Percentile Classes

Probability

Probability Theory:

Mutually Exclusive Events: Let S be the sample space associated with a random experiment and let E_1 and E_2 be the two events. Then E_1 and E_2 are mutually exclusive events if $E_1 \cap E_2 \neq \emptyset$,

Mutually Exclusive and Exhaustive System of Events: Let S be the sample space associated with a random experiment, Let E_1, E_2, ... E_n be the subsets of S such that

(i) $E_1 \cap E_j = \phi$ for i ≠ j and

(ii) $E_1 \cup E_2 \cup E_3 \cup .... \cup E_n = S$

When the set of events E_1, E_2, E_3.....E_n is said to form a mutually exclusive and exhaustive system of events.

Definition of Probability: In a random experiment, let S be the sample space and let E ⊆ S.

Where E is a an event.

The probability of occurrence of the event E is defined as

P(E) = $\frac{number\ of\ favourable\ outcomes}{number\ of\ possible\ outcomes}$

= $\frac{number\ of\ elements\ E}{number\ of\ elements\ S}$ = $\frac{n(E)}{n(S)}$

= $\frac{number\ of\ elementary\ events\ in\ E}{number\ of\ elementary\ events\ in\ S}$

From the above definitions it is clear that

(i) 0 ≤ P(E) ≤ 1

(ii) P (∅) = 0

(iii) P(S) = 1

Also, P($\bar{E}$) = $\frac{number\ of\ elementary\ events\ in\ \bar{E}}{number\ of\ elementary\ events\ in\ S}$

= $\frac{n(S)-n(E)}{n(S)}$

= 1- $\frac{n\ (E)}{n(S)}$

= 1- P(E)

➔ P($\bar{E}$) = 1- P (E)

∴ P(E) = + P ($\bar{E}$) = 1

Odd in favour of An event and odds against an event

In m be the number of ways in which an event occurs and n be the number of ways in which it does not occur, then

(i) odds in favour of the events = $\frac{m}{n}$ (or m:n)

(ii) odds against the event = $\frac{n}{m}$ (or n:m)

Some important results:

(A) If, A, B and C are three events, then
(i) P [Exactly one of A, B, C occurs]
= P(A) + P(B) + P(C) – 2[A∩B] + (B∩C) + (A∩C)] +3P (A∩B∩C)

(ii) P (Atleast two of A, B, C occur)
= P (A∩B) + P(B∩C) + P(A∩C) – 2P (A∩B∩C)

(B) If A and B are two events, then P (exactly one of A, B occurs)
= P(A) + P(B) + 2P(A∪B)
= P(A∪B) – P (A∩B)

Conditional Probability: Let A and B be two events associated with a random experiment, then, the probability of occurrence of A under the condition that B has already occurred and P(B)≠ 0 is called the conditional probability and it is denoted by P$\left(\frac{A}{B}\right)$

Thus, P$\left(\frac{A}{B}\right)$ = Probability of occurrence of A given that B has already occurred.

Similarly, P $\left(\frac{B}{A}\right)$ = Probability of occurrence of B given that A has already occurred.

NOTE: (i) Sometimes P$\left(\frac{A}{B}\right)$ is used to denote the probability of occurrence of A when B occurs.
(ii) Similarly P$\left(\frac{B}{A}\right)$ is used to denoted the probability of occurrence of B when A occurs.

The above two cases happens due to the simultaneous occurrence of two events since the two events are the subsets of the same sample space.

Multiplication Theorem:

Let A and B be two events associated with the same random experiment then

P(A∩B) = P(A)P$\left(\frac{B}{A}\right)$ if P(A) ≠ 0 …(i)

Or P(A∩B) = P(B) P$\left(\frac{A}{B}\right)$, P(B) ≠ … (ii)

NOTE: P$\left(\frac{B}{A}\right)$ = $\frac{P(A\cap B)}{P(A)}$ from (i)

And P$\left(\frac{A}{B}\right)$ = $\frac{P(A\cap B)}{P(B)}$ from (ii)

In general, if A_1, a_2, $a_3 \ldots a_n$ are events associated with the same random experiment, then

P$(A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_n)$

= P(A_1) P $\left(\frac{A_3}{A_1}\right)$ P $\left(\frac{A_3}{A_1 \cap A_2}\right)$ …P $\left(\frac{A_n}{A_1 \cap A_2 \cap \ldots \cap A_{n-1}}\right)$

Independent Events: Events are said to be independent, if the occurrence of one does not depend upon the occurrence of the other

Suppose an urn contains m red balls and n green balls. Two balls are drawn from the urn one after the other. If the ball drawn in the first draw is not replaced back in the bag, then two events of drawing the ball are dependent because first draw of the ball determine the probability of drawing the second ball.

If the ball drawn in the first draw is replaced back in the bag, then two events are independent because first draw of a ball has no effect on the second draw.

Theorem I: Two events A and B associated with the same sample space of a random experiment are independent if and only if
P (A∩B) = P (A) . P(B)

Theorem 2. If A_1, A_2, A_3…. A_n are independent events associated with a random experiment, then
P($A_1 \cap A_2 \cap A_3 \ldots \cap A_n$) = P($A_1$) P ($A_2$)…P($A_n$)

Theorem 3. If A_1, A_2, ….A_n are n independent events associated with a random experiment, then
P($A_1 \cup A_2 \cup \ldots \cup A_n$) = 1-P($\overline{A_1}$) P ($\overline{A_2}$) ….P($\overline{A_n}$)

Important results:

If A and B are independent events then the following events are also independent.

(i) A ∩ $\bar{B}$ (ii) $\bar{A}$ ∩ B (iii) $\bar{A} \cap \bar{B}$

Law of Total probability:

Let E_1, E_2, …E_n be n mutually exclusive and exhaustive events associated with a random experiment. If A is an event which occurs with E_1 or E_2 or ….or E_4, then

P(A) = P(E_1) P $\left(\frac{A}{E_1}\right)$ + P (E_2)P $\left(\frac{A}{E_2}\right)$ + ….P(E_n)P($\frac{A}{E_n}$)

Bayes Rule: Let E_1, E_2, …E_n be n mutually exclusive and exhaustive events associated with a random experiment if A is an event which occurs with E_1 or E_2, or …E_n then,

P$\left(\frac{E_1}{A}\right)$ = $\frac{P(E_1)P(\frac{A}{E_1})}{\sum_{i=1}^{n} P(E_1)P\,(\frac{A}{E_1})}$ i.1, 2…..n

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Note: Although we have discussed above many ways to solve the Probability but the best way to solve Probability by using Permutation and Combination. Probability is simply total number of condition events divide by total events without condition.

Exercise – 01

1. A three-digit number is to be formed by using the digits 1,2,3,..., 9. What is the probability that the number formed is greater than 500, if repetition is not allowed?
(a) 280/504 (b) 54/280 (c) 58/204 (d) 24/504

2. A bag contains 4 red and 7 green balls. If three balls are drawn from the bag, replaced, and once again three balls are drawn from the bag, then what is the probability of obtaining 3 red balls on the first drawing and 3 green balls on the second drawing?
(a) 14/5445 (b) 14/27225 (c) 28/5445 (d) None of these

3. One number is selected at random from the first 25 natural numbers. What is the probability that it is a multiple of either 5 or 7?
(a) 2/12 (b) 8/25 (c) 4/25 (d) None of these

4. A bag contains 5 green apples and 7 red apples. If two apples are drawn from the bag, then what is the probability that one is red and the other is green?
(a) 12/66 (b) 35/66 (c) 2; 12 (d) 2/35

5. Find the chance of drawing 2 blue balls in secession from a bag containing 5 red and 7 blue. Balls, if the balls are not being replaced.
(a) $\frac{3}{13}$ (b) $\frac{21}{64}$ (c) $\frac{7}{22}$ (d) $\frac{21}{61}$

6. From a pack of 52 cards, two are drawn at random. Find the chance that one is a knave and the other a queen.
(a) $\frac{8}{663}$ (b) $\frac{1}{6}$ (c) $\frac{1}{9}$ (d) $\frac{1}{12}$

7. Three coins tossed, the probability that there is at least one tail is:
(a) $\frac{2}{3}$ (b) $\frac{7}{8}$ (c) $\frac{3}{8}$ (d) $\frac{1}{2}$

8. 100 students appeared for two examinations 60 passed the first, 50 passed the second and 30 passed both. Find the probability that a student selected at random has failed in both the examinations?
(a) $\frac{1}{5}$ (b) $\frac{1}{7}$ (c) $\frac{5}{7}$ (d) $\frac{5}{6}$

9. What is the probability of throwing a number greater than 2 with a fair dice?
(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) 1 (d) $\frac{3}{5}$

Directions(Q10 to Q13): Two fair coins are tossed simultaneously. Find the probability of

10. Getting only one head.
(a) ½ (b) 1/3 (c) 2/3 (d) ¾

11. Getting atleast one head.
(a) 1/4 (b) 3/4 (c) 2/3 (d) 1/3

12. Getting two heads
(a) $\frac{2}{7}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{4}{5}$

13. Getting atleast two heads:
(a) ¾ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 1

Directions(Q14 to Q20): Three fair coins are tossed simultaneously. Fund the probability of

14. Getting one head:
(a) 0 (b) 3/4 (c) 5/8 (d) $\frac{3}{8}$

15. Getting one tail.
(a) 1 (b) $\frac{1}{4}$ (c) $\frac{5}{8}$ (d) $\frac{3}{8}$

16. Getting atleast one head.
(a) $\frac{7}{8}$ (b) $\frac{1}{8}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

17. Getting two heads.
(a) $\frac{3}{5}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{2}{5}$

18. Getting atleast two heads.
(a) $\frac{3}{8}$ (b) $\frac{7}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

19. Getting atleast one head and one tail.
(a) $\frac{2}{8}$ (b) $\frac{1}{2}$ (c) $\frac{3}{10}$ (d) $\frac{3}{4}$

20. Getting more heads than the number of tails.
(a) 2 (b) $\frac{7}{8}$ (c) $\frac{5}{8}$ (d) $\frac{1}{2}$

Directions (Q21 to Q29): Two dice are rolled simultaneously. Find the probability of

21. Getting a total of 9.
(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{8}{9}$ (d) $\frac{9}{10}$

22. Getting a sum greater than 9.
(a) $\frac{10}{11}$ (b) $\frac{5}{6}$ (c) $\frac{1}{6}$ (d) $\frac{8}{9}$

23. Getting a total of 9 or 11.
(a) $\frac{2}{99}$ (b) $\frac{20}{99}$ (c) $\frac{1}{6}$ (d) $\frac{1}{10}$

24. Getting a doublet.
(a) 1/12 (b) 0 (c) 5/8 (d) 1/6

25. Getting a doublet of even numbers.
(a) 5/8 (b) 1/12 (c) 3/4 (d) ¼

26. Getting a multiple of 2 on one die and a multiple of 3 one the other.
(a) $\frac{15}{36}$ (b) $\frac{25}{36}$ (c) $\frac{11}{36}$ (d) $\frac{5}{6}$

27. Getting the sum of numbers on the two faces divisible by 3 or 4
(a) 4/9 (b) $\frac{1}{7}$ (c) $\frac{5}{9}$ (d) $\frac{7}{12}$

28. Getting the sum as a prime number.
(a) $\frac{3}{5}$ (b) $\frac{5}{12}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

29. Getting atleast one "5".
(a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) $\frac{5}{36}$ (d) $\frac{11}{36}$

Directions (Q30 to Q38): One card is drawn from a pack of 52 cards. Each of the 52 cards being equally likely to be drawn. Find the probability that

30. The card drawn is black.
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{8}{13}$ (d) can't be determine

31. The card drawn is a queen.
(a) $\frac{1}{12}$ (b) $\frac{1}{13}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

32. The card drawn is black and a queen.
(a) $\frac{1}{13}$ (b) $\frac{1}{52}$ (c) $\frac{1}{26}$ (d) $\frac{5}{6}$

33. The card drawn is either black or a queen.
(a) $\frac{15}{26}$ (b) $\frac{13}{17}$ (c) $\frac{7}{13}$ (d) $\frac{15}{26}$

34. The card drawn is either king or a queen.
(a) $\frac{5}{26}$ (b) $\frac{1}{13}$ (c) $\frac{2}{13}$ (d) $\frac{12}{13}$

35. The card drawn is either a heart, a queen or a king.
(a) $\frac{17}{52}$ (b) $\frac{21}{52}$ (c) $\frac{19}{52}$ (d) $\frac{9}{26}$

36. The card drawn is neither a spade nor a king.
(a) 0 (b) $\frac{9}{13}$ (c) $\frac{1}{2}$ (d) $\frac{4}{13}$

37. The card drawn is neither an ace nor a king
(a) $\frac{11}{13}$ (b) $\frac{1}{2}$ (c) $\frac{2}{13}$ (d) $\frac{11}{26}$

38. The odds in favour of an event are 2:7. Find the probability of occurrence of this event.
(a) $\frac{2}{9}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{2}{5}$

39. The odds against of an event are 5:7. Find the probability of occurrence of this event.
(a) $\frac{3}{8}$ (b) $\frac{7}{12}$ (c) $\frac{2}{7}$ (d) $\frac{5}{12}$

40. From a group of 3 men and 2 women, two persons are selected at random. Find the probability that atleast one woman is selected.

(a) $\frac{1}{5}$ (b) $\frac{7}{10}$ (c) $\frac{2}{5}$ (d) $\frac{5}{6}$

41. The probability of occurrence of two events A and B are ¼ and ½ respectively. The probability of their simultaneous occurrence is $\frac{7}{50}$. Find the probability that either A or B must occur.
(a) $\frac{61}{100}$ (b) $\frac{29}{100}$ (c) $\frac{39}{100}$ (d) $\frac{56}{99}$

42. If A and B are two events such that P(A) = 0.4, P(B) = 0.8 and P $\left(\frac{B}{A}\right)$ = 0.6, find P (A∪B)
(a) 0.24 (b) 0.96 (c) 0.04 (d) none of these

43. Three fair coins are tossed. Find the probability that they are all tails, if one of the coins shows a tail.
(a) $\frac{2}{7}$ (b) $\frac{5}{14}$ (c) $\frac{1}{7}$ (d) none of these

44. A die is thrown twice and the sum of the numbers appearing is observed to be 9. What is the conditional probability that the number 4 has appeared atleast once?
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) none of these

45. A die is rolled. If the outcome is an odd number, what is the probability that it is a number greater than 1?
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{8}$ (d) $\frac{5}{6}$

46. In a class 45% students read English, 30% read French and 20% read both English and French. One student is selected at random, Find the probability that he reads English, if it is known that he reads French.
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{5}{6}$ (d) none of these

47. Two balls are drawn from a bag containing 2 white, 3 red and 4 black balls one by one without replacement. What is the probability that atleast one ball is red?
(a) $\frac{7}{12}$ (b) $\frac{5}{12}$ (c) $\frac{3}{10}$ (d) none of these

EXERCISE – 01 (Solutions)

1. Ans. (a)
Solution: Total number of three digit numbers that can be formed without repetition = 9x9x8
Total number of three digit numbers greater than 500 that can be formed without repetition = 5x9x8
Therefore, the required probability = $\frac{5x9x8}{9x9x8}$
hence, option (a) is the answer.

2. Ans. (c)
Solution:
The required probability = $\frac{4C_3 \times 7C_3}{11C_3 \times 11C_3}$
$= \frac{140}{165 \times 165}$

3. Ans. (b)
Solution: Total multiples of 5 = 5, 10, 15, 20, 25 = 5 numbers
Total multiples of 7 = 7,14,21=3 numbers
Hence, total required number = 8 numbers
The required probability = $\frac{8}{25}$

4. Ans. (b)
Solution: The required probability = $\frac{5C_1 \times 7C_1}{12C_2}$
$= \frac{35}{66}$

5. Ans. (c)
Solution: Event definitions: First is blue and second is blue
=7/12 x 6/11
= 7/22

6. Ans. (a)

Solution: Knave and queen or Queen and Knave
4/52 x 4/51 + 4/52 x 4/51
= 8/663

7. Ans. (b)
Solution: At least one tail is the non – event for all heads.
Thus, P (at least 1 tail) = 1 – P(all heads)
= 1-1/8
= 7/8

8. Ans. (a)
Solution:
it is evident that 80 student passed at least 1 exam. Thus 20 failed both and the required probability is 20/100 = 1/5.

9. Ans. (a)
Solution: 3 or 4 or 5 or 6 = 4/6
= 2/3

10. Ans. (a)
Solution:
S = [HH, HT, TH, TT]
n(s) = 4

E = [HT, TH]
n(E) = 2
P(E) = $\frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

11. Ans. (b)
Solution:
S = [HH, HT, TH, TT]
n(s) = 4
E = [HH, HT, TH]
n(E) = 3

12. Ans. (b)
Solution:
S = [HH, HT, TH, TT]
n(s) = 4
E = [H,H]
n(E) = 1
P(E) = $\frac{1}{4}$

13. Ans. (c)
Solution:
S = [HH, HT, TH, TT]
n(s) = 4
E = [H,H]
n(E) = 1
P(E) = $\frac{1}{4}$

Hint (Q14 to Q20):
S = [HHH, HHT, HTH, HTT, THH, THT, TTH, TTT]
n(s) = 8

14. Ans. (d)
Solution: E = [HT T, THT, T TH]
n(E) = 3
P(E) = $\frac{3}{8}$

15. Ans. (d)
Solution: E = [HHT, HTH, THH]
n(E) = 3
P(E) = $\frac{3}{8}$

16. Ans. (a)
Solution: E = [HHH, HHT, HTH, HTT, THH, THT, TTH,]
n(E) = 7
P(E) = $\frac{7}{8}$

17. Ans. (b)
Solution: E = [HHT, HTH, THH]
n(E) = 3
P(E) = $\frac{3}{8}$

18. Ans. (c)
Solution: E = [HHH, HHT, HTH, THH]
n(E) = 4
P(E) = $\frac{4}{8}$
= $\frac{1}{2}$

19. Ans. (d)
Solution: E = [HHT, THT, HTT, THT, TTH]
n(E) = 6
P(E) = $\frac{6}{8}$
= $\frac{3}{4}$

20. Ans. (d)
Solution: E = [HHH, HHT, HTH, THH]
n(E) = 4
P(E) = $\frac{4}{8}$
= $\frac{1}{2}$

Hint (Q21 to Q29):
S = [(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),……
…(6,5), (6,6)]
n(S) = 6x6=36

21. Ans. (b)
Solution: E = {(6,3),(5,4),(4,5),(3,6)}
n(E) = 4
P(E) = $\frac{4}{36}$
= $\frac{1}{9}$

22. Ans. (c)
Solution: E = {(6,4),(5,5),(4,6),(6,5),(5,6),(6,6)}
n(E) = 6

P(E) = $\frac{6}{36}$

= $\frac{1}{6}$

23. Ans. (c)

Solution: E = {(6,3),(5,4),(4,5),(3,6)(6,5),(5,6)}

n(E) = 6

P(E) = $\frac{6}{36}$

= $\frac{1}{6}$

24. Ans. (d)

Solution: E = {(1,1),(2,2),(3,3),(4,4), (5,5),(6,6)}

n(E) = 6

P(E) = $\frac{6}{36}$

= $\frac{1}{9}$

25. Ans. (b)

Solution: E = {(2,2),(4,4),(6,6)}

n(E) = 3

P(E) = $\frac{3}{36}$

= $\frac{1}{12}$

26. Ans. (c)

Solution: E = {(2,3),(2,6),(4,3),(4,6),(6,3),(6,6,),(3,2),(6,2),(3,4),(6,4),(3,6),}

n(E) = 11

P(E) = $\frac{11}{36}$

= $\frac{11}{36}$

27. Ans. (c)

Solution: E = {(1,2),(1,5),(2,1),(2,4),(3,3,),(3,6),(4,2),(4,5),(5,1),(5,4),(6,3),(6,6), (1,3),(2,2),(2,6),(3,1),(3,5),(4,4),(5,3),(6,2)}

n(E) = 20

P(E) = $\frac{20}{36}$

= $\frac{5}{9}$

28. Ans. (b)

Solution: E = {(1,1),(1,2),(1,4),(1,6),(2,1),(2,3),(2,5),(3,2),(3,4), (4,1),(4,3),(5,2),(5,6),(6,1),(6,5)}

n(E) = 15

P(E) = $\frac{15}{36}$

= $\frac{5}{12}$

29. Ans. (d)

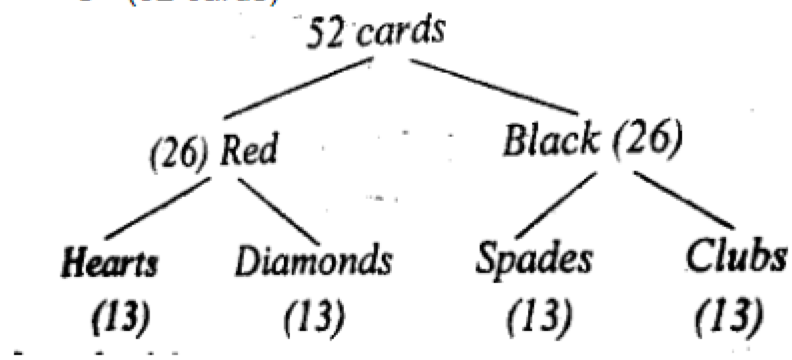
Solution: E = {(1,5),(2,5),(3,5),(4,5),(5,5),(6,5),(5,1),(5,2),(5,3), (5,4),(5,6) }

n(E) = 11

P(E) = $\frac{11}{36}$

Hint(Q30 to Q37)

S =(52 cards)



In each of the four there is one ace, one king, one queen and one jack (or knave) and rest 9 cards are numbered.

30. Ans. (a)

Solution:

N(S) = 52

n(E) = 26

P(E) = $\frac{26}{52}$

= $\frac{1}{2}$

31. Ans. (b)

Solution:

N(S) = 52

n(E) = 4

P(E) = $\frac{4}{52}$

= $\frac{1}{13}$

32. Ans. (c)

Solution:

N(S) = 52

Since drawn card must be black so there are only two queens.

Hence

n(E) = 2

P(E) = $\frac{2}{52}$

= $\frac{1}{26}$

33. Ans. (c)

Solution:

N(S) = 52

There are 26 black cards (including two queens).

Besides it there are two more queens (in red colours)

Thus

n(E) = 26+2=28

P(E) = $\frac{28}{52}$

= $\frac{7}{13}$

34. Ans. (c)

Solution:

N(S) = 52
There are 4 kings and 4 queens
E = K∪Q
n(E) =4+4=8
P(E) = $\frac{8}{52} = \frac{2}{13}$

35. Ans. (c)
Solution:
N(S) = 52
There are 13 hears (including one queen and one king). Besides it there are 3 queens and 3 kings in remaining 3 suits each.
Thus
n(E) = 13+3+3=19
P(E) = $\frac{19}{52}$

36. Ans. (b)
Solution:
N(S) = 52
There are 13 spades (including one king). Besides there are 3 more kings in remaining 3 suits).
Thus
n(E) = 13+3=16
P($\bar{E}$) =52-16=36
P($\bar{E}$) =$\frac{36}{52} = \frac{11}{13}$

37. Ans. (a)
Solution:
N(S) = 52
There are 4 aces and 4 kings
n(E) =4+4=8
P($\bar{E}$) =52-8=44
P($\bar{E}$) =$\frac{44}{52} = \frac{11}{13}$

38. Ans. (a)
Solution: Total number of outcomes = 2+7=9
Favourable number of cases = 2
P(E) = $\frac{2}{9}$

39. Ans. (b)
Solution: Total number of outcomes = 5+7 = 12
Number of cases against the occurrence of event = 5
Number of cases in favour of event = 12-5=7
P(E) = $\frac{7}{12}$

40. Ans. (b)
Solution: n(S) = $^5C_2$=10
n(E) = ($^2C_1$X$^3C_1$)+($^2C_2$) = 7
P(E) = $\frac{7}{10}$

41. Ans. (a)
Solution: P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{2}$and P (A∩B) = $\frac{7}{50}$
P (A or B) = P(A∪B)
= P (A)+P(B) - P (A∩B)
= $\frac{1}{4} + \frac{1}{2} - \frac{7}{50} = \frac{61}{100}$

42. Ans. (b)
Solution: P(A∪B) =0.96

43. Ans. (c)
Solution: Here S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
Let A be the event that one of the coins shows a tail
A = {HHT, HTH, THH, HTT, THT, TTH, TTTT}
P(A) = $\frac{7}{8}$
Now, let B be the event that they are all tails
B = {TTT}
P(B) = $\frac{1}{8}$
$(A \cap B) = \{TTT\}$
$(A \cap B) = \frac{1}{8}$
$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1/8}{7/8} = \frac{1}{7}$$

44. Ans. (a)
Solution: Let A be the event of getting the sum 9 and B be the event of getting atleast on 4.
Then A= {(3, 6), (4,5),(5,4), (6,3)}
B = {(1,4), (2,4), (3,4), (4,4),()5,4}, (6,4), (4,1),(4,2), (4,3), (4,5),(4,6)}
Then A∩B = {(4,5)(5,4)}
Required probability = P $\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{p(B)}$
$= \frac{n(A\cap B)}{n(A)} = \frac{2}{4} = \frac{1}{2}$

45. Ans. (a)
Solution: Let A = event of getting an odd number and B = the event of getting a number greater than 1.
A = {1,3,5}, B = {3,5}, A∩B = {3,5}
∴ Required probability = P $\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{p(B)} = \frac{n(A\cap B)}{n(A)}$
$= \frac{2}{3}$

46. Ans. (b)
Solution: Let A be the event of reading English and B be the event of reading French.
Then P(A) = $\frac{45}{100} = \frac{9}{20}$, P(B) = $\frac{30}{100} = \frac{3}{10}$
And P (A∩B) = $\frac{20}{100} = \frac{1}{5}$
Required probability = P $\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{p(B)} = \frac{\frac{1}{5}}{\frac{3}{10}} = \frac{2}{3}$

47. Ans. (a)
Solution: Let A be the event of not getting a red ball in first draw and B be the event of getting a

red ball in second draw. Then required probability
= Probability that atleast on ball is red
= 1-Probability that none is red
= 1-P(A and B)
= 1-P(A∩B)
= 1-P(A).P$(\frac{B}{A})$

= 1- $\left(\frac{2}{3}x\frac{5}{8}\right)$ = $\frac{7}{12}$
Here P(A) = $\frac{6}{9}$ = $\frac{2}{3}$
And P $(\frac{B}{A})$ = $\frac{5}{8}$ [There are 5 balls (excluding 3 red balls)after the selection of one non-red ball]

Exercise – 02

1. It is known that at noon, the sun is hidden by clouds on an average of two days out of every three days. Find the probability that at noon on at least four out of five days the sun will be shining.
(a) 9/11 (b) 11/243
(c)11/81 (d) None of these

2. Two fair dice are thrown. What is the probability that the number of dots on the first dice exceeds 3 and that on the second exceeds 4?
(a) 26 (b) 3/6 (c) 1/6 (d) 5/6

3. What is the probability that there are 53 Sundays and 53 Tuesdays in a leap year?
(a) 0 (b) 1
(c) 0.5 (d) None of these

4. What is the probability that there are at least 52 Sundays in a leap year?
(a) 0 (b) 1 (c) 0.5 (d) None of these

5. What is the probability that there are 53 Sundays and 53 Saturdays in a leap year?
(a) 1/7 (b) 2/7 (c) 0 (d) None of these

6. What is the probability that there are 53 Sundays in a leap year?
(a) 2/7 (b) 1/7 (c) 0 (d) None of these

7. What is the chance of throwing a number greater than 4 with an ordinary dice whose faces are numbered from 1 to 6?

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{9}$ (d) $\frac{1}{8}$

8. Amit throws three dice in a special game of Ludo. If it is known that he needs 15 or higher in this throw to win then find the chance of his winning the game.
(a) $\frac{5}{54}$ (b) $\frac{17}{216}$ (c) $\frac{13}{216}$ (d) $\frac{15}{216}$

9. In a horse race there were 18 horses numbered 1-18. The probability that horse I would win is 1/6, that 2 would win is 1/10 and that 3 would win is 1/8. Assuming that a tie is impossible, find the chance that one of the three will win.
(a) $\frac{47}{120}$ (b) $\frac{119}{120}$ (c) $\frac{11}{129}$ (d) $\frac{1}{5}$

10. Two balls are to be drawn from a bag containing 8 grey and 3 blue balls. Find the chance that they will both be blue.
(a) $\frac{1}{5}$ (b) $\frac{3}{55}$ (c) $\frac{11}{15}$ (d) $\frac{14}{45}$

11. In a certain lottery the prize is 1 crore and 5000 tickets have been sold. What is the expectation of a man who holds 10 tickets?
(a) 20,000 (b) 25,000 (c) 30,000 (d) 15,000

12. If a number of two digits is formed with the digits 2,3,5,7,9 without repetition of digits, what is the probability that the number formed is 35?
(a) $\frac{1}{10}$ (b) $\frac{1}{20}$ (c) $\frac{2}{11}$ (d) $\frac{1}{11}$

13. A bag contains 20 balls marked 1 to 20. One ball is drawn at random. Find the probability that it is marked with a number multiple of 5 or 7.
(a) $\frac{3}{10}$ (b) $\frac{7}{10}$ (c) $\frac{1}{11}$ (d) $\frac{2}{3}$

14. A bag contains 3 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that both are black?
(a) $\frac{1}{8}$ (b) $\frac{7}{40}$ (c) $\frac{12}{40}$ (d) $\frac{13}{40}$

15. A box contains 5 defective and 15 non-defective bulbs. Two bulbs are chosen at random. Find the probability that both the bulbs are non-defective.
(a) $\frac{5}{19}$ (b) $\frac{3}{20}$ (c) $\frac{21}{38}$ (d) none of these

16. A die is thrown twice, what is the probability that atleast one of the two throws come up with the number 5?
(a) $\frac{11}{36}$ (b) $\frac{5}{6}$ (c) $\frac{15}{36}$ (d) none of these

17. A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of its being a heart or a king.
(a) $\frac{4}{13}$ (b) $\frac{9}{13}$ (c) $\frac{8}{13}$ (d) $\frac{11}{26}$

18. A card is drawn from a deck of 52 cards. Find the probability of getting a red card or a heart or a king.
(a) $\frac{6}{13}$ (b) $\frac{7}{13}$ (c) $\frac{11}{26}$ (d) $\frac{15}{26}$

19. Four cards are drawn at a time from a pack of 52 playing cards. Find the probability of getting all the four cards of the same suit.
(a) $\frac{5}{13}$ (b) $\frac{12}{65}$ (c) $\frac{44}{4165}$ (d) $\frac{44}{169}$

20. A natural number is chosen at random from amongst the first 300. What is the probability that the number so chosen is divisible by 3 or 5?
(a) $\frac{48}{515}$ (b) $\frac{4}{150}$ (c) $\frac{1}{2}$ (d) none of these

21. In a class 40% of the students offered Physics 20% offered Chemistry and 5% offered both. If a student is selected at random, find the probability that he has offered Physics or Chemistry only.
(a) 45% (b) 55% (c) 36% (d) none of these

22. An urn contains 4 white 6 black and 8 red balls. If 3 balls are drawn one by one without replacement, find the probability of getting all white balls.
(a) $\frac{5}{204}$ (b) $\frac{1}{204}$ (c) 13/204 (d) none of these

23. A box contains 25 tickets, numbered 1, 2, 3,…25. A ticket is drawn and them another ticket is drawn without replacement. Find the probability that both tickets will show odd numbers.
(a) $\frac{37}{50}$ (b) $\frac{13}{50}$ (c) $\frac{13}{25}$ (d) none of these

24. Two persons A and B throw a die alternatively till one them gets a three and wins the game, Fine the respective probabilities of winning.
(a) $\frac{6}{11}.\frac{5}{11}$ (b) $\frac{5}{11}.\frac{8}{11}$ (c) $\frac{3}{11}.\frac{7}{11}$ (d) $\frac{8}{11}.\frac{3}{11}$

25. Two persons A and B throw a coin alternatively till one of them gets head and wins the game, Find their respective probabilities of winning.
(a) $\frac{1}{3},\frac{5}{6}$ (b) $\frac{3}{5},\frac{4}{5}$ (c) $\frac{2}{3},\frac{1}{3}$ (d) $\frac{1}{6},\frac{5}{6}$

26. From a pack of 52 cards, two are drawn one by without replacement. Find the probabilities that both them are kings.
(a) $\frac{11}{21}$ (b) $\frac{13}{121}$ (c) $\frac{1}{221}$ (d) $\frac{1}{121}$

27. The probability that A hits a target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$,What is the probability that the target will be hit, it each one of A and B shoots the target?
(a) $\frac{5}{6}$ (b) $\frac{3}{5}$ (c) $\frac{11}{15}$ (d) $\frac{1}{6}$

28. A problem is given to three students whose chances of solving it are $\frac{1}{2},\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved?
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{7}{12}$

29. An air gun can take a maximum of 4 shots at a balloon at some distance, The probabilities of hitting the balloon at the first, second, third and fourth shot are 0.1, 0.2, 0.3 and 0.4 respectively. What is the probability that the balloon is hit?
(a) 0.6976 (b) 0.6576 (c) 0.786 (d) none of these

30. A speaks truth in 60% and B is 80% of the cases. In what percentage of cases are they likely to contradict each other narrating the same incident?
(a) 44% (b) 36% (c) 64% (d) 48%

31. A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls. One bag is selected at random. From the selected bag, one ball is drawn. Find the probability that the ball drawn is red.
(a) $\frac{23}{42}$ (b) $\frac{19}{42}$ (c) $\frac{7}{32}$ (d) $\frac{16}{39}$

32. A box contains 20 bulbs. The probability that the box contains exactly 2 defective bulbs is0.4 and the probability that the box contains exactly 3 defective bulbs is 0.6. Bulbs are drawn at random one by one without replacement and tested till the defective bulbs are found, What is the probability that the testing procedure ends at the twelfth testing?
(a) 0 (b) 1
(c) can't be determined (d) none of these

33. There are 3 boxes each containing 3 red and 5 green balls, Also there are 2 boxes, each containing 4 red and 2 green balls, A green ball is selected at random. Find the probability that this green ball is from a box of the first group.
(a) $\frac{54}{61}$ (b) $\frac{45}{61}$ (c) $\frac{8}{31}$ (d) none of these

34. A man speaks truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that is actually a six.
(a) $\frac{3}{8}$ (b) $\frac{5}{8}$ (c) $\frac{7}{8}$ (d) $\frac{1}{12}$

35. The digits 1,2,3,4,5,6,7,8,9, are written in random order to form a nine digit number. Find the probability that this number is divisible by 4:
(a) $\frac{4}{9}$ (b) $\frac{2}{9}$ (c) $\frac{17}{81}$ (d) none of these

36. If from each of the three boxes containing 3 white and 1 black 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is:
(a) $\frac{13}{32}$ (b) $\frac{27}{32}$ (c) $\frac{19}{32}$ (d) none of these

37. There are four calculators and it is known that exactly two of them are defective. They are tested one by one in a random order till both the defective calculators are identified. Then the probability that only two tests are required is
(a) $\frac{5}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

38. 20 girls, among whom are A and B sit down at a round table. The probability that there are 4 girls between A and B is:
(a) $\frac{17}{19}$ (b) $\frac{2}{19}$ (c) $\frac{13}{19}$ (d) $\frac{6}{19}$

EXERCISE – 02 (Solutions)

1. Ans. (b)
Solution: Probability that the sun is hidden = 2/3; so, the probability that the sun is not hidden by clouds = Sun will be shining = 1/3

At least four out of five days, sun will be shining = Probability of exactly four days = probability of exactly five days

$= {}^5C_4 \times (1/3)^4 \times (2/3)^1 + {}^5C_5 \times (1/3)5 \times (2/3)^6 =$
$5\times\frac{2}{243} + 1\times\frac{1}{243} = \frac{11}{243}$

2. Ans. (c)
Solution: Ways in which number of dots on the first dice exceed 3 = 4,5,6=3 ways
Ways in which number of dots on the second dice exceed 4=5, 6=2 ways
Hence, the required probability = $\frac{3\times2}{6\times6}$
= 1/6
3. Ans. (a)
Solution: In a leap year, there are 366 days = 52 weeks + 2 days extra
If there are 53 Sundays, then the other extra day will be either a Saturday or Monday. Hence, the required probability = 0.
4. Ans. (b)
Solution: All the days will occur atleast 52 times. Hence, the required probability = 1.
5. Ans. (a)
Solution: There are 7 different possibilities. Hence, the required probability = 1/7.
6. Ans. (a)
Solution: There are two extra days and seven different possibilities viz. (Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), and (Saturday, Sunday). Hence, the required probability = 2/7.
7. Ans. (a)
Solution: 5 or 6 out of a sample space of 1,2,3,4,5 or 6 = 2/6 = 1/3
8. Ans. (a)
Solution: Event definition is: 15 or 16 or 17 or 18.
15 can be got as: 5 and 5 and 5 (one way)
Or
6 and 5 and 4 (Six ways)
Or
6 and 6 and 3 (3 ways)
Total 10 ways.
16 can be got as. 6 and 6 and 4 (3 ways)
Or
6 and 5 and 5 (3 ways)
Total 6 ways.
17 has 3 ways and 18 has 1 way of appearing.
Thus, the required probability is:
(10+6+3+1)/216
= 20/216
= 5/54.
9. Ans. (a)
Solution: 1/6 + 1/10 + 1/8 = 47/120
10. Ans. (b)
Solution: The event definition would be given by:
First is blue and second is blue is blue = 3/11 x 2/10
= 3/55
11. Ans. (a)
Solution: Expectation = Probability of winning x Reward of winning = (10/5000) x 1 crore = (1 crore/500)
= 20000.
12. Ans. (b)
Solution: $1/{}^5P_2 = 1/20$.
13. Ans. (a)
Solution: Positive Outcomes are: 5,7,10,14,15 or 20
Thus, 6/20 = 3/10
14. Ans. (b)
Solution: Black and black = (7/16) x 6/15
= 7/40
15. Ans. (c)
Solution: $n(S) = {}^{20}C_2=190$
$n(E) = {}^{15}C_2 = 105$
$P(E) = \frac{105}{190} = \frac{21}{38}$
16. Ans. (a)
Solution: A = {(5,),(5,2),(5,3),(5,4),(5,5),(5,6),}
B = {(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)}
A∩B = {(5,5)}
Also
n(S) = 36
$P(A) = \frac{6}{36} = \frac{1}{6}$
$P(B) = \frac{6}{36} = \frac{1}{6}$
And
$A\cap B = \frac{1}{36}$
Required probability = P(A)+P(B)-P(A∩B)
$= \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}$
17. Ans. (a)
Solution: n(S) = 52
A → The event of getting a heart
B → The event of getting a king
Then A∩B → The event of getting a king of heart.
$P(A) = \frac{13}{52} = \frac{1}{4}$, $P(B) = \frac{4}{52} = \frac{1}{13}$
and
$(A\cap B) = \frac{1}{52}$
P (a heart or a king) = P (A or B) = P(A∩B)

= P(A) + P(B) - P(A∩B)

$= \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{4}{13}$

18. Ans. (b)

Solution: n(S) = 52

Let A, B, C be the events of getting a red card, a heart and a king respectively.

Then

n(A) = 26, n(B) = 13, n(c) = 4

Clearly n (A∩B) = 13, n(B∩C) = 1

N (A∩C) = 2, n(A∩B∩ C) = 1

P(A) $= \frac{26}{52} = \frac{1}{2}, P(B) = \frac{13}{52} = \frac{1}{4}, P(C) = \frac{4}{52} = \frac{1}{13}$

P(A∩B) $= \frac{13}{52} = \frac{1}{4}, P(\mathrm{B} \cap \mathrm{C}) = \frac{1}{52}$

$P(\mathrm{A} \cap \mathrm{C}) = \frac{2}{52} = \frac{1}{26}$

$P(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}) = \frac{1}{52}$

P (a red card, or a heart or a king) = P(A∪B∪C)

= P(A)+P(B)+P(C)- $P(\mathrm{A} \cap \mathrm{B}) + P(\mathrm{B} \cap \mathrm{C}) + P(\mathrm{A} \cap \mathrm{C}) + P(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C})$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{13} - \left(\frac{1}{4} + \frac{1}{52} + \frac{1}{26}\right) + \frac{1}{52} = \frac{7}{13}$

19. Ans. (c)

Solution: n(S) = ${}^{52}C_4$

Let E_1, E_2, E_3, E_4, be the event of getting all spades, all clubs, all hearts and all diamonds respectively.

Then

n(E_1) = ${}^{13}C_4$

n(E_2) = ${}^{13}C_4$

n(E_3) = ${}^{13}C_4$

n(E_4) = ${}^{13}C_4$

n(E_1) = $\frac{13C_4}{52C_4}$, P(E_2) = $\frac{13C_4}{52C_4}$

n(E_3) = $\frac{13C_4}{52C_4}$, P(E_4) = $\frac{13C_4}{52C_4}$

Since E_1, E_2 E_3, and E_4 are mutually exclusive events.

P(getting all the 4 cards of the same suit)

P(E_1 or E_2 or E_3 or E_4) = P(E_1)+ P(E_2)+ P(E_3)+ P(E_4)

$= 4 \times \left(\frac{13C_4}{52C_4}\right) = \frac{44}{4165}$

20. Ans. (c)

Solution: n(S) = 300

Let A be the event of getting a number divisible by 3 and B be the event of getting a number divisible by 5 and $(\mathrm{A} \cap \mathrm{B})$ be the event of getting a number divisible by both 3 and 5 both

N(A) = 100, n(B) = 60, n$(\mathrm{A} \cap \mathrm{B})$=20

P(A) $= \frac{100}{300} = \frac{1}{3}, P(B) = \frac{60}{300} = \frac{1}{5}, P(\mathrm{A} \cap \mathrm{B}) = \frac{20}{600} = \frac{1}{30}$

P(A∪B) = P(A)+P(B)-P$(\mathrm{A} \cap \mathrm{B})$

$= \frac{1}{3} + \frac{1}{5} - \frac{1}{30} = \frac{1}{2}$

21. Ans. (b)

Solution: n(S) = 100

n(A) = 40, n(B) = 20, n$(\mathrm{A} \cap \mathrm{B})$ = 5

P(A∪B) = P(A)+P(B)-P$(\mathrm{A} \cap \mathrm{B})$

$= \frac{40}{100} + \frac{20}{100} - \frac{5}{100}$

P(A∪B) $= \frac{55}{100} = 55\%$

22. Ans. (b)

Solution: Let A, B, C be the events of getting a white ball in first, second and third fraw respectively, then

Required probability = P(A∩B∩C)

= P(A)P$(\frac{B}{A})$P $\left(\frac{C}{A \cap B}\right)$

Now P(A) = probability of drawing a white ball in first draw = $\frac{4}{18} = \frac{2}{9}$

When a white ball is drawn in the first draw there are 17 balls left in the urn, out of which 3 are white

P$\left(\frac{B}{A}\right) = \frac{3}{17}$

Since the ball drawn is not replaced, therefore after drawing a white ball in the second draw there are 16 balls left in the run, out of which are white.

P$\left(\frac{C}{A \cap B}\right) = \frac{2}{16} = \frac{1}{8}$

Hence the required probability $= \frac{2}{9} x \frac{3}{17} \times \frac{1}{8} = \frac{1}{204}$

23. Ans. (b)

Solution: Let A be the event of drawing an odd numbered ticket in the first draw and B be the event of drawing an odd numbered ticket in the second draw. Then

Required probability = P(A∩B) = P(A)P$\left(\frac{B}{A}\right)$

P(A) = $\frac{13}{25}$, since there 13 odd number 1, 3, 5, …25.

Since the ticket drawn in the first draw is not replaced, therefore second ticket drawn is from the remaining 24 tickets, out of which 12 are odd numbered.

$P\left(\frac{B}{A}\right) = \frac{12}{24} = \frac{1}{2}$

Hence, required probability = $\frac{13}{25}$ x $\frac{1}{2}$ = $\frac{13}{50}$

24. Ans. (a)
Solution: Let E = the event that A gets a three and F = the event that B gets a three
Then, P(E) = $\frac{1}{6}$, P(F) = $\frac{1}{6}$
$P(\bar{E}) = \frac{5}{6}$, $P(\bar{F}) = \frac{5}{6}$
Suppose A wins then, he gets a three in 1st or 3rd of 5th throw etc.
∴ P(A wins) = P[E or $(\bar{E}\ \bar{F}\ E)$ or $(\bar{E}\ \bar{F}\ \bar{E}\ \bar{F}\ E)$ or ….∞]
= P [E or $(\bar{E}\ AND\ \bar{F}\ and\ E)$ + P($\bar{E}$ and $\bar{F}$ and $\bar{E}\ and\ \bar{F}$ and E) + …∞]
P [E or $(E\ and\ F\ and\ E) or\ (E\ and\ F\ and\ E\ and\ F\ and\ E) or\ \dots\dots$]
= P(E) + P(E and F and E) + P(E and F and E and F and E)+….∞
=P(E) + P(E) P(F) P(E) + P(E) P(F) P(E) P(F).P(E)+…..∞
$= \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \cdots..\infty$
$= \frac{1}{6} + \frac{1}{6}\left(\frac{5}{6}\right)^2 + \frac{1}{6}\left(\frac{5}{6}\right)^4 + \cdots.\infty$
$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \cdots.\infty\right]$
$= \frac{1}{6}\frac{1}{\left[1-\left(\frac{5}{6}\right)^2\right]} = \left(\frac{1}{6}.\frac{36}{11}\right) = \frac{6}{11}$
Thus, P (A wins) = $\frac{6}{11}$ and P(B wins) = $\frac{5}{11}$

25. Ans. (c)
Solution: We have, P(H) = $\frac{1}{2}$ and P(T) = $\frac{1}{2}$
Now, A wins if he throws a head in 1st, or 3rd or 5th or. Draw
∴ P(A wins) = P[H or (T TH) or (T T T TH) or (TTTTT TH) or….
= P(H)+ P(T TH)+ P(TTT TH)+….
= P(H)+ P(T)P(T)P(H)+ P(T) P(T) P(T) P(T) P(H)+
$$= \frac{1}{2} + \left(\frac{1}{2}\right)^2 \frac{1}{2} + \left(\frac{1}{2}\right)^4 \frac{1}{2} + \cdots..\infty$$
$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \cdots..\infty$$
$$= \frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \cdots..\infty\right]$$
$= \frac{1}{2}\frac{1}{\left[1-\left(\frac{1}{2}\right)^2\right]} = \left(\frac{1}{2} \times \frac{4}{3}\right) = \frac{2}{3}$

Thus P (A wins) = $\frac{2}{3}$ and P(B wins) = $\left(1 - \frac{2}{3}\right) = \frac{1}{3}$

26. Ans. (c)
Solution: Required probability = $\frac{4}{52}$ x $\frac{3}{51}$ x $\frac{1}{221}$

27. Ans. (b)
Solution: Let A = the event that A hits the target.
And B = the event that B hits the target
As given we have P(A) = $\frac{1}{3}$ and P(B) = $\frac{2}{5}$
Clearly A and B are independent events
P (A∩B) = P(A). P(B) = $\frac{1}{3}$ x $\frac{2}{5}$ = $\frac{2}{15}$
P (target is hit) = P (A hits or B hits)
= P(A∪B) = P (A) + P(B) – P(A∩B)
= $\frac{1}{3}$ + $\frac{2}{5}$ - $\frac{2}{15}$ x $\frac{3}{5}$

28. Ans. (c)
Solution: Let A, B, C be the respective events of solving the problem and $\bar{A}$, $\bar{B}$, $\bar{C}$ be the respective events of not solving the problem.
Then A, B, C are independent events
∴ $\bar{A}$, $\bar{B}$, $\bar{C}$ are independent events

Now, P(A) = $\frac{1}{2}$ P(B) = $\frac{1}{3}$ and P(C) = $\frac{1}{4}$
$P(\bar{A}) = \frac{1}{2}$ $P(\bar{B}) = \frac{2}{3}$ and $P(\bar{C}) = \frac{3}{4}$

∴ P (None solves the problem)
= P(not A) and (not B) and (not C)
= $P(\bar{A}) \cap \bar{B} \cap \bar{C})$
= $P(\bar{A})P(\bar{B})P(\bar{C})$ (∴$\bar{A}, \bar{B}$ AND $\bar{C}$ are independent)

= $\frac{1}{2}$ x $\frac{2}{3}$ x $\frac{3}{4}$ = $\frac{1}{4}$
Hence, P (the problem will be solved)
= 1- P (None solves the problem)
= 1- $\frac{1}{4}$ = $\frac{3}{4}$

29. Ans. (a)
Solution: Let P_1 = 0.1, P_2 = 0.2, P_3 = 0.3, P_4= 0.4
∴ P (The balloon is hit) = P (the balloon is hit atleast once)
= 1 – P(the balloon is hit in none of the shots)
= 1 – (1-P_1) (1-P_2) (1-P_3) (1-P_4)
= 1-(0.9)(0.8)(0.7)(0.6) = 0.6976

30. Ans. (a)
Solution: Let E = the event that A speaks the truth
And F = the event that B speaks the truth
Then $\bar{E}$ = the event that A tells a lie.

And $\bar{F}$ = the event that B tells a lie.
Clearly E and F are independent events, so E and $\bar{F}$ and well as $\bar{E}$ and F are independent.

Now, P(E) = $\frac{60}{100} = \frac{3}{5}$, P(F) = $\frac{80}{100} = \frac{4}{5}$

P($\bar{E}$) = $\frac{2}{5}$, P($\bar{F}$) = $\frac{1}{5}$

∴ P(A and B contradict each other = P(A speaks the truth and B tells a lie)
Or (A tells a lie and B speaks the truth)
= P [E ∩ $\bar{F}$] ∪ ($\bar{E}$ ∪ F)]
= P (E ∩ $\bar{F}$) + P($\bar{E}$ ∩ F)
= P(E) P($\bar{F}$) + P($\bar{E}$) P(F)
= $\frac{3}{5} \times \frac{1}{5} + \frac{2}{5} \times \frac{4}{5}$
= $\frac{11}{25}$ = 44%
So, A and B contradict each other in 44% cases.

31. Ans. (b)
Solution: A red ball can be drawn in to mutually exclusive ways
(i) Selecting bag I and then drawing a red ball from it.
(ii) Selecting bag II and then drawing a red ball from it.
Let E_1 E_2 and A denote the events defined as follow.
E_1 = Selecting bag I,
E_2 = Selecting bag II
A = drawing a red ball.
Since one of the two bags is selected randomly, therefore
P(E_1) = $\frac{1}{2}$ and PE_2 = $\frac{1}{2}$
Now, P($\frac{A}{E_1}$) = Probability of drawing a red ball when the first bag has been chosen = $\frac{4}{7}$
P($\frac{A}{E_2}$) = Probability of drawing a red ball when the second bag has been selected = $\frac{2}{6}$

Using the law of total probability we have
P (red ball) = P(A) = P(E_1)P($\frac{A}{E_1}$) + P(E_2)P($\frac{A}{E_2}$)
= $\frac{1}{2} \times \frac{4}{7} + \frac{1}{2} \times \frac{2}{6} = \frac{19}{42}$

32. Ans. (d)
Solution: The testing procedure may terminate at the twelfth testing in two mutually exclusive ways.
(i) When lot contain 2 defective bulbs
(ii) When lot contains 3 defective bulbs.
Consider the following events:
A = Testing procedure ends the twelfth testing
E_1 = lot contains 2 defective bulbs
E_2 = lot contains 3 defective bulbs
Required probability = P(A)
= P(A∩E_1) ∩P(A∩E_2)
= P(A∩E_1) + P (A∩E_2)
= P(E_1) P($\frac{A}{E_1}$) + P(A_2)P($\frac{A}{E_2}$)

Now P($\frac{A}{E_1}$)= probability that first 11 draws contain 10 non defective and one defective and 12[th] draw contains a defective article.
= $\frac{{}^{18}C_{10} \times {}^{2}C_{1}}{{}^{20}C_{11}} \times \frac{1}{9}$
And P($\frac{A}{E_2}$)= probability that first 11 draws contain 9 non defective and 2 defective and 12[th] draw contains a defective article.
= $\frac{{}^{17}C_{9} \times {}^{3}C_{2}}{{}^{20}C_{11}} \times \frac{1}{9}$

Hence, Required probability
= 0.4 x $\frac{{}^{18}C_{10} \times {}^{2}C_{1}}{{}^{20}C_{11}} \times \frac{1}{9}$ + 0.6 x $\frac{{}^{17}C_{9} \times {}^{3}C_{2}}{{}^{20}C_{11}} \times \frac{1}{9}$

33. Ans. (b)
Solution: Let E_1, E_2 and A be the events defined as follows:
E_1 = selecting a box from the first group
E_2 = selecting a box from the second group
and
A = ball drawn is green
Since there are 5 boxes out of which 3 boxes belong the fire group and 2 boxes belong the second group.
Therefore
P(E_1) = $\frac{3}{5}$, P(E_2) = $\frac{2}{5}$
If E_1 has already occurred then a box from the first group x chosen. Then box chosen contains 5 green balls and 3 red balls.

Therefore the probability of drawing a green ball from it is = $\frac{5}{8}$
So, $P\left(\frac{A}{E_1}\right) = \frac{5}{8}$

Similarly $P\left(\frac{A}{E_2}\right) = \frac{2}{6} = \frac{1}{3}$

Now, we have to find $P\left(\frac{A}{E_1}\right)$
By Bay'e rule, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{3}{5}\times\frac{5}{8}}{\frac{3}{5}\times\frac{5}{8}+\frac{2}{5}\times\frac{1}{3}} = \frac{45}{61}$$

34. Ans. (a)

Solution: Let E_1 E_2 and A be the events defined as follows:

E_1 = six occurs, E_2 = six does not occur

And A = the man reports that it is a six.

We have, P(E_1) = $\frac{1}{6}$, P(E_2) = $\frac{5}{6}$

Now P$\left(\frac{A}{E_1}\right)$ = probability that the man reports that there is a six on the die given that six has occurred on the die.

= probability that the man speaks truth = $\frac{3}{4}$

And P$\left(\frac{A}{E_2}\right)$ = probability that the man reports that there is six on the die given that six has not occurred on the die.

= Probability that the man does not speak truth

= 1 - $\frac{3}{4}$ = $\frac{1}{4}$

We have to find $P\left(\frac{E_1}{A}\right)$

By Bayes rule, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}} = \frac{3}{8}$$

35. Ans. (b)

Solution:

Total possible number of 4 digits = 4! = 24

The number is divisible by 5 if unit digit itself is 5. Therefore we fix 5 at unit place and then remaining 3 places can be filed up in 3! Ways.

Hence, the required probability = $\frac{3!}{4!} = \frac{6}{24} = \frac{1}{4}$

36. Ans. (a)

Solution:

Box 1	Box 2	Box 3
3W	2W	1W
1B	2B	3B

There can be three manually exclusive cases of drawing 2 white balls and 1 black ball.

	Box 1	Box 2	Box 3
Case 1	1W	1W	1B
Case 2	1W	1B	1W
Case 3	1B	1W	1W

= P$(W_1 \cap W_2 \cap B_3) \cup (W_1 \cap B_2 \cap W_3) \cup (B_1 \cap W_2 \cap W_3)$

= P(W_1) P(W_2) P(B_3)+ P(W_1) P(B_2) P(W_3)

$= \frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{3}{4}\times\frac{2}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4} = \frac{26}{64} = \frac{13}{32}$

37. Ans. (b)

Solution:

The total number of ways in which two calculators can be chosen out of four calculators is $^4C_2$ = 6.

If only two tests are required to identify defective calculators, then in first two tests defective calculators are identified. This can be done in one way only.

Required probability = $\frac{1}{6}$

38. Ans. (a)

Solution:

20 girls can be seated around a round table in 19! ways.

So, exhaustive number of cases = 19!

Excluding A and B, out of remaining 18 girls, 4 girls can be selected $^{18}C_4$ ways which can be arranged in 4! ways.

Remaining 20 - (4 - 2) = 14 girls can be arranged in 14! ways.

Also A and B mutually can be arranged in 2! ways.

:. Required number of arrangements = $^{18}C_4$ x 4! x 2! x !4!

=

18! x 2

Required probability $= \frac{18!\times 2}{19!} = \frac{2}{19}$

Exercise – 03

1. In a convex hexagon, two diagonals are drawn at random. The probability that the diagonals intersect at an interior point of the hexagon is:

   (a) $\frac{1}{56}$ (b) $\frac{1}{8}$ (c) 3/28 (d) None of these

2. Seven white balls and 3 black balls are placed in a row at random. The probability that no two black balls are adjacent is.

   (a) $\frac{1}{3}$ (b) $\frac{7}{15}$ (c) $\frac{2}{15}$ (d) $\frac{1}{3}$

3. Three dice are thrown simultaneously. The probability of getting a sum of 15 is:

   (a) $\frac{1}{72}$ (b) $\frac{5}{36}$ (c) $\frac{5}{72}$ (d) None of these

4. A box contains 6 red balls, 7 green balls, and 5 blue balls. Each ball is of a different size The probability that the red ball being selected is:

   (a) $\frac{1}{18}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$

5. A dice is thrown 2n + 1 times, $n \epsilon N$. The probability that the faces with even numbers show odd number of times is:

   (a) $\frac{2n+1}{4n+1}$ (b) Lem than $\frac{1}{2}$

   (c) Greater than $\frac{1}{2}$ (d) None of these

6. Let A = {2, 3, 4, ... 20, 21}. A number is chosen at random from the set A and it is found to be a prime number. The probability that H is more than 10 is

   (a) $\frac{9}{10}$ (b) $\frac{1}{10}$

   (c) $\frac{1}{5}$ (d) None of these

7. Triangles are formed by joining vertices of an octagon Any one of those triangle is selected at random. What is the probability that the selected triangle has no side common with the octage.

   (a) 3/7 (b) 2/7 (c) 5/7 (d) 1/7

8. A month is randomly selected from the months in a non-leap year and it is found that it has five Sundays. What is the probability that it has five Mondays?

   (a) 1/7 (b) 2/7 (c) 2/3 (d) 20/33

9. Manoj throws a fair dice. He is promised an amount thrice the value of the number showing up if the number thrice the value of the number showing up if the number showing up is odd and an amount twice the value of the number showing up if it is even. What is the maximum amount Manoj is willing to pay each time to throw the dice, if in the long run he wants to make an average profit of 5 per throw?
(a) 3.50 (b) 8.5 (c) 5 (d) None of these

10. There is a frame of a cuboid of length 6 units, breadth 5 units, and height 7 units. The cuboid is only composed of skeleton of 210 cubes of side 1. An insect is on one corner of the cube and it wants to travel to the opposite end of the longest diagonal. It can only travel along the sides of the small cube and it always takes the shortest possible route. Find the probability that it passes through at least one of the corners.
(a) $\frac{1}{6}$ (b) $\frac{7}{12}$ (c) $\frac{5}{18}$ (d) None of these

11. A natural number x is chosen at random from the first one hundred natural numbers. What is the probability that x + $\frac{100}{x}$ > 50?
(a) 13/20 (b) 3/5 (c) 9/20 (d) 11/20

12. If 8 coins are tossed, what is the chance that one and only one will turn up Head?
(a) $\frac{1}{16}$ (b) $\frac{3}{35}$ (c) $\frac{3}{32}$ (d) $\frac{1}{32}$

13. A speaks the truth 3 out of 4 times, and B 5 out of 6 times. What is the probability that they will contradict each other in starting the same fact?
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$ (d) None of these

14. A party of n persons sit at a round table. Find the odds against two specified persons sitting next to each other.
(a) $\frac{n+1}{2}$ (b) $\frac{n-3}{2}$ (c) $\frac{n+3}{2}$ (d) None of these

15. In four throws with a pair of dices what is the chance of throwing a double twice?
(a) $\frac{11}{216}$ (b) $\frac{25}{216}$ (c) $\frac{35}{126}$ (d) $\frac{41}{216}$

16. A fair coin is tossed repeatedly. If Head appears on the first four tosses then the probability of appearance of tail on the fifth toss is
(a) $\frac{1}{7}$ (b) $\frac{1}{2}$ (c) $\frac{3}{7}$ (d) $\frac{2}{3}$

17. A team of 4 is to be constituted out of 5 girls and 6 boys. Find the probability that the team may have 3 girls.
(a) $\frac{4}{11}$ (b) $\frac{3}{11}$ (c) $\frac{5}{11}$ (d) $\frac{2}{11}$

18. A bag contains 5 red, 4 green and 3 black balls. If three balls are drawn out of it at random, find the probability of drawing exactly 2 red balls.
(a) $\frac{7}{22}$ (b) $\frac{10}{33}$ (c) $\frac{7}{12}$ (d) $\frac{7}{11}$

19. Sanjay writes a letter to his friends from IIT, Kanpur. It is known that one out of ‘n’ letters that are posted does not reach its destination. If Sanjay does not receive the reply to his letter, then what is he probability that Keasari did not receive Sanjay’s letter? It is certain that Kesari will definitely reply to Sanjay’s letter if he receives it.

(a) $\frac{n}{(2n-1)}$ (b) $\frac{n-1}{n}$ (c) $\frac{1}{n}$ (d) None of these

20. A number is chosen at random from the numbers 10 to 99. By seeing the number, a man will sing if the product of the digits is 12.If he chooses three numbers with replacement, then the probability that he will sing at least once is:
(a) $1-\left(\frac{43}{45}\right)^3$ (b) $\left(\frac{43}{45}\right)^3$ (c) $1-\frac{48\times 86}{90^3}$ (d) None of these

21. If the integer's m and n are chosen at random from 1 to 100, then the probability that a number of the form 7^m+7^n is divisible by 5 is
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{16}$ (d) $\frac{1}{6}$

22. There are 5 envelopes corresponding to 5 letters. If the letters are placed in the envelopes at random, what is the probability that all the letters are not placed in the right envelopes?
(a) $\frac{119}{120}$ (b) $\frac{59}{60}$ (c) $\frac{23}{24}$ (d) $\frac{4^5}{5^5}$

23. Two persons A and B toss a coin alternately till one of them gets. Head and wins the game. Find B's chance of winning if A tosses the coin first.
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) None of these

24. There are 10 pairs of socks in a cupboard from which 4 individual socks are picked at random. The probability that there is at least one pair is.
(a) $\frac{195}{323}$ (b) $\frac{99}{323}$ (c) $\frac{198}{323}$ (d) $\frac{185}{323}$

25. Two small squares on a chess board are chosen at random. Find the probability that they have a common side.
(a) $\frac{1}{12}$ (b) $\frac{1}{18}$ (c) $\frac{2}{15}$ (d) $\frac{3}{14}$

26. Four numbers are multiplied together. Then the probability that the product will be divisible by 5 or 10 is:
(a) $\frac{169}{625}$ (b) $\frac{369}{625}$ (c) $\frac{169}{1626}$ (d) none of these

27. 8 couples (husband and wife) attend a dance show 'Nach Baliye' in a popular TV channel: A lucky in which 4 persons picked up for a prize is held, then the probability that there is atleast one couple will be selected is
(a) $\frac{8}{39}$ (b) $\frac{15}{39}$ (c) $\frac{12}{13}$ (d) none of these

28. A committee of five persons is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is:
(a) 4/9 (b) 5/9 (c) 13/18 (d) none of these

29. A speaks truth in 60% cases and B speaks truth in 80% cases. The probability that they will say the the same thing while describing a single event is:
(a) 0.36 (b) 0.56 (c) 0.48 (d) 0.20

30. Nine squares are chosen at random on a chessboard. What is the probability that they form a square of size 3x3?
(a) $\frac{9}{64C_9}$ (b) $\frac{36}{64C_9}$ (c) $\frac{6}{64C_9}$ (d) none of these

31. Seven digits from the numbers 1,2,3,4,5,6,7,8 and 9 are written in random order. The probability that this seven digit number is divisible by 9 is:
(a) $\frac{7}{9}$ (b) $\frac{1}{9}$ (c) $\frac{2}{9!}$ (d) $\frac{4}{9}$

32. What is the probability that four S's come consecutively in the word MISSISSIPPI?
(a) $\frac{4}{165}$ (b) $\frac{4}{135}$ (c) $\frac{24}{165}$ (d) none of these

33. Each coefficient in the equation $ax^2+bx+c=0$ is determined by throwing ordinary six faced die. Find the probability that the equation will have real roots.
(a) $\frac{34}{161}$ (b) $\frac{43}{216}$ (c) $\frac{25}{36}$ (d) none of these

34. A consignment of 15 wristwatches contains 4 defectives. The wristwatches are selected at random, one by one and examined. The ones examined are not put back. What is the probability that ninth one examined is the last defective?
(a) $\frac{11}{195}$ (b) $\frac{17}{195}$ (c) $\frac{8}{195}$ (d) $\frac{16}{195}$

35. Given that the sum of two non-negative quantities is 200, the probability that their product is not less than $\frac{3}{4}$ times their greatest product value is:
(a) $\frac{99}{200}$ (b) $\frac{101}{200}$ (c) $\frac{87}{100}$ (d) none of these

36. Three numbers are to be selected at random without replacement from the set of numbers (1, 2, ….n). The conditional probability that the third number lies between the first two, if the first number is known to be smaller than the second is:
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{5}{6}$ (d) $\frac{7}{12}$

37. A letter is taken out at random from 'ASSISTANT' and another is taken out from 'STATISTICS' The probability that they are the same letters is:
(a) $\frac{35}{96}$ (b) $\frac{19}{90}$ (c) $\frac{19}{96}$ (d) none of these

38. Two numbers a and b are chosen at random from the set of first 30 natural numbers. The probability that a^2+b^2 is divisible by 3 is:
(a) $\frac{37}{87}$ (b) $\frac{47}{87}$ (c) $\frac{17}{29}$ (d) none of these

39. The digits 1,2,3,…..,9 are written in random order of form a nine digit number. Find the probability that this number is divisible by 11.
(a) $\frac{11}{63}$ (b) $\frac{11}{81}$ (c) $\frac{11}{126}$ (d) none of these

Exercise – 03 (Solutions)

1. Ans (a)
2. Ans (b)
3. Ans (d)
4. Ans (c)
5. Ans. (d)

Solution: Required probability is simply $\frac{1}{2}$.

Hence, the correct option is (d).

6. Ans. (d)

Solution: Total number of primes = 8 and number of numbers more than 10 = 11.

7. Ans. (b)

Solution: Tolal number of triangles formed = ${}^8C_3$ = 56

Triangles having three sides common = 8

Triangles having no side common = ${}^8C_1 \times {}^4C_1$ = 32

Triangles having three sides common = 0

Triangles having no side common = 56-40 = 16

So, probability =16/56 = 2/7

8. Ans. (d)

Solution: In a non-leap year, February has 28 days, and so. it must have each day of week exactly 4 times. Now, we know 7 months have 31 days and 4 months have 30 days.

If a month has 31 days and it has 5 Sundays, then it is possible for 5 Fridays, Saturday, and Sunday, or 5 Saturday, Sundays, and Monday, or 5 Sunday, Monday, and Tuesday. So, the probability of having 5 Tuesday if it has 5 Sundays is 2/3.

If a month has 30 days and it has 5 Sundays, then it is possible for 5 Saturdays and Sundays or 5 Sundays and Mondays. So, the probability of having 5 Tuesdays if it has 5 Sundays is 1/2.

So, if a month is selected randomly, then probability of having 5 Mondays if it has 5 Sundays must be:

(7/11x2/3) + (4/1 x l/2) = 20/33

Hence, the correct option is (d).

9. Ans. (a)

Solution: The average earning per throw for Manoj can be calculated by summing the multiplication of probability of showing up of each number and the earning it will result into. As, die is fair, the probability of showing of each number is 1/6.

Average earning per throw is (1/6 x 3) + (1/6 x 4) + (1/6 x 9) + (1/6 x 8) + (1/6 x 15) + (1/6 x 12) = 8.5

So, to earn average profit of 5 per throw, he must be willing to pay 3.5 per throw.

Hence, the correct option is (a).

10. Ans. (d)

11. Ans. (d)

Solution: The given condition is satisfied for all numbers from 51 to 100. It is also satisfied for 50, 49, 48, 1, and 2. So, there are total 55 numbers from first 100 natural numbers for which the given condition is satisfied. Therefore, the required probability = 55/100 = 11/20

Hence, the correct option is (d).

12. Ans. (d)

Solution: One head and seven tails would have eight positions where the head can come.

Thus, 8 x $(1/2)^8$ = (1/32)

13. Ans. (b)

Solution: They will contradict each other if: A is true and B is false or A is false and B is true.

(3/4)x(1/6)+(1/4)x(5/6) = 1/3

14. Ans. (b)

Solution: For the counting of the number of events, think of it as a circular arrangement with n-1 people (by considering the two specified persons as one). This will give you n(E) = (n-2)! x (2)!

15. Ans. (b)

Solution: ${}^4C_2$ x $(6/36)^2$ x $(30/36)^2$

= 6 x (1/36) x (25/36)

= 25/216.

16. Ans. (b)

Solution: The appearance of head or tail on a toss is independent of previous occurrences.

Hence, ½.

17. Ans. (d)

Solution: There can be three girls and one boy.

18. Ans. (a)

Solution: The event definition is Red AND Red AND Not Red OR Red AND Not Red AND Red OR Not Red AND Red AND Red.

19. Ans. (a)

Solution: The required answer will be given by.

$$\frac{P(Kesari\ does\ not\ receive\ the\ l}{P\ (Kesari\ does\ not\ receive\ the\ letter) + P(Kesari\ replied\ an}$$

20. Ans. (a)

Solution: The number of events for the condition that he will sing

= 4, [34,43,26,62]

The number of events in the sample = 90.

Probability that he will sing at least once

= 1 – Probability that he will not sing.

21. Ans. (a)

Solution: For divisibility by 5 we need the units digit to be either 0 or 5.

The units digit in the powers of 7 follow the pattern

7,9,3,1,7,9,3,1,7,9,........

Hence, divide 1 to 100 into four groups of 25 element each as follows.

A = 1,5,9,......... → 25 elements

B = 2,6,10,......... → 25 elements

C = 3,7,11,......... → 25 elements

D = 4,8,12,......... → 25 elements

Check the combination values of m and n to that $7^m + 7^n$ is divisible by 5.

22. Ans. (a)

Solution: All four are not in the correct envelopes means that at least one of them is in a wrong envelope. A little consideration will show that one letter being placed in a wrong envelope is not possible, since it will have to be interchanged with some other letter.

Since, there is only one way to put all the letters in the correct is only one way to put all the letters in the correct envelopes, we can say that the event of not all four letters going into the correct envelopes will be given by

5! – 1 = 119

23. Ans. (a)

Solution: Q.37 Are similar to Question No. 2 of LOD III.

24. Ans. (b)

25. Ans. (b)

Solution: The common side could be horizontal or vertical.

Accordingly, the number of ways the event can occur is.

N(E) = 8 x 7 + 8 x 7 = 112

N(S) $64C_2$

$= \frac{2\times8\times7\times2}{64\times63} = \frac{1}{18}$

26. Ans. (b)

Solution:

The divisibility of the product of four numbers depends upon the value of the last digit of each number.

The last digit of a number can be any of the 10 digits 0,1,2....9.

So, the total number of ways of selecting last digits of four numbers is $10\text{x}10\text{x}10\text{x}10\text{x}=10^4$

If the product of the 4 numbers is not divisible by 5 or 10.

Then the number of choices for the last digit of each number is 8 (excluding 0 or 5).

So, favourabe number of ways = $8\text{x}8\text{x}8\text{x}8 = 8^4$

The probability that the product is not divisible by 5 or 10

$=\frac{8^4}{10^4} = \left(\frac{8}{10}\right)^4$

Hence, Required probability = $1 - \left(\frac{8}{10}\right)^4 = \frac{369}{625}$

27. Ans. (b)

Solution:

P(selecting atleast one couple) = 1 – P (selecting none of the couples for the prize.)

$= 1 - \left(\frac{16C_1\times14C_1\times12C_1\times10C_1}{16C_4}\right) = \frac{15}{36}$

28. Ans. (a)

Solution: Total number of ways in which S people can be chosen out 9 people = ${}^9C_5 = 126$

Number of ways in which the couple serves the committee

$= {}^7C_3\text{x}{}^2C_2 = 35$

Number of ways in which the couple does not serve committee = ${}^7C_5 = 21$

Favourable number of cases = 35 + 21 = 56

Hence, the required probability = $\frac{56}{126} = \frac{4}{9}$

29. Ans. (b)

Solution: E_1 = The event in which A speaks truth

E_2 = The event in which B speaks truth

Then $P(E_1) = \frac{60}{100}$=3/5

and $P(E_1) = \frac{2}{5}$, $P(E_2) = \frac{1}{5}$

Required probability = $P\,[(E_1 \cap E_2) \cup ((\bar{E}_2 \cap \bar{E}_1)]$

$= P\,[(E_1 \cap E_2) + P\,(\bar{E}_2 \cap \bar{E}_1)$

$= P\,(E_1).\,P\,(E_2) + P\,(\bar{E}_1).P(\bar{E}_2)$

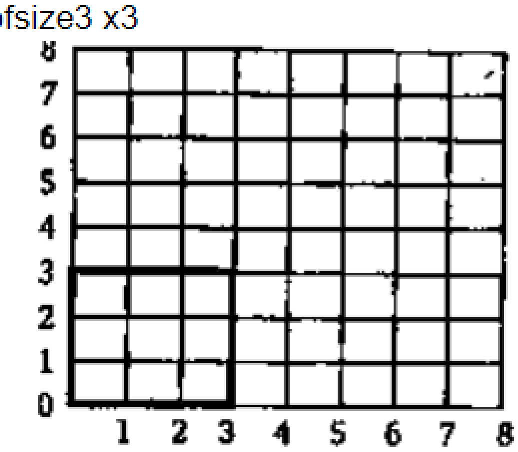
$\left(\frac{3}{5}\times\frac{4}{5}\right)+\left(\frac{2}{5}\times\frac{1}{5}\right) = \frac{14}{25} = 0.56$

30. Ans. (b)

Solution: We can choose 9 squares out of 64 squares in ${}^{64}C_9$ ways.

Hence, exhaustive number of cases = ${}^{64}C_9$

From the figure it is clear that the given square ofsize3 x3



can be formed by using four consecutive horizontal and 4 consecutive vertical lines, which can be done in

${}^6C_1 \times {}^6C_1$=36ways

Basically you can make 6 squares of size 3 x 3 in vertical direction and 6 squares of the size 3 x 3 in horizontal direction. Hence total 6 x 6 = 36 squares can be chosen.

The required probability = $\frac{36}{64C_9}$

31. Ans. (b)

Solution: Total 7 digit numbers can be formed from the 9 digits = ${}^9P_7$

There are four exclusive cases of selecting 7 digits out of 9 digits which can form 7 digit numbers which are divisible by 9.

2, 3, 4, 5, 6, 7, 9 } 36removing 1 and 8

1, 3, 4, 5, 6, 8, 9 } 36 removing 2 and 7

1, 2, 4, 5, 7, 8,9} 36 removing 3 and 6

1, 2, 3, 6, 7, 8, 9 } 36 removing 4 and 5

All the 7 numbers of each of the 4 sets can be arranged in 7! ways.

Hence the favourable number of numbers = 4x7!

Required probability = $\frac{4\times7!}{9P_7} = \frac{1}{9}$

32. Ans. (a)

Solution: Total number of words that can be formed from the letters of the word MISSISSIPPI is $\frac{11!}{4!4!2!}$

When all the S's are together then the number of words can be formed = $\frac{8!}{4!2!}$

Required probability = $\frac{\frac{8!}{4!2!}}{\frac{11!}{4!4!2!}} = \frac{4}{165}$

33. Ans. (b)

Solution: Since each of the coefficients a, b and c can take values from 1 to 6. Therefore the total number of equations

= 6x6x6 = 216

Hence the exhaustive number of cases = 216

Now, the roots of the equation $ax^2 + bx + c = 0$ will be real if b2 - 4ac > 0 => b2 > 4ac

Following are the number of favourable cases

a	c	ac	$4ac$	$b^2\ (\geq 4ac)$	b	Number of cases
1	1	1	4	4, 9, 16, 25, 36	2, 3, 4, 5, 6	$1\times5=5$
1 2	2 1	2	8	9, 16, 25, 36	3, 4, 5, 6	$2\times4=8$
1 3	3 1	3	12	16, 25, 36	4, 5, 6	$2\times3=6$
1 2 4	4 2 2	4	16	16, 25, 36	4, 5, 6	$3\times3=9$
1 5	5 1	5	20	25, 36	5, 6	$2\times2=4$
1 2 3 6	6 3 2 1	6	24	25, 36	5, 6	$4\times2=8$
2 4	4 2	8	32	36	6	$2\times1=2$
3	3	9	36	36	6	$1\times1=1$
					Total	= 43

Note → ac = 7 is not possible

Since b^2 36 and 4ac ≤ b^2 hence ac = 10, 11, 12....etc. is not possible.

Hence, Total number of favourable cases = 43

So, the required probability = $\frac{43}{216}$

34. Ans. (c)

Solution: Let A be the event of getting exactly 3 defectives in the examination of 8 wristwatches. And B be the event of getting ninth wristwatch defective

Then

Requited probability = P(A∩B) = P(A)P$\left(\frac{B}{A}\right)$

Now. P(A) $\frac{4C_3\times11_5}{15C_4}$

And P $\left|\frac{B}{A}\right|$ Probability that the nineth examined wrisrward is defective given that there were 3 defectives in the first prices examined $\frac{1}{7}$

Hence, requited probability = $\frac{4C_3\times11C_5}{15C_4}$

$$= \frac{1}{7} = \frac{8}{195}$$

35. Ans. (b)

Solution: Let x and y be the two non-negative integers

since x + y = 200

$(xy)_{max}$= 100x100 = 10000 $(xy_{max}$atx=y)

Now, xy ≤ 10000

=> xy ≥ $\frac{3}{4}$ ×10000

= xy ≥7500 => x (200-x) ≥ 7500

$$x^2 - 200x + 7500 \leq 0$$

50 ≤x ≤ 150

So favourable number of ways = 150 - 50 + 1 = 101

Total number of ways = 200

Hence, required probability = $\frac{101}{200}$

36. Ans. (a)

Solution: Consider the following events

A = The first number is less than the second number

B = The third number lies between the first and the second.

Now, we have to find $P\left(\frac{B}{A}\right)$

Also, we have $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

Any 3 numbers can be chosen out of n numbers in ${}^nC_3$ ways.

Let the selected numbers be x_1, x_2, x_3. Then they satisfy exactly one of the following inequalities.

$x_1 < x_2 < x_3$, $x_1 < x_3 < x_2$, $x_2 < x_1 < x_3$,
$x_2 < x_3 < x_1$, $x_3 < x_1 < x_2$, $x_3 < x_2 < x_1$,

The total number of ways of selecting three numbers and then arranging them

= ${}^nC_3$ x 3! = ${}^nP_3$

P(A) = $\frac{nC_3 \times 3}{nC_3 \times 3!}$

and

P(A∩B) = $\frac{nC_3}{nC_3 \times 3!}$

Hence

$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \frac{1}{3}$

37. Ans. (b)

Solution: ASSISTANT → AA I N SSS TT

STATISTICS → A II C SSS TTT

Here N and C are not common and same letters can be A, I, S, T. Therefore

Probability of choosing A = $\frac{2C_1}{9C_1} \times \frac{1C_1}{10C_1} = \frac{1}{45}$

Probability of choosing I = $\frac{1}{9C_1} \times \frac{2C_1}{10C_1} = \frac{1}{45}$

Probability of choosing S = $\frac{3C_1}{9C_1} \times \frac{3C_1}{10C_1} = \frac{1}{10}$

Probability of choosing T = $\frac{2C_1}{9C_1} \times \frac{3C_1}{10C_1} = \frac{1}{15}$

Hence, required probability = $\frac{1}{45} + \frac{1}{45} + \frac{1}{10} + \frac{1}{15} = \frac{19}{90}$

38. Ans. (b)

Solution: Out of 30 numbers 2 numbers can be chosen in ${}^{30}C_2$ ways.

So, exhaustive number of cases = ${}^{30}C_2$ = 435

Since a^2 - b^2 is divisible by 3 if either a and b are divisible by 3 or none of a and b is divisible by 3.

Thus, the favourable numbers, of cases = ${}^{10}C_2$ + ${}^{20}C_2$ = 235

Hence, required probability = $\frac{235}{435} = \frac{47}{87}$

39. Ans. (c)

Solution: A number is divisible by 11 only if the difference of the sum of the digits at odd places and sum of the digits at even places is divisible by 11 i.e, 0, 11, 22, 33....

Here the sum of all the 9 digits (1, 2, 3, ...9) is 45.

We cannot create the difference of zero since x + y = 45, which is odd hence cannot be broken into two equal parts in integers.

Now, we will look for the possibilities of 11 which are as follows:

{1,2, 6, 8}{1,2, 5, 9}{1, 3,6, 7}
{1, 3, 5, 8} {1,3, 4, 9}{1,4, 5, 7}
{2, 3, 5, 7}{2,3, 4, 8}{2, 4, 5, 6}
and {4, 7, 8, 9}{5, 6, 8, 9} '

The above set of values either gives the sum of 17 or 28. Since if the sum of 4 digits at even places be 17 or 28 then the sum of rest of the digits (i.e., digits at odd places) be 28 or 17 respectively and thus we can get the difference of 11.

Further we cannot get the difference of 22 or 33...

So there is only possible difference that can be created is 11

and there are only 11 set of values given above containing 4

digits which can be arranged in 4! ways and the remaining 5
digits can be arranged in 5! ways.
Thus the favourable number of numbers = 11 x 41 x 5!
But the total number of ways of arranging a nine digit number is $^9P_9$ =9!

Exclusive number of cases = 9!
Required probability = $\frac{11\times4!\times5!}{9!} = \frac{11}{126}$.