Percentile Classes

Functions – Graphs – Maxima/Minima

Maxima Minima

Finding Maxima and Minima in Some typical cases

1. Maxima and minima of quadratic equation

The standard methods of finding the maxima and the minima of all the quadratic equations are as follows:

Let us discuss these methods with respect to a particular equation y = x^2 – 5x + 6.

(a) Graphical method

As we have seen in the concepts of quadratic equation, graph of y = ax^2 + bx + c = 0 will have its

Minimum value at x = $\frac{-b}{2a}$ and y_{min} = $\frac{-D}{4a}$, when a is positive

Maximum value at x = $\frac{-b}{2a}$ and y_{max} = $\frac{-D}{4a}$, when a is negative

So, the minimum value of y = x^2 – 5x + 6 will be $y_{min} = \frac{-D}{4a} = \frac{-1}{4}$.

(b) Quadratic equation method

Suppose we have found the minimum value of y = $x^2 - 5x + 6, then\ we\ can$ find the minimum vale by breaking this equation into the sum of a whole square and whatever is left, i.e., P^2 + Q.

x^2 – 5x + 6= $\left[x^2 - 2.x.\frac{5}{2} + (\frac{5}{2})^2\right] - \frac{1}{4} = (x - \frac{5}{2})^2$ - $\frac{1}{4}$

Who know that the minimum value of $(x - \frac{5}{2})^2$ is equal to zero, so the minimum value of y = $-\frac{1}{4}$.

2. Maxima and minima of any modulus

Here, we will discuss some more situations involving modulus. Let us see this with the help of an example:

Finding the maximum and the minimum value of y = y (x) = |x+3| + |x-5| + |x-7|

Maximum value: Obviously, the maximum value can be extended upto +∝.

Minimum value: In this case, all the three parts of f(x), i.e, |x+3|, |x-5|, and |x-7| cannot be simultaneously equal to zero. So, we are required to find the critical points here at first, and then these points in f(x) to see that which one gives us the minimum value.

To obtain critical points, put all the three components |x+3|, |x-5| and |x-7| one by one equal to zero.

|x+3| = 0 so x=-3
|x-5| = 0 so x = 5
|x-7| = 0 so x = 7

Now, putting these values in f(x) gives us the following result:

At x = -3, f(x) = 18
At x = 5, f(x) = 10
At x = 7, f(x) = 12

So, value the minimum value of f(x) = 10

3. Finding the maximum or the minimum value of the product/sum of two or more than two variables when the sum/product of these variables is given.

1. **If sum of two or more than two variables is given, then the product will be maximum when the value of all the variables are equal.**

Example. Find the maximum and the minimum value of xy subject to x + y = 8.

(a) 8 (b) 16 (c) 20 (d) 24

Solution: The maximum value of xy will occur then x = y = 4.

So, the maximum value of xy = 4 x 4 = 16.

To obtain the minimum value, we can take either x or y to be negative, and then the product of xy will be negative. This process continues till -∝ so, the minimum value of xy = -∝.

2. **If the product of two or more than two positive variables is given, the n there sum will be the minimum when the value of all the variable are equal.**

Example: What is the minimum value of f(x) = $\frac{x^3+x+2}{x}$ = x > 0?

Solution Since $\frac{x^2+x+2}{x} = x^2 + 1 + \frac{1}{x} + \frac{1}{x}$

The product of all the terms $x^2, 1, \frac{1}{x}$, and $\frac{1}{x}$ is 1, the sum of these terms will have the minimum value for x^2 = 1 = $\frac{1}{x}$, that is, for x = 1. Hence, the minimum value of f(x) is 4.

3. **Using arithmetic mean, geometric mean, and harmonic mean to find the maxima and the minima**

(i) AM ≥ GM ≥ HM
(ii) $(GM)^2$= AM x HM (for two numbers only)

Example: If a, b, c, and d are positive real numbers such that a + b+ c+ d = 2, then which of the following is true regarding the values of N = (a+b) (c+d)?

Solution: Using AM ≥ GM

$\frac{1}{2}$[(a+b) + (c+d)] ≥ $[(a+b)]^{1/2} = \frac{1}{2}(2) \geq (N)^{1/2}$

Also, (a+b) (c+d) ≥ 0
So, 0 ≤ N ≤ 1

Some Special Cases:

(a) Given that a + b+ c + d… = K (constant) Maximum value of ($a^x, b^x, c^z . d^t$…..) will be obtained if $\frac{a}{x} = \frac{b}{y} = \frac{c}{z} = \frac{d}{t}$ = ….

(b) Given that ax + by = K (constant) Maximum value of $x^p y^q$ is obtained when $\frac{ax}{p} = \frac{by}{q}$, converse of the above expression is also true.

Value of ax + by is obtained when $\frac{ax}{p} = \frac{by}{q}$,

Graphs and Domain

Domain & Graphs different types of functions

A function f: A → B is called a real valued if the image of every element of A, under F is a real number i.e, if f(x) ∈ R ∀ x ∈ A

y = f(x)

x → independent variable
y → dependent variable
Domains: the values of x must be such that for every x, y must be real is called the domains.

In other words the value of x for which the values of f(x) in not a real number cannot be included in the domain of the function y = f(x), there are five different common situations where the domain of f(x) for some particular value (s) is not defined.

(a) When base of $(1/2n)^{th}$ power is negative for any n ∈ *I:*

If $x^{1/2}$ = k ➔ x = k^2 since k^2 is always positive even when x is negative, therefore this equality is not satisfied.

Hence the domain of definition of y = $\sqrt{x}$ is defined for only x ≥ 0 i.e., not defined for x < 0

(b) When denominator of a function is zero :
If $\frac{x^2+1}{2x}$ is such that x = 0, then $\frac{x^2+2}{2x}$ is not defined i.e., then function y = $\frac{1}{x}$ is defined only for x ≠ 0 (i.e., non zero real numbers) therefore the domain of function f(x) = $\frac{1}{x}$ is (-∞, 0)∪(0,∞) or (-∞, ∞) – {0} or -∞ < x 0 ∪ 0 < x < ∞.

(c) When function becomes 0^0.
Example y = $(|x| - 3)^{(x-3)}$ is undefined for x = 3

(d) When the value of x in $log_a x$ becomes nonpositive (I.e, negative or zero)
∴ $log_0(-3)$ is not defined.

(e) when the value of a in $log_a x$ becomes 1 or 0 or negative.
∴ $log_1 5$ *is not* detined.
Similarly, $log_{(-3)}$ 10 is not defined.

Range: The set of all images of all the elements of domain.
It can be obtained by one of the following ways.

(i) **obtaining x in terms of y and then finding the values of y for which we get real x.**

e.g., f(x) = $\frac{x+2}{x+3}$ = y

⇨ yx + 3y = x +2
⇨ x(y-1) = (2-3y)

⇨ x = $\frac{2-3y}{y-1}$

For y = 1, there is no real value of x for the above equation is satisfied hence.

Range = R – {1}   R → Set of real numbers

(ii) **If the given function is continuous, then range = [*l, m*] where l is the least value of the function in its domain and m is the maximum value in its domain.**

Algebraic Functions:

A function which contains finite number of terms having different powers of independent variable (x) and the operations +, -, x, + is called algebraic function e.g.,

$4x^2 - 3x^{1/2} + 7$, $5x^2 + 13x + 8$, $\frac{x^2+1}{x^2-2}$ etc.

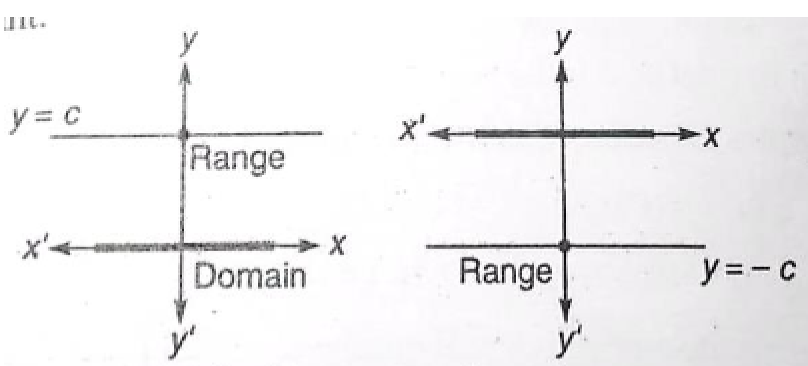
(A-1) polynomial functions: A function f(x) of the following form is known as polynomial function.

f(x) = $a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$

Where n ∈ N and $a_1, a_2, a_3 \ldots . a_n \in$ R

Domains: It is the set of real numbers
Range: It is the set of real numbers

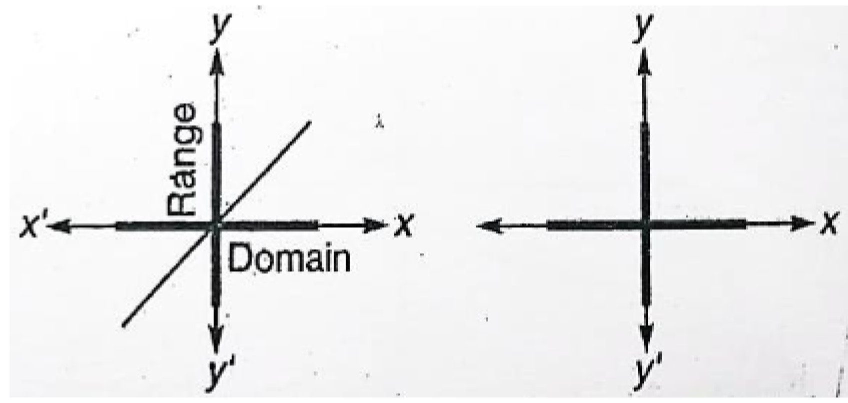
Constant function: f(x) = c, c ∈ R, c is constant.



Domain: it is the set of real numbers
Range: it is a particular real number {c}

Identity function

y = x i.e., f (x) = x



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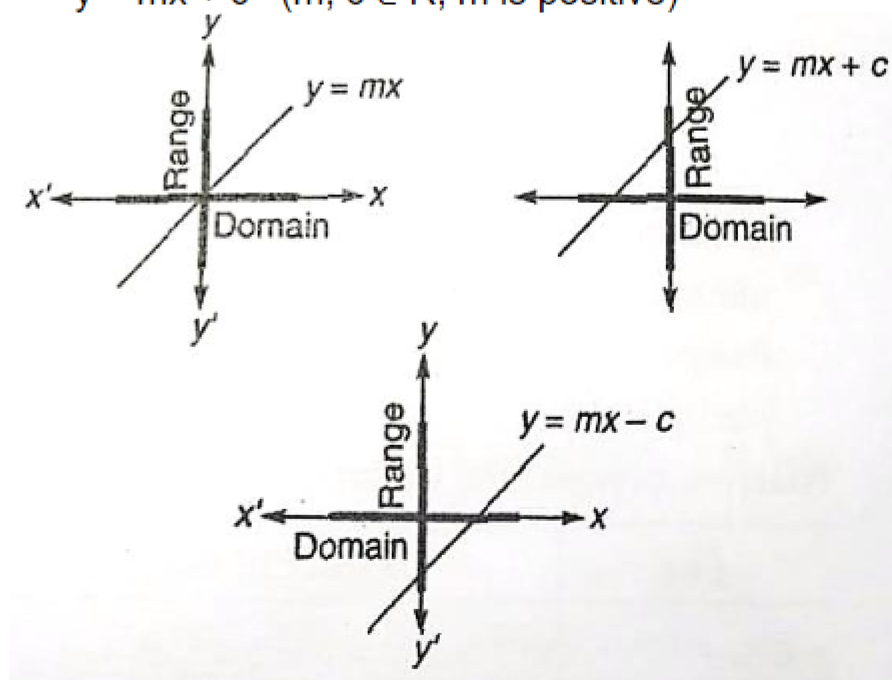
Domain → It is the set of real numbers.
Range → It is also the set of real numbers.

Linear function

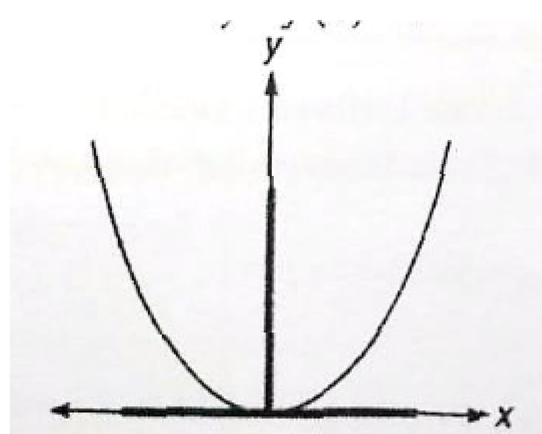
y = mx + c (m, c ∈ R, m is positive)



Domain → It is the set of real number.
Range → It is also the set of real number.

Quadratic function

y = f (x) = x^2

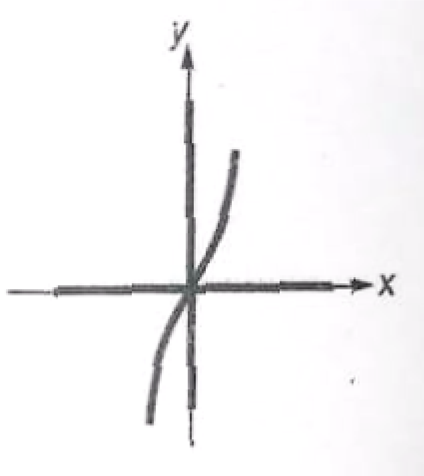


∴ $f(x) = f(-x)$

∴ Graph of f(x) is symmetric about y axis.
Domain → It is the set of real values.
Range → $R^+ \cup \{0\}$ i.e., non-negative real numbers.

Cubic equation

y = f(x) = x^3

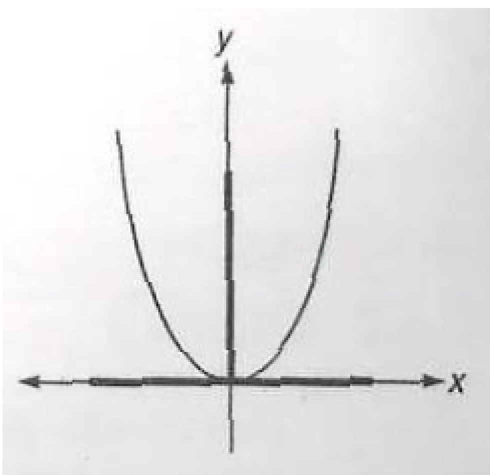


∴ Graph of f(x) is symmetric about origin
Domain → R (Set of real numbers)
Range → R (Set of real numbers)

Biquadratic function

$$y = f(x) = x^4$$

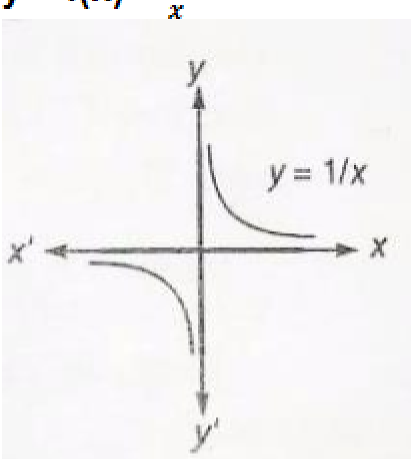
∴ f(x) = f(-x)



∴ f (x) = f (-x) is symmetric about y axis.
Domain → R (Set of real numbers)
Range → $R^+ \cup \{0\}$ i.e, set of non negative real numbers.

Graph and some Rational Function

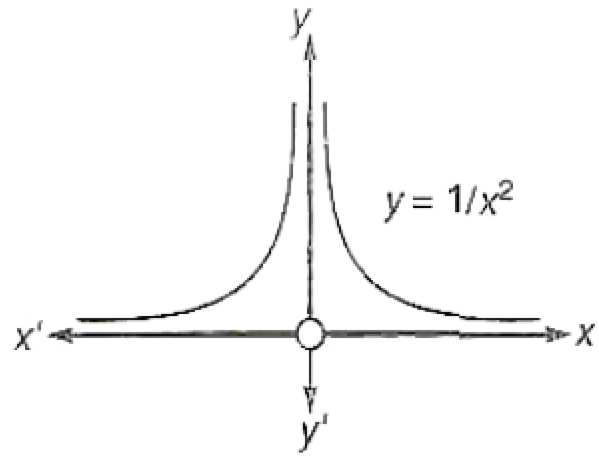
1. **y = f(x) = $\frac{1}{x}$**



y(x) = - f (-x)
Domain → R – {0}; Set of none zero rational numbers
Range → R – {0}; set of non-zero rational numbers

2. **y = f (x) = $\frac{1}{x^2}$**

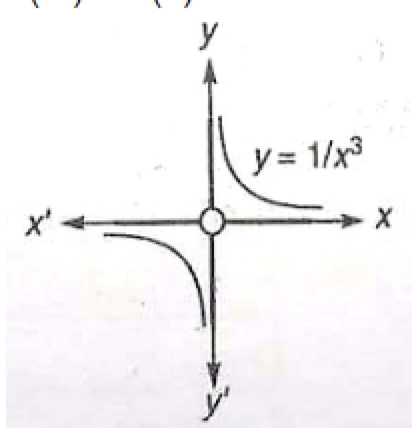
⇨ f (x) = f (-x)



Domain → R – {0} i.e., set of non-zero rational numbers
Range → R^+i.e., set of positive rational numbers

2. **y = f(x) = $\frac{1}{x^3}$**

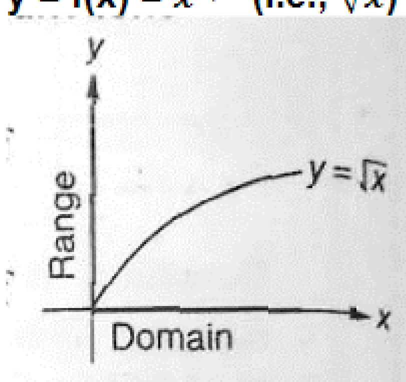
f (-x) = -f(x)



Domain → R – {0} i.e., set of non-zero rational numbers
Range → R – {0} i.e., set of non-zero rational number.

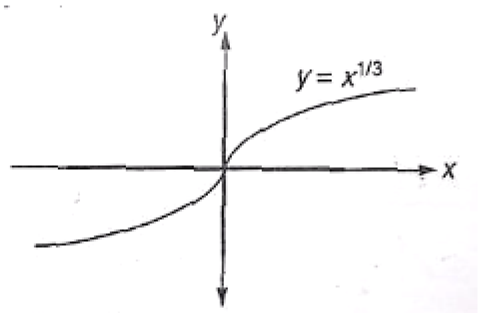
Graphs of some simple irrational functions

1. **y = f(x) = $x^{1/2}$ (i.e., $\sqrt{x}$)**



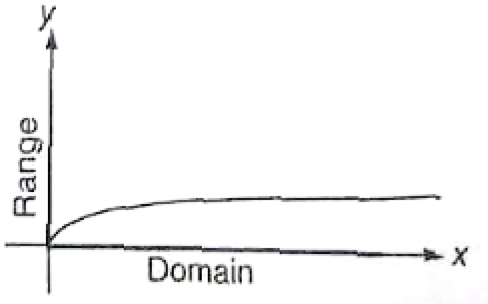
Domain → $R^+ \cup \{0\}$ i.e, set of non-negative real numbers.
Range → $R^+ \cup \{0\}$ i.e., set of non-negative real numbers

2. **y = f(x) = $x^{1/3}$**



Domain → (i.e., set of real numbers)
Range → R (i.e., set of real numbers)

3. **y = f (x) = $x^{1/4}$**



Domain → $R^+ \cup \{0\}$ i.e, set of non-negative real numbers
Range → $R^+ \cup \{0\}$ i.e., set of non-negative real numbers

Functions

Even Function

Let a function $y = f(x)$ be given in a certain interval. The function is said to be even if for any value of x.

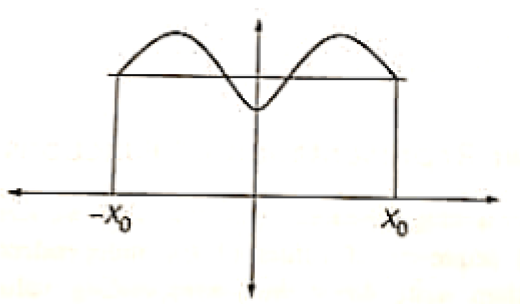
→ f(x) = f(-x)

(a) The sum difference, product and quotient of an even function is also an even function.
(b) The graph of an even function is symmetrical about the y-axis.

However, when ,y is the independent variable, it is symmetrical about the y axis. In other words. if x = (f(y) is an even function, then the graph of this function will be symmetrical about the.x-axis. Example: x = y^2.

Examples of even functions: y = $x^2, y = x^4, y = -3x^8$, y = x^2 + 3, y = $\frac{x^4}{5}, y = |x|$ are all even functions.

The symmetry about the y-axis of an even function is illustrated below.



Odd Functions

Let a function y = f(x) be given in a certain interval. The function is said to be odd if for any value of x

f(x) = -f(-x)

Properties of odd functions.
(a) The sum and difference of an odd function is an odd function.
(b) The product and quotient of an odd function is an even function.
(c) The graph of an odd function is symmetrical abo the origin.

Inverse of a Function

Let there be a function y = f(x), which is defined for the domain D and has a range R.

Then, by definition, for every value of the independent variable x in the domain D, there exists a certain value of the dependent variable v. In certain cases the same value of the dependent variable y can be got for different values of x. For example, if y = x^2, then for x = 2 and x = -2 give the value of y as 4.

In such a case, the inverse function of the function y = f(x) *does not exist.*

However, if a function y = f(x) is such that for every value of y (from the range of the function R) (here corresponds one and only one value of x from the domain D, then the inverse function of y = f(x) exists and is given by x = g(y). Here it can be noticed that x becomes the dependent variable and y becomes the independent variable. Hence, this function has a domain R and a range D.

Under the above situation, the graph of y = f(x) and x = g(y) are one and the same.

However, when denoting the inverse of the function, we normally denote the independent variable by y and, hence, the inverse function of y =f(x) is denoted by y = g(x) and not by x = g(y)

The graphs of two inverse functions when this change is used are symmetrical about the line y = x (which is the bisector of the first and the third quadrants).

Shifting of Graphs

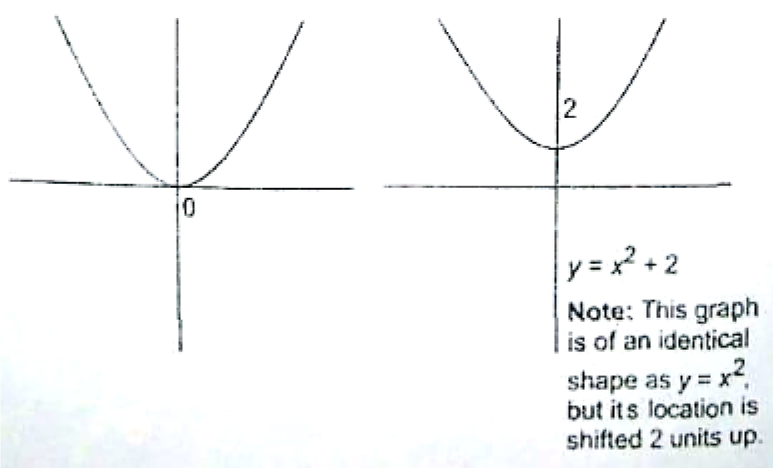
The ability to visualize how graphs shift when the basic analytical expression is changed is a very important skill. For instance if you knew how to visualize the graph of $(x + 2)^2$ - 5. It will definitely add a lot of value to your ability to solve questions of functions and all related chapters of block 5 graphically.

In order to be able to do so, you first need to understand the following points clearly:

(1) The relationship between **the graph of y = f(x) and y = f(x) +** c (where c represents a positive constant): The shape of the graph of y =f(x) + c will be the same as that of the y = f(x) graph. The only difference would be in terms of the fact that fix) + c is shifted c units up on the x - y plot.

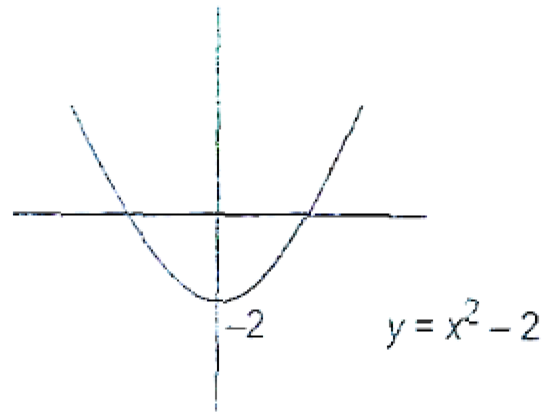
The following figure will make it clear for you:

Example: Relationship between y = x^2 and y = f(x) - c;



(2) The relation-ship between **y =f(x) and y = f(x) – c;**

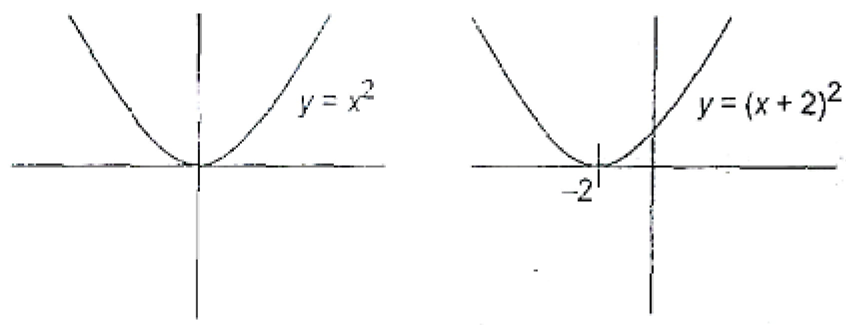
In tins case while the shape remains the same, the position of the graph gets shifted c units down.



(3) The relationship between **y = f(x) and y = f(x+c);**

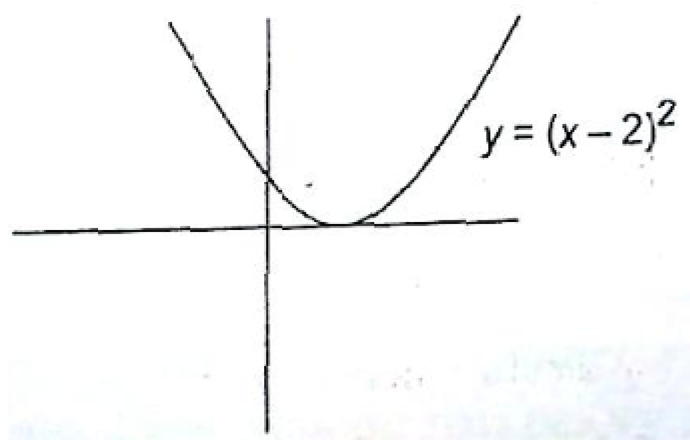
In this case the graph will get shifted c units to the left. (Remember, c was a positive constant)

Example:



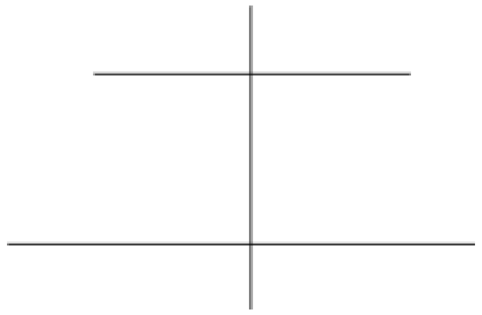
(4) The relationship between **y =f(x) and y = f(x-c);**

In this case the graph will get shifted c units to the right on the x - y plane.

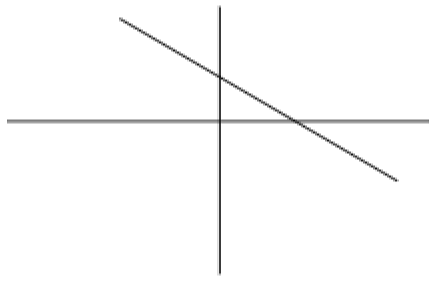


Exercise 01
Graphs

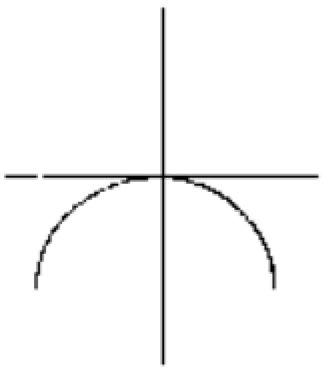
1. Mark a if $f(-x) = f(x)$
   Mark b if $f(-x) = -f(x)$
   Mark c if neither a nor b is true
   Mark d if $f(x)$ does not exist at at least one point of the domain.



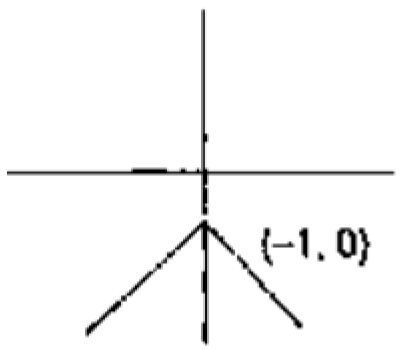
2. Mark a if $f(-x) = f(x)$
   Mark b if $f(-x) = -f(x)$
   Mark c if neither a nor b is true
   Mark d if $f(x)$ does not exist at at least one point of the domain.



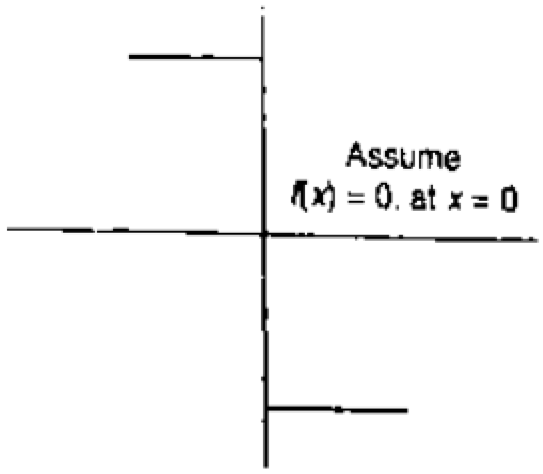
3. Mark a if $f(-x) = f(x)$
   Mark b if $f(-x) = -f(x)$
   Mark c if neither a nor b is true
   Mark d if $f(x)$ does not exist at at least one point of the domain.



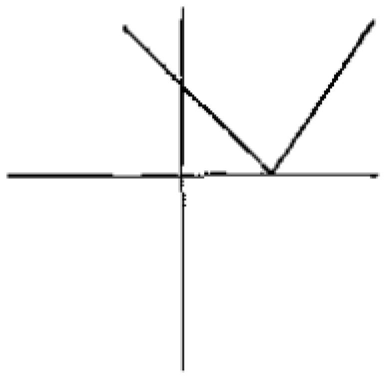
4. Mark a if $f(-x) = f(x)$
   Mark b if $f(-x) = -f(x)$
   Mark c if neither a nor b is true
   Mark d if $f(x)$ does not exist at at least one point of the domain.



5. Mark a if f(-x) = f(x)
Mark b if f(-x) = -f(x)
Mark c if neither a nor b is true
Mark d if f(x) does not exist at at least one point of the domain.

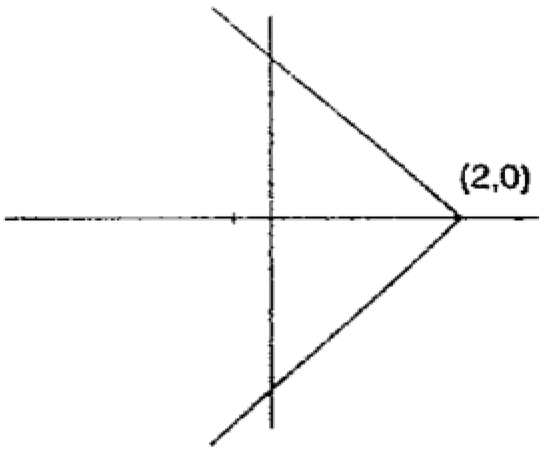


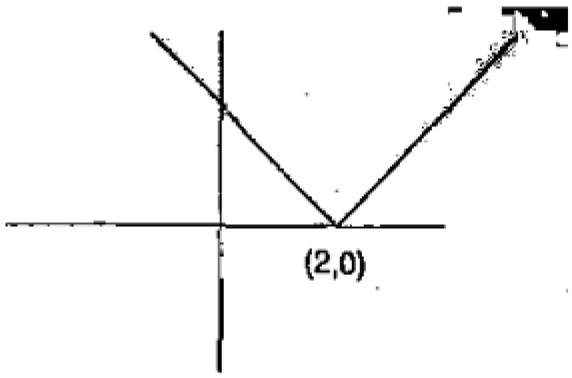
6. Mark a if f(-x) = f(x)
Mark b if f(-x) = -f(x)
Mark c if neither a nor b is true
Mark d if f(x) does not exist at at least one point of the domain.

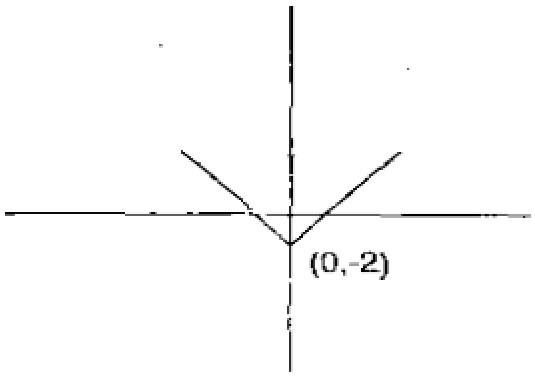


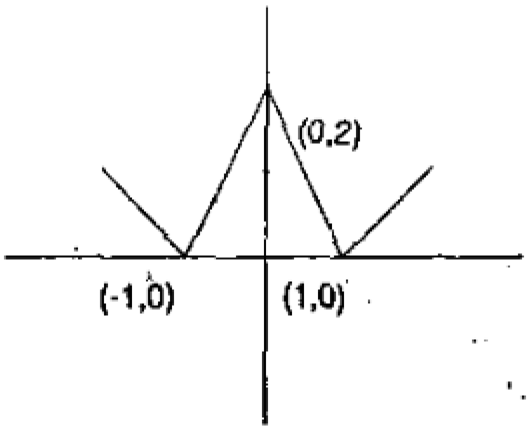
7. Which of the following curves correctly represents
y = |x-2|

(a)

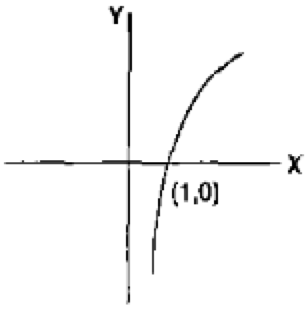


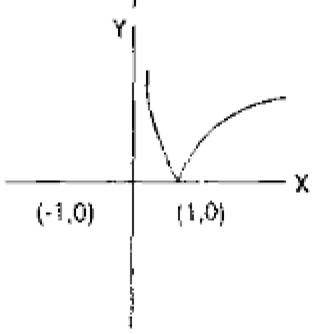
(b) 

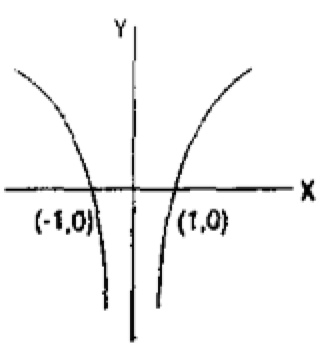
(c) 

(d) 

8. Which of the following represents the curve of y = log |x|?

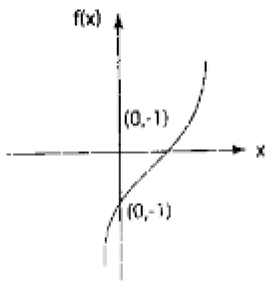
(a) 

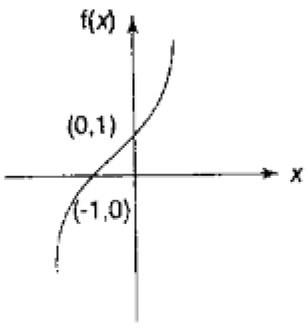
(b) 

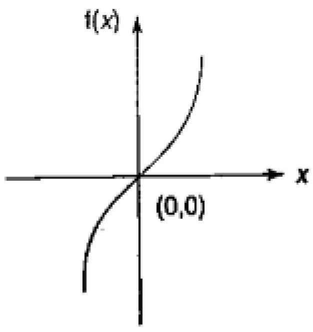
(c) 

(d) None of these

9. Which of the following options correctly represents the curve of f(x) = $(x-1)^3$

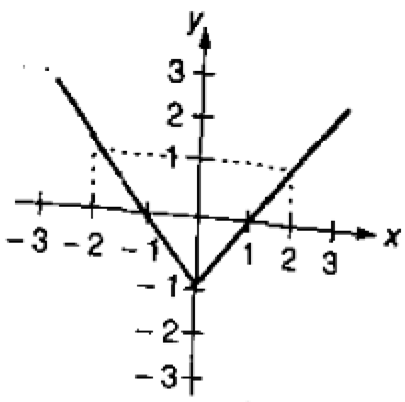
(a) 

(b) 

(c) 

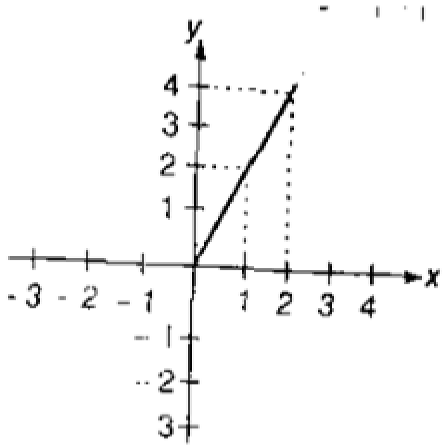
(d) None of these

10. In the question a graph of a function is being shown select the correct equation of the function of the graph.



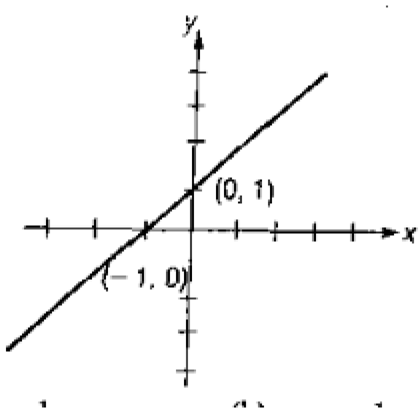
(a) $y = |x-1|$ (b) $x = |y| + 1$
(c) $y = |x| - 1$ (d) $y = |x| - 2$

11. In the question a graph of a function is being shown select the correct equation of the function of the graph.



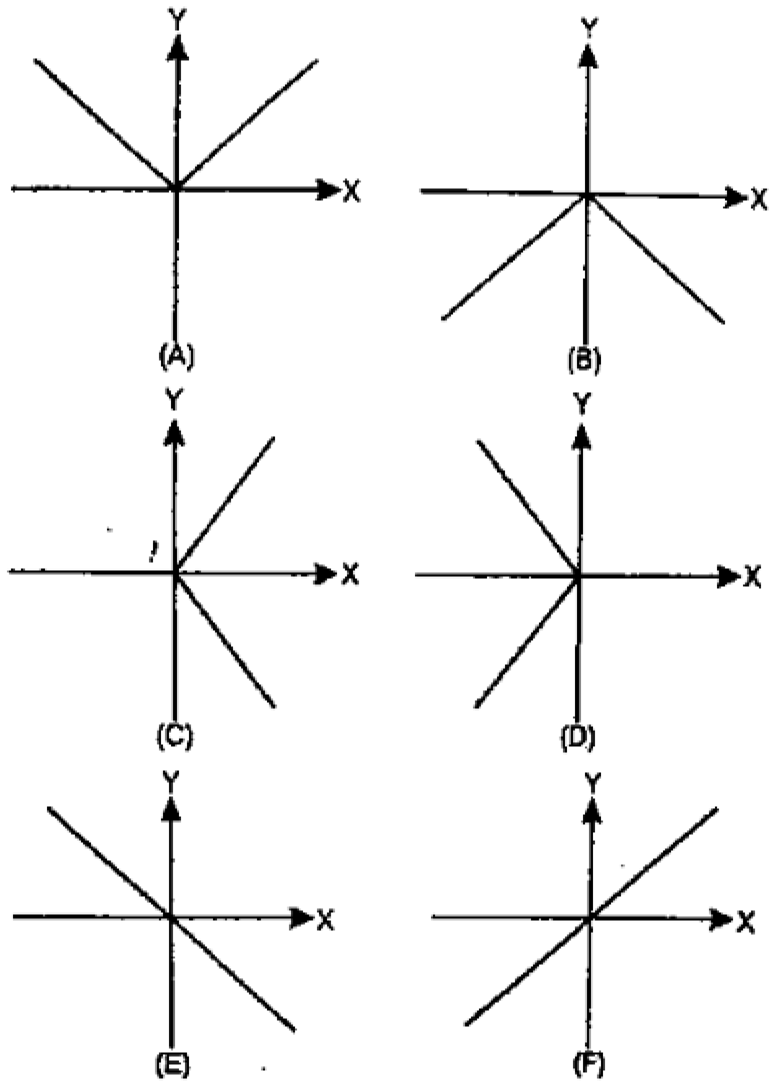
(a) $x = |y| + y$ (b) $y = |x| + x$
(c) $y = |x|$ (d) $x = |y|$

12. In the question a graph of a function is being shown select the correct equation of the function of the graph.



(a) x = y +1
(b) y = x -1
(c) x = y -1
(d) none of these

Directions(13 - 16): Followings graphs represent various functions. Match the figure with the appropriate function.



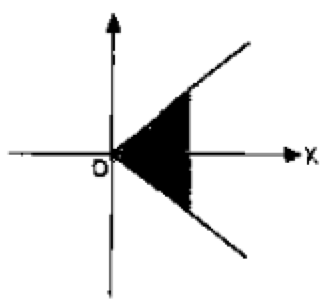
13. f(x) = -x
(a) B
(b) C
(c) E
(d) D

14. f (x) = |x|
(a) A
(b) B
(c) C
(d) D

15. f(x) = -|x|
(a) E
(b) B
(c) A
(d) F

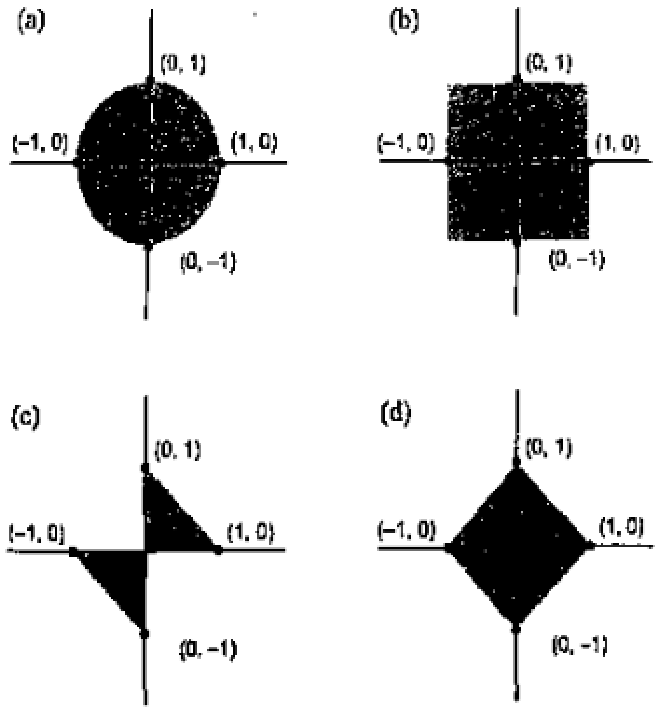
16. f(x) = x
(a) E
(b) F
(c) A
(d) B

17. The shaded region in the diagram represents the relation:



(a) $y \le x$ (b) $|y| < |x|$ (c) $y \le |x|$ (d) $|y| \le x$

18. The set $\{(x, y): |x| + |y| \le 1\}$ is represented by shaded region in one of the four figures. Which one is it?



Exercise 01 Solutions

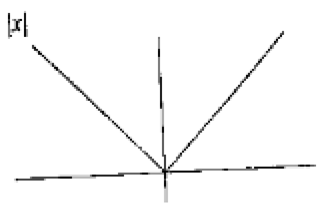
1. Ans. (a)
Solution: You essentially have to mark (a) if it is an even function, mark (b), if it is an odd function, mark (c) if the function is neither even nor odd.
Also, option (d) would occur if the function does not exist atleast one point of the domain. This means one of two things.
Either the function is returning two values for one value of x.

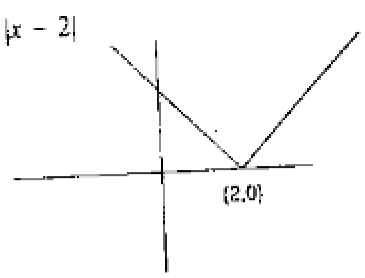
2. Ans. (c)
Solution: You essentially have to mark (a) if it is an even function, mark (b), if it is an odd function, mark (c) if the function is neither even nor odd.
Also, option (d) would occur if the function does not exist atleast one point of the domain. This means one of two things.
Either the function is returning two values for one value of x. (as in question 26, 30, 37 and 38) or the function has a break in between (not seen in any of these questions).

3. Ans. (a)
Solution: You essentially have to mark (a) if it is an even function, mark (b), if it is an odd function, mark (c) if the function is neither even nor odd.
Also, option (d) would occur if the function does not exist atleast one point of the domain. This means one of two things.
Either the function is returning two values for one value of x. (as in question 26, 30, 37 and 38) or the function has a break in between (not seen in any of these questions).

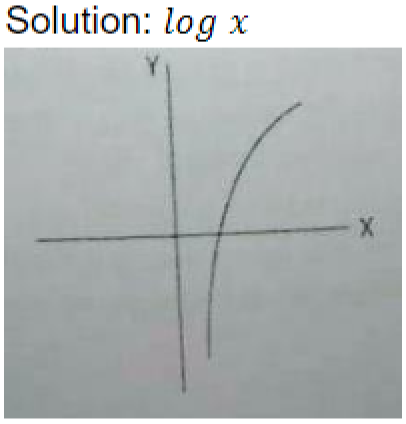
4. Ans. (a)
Solution: You essentially have to mark (a) if it is an even function, mark (b), if it is an odd function, mark (c) if the function is neither even nor odd.
Also, option (d) would occur if the function does not exist atleast one point of the domain. This means one of two things.
Either the function is returning two values for one value of x. (as in question 26, 30, 37 and 38) or the function has a break in between (not seen in any of these questions).

5. Ans. (b)
   Solution: You essentially have to mark (a) if it is an even function, mark (b), if it is an odd function, mark (c) if the function is neither even nor odd.
   Also, option (d) would occur if the function does not exist atleast one point of the domain. This means one of two things.
   Either the function is returning two values for one value of x. (as in question 26, 30, 37 and 38) or the function has a break in between (not seen in any of these questions).
6. Ans. (c)
   Solution: You essentially have to mark (a) if it is an even function, mark (b), if it is an odd function, mark (c) if the function is neither even nor odd.
   Also, option (d) would occur if the function does not exist atleast one point of the domain. This means one of two things.
   Either the function is returning two values for one value of x. (as in question 26, 30, 37 and 38) or the function has a break in between (not seen in any of these questions).
   We see even functions in 25, 31, 32, 33 and 34 [symmetry about the y axis].
7. Ans. (b)
   Solution:

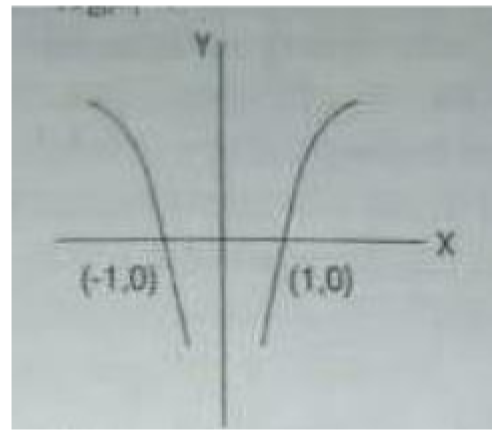




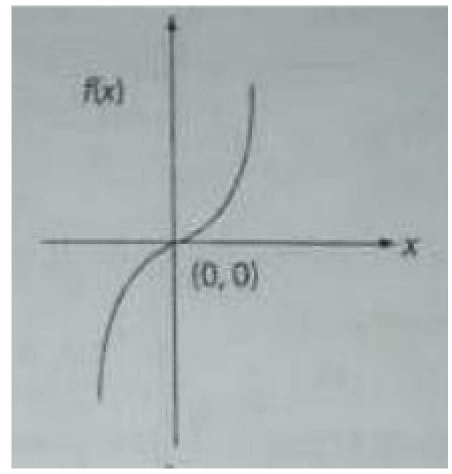
8. Ans. (c)
   Solution: $log\ x$



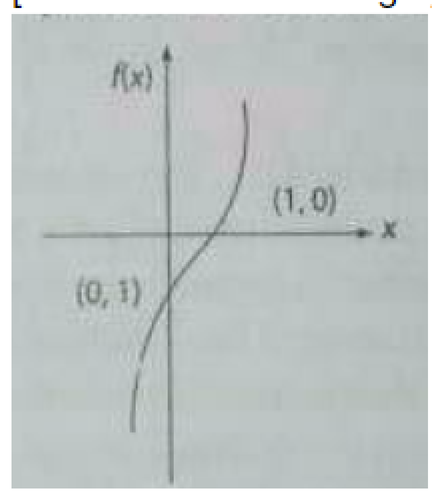
f (x) → f(|x|)
Take mirror image about y-axis
Log |x| →



9. Ans. (a)
   Solution: x^3 →



$(x-1)^3$ →
[Shift curve one unit right]



10. Ans. (c)

Solution. at x = 1	y = 0
x = -1	y = 0
x = 0	y = -1
x = 2	y = 1
x = -2	y = 1

11. Ans. (b)

Solution.	y = |x| + x
At x = 0	y = 0
x = 1	y = 2
x = 2	y = 4
x = -1,	y = 0
x = -2	y = 0

12. Ans. (c)
    Solution.
    x = y – 1
    ⇨ y = x + 1
    ⇨ x = 0, y = 1

⇨ x = -1, y = 0
⇨ x = 1, y = 2
⇨ x = -2, y= 1

13. Ans. (c)
Solution: y = f(x) = -x
Here if x = 1, then y = -1
If x = 0, then y = 0
If x = -1 then y = 1
Put these coordinates on graph, which is graph (E).
So, the answer is option (c).

14. Ans. (a)
Solution: y = f(x) = (x)
Here if x = -1, then y = 1
If x = 0, then y = 1
If x = 1 then y = 1
These are the coordinates of graph (A)
So, the answer is option (a).

15. Ans. (b)
Solution: y = f(x) = -|x|
Here if x = 1, then y = -1
If x = 0, then y = 0
If x = -1 then y = -1
Therefore, these are the coordinates of graph (B).
So, the answer is option (b).

16. Ans. (b)
Solution: y = f(x) = x
Here if x = 1, then y = 1
If x = 0, then y = 0
If x = -1 then y = -1
Therefore, these are the coordinate of graph (F)
So, the answer is option (b).

17. Ans. (d)

18. Ans. (d)

Exercise 02

Domain of Function

1. Find the domain of the definition of the function y = |x|.
(a) 0 ≤ x (b) - ∞ < x < + ∞ (c) x < + ∞ (d) 0 ≤ x < + ∞

2. Find the domain of the definition of the function y = $|\sqrt{x}|$
(a) x ≥ 0 (b) - ∞ < x < + ∞ (c) x > 0 (d) x < + ∞

3. Find the domain of the definition of the function y = $(9-x^2)^{1/2}$.
(a) - 3 ≤ x ≤ 3 (b) (-∞, -3] ∪ [3, ∞)
(c) - 3 ≤ x (b) x ≤ 3

4. The domain of definition of y = $\left[log_{10}\left(\frac{5x-x^2}{4}\right)\right]^{1/2}$ is
(a) [1, 4] (b) [-4, -1] (c) [0, 5] (d) [-1, 5]

5. The domain of definition of the function
y = $\frac{1}{\{log_{10}(3-x)\}}+\sqrt{x+7}$
(a) (-7,3] – {2} (b) [-7,3]-{1}
(c) (-7,3) – {0} (d) (7,3)

6. Find the domain of the definition of the function y = $1/(x-|x|)^{1/2}$
(a) -∞ < x < ∞ (b) - ∞ < x < 0
(c) 0 < x < ∞ (d) no where

7. Find the domain of the definition of the function y = $(2x^2+x+1)^{-3/4}$
(a) x ≥ 0 (b) All x except x = 0
(c) - 3 ≤ x ≤ 3 (d) nowhere

8. Find the domain of the definition of the function y = $log_{10}[1 - log_{10}(x^2 - 5x + 16)]$.
(a) (2,3] (b) [2,3) (c) [2,3 (d) none of these

9. The domain of the function f(x) = $\frac{1}{\sqrt{x}}$ is:
(a) $-\infty < x < \infty$ (b) $0 < x < \infty$
(c) R – {0} (d) $0 \le x < \infty$

10. The domain of the function f(x) = ($\sqrt{x^2}$)is:
(a) $-\infty < x < \infty$ (b) O < x < ∞
(c) R – {0} (d) $R^+ \cup \{0\}$

11. The domain of the function f(x) = logx^2 *is*:
(a) R (b) R^+ (c) R – {0} (d) $R^+ \cup \{0\}$

12. The domain of y = $\frac{1}{\sqrt{|x|-x}}$ *is*
(a) [0, ∞) (b) (0, ∞) (c) (-∞, 0) (d) (-∞, 0)

13. The domain of y = $log_{10}(\sqrt{6-x} + \sqrt{x-4})$ is:
(a) R (b) R – {0} (c) [6, ∞) (d) (4, 6)

14. The domain of the function f (x) = $\sqrt{4x-3} + \sqrt{2x-6}$ is:
(a) [0, ∞] (b) $\left[\frac{3}{4}, \infty\right]$ (c) $\left[\frac{4}{3}, \infty\right]$ (d) [3, ∞]

15. The domain of the function f(x) = log (5x-6-x^2) *is*:
(a) (5,6) (b) 2,3 (c)2, ∞ (d) none of these

16. The domain of the function f(x) = $log_{10}log_{10}(1 + x^3)$ is:
(a) (-1, +∞) (b) (0, +∞) (c) [0, +∞) (d) (-1, 0)

17. The domain of the real valued function f(x) = $log_e|log_e x|$*is*:
(a) (1, +∞) (b) (0, +∞) (c) (e, +∞) (d) None of these

Exercise 02 Solutions

1. Ans. (b)
Solution: y = |x| will be defined for all values of x. from = - ∞ to + ∞

2. Ans. (a)
Solution: Since the function contains $\sqrt{x}$ in it, x ≥ 0 would be the domain.

3. Ans. (a)
Solution: (9-x^2) ≥0, -3 ≤ x ≤ 3.

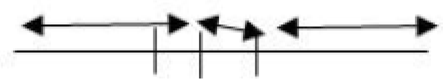
4. Ans. (a)
Solution: $\frac{5x-x^2}{4} \ge 1 \rightarrow 1 \le x \le 4$.

5. Ans. (a)
Solution: The function would be defined when the term $\frac{1}{\{log_{10}(3-x)\}}$ is real , which will occur when x < 3. However if x = 2, then the denominator of the term becomes 0. Which should not be allowed. The other limit of the function gets defined by the constraint defined by the term $\sqrt{x+7}$, for $\sqrt{x+7}$ to be real, x ≥ -7 is the requirement.
Hence, the required domain is;
Required domain = -7 ≤ x <3, x ≠ 2
i.e., x ∈ [-7,3) – {2}

6. Ans. (d)
Solution: x - |x| is either negative for x < 0 or 0 for x ≥ 0.
Thus, option (d) is correct.

7. Ans. (d)
8. Ans. (d)
9. Ans. (b)
Solution. $\sqrt{x}$ is defined only when x ≥ 0
But $\frac{1}{\sqrt{x}}$ is defined only when x > 0
Since rational expressions are not defined for zero denominator so, $\sqrt{x} \neq 0$
∴ Domain of $\frac{1}{\sqrt{x}}$ $is\ (0, \infty)$ i.e. R^+
10. Ans. (a)
Solution. Since x^2 is positive for all values of x
i.e., $x^2 \geq 0\ \forall\ x \in R$
∴ $\sqrt{x^2}$ is defined for all values of R
Hence domain of $\sqrt{x^2}$ is R.
11. Ans. (c)
Solution. x^2 is positive for all values of x except at x = 0
Since $log_a b$ is defined only if b> 0
∴ domain of $log_{10}x^2$is R – {0}
12. Ans. (c)
Solution. |x| - x > 0 (∴ |x| - x ≠ 0)
⇨ |x| > x also |x| - x > 0 for the condition of square root. Since |x| is positive value and |x|, x are numerically equal.
⇨ Therefore x < 0
Hence the domain of $f(x) = \frac{1}{\sqrt{|x|-x}}$ is (-∞, 0)
13. Ans. (d)
Solution. $(\sqrt{6-x} + \sqrt{x-4}) > 0$
Also (6-x) > 0 and (x-4) > 0
∴ 6 > x and x > 4
4 < x < 6
∴ domain of given function is (4,6).
14. Ans. (d)
Solution: For $\sqrt{4x-3}, 4x-3\ \geq 0$
[since we cannot find under root of negative values.]
Or 4x ≥ 3. So, x ≥ $\frac{3}{4}$
For $\sqrt{2x-6}, 2x-6 \geq 0$ → x ≥ 3
So, for both the equation, x should be greater than or equal to 3. The domain is [3, ∞). Hence, the domain is 3 ≤ x < ∞.
15. Ans. (b)
Solution: f (x) = log (5x – 6 - x^2)
Logarithmic function is defined only for positive values.
So, 5x – 6 x^2 > or > $x^2 - 5x + 6 < 0$

Root of this equation are 2 and 3.
Put them on a number line.



- ∞ 0 2 3 + ∞
Take a value from the internal 2 to 3 (say 2.5), it satisfies the inequality.
It can be seen that value of x lies in between 2 and 3. So, the domain is (2,3) =2 < x < 3.
16. Ans. (b)
Solution: f(x) = $log_{10}log_{10}(1+x^3)$
Here x, cannot be negative number and zero.
Because f(x) = $log_{10}log_{10}$(1+(0)^3) = $log_{10}log_{10}$(1)= log_{10} (0)
[since $log_{10}1$= 0]
But we cannot find log of zero.
So, domain is (0, +∞)
17. Ans. (d)
Solution: f(x) = $log_e|log_e x|$
For any logarithmic expression, value of x should be a positive number.
Hover, for this expression, we cannot take 1 as a value. Of x [Because I we take x = 1, then f(x) = $log_e|log_e 1|$ = $log_e|0|$ = $log_e 0$]

Exercise 03
Maxima Minima

1. For what value of x, x^2 + 10 x + 11 will be the minimum value?
(a) 5 (b) +10 (c) -5 (d) -10

2. Find the maximum value of the function 1/($x^2 - 3x + 2$).
(a) 11/4 (b) ¼ (c) 0 (d) none of these

3. Find the minimum value of the function f(x) = $log_2\,(x^2 - 2x + 5)$.
   (a) -4 (b) 2 (c) 4 (d) -2

4. If the function R(x) = max $(x^2 - 8, 3x, 8)$, then what is the max value of R(x)?
   (a) 4 (b) $\frac{1+\sqrt{5}}{2}$ (c) ∞ (d) 0

5. If $p^2 + q^2 + r^2$ = 1, then the maximum value of pqr is:
   (a) 1 (b) $\frac{1}{3\sqrt{3}}$ (c) 3 (d) None of these

6. If f(x) = $2x^2 + 3x + 4$ *and* g (x) = 5 - x^2, then the minimum [f(x)] – maximum [g(x)] is:
   (a) 0 (b) 5 (c) $\frac{17}{8}$ (d) $-\frac{17}{8}$

7. What is the minimum value of the expression $x^2 + 8x + 10$?
   (a) 0 (b) +∞ (c) -6 (d) +6

8. What is the maximum value of the expression 1 – 6x - x^2?
   (a) 0 (b) 10 (c) -∞ (d) None of these

9. What is the maximum value of f(x) = minimum (4-5x, x-3) for every x ∈ (0, 4)?
   (a) -1 (b) 2 (c) 4 (d) None of these

10. If x + y + z = 24, then the maximum value of xyz is:
    (a) 215 (b) 512 (c) 125 (d) 576

11. The minimum value of $4^x + 4^{1-x}, x \in R$, is:
    (a) 2 (b) 4 (c) 1 (d) none of these

12. What is the maximum value of y = |x-5| + |x-7|?
    (a) 2- (b) 2 (c) 0 (d) ∞

13. If y = |x| - 5, then the minimum possible value of y is:
    (a) 5 (b) -5 (c) 0 (d) both a and b

14. Let x, y and z be three positive numbers such that x + t1+ z = 1.
    Then minimum value of
    $(x+\frac{1}{x})^2 + (y+\frac{1}{y})^2 + (z+\frac{1}{z})^2$ will be:
    (a) 16 (b) 24 (c) 33 (d) None of these

15. If a, b and c are the positive real numbers, then find the greatest value of $a^2\,b^3c^4$ subject to the condition a + b +c = 18.
    (a) +∞ (b) $4^2.6^3.8^4$ (c) 16686 (d) None of these

16. Consider the following functions:
    f(x) = $x^2 + 3x$
    g(x) = 3x + 4
    For what value of x will f[g(x)] have its minimum value?
    (a) 0 (b) $\frac{-11}{2}$ (c) $\frac{-11}{6}$ (d) None of these

17. The minimum value of 3x + 4y, subject to the conditions x^2y^3 = 6 and x and y are positive, is:
(a) 10 (b) 14 (c) 13 (d) 13

18. If f(x) = $2x^2 + 3x + 4$ And g(x) = 5 - x^2, then the minimum [f(x)] – maximum [g(x)] is:
(a) 0 (b) 5 (c) $\frac{17}{8}$ (d) $\frac{17}{8}$

19. Given y = minimum (x-5, 7-x). what is the minimum value of y?
(a) 0 (b) -∝ (b) +∝ (d) 1

20. Given y = minimum (x-5, 7-x) what is the maximum value of y?
(a) 0 (b) -∝ (b) +∝ (d) 1

21. The circumference of a circle and the perimeters of a pentagon and an octagon are the same. Their area are denoted by c, P and O, respectively. Which of the following is true for their areas?
(a) C > P > O (b) O > P > C
(c) O > C > P (d) C > O > P

Exercise 03 Solutions

1. Ans. (c)
Solution: dy/dx = 2x + 10 = 0 ➔ x = -5

2. Ans. (d)
Solution: Since the denominator x^2 – 3x + 2 has real roots, the maximum value would be infinity.

3. Ans. (b)
Solution: The minimum value of the function would occur at the minimum value of $(x^2 - 2x + 5)$ as this quadratic
For t = $x^2 - 2x + 5$
dt/dx = 2x – 2 = 0 ➔ x = 1
⇨ $x^2 - 2x + 5$ = 4.
Thus, minimum value of the argument of the log is 4.
So, minimum value of the function is $log_2 4$ =2.

4. Ans. (c)
Solution: Since R(x) is the maximum amongst the three given function, its value would always be equal to the highest amongst the three, it is easy to imagine that $x^2 - 8$ and 3x are increasing functions, therefore the value of the function is continuously increasing as you increase the value of x, similarly $x^2 - 8$ would be increasing continuously as your go farther and farther down on the negative side of the axis. Hence, the maximum value of R(x) would be infinity. Option (c) is the correct answer.

5. Ans. (b)
Solution. p^2 x q^2 x r^2 is maximum when $p^2 = q^2$ = r^2= $\frac{1}{3}$

$(pqr)^2{}_{max}$ = $\frac{1}{27}$

= $(pqr)_{max}$ = $\frac{1}{3\sqrt{3}}$

6. Ans. (d)
Solution: f(x) = $2x^2 + 3x + 4$
Then, minimum of f(x) = - $\frac{D}{4a}$ = $\frac{-(b^2-4ac)}{4a}$
= $\frac{-(9-32)}{4 \text{ x } 2}$ = $\frac{23}{8}$ and g (x) = 5- x^2
= Then maximum of g(x) = $5 - (0)^2 = 5$ *since* x^2 is a positive number or zero.
The smallest value of x^2 should be zero
Then minimum f(x) – maximum g(x) = $\frac{23}{8} - 5$ = $\frac{23-40}{8} = -\frac{17}{8}$

7. Ans. (c)
Solution: Minimum value of the expression $x^2 + 8x + 10$ *is* $\frac{-d}{4a}$
= $\frac{-(b^2-4ac)}{4a}$ = $\frac{-[8^2-4 \text{ x } 1 \text{ x } 10]}{4 \text{ x } 1}$ = $\frac{-24}{4}$ = -6

8. Ans. (b)
Solution: For expression like -x^2 – 6x + 1,
Maximum value = $\frac{-D}{4a}$ = $\frac{(-6)^2(4 \text{ x} -1 \text{ x } 1)}{4 \text{ x} -1}$ = - $\frac{36+4}{-4}$
= 10

9. Ans. (d)
Solution: for maximum value of f(x) = minimum (4-5x, x-3), where x ∈ (0,4)
x should be 1.

So, maximum value of f(x) = minimum (4-5 x 1, 1-3) = minimum (-1, -2) = -2 which is the maximum value of f(x).

10. Ans. (b)
Solution: x + y + z = 24
If x, y and z all are equal , the then value of xyz will be maximum.
So, x = y = z = 8

Then, x y z = 8 x 8 x 8 = 512

11. Ans. (b)
Solution: We know that A ≥ GM
$\frac{4^x+4^{1-x}}{2} \geq [4^x \, x \, 4^{1-x}]^{1/2} = [4^{(x+1-x)}]^{1/2}$

or $\frac{4^x+4^{1-x}}{2} \geq [4^{(x+1-x)}]^{1/2}$

or $\frac{4^x+4^{1-x}}{2} \geq 2$, or $4^x + 4^{1-x} \geq 4$

Hence minimum value = 4

12. Ans. (d)
Solution: It is clearly seen that if we increase the value of x in the function, then the value of y will also increase. Thence, the maximum value of the function should be (∞)

13. Ans. (c)
Solution: y = |x| - 5, then, the minimum value of y will be -5 because the minimum value of |x| = 0.

14. Ans. (d)

15. Ans. (b)

16. Ans. (c)
Solution: f(x) = $x^2 + 3x$
g (x) = 3x + 4
Then, f[g(x)] = $(3x+4)^2 = 3(3x+4)$
= $9x^2$ + 24x + 16 + 9x + 12
= $9x^2$ + 33x + 28
Then, the value of x for which f[g(x)] has its minimum value = $\frac{b}{2a} = \frac{-33}{2\,x\,9} = \frac{-11}{6}$

17. Ans. (a)

18. Ans. (d)
Solution: Minimum [f(x)] = minimum $(2x^2 + 3x + 4)$ = 46/ 16
Maximum [g(x)] = 5

19. Ans. (b)
Solution: Drawing the graphs of x – 5 and 7 –x tell us that both the graphs are heading towards +∝ and -∝, depending upon the values. So, the minimum of y will be -∝.

20. Ans. (d)
Solution: Now, the direction of movement of both the graphs are opposite (one is going towards +∝ and other one is going towards -∝ for a particular set of values and vice versa)
For x < 6, x -5 will provide us the minimum value, and for x > 6, 7 –x will provide us with the minimum value, and at x = 1, we will get the values equal. This x = 6 is the point for which y_{max} = minimum (x-5, 7-x) is obtained.

21. Ans. (d)

Exercise 04
Function/Graph/Maxima/Minima

1. If $f(x) = \sqrt{x^3}$, then $f(3x)$ will be equal to
(a) $\sqrt{3x^3}$ (b) $3\sqrt{x^3}$ (c) $3\sqrt{(3x^3)}$ (d) $3\sqrt{x^5}$

2. If f(x) is an even function, then the graph y = f (x) will be symmetrical about
(a) x-axis (b) y-axis (c) both the axes (d) none of these

3. If f(x) is an odd function, the then graph y = f (x) will be symmetrical about
(a) x-axis (b) y-axis (c) both the axes (d) origin

4. Which of the following is an even function?
(a) x^{-8} (b) x^3 (c) x^{-33} (d) x^7

5. $f(x) = \frac{1}{x}$, g(x) = |3x-2|
Then f(g(x)) = ?
(a) $\frac{1}{(|3x-2|}$ (b) $|\frac{1}{3x} - 2|$ (c) $\frac{1}{|3x|} - 2$ (d) none of these

6. Which of the following statements is true:
(a) If f(x) and g(x) are odd functions then their sum is an even function.
(b) If f(x) and g(x) are even functions then their sum is an odd function.
(c) If (x) and g(x) are odd functions then their product is an even function
(d) none of these

7. If u (t) = 4t – 5, v(t) = t^2 and f(t) = 1/t, then the formula for u (f(v(t)) is
(a) $\frac{1}{(4t-5)^2}$ (b) $\frac{4}{(t-5)}$ (c) $\frac{4}{t^2} - 5$ (d) none of these

8. If f(t) = $\sqrt{t}$, g(t) = $\frac{t}{4}$ and h (t) = 4t – 8, then the formula for g(f(h(t))) will be
(a) $\frac{\sqrt{t-2}}{4}$ (b) $2\sqrt{t} - 8$ (c) $\frac{\sqrt{(4t-8)}}{4}$ (d) $\frac{\sqrt{(t-8)}}{4}$

9. Which of the following is an even function?
(a) e^x (b) e^{-x} (c) $e^x + e^{-x}$ (d) $\frac{e^x + e^{-x}}{e^x - e^{-x}}$

10. h[f(2,3,1), g(3,4,2), h(1/3,1/3,3)]
f(x,y,z) = xy + yz + zx
g(x,y,z) = $x^2y + y^2z + z^2x$ and
h (x, y, z) = 3 xyz
(a) 0 (b) 23760 (c) 2640 (d) none of these

11. g[f(1,0,0), g(0,1,0), h(1,1,1)]
f(x,y,z) = xy + yz + zx
g(x,y,z) = $x^2y + y^2z + z^2x$ and
h (x, y, z) = 3 xyz
(a) 0 (b) 9 (c) 12 (d) none of these

12. f[f(1,1,1), g(1,1,1), h(1,1,1)]
f(x,y,z) = xy + yz + zx
g(x,y,z) = $x^2y + y^2z + z^2x$ and
h (x, y, z) = 3 xyz
(a) 9 (b) 18 (c) 27 (d) none of these

13. f(1,2,3) – g(1,2,3) + h(1,2,3)
f(x,y,z) = xy + yz + zx
g(x,y,z) = $x^2y + y^2z + z^2x$ and
h (x, y, z) = 3 xyz
(a) -6 (b) 6 (c) 12 (d) 8

14. If |x| denotes the greatest integer ≤ x, then
$\left[\frac{1}{3}\right] + \left[\frac{1}{3} + \frac{1}{99}\right] + \left[\frac{1}{3} + \frac{2}{99}\right] + \ldots \left[\frac{1}{3} + \frac{98}{99}\right] = ?$

15. What is the maximum possible value of (21 sin X + 72 cos X)?

(a) 21 (b) 57 (c) 63 (d) 75

16. If f(t) = (t-1)/(t+1), then f(f(t))) will be equal to
(a) 1/t (b) -1/t (c) t (d) -t

17. If f(x) = e^x and $g(x) = log_e x$ then value of fog will be
(a) x (b) 0 (c) 1 (d) e

18. If 3f (x+2) + 4$f\left(\frac{1}{x+2}\right)$ = 4x, x ≠ -2, then f(4) =
(a) 7 (b) -52/7 (c) 8 (d) none of these

19. if $f\ (x) = \frac{x}{\sqrt{1+x^2}}$, then f(f(f(x))) is :
(a) $\frac{3x}{\sqrt{1+x^2}}$ (b) $\frac{x}{\sqrt{1+3x^2}}$ (c) $\frac{x^3}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{3+3x^2}}$

20. If f(x) = $\frac{4^x}{4^x+2}$ then the value of f(x) + f(1-x) is :
(a) 0 (b) -1 (c) 1 (d) can't be determined

21. Let f(x) = $\frac{x}{x+3}$ then f(x+1) = ?
(a) $\frac{3x+2}{x+2}$ (b) $\frac{x+1}{x+4}$ (c) $\frac{x+1}{x+3}$ (d) $\frac{2x+3}{x+3}$

22. If f(x) = $x^3 - x^2, then\ f(x+1)$ is :
(a) $x^3 - x^2 + 1$
(b) x $(x+1)^2$
(c) $x^2\ (x-1)$
(d) none of these

23. Let a function is defined as f(x) = $\frac{a^x + a^x}{2}$; a > 0, what is the value of f(x+y) + f(x-y) ?
(a) f (x) + f(y)
(b) f (x) f(y)
(c) 2 f(x) f(y)
(d) 4f(x) f(y)

24. If $\left|\frac{30-x}{5}\right|$ < 4, then find x:
(a) - 6 < x < 6
(b) 10 < x < 50
(c) x > 10
(d) None of these

25. If f(x) = $\frac{x+1}{x-1}$, x ≠ 1 then find f(f(f(f(2)))).
(a) 1 (b) 2 (c) 3 (d) 4

26. Which of the following function is an even function?
(a) f(x) = log$\frac{1-x}{1+x}$
(b) f(x) = x$\frac{a^x+1}{a^{x-1}}$
(c) f(x) = log$\left[x + \sqrt{1+x^2}\right]$
(d) f(x) = $\sqrt{(1+x+x^2)} - \sqrt{(1-x+x^2)}$

27. If f(x) = $x^2 + 2, then\ f^{-1}$(x) is:
(a) x^5 (b) $\sqrt{x+2}$ (c) $\sqrt{x} + 2$ (d) none of these

28. If f(x) $\frac{x-1}{x+1}$ then f^{-1}(x) is:

(a) $\frac{x+1}{1-x}$ (b) $\frac{x+1}{1-x}$ (c) $\frac{1}{x+1}$ (d) none of these

29. Given that f(x) = x^2+4x+4 and g(x) = $x^2+4x+3, then\ find\ the\ values\ of\ x\ such\ that\ f(g(x)$ = g(f(x)).
(a) x = -1 (b) x = -2 (c) x = -3 (d) none of these

30. For any y = f(x),
If f(-x) = f(x), then this function is known as an even function.
If f(-x) = -f(x), then this function is known as an odd function.
The sum of an odd function and an even function is always
(a) an odd function. (b) an even function.
(c) neither odd nor even function. (d) nothing can be said

31. If f(x) = $2^x - 2^{-x}$, then the value of 2 f(x) – 5 f(x-1) + 2f(x-2) is:
(a) 1 (b) -3 (c) 15 (d) none of these

32. If y = f(x) = (x+2)/(x-1), then which of the following is equal to f(y) ?
(a) x (b)2x (c) x/2 (d) x^2

33. The set of real values of x satisfying |x-1|≤ 3 and |x-1| ≥ 1 is:
(a) [2,4] (b) (-∞,2) ∪ [4, +∞]
(c) [-2, 0] ∪ [2, 4] (d) none of these

34. Let f(n) = $\left[\frac{1}{2}+\frac{n}{100}\right]$, where [x] denotes the integral part of x. then, the values of $\sum_{n=1}^{100} f(n)\ is$:
(a) 50 (b) 51 (c) 1 (d) none of these

35. The domain of the function f(x) = log_e(x-[x]), where [.] denotes the greatest integer function, is:
(a) R (b) R – Z (c) (0, +∞) (d) None of these

36. Let f(x) = |x-2| + |x-3| + |x-4| and g(x) = f(x+1). Then,
(a) g(x) is an even function
(b) g(x) is an odd function
(c) g(x) is neither even nor odd.
(d) None of these

37. The inverse function of the function f(x) = f(x) = $\frac{e^x-e^{-x}}{e^x+e^{-x}}$ is:
(a) $\frac{1}{2}\log\frac{1+x}{1-x}$ (b) $\frac{1}{2}\log\frac{2+x}{2-x}$
(c) $\frac{1}{2}\log\frac{1-x}{1+x}$ (d) None of these

38. If f(x) = log $\frac{(1+x)}{1-x}$ and g(x) $\frac{3x+x^3}{1+3x^2}$, then f[g(x)] is equal to :
(a) [f(x)]3 (b) -f(x) (c) $[f^3(x)]$ (d) 3[f(x)]

39. If f(x) = $\frac{4^x}{4^x+2}$, then find the value of f $\left(\frac{1}{1999}\right)$ + f $\left(\frac{2}{1999}\right)$ + …..+ f $\left(\frac{1998}{1999}\right)$
(a) 1998 (b) 1999 (c) 998 (d) 999

40. If f(x) = log $\frac{1+x}{1-x}$, then f(x) + f(y) is:

(a) f (x+y) (b) $f\left(\frac{x+y}{1+xy}\right)$ (c) $(x+y)f\left(\frac{1}{1+xy}\right)$ (d) None of these

41. If f(x) = maximum (4x+3, 3x + 6) for x ∈ [-6, 10], then find the maximum value of f(x)?
(a) +∞ (b) 36 (c) 43 (d) none of these

42. f(x) = |x| - x
(a) A (b) B (c) C (d) none of these

43. a, b and c are three positive numbers and abc^2 has the greatest value $\frac{1}{64}$ then;
(a) 1 = b = $\frac{1}{2}$, c = $\frac{1}{4}$ (b) a = b = $\frac{1}{4}$, c = $\frac{1}{2}$
(c) a = b = c = $\frac{1}{3}$ (d) None of these

44. If ab = 2a + 3b, a>0, b> 0, then the minimum value of ab is:
(a) 12 (b) 24 (c) $\frac{1}{4}$ (d) None of these

45. If a > 1 and b > 1, then find the minimum value of $log_b a + log_a b$.
(a) 0 (b) 1 (c) 2 (d) none of these

46. Given that $(x-2)^2 = 9$, and $(y-3)^2 = 25$. What is the minimum value of $\frac{x}{y}$?
(a) $\frac{1}{2}$ (b) $\frac{5}{8}$ (c) $-\frac{1}{8}$ (d) $-\frac{5}{2}$

47. |x+y| = 10. Where x and y are integers, then what is the minimum value of $x^2 + y^2$? .
(a) 75 (b) 100 (c) 50 (d) none of these

48. If |x| - |y| = 13, then which of the following cannot be the value of x – y ?
(a) -19 (b) -9 (c) -17 (d) None of these

49. What is the least value of (x-2)$(x-4)^2$ $(x-6)$ (x+6), for real values of x?
(a) 6 (b) $\frac{4}{3}$ (c) 4 (d) none of these

50. The minimum of f(x) = |x-1| + |x-4| + |x-5| is attained at x =
(a) 1 (b) 3 (c) 4 (d) 5

51. Let y = k^2 (x^2+1) + 2k (3x + 4) + 27. If k and x are real numbers, what is the minimum value of y?
(a) 1 (b) 3 (c) 4 (d) None of these

52. The least integral value of k for which (k-2) $x^2 + 8x + k + 4 > 0$ for all x ∈ R, is:
(a) -5 (b) 4 (c) 3 (d) None of these

53. If a + b+ c = 20 and $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ = 30, then the value of $\frac{a}{b}+\frac{b}{c}+\frac{a}{c}+\frac{c}{a}+\frac{b}{c}+\frac{c}{b}$ is:
(a) 597 (b) 8 (c) 350 (d) 441

54. A rectangle of largest area is inscribed in a semicircle of radius r. what is the area of the rectangle?
(a) 1.5 x r^2 (b) r^2 x √2 (c) r^2 (d) $r^2\sqrt{3}$

55. The altitude of a right circular cone of the minimum volume circumscribed about a sphere of the radius r is.
(a) 2r (b) 3r (c) 4r (d) None of these

Exercise 04 Solutions

1. Ans. (c)
Solution: f (x) = $\sqrt{x^3}$ ➔ f (3x) = $\sqrt{(3x)^3}$ ➔ $3\sqrt{3x^2}$
Option (c) is correct
2. Ans. (b)
Solution: y – axis by definition.
3. Ans. (d)
Solution: origin by definition.
4. Ans. (a)
Solution: x^{-8} *is even since* $f(x)$ = f(-x) in this case.
5. Ans. (a)
Solution: f(g(x)) = f(|3x-2|) = $\frac{1}{|3x-2|}$
Option (a) is correct.
6. Ans. (c)
Solution: When f(x) and g(x) both are odd then
S(x) = f(x) + g(x)
S(-x) = f(x) + g(-x) = - |f(x) + g(x)|, S (x) is an odd function, this conclusion reject option (a).
Their product P (x) = f(x) . g(x)
P (-x) = f(-x) . g(-x) = [-f(x)] [-g(x)] = f(x) g (x).
P(x)
Is an even function. This is what is being said by the option (c). hence it is the correct answer.
If we check for option (b) we can see that; when f(x) and g(x) both are even then S(x) = f(x) + g(x) S(-x) = f(-x) + g(-x) = [f(x) + g(x)], S(s) is an even function,
Hence only option (c) is true.
7. Ans. (c)
Solution: u (fv(t))) = u (f(t^2)) = u (1/t^2) = $\left(\frac{4}{t^2}\right)$ – 5.
8. Ans. (c)
Solution: g(f(h(t))) = g (f(4t-8))= g($\sqrt{4t\ -8}$)
= $\frac{(\sqrt{4t\ -8})}{4}$
9. Ans. (c)
Solution: $e^x + \ e^{-x} = e^{-x} + e^x$
10. Ans. (c)
Solution: The given function would become h [11, 8, 1] = 2640.
11. Ans. (a)
Solution: The given function would become g[0, 0, 3] =0
12. Ans. (c)
Solution: the given function would become f[3, 3, 3] = 27.
13. Ans. (b)
Solution: f(1, 2, 3) – g(1, 2, 3) + h(1, 2, 3) = 11 – 23 + 18 = 6.
14. Ans. 33
Solution: $\left[\frac{1}{3}\right] + \left[\frac{1}{3}+\frac{1}{99}\right] + \left[\frac{1}{3}+\frac{2}{99}\right] + \left[\frac{1}{3}+\frac{65}{99}\right] = 0$

$\left[\frac{1}{3}+\frac{66}{99}\right] + \left[\frac{1}{3}+\frac{67}{99}\right] + \ldots\left[\frac{1}{3}+\frac{98}{99}\right] = 33$

$\left[\frac{1}{3}\right] + \left[\frac{1}{3}+\frac{1}{99}\right] + \left[\frac{1}{3}+\frac{2}{99}\right] + \cdots \left[\frac{1}{3}+\frac{98}{99}\right]$ = 0 + 33 = 33
15. Ans. (d)
Solution: Let f(X) = 21 sin X + 72 cos X

⇨ f (X) = 21 con X – 72 sin X
⇨ If f(X) = 0, 21 cos X = 72 sin X.

∴ tan X = 21/72 therefore sin X = 21/75, cos X = 72/75 (since, from the value of tan X we can think of a right angled triangle with the legs as 21 and 72 respectively. This would give us the hypotenuse length of the triangle as 75 – using the Pythagoras theorem)

Since f(x) = -21 sin X < 0 therefore f(X) has a maximum at f(X) = 0. Thus , we can use the values of sin X = 21/75 & cos X = 72/75.

∴ Maximum value of
f(x) = $\frac{21.21}{75} + \frac{72.72}{75} = \frac{75^2}{75}$ = 75

Option (d) is correct.
16. Ans. (b)
Solution: f(f(t)) = f((t-1)/(t+1))

= $\left[\left(\frac{t-1}{t+1}\right) - 1\right] / \left[\left(\frac{t-1}{t+1}\right) + 1\right] \ = \frac{t-1-t-1}{t-1+t+1}$

= -2/2t = -1/t.
17. Ans. (a)
Solution: fog = f($log_e x$) = $e^{log_e x}$ = x.
18. Ans. (b)
Solution: 3 f(x+2) + 4f $\left(\frac{1}{x+2}\right)$ = 4x

Let x + 2 = t

$3f(t) + 4f\left(\frac{1}{t}\right) = 4t - 8$ or $\frac{8}{3}f(t) = + f\left(\frac{1}{t}\right)$ ....(1)

Now replacing t with $\frac{1}{t}$ in the above equation, we get

$3f\left(\frac{1}{t}\right) + 4f(t) = \frac{4}{t} - 8$ or $f\left(\frac{1}{t}\right) + \frac{4}{3}f(t)$

$= \frac{4}{3t} - \frac{8}{3}$ ...(2)

From (1) and (2)

$f(t) = \frac{12}{7}\left\{\frac{4}{3t} - \frac{8}{3} - t + 2\right\}$

$f(4) = \frac{12}{7}\left\{\frac{1}{3} - \frac{8}{3} - 4 + 2\right\} = \frac{-52}{7}$

19. Ans. (b)

Solution. $f(x) = \frac{x}{\sqrt{1+x^2}}$

$\therefore f(f(x)) = f(y) = \frac{y}{\sqrt{1+y^2}} = \frac{x/\sqrt{1+x^2}}{\sqrt{1+\left(\frac{x}{\sqrt{1+x^2}}\right)^2}}$

$= \frac{x}{\sqrt{1+2x^2}} = z\ (say)$

$\therefore f(f((f(x))) = f(z) = \frac{z}{\sqrt{1+z^2}} = \frac{x/\sqrt{1+2x^2}}{\sqrt{1+\left(\frac{x}{\sqrt{1+2x^2}}\right)^2}}$

$= \frac{x}{\sqrt{1+3x^2}}$

20. Ans. (c)

Solution. $f(x) = \frac{4^x}{4^x+2}$

$\therefore f(1-x) = \frac{4^{1-x}}{4^{1-x}+2} = \frac{2}{2+4^x}$

$\therefore f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{2}{4^x+2} = 1$

Hence (c) is correct.

21. Ans. (b)

Solution. $f(x+1) = \frac{(x+1)}{(x+1)+3} = \frac{x+1}{x+4}$

22. Ans. (b)

Solution. $f(x) = x^3 - x^2$

$f(x+1) = (x+1)^3 - (x+1)^2$

$= (x+1)^2(x+1-1) = (x+1)^2(x)$

23. Ans. (c)

Solution. $f(x) = \frac{a^x + a^{-x}}{2}$, $f(y) = \frac{a^t + a^{-y}}{2}$;

$= f(x+y) = \frac{a^{x+y} + a^{-(x+y)}}{2}$ .....(1)

And $f(x-y) = \frac{a^{x-y} + a^{-(x-y)}}{2}$ ..... (2)

Adding (1) and (2), we get

$f(x+y) + f(x-y) = \frac{a^{x+y} + a^{-x-y}}{2} + \frac{a^{x-y} + a^{-x+y}}{2}$

$= \frac{1}{2}[a^x(a^y + a^{-x}) + a^{-x}](a^y + a^{-y})]$

$= \frac{1}{2}(a^x + a^{-x})(a^y + a^{-y})$

$= 2f(x)f(y)$

24. Ans. (b)

Solution. $\left|\frac{30-x}{5}\right| < 4$

⇨ $|30-x| < 20$

⇨ $-20 < (30-x) < 20$

⇨ $-20 < 30 - x < 20$

⇨ $-50 < -x < -10$

⇨ $50 > x > 10$

⇨ $10 < x < 50$

25. Ans. (c)

Solution: $f(x) = \frac{x+1}{x-1}$

Then, $f(f(f(f(2)))) = f(f(f(f(\frac{2+1}{2-1}))))$ (put x = 2)

➔ $f(f(f(f(\frac{4}{2})))) = f(f(f(f(\frac{3+1}{3-1}))))$ (put x = 2)

➔ $f(f(f(\frac{4}{2}))) = f(f(f(2)))$

➔ f(f(3)) (since f(f(2))) = f(3). We have found it]

➔ f(f(3)) = f(2) [since f(f(3)) = f(2) we have found it]

➔ f (2) = 3

26. Ans. (a)

Solution: if f(-x) = f(x) then it is a even function, after verifying the options, we can see that option (b) is an even function.

Because $f(x) = x = \frac{e^{ax}-1}{e^{ax}+1}$

Then, $f(-x) = (-x) = \frac{e^{a(-x)}-1}{e^{a(-x)}+1} = -x\ \frac{e^{-ax}-1}{e^{-ax}+1}$

$= -x\frac{\frac{1}{e^{ax}-1}}{\frac{1}{e^{ax}+1}} = -x\frac{(1-e^{ax})/e^{ax}}{1+e^{ax})/e^{ax}}$

$= -x\frac{(1-e^{ax})}{(1+e^{ax})}$

(Take "-" sign out of bracket)

$= -x\frac{-(e^{ax}-1)}{e^{ax}+1} = x\frac{e^{ax}-1}{e^{ax}+1} = f(x)$

27. Ans. (d)

Solution: $f(x) = x^2 + 2$

Let $y = x^2 + 2 - t - 2 = x^2$ ➔ $x = \sqrt{y-2}$

Then put y =x

$f^1(x) = \sqrt{x-2}$

This is not matching with the given conditions. So, the answer is none of these.

28. Ans. (a)

Solution: f(x) = y = $\frac{x-1}{x+1}$

y – 1 = $\frac{x-1}{x+1}$ = -1 [subtract 1 from both sides]

y – 1 = $\frac{x-1-x-1}{x+1}$ = $\frac{2}{x+1}$

x+1 = - $\frac{2}{y-1}$ = $\frac{2}{1-y}$

x = - $\frac{2}{1-y}$ – 1 = $\frac{2-1+y}{1-y}$ = $\frac{1+y}{1-y}$

So, f^1 (x)=$\frac{1+x}{1-x}$

29. Ans. (d)

Solution: f(x) = $x^2 + 4x + 4$ (i)

g (x) = $x^2 + 4x + 3$ (ii)

Then, f(g(x)) = $(x^2 + 4x + 3)^2 + 4(x^2 + 4x + 3) +$ 4 [put x = $x^2 + 4x + 3$ in equation (i)]

= $(x^4$ + 8 x^3 + 22 x^2 + 24 x + 9) + $(4x^2 + 16x +$ $12)$ + 4

= x^4 + 8 x^3 + $26x^2$ + 40x + 25 (iii)

g(f(x)) = g($x^2 + 4x + 4$)

= $(x^2 + 4x + 4)^2 + 4(x^2 + 4x + 4) + 3$

= $(x^4 + 8x^3 + 24\,x^2 + 32x + 16) + (4x^2 + 16x +$ $16)$ + 3

= x^4 + 8 x^3 + 26 x^2 + 48x + 35 (iv)

Then f(g(x)) = g(f(x))

x^4 + 8 x^3 + $26x^2$ + 40x + 25 = x^4 + 8 x^3 + 26 x^2 + 48x + 35

$2x^2 + 8x + 10 = 0$ → x^2 + 4x + 5 = 0

30. Ans. (c)

Solution: The answer is option (c) neither odd nor even. See the properties of odd and even function.

31. Ans. (d)

Solution: f(x) = $2^x - 2^{-x}$

Then, 2f(x) = - 5f(x-1)+ 2f (x-2)

= 2[$2^x - 2^{-x}$] – 5 [$2^{x-1} - 2^{1-x}$] + 2 [$2^{x-2} - 2^{2-x}$].

Hence the answer is option (d).

32. Ans. (a)

Solution: f(x) = $\frac{x+2}{x-1}$

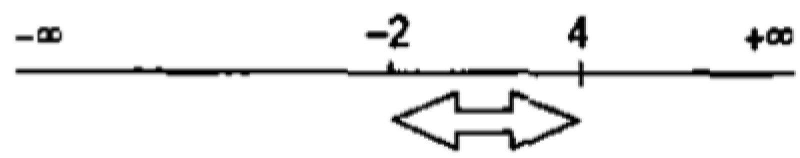
Then f(y) = f[f(x)] = f$\left[\frac{x+2}{x-1}\right]$

= $\left(\frac{x+2}{x-1} + 2\right) / \left(\frac{x+2}{x-1}\right) - 1$

= (x+2 + 2x - 2)/ (x + 2 –x + 1) = 3x /3 = x.

33. Ans. (c)

Solution: |x-1| ≤ 3 or -3 ≤ (x-1) ≤ 3 or (-3 + 1) ≤ (x-1+1) ≤ (3+1) or -2 ≤ x ≤4

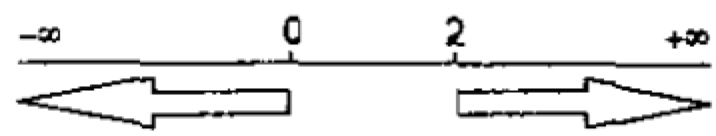


For |x-1| ≥ 1

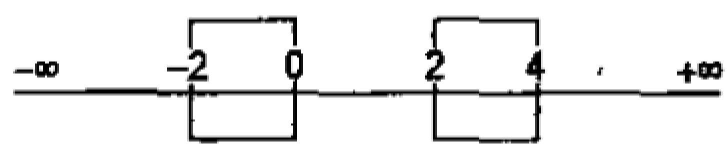
Solving this, we get

(x-1) ≤ -1 or (x-1) ≥ 1 or

x ≤ or x ≥ 2



Combine both, we get



So, common set is [-2, 0] U [2,4].

34. Ans. b

Solution: For all the values of n < 50. F(n) = 0 And all the n ≥ 50, f(n) = 1 Hence, 51 such values are there.

35. Ans. (b).

36. Ans. (c)

Solution: g(x) = f(x+1) = |x-2+1| + |x-3+1| + |x-4+1| = |x-1| + |x-2| + |x-3|

Obviously this is neither odd nor even. Alternatively we know the graph of this function will neither be symmetrical to axis or origin [see the topic graph and maxima minima]

37. Ans. (a)

Solution: f(x) =y = $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ = f(x) = $\frac{e^x - (1/e^x)}{e^x + (1/e^x)}$

= y = $\frac{(e^{2x}-1)/e^x}{(e^{2x}+1)/e^x}$ = $\frac{e^{2x}-1}{e^{2x}+1}$

Subtract 1 fro both sides.

y – 1 = $\frac{e^{2x}-1}{e^{2x}+1}$ – 1 = $\frac{-2}{e^{2x}+1}$

$e^{2x} + 1 = \frac{-2}{y-1} = \frac{2}{1-y}$

$e^{2x} = \frac{2}{1-y}$ -1 = $\frac{1+y}{1-y}$

2x = log $\frac{1+y}{1-y}$

x = $\frac{1}{2}$log$\frac{1+y}{1-y}$

then $f^{-1}(x)$ = $\frac{1}{2}$log $\frac{1+x}{1-x}$

38. Ans. (d)

Solution: f(x) = log $\frac{1+x}{1-x}$ and g(x) = $\frac{3x+x^3}{1+3x^2}$

Then, f[g(x)] = f $\left[\frac{3x+x^3}{1+3x^2}\right]$

= log $\frac{1+(3x+x^3)/(1x+3x^2)}{1-(3x+x^3)/(1+3x^3)}$

= log $\frac{1+(3x^2+3x+x^3)/(1+3x^2)}{1+(3x^2-3x-x^3)/(1+3x^2)}$

= log $\frac{(1+x)^3}{(1-x)^3}$ = log $(\frac{1+x}{1-x})^3$

= 3log $\frac{1+x}{1-x}$ = 3f(x)

39. Ans. (d)

40. Ans. (b)

Solution: f(x) = log $\frac{1+x}{1-x}$

Then, f(x) + f(y) = log $\frac{1+x}{1-x}$ + log $\frac{1+y}{1-y}$

= log $\left[\frac{(1+x)(1+y)}{(1-x)(1-y)}\right]$

= log $\left[\frac{1+x+y+xy}{1- -y+xy}\right]$ (i)

Then, check the options.

Option (b) is the answer because

f$\left(\frac{x+y}{1+xy}\right)$ = log $\frac{1+(x+y)/(1/xy)}{1-(x+y)(1+xy)}$

= log $\left[\frac{1+x+y+x\,y}{1-x-y+xy}\right]$

= f(x) = f(y)

[from equation (i)]

41. Ans. (c)

Solution: f(x) = maximum (4x + 3, 3x + 6) for x ∈ [-6, 10] It is clear from the function that for the maximum value of [4x+3, 3x + 6] the value of x should be the biggest positive number, which is + 10.

So, for x = 10, y(x) = maximum (4x 10 + 3, 3 x 10 + 6) = maximum (43, 36) = 43

42. Ans. (d)

Solution: y = f(x) = |x| -x

if x = 1, then y = 0

If x = 2, then y = 0

If x = -1 then y = 1

Here, these coordinates are not of any graphs.

So, the answer is option (d).

43. Ans. (b)

Solution: Assume a =x , b=x, c = 2x

Then abc^2 = $\frac{1}{64}$

x x x x $(2x)^2$ = $\frac{1}{64}$ → x x x x $4x^2$ = $\frac{1}{64}$

x^4 = $\frac{1}{512}$ or x = $\frac{1}{4}$

So, a = $\frac{1}{4}$, b = $\frac{1}{4}$, and c =2x = $\frac{1}{2}$

44. Ans. (b)

Solution: For minimum value of ab, 2a should equal to 3b.

Then, 2a = 3b = $\frac{a}{b} = \frac{3}{2}$

Then, a = 3k and b = 2k

Put k = 1, then a = 3 and b = 2

However, for these values, ab ≠ 2a + 3b

Then put k = 2, then a = 6 and b = 4

For a = 6 and b = 4, ab = =2a + 3b

So, the minimum value of ab = 4 x 6 = 24

45. Ans. (c)

Solution: Method 1

$log_a b = 1/log_b a$

Minimum value of x + $\frac{1}{x}$ = 2 if x > 0

Method 2

log_bb + $log_a b$ = ? when a > 1, b = 1

Put a = b (>1)

Then, log_bb + log_bb = 1 + 1 = 2

If we take a > b or b > a, then the value of the function will b greater than 2.

So, the minimum value of log_ba + $log_a b$ = 2

46. Ans. (b)

Solution: $(x-2)^2$ = 9 = $(\pm 3)^2$

x – 2 = ±3

x = 5 or -1 and

$(y-3)^2$ = 25 = $(\pm 5)^2$

y – 3 = ±5

y = 8 or -2

Then minimum value of $\frac{x}{y} = \frac{5}{-2} = -\frac{5}{2}$ (which is minimum).

47. Ans. (c)
Solution: |x+y| = 10
minimum value of $x^2 + y^2$will bethe option , when x = y = 5.
So, $x^2 + y^2 = (5)^2 = (5)^2$ = 50
The maximum value of $x^2 + y^2 = could\ be\ (\infty)$

48. Ans. (b)

49. Ans. (d)
Solution: Least value of (x-2) $(x-4)^2\ (x+6)$ = ?
Take x as a negative number.
If we decrease the value of x, then he value of this function will also decrease.
Hence the least value of this function is (-∞)

50. Ans. (c)
Solution: Minimum value of f(x) will be attained at the median of the critical points f(x), i.e., x = 4.

51. Ans. (d)

52. Ans. (a)
Solution: D = $8^2 - 4\ (k-2)(k+4)$ > 0
Or (k+6) (k-4) < 0
Or -6 < k < 4. So, k = 5.

53. Ans. (a)
Solution: Use AM ≥ GM ≥ HM for all the four fractions.

54. Ans. (c)

55. Ans. (d).

Exercise 05

TITA – Short Answer

1. A function f(x) is defined for all real values of x as f(x) = ax^2 + bx + c, if f(3) = = f(-3) = 18, f(0) = 15, then what is the value of f(12)?

2. If f(x) = $\frac{x^2+1}{x-1}$ then f(f(f(2))) = ?

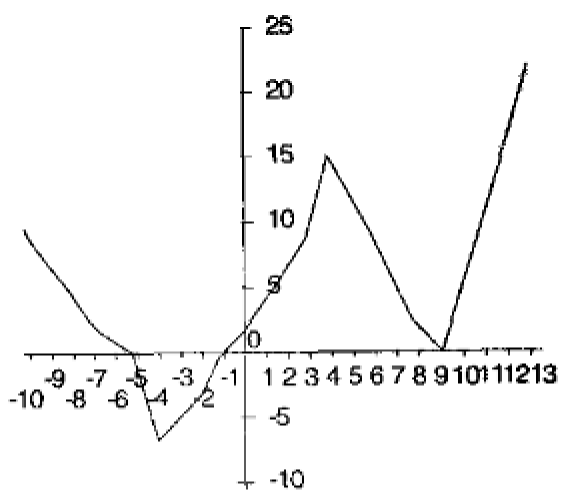
3. Given f(t) = kt + 1 and g(t) = 3t + 2. If fog = gof, find k.

4. if x > 0, the minimum value of $\frac{\left(x+\frac{1}{x}\right)^6-\left(x^6+\frac{1}{x^6}\right)-2}{(x+\frac{1}{x})^3+\left(x^3+\frac{1}{x^3}\right)}$ is ______?

5. If x and y are real numbers, then the minimum values of $x^2 + 4xy$ + 6y^2 – 4y + 4 is

6. The sum of the possible values of X in the equation |x+7| + |x-8| = 16 is:

7. The figure below shows the graph of a function f(x), how many solutions does the equation f(f(x)) = 15 have



8. If f(x) = 1 – f(1-x), then the value of

$f\left(\frac{1}{999}\right) + f\left(\frac{2}{999}\right) + \ldots + f\left(\frac{998}{999}\right)$ is:

9. If $f(x) = 2x^2 + 7x - 9$ and g (x) = 2x + 3, then find the value of g(f(x)) at x = 2

10. If y = min $(x^2 + 2, 6 - 3x)$, then the greatest value of y for x > 0

11. If f (x) = $\begin{cases} 1-x, & 0 \le x \le 2 \\ x-1, & 2 \le x \le 4 \\ 1, & 4 \le x \le 6 \end{cases}$

Then find, f(0) + $f\left(\frac{1}{2}\right)$ + f(1) + $f\left(\frac{45}{18}\right)$:

12. If f(x, y) = $3x^2 - 2xy$ - y^2 + 4, then f(f(2, 3), f(1-, 1) is equal to:

13. If f(x) = $2x^2 + 6x - 1$ and g(x) = |x+5|, then the value off [g{f(g(-6)))}] is:

14. If |x| + |y| = 7, then what is the sum of the minimum and the maximum values of x + y?

15. What is the minimum value of $\frac{(a+1)^2}{a} + \frac{(b+1)^2}{b} + \frac{(c+1)^2}{c}$?
(a) 1 (b) 4 (c) 8 (d) 12

Exercise 05 Solutions

1. Ans. 63
Solution: Since f(0) = 15, we get c = 15.
Next, we have f(3) = = f(-3) = 18. Using the information we get:
9a + 3b + c = 9a - 3b + c → 3b = -3b

∴ 6b = 0, b = 0.
Also, since
f(3) = 9a + 2b + c = 18 → we get: 9a + 15 = 18
→ a = 1/3
The quadratic function becomes f(x) = $\frac{x^2}{3}$ + 15.
F(12) = 144/3 + 15 = 63.

2. Ans. (7.86)
Solution: f(2) = $\frac{2^2+1}{2-1}$ = 5

$f(f(2)) = f(5) = \frac{5^2+1}{5-1} = \frac{26}{4}$

$f(f(f(2))) = f\left(\frac{26}{4}\right) = \frac{\left(\frac{26}{4}\right)^2+1}{\frac{26}{4}-1} = \frac{\frac{676+16}{16}}{\frac{22}{4}}$

$= \frac{692}{16} \times \frac{4}{22} = 7.86$

3. Ans. 2
Solution: fog = f(3t + 2) = k(3t + 2) + 1
gof = f(kt + 1) = 3 (kt + 1) + 2
k (3t+2) + 1 = 3 (kt + 1) + 2
⇨ 2 k + 1 = 5
⇨ k = 2.

4. Ans. 6
Solution: For x > 0.x + $\frac{1}{x}$ has a minimum value of 2, when x is taken as 1. Why we would need to minimize x + $\frac{1}{x}$ is because it is raised to the power 6 in the numerator, so allowing x + $\frac{1}{x}$ to become greater than its minimum would increase the value of the expression. Also, the value of any expression of the form $x^n + \frac{1}{x^n}$ would also give us a value of 2.
Hence the value of the expression would be:

$$\frac{\left(x+\frac{1}{x}\right)^6 - \left(x^6+\frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)} = \frac{2^6-2-2}{2^3+2} = 6$$

5. Ans. 2
Solution: x^2 + 4xy + y^2 – 4y + 4
= x^2 + $4y^2$ + 4xy + $2y^2$ – 4y + 2 +2
= $(x + 2y)^2 + 2\,(y^2 - 2y + 1) + 2$
The above expression is minimum for y = 1, x = -2
So minimum value of the given expression = 0 + 0 + 2 = 2.

6. Ans. 1
Solution: For x < -7

|x+7| + |x+8| = -(x+7) – (x-8) - (x+7) – (x-8) = 16

-2x + 1 = 16
x = -7.5
Fro -7 ≤ x ≤ 8

|x +7| + |x-8| = x + 7 – x + 8 = 2x – 1
Therefore the given equation has no solution in this range.
For x ≥ 8
|x+7| + |x-8| = x + 7 + x – 8 = 2x – 1
2x – 1 = 16
⇨ x = $\frac{17}{2}$ = 8.5

7. Ans. 7
Solution: According to the graph, f(4) = 15 and f(12) = 15.
So f(f(x)) = 15 for f(x) = 4, 12.
According to the graph f(x) = 4. Has four solutions.
According to the graph f(x) = 12 has three solutions.
Hence the given equation has 7 solution.

8. Ans. 499
Solution. f (x) = 1- f(1-x)
⇨ f(x) + f(1-x) = 1
Now, f$\left(\frac{1}{999}\right)$ + $\left(\frac{2}{999}\right)$ + … $\left(\frac{997}{999}\right)$ + f $\left(\frac{998}{999}\right)$

= $\left[f\left(\frac{1}{999}\right) + f\left(\frac{998}{999}\right)\right]$ + $\left[f\left(\frac{2}{999}\right) + f\left(1-\frac{2}{999}\right)\right]$ + ….
= 1 + 1 + 1 + … 499 times = 499

9. Ans. 29
Solution. g (f(x)) = g(f(2)) = g(f(2)) = g(13) = 29

∴ f(2) = 2. $(2)^2 + 7(2) - 9$ = 13
And g(13) = 2 x 13 + 3 = 29

10. Ans. 3
Solution. y is maximum when $x^2 + 2 = 6 - 3x$
i.e.,$x^2 + 3x - 4$ = 0
$x^2 + 4x - x - 4$ = 0
x (x-4) – 1 (x+4) = 0
x = 1 or x = -4
∴ at x = 1, y = 3 (∴ x > 0)

11. Ans. 3
Solution. f (0) = 1 – 0 = 1, f($\frac{1}{2}$) = 1 - $\frac{1}{2}$ = $\frac{1}{2}$ = 0.5
f(1) = 1-1 = 0, f$\left(\frac{45}{18}\right)$ = 2.5 – 1 = 1.5
∴ f(0) + f($\frac{1}{2}$) + f(1) + f$\left(\frac{45}{18}\right)$ = 1 + 0.5 + 0 + 1.5 = 3

12. Ans. 95
Solution: f(x) = $3x^2 - 2xy - y^2 + 4$
Then, f((f(2, 3), f(-1, 1)]
Then value of f(2,3) = 3(2^2) – 2 x 2 x 3 – $(3)^2$ + 4
= 12 – 12 – 9 + 4 = - 5 (i)
f(-1, 1) = 3 $(-1)^2$ – 2 x -1 x 1 = $(1)^2$ + 4
= 3 + 2 – 1+ 4 = 8 (ii)
Then f(f(2,3), f(-1,1) = f(-5, 8)
= 3$(-5)^2$ – 2 x -5 x 8 – $(8)^2$ + 4
= 75 + 80 – 64 + 4 = 95

13. Ans. 359
Solution: f(x) = $22x^2 + 6x - 1$ and g(x) = |x+5|

Then the value of f|f(f(-6))}]
= f[g(f(-6)+5|)}] [Since g(x) = |x+5|]
= f[g{f(1)}] = f[g{$2(1)^2 + 6(1) - 1$}] = f(g){2 + 6 – 1}] = 1
f[g{7}]
= f[1|75|] = f[12]
= 2 $(12)^2 + 6\ (12)$ – 1 = 2 x 144 + 72 – 1 = 288 + 71 = 359

14. Ans. 0
Solution: |x| + |y| = 7
The minimum value of x + y will be attained, when both are negative.
So, minimum of x + y = -7
The maximum value of x + y will be attained when both are positive.
So, maximum of x + y = 7
Then, maximum of (x+y) and minimum of (x+y) = 7 + (-7) = 0

15. Ans. 12