Percentile Classes

Permutation and Combination

Fundamental Principle of Counting

(i) Multiplication: If one operation can be performed in W ways and corresponding to each way of performing the first operation a second operation can be performed in n ways, then the two operations together can be performed in m x n ways.

If after two operations are performed in any one of the m x n ways a third operation can be performed in p ways, then the three operations together can be performed in m x n x p ways and so on.
In general if there are n jobs (works/operations) j_1, j_2 j_3 ……j_n such that j_i can be performed to dependently in m_1 ways. (i = 1,2,3,..n). Then the total number of ways in which all the jobs can be performed is m_1 x m_2 x w_3 x...x m_n.

Here the different jobs/operations are mutually inclusive. It implies that all the jobs are being done in succession. In this case we use the word 'and' to complete the all stages of operation and the meaning of 'and' is multiplication.
e.g., A student has to select a letter from vowels and another letter from consonants, then in how many ways can he make this selection?

(ii) Addition: If there are two jobs such that they can be performed independently in m and n ways respectively, then either of the two jobs can be performed in (m + n) ways.

In general if there are various jobs which are mutually exclusive, then they can be performed in $m_1 + m_2 + m_3 + \ldots + m_n$ ways.

In this case we use the word 'or' between various jobs and the meaning of 'or' is addition.
e.g. A student has to select a letter either from vowels 'or' from consonants, then in how many ways can he make this selection?

Permutation

Each of the different "arrangements" that can be made out of a given number of things by taking some or all of them at a time is called "permutation".

Thus the permutations of three letters (say) a, b, c taken two at a time are
ab, ba, bc, cb, ac, ca

Therefore the number of permutations of three different things taken two at a time in ${}^3P_2 = 6$

- Permutations of n different things taken r at a time = ${}^nP_r = \frac{n!}{(n-r)!}$
- Permutations of n different things taken all at a time ${}^nP_n = n!$
- Permutations of n different things taken r at a time, when a particular things always occurs = $r\,.{}^{(n-1)}P_{(r-1)}$
- Permutations of n different things taken r at a time when a particular thing never occurs = ${}^{(n-1)}P_{(r-1)}$

Permutations of n things not all different:

- Number of permutations of n things, take all at a time, of which p are alike of one kind, q are alike of second kind , r are alike of third kind and rest are different

$= \frac{n!}{p!q!r!}$

- Number of permutations of n things, of which P_1 are alike of one kind, P_2 are alike of second kind, P_3 are alike of third kind ….P_r are alike of r^{th} kind such that

$P_1 + P_2 + P_3 + \ldots + P_r = n$ is $\frac{n!}{P_1!\,P_2!\,P_3!\ldots P_r!}$

Permutations where repetitions are allowed

- The number of permutations of n different thing taken r at a time, when each may be repeated any number of times in each arrangement is ‘n’.

Circular permutations:

In the previous articles we have learned linear permutations, Now we have to know about circular permutations, in circular permutation things are to be arranged in the form of a ring or a circle, e.g., arrangements of persons around circular dinning table, conferencing table, garland etc,
Since in the circular permutation there is no any each point i.e, we can not say which one is the beginning article or ending article in the possible arrangements.

Hence the number of circular permutations of n objects = $\frac{n!}{n}$ = (n-1)!

Thus in a circular permutation one thing is kept fixed and the remaining (n-1) things are arranged in (n-1) ways.

- If the clockwise and counter clockwise orders are not distinguishable, then the number of ways = $\frac{1}{2}\{(n-1)!\}$

Combination:

Let A, B, C be three letters, then we can combine any two of them in the following ways:

AB, BC, AC
Similarly if A, B, C, D are four letters, then we can combine any two of them in the following manner:
AB, AC, AD, BC, BD, CD
Similarly we can combine any 3 of A, B, C, D as:
ACB, ABD, ACD, BCD

Note: In the combinations order of the letters (or things) is not considered.
Here AB, BA are same and BC, CB are also same and so on.
Hence ABC, ACB, BAC, BCA CAB and CBA are same and counted as a single combination of A, B, C.

Note: The word combination is generally used form selection of thing and the world permutation is used for arrangements of things.

Combinations of ‘n’ different things taken ‘r’ at a time

The number of all combinations of n distinct things taken r at a time (r $\leq$ n) is

$^nC_r = \frac{n!}{(n-r)!r!}$, **C denotes combination,**

Properties

1. $^nC_r = (^nP_r)/(r!)$
2. $^nC_0 = {^nC_n} = 1$
3. $^nC_r = {^nC_{n-r}}\ (0 \leq r \leq n)$
4. $^nC_x = {^nC_y}$ if $x + y = n$ or $x = y$ $(x, y \epsilon W)$
5. $^nC_{r-1} + {^nC_{n=r}} = {^{n+1}C_r}$
6. If n is even, the greatest value of $^nC_r = {^nC_m}$ when $m = \frac{n}{2}$.
7. If n is odd, the greatest value of $^nC_0 = {^nC_m}$ when $m = \frac{(n-1)}{2}$ or $= \frac{(n+1)}{2}$
8. $^rC_r + {^{r+1}C_r} + {^{r+2}C_r} + \ldots.. + {^nC_r} = {^{n+1}C_{r+1}}; r \leq 1$

9. $^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 = {}^nC_m + \ldots + {}^nC_n = 2^n$

10. $^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n{}_{-1}$

11. $^nC_0 + {}^nC_2 + {}^nC_4 = {}^nC_m + \ldots = 2^{2n}$

Restricted Combination

1. Number of combinations of n thing taken 'r' at a time in which x particular always occur is $^{n-1}C_{r-1}$.

2. Number of combinations of n things take 'r' at a time in which x particular ting never occur is $^{n-1}C_r$.

3. Number of ways of selections of zero or more things from a group of 'n' distinct things is 2^n.
   i.e, "$C_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n$

4. Number of ways of selections of one or more things from a group of 'n' distinct things is 2^n-1.
   i.e, $^nC_1 + {}^nC_1 + {}^nC_3 + \ldots + {}^nC_n = 2^n - 1$
   ($^nC_0 = 1$)

5. Number of ways of selection of 'r' things out of 'n' identical things is 1. ($1 \leq n$)

6. Number of ways of selections of zero or more thing from a group of 'n' identical things is (n+1).

7. Number of ways of selection of some of all things i.e. atleast 1 out of (p+q+r+…) things of which p are alike of one kind, q are alike of second kind r are alike of third kind and so on, is [(p+1) (q+1) (r+1)…)]-1.

8. Total number of ways of selecting one or more things from 'p' identical things of one kind, 'q' identical thing of second kind, 'r' identical things of third kind and n different things is $((p+1)(q+1)(r+1)2^n)-1$

9. Number of selections of k consecutive things out of n things in a row is (n-k+1).

10. Number of selections of k consecutive things out of n thing in a circle = $\begin{cases} n & when \quad k < n \\ 1 & when \quad k = n \end{cases}$

Division of items into Groups of Unequal Size

1. The number of ways on which (m+n) different things can be divided into two groups containing m and n things respectively is $\frac{(m+n)!}{m!n!}$

   Since m things can be selected out of (m+n) things in $^{m+n}C_m$ ways and each time when m things are taken, n things left out to form the other group of n things in ^nC_n ways i.e., in 1 way only.

   Hence the required number of ways

   $= {}^{m+n}C_m = \frac{(m+n)!}{m!n!}$

2. The number of ways in which (m+n+p) different things can be divided into three groups containing m,
   Respectively is $\frac{(m+n+p)!}{m!n!p!}$

   Similarly: If (m+n+p) things are distributed among three persons in the groups of m, n and p things then then number of ways

$= \frac{(m+n+p)!}{m!n!p!} \times 3!$

3. **The number of ways in which 'n' distinct thing can be distributed to r different persons = r^n.**

Division of items into Groups of Equal Size.

The number of ways on which mn different things can be divided equally into groups, each group containing n thing

$=\left(\frac{(mn)!}{n!}\right)\frac{1}{m!}$

The number of ways on which mn different things can be distributed equally into m groups, each group containing things $= \frac{(mn)!}{(n!)^m}$

Note: The distribution order is important hence the divisible thing can be arranged in m ways. Since things are divided into m groups.

1. **The number of ways of dividing n identical thing among r person (or groups) each of whom, can receive zero or more things is $^{n+r-1}C_{r-1}$ where $0 \le r \le n$**

2. **The number of ways of dividing n identical thing among r persons, each one of whom, receives atleast one item is $^{n+r-1}C_{r-1}$ where $0 \le r \le n$.**

3. **The number of ways in which 'n' identical thing can be divided into r groups so that no group contains less then m items and more than k (where m<k) is coefficient of x^n in the expansion of $(x^n+x^{m+1}+\dots+x^k)^r$.**

Some Important Results

1. Number of squares in a square of having 'n' columns and 'n' rows

   $=1^2+2^2+3^2+\dots+n^2$

   $= \frac{n(n+1)(2n+1)}{6}$

2. Number of rectangles in a square having 'n' columns and 'n' rows

   $= 1^3+2^3+3^3+\dots+n^3= \left[\frac{n(n+1)}{2}\right]^2$

3. Number of squares in a rectangle having 'm' columns and 'n' rows

   $= m.n + (m-1)(n-1)+(m-2)(n-2)+\dots\dots0.$

4. Number of rectangles in a rectangle having 'm' columns and 'n' rows.

   $=(1+2+3+\dots+m).(1+2+3+\dots+n).$

5. Number of quadrilaterals if m parallel lines intersect 'n' parallel lines = $^mC_2 \times {^nC_2}$.

6. Number of terms in $(a_1+a_2+\dots+a_n)^m$ is $^{m+n-1}C^{n-1}$.

7. Number of terms in $(1+x+x^2+\dots+x^n)^m$ is m+n+1.

Permutation and Combination in Geometry

It is quite difficult to quantify the importance of P and C in Geometry. A considerable number of P and C questions that use the concepts of geometry (and vice versa) have been asked in the CAT and other premier B-school exams.

Example. How many diagonals will be there in an n sided regular polygon?
Solution: An n-sided regular polygon will have n vertices. When we join any of these two vertices (${}^nC_2$)we get a straight line, which will be either a side or a diagonal.

So, ${}^nC_2$ = number of sides + number of diagonals = n + number of diagonals

Hence the number of diagonals = ${}^nC_2 - n = \frac{n(n-3)}{2}$

The above result can be used as a formula also.

Example. Ten points are marked on a straight line and 11 points are marked on another parallel straight line. How many triangles can be constructed with vertices among these points?

Solution: Triangles will be constructed by taking one point from the 1st straight line and two more points from the 2nd straight line, and vice versa.

So, the total number of A formed = ${}^{10}C_2 \times {}^{11}C_1 \times {}^{11}C_2 \times {}^{10}C_1 = 1045$

Example: There is an n sided polygon (n>5). Triangles are formed by joining the vertices of the polygon. How many triangles can be constructed that will have no side common with the polygon?

Solution: An n sided polygon will have n vertices. Triangles constructed out of these n vertices will be of the following three types:

(i) Having two sides common with the polygon
(ii) Having one side common with the polygon
(iii) Having no side common with the polygon and the total number of triangles formed will be ${}^nC_3$
   a. Having two sides common with the polygon-out of total n vertices any combination of three c consecutive vertices will give us the triangle that has two sides common with polygon = n.
   b. Having one side common with the polygon Number of selection of three vertices out of which two are consecutive (if we select A_5 and A_6 as the two vertices then A_7 or A_4 should not be the third vertex because it will constitute the two sides of the common triangles) = n x ${}^{(n-4)}C_1$
   c. So, the total number of triangles having no side common with polygon = ${}^nC_3 - n \times {}^{(n-4)}C_1 - n$

Some Important Results
1. Maximum number of points of intersection among n straight lines = ${}^nC_2$
2. Maximum number of points of intersection among n circles = ${}^nP_2$

Some more results
1. Number of terms in $(a_1 + a_2 + \ldots + a_n)^m$ is ${}^{m+n-1}C_{m-1}$
   Illustration: Find the number of terms in $(a+b+c)^2$.

Solution: n = 3, m = 2
${}^{m+n-1}C_{m-1} = {}^4C_2 = 6$

Corollary Number of terms in

$(1+ x + x^2 + \ldots\ldots x^n)^m$ is mn + 1

Exercise – 01
Combination

1. Find the value of $^8C_3$.
(a) 56 (b) 8! (c) 65 (d) 3^8

2. Find the value of $^{10}C_5$.
(a) 525 (b) 126 (c) 252 (d) 50

3. $^{17}C_r = {}^{17}C_{r+3}$. Find the value of r.
(a) 17 (b) 6 (c) 7 (d) 13

4. What is the value of x when $^{11}C_x$ is maximum ?
(a) 7 (b) 6 (c) 5 (d) both (b) and (c)

5. For what value of x, $^{14}C_X$ is maximum?
(a) 5 (b)6 (c)7 (d)8

6. How many different committees of 5 members may be formed from 6 gentlemen and 4 ladies?
(a) 181 (b) 357 (c)603 (d)252

7. Droupdi has 5 friends. In how many ways can she invite one or more of them to a dinner?
(a) 31 (b)5^5 (c) 13 (d)25

8. In How many ways can a committee of 6' members be formed from 7 men and 6 ladies consisting of 4 men and 2 ladies?
(a) 252 (b) 525 (c) 625 (d) 256

9. A committee of 5 persons is to be formed from a group of 6 gentlemen and 4 ladies. In how many ways can this be done if the committee is to be included atleast one lady?
(a) 123 (b)113 (c) 246 (d) 945

10. A committee of 7 persons is to be chosen from 13 persons of whom 6 are Americans and 7 are Indians. In how many ways can the selection be made so as to retain a majority of Indians?
(a) 945 (b) 1057 (c)923 (d)1056

11. In how many ways 7 members forming a committee out of 11 be selected so that 3 particular members must be included.
(a) 60 (b) 130 (c) 80 (d) 70

12. A committee of 3 experts is to be selected out of a panel of 7 persons, three of them are engineers, three of them are managers and one is both engineer and manager. In how many ways can the committee be selected if it must have at least an engineer and a manager?
(a) 33 (b) 22 (c) 11 (d) 66

13. A committee of 5 persons is to be formed out of 6 gents and 4 ladies. In how many ways this can be done, when at most two ladies are included?
(a) 186 (b) 168 (c)136 (d)169

14. In an election, a voter may vote for any number of candidates not greater than the number to be chosen. There are 7 candidates and 4 members are to be chosen. In how many ways can a person vote?
(a) 89 (b) 98 (c) 79 (d) 101

15. A question has two parts, Part A and Part B, each containing 8 questions. If the students has to choose 6 from part A and 5 questions from part B, in how many ways can he choose the questions?
(a) 1268 (b) 1788 (c) 1024 (d) 1568

16. An examination paper consists of 12 questions divided into two parts, part A and part B. Part A contains 7 questions and part B contains 5 questions, A candidate is required to attempt 8 questions, selecting atleast 3 from each part. In how many ways can he select the questions?
(a) 240 (b) 60 (c) 420 (d) 480

17. Mr. Daruwala has 18 acquaintances of whom 13 are relatives. In how many ways he may invite 10 guests of that 8 of whom are relatives?
(a) 12870 (b) 22022 (c) 20222 (d) 12780

18. Amitabh has a list of 24 friends. He wishes to invite some of them in such a manner that he can enjoy maximum number of parties, but in each party the number of friends (i.e., invitees) be same and each party must have different set of persons. Then how many parties can Amitabh enjoy?
(a) 2704156 (b) 357600 (c) 235763 (d) 270156

19. In a meeting everyone had shaken hands with everyone else, it was found that 66 handshakes were exchanged. How many members were present in the meeting?
(a) 10 (b) 14 (c)12 (d)8

20. A cricket team of 11 players is to be formed from 16 players including 4 bowlers and 2 wicket keepers. In how many different ways can a team be formed so that the team has atleast 3 bowlers and atleast one wicket keeper?
(a) 2472 (b) 2274 (c) 2427 (d) 1236

21. A cricket team of 11 players is to be formed from 20 players including 6 bowlers and 3 wicket keepers. In how Many different ways can a team be formed so that the team has exactly 2 wicket keepers and atleast 4 bowers?
(a) 22725 (b) 27225 (c) 22275 (d) none of (a), (b), (c)

22. Out of 3 books on Economics, 4 books on Corporate Strategy and 5 books on Philosophy, how many collection consists of exactly one book on each subject?
(a) 40 (b) 36 (c) 60 (d) 120

23. A box contains 7 red, 6 white and 4 blue balls. How many selection of three balls can be made so that all three are red balls?
(a) 35 (b) 70 (c)42 (d)17

24. An urn contains 5 different red and 6 different green balls. In how many ways can 6 balls be selected so that there are atleast two balls of each colour?
(a) 425 (b) 245 (c) 125 (d) 625

25. How many different straight lines can be formed by joining 12 different points on a plane of which 4 are collinear and the rest are non-collinear?

(a) 16 (b) 32 (c) 61 (d) 64

26. Find the number of diagonals in a decagon,
(a) 25 (b) 35 (c) 45 (d) ${}^{n}C_2 – n$

27. Find the number of diagonals in an n-sided polygon.
(a) n^2 (b) $\frac{n(n-3)}{2}$ (c) n! (d) 2^n

28. A polygon has 54 diagonals. Find the number of sides.
(a) 6 (b) 12 (c) 27 (d) 18

29. Find the, number or Triangles that can be joining the 6 non-collinear points on a plane,
(a) 18 (b) 20 (c) 24 (d) 36

30. Find the number of triangles formed by joining 12 different points on a plane, no three of them being collinear (with the exception of 4 points which are collinear).
(a) 126 (b) 216 (c) 220 (d) 222

31. Two parallel lines each have a number of distinct points marked on them. On one line there are 2 points P and 0. On the other line there are 8 points. Find the number of different triangles which could be formed having 3 of the 10 points as vertices.
(a) 216 (b)126 (c) 64 (d)32

32. If 20 straight lines be drawn in a plane, no two of them being parallel and no three of them concurrent, how many points of intersection will there be?
(a) 95 (b) 380 (c) 400 (d) 190

33. Find the number of different straight lines obtained by joining n points on a plane, no three of which are collinear.
(a) n^3 (b) $\frac{n(n-1)}{2}$ (c) n! (d) 2^4

34. There are n points in a plane no three of which are in the same straight line with the exception of them m(m < n) points which are all in the same straight line. Find the number of triangles formed by joining them,
(a) ${}^{n-m}C_3$ (b) ${}^{n}C_3$ - ${}^{m}C_3$ (c) ${}^{n}C_m$ (d) none of (a), (b), (c)

35. If m parallel lines in plane are intersected by a family of n parallel lines. Find the number of parallelograms formed
(a) m^n (b) (m+1) (n+1) (c) $\frac{(m-n)}{n!}$ (d) $\frac{mn(m-1)(m-1)}{4}$

36. In a plane there are 37 straight lines, of which 13 pass through the point A and 11 pass through the point B. Besides, no three lines pass through one point, no line passes through both points A and B, and no two are parallel. Find the number of points of intersection of the straight lines.
(a) 535 (c) 235
(b) 525 (d) 355

37. Find the number of ways of selecting 4 letters from the word EXAMINATION.
(a) 136 (b) 126 (c) 252 (d) 525

38. Eighteen guests have to be seated half on each side of a long table. Four particular guests desire to sit on one particular side and three others on other side. Determine the number of ways in which the sitting, arrangements can be made.
(a) 18! (b) $462 \times (9!)^2$ (c) $(9!)^2$ (d) $7! \times (9!)^2$

39. Given 3 different red dyes, 4 different blue dyes, and 5 different green dyes, how many combinations of dyes can be made taking atleast one green and one blue dye?
(a) 2300 (b) 31 (c) 3720 (d) 3560

40. A bag contains 4 mangoes and 5 oranges. In how many ways can I make a selection so as to take atleast one mango and one orange?
(a) 465 (b) 365 (c) 8 (d) 19

41. In how many ways can a pack of 52 cards' be divided equally among four players in order?
(a) $(52!)^4$ (b) $4 \times (13!)$. (c) $\frac{52!}{(13!)^4}$ (d) none of these

42. In how many ways can a pack of 52 cards be divided equally into four groups?
(a) $\frac{52!}{(13!)^4 \times 4!}$ (b) $4!\left[\frac{52!}{(13!)^4}\right]$ (c) $\frac{52!}{(13!)^4}$ (d) None of these

43. Find the number of ways of distributing 7 identical balls into three boxes so that no box is empty and each box being large enough to accommodate all balls,
(a) 15 (b) 7^3 (c) 3^7 (d) $\frac{7!}{3!}$

44. Find the number of non negative integral solutions of:
$x_1 + x_2 + x_3 + x_4 = 44$
(a) ${}^{44}C_4$ (b) ${}^{43}C_3$ (c) ${}^{47}C_3$ (d) none of these

45. How many integral solutions are there to a + b + c = 18 when $a \geq 1$, $b \geq 2$, $c \geq 3$?
(a) ${}^{19}C_3$ (b) ${}^{17}C_2$ (c) ${}^{15}C_3$ (d) ${}^{14}C_2$

46. There are 4 qualifying examinations to enter into Oxford University: RAT, BAT, SAT, and PAT. An Engineer cannot go to Oxford University through BAT or SAT. A CA on the other hand can go to the Oxford University through the RAT, BAT & PAT but not through SAT. Further there are 3 ways to become a CA(viz., Foundation, Inter & Final). Find the ratio of number of ways in which an Engineer can make it to Oxford University to the number of ways a CA can make it to Oxford University,
(a) 3:2 (b) 2:3 (c) 2:9 (d) 9:2

47. If ${}^nC_3 = {}^nC_8$, find n.
(a) 11 (b) 12 (c) 14 (d) 10

48. If ${}^nC_4 = 70$, find n.
(a) 5 (b) 8 (c) 4 (d) 7

49. In how many ways can 10 identical presents be distributed among 6 children so that each child gets at least one present?
(a) ${}^{15}C_5$ (b) ${}^{16}C_6$ (c) ${}^{9}C_5$ (d) 6^{10}

50. A captain and a vice-captain are to be chosen out of a team having eleven players. How many ways are there to achieve this?
(a) 10.9 (b) ${}^{11}C_2$ (c) 110 (d) 10.9!

51. In how many ways can Ram choose a vowel and a consonant from the letters of the word ALLAHABAD?
(a) 4 (b) 6 (c) 9 (d) 5

52. There are three rooms in a motel: one single, one double and one for four persons. How many ways are there to house seven persons in these rooms?
(a) 7!/1!2!4! (b) 7! (c) 7!/3 (d) 7!/3!

53. There are 8 consonants and 5 vowels in a word jumble. In how many ways can we form 5-letter words having three consonants and 2 vowels?
(a) 67200 (b) 8540 (c) 720 (d) None of these

54. There are 5 different Jeffrey Archer books, 3 different Sidney Sheldon books and 6 different John Grisham books. The number of ways in which at least one book can be given away is
(a) 2^{10} - 1 (b) 2^{11}- 1 (c) 2^{12} -1 (d) 2^{14}-1

55. A cricket team of 11 is to be chosen from 8 batsmen, 6 bowlers, and 2 wicket-keepers. In how many ways can the team be chosen if there must be at least four batsmen,, at least four bowlers, and exactly one wicket-keeper?
(a) 1681 (b)5304 (c) 1652 (d) None of these

56. From a group of persons, the number of ways of selecting 5 persons is equal to that of 8 persons. The number of persons in the group is:
(a) 13 (b) 40 (c) 18 (d) 21

Exercise – 2
Permutation

1. Find the value of ${}^{9}P_3$
(a) 504 (b) 309 (c) 405 (d) 600

2. If ${}^{56}P_{r+6}$: ${}^{54}P_{r+3}$= 30800 : 1 find r,
(a) 14 (b) 20 (c) 41 (d) 21

3. How many even numbers less than 10,000 can be formed with the digits 3,5, 7,8, 9 without any repetition?
(a) 32 (b) 16 (c) 44 (d) 41

4. How many numbers with different digits each greater than 4000 can be formed from the digits 0,2, 5, 7,8 ?
(a) 160 (b) 168 (c) 320 (d) 270

5. How many 6 digit telephone numbers can be constructed with the digits 0, 1,2, 3, 4, 5, 6, 7, 8, 9 if each number starts with 35 and no digit appears more than once?
(a) 1600 (b) 1680 (c) 900000 (d) 9000

6. Find the sum of all the four digit numbers which are formed by the digits 1, 2, 5, 6.
(a) 933510 (b) 93324 (c) 65120 (d) 8400

7. How many numbers each lying between 100 and 1000 ' can be formed with the digits 0, 2, 3, 4, 5 (no digit being repeated)?
(a) 24 (b) 48 (c) 72 (d) 96

8. If all the words formed by the letters of word RAINBOW are arranged in a dictionary form, then what is the position of the word RAINBOW in that dictionary?
(a) 3136 (b) 3631 (c) 3361 (d) 1363

9. How many different words can be formed with the letters of the word RAINBOW so that the vowels occupy odd paces?
(a) 676 (b) 625 (c) 343 (d) 576

10. In how many ways can the letters of the word RAINBOW be arranged so that only two vowels always remain together?
(a) 2880 (b) 1440
(c) 3200 (d) none of these

11. Four men and three women are to be seated for a dinner such that no two women sit together and no two men sit together. Find the number of ways in which this can be arranged.
(a) 144 (b) 72
(c) 36 (d) none of these

12. In how many ways 6 students and 4 teachers be arranged in a row so that no two teachers are together?
(a) 604800 (b) 24680 (c) 25860 (d) None of these

13. In how many different ways can 8 examination papers be arranged in a row, so that the best and the worst papers may never come together?
(a) 30240 (b) 23400 (c) 12340 (d) None of these

14. A person has 3 shirts 4 coats and 6 ties. In how many ways can he wear them?
(a) 27 (b) 36 (c) 72 (d) 130

15. How many different signals can be made by taking 3 different coloured flags at a time from 5 different coloured flags?
(a) 120 (b) 60 (c) 15 (d) 125

16. There are 10 stations on. a certain railway line many different kinds of tickets of class IInd must be printed in order that a passenger may go from one station to any other by purchasing ticket?
(a) 90 (b) 45 (c) 135 (d) 100

17. Find the number of ways in which 12 different books can be arranged on a shelf so that three particular book shall not be together,
(a) 126 x 10! (b) 125000 (c) 357500 (d) 123040

18. In how many ways can the letters of the word REPEAT be arranged ?
(a) 2240 (b) 232230 (c) 360 (d) 235760

19. In how many ways can the letters of the word ASSASSINATION be arranged?
(a) 181800 (b) 818100 (c) 108108 (d) 10810800

20. How many arrangements can be made out of the letters of the word COMMITTEE, taken all at a time, such that the four vowels do not come together?

(a) 216 (b) 45360 (c) 1260 (d) 43200

21. How many words can be made from the word MATHEMATICS in which vowels are together?
(a) 12960 (b) 120960 (c) 15400 (d) none of these

22. If all the letters of the word SEQUESTERED be arranged as in a dictionary, what is 50th word?
(a) DEEEEQURSTS (b) RUQDESTESEE
(c) ESSTREEUQDE (d) DQUESTREEES

23. How many 6 digit numbers can be formed out of the number 567724, which are even?
(a) 360 (b) 480 (c) 180 (d) 220

24. How many numbers of 5 digits can be formed with the digits 0, 2, 3, 4 and 5 if the digits may repeat?
(a) 2500 (b) 250 (c) 120 (d) 2400

25. How many of the numbers from 1000 to 9999 (both inclusive) do not have four different digits?
(a) 4446 (b) 4664 (c) 4464 (d) 6444

26. In how many ways can 5 prizes be given away to 4 boys, when each boy is eligible for all the prizes?
(a) 1024 (b) 20 (c) 625 (d) 540

27. In how many ways can n balls be randomly distributed in n cells?
(a) n! (b) n^n (c) n(n-1) (d) 2^n

28. In how many ways can 5 letters be posted in 4 letter boxes?
(a) 512 (b) 1024 (c) 625 (d) 20

29. In how many ways can 6 boys form a ring?
(a) 120 (b) 720 (c) 119 (d) none of (a), (b), (C)

30. In how many ways can 6 beads be strung into a necklace?
(a) 60 (b) 360 (c) 720 (d) 120

31. In how many ways can 5 men and 2 ladies be arranged at a round table if two ladies are never together?
(a) 5040 (b) 480 (c) 240 (d) 720

32. 7 sisters dine at a round table. They dine together ti|| each of them dine with different neighbours i.e., they do not like to dine with same neighbours in any two arrangements. In a year how many days they dine together?
(a) 180 (b) 504 (c) 720 , (d) 360

33. Find the number of ways in which 10 different flowers can be strung to form a garland so that 3 particular flowers are always together.
(a) 30240 (b) 30420 (c) 23400 (d) none of (a), (b), (c)

34. In how many ways can 7 Indians, 5 Pakistanis and 6 Dutch be seated in a row so that all persons of the same nationality sit together?
(a) 3! (b) 7!5!6! (c) 3!.7!.5!.6! (d) 182

35. In how many ways can the letters of the word PATNA be rearranged?
(a) 60 (b) 120 (c) 118 (d) 59

36. If ${}^{10}P_r = 720$, find r.
(a) 4 (b) 5 (c) 3 (d) 6

37. How many numbers of four digits can be formed with the digits 0, 1, 2, 3 (repetition of digits being allowed?
(a) 12 (b) 108 (c) 256 (d) 192

38. How many new words are possible from the letters of the word PERMUTATION?
(a) 11!/2! (b) (11!/2!) - 1 (c) 11!-1 (d) None of these

39. In how many ways can 12 papers be arranged if the best and the worst paper never come together?
(a) 12!/2! (b) 12!-11! (c) (12!- 11!)/2 (d) 12!-2.11!

40. In how many ways can the letters of the word 'EQUATION' be arranged so that all the vowels come together?
(a) ${}^9C_4$, ${}^9C_5$ (b) 4!.5! (c) 9!/5! (d) 9!-4!5!

41. How many 7-digit numbers are there having the digit 3 three times & the digit 5 four times?
(a) 7!/(3!)(5!) (b) $3^5 x 5^5$ (c) 77 (d) 35

42. From 4 gentlemen and 4 ladies a committee of 5 is to be formed. Find the number of ways of doing so if the committee consists of a president, a vice-president and three secretaries?
(a) ${}^8P_5$ (b) 1120 (c) ${}^4C_2$ x ${}^4C_3$ (d) None of these

43. In a letter lock, each of the three rings is marked with 15 letters. What is the maximum number of unsuccessful attempts that one has to make before the lock is opened?
(a) 3374 (b) 5284 (c) 8457 (d) 8101

44. From 3 different soft drinks, 4 Chinese dishes, and 2 ice-creams, how many different meals are possible if at least one of each of the three items is to be included, depending upon the number of people likely to turn up?
(a) 315 (b) 282 (c) 864 (d) None of these

45. Akshay is planning to give a birthday party at his place. In how many ways can he invite one or more of five friends and seat them at a circular table?
(a) 84 (b) 89 (c) 78 (d) 81

46. How many new words can be formed from the letters of the word 'Circle' taken together?
(a) 720 (b) 719 (c) 360 (d) 359

47. How many words can be formed out of the letters of the word 'Article' so that the vowels occupy the even places?
(a) 72 (b) 144 (c) 288 (d) 36

48. How many signals can be made by hoisting 2 blue, 2 red, and 5 black flags on a pole at the same time?
(a) 378 (b) 1512 (c) 756 (d) None of these

49. Five men, 6 boys, and 7 women are to be seated in a row so that the men, women, and boys may not be separated in:
(a) 5! x 6! x 7! ways (b) 6!6!7! ways
(c) 2!5!6!7! ways (d) 3!5!6!7! ways

50. How many numbers between 100 and 1000 can be formed with the digits, 1, 2, 3, 4, 5, 6, and 7, with no digit being repeated?
(a) 210 (b) 420 (c) 105 (d) 320

Exercise – 3

1. If a + b + c = 21, what is the total number of non-negative integral solutions?
(a) 123 (b) 253 (c) 321 (d) 231

2. If a + b + c = 21 what is the total number of positive integral solutions?
(a) 109 (b) 190 (c) 901 (d) 910

3. In how many ways can 8 identical apples be divided among 3 sisters?
(a) 25 (b) 65 (c) 45 (d) 24

4. In how many ways can 100 soldiers be divided into 4 squads of 10, 20, 30, 40 respectively?
(a) 1700 (b) 18! (c) 190 (d) none of these

5. In how many- ways can 16 books on different subjects be divided equally into 4 groups?
(a) $\frac{16!}{(4!)}$ (b) 4! x (16!) (c) $\frac{16!}{(4!)^5}$ (d) none of these

6. The number of squares on a chessboard is:
(a) 102 (b) 108 (c) 216 (d) 204

7. If P_r stands for ${}^rP_r$, then the value of
$1 + 1.\ P_1 + 2.\ P_2 + 3.\ P_3 + \ldots + n.\ P_n$ is:
(a) $\frac{(n-1)!}{2}$ (b) $\frac{(n+1)!}{2}$ (c) 2(n-1)! (c) (n+1)!

8. The number of integral solutions for the equal a+b + c + d = 12, where (a, b, c, d)≥-1 is:
(a) ${}^{19}C_3$ (b) ${}^{18}C_4$ (c) ${}^{20}C_4$ (d) none of these

9. If, If n = ${}^kC_2$, the value of ${}^nC_2$ is:
(a) $2({}^{k+2}C_4)$ (b) (n-2)! (c) (k-2)! (d) $3({}^{k+3}C_4)$

10. How many subsets containing at most n elements from the set of (2n + 1) elements can be selected?
(a) 2^n (b) 2^{n-1} (c) 2^{n+1} (d) 2^{2n}

11. The number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 \leq$ n (where n is a positive integer) is:
(a) ${}^{n+3}C_3$ (b) ${}^{n+2}C_3$ (c) ${}^{n+4}C_4$ (d) ${}^{n+4}C_3$

12. 2m people are arranged along two sides of a long table with m chairs each side, r men wish to sit on one particular side and x on the other. In how many ways can they be seated? (r, 5 ≤ mi)
(a) ${}^{48}C_r$ (b) 68 m (c) $\frac{(2m+r)!}{r-s!}$ (d) none of these

13. Management city has m parallel roads running East-West and n parallel roads running North-South. How many shortest possible routes are possible to go from one corner of the city to its diagonally opposite corner?

(a) $^{m+n}C_{m-1}$ (b) $^{m+n}C_n$ (c) $^{(m+n-2)}C_{(m-1)}$ (d) none of these

14. The sum of the numbers of the n^{th} term of the series (1)+(1 + 2)+(1 + 2+3)+(1 + 2+3+4)+...(1 + 2+3+...n):
(a) $^{n+1}C_3$ (b) $^{n+1}C_2$ (c) $^{n}C_2$ (d) $^{n+2}C_3$

15. Maximum number of points of intersection of 6 circles, is:
(a) 30 (b) 28 (c) 15 (d) none of these

16. An examination consists of 4 papers. Each paper has a maximum of 'n' marks. Find the number of ways in which a students can get 2n marks in the examination.
(a) $\frac{1}{3}(n^2-5n + 4)$ (b) $\frac{1}{3}(n+1)(2n^2 + 4n +3)$
(c) $\frac{1}{6}(n+1)(n+4)$ (d) none of the above

17. The letters of the word ALLAHABAD are rearranged to form new words and put in a dictionary. If the dictionary has only these words and one word on every page in alphabetical order then what is the page number on which the word LABADALAH comes?
(a) 6089 (b) 6088 (c) 6087 (d) 6086

18. If x, y and z can only take the values 1, 2, 3, 4, 5, 6, 7 then find the number of solutions of the equation x + y + z= 12.
(a) 36 (b) 37 (c) 38 (d) 31

19. There are nine points in a plane such that no three are collinear. Find the number of triangles that can be formed using these points as vertices,
(a) 81 (b) 90 (c) 9 (d) 84

20. There are nine points in a plane such that exactly three points out of them are collinear. Find the number of triangles that can be formed using these points as vertices.
(a) 81 (b) 90 (c) 9 (d) 83

21. If xy is a 2-digit number and u, v, x, y are digits, then find the number of solutions of the equation:
$(xy)^2 = a! + v$
(a) 2 (b) 3 (c) 0 (d) 5

22. There is a number lock with four rings. How many attempts at the maximum would have to be made before getting the right number?
(a) 104 (b) 255 (c) 10^4-1 (d) None of these

23. How many rectangles can be formed out of a chessboard?
(a) 204 (b) 1230
(c) 1740 (d) None of these

24. How many distinct 6-digit numbers are there having 3 odd and 3 even digits?
(a) 55 (b) $(5.6)^3.(4.6)^3.3$ (c) 281250 (d) None of these

25. The sides AB, BC, CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. The total number of triangles that can be constructed by using these points as vertices are
(a) 212 (b) 210 (c) 205 (d) 190

26. The number of positive integral solutions of x + y + z = n, n ϵ N, n > 3 is:

(a) $^{n-1}C_2$ (b) $^{n-1}P_2$ (c) n(n - 1) (d) None of these

27. Triplet x, y, and z are chosen from the set {1, 2,3,.... 24,25} such that x < y < z. How many such triplets are possible?
(a) 25_{C_2} (b) 600 (c) $25_{C_2} + 25_{C_3}$ (d) 1200

28. The total number of nine-digit numbers of different digits is:
(a) 10(9!) (b) 8(9!) (c) 9(9!) (d) None of these

29. ABCD is a convex quadrilateral. 3,4, 5, and 6 points are marked on the sides AB, BC, CD, and DA, respectively. The number of triangles with vertices on different sides is:
(a) 270 (b) 220 (c) 282 (d) 342

30. If x, y, and z are integers and $x \geq 0$, $y \geq 1$, $z \geq 2$, x+y+z=15, then the number of values of the ordered triplet (x,y,z) is:
(a) 91 (b) 455 (c) $^{17}C_{15}$ (d) None of these

31. If a, b, and c are positive integers such that $a+b+c \leq 8$, then the number of possible values of the ordered triplet (a, b, and c) is:
(a) 84 (b) 56 (c) 83 (d) None of these

32. The product of r consecutive integers is necessarily divisible by:
(a) r (b) $\sum_{k=1}^{r-1} k$ (c) (r+1)! (d) None of these

33. Find the number of integral solutions of equation x+y + z + t = 29, x>0,y>1, z>2 and $t \geq 0$.
(a) $^{27}C_3$ (b) $^{28}C_3$ (c) 2600 (d) $^{29}C_4$

34. Find the number of non-negative integer solutions to the system of equations *a + b + c + d + e = 20 and a + b + c = 5.*
(a) 240 (b) 336 (c) 672 (d) 1008

Exercise – 4

1. A letter lock consists of 4 rings, each ring contains 9 non-zero digits. This lock can be opened by setting a 4 digit code with the proper combination of each of the 4 rings. Maximum how many codes can be formed to open the lock?
(a) 4^9 (b) $^9P_4$ (c) 9^4 (d) none of these

2. A group consists of 4 couples in which each of the 4 persons have one wife each. In how many ways could they be arranged in a straight line such that the men and women occupy alternate positions?
(a) 1152 (b) 1278 (c) 1296 (d) none of these

3. If all C's occur together and all U's also occur together, then how many arrangements are possible of the word SUCCESSFUL?

(a) 5745 (b) 2760 (c) 6720 (d) 5432

4. What is the sum of all 5 digit numbers which can be formed with the digits 0,1, 2, 3,4 without repetition?
(a) 2599980 (b) 235500 (c) 923580 (d) 765432

5. There are 16 executives including two brothers, Lehman and Mckinsey. In how many ways can they be arranged around the circular table if the two brothers can not be seated together?
(a) (14!) .13 (b) (14!) (c) $\frac{14!}{3!}$ (d) none of these

6. What is the total number of ways of selecting atleast one item from each of the two sets containing 6 different items each?
(a) 2856 (b) 3969 (c) 489 (d) none of these

7. In how many ways can 4 books be arranged out of 16 books on different subjects?
(a) 34650 (b) 43680 (c) 43890 (d) none of these

8. Four dice are rolled. The number of possible outcomes in which atleast one die shows 4 is:
(a) 671 (b) 168 (c)176 (d) none of these

9. How many 5 digit numbers divisible by 3 can be formed using the digits 0,1, 2, 3, 4 and 5, without repetition?
(a) 108 (b) 216 (c) 810 (d) 180

10. In how many ways can a committee of 4 women and 5 men be chosen from 9 women and 7 men, if Mr, A refuses to serve on the committee if Ms. B is a member?
(a) 1608 (b) 1860 (c) 1680 (d) 1806

11. In how many ways can the following prizes be given away to a class of 30 students, first and second in Mathematics, first and second in Physics, first in Chemistry and first in English?
(a) $\frac{30!}{4!}$ (b) $(30)^4 \times (29)^2$ (c) $(30)^3 - 1$ (d) $(30)^4 \times (29)^4$

12. How many 4 digit numbers divisible by 4 can be formed without using the digits 0,6,7,8,9 if the repetition of digits is not allowed?
(a) 67 (b) 68 (c) 24 (d) 48

13. The number of ways in which 4 pictures can be hung from 6 picture nails on the wall is :
(a) 4^6 (b) ${}^4P_6$ (c) ${}^6P_4$ (d) 6^4

14. The number of ways in which a team of eleven players can be selected from 22 players including 2 of them and excluding 4 of them is:
(a) ${}^{15}C_{10}$ (b) ${}^{16}C_{10}$ (c) ${}^{16}C_9$ (d) none of these

15. How many 10 digits numbers can be formed by using die digits 2 and 3?
(a) 2^{10} (b) 10^2 (c) 10! (d) none of these

16. How many different four digit numbers can be formed with die digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 such that the digit 5 appears exactly once?
(a) 1024 (b) 2048 (c) 4096 (d) none of these

17. How many different eight digit numbers can be formed using only four digits 1, 2, 3,4 such that the digit 2 occurs twice?

(a) 20412 (b) 12042 (c) 25065 (d) none of these

18. Given that n is the odd, the number of ways in which three numbers in AP can be selected from 1, 2, 3, 4, ... n is :
(a) $\frac{(n-1)^2}{4}$ (b) n^2 (c) n^3 (d) $(n-2)^2$

19. Ajay has 7 relatives, 3 men and 4 women. His wife Kajol also has 7 relatives, 3 women and 4 men. In how many ways can they invite 3 men and 3 women so that 3 of them are the Aja/s relatives and 3 his wife Kajol's?
(a) 485 (b) 458 (c) 365 (d) none of these

20. An urn contains 5 boxes. Each box contains 5 balls of different colours red, yellow, white, blue and black. Rangeela wants to pick up 5 balls of different colours, a different coloured ball from each box. If from the first box in the first draw, he has drawn a red ball and from the second box he has drawn a black ball, find the maximum number of trials that are needed to be made by Rangeela to accomplish his task if a ball picked is not replaced,
(a) 12 (b) 11 (c) 20 (d) 60

21. There are *N* men sitting around a circular table at N distinct points. Every possible pair of men except the ones sitting adjacent to each other sings a 2 minute song one pair after other. If the total time taken is 88 minutes, then what is the value of N?
(a) 8 (b) 9 (c) 10 (d) 11

22. There are 6 pups and 4 cats. In how many ways can they be seated in a row so that no cats sit together?
(a) 6^4 (b) 10!/(4!).(6!) (c) $6!\text{x}^7P_4$ (d) None of these

23. How many new words can be formed with the word MANAGEMENT all ending in G?
(a) $10!/(2!)^4$-1 (b) $9!/(2!)^4$ (c) $10!/(2!)^4$ (d) None of these

24. In how many ways 5 MBA students and 6 Law students can be arranged together so that no two MBA students are side by side?
(a) $\frac{7!6!}{2!}$ (b) 6!.6! (c) 5!.6! (d) $^{11}C_5$

25. Find the number of ways of selecting 'n' articles out of 3n + 1, out of which n are identical,
(a) 2^{2n-1} (b) $^{3n+1}C_n$ (c) $^{3n+1}P_n/n!$ (d) None of these

26. How many 10-digit numbers have at least 2 equal digits?
(a) 9 x $^{10}C_2$ x 8! (b) 9.10^9 - 9.9! (c) 9x9! (d) None of these

27. How many different 7-digit numbers can be written using only three digits 1, 2 and 3 such that the digit 3 occurs twice in each number?
(a) $^7C_2.2^5$ (b) 7!/(2!) (c) $7!/(2!)^3$ (d) None of these

28. Seven different objects must be divided among three people. In how many ways can this be done if one or two of them must get no objects?
(a) 381 (b) 36 (c) 84 (d) 180

29. How many 4-digii number, that are divisible by 4 can be formed from the digits 1. 2, 3, 4 and 5.
(a) 36 (b) 72 (c) 24 (d) None of these

30. How many 8-digit numbers are there the sum of whose digits is even?

(a) 14400 (b) 4.5^5 (c) 45.10^6 (d) None of these

31. How many natural numbers not more than 4300 can be formed with the digits 0,1, 2, 3, 4 (if repetitions are allowed)?
(a) 574 (b) 570 (c) 575 (d) 569

32. How many even natural numbers divisible by 5 can be formed with the digits 0, 1, 2, 3, 4, 5, 6 (if repetitions of digits not allowed)?
(a) 1957 (b) 1956 (c) 1236 (d) 1235

33. The number of ways to give 16 different things to three persons A, B, and C, so that B gets 1 more than A, and C gets 2 more than B, is:
(a) $\frac{16!}{4!5!7!}$ (b) 4!5!7! (c) $\frac{16!}{3!5!8!}$ (d) None of these

34. If ${}^nC_4$, ${}^nC_5$, and ${}^nC_6$ are in an AP, then find n is :
(a) 8 (b) 9 (c) 14 (d) 10

35. A tea party is arranged for 16 people along the two sides of a long table with 8 chairs on each side. Four men wish to sit on one particular side and two on the other side. In how many ways can they be seated?
(a) ${}^{10}C_4 \times 8!$ (b) $10P4 \times (8!)^2$
(c) ${}^{10}C_4 \times (8!)^2$ (d) $4! \times 2! \times (8!)^2$

36. There are 4 letters and 4 directed envelopes. The number of ways in which all the letters can be put in the wrong envelope is:
(a) 8 (b) 9 (c) 16 (d) None of these

37. If each of the m points on the straight line be joined to each of the n points on the other straight line, then excluding the points on the given two lines, the number of points of intersection of these lines is:
(a) $\frac{1}{4}mn(m - 1)(n-1)$ (b) ${}^{m+n}C_2$
(c) ${}^{mn}C_2$ (d) None of these

38. Consider *S* =(1,2, 3,..., 10). In how many ways two numbers from *S* can be selected so that the sum of the numbers selected is a double-digit number?
(a) 36 (b) 16 (c) 29 (d) ${}^9C_2$, ${}^5C_2$

Exercise – 05
TITA / Short Answer

1. Find the number of ways in which 21 balls can be distributed among 3 persons such that each person does not receive less than 5 balls,

2. How many different words can be made using the letters of the word 'HALLUCINATION1 if all consonants are together?

3. In bow many different ways can 6 different balls be distributed to 4 different boxes, when each box can hold any number of bail?

4. How many 5 digit numbers contain exactly two 7 in them?

ede

5. There are 4 different monitors and 6 different mother boards. How many different arrangements can be made to purchase a monitor and a motherboard?

6. If B_1 and B_5 have the caps C_1 and C_5 among themselves in how many ways can you arrange the caps among the 5 boxes ?

7. Find the total number of factors of 1680.

8. The exponent of 3 in 33! is :

9. The sides AB, BC, CA of a triangle ABC have 3,4 and 5 interior points respectively on them. The total number of triangles that can be constructed by using these points as vertices is:

10. The sum of the divisors of $2^3 . 3^4 . 5^2$ is:

11. Maximum number of points of intersection of 6 straight lines is:

12. Maximum number of points into which 3 circles and 3 lines intersect is :

13. The number of rectangles excluding squares from a rectangle of size of 12 x 8 is :

14. The number of ways in which 9 identical balls can be placed in three identical boxes is :

15. The number of different selection of 5 letters from 1A, 2 B's, 3C's, 4D's and 5E's is:

16. How many straight lines can be formed from 8 non-collinear points on the X-Y plane?

17. Let n be the number of different 5-digit numbers, divisible by 4 that can be formed with the digits 1, 2, 3, 4, 5 and 6, with no digit being repeated. What is the value of n?

18. How many numbers can be formed with odd digits 1, 3, 5, 7, 9 without repetition?

19. How many different 4-digit numbers can be written using the digits 1, 2, 3, 4, 5, 6, 7 and 8 only once such that the number 2 is contained once,

20. The number of natural numbers of two or more than two digits in which digits from left to right are in increasing order is

21. In how many ways a cricketer can score 200 runs with fours and sixes only?

22. Five boys and three girls are sitting in a row of eight seats. In how many ways can they be seated so that not all girls sit side by side?

23. Six white and six black balls of the same size are distributed among ten urns so that there is at least one ball in each urn. What is the number of different distributions of the balls?

24. There are 8 different locks, with exactly one key for each lock. All the keys have been mixed up. What is the maximum number of trials required in order to determine which key belongs to which lock?

25. The number of ways of painting the faces of a cube with six different colour is:

Answer Key & Explanations

Exercise – 1

1. Ans. (a)

Solution: $^8C_3 = \frac{8!}{3!5!} = \frac{8 \times 7 \times 6}{6} = 56$

2. Ans. (c)

Solution: $^{10}C_5 = \frac{10 \times 9 \times 8\ 7 \times 6 \times 5!}{5 \times 4 \times 3 \times 2 \times 1 \times (5!)} = 252$

3. Ans. (c)

Solution:

➔ Either r = r + 3 or r + r + 3 = 17

But since r ≠ r + 3

∴ r + r + 3 = 17 ➔ r = 7

4. Ans. (d)

Solution: If n is odd, the greatest value of ^nC_r = ^nC_m

When m = $\frac{(n-1)}{2}$ or m = $\frac{(n+1)}{2}$

x = $\frac{(11-1)}{2}$ = 5 or x = $\frac{(11+1)}{2}$ = 6

Hence (d) is the correct choice

5. Ans. (c)

Solution: $^{14}C_x$ is maximum when x = $\frac{14}{2}$ = 7

Alternative: Go through options.

6. Ans. (d)

Solution: Since there are total 10 persons and out of these 10 persons we have to select any 5 persons which can be done in $^{10}C_5$ ways.

$^{10}C_5 = \frac{10!}{5! \times 5!} = 252$

7. Ans. (a)

Solution: She may invite one or more friends by selecting either 1 or 2 or 3 or 4 or 5 friends out of 5 friends.

∴1 friend can be selected out of 5 in $^5C_1$ ways

2 friends can be selected out of 5 in $^5C_2$ ways

3 friends can be selected out of 5 in $^5C_3$ ways

4 friends can be selected out of 5 in $^5C_4$ ways

5 friends can be selected out of 5 in $^5C_5$ ways

Hence the required number of ways

= $^5C_1 + {^5C_2} + {^5C_3} + {^5C_4} + {^5C_5}$

= 5+10 + 10 + 5+1 = 31

Alternatively: $^5C_1 + {^5C_2} + {^5C_3} + {^5C_4} + {^5C_5} = 2^5 - 1 = 32$

Since, $^nC_1 + {^nC_2} + {^nC_3} + ... + {^nC_n} = 2^n - 1$

8. Ans. (b)

Solution: 4 men can be selected out of 7 men in $^7C_4$ ways

and 2 ladies can be selected out of 6 ladies in $^6C_2$ ways Hence, the required number of ways = $^7C_4 \times {^6C_2}$

= 35 x 15 = 525

9. Ans. (c)

Solution: A committee of 5 persons is to be formed from 6 gentlemen and 4 ladies by taking.

(i) 1 lady out of 4 and 4 gentlemen out of 6.

(ii) 2 ladies out of 4 and 3 gentlemen out of 6.

(iii) 3 ladies out of 4 and 2 gentlemen out of 6.

(iv) 4 ladies out of 4 and 1 gentleman out of 6.

In case II the number of ways = $^4C_1 \times {^6C_4}$ = 4 x 15 = 60.

In case II the number of ways = $^4C_2 \times {^6C_3}$ = 6 x 20 = 120.

Incase III the number of ways = $^4C_3 \times {^SC_2}$ =4x15=60

In case IV the number of ways = $^4C_4 \times {^6C_1}$ = 1 x 6 = 6

Hence, the required number of ways

= 60 + 120 + 60 + 6 = 246

10. Ans. (b)

Solution: A committee of 7 persons retaining a majority of Indians can be made from 6 Americans and 7 Indians by taking

(i) 1 American out of 6 and 6 Indians out of 7.

(ii) 2 Americans out of 6 and 5 Indians out of 7.

(iii) 3 Americans out of 6 and 4 Indians out of 7.

In case I the number of ways = $^6C_1 \times {^7C_6}$ = 6 x7 = 42

In case II the number of ways = $^6C_2 \times {^7C_5}$ = 15 x 21 = 315

In case III the number of ways = $^6C_3 \times {^7C_4}$ = 20x35=700

Hence, the required number of ways

= 700 + 315+42=1057

11. Ans. (d)

Solution: Since 3 particular members are already selected, then we are required to select only 4 members out of the remaining 8 members.

It can be done in $^8C_4$ ways.

$^8C_4$=70

12. Ans. (a)

Solution: 3 experts including atleast an engineer and a manager can be selected by taking.

(i) 2 managers out of 3 and 1 engineer out of 3.

(ii) 1 manager out of 3 and 2 engineer out of 3.

(iii) 2 persons out of 6 (3 managers and 3 engineers) and 1 person out of one who is both engineer and manager.

In case I, the number of ways = ${}^3C_2 \times {}^3C_1 = 9$

In case II, the number of ways = ${}^3C_1 \times {}^3C_2 = 9$

In case III, the number of ways = ${}^6C_1 \times {}^1C_1 = 15$

Hence, the required number of ways = 9 + 9+15=33

13. Ans. (a)

Solution: A committee of 5 persons, consisting of at most two ladies, can be formed in the following ways.

(i) selecting 5 gents only out of 6.

(ii) selecting 4 gents only out of 6 and one lady out of 4.

(iii) selecting 3 gents only out of 6 and two ladies out of 4.

In case I, the number of ways = ${}^6C_5$

In case II, the number of ways = ${}^6C_4 \times {}^4C_1$

In case III, the number of ways = ${}^6C_3 \times {}^4C_2$

Required number of ways $= {}^6C_5 + {}^6C_4 \times {}^4C_1 + {}^6C_3 \times {}^4C_2$

= 6 + 60 + 120 = 186

14. Ans. (b)

Solution: A voter may cast vote for either 1 candidate or 2 candidates or 3 candidates or 4 candidates out of 7.

The voter may cast vote for 1 candidate in ${}^7C_1$ ways

The voter may cast vote for 2 candidates in ${}^7C_2$ ways

The voter may cast vote for 3 candidates in ${}^7C_3$ ways

The voter may cast vote for 4 candidate in ${}^7C_4$ ways

Hence, the required number of ways

$= {}^7C_1 + {}^7C_2 + {}^7C_3 + {}^7C_4$

=7 + 21 + 35 + 35= 98

15. Ans. (d)

Solution: Required number of ways = ${}^8C_6 \times {}^8C_5$

= 28 x 56 = 1568

16. Ans. (c)

Solution: Required number of ways

$= {}^7C_3 \times {}^5C_5 + {}^7C_4 \times {}^5C_4 + {}^7C_5 \times {}^5C_3$

= 35 x 1 + 35 x 5 + 21 x 10

= 35+175+210=420

17. Ans. (a)

Solution: Required number of ways = ${}^{13}C_8 \times {}^5C_2$

= 1287x10

= 12870

18. Ans. (a)

Solution: If n is even, ${}^nC_r$ is maximum when $r = \frac{n}{2}$

∴ Number o invitees in a party $= \frac{24}{2} = 12$

And maximum possible number of parties = ${}^{24}C_{12}$

= 2704156

19. Ans. (c)

Solution: Let there were n persons in the meeting, then number of handshakes = ${}^nC_2$

${}^nC_2 = 66$

20. Ans. (a)

Solution: Total number of bowlers = 4

Total number of wicket keepers = 2

Rest (normal) players = 10

Possible Combinations :

Bowlers	Wicket Keepers	Normal Player
3	1	7
3	2	6
4	1	6
4	2	5

∴ Required number of ways

$= ({}^4C_3 \times {}^2C_1 \times {}^{10}C_7) + ({}^4C_3 \times {}^2C_2 \times {}^{10}C_6) + ({}^4C_4 \times {}^2C_1 \times {}^{10}C_6) + ({}^4C_4 \times {}^2C_2 \times {}^{10}C_5)$

= (3x 1 x 7) + (3 x 2 x 6) + (4 x 1 x 6) + (4 x 2 x 5)

= 960 + 840 + 420 + 252 = 2472

21. Ans. (b)

Solution: Total number of bowlers = 6

Total number of wicket keepers = 3

Total number of normal players = 11 [20 - (6 + 3)]

Possible combinations:

Bowlers	**Wicket**	Normal
4	2	5
5	2	4
6	2	3

Required number of ways

$= ({}^6C_4 \times {}^3C_2 \times {}^{11}C_5) + ({}^6C_5 \times {}^3C_2 \times {}^{11}C_4) + ({}^6C_6 \times {}^3C_2 \times {}^{11}C_3)$

= 20790 + 5940 + 495 = 27225

22. Ans. (c)

Solution: 1 book on Economics can be collected out of 3 in ${}^3C_1$ ways

1 book on Corporate strategy can be collected out of 4 in $^4C_1$ ways 1 book on Philosophy can be collected out of 5 in $^5C_1$ ways Hence the required number of collections each of which consists of exactly one book on each subject

$$= {}^3C_1 \times {}^4C_1 \times {}^5C_1$$
$$= 3 \times 4 \times 5 = 60$$

23. Ans. (a)

Solution: Since all the three balls are red which can be selected from 7 red balls only.

∴ Required number of selections = $^7C_3 = 35$

24. Ans. (a)

Solution: Total number of red balls = 5

Total number of green balls = 6

Possible combinations are:

Red	Green
2	4
3	3
4	2

∴ Required number of selections

$= ({}^5C_2 \times {}^6C_4) + ({}^5C_3 \times {}^6C_3) + ({}^5C_4 \times {}^6C_2) = 425$

25. Ans. (c)

Solution: Total number of lines formed by 12 points = $^{12}C_2$

Number of lines formed by 4 points = $^4C_2$

∴ Required number of lines = Total lines formed by 12 points

∴ number of lines formed by 4 collinear points + 1

$= {}^{12}C_2 - {}^4C_2 + 1 = 61$

Explanation: If no three points are collinear, a straight line can be formed by joining any two points out of given non- collinear points. But since if some points are collinear then some lines become overlapped.

Here 12 points can make $^{12}C_2$ lines, but since 4 points are collinear, we can find only one line by joining these 4 collinear points where as we have considered $^4C_2$ lines # $^{12}C_2$ lines.

Hence the required number of straight lines

$= {}^{12}C_2 - {}^4C_2 + 1 = 61$

26. Ans. (b)

Solution: A decagon has 10 vertices i.e., 10 non-collinear points. Hence the total number of lines formed by 10 non-collinear point' = $^{10}C_2 = 45$

But since out of 45 lines 10 lines are the sides of the decagon and the remaining 35 lines are diagonals of a decagon.

Alternatively : Number of diagonals = $^nC_2$-n

27. Ans. (b)

Solution: Number of diagonals in an n sided polygon

= Number of total lines – number of sides

$= {}^nC_2 - n$

$= \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$

28. Ans. (b)

Solution: Number of diagonals = $^nC_2 - n = 54$

$= \frac{n(n-1)}{2} - n = 54$

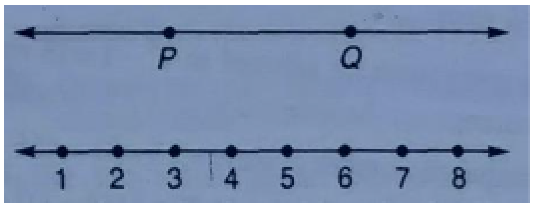
$= \frac{n(n-3)}{2} = 54$

$= n = 12$

Alternatively: Go through options.

29. Ans. (b)

Solution:



A triangle can be formed by joining 3 non-collinear points. Hence number of triangles is equal to the number of ways in which 3 points can be selected out of 6 non-collinear points.

Number of triangles = $^6C_3 = 20$

30. Ans. (b)

Solution: Number of triangles that can be formed if no three points are collinear = $^{12}C_3$ = 220.

But since 4 points are collinear, we cannot form any triangle from these 4 points by joining any three of them where as we have taken $^4C_3$ triangles in the above result (i.e.,$^{12}C_3$).

Hence the required number of triangles = $^{12}C_3 - {}^4C_3$

= 220 - 4 = 216

31. Ans. (c)

Solution: A triangle can be formed by selecting 1 point out of *P* and Q and selecting 2 points out of 8 points on the other parallel line.

or a triangle can be formed by selecting 2 points out of 2 points (P and Q) and selecting 1 point out of 8 points.

∴ Required number of triangles = $({}^2C_1 \times {}^8C_2) + ({}^2C_2 \times {}^8C_1)$

$= (2 \times 28) + (1 \times 8) = 64$

32. Ans. (d)

Solution: An intersection point is formed by the intersection of two lines. Hence number of

intersection points is equal to the number of ways of selecting 2 lines out of the given 20 non-parallel and non-concurrent lines.
i.e, Required number of points = ${}^{20}C_2 = 190$.

33. Ans. (b)
Solution: Number of straight line formed by ‘n’ non-collinear points
$= {}^{n}C_2 = \frac{n(n-1)}{2}$

34. Ans. (b)
Solution: Number of triangles formed by ‘n’ non-collinear points is ${}^{n}C_3$.
But since m points are in a straight line therefore from these m points ${}^{m}C_3$ triangles can not be formed. Hence required number of triangles = ${}^{n}C_3 - {}^{m}C_3$.

35. Ans. (d)
Solution: A parallelogram is formed by choosing two straight lines. From the set of m parallel lines and two straight lines from the set of n parallel lines.
Two straight lines from the set of m parallel lines can be chosen in ${}^{m}C_2$ ways and two straight lines from the set of n parallel lines can be chosen in ${}^{n}C_2$ ways. Hence the number of parallelograms formed
$= {}^{m}C_2 \times {}^{n}C_2 = \frac{m(m-1)}{2} \times \frac{n(n-1)}{2}$

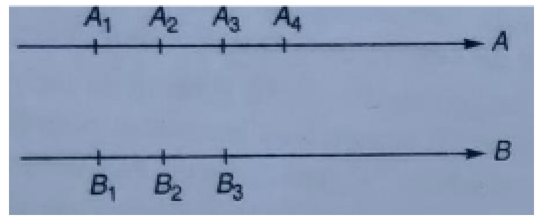
$= \frac{mn\,(m-1)(n-1)}{4}$

36. Ans. (a)
Solution: The number of points of intersection of 37 lines is ${}^{37}C_2$. But 13 straight lines out of the given 37 straight lines pass through the same point A. Therefore instead of getting ${}^{13}C_2$ points, we get only one point A. Similarly 11 straight lines out of the given 37 straight lines intersect at point *B*. Therefore instead of getting ${}^{11}C_2$ points, we get only one point *B*.
Hence the number of intersection points of die lines is

$${}^{37}C_2 - {}^{13}C_2 - {}^{11}C_2 + 2 = 535$$

37. Ans. (a)
Solution: There are 11 letters in the given word of which 2 are A’s. 2 are I’s, 2 are N’s and the remaining 5 letters are different, thus we have 11 letters of 8 different kinds (A, A), (I, I), (N, N), *E*, *X*, *M*, *T*, *O*.
A group of 4 letters can be classified as follows:
(i) Two alike of one kind and two alike of another find.
(ii) Two alike and the other two different.
(iii) All four different.
In case I, the number of ways = ${}^{3}C_2 = 3$
In case II, the number of ways = ${}^{3}C_1 \times {}^{7}C_2 = 63$
In case III, the number of ways = ${}^{8}C_4 = 70$
Hence, the required number of ways = 3 + 63 + 70 = 156

38. Ans. (b)



Solution: Hence we are left with 11 guests only out of which we choose 5 guests for the side A in ${}^{11}C_5$ ways and remaining 6 guest can be selected for the side B in ${}^{6}C_6$ ways.
Further in each side 9 guests can be arranged in 9! ways.
Hence the required number of arrangements
$= {}^{11}C_5 \times {}^{6}C_6 \times 9! \times 9! = 462 \times (9!)^2$

39. Ans. (c)
Solution: Atleast 1 green dye can be selected out of 5 green dyes in $(2^5 - 1)$ i.e., 31 ways.
Similarly, atleast on blue dye can be selected out of 4 in $(2^4 - 1)$ i.e., 15 ways.
And atleast 1 red or no red dye can be selected out od 3 red dyes in 2^3 i.e, 8 ways.
Hence the required number of ways = 31 x 15 x 8 = 3720.

40. Ans. (a)
Solution: Atleast one mango can be selected in $2^4 - 1 = 15$ ways and
atleast one orange can be selected in $2^5 - 1 = 31$ ways.
Hence the required number of ways = 15 x 31 = 465

41. Ans. (c)
Solution: It mean each player can receive 13 cards. .
∴ First player can get 13 cards in ${}^{52}C_{13}$ways.
Second player can get 13 cards in ${}^{39}C_{13}$ ways.
Third player can get 13 cards in ${}^{26}C_{13}$ ways
Fourth player can get 13 cards in ${}^{13}C_{13}$ ways.
Hence the total number of ways
$= {}^{52}C_{13} \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13} = \frac{52!}{(13!)^4}$

Alternatively: 52 cards can be divided equally among 4 players in $\frac{51!}{(13!)^4 \times (4!)}$ ways.

But since order is important, hence the required number of ways

$= \frac{52! \times 4!}{(13!)^4 \times (4!)} = \frac{52!}{(13!)^4}$

42. Ans. (a)

Solution: Required number of ways = $\frac{52!}{(13!)^4 \times (4!)}$

43. Ans. (a)

Solution: Number of ways of distributing 7 identical balls among 3 boxes so that a box receives atleast one ball

$= {}^{7-1}C_{3-1} = {}^{6}C_2 = 15$

44. Ans. (c)

Solution: Total number of non-negative integral Solutions of the given equations is same as the number of ways of distributing 44 items among 4 persons such that each person can receive any number of items.

Hence, total number of solutions = ${}^{44+4-1}C_{4-1} = {}^{47}C_3$.

45. Ans. (d)

Solution: Let u ≥ 0, v ≥ 0, w ≥ 0, then

$a \geq u + 1,\ b \geq v + 2,\ c \geq w + 3$

$\therefore a + b + c = 18$

$\Rightarrow u + 1 + v + 2 + w + 3 = 18$

$\Rightarrow u + v + w = 12$

$\therefore$ Total number of solutions of this equations is

$${}^{12+3-1}C_{3-1} = {}^{14}C_2$$

46. Ans. (b)

Solution: An IITian can make it to IIMs in 2 ways, while a CA can make it through in 3 ways. Required ratio is 2:3. Option (b) is correct.

47. Ans. (a)

Solution: Use the property ${}^{n}C_r = {}^{n}C_{n-r}$ to see that the two values would be equal at n = 11 since ${}^{n}C_3 = {}^{n}C_8$.

48. Ans. (b)

Solution: Trial and error would give us ${}^{8}C_4$ as the answer. ${}^{8}C_4 = 8 \times 7 \times 6 \times 5/4 \times 3 \times 2 \times 1 = 70$.

49. Ans. (c)

Solution: This is a typical case for the use of the formula ${}^{n-1}C_{r-1}$ with n = 10 and r = 6. So the answer would be given ${}^{9}C_5$.

50. Ans. (c)

Solution: ${}^{11}C_1 \times {}^{10}C_1 = 110$. Alternately, ${}^{11}C_2 \times 2!$

51. Ans. (a)

52. Ans. (a)

Solution: Choose 1 person for the single room & from the remaining choose 2 people for the double room & from the remaining choose 4 people for the 4 persons room $\rightarrow {}^{7}C_1 \times {}^{6}C_2 \times {}^{4}C_4$.

53. Ans. (a)

Solution: ${}^{8}C_3 \times {}^{5}C_2 \times 5! = 67200$

54. Ans. (d)

Solution: For each book we have two options, give or not give. Thus, we have a total of 2^{14} ways in which the 14 books can be decided upon. Out of this, there would be 1 way in which no book would be given. Thus, the number of ways is $2^{14} - 1$.

Hence, Option (d) is correct.

55. Ans. (d)

56. Ans. (a)

Exercise – 2

1. Ans. (a)

Solution: ${}^{9}P_3 = \frac{9!}{6!}$ $\quad (\therefore {}^{n}P_r = \frac{n!}{(n-r)!})$

$= \frac{9 \times 8 \times 7 \times 6!}{6!}$

$= 9 \times 8 \times 7 = 504$

2. Ans. (c)

4	1

Solution: $\frac{{}^{56}P_{r+6}}{{}^{56}P_{r+3}} = \frac{30800}{1}$

$\frac{\frac{56!}{(50-r)!}}{\frac{54!}{(51-r)!}} = \frac{30800}{1}$

4	3	1

$\Rightarrow \frac{56 \times 55 \times (51-r)!}{(50-r)!} = \frac{30800}{1}$

$\Rightarrow \frac{(51-r) \times (50-r)!}{(50-r)!} = \frac{10}{1}$

$= 51 - r = 10$

$r = 41$

Alternatively: Go through options.

3. Ans. (d)

Solution: Case 1. There is only one even number of one digit.

Case 2:

$= 4 \times 1 = 4$

There are only 4 even numbers of two digit (Numbers are 38, 58, 78 and 98)

Case 3.

$= 4 \times 3 \times 1 = 12$

There are only 12 even numbers of 3 digit.

Case 4.

4	3	2	1

= 4 x 3 x 2 x 1 = 24

There are 24 even numbers of 4 digits thus ther are total 1 + 4 + 12 + 24 = 1 even numbers.

Note: In each case 8 is fixed at unit place, since if the unit digit is an even digit then the whole number is an even number.

4. Ans. (b)

Solution:

3	4	3	2

= 3 x 4 x 3 x 2 = 72

Thousands place can assume only 3' values viz., 5,7,8. Since required numbers are greater than 4000.

4	4	3	2	1

= 4 x 4 x 3 x 2 x 1 = 96

and 98)

Ten thousands place can assume all the remaining non-zero digits and thousands place can assume zero also except the digit which has been filled up at thousands placed Therefore total required numbers

= number of 4 digit numbers + number of 5 digit numbers = 72+ 96 = 168

5. Ans. (b)

Solution:

8	7	6	5

= 8 x 7 x 6 x 5 = 1680

3 and 5 have been already used, so we have 8 digits for thousands place, then 7 digits for hundreds place, 6 digits for tens digit and remaining 5 digits for unit place.

Alternative:

$^8P_4 = \frac{8!}{4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!}$ = 8 x 7 x 6 x 5 = 1680

6. Ans. (b)

Solution:

4	3	2	1

= 4 x 3 x 2 x 1 = 24

There are total 24 numbers of 4 digit.

Since we have only 4 digits it means we use each of the digit 6 $\left(=\frac{24}{4}\right)$ times in each of the unit, tens hundreds and thousands place.

7. Ans. (b)

Solution:

= 48

Hundred's place can not assume 0.

8. Ans. (b)

Solution: The correct order of the letters is as follows:

A, B, I, N,O,R,W

Number of words begin with A = 6!

Number of words begin with B ~ 61

Number of words begin with I = 6!

Number of words begin with N = 6!

Number of words begin with 0 = 6!

Number of words begin with RAR = 4!

Number of words begin with RA/B = 3!

Now the next word is RAINBOW (it is the first word which begins with RAIN)

So the ranking of the word RAINBOW

=5 x 6!+ 4!+ 3!+1 = 3631

9. Ans. (d)

Solution: <u>1</u> 2 <u>3</u> 4 <u>5</u> 6 <u>7</u>

First of all arrange any 3 consonants at even places in $^4P_3$ ways. Now the newly created four odd places can be filled by the remaining letters which includes 3 vowels and 1 consonants, which can be done in $^4P_4$ ways. Hence the required number of permutation

= $^4P_3$ x $^4P_4$ = 24 x 24 = 576

10. Ans. (a)

Solution: First of all arrange all the four consonants *R, N, B, W* in 4! ways.

Then there are 5 places to be filled up by the vowels. But any two vowels are always together then we assume that there are only two vowels which can be filled in 5 places in $^5P_2$ ways. But we have to take any two vowels together out of 3 vowels then this can be done in $^3P_2$ ways.

Hence the total number of permutation = $^5P_2$ x $^3P_2$ x 4!

= 20 x 6 x 24 = 2880

Alternatively: Since there can be only 3 cases

(i) When A, *I* and *0* are separate from each other

(ii) When A, *I* and *0* can always together

(iii) When any two vowels out of three vowels A, *I* and *0* are together.

Now we need to calculate the value of Case (iii).

Required number of permutations

= Total number of permutations - (Case (i) + Case (ii)]

7! - (1440 + 720) = 5040 - (2160) = 2880

11. Ans. (a)

Solution: Since no two men or two women sit together it means they sit on the alternate positions.

Therefore first of all we arrange 3 women in 3! ways then we arrange 4 men in newly created 4 places in $^4P_4$ ways.

Thus the total number of arrangements = 3! x $^4P_4$

= 6x24=144 '

12. Ans. (a)

Solution: First of all the 6 students can be arranged in 6! ways then 4 teachers can be arranged in 7 places in $^7P_4$ ways.

Hence, the required number of arrangements = 6! x $^7P_4$

= 720 x 840 = 604800

13. Ans. (a)

Solution: Total number of arrangements = 81=40320

Number of arrangements when best and worst papers are together = 71x21 =10080

Number of arrangements in which best and worst papers are not together = 40320 - 10080 = 30240

14. Ans. (c)

Solution: The required number of permutation = 3x4x6 = 72

15. Ans. (b)

Solution: Required number of signals = $^5P_3$ = 60

16. Ans. (a)

Solution: A passanger from any station may purchase ticket for anyone of the other 9 stations. Therefore, there must be 9 tickets in each station. Therefore total number of different tickets = 9 x 10 = 90

17. Ans. (a)

Solution: Total number of permutations = 12!

Number of permutations when three particulars books are1 together = 10! x 3!

Number of permutations when three particular books are not together=12! -10! x 3!

= 10! (12 x 11-6)

= 10! X 126 = 457228800

18. Ans. (c)

Solution: In the given word there are 6 letters of which E occurs 2 times.

Hence the required number of ways = $\frac{6!}{2!}$

= $\frac{6 \times 5 \times 4 \, 3 \times 2 \times 1}{2}$ = 360

19. Ans. (d)

Solution: Required number of permutations = $\frac{13!}{3!.4!.2!.2!}$

= 10810800

20. Ans. (d)

Solution: There are total 9 letters in the word COMMITTEE in which there are 2M's 2T's, 3E's.

∴ The number of ways in which 9 letters can be arranged

= $\frac{9!}{2!x\ 2!x\ 2!}$ = 45360

There are 4 vowels O,I, E, E in the given word. If the four vowels always come together, taking them as one letter we have to arrange 5+1 = 6 letters which include 2Ms and 2Ts 6!

and this be done in $\frac{6!}{2! \times 2!}$ = 180 ways.

21. Ans. (b)

Solution: $\underbrace{M, M, T, T, H, C, S}\ \underbrace{A, A, E, I}$

When all the vowels are together then n = 7 + 1 = 8

∴ required number of permutation = $\frac{8!x\ 4!}{2!x\ 2!x\ 2!}$

= 120960

Note: (M, T and A occur 2 times and A A E I can be arranged mutually in $\frac{4!}{2!}$ ways)

22. Ans. (a)

Solution: The correct order of letters is D, E, E, E, E, Q, R, S, S, T, U.

Number of words begin with DEEEEQR= $\frac{4!}{2!}$ = 12

Number of words begin with DEEEEQS = 4! = 24

Number of words begin with DEEEEQT = $\frac{4!}{2!}$ = 12

Now, the next two words are DEEEEQURSST and DEEEEQURSTS

Hence the 50th word is DEEEEQURSTS

23. Ans. (c)

Solution:

5	4	3	2	1	3

= 3.(5!)

Unit place can assume any of the three even digits viz,2,4,6 and rest of the places can be filled up in 5! ways.

But since digit 7 occurs 2 times.

Required number of numbers = $\frac{3.(5!)}{2!}$ = 180

24. Ans. (a)
Solution: Ten thousands place can assume only non zero digits hence ten thousands place can be filled up in 4 ways and thousands place can be filled up in 5 ways since repetition is allowed (and 0 can be filled up in this place). Similarly hundreds, tens and unit places can be filled up in 5 ways each.
. .The required number of numbers = 4 x 5 x5 x 5 x'5= 2500

25. Ans. (c)
Solution: Number of numbers of 4 digit in which repetition allowed
= 9x10x10x10 = 9000
Number of numbers of 4 digit in which repetitions is not allowed
= 9 x 9 x 8 x 7 = 4536
Hence the required number of numbers of 4 digit
= 9000 - 4536 = 4464

26. Ans. (a)
Solution: First prize can be given away to 4 boys in 4 ways.
Similarly second, third, fourth and fifth prizes can also be given away to four boys in 4 ways.
Hence die required number of way in which all the 5 prizes can be given away to 4 boys
= 4 x 4 x 4 x 4x4 = 1024

27. Ans. (b)
Solution: The first ball can be placed in any one of then cells inn ways. The second ball can also be placed in any one of die n cells in n ways.
∴ The first and second balls can be placed in n cells in n x n.... n^2 ways.
Similarly each of the rest balls can be placed in n ways. Hence the required number of ways = n x n x... x n times = n^n

28. Ans. (b)
Solution: First letter can be posted in 4 letter boxes in 4 ways.
Similarly second letter can be posted in 4 letter boxes in 4 ways and so on.
Hence all the 5 letters can be posted in
= 4 x 4 x 4 x 4 x 4 = 1024

29. Ans. (a)
Solution: Required number of permutations = 5! = 120

30. Ans. (a)

31. Ans. (b)
Solution: Total number of arrangements = 6! = 720
Number of arrangements in which two ladies are together = 2x5!=240
∴ Number of arrangements in which two ladies are never together = 720 - 240 = 480
Alternatively: First of all place the five men in 4! ways, then place the two ladies in any 5 spaces in ${}^5P_2$ ways.
Hence, the required number of ways = 4! x ${}^5P_2$ = 480

32. Ans. (d)
Solution: Clearly 7 sisters can sit around a table in 6! ways. But in clockwise and anticlockwise arrangements each of the ladies have the same neighbours.
So, the required number of ways = $\frac{1}{2}$ x (6!) = 360
Hence they dine together only 360 days.

33. Ans. (a)
Solution: Since 3 particulars flowers are together
Hence there can be total (10 - 3) +1 = 8 flowers.
These 8 flowers can be arranged 7! ways but the 3 flowers which are together can be arranged mutually in 3! ways.
Hence, the required number of ways = 3!x7!
= 6 x 5040
= 30240

34. Ans. (c)
Solution: We need to assume that the 7 Indians are 1 person, so also for the 6 Dutch and the 5 Pakistanis. These 3 groups of people can be arranged amongst themselves in 3! ways. Also, within themselves the 7 Indians the 6 Dutch and the 5 Pakistanis can be arranged in 7!, 6! And 5! ways respectively. Thus, the answer is 3! x 7! x 6! x 5!.

35. Ans. (d)
Solution: Rearrangements do not count the original arrangements. Thus, 5!/2! - 1 =59 ways of rearranging the letters of PATNA.

36. Ans. (c)
Solution: ${}^{10}P_3$ would satisfy the value given as ${}^{10}P_3$ = 10 x 9 x 8 = 720.

37. Ans. (d)
Solution: 3 x 4 x 4 x 4 = 192

38. Ans. (b)
Solution: Number of 11 letter words formed from the letters P, E, R, M, U, T, A, T, I, O, N = 11/2!.
Number of new words formed = total words -

1 = 11!/2!-1

39. Ans. (d)
Solution: All arrangements - Arrangements with best and worst paper together = 12! - 2! x 11!.

40. Ans. (b)
Solution: The vowels EUAIO need to be considered as 1 letter to solve this. Thus, there would be 4! ways of arranging Q, T and N and the 5 vowels taken together. Also, there would be 5! ways of arranging the vowels amongst themselves. Thus, we have 4! x 5!.

41. Ans. (d)
Solution: 7!/3! X 4! = 35

42. Ans. (b)
Solution: From 8 people we have to arrange a group of 8 in
Which three are similar $\frac{8P_5}{3!}$ or $\frac{8C_5 \times 5!}{3!}$

43. Ans. (a)
44. Ans. (a)
45. Ans. (b)
46. Ans. (d)
47. Ans. (b)
48. Ans. (c)
49. Ans. (d)
50. Ans. (a)

Exercise – 3

1. Ans. (b)
Solution: Number of non-negative integral solutions = $^{n+r-1}C_{r-1}$
$= {}^{21+3-1}C_{3-1}$
$= {}^{23}C_2 = 253$

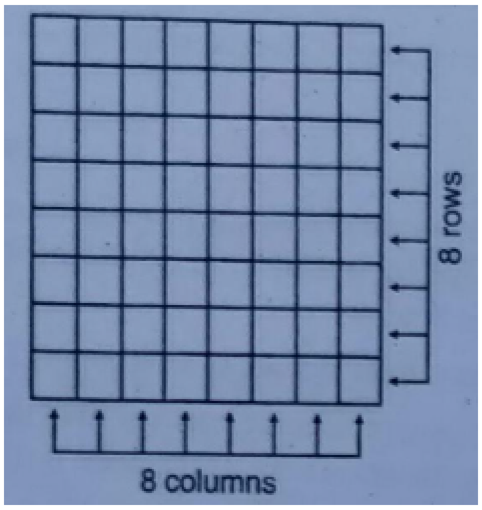
2. Ans. (b)
Solution: Number of positive integral solutions = $^{n-1}C_{r-1}$
$= {}^{21-1}C_{3-1} = {}^{20}C_2$
$= {}^{20}C_2$
$= 190$

3. Ans. (c)
Solution: Using $^{n+r-1}C_{r-1}$, we get $^{8+3-1}C_{3-1}$, $= {}^{10}C_2 =$ 45

4. Ans. (d)
Solution: $^{100}C_{10} \times {}^{90}C_{20} \times {}^{70}C_{30} \times {}^{40}C_{40} = \frac{100!}{10! \times 20! 30! \times 40!}$

5. Ans. (c)
Solution: $^{16}C_4 \times {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4 \times \frac{1}{4!} = \frac{16!}{(4!)^5}$

6. Ans. (d)
Solution: In a chess board there are 8 columns and 8 rows.



Since number of columns = number of rows
Hence it is considered as a square.
∴ Hence required number of squares
$= 1^2 + 2^2 + 3^2 + \ldots + 8^2$
$= \frac{8 \times 9 \times 17}{6} = 204$
$\left[\therefore \sum n^2 = \frac{n(n+1)(2n+1)}{6}\right]$

7. Ans. (d)
Solution: 1 + 1. P_1 + 2.P_2 + 3.P_3 + 4.P_4. + …. + n.P_n
= 1 + 1 + 2.2! + 3.3! + 4.4! + …+n.n!
= 1 + [(2! – 1!) + (3! – 2!) + (4! – 3!) + ….+ ((n+1)! – n!]
= 1 + [(n+1)! – 1!]
= (n+1)!
Alternatively: Consider n = 1, 2, 3 … etc. and then verify the result.

8. Ans. (a)
Solution: ∴ a ≥ - 1, b ≥ - 1, c ≥ -1, d ≥ - 1.
Let u ≥ 0, v ≥ 0, w ≥ 0, x ≥ 0.
∴ a + b + c + d = 12
➔ (u-1) + (v-1) + (w-1) + (x-1) = 12
➔ u + v + w + x = 16

∴ Required number of solutions = ${}^{16+4-1}C_{4-1}$ = ${}^{19}C_3$

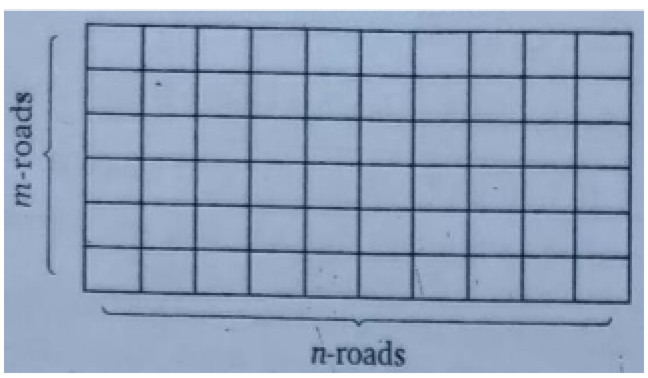
9. Ans. (d)
Solution: Assume some value of k then find 'n' and hence ${}^nC_2$.
Now, verify the correct option.

10. Ans. (d)
Solution: Consider some value of n then verify the result.

11. Ans. (c)
Solution: Let x_5 be such that
$x_1 + x_2 + x_3 + x_4 + x_5 = n$
The number of required solutions = ${}^{n+5-1}C_{5-1}$ = ${}^{n+4}C_4$

12. Ans. (d)
Solution: We can arrange r persons on m chairs on a particular side in ${}^mP_r$ ways and s persons on m chairs on the other side in ${}^mP_s$ ways. Now, we can arrange (2m-r-s) persons on the remaining (2m-r-s) chairs in ${}^{(2m-r-s)}P_{(2m-r-s)}$
Thus, the required number of ways of arranging the persons
= $({}^mP_r)\ ({}^mP_s)\ ({}^{2m-r-s}P_{2m-r-s})$

13. Ans. (c)
Solution: The diagram of roads can be as follows.



Suppose the distance between two successive parallel roads be one unit then a person can travel (m - 1)
steps in north-south direction and he can travel (n - 1) steps in east-west direction.
Thus he has to travel {(m - 1) + (n - 1)} = (m + n - 2) steps to reach from one corner to diagonally opposite
corner. Thus he can arrange his steps in ${}^{(m+n-2)}C_{(m-1)}$ x ${}^{(n-1)}C_{(n-1)}$ ways
${}^{m+n-2}C_{m-1}$ ways.

14. Ans. (b)
Solution: Go through options.

15. Ans. (a)
Solution: ${}^6P_2$ = 30.

16. Ans. (b)
Solution: Note: The general solution of this type of problems involves higher mathematics. But we have given in equation number 37 a very simple , lucid and a novel solution for the lay students.

17. Ans. (a)
Solution: No. of words starting with A = 8!/ 2!3! = 3360. No. of words starting with B = 8!/ 214! = 840 No. of words starting with D = 8!/2!4! = 840 No. of words starting with H = 8!/2!4! = 840 Now words with L start.
No. of words starting with LAA = 6!/2! = 360 Now LAB starts and first word starts with LABA. No. of words starting with LABAA = 4! = 24 After this the next words will be LABADAAHL, LABADAALH, LABADAHAL, LABADAHLA and hence, Option (a) is the answer.

18. Ans. (b)
Solution: We will consider x = 7 to x = 1.
For x = 7, y + z = 5. No. of solutions = 4
For x = 6, y + z = 6. No. of solution = 5
For x = 5, y + z = 7. No. of solutions = 6
For x = 4, y + z = 8. No. of solutions = 7
For x = 3, y = z = 9. No. of solutions = 6
For x = 2, y + z = 10. No. of solutions = 5
For x = 1, y + z = 11. No. of solutions = 4
Hence number of solutions = 37
Hence, Option (b) is the answer.

19. Ans. (d)
Solution: As no three points are collinear, therefore every combination of 3 points out of the nine points will give us a triangle. Hence, the answer is ${}^9C_3$ or 84. Hence, Option (d) is correct.

20. Ans. (d)
Solution: The number of combinations of three points picked from the nine given points is ${}^9C_3$ or 84. All these combinations will result in a triangle except the combination of the three collinear points. Hence number of triangles formed will be 84 - 1 = 83.
Hence, Option (d) is the answer.

21. Ans. (b)
Solution: $(xy)^2$ = a! + v
Here xy is a two-digit number and maximum value of its square is 9801. 8! is a five-digit number => *u* is less than 8 and 4! is 24 which when added to a single digit will never give the square of a two- digit number. Hence *u* is greater than 4. So, possible values of *u* can be 5, 6 and 7.

If u = 5, u!= 120 => $(xy)^2$ = a! + v => $(xy)^2$ = 120 + v= 120 + 1 = 121 = 11^2

If a = 6, a! - 720 => $(xy)^2$ = a! + v => $(xy)^2$ = 720 + v = 720 + 9 = 729 = 27^1

If a = 7, a!= 5040 => $(xy)^2$ = a! + v => $(xy)^2$ = 5040 + v = 5040 + 1 - 5041 = 71^2

So there are three cases possible. Hence, 3 solutions exist for the given equation.

Hence, Option (b) is the correct answer.

22. Ans. (c)

Solution: Total number of attempts = 10^4 out of which one is correct.

23. Ans. (d)

Solution: A chess board consists 9 parallel lines X 9 parallel lines. For a rectangle we need to select 2 parallel lines and two other parallel lines that are perpendicular to the first set.

Hence, ${}^9C_2$ x ${}^9C_2$

24. Ans. (c)

Solution: Total number of 6 digit numbers having 3 odd and 3 even digits (including zero in the left most place) = 5^3 x 5^3.

From this subtract the number of 5 digit numbers with 2 even digits and 3 odd digits (to take care of the extra counting due to zero)

25. Ans. (c)

26. Ans. (a)

27. Ans. (b)

28. Ans. (c)

Solution: it will be a number of 9 digits with the following box diagram:

9 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 = 9 x 9!

29. Ans. (d)

Solution: The triangles must be formed by the 1 marked point, each on any 3 consecutive sides, for examples, AB, BC, and CD (and so on). Points A, B, C, and D cannot be included in this case as we cannot form triangle using any of these points. So, total number of triangles that can be formed =

3 x 4 x 5 + 4 x 5 x 6 + 5 x 6 x 3 + 6 x 3 x 4 = 60 + 120 + 90 + 72 = 342

Hence, the correct option is (d).

30. Ans. (a)

Solution: This question can be rewritten as x > 0, y > 0, z > 0, x +y +z = 12. So, we have already given 1 to y and 2 to z. Now, number of non-negative solutions is ${}^{n+r-1}C_{r-1}$ p where a =12 and r = 3. So, number of possible triplets is ${}^{12+3-1}C_{3-1}$= ${}^{14}C_2$ = 91

Hence, the correct option is (a).

31. Ans. (b)

Solution: As the minimum value of a, b, and c is 1, the minimum value of a + b+c =n, which is 3. Here, r=3, and n could have values 3,4,5, 6, 7, and 8.

As we know positive integral value of a + b + c - n. which is ${}^{n-1}C_r$ Now, we have r = 2 and n varies from 3 to 8. So, total number of possible solutions =${}^{3-1}C_{3-1}$ + ${}^{4-1}C_3$ + ${}^{3-1}C_3$ + ${}^{6-1}C_3$ +${}^{7-1}C_3$ + ${}^{n-1}C_{3-1}$

= ${}^2C_2$ + ${}^3C_2$ + ${}^4C_2$ + ${}^5C_2$ + ${}^6C_2$ = 1 + 3 + 6 + 10 + 15 + 21 = 56

Hence the correct answer is (b).

32. Ans. (a)

33. Ans. (c)

Solution: It is similar to x+y + z + r = 26 where x>0,y>0,z>0, and t > 0. Now, we can first find the number of cases;

when all of x, y, z, and t are more than 0. Then, we can add the cases when t = 0.

Case 1: x+y+z+1=26 and it must be positive integral ; solution, that is, all of the variables are greater than 0.

Number of possible solutions =${}^{n-1}C_{r-1}$ (where r = 4 and n = 26) = ${}^{26-1}C_{4-1}$_ =${}^{25}C_3$ = 2300

Case II t = 0. Then, x+y+z=26, where all are positive integers = ${}^{n-1}C_{r-1}$ = ${}^{26-1}C_{3-1}$ = ${}^{25}C_2$ = 300

So, total number of possible cases = 2300 + 300 = 2600;

Hence, the correct option is (c). :

34. Ans. (b)

Solution: Given a + b + c + d + e = 20 (i)

a + b + c = 5 (ii)

Given system of equation is equivalent to a + b + c + = 5 (iii)

And d + e = 15 (iv)

Number non-negative integral solutions of equation (iii)

= ${}^{n+r-1}C_r$ = ${}^{3+5-1}C_5$ = ${}^7C_5$

Number of non-negative integral solution of equation (iv)

= ${}^{n+r-1}C_r$ = ${}^{2+15-1}C_{15}$ = ${}^{16}C_{15}$

∴ Required number = ${}^7C_5$. ${}^{16}C_{15}$ = 336

Exercise – 4

1. Ans. (c)
Solution: 9 x 9 x 9 x 9 = 9^4

2. Ans. (a)
Solution: **Case I.** MW MW MW MW
Case II. WM WM WM WM
Let us arrange 4 men in 4! ways, then we arrange 4 women in $^4P_4$ ways at 4 places either left of the men or right of the men. Hence required number of arrangements
= 4!x $^4P_4$ + 4! x $^4P_4$ = 2 x 576 = 1152

3. Ans. (c)
Solution: (C, C), (U, U), S, S, S, E, F, L
These 8 letters now can be arranged in $\frac{8!}{3!}$ ways.

4. Ans. (a)
Solution: Total number of 5 digit numbers (including which begins with zero) = 5! = 120.
Number of 5 digit numbers which begin with zero = 4! = 24.
Sum of all 5 digit numbers.
= (0 + 1 + 2 + 3 + 4) x 4! X (11111)
= 240 x 11111 = 2666640
Sum of all five digit numbers which begin with zero
= (1+2+3+4) x 3! X (1111) = 66660
Hence the sum of the required numbers
= sum of all 5 digit numbers including those numbers which begin with zero – sum of all 5 digit numbers which begin with zero = 2666640 – 66660 = 2599980

5. Ans. (a)
Solution: Total number of ways of arranging 16 people = 15! Ways
Number of ways in which two brothers are together = 14! x 2
∴ Number of ways in which two brothers are never together
= 15! – 14! X 2
= 14! (15-2) = 14! X 13

6. Ans. (b)
Solution: We can select atleast one item from 6 different items = $(2^6\text{-}1)$
Similarly we can select atleast one item from other set of 6 different items in $(2^6 - 1)$ ways.
Required number of ways = $(2^6 -1)$ $(2^6 -1)$6!
= $(2^6 - 1)^2$ = 3969

7. Ans. (b)
Solution: $^{16}P_4$ = 43680

8. Ans. (a)
Solution: Total number of possible outcomes = 6^4.
The number of possible outcomes in which 4 does not appear on any die is 5^4.
Therefore the number of possible outcomes in which atleast one die shows digit 4 = 6^4 – 5^4 = 671

9. Ans. (b)
Solution: Case I: 0,1, 2, 4, 5
Case II: 1, 2, 3, 4, 5
Since we know that a number is divisible by 3, if and only if the sum of its digits is divisible by 3.
Case I, number of 5 digit numbers = 4 x 4 x 3 x 2 = 96
Case II, number of 5 digit numbers = 5 x 4 x 3 x 2 = 120
Total required numbers = 216

10. Ans. (d)
Solution: The number of ways of choosing the committee when Ms, Bs a member (when Mr. A refuses to serve)
= $^8C_3$ x $^6C_5$
= 56 x 6 = 336
The number of ways of choosing the committee when Ms. Bs not a member (when Mr. A can seve)
= $^8C_4$ x $^7C_5$
= 70 x 21 = 1470
Thus the required number of ways = 336 + 1470 = 1806.
Alternatively: Total number of ways of selecting 4 women and 5 men = $^9C_4$ x $^7C_5$ = 2646.
Number of ways of selecting 4 women and 5 men in which Ms. B and Mr. A re not present together
= $^8C_3$ x $^6C_4$ = 840
Hence the number of ways in which Ms. B and Mr. A do not serve together = 2646 – 840 = 1806

11. Ans. (b)
Solution: First and second prizes can be given in Mathematics in (30 x 29) ways.
First and second prizes can be given in Physics in (30 x 29) ways.
First prize can be given Chemistry in 30 ways.
First prize can be given in English in 30 ways.
Hence, the number of ways to give prizes in all the four subjects
(30 x 29) x (30 x 29) x (30 x 30) = $(30)^4$ x $(29)^2$

12. Ans. (c)

Solution: Available digits are 1, 2, 3, 4 and 5. Now since we know that a number is divisible by 4 if and only if the number formed by last two digits is divisible by 4. So the following cases are possible

Thousands	Hundreds	Tens	Unit
x	y	1	2
x	y	2	4
x	y	3	5
x	y	5	2

In each case thousands and hundreds place can be filled up in ${}^3P_2$ ways.

Hence the required number of ways = ${}^3P_2$ x 4 = 24.

13. Ans. (c)

Solution: Required number of ways = ${}^6P_4$.

14. Ans. (c)

Solution: Required number of ways = ${}^{16}C_9$.

15. Ans. (a)

Solution: Since each digit of a 10 digit number can be written as either 2 or 3, therefore required number of 10 digit number is 2^{10}.

16. Ans. (b)

Solution: Digit 5 can be placed in any one of the 4 places in 4 way Now the remaining 3 places can be filled up with remaining 8 digits in 8^3 ways.

Hence, the required number of ways = 4 x 8^3 = 2048.

17. Ans. (a)

Solution: Digit 2 can be arranged in two places out of 8 places in $\frac{{}^8P_2}{2!}$ Ways.

Now, the remaining 6 places can be filled by the rest 3 digits in 3^6 ways.

Hence, the required number of ways = $\frac{{}^8P_2}{2!}$ X 3^6

= ${}^8C_2$ x 3^6 = 20412

18. Ans. (a)

Solution: Let n = 2m + 1, for the three numbers are in AP we have the following patterns: ,'

Common differences	Numbers	Number of ways
1	(1, 2,3)(2,3,4)...(n-2,n-l,a)	(n-2)
2	a,3,5),(2,4,6)...(n-4,n-2,n)	(n-4)
3	(l,4,7),(2,5,8)...(n-6,n-3,n)	(n-6)
	… …..	
--	… …..	
--		
m	(1, m + 1, 2m + 1)	1

∴ Favourable number of ways

= (n-2)+(n-4)+(n-6)+ ...+ 3+1 (total m terms)

= $\frac{m}{2}$ (n-1) (n-1) $(n-1)^2$

Alternatively: Consider some proper value of n and verify the result.

19. Ans. (a)

Solution: There are four possible cases.

$H \to$ Husband's Relatives

$W \to$ Wife's relatives

	H	W	H	W	H	W	H	W
M	0	3	1	2	2	1	3	0
F	3	0	2	1	1	2	0	3

Hence, the required number of ways.

= $({}^4C_3 \times {}^4C_3)$ + $({}^3C_1 \times {}^4C_2)$ $({}^4C_2 \times {}^3C_1)$ + $({}^4C_2 \times {}^4C_1)$ $({}^4C_1 \times {}^3C_2)$ + $({}^3C_3 \times {}^3C_3)$

= (4 x 4) (3 x 6 x 6 x 3) + (3 x 4 x 4 x 3) + (1 x 1)

= 485

20. Ans. (a)

Solution: As we need to find the maximum number of trials, so we have to assume that the required ball in every box is picked as late as possible. So in the third box, first two balls will be red and black. Hence third trial will give him the required ball. Similarly, in fourth box, he will get the required ball in fourth trial and in the fifth box, he will get the required ball in fifth trial. Hence maximum total number of trials required is 3 + 4 + 5 = 12.

Hence, Option (a) is the answer.

21. Ans. (d)

Solution: Total number of pairs of men that can be selected if the adjacent ones are also selected is ${}^NC_r$ Total number of pairs of men selected if only die adjacent ones are selected is N. Hence total number of pairs of men selected if the adjacent ones are not selected is ${}^NC_2$-N.

Since the total time taken is 88 min, hence the number of pairs is 44.

Hence, ${}^nC_2$ - N = 44 → N = 11.

Hence, Option (d) is the answer.

22. Ans. (c)
Solution: First arrange 6 pups in 6 places in 6! ways.
This will leave us with 7 places for 4 cats. Answer = 6! x $7p_4$

23. Ans. (b)
Solution: Arrangement of M, A, N, A, E, M, E, N, T is
$= \frac{9!}{2!x\ 2!\ x\ 2!x\ 2!}$

24. Ans. (a)
Solution: First make the six law students sit in a row. This can be done in 6! Ways. Then, there would be 7 places for the MBA students. We need to select 5 of these 7 places for 5 MBA students and then arrange these 5 students in those 5 places. This can be done in $^7C_5$ x 5! Ways.
Thus, the answer is: 6! x $^7C_5$ x 5! = 7! x 6!/2!

25. Ans. (d)
Solution: Divide 3n + I articles in two groups.
(i) n identical articles and the remaining
(ii) 2n + 1 non-identical articles
We will select articles in two steps. Some from the first group and the rest from the second group.

Number of articles from first group	Number of articles from second group	Number of ways.
0	N	1 x $^{2n+1}C_n$
1	n-1	1 x $^{2n+1}C_{n-1}$
2	n-2	1 x $^{2n+1}C_{n-2}$
3	n-3	1 x $^{2n+1}C_{n-3}$
....	....	.....
n-1	1	1 x $^{2n+1}C_1$
N	0	1 x $^{2n+1}C_0$

Total number of ways = $^{2n+1}C_n$ + $^{2n+1}C_{n-1}$ + $^{2n+1}C_{n-2}$ + $^{2n+1}C_{n-3}$ ..... $^{2n+1}C_1$ + $^{2n+1}C_0$
$= \frac{2^{2n-1}}{2} = 2^{2n}$

26. Ans. (b)
Solution: "Total number of all 10-digits numbers - Total number of all 10-digits numbers with no digit repeated" will give the required answer.
= 9 x $10^{9'}$ - 9 x $^9P_8$

27. Ans. (a)
Solution: Select the two positions for the two 3's. after that the remaining 5 place have to be filled using either 1 or 2.
Thus, $^7C_2$ x 2^5

28. Ans. (a)
Solution: Let the three people be *A*, *B* and C.
If 1 person gets no objects, the 7 objects must be distributed such that each of the other two get 1 object at least.
This can be done as 6 & I, 5 & 2, 4 & 3 and their rearrangements.
The answer would be
($^7C_6$ + $^7C_5$ + ^7Q x 3! = 378
Also, two people getting no objects can be done in ways,
Thus, the answer is 378 + 3 = 381

29. Ans. (c)
Solution: Natural numbers which consist of the digits 1,2,3, 4. and 5 and are divisible by 4 must have either 12, 24,32 or 52 in the last two places. For the other two places we have to arrange three digits in two places.

30. Ans. (c)
Solution: There will be 5 types of numbers, viz.
numbers which have
All eight digits even or six even and two odd digits or four even and four odd digits or two even and six odd digits or all eight odd digits.
This will be further solved as below:
Eight even digits -> 5^8 - 5^7 = 4 x 5^7
Six even and two odd digits —>
when the left most digit is even —> 4 x $^7C_5$ x 5^5 x 5^5
when the left most digit is odd -> 5 x $^1C_1$ x 5^6 x 5^6
Four even and four odd digits —>
when the left most digit is even -> 4 x $^7C_5$ x 5^s x 5
when the left most digit is odd → 5 x $^7C_4$. x 5^4 x 5^1
Two even and six odd digits →
when the left most digit is even -> 4 x ^1C x 5 x 5^6
when the left most digit is odd —> 5 x $^7C_2$ x 5^2 x 5^s Eight odd digits —> 58

31. Ans. (c)
Solution: The condition is that we have to count the number of natural numbers not more than 4300.
The total possible numbers with the given digits = 5 x 5 x 5 x 5 = 625- 1 = 624.

Subtract form this the number of natural number greater than 4300 which etui be formed from the given digits
=1 X 2 X 5 X 5 - 1= 49.
Hence, the required number of numbers = 624 - 49 = 575.

32. Ans. (b)
33. Ans. (a)
34. Ans. (c)
35. Ans. (c)

Solution: Four persons have chosen to sit on one particular side (assume side A) and 2 persons on the other side (assume side B). So, we are supposed to select 4 persons for side A from the remaining 10 persons and remaining 6 per sons will be sitting on side B.
Number of ways 4 persons can be selected from 10 per sons = ${}^{10}C_4$
Number of ways 6 persons can be selected from the remaining 6 persons = ${}^{6}C_6$
Number of ways 8 persons can be arranged on side A = 8!
Number of ways 8 persons can be arranged on side B = 8!
Total number of ways = ${}^{10}C_4 \times {}^{6}C_6 \times 8! \times 8!$

36. Ans. (b)

Solution: It is the simple case of derangement with n = 4.
Use the following formula:

$$n(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots.+(-1)^n\frac{1}{n!})$$

Hence answer = 4! (1-1 + ½ -1/6 + 1/24) = 9

37. Ans. (a)

Solution: Each of the m points of 1st straight line will form Intersection points with the n - 1 points of 2nd line. Similarly each of n points of 2nd line will make intersection point with the m - 1 points on other line (here, we haw w and n - 1 as each point cannot make intersection pc with the point it already joined with. However. due to repetition of these points twice, we need to divide both by 2.
So, total number of points of intersection of [(1/2) x m(n - 1)] x [(1/2) x n(m- 1)] = ¼ mn(m-1) x (n-1)
Hence, the correct option is (a).

38. Ans. (c)

Solution: If the sum is a two-digit number, then both the values simultaneously cannot be less than 5. Now, start making the sets by taking different values greater than or equal to at least one of the numbers.

Exercise – 5

1. 28

Solution: Let x, y , z be the number of balls received by the three persons, then
$x \geq 5, y \geq 5, z \geq 5$ and x + ty+ z = 21
Let $u \geq 0, v \geq 0, w \geq 0$, then
∴ x + y + z = 21
➔ u + 5 + v + 5 + w + 5 = 21
➔ u + v + w = 6
∴ Total number of solutions = ${}^{6+3-1}C_{3-1} = {}^{8}C_2$ = 28

2. Ans. 1587600

Solution: H L C N T A U I O
L N A I
There are total 13 letters out of which 7 are consonants and 6 are vowels. Also there are 2L's, 2N's, 2A's, and 21' s.
If all the consonants are together then the number of arrangements = $\frac{7!}{2!.2!}$
But the 7 consonants can be arranged themselves in = $\frac{7!}{2!.2!}$ *ways*
= $(1260)^2$ = 1587600

3. Ans. 4096

Solution: Every ball can be distributed in 4 ways.
Hence the required number of ways = 4 x 4 x 4 x 4 x 4 x 4 = 4^6 = 4096

4. Ans. 6804

Solution:

When 7 is at ten thousands place →	1	1	9	9	9	→ 9^3
	1	9	1	9	9	→ 9^3
	1	9	9	1	9	→ 9^3
	1	9	9	9	1	→ 9^3
When 7 is not at the ten thousands place →	8	1	1	9	9	→ 8×9^2
	8	1	9	1	9	→ 8×9^2
	8	1	9	9	1	→ 8×9^2
	8	9	1	1	9	→ 8×9^2
	8	9	1	9	1	→ 8×9^2
	8	9	9	1	1	→ 8×9^2

Hence the required number of ways = 4 x 9^3 + 6 x 8 x 9^2
= 9^2 (36 + 48)

= 81 x 84 = 6804

5. Ans. 24

Solution: 4 x 6 = 24

6. Ans. 500

Solution: B_1 and B_5 can be filled up in 2 x 2 = 4 ways and B_2, B_3, B_4, can be filled up in = 5 x 5 x 5 = 125 ways.

Hence the required number of ways = 4 x 125 = 500

7. Ans. 40

Solution: $1680 = 2^4 \times 3 \times 5 \times 7$

Since if $N = a^p - b^q - c^r \ldots$

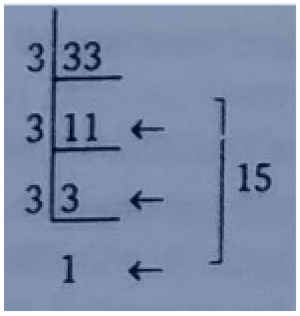
We have number of factors of N = (p + 1) (q + 1) (r + 1) ....

Hence the total number of factors of the given number

= (4 + 1) (1+1) (1+1) (1+1) = 40

8. Ans. 15

Solution: ∴ The exponent of 3 in 33! Is 15.



9. Ans. 205

Solution: Total number of points = 12

Total number of triangles = $^{12}C_3$

But there are 3 cases which must be excluded for the required number of triangles.

Case 1. The number of triangles formed by 3 points on

AB = $^3C_3$=1

Case 2. The number of triangles formed by 4 points on

BC = $^4C_3$ = 4

Case 3. The number of triangles formed by 5 points on

CA = $^5C_3$ = 10

Hence, the required number of triangles = 220 - (10 + 4 + 1)

= 205

10. Ans. 56265

Solution: Any divisor of $2^3 . 3^4 . 5^2$ is of the form $2^a . 3^b . 5^c$ where $0 \le a \le 3$, $0 \le b \le 4$ and $0 \le c \le 2$ thus, the sum of the divisors of $2^3 . 3^4 . 5^2$ is

$(1 + 2 + \ldots + 2^3)(1+3+\ldots 3^4)(1 + 5 + 5^2)$

$= \frac{(2^4-1)(3^5-1)(5^3-1)}{(2-1)(3-1)(5-1)}$

$= \frac{15 \times 242 \times 124}{1 \times 2 \times 4}$ = 56265

11. Ans. 15

Solution: $^6C_2$ = 15.

12. Ans. 27

Solution: 3 lines intersect each other in 3C2 = 3 points.

3 circles intersect each other in $^3P_2$ = 6 points.

Every line cuts 3 circles into 6 points. Therefore 3 lines cuts 3 circles into 18 points.

Therefore, the maximum number of points = 3 + 6 + 18 = 27

13. Ans. 2460

Solution: Total number of rectangles

= (1 + 2 + 3 + ... + 12) x (1 + 2 + ... + 8)

$= \frac{12 \times 13}{2} \times \frac{8 \times 9}{2}$ = 2808

Total number of squares

= (12x8) +.(11x7) + (10 x 6) +. .. + (5 x 1) = 348

∴ Required number of rectangles = 2808 - 348 = 2460

Hint: Number of squares = $\sum_{r=1}^{8}(13 - r)(9 - r)$

14. Ans. 12

Solution: There are 12 ways' as follows :

(9, 0, 0), (8,1, 0), (7, 2,0), (6, 3, 0), (5,4,0), (7,1,1), (6, 2,1), (5, 3,1), (5,2, 2), (4,4,1) (4, 3, 2), (3,3,3).

15. Ans. 71

Solution:

Number of Similar letters	Number of Different Letters	Number of Selections
5	0	$^1C_1 = 1$
4	1	$^4C_1 \times {}^2C_1 = 8$
3	2	$^3C_1 \times {}^4C_2 = 18$
3 2	0	$^3C_1 \times {}^3C_1 = 9$
2 2	1	$^4C_2 \times {}^3C_1 = 18$
2	3	$^4C_1 \times {}^4C_3 = 16$
0	5	$^5C_5 = 1$

Hence the total number of selections

= 1 + 8 + 18 + 9 + 18 + 16 + 1 = 71

16. Ans. 28

Solution: For a straight line we just need to select 2 points out of the 8 points available. ${}^{8}C_2$ would be the number of ways of doing this.

17. Ans. 192

Solution: With the digits 1, 2, 3, 4, 5 and 6 the numbers divisible by 4 that can be formed are numbers ending in: 12, 16, 24, 32, 36, 52, 56 and 64.

Number of numbers ending in 12 are: 4 x 3 x 2 = 24

Thus the number of numbers is 24 x 8 = 192

18. Ans. 325

Solution: One digit no. = 5;

Two digit no = 5 x 4 = 20;

Three digit no = 5x4x3 = 60;

four digit no = 5 x 4 x 3x2 = 120;

Five digit no.= 5 x 4 x 3 x 2 x 1 = 120

Total number of no = 325.

19. Ans. 1260

Solution: ${}^{4}C_1 \times {}^{7}C_3 \times 3! = 840$

20. Ans. 502

Solution: We cannot take '0' since the smallest digit must be placed at the left most place. We have only 9 digits from which to select the numbers. First select any number of digits. Then for any selection there is only one possible arrangement where the required condition is met This can be done in ${}^{9}C_1 + {}^{9}C_2 + {}^{9}C_3 + \ldots + {}^{9}C_9$ ways = $2^9 - 1 = 511$ ways.

But we can't take numbers which have only one digit, hence the required answer is 511 - 9.

21. Ans. 17

Solution: 200 runs can be scored by scoring only fours or through a combination of fours and sixes. Possibilities are 50 x 4, 47 x 4 + 2 x 6, 44 x 4 + 4 x 6 … A total of 17 ways.

22. Ans. 36000

Solution: Required permutations = Total permutations with no condition - permutations with the conditions which we do not have to count.

23. Ans. 26250

24. Ans. 28

:

25. Ans. 1