Percentile Classes

Inequalities

Properties of Inequalities

1. If a> b then b < a and vice versa.
2. If a > b and B > c then a > c.
3. If a > b then for any c, a+b > b + c. In other words, an inequality remains true if the same number is added on both sides of the inequality.
4. Any number can be transposed from one side of an inequality with the sign of the number reversed. This does not change the sense of the inequality.
5. If a > b and c > 0 then ac > bc. Both sides of an inequality may be multiplied (or divided) by the same positive number without changing the sense of the inequality.
6. If a > b and c < 0 then ac < be. That is, both sides of an inequality may be multiplied (or divided) by the same negative number but then the sense of the inequality is reversed.
7. If a>b and C > D then a + c>b + d. (Two inequalities having the same sense may be added term wise.)
8. If a>b and c<d then a-c> b - d
   From one inequality it is possible to subtract term wise another inequality of the opposite sense, retaining the sense of the inequality from which the other was subtracted.

Certain important inequalities

1. $a^2 + b^2 \geq 5$ (Equality for a = b)
2. |a+b| ≤ |a| + |b| (Equality reached if both a and b are of the same sign or if one of them is zero.)
   This can be generalized as $|a_1 + a_2 + a_3 \ldots\ldots a_n| \leq |a_1| + |a_2| + |a_3| + \ldots\ldots |a_n|$
3. |a-b| ≥ |a| - |b|
4. ax^2 + bx + c ≥ 0 if a > 0 and D = b^2 – 4ac ≤ 0. The equality is achieved only if D = 0 and x = -b/2a.
5. Arithmetic mean ≥ Geometric mean that is, $\frac{(a+b)}{2}$ ≥ ab
6. a/b + b/a ≥2 if a > 0 and b > 0 or if a < 0
7. $a^3 + b^3 \geq$ ab(a+b) if a > 0 and b > 0. Then equality being obtained only when a = b.
8. $a^2 + b^2 + c^2 \geq$ ab +ac + bc
9. (a + b) (b+c) (a+c) ≥ 8 abc if a ≥ 0, b ≥ 0 and c ≥ 0, the equation being obtained when a = b =c
10. For any 4 numbers x_1, x_2, y_1, y_2 satisfying the conditions

    $$x_1^2 + x_2^2 = 1$$
    $$y_1^2 + y_2^2 = 1$$

    The inequality $|x_1y_1 + x_2y_2| \leq 1$ is true.
11. $\frac{a}{b^{1/2}} + \frac{b}{a^{1/2}} \geq a^{1/2} + b^{1/2}$ where a ≥ 0 and b ≥ 0
12. If a + b = , then $a^4 + b^4 \geq 2$
13. The inequality |x| ≤ a, means that
    - a ≤ x ≤ a for a > 0
14. $2^n > n^2$ for n ≥ 5

Some important Results

1. If a > b , then it is evident that
   a+c>b+c
   a-c> b-c
   ac > be
   a/c > b/c; that is,

an inequality will still hold after each side has been increased, diminished, multiplied, or divided by the same positive quantity,
By adding c to each side,
a > b + c', which shows that
in an inequality any term may be transposed from one side to the other if its sign is changed.

2. If a > b, then evidently b < a; that is, if the sides of an inequality be transposed, the sign of inequality must be reversed.
3. if a>b, then a-b is positive, and b-a is negative;
that is, -a-(-b) is negative, and therefore -a < -b; hence.
If the signs of all the terms of an inequality be changed, the sign of inequality must be reversed. Again,
If a>b, then -a < -b and, therefore, -ac -bc; that is,
if the sides of an inequality be multiplied by the same negative quantity, the sign of inequality must be reversed.
If $a_1 > b_1$, $a_2 > b_2 > a_3 > b_3 \ldots\ldots a_m > b_m$ it is clear that $a_1 + a_2 + a_3 + \ldots + a_m > b_1 + b_2 + b_3 + \ldots + b_m$; and $a_1 a_2 a_3 \ldots a_m > b_1 b_2 b_3 \ldots b_m$.

4. If a > b, and if p, q are positive integers, then or $a^{1/q} > b^{1/q}$ and, therefore, $a^{p/q} > b^{p/q}$ that is, $a^n > b^n$, where n is any positive quantity. Further,
$1/a^n < 1/b^n$; that is $a^{-n} < b^{-n}$
The square of every real quantity is positive, and therefore greater than zero. Thus $(a - b)^2$ is positive.
Let a and b be two positive quantities, S their sum and P their product. Then from the identity
4ab = $(a + b)^2 - (a - b)^2$
We have
4P = $S^2 - (a - b)^2$, and $S^2 = 4P + (a + b)^2$
Hence, if S is given, P is greatest when a = b; and if P is given, S is least when a-b;
That is, if the sum of two positive quantities is given, their product is greatest when they are equal; and if the product of two positive quantities is given, their sum is least when they are equal.

Notation of Ranges

1. Ranges where the ends are excluded: If the value of x is denoted as (1,2) it means 1 < x 2 i.e. x is greater than 1 but smaller than 2
Similarly if we denote the range of values of x as – (7,-2) *U* (3, 21), this means that the value of x can be denoted as -7 < x < -2 and 3 < x < 2l. this would mean that the inequality is satisfied between the two ranges and is note satisfied outside these two ranges.
Based on this notation write the ranges of x for the following representation:

   (1, +∞)*U*(-∞, -7)
   (-∞, 0) *U* (4, + ∞), (-∞, 50) *U* (-50, +∞)

2. Ranges where the ends are included
   [2.5] means 2 ≤ x ≤ 5
3. Mixed ranges
   (3, 21)] means 3 < x ≤ 21

Exercise 01

Directions(Q1-Q24): solve the following inequalities:

1. $3x^2 - 7x - 6 < 0$
   (a) -0.66 < x <3 (b) x < -0.66 or x > 3
   (c) 3 < x < 7 (b) -2 < x < 2

2. $x^2 - 14x - 15 > 0$
   (a) x < -1 (b) 15 <x
   (c) both (a) and (b) (d) -1 < x < 15

3. $2 - x - x^2 \geq 0$
   (a) -2 ≤ x ≤ 1 (b) -2 < x < 1
   (c) x < -2 (d) x > 1

4. $|x^2 - 4x| < 5$
   (a) -1 ≤ x ≤ 5 (b) 1 ≤ x ≤5
   (c) -1 ≤ x ≤ 1 (d) -1 < x <5

5. $|x^2 - 5x| < 6$
   (a) -1 < x <2 (b) 3 < x < 6
   (c) both (a) and (b) (d) -1 < x < 6

6. $|x^2 - 2x| < x$
   (a) 1 < x <3 (b) -1 < x < 3
   (c) 0 < x <4 (d) x > 3

7. $x^2 - 7x + 12 < |x-4|$
   (a) x < 2 (b) x > 4 (c) 2 < x < 4 (d) 2 ≤x ≤ 4

8. $|x-6| > x^2 - 5x + 9$
   (a) 1 ≤ x ≤ 3 (b) 1 < x < 3
   (c) 2 < x < 5 (d) -3 < x < 1

9. $x^2 - |5x + 8| > 0$
   (a) $x > (5-\sqrt{57})/2$ (b) $x < (5+\sqrt{57})/2$
   (c) $x > (5+\sqrt{57})/2$ (d) Both (a) and (b)

10. $|x^2 - 2x - 8| > 2x$
    (a) $x < 2\sqrt{2}$ (b) $x < 3 + 3\sqrt{5}$
    (c) $x > 2 + 2\sqrt{3}$ (d) both (a) and (c)

11. $\sqrt{\frac{x-2}{1-2x}} > -1$
    (a) 0.5 > x (b) x > 2 (c) both (a) and (b) (d) 0.5 < x ≤ 0

12. $\sqrt{3x - 10} > \sqrt{6 - x}$

(a) $4 < x \leq 6$ (b) $x < 4$ or $x > 6$
(c) $x < 4$ (d) $x > 8$

13. $\sqrt{x^2-2x-3} < 1$
(a) $(-1-\sqrt{5} < x < -3)$ (b) $1 \leq x < (\sqrt{5}-1)$
(c) $x > 1$ (d) None of these

14. $\sqrt{9x-20} < x$
(a) $4 < x < 5$ (b) $20/9 \leq x < 4$
(c) $x > 5$ (d) both (b) and (c)

15. $\sqrt{x+78} < x + 6$
(a) $x < 3$ (b) $x > 3$ or $x < 2$ (c) $x > 3$ (d) $3 < x < 10$

16. $x + 3 < \sqrt{x+33}$
(a) $x > 3$ (b) $x < 3$ (c) $-3 < x < 3$ (d) $-33 < x < 3$

17. $x + 2 < \sqrt{x+14}$
(a) $-14 \leq x < 2$ (b) $x > -14$
(c) $x < 2$ (d) $-11 < x < 2$

18. $\sqrt{x+2} > x$
(a) $-2 \leq x < 2$ (b) $-2 \leq x$ (c) $x < 2$ (d) $x = -2$ or $x > 2$

19. $(x-1)(3-x)(x-2)^2 > 0$
(a) $1 < x < 3$ (b) $1 < x < 3$ but $x \neq 2$
(c) $0 < x < 2$ (d) $-1 < x < 3$

20. $\frac{2x-3}{3x-7} > 0$
(a) $x < 3/2$ or $x > 7/3$ (b) $3/2 < x < 7/3$
(c) $x > 7/3$ (d) none of these

21. $\frac{3}{x-2} < 1$
(a) $2 < x < 5$ (b) $x < 2$
(c) $x > 5$ (d) $x < 2$ or $x > 5$

22. $\frac{4x+3}{2x-5} < 6$
(a) $x < 2.5$ (b) $x < 33/8$ (c) $x \geq 2.5$ (d) $x < 2.5$ or $x > 33/8$

23. $\frac{7x-5}{8x+3} > 4$
(a) $-17/25 < x < -3/8$ (b) $x > -17/25$
(c) $0 < x < 3/8$ (d) $-17/25 < x < 0$

24. $\frac{4}{x+2} > 3-x$
(a) $-2 < x < -1$ or $x > 2$ (b) $-2 < x < 2$
(c) $-2 < x < -1$ (d) $0 < x < 3$

Directions(Q25 – Q55): Solve the following polynomial inequalities

25. (x-1)(3-x)$(x-2)^2$ > 0
(a) 1 < x < 2 (b) -1 < x < 3 (c) -3 < x < -1 (d) 1 < x < 3, x ≠ 2

26. $\frac{x^2-5x+6}{x^2+x+1} < 0$
(a) x < 2 (b) x > 3 (c) 2 < x < 3 (d) x < 2 or x > 3

27. $\frac{x^2+2x-3}{x^2+1} < 0$
(a) x < -3 (b) -7 < x < -3 (c) -3 < x < 1 (d) -7 < x < 1

28. $\frac{s^2+4x+4}{2x^2-x-1} > 0$
(a) x < -2 (b) x > 1 (c) x ≠ 2 (d) None of these

29. $x^4 - 5x^2 + 4 < 0$
(a) -2 < x < 1
(b) -2 < x < 2
(c) -2 < x < -1 or 1 < x < 2
(d) 1 < x < 2

30. $\frac{x-2}{x^2+1} < -\frac{1}{2}$
(a) - 3 < x < 3
(b) x < -3
(c) - 3x < x < 6
(d) -3 < x< 1

31. $\frac{x^2-7x+12}{2x^2+4x+5} > 0$
(a) x < 3 o x > 4
(b) 3 < x < 4
(c) 4 < x < 24
(d) 0 < x < 3

32. $\frac{x^4+x^2+1}{x^2-4x-5} < 0$
(a) x < -1 or x > 5
(b) -1 < x < 5
(c) x > 5
(d) - 5 < x < -1

33. $\frac{1+3x^2}{x^2-4x-5} < 0$
(a) 0 < x < 8
(b) 2.5 < x < 8
(c) -8 < x < 8
(d) 3 < x < 8

34. $\frac{1+x^2}{x^2-5x+6} < 0$
(a) -1 <2 (b) x > 3 (c) both a and b (d) 2 < x < 3

35. $\frac{1-2x-3x^2}{3x-x^2-5} > 0$
(a) x < -1 or x > 1/3
(b) x < -1 or x = 1/3
(c) - 1 < x < 1/3
(d) x < 1/3

36. $\frac{2x^2-3x-459}{x^2+1} > 1$
(a) x > -20 (b) x < 0 (c) x < -20 (d) -20 < x < 20

37. $\frac{1-2x-3x^2}{3x-x^2-5} > 0$
(a) 1 < x < 3 (b) 1 < x < 7 c) -3 < x < 3 (d) None of these

38. $\frac{x}{x^2-3x-4} > 0$
(a) -1 < x < 0 (b) 4 < x (c) both a and b (d) -1 < x < 4

39. $\frac{17-15x-2x^2}{x+3} < 0$
(a) -8.5 < x ≤ -3 (b) -17 < x < -3
(c) -8.5 < x < -3 or x > 1 (d) -8.5 < x < 1

40. $\frac{x^2-9}{3x-x^2-24} < 0$
(a) -3 < x < 3 (b) x < -3 or x > 3
(c) x < -5 or x > 5 (d) x < -7 or x > 7

41. $2x^2 + \frac{1}{x} > 0$
(a) x > 0 (b) -2 ≤ x < 0 (c) x > 3 (d) all of these

42. $\frac{x^2-5x+6}{4x^2-4x+1} < 0$
(a) x < 1 or x > 7 (b) 1 < x < 7 (c) -7 < x < 1 (d) -7 < x < 7

43. $\frac{x^2-6x+9}{5-4x-x^2} \geq 0$
(a) -5 < x < 1 or x = 3 (b) -5 ≤ x < 1 or x = 3
(c) -5 < x ≤ 1 or x = 3 (d) -5 ≤ x ≤ -1

44. $\frac{1}{x+2} < \frac{3}{x-3}$
(a) -4.5 < x < -2 (b) -.5 < x < -2 or 3 < x
(c) -4.5 < x < -2, x > 3 (d) b or c

45. $\frac{14x}{x+1} - \frac{9x-30}{x-4} < 0$
(a) -1 < x < 1 or 4 < x < 6 (b) -1 < x < 4, 5< x < 7
(c) 1 < x < 4 or 5 < x < 7 (d) -1 < x < 1 or 5 < x < 7

46. $\frac{5x^2-2}{4x^2-x+3}$ - < 1
(a) x < 1 (b) - 2 < x < 2
(c) -2.7 < x < 1.75 (d) $(-(1+\sqrt{21})/2 < x < (\sqrt{21}-1)/2$

47. $\frac{x^2-1}{2x+5} < 3$
(a) x < -2.5 or -2 < x < 8 (b) -2.5 < x < -2
(c) -2.5 < x < 8 (d) both (a) and (b)

48. $\frac{x^2+2}{x^2-1} < -2$
(a) -1 < x < 2 (b) -1 < x < 1
(c) -1 < x < 0, 0 < x < 1 (d) -2 < x < 2

49. $\frac{2x+3}{x^2+4x-5} > \frac{1}{2}$
(a) x < -5 (b) x > 1 (c) -5 < x < 1 (d) -5 < x < 5

50. $\frac{15-4x}{x^2-x-12} < 4$
(a) $x < -\sqrt{63}/2, -3 < x < \sqrt{63}/2$
(b) x > 4
(c) both a and b
(d) x > 4, < < -63/2
(e) none of these

51. $\frac{5-4x}{3x^2-x-4} < 4$
(a) $x < -\frac{\sqrt{7}}{2}$
(b) $-1 < x < \frac{\sqrt{7}}{2}$
(c) x > 4/3
(d) all of these

52. $\frac{4}{1+x} + \frac{2}{1-x} < 1$
(a) -1 < x < 1
(b) x < -1
(c) x > 1
(d) both b and c

53. $\frac{x^4-3x^3+2x^2}{x^2-x-30} > 0$
(a) x < -5
(b) 1 < x < 2
(c) x > 6
(d) all of these

54. $\frac{2x}{x^2-9} \leq \frac{1}{x+2}$
(a) x < -3
(b) -2 < x < 3
(c) all except (a) and (b)
(d) both (a) and (b)

55. $\frac{1}{x-2} + \frac{1}{x-1} > \frac{1}{x}$
(a) $-\sqrt{2} < x < 0$ or $2 < x$
(b) $\sqrt{2} < x$
(c) $1 < x < \sqrt{2}$
(d) both (a) and (c)

Directions(Q56 – Q59): solve inequalities based on modulus

56. $\frac{|x+2|-x}{x} < 2$
(a) $-5 \leq x < 0$
(b) $0 \leq x \leq 1$
(c) both (a) and (b)
(d) always except (b)

57. $\frac{|x-3|}{x^2-5x+6} \geq 2$
(a) [3/2, 1]
(b) [1, 2]
(c) [1.5, 2]
(d) None of these

58. $|x| < \frac{9}{x}$
(a) x < -1
(b) 0 < x < 3
(c) 1 < x < 3; x < -1
(d) $-\infty < x < 3$

59. $\frac{(x^2-4x+5)}{x^2+5x+6} \geq 0$
(a) $-\infty < x < \infty$
(b) x < -3
(c) x > -2
(d) $-\infty < x < 3$

Directions (Q60 – Q62): Solve the following irrational inequalities.

60. $(x-1)\sqrt{x^2-x-2} \geq 0$
(a) x < 2
(b) $3 \leq x < \infty$
(c) always except (a)
(d) both (a) and (b)

61. $\frac{\sqrt{x-3}}{x-2} > 0$

(a) $0 \le x < 2$ (b) $x > 3$

(c) $0 < x < 1$ (d) both (b) and (c)

62. If x satisfies the inequality $|x-1| + |x-2| + |x-3| \ge 6$, then:

(a) $0 \le x \le 4$ (b) $x \le 0$ or $x \ge 4$

(c) $x \le -2$ or $x \ge 3$ (d) $x \ge 3$

63. If $f(x) = \frac{x}{2x^2+5x+8}$ for all x > 0 what is the greatest value of f(x)?

(a) ¼ (b) 1/8 (c) 1/9 (d) 1/5

FILL IN THE BLANKS

Directions for 64 and 65: If $\frac{x^2-5x+6}{|x|+5} \le 0$

64. Find the minimum value of x, for which the above inequality is true.

___________.

65. For how many integer values of x, the above inequality is true.

___________.

66. For how many integer values of x is:

$\frac{x^2+6x-7}{x^2+1} > 2$

___________.

67. Maximum value of x, for which $\frac{x^2-9}{x^2+x+1} \le 0$

___________.

Directions for 68 and 69: $\frac{x^2-7|x|+10}{x^2-8x+16} < 0$

68. For how many negative integral values of x, is the above inequality true?

___________.

69. Find the sum of all integer values for which the above inequality is true.

___________.

70. If $f(x) = x^2 + 2|x| + 1$, then for how many real values of x is: $f(x) \le 0$.

___________.

___________.

Directions for 71 and 72: $|4x-3| \le 8$ and $|3y+4| \le 17$ then answer the following question:

71. Minimum value of |x| + |y| = ___________

72. Maximum value of |x| - |y| = ___________

73. If $f(x)$ = min (3x+4, 6-2x) and $f(x) < p$ where p is an integer then the minimum possible value of p = ______?_____

ANSWER KEY & EXPLANATIONS

1. Ans.: (d)
   Solution: At x = 0 inequality is not satisfied.
   Hence, option (b), (c) and (d) are rejected. At x = 2, inequality is not satisfied. Hence option (a) is rejected
2. Ans.: (c)
   Solution: at x = 0 inequality is not satisfied. Thus option (d) is rejected.
   X = -1 and x = 15 are the roots of the quadratic equation. Thus, option (c) is correct.
3. Ans.: (a)
   Solution: At x = 0, inequality is satisfied.
   Thus, option, (c) and (d) are rejected.
   At x = 1, inequality is satisfied
   Hence, we choose option (a).
4. Ans.: (d)
   Solution: At x = 0 inequality is satisfied, option (b) is rejected
   At x = 2, inequality is satisfied, option (c) is rejected.
   At x = 5, LHS = RHS
   At x = -1, LHS = RHS.
   Thus, option (d) is correct.
5. Ans.: (c)
6. Ans.: (a)
   Solution: At x = 1 and x = 3 LHS = RHS
   At x = 2 inequality is satisfied.
   At x = 0.1 inequality is not satisfied.
   At x = 2.9 inequality is satisfied.
   At x = 3.1 inequality is not satisfied.
   Thus, option (a) is correct.
7. Ans.: (c)
   Solution: at x = 0, inequality is not satisfied, option (a) is rejected,
   At x = 5, inequality is not satisfied, option (b) is rejected.
   At x = 2 inequality is not satisfied.
   Options (d) are rejected.
   Option (c) is correct.
8. Ans.: (b)
   Solution: at x = 2, inequality is satisfied.
   At x = 0, inequality is not satisfied.
   At x = 1, inequality is not satisfied but LHS = RSH
   At x = 3, inequality is not satisfied but LHS = RSH.
   Thus, option (b) is correct.
9. Ans.: (d)
10. Ans.: (d)
11. Ans.: (d)
12. Ans.: (a)
13. Ans.: (d)
14. Ans.: (d)
15. Ans.: (c)
16. Ans.: (d)
17. Ans.: (a)
18. Ans.: (a)
19. Ans.: (b)
20. Ans.: (a)
21. Ans.: (d)
22. Ans.: (d)
23. Ans.: (a)
24. Ans.: (a)
25. Ans. (d)
26. Ans. (c)
27. Ans. (c)
28. Ans. (d)
29. Ans. (c)
30. Ans. (d)
31. Ans. (a)
32. Ans. (b)
33. Ans. (b)
34. Ans. (d)
35. Ans. (a)
36. Ans. (c)
37. Ans. (d)
38. Ans. (c)
39. Ans. (c)
40. Ans. (b)
41. Ans. (d)
42. Ans. (b)
43. Ans. (a)
44. Ans. (b)
45. Ans. (a)
46. Ans. (d)
47. Ans. (a)
48. Ans. (c)
49. Ans. (c)
50. Ans. (c)
51. Ans. (d)
52. Ans. (d)
53. Ans. (d)
54. Ans. (d)
55. Ans. (d)
56. Ans. (d)
57. Ans. (d)
58. Ans. (b)
59. Ans. (d)
60. Ans. (c)
61. Ans. (b)
62. Ans. (b) If we put x = 3

Then |3-1| + |3-2| + |3-3| = 3 ≤ 6
Therefore option (a), (c), (d) are not correct.
Hence only option (b) is correct.

63. Ans. (c)

Solution: $f(x) = \frac{x}{2x^2+5x+8} = \frac{1}{2x+\frac{8}{x}+5}$

$f(x)$is maximum where denominator is minimum

$2x + \frac{8}{x} \geq \sqrt{2x \times \frac{8}{x}}$

$2x + \frac{8}{x} \geq 4$

$\left(2x + \frac{8}{x}\right)_{min} = 4$

$(f(x))_{max} = \frac{1}{4+5} = \frac{1}{9}$

64. Ans.: (2)

$\frac{x^2-5x+6}{|x|+5} \leq 0$

$\frac{(x-2)(x-3)}{|x|+5} \leq 0$

Case I: x > 0
In this case the expression would become:

➔ $\frac{(x-2)(x-3)}{x+5} \leq 0$

➔ $2 \leq x \leq 3$

Case II: x < 0
In this case the expression would become:

➔ $\frac{(x-2)(x-3)}{x-5} \leq 0$

➔ $\frac{(x-2)(x-3)}{x-5} \geq 0$

The above inequality is not satisfied for any negative value of x.
Therefore solution of the above inequality ➔ $2 \leq x \leq 3$
Minimum value of x = 2

65. Ans.: (2)

$\frac{x^2-5x+6}{|x|+5} \leq 0$

$\frac{(x-2)(x-3)}{|x|+5} \leq 0$

Case I: x > 0
In this case the expression would become:

➔ $\frac{(x-2)(x-3)}{x+5} \leq 0$

➔ $2 \leq x \leq 3$

Case II: x < 0
In this case the expression would become:

➔ $\frac{(x-2)(x-3)}{x-5} \leq 0$

➔ $\frac{(x-2)(x-3)}{x-5} \geq 0$

The above inequality is not satisfied for any negative value of x.
Therefore solution of the above inequality ➔ $2 \leq x \leq 3$
The above inequality is true for two integral values of x.

66. Ans.: (0)

Solution: $\frac{x^2+6x-7}{x^2+1} - 2 > 0$

$\frac{-x^2+6x-9}{x^2+1} > 0$

$\frac{(x-3)^2}{x^2+1} < 0$

$(x-3)^2 > 0,\ x^2+1 > 0$

So the above inequality is not true for any real value of x.

67. Ans.: (3)

Solution: $x^2 + x + 1 > 0$ (It is a quadratic equation with negative discriminant)
Hence, for the expression to be non-positive, the numerator has to be non-positive. i.e. $x^2 - 9 \leq 0$
(x-3) (x+3) ≤ 0
➔ $-3 \leq x \leq 3$
Required maximum value of x = 3

68. Ans.: (2)

$\frac{(|x|)^2-7|x|+10}{x^2-2\times4\times x+4^2} < 0$

$\frac{(|x|-5)(|x|-2)}{(x-4)^2} < 0$

➔ 2 < |x| < 5, but x ≠ 4
The upper limit defines the rage of x as: -5 < x < 5
The lower limit defines the range of x as: x < -2, x > 2
To obey both he limits we will get: -5 < x < -2.
2 < x < 5 & x ≠ 4.
The above inequality is true for x = -4, -3.
Therefore fore two negative integral values of x the given inequality is true.

69. Ans.: (-4)

Solution: $\frac{(|x|)^2-7|x|+10}{x^2-2\times4\times x+4^2} < 0$

$\frac{(|x|-5)(|x|-2)}{(x-4)^2} < 0$

➔ 2 < |x| < 5, but x ≠ 4
The upper limit defines the rage of x as: -5 < x < 5
The lower limit defines the range of x as: x < -2, x > 2
To obey both he limits we will get: -5 < x < -2.
2 < x < 5 & x ≠ 4.
Required sum = -4 -3 + 3 = -4

70. Ans.: (0)

Solution: $x^2 + 2|x| + 1 \leq 0$

$|x|^2 + 2|x| + 1 \leq 0$

$(|x|+1)^2 \leq 0$

This is not possible for any real value of x.

71. Ans. (0)

Solution: -8 ≤ 4x – 3 ≤ 8	|3y+4|≤ 17
-5 ≤ 4x ≤ 11	-17 ≤ 37 + 4 ≤ 17
$-\frac{5}{4} \leq x \leq \frac{11}{4}$	$-7 \leq y \leq \frac{13}{3}$

X and y both can be zero, therefore minimum value of |x| + |y| = 0 + 0 = 0

72. Ans. (2.75)

Solution: |x| - |y| will be maximum when |x| is maximum and |y| is minimum

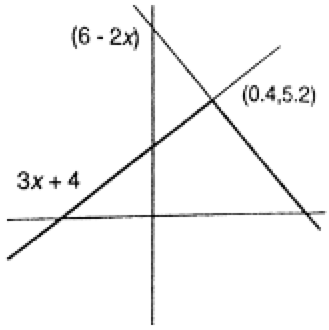
$(|x| - |y|)_{max} = |x|_{max} - |y|_{min}$

$= \frac{11}{4} - 0$

$= \frac{11}{4} = 2.75$

73. Ans. (6)

Solution: $3x + 4$ is an increasing while $6-2x$ is a decreasing function.
Graph of min $(3x+4, 6-2x)$



From the graph is clear that maximum value of $f(x) = 5.2$, which occurs at $x = 0.4$
$F(x) < p$ (where p is an integer least value of p must be 6.)