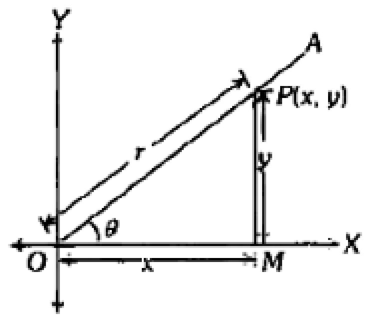
Percentile Classes

Trigonometry / Height n Distance

Trigonometrically ratios or functions

In the right angled triangle OMP, we have base $= OM = x$, perpendicular $= PM = y$ and hypotenuse $= OP = r$.

We define the following trigonometric ratio which are also known as trigonometric function.



$$\sin\theta = \frac{\text{perpendicular}}{\text{Hypotenues}} = \frac{y}{r}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenues}} = \frac{x}{r}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{y}{x}$$

$$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{x}{y}$$

$$\sec\theta = \frac{\text{Hypotenues}}{\text{Base}} = \frac{r}{x}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenues}}{\text{Perpendicular}} = \frac{r}{y}$$

(1) Relation between trigonometric ratios (functions)

(i) $\sin\theta . \text{cosec}\,\theta = 1$

(ii) $\tan\theta . \cot\theta = 1$

(iii) $\cos\theta . \sec\theta = 1$

(iv) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

(v) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

(2) Fundamental trigonometric identities

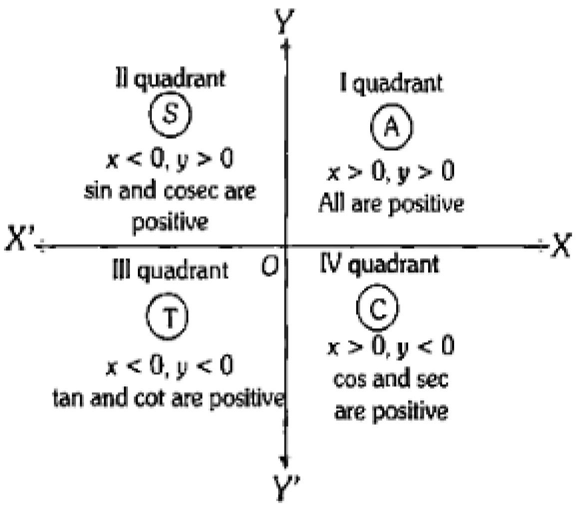
(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \text{cosec}^2\,\theta$

(3) Sign of trigonometrically ratios or functions : Their signs depends on the quadrant in which the terminal side of the angle lies.

In brief: A crude aid to memories the signs of trigonometrically ratio in different quadrant. **"Add Sugar To Coffee".**



Trigonometrically ratios for various angles

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\sin\theta$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	-1	0
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0	-1	0	1
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0	∞	0

Formulae for the trigonometric ratios of sum and differences of two angles

(1) $\sin(A+B)=\sin A\cos B+\cos A\sin B$

(2) $\sin(A-B)=\sin A\cos B-\cos A\sin B$

(3) $\cos(A+B)=\cos A\cos B-\sin A\sin B$

(4) $\cos(A-B)=\cos A\cos B+\sin A\sin B$

(5) $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$

(6) $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

(7) $cat(A+B)=\dfrac{\cot A\cot B-1}{\cot A+caotB}$

(8) $cat(A-B)=\dfrac{\cot A\cot B+1}{\cot A-caotB}$

(9) $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B=\cos^2 B-\cos^2 A$

(10) $\cos(A+B)\cos(A-B)=\cos^2 A-\sin^2 B=\cos^2 B-\sin^2 A$

Formulae to transform the product into sum or difference

(1) $2\sin A\cos B=\sin(A+B)+\sin(A-B)$

(2) $2\cos A\sin B=\sin(A+B)-\sin(A-B)$

(3) $2\cos A\cos B=\cos(A+B)+\cos(A-B)$

(4) $2\cos A\cos B=\cos(A-B)-\cos(A+B)$

Trigonometric ratio of multiple of an angle

(1) $\sin 2A=2\sin A\cos A=\dfrac{2\tan A}{1+\tan^2 A}$

(2) $\cos 2A=2\cos^2 A-1=1-2\sin^2 A$

$=\cos^2 A-\sin^2 A=\dfrac{1-\tan^2 A}{1+\tan^2 A}$; where $A\neq(2n+1)\dfrac{\pi}{4}$.

(3) $\tan 2A=\dfrac{2\tan A}{1-\tan^2 A}$

$$= 4\sin(60^\circ - A).\sin A.\sin(60^\circ + A)$$

(5) $\cos 3A = 4\cos^3 A - 3\cos A$

$$= 4\cos(60^\circ - A).\cos A.\cos(60^\circ + A)$$

(6) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} = \tan(60^\circ - A).\tan A \tan(60^\circ + A)$

where $A \neq n\pi + \pi/6$

Maximum and minimum value of a

$a\cos\theta + b\sin\theta$

Let $a = r\cos\alpha .....(i)$ and $b = r\sin\alpha .....(ii)$

Squaring and adding (i) and (ii),then $a^2 + b^2 = r^2 \quad or,\ r = \sqrt{a^2 + b^2}$

$\therefore a\sin\theta + b\cos\theta = r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) = r\sin(\theta + \alpha)$

But $-1 \leq \sin\theta < 1$ So, $-1 \leq \sin(\theta + \alpha) \leq 1;$

Then $-r \leq r\sin(\theta + \alpha) \leq r$

Hence, $-\sqrt{a^2 + b^2} \leq a\sin\theta + b\cos\theta \leq \sqrt{a^2 + b^2}$

Exercise 01

Trigonometry Ratios

1. If $\sin\theta = \dfrac{\sqrt{3}}{2}$ and $\cos\theta = 1/2$ then tan θ is equal to

   (a) $\sqrt{3}$ (b) $\dfrac{1}{\sqrt{3}}$ (c) 1 (d) 2

2. If $\sin\theta = \dfrac{1}{\sqrt{2}}$ and $\cos\theta = \dfrac{1}{\sqrt{2}}$ then cot θ is equal to

   (a) 2 (b) $\dfrac{1}{\sqrt{2}}$ (c) 1 (d) $\sqrt{2}$

3. If $\tan\theta = 5/12$ and θ is acute. Then cosec θ is equal to

   (a) 13/5 (b) 5/13 (c) 12/5 (d)12/13

4. If $\operatorname{cosec}\theta = \dfrac{17}{8}$ and θ is acute, then $\sec\theta$ is equal to

   (a) 15/17 (b) 8/15 (c) 15/8 (d) 17/15

5. If $\sin\theta = \dfrac{1}{2}$ and then the value of $(\tan\theta + \sec\theta)$ is equal to

   (a) $\sqrt{3}$ (b) $\dfrac{\sqrt{3}}{2}$ (c) $\dfrac{2}{\sqrt{3}}$ (d) $\dfrac{1}{\sqrt{3}}$

6. If $\sin 30° + \tan 45° - \cos 60°$ is equal to

(a) 2 (b) 1/2 (c) 1 (d) 1/4

7. If $\sin^2 45° + \tan 45° - \cos 45°$ is equal to

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) 2 (d) $\sqrt{2}$

8. If $\sin\theta = \frac{15}{17}$ and θ is acute, find $\left(\frac{8\sin\theta - 3\cos\theta}{8\sin\theta + 3\cos\theta}\right)$

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{3}{5}$

9. If $\tan\theta = \frac{1}{2}$ and θ is acute, find $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$ is equal to

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) $\frac{2}{5}$ (d) $\frac{4}{5}$

10. If $\tan\theta + \cot\theta = 4$ Find $\left(\tan^2\theta + \cot^2\theta\right)$

(a) 18 (b) 12 (c) 16 (d) 14

11. Find the value of $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

(a) 2 (b) 1 (c) 0 (d) 4

12 If $x = a\sin\theta$ and $y = a\cos\theta$, then .

(a) $x^2 + y^2 = a^2$ (b) $x^2 - y^2 = a^2$ (c) $x + y = a^2$ (d) $x - y = a^2$

13 If $A = 60^\circ$ *and* $B = 45^\circ$ Find the value of tan 15°.

(a) $1 + 2\sqrt{3}$ (b) $2 + \sqrt{3}$ (c) $2 - \sqrt{3}$ (d) $1 - 2\sqrt{3}$

14. If tan $(A - B) = \frac{1}{\sqrt{3}}$ *and* $A = \frac{1}{\sqrt{2}}$ A and B are acute . Find B

(a) $45°$ (b) $15°$ (c) $30°$ (d) $60°$

15. Evaluate $\frac{1 - \tan^2 30°}{1 + \tan^2 30°}$

(a) 1/2 (b) 2 (c) 1/3 (d) 3

16. $\sqrt{2}\sin 45^\circ - \sqrt{2}\cos 45^\circ$ is equal to

(a) 1 (b) 0 (c) $\sqrt{2}$ (d) 2

17. $\sin 30° \cos 60° + \cos 30° \sin 60°$ is equal to

18. $\cos 60^\circ \sin 30^\circ - \sin 60^\circ \sin 30^\circ$ is equal to

(a) $\sqrt{2}$ (b) 2 (c) 1 (d) 0

19. $\sin 60^\circ \cos 60^\circ \tan 60^\circ$ is equal to

(a) 3/2 (b) 1/4 (c) 3/4 (d) 1/2

20. $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$ is equal to

(a) 0 (b) $\sqrt{2}$ (c) 1 (d) $\sqrt{3}$

21. $\sqrt{\frac{1-\cos^2 45^\circ}{1-\sin^2 45^\circ}}$ is equal to

(a) 2 (b)1 (c) 4 (d) $\sqrt{2}$

22. If $15\sin\theta = 8\cos\theta$ find the value of $15\sin\theta = 8\cos\theta$

(a) $\frac{1}{17}$ (b) $\frac{1}{15}$ (c) $\frac{1}{8}$ (d) $\frac{1}{12}$

23. If $\sec\theta + \tan\theta = 4$, then the find the value $\sin\theta$.

(a) $\frac{14}{17}$ (b) $\frac{12}{17}$ (c) $\frac{15}{17}$ (d) $\frac{16}{17}$

24. If $\sin\theta = \frac{a}{b}$ *and* $\cos\theta = \frac{c}{d}$, then $\cot\theta$ is equal to.

(a) $= \frac{ad}{bc}$ (b) $= \frac{bc}{ad}$ (c) $= \frac{ac}{bd}$ (d) $= \frac{bd}{ac}$

25. If $\cos ec\theta = \sec\theta$, then the value of θ is equal to.

(a) 90° (b) 60° (c) 30° (d) 45°

26. Find $\frac{\cos ec\theta}{\sqrt{\cos ec\theta - 1}}$

(a) $\sec\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) $\cot\theta$

27. If $x = 2\sec\theta$ *and* $y = 2\tan\theta$, *then* $x^2 - y^2$ is equal to

(a) 4 (b) 2 (c) 1 (d) 8

28. Find $\frac{\sqrt{1-\cos^2}}{\cos\theta}$

(a) $\cot\theta$ (b) $\tan\theta$ (c) $\sin\theta$ (d) $\cos ec\theta$

29. Find $\frac{\tan\theta \times \cos\theta}{\sin\theta}$

(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) 1

30. Find $\left[1+\tan\theta^2 60^\circ\right]^3$

(a) 8 (b) 256 (c) 16 (d) 64

31. Find $\frac{\sin 20^\circ}{\cos 70^\circ}+\frac{\cos 75^\circ}{\sin 25^\circ}$

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

32. Find $\frac{\tan 18^\circ}{\cot 72^\circ}-\frac{\cot 72^\circ}{\tan 18^\circ}$

(a) 2 (b) 1 (c) 0 (d) $\sqrt{2}$

33. $\sin^2 24^\circ+\sin^2 66^\circ$ is equal to.

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) 2

34. $\cos^2 40^\circ+\cos^2 50^\circ$ is equal to

(a) 1 (b) $\sqrt{3}$ (c) 3 (d) $3\sqrt{3}$

35. $\sin^2 10^\circ+\sin^2 80^\circ-\tan^2 45^\circ$

(a) 0 (b) 1 (c) 2 (d) 4

36. $\cos 36^\circ-\sin 54^\circ$ is equal to

(a) $\sqrt{2}$ (b) 1 (c) 0 (d) $2\sqrt{2}$

37. If $(\cos ec\theta+\cot\theta)=3$, then $(\cos ec\theta-\cot\theta)$ is equal to

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\sqrt{3}$

38. If $(\cos\theta+\sec\theta)=2, then \left(\cos\theta^2+\sec\theta^2\right)$ is equal to.

(a) $\frac{1}{2}$ (b) 2 (c) 4 (d) $\frac{1}{4}$

39. Find the value of $\left(32\cot^2 45^\circ-8\sec^2 60^\circ+4\cot^3 30^\circ\right)$

(a) $2\sqrt{3}$ (b) $6\sqrt{3}$ (c) $4\sqrt{3}$ (d) $12\sqrt{3}$

40. Find then value of $\sin 0^\circ+\cos 30^\circ+\cos ec 60^\circ+\cot 90^\circ$

Exercise 01

Solutions

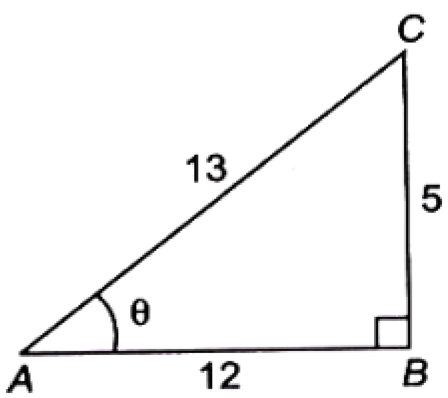
1. **Sol. (a)**

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) = \left(\frac{\sqrt{3}}{2}\times\frac{2}{1}\right) = \sqrt{3}$$

2. **Sol. (a)**

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right) = \left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{1}\right) = 1$$

3. **Sol. (a)**



$$\tan\theta = \frac{5}{12} = \frac{BC}{AB}$$

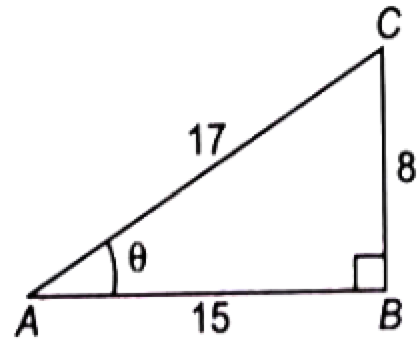
$$AC^2 = AB^2 + BC^2 = (12)^2 + (5)^2$$

$$= 144 + 25 = 169 = 13^2$$

$$\Rightarrow AC = 13$$

$$cpses\theta = \frac{AC}{BC} = \frac{13}{5}$$

4. **Sol. (d)**



$$\cos ec\theta = \frac{17}{8} = \frac{AC}{BC}$$

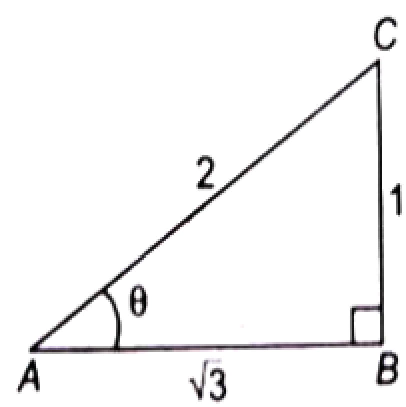
$$= (AB)^2 = \left(AC^2 - BC^2\right)$$

$$= \left(17^2 - 8^2\right) = 225 = (15)^2$$

$$\Rightarrow AB = 15$$

$$\therefore \sec\theta = \frac{AC}{AB} = \frac{17}{15}$$

5. **Sol. (a)**



$$\sin\theta = \frac{1}{2} = \frac{BC}{AC}$$

$$AB^2 = \left(AC^2 - BC^2\right)$$

$$= \left(2^2 - 1^2\right)$$

$AB=\sqrt{3}$

$$\tan\theta+\sec\theta=\left(\frac{1}{\sqrt{3}}+\frac{2}{\sqrt{3}}\right)=\frac{3}{\sqrt{3}}=\sqrt{3}$$

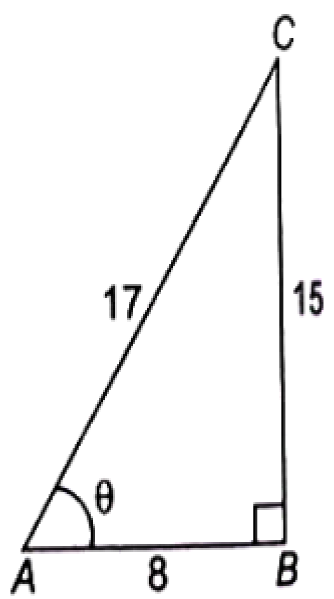
6. **Sol.** **(c)** Given expression $=\frac{1}{2}+1=1$

7. **Sol.** **(a)** Given expression

$$=\left(\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}+1\times-\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right)$$

$$=\left(\frac{1}{2}+1-\frac{1}{2}\right)=1$$

8. **Sol.** **(b)**



$$\sin\theta=\frac{15}{17}=\frac{BC}{AC}$$

$$AB^2=(AC)^2-(BC)^2=(17)^2-(15)^2$$

$$=289-225=64=8^2$$

$$\Rightarrow AB=8$$

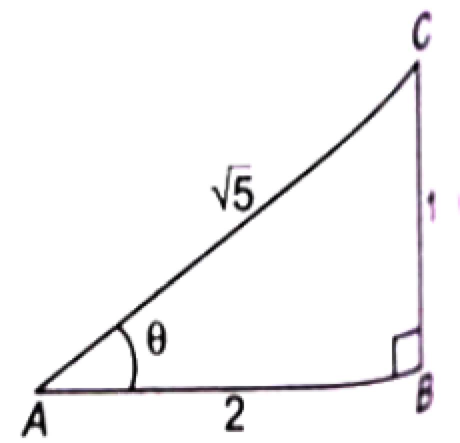
Now, $$\frac{8\sin\theta-3\cos\theta}{8\sin\theta+3\cos\theta}=\frac{8\frac{\sin\theta}{\cos\theta}-\frac{3\cos\theta}{\cos\theta}}{\frac{8\sin\theta}{\cos\theta}+\frac{3\cos\theta}{\cos\theta}}$$

$$=\frac{8\tan\theta-3}{8\tan\theta+3}$$

$$=\frac{8\times\frac{15}{8}-3}{8\times\frac{15}{8}+3}$$

$$=\frac{15-3}{15+3}=\frac{12}{18}=\frac{2}{3}$$

9. **Sol.** **(b)**



$$\tan\theta=\frac{1}{2}=\frac{BC}{AB}$$

$$\Rightarrow BC=1 \text{ and } AB=2$$

$$AC^2=(AB)^2+(BC)^2=(2)^2+(1)^2=5$$

$$\Rightarrow AC=\sqrt{5}$$

$$\operatorname{cosec}\theta=\frac{AC}{BC}=\sqrt{5}$$

$$\sec\theta=\frac{AC}{AB}=\frac{\sqrt{5}}{2}$$

$$\therefore\frac{\operatorname{cosec}^2\theta-\sec^2\theta}{\operatorname{cosec}^2\theta+\sec^2\theta}=\frac{(\sqrt{5})^2-\left(\frac{\sqrt{5}}{2}\right)^2}{(\sqrt{5})^2+\left(\frac{\sqrt{5}}{2}\right)^2}$$

$$=\frac{5-\frac{5}{4}}{5+\frac{5}{4}}=\frac{\frac{15}{4}}{\frac{25}{4}}$$

$$=\frac{15}{25}=\frac{3}{5}$$

10. **Sol.** **(d)** Given expression

$=\tan\theta+\cot\theta=4$

Squaring both side

$$(\tan\theta+\cot\theta)^2=4^2$$

$$\Rightarrow \tan^2\theta + \cot^2\theta = 16 - 2 = 14$$

11. **Sol.** **(a)** Given expression

$(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$$\left(\sin\theta^2 + \cos\theta^2 + 2\sin\theta\cos\theta\right)$$

$$+\left(\sin\theta^2 + \cos\theta^2 - 2\sin\theta\cos\theta\right)$$

$$2\left(\sin\theta^2 + \cos\theta^2\right) = 2\times 1 = 2$$

12 **Sol.** **(a)** Given expression $x = a\sin\theta$ and $y = a\cos\theta$

$$\sin\theta = \frac{x}{a} \textit{ and } \cos\theta = \frac{y}{a}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 = 1$$

$$\Rightarrow x^2 + y^2 = a^2$$

13 **Sol.** **(c)** We know that

$$\tan(A - B) = \frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ}\left[A = 60^\circ, B = 45^\circ\right]$$

$$\tan 15^\circ = \frac{\sqrt{3} - 1}{1 + \sqrt{3}\times 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}\times\frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{\left(\sqrt{3} - 1\right)^2}{2}$$

$$= \frac{3 + 1 - 2\sqrt{3}}{2}$$

$$= \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

$$\therefore \tan 15^\circ = 2 - \sqrt{3}$$

14. **Sol.** **(b)** $\tan 30^\circ = \dfrac{1}{\sqrt{3}}$

$$\Rightarrow A - B = 30^\circ \ldots (i)$$

1

$$\Rightarrow A = 45^\circ \ldots\text{(ii)}$$

From Eqs. (i) and (ii) we get $B = 15^\circ$

Hence, $B = 15^\circ$

15. **Sol.** **(b)** Given expression $= \dfrac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$

$$= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} \quad \left(\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right)$$

$$= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{2}{3}\times\frac{3}{4} = \frac{1}{2}$$

16. **Sol.** **(b)** Given expression

$\sqrt{2}\sin 45^\circ - \sqrt{2}\cos 45^\circ$

$$= \sqrt{2}\times\frac{1}{\sqrt{2}} - \sqrt{2}\times\frac{1}{\sqrt{2}}\left[\sin 45^\circ = \frac{1}{\sqrt{2}} \textit{ and } \cos 45^\circ = \frac{1}{\sqrt{2}}\right]$$

$= 1 - 1 = 0$

17. **Sol.** **(a)**

$\sin A\cos B + \cos A\sin B = \sin(A + B)$

$$\sin 30^\circ\cos 60^\circ + \cos 30^\circ\sin 60^\circ$$
$$= \sin(30^\circ + 60^\circ)$$

$$\left[A = 30^\circ \textit{ and } B = 60^\circ\right]$$

$$= \sin 90^\circ = 1$$

18. **Sol.** **(a)**

$\cos A\cos B + \sin A\sin B = \cos(A + B)$

$$\cos 60^\circ\sin 30^\circ - \sin 60^\circ\sin 30^\circ =$$
$$\cos(60^\circ + 30^\circ)$$

$$\left[A = 60^\circ \textit{ and } B = 30^\circ\right]$$

$$= \cos 90^\circ = 1$$

19. **Sol.** **(a)**

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \cos = 60^\circ = \frac{1}{2} \textit{ and } \tan = 60^\circ = \sqrt{3}$$

Hence, $\sin 60^\circ \cos 60^\circ \tan 60^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} \times \sqrt{3} = \frac{3}{4}$$

20. **Sol.** **(d)** $\frac{2\tan A}{1+\tan^2 A} = \tan 2A$

$$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \tan 60^\circ$$

$[A = 30^\circ]$

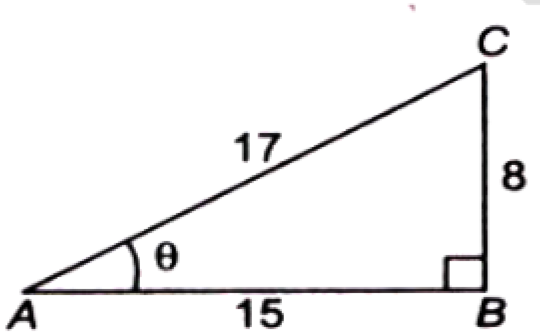
$= \sqrt{3}$

21. **Sol.** **(b)** $\sin^2\theta + \cos^2\theta = 1$

$1-\cos^2\theta = \sin^2\theta$ *and* $1-\sin^2\theta = \cos^2\theta$

$$\therefore \sqrt{\frac{1-\cos^2 45^\circ}{1-\sin^2 45^\circ}} = \sqrt{\frac{\sin^2 45^\circ}{\cos^2 45^\circ}} = \sqrt{\frac{\left(\frac{1}{\sqrt{2}}\right)^2}{\left(\frac{1}{\sqrt{2}}\right)^2}} = \sqrt{1} = 1$$

22. **Sol.** **(a)**



$$15\sin\theta = 8\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{8}{15} \Rightarrow \tan\theta = \frac{8}{15}$$

$$\tan\theta = \frac{8}{15} = \frac{BC}{AB}$$

$$AC^2 = AB^2 + BC^2 = (15)^2 + (8)^2$$

$$= 225 + 64 = 289 = (17)^2$$

$$\Rightarrow AC = 17$$

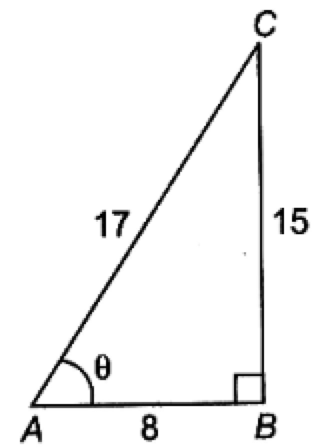
$$\sin\theta = \frac{BC}{AC} = \frac{8}{17} \text{ and}$$

$$\cos\theta = \frac{AB}{AC} = \frac{15}{17}$$

Now,

$$2\sin\theta - \cos\theta = \left(2\times\frac{8}{17}\times\frac{15}{17}\right) = \left(\frac{16}{17}-\frac{15}{17}\right) = \frac{1}{17}$$

23. **Sol.** **(c)**



We know that $1+\tan^2\theta = \sec^2\theta$

$$\therefore \sec^2\theta - \tan^2\theta = 1$$

$$\Rightarrow (\sec\theta+\tan\theta)(\sec\theta-\tan\theta) = 1$$

$$\Rightarrow (\sec\theta-\tan\theta) = \frac{1}{(\sec\theta+\tan\theta)} = \frac{1}{4}$$

$$\Rightarrow (\sec\theta-\tan\theta) = \frac{1}{4} \text{ ....(i)}$$

And $\sec\theta + \tan\theta = 4$ ....(ii)

Adding Eqs. (i) and Eqs. (ii) ,we get =

$$\sec\theta = \frac{17}{8}$$

$$\sec\theta = \frac{17}{8} = \frac{AC}{AB}$$

$$\Rightarrow BC^2 = AC^2 - AB^2 = (17)^2 - (8)^2$$

= 289 – 64 = 225

$$\Rightarrow BC^2 = (15)^2 \Rightarrow BC = 15$$

$$\sin\theta = \frac{BC}{AC} = \frac{15}{17}$$

24. **Sol.** **(b)** $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{c}{d}}{\frac{a}{b}} = \frac{c}{d}\times\frac{b}{a} = \frac{bc}{ad}$

25. **Sol.** **(d)** Given expression $\cos ec\theta = \sec\theta$

$$\Rightarrow \frac{1}{\sin\theta} = \frac{1}{\cos\theta}$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = 1$$

$$\tan\theta = 1 \Rightarrow \theta = 45°$$

26. **Sol.** **(a)** We know that

$1+\cot^2\theta = \cos ec^2\theta$

$$\Rightarrow \cos ec^2\theta - 1 = \cot^2$$

Given expression $\dfrac{\cos ec\theta}{\sqrt{\cos ec\theta - 1}}$

$$\frac{\cos ec\theta}{\sqrt{\cot^2\theta}} = \frac{\cos ec\theta}{\cot\theta}$$

$$= \frac{\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} = \frac{1}{\sin\theta} \times \frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta$$

27. **Sol.** **(a)** Given,

$x = 2\sec\theta \Rightarrow x^2 = 4\sec^2$ *and*

$$y = 2\tan\theta$$

$$\Rightarrow y^2 = 4\tan^2\theta$$

Then,

$$x^2 - y^2 = 4\sec^2\theta - 4\tan^2\theta = 4\left(\sec^2 - \tan^2\right)$$

$$= 4 \times 1 = 4\left[\sec^2 - \tan^2 = 1\right]$$

28. **Sol.** **(b)** We know that $= \sin^2 + \cos^2 = 1$

$$\Rightarrow 1 - \cos^2\theta = \sin^2\theta$$

Given expression

$$= \frac{\sqrt{1 - \cos^2\theta}}{\cos\theta} = \frac{\sqrt{\sin^2\theta}}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

29. **Sol.** **(d)** Given expression

$$= \frac{\tan\theta \times \cos\theta}{\sin\theta} = \frac{\frac{\sin\theta}{\cos\theta} \times \cos\theta}{\sin\theta} = \frac{\sin\theta}{\sin\theta} = 1$$

$$\Rightarrow 1 + \tan^2 60^\circ = \sec^2 60^\circ = (2)^2 = 4\left[\therefore \sec 60^\circ = 2\right]$$

Given expression $\left[1 + \tan^2 60^\circ\right]^3 = 4^3 = 64$

31. **Sol.** **(b)**

$$\frac{\sin 20°}{\sin 70°} = \frac{\sin\left(90^\circ - 60^\circ\right)}{\cos 70°} = \frac{\cos 70°}{\sin 70°} = 1$$

$$\frac{\cos 75°}{\sin 25°} = \frac{\cos\left(90^\circ - 25°\right)}{\sin 25°} = \frac{\sin 25°}{\sin 25°} = 1$$

$$\Rightarrow \frac{\sin 20°}{\cos 20°} + \frac{\cos 75°}{\sin 25°} = (1+1) = 2$$

32. **Sol.** **(c)**

$$= \frac{\tan 18^\circ}{\cot 72°} = \frac{\tan\left(90° - 72°\right)}{\cot 72°} = \frac{\cot 72°}{\cot 72°} = 1$$

$$= \frac{\cot 72^\circ}{\tan 18°} = \frac{\cot\left(90° - 18°\right)}{\tan 18°} = \frac{\tan 18°}{\tan 18°} = 1$$

$$\Rightarrow \frac{\tan 18°}{\cot 72°} - \frac{\cot 72°}{\tan 18°} = (1-1) = 0$$

33. **Sol.** **(c)** $\sin^2 24° + \sin^2 66°$

$$= \sin^2\left(90° - 60°\right) + \sin^2 66°$$

$$= \cos^2 66° + \sin^2 66°$$

= 1 **[**$\therefore \sin^2\theta + \cos^2\theta = 1$**]**

34. **Sol.** **(a)**

$$\cos^2 40° + \cos^2 50° = \cos^2\left(90° - 50°\right) + \cos^2 50°$$

$$= \cos^2 66° + \sin^2 50$$

= 1 **[**$\therefore \sin^2\theta + \cos^2\theta = 1$**]**

35. **Sol.** **(a)** $\sin^2 10° + \sin^2 80° - \tan^2 45°$

$$= \sin^2\left(90° - 80°\right) + \sin^2 80° - \tan^2 45°$$

$$= \left(\cos^2 80° - \sin^2 80°\right) - \tan^2 45°$$

$$= \left(\sin^2 80° + \cos^2 80°\right) - \tan^2 45°$$

$= 1 - 1 = 0 \quad [\because \sin^2\theta + \cos^2\theta = 1\, and\, \tan 45° = 1]$

36. **Sol.** **(c)**

$\cos 36° - \sin 54° = \cos 36° - \sin(90° - 36°)$

$= \cos 36° - \cos 36° = 0$

37. **Sol.** **(b)** We know that

$\cos ec^2\theta - \cot^2\theta = 1$

$\Rightarrow (\cos ec\theta + \cot\theta)(\cos ec\theta - \cot\theta) = 1$

$\Rightarrow 3 \times (\cos ec\theta - \cot\theta) = 1$

$\Rightarrow \cos ec\theta - \cot\theta = \frac{1}{3}$

38. **Sol.** **(b)** Given, $(\cos\theta + \sec\theta)^2 = 2$

$(\cos\theta + \sec\theta)^2 = 2^2 = 4$

$\Rightarrow \cos^2\theta + \sec^2\theta + \cos\theta \times \sec\theta = 4$

$\Rightarrow \cos^2\theta + \sec^2\theta + 2\cos\theta \times \frac{1}{\cos\theta} = 4$

$\Rightarrow \cos^2\theta + \sec^2\theta - 2 = 4$

$\Rightarrow \cos^2\theta + \sec^2\theta = 2$

39. **Sol.** **(d)**

$\cot 45° = 1, \sec 60° = 2, \cot 30° = \sqrt{3}$

Given $32\cot^2 45° - 8\sec 60° + 4\cot^3 30°$

$= 32(1)^2 - = (2)^2 + 4\left(\sqrt{3}\right)^3$

$= 32 - (8 \times 4) + \left(4 \times 3\sqrt{3}\right)$

$= 32 - 32 + 12\sqrt{3} = 12\sqrt{3}$

40. **Sol.** **(c)**

$\sin 0°, \cos 30° = \frac{\sqrt{3}}{2}, \cos ec60° = \frac{2}{\sqrt{3}}\, and\, \cot 90° = 0$

Given, $\sin 0° + \cos 30° + \cos ec60° + \cot 90°$

$= 0 + \frac{\sqrt{3}}{2} + \frac{2}{\sqrt{3}} + 0 = \frac{\sqrt{3}}{2} + \frac{2}{\sqrt{3}}$

$= \frac{7}{2\sqrt{3}} = \frac{7}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{6}$

Exercise 02

(Heights and Distances)

1. A tower stands vertically on the ground. From a point on the ground, which is 18 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be $30°$. Find the height of the tower,

(a) m $6\sqrt{3}$ (b) $4\sqrt{3}$ m (c) $8\sqrt{3}$ m (d) $2\sqrt{3}$ m

2. A circus artist is climbing a 15 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is $60°$.

(a) 4.5 m (b) 7.5 m (c) 2.5 m (d) 9.5 m

3. Find the angle of elevation of the top of a tower from a point on the ground which is 10 m away from the foot of the tower whose height is $10\sqrt{3}$ m.

(a) $45°$ (b) $60°$ (c) $30°$ (d) $75°$

(a) $18\sqrt{3}$ m (b) $40\sqrt{3}$ m (c) $20\sqrt{3}$ m (d) $36\sqrt{3}$ m

5. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is $45°$. From another point 30 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is $30°$. Find the width of the canal. (Take $\sqrt{3}$ = 1.73)

(a) 46.15 m (b) 40.95 m (c) 42.35 m (d) 44.25 m

6. As observed from the top of a 54 m high lighthouse from the sea-level, the angle of depression of a ship is $60°$. Find the distance between the ship and the light house.

(a) $18\sqrt{3}$ m (b) $18\sqrt{2}$ m (c) $20\sqrt{3}$ m (d) $15\sqrt{3}$ m

7. From the top of a 6 m high building the angle of elevation of the top of a cable tower is $30°$ and the angle of depression of its foot is $60°$. Find the height of the tower,

(a) 16 m (b) 9 m (c) 12 m (d) 8 m

8. Over a tower AB of height 10 m, there is a flag staff BC. A B and BC subtend equal angles of magnitude, $30°$ at a point 'P' certain distance from the foot 'A' of the tower. Find the height of the flag staff,

(a) 15 m (b) 20 m (c) 24 m (d) 12 m

9. From the top of a tower 300 m high, the angles of depression of the top of the building and its bottom are observed to be $45°$ and $60°$ respectively. Find the height of the building. (Take $\sqrt{3}$ = 1.73)

(a) 145m (b) 127 m (c)110 m (d) 160 m

10. The upper part of a tree is broken by wind into two parts makes an angle of $30°$ with the ground. The top of the tree touches the ground at a distance of 15 m from the foot of the tree. Find the height of the tree before it was broken. (Take $\sqrt{3}$ = 1.73)

(a) 21.45m (b) 25.95 m (c) 27.25m (d) 28.15m

11. An aeroplane at an altitude of 3000 m observes the angles of depression of opposite points on the two banks of a river to be $45°$ and $60°$ respectively. Find the width of the river in metre,

(a) 4730 (b) 4430 (c) 4150 (d) 4650

12. Find the height of a chimney, when it is observed that on walking towards it 60 m in a horizontal line through its base, the elevation of its top changes from $30°$ to $45°$.

(a) $18\left(\sqrt{3}+1\right)$ m (b) $24\left(\sqrt{3}+1\right)$ m (c) $30\left(\sqrt{3}+1\right)$ m (d) $36\left(\sqrt{3}+1\right)$ m

13. From the ground and first floor of a building, the angles of elevation of the top of the spire of a church was found to be $60°$ and $45°$ respectively. The first floor is 6 m high. Find height of the spire. (Take $\sqrt{3}$ = 1,73)

(a) 12.24 m (b) 15.29 m (c) 18.24 m (d) 14.19 m

14. A glider is flying at an altitude of 3600 m. The angle of depression of the control tower of the airport from the glider is $30°$. What is the horizontal distance between the glider and the control tower?

(a) 1200 $\sqrt{3}$ m (b) 1200 m (c) 600 $\sqrt{3}$ m (d) 600 m

15. Two towers are standing on a level ground. From a point on the ground midway between them, the angles of elevation are $60°$ and $30°$ respectively. If the height of the first tower is 45 m. then the height of the second tower is (a) 15m (b) 30m (c) $15\sqrt{3}$ m (d) $30\sqrt{3}$ m

16. The angle of elevation of the top of an unfinished tower at a point 80 m from its base is $45°$. Then the height of the tower must be raised so that, angle of elevation at the same point is $60°$, Find the new height of the tower. (Take $\sqrt{3}$ = 1.73)

(a) 138.4 m (b) 126.8 m (c) 116.2m (d) 142.6m

17. From the top of a hill 240 m high, the angles of depression of the top and bottom of a pillar are $30°$ and $60°$ respectively. Find the height of the pillar,

(a) 200 m (b) 180 m (c) 160 m (d) 120 m

18. There is a flag staff on top of a building. The height of the building being 10 m. At a point certain distance away from the foot of the building the angles of elevation to the top and bottom of flag staff are $60°$ and $30°$ respectively. Find the height of the flag staff,

(a) 20 m (b) 5 m (c) 15 m (d) 25 m

19. The angle of elevation of a tower from a point 300 m above a lake is $30°$ and the angle of depression of its reflection in the lake is $60°$. Find the height of the tower,

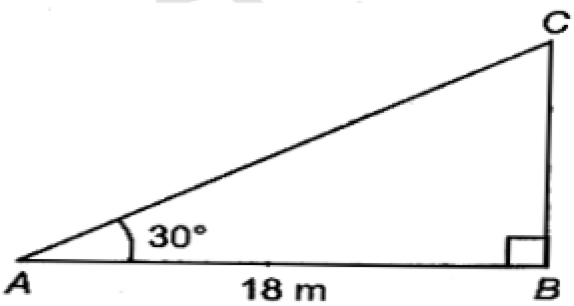
(a) 600 m (b) 450 m (c) 200 m (d) 750 m

Exercise 02 (Solutions)

1.

Sol. **(a)** Let BC be the height of the tower. Then AB = 18 m

And $\angle BAC = 30°$. Let BC = h meters.

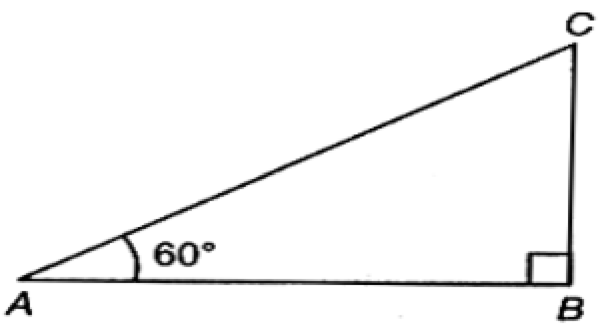


$$\tan 30° = \frac{BC}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{18}$$

$$\Rightarrow h = \frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow h = 6\sqrt{3}m$$

2. **Sol.** **(b)**



Let BC be the pole and AC is the rope

Then, AC = 15 m

And $\angle BAC = 60°$

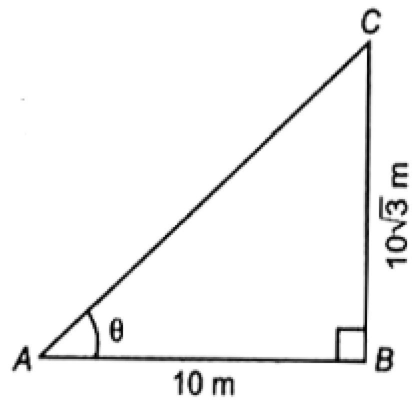
Let BC = h meters.

In right ΔABC,

$$\Rightarrow \frac{1}{2} = \frac{h}{15} \Rightarrow h = \frac{15}{2} = 7.5m$$

Hence, the height of pole is 7.5 m.

3. **Sol. (b)**



Let BC be the height of the tower and AB be the point

On the ground away from the tower. Let θ be the angle of

Elevation. Then AB = 10 m,

$BC = 10\sqrt{3}m$ and $\angle BAC = \theta$

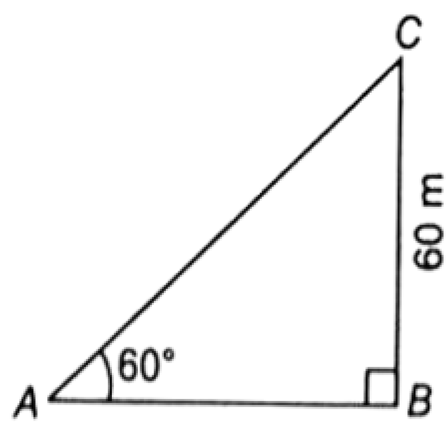
In right ΔABC

$$\Rightarrow \tan\theta = \frac{BC}{AB} = \frac{10\sqrt{3}}{10} = \sqrt{3}$$

$$\Rightarrow \theta = 60°$$

Hence, the angle of elevation is $60°$

4. **Sol. (b)**



Let AC be the length of the string and BC be

The height of the kite above the ground

Then $BC = 60m, \angle BAC = 60°$ and

Let AC = x meters

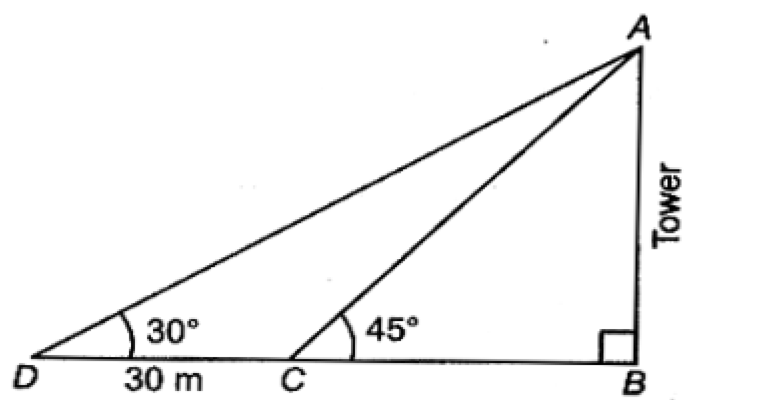
In right $\Delta ABC = \sin 60° = \frac{BC}{AC}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{x}$$

$$\Rightarrow x = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 40\sqrt{3}m$$

Hence, the length of the string is $40\sqrt{3}m$

5. **Sol. (b)** Let AB be the TV tower and BC the width of th canal.



Let, $DC = 30m, \angle BCA = 45°\, and\, \angle CDA = 30°$

Let $AB = h\, metre$

In right $\Delta ABC, \tan 45° = \frac{AB}{BC}$

$$\Rightarrow 1 = \frac{h}{BC} \Rightarrow BC = h$$

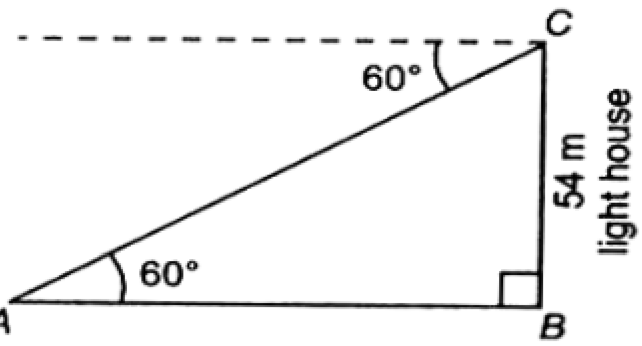
In right

$$\Delta ABD, \tan 30° = \frac{AB}{BD} = \frac{AB}{BC + CD} = \frac{h}{30 + h}$$

$$\Rightarrow h = \frac{30}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = 15(\sqrt{3}+1) = 40.90m$$

Hence, the width of the canal = BC = 40.90m

6. **Sol.** **(a)** Let BC be the light house and AB the distance between the ship and the light house.



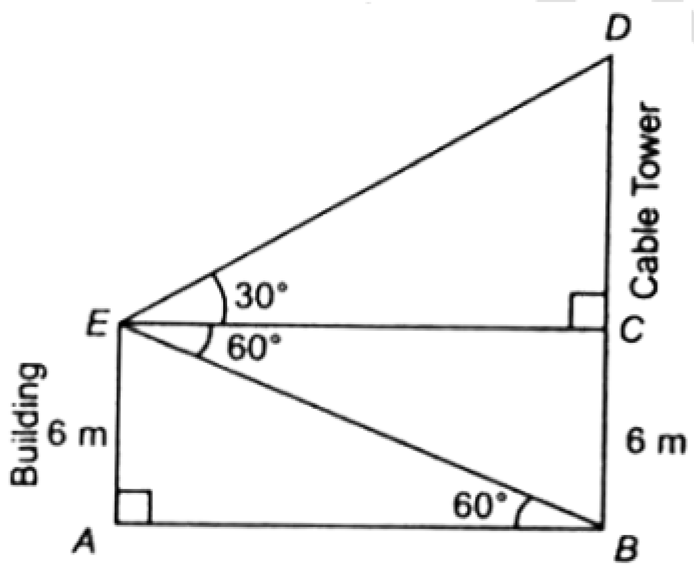
Then, BC = 54 m and $\angle BAC = 60°$ Let AB = x metres.

In right $\Delta ABC, \tan 60° = \frac{BC}{AB}$

$\Rightarrow \sqrt{3} = \frac{54}{x} \times \frac{\sqrt{3}}{\sqrt{3}} = 18\sqrt{3}m$

Hence, the distance between the ship and light house $18\sqrt{3}m$

7. **Sol.** **(d)** Let AE be the building and BD be the cable tower.



Draw $EC \perp BD$.

Then, $AE = BC = 6m, \angle ABE = 60°$

And $\angle CED = 30°$

In right $\Delta EAB,$

$\Rightarrow AB = 2\sqrt{3}m$

$AB = EC = 2\sqrt{3}$ m

In right $\Delta DCE, \tan 30° = \frac{DC}{EC}$

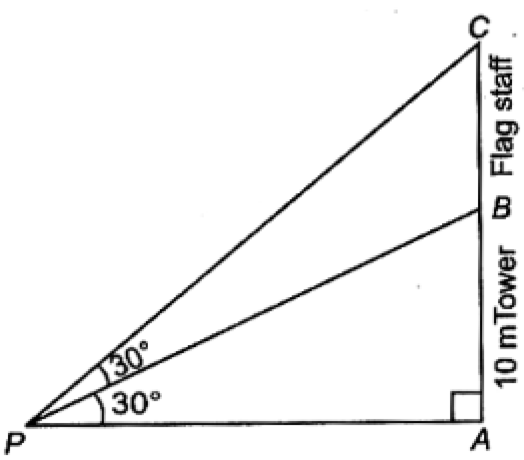
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{DC}{2\sqrt{3}} \Rightarrow DC = 2m$

Now, height of the tower = BD = BC + CD

= (6 + 2) m = 8 m

8. **Sol.** **(b)** Let AB be the tower and BC be the flag staff.

P is a point at a certain distance from the foot of the tower. Then,



$AB = 10m, \angle APB = 30° \, and \, \angle APC = 60°$

Let BC= h metres

In right $\Delta PAB, \tan 30° = \frac{AB}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{AP}$

$\Rightarrow AP = 10\sqrt{3}m$

In right $\Delta PAC, \tan 60° = \frac{AC}{AP}$

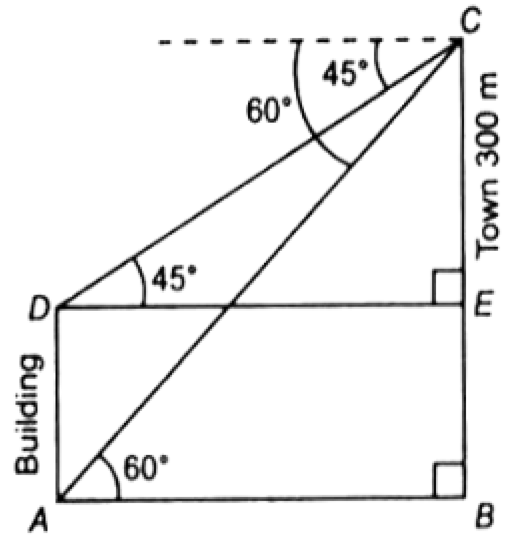
$\Rightarrow \sqrt{3} = \frac{AB + BC}{10\sqrt{3}} = \frac{10 + h}{10\sqrt{3}}$

$\Rightarrow 10 + h = 30 \Rightarrow h = 20m$

9. **Sol.** **(b)** Let BC be the tower and AD be the building. Draw

$DE \perp BC$. then BC = 300 m. $\angle EDC = 45°$ and

$\angle BAC = 60°$. Let CE = x metres.



From right ΔEDC,

$$\tan 45° = \frac{CE}{DE}$$

$$1 = \frac{CE}{DE}$$

$$\Rightarrow CE = DE = AB = x$$

From right ΔABC,

$$\tan 60° = \frac{BC}{AB} = \frac{300}{x}$$

$$\sqrt{3} = \frac{300}{x} \Rightarrow x = \frac{300}{\sqrt{3}} m$$

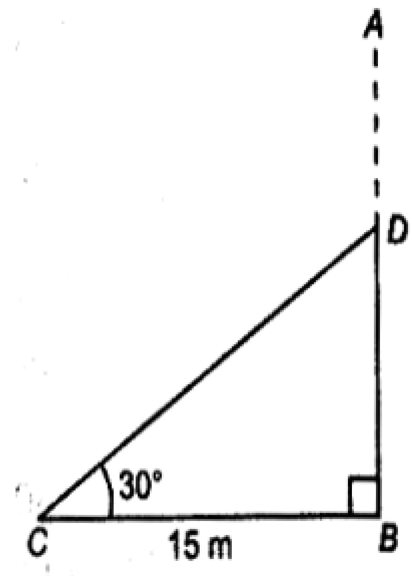
$$\therefore BE = BC - BE = \left(300 - \frac{300}{\sqrt{3}}\right) m = \left(300 - 100\sqrt{3}\right) m$$

But, AD = BE

$\Rightarrow$ AD = 127 m

Hence, the height o the building is 127 m.

10. **Sol.** **(a)**



Let AB be the tree bent at point, D, so that DA takes

The position DC. Then. DA = DC.

$BC = 15m \, and \, \angle BCD = 30°$

In right $\Delta CBD, \tan 30° = \frac{BD}{BC}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BD}{15} \Rightarrow BD = \frac{15}{\sqrt{3}} m$$

In right $\Delta CBD, \cos 30° = \frac{BC}{CD}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{15}{CD} \Rightarrow CD = \frac{30}{\sqrt{3}} m$$

Total length of the tree = BD + DC

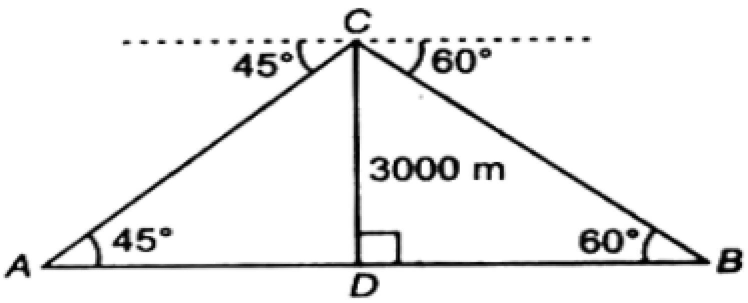
$$= \frac{15}{\sqrt{3}} + \frac{30}{\sqrt{3}} = \frac{45}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 15\sqrt{3}m$$

$$= (15 \times 1.73) m = 25.95m$$

11. **Sol.** **(a)** Let CD be the altitude of the aeroplane and A and B

Be opposite points on the two banks of the river. Then

CD = 3000 m, $\angle DAC = 45° \, and \, \angle DBC = 60°$



In right ΔADC

$$\tan 45 = \frac{DC}{AD}$$

$$\Rightarrow 1 = \frac{3000}{AD} \Rightarrow AD = 3000m$$

In right $\Delta BDC, \tan 60° = \frac{CD}{BD}$

$$\Rightarrow \sqrt{3} = \frac{3000}{BD} \Rightarrow BD = \frac{300}{\sqrt{3}}m$$

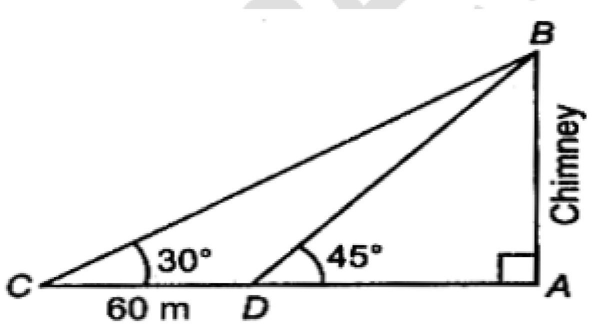
Width of the river = AD + BD

$$= \left(3000 + \frac{3000}{\sqrt{3}}\right)m = 4730m$$

12. **Sol.** **(c)** Let AB be the chimney. Then,

$CD = 60m, \angle ADB = 45°$ and $\angle DCB = 30°$

Let AB = h metres.



In right ΔCAB,

$$\tan 45° = \frac{AB}{AD}$$

$$1 = \frac{AB}{AD} = \frac{AB}{AD} \Rightarrow AB = AD = h$$

In right ΔCAB,

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{60+h}$$

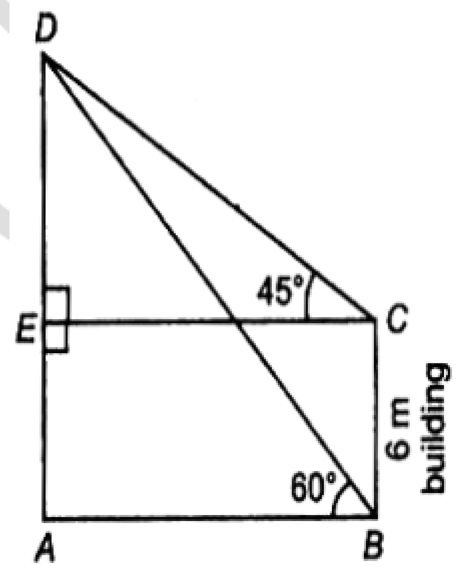
$$\Rightarrow h = \frac{60}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\frac{60\left(\sqrt{3}+1\right)}{2}$$

$$= 30\left(\sqrt{3}+1\right)m$$

Hence, the height of the chimney is $= 30\left(\sqrt{3}+1\right)m$

13. **Sol.** **(d)**



Let BC be the building and

AD be the spire.

Then, Draw, $CE \perp AB$. Then

$BC = AE = 6m, \angle ABD = 60°$

And $\angle ACE = 45°$

Also, AB = EC

In right ΔCED.

$$\tan 45° = \frac{ED}{EC} \Rightarrow 1 = \frac{ED}{EC}$$

$$\Rightarrow ED = EC$$

In right ΔBAD,

$$\Rightarrow \sqrt{3} = \frac{6+ED}{ED} \quad [\therefore AB = EC = ED]$$

$$\Rightarrow ED = \frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{6\left(\sqrt{3}+1\right)}{2}$$

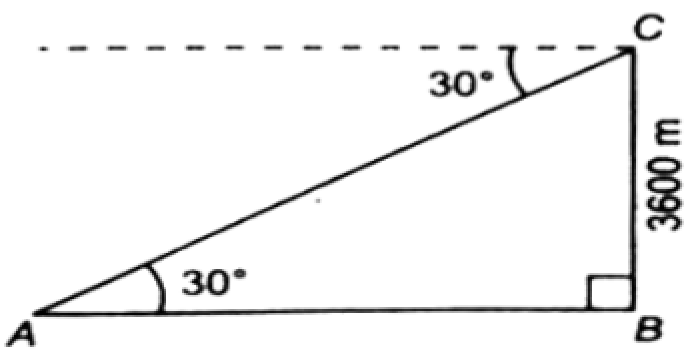
$$= 3\left(\sqrt{3}+1m\right) = 8.19m$$

$$AD = AE + ED = (6+8.19)m = 14.19$$

Thus, the height of the spire is 14.19 m

14. **Sol.** **(a)** Let the height of the spire is 14.19 m

Distance between the control tower and airport. Then



$$BC = 3600m \, and \, \angle BAC = 30°$$

In right ΔABC,

$$\tan 30° = \frac{BC}{AB}$$

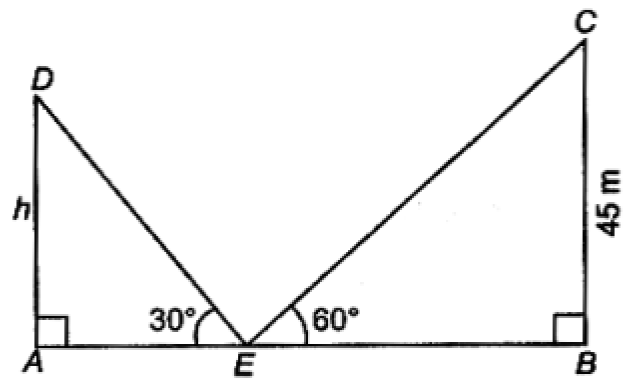
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3600}{AB}$$

$$\Rightarrow AB = \frac{3600}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 1200\sqrt{3}m$$

15. **Sol.** **(a)** Let BC and AD be the two towers. Let AB be the

Distance between the two towers such that AE = EB

Then, BC = 45 m, $\angle BEC = 60° \, and \, \angle AED = 30°$



In right ΔEBC,

$$\tan 60° = BC / BE$$

$$\Rightarrow \sqrt{3} = \frac{45}{BE} \Rightarrow BE = \frac{45}{\sqrt{3}}m$$

In right $\Delta DAE, \tan 30° = \frac{AD}{AE}$

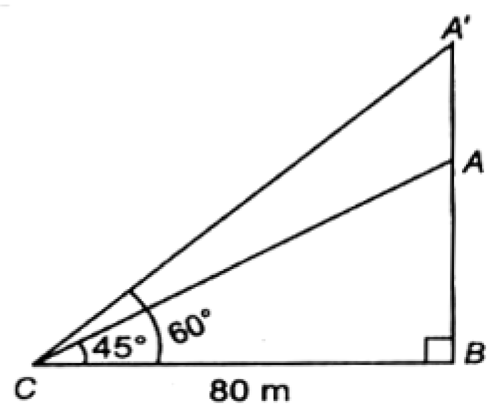
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{45/\sqrt{3}} = \frac{\sqrt{3}h}{45}$$

$$\Rightarrow h = \frac{45}{\sqrt{3} \times \sqrt{3}} = 15m$$

Hence height of the second tower is 15 m.

16. **Sol.** **(a)** Let AB be the initial height of the tower and A' B E

The new height of the tower. Then



$$BC = 80m, \angle BCA = 45° \, and \, \angle BCA' = 60°$$

In right ΔABC

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{AB}{BC} \Rightarrow AB = BC = 80m$$

$\tan 60° = \frac{A'B}{BC}$

$\Rightarrow \sqrt{3} = \frac{A'A + AB}{BC}$

$= \frac{A'A + 80}{80}$

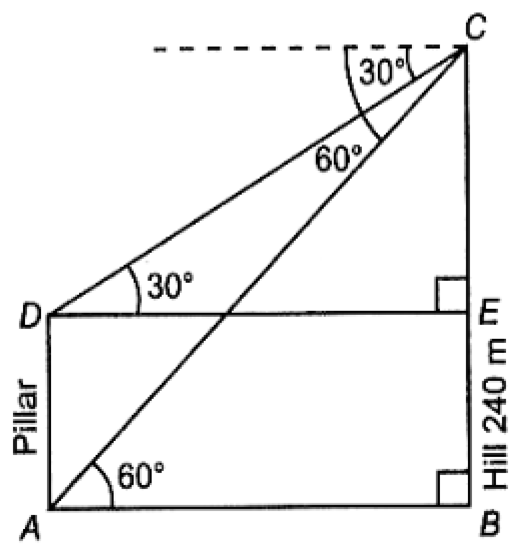
$\Rightarrow A'A = 80\left(\sqrt{3} - 1\right)m$

New height of the tower = A'B = A'A + AB

= (584 + 80) m = 1384m

17. **Sol.** **(c)** Let BC be the hill and AD be the pillar.

Draw $DE \perp BC$ then,



AD = BE

And AB = DE, BC = 240 m,

$\angle CAB = 60°$

And $\angle CDB = 30°$

In right ΔDEC,

$\tan 30° = \frac{CE}{DE}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{CE}{DE}$

In right $\Delta ABC, \tan 60° = \frac{BC}{AB}$

$\Rightarrow \frac{1}{\;} = \frac{240}{\;}$

$\Rightarrow AB = \frac{240}{\sqrt{3}}m$

But, DE = AB

$\Rightarrow CE\sqrt{3} = \frac{240}{\sqrt{3}}$

$\Rightarrow CE = 240/3 = 80m$

Now, BC = BE + EC

$\Rightarrow$BE = BC 0 EC

= (240 - 80) m

= 160 m

$\Rightarrow$ AD = 160 m

Hence, the height of the pillar is 160 m

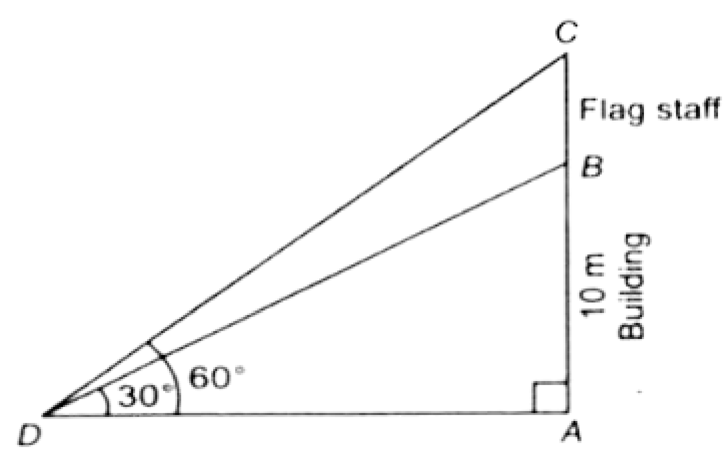
18. **Sol.** **(a)** Let AB be the height of the building and BC the

Height of the flag staff. Then,

AB = 10 m,

$\angle ADB = 30°$

And $\angle ADC = 60°$



In right ΔBAD,

$\tan 30° = \frac{AB}{AD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{AD}$

$\Rightarrow AD = 10\sqrt{3}m$

In right $\Delta CAD, \tan 60° = \frac{AC}{AD} = \frac{AB+BC}{AD}$

$= \frac{10+BC}{10\sqrt{3}}$

$= \sqrt{3} = \frac{10+BC}{10\sqrt{3}}$

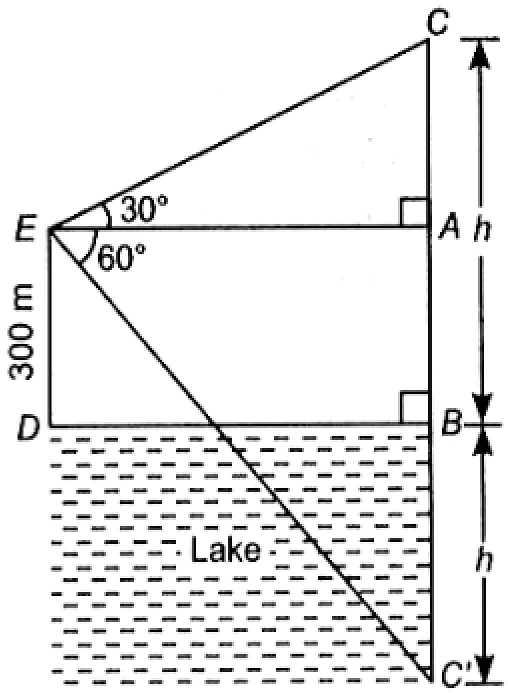
$\Rightarrow BC = (30-10)m = 20m$

19. **Sol.** **(a)** Let BC be the tower and E the point of observation

300 m above the lake surface. Draw $AE \perp BC$ is the

Reflection of the tower BC in lake such that

$BC = BC = h$ metre.



Then,
$DE = AB = 300m, \angle AEC = 30°, \angle AEC = 60°$

Also, $AE = BD = x$ metre

$AC = BC - BA = (h-300)m$

$AC = BC + BA = (h+300)m$

In right $\Delta CAE \tan 30° = \frac{AC}{AE}$

$\Rightarrow \sqrt{3} = \frac{h+300}{x}$ ..... (i)

In right $\Delta C\,AE, \tan 60° = \frac{AC}{AE}$

$\Rightarrow \sqrt{3} = \frac{h+300}{x}$ ..... (ii)

On dividing $= \frac{h-300}{h+300} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3}$

$\Rightarrow 3(h-300) = (h+300)$

$\Rightarrow h = 600m$

Hence, the height of the tower is 600 m.

Exercise 03
Trigonometry (High Level)

Fundamental trigonometrically ratios and functions, Trigonometrically ratio of allied angles

1. The angle subtended at the centre of a circle of radius 3 metres by an arc of length 1 metre is equal to

   (a) 20° (b) 60° (c) $\frac{1}{3}$ radian (d) 3 radians

2. A circular wire of radius 7 *cm* is cut and bend again into an arc of a circle of radius 12cm . The angle subtended by the arc at the centre is

3. The radius of the circle whose arc of length 15cm makes an angle of $3/4$ radian at the centre is

(a) 10 cm (b) 20 cm (c) $11\frac{1}{4}cm$ (d) $22\frac{1}{2}cm$

4. $\tan 1^\circ \tan 2^\circ \tan 4^\circ .... \tan 89^\circ =$

(a) 1 (b) 0 (c) ∞ (d) $1/2$

5. If $\sin x = \frac{-24}{25}$ then the value of $\tan x$ is

(a) $\frac{24}{25}$ (b) $\frac{-24}{7}$ (c) $\frac{25}{24}$ (d) None of these

6. The expression $\cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} + \frac{3\pi}{13} + \cos\frac{5\pi}{13}$ is equal to

(a) -1 (b) 0 (c) 1 (d) None of these

7. $\frac{1+\sin A - \cos A}{1+\sin A + \cos A}$

(a) $\sin\frac{A}{2}$ (b) $\cos\frac{A}{2}$ (c) $\tan\frac{A}{2}$ (d) $\cot\frac{A}{2}$

8. $\cos^4\theta - \sin^4\theta$ is equal to

(a) $1-2\sin^2\left(\frac{\theta}{2}\right)$ (b) $2\cos^2\theta - 1$

(c) $1+2\sin^2\left(\frac{\theta}{2}\right)$ (d) $1+2\cos^2\theta$

9. If $(\sec\alpha + \tan\alpha)(\sec\beta + \tan\beta)(\sec\gamma + \tan\gamma) = \tan\alpha \tan\beta \tan\gamma$, then $(\sec\alpha + \tan\alpha)(\sec\beta + \tan\beta)(\sec\gamma - \tan\gamma) =$

(a) $\cot\alpha \cot\beta \cot\gamma$

(b) $\tan\alpha \tan\beta \tan\gamma$

(c) $\cot\alpha \cot\beta \cot\gamma$

(d) $\tan\alpha + \tan\beta + \tan\gamma$

10. $\cos 1^\circ \cos 2^\circ \cos 3^\circ .... \cos 179^\circ =$

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

11. The value of $\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ}$ is

(a) 2 (b) 3 (c) 1 (d) 0

12. The value of $\sin 10^\circ + \sin 20^\circ + \sin 30^\circ + ... + \sin 360^\circ$ is

(a) 1 (b) 0 (c) -1 (d) None of these

13. $\cos 1^0 + \cos 2^0 + \cos 3^0 + ... \cos 180^0 =$

(a) 0 (b) 1 (c) -1 (d) 2

Trigonometrically ratios of sum and difference of two and three angles

14. The value of $\tan 20^\circ + 2\tan 50^\circ - \tan 70^\circ$ is equal to
    (a) 1 (b) 0 (c) $\tan 50^\circ$ (d) None of these

15. $\tan 5x \tan 3x \tan 2x =$
    (a) $\tan 5x - \tan 3x - \tan 2x$ (b) $\dfrac{\sin 5x - \sin 3x - \sin 2x}{\cos 5x - \cos 3x - \cos 2x}$
    (c) 0 (d) None of these

16. $\tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ =$
    (a) 1 (b) 2 (c) 3 (d) $\sqrt{3}/2$

17. $\cos\dfrac{2\pi}{15}\cos\dfrac{4\pi}{15}\cos\dfrac{8\pi}{15}\cos\dfrac{16\pi}{15} =$
    (a) $1/2$ (b) $1/4$ (c) $1/8$ (d) $1/16$

18. $\sin 36^\circ \sin 72^\circ \sin 108^\circ \sin 144^\circ =$
    (a) $1/4$ (b) $1/16$ (c) $3/4$ (d) $5/16$

Maximum & value of trigonometrical function, Conditional identities.

19. The minimum value of $f(x) = \text{Sin}^4 x + \text{Cos}^4 x$
    (a) $1/4$ (b) $\dfrac{1}{2\sqrt{2}}$ (c) $\dfrac{-1}{2}$ (d) $\dfrac{1}{2}$

20. The maximum value of $3\cos\theta + 4\sin\theta$ is
    (a) 3 (b) 4 (c) 5 (d) None of these

21. In the graph of the function $\sqrt{3}\sin x + \cos x$ the maximum distance of a point from $X-$axis is
    (a) 4 (b) 2 (c) 1 (d) $\sqrt{3}$

22. The value of x for which of $\sqrt{3}\cos x + \sin x$ is maximum
    (a) 30° (b) 45° (c) 60° (d) 90°

23. The maximum value of $a\cos x + b\sin x$ is
    (a) $a+b$ (b) $a-b$ (c) $|a|+|b|$ (d) $\left(a^2+b^2\right)^{1/2}$

24. The minimum value of $3\cos x - 4\sin x + 5$ is
    (a) 5 (b) 9 (c) 7 (d) 0

25. The greatest and least value of $\sin x \cos x$. are
    (a) $1-1$ (b) 1/2 , -1/2 (c) $\dfrac{1}{4},\dfrac{1}{4}$ (d) $2-2$

26. Maximum value of $f(x) = \sin x + \cos x$ is
    (a) 1 (b) 2 (c) $\dfrac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

Exercise 03
Solutions

1. **Sol** **(c)** Given that radius $(r)=3m\,and\,arc(d)=1m$

We know that Angle $=\dfrac{\text{arc}}{\text{radius}}=\dfrac{1}{3}\text{radian}.$

2. **Sol** **(b)** Given that diameter of circular wire $=14cm$

Therefore length of circle wire $=14\pi cm.$

$\therefore$ Required angle $=\dfrac{arc}{radius}=\dfrac{14\pi}{12}=\dfrac{7\pi}{6}$

$=\dfrac{7}{6}\pi.\dfrac{180°}{\pi}=210°!$

3. **Sol** **(b)** Angle

$=\dfrac{arc}{radius}=\dfrac{15}{(3/4)}cm$

Radius = 20 cm.

4. **Sol** **(a)**

$\tan 1°\tan 2°.........\tan 89°$

$=(\tan 1°\tan 89°)(\tan 2°\tan 88°)........$

$=(\tan 1°\tan(90°-1°))(\tan 2°\tan(90°-2°))........$

$=(\tan 1°.\cot 1°)(\tan 2°.\cot 2°)........[\therefore \tan(90-\theta)=\cot$

$=1.1=1.$

5. **Sol** **(b)**

$\cos z=\sqrt{1-\sin^2 x}=\sqrt{1-\left(\dfrac{-24}{25}\right)^2}=\dfrac{7}{25}$

$\Rightarrow \tan x=\dfrac{\sin x}{\cos x}=\dfrac{-24}{7}.$

6. **Sol** **(b)** Given expression =

$=\cos\dfrac{10\pi}{13}+\cos\dfrac{8\pi}{13}+\dfrac{3\pi}{13}+\cos\dfrac{5\pi}{13}$

$\left(\cos\dfrac{10\pi}{13}+\cos\dfrac{3\pi}{13}\right)+\left(\cos\dfrac{8\pi}{13}+\cos\dfrac{5\pi}{13}\right)$

$2\cos\left(\dfrac{13\pi}{2\times 13}\right)\cos\left(\dfrac{7\pi}{2\times 13}\right)$

$+2\cos\left(\dfrac{13\pi}{2\times 13}\right)\cos\left(\dfrac{3\pi}{2\times 13}\right)$

$=2\cos\dfrac{\pi}{2}\left(\cos\dfrac{7\pi}{26}+\cos\dfrac{3\pi}{26}\right)\qquad\left[\therefore \cos\dfrac{\pi}{2}=0\right]$

$=0.$

7. **Sol** **(c)** $\dfrac{1+\sin A-\cos A}{1+\sin A+\cos A}$

$\dfrac{2\sin^2\dfrac{A}{2}+2\sin\dfrac{A}{2}2\cos\dfrac{A}{2}}{2\cos^2\dfrac{A}{2}+2\sin\dfrac{A}{2}2\cos\dfrac{A}{2}}$

$\dfrac{2\sin\dfrac{A}{2}+\left(\sin\dfrac{A}{2}\cos\dfrac{A}{2}\right)}{2\cos\dfrac{A}{2}+\left(\cos\dfrac{A}{2}\sin\dfrac{A}{2}\right)}=\tan\dfrac{A}{2}.$

Trick : Put $A=60°.$

Then $\dfrac{1+(\sqrt{3}/2)-(1/2)}{1+(\sqrt{3}/2)+(1/2)}=\dfrac{1+\sqrt{3}}{3+\sqrt{3}}=\dfrac{1}{\sqrt{3}}$

which is given by option (c), i.e., $\tan\dfrac{60°}{2}=\dfrac{1}{\sqrt{3}}$

Note : Students should remember at the time of assuming the values of $A,B,\theta,......$ *etc.* that, for the assumed values, the options must have different values.

8. **Sol** **(b)** $\cos^4\theta-\sin^4\theta$

$=(\cos^2\theta+\sin^2\theta)(\cos^2\theta-\sin^2\theta)$

$=\cos^2\theta-\sin^2\theta=\cos^2\theta-(1-\cos^2\theta)$

$=2\cos^2\theta-1.$

9. **Sol** **(a)** Given,

$(\sec\alpha+\tan\alpha)(\sec\beta+\tan\beta)(\sec\gamma+\tan\gamma)$

$=\tan\alpha\tan\beta\tan\gamma\qquad .....(i)$

Let

$x=(\sec\alpha-\tan\alpha)(\sec\beta-\tan\beta)(\sec\gamma-\tan\gamma)..........($

$$\left(\sec^2\alpha-\tan^2\alpha\right)\left(\sec^2\beta-\tan^2\beta\right)\left(\sec^2\gamma-\tan^2\gamma\right)$$
$$=x.\left(\tan\alpha\tan\beta\tan\gamma\right)$$

$$=x=\frac{1}{\left(\tan\alpha\tan\beta\tan\gamma\right)}\quad\therefore x=\left(\cot\alpha\cot\beta\cot\gamma\right)$$

10. **Sol** **(a)** We know that one of the factor of the given expression.

is $\cos 90^\circ=0$.

Therefore $\cos 1^\circ.\cos 2^\circ.\cos 3^\circ.....\cos 179^\circ=0$

11. **Sol** **(a)**

$\tan(90-\theta^\circ)=\cot\theta, \cot(90-\theta^\circ)=\tan\theta.$

Therefore $\frac{\cot 54^\circ}{\tan 36^\circ}+\frac{\tan 20^\circ}{\cot 70^\circ}$

$$=\frac{\cot 54^\circ}{\tan(90^\circ-54^\circ)}+\frac{\tan 20^\circ}{\cot(90^\circ-20^\circ)}$$

$$=\frac{\cot 54^\circ}{\cot 54^\circ}+\frac{\tan 20^\circ}{\tan 20^\circ}=1+1=2.$$

12. **Sol** **(b)** Since,

$\sin 190^\circ=-\sin 10^\circ, \sin 200^\circ=-\sin 20^\circ,$

$\sin 210^\circ=-\sin 30^\circ, \sin 360^\circ=\sin 180^\circ=0$ *etc.*

13. **Sol** **(c)**

$$\left(\cos 1^\circ+\cos 179^\circ\right)+\left(\cos 2^\circ+\cos 178^\circ\right)+......$$
$$+\left(\cos 89^\circ+\cos 91^\circ\right)+\cos 90^\circ+\cos 180^\circ=-1.$$

14. Sol **(b)**

$\tan 20^\circ+2\tan 50^\circ-\tan 70^\circ$

$$=\frac{\sin 20^\circ}{\cos 20^\circ}-\frac{\sin 70^\circ}{\cos 70^\circ}+2\tan 50^\circ$$

$$=\frac{\sin 20^\circ\cos 70^\circ-\cos 20^\circ\sin 70^\circ}{\cos 70^\circ\cos 70^\circ}+2\tan 50^\circ$$

$$=\frac{\sin\left(20^\circ-70^\circ\right)}{\frac{1}{2}\left[\cos\left(70^\circ+20^\circ\right)+\cos\left(70^\circ-20^\circ\right)\right]}+2\tan 50^\circ$$

$$=\frac{2\sin\left(-50^\circ\right)}{\cos 90^\circ\cos 50^\circ}+2\tan 50^\circ$$

$$=\frac{-2\sin\left(50^\circ\right)}{0+\cos 50^\circ}+2\tan 50^\circ$$

$$=-2\sin\left(50^\circ\right)+2\tan 50^\circ=0.$$

15. **Sol** **(a)** We have,

$$\Rightarrow \tan 5x=\frac{\tan 3x+\tan 2x}{1-\tan 3x\tan 2x}$$

$$\Rightarrow \tan 5x-\tan 5x\tan 3x\tan 2x=\tan 3x+\tan 2x$$

$$\Rightarrow \tan 5x\tan 3x\tan 2x=\tan 5x-\tan 3x-\tan 2x.$$

16. **Sol** **(c)**

$\tan 20^\circ\tan 40^\circ\tan 60^\circ\tan 80^\circ=$

$$=\frac{\sin 20^\circ\sin 40^\circ\sin 80^\circ\tan 60^\circ}{\cos 20^\circ\cos 40^\circ\cos 80^\circ}$$

Here,

$N^r=\left(\sin 20^\circ\sin 40^\circ\sin 80^\circ\right)$

$$=\frac{\sin 20^\circ}{2}\left(2\sin 40^\circ\sin 80^\circ\right)$$

$$=\frac{\sin 20^\circ}{2}\left(\cos 40^\circ-\cos 120^\circ\right)$$

$$=\frac{1}{2}\sin 20^\circ\left(1-2\sin^2 20^\circ+\frac{1}{2}\right)$$

$$=\frac{1}{2}\sin 20^\circ\left(\frac{3}{2}-2\sin^2 20^\circ\right)=\frac{\sin 60^\circ}{4}=\frac{\sqrt{3}}{8}$$

Now, we take $D^r=\cos 20^\circ\cos 40^\circ\cos 80^\circ$

$$=\frac{\sin 2^3 20^\circ}{2^3\sin 20^\circ}=\frac{\sin 160^\circ}{8\sin 20^\circ}=\frac{\sin 20^\circ}{8\sin 20^\circ}=\frac{1}{8}$$

$\therefore$ Hence $\tan 20^\circ\tan 40^\circ\tan 80^\circ=\frac{\sqrt{3}/8}{1/8}$

Therefore

$\tan 20^\circ\tan 40^\circ\tan 60^\circ\tan 80^\circ=\sqrt{3}.\sqrt{3}=3.$

17. Sol **(d)**

$$\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}=$$

$$=\frac{\sin 2^4\frac{2\pi}{15}}{2^4\sin\frac{2\pi}{15}}=\frac{\sin\frac{32\pi}{15}}{16\sin\frac{2\pi}{15}}=\frac{1}{16}\frac{\sin\frac{2\pi}{15}}{\sin\frac{2\pi}{15}}=\frac{1}{16}.$$

18. **Sol** **(d)**

$\sin 36^\circ\sin 72^\circ\sin 108^\circ\sin 144^\circ$

$$=\sin^2 36^\circ\sin^2 72^\circ=\frac{1}{4}\left\{\left(2\sin^2 36^\circ\right)\left(2\sin^2 72^\circ\right)\right\}$$

$$=\frac{1}{4}\left\{\left(1-\cos 72^\circ\right)\left(1-\cos 144^\circ\right)\right\}$$

$$=\frac{1}{4}\left\{\left(1-\sin 18^\circ\right)\left(1+\cos 36^\circ\right)\right\}$$

$$=\frac{1}{4}\left[\left(1-\frac{\sqrt{5}-1}{4}\right)\left(1+\frac{\sqrt{5}+1}{4}\right)\right]$$

$$=\frac{20}{16}\times\frac{1}{4}=\frac{5}{16}.$$

19. Sol **(d)**

f(x) = $\text{Sin}^4x + \text{Cos}^4x$

$= 1 - 2\sin^2 x\cos^2 x$

$f(x) = 1 - \frac{1}{2}\sin^2 2x\left[\because 0 \le \sin^2 2x \le 1\right]$

$f_{\min}(x) = \frac{1}{2}.$

20. **Sol** **(c)** Let $3 = r\cos\alpha, 4 = r\sin\alpha$, so $r = 5$

$f(\theta) = r.(\cos\alpha\cos\theta + \sin\alpha\sin\theta) = 5.\cos(\theta - \alpha)$

$\therefore$ The maximum value of $f(\theta) = 5.1 = 5.$

$\{\textit{Since the maximum value of } \cos(\theta-\alpha) = 1\}.$

Alter : As we know that, the maximum value of $a\sin\theta + b\cos\theta + \sqrt{a^2+b^2}$ and the minimum value is $-\sqrt{a^2+b^2}$ Therefore, the maximum value is $(3\cos\theta + 4\sin\theta) = +\sqrt{3^2+(-4)^2} = 5$ and the minimum value is -5.

21. **Sol** **(b)** Maximum distance $= \sqrt{\left(\sqrt{3}\right)^2 + (1)^2} = 2.$

Hence, in the graph of function $\sqrt{3}\sin x + \cos x,$

maximum distance of a point from x - axis is 2.

22. **Sol** **(a)** Let

$\Rightarrow f(x) = 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right) = 2\sin\left(x + \frac{\pi}{3}\right)$

But $-1 \le \sin\left(x + \frac{\pi}{3}\right) \le 1$

Hence, $f(x)$ is maximum , if $x + \frac{\pi}{3} = 90 \Rightarrow x = 30°.$

Trick : Check from options.

23. **Sol** **(d)** We know that the maximum value of $a\cos\theta + b\sin\theta$ is $\sqrt{a^2+b^2}$.

Hence maximum value of a $a\cos x + b\sin x$ will be $\left(a^2+b^2\right)^{1/2}$.

24. **Sol** **(d)** The minimum value of $3\cos c + 4\sin x$ is $-\sqrt{3^2+4^2} = -5$

Hence the minimum value of $3\cos x - 4\sin x + 5$

$= -5 + 5 = 0.$

25. **Sol** **(b)** Let

$f(x) = \sin x\cos x = \frac{1}{2}\sin 2x$

We know $-1 \le \sin 2x \le 1 \Rightarrow \frac{-1}{2} \le \frac{1}{2}\sin 2x \le \frac{1}{2}$

Thus the greatest and least value of $f(x)$ are $\frac{1}{2}$ *and* $-\frac{1}{2}$ respectively.

26. Sol **(d)** Maximum value of

$f(x) = \sqrt{1^2+1^2} = \sqrt{2}.$