Percentile Classes

Determinants and Matrices

Determinants

Definition:-

Let us consider three homogeneous linear equations

$a_1x+b_1y+c_1z=0$, $a_2x+b_2y+c_2z=0$ and

$a_3x+b_3y+c_3z=0$

Eliminating x, y, z from above three equations, we obtain

$a_1(b_2c_3-b_3c_2)-b_1(a_2c_3-a_3c_2)+c_1(a_2b_3-a_3b_2)=0......(i)$

The L.H.S. of (i) represented by $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$=a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}-b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}+C_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

Its contains three rows and three columns, it is called a determinant of third order.

The number of elements in a second order is $2^2=4$ and the number of elements in a third order determinant is $3^2=9$.

Rows and columns of a determinant : In a determinant horizontal lines counting from top $1^{st}, 2^{nd}, 3^{rd}$..... respectively known as rows and are denoted by $R_1, R_2, R_3, ...$ and vertical lines counting left to right. $1^{st}, 2^{nd}, 3^{rd}$..... respectively known as columns and are denoted by $C_1, C_2, C_3, ...$

Properties of determinants

P - 1: The value of determinant remains unchanged, if the rows and the columns are interchanged.

Since the determinant remains unchanged when rows and columns are interchanged, it is obvious that any theorem which is true for 'rows' must also be true for 'columns'.

P - 2: If any two rows (or columns) of a determinant be interchanged, the determinant is unaltered in numerical value but is changed in sign only.

P - 3: If a determinant has two rows (or columns) identical, then its value is zero.

P - 4: If all the elements of any row (or column) be multiplied by the same number, then the value of determinant is multiplied by that number.

P - 5: If each element of any row (or column) can be expressed as a sum of two terms, then the determinant can be expressed as the sum of the determinants.

$e.g., \begin{vmatrix} a_1+x & b_1+y & c_1+z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}+\begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

P - 6: The value of a determinant is not altered by adding to the elements of any row (or column) the same multiples of the corresponding elements of any other row (or column)

$e.g., \quad D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

and $D'=\begin{vmatrix} a_1+ma_2 & b_1+mb_2 & c_1+mc_2 \\ a_2 & b_2 & c_2 \\ a_3-na_1 & b_3-nb_1 & c_3-nc_1 \end{vmatrix}$

Then $D'=D$.

P - 7: If all elements below leading diagonal or above leading diagonal or except leading diagonal elements are zero then the value of the determinant equal to product of elements on diagonal.
P - 8: If a determinant D becomes zero on putting x = *a* . then we say that (x - *a*) is factor of determinant.
P - 9: It should be noted that while applying operations on determinants at least one row (or column) must remain unchanged or. Maximum number of operations = order of determinant -1.
P - 10: If any row (or column) is multiplied by a non-zero number, then the determinant will be divided by that number.

(1) Minor of an element: If we take the element of the determinant and delete (remove) the row and column containing that element, the determinant left is called the minor of that element. It is denoted by M_{ij} .

Consider the determinant $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

the determinant of minors $M = \begin{vmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{vmatrix}$

where $M_{11} = \text{minor of } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

$M_{12} = \text{minor of } a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

$M_{13} = \text{minor of } a_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Similarly, we can find the minors of other elements, Using thisconcept the value of determinant can be
$\Delta = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$
or, $\Delta = -a_{21}M_{21} + a_{22}M_{22} - a_{23}M_{23}$
or, $\Delta = a_{31}M_{31} - a_{32}M_{32} + a_{33}M_{33}$

(2) Cofactor of an element: The cofactor of an element a_{ij} (i.e. the element in the i^th row and j^th column) is defined as $(-1)^{i+j}$ times the minor of that element. Its denoted by C_{ij} or A_{ij} or F_{ij}.$C_{ij} = (-1)^{i+j} M_{ij}$

If $\Delta = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then determinant of cofactors is

$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

Where $C_{11} = (-1)^{1+1} M_{11} = +M_{11}$, $C_{12} = (-1)^{1+2} M_{12} = -M_{12}$ and $C_{13} = (-1)^{1+3} M_{13} = +M_{13}$
Similarly, we can find the cofactors of other elements.

Product of two determinants
Let the two determinants of third order be,

$D_1 \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ and $D_2 \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{bmatrix}$

Product $= \begin{bmatrix} a_1\alpha_1 + b_1\beta_1 + c_1\gamma_1 & a_1\alpha_2 + b_1\beta_2 + c_1\gamma_2 & a_1\alpha_3 + b_1\beta_3 + c_1\gamma_3 \\ a_2\alpha_1 + b_2\beta_1 + c_2\gamma_1 & a_2\alpha_2 + b_2\beta_2 + c_2\gamma_2 & a_2\alpha_3 + b_2\beta_3 + c_2\gamma_3 \\ a_3\alpha_1 + b_3\beta_1 + c_3\gamma_1 & a_3\alpha_2 + b_3\beta_2 + c_3\gamma_2 & a_3\alpha_3 + b_3\beta_3 + c_3\gamma_3 \end{bmatrix}$
We can also multiply rows by columns or columns by rows or columns by columns.

Application of determinants in solving a system of linear equations

(1) Solution of system of linear equations in three variables by Cramer's rule: The solution of the system of linear equations

$a_1x+b_1y+c_1z=d_1$ .... (i)

$a_2x+b_2y+c_2z=d_2$ .... (ii)

$a_3x+b_3y+c_3z=d_3$ .... (iii)

Is given by $x=\frac{D_1}{D}$, $y=\frac{D_2}{D}$ and $z=\frac{D_3}{D}$

where, $D=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $D_1=\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

$D_2=\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$, and $D_3=\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

Provided that $D \neq 0$

(2) Conditions for consistency: For a system of 3 simultaneous linear equations in three unknown variable.

(i) If $D \neq 0$, then the given system of equation is consistent and has a unique solution given by $x=\frac{D_1}{D}$, $y=\frac{D_2}{D}$ and $z=\frac{D_3}{D}$

(ii) If $D=0$ and $D_1=D_2=D_3=0$, then the given system of equation is consistent with infinitely many solution.

(iii) If $D=0$ and at least one of the determinants D_1, D_2, D_3 is non-zero, then given system of equations is inconsistent and has no solution,

Some special determinants

(1) Symmetric determinant

A determinant is called symmetric determinant if for its every

Element $a_{ij}=a_{ji} \forall i,j$ e.g., $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

(2) Skew-symmetric determinant: A determinant is called skew symmetric determinant if for its every element

$a_{ij}=a_{ji} \forall i,j$ e.g., $\begin{vmatrix} 0 & 3 & -1 \\ -3 & 0 & 5 \\ 1 & -5 & 0 \end{vmatrix}$

- Every diagonal element of a skew-symmetric determinant is always zero.
- The value of a skew symmetric determinant of even order is always a perfect square and that of odd order is always zero.

(3) Cyclic order: If elements of the rows (or columns) are in cyclic order, i.e, (i) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(ii) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

$= (a-b)(b-c)(c-a)(ab+bc+ca)$

(iii) $\begin{vmatrix} a & bc & abc \\ b & ca & abc \\ c & ab & abc \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$

(iv) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

(v) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -\left(a^3+b^3+c^3-3abc\right)$

Matrices

Definition:- A rectangular arrangement of numbers (which may be real or complex numbers) in rows and columns, is called a matrix. This arrangement is enclosed by small ( ) or big [ ] brackets. The numbers are called the elements of the matrix or entries in the matrix.

Order of a matrix

A matrix having m rows and n columns is called a matrix of order $m \times n$ or simply $m \times n$ matrix (read as an m by n matrix). A matrix A of order $m \times n$ is usually written in the following manner

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots a_{1J} & \dots a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots a_{2j} & \dots a_{2n} \\ \dots\dots & \dots\dots & \dots\dots & \dots\dots & \dots\dots \\ a_{i1} & a_{i2} & a_{i3} & \dots a_{3j} & \dots a_{in} \\ \dots\dots & \dots\dots & \dots\dots & \dots\dots & \dots\dots \\ a_{m1} & a_{m2} & a_{mj} & \dots a_{mj} & \dots a_{mn} \end{bmatrix} \text{ or } A = \left[a_{ij}\right]_{m \times n}$$

$$\text{where } \begin{array}{l} i = 1,2\dots\dots m \\ j = 1,2\dots\dots n \end{array}$$

Here a_{ij} denotes the element of ith row and j^{th} column.

Example: order of matrix $\begin{bmatrix} 3 & -1 & 5 \\ 6 & 2 & -7 \end{bmatrix}$ is 2×3.

A matrix of order $m \times n$ contains mn elements. Every row of such a matrix contains n elements and every column contains m elements.

Equality of matrices

Two matrix A and B are said to be equal matrix if they are of same order and their corresponding elements are equal.

Types of matrices

(1) Row matrix: A matrix is said to be a row matrix or row vector if it has only one row and any number of columns. *Example* : [5 0 3] is a row matrix of order 1×3 and [2] is a row matrix of order 1×1.
(2) Column matrix: A matrix is said to be a column matrix or column vector if it has only one column and any number of rows.

Example: $\begin{bmatrix} 2 \\ 3 \\ -6 \end{bmatrix}$ is a column matrix of order 3×1 and [2] is a column matrix of order 1×1. Observe that [2] is both a row matrix as well as column matrix.

(3) Singleton matrix: If in a matrix there is only one element then it is called singleton matrix.
Thus, $A=\left[a_{ij}\right]_{m\times n}$ is a singleton matrix, if m = n = 1

Example: [2], [3], [a], [-3] are singleton matrices.

(4) Null or zero matrix : If in a matrix all the elements are zero then it is called a zero matrix and it is generally denoted by 0. Thus $A=\left[a_{ij}\right]_{m\times n}$ is a zero matrix if $a_{ij}=0$ for all i and j.

Example: [0], $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \end{bmatrix}$ are all zero matrices, but of different orders.

(5) Square matrix : If number of rows and number of columns in a matrix are equal, then it is called a square matrix.
Thus $A=\left[a_{ij}\right]_{m\times n}$ is a square matrix if $m=n$.

Example: $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a square matrix of order 3×3

(i) If $m\neq n$ then matrix is called a rectangular matrix.
(ii) The elements of a square matrix A for which $i=j, i.e. a_{11}, a_{22}, a_{33}....a_{nn}$ are called diagonal elements and the line joining these elements is called the principal diagonal or leading diagonal of matrix A.

(6) Diagonal matrix: If all elements except the principal diagonal in a square matrix are zero, it is called a diagonal matrix. Thus a square matrix $A\left[a_{ij}\right]$ is a diagonal matrix if $a_{ij}=0$, when $i\neq j$.

Example: $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a diagonal matrix of order 3×3 which can be denoted by diag [2, 3, 4].

(7) Identity matrix: A square matrix in which elements in the main diagonal are all '1' and rest are all zero is called an identity matrix or unit matrix. Thus, the square matrix $a=\left[a_{ij}\right]$ is an identity matrix, if $a_{ij}=\begin{cases} 1, if\ i=j \\ 0, if\ i\neq j \end{cases}$

We denote the identity matrix of order n by I_n.

Example: $[1] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are identity matrices of order 1, 2 and 3 respectively.

(8) Scalar matrix: A square matrix whose all non diagonal elements are- zero and diagonal elements are equal is called a scalar matrix. Thus, if $A=\left[a_{ij}\right]$ is a square matrix and

$a_{ij} = \begin{cases} \alpha, \text{if } i = j \\ 0, \text{if } i \neq j \end{cases}$, then A is a scalar matrix.

Unit matrix and null square matrices are also scalar matrices.

(9) Triangular matrix: A square matrix $\left[a_{ij}\right]$ is said to be triangular matrix if each element above or below the principal diagonal is zero. It is of two types

(i) Upper triangular matrix: A square matrix $\left[a_{ij}\right]$ is called the upper triangular matrix, if $a_{ij} = 0$ when $i > j$.

Example: $\begin{bmatrix} 3 & 1 & 2 \\ 0 & 4 & 3 \\ 0 & 0 & 6 \end{bmatrix}$ is an upper triangular matrix of order 3×3

(ii) Lower triangular matrix: A square matrix $\left[a_{ij}\right]$ is called the lower triangular matrix, if $a_{ij} = 0$ when $i < j$.

Example: $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 2 \end{bmatrix}$ is a lower triangular matrix of order 3×3

Trace of a matrix

The sum of diagonal elements of a square matrix. A is called the trace of matrix A , which is denoted by *tr A*.

$$i.e. trA = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \ldots a_{nn}$$

Addition and subtraction of matrices

If $A = \left[a_{ij}\right]_{n \times n}$ and $B = \left[b_{ij}\right]_{n \times n}$ are two matrices of the same order then their sum $A + B$ is a matrix whose each element is the sum of corresponding elements *i.e.*, $A + B = \left[a_{ij} + b_{ij}\right]_{m \times n}$.

Similarly, their subtraction A - *B* is defined as

$$A - B = \left[a_{ij} - b_{ij}\right]_{m \times n}$$

Matrix addition and subtraction can be possible only when matrices are of the same order.

Properties of matrix addition: If *A*, *B* and C are matrices of same order, then

(i) $A + B = B + A$ (Commutative law)

(ii) $(A + B) + C = A + (B + C)$ (Associative law)

(iii) $A + O = O + A = A$, where O is zero matrix which is additive identity of the matrix.

(iv) $A + (-A) = 0 = (-A) + A$, where (-A) is obtained by changing the sign of every element of A, which is additive inverse of the matrix.

(v) $\left.\begin{matrix} A + B = A + C \\ B + A = C + A \end{matrix}\right\} \Rightarrow B = C$ (Cancellation law)

Scalar multiplication of matrices

Let $A = \left[a_{ij}\right]_{m \times n}$ be a matrix and *k* be a number, then the matrix which is obtained by multiplying every element of A by k is called scalar multiplication of A by *k* and it is denoted by *kA*.

Thus, if $A = \left[a_{ij}\right]_{m \times n}$, then $kA = Ak\left[ka_{ij}\right]_{m \times n}$.

Properties of scalar multiplication

If A, B are matrices of the same order and λ, μ are any-two scalars then

(i) $\lambda(A + B) = \lambda A + \lambda B$ (ii) $(\lambda + \mu)A = \lambda A + \mu A$

(iii) $\lambda(\mu A)=(\lambda\mu A)=\mu(\lambda A)$ (iv) $(-\lambda A)=-(\lambda A)=\lambda(-A)$

• All the laws of ordinary algebra hold for the addition or subtraction of matrices and their multiplication by scalars.

Multiplication of matrices

Two matrices A and B are conformable for the product AB if the number of columns in A (pre-multiplier) is same as the number of rows in B (post multiplier). Thus, if $A\left[a_{ij}\right]_{m\times n}$ and $B\left[b_{ij}\right]_{n\times p}$ are two matrices of order $m\times n$ and $n\times p$ respectively, then their product AB is of order $m\times p$ and is defined as

$$(AB)_{ij}=\sum_{r=1}^{n}a_{ir}b_{rj}=[a_{i1}a_{i2}...a_{in}]\begin{bmatrix}b_{1j}\\b_{2j}\\\vdots\\b_{nj}\end{bmatrix}=(i^{th}\text{rowof}A)(j^{th}\text{columnof}B)$$

where $i=1,2...,m$ and $j=1,2,...p$

Now we define the product of a row matrix and a column matrix.

Let $A[a_1a_2.....a_n]$ be a row matrix and $B=\begin{bmatrix}b_1\\b_2\\\vdots\\b_n\end{bmatrix}$ be a column matrix.

Then $AB=[a_1b_1+a_2b_2+....+a_nb_n]$ ....(ii)

Thus, from (i), $(AB)_{ij}=$ Sum of the product of elements of ith row of A with the corresponding elements of jth column of B.

Properties of matrix multiplication

If A,B and C are three matrices such that their product is defined, then

(i) $AB\neq BA$, (Generally not commutative)

(ii) $(AB)C=A(BC)$, (Associative law)

(iii) $IA=A=AI$, where I is identity matrix for matrix multiplication.

(iv) $A(B+C)=AB+AC$, (Distributive law)

(v) If $AB=AC\not\Rightarrow B=C$, (Cancellation law is not applicable)

(vi) If $AB=0$, it does not mean that $A=0$ or $B=0$, again product of two non zero matrix may be a zero matrix.

Remember that if A and B are two matrices of the same order, then

(i) $(A+B)^2=A^2+B^2+AB+BA$

(ii) $(A-B)^2=A^2+B^2-AB-BA$

(iii) $(A-B)(A+B)=A^2-B^2+AB-BA$

(iv) $(A+B)(A-B)=A^2-B^2-AB+BA$

(v) $A(-B)=(-A)(B)=-AB$

Positive integral powers of a matrix

The positive integral powers of a matrix A are defined only when A is a square matrix.

Also then $A^2=A.A,\ A^3=A.A.A.=A^2A.$

Also for any positive integers m and n,

(i) $A^mA^n=A^{m+n}$

(ii) $\left(A^m\right)^n=A^{mn}=\left(A^n\right)^m$

(iii) $I^n=I, I^m=I$

(iv) $A^0=I_n$, where A is a square matrix of order n.

Transpose of a matrix

The matrix obtained from a given matrix A by changing its rows into columns or columns into rows is called transpose of matrix A and is denoted by A^T or A'.

From the definition it is obvious that if order of A is $m \times n$ then order of A^T is $n \times m$.

Example:

Transpose of matrix $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}_{2\times3}$ is $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}_{3\times2}$

Properties of transpose: Let A and B be two matrices

then, (i) $\left(A^T\right)^T = A$

(ii) $(A+B)^T = A^T + B^T$, A and B being of the same order

(iii) $(kA)^T = kA^T$, k be any scalar (real or complex)

(iv) $(AB)^T = B^T A^T$, A and B being conformable for the product AB

(v) $(A_1A_2A_3.....A_{n-1}A_n)^T = A_n^T A_{n-1}^T.....A_3^T A_2^T A_1^T$

(vi) $I^T = I$

Special types of matrices

(1) Symmetric matrix : A square matrix $A = \left[a_{ij}\right]$ is called symmetric matrix if $a_{ij} = aji$ for all i, j *or* $A^T = A$..

Example: $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

(2) Skew-symmetric matrix : A square matrix $A = \left[a_{ij}\right]$ is called skew-symmetric matrix if $a_{ij} = -a_{ij}$ for all i, j or $A^T = -A$.

Example: $\begin{bmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & 0 \end{bmatrix}$

All principal diagonal elements of a skew- symmetric matrix are always zero because for any diagonal element.
$a_{ij} = -a_{ij} \Rightarrow a_{ij} = 0$

Properties of symmetric and skew-symmetric matrices

(i) If A is a square matrix, then $A + A^T, AA^T, A^T A$ are symmetric matrices, while $A - A^T$ is skew-symmetric matrix.

(ii) If A is a symmetric matrix, then $-A, KA, A^T, A^n, A^{-1}, B^T AB$ are also symmetric matrices, where $n \in N, K \in R$ and B is a square matrix of order that of A.

(iii) If A is a skew-symmetric matrix, then

(a) A^{2n} is a symmetric matrix for $n \in N$.

(b) A^{2n+1} is a skew-symmetric matrix for $n \in N$.

(c) kA is also skew-symmetric matrix, where $k \in R$..

(d) $B^T AB$ is also skew- symmetric matrix where B is a square matrix of order that of A.

(iv) If A, B are two symmetric matrices, then

(a) $A \pm B, AB + BA$ are also symmetric matrices,

(b) $AB - BA$ is a skew- symmetric matrix,

(c) AB is a symmetric matrix, when AB = BA .

(v) If A, B are two skew-symmetric matrices, then
(a) $A \pm B, AB - BA$ are skew-symmetric matrices,
(b) $AB + BA$ is a symmetric matrix.
(vi) If A a skew-symmetric matrix and C is a column matrix, then $C^T AC$ is a zero matrix.
(vii) Every square matrix A can unequally be expressed as sum of a symmetric and skew-symmetric matrix
$i.e., A = \left[\frac{1}{2}\left(A + A^T\right)\right] + \left[\frac{1}{2}\left(A - A^T\right)\right].$

(3) Singular and Non-singular matrix : Any square matrix A is said to be non-singular if $|A| \neq 0$. and a square matrix A is said to be singular if $|A| = 0$.

Example : $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ *then* $|A| = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 10 - 12 = -2 \Rightarrow A$

is a non-singular matrix,

(4) Hermitian and Skew-hermitian matrix : A square matrix $A = \left[a_{ij}\right]$ is said to be hermitian matrix if

$a_{ij} = \bar{a}_{ij;} \forall ji.e., A = A^{\theta}.$

Example:

$$\begin{bmatrix} a & b+ic \\ b-ic & d \end{bmatrix}, \begin{bmatrix} 3 & 3-4i & 5+2i \\ 3+4i & 5 & -2+i \\ 5-2i & -2-i & 2 \end{bmatrix}$$

are Hermitian matrices. If A is a Hermitian matrix then $aij\ f\ \ \bar{a}_{ij} \Rightarrow a_{ij}$ is real $\forall i,$, thus every diagonal element of a Hermitian matrix must be real.

A square matrix, $A = | a_{ij} |$ is said to be a Skew-Hermitian if $a_{ij} = -\bar{a}_{ij}. \forall i, ji.e. A^{\theta} = -A.$. If A is a skew-Hermitian matrix, then $a_{ij} = -\bar{a}_{ij} \Rightarrow a_{ij} + \bar{a}_{ij} = 0\, i.e. a_{ij}$ must be purely imaginary or zero.

Example: $\begin{bmatrix} 0 & -2+i \\ 2-i & 0 \end{bmatrix}, \begin{bmatrix} 3i & -3+2i & -1-i \\ 3+2i & -2i & -2-4i \\ 1-i & 2-4i & 0 \end{bmatrix}$

are skew-hermitian matrices.

(5) Orthogonal matrix: A square matrix A is called orthogonal if $AA^T = I = A^T A$ i.e. if $A^{-1} = A^T$

Example: $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

is orthogonal because $A^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = A^T$

In fact every unit matrix is orthogonal. Determinant of orthogonal matrix is - 1 or 1.

(6) Idempotent matrix: A square matrix A is called an idempotent matrix if $A^2 = A$.

Example: $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ is an idempotent matrix, because

$$A^2 = \begin{bmatrix} 1/4+1/4 & 1/4+1/4 \\ 1/4+1/4 & 1/4+1/4 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} = A$$

Also, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are idempotent matrices because $A^2 = A$ and $B^2 = B$

In fact every unit matrix is idempotent.

(7) Involutory matrix: A square matrix A is called aninvolutory matrix if $A^2 = I$ or $A^{-1} = A$

Example:

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an involutory matrix because $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

In fact every unit matrix is involutory.

(8) Nilpotent matrix: A square matrix A is called anilpotent matrix if there exists a $p \in N$ such that $A^p = 0$, where p is index of matrix.

Example: $A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ is a nilpotent matrix because $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ (Here P = 2)

Determinant of every nilpotent matrix is 0.

(9) Unitary matrix: A square matrix is said to be unitary, if $\overline{A}'A = I$ since $|\overline{A}'| = |A|$ and $|\overline{A}'A| = |\overline{A}'||A|$ therefore if $\overline{A}'A = I$, we have $|\overline{A}'||A| = 1$

Thus the determinant of unitary matrix is of unit modulus. Fora matrix to be unitary it must be non-singular.

Hence $\overline{A}'A = I \Rightarrow A\overline{A}' = I$

(10) Periodic matrix: A matrix A will be called a periodic matrix if $A^{k+1} = A$ where k is a positive integer. If, however k is the least positive integer for which $A^{k+1} = A$, then k is said to be the period of A.

(11) Differentiation of a matrix: If $A = \begin{bmatrix} f(x) & g(x) \\ h(x) & l(x) \end{bmatrix}$ then

$\frac{dA}{dx} = \begin{bmatrix} f'(x) & g'(x) \\ h'(x) & l'(x) \end{bmatrix}$ is a differentiation of matrix A.

Example: If $A = \begin{bmatrix} x^2 & \sin x \\ 2x & 2 \end{bmatrix}$ then $\frac{dA}{dx} = \begin{bmatrix} 2x & \cos x \\ 2 & 0 \end{bmatrix}$

(12) Conjugate of a matrix: The matrix obtained from any given matrix A containing complex number as its elements, on replacing its elements by the corresponding conjugate complex numbers is called conjugate of A and is denoted by A.

Example: $A = \begin{bmatrix} 1+2i & 2-3i & 3+4i \\ 4-5i & 5+6i & 6-7i \\ 8 & 7+8i & 7 \end{bmatrix}$

Then $\overline{A} = \begin{bmatrix} 1-2i & 2+3i & 3-4i \\ 4+5i & 5-6i & 6+7i \\ 8 & 7-8i & 7 \end{bmatrix}$

Properties of conjugates

(i) $\overline{(\overline{A})} = A$

(ii) $\overline{(A+B)} = \overline{A} + \overline{B}$,

(iii) $\overline{(\alpha A)} = \overline{\alpha}\overline{A}, \alpha$ being any number

(iv) $\overline{(AB)} = \overline{A}\overline{B}$, A and B being conformable for multiplication

(13) Transpose conjugate of a matrix: The transpose of the conjugate of a matrix A is called transposed conjugate of A and is denoted by A^θ. The conjugate of the transpose of A is the same as the transpose of the conjugate of A i.e. $\overline{(A')} = (\overline{A})' = A^\theta$

If $A = [a_{ij}]_{m \times n}$ then $A^\theta = [b_{ji}]_{n \times m}$ where $b_{ji} = \overline{a}_{ij}$

i.e., the $(j,i)^{th}$ element of A^θ = the conjugate of $(i,j)^{th}$ element A.

Example: if $A = \begin{bmatrix} 1+2i & 2-3i & 3+4i \\ 4-5i & 5+6i & 6-7i \\ 8 & 7+8i & 7 \end{bmatrix}$

then $A^{\theta} = \begin{bmatrix} 1-2i & 4+5i & 8 \\ 2+3i & 5-6i & 7-8i \\ 3-4i & 6+7i & 7 \end{bmatrix}$

Properties of transpose conjugate

$(i)\ \left(A^{\theta}\right)^{\theta} = A$

$(ii)\ (A+B)^{\theta} = A^{\theta} + B^{\theta}$

$(iii)\ (kA)^{\theta} = \overline{K}A^{\theta}, K$ being any number

$(iv)\ (AB)^{\theta} = B^{\theta}, A^{\theta}$

Adjoint of a square matrix

Let $A = \left[a_{ij}\right]$ be a square matrix of order n and let c_{ij} be cofactor of a_{ij} in A. Then the transpose of the matrix of cofactors of elements of A is called the adjoint of A and is denoted by adj A

Thus, $adj = A = \left[C_{ij}\right]^{T} \Rightarrow (adjA)_{ij} = C_{ji} =$ cofactor of a_{ji} in A.

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

then $adjA = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^{T} = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix};$

where C_{ij} denotes the cofactor of a_{ij} in A.

Example: $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}, C_{11} = s, C_{12} = -r, C_{21} = q, C_{22} = p$

$\therefore adj\ A = \begin{bmatrix} s & -r \\ -q & p \end{bmatrix}^{T} = \begin{bmatrix} s & -q \\ -r & p \end{bmatrix}$

Properties of adjoint matrix: If A, B are square matrices of order n and I_n is corresponding unit matrix, then

(i) $A(adjA) = |A| I_n = (adjA)A$

$(Thus\ A(adjA)\ is\ always\ a\ scalar\ matrix)$

(ii) $|adjA| = |A|^{n-1}$

(iii) $adj(adjA) = |A|^{n-2} A; |A| \neq 0.$

(iv) $|adj(adjA)| = |A|^{(n-1)^2}$

(v) $adj\left(A^{T}\right) = (adjA)^{T}$

(vi) $adj(AB) = (adjB)(adjA)$

(vii) $adj\left(A^{m}\right) = (adjA)^{m}, m \in N$

(viii) $adj(kA) = k^{n-1}(adjA), k \in R$

(ix) $adj(I_n) = I_n$

(x) $adj(O) = O$

(xi) A is symmetric matrix $\Rightarrow$ A is also symmetric matrix.
(xii) A is diagonal matrix $\Rightarrow$ adj A is also diagonal matrix.
(xiii) A is triangular matrix $\Rightarrow$ adj A is also triangular matrix.
(xiv) A is singular $\Rightarrow |adjA| = 0$

A non-singular square matrix of order n is invertible if there exists a square matrix B of the same order suchthat $AB = I_n = BA.$

In such a case, we say that the inverse of A is B and we write $A^{-1} = B.$ The inverse of A is given by $A^{-1} = \frac{1}{|A|}adjA.$

The necessary and sufficient condition for the existence of the inverse of a square matrix A is that $|A| \neq 0.$ i.e., matrix should be non-singular.

Properties of inverse matrix

If A and B are invertible matrices of the same order, then

(i) $\left(A^{-1}\right)^{-1} = A$

(ii) $\left(A^{T}\right)^{-1} = \left(A^{-1}\right)^{T}$

(iii) $(AB)^{-1} = B^{-1}A^{-1}$

(iv) $\left(A^{k}\right)^{-1} = \left(A^{-1}\right)^{k}, k \in N$ [In particular $\left(A^{2}\right)^{-1} = \left(A^{-1}\right)^{2},$]

(v) $adj\left(A^{-1}\right) = (adjA)^{-1}$

(vi) $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

(vii) A = diag$(a_1 a_2 \ldots . a_n) \Rightarrow A^{-1} = diag\left(a_1^{-1} a_2^{-1} \ldots a_n^{-1}\right)$

(viii) A is symmetric matrix $\Rightarrow A^{-1}$ is also symmetric matrix.

(ix) A is diagonal matrix, $|A| \neq 0 \Rightarrow A^{-1}$ is also diagonal matrix

(x) A is a scalar matrix $\Rightarrow A^{-1}$ is also a scalar matrix.

(xi) A is triangular matrix, $|A| \neq 0 \Rightarrow A^{-1}$is also triangular matrix .

(xii) Every invertible matrix possesses a unique inverse.

(xiii) **Cancellation law respect to multiplication**

If A is a non-singular matrix i.e., if $|A| \neq 0,$ then A^{-1} exists and

$$AB = AC \Rightarrow A^{-1}(AB) = A^{-1}(AC)$$

$$\Rightarrow \left(A^{-1}A\right)B = \left(A^{-1}A\right)C$$

$$\Rightarrow IB = IC \Rightarrow B = C$$

$$\therefore AB = AC \Rightarrow B = C \Leftrightarrow |A| \neq 0$$

Rank of matrix

Definition: Let A be a $m \times n$ matrix. If we retain any r rows and r columns of A we shall have a square sub-matrix of order r .The determinant of the square sub-matrix of order r is called a minor of A order r . Consider any matrix A which is of the order of

3×4 say, $A = \begin{bmatrix} 1 & 3 & 4 & 5 \\ 1 & 2 & 6 & 7 \\ 1 & 5 & 0 & 1 \end{bmatrix}$ It is 3×4 matrix so we can have

minors of order 3, 2 or 1. Taking any three rows and three columns minor of order three. Hence minor of order 3

$$=\begin{bmatrix}1 & 3 & 4\\1 & 2 & 6\\1 & 5 & 0\end{bmatrix}=0$$

Making two zeros and expanding above minor is zero. Similarly we can consider any other minor of order 3 and it can be shown to be zero. Minor of order 2 is obtained by taking any two rows and any two columns.

Minor of order $2=\begin{bmatrix}1 & 3\\1 & 2\end{bmatrix}=2-3=1\neq 0$. Minor of order 1 is every element of the matrix.

Rank of a matrix: The rank of a given matrix A is said to be r if

(1) Every minor of A of order $r+1$ is zero.

(2) There is at least one minor of A of order r which does not vanish. Here we can also say that the rank of a matrix A is said to be r ,if

(i) Every square sub matrix of order $r+1$is singular.

(ii) There is at least one square sub matrix of order r which is non-singular.

The rank r of matrix A is written as $p(A)=r$.

Working rule for finding the rank of a matrix

Let $A=\begin{bmatrix}a_{11} & a_{12} & a_{13} & \cdots & a_{1n}\\a_{21} & a_{22} & a_{23} & \cdots & a_{2n}\\a_{31} & a_{32} & a_{33} & \cdots & a_{3n}\\\vdots & \vdots & \vdots & & \vdots\\a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn}\end{bmatrix}$

(1) Make $a_{11}=1$ by elementary row transformations.

(2) Make $a_{21},a_{31}...,a_{m1}$all zeros using the transformations

$R_2\rightarrow R_2-a_{21}R_1R_3\rightarrow R_3-a_{31}R_1..R_m\rightarrow R_m-a_{m1}R_1$

(3) Make $a_{22}=1$by elementary row transformations.

(4) Make $a_{32},a_{42}...a_{m2}$all zeros using the transformations

$R_3\rightarrow R_3-a_{32}R_2,R_4\rightarrow R_4-a_{42}R_2...$

(5) Repeat this process until the matrix A is reduced to Echelon form.

(6) Count the number of non-zero rows in it which will be the rank of the matrix A.

Echelon form of a matrix

A matrix A is said to be in Echelon form if either A is the null matrix or A satisfies the following conditions:

(1) Every non- zero row in A precedes every zero row.

(2) The number of zeros before the first non-zero element in a row is less than the number of such zeros in the next row.

If can be easily proved that the rank of a matrix in Echelon form is equal to the number of non-zero row of the matrix.

Rank of a matrix In Echelon form: The rank of a matrix in Echelon form is equal to the number of non-zero rows in that matrix.

Homogeneous and non-homogeneous systems of linear equations

A system of equations $AX=B$is called a homogeneous system if$B=O$. If$B\neq O$, it is called a non-homogeneous system of equations.

e.g., $2x+5y=0$

$3x-2y=0$

is a homogeneous system of linear equations whereas the system of equations given by

e.g., $2x+3y=5$

$x+y=2$

is a non-homogeneous system of linear equations.

(1) Solution of Non-homogeneous system of linear equations

(i) Matrix method: If $AX = B$, then $X = A^{-1}B$ gives a unique solution, provided A is non-singular.
But if A is a singular matrix *i.e.*, if $|A| = 0$, then the system of equation $AX = B$ may be consistent with infinitely many solutions or it may be inconsistent.
(ii) Rank method for solution of Non-Homogeneous system $AX = B$
(a) Write down A, B
(b) Write the augmented matrix [A : B]
(c) Reduce the augmented matrix to Echelon form by using elementary row operations.
(d) Find the number of non-zero rows in A and [A : B] to find the ranks of A and [A : B] respectively.
(e) If $p(A) \neq p(A:B)$, then the system is inconsistent.
(f) $p(A) = p(A:B) =$ the number of unknowns, then the system has a unique solution.
If $p(A) = p(A:B) <$ number of unknowns, then the system has an infinite number of solutions.

(2) Solutions of a homogeneous system of linear equations: Let $AX = O$ be a homogeneous system of 3 linear equations in 3 unknowns.
(a) Write the given system of equations in the form $AX = O$ and write A.
(b) Find $|A|$.
(c) If $|A| \neq 0$ then the system is consistent and $x = y = z = 0$ is the unique solution.
(d) If $|A| = 0$, then the systems of equations has infinitely many solutions. In order to find that put $z = K$ (any real number)and solve any two equations for x and y so obtained with $z = K$ give a solution of the given system of equations.

Consistency of a system of linear equation *AX* = *B*, where *A* is a square matrix

In system of linear equations $AX = B, A = (a_{ij})_{n\times n}$ is said to be
(i) Consistent (with unique solution) if $|A| \neq 0$.
i.e., if A is non-singular matrix.
(ii) Inconsistent (It has no solution) if $|A| = 0$ and $(adjA)B$ isa non-null matrix.
(iii) Consistent (with infinitely m any solutions) if $|A| = 0$ and$(adjA)B$ is a null matrix.

Cayley-Hamilton theorem

Every matrix satisfies its characteristic equation e.g. let A be a square matrix then $|A - xI| = 0$ is the characteristics equation of A. If $x^3 - 4x^2 - 5x - 7 = 0$ is the characteristic equation for A, then
$A^3 - 4A^2 - 5A - 7I = 0$.
Roots of characteristic equation for A are called Eigen values of A or characteristic roots of A or latent roots of A.
If λ is characteristic root of A, then λ^{-1} is characteristic root of A^{-1}

Geometrical transformations

(1) Reflection in the x-axis : If $P'(x', y')$ is the reflection of the point $P'(x, y)$ on the x-axis, then the matrix
$\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}$ describes the reflection of a point $P(x, y)$ in the x-axis.

(2) Reflection in the y-axis

Here the matrix is $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(3) Reflection through the origin

Here the matrix is $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(4) Reflection in the line y = x

Here the matrix is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(5) Reflection in the line $y = -x\tan\theta$ **tan *0***

Here the matrix is $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(6) Reflection in $y = x\tan\theta$

Here matrix is $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

(7) Rotation through an angle θ

Here the matrix is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Exercise-01

Determinants

Expansion, Solution of Equation, Properties

1. $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix} =$

(a) $a(x+y+z)+b(p+q+r)+c$ (b) 0
(c) $abc+xyz+pqr$ (d) None of these

2. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix} =$

(a) 0 (b) $a^3+b^3+c^3-3abc$
(c) $3abc$ (d) $(a+b+c)^3$

3. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} =$

(a) 1 (b) 0 (c) x (d) xy

4. If $\begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ z-y & z-x & x+y \end{vmatrix} = kxyz,$ then the value of k is

(a) 2 (b) 4 (c) 6 (d) 8

5. The roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ are

(a) $-1,-2$ (b) $-1,2$ (c) $1,-2$ (d) $1,2$

6. $\begin{vmatrix} 1 & 5 & \pi \\ \log_e e & 5 & \sqrt{5} \\ \log_{10} 10 & 5 & e \end{vmatrix} =$

(a) $\sqrt{\pi}$ (b) e (c) 1 (d) 0

7. If $a \neq b \neq c$, the value of x which satisfies the equation

$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0,$ is.

(a) $x=0$ (b) $x=a$ (c) $x=b$ (d) $x=c$

8. If $a+b+c=0$, then the solution of the equation

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix}=0 \text{ is}$$

(a) 0

(b) $\pm\frac{3}{2}\left(a^2+b^2+c^2\right)$

(c) $0,\pm\sqrt{\frac{3}{2}\left(a^2+b^2+c^2\right)}$

(d) $0,\pm\sqrt{a^2+b^2+c^2}$

9. If $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix}=0$, then $x=$

(a) $1,9$ (b) $-1,9$ (c) $-1,-9$ (d) $1,-9$

10. $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}=$

(a) abc (b) $2abc$ (c) $3abc$ (d) $4abc$

11. The roots of the equation $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}=0$ are

(a) $0,-3$ (b) $0,0,-3$ (c) $0,0,0,-3$ (d) None of these

12. One of the roots of the given equation $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix}=0$ is.

(a) $-(a+b)$ (b) $-(b+c)$ (c) $-a$ (d) $-(a+b+c)$

13. $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}=$

(a) 2 (b) -2 (c) x^2-2 (d) None of these

14. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}=$

(a) $a^3+b^3+c^3-3abc$

(b) $a^3+b^3+c^3+3abc$

(c) $(a+b+c)(a-b)(b-c)(c-a)$

(d) None of these

15. If $f(x)=\begin{vmatrix} x-3 & 2x^2-18 & 3x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix}$ then

$f(1).f(3)+f(3).f(5)+f(5).f(1)=$

(a) $f(1)$ (b) $f(3)$

(c) $f(1)+f(3)$ (d) $f(1)+f(5)$

(e) $f(1)+f(3)+f(5)$

16. $\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix}=$

(a) $abc(a+b+c)$ (b) $3a^2b^2c^2$

(c) 0 (d) None of these

17. $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix}=$

(a) abc (b) $1/abc$ (c) $ab+bc+ca$ (d) 0

18. If $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix}=k(x+y+z)(x-z)^2$, then $k=$

(a) $2xyz$ (b) 1 (c) xyz (d) $x^2y^2z^2$

19. If -9 is a root of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}=0$, then the other two roots are

(a) $2,7$ (b) $-2,7$ (c) $2,-7$ (d) $-2,-7$

20. If $A=\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^2 & c^3 \end{vmatrix}, B=\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}, C=\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$ then which of the relation is correct.

(a) $A=B$ (b) $A=C$ (c) $B=C$ (d) None of these

21. The solutions of the equation $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix}=0$ are

(a) $3,-1$ (b) $-3,1$ (c) $3,1$ (d) $-3,-1$

22. If $\begin{vmatrix} x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \\ x+a & x+b & x+c \end{vmatrix}=0$, then a,b,c are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

23. The value of the determinant $\begin{vmatrix} 4 & -6 & 1 \\ -1 & -1 & 1 \\ -4 & 11 & -1 \end{vmatrix}$ is

(a) -75 (b)25 (c) 0 (d) -25

24. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}=0$, a,b,c are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

25. The determinant $\begin{vmatrix} 4+x^2 & -6 & -2 \\ -6 & 9+x^2 & 3 \\ -2 & 3 & 1+x^2 \end{vmatrix}$ is not divisible by

(a) x (b) x^3 (c) $14+x^2$ (d) x^5

26. If $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -1 \end{vmatrix}=0$, then the value of k is

(a) -1 (b) 0 (c) 1 (d) None of these

27. $\begin{vmatrix} a-1 & a & bc \\ b-1 & b & ca \\ c-1 & c & ab \end{vmatrix}=$

(a) 0 (b) $(a-b)(b-c)(c-a)$

(c) $a^3+b^3+c^3-3abc$ (d) None of these

28. The value of $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix}$ is equal to

(a) 0. (b) 679 (c) 779. (d) 1000

29. The value of $\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix}$ is

(a) $441\times446\times451$ (b) 0

(c) -1 (d) 1

30. If $p+q+r=0=a+b+c$, then the value of the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ is

(a) 0 (b) $pa+qb+rc$ (c) 1 (d) None of these

31. The value of $\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ b+c & c+a & a+b \end{vmatrix}$ is

(a) 1 (b) 0

(c) $(a-b)(b-c)(c-a)$ (d) $(a+b)(b+c)(c+a)$

32. $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = K\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$, then $K =$

(a) 1 (b) 2 (c) 3 (d) 4

33. $\begin{vmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{vmatrix} =$

(a) 0 (b) $(p-q)(q-r)(r-p)$
(c) pqr (d) $3pqr$

34. $\begin{vmatrix} x & 4 & y+z \\ y & 4 & z+x \\ z & 4 & x+y \end{vmatrix} =$

(a) 4 (b) $x+y+z$
(c) xyz (d) 0

35. The value of x, if $\begin{vmatrix} -x & 1 & 0 \\ 1 & -x & 1 \\ 0 & 1 & -x \end{vmatrix} = 0$ is equal to

(a) $\pm\sqrt{6}$ (b) $\pm\sqrt{2}$ (c) $\pm\sqrt{3}$ (d) $\sqrt{2}, \sqrt{3}$

36. A root of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$ is

(a) 6 (b) 3 (c) 0 (d) None of these

37. $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} =$

(a) 0
(b) $12\cos^2 x - 10\sin^2 x$
(c) $12\sin^2 x - 10\cos^2 x - 2$
(d) $10\sin 2x$

38. The roots of the equation $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$ are

(a) 1,2 (b) $-1,2$ (c) $1,-2$ (d) $-1,-2$

39. If $D_p = \begin{vmatrix} p & 15 & 8 \\ p^2 & 35 & 9 \\ p^3 & 25 & 10 \end{vmatrix}$, then, $D_1 + D_2 + D_3 + D_4 + D_5 =$

(a) 0 (b) 25 (c) 625 (d) None of these

40. The value of $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$ is equal to

(a) $9a^2(a+b)$

(b) $9b^2(a+b)$

(c) $a^2(a+b)$

(d) $b^2(a+b)$

41. If a,b,c are different and $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$ then

(a) $a+b+c=0$
(b) $abc=1$
(c) $a+b+c=1$
(d) $ab+bc+ca=0$

42. If $\begin{vmatrix} x-1 & 3 & 0 \\ 2 & x-3 & 4 \\ 3 & 5 & 6 \end{vmatrix} = 0,$ then $x=$

(a) 0 (b) 2 (c) 3 (d) 1

43. If a,b,c are in A.P., then the value of $\begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$ is

(a) $x-(a+b+c)$

(b) $9x^2+a+b+c$
(c) $a+b+c$
(d) 0

44. If $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, then a,b,c are

(a) Equal (b) In A.P. (c) In G.P. (d) In H.P.

45. The value of $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$ is

(a) 8 (b) –8 (c) 400 (d) 1

46. If $\begin{vmatrix} a & b & a+b \\ b & c & b+c \\ a+b & b+c & 0 \end{vmatrix} = 0;$ then a,b,c are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

47. If $A=\begin{vmatrix}-1 & 2 & 4\\ 3 & 1 & 0\\ -2 & 4 & 2\end{vmatrix}$ and $B=\begin{vmatrix}-2 & 4 & 2\\ 6 & 2 & 0\\ -2 & 4 & 8\end{vmatrix}$ then B is given by

(a) $B=4A$ (b) $B=-4A$ (c) $B=-A$ (d) $B=6A$

48. Solution of the equation $\begin{vmatrix}1 & 1 & x\\ p+1 & p+1 & p+x\\ 3 & x+1 & x+2\end{vmatrix}=0$ are

(a) $x=1,2$ (b) $x=2,3$ (c) $x=1,p,2$ (d) $x=1,2,-p$

49. The value of $\begin{vmatrix}5^2 & 5^3 & 5^4\\ 5^3 & 5^4 & 5^5\\ 5^4 & 5^5 & 5^7\end{vmatrix}$ is

(a) 5^2 (b) 0 (c) 5^{13} (d) 5^9

50. The value of $\begin{vmatrix}41 & 42 & 43\\ 44 & 45 & 46\\ 47 & 48 & 49\end{vmatrix}=$

(a) 2 (b) 4 (c) 0 (d) 1

51. The determinant $\begin{vmatrix}a & b & a-b\\ b & c & b-c\\ 2 & 1 & 0\end{vmatrix}$ is equal to zero if a,b,c, are in

(a) G.P. (b) A.P. (c) H.P. (d) None of these

Exercise-01

Solutions

1.

Sol (b) $\begin{vmatrix}a-b & b-c & c-a\\ x-y & y-z & z-x\\ p-q & q-r & r-p\end{vmatrix}=\begin{vmatrix}0 & b-c & c-a\\ 0 & y-z & z-x\\ 0 & q-r & r-p\end{vmatrix}=0$

[by] $c_1 \rightarrow c_1+c_2+c_3$

2. Sol (a) $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix} = \begin{vmatrix} 0 & a-b & (a-b)(a+b+c) \\ 0 & b-c & (b-c)(a+b+c) \\ 1 & c & c^2-ab \end{vmatrix}$

$by \begin{bmatrix} R_1 \to & R_1 - & R_2 \\ R_2 \to & R_2 - & R_3 \end{bmatrix}$

$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b+c \\ 0 & 1 & a+b+c \\ 1 & c & c^2-ab \end{vmatrix} = 0 \; [\because R_1 \cong R_2]$

Alter: Put $a=1, b=2, c=3$ and solve.

3. Sol (d) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ -x & x & 1 \\ 0 & -y & 1+y \end{vmatrix} = xy, \begin{bmatrix} C_1 \to & C_1 - & C_2 \\ C_2 \to & C_2 - & C_3 \end{bmatrix}$

4.Sol (d) $\begin{vmatrix} y+z & x-z & x-y \\ y-z & z+x & y-x \\ z-y & z-x & x+y \end{vmatrix} = \begin{vmatrix} y+z & x-z & x-y \\ 2y & 2x & 0 \\ 2z & 0 & 2x \end{vmatrix}$

$R_2 \to R_2 + R_1$ and $R_3 \to R_3 + R_1$

$= 4\begin{vmatrix} y+x & x-z & x-y \\ y & x & 0 \\ z & 0 & x \end{vmatrix}$

$= 4\left[(y+z)\left(x^2\right) - (x-z)(xy) + (x-y)(-zx)\right]$

$= 4\left[x^2y + zx^2 - x^2y + xyz - zx^2 + xyz\right] = 8xyz$

Hence, $k = 8$.

Trick: Check by substituting $x=1, y=2, z=3$.

5.Sol (b) $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 0 & 6 & 15 \\ 0 & -2-2x & 5\left(1-x^2\right) \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

$\begin{bmatrix} R_1 \to & R_1 - & R_2 \\ R_2 \to & R_2 - & R_3 \end{bmatrix}$

$\Rightarrow 3.2.5\begin{vmatrix} 0 & 1 & 1 \\ 0 & -(1+x) & 1-x^2 \\ 1 & x & x^2 \end{vmatrix} = 0$

$\Rightarrow (1+x)\begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1-x^2 \\ 1 & x & x^2 \end{vmatrix} = 0$

$\Rightarrow x = 1 = 0 \, or \, x-2 = 0 \Rightarrow x = -1, 2.$

Trick : Obviously by inspection $x = -1, 2$ satisfy the equation.

At $x=-1\begin{vmatrix}1 & 4 & 20\\ 1 & -2 & 5\\ 1 & -2 & 5\end{vmatrix}=0\,as\,R_2\equiv R_3$

At $x=2\begin{vmatrix}1 & 4 & 20\\ 1 & -2 & 5\\ 1 & 4 & 20\end{vmatrix}=0\;asR_1\equiv R_3$

6.Sol (d) $\Delta=\begin{vmatrix}1 & 5 & \pi\\ 1 & 5 & \sqrt{5}\\ 1 & 5 & e\end{vmatrix}=0$ $\quad[\because \log_a a=1 \text{and} 5C_1=C_2]$

7.**Sol (a)**Obviously on putting $x=0$, we observe that the determinant becomes

$$\Delta_{x=0}=\begin{vmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{vmatrix}=a(bc)-b(ac)=0$$

$\therefore x=0$ is a root of the given equation.

Alter: Expanding Δ, we get

$\Delta\equiv\sim-(x-a)\left[-(x+b)(x-c)\right]+(x-b)\left[(x+a)(x+c)\right]=0$

$\Rightarrow 2x^3-\left(2\sum ab\right)x=0$

$\Rightarrow$ Either $x=0\,or\,x^2=\sum ab\ \left(i.e.,\ x=\pm\sum ab\right)$

Again $x=0$ satisfies the given equation.

8.Sol (c) $\begin{vmatrix}a-x & c & b\\ c & b-x & a\\ b & a & c-x\end{vmatrix}=0$

$\Rightarrow\begin{vmatrix}a+b+c-x & c & b\\ a+b+c-x & b-x & a\\ a+b+c-x & a & c-x\end{vmatrix}=0$

$\Rightarrow\left(x-\sum a=0\right)\begin{vmatrix}1 & c & b\\ 1 & b-x & a\\ 1 & a & c-x\end{vmatrix}=0$ $\quad$ [by hypothesis]

or $1\left\{(b-x)(c-x)-a^2\right\}-c\{c-x-a\}+b\{a-b+x\}=0$

by expanding the determinant.

or $x^2-\left(a^2+b^2+c^2\right)+(ab+bc+ca)=0$

or $x^2-\left(\sum a^2\right)-\frac{1}{2}\left(\sum a^2\right)=0$

$\left[\because a+b+c=0\Rightarrow(a+b+c)^2\Rightarrow\sum a^2+2\sum ab=0\Rightarrow\sum ab=-\frac{1}{2}\sum a^2\right]$

or $x=\pm\sqrt{\frac{3}{2}\sum a^2}$

$\therefore$ the solution $x=0\,\text{or}\pm\sqrt{\frac{3}{2}\sum a^2}$

Trick: Put $a=1, b=-1$ and $c=0$ so that they satisfy the condition $a+b+c=0$ Now the determinant

becomes $\begin{vmatrix}1-x & 0 & -1\\ 0 & -1-x & 1\\ -1 & 1 & -x\end{vmatrix}=0$

$\Rightarrow(1-x)\{x(1+x)-1\}+1(1+x)=0$

$\Rightarrow(1-x)\left\{x^2+x-1\right\}+x+1=0$

$\Rightarrow x\left(x^2-3\right)=0$

Now putting these in the options, we find that option (c) gives the same values *i.e.* $0,\pm\sqrt{3}$

9.**Sol (d)**By $C_1 \to C_1+C_2+C_3$, we have

$$(9+x)\begin{vmatrix}1 & 3 & 5\\ 1 & x+2 & 5\\ 1 & 3 & x+4\end{vmatrix}=0$$

$$\Rightarrow (x+9)\begin{bmatrix}0 & 1-x & 0\\ 0 & -(1-x) & 1-x\\ 1 & 3 & x+4\end{bmatrix}=0\begin{bmatrix}R_1 \to R_1-R_2\\ R_2 \to R_2-R_3\end{bmatrix}$$

$$\Rightarrow (x+9)(1-x)^2\begin{vmatrix}0 & 1 & 0\\ 0 & -1 & 1\\ 1 & 3 & x+4\end{vmatrix}=0$$

$\Rightarrow x=1,1,-9$ [Since the determinant = 1]

10. Sol (d) $\begin{vmatrix}b+c & a & a\\ b & c+a & b\\ c & c & a+b\end{vmatrix}=\begin{vmatrix}0 & -2c & -2b\\ b & c+a & b\\ c & c & a+b\end{vmatrix}$

$\left[by\, R_1 \to R_1-\left(R_2+R_3\right)\right]$

$=2c.b(a+b-c)-2b.c(b-c-a)=4abc.$

11. Sol (b) by $R_1 \to R_1-R_2$ and $R_2 \to R_2-R_3$

$$\begin{vmatrix}x & -x & 0\\ 0 & x & -x\\ 1 & 1 & 1+x\end{vmatrix}=0 \Rightarrow x^2\begin{vmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ 1 & 1 & 1+x\end{vmatrix}=0$$

$\Rightarrow x^2(x+3)=0$

$\Rightarrow x=0,0,-3$

Trick: Obviously the equation is of *degree* three, therefore it must have three solutions. So check for option (b).

12. Sol (d) $\begin{vmatrix}x+a & b & c\\ b & x+c & a\\ c & a & x+b\end{vmatrix}=0$

$$\Rightarrow (x+a+b+c)\begin{vmatrix}1 & b & c\\ 1 & x+c & a\\ 1 & a & x+b\end{vmatrix}=0,$$

$\Rightarrow x=-(a+b+c)$ is one of the root of the equation.

13. Sol (b) $\Delta=\begin{vmatrix}-1 & -2 & x+4\\ -2 & -3 & x+8\\ -3 & -4 & x+14\end{vmatrix} by \begin{matrix}C_1 \to C_1-C_1\\ C_2 \to C_2-C_3\end{matrix}$

$$=\begin{vmatrix}-1 & -1 & x\\ -2 & -1 & x\\ -3 & -1 & x+2\end{vmatrix}, \begin{vmatrix}by \begin{matrix}C_2 \to & C_2- & C_1\\ C_3 \to & C_3+ & 4C_1\end{matrix}\end{vmatrix}$$

$=-(-x-2+x)+1.(-2x-4+3x)+x(2-3)$

$=2+x-4-x=-2$

Trick: Put $x=1$ then $\begin{vmatrix} 2 & 3 & 5 \\ 4 & 6 & 9 \\ 8 & 11 & 15 \end{vmatrix} = -2$

Note: Since there is a option "None of these", therefore we should check for one more different value of x.
Put x = -1.

$$\begin{vmatrix} 0 & 1 & 3 \\ 2 & 4 & 7 \\ 6 & 9 & 13 \end{vmatrix} = -1(26-42)+3(18-24) = -2$$

Therefore answer is (b).

14. **Sol (c)** $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ vanishes when $a=b, b=c, c=a$.

Hence $(a-b),(b-c),(c-a)$ are factors of Δ. Since Δ is symmetric in a,b,c and of 4th degree $(a+b+c)$ is also factor, so that we can write

$\Delta = k(a-b)(b-c)(c-a)(a+b+c)$ .... (i)

Where by comparing the coefficients of the leading term bc^3 on both the sides of identity (i). We get $1 = k(-1)(-1) \Rightarrow k = 1$

$\therefore \Delta = (a-b)(b-c)(c-a)(a+b+c)$

Trick: Put $a=1, b=2, c=3,$ so that determinant

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 8 & 27 \end{vmatrix} = 1(30)-1(24)+1(8-2) = 12$ which is

given by (c). i.e., $(1+2+3)(1-2)(3-1) = 12.$

15. Sol (b) $f(x) = 2(x-3)(x-5); \begin{vmatrix} 1 & x+3 & 3(x^2+3x+9) \\ 1 & x+5 & 4(x^2+5x+25) \\ 1 & 1 & 3 \end{vmatrix}$

(Taking out $(x-3),(x-5)$ and 2 from Ist row, IInd row and IInd column respectively)

$f(x) = 2(x-3)(x-5)$

$$\begin{vmatrix} 0 & (x+2) & 3(x^2+3x+8) \\ 0 & 2 & x^2+11x+73 \\ 1 & 1 & 3 \end{vmatrix}$$

$[R_1 \to R_1 - R_3 \text{ and} R_2 \to R_2 - R_1]$

$= 2(x-3)(x-5)\left[1(x+2)(x^2+11x+73) - 6(x^2+3x+8)\right]$

$= 2(x^2-8x+15)(x^3+13x^2+95x+146-6x^2-18x-48)$

$= 2(x^2-8x+15)(x^3+7x^2+77x+98)$

$= 2(x^5-x^4+36x^3-413x^2+371x+1470)$

$f(1) = 2928, f(3) = 0, f(5) = 0$

$\therefore f(1).f(3)+f(3).f(5)+f(5).f(1) = 0+0+0 = 0 = f(3)$.

16. Sol (c) $\Delta = (b-a)(b-a)\begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix}$

$= (b-a)^2 \begin{vmatrix} b & b & c \\ a & a & b \\ c & c & a \end{vmatrix} = 0, [\text{by } C_2 \to C_2 + C_3]$

Trick : Put $a = 1, b = 2, c = 3$ and check.

17. **Sol (d)** $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix} = \dfrac{1}{abc}\begin{vmatrix} 1 & a^3 & abc \\ 1 & b^3 & abc \\ 1 & c^3 & abc \end{vmatrix} = \dfrac{abc}{abc}\begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix} = 0$

18. Sol (b) $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)\begin{vmatrix} 2 & 1 & 1 \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$

$[by\, R_1 \to R_1 + R_2 + R_3]$

$= (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ x & z & x \\ x & y & z \end{vmatrix}; \ [\text{by } C_1 \to C_1 - C_2]$

$(x+y+z).\left\{\left(z^2 - xy\right) - \left(xz - x^2\right) + (xy - xz)\right\}$

$= (x+y+z)(x-z)^2 \Rightarrow k = 1$

Trick: Put $x = 1, y = 2, z = 3$ then

$\begin{vmatrix} 5 & 1 & 2 \\ 4 & 3 & 1 \\ 3 & 2 & 3 \end{vmatrix} = 5(7) - 1(12-3) + 2(8-9)$

$= 35 - 92 = 24 \ \& \ (x+y+z)(x-z)^2 = (6)(-2)^2 = 24$

$\therefore k = \dfrac{24}{24} = 1$

19. Sol (a) $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0 \Rightarrow (x+9)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0 \ [by\, R_1 \to R_1 + R_2 + R_3]$

$\Rightarrow (x+9)\left\{\left(x^2 - 12\right) - (2x - 14) + (12 - 7x)\right\} = 0$

$\Rightarrow (x+9)\left(x^2 - 9x + 14\right) = 0$

$\Rightarrow (x+9)(x-2)(x-7) = 0$

Hence, the other two roots are $x = 2, 7$

20. **Sol (d)**It is obvious.

21. Sol (a) $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0 \Rightarrow x(5x - 2x) - 2(2x + x) - 1(4 + 5) = 0$

$\Rightarrow 3x^2 - 6x - 9 = 0 \Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x+1)(x-3) = 0 \Rightarrow x = -1, 3$

22. Sol (a) $\begin{vmatrix} x & x & x \\ x & x & x \\ x & x & x \end{vmatrix} + \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ a & b & c \end{vmatrix} = 0$

As second determinant is zero then either a, *b,* c must be multiple of 2, 3 and 4 or 1, 2, 3. Both of them are in A.P. Hence *a, b, c* must be in A.P. Trick : In such type of problem, put any suitable value of x *i.e.* 0, so that the determinant.

$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ a & b & c \end{vmatrix} = 0$

$\Rightarrow 1(3c-4b)-2(2c-4a)+3(2b-3a)=0$

$\Rightarrow -c+2b-a=0 \Rightarrow 2b=a+c$. Hence the result.

Aliter: Check by option. If a,b,c are in A.P. then

Put $a=1, b=2, c=3$ we get $\Delta=0$

23. Sol (d) $\begin{vmatrix} 4 & -6 & 1 \\ -1 & -1 & 1 \\ -4 & 11 & -1 \end{vmatrix} = \begin{vmatrix} 0 & 5 & 0 \\ -1 & -1 & 1 \\ -4 & 11 & -1 \end{vmatrix} \{\text{Apply } R_1 \to R_1 + R_3\}$

$= -5(1+4) = -25$

24. Sol (b) $\Delta \begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$

$= \begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ 0 & 0 & -(a\alpha^2 2b\alpha+c) \end{vmatrix}, [\text{by } R_3 \to R_3 - aR_1 - R_2]$

$= a\{-c(a\alpha^2+2b\alpha+c)-0\} - b\{-b(a\alpha^2+2b\alpha+c)-0\}$

by expanding along C_1

$= (b^2-ac)(a\alpha^2+2b\alpha+c)$

Thus, $\Delta=0$, if either $b^2-ac=0$ or $a\alpha^2+2b\alpha+c=0$

i.e., a,b,c in G.P. or $a\alpha^2+2b\alpha+c=0$

Trick: Put $\alpha=0$, then the determinant

25. **Sol (d)**Applying $C_2 \to C_2 + C_3$

$\begin{vmatrix} 4+x^2 & -6 & -2 \\ -6 & 9+x^2 & 3 \\ -2 & 3 & 1+x^2 \end{vmatrix} = x^4(14+x^2) = x, x^3(14+x^2)$

Hence, the determinant is divisible by x, x^3 and $(14+x^2)$

but not divisible by x^5

26. **Sol (d)** $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -1 \end{vmatrix} = 0 \Rightarrow k = \frac{33}{8}$

27. Sol (d) $\begin{vmatrix} a-1 & a & bc \\ b-1 & b & ca \\ c-1 & c & ab \end{vmatrix} = \begin{vmatrix} a & a & bc \\ b & b & ca \\ c & c & ab \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

$= -\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = -\begin{vmatrix} a & a^2 & 1 \\ b-a & b^2-a^2 & 0 \\ c-a & c^2-a^2 & 0 \end{vmatrix}$

[by $R_2 \to R_2 - R_1; R_3 \to R_3 - R_1$]

$= -(a-b)(b-c)(c-a)$

28. **Sol (a)** $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix} = \begin{vmatrix} 25 & 21 & 219 \\ 15 & 27 & 198 \\ 21 & 17 & 181 \end{vmatrix}$

[Applying $C_1 \to C_1 - C_2; C_3 \to C_3 - 10C_2$]

$= \begin{vmatrix} 4 & 21 & 9 \\ -12 & 27 & -72 \\ 0 & -4 & 2 \end{vmatrix}$ [Applying $R_3 \to R_3 - R_1$]

$= 4\begin{vmatrix} 1 & 2 & 9 \\ -3 & 3 & -72 \\ 0 & -4 & 2 \end{vmatrix} 4 \begin{vmatrix} 1 & 21 & 9 \\ 0 & 90 & -45 \\ 0 & -4 & 2 \end{vmatrix}$ [by $R_3 \to 3R_1 + R_2$]

$= 4(90 \times 2 - 45 \times 4) = 0$

29. Sol (b) $\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix} = \begin{vmatrix} -1 & -1 & 443 \\ -1 & -1 & 447 \\ -1 & -1 & 451 \end{vmatrix} = 0$

$[C_1 \to C_1 - C_2, C_2 C_2 \to C_2 - C_3]$

30. **Sol (a)**We have $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$

$= pqr\left(a^3 + b^3 + c^3\right) - abc\left(p^3 + q^3 + r^3\right)$

$= pqr(3abc) - abc(3pqr) = 0,$

$\begin{bmatrix} \because p + q + r = 0, \therefore p^3 + q^3 + r^3 = 3pqr \\ \because a + b + c = 0, \therefore a^3 + b^3 + c^3 = 3abc \end{bmatrix}$

31. Sol (c) $\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ b+c & c+a & a+b \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ c(b-a) & a(c-b) & ab \\ b-a & c-b & a+b \end{vmatrix}$

$[C_1 \to C_1 - C_2, C_2 \to C_2 - C_3]$

$= (b-a)(c-b)\begin{vmatrix} 0 & 0 & 1 \\ c & a & ab \\ 1 & 1 & a+b \end{vmatrix} = (b-a)(c-a)(c-b)$

$= (a-b)(b-c)(c-a).$

Trick: Put $a = 1, b = 2, c = 3$ and check by option.

32.

Sol (b) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} + \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix} k \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$$\Rightarrow 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = k\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \Rightarrow k = 2$$

33. **Sol (a)**Since determinant of a skew-symmetric matrix of cdd order is zero

34. **Sol (d)** Apply $C_1 \to C_1 + C_3$ and take $x+y+z$ common from C_1 and 4 from C_2 to make first two columns identical.

35. **Sol (b)**We have $\Delta = \begin{vmatrix} -x & 1 & 0 \\ 1 & -x & 1 \\ 0 & 1 & -x \end{vmatrix}$

$$\Delta = -x\left(x^2 - 1\right) - 1(-x - 0) = 0 \Rightarrow x = \pm\sqrt{2}$$

36. **Sol (c)** Applying $C_1 \to C_1 + C_2 + C_3$, we obtain

$$-x\begin{vmatrix} 1 & -6 & 3 \\ 0 & 3-x & 3 \\ 0 & 3 & -6-x \end{vmatrix} = 0$$

$$\Rightarrow -x\begin{vmatrix} 1 & -6 & 3 \\ 0 & 9-x & 0 \\ 0 & 9 & -9-x \end{vmatrix} = 0$$

$$\Rightarrow -x(9-x)(9-x) = 0 \Rightarrow x = 0, 9, -9$$

Trick: Check by assuming the value of x from the given options.

37. **Sol (a)** Applying $C_1 \to C_1 + C_2$, we get

$$\begin{vmatrix} 1 & \cos^2 x & 1 \\ 1 & \sin^2 x & 1 \\ 2 & 12 & 2 \end{vmatrix} = 0$$

38. **Sol (b)** we have $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x+1 & 1 & 1 \\ x+1 & x-1 & 1 \\ x+1 & 1 & x-1 \end{vmatrix} = 0$$

$$\left[Applying\, C_1 \to C_1 + C_2 + C_3\right]$$

$$\Rightarrow (x+1)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow (x+1)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x-2 & 0 \\ 0 & 1 & x-2 \end{vmatrix} = 0$$

$$\left[Applying\, R_2 \to R_2 - R_1, R_3 \to R_3 - R_1\right]$$

$\Rightarrow (x+1)(x-2)^2 = 0 \Rightarrow x = -1, 2$

39. Sol (d) $D_1 = \begin{vmatrix} 1 & 15 & 8 \\ 1 & 35 & 9 \\ 1 & 25 & 10 \end{vmatrix} D_2 = \begin{vmatrix} 2 & 15 & 8 \\ 4 & 35 & 9 \\ 8 & 25 & 10 \end{vmatrix}$

$D_3 = \begin{vmatrix} 3 & 15 & 8 \\ 9 & 35 & 9 \\ 27 & 25 & 10 \end{vmatrix} D_4 = \begin{vmatrix} 4 & 15 & 8 \\ 16 & 35 & 9 \\ 64 & 25 & 10 \end{vmatrix}$

$D_5 = \begin{vmatrix} 5 & 15 & 8 \\ 25 & 35 & 9 \\ 125 & 25 & 10 \end{vmatrix}$

$\Rightarrow D_1 + D_2 + D_3 + D_4 + D_5 = \begin{vmatrix} 15 & 15 & 8 \\ 55 & 35 & 9 \\ 225 & 25 & 10 \end{vmatrix}$

$= 15(125) + 15(1475) - 8(6500)$

$= 1875 + 22125 - 52000 = -28000$

40. **Sol (b)** Operating $C_1 \to C_1 + C_2 + C_3$ We get the value of

Given determinant as $\begin{vmatrix} 3a+3b & a+b & a+2b \\ 3a+3b & a & a+b \\ 3a+3b & a+2b & a \end{vmatrix}$

$= 3(a+b)\begin{vmatrix} 1 & a+b & a+2b \\ 1 & a & a+b \\ 1 & a+2b & a \end{vmatrix}$

Operate $R_3 \to R_3 - R_1, R_2 \to R_2 - R_1$

$= 3(a+b)\begin{vmatrix} 1 & a+b & a+2b \\ 0 & -b & -b \\ 0 & b & -2b \end{vmatrix}$

$= 3(a+b)\left(2b^2 + b^2\right) = 9b^2(a+b)$

Trick: Put $a = 1, b = 2, c = 3$ and check by options.

41. **Sol (b)** $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} - \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$

$\Rightarrow abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$

$\Rightarrow (abc-1)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$

Since a,b,c are different, so $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$

Hence, $abc - 1 = 0$ i.e., $abc = 1$

42. **Sol (d)** $\because$ Given determinant reduces to $(x-1)(6x-38) = 0$

$\Rightarrow x = 1, 19/3$

43. **Sol (d)** Let $A = \begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & a+b \\ x+6 & x+7 & x+c \end{vmatrix}$

$[\text{Applying } C_2 \to C_2 - C_1, \text{we get}]$

$\Rightarrow A = \begin{vmatrix} x+2 & 1 & x+a \\ x+4 & 1 & x+b \\ x+6 & 1 & x+c \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow A = \begin{vmatrix} x+2 & 1 & x+a \\ 2 & 0 & b-a \\ 4 & 0 & c-a \end{vmatrix} = -1(2c - 2a - 4b + 4a)$

$= 2(2b - c - a)$

$\because a,b,c,$ are in A.P. $\Rightarrow A = 0$

Trick: Let a,b,c are in A.P. Put $a = 4, b = 5, c = 6$

$\begin{vmatrix} x+2 & x+3 & x+4 \\ x+4 & x+5 & x+6 \\ x+6 & x+7 & x+8 \end{vmatrix}$

$\begin{vmatrix} x+2 & 1 & 1 \\ x+4 & 1 & 1 \\ x+6 & 1 & 1 \end{vmatrix} = 0 [C_2 \to C_2 - C_1, C_3 \to C_3 - C_2]$

44. Sol (b) $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_2$

$\begin{vmatrix} x+1 & x+2 & x+a \\ 1 & 1 & (b-a) \\ 1 & 1 & (c-b) \end{vmatrix} = 0$

$C_1 \to C_1 - C_2$

$\Rightarrow \begin{vmatrix} -1 & x+2 & x+a \\ 0 & 1 & (b-a) \\ 0 & 1 & (c-b) \end{vmatrix} = 0$

$\Rightarrow -1(c - b - b + a) = 0 \Rightarrow 2b = a + c$

$\Rightarrow a,b,c$ are in A.P.

45. Sol (b) $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix} \{Operate\, R_3 \to R_3 - R_2, R_2 \to R_2 - R_1\}$

$= \begin{vmatrix} 1 & 4 & 9 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{vmatrix} = 1(45-49) - 4(27-35) + 9(21-25)$

$= -4 + 32 - 36 = -8$

46. **Sol (b)**Applying $R_3 \to R_3 - R_1 - R_2$

$\begin{vmatrix} a & b & a+b \\ b & c & b+c \\ 0 & 0 & -a-2b-c \end{vmatrix} = 0$

$\Rightarrow (-a-2b-c)\left(ac - b^2 0\right) = 0$

$\Rightarrow ac = b^2 \; or\; a + 2b + c = 0$

$\Rightarrow a, b, c$ are in G.P.

47. **Sol (b)**B is obtained from A by the operations

$R_1 \leftrightarrow R_3, R_3 \to 2R_3 \,\text{and}\, R_2 \to 2R_2$

Hence, $B = (-1)4A = -4A$

48. Sol (a) $A = \begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix}$

$|A| = 0$ for $x = 1$ and 2. So option (a) is correct.

49. **Sol (b)**Taking out 5 from R_2 makes $R_2 = R_1$

Q The value of determinant is equal to zero.

50. **Sol (c)**Given $\Delta \begin{vmatrix} 41 & 42 & 43 \\ 44 & 45 & 46 \\ 47 & 48 & 49 \end{vmatrix}$

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$ we get

$\Delta \begin{vmatrix} 41 & 1 & 1 \\ 44 & 1 & 1 \\ 47 & 1 & 1 \end{vmatrix}$. Since two columns $(C_2 \, and \, C_3)$ are identical, therefore $\Delta = 0$

51. **Sol (a)** On expanding, $-a(b-c) + 2b(b-c) + (a-b)(b-2c) = 0$

$\Rightarrow -ab + ac + 2b^2 - 2bc + ab - 2ac - b^2 + 2bc = 0$

$\Rightarrow b^2 - ac = 0 \Rightarrow b^2 = ac$

Exercise-02

Determinants

Minor, Co-factor, Product, Linear Equation

1. The co-factor of the element '4' in the determinant $\begin{vmatrix} 1 & 3 & 5 & 1 \\ 2 & 3 & 4 & 2 \\ 8 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \end{vmatrix}$ is

(a) 4 (b) 10 (c) -10 (d) -4

2. The value of $\begin{vmatrix} x+y & y+z & z+x \\ x & y & z \\ x-y & y-z & z-x \end{vmatrix} =$

(a) $2(x+y+z)^2$

(b) $2(x+y+z)^3$

(c) $(x+y+z)^3$

(d) 0

3. If $\Delta_1 = \begin{vmatrix} 1 & 0 \\ a & b \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 1 & 0 \\ c & d \end{vmatrix}$, then $\Delta_2\Delta_1$ equal to

(a) ac (b) bd (c) $(b-a)(d-c)$ (d) None of these

4. $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix} =$

(a) 7 (b) 10 (c) 13 (d) 17

5. If $\begin{vmatrix} 5 & 6 & 3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{vmatrix}$, then co-factor of the element of 2nd row are

(a) $39,-3,11$ (b) $-39,3,11$
(c) $-39,27,11$ (d) $-39,-3,11$

6. The minors of - 4 and 9 and the co-factors of - 4 and 9 in determinant $\begin{vmatrix} -1 & -2 & 3 \\ -4 & -5 & -6 \\ -7 & 8 & 9 \end{vmatrix}$ are respectively

(a) $42,3;-42,3$
(b) $-42,-3;42,-3$
(c) $42,3;-42,-3$
(d) $42,3;42,3$

7. The following system of equation $3x-2y+z=0, \lambda x-14y+15z=0, x+2y-3z=0$ has a solution other than $x=y=z=0$ for λ equal to

(a) 1 (b) 2 (c) 3 (d) 5

8. $x+ky-z=0, 3x-ky-z=0$ and $x-3y+z=0$ has non-zero solution for k =

(a) -1 (b) 0 (c) 1 (d) 2

9. The system of equations

$\alpha x+y+z=\alpha-1$

$x+\alpha y+z=\alpha-1$

$x+y+\alpha z=\alpha-1$

has no solution, if α is

(a) Not -2 (b) 1 (c) -2 (d) Either -2 or 1

10. The number of solution of the equations $x+4y-z=0,\ 3x-4y-z=0,\ x-3y+z=0$ is

(a) 0 (b) 1 (c) 2 (d) Infinite

11. If $a_1x+b_1y+c_1z=0,\ a_2x+b_2y+c_2z=0,\ a_3x+b_3y+c_3z=0$ and $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$

(a) One trivial and non - trivial solution
(b) No solution
(c) One solution
(d) Infinite solution

12. If the system of equations, $x+2y-3z=1, (k+3)z=3,$
$(2k+1)x+z=0$ is inconsistent, then the value of k is

(a) -3 (b) $1/2$ (c) 0 (d) 2

13. If the system of equation $x-ky-z=0,\ kx-y-z=0$ and $x+y-z=0$ has a non zero solution, then the possible value of k are

(a) $-1,2$ (b) $1,2$ (c) $0,1$ (d) $-1,1$

14. Set of equation $a+b-2c=0,\ 2a-3b+c=0,$ and $a-5b+4c=\alpha$ is consistent for α equal to

(a) 1 (b) 0 (c) -1 (d) 2

15. If the system of linear equation $x+2ay+az=0,\ x+3by+bz=0,\ x+4cy+cz=0$ has a non zero solution, then a, b, c
(a) Are in A.P (b) Are in G.P. (c) Are in H.P. (d) Satisfy $a+2b+3c=0$

16. If a,b,c are respectively the p^{th}, q^{th}, r^{th} terms of an A.P., then $\begin{vmatrix} a & p & 1 \\ b & q & 1 \\ c & r & 1 \end{vmatrix} =$
(a) 1 (b) -1 (c) 0 (d) pqr

17. The system of linear equations $x+y+z=2,\ 2x+y-z=3,\ 3x+2y+kz=4$ has a unique solution if
(a) $k \neq 0$ (b) $-1<k<1$ (c) $-2<k<2$ (d) $k=0$

18. The system of equations $x_1-x_2+x_3=2,\ 3x_1-x_2+2x_3=-6$ and $3x_1+x_2+x_3=-18$ has
(a) No solution` (b) Exactly one solution (c) Infinite solutions (d) None of these

19. The system of equations $x+y+z=2,\ 3x-y+2z=6$ and $3x+y+z=-18$ has
(a) A unique solution
(b) No solutions
(c) An infinite number of solutions
(d) Zero solution as the only solution

20. For What value of λ, the system of equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=12$ is inconsistent
(a) $\lambda=1$ (b) $\lambda=2$ (c) $\lambda=-2$ (d) $\lambda=3$

21. If x is a positive integer, then $\Delta = \begin{vmatrix} x! & (x+1)! & (x+2)! \\ (x+1)! & (x+2)! & (x+3)! \\ (x+2)! & (x+3)! & (x+4)! \end{vmatrix}$ is equal to
(a) $2(x!)(x+1)!$ (b) $2(x!)(x+1)!(x+2)!$
(c) $2(x!)(x+3)!$ (d) None of these

22. If the system of equations $x+ay=0,\ az+y=0$ and $ax+z=0$ has infinite solutions, then the value of a is
(a) -1 (b) 1 (c) 0 (d) No real values

23. The value of x,y,z in order of the system of equations $3x+y+2z=3,\ 2x-3y-z=-3,\ x+2y+z=4$, are
(a) $2,1,5$ (b) $1,1,1$ (c) $1,-2,-1$ (d) $1,2,-1$

24. The value of λ for which the system of equations $2x-y-z=12,\ x-2y+z=-4,\ x+y+\lambda z=4$ has no solution is
(a) 3 (b) -3 (c) 2 (d) -2

25. The system of equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$, has no solution for
(a) $\lambda \neq 3, \mu=10$ (b) $\lambda=3, \mu \neq 10$ (c) $\lambda \neq 3, \mu \neq 10$ (d) None of these

Exercise-02

Solution

1.

Sol **(b)** The co - factor of element 4, in the 2^{nd} row and 3^{rd} column is

$$=(-1)^{2+3}\begin{vmatrix}1 & 3 & 1\\ 8 & 0 & 1\\ 0 & 2 & 1\end{vmatrix}=-\{1(-2)-3(8-0)+1.16\}=10$$

2. **Sol** **(d)** $\begin{vmatrix}x+y & y+z & z+x\\ x & y & z\\ x-y & y-z & z-x\end{vmatrix}=2\begin{vmatrix}x & y & z\\ x & y & z\\ x-y & y-z & z-x\end{vmatrix}$

$[\text{applying } R_1\to R_1+R_3]$

$=0$

3. Sol **(b)** $\Delta_2\Delta_1=\begin{vmatrix}1 & 0\\ c & d\end{vmatrix}\begin{vmatrix}1 & 0\\ a & b\end{vmatrix}$

$\begin{vmatrix}1 & 0\\ c+ad & bd\end{vmatrix}=bd$

4. **Sol** **(b)** $\begin{vmatrix}\log_3 512 & \log_4 3\\ \log_3 8 & \log_4 9\end{vmatrix}\times\begin{vmatrix}\log_2 3 & \log_8 3\\ \log_3 4 & \log_3 4\end{vmatrix}$

$$=\left(\frac{\log 512}{\log 3}\times\frac{\log 9}{\log 4}-\frac{\log 3}{\log 4}\times\frac{\log 8}{\log 3}\right)$$

$$\times\left(\frac{\log 3}{\log 2}\times\frac{\log 4}{\log 3}-\frac{\log 3}{\log 8}\times\frac{\log 4}{\log 3}\right)$$

$$=\left(\frac{\log 2^9}{\log 3}\times\frac{\log 3^2}{\log 2^2}-\frac{\log 2^3}{\log 2^2}\right)\times\left(\frac{\log 2^2}{\log 2}-\frac{\log 2^2}{\log 2^3}\right)$$

$$=\left(\frac{9\times 2}{2}-\frac{3}{2}\right)\left(2-\frac{2}{3}\right)=\frac{15}{2}\times\frac{4}{3}=10$$

5.Sol **(c)** $C_{21}=(-1)^{2+1}(18+21)=-39$

$C_{22}=(-1)^{2+2}(15+12)=27$

$C_{23}=(-1)^{2+3}(-35+24)=11$

6.**Sol** **(b)** Minor of $-4=\begin{vmatrix}-2 & 3\\ 8 & 9\end{vmatrix}=-42$

Minor of $9=\begin{vmatrix}-1 & -2\\ -4 & -5\end{vmatrix}=-3$

and cofactor of $-4=(-1)^{2+1}(-42)=42,$

cofactor of $9=(-1)^{3+3}(-3)=-3.$

7.**Sol** **(d)** The system of equations has infinitely many (non - trivial) solution, if

$$\Delta = 0 \; i.e., \text{ if } \begin{vmatrix} 3 & -2 & 1 \\ \lambda & -14 & 15 \\ 1 & 2 & -3 \end{vmatrix} = 0$$

$$\Rightarrow 3(42-30) - \lambda(6-2) + 1(-30+14) = 0 \Rightarrow \lambda = 5$$

8.**Sol (c)** It has a non - zero solution if

$$\begin{vmatrix} 1 & k & -1 \\ 3 & -k & -1 \\ 1 & -3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow -6k + 6 = 0 \Rightarrow k = 1.$$

9.**Sol (c)** For no solution or infinitely many solutions

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0 \Rightarrow \alpha = 1, \alpha = -2.$$

But for $\alpha = 1$, dearly there are infinitely many solutions and when we put $\alpha = -2$, in given system of equations and adding them together $L.H.S. \neq R.H.S.$ i.e, No solution.

10. **Sol (b)** The given system of homogeneous equations has

$$\Delta = \begin{vmatrix} 1 & 4 & -1 \\ 3 & -4 & -1 \\ 1 & -3 & 1 \end{vmatrix} = 1(-4-3) - 4(3+1) - 1(-9+4)$$

$$= -7 - 16 + 5 \neq 0$$

There exists only one trivial solution.

11. **Sol (d)** It is based on fundamental concept.

12. **Sol (a)** For the equations to be inconsistent $D = 0$

$$\therefore D = \begin{vmatrix} 1 & 2 & -3 \\ 0 & 0 & k+3 \\ 2k+1 & 0 & 1 \end{vmatrix} = 0 \Rightarrow k = -3$$

and $D_1 = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \neq 0$

13. **Sol (d)** For non trivial solution $\Delta = 0$

$$\Rightarrow \begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0 \Rightarrow k = 1. -1$$

14. **Sol (b)** $a + b - 2c = 0$

$2a - 3b + c = 0$

$a - 5b + 4c = \alpha$

System is consistent, if $D = \begin{vmatrix} 1 & 1 & -2 \\ 2 & -3 & 1 \\ 1 & -5 & 4 \end{vmatrix} = 0$

and $D_1 = \begin{vmatrix} 0 & 1 & -2 \\ 0 & -3 & 1 \\ \alpha & -5 & 4 \end{vmatrix} = 0$ and D_2 also zero.

Hence, value of α is zero.

15. **Sol (c)** $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0,$ $\quad [C_2 \to C_2 - 2C_3]$

$= \begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} = 0,$ $\quad [R_3 \to R_3 - R_2, R_2 \to R_2 - R_1]$

$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c-b & c-b \end{vmatrix} = 0;$

$b(c-b) - (b-a)(2c-b) = 0$

On simplification, $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore a, b, c$ are in Harmonic progression.

16. **Sol (c)** Let first term = A and common difference $= D$

$\therefore a = A + (p-1)D, c = A + (r-1)D$

$\begin{vmatrix} a & p & 1 \\ b & q & 1 \\ c & r & 1 \end{vmatrix} = \begin{vmatrix} A+(p-1)D & p & 1 \\ A+(q-1)D & q & 1 \\ A+(r-1)D & r & 1 \end{vmatrix}$

$[Opreate\ C_1 \to C_1 - DC_2 + DC_3]$

$= \begin{vmatrix} A & p & 1 \\ A & q & 1 \\ A & r & 1 \end{vmatrix} = A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} = 0$

17. **Sol (a)** The given system of equation has a unique solution if

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \Rightarrow k \neq 0$

18. Sol (c) $D = \begin{vmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = 1[-1-2] - 1[6-3] + 1[3+3] = 0$

and $D_1 = \begin{vmatrix} 2 & -1 & 1 \\ -6 & -1 & 2 \\ -18 & 1 & 1 \end{vmatrix} = 2[-1-2] - 1[36+6] + 1[-6-18]$

$= -6 + 30 - 24 = 0$

Also, $= -6 + 30 - 24 = 0$

So the system is consistent $(D = D_1 = D_2 = D_3 = 0)$

i.e., system has infinite solution .

19. **Sol (a)** Given system of equation can be written as

$$\begin{bmatrix}1 & 1 & 1\\3 & -1 & 2\\3 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\6\\-18\end{bmatrix}$$

As, $\begin{bmatrix}1 & 1 & 1\\3 & -1 & 2\\3 & 1 & 1\end{bmatrix}\neq 0$ So, unique solution exists.

20. **Sol (d)** Given system will be inconsistent when $D=0$

$$\begin{vmatrix}1 & 1 & 1\\1 & 2 & 3\\1 & 2 & \lambda\end{vmatrix}=0$$

Applying $C_1 \to C_1$ and $C_2 \to C_2 - C_3$

$$\Rightarrow \begin{vmatrix}0 & 0 & 1\\-1 & -1 & 3\\-1 & 2-\lambda & \lambda\end{vmatrix}=0 \Rightarrow -1(2-\lambda)-1=0 \Rightarrow \lambda=3.$$

21. **Sol (b)** $\nabla = \begin{vmatrix}x! & (x+1)! & (x+2)!\\(x+1)! & (x+2)! & (x+3)!\\(x+2)! & (x+3)! & (x+4)!\end{vmatrix}$

$$= x!(x+1)!(x+2)!\begin{vmatrix}1 & (x+1) & (x+2)(x+1)\\1 & (x+2) & (x+3)(x+2)\\1 & (x+3) & (x+4)(x+3)\end{vmatrix}$$

Applying $R_1 \to R_2 - R_1 R_2 \to (R_3 - R_2)$ we get

$$= x!(x+1)!(x+2)!\begin{vmatrix}0 & 1 & 2(x+2)\\0 & 1 & 2(x+3)\\1 & (x+3) & (x+4)(x+3)\end{vmatrix}$$

$= 2x!(x+1)!(x+2)!$. (on simplification)

Trick: Put $x=1$ and then match the ultimate.

22. **Sol (a)** $\begin{vmatrix}1 & a & 0\\0 & 1 & a\\a & 0 & 1\end{vmatrix}=0 \Rightarrow 1+a\left(a^2\right)=0 \Rightarrow a^3=-1 \Rightarrow a=-1.$

23. **Sol (d)** Put the value $(x,y,z)=(1,2,-1)$, which satisfies equation. Hence,(d) is correct.

24. **Sol (d)** The coefficient determinant $D=\begin{vmatrix}2 & -1 & -1\\1 & -2 & 1\\1 & 1 & \lambda\end{vmatrix}$

$=-3\lambda-6$

For no solution, the necessary condition is $D=0$

i.e., $-3\lambda-6=0 \Rightarrow \lambda=-2$

It can be seen that for $\lambda=-2$, there is no solution the given system or equation.

25. **Sol (b)** This will have no solution, if $\nabla=0$ which gives $\lambda=3$. Also at least $\nabla x, \nabla y, \nabla z \neq 0$ which gives $\mu \neq 10$

Exercise-03

Matrices

Types and Algebra

1. In a skew symmetric matrix, the diagonal elements are all

(a) Different from each other (b) Zero

(c) One (d) None of these

2. If $M=\begin{bmatrix}1 & 2\\ 2 & 3\end{bmatrix}$ and $M^2-\lambda M-I_2=0$, then $\lambda=$

(a) -2 (b) 2 (c) -4 (d) 4

3. If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ and $B=\begin{bmatrix}\cos\beta & -\sin\beta\\ \sin\beta & \cos\beta\end{bmatrix}$, then the correct relation is

(a) $A^2=B^2$ (b) $A+B=B-A$

(c) $AB=BA$ (d) None of these

4. If A $=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 0 & 0\end{bmatrix}$, than A is

(a) Symmetric (b) Skew Symmetric (c) Non - singular (d) Singular

5. If A = $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ a & b & -1\end{bmatrix}$, then $A^2=$

(a) Unit matrix (b) Null matrix (c) A (d) $-A$

6. If $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ then $A^n=$

(a) $\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}n & n\\ 0 & n\end{bmatrix}$

(c) $\begin{bmatrix}n & 1\\ 0 & n\end{bmatrix}$ (d) $\begin{bmatrix}1 & 1\\ 0 & n\end{bmatrix}$

7. If $2A+3B=\begin{bmatrix}2 & -1 & 4\\ 3 & 2 & 5\end{bmatrix}$ and $A+2B=\begin{bmatrix}5 & 0 & 3\\ 1 & 6 & 2\end{bmatrix}$, then $B=$

(a) $\begin{bmatrix}8 & -1 & 2\\ 1 & 10 & -1\end{bmatrix}$ (b) $\begin{vmatrix}8 & 1 & 2\\ -1 & 10 & -1\end{vmatrix}$

(c) $\begin{bmatrix}8 & 1 & -2\\ -1 & 10 & -1\end{bmatrix}$ (d) $\begin{bmatrix}8 & 1 & 2\\ 1 & 10 & 1\end{bmatrix}$

8. If $[m\ \ n]\begin{bmatrix}m\\ n\end{bmatrix}=[25]$ and $m>n$, then $(m,n)=$

(a) $(2,3)$ (b) $(3,4)$ (c) $(4,3)$ (d) None of these

9. If the matrix $\begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ is singular, then λ

(a) -2 (b) 4 (c) 2 (d) -4

10. If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ and $A^n = O$, then the minimum value of n is

(a) 2 (b) 3 (c) 4 (d) 5

11. If $A = \begin{bmatrix} 1/3 & 2 \\ 0 & 2x-3 \end{bmatrix}, B = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ and $AB = I$, then $x =$

(a) -1 (b) 1 (c) 0 (d) 2

12. If $A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$, then for what value of $\lambda, A^2 = O$

(a) 0 (b) ± 1 (c) -1 (d) 1

13. If $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 5 \\ 2 & -5 & 0 \end{bmatrix}$, then

(a) $A' = A$ (b) $A' = -A$ (c) $A' = 2A$ (d) None of these

14. If A^T, B^T are transpose matrices of the square matrices A, B respectively, then $(AB)^T$ is equal to

(a) $A^T B^T$ (b) AB^T (c) BA^T (d) $B^T A^T$

15. If $A = [1 \quad 2 \quad 3]$ and $B = \begin{bmatrix} -5 & 4 & 0 \\ 0 & 2 & -1 \\ 1 & -3 & 2 \end{bmatrix}$, then $AB =$

(a) $\begin{bmatrix} -5 & 4 & 0 \\ 0 & 4 & -2 \\ 3 & -9 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

(c) $[-2 \quad -1 \quad 4]$ (d) $\begin{bmatrix} -5 & 8 & 0 \\ 0 & 4 & -3 \\ 1 & -6 & 6 \end{bmatrix}$

16. If $\begin{bmatrix} 2 & -3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & -5 \end{bmatrix}$, then $[a,b,c,d] =$

(a) $(1,6,2,5)$ (b) $(1,2,7,5)$

(c) $(1,2,-7,5)$ (d) $(-1,-2,7,-5)$

17. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then $A^2 - 4A$ is equal to

(a) $2I_3$ (b) $3I_3$ (c) $4I_3$ (d) $5I_3$

18. If $A=\begin{bmatrix}0&2&0\\0&0&3\\-2&2&0\end{bmatrix}$ and $B=\begin{bmatrix}1&2&3\\3&4&5\\5&-4&0\end{bmatrix}$, then the element of third row and third column in AB will be

(a) -18 (b) 4 (c) -12 (d) None of these

19. If $A=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$, then $A^5=$

(a) $5A$ (b) $10A$ (c) 16A (d) 32A

20. If $A=\begin{bmatrix}0&1\\0&0\end{bmatrix}$ and $AB=O$, then $B=$

(a) $\begin{bmatrix}1&1\\1&1\end{bmatrix}$ (b) $\begin{bmatrix}0&1\\-1&0\end{bmatrix}$

(c) $\begin{bmatrix}0&-1\\1&0\end{bmatrix}$ (d) $\begin{bmatrix}-1&0\\0&0\end{bmatrix}$

21. If A and B are square matrices of order 2,then $(A+B)^2=$

(a) $A^2+2AB+B^2$ (b) $A^2+AB+BA+B^2$

(c) $A^2+2BA+B^2$ (d) None of these

22. If $\begin{bmatrix}1&0&0&0\\2&3&0&0\\4&5&6&0\\7&8&9&10\end{bmatrix}$, then A is

(a) An upper triangular matrix (b) A null matrix

(c) A lower triangular matrix (d) None of these

23. If A = $\text{diag}(2,-1,3)$, B = $\text{diag}(-1,3,2)$, then $A^2B=$

(a)$\text{diag}(5,4,11)$ (b)$\text{diag}(-4,3,18)$ (c) $\text{diag}(3,1,8)$ (d) B

24. $\cos\theta\begin{bmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta&-\cos\theta\\\cos\theta&\sin\theta\end{bmatrix}=$

(a) $\begin{bmatrix}0&0\\0&0\end{bmatrix}$ (b) $\begin{bmatrix}1&0\\0&0\end{bmatrix}$

(c) $\begin{bmatrix}0&1\\1&0\end{bmatrix}$ (d) $\begin{bmatrix}1&0\\0&1\end{bmatrix}$

25. If I is a unit matrix, then 3I will be

(a) A unit matrix (b) A triangular matrix

(c) A scalar matrix (d) None of these

26. If $A=\begin{bmatrix}1&a\\0&1\end{bmatrix}$, then A^4 is equal to

(a) $\begin{bmatrix}1&a^4\\0&1\end{bmatrix}$ (b) $\begin{bmatrix}4&4a\\0&4\end{bmatrix}$

(c) $\begin{bmatrix} 4 & a^4 \\ 0 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4a \\ 0 & 1 \end{bmatrix}$

27. If $\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix} X = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$, then $X =$

(a) $\begin{bmatrix} -3 & 4 \\ 14 & -13 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -4 \\ -14 & 13 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 4 \\ 14 & 13 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 4 \\ -14 & 13 \end{bmatrix}$

28. The order of $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is

(a) 3×1 (b) 1×1

(c) 1×3 (d) 3×3

29. If A and B are two matrices and $(A+B)(A-B) = A^2 - B^2$, then

(a) $AB = BA$ (b) $A^2 + B^2 = A^2 - B^2$

(c) $A'B' = AB$ (d) None of these

30. $A = \begin{bmatrix} 5 & -3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -4 \\ 3 & 6 \end{bmatrix}$, then $A - B =$

(a) $\begin{bmatrix} 11 & -7 \\ 5 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 1 \\ -1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 11 & 7 \\ 5 & 10 \end{bmatrix}$ (d) $\begin{bmatrix} 12 & -7 \\ 5 & -10 \end{bmatrix}$

31. The value of x for which the given matrix $\begin{bmatrix} -x & x & 2 \\ 2 & x & -x \\ x & -2 & -x \end{bmatrix}$ will be non-singular are

(a) $-2 \le x \le 2$ (b) For all x other than 2 and -2

(c) $x \ge 2$ (d) $x \le -2$

32. If A and B are square matrices of same order , then

(a) $A + B = B + A$ (b) $A + B = A - B$

(c) $A - B = B - A$ (d) $AB = BA$

33. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $A^2 =$

(a) $\begin{bmatrix} 8 & -5 \\ -5 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & -5 \\ 5 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & -5 \\ -5 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

34. If $A=\begin{bmatrix}1&3&0\\-1&2&1\\0&0&2\end{bmatrix}, B=\begin{bmatrix}2&3&4\\1&2&3\\-1&1&2\end{bmatrix}$, then $AB=$

(a) $\begin{bmatrix}5&9&13\\-1&2&4\\-1&2&4\end{bmatrix}$
(b) $\begin{bmatrix}5&9&13\\-1&2&4\\-2&2&4\end{bmatrix}$
(c) $\begin{bmatrix}1&2&4\\-1&2&4\\-2&2&4\end{bmatrix}$
(d) None of these

35. If $A=\begin{bmatrix}1&1\\0&1\end{bmatrix}, B=\begin{bmatrix}0&1\\1&0\end{bmatrix}$ then $AB=$

(a) $\begin{bmatrix}0&0\\0&0\end{bmatrix}$
(b) $\begin{bmatrix}1&1\\1&0\end{bmatrix}$
(c) $\begin{bmatrix}1&0\\0&1\end{bmatrix}$
(d) $\begin{bmatrix}1&1\\0&1\end{bmatrix}$

36. If $A=\begin{bmatrix}i&0\\0&i\end{bmatrix}$ and $B=\begin{bmatrix}0&-i\\-i&0\end{bmatrix}$, then $(A+B)(A-B)$ is equal to

(a) A^2-B^2
(b) A^2+B^2
(c) $A^2-B^2+BA+AB$
(d) None of these

37. If $A=\begin{bmatrix}1&-2\\3&0\end{bmatrix}, B=\begin{bmatrix}-1&4\\2&3\end{bmatrix}, C=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$ then $5A-3B-2C=$

(a) $\begin{bmatrix}8&20\\7&9\end{bmatrix}$
(b) $\begin{bmatrix}8&-20\\7&-9\end{bmatrix}$
(c) $\begin{bmatrix}-8&20\\-7&9\end{bmatrix}$
(d) $\begin{bmatrix}8&7\\20&-9\end{bmatrix}$

38. If $\begin{bmatrix}x&0\\1&y\end{bmatrix}+\begin{bmatrix}-2&1\\3&4\end{bmatrix}=\begin{bmatrix}3&5\\6&3\end{bmatrix}-\begin{bmatrix}2&4\\2&1\end{bmatrix}$, then

(a) $x=-3, y=-2$
(b) $x=3, y=-2$
(c) $x=3, y=2$
(d) $x=-3, y=2$

39. If $A=\begin{bmatrix}1&2\\-3&0\end{bmatrix}$ and $B=\begin{bmatrix}-1&0\\2&3\end{bmatrix}$, then

(a) $A^2=A$
(b) $B^2=B$
(c) $AB\neq BA$
(d) $AB+BA$

40. In order that the matrix $\begin{bmatrix}1&2&3\\4&5&6\\3&\lambda&5\end{bmatrix}$ be non-singular, λ should not be equal to

(a) 1
(b) 2
(c) 3
(d) 4

41. If $U=[2\ -3\ 4], X=[0\ 2\ 3], V=\begin{bmatrix}3\\2\\1\end{bmatrix}$ and $Y=\begin{bmatrix}2\\2\\4\end{bmatrix}$, then $UY+XY=$

(a) 20 (b) $[-20]$ (c) -20 (d) $[20]$

42. If $A=[1\ 2\ 3]$, $B=\begin{bmatrix}2\\3\\4\end{bmatrix}$ and $C=\begin{bmatrix}1 & 5\\0 & 2\end{bmatrix}$, then which of the following is defined

(a) AB (b) BA (c) $(AB).C$ (d) $(AC).B$

43. The matrix product $AB=O$, then

(a) $A=O$ and $B=O$

(b) $A=O$ or $B=O$

(c) A is null matrix

(d) None of these

44. If $A=\begin{bmatrix}1 & 0\\2 & 0\end{bmatrix}, B=\begin{bmatrix}0 & 0\\1 & 12\end{bmatrix}$, then

(a) $AB=O, BA=O$

(b) $AB=O, BA\neq O$

(c) $AB\neq O, BA=O$

(d) $AB\neq O, BA\neq O$

45. The matrix $\begin{bmatrix}2 & 5 & -7\\0 & 3 & 11\\0 & 0 & 9\end{bmatrix}$ is known as

(a) Symmetric matrix (b) Diagonal matrix

(c) Upper triangular matrix (d) Skew symmetric matrix

46. If the matrix $\begin{bmatrix}a & b\\c & d\end{bmatrix}$ is commutative with the matrix $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$, then

(a) $a=0, b=c$ (b) $b=0, c=d$

(c) $c=0, d=a$ (d) $d=0, a=b$

47. $\begin{bmatrix}1\\-1\\2\end{bmatrix}[2\ 1\ -1]=$

(a) $[-1]$ (b) $\begin{bmatrix}2\\-1\\-2\end{bmatrix}$

(c) $\begin{bmatrix}2 & 1 & -1\\-2 & -1 & 1\\4 & 2 & -2\end{bmatrix}$ (d) Not defined

48. If $[1\ \ x\ \ 1]\begin{bmatrix}1 & 2 & 3\\0 & 5 & 1\\0 & 3 & 2\end{bmatrix}\begin{bmatrix}x\\1\\-2\end{bmatrix}=0$, then $x=$

(a) 3/4 (b) 1 (c) 5/4 (d) 1/4

49. If $A=\begin{bmatrix}1 & -2\\ 4 & 5\end{bmatrix}$ and $f(t)=t^2-3t+7$, then $f(A)+\begin{bmatrix}3 & 6\\ -12 & -9\end{bmatrix}$ is equal to

(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ (c) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$

50. Let $A=\begin{pmatrix}0 & 0 & -1\\ 0 & -1 & 0\\ -1 & 0 & 0\end{pmatrix}$, the only correct statement about the matrix A is.

(a) $A^2=I$
(b) $A=(-1)I$, where I is a unit matrix
(c) A^{-1} does not exist
(d) A is a zero matrix

51. If $A=\begin{bmatrix}0 & 2\\ 3 & -4\end{bmatrix}$ and $kA=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}$, then the value of k,a,b are respectively

(a) $-6,-12,-18$ (b) $-6,4,9$ (c) $-6,-4,-9$ (d) $-6,12,18$

52. If $\begin{bmatrix}2+x & 3 & 4\\ 1 & -1 & 2\\ x & 1 & -5\end{bmatrix}$ is a singular matrix, then x is.

(a) $\frac{13}{25}$ (b) $-\frac{25}{13}$ (c) $\frac{5}{13}$ (d) $\frac{25}{13}$

53. For the matrix $A=\begin{bmatrix}1 & 1 & 0\\ 1 & 2 & 1\\ 2 & 1 & 0\end{bmatrix}$, which of the following is correct

(a) $A^3+3A^2-I=O$
(b) $A^3-3A^2-I=O$
(c) $A^3+2A^2-I=O$
(d) $A^3-A^2+I=O$

54. If $A=\begin{bmatrix}2 & \lambda & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$ is non singular, if

(a) $\lambda\neq-2$ (b) $\lambda\neq2$ (c) $\lambda\neq3$ (d) $\lambda\neq-3$

55. What must be the matrix X if 2X + $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}3 & 8\\ 7 & 2\end{bmatrix}$

(a) $\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}$ (b) $\begin{bmatrix}1 & -3\\ 2 & -1\end{bmatrix}$ (c) $\begin{bmatrix}2 & 6\\ 4 & -2\end{bmatrix}$ (d) $\begin{bmatrix}2 & -6\\ 4 & -2\end{bmatrix}$

56. If matrix $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$, then $A^{16}=$

(a) $\begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

57. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then $A^{100} =$

(a) $2^{100} A$ (b) $2^{99} A$ (c) $2^{101} A$ (d)None of these

58. Matrix $A = \begin{bmatrix} 1 & 0 & -k \\ 2 & 1 & 3 \\ k & 0 & 1 \end{bmatrix}$ is invertible for

(a) $k = 1$ (b) $k = -1$ (c) $k = 0$ (d) All real k

59. The value of a for which the matrix $A = \begin{pmatrix} a & 2 \\ 2 & 4 \end{pmatrix}$ is singular if

(a) 2,2,3,4 (b) 2,3,1,2 (c) 3,3,0,1 (d) None of these

60. If $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ and I is the unit matrix of order 2, then A^2 equals

(a) $4A - 3I$ (b) $3A - AI$ (c) $A - I$ (d) $A + I$

61. If $P = \begin{pmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{pmatrix}$ and $Q = \begin{pmatrix} -i & i \\ 0 & 0 \\ i & -i \end{pmatrix}$, then PQ is equal to

(a) $\begin{pmatrix} -2 & 2 \\ 1 & -1 \\ 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & -2 \\ -1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

62. $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ is equal to

(a) $\begin{bmatrix} 43 \\ 44 \end{bmatrix}$ (b) $\begin{bmatrix} 43 \\ 45 \end{bmatrix}$ (c) $\begin{bmatrix} 45 \\ 44 \end{bmatrix}$ (d) $\begin{bmatrix} 44 \\ 45 \end{bmatrix}$

Exercise-03

Solutions

1. **Sol (b)**

2. **Sol (d)** $M^2 - \lambda M - I_2 = 0,$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 2\lambda \\ 2\lambda & 3\lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - \begin{bmatrix} \lambda & 2\lambda \\ 2\lambda & 3\lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 5-\lambda & 8-2\lambda \\ 8-2\lambda & 13-3\lambda \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 5-\lambda = 1, 8-2\lambda = 0 13-3\lambda = 1$$

$$\Rightarrow \lambda = 4,$$

Which satisfies all the three equations?

3.**Sol (c)** Clearly, $AB = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$

$$= \begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = BA(verify).$$

4.**Sol (c)** $\Delta = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = -1 \neq 0,$ hence matrix is non - singular.

5.**Sol (a)** $A^2 = A.A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

6.**Sol (a)** $A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

and $A^3 = A^2.A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow A^n = A^{n-1}.A = \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$$

7.**Sol (b)**Given, $2A+3B = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$ ...... (i)

and $A+2B = \begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix}$

$\Rightarrow 2A+4B = \begin{bmatrix} 10 & 0 & 6 \\ 2 & 12 & 4 \end{bmatrix}$ ...... (ii)

Solving (i) and (ii), we get

$$\Rightarrow -B = \begin{bmatrix} -8 & -1 & -2 \\ 1 & -10 & 1 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$$

8.**Sol (b)**Here $m^2+n^2 = 25$ and $m < n$. Hence, $(m,n) = (3,4)$

9.**Sol (b)**The matrix $\begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ is singular

If $\begin{vmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{vmatrix} = 0$

$$\Rightarrow 1(40-40)-3(20-24)+(\lambda+2)(10-12) = 0$$

$$\Rightarrow 2(\lambda+2) = 12 \Rightarrow \lambda = 4.$$

10. Sol (a) $A^2 = A.A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$

$$=\begin{bmatrix} a^2b^2-a^2b^2 & ab^3-ab^3 \\ -a^3b+a^3b & -a^2b^2+a^2b^2 \end{bmatrix}=0$$

$\Rightarrow A^3 = A.A^2 = 0$ and $A^n = 0$, for all $n \geq 2$

11. Sol (b) $AB=\begin{bmatrix} 1/3 & 2 \\ 0 & 2x-3 \end{bmatrix}\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 3-2x \end{bmatrix}=I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Leftrightarrow 3-2x=1$ or $x=1$.

12. Sol (b) $A^2 = A.A=\begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}\begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}=\begin{bmatrix} \lambda^2-1 & 0 \\ 0 & -1+\lambda^2 \end{bmatrix}=0$

$\Rightarrow \lambda^2-1=0 \Rightarrow \lambda=\pm 1$

13. **Sol (b)** $A'=\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -5 \\ -2 & 5 & 0 \end{bmatrix}=-A.$

14. **Sol (d)**By property.

15. **Sol (c)** $AB=\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} -5 & 4 & 0 \\ 0 & 2 & -1 \\ 1 & -3 & 2 \end{bmatrix}=\begin{bmatrix} -2 & -1 & 4 \end{bmatrix}.$

16. **Sol (c)** $\begin{bmatrix} 2 & -3 \\ 4 & 0 \end{bmatrix}-\begin{bmatrix} a & c \\ b & d \end{bmatrix}=\begin{bmatrix} 1 & 4 \\ 2 & -5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix}=\begin{bmatrix} 2 & -3 \\ 4 & 0 \end{bmatrix}-\begin{bmatrix} 1 & 4 \\ 2 & -5 \end{bmatrix}=\begin{bmatrix} 1 & -7 \\ 2 & 5 \end{bmatrix}$

$\Rightarrow (a,b,c,d)=(1,2,-7,5)$

17. Sol (d) $A=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$\therefore A^2=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}=\begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$

$\therefore A^2-4A=\begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}-\begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}=\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}=5I_3.$

18. **Sol (b)**In the product AB, the required element
$C_{33}=(-2)3+2.5+0.0=-6+10=4.$

19.

Sol (c) $A=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \Rightarrow A^5=\begin{bmatrix} 2^5 & 0 & 0 \\ 0 & 2^5 & 0 \\ 0 & 0 & 2^5 \end{bmatrix}=2^4\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$=16A.$

20. **Sol (d)**Since $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}=0=AB$

$B=\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$

21. **Sol (b)** It is obvious.

22. **Sol (c)** Since a square matrix A whose elements $a_{ij}=0$ for $i<j$. Then A is the Lower triangular matrix.

23. **Sol (b)** $A^2B=(A.A)B=\begin{bmatrix}2&0&0\\0&-1&0\\0&0&3\end{bmatrix}\begin{bmatrix}2&0&0\\0&-1&0\\0&0&3\end{bmatrix}.\begin{bmatrix}-1&0&0\\0&3&0\\0&0&2\end{bmatrix}$

$=\begin{bmatrix}4&0&0\\0&1&0\\0&0&9\end{bmatrix}\begin{bmatrix}-1&0&0\\0&3&0\\0&0&2\end{bmatrix}=\begin{bmatrix}-4&0&0\\0&3&0\\0&0&18\end{bmatrix}$

24. **Sol (d)** $\cos\theta\begin{bmatrix}\cos\theta&\sin\theta\\-\sin\theta&\cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta&-\cos\theta\\\cos\theta&\sin\theta\end{bmatrix}$

$=\begin{bmatrix}\cos^2\theta+\sin^2\theta&0\\0&\cos^2\theta+\sin^2\theta\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

25. **Sol (c)** It is based on fundamental concept.

26. **Sol (d)** $A^2=A.A=\begin{bmatrix}1&a\\0&1\end{bmatrix}\begin{bmatrix}1&a\\0&1\end{bmatrix}=\begin{bmatrix}1&2a\\0&1\end{bmatrix}$

$A^3=A.A^2=\begin{bmatrix}1&a\\0&1\end{bmatrix}\begin{bmatrix}1&2a\\0&1\end{bmatrix}=\begin{bmatrix}1&3a\\0&1\end{bmatrix}$

$A^4=A.A^3=\begin{bmatrix}1&a\\0&1\end{bmatrix}\begin{bmatrix}1&3a\\0&1\end{bmatrix}=\begin{bmatrix}1&4a\\0&1\end{bmatrix}$

27. Sol (a) $\begin{bmatrix}3&1\\4&1\end{bmatrix}X=\begin{bmatrix}5&-1\\2&3\end{bmatrix}\Rightarrow X=\begin{bmatrix}-3&4\\14&-13\end{bmatrix}$

Since $\begin{bmatrix}3&1\\4&1\end{bmatrix}\begin{bmatrix}-3&4\\14&-13\end{bmatrix}=\begin{bmatrix}5&-1\\2&3\end{bmatrix}$

28. **Sol (b)** Order will be $(1\times3)(3\times3)(3\times1)=(1\times1)$

29.

Sol (a) Since $(A+B)(A-B)=A^2-B^2$,

By matrix distribution law,

$\Rightarrow A^2-AB+BA-B^2=A^2-B^2$

$\Rightarrow BA-AB=0\Rightarrow BA=AB.$

30. Sol (b) $A=\begin{bmatrix}5&-3\\2&4\end{bmatrix}$ and $B=\begin{bmatrix}6&-4\\3&6\end{bmatrix}$,

$\therefore A-B=\begin{bmatrix}-1&1\\-1&-2\end{bmatrix}$.

31. **Sol (b)** Given Matrix $\begin{bmatrix}-x&x&2\\2&x&-x\\x&-2&-x\end{bmatrix}$ will be non-singular,

If $\begin{vmatrix}-x&x&2\\2&x&-x\\x&-2&-x\end{vmatrix}\neq0$

$\Rightarrow -x\left(-x^2-2x\right)-x\left(x^2-2x\right)+\left(-4-x^2\right)\neq0$

$\Rightarrow 2x^2 - 8 \geq 0 \Rightarrow x^2 \geq 4$
$\therefore x \neq \pm 2$.

32. **Sol (a)** It is obvious. (By commutative law).

33. **Sol (d)** $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

34. **Sol (b)** $AB = \begin{bmatrix} 5 & 9 & 13 \\ -1 & 2 & 4 \\ -2 & 2 & 4 \end{bmatrix}$

35. **Sol (b)** $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

36. **Sol (a)**Here $AB = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

and $BA = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Since $AB = BA$, therefore $(A+B)(A-B) = A^2 - B^2$.

37. **Sol (b)** $5A - 3B - 2C = \begin{bmatrix} 5 & -10 \\ 15 & 0 \end{bmatrix} - \begin{bmatrix} -3 & 12 \\ 6 & 9 \end{bmatrix} - \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

$\begin{bmatrix} 5+3-2 & -10-12+2 \\ 15-6-2 & 0-9 \end{bmatrix} = \begin{bmatrix} 8 & -20 \\ 7 & -9 \end{bmatrix}$

38. **Sol (b)**Since $x - 2 = 3 - 2 \Rightarrow x = 3$
and $y + 4 = 3 - 1 \Rightarrow y = -2$

39. **Sol (c)**Since, $A^2 = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ -3 & -6 \end{bmatrix} \neq A$

$B^2 = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 9 \end{bmatrix} \neq B$

Now, $AB = \begin{bmatrix} 1 & 2 \\ -.3 & 0 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 3 & 0 \end{bmatrix}$

and $BA = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -7 & 4 \end{bmatrix}$

Obviously, $AB \neq BA$.

40. **Sol (d)**Matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & \lambda & 5 \end{bmatrix}$ be non singular,

only if $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & \lambda & 5 \end{vmatrix} \neq 0$

$\Rightarrow 1(25 - 6\lambda) - 2(20 - 18) + 3(4\lambda - 15) \neq 0$
$\Rightarrow 25 - 6\lambda - 4 + 12\lambda - 45 \neq 0$
$\Rightarrow 6\lambda - 24 \neq 0 \Rightarrow \lambda \neq 4$

41. **Sol (d)** $UV = [4]$ and $XY = [16]; \therefore UV + XY = [20]$

42. **Sol (a,b)** Since order of A is 1×3 and that of B is 3×1 therefore AB and BA must be defined and their order will be 1×1 and 3×3 respectively.

43. **Sol (d)**It is property.

44. Sol (b) $AB=\begin{bmatrix}1&0\\2&0\end{bmatrix}\begin{bmatrix}0&0\\1&12\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}=0$

While $AB=\begin{bmatrix}1&0\\2&0\end{bmatrix}\begin{bmatrix}0&0\\1&12\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}=0$

45. **Sol (c)**We know that if all the elements below the diagonal in the matrix are zero, then it is an upper triangular matrix.

46. **Sol (c)**Matrix $\begin{bmatrix}a&b\\c&d\end{bmatrix}$ is commutative with $\begin{bmatrix}1&1\\0&1\end{bmatrix}$

$\Rightarrow AB=BA\Rightarrow\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}=\begin{bmatrix}1&1\\0&1\end{bmatrix}\begin{bmatrix}a&b\\c&d\end{bmatrix}$

$\Rightarrow\begin{bmatrix}a&a+b\\c&c+d\end{bmatrix}=\begin{bmatrix}a+c&b+d\\c&d\end{bmatrix}$

$\therefore a=a+c\Rightarrow c=0$ and $a+b=b+d\Rightarrow a=d$

47. **Sol (c)** $\begin{bmatrix}1\\-1\\2\end{bmatrix}\begin{bmatrix}2&1&-1\end{bmatrix}=\begin{bmatrix}2&1&-1\\-2&-1&1\\4&2&-2\end{bmatrix}$

48. Sol (c) $[1x1]\begin{bmatrix}1&2&3\\0&5&1\\0&3&2\end{bmatrix}\begin{bmatrix}x\\1\\-2\end{bmatrix}=0$

$\Rightarrow[12+5x+33+x+2]\begin{bmatrix}x\\1\\-2\end{bmatrix}=0$

$\Rightarrow x+(2+5x+3)+(-2)(3+x+2)=0\Rightarrow x=\frac{5}{4}.$

49. **Sol (b)**Given that, $A=\begin{bmatrix}1&-2\\4&5\end{bmatrix}$ and $f(t)=t^2-3t+7$

Now, $A^2=\begin{bmatrix}1&-2\\4&5\end{bmatrix}\begin{bmatrix}1&-2\\4&5\end{bmatrix}=\begin{bmatrix}-7&-12\\24&17\end{bmatrix}$

Now, $f(A)=A^2-3A+7$

$=\begin{bmatrix}-7&6\\-12&-9\end{bmatrix}-3\begin{bmatrix}1&-2\\4&5\end{bmatrix}+7\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}-3&-6\\12&9\end{bmatrix}$

$\therefore f(A)+\begin{bmatrix}3&6\\-12&-9\end{bmatrix}=\begin{bmatrix}-3&-6\\12&9\end{bmatrix}+\begin{bmatrix}3&6\\-12&-9\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}.$

50. **Sol (a)**Let $A=\begin{pmatrix}0&0&-1\\0&-1&0\\-1&0&0\end{pmatrix}$

Check by options

(i) $A^2=\begin{pmatrix}0&0&-1\\0&-1&0\\-1&0&0\end{pmatrix}\begin{pmatrix}0&0&-1\\0&-1&0\\-1&0&0\end{pmatrix}$

$A^2=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=I$

(ii) $(-1)I=\begin{pmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{pmatrix}\neq A$

(iii) $|A|=1\neq 0\Rightarrow A^{-1}$ exists.

(iv) Clearly A, is not a zero matrix.

51. **Sol (c)**Given, $kA=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}\Rightarrow k\begin{bmatrix}0 & 2\\ 3 & -4\end{bmatrix}=\begin{bmatrix}0 & 3a\\ 2b & 24\end{bmatrix}$

$\Rightarrow 2k=3a, 3k=2b, -4k=24$

$\Rightarrow a=\frac{2k}{3}, b=\frac{3k}{2}, k=-6$

$\Rightarrow k=-6, a=-4, b=-9$

52. **Sol (b)**Given $\begin{vmatrix}2+x & 3 & 4\\ 1 & -1 & 2\\ x & 1 & -5\end{vmatrix}=0$

$\Rightarrow (2+x)(5-2)-3(-5-2x)+4(1+x)=0$

$\Rightarrow 6+3x+15+6x+4+4x=0$

$\Rightarrow 13x=-25\Rightarrow x=-\frac{25}{13}$

53. Sol (b) $A^2=AA=\begin{bmatrix}1 & 1 & 0\\ 1 & 2 & 1\\ 2 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 1 & 0\\ 1 & 2 & 1\\ 2 & 1 & 0\end{bmatrix}=\begin{bmatrix}2 & 3 & 1\\ 5 & 6 & 2\\ 3 & 4 & 1\end{bmatrix}$

$\Rightarrow A^3=A^2A=\begin{bmatrix}2 & 3 & 1\\ 5 & 6 & 2\\ 3 & 4 & 1\end{bmatrix}\begin{bmatrix}1 & 1 & 0\\ 1 & 2 & 1\\ 2 & 1 & 0\end{bmatrix}=\begin{bmatrix}7 & 9 & 3\\ 15 & 19 & 6\\ 9 & 12 & 4\end{bmatrix}$

Here, $A^3-3A^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I\Rightarrow A^3-3A^2-I=0$

54. **Sol (a)**The given matrix $A=\begin{bmatrix}2 & \lambda & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$ is non singular,

if $|A|\neq 0$

$|A|=\begin{vmatrix}2 & \lambda & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{vmatrix}=\begin{vmatrix}1 & \lambda+3 & 0\\ -1 & 3 & 4\\ 1 & -2 & -3\end{vmatrix}, [R_1\to R_2+R_1]$

$=\begin{vmatrix}1 & \lambda+3 & 0\\ 0 & 1 & 1\\ 0 & -\lambda-5 & -3\end{vmatrix}\quad \begin{bmatrix}R_2\to R_2+R_3\\ R_3\to R_3-R_1\end{bmatrix}$

$=1(-3+\lambda+5)\neq 0\Rightarrow \lambda+2\neq 0\Rightarrow \lambda\neq -2$

55. Sol (a) $2x=\begin{bmatrix}3 & 8\\ 7 & 2\end{bmatrix}-\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}$

$2x=\begin{bmatrix}2 & 6\\ 4 & -2\end{bmatrix}\Rightarrow X=\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}$

56. Sol (d) Given, Matrix $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

We know that $A^2 = A.A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Therefore

$$A^{16} = \left(A^2\right)^8 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}^8 = \begin{bmatrix} (-1)^8 & 0 \\ 0 & (-1)^8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

57. Sol (b) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^3 = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2^2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^n = 2^{n-1}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow A^{100} = 2^{99}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2^{99}.A.$$

58. **Sol (d)** $\therefore$ On expansion, $|A| = k^2 + 1$, Which can be never zero . Hence matrix A is invertible for all real k.

59. **Sol (b)**Put $a = 1; \therefore |A| = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 4 - 4 = 0$

Hence, A is a singular matrix for $a = 1$.

60. $\Rightarrow 4A - 3I = \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}.$

61. **Sol (b)** First note that PQ must be of order 3×2 and its $(1,1)^{th}$ entry is $i(-i) + 0 - i(i) = 2.$

62.

Sol (a) $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 35 \\ 40 \end{bmatrix}; \therefore \begin{bmatrix} 35 \\ 40 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 43 \\ 44 \end{bmatrix}.$

Exercise-04

Matrices

Transpose, Adjoint, Inverse, Solution of Equation

1. Inverse of the matrix $\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 3 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 & 5 \\ 7 & 4 & 6 \\ 4 & 2 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & -4 \\ 8 & -4 & -5 \\ 3 & 5 & 2 \end{bmatrix}$

2. If A and B are non-singular matrices, then

(a) $(AB)^{-1} = A^{-1}B^{-1}$

(b) $AB = BA$

(c) $(AB)' = A'B'$

(d) $(AB)^{-1} = B^{-1}A^{-1}$

3. Ad joint of the matrix $\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ is

(a) N (b) $2N$ (c) $-N$ (d) None of these

4. From the following find the correct relation

(a) $(AB)' = A'B'$

(b) $(AB)' = B'A'$

(c) $A^{-1} = \frac{adjA}{A}$

(d) $(AB)^{-1} = A^{-1}B^{-1}$

5. If k is a scalar and I is a unit matrix of order 3, then adj (kI) =

(a) k^3I (b) k^2I (c) $-k^3I$ (d) $-k^2I$

6. Suppose A is a matrix of order 3 and $B = |A|A^{-1}$. If $|A| = -5$ then $|B|$ is equal to

(a) 1 (b) -5 (c) -1 (d) 25

7. Let $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and $A^{-1} = xA + yI$, then the value of x and y are

(a) $x = \frac{-1}{11}, y = \frac{2}{11}$

(b) $x = \frac{-1}{11}, y = \frac{-2}{11}$

(c) $x = \frac{1}{11}, y = \frac{2}{11}$

(d) $x = \frac{1}{11}, y = \frac{-2}{11}$

8. The element of second row and third column in the inverse of $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is.

(a) -2 (b) -1 (c) 1 (d) 2

9. If A and B be symmetric matrices of the same order, then $AB - BA$ will be a
(a) Symmetric matrix
(b) Skew symmetric matrix
(c) Null matrix
(d) None of these

10. An orthogonal matrix is

(a) $\begin{bmatrix} \cos\alpha & 2\sin\alpha \\ -2\sin\alpha & \cos\alpha \end{bmatrix}$ (b) $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

(c) $\begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

11. If A and B are square matrices of the same order and $AB = 3I$, then A^{-1} is equal to

(a) $3B$ (b) $\frac{1}{3}B$ (c) $3B^{-1}$ (d) $\frac{1}{3}B^{-1}$

12. If A and B are square matrices of the same order such that $(A+B)(A-B) = A^2 - B^2$, then $\left(ABA^{-1}\right)^2 =$

(a) A^2B^2 (b) A^2 (c) B^2 (d) I

13. The inverse of $\begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$ is

(a) $\frac{-1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ (b) $\frac{-1}{8}\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$ (c) $\frac{1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ (d) $\frac{1}{8}\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$

14. If A is a square matrix, A' its transpose, then $\frac{1}{2}(A - A')$ is
(a) A symmetric matrix
(b) A skew symmetric
(c) A unit matrix
(d) An elementary matrix

15. The adjoint of the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is

(a) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ (b) $\begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ (d) $\begin{bmatrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

16. If A is a square matrix, then which of the following matrix is not symmetric
(a) $A + A'$ (b) AA' (c) $A'A$ (d) $A - A'$

17. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and $A adj\ A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then k is equal to

(a) 0 (b) 1 (c) $\sin\alpha\cos\alpha$ (d) $\cos 2\alpha$

(a) $(6,11)$ (b) $(6,-11)$ (c) $(-6,11)$ (d) $(-6,-11)$

34. If $A=\begin{pmatrix}3 & 2\\ 0 & 1\end{pmatrix}$, then $\left(A^{-1}\right)^3$ is equal to

(a) $\frac{1}{27}\begin{pmatrix}1 & -26\\ 0 & 27\end{pmatrix}$ (b) $\frac{1}{27}\begin{pmatrix}-1 & 26\\ 0 & 27\end{pmatrix}$

(c) $\frac{1}{27}\begin{pmatrix}1 & -26\\ 0 & -27\end{pmatrix}$ (d) $\frac{1}{27}\begin{pmatrix}-1 & -26\\ 0 & -27\end{pmatrix}$

35. The matrix $\begin{pmatrix}1 & a & 2\\ 1 & 2 & 5\\ 2 & 1 & 1\end{pmatrix}$ is not invertible, if 'a' has the value

(a) 2 (b) 1 (c) 0 (d) -1

36. If matrix $A=\begin{bmatrix}3 & 2 & 4\\ 1 & 2 & -1\\ 0 & 1 & 1\end{bmatrix}$ and $A^{-1}=\frac{1}{K}adj(A)$, then K is

(a) 7 (b) -7 (c) $\frac{1}{7}$ (d) 11

37. If $A=\begin{bmatrix}-2 & 6\\ -5 & 7\end{bmatrix}$ then adj (A)

(a) $\begin{bmatrix}7 & -6\\ 5 & -2\end{bmatrix}$ (b) $\begin{bmatrix}2 & -6\\ 5 & -7\end{bmatrix}$

(c) $\begin{bmatrix}7 & -5\\ 6 & -2\end{bmatrix}$ (d) None of these

.

38. The multiplicative inverse of matrix $\begin{bmatrix}2 & 1\\ 7 & 4\end{bmatrix}$ is

(a) $\begin{bmatrix}4 & -1\\ -7 & -2\end{bmatrix}$

(b) $\begin{bmatrix}-4 & -1\\ 7 & -2\end{bmatrix}$

(c) $\begin{bmatrix}4 & -7\\ 7 & 2\end{bmatrix}$

(d) $\begin{bmatrix}4 & -1\\ -7 & 2\end{bmatrix}$

39. The element in the first row and third column of the inverse of the matrix $\begin{bmatrix}1 & 2 & -3\\ 0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}$ is

(a) -2 (b) 0 (c) 1 (d) 7

40. For any square matrix A, AA^T is a
(a) unit matrix
(b) Symmetric matrix
(c) Skew symmetric matrix

(d) Diagonal matrix

41. The matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is invertible, it

(a) $\lambda \neq -15$ (b) $\lambda \neq -17$ (c) $\lambda \neq -16$ (d) $\lambda \neq -18$

42. If $X = \begin{bmatrix} -x & -y \\ z & t \end{bmatrix}$ then transpose of adj X is

(a) $\begin{bmatrix} t & z \\ -y & -x \end{bmatrix}$ (b) $\begin{bmatrix} t & y \\ -z & -x \end{bmatrix}$

(c) $\begin{bmatrix} t & -z \\ y & -x \end{bmatrix}$ (d) None of these

43. The matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is known is

(a) Upper triangular matrix (b) Skew symmetric matrix
(c) Symmetric matrix (d) diagonal matrix

44. If $A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{pmatrix}$, then $(AB)^T$ is equal to

(a) $\begin{pmatrix} -3 & -2 \\ 10 & 7 \end{pmatrix}$ (b) $\begin{pmatrix} -3 & 10 \\ -2 & 7 \end{pmatrix}$

(c) $\begin{pmatrix} -3 & 7 \\ 10 & 2 \end{pmatrix}$ (d) None of these

45. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ and $(10)B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inverse of matrix A, then α is.

(a) 5 (b) –1 (c) 2 (d) –2

46. The inverse of a matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is

(a) $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ (b) $\frac{1}{(ad - bc)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

(c) $\frac{1}{|A|}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} b & -a \\ d & -c \end{pmatrix}$

47. If $A = \begin{bmatrix} \cos\theta & -\sin g\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then which of the following statement is not correct?

(a) A is orthogonal matrix (b) A' is orthogonal matrix
(c) Determinant A = 1 (d) A is not invertible

48. If $A^2 - A + I = 0$ then $A^{-1} =$

(a) A^{-2} (b) $A + I$ (c) $I - A$ (d) $A - I$

49. If $A=\begin{bmatrix}2 & 2\\ -3 & 2\end{bmatrix}$ and $B=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ then $\left(B^{-1}A^{-1}\right)^{-1}=$

(a) $\begin{bmatrix}2 & -2\\ 2 & 3\end{bmatrix}$ (b) $\begin{bmatrix}3 & -2\\ 2 & 2\end{bmatrix}$

(c) $\frac{1}{10}\begin{bmatrix}2 & 2\\ -2 & 3\end{bmatrix}$ (d) $\frac{1}{10}\begin{bmatrix}3 & 2\\ -2 & 2\end{bmatrix}$

50. A square matrix $A=\left[a_{ij}\right]$ in which $a_{ij}=0$ for $i\neq j$ and $a_{ij}=k$ (constant) for $i=j$ is called a

(a) Unit matrix (b) Scalar matrix (c) Null matrix (d) Diagonal matrix

51. If $A=\begin{bmatrix}-1 & 2\\ 2 & -1\end{bmatrix}$ and $B=\begin{bmatrix}3\\ 1\end{bmatrix}$, $AX=B$, then $X=$

(a) $[5\ 7]$

(b) $\frac{1}{3}\begin{bmatrix}5\\ 7\end{bmatrix}$

(c) $\frac{1}{3}[5\ 7]$

(d) $\begin{bmatrix}5\\ 7\end{bmatrix}$

52. $A\begin{bmatrix}0 & 3\\ 2 & 0\end{bmatrix}$ and $A^{-1}=\lambda(adj(A))$ then $\lambda=$

(a) $\frac{-1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{-1}{3}$ (d) $\frac{1}{6}$

53. If $A=\begin{bmatrix}4 & 2\\ 3 & 4\end{bmatrix}$, then |adj A| is equal to

(a) 16 (b) 10 (c) 6 (d) None of these

54. The matrix $A=\begin{bmatrix}1 & -3 & -4\\ -1 & 3 & 4\\ 1 & -3 & -4\end{bmatrix}$ is nilpotent of index

(a) 2 (b) 3 (c) 4 (d) 6

55. If $A=\begin{bmatrix}2 & 5\\ 3 & 7\end{bmatrix}$ and $B=\begin{bmatrix}0 & 3\\ 4 & 1\end{bmatrix}$, then

(a) $|AB|=|A||B|$

(b) $|AB|=|A|$

(c) $|AB|=|B|$

(d) $|AB|=-|A||B|$

56. Rank of matrix $\begin{bmatrix}4 & 1 & 0 & 0\\ 3 & 0 & 1 & 0\\ 6 & 0 & 2 & 0\end{bmatrix}$ is

(a) 4 (b) 3 (c) 2 (d) 1

57. Let A be a skew-symmetric matrix of odd order, then |A| is equal to

(a) 0 (b) 1 (c) –1 (d) None of these

58. The equation $x+2y=3z=1,\ 2x+y+3z=2,\ 5x+5y+9z=4,$ have

(a) Unique solution
(b) Infinitely many solutions
(c) Inconsistent
(d) None of these

59. If $A=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$ and $B=\begin{bmatrix}1&2&3\\0&1&3\\0&0&2\end{bmatrix}$, then |AB| is equal to

(a) 4 (b) 8 (c) 16 (d) 32

60. Equations $x+y=2,\ 2x+2y=3$ will have

(a) Only one solution
(b) Many finite solution
(c) No solution
(d) None of these

61. The number of solution of the following equations
$x_2-x_3=1,\ -x_1+2x_3=-2,\ x_1+2x_2=3$ is.

(a) Zero (b) One (c) Two (d) Infinite

62. If $A=\begin{bmatrix}1&0&1\\2&1&0\\3&2&1\end{bmatrix}$, then det $A=$

(a) 2 (b) 3 (c) 4 (d) 5

63. The number of solutions of the system of equations is
$x-y+z=2;\ 2x+y-z=5;\ 4x-y+z=10$

(a) ∞ (b) 1 (c) 2 (d) 0

64. If $A=\begin{bmatrix}2&4&5\\4&8&10\\-6&-12&-15\end{bmatrix}$. Then rank of A is equal to

(a) 0 (b) 1 (c) 2 (d) 3

65. if $A=\begin{bmatrix}\alpha&2\\2&\alpha\end{bmatrix}$ and $|A^3|=125$ then $\alpha=$

(a) ±3 (b) ±2 (c) ±5 (d) 0

66. If $\begin{bmatrix}1&1&1\\1&-2&-2\\1&3&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\3\\4\end{bmatrix}$, then $\begin{bmatrix}x\\y\\z\end{bmatrix}$ is equal to

(a) $\begin{bmatrix}1\\1\\1\end{bmatrix}$ (b) $\begin{bmatrix}1\\-2\\3\end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

67. Let $A = \begin{bmatrix} 5 & 5a & a \\ 0 & a & 5a \\ 0 & 0 & 5 \end{bmatrix}$, If $|A^2| = 25$, then $|a|$ equal

(a) 5^2 (b) 1 (c) 1/5 (d) 5

68. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, then determinant of $A^2 - 2A$ is

(a) 5 (b) 25 (c) –5 (d) –25

69. If I is a unit matrix of order 10, then the determinant of I is equal to

(a) 10 (b) 1 (c) 1/10 (d) 9

70. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$, then the value of $|adj\, A|$ is

(a) 36 (b) 72 (c) 144 (d) None of these

Exercise-04

Solutions

1. **Sol (c)** Let $A = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

then $|A| = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{vmatrix} = 1$

The matrix of cofactors of A

$= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$

Therefore, adj $(A) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}, adj\, A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}, \quad [\because |A| = 1]$

2. **Sol (d)** $(AB)^{-1} = B^{-1}A^{-1}$.

3. **Sol (a)** The cofactor of $N = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ are

$c_{11} = -4, c_{12} = 1, c_{13} = 4; c_{21} = -3, c_{22} = 0, c_{23} = 4$

$c_{31} = -3, c_{32} = 1, c_{33} = 3$

$\therefore adj\, N = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} = N$

4. **Sol (b)** It is obvious.

5. **Sol (b)** Let $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $kI = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$

$\Rightarrow adj(kI) = \begin{bmatrix} k^2 & 0 & 0 \\ 0 & k^2 & 0 \\ 0 & 0 & k^2 \end{bmatrix} = k^2 I$

6. **Sol (d)** Here, $|B| = |A||A^{-1}| = 25$

$\left[\because |A| = |A|^{-1}\right]$

7. **Sol (a)** $\because A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$

$\therefore A^{-1} = \frac{adjA}{|A|} = \frac{1}{11}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$

Given $A^{-1} = xA + yI$

$\Rightarrow \frac{1}{11}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} x & 2x \\ -5x & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}$

$\Rightarrow x + y = \frac{1}{11}, 2x = \frac{-2}{11} \Rightarrow x = \frac{-1}{11}, y = \frac{2}{11}$

8. **Sol (b)** In A^{-1}, the element of 2nd row and 3rd column is the c_{32} element of the matrix (c_{ij}) of cofactors of element of A, (due to transposition) divided by $\Delta = |A| = -2$

$\therefore$ Required element $= \frac{(-1)^{3+2} M_{32}}{-2} = \frac{-(-2)}{-2} = -1$

where M_{32} = minor of c_{32} = in $A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = 0 - 2 = -2$

9. **Sol (b)** Since A,B are symmetric $\Rightarrow A = A'$ and $B = B'$

$\therefore (AB - BA)' = (AB)' = B'A - A'B$

$= -(A'B' - B'A') = -(AB - BA)$

$\Rightarrow (AB - BA)$ is skew-symmetric.

10. **Sol (b)** A square matrix is to be orthogonal matrix if $A'A = I = AA'$

$\Rightarrow A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}, A'=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$\Rightarrow AA'=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, A'A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\therefore AA'A'A=I$

11. **Sol (b)**Given, $AB=3I$

$\Rightarrow A^{-1}(AB)=A^{-1}(3I)$ [Per multiplication by A^{-1}]

$\Rightarrow A^{-1}AB=3A^{-1}\Rightarrow IB=3A^{-1}\left[\because A^{-1}A=I\right]$

$\Rightarrow B=3A^{-1}\Rightarrow A^{-1}=\frac{1}{3}B.$

12. Sol (c) $(A+B)(A-B)=A^2-B^2$

$\Rightarrow AB=BA$

$\therefore\left(ABA^{-1}\right)^2=\left(BAA^{-1}\right)^2=(BI)^2=B^2$

13. **Sol (a)**Let $A=\begin{bmatrix}2 & -3\\ -4 & 2\end{bmatrix}, \therefore |A|=\begin{bmatrix}2 & -3\\ -4 & 2\end{bmatrix}=4-12=-8\backslash$

The matrix of cofactors of the element of A viz.

$\begin{bmatrix}c_{11} & c_{12}\\ c_{21} & c_{22}\end{bmatrix}=\begin{bmatrix}2 & -(-4)\\ -(-3) & 2\end{bmatrix}=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}$

$\therefore adjA=$ transpose of the matrix of cofactors of

element of $A=\begin{bmatrix}2 & 3\\ 4 & 2\end{bmatrix}$

$\therefore A^{-1}=\frac{1}{\Delta}adjA=\frac{1}{-8}\cdot\begin{bmatrix}2 & 3\\ 4 & 2\end{bmatrix}$

14. **Sol (b)**Taking $\left[\frac{1}{2}(A-A')\right]=\frac{1}{2}(A-A)'=\frac{1}{2}(A'-(A'))$

Since $(A')'=A$

$=\frac{1}{2}(A'-A)=\frac{1}{2}(A-A')$

$\Rightarrow\frac{1}{2}(A'-A)$ is a skew symmetric matrix.

15. **Sol (a)**Let $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$adj\ A=\begin{bmatrix}\cos\theta & +\sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}^T=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

16. **Sol (d)**Since $(A+A')'=A'+A=A+A'$, so it is symmetric.

$(AA)'=(A')'A'=AA'$, so it is symmetric.

$(AA)'=A'(A')'=A'A$, so it is symmetric

But $(A-A')=A'-A\neq A-A'$. Hence it is not symmetric.

17. **Sol (b)**Let $A=\begin{bmatrix}\cos\alpha & -(-\sin\alpha)\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

The matrix of cofactors of the elements of A

$=\begin{bmatrix}\cos\alpha & -(-\sin\alpha)\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$\therefore AadjA = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ (as given) $\Rightarrow k = 1$

Alter : A (adj A)= |A| $I_n = 1.I_n$.

Hence $k = 1$

18. **Sol (a)** $3A^3 + 2A^2 + 5A + I = 0 \Rightarrow = -3A^3 - 2A^2 - 5A$

$\Rightarrow IA^{-1} = -3A^2 - 2A - 5I \Rightarrow A^{-1} = -\left(3A^2 + 2A + 5I\right)$

19. **Sol (b)** It is obvious.

20. **Sol (a)** let A be a symmetric matrix.

Then $AA^{-1} = I \Rightarrow \left(AA^{-1}\right)^T = I$

$\Rightarrow \left(A^{-1}\right)^T A^T = I \Rightarrow \left(A^{-1}\right)^T = \left(A^T\right)^{-1}$

$\Rightarrow \left(A^{-1}\right)^T = (A)^{-1}, \left(\because A^T = A\right)$

$\Rightarrow A^{-1}$ is a symmetric matrix.

21. **Sol (c)** $A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}; AdjA = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}^T$

$c_{11} = 8 - 6 = 2$ $\quad c_{12} = -(0+9) = -9$

$c_{21} = -(-8+4) = 4$ $\quad c_{22} = 4 - 6 = -2$

$c_{31} = +6 - 4 = 2$ $\quad c_{32} = -(-3-0) = 3$

$c_{13} = 0 - 6 = -6$

$c_{23} = -(-2+6) = -4$

$c_{33} = 2 - 0 = 2$

$\therefore AdjA = \begin{bmatrix} 2 & -9 & -6 \\ 4 & -2 & -4 \\ 2 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 2 \\ -9 & -2 & 3 \\ -6 & -4 & 2 \end{bmatrix}$

$\therefore A.AdjA = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 4 & 2 \\ -9 & -2 & 3 \\ -6 & -4 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

Alter: $A(AdjA) = |A|I = 8I = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

22. **Sol (d)**Let $A = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}, |A| = 1$

$adj(A) = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$A^{-1} = \dfrac{adj(A)}{|A|} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

23. **Sol (b)**Since the given matrix is symmetric, therefore

$a_{12} = a_{21} \Rightarrow x + 2 = 2x - 3 \Rightarrow x = 5$.

24. **Sol (d)** $|A| = \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{vmatrix} = 1[3] + 1(6) + 1[-4] = 5$

$B = adjA = \begin{bmatrix} 3 & 1 & 1 \\ -6 & -2 & 3 \\ -4 & -3 & 2 \end{bmatrix}$

$adj\,B = \begin{bmatrix} 5 & -5 & 5 \\ 0 & 10 & -15 \\ 10 & 5 & 0 \end{bmatrix} = 5A \text{ and } C = 5A$

$C = adjB; |C| = |adjB|; \therefore \dfrac{|adjB|}{|C|} = 1$

Trick: $\dfrac{|adjB|}{|C|} = \dfrac{|adj(adjA)|}{|C|}$

$= \dfrac{|A|^{(n-1)^2}}{5^n\,|A|} = \dfrac{5^4}{5^3 \times 5} = 1$

25. **Sol (c)** $\because A^T = -A$. It is skew- symmetric.

26. **Sol (a)** $A = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}, A^T = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}, A + A^T = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$

27. **Sol (a)** $A^{-1} = A^2$, because $A^3 = I$.

28. **Sol (a)** Since $|adjA| = |A|^{n-2}$ therefore $|A| = 0$

$\Rightarrow |adjA| = 0 \Rightarrow adjA$ is also singular

29. **Sol (a)** Given $Q = PAP^T$

$\Rightarrow P^T Q = AP^T, (\therefore PP^T = I)$

$\Rightarrow P^T Q^{2005} P = AP^T Q^{2004} P = AP^T Q^{2003} PA$

$(\therefore Q = PAP^T \Rightarrow QP = PA)$

$= AP^T Q^{2002} PA^2 = AP^T PA^{2004}$

$= AIA^{2004} = A^{2005} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

30. **Sol (c)** $A.\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 2I$

31. **Sol (c)** $A[adj(A)] = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 7 & -4 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = |A|\,I$

$[\therefore |A| = 21 - 20 = 1]$

32. **Sol (b)** It is a concept.

33. **Sol (c)** Given $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}, A^{-1} = \dfrac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$

$$cA=\begin{bmatrix}c&0&0\\0&c&c\\0&-2c&4c\end{bmatrix}, dI=\begin{bmatrix}d&0&0\\0&d&0\\0&0&d\end{bmatrix}$$

$\therefore$ By $A^{-1}=\frac{1}{6}\left[A^2+cA+dI\right]$

$\Rightarrow 6=1+c+d,$ [by equality of matrices]

$\therefore (-6,11)$ satisy the relation.

34. **Sol (a)** $|A|=3, AdjA=\begin{pmatrix}1&-2\\0&3\end{pmatrix}; \therefore A^{-1}=\frac{1}{3}\begin{pmatrix}1&-2\\0&3\end{pmatrix}$

$\Rightarrow \left(A^{-1}\right)^3=\frac{1}{27}\begin{pmatrix}1&-2\\0&3\end{pmatrix}^3=\frac{1}{27}\begin{pmatrix}1&-26\\0&27\end{pmatrix}.$

35. **Sol (b)** The matrix is not invertible if $\begin{vmatrix}1&a&2\\1&2&5\\2&1&1\end{vmatrix}=0$

$\Rightarrow 1(2-5)-a(1-10)+2(1-4)=0$

$\Rightarrow -3+9a-6=0 \Rightarrow a=1.$

36. **Sol (d)** $K=|A|;|A|=\begin{vmatrix}3&2&4\\1&2&-1\\0&1&1\end{vmatrix}=11$

37. **Sol (a)** It is obvious.

38. **Sol (d)** From option, check $AA^{-1}=I$

39. **Sol (d)** Let $A=\begin{bmatrix}1&2&-3\\0&1&2\\0&0&1\end{bmatrix} \Rightarrow |A|=1$

$$adj(A)=\begin{bmatrix}1&2&-1\\-2&1&1\\7&-2&1\end{bmatrix}^T$$

Hence, $A^{-1}=\frac{adj(A)}{|A|}$

$\Rightarrow A^{-1}=\begin{bmatrix}1&-2&7\\2&1&-2\\-1&1&1\end{bmatrix}$. Hence, element $A_{13}=7$

40. **Sol (b)** We have, $\left(AA^T\right)=\left(A^T\right)^T A^{T=AA^T}$ [by reversal law]

$\therefore AA^T$ is symmetric matrix.

41. **Sol (b)** $\begin{vmatrix}\lambda&-1&4\\-3&0&1\\-1&1&2\end{vmatrix}\neq 0 \Rightarrow \lambda \neq -17$

42. **Sol (c)** $X=\begin{bmatrix}-x&-y\\z&t\end{bmatrix}; adjX=\begin{bmatrix}t&y\\-z&-x\end{bmatrix}$

$X=\begin{bmatrix}-x&-y\\z&t\end{bmatrix}; adjX=\begin{bmatrix}t&y\\-z&-x\end{bmatrix}$

$\therefore$ Transpose of $adj(X)=\begin{bmatrix}t&-z\\y&-x\end{bmatrix}$

43. **Sol (b)** The given matrix is a skew-symmetric matrix

$[\because A' = -A]$

44. **Sol (b)** $AB = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -3 & -2 \\ 10 & 7 \end{pmatrix}$

$(AB)^T = \begin{pmatrix} -3 & 10 \\ -2 & 7 \end{pmatrix}$

45. **Sol (a)** $A^1 = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

Given $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

Hence, $\alpha = 5$

46. **Sol (b)** $|A| = (ad - bc)$

$\therefore A^{-1} = \frac{1}{(ad - bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

47. **Sol (d)** $|A| = 1 \neq 0$, therefore A is invertible.

Thus (d) is not correct

48. **Sol (c)** $A^2 - A + I = 0$

$\Rightarrow I = A - A^2 \Rightarrow I = A(I - A)$

$\Rightarrow A^{-1}I = A^{-1}(A(I - A)) \Rightarrow A^{-1} = I - A.$

49. **Sol (a)** $\left(B^{-1}A^{-1}\right)^{-1} = \left(A^{-1}\right)^{-1}\left(B^{-1}\right)^{-1} = AB$

[Reversal law of inverses]

$= \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$

50. **Sol (b)** When $a_{ij} = 0$ for $i \neq j$ and a_{ij} is constant for $i \neq j$,

then the matrix $\left[a_{ij}\right]_{n\times n}$ is called a scalar matrix.

51. **Sol (b)** $A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B; A^{-1} = \frac{adjA}{|A|}$

$A^{-1} = \frac{-1}{3}\begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$and\ X = A^{-1}B = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}; X = \frac{1}{3}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$

52. **Sol (a)** $\lambda = \frac{1}{|A|} = \frac{-1}{6}\ \left(\because A^{-1} = \frac{Adj(A)}{|A|}\right)$

53. **Sol (b)** $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$

$adj\ A = \begin{bmatrix} 4 & -2 \\ -3 & 4 \end{bmatrix}$

$|adjA| = (4\times4)-(-3\times-2) = 16-16$

$|adjA| = 10$

54. **Sol (a)**Since $A^2 = O$ (Zero matrix) and 2 is the least $+ve$ integer for which $A^2 = O$. Thus, A is nilpotent of index 2.

55. **Sol (a)**We know that if A, B are n square matrices, then $|AB| = |A||B|$

56. **Sol (c)** $\begin{bmatrix} 4 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $[By\, R_3 \to R_3 - 2R_2]$

$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $[By\, c_1 \to c_1 - 4c_2 - 3c_3]$

$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

[Replace C_1 by C_2 and then Replace C_2 by C_3]

Hence rank of matrix is 2.

57. **Sol (a)**Let A be a skew-symmetric matrix of odd order, say $(2n+1)$ .Since A is skew-symmetric, therefore

$A^T = -A$

$\Rightarrow |A^T| = |-A| \Rightarrow |A^t| = (-1)^{2n+1}|A|$

$\Rightarrow |A^T| = -|A| \Rightarrow |A| = -A|$

$\Rightarrow 2|A| = 0 \Rightarrow |A| = 0$

58. **Sol (a)**Here $|A| \neq 0$. Hence unique solution.

59. **Sol (c)** $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 2I$

$\therefore AB = 2IB = 2B = \begin{bmatrix} 2 & 4 & 6 \\ 0 & 2 & 6 \\ 0 & 0 & 4 \end{bmatrix}$

Therefore $|AB| = \begin{vmatrix} 2 & 4 & 6 \\ 0 & 2 & 6 \\ 0 & 0 & 4 \end{vmatrix} = 2(8) = 16$

$|AB| = 2\times2\times2 = 8, |B| = 1\times1\times2 = 2$

Alter: $\therefore |AB| = |A||B| = 2\times8 = 16$

60. **Sol (c)**From (iii) equation $2(x+y) = 3 or 2.2 = 3 or 4 = 3$ which is not feasible, so given equation has no solution.

61. **Sol (a)** $D = \begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & -2 & 0 \end{vmatrix} = 0$

$D_1 = \begin{vmatrix} 1 & 1 & -1 \\ -2 & 0 & 2 \\ 3 & -2 & 0 \end{vmatrix} = 14 \Rightarrow D_1 \neq 0$

$\therefore D = 0 \, and \, D_1 \neq 0$

hence the system is inconsistent, so it has no solution.

62. **Sol (a)** $|A|=\begin{bmatrix}1&0&1\\2&1&0\\3&2&1\end{bmatrix}=1(1-0)+0+1(4-3)=2$

63. **Sol (d)** $\Delta=\begin{bmatrix}1&-1&1\\2&1&-1\\4&-1&1\end{bmatrix}$

$=1(1-1)+1(2+4)+1(-2-4)=0+6-6=0$

Hence, number of solutions is zero.

64. **Sol (b)** $A=\begin{bmatrix}2&4&5\\4&8&10\\-6&-12&-15\end{bmatrix}_{3\times3}$

$|A|=0,$ then rank cannot be 3.

Considering 2×2 minor, $\begin{bmatrix}2&4\\4&8\end{bmatrix}$ is determinant is zero

Similarly considering

$\begin{bmatrix}4&5\\8&10\end{bmatrix},\begin{bmatrix}4&8\\-6&12\end{bmatrix},\begin{bmatrix}8&5\\-12&10\end{bmatrix},\begin{bmatrix}4&10\\-6&-15\end{bmatrix}$

their determinants is zero. Each rank can not be 2.

Thus rank = 1.

65. **Sol (a)** $|A^3|=125; |A|^3=125=5^3$

$\Rightarrow |A|=5\Rightarrow \alpha^2-4=5\Rightarrow \alpha=\pm3.$

66. **Sol (d)** $\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1&1&1\\1&-2&-2\\1&3&1\end{bmatrix}^{-1}\begin{bmatrix}0\\3\\4\end{bmatrix}$

$$\begin{bmatrix}1&1&1\\1&-2&-2\\1&3&1\end{bmatrix}^{-1}=\frac{1}{16}\begin{bmatrix}4&2&0\\-3&0&3\\5&-2&-3\end{bmatrix}$$

$$\therefore \begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{16}\begin{bmatrix}4&2&0\\-3&0&3\\5&-2&-3\end{bmatrix}\begin{bmatrix}0\\3\\4\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\2\\-3\end{bmatrix}$$

67. **Sol (c)** $A^2=\begin{bmatrix}5&5\alpha&\alpha\\0&\alpha&5\alpha\\0&0&5\end{bmatrix}\begin{bmatrix}5&5\alpha&\alpha\\0&\alpha&5\alpha\\0&0&5\end{bmatrix}$

$$\Rightarrow A^2=\begin{bmatrix}25&25\alpha+5\alpha^2&5\alpha+25\alpha^2+5\alpha\\0&\alpha^2&5\alpha^2+25\alpha\\0&0&25\end{bmatrix}$$

$\therefore |A^2|=625\alpha^2\Rightarrow 625\alpha^2=25\Rightarrow |\alpha|=1/5.$

68. **Sol (b)** $A=\begin{bmatrix}1&3\\2&1\end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 4 & 7 \end{bmatrix}$

and $A^2 - 2A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$. det $\left(A^2 - 2A\right) = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 25.$

69. **Sol (b)** Determinants of unit matrix of any order. =1.

70. **Sol (c)** $|A| = 12, |AdjA| = |A|^{n-1}$

$= |A|^{3-1} = |A|^2 = (12)^2 = 144$