{5}$ (b) $\frac{12n-7}{16(5^{n-1})}$

(c) $\frac{35}{16}-\left[\frac{12n+7}{16(5^{n-1})}\right]$ (d) none of these

12. Find the sum of the series

$1.3^2 + 2.5^2 + 3.7^2 + \ldots$. To 20 terms:

(a) 12896 (b) 187898 (c) 98970 (d) 188090

13. Find the sum of the series

$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$ to 16 terms:

(a) 224 (b) 446 (c) 2356 (d) none of these

14. Find the sum of the series $\frac{1}{4}-\frac{3}{16}+\frac{9}{64}-\frac{27}{256}+\ldots$

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (d) $\frac{1}{7}$ (d) none of these

15. The sum to n terms of the series 1 + (1+3) + (1+3+5) + … is:

(a) $[\frac{n(n+1)}{2}]^2$ (b) n^2 (c) $\frac{n(n+1)(2n+1)}{6}$ (d) none of these

16. The sum to n terms of the series.

$1^2+(1^2+3^2)+(1^2+3^2+5^2)+\cdots$ is;

(a) $\frac{1}{3}(n^4+2n^2)$ (b) $\frac{1}{3}(n^3+3n^2\ n)$

(c) $\frac{1}{6}n\,(n+1)\,(2n^2+2n-1)$ (d) none of these

17. The sum of first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\ldots$

(a) 2^n-1 (b) $1-2^{-n}$

(c) 2^n-n+1 (d) $n+2^{-n}-1$

18. The sum to n terms of the series, where n is an even number:

$1^2-2^2+3^2-4^2+5^2-6^2+\ldots$:

(a) n(n+1) (b) $\frac{n(n+1)}{2}$

(c) $-\frac{n(n+1)}{2}$ (d) none of these

19. The sum to n terms of the series

$\frac{1}{\sqrt{1}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\ldots$ is:

(a) $\sqrt{2n+1}$ (b) $\frac{1}{2}\sqrt{2n+1}$ (c) $\sqrt{2n-1}$ (d) $\frac{1}{2}\{\sqrt{2n+1}-1\}$

20. Find the sum to n terms of the series 3 + 6 + 10 + 16 + …

(a) $\frac{n(n-1)}{2}-1$ (b) $n(n+1)+2^n-1$

(c) n(n+2)+1 (d) $3(2n+1) - 2^n$

21. Find the sum to n terms of the series

$1 + (1+2) + (1+2+3) + (1+2+3+4) + \ldots$

(a) $\frac{n(n+1)(2n+1)}{6}$ (b) $\frac{n(n+1)(n+2)}{6}$

(c) $\frac{n(n+1)(n+2)}{12}$ (d) $\frac{n(n+1)}{2}$

22. The sum to n terms of the series

$1^2 + (1^2 + 3^2) + (1^2 + 3^2 + 5^2) + \ldots$ is

(a) $\frac{1}{3}(n^3 + n^2 + 1)$ (b) $\frac{1}{6}n(n+1)(2n^2 + 2n - 1)$

(c) $\frac{1}{3}(2n^2 + 2n - 1)$ (d) none of these

23. The sum to n terms of the series

$\frac{3}{1^2.2^2} + \frac{5}{2^2\,3^2} + \frac{7}{3^2\,.4^2} + \cdots is:$

(a) $\frac{(n^2-2n)}{(n-1)^2}$ (b) $\frac{n^2+2n}{(n+1)^2}$

(c) $\frac{2n^2+1}{n}$ (d) none of these

24. The sum of the infinite series

$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \cdots\infty$ is equal to:

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{38}{27}$ (d) none of these

25. What is the sum of n terms of the series $-1 + 1^2 - 2 + 2^2 - 3 + 3^2 \ldots$?

(a) $\frac{n(n^2-1)}{3}$ (b) $n^2 + n$

(c) $\frac{n(n^2-1)}{3}$ (d) none of these

26. Find the sum of $1 + \frac{2}{3} + \frac{4}{9} + \frac{6}{27} + \frac{8}{81} + \frac{10}{243} + \ldots\ \infty$:

(a) $\frac{2}{3}$ (b) $\frac{5}{2}$ (c) $\frac{19}{45}$ (d) $\frac{81}{17}$

27. $If\ (x)$ is the least integer greater than or equal to x, then find the value of the following series:

$\{\sqrt{1}\} + \{\sqrt{2}\} + \{\sqrt{3}\} + \{\sqrt{4}\} + \ldots + \{\sqrt{99}\} + \{\sqrt{100}\}$

(a) 715 (b) 55 (c) 157 (d) 835

28. If [x] is the greatest integer less than or equal to x, then find the value of the following series.

$[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + [\sqrt{4}] + \ldots + [\sqrt{323}]$

(a) 3237 (b) 2373 (c) 3723 (d) none of these

29. Find the value of the expression: 1 - 3+ 5 - 7... to 100 terms.

(a) -150 (b) -100 (c) -50 (d) 50

30. The sum of the series $\frac{1}{1\times5} + \frac{1}{5\times9} + \frac{1}{9\times13} \ldots + \frac{1}{221\times225}$ is

(a) 28/221 (b) 56/221 (c) 56/225 (d) none of these

31. The sum of the series $\frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \cdots + \frac{1}{\sqrt{224}+\sqrt{225}}\ is:$

(a) $15-\sqrt{3}$ (b) $\sqrt{15} - 2$ (c) 12 (d) none of these

32. Find the sum to n terms of the series 11 + 103+ 1005 + …

(a) $\frac{10(10^n-1)}{9}+1$ (b) $\frac{10\,(10^n-1)}{9}+n$

(c) $\frac{10\,(10^n-1)}{9}+n^2$ (d) $\frac{10(10^n+1)}{11}+n^2$

33. In a certain colony of cancerous cells, each cell breaks into two new cells every hour. If there is a single productive cell at the start and this process continues for 9 hours, how many cells will the colony have at the end of 9 hours? It is known that the life of an individual cell is 20 hours,

(a) 2^9 - 1 (b) 2^{10} (c) 2^9 (d) 2^{10}- 1

Exercise – 03

Solutions

1. Ans. (d)
Solution: Solve through options

2. Ans. (d)
Solution: Sum = (n-1)2^n+1

3. Ans. (c)
Solution: 7 + 26 + 63 + 124 + …..4 + 999
$1^3-1, 2^3-1, 3^3-1, 4^3-1, \ldots 10^3-1$
$T_n = n^3-1$
Sum = $\sum_{n=1}^{10} T_n = \sum_{n=1}^{10} n^3 - 1$
We know that $\left[1^3+2^3+3^3+\cdots..n^3 = \left(\frac{n(n+1)}{2}\right)^2\right]$
So, sum = $\sum_{n=1}^{10} n^3 - 1 = \left(\frac{n(n+1)}{2}\right)^2$ – n = $\left(\frac{10\,x\,11}{2}\right)^2 - 10$
= $55^2 - 10 = 3025 - 10 = 3015$

4. Ans. (d)
Solution: Break the series in two sequence 1, 2, 3, 4, …n and $1^2, 2^2, 3^2, \ldots., n^2$ and solve or Check the options

5. Ans. (b)
Solution: $S_n = 7+77+777+\cdots to\ n\ terms$
= 7 (1+11+111+…to n terms)
= $\frac{7}{9}$(9+99+999+… to n terms)
= $\frac{7}{9}((10-1)+(100-1)+\cdots to\ n\ terms$
= $\frac{7}{9}$((10+100+1000+… to n terms)) - (1 + 1+1+…n terms)
= $\frac{7}{9}\left\{\frac{10.(10^n-1)}{10-1}-n\right\}$
= $\frac{7}{9}\left\{\frac{10}{9}(10^n-1)-n\right\}$
Alternatively: best way is to go through options.
Consider n = 2, then $S_2 = 7+77 = 84$
Now, put n = 2 in choice (b).
Note: This type of questions necessarily be solved by option and must take at least two values for n = 2, 3 etc as a measure of safety.

6. Ans. (c)
Solution: Go by options

7. Ans. (b)
Solution: S = $\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\frac{7}{2^4}+\cdots.+\frac{(2n-3)}{2^{n-1}}+\frac{2n-1}{2^n}$
And $\frac{1}{2}S = \frac{1}{2^2}+\frac{3}{2^2}+\frac{5}{2^3}+\cdots.+\frac{(2n-3)}{2^n}+\frac{2n-1}{2^{n+1}}$
Subtracting we get,
$\frac{1}{2}S = \frac{1}{2}+\frac{2}{2^2}+\frac{2}{2^3}+\cdots+\frac{2}{2^n}-\frac{2n-1}{2^{n+1}}$
$= \frac{1}{2}+2\left(\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\cdots+\frac{1}{2^n}\right)-\frac{2n-1}{2^{n+1}}$
$= \frac{1}{2}+\frac{2.\frac{1}{2^2}\left\{1-\left(\frac{1}{2}\right)^{n-1}\right\}}{1-\frac{1}{2}}-\frac{2n-1}{2^{n+1}}$
= S = 3 - $\frac{2n+3}{2^n}$

8. Ans. (b)
Solution: Go through options.
Alternatively: 11 + 102 + 1003 + …
= (10+1) + (100+2) + (1000+3)
= (10+100+1000+…) + (1+2+3+…)

9. Ans. (b)
Solution: go through option
Alternatively: T_n = 1 + 3 + 9 + …
or $T_n = \frac{1}{2}(3^n-1)$

$S_n = t_1 + t_2 + t_3 + \ldots + t_n$

$S_n = t_1 + t_2 + t_3 \ldots\ldots + t_n$

$S_n = \frac{1}{2}[(3^1 + 3^2 + 3^3 + \cdots. + 3^n) - n]$

$= \frac{1}{2}\left[\frac{3(3^n-1)}{2} - n\right]$

$= \frac{1}{4}\,[3(3^n - 1) - 2n]$

$= \frac{3}{4}(3^n - 1) - \frac{n}{2}$

10. Ans. (b)

Solution: Let $S_n = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \cdots + \frac{n}{3^{n-1}}$

$\therefore\ \frac{1}{3}S_n = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \cdots + \frac{n-1}{3^{n-1}} + \frac{n}{3^n}$

Subtracting we get,

$\frac{2}{3}S_n = \left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \cdots\right) - \frac{n}{3^n}$

$\frac{2}{3}S_n \left[\frac{1\left\{1-\left(\frac{1}{3}\right)^n\right\}}{1-\frac{1}{3}}\right] - \frac{n}{3^n}$

⇨ $\frac{2}{3}S_n = \frac{3}{2}\left\{1 - \left(\frac{1}{3}\right)^n\right\} - \frac{n}{3^n}$

⇨ $S_n = \frac{9}{4} - \frac{3}{4}\left(\frac{3+2n}{3^n}\right)$

11. Ans. (c)

Solution: Go through the options

12. Ans. (d)

Solution: $t_n = n\,(2n+1)^2$

$= 4n^3 + 4n^2 + n$

$S_n = \sum t_n = 4\sum n^3 + 4\sum n^2 + \sum n$

Then put n= 20

13. Ans. (b)

Solution: $t_n = \frac{n^2\,(n+1)^2}{4n^2} = \frac{1}{4}\,(n^2 + 2n + 1)$

$s_n = \sum t_n = \frac{1}{4}[\sum n^2 + 2\sum n + n\,]$

Then put n = 16.

14. Ans. (c)

Solution: $S_\infty = \frac{1}{4} - \frac{3}{16} + \frac{9}{64} - \frac{27}{256} + \cdots$

$S_\infty = \frac{\frac{1}{4}}{1-\left(-\frac{3}{4}\right)} = \frac{\frac{1}{4}}{\frac{7}{4}}$

$\left(\therefore r = \frac{-\frac{3}{16}}{\frac{1}{4}}\right)$

$S_\infty = \frac{1}{7}$

15. Ans. (c)

Solution: $1 + 4 + 9 + 16 + \ldots + n^2$

$= 1^2 + 2^2 + 3^2 + 4^2 + \cdots. + n^2 = \frac{n(n+1)(2n+1)}{6}$

16. Ans. (c)

Solution: Put n = 2 and 3 and then check for the correct choice

Sum of 2 terms = 11

And sum of 3 terms = 46

At n = 2 $\frac{1}{6}$ x 2 (3) (11) = 11

And at n = 3

$\frac{1}{6}$ x 3 x 4 x 23 = 46

Hence choice (c) is correct.

17. Ans. (d)

Solution: n = 1 $S_n = \frac{1}{2}$

n = 2, $S_n = \frac{5}{4} = \left(\frac{1}{2} + \frac{3}{4}\right)$

n = 3, $S_n = \frac{17}{8} = \left(\frac{1}{2} + \frac{3}{4} + \frac{7}{8}\right)$

choice (a is wrong

since at n = 2, $S_2 = 3 \neq \frac{5}{4}$

Choice (b) is also wrong

Since at n = 2

$S_2 = \frac{3}{4} \neq \frac{5}{4}$

Choice (c) is also wrong

Since at n = 2 $S_2 = 3 \neq \frac{5}{4}$

Choice (d) is correct

Since at n = 1, $S_1 = \frac{1}{2}$

At n = 2, $S_2 = \frac{5}{4}$

At n = 3, $S_3 = \frac{17}{18}$

18. Ans. (c)

Solution: $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + \ldots$

= (1-2) (1+2) + (3-4) (3+4) + (5-6) (5+6) + (7+8) (7-8) + …

= (1+2) – (3+4)- (5+6)…

= - [(1+2) + (3+4) + (5+6) + …]

= - [1 + 2+ 3+ 4+ 5+ 6+….] = - $\left[\frac{n(n+1)}{2}\right]$

Alternately: go through options

19. Ans. (d)

Solution: $S_n = \frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \dots \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}}$

$\frac{1}{2}[(\sqrt{3}-\sqrt{1}) + (\sqrt{5}-\sqrt{3}) + (\sqrt{7}-\sqrt{5}) + \dots +$
$\dots ((\sqrt{2n+1}-\sqrt{2n-1}))]$

$= \frac{1}{2}(\sqrt{2n+1}-1)$

Alternatively:
Substitute n = 1, 2, 3 etc, and check the correct answer.

20. Ans. (b)
Solution: Go through options
Let n = 2, then $s_n = 3 + 6 = 9$
$\therefore \quad s_n = 2\,(3) + 2^2 - 1 = 9$

At n = 3,
$S_n = 3\,x\,4\; + \;2^3 - 1 = 19$

Hence choice (b) is correct.
Alternatively: 3 + 6 + 10 + 16 + …
= (2 + 4 + 6+ 8 + …) + (1+ 2+ 4+ 8+…)
= (n (n+1) + (2^n – 1))

21. Ans. (b)
Solution: Best way is to go through optons
Let n = 2, then $s_n = S_2 = 1 + (1+2) = 4$
From choice (b)
$s_2 = \frac{2\,x\,3\,x\,4}{6} = 4$

And for n = 3
$S_n = s_3$ = 1 + (1+2) + (1+2+3) = 10

$\therefore$ from choice (b)
$s_3 = \frac{3\,x\,4\,x\,5}{6}$ = 10

Hence choice (b) is correct.

Alternatively: $T_n = \frac{n(n+1)}{2}$

$\therefore \quad Sn = \sum\left[\frac{n(n+1)}{2}\right] = \sum\left[\frac{n^2+n}{2}\right]$

$= \frac{1}{2}[\sum n^2 + \sum n]$

$= \frac{1}{2}[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}]$

$= \frac{n(n+1)}{4}\left[\frac{2n+4}{3}\right] = \frac{n(n+1)(n+2)}{6}$

22. Ans. (b)
Solution: $S_1 = 1^2 = 1$
$S_2 = 1^2 + (1^2 + 3^2) = 11$
$S_3 = 1^2 + (1^2 + 3^2) + (1^2 + 3^2 + 5^2) = 46$

Now put n = 1, 2, 3 in choice (b) you will get
$S_1 = \frac{1}{6}\,x\,1\,x\,2\,x\,3 = 1$
$S_2 = \frac{1}{6}\,x\,2(3)(11) = 11$

$S_3 = \frac{1}{6}\,x\,3(4)23 = 46$

Hence choice (b) is correct.
Alternatively:
$t_r = 1^2 + 3^2 + 5^2 + \dots + (2r-1)^2$
$= \sum_{k=1}^{r}(2k-1)^2 = \sum_{k=1}^{r} 4k^2 - 4k + 1$
$= 4\left\{\frac{1}{6}r\,(r+1)(2r+1)\right\} - \frac{4}{2}r\,(r+1) + r$

$= \frac{1}{3}\,(4r^3 - r)$

$\therefore S_n = \frac{4}{3}\sum_{r=1}^{n} r^3 - \frac{1}{3}\sum_{r=1}^{n} r$

$= \frac{4}{3}.\frac{1}{4}n^2\,(n+1)^2 - \frac{1}{3}.\frac{1}{2}$ n(n+1)

$= \frac{1}{6}n(n+1)\,(2n^2 + 2n - 1)$

23. Ans. (b)
Solution: Best way is to go through options by substituting the values of n = 1, 2, 3, …
Alternatively: $\frac{3}{1^2.2^2} + \frac{5}{2^2.3^2} + \frac{7}{3^2.4^2} + \cdots$

$= \left(1 - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \dots \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$
$= 1 - \frac{1}{(n+1)^2} = \frac{n^2+2n}{(n+1)^2}$

24. Ans. (a)
Solution: $S = \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \cdots\infty$

⇨ $S = \frac{1}{3}\left[\frac{3}{1.4} + \frac{3}{4.7} + \frac{3}{7.10} + ..\infty\right]$
⇨ $S = \frac{1}{3}\,[(\frac{1}{1} - \frac{1}{4}) + (\frac{1}{4} - \frac{1}{7}) + (\frac{1}{7} - \frac{1}{10}) + \cdots\infty$

⇨ $S = \frac{1}{3}[1] \quad \therefore S = \frac{1}{3}$

25. Ans. (d)
Solution: Go through options
Let n = 4, then

$S_n = -1 + 1^2 - 2 + 2^2 = 2$

From option (a) $\frac{4 x 17}{3} = \frac{68}{3} \neq 2$

From option (b) 16 + 4 = 20 ≠ 2

From option (c) $\frac{4 x 15}{3}$ = 20 ≠ 2

Hence choice (d) is correct.

26. Ans. (b)

Solution: let

$S = 1 + \frac{2}{3} + \frac{4}{9} + \frac{6}{27} + \frac{8}{81} + \frac{10}{243} + \infty$

$\therefore 3(S) = 3 + 2 + \frac{4}{3} + \frac{6}{9} + \frac{8}{27} + \frac{10}{81} + \ldots \infty$

$\therefore 2(s) = 4 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \ldots \infty$

$\therefore 2(s) = 4 + 2\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots\right)$

➔ $2(s) = 4 + 2\left[\frac{\frac{1}{3}}{1-\left(\frac{1}{3}\right)}\right]$

➔ $2(S) = 4 + 1$

➔ $S = \frac{5}{2}$

27. Ans. (a)

Solution: Let S = $\{\sqrt{1}\} + \{\sqrt{2}\} + \{\sqrt{3}\} + \{\sqrt{4}\} + \ldots + \{\sqrt{99}\} + \{\sqrt{100}\}$

⇨ S = 1 + 2 + 2 + 2 + 3 + 3 + 3 + 3 + 3 + … + 10

⇨ S = 1 + 3 (2) + 5(3) + 7(4) + 9(5) + …+ 19(10)

Since T_n = (2n -1)n = $2n^2 - n$

$S_n = \sum(2n^2 - n) = 2\sum n^2 - n$

$= \frac{2n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2}\left[\frac{2}{3}(2n+1) - 1\right]$

$= \frac{n(n+1)}{2}\left[\frac{4n+2-3}{3}\right]$

$= \frac{n(n+1)(4n-1)}{6}$

$S_{10} = \frac{10 x 11 x 39}{6}$ = 715

28. Ans. (c)

Solution: Let S = $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \ldots [\sqrt{323}]$

⇨ S = 1 + 1 + 1 + 1 + 2 + 2+ 2 + 2 + 2 + ….+ 17 + 17

⇨ S = 1(3) + 2(5) + 3(7) + 4 (9) + … + 17 (35)

$\therefore T_n = n\ (2n + 1)$

$\therefore S_n = \sum n(2n+1) = 2\sum n^2 + \sum n$

$= \frac{2.n\ (n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2}\left[\frac{2}{3}\ (2x + 1\ ) + 1\right]$

$= \frac{n(n+1)}{2}\left[\frac{4n+5}{3}\right]$

$= \frac{n(n+1)(4n+5)}{6}$

$\therefore$ S = $\frac{17 x 18 x 73}{6}$ = 3723

29. Ans. (b)

Solution: View: 1 - 3 + 5 - 7 + 9-11 ..... 100 terms as (1- 3) + (5 - 7) + (9 – 11) …. 50 terms. Hence, -2 + -2 + -2 ... 50 terms = 50 x -2 =….100. Option (b) is correct.

30. Ans. (c)

Solution: Questions such as these have to be solved on the basis of a reading of the pattern of the question. The sum upto the first term is: 1/5. Upto the second term it is 2/9 and upto the third term it is 3/12. It can be easily seen that for the first term, second term and third term the numerators are 1, 2 and 3 respectively. Also, for 1/5 — the 5 is the second value in the denominator of 1/1 x 5 (the first term); for 2/9 also the same pattern is followed— as 9 comes out of the denominator of the second term of series and tar 3/13 the 13 comes out of the denominator of the third term of the series and so on. The given series has 56 terms and hence the correct answer would be 56/225.

31. Ans. (a)

Solution: Solve this on the same pattern as Question 31 and you can easily see that for the first term sum of the series is 2 - $\sqrt{3}$. for 2 terms we have the sum as $\sqrt{5}$ - $\sqrt{3}$ and so on. For the given series of 120 terms the sum would be $\sqrt{225} - \sqrt{3}$ = 15 - $\sqrt{3}$. Option (a) is correct.

32. Ans. (c)

Solution: Solve this one using options to check the correct answer.

33. Ans. (c)

Solution: Since, the problem says that the cell breaks into two new cells, it means that the original cell no longer exists. Hence, after 1 hour there would be 2^1 cells, after 2 hours there would be 2^2 cells and so on. After 9 hours there would be 2^9 cells. Hence, option (c) is correct.

Exercise – 04

Advanced Level

1. If x = 1 + a + a^2 + a^3 + ... to ∞ (|a|<1),
y = 1 + b + b^2 + b^3 + ... to ∞ (|b|<1),
Then, find 1 +ab+ a^2b^2+ a^3b^3 + ...to ∞
(a) x+y/x+y-1 (b) xy/x+y (c) xy/x+y-1 (d) None of these

2. If the (n+1)th term of a harmonic progression is twice the (3n+1) th term, find the ratio of the first term to the (n+1)th term.
(a) 1 (b) 2 (c) 3 (d) 4

3. If the sum of the reciprocals of the first seven terms of a harmonic progression is 70, find the fourth term of the HP.
(a) 2/15 (b) 1/10 (c) 3/7 (d) 5/12

4. If the *m*th term of a HP is n and the *n*th term is m, what is the value of the (m+n)the term?
(a) m/(m+n) (b) mn/m+n (c) n/m=n (d) none of these

5. Find the sum of all the numbers divisible by 6 in between 100 to 400
(a) 12,500 (b) 12,450 (c) 11,450 (d) 11,550

6. Let *f*(x) = 2x +1. Then, the number of real values of x for which the three unequal numbers f(x), f(2x), and f(4x) are in a GP is:
(a) 1 (b) 2 (c) 0 (d) none of these

7. If a, b and c are in AP, then $\frac{a}{bc}, \frac{1}{c}, \frac{2}{b}$ are in:
(a) AP (b) GP (c) HP (d) none of these

8. The 288th term of the series a, b, b, c, ,c ,c ,d,d,d,d,e,e,e,e,e… is:
(a) u (b) v (c) w (d) x

9. The difference between the two numbers is four and the AM between them is six. The product of the numbers is:
(a) 24 (b) 12 (c) 32 (d) 48

10. If log $\left(\frac{5c}{a}\right)$, log$\left(\frac{3b}{5c}\right)$ and log $\left(\frac{a}{3b}\right)$ are in an AP, where a, b, and c are in a GP, then a, b, and c, are the lengths of sides of:
(a) An isosceles triangle (b) An equilateral triangle
(c) A scalene triangle (d) none of these

11. The coefficient of x^{15} in the product (1-x) (1-2x) (1-$2^2.x$)…(1-$2^{15}.x$) is equal to:
(a) 2^{105}-2^{121} (b) 2^{121}- 2^{105} (c) $2^{120} - 2^{104}$ (d) none of these

12. The coefficient of x^{49} in the product (x-1) (x-3)…. (x-99) is:
(a) -99^2 (b) 1 (c) -2500 (d) none of these

13. Let $t_n = n.(n!)$. then, $\sum_{n=1}^{15} t_n$ is equal to:
(a) 15!-1 (b) 15!+1 (c) 16!-1 (d) None of these

14. The harmonic mean of the roots of the equation $(5+\sqrt{2})\ x^2$- $(4+\sqrt{5})x$ + 8 +$2\sqrt{5}$ =0 is:
(a) 2 (b) 4 (c) 6 (d) 8

15. f(x) = 2x+1, where x = 1 , 2, …., 100. g(x) = 3x-2, where = 1, 2,….,100. For how many values of x, f(x) = g(x)?
(a) 31 (b) 32 (c) 33 (d) 34

16. If a, b, c be the pth qth and rth term of an AP, then $\frac{a}{p}\ (q-r)$ + $\frac{b}{q}$ (r-p) + $\frac{c}{r}$(p-q)
(a) pqr (b) ap+bq + cr
(c) 0 (d) p + q+ r

17. The sum of four integers of A.P. is 24 and their products is 945. Find the product of the smallest and greatest integers:
(a) 30 (b) 27 (c) 35 (d) 39

18. If the A.M. of two positive numbers a and b, (a>b), is twice their G.M. then a : b is:
(a) 2 : $\sqrt{3}$ (b) 2 : 7 + $4\sqrt{3}$
(c) 2 + $\sqrt{3}$: 2 - $\sqrt{3}$ (d) 7 + $4\sqrt{3}$: 7 - $4\sqrt{3}$

19. The number of divisors of 187, 637 and 1001 are in:
(a) A.P. (b) G.P. (c) H.P. (d) none of these

20. The number $\underbrace{1111 \dots 1}_{119\ times}$ is a;
(a) prime number (b) composite number
(c) multiple of 3 (d) none of these

21. If n is a positive integer , the n $\underbrace{111 \dots 1}_{2n\ times}$ - $\underbrace{222 \dots 2}_{n\ times}$ is:
(a) a perfect square (b) a perfect cube
(c) prime number (d) none of these

22. Find the sum to n terms of the series.

$$\log a + \log \frac{a^3}{b} + \log \frac{a^5}{b^2} + \log \frac{a^7}{b^3} + \cdots$$

(a) $\log \left(\frac{a^{2n}}{b^{n-1}}\right)^{n/2}$ (b) $\log \frac{a^{2n-1}}{b^{n-1}}$
(c) $\log \frac{a^{2n}}{b^n}$ (d) none of these

23. If $log_5 2\ log_5(2^x-3)$ and $log_5\left(\frac{17}{2}+\ 2^{x-1}\right)$ are in A.P. then the value of x is:
(a) 2 (b) 3 (c) $log_2 3$ (d) $log_2 5$

24. If $log_2\ (5.2^x+1), log_4\ (2^{1-x}+1)$ and 1 are in A.P. then x equals:
(a) $log_2 5$ (b) $1-log_5 2$
(c) $log_5 2$ (d) none of these

25. X and Y are two numbers whose A.M. is 41 and G.M. is 9. Which of the following may be a value of X?
(a) 125 (b) 49 (c) 81 (d) 25

26. Find the sum of the series till 23rd terms for the series: 1, 4, 5, 8, 9, 12, 13, 16, 17,....
(a) 585 (b) 560 (c) 540 (d) 520

27. The sum of the first two terms of an infinite geometric series is 36. Also each term of the series is equal to the sum of all the terms that follow, find the sum of the series
(a) 48 (b) 54 (c) 72 (d) 96

28. A number 28 is divided into four parts that are in AP such that the product of the first and fourth is to the product of the second and third is 5 : 6 find the smallest part.
(a) 2 (b) 4 (c) 8 (d) 6

29. Find the sum of the integers between 100 and 300 that are multiples of 7.
(a) 10512 (b) 5586 (c) 10646 (d) 10546

30. Find the sum of all odd numbers lying between 1000 and 2000.
(a) 7,50,000 (b) 7,45,000 (c) 7,55,000 (d) 7,65,000

31. The first and the last terms of an A.P. are 113 and 253. If there are six terms in this sequence, find the sum of sequence.
(a) 980 (b) 910 (c) 1098 (d) 920

32. What is the 13th term of 2/9. 1/4,2/7, 1/3 ....?
(a) -2 (b) 1 (c) -3/13 (d) -2/3

33. How many terms of an A.P. must be taken for their sum to be equal to 200 if its third term is 16 and the difference between the 6th and the 1st term is 30?
(a) 6 (b) 9 (c) 7 (d) 8

34. Four numbers are inserted between the numbers 4 and 34 such that an A.P. results. Find the smallest of these four numbers.
(a) 11.5 (b) 11 (c) 12 (d) 10

35. If A is the sum of the n terms of the series 2 + 1/2 + 1/8 + ... and B is the sum of 2n terms of the series 2+1+ 1/2 + ... then find the value of B/A.
(a) 1/3 (b) 2 (c) 2/3 (d) 3/2

36. Find the infinite sum of the series 1/ + 1/3 + 1/6 + 1/10 + 1/15 ..
(a) 2 (b) 2.25 (c) 3 (d) 4

37. The sum of the series: 1/3 + 4/15 + 4/35 + 4/63 + ... upto 6 terms is:
(a) 12/13 (b) 13/14 (c) 14/13 (d) None of these

38. If $f(4x) = 8x + 1$. Then for how many positive real values of x, $f(2x)$ will be G.M. of $f(x)$ and $f(4x)$:

39. If x, y, z, w are positive real numbers such that x,y,z w form an increasing A.P. and x, y, w form an decreasing G.P. then w/x = ?
(a) 1 (b) 2 (c) 3 (d) 4

40. sum of 16 terms of the series 1 + 1 + 3 + 1 + 3 + 5 + 1 + 3+ 5 + 7 + ...

41. The sum of all terms of the arithmetic progression having ten terms except for the first term, is 99. and except for the sixth term, 89. Find the third term of the progression if the sum of the first and the fifth term is equal to 10.
(a) 15 (b) 5 (c) 8 (d) 10

42. A geometric progression consists of 500 terms. Sum of the terms occupying the odd places is P_1 and the sum of the terms occupying the even places is P_2. Find the common ratio.
(a) P_2/P_1 (b) P_1/P_2 (c) $P_2 + P_1/ P_1$ (d) $P_2 + P_1/P_2$

43. If(2+4 + 6+...50 terms) /(1 +3 + 5 +... n terms) = 51/2, then find the value of n.
(a) 12 (b) 13 (c) 9 (d) 10

44. The interior angles of a polygon are in AP. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon.
(a) 7 (b) 8 (c) 9 (d) 10

45. The sum of the third and the ninth term of an AP is 10. Find a possible sum of the first 11 terms of this .AP
(a) 55 (b) 44 (c) 66 (d) 48

46. If the ratio of harmonic mean of two numbers to their geometric mean is 12 : 13. find the ratio of the
(a) 4/9 or 9/4 (b) 2/3 or 32 (c) 2/5 or 5/2 (d) None of these

47. If three positive real numbers x, y , z are in AP such that xyz = 4, then what will be the minimum value of y?
(a) $2^{1/3}$ (b) $2^{2/3}$ (c) $2^{1/4}$ (d) $2^{3/4}$

Exercise – 04
Solutions

1. Ans. (c)
Solution: Since $S_\infty = \frac{a}{1-r}$
x = $\frac{1}{1-a}$; $y = \frac{1}{1-b}$

$\therefore$ a = $\frac{x-1}{x}$, b = $\frac{y-1}{y}$

S = 1 + $\frac{1}{ab} + \frac{1}{a^1b^1} + \ldots\infty$

S = $\frac{1}{1-ab}$
Substitute the values of a and b
We get S = $\frac{xy}{x+y-1}$

2. Ans. (b)
Solution: $T_n = \frac{1}{nth\ term\ of\ AP}$
$T_n = \frac{1}{a+(n-1)d}$
Since T_{n+1} = 2 T_{3n+1}

$\therefore \frac{1}{a+nd} = \frac{1}{a+3nd}$

a-nd = 0 → a = nd
$\frac{T_1}{T_{n+1}} = \frac{a+nd}{a} = \frac{2a}{a}$ → '2'

3. Ans. (b)
Solution: Since S_7 = 70, $S_n = \frac{n}{2}$[2a+(n-1)d]}
Given that $\frac{7}{2}\ (2a + 6d)$ = 70, or, 2a + 6d = 20,
or, a + 3d = 10. Hence, T_4 = 10

4. Ans. (b)
Solution: T_m = n, T_n = m
$\frac{1}{a+(m-1)d}$ = n, $\frac{1}{a+(n-1)d} = m$

a+ (m-1)d = $\frac{1}{n}$...(i), a + (n-1)d = $\frac{1}{m}$ ....(ii)

On solving (i) and (ii)

We get a = 1/mn, d = $\frac{1}{mn}$, $\left(T_{m+n} = \frac{mn}{m+n}\right)$

5. Ans. (b)

Solution: First number = 102 and last number = 396. There will be 50 terms in this series.

Then, sum = $\frac{n}{2}$(first term + Last term) = $\frac{50}{2}$ [102+396] = 12450

6. Ans. (a)

Solution: $f(x)$ = 2x + 1, $f(2x) = 4x + 1, f(4x)$ = 8x +1

Since, f(x), f(2x) and f(4x) are in GP.

Therefore $(4x+1)^2$ = (8x+1) (2x+1)

On solving, we get x = 1.

7. Ans. (d)

Solution: Since a, b, c are in AP

Consider a, b, c as 2, 4, 6.

Then, $\frac{a}{bc}, \frac{1}{c}, \frac{2}{b}$ will be $\frac{1}{12}, \frac{1}{6}, \frac{1}{2}$

8. Ans. (c)

Solution: a, b, ,b, c, c ,c, d, d, ,d, d

1a, 2b, 3c, 4d, 5e….

$T_n = \frac{n(n+1)}{2}$

T_{23} = 276

9. Ans. (c)

Solution: Since a – b = 4 and $\frac{a+b}{2}$ = 6, thus, a = 8, b = 4

Hence ab = 32

10. Ans. (d)

Solution: For a, b, c to be the length of the sides of the triangle , it is AM should not be 0.

11. Ans. (a)

Solution: (1-x) (1-2x) (1-$2^2.x$)…(1-$2^{15}.x$)

Let us consider (1-x) (1-2x) = 1-3x + $2x^2$

Coefficient of x is -3.

i.e., = $2^0 - 2^2$

For (1-x) (1-2x) (1-4x)

Coefficient of x^2 is 14.

i.e., = 2^4 - 2^1 → ($2^1 - 2^4$)

Therefore coefficient of x is $2^0 - 2^2$

Coefficient of x^2 $is - (2^1 - 2^4)$

Coefficient of x^3 is (2^3- 2^7)

Coefficient of x^4 is - (2^5- 2^{11})

.

Coefficient of x^{15} is (2^{105}- 2^{121})

Hence option (a) is correct.

12. Ans. (c)

Solution: (x-1) (x-3) … (x-99)

Coefficient of x^{49} will be sum of

-1, -3, -5, …-99 = - 2500

Hence, option (c) is the answer.

13. Ans. (c)

Solution: Solve through options.

14. Ans. (b)

Solution: We can easily find out the values : 2ab and a+b, where a and b are the roots of the equation given.

15. Ans. (c)

Solution: Get the terms of sets $f(x)$ and $g(x)$ and check how many terms are common.

16. Ans. (c)

Solution: Let an AP be 2, 5, 8, 11, 14, 17, 20, 23, …

And let = =3, q=5, r = 6

Then $S_p = a = (2+5+8) = 15$

And $S_q = b = (2+5+8+11+14) = 40$

And $S_r = c = (2+5+\cdots.+14+17) = 57$

$\therefore \frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}$(p-q)

⇨ $\frac{15}{3}(5-6) + \frac{40}{5}(6-3) + \frac{57}{6}(3-5)$

⇨ –5 + 4 – 19 = 0

Hence (c)is the correct.

17. Ans. (b)

Solution: Let a, b, c ,d be four integers in AP

Then

a +d = b + c = 12.

Since a, b, ,c ,d = 945, it means one of the a, b, c and d has its unit digit 5.

Therefor the obvious possibility is b = 5

So, if b = 5, then c = 7

a = 3 and d = 9

a.b.c.d = 3 x 5 x 7 x 9 = 945

Hence the values of a, b, c and d are correct.

Therefore a x d = 3 x 9 = 27

Alternatively: Option a is not suitablbe as

30 = 1 x 30 x

30 = 2 x 15 x

30 = 3 x 10 x

30 = 5 x 6 x

Since difference between a and d must be divisible by 3.

Option (c) is not suitable as

35 = 1 x 35 x

35 = 5 x 7 x

Since the difference between a and d is not divisible by 3.

Option (d) is also wrong as

39 = 1 x 39 x

= 3 x 13 x

Since the difference between a and d is not divisible by 3.

Choice (b) is correct as.

27 =- 1 x 27 x

27 = 3 x 9 √

If a = 3 and d = 9

Then b = 5 and c = 7

A .b . c. d = 945

Note: Choices (a) and (d) are wrong since 30 and 39 are not the factors of 945

Alternatively: Find the factors of 945 and then find the required values of b and c under the condition a + b + c +d = 24.

18. Ans. (c)

Solution: Given that A.M. = 2 G.M.

○ $\frac{a+b}{2} = 2\sqrt{ab}$

➔ a+ b = $4\sqrt{ab}$

Now, let use consider choice (c)

$\therefore (2+\sqrt{3}) + (2-\sqrt{3}) = 4\sqrt{(2+\sqrt{3})(2-\sqrt{3})}$

4 = 4.1

4 = 4

Hence choice (c) is correct

19. Ans. (a)

Solution: Number of divisors of 187 = 4

Number of divisors of = 637 = 6

Number of divisors = 1001 = 8

Hence they are in A.P.

HINT: to know more about how to find the number of divisors refer to chapter "Number System",

20. Ans. (b)

Solution: We have

$\underbrace{1111\ldots1}_{119\ times} = 10^{118} + 10^{117} + .. + 10^2 + 10^1 + 1$

$= \frac{(10^{119}-1)}{10-1} = \left(\frac{(10^{119}-1)}{10^7-1}\right)\left(\frac{10^7-1}{10-1}\right)$

$= (10^{112} + 10^{105} + 10^{98} + \ldots 1)(10^6 + 10^5 + \cdots 10 + 1)$

Thus $\underbrace{1111\ldots1}_{119\ times}$ is not a prime number.

21. Ans. (a)

Solution: 11 - 2 = 9 = 3^2

1111-22 = 1089 = 33^2

111111-222 = 110889 = 333^2

11111111 – 2222 = 11108889 = 3333^2 *etc*

22. Ans. (a)

Solution: $\log a + \log\frac{a^3}{b} + \log\frac{a^5}{b^2} + \log\frac{a^7}{b^3} + \ldots + \log\frac{a^{2n-1}}{b^{n-1}}$

$= \log a \times \frac{a^3}{b} \times \frac{a^5}{b^2}\, x\, \frac{a^7}{b^3}\, x \ldots \times \frac{a^{2n-1}}{b^{n-1}}$

$= log\ \frac{a^{(1+3+5+7+\cdots+2n-1)}}{b^{1+2+3+4+\cdots+n-1}}$

$= \log\frac{a^{n^2}}{b^{(n-1)n/2}}$

$= \log\left(\frac{a^{2n}}{b^{n-1}}\right)^{n/2}$

23. Ans. (b)

Solution: Go through options.

Remember if log a, log b, log c are in A.P. then a, b, c are in G.P.

$\therefore 2, (2^x - 3)$ and $\left(\frac{17}{2} + 2^{x-1}\right)$ must be in G.P.

Let x = 3, then the value of $2^x - 3 = 5$

And $\frac{17}{2} + 2^{x-1} = \frac{25}{2}$

Since 2, 5, $\frac{25}{2}$ are in G.P.

24. Ans. (b)

Solution: Go through options.
Let us consider choice (b)
x = 1 - $log_2 5 = log_2 2 - log_2 5 = log_2\left(\frac{2}{5}\right)$

$\therefore log_2(5.2^x + 1), log_4(2^{1-x} + 1)$, 1
$= log_2(5.2^{\frac{log_2 2}{5}} + 1),\ log_4\left(\frac{2}{2^{\frac{log_2 2}{5}}} + 1\right)$, 1
$= log_2 3, log_4\left(\frac{2}{\frac{2}{5}} + 1\right), 1$
$= log_2 3, log_4 6, 1 = log_4 9, log_4 6, log_4 4$
$\therefore$ 9, 6, 4 are in G.P.
Hence the assumed choice (b) is correct.
Hint: If a, b, c re in G.P. then log a, log b, log c are in A.P.

25. Ans. (c)
Solution: AM = 41 means that their sum is 82 and GM = 9 means their product is 81. The numbers can only be 81 and 1. Option (c) is correct.

26. Ans. (c)
Solution: The sum to 23 terms of the sequence would be: The sum to 12 terms of the sequence 1, 5, 9, 13, ... +The sum to 11 terms of the sequence 4, 8, 12, 16,.... The required sum would be

$\frac{12}{2}(2.1 + (12-1)4) + \frac{11}{2}[2.4 + (11-1)4] =$
6 x 46 + 11 x 24 = 276 + 264 = 540.

Option (c) is correct.

27. Ans. (a)
Solution: a = $\frac{ar}{1-r}$ or 1 – r = r or r = ½ a + ar = 36 or a = 24

Required sum = $\frac{24}{1-\frac{1}{2}} = 48. option\ (a) is\ correct.$

28. Ans. (b)
Solution: Since the four parts of the number are in AP and their sum is 28, the average of the four parts must be 7. Looking at the options for the smallest pan, only the value of 4 fits in, as it leads us to mink of the AP 4, 6, 8, 10. In this case, the ratio of the product of the first and fourth (4x10) to the product of the first and second (6 x 8) are in 5: 6 ratio.

29. Ans. (b)
Solution: The sum of the required series of integers would be given by 105+ 112+ 119 + ...294 = 28 x 199.5 = 5586. Option (b) is correct.

30. Ans. (a)
Solution: 1001 + 1003 + 1005 + ... 1999 = 1500 x 500 = 750000.

31. Ans. (c)
Solution: 6 x average of 113 and 253 = 6 x 183 = 1098.
option (c) is correct.

32. Ans. (d)
Solution: 2/9, 1/4, 2/7,1/3.........
This is an HP series. The corresponding AP will be:
9/2, 4/1, 7/2, 3/1.......
Or 4.5, 4, 3.5, 3 ......
i.e, this in an AP with first term 4.5 and common difference – 0.5.
Hence T_{13} = 4.5 + 12 (-0.5) = -1.5
The corresponding T_{13} = HP is 1/-1.5 = 1 x -2/3 = - 2/3

33. Ans. (d)
Solution: If the difference between the 6th and the 1st term is 30, it means that the common difference is equal to 6. Since, the third term is 16, the AP would be 4, 10, 16, 20, 26, 32, 38, 46 and the sum to 8 terms for this AP would be 200. Thus, option (d) is correct.

34. Ans. (d)
Solution: 5d = 30 → d = 6. Thus, the numbers are 4, 10, 16, 22, 28, 34. The smallest number is 10. Option (d) is correct.

35. Ans. (d)
Solution: Solve this question by looking at hypothetical values for n and 2n terms.
Suppose, we take the sum to 1 (n = 1) term of the first series and the sum to 2 terms (2n = 2) of the second series we would get B/A as 3/2

For n = 2 and 2n = 4 we get, A = 5/2 and B = 15/4 and B/A - 15/4/5/2 = 3/7
Thus, we can conclude that the required ratio is always constant at 3/2 and hence the correct option is (d).

36. Ans. (a)
Solution: If you look for a few more terms in the series, the series is: 1, 1/3, 1/6, 1/10, 1/15, 1/21, 1/28, 1/36, 1/45, 1/55, 1/66, 1/78, 1/91. 1/105, 1/120, 1/136. 1/153 and so on. If you estimate the values of the individual terms it can be seen that the sum would tend to 2 and would not he good enough to reach oven 2.25. Thus, option (a) is correct.

37. Ans. (d)

Solution: For this question too you would need to read the pattern of the value being followed.
The given sum has 6 terms.
It can be seen the sum to 1 term = 1/3
Sum to 2 term = 3/5
Sum to 3 term = 5/7
Hence, the sum to 6 terms would be 11/13.

38. Ans. (0)
Solution: If f (4x) = 8x + 1 then f(x) = 2x + 1 & f (2x) = 4x + 1
$(4x+1)^2$ = (8x + 1) (2x + 1)
x = 0
So for no positive value of x, f(2x) is the G.M. of f(x), f(4x).

39. Ans. (d)
Solution: if y = x + d, z = x + 2d, w = x +3d then
$(x+d)^2$ = x (x+3d) or d = x
w/x = 4x/x = 4

40. Ans. (56)
Solution: $S_{16} = 1 + (1+3) + (1+3+5) + (1+3+5+7) + (1+3+5+7+9) + 1$
$S_{16} = \frac{1}{6}\, 5\, (5+1)(2.5+1) + 1 = 56$

41. Ans. (b)
Solution: Any sub-part of an AP is also an A P. Thus, the third term would be the average of the first and the fifth term. Hence, the third term would be 5.

42. Ans. (a)
Solution: The answer would directly be P_2/P_1, Assume a series having a few number of terms e.g. 1,2,4,8,16,32. The value of P_2 here = 42, while P_1 = 21. The common ratio can be seen to be P_2/P_1, = 2.

43. Ans. (d)
Solution: Use the options to get the answer. for n=10, we get the required ratio as 51/2.

44. Ans. (c)
Solution: The sum of the interior angles of any polygon of n sides is given by
(n – 2) 180. This needs to match the sum of the AP 120 + 125 + 130 +..n terms. For n = 9, we get the two sums equal and hence option (c) is correct.

45. Ans. (a)
Solution: The third and ninth terms of an 11 tem AP are a pair of corresponding terms of the A.P. hence, their average would be the average of the A.P. this gives us the required sum of the AP as 11 x 5 = 55.

46. Ans. (d)
Solution: Solve using options, none of the given options matches, hence option (d) is correct.

47. Ans. (b)
Solution: The minimum value of y would occur when all the three values are equal. Thus, $y^3 = 4 \longrightarrow$ y = $2^{2/3}$

Exercise – 05
TITA/ Short Answer

1. How many terms are common in two arithmetic progression 1, 4, 7, 10… upto 63 terms and 3, 7, 11, 15, … upto 47 term
2. Let a_n be the nth term of an AP and a_7 = 22, then the value of the common difference (d) that would make $a_3\ a_7\ a_{11}$ greatest is:
3. The value of $(0.2)^{log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+..\right)}$:
4. The sum of the series
$$\frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + \frac{5555}{(13)^4} + \cdots .\infty$$
5. Find the 23rd term of the sequence: 1, 4, 5, 8, 9, 12, 13, 16, 17, ….
6. In an infinite geometric progression, each term is equal to 3 times the sum of the terms that follow. If the first term of the series is 4, find the product of first three terms of the series?
7. A student takes a test consisting of 100 question with differential marking is told that each question after the first is worth 5 marks more than the preceding question. If the 5th question of the test is worth 25 marks, what is the maximum score that the student can obtain by attempting 90 questions?

8. The internal angles of a plane polygon are in AP. The smallest angle is 100^0 and the common difference is 10^0. Find the number of sides of the polygon.
9. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the arithmetic mean of a and b then find the value of n.
10. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the harmonic mean of a and b then find the value of n.
11. If a, b are two numbers such that a, b > 0. If harmonic mean of a, b is equals to geometric mean of a, b then what can be said about the relationship between a and b.
12. Product of 36 positive integers is 1. Their sum is ≥
13. If we have two numbers a, b. A.M. of a, b is 12 and H.M. is 3. Find the value of ab
14. The sum to 16th groups of the series (1)+ (1 + 3) + (1 +3 + 5) + (1 +3 + 5 + 7)+ .....
15. If the sum of n terms of a progression is $2n^2 + 3.\,then\ which\ term\ is\ equals\ to\ 78?$
16. Sum of 7 terms of the series $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots$
17. find the sum of 20 terms of the series 3 + 6 + 10 + 16 + …
18. Find the sum of all three-digit whole numbers less than 500 that leave a remainder of 2 when they are divided by 3.
19. $S = \frac{1}{1!+2!} + \frac{1}{2!+3!} + \frac{1}{3!+4!} + \ldots + \frac{1}{19!+20!}\ then\ S = ?$
20. The sum to 17 terms of the series.
    $\frac{3}{1^2.2^2} + \frac{5}{2^2.3^2} + \frac{7}{3^2.4^2} + \ldots$ is:
21. Find the value of S if
    $S = \frac{4}{11} + \frac{44}{11^2} + \frac{444}{11^3} + \frac{4444}{11^4} + \cdots \infty$
22. $\frac{9}{2} + \frac{25}{6} + \frac{49}{12} + \ldots + \frac{9801}{2450} = ?$
23. From the 1st 12 natural numbers how many Arithmetic Progressions of 4 terms can be formed such that the common difference is a factor of the 4th term?
24. The product of 1st five terms of an increasing A.P.is 3840, If the 1st 2nd and 4th terms of the AP are in GP find 10th term of the series.
25. If

$$S = \left[1 + \left(-\frac{1}{3}\right)\right]\left[1 + \left(-\frac{1}{3}\right)^2\right]\left[1 + \left(-\frac{1}{3}\right)^4\right]\left[1 + \left(-\frac{1}{3}\right)^8\right]\ldots$$

    Then S =
26. Sum of the series $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$ to 10 terms

Exercise – 05
Solutions

1. Ans. 16
   Solution: 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34,37, …,187
   3, 7, 11, 15, 19, 23, 27, 31, 35, 39, …, 187

   The common terms are 7, 19, 31, 43, …187
   Therefore number of such terms = $\left(\frac{187-7}{12}\right) + 1$ = 16
2. Ans. 0
   Solution: Let d be the common difference of the AP
   Then $a_3.a_7.a_{11} = (22 - 4d).22.(22 + 4d)$
   = 88 $(121\text{-}4d^2)$
   Obviously R.H.S. is greatest for d = 0
3. Ans. 4
   Solution: $_{(0.2)}log_{\sqrt{5}}\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ..\right) = {}_{(0.2)}log_{\sqrt{5}}\left(\frac{1}{2}\right)$

   = (5^{-1}) log $\sqrt{5}$ $\left(\frac{1}{2}\right)$

   = $_5log_{\sqrt{5}}\,2$ $(\therefore$ m log n = $\log(n)^m)$

= $\sqrt{5}^2 \log \sqrt{5}\,(4)$ $(\therefore {}_5 log_a b = b)$
= 4

4. Ans. s = $\frac{65}{36}$
Solution: $\frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + \frac{5555}{(13)^4} + \cdots . \infty$
$\frac{S}{13} = \frac{5}{(13)^2} + \frac{55}{(13)^3} + \frac{555}{(13)^4} + \frac{5555}{(13)^5} + \cdots \infty$

$\therefore (s - \frac{s}{13}) = \frac{5}{13} + \frac{50}{(13)^2} + \frac{500}{(13)^3} + \frac{5000}{(13)^4} + \cdots \infty$
S $\left(1 - \frac{1}{13}\right) = \frac{5}{13} + \frac{5}{13}(\frac{10}{13}) + \frac{5}{13}(\frac{10}{13})^2 + \frac{5}{13}(\frac{10}{13})^3 + \ldots$
∞

$\frac{12}{13}S = \frac{5/13}{3/13}$
$\left(\therefore\ S_\infty = \frac{a}{1-r}\right)$

$\frac{12}{13}S = \frac{5}{3}$

⇨ s = $\frac{65}{36}$

5. Ans. 45
Solution: The 23rd term of the sequence would be the 12th
term of the sequence 1. 5. 9, 13. ..
The 12th term of the sequence would be 1+ 4 x 11 = 45

6. Ans. (1)
Solution: Let the series be a, ar, ar^2, ar^3, ….
According to the question a = 3ar/(1 - r) or r = 1/4. The series would be 4, 4/4, 4/16,...... and so on. The
product of first three terms of the series would be 4.1 . $\frac{1}{4}$ = 1

7. Ans. (d)
Solution: 5th term = 25. .1st term = 25 - (4 x 5) = 5, 11th term = 5 + 10 x 5 = 55, 100th term = 5 + 99x5 = 500 Student will score maximum marks if he attempts question 11 to 100. The maximum score would be the sum of the series 55+60+ .... + 495 + 500 = (90 x 555)/2 = 24975.

8. Ans. (8)
Solution: Smallest interior angle = 100, largest exterior angle = 100 – 100 = 80
Similarly other exterior angles are 70, 60, 50, …
Sum of all the exterior angles = 360

So, $\frac{n(2.80+(n-1)(-10))}{2}$ = 360 or n(17-n) = 72

We can see that the above equation is true for both n = 8, 9 but for n = 9, the 9th exterior angle must be 0 which is not possible so only 8 is possible

9. Ans. (0)
Solution: $\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{a+b}{2}$

For n = 0 the above equality is true. So n must be 0

10. Ans. (-1)
Solution: $\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{2ab}{a+b}$

For n = -1 the above equality is true so n = -1

11. Ans. (a=b)
Solution: According to the question $\frac{2ab}{a+b} = \sqrt{ab},$ this equality will be true only for a = b, hence , a = b.

12. Ans. (n)
Solution: A.M of n positive integers is always greater than or equals to G.M. of the numbers.
Then according to the equation,
Sum of the numbers ≥ n
$(product\ of\ n\ positive\ integers)^{1/n}$
Sum of the numbers ≥ n

13. Ans. (36)
Solution: $G.M^2$ = A.M H.M. = 12 x 3 = 36.

14. Ans. (1496)

15. Ans. (20)
Solution: Going through the trial and error = $2(20)^2 + 3 - 2(19)^2 - 3 = 78$

16. Ans. (153)
Solution: $1^1 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots =$ (1-2) (1+2) + (3-4) (3+4) + … + (15-16) (15+16) + $17^2 = -(1 + 2 + 3 + 4 + \cdots + 16) + 17^2$ = 289-136 = 153

17. Ans. (1539)
Solution: 3 + 6 + 10 + 16 + ….
(1+2) + (1+2+3) + (1+2+3+4) + … = 1 + (1+2) + (1+2+3) + (1+2+3+4) …. – 1
Required sum = sum of 21 terms of series 1 + (1+2) + (1+2+3) + …..-1 = $\sum_{n=21} \frac{(n+1)n}{2} - 1 = 1540 - 1 = 1539$

18. Ans. 39797
Solution: We need the sum of the AP; 101, 104, 107, …497 = 133 x 299 = 39767.

19. Ans. $\frac{1}{2!} - \frac{1}{21!}$

Solution: mth term of the series = $\frac{1}{m!+(m+1)!}$ = $\frac{1}{m!(m+2)!}$

S = $\frac{1}{2!}$ - $\frac{1}{3!}$ + $\frac{1}{3!}$ - $\frac{1}{4!}$ + ...$\frac{1}{20!}$ − $\frac{1}{21!}$ = $\frac{1}{2!}$ - $\frac{1}{21!}$

20. Ans. $\left(\frac{323}{324}\right)$

Solution: S = $\frac{3}{1^2.2^2}$ + $\frac{5}{2^2.3^2}$ + $\frac{7}{3^2.4^2}$ + ...

= $\left(1-\frac{1}{2^2}\right)+\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+\left(\frac{1}{3^2}-\frac{1}{4^2}\right)+ \ldots +\left(\frac{1}{17^2}-\frac{1}{18^2}\right)$

= 1 - $\frac{1}{18^2}$ = $\frac{323}{324}$

21. Ans. $\left(\frac{22}{5}\right)$

Solution: S = $\frac{4}{11}+\frac{44}{11^2}+\frac{444}{11^3}+\frac{4444}{11^4}+\ldots$

$\frac{S}{11}=\frac{4}{11^2}+\frac{44}{11^3}+\frac{444}{11^3}+\ldots$

S - $\frac{S}{11}=\frac{4}{11}+\frac{40}{11^2}+\frac{400}{11^3}+\ldots$

$\frac{10S}{11}=\frac{4}{11}\left(\frac{1}{1-\frac{10}{11}}\right)$ = 4

S = 44/10 = 22/5

22. Ans. $\left(\frac{9849}{50}\right)$

Solution: 4 + $\frac{1}{2}$ + 4 + $\frac{1}{6}$ + 4 + $\frac{1}{12}$ + ... 4 + $\frac{1}{2450}$

= 4 + $\left(1-\frac{1}{2}\right)$ + 4 + $\left(\frac{1}{2}-\frac{1}{3}\right)$ + 4 + $\left(\frac{1}{3}-\frac{1}{4}\right)$ + ... + 4 + $\left(\frac{1}{49}-\frac{1}{50}\right)$

= 49 x 4 + $\left(1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\cdots+\frac{1}{49}-\frac{1}{50}\right)$

= 196 + $\frac{49}{50}$ = $\frac{9849}{50}$

23. Ans. (13)

Solution: If a be the 1st term and d be the common difference of the A.P. the 4th term of the series will be a + 3d. If a + 3d is divisible by d then a should be divisible by d. hence the cases are:

d= 1, a= 1, 2,3.4,5,6,7, 8,9

d=2, a = 2, 4, 6

d = 3, a = 3

So the required answer is 9 + 3 + 1 = 13

24. Ans. (20)

Solution: If a – 2d be the first term and d be the common difference of A.P. then according to the question:

(a-2d) (a-d)a(a+d) (a+2d) = 3840 ....(1)

$\frac{a-d}{a-2d}=\frac{a+d}{a-d}$

d (3d-a) = 0

a = 3d

by putting a = 3d in equation 1 we get;

d x 2d x 3d x 4d x 5d = 3840

by solving we get d = 2 & a = 6

10th term = 2 + 9.2 = 20.

25. Ans. $\left(\frac{3}{4}\right)$

Solution: Let x = -1/3

S = (1+x) $(1+x^2)$ $(1+x^4)$ ...

(1-x) S = (1-x)(1+x) $(1+x^2)$ $(1+x^4)$....

(1-x)S = $(1-x^2)$ $(1+x^2)$ $(1+x^4)$ $(1+x^8)$...

(1-x)S = $(1-x^4)$ $(1+x^4)$ $(1+x^8)$.....

Since x < 0 & |x| < 1 so the value of RHS would be equals to 1.

(1-x)S = 1 or S = 1/(1-x) Or 1/(1-(-1/3)) = 1/4

26. Ans. (126.5)

Solution: nth term = $\frac{\sum n^3}{\sum(2n-1)}=\frac{1}{4}(n^2+2n+1)$

Sum of n terms of the given series = $\frac{1}{4}\left(\frac{1}{6}n(n+1)(2n+1)+n(n+1)+n\right)$

For n = 10 the required sum = $\frac{1}{4}$ [505] = 126.5