Solutions

Exercise 01

1. Solution: (a) and (b) are only two quadratic equations. On simplifying (b) gives a liner equation and (c) is a cubic equation.

2. Solution:
On substituting $x = 3$, we get
$$3(3)^2 + 5(3) + 2 = 44 \neq 0$$
Therefore $x = 3$ is not a solution of the given equation.
Again, substitute $x = -1, we\ get$
$$3(-1)^2 + 5(-1) + 2 = 0$$
Therefore $x = -1$ is the solution (i.e., root) of the given equation.

3.
$$6x^2 - x - 2 = 0$$
$$\Rightarrow 6x^2 - 4x + 3x - 2 = 0$$
$$\Rightarrow 2x(3x - 2) + 1(3x - 2) = 0$$
$$\Rightarrow (3x - 2)(2x + 1) = 0$$
Either $(2x + 1) = 0\ or\ (3x - 2) = 0$
$$\Rightarrow x = -\frac{1}{2}\ or\ x = \frac{2}{3}$$
$$x = -\frac{1}{2}, \frac{2}{3}$$

4. If $x = 2$ is a root of the given equation, then $x = 2$ must satisfy it.
$$\therefore 3(2)^2 - 2K(2) + 5 = 0$$
$$\Rightarrow 12 - 4K + 5 = 0$$
$$\Rightarrow -4K = -17 \Rightarrow K = \frac{17}{4}$$

5.
$$8x - 2x^2 = 4 \Rightarrow 2x^2 - 8x + 5 = 0$$
Here, $a = 2, b = -8\ and\ c = 5$
Substituting the values of $a, b\ and\ c\ in\ the\ formula$
$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$
$$\alpha = \frac{8 + \sqrt{64 - 40}}{4} \quad \text{and} \quad \beta = \frac{8 - \sqrt{64 - 40}}{4}$$
$$\alpha = \frac{8 + 2\sqrt{6}}{4} \quad \text{and} \quad \beta = \frac{8 - 2\sqrt{6}}{4}$$
$$\alpha = \frac{4 + \sqrt{6}}{2} \quad \text{and} \quad \beta = \frac{4 - \sqrt{6}}{2}$$

6. $9y^4 - 29y^2 + 20 = 0$
Put $y^2 = x$
$$9x^2 - 29x + 20 = 0$$
$$\Rightarrow 9x^2 - 20x - 9x + 20 = 0$$
$$\Rightarrow (x - 1)(9x - 20) = 0$$
$$\Rightarrow x = 1\ or\ x = \frac{20}{9}$$
$$\Rightarrow y^2 = 1\ or\ y^2 = \frac{20}{9}$$
$$\Rightarrow y = \pm 1\ and\ y = \pm\frac{2\sqrt{5}}{3}$$

7. $x^6 - 26x^3 - 27 = 0$
Let $\quad x^3 = y$
Then, $\quad y^2 - 26y - 27 = 0$
$\Rightarrow \quad y^2 - 27y + y - 27 = 0$
$\Rightarrow \quad (y + 1)(y - 27) = 0$
$\Rightarrow \quad y = -1\ or\ y = 27$

$\Rightarrow \quad x^3 = -1 \text{ or } x^3 = 27$
$\Rightarrow \quad x = -1 \text{ or } x = 3$

8. $2x - \frac{3}{x} = 5$
$\Rightarrow 2x^2 - 3 = 5x$
$\Rightarrow (2x+1)(x-3) = 0$
$\Rightarrow x = -\frac{1}{2} \text{ or } x = 3.$

9. $\sqrt{2x+9} + x = 13$
$\Rightarrow \sqrt{2x+9} = 13 - x$
Squaring both sides
$2x + 9 = (13-x)^2$
$\Rightarrow x^2 - 28x + 160 = 0$
$\Rightarrow (x-8)(x-20) = 0$
$\Rightarrow x = 8, x = 20$

10. $\sqrt{2x+9} - \sqrt{x-4} = 3$
$\Rightarrow \sqrt{2x+9} = 3 + \sqrt{x-4}$
Squaring both sides and simplifying, we get
$x + 4 = 6\sqrt{x-4}$
Again squaring both sides,
$(x+4)^2 = 36(x-4)$
$\Rightarrow x^2 - 28x + 160 = 0$
$\Rightarrow (x-8)(x-20) = 0$
$\Rightarrow x = 8, x = 20$
Verification: $2x + 9 \geq 0 \text{ and } x - 4 \geq 0$
$\Rightarrow x \geq \frac{9}{2} \text{ and } x \geq 4$
Since the values $x = 8$ and 20 satisfy both these conditions
$\therefore \quad x = 8, \; x = 20$

11. Put $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$
$\therefore \quad 2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$
$\Rightarrow 2\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 9\left(x + \frac{1}{x}\right) + 14 = 0$
Substitute; $\quad x + \frac{1}{x} = y$
$\therefore \quad 2(y^2 - 2) - 9y + 14 = 0$
$\Rightarrow 2y^2 - 4 - 9y + 14 = 0$
$\Rightarrow (y-2)(2y-5) = 0$
$\Rightarrow y = 2 \text{ or } y = \frac{5}{2}$
Since $\quad x + \frac{1}{x} = 2 \Rightarrow x^2 - 2x + 1 = 0$
$\Rightarrow \quad (x-1)^2 = 0 \Rightarrow x = 1$
Also, $\quad y = \frac{5}{2} \Rightarrow x + \frac{1}{x} = \frac{5}{2}$
$\Rightarrow \quad 2x^2 - 5x + 2 = 0$
$\Rightarrow \quad x = 2 \text{ or } x = \frac{1}{2}$
$\therefore \quad x = \frac{1}{2}, 1, 2$

12. Put $\quad x^2 + \frac{1}{x^2} = (x - \frac{1}{x})^2 + 2$
$\therefore \quad 6\left[\left(x - \frac{1}{x}\right)^2 + 2\right] - 25\left(x - \frac{1}{x}\right) + 12 = 0$
Let $\quad x - \frac{1}{x} = y$
$\Rightarrow \quad 6(y^2 + 2) - 25y + 12 = 0$

$\Rightarrow$ $y=\frac{3}{2}, y=\frac{8}{3}$

Since $y=x-\frac{1}{x}$ and $x-\frac{1}{x}=\frac{8}{3}$

$\therefore$ $x-\frac{1}{x}=\frac{3}{2}$ and $x=-\frac{1}{3}, x=3$

$\Rightarrow$ $x=-\frac{1}{2}$ and $x=2$

$\therefore$ $x=-\frac{1}{3},-\frac{1}{2},2,3$

13. $\sqrt{x^2+x-6}-x+2=\sqrt{x^2-7x+10}$

$\sqrt{(x+3)(x-2)}-(x-2)=\sqrt{(x-5)(x-2)}$

$\Rightarrow \sqrt{(x-2)}[\sqrt{(x+3)}-\sqrt{(x-2)}-\sqrt{(x-5)}=0]$

Either; $\sqrt{(x-2)}=0 \Rightarrow x=2$

Or $\sqrt{(x+3)}-\sqrt{(x-2)}-\sqrt{(x-5)}=0$

$\Rightarrow \sqrt{(x+3)}-\sqrt{(x-2)}=\sqrt{x-5}$

Squaring both sides

$x^2+12x+36=4(x^2+x-6)$

$\Rightarrow x=6, x=-\frac{10}{3}$

Since the equation involves radical therefore substitution $r=2,6$ *and* $-\frac{10}{3}$ in the original equation, we find that $x=-\frac{10}{3}$ does not satisfy the equation.

$\therefore x=2,6$

14. $3^{x+2}+3^{-x}-10=0$

$\Rightarrow 3^x.3^2+\frac{1}{3^x}-10=0$

Let, $3^x=y \Rightarrow 9y+\frac{1}{y}-10=0$

$\Rightarrow 9y^2-10y+1=0$

$\Rightarrow (9y-1)(y-1)=0 \Rightarrow y=\frac{1}{9}$ *or* $y=1$

When $y=\frac{1}{9} \Rightarrow 3^x=1/3^2$

$\Rightarrow x=-2$

When $y=1 \Rightarrow 3^x=1$

$\Rightarrow x=0$

$\therefore x=-2,0$

15. $(x+1)(x+2)(x+3)(x+4)=24$

$\Rightarrow [(x+1)(x+4)][(x+2)(x+3)]=24$

$\Rightarrow (x^2+5x+4)(x^2+5x+6)=24$

Let $x^2+5x=y$

$\therefore (y+4)(y+6)=24$

$\Rightarrow y^2+10y=0$

$\Rightarrow y=0$ *and* $y=-10$

Now, $y=x^2+5x$

When $y=0, x^2+5x=0 \Rightarrow x=0, x=-5$

Again when $y=-10, x^2+5x=-10$

$\Rightarrow x^2+5x+10=0$

Since LHS expression cannot be factorized, therefore we should use the formula for finding the value of x.

Here,. $D=b^2-4ac$

$=25-4\times 1\times 10=-15$

Since $D<0$, the equation $x^2+5x+10=0$ has no real solution.

$\therefore x=0,-5$

16. Let α,β be the two roots of the equation then,

$\alpha+\beta=-\frac{b}{a}=-\frac{2}{3}$

and $\alpha\beta=\frac{c}{a}=-\frac{1}{3}$

17. Method (I)

$$\alpha+\beta=-3+5=2$$

And $\alpha\beta=-3\times 5=-15$

$\therefore$ The required quadratic equation

$x^2-(\alpha+\beta)x+(\alpha\beta)=0$

$\Rightarrow x^2-(2)x+(-15)=0$

$\Rightarrow x^2-2x-15=0$

Method (II): Let $\alpha=-3$ *and* $\beta=5$, *then the required equation*

$(x-\alpha)(x-\beta)=0$

$\Rightarrow (x+3)(x-5)=0$

$\Rightarrow x^2-2x-15=0$

18. $2x^2-3x+2=0$, and $\alpha,\beta=1$

$\therefore\ \alpha+\beta=\frac{3}{2}, \alpha\beta=1$

For the new equation, roots are α^2 *and* β^2

$\therefore$ Sum of the roots

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\left(\frac{3}{2}\right)^2-2(1)$

$=\frac{9}{4}-2=\frac{1}{4}$

And product of the roots $=\ \alpha^2\beta^2=(\alpha\beta)^2=(1)^2=1$

$\therefore$ the required equation is

$x^2-(sum\ of\ roots)x+product\ of\ roots=0$

$\Rightarrow x^2-\frac{1}{4}x+1=0$

$\Rightarrow 4x^2-x+4=0$

19. Let the roots be α *and* α

$\therefore$ Sum of roots $=\alpha+\alpha=2\alpha=-\frac{2k}{9}$

$\Rightarrow \alpha=-\frac{k}{9}$

And product of the roots $=\alpha^2=\frac{4}{9}$

$\therefore\ (-\frac{k}{9})^2=\frac{4}{9}\Rightarrow\frac{k^2}{81}=\frac{4}{9}$

$\Rightarrow k^2=36\Rightarrow k=\pm 6.$

Alternatively: In order that roots of a quadratic equation are equal, its discriminant must be zero.

i.e., $b^2-4ac=0$

$\therefore$ $(2k)^2-4\times 9\times 4=0$

$\Rightarrow$ $k=\pm 6.$

20. For any quadratic polynomial to have real linear factors, we must have $D\geq 0$

$\therefore\ b^2-4ac\geq 0$

$p^2-4\times 9\times 4\geq 0$

$\Rightarrow p^2-144\geq 0$

$\Rightarrow p^2\geq 144$

$\Rightarrow p\geq\pm 12$

Either $p\leq -12$ *or* $p\geq 12.$

21. Let width of the rectangle $=x$

$\therefore$ Length of rectangle $=(x+7)cm.$

$\therefore$ Area of rectangle $=(x+7)\times x$

$\Rightarrow (x+7)(x)=60$

$\Rightarrow x^2+7x-60=0$

$\Rightarrow (x+12)(x-5)=0$

$\Rightarrow x=-12$ *and* $x=5$

Since width can never be negative, therefore $x=5cm$ and $x+7=12\ cm$ are the required values.

i.e., $Length=12\ cm\ and\ width=5\ cm.$

22. Let the common root be a, *then*,

$$a^2 - ka - 21 = 0 \qquad \ldots\text{(i)}$$
$$a^2 - 3ka + 35 = 0 \qquad \ldots\text{(ii)}$$

Solving by the rules of cross multiplication,

$$\frac{a^2}{-34k-63k} = \frac{a}{-21-36} = \frac{1}{-3k+k}$$
$$\therefore \quad a = \frac{-98k}{-56} = \frac{7k}{4}$$

And $\therefore \frac{7k}{4} = \frac{28}{k} \Rightarrow 7k^2 = 28 \times 4 \Rightarrow k = \pm 4$

23.

When $a < 0$, we get maxima otherwise if $a > 0$ we get minima.

As we know, at $x = \frac{-b}{2a}$, we get the maxima,

$$\therefore \quad y = \frac{4ac - b^2}{4a} = \frac{4 \times (-5) \times 7 - (10)^2}{4 \times (-5)}$$
$$= \frac{-140 - 100}{-20} = \frac{-240}{-20} = 12$$

Thus the maximum value of the given quadratic equation is 12.

24. Let, $y = \frac{x+2}{2x^2+3x+6}$

Then, $2x^2y + (3y-1)x + 6y - 2 = 0$

For x to be real, $(3y-1)^2 - 8y(6y-2) \geq 0$

Or $(1+13y)(1-3y) \geq 0$

Or $(13y+1)(3y-1) \leq 0$

Putting each factor equal to zero, we get

$$y = -\frac{1}{13}, \frac{1}{3}$$

If $y < -\frac{1}{13}, (1+13y)(1-3y) < 0$

If $-\frac{1}{13} < y < \frac{1}{3}, (1+13y)(1-3y) > 0$

If $y > \frac{1}{3}, \qquad (1+13y)(1-3y) < 0$

Thus, y will lie between $-\frac{1}{13}$ *and* $\frac{1}{3}$.

Hence the maximum value of y *is* $\frac{1}{3}$ *and minimum value is* $-\frac{1}{13}$.

25. $x^2 - 5x + 4 = (x-1)(x-4)$

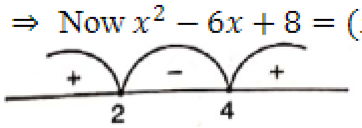
Since $\qquad f(x) = x^2 - 5x + 4 > 0$

therefore the required range is $x < 1$ and $x > 4$.

26. $-x^2 + 6x - 8 > 0$

$\Rightarrow x^2 - 6x + 8 < 0$

$\Rightarrow$ Now $x^2 - 6x + 8 = (x-2)(x-4)$



Since $\qquad f(x) = x^2 - 6x + 8 < 0.$

Therefore the required range is $2 < x < 4$.

27.

SOLUTION The range of the variable x in this inequality consists of all values of x except $x = -2$ and $x = 1/4$. Hence we cannot cross multiply. So we adopted another method

$$\frac{x-2}{x+2} - \frac{2x-3}{4x-1} > 0 \quad \text{...(i)}$$

or $$\frac{(x-2)(4x-1)-(x+2)(2x-3)}{(x+2)(4x-1)} > 0$$

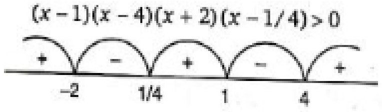
or $$\frac{2(x^2-5x+4)}{(x+2)(4x-1)} > 0$$

or $$\frac{(x-1)(x-4)}{(x+2)(x-1/4)} > 0 \quad \text{...(ii)}$$

Now multiply both sides of (ii) by the expression $(x+2)^2\left(x-\frac{1}{4}\right)^2$, which is positive for the x under consideration.

$\therefore$ $$\frac{(x-1)(x-4)(x+2)^2\left(x-\frac{1}{4}\right)^2}{(x+2)\left(x-\frac{1}{4}\right)} > 0$$

or $$(x-1)(x-4)(x+2)(x-1/4) > 0 \quad \text{...(iii)}$$



Thus, the range is $x < -2$ or $\frac{1}{4} < x < 1$ or $x > 4$

28.

$$\frac{x^2-2x+5}{3x^2-2x-5} > \frac{1}{2}$$

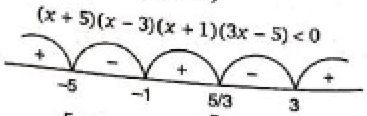
or $$\frac{x^2-2x+5}{3x^2-2x-5} - \frac{1}{2} > 0$$

or $$\frac{x^2+2x-15}{3x^2-2x-5} < 0$$

or $$\frac{(x+5)(x-3)}{(x+1)(3x-5)} < 0$$

or $$\frac{(x+5)(x-3)(x+1)^2(3x-5)^2}{(x+1)(3x-5)} < 0$$

or $$(x+5)(x-3)(x+1)(3x-5) < 0$$



$\therefore$ $-5 < x < -1$ or $\frac{5}{3} < x < 3$

29. Ans: a

$$2\left(x^2+\frac{1}{x^2}\right) - 3\left(x+\frac{1}{x}\right) - 1 = 0$$

$$\Rightarrow \quad 2\left[x^2+\frac{1}{x^2}+2-2\right] - 3\left(x+\frac{1}{x}\right) - 1 = 0$$

$$\Rightarrow \quad 2\left[\left(x+\frac{1}{x}\right)^2-2\right] - 3\left(x+\frac{1}{x}\right) - 1 = 0$$

$$\Rightarrow \quad 2\left(x+\frac{1}{x}\right)^2 - 3\left(x+\frac{1}{x}\right) - 5 = 0$$

$$\Rightarrow \quad 2t^2 - 3t - 5 = 0 \quad \text{(Substituting } x+\tfrac{1}{3} = t)$$

Now solve it and you will get

$$t = -1 \quad \text{and} \quad t = \frac{5}{2}$$

Now if $t = -1$, then $x + \frac{1}{x} = -1$.

$$\Rightarrow \quad x^2 + 1 + x = 0$$

$$\Rightarrow \quad x^2 + x + 1 = 0$$

$$\therefore \quad x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$$

and if $t = \frac{5}{2}$ then $x + \frac{1}{x} = \frac{5}{2}$

$$\Rightarrow \quad 2x^2 - 5x + 2 = 0$$

$$\Rightarrow \quad x = \frac{1}{2}, 2$$

30. $\sqrt{2x^2 - 2x + 1} = 2x - 3$
Square on both sides and simplify

Exercise 02 (MCQs)

1. Ans: d
Has no maximum

2. Ans: c
The product of the roots is given by: $(a^2 + 18a + 81)/1$
Since product is unity we get: $a^2 + 18a + 81 = 1$
Thus, $a^2 + 18a + 80 = 0$
Solving, we get: $a = -10$ *and* $a = -8$.

3. Ans: d
To solve this take any expression whose roots differ by 2.
Thus, $(x-3)(x-5) = 0$
$\Rightarrow x^2 - 8x + 15 = 0$
In this case, $a = 1, b = -8$ *and* $c = 15$.
We can see that $b^2 = 4(c+1)$.

4. Ans: b
$y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}}$
$\Rightarrow y = \sqrt{x+y}$
$\Rightarrow y^2 = x + y$
$y^2 - y - x = 0$
Solving quadratically, we have option (b) as the root of this equation.

5. Ans: a
$mn = \frac{r}{p}$ (i)
$(mk)(nk) = mnk^2 = \frac{c}{a}$ (ii)
Equation (ii) ÷ equation (i)
$k^2 = \frac{c}{a} \times \frac{p}{r}$
$k = \sqrt{\frac{cp}{ar}}$

6. Ans: c
From (i) we have sum of roots = 14
And from (ii) we have product of roots = 48.
Option (c) is correct

7. Ans: b
$x^2 - 3x + 2 = 0$ gives its roots as $x = 1, 2$.
Put these values in the equation and then use the options

8. Ans: b

9. Ans: a

10. $p(p-1)/3 < 0$ (Product of roots should be negative).
$\Rightarrow p(p-1) < 0$

$p^2 - p < 0.$
This happens for $0 < p < 1.$
Option (b) is correct.

11. $\Rightarrow \gamma + \delta = -n \; and \; \gamma\delta = 1$
$(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) = (\alpha - \gamma)(\beta - \delta)(\beta - \delta)(\beta - \gamma)(\alpha + \delta)$
$= [\alpha\beta + \alpha\delta - \gamma\beta - \gamma\delta][\alpha\beta + \beta\delta - \alpha\gamma - \gamma\delta]$
$= [1 + \alpha\delta - \gamma\beta - 1][1 + \beta\delta - \gamma a - 1]$
$= (\alpha\delta - \gamma\beta)(\beta\delta - \gamma\alpha)$
$= 1.\delta^2 - a^2.1 - \beta^2.1 + \gamma^2.1 = (\delta^2 + \gamma^2) - (a^2 + \beta^2)$
$= [(\delta + \gamma)^2 - 2\delta\gamma] - [(a + \beta)^2 - 2a\beta]$
$= [(-n)^2 - 2.1] - [(-m)^2 - 2.1] = n^2 - m^2$
Option (a) is correct

12. Roots of the given equation $= \frac{2a \pm \sqrt{4a^2 - 4ab}}{2b}$
$= \frac{a \pm \sqrt{a^2 - ab}}{b}$
$= \frac{\sqrt{a}(\sqrt{a} \pm \sqrt{a-b})}{b} \times \frac{\sqrt{a} \mp \sqrt{a-b}}{\sqrt{a} \mp \sqrt{a-b}} = \frac{\sqrt{a}}{\sqrt{a} \mp \sqrt{a-b}}$

13. Ans: b
$K + 6 = 2K - 1$
$K = 7$

14. Ans:b
Let roots $= \alpha, \beta$
Therefore, $a^2 + \beta^2 = (a + \beta)^2 - 2a\beta = (P - 2)^2 + 2\,(p + 1)$
$= p^2 - 4p + 4 + 2p + 2 = (P - 1)^2 + 5$
Hence, value of p for the least value = 1

15. Ans: d
$a - \beta = -2(p + 1); \qquad \alpha\beta = 9p - 5$
$\therefore \; a > 0 \; and \; \beta > 0$
$\therefore \; \alpha + \beta > 0 \Rightarrow -2(p - 1) > 0 \Rightarrow p < -1$
$\therefore \; \alpha\beta > 0 \Rightarrow 9p - 5 > 0 \; \Rightarrow p > 5/9$
Hence, option (d) is the answer.

16. Ans: c
Let common root = α
$\therefore \; a^2 - \text{a}\,\alpha - 21 = 0$
$a^2 - 3\text{a}\,\alpha + 35 = 0$
Solving the two equations, we get a =4

17. Ans: d
Sum roots = $-2/3(-b/a)$

18. Ans: a

19. Ans: c

20. Ans: c

21. Ans: b

22. Ans: b
Take a quadratic equation: $x^2 + 3x + 2 = 0$
Therefore; Sum of roots = - 3
Product of roots = 2

Roots = -1, -2
Now, new quadratic equation: $2x^2 + 3x + 1 = 0$
Therefore; Sum of roots: $-\frac{3}{2}$
Product of roots: $\frac{1}{2}$
Roots = $-1, -\frac{1}{2}$
Therefore, sum, product, and roots will change. Hence option (d) is the answer.

23. Ans: b
$x^4 + x^2 = 0$ can be written as $\Rightarrow x^2(x^2 + 1) = 0$
Therefore, either $x^2 = 0\ or (x^2 + 1) = 0$
Case I: If $x^2 = 0, then\ x = 0, 0\ (two\ solutions)$
Case II: $(x^2 + 1) = 0 \Rightarrow x^2 = -1$
Minimum value of $x^2 = 0, and\ x^2 = -1$ is not possible.
Hence, a total of two real solutions are possible.

24. Ans: c
$as^4 + bs^3 + cs^2 + ds + e = 0$
$\Rightarrow a(s - s_1)(s - s_2)(s - s_3)(s - s_4) = 0 (because\ S_1, S_2, S_3\ and\ S_4\ are\ roots)$
Now, putting $s = 1\ in\ LHS$
$\Rightarrow P(1) = a(1 - s_1)(1 - s_2)(1 - s_3)(1 - s_4)$
$\Rightarrow (1 - s_1)(1 - s_2)(1 - s_3)(1 - s_4) = (a + b + c + d + e)/a$

25. Ans: c
$x^2 - 5x + 6 = 0 \Rightarrow Roots = 2, 3$
When 2 is the common root, then $p(2) = 0 \Rightarrow 2^2 + 2m + 3 = 0$
$\Rightarrow m = -7/2$
When 3 is the common root, then $P(3) = 0 \Rightarrow 3^2 + 3m + 3 = 0$
$\Rightarrow m = -4$

Exercise 03 (MCQs)

1. Ans: c

2. Ans: d

3. Ans: b

4. Ans: c

5. Ans: a

6. Ans: d

7. Ans: d

8. Ans: d
$x^3 + x^2 + 2x - 17 = 0$
Let roots be $\alpha, \beta, and\ \gamma$
Therefore; $\alpha\ \beta\ \gamma = 17$
Now, 17 is having only two factors 1 and 17, so the only possible integral roots = 1 and 17. Checking for these two values we find no integral root possible.

9. Ans: a
$x - \frac{1}{x^2-4} = 2 - \frac{1}{x^2-4}$

$\Rightarrow x = 2$. Although we can see that for $x = 2, 1/(x^2 - 4)$ will not hold.
Hence, no value of x is possible.

10. Ans: b
Product of roots =1
$\Rightarrow \frac{4k}{k^2+1} = 1 \Rightarrow k = -2 \pm \sqrt{3}$

11. Ans: c
$(x-a)(x-b) = x$
$\Rightarrow x^2 - (a+b)x + ab - c = 0; roots = \alpha, \beta$
$(x-\alpha)(x-\beta) + c = 0$
$\Rightarrow x^2 - (\alpha+\beta)x + \alpha\beta + c = 0$
$\Rightarrow x^2 - (a+b) + ab - c + c = 0$
(putting values of $(\alpha + \beta) and \alpha\ \beta$ from previous equation)
Hence, new equation = $x^2 - (a+b) + ab = 0$. Therefore, the roots are $a\ and\ b$.

12. Ans: c
$x^2 + px + q = 0$
Given roots are (a, b)
Then, $a + b = -p\ and\ ab = q$
And $x^2 + px - r = 0$ and the roots are (g, d)
Then, $g + d = -p\ and\ gd = -r$
But, in first equation, $-p = a + b$
Then, $a + b = g + d$
Then, $(a-g)(a-d) = a^2 - a[g+d] + gd$
$= a^2 - a[a+b] + gd[\therefore a + b = g + d]$
$= a^2 - a^2 - ab + gd = -q - r = (q + r)$
Hence, option (c) is the answer.

13. Ans: c

14. Ans: b
As $(x-1)^3$ is a factor of the polynomial, 1 is a repeated root (3 times) to the given equation.
Let the fourth root be x, Therefore, $1.1.1. x = -1$
$\Rightarrow x = -1 \Rightarrow -1\ is\ another\ root$
Hence, $(x+1) is\ a\ factor.$

15. Ans: c
$a^3 + \beta^3 = (a+\beta)^3 - 3a\beta(a+\beta)$
$\left(\frac{3}{2}\right)^2 + 3 \times 1 \times \frac{3}{2} = \frac{63}{8}$

16. Ans: a
Alternatively: Go through options.

17. Ans: c
For equal roots $D = 0$
i.e., $b^2 - 4ac = 0$
$\Rightarrow [-2(1 + 3k)]^2 - 4 \times 1 \times 7 \times (3 + 2k) = 0$
Solve it and get the value of k.

18. Ans: a
$D = b^2 - 4ac$
$= 4 - 4 \times (-3)x(-8) = -92$

19. Ans: a
$D = b^2 - 4ac = 25 - 4 \times 1 \times 7 = -3$

Since $D < 0$, therefore roots are not real, i.e., roots will be imaginary.

20. Ans: a

$\alpha + \beta = \alpha\beta$

$\Rightarrow -\frac{b}{a} = \frac{c}{a} \Rightarrow -b = c$

$\therefore \; -2k = 4$

$\Rightarrow \; k = -2$

21. Ans: b

Let $x = \sqrt{6 + \sqrt{6 + + \ldots \infty}} \Rightarrow x^2 = x + 6 \Rightarrow x^2 - x - 6 = 0 \Rightarrow x = 3, -2, but x > 0.$

22. Ans: b

$$k = \frac{x^2 - x + 1}{x^2 + x + 1}$$

$\Rightarrow \quad kx^2 + kx + k = x^2 - x + 1$

$\Rightarrow \quad (k-1)x^2 + (k+1)x + k - 1 = 0$

Since x is real, the discriminant

$D = (k+1)^2 - 4(k-1)^2 \geq 0$

$\Rightarrow \quad (3k-1)(-k+3) \geq 0$

$\Rightarrow \quad \left(k - \frac{1}{3}\right)(k-3) \leq 0$

$\Rightarrow \quad \frac{1}{3} \leq k \leq 3$

23. Ans: c

Let a be a common root of the two given equations, **then** $a^2 - 3aa + 35 = 0$ and $a^2 - aa - 21 = 0$.

On subtracting we get $-2aa + 56 = 0$ or $a = \frac{28}{a}$.

As a is a root of $x^2 - ax - 21 = 0$,

$\therefore \; \left(\frac{28}{a}\right)^2 - a\left(\frac{28}{a}\right) - 21 = 0$

or $a^2 = 4^2$ or $a = \pm 4$

$\therefore \; a > 0$, we get $a = 4$

24. Ans: c

$2^{3x^2 - 7x + 4} = 1 = 2^{\circ}$

$\Rightarrow \; 3x^2 - 7x + 4 = 0$

$\Rightarrow \; 3x^2 - 3x - 4x + 4 = 0$

$\Rightarrow \; 3x(x-1) - 4(x-1) = 0$

$\Rightarrow \; x = 1 \; or \; x = \frac{4}{3}$

$\therefore \; x = 1, \frac{4}{3}$

25. Ans: a

$(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$ ....(i)

It is possible only when $x = 1, x = 2, x = 3$.

But $x = 1, 2, 3$ $do\ not\ satisfy\ eq.(i)$

Exercise 04 (MCQs)

1. Ans: d

Assume that roots of the equation $3ax^2 +$

Assume that roots of the equation $3ax^2 + 2bx + c = 0$ are α, β.

$\alpha+\beta = -\frac{2b}{3a}, \alpha\beta = \frac{c}{3a}$ and $\frac{\alpha}{\beta} = \frac{2}{3}$ (given)

$\therefore \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{2}{3}+\frac{3}{2}$

$\frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{13}{6} \Rightarrow \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{13}{6}$

Now putting the values of $(\alpha+\beta)$ and $\alpha\beta$ and then solving,

We get, $8b^2 = 25ac$.

2. Ans: b

Let the number of chairs bought initially $= n$.

$$\therefore \frac{2400}{n} - \frac{2400}{n+10} = 20$$

$$\Rightarrow 120\left[\frac{10}{n(n+10)}\right] = 1$$

$$\therefore n(n+10) = 1200 \Rightarrow n = 30$$

3. Ans: c

4. Ans: b

$x^4 \frac{1}{x^4} = 47$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 = 47 \Rightarrow x^2+\frac{1}{x^2} = 7$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 - 2 = 7$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 = 9$

$\Rightarrow x - \frac{1}{x} - 3 \Rightarrow \left(x+\frac{1}{x}\right)^3 = 27$

$\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x+\frac{1}{x}\right) = 27$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\times 3 = 27 \Rightarrow x^3 + \frac{1}{x^3} = 18$

5. Ans: c

Let the roots be 3, α, and β.

$\therefore 3\,\alpha\,\beta = 6 \Rightarrow \alpha\,\beta = 2$

and $3x + 3\,\beta + \alpha\,\beta = 11$

$\Rightarrow 3(\alpha+\beta) + 2 = 11$

$\therefore \alpha+\beta = 3$

$\therefore \alpha = 1, \beta = 2$

6. Ans: c

$(m+n)^{100} = m^{100} + {}^{100}c_1\, m^{99}.\, n + {}^{100}c_2\, m^{98}.\, n + \ldots + n^{100}$

$= m^{100} + n^{100} + k$

[where $k = {}^{100}C_1\, m^{99} n + {}^{100}C_2\, m^{98}.\, n + \ldots + {}^{100}C_{99}\, mn^{99}$]

$\therefore$ $k > 0$ for m and n belonging to natural number

$\therefore$ $(m+n)^{100} > m^{100} + n^{100}$

7. Ans: a

$x^2 + 5|x| + 6 = 0$

$\because$ All the terms in LHS are positive.

Hence, no real root is possible.

8. Ans: d

Given that x_1, x_2, and x_3 are in AP.

Then, $2x_2 = x_1 + x_3$ (i)

It is also given that sum of the roots

$x_1 + x_2 = 4$ (ii)

Here, with both equations, we can find neither x_1 nor x_2.

Then, answer is (d)

9. Ans: c

Let the roots be α, 2α. Where $\alpha > 0$

$\therefore \alpha + 2\alpha = -m \Rightarrow m = -3\alpha$

and, $2\alpha^2 = C$

Now, since $m + c = 2 \Rightarrow 2\alpha^2 - 3\alpha = 2$

$\alpha = -1/2, 2$

$\because \alpha > 0$

$\therefore \alpha = 2 \therefore m = -3\alpha = -6$

10.

$(ax^2 + bx + c)(ax^2 - dx - c) = 0$

$\therefore$ Either $ax^2 + bx + c = 0$ or, $ax^2 - dx + c = 0$ or both

$\therefore$ Roots of $ax^2 + bx + c = 0$ will be real, if

$b^2 - 4ac > 0$

Similarly, for $ax^2 - dx - c$, roots will be real, if

$d^2 + 4ac > 0$

Now, at least one of the two conditions will hold true since either $4ac$ will be greater than zero or less than zero or equal to zero.

$\therefore$ At least 2 real zeroes will be there.

11. Ans: b

$(x+y)\left(\frac{x}{y}\right) = \frac{1}{2}$ and $(x+y)\frac{x}{y} = \frac{-1}{2}$

Solving these two equations, the values of

$(x+y)$ and $\left(\frac{x}{y}\right)$ will be $(1, -1/2)$

When $x + y = 1$ and $\frac{x}{y} = -1/2$

$(x, y) = (2, -1)$

When $x + y = -1/2$ and $\frac{x}{y} = 1$

$(x, y) = \left(\frac{-1}{4}, -\frac{1}{4}\right)$

$\therefore$ Number of possible pairs = 2

12. Ans: a

$\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$

$\Rightarrow (x+1) + (x-1) - 2\sqrt{x^2-1} = 4x - 1$

$\Rightarrow -2\sqrt{x^2-1} = 2x - 1$

$\Rightarrow 4(x^2 - 1) = 4x^2 + 1 - 4x$

$\Rightarrow x = \frac{5}{4}$ which when put in the main equation does not satisfy it.

Hence, no solution is possible.

13. Ans: c

Let $\frac{x^2-x+1}{x^2+x+1}=y$

$x^2-x+1=y[x^2+x+1]$

$x^2-x+1=yx^2+yx+y$

$yx^2-x^2+yx+x+y-1=0$

$x^2[y-1]+x[y+1]+y-1=0$

For real values of $D^2 \geq 0$

Then, $b^2-4ac \geq 0 \rightarrow$

$(y+1)^2-4(y-1)^2 \geq 0\ (y^2+2y+1)-4(y^2-2y+1) \geq 0$

Or, $y^2+2y+1-4y^2+8y-4 \geq 0 - 3y^2+10y-3 \geq 0$

Or, $3y^2-10y+3 \leq 0$

Or, $3y^2-9y-y+3 \leq 0$

Or, $3y[y-3]-1[y-3] \leq 0\ (3y-1)(y-3) \leq 0$

Hence, $3y-1 \leq 0$ and $y-3 \leq 0$

$y \leq \frac{1}{3}$ and $y \leq 3$

Hence, maximum value of y is 3 and minimum value of y is 1/3.

14. Ans: a

$2\left[a^{1/3}+\frac{1}{a^{1/3}}\right]=5$

$\Rightarrow\ 2a^{2/3}-5a^{1/3}+2=0$

$\Rightarrow\ (a^{2/3}-2)(2a^{1/3}-1)=0$

$\therefore\ a^{1/3}=2, a^{1/3}=1/2$

$\Rightarrow\ a=8, a=1/8$

15. Ans: a

If roots are real and equal, then $D=0$

$D=[\sqrt{2}\,(p+q)]^2-4\,(p^2+q^2)\times 1$

$=2\,(p^2+q^2+2pq)-4\,(p^2+q^2)=-2\,(p^2+q^2-2pq)$

$=-2\,[(p-q)^2]=0$

Hence, $p=q$

16. Ans: c

Due to symmetry, we can say that the maximum value of $xy+yz+zx$ will be at $x=y=x$

Now, $x^2+y^2+z^2=1$

$\Rightarrow x=y=z=1/\sqrt{3}$

$\therefore xy+yz+zx \leq 1$ which is present only in one option.

17. Ans: a

Let the roots of the given equation be α and β.

Now, for roots $(\alpha-\beta)$, $(\beta-2)$, the equation can be deduced by replacing x with $(x+2)$.

$\therefore$ The deduced equation would be

$\Rightarrow (x+2)^2-(p+1)\,(x+2)+p^2+p-8=0$

$\Rightarrow\ x^2+(3-p)\,x+p^2-p-6=0$

$\Rightarrow\ x^2+(p-3)\,x+(p+2)\,(p-3)=0$

Now, $\alpha>2$ and $\beta<2$

$\therefore\ (\alpha-2)>0$ and $(\beta-2)<0$

$\therefore\ (\alpha-2)\,(\beta-2)<0 \Rightarrow (p+2)\,(p-3)<0$

$\therefore\ (\alpha-2)\,(\beta-2)<0 \Rightarrow (p+2)\,(p-3)<0$

$\therefore\ -2<p<3$

18. Ans: b

$$\alpha + \beta = \frac{3}{8}, \quad \alpha\beta = \frac{27}{8}$$

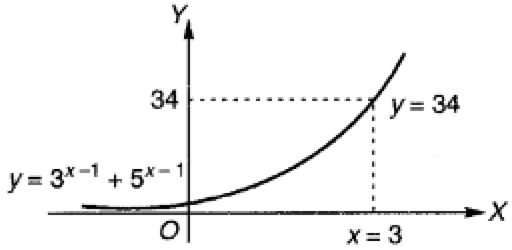
$$\therefore \quad \left(\frac{\alpha^2}{\beta}\right)^{1/3} + \left(\frac{\beta^2}{\alpha}\right)^{1/3} = \left(\frac{\alpha^3}{\alpha\beta}\right)^{1/3} + \left(\frac{\beta^3}{\alpha\beta}\right)^{1/3}$$

$$= \frac{\alpha + \beta}{(\alpha\beta)^{1/3}} = \frac{3/8}{(27/8)^{1/3}}$$

$$= \frac{3/8}{3/2} = \frac{1}{4}$$

19. Ans: b

It is very obvious that at $x = 3$ the given expression satisfies. Now, $y = 3^{x-1}$ and $y = 5^{x-1}$ are both increasing function of x (exponential functions with base greater than 1). Therefore their sum $y = 3^{x-1} + 5^{x-1}$ is also an increasing function of x. It means for $x < 3$, $y = 3^{x-1} + 5^{x-1} < 34$ and for $x > 3$, $y = 3^{x-1} + 5^{x-1} > 34$.



Thus, the equation has no other solution.

20. Ans: b

Let $f(x) = ax^2 + bx + c$. Since 1 lies outside the roots of $f(x) = 0$, So,

$af(1) > 0 \Rightarrow f(1) > 0 \qquad (\because a > 0)$

$\Rightarrow a + b + c > 0$

Exercise 05 (TITA or Short Answers)

1. Ans: c

The minimum value of $(p + 1/p)$ *is at* $p = 1$. The value is 2.

2. Ans: 2

$|x|^2 - 2|x| - 3 = 0$

$\Rightarrow (|x| - 3)(|x| + 1) = 0$

$\Rightarrow |x| = 3, -1$

$|x| = -1$ *is not possible*

$\Rightarrow |x| = 3$

$\Rightarrow x = \pm 3$

Therefore for the given equation only two real roots are possible.

3. Ans: c

Taking the values of A, B and C as 1, 2 and – 1.

We get $A^4 + B^4 + C^4 = 18$.

4. Ans: (b)

$x^2 + |x| - 6 = 0 \Rightarrow x^2 + x - 6 = 0$ *where* ≥ 0, *Therefore root* $= 2$

Else $x^2 - x - 6 = 0$ *if* $x < 0$, *Therefore* $= -2$

Hence, sum of roots = 0

5. Ans: a

$\alpha+\beta=\frac{1}{\alpha^2}+1/\beta^2$

$\Rightarrow \frac{-b}{a}=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}\Rightarrow -\frac{b}{a}=\frac{(\alpha+\beta)^2-2\propto\beta}{\alpha^2\beta^2}$

$\Rightarrow -\frac{b}{a}=\frac{\left(\frac{b^2}{a^2}\right)-\frac{2c}{a}}{\frac{c^2}{a^2}}$

$\Rightarrow \frac{b^2}{ac}+\frac{bc}{a^2}=2$

6. Ans: b

$p(x)=x^3-ax^2+bx+10$; since it is divisible by $(x+5)$

$\therefore\ p(-5)=0$

$\Rightarrow (-5)^3-25a-5b+10=0$

$\Rightarrow\ 5a+b=-23$ (i)

$\Rightarrow\ Q(x)=x^4+x^3+bx^2-ax+42=0$

$\therefore\ Q(3)=0$

$\Rightarrow\ 81+27+9b-3a+42=0$

$\Rightarrow\ a-3b=50$

7. Ans: c

$x^2+x+2=0$

$\therefore\ \alpha+\beta=-1$

$\alpha\beta=2$

Now, $\frac{\alpha^{10}+\beta^{10}}{\alpha^{-10}+\beta^{-10}}=\frac{\alpha^{10}+\beta^{10}}{\frac{\alpha^{10}+\beta^{10}}{(\alpha\beta)^{10}}}=(\alpha\beta)^{10}=2^{10}=1024$

8. Ans: b

$S-2=2\frac{1}{3}+2^{2/3}\Rightarrow (S-2)^2=2^{2/3}+2.2^{1/3}+2.2$

Now, required $=2^{2/3}-2.2^{1/3}+2.2-(2.2^{1/3}+2.2^{2/3})$
$-2=2$

9. Ans: b

$\text{Log}_4(x-1)=\log_2(x-3)$

$\Rightarrow\frac{1}{2}\log_2(x-1)=\log_2(x-3)$

$\Rightarrow (x-1)=(x-3)^2$

$\therefore\ x=5,2$

Now, $x=2$ is not possible as $\log(x-3)=\log(-1)$ is not possible.

10. Ans: a

$P(x)=x^4+2x^3+mx^2+nx+3$

Now, $P(2)=0$

$\Rightarrow 16+16+4m+2n+3=0$ (i)

$\Rightarrow 4m+2n+35=0$

and, $P(4)=0$

$\Rightarrow 256+128+16m+4n+3=0$

$\Rightarrow 16m+4n+387=0$ (ii)

Multiplying 5 in equation (i) and then subtracting from equation (ii)

$4m+6n-212=0$

$\therefore\ 2m+3n=106$

11. Ans: a

For the equations to have same pair of roots

$\frac{2p-1}{q+1} = \frac{2p+1}{4q+1} = \frac{c}{3c}$

$\therefore 3(2p-1) = q+1 \Rightarrow 6p - q = 4$

and, $3(2p+1) = 4q+1 \Rightarrow 6p - 4q = -1$

Solving two equations $q = 2$ and $p = 1$

$\therefore (p+q) = 3$

12. Ans: c

$x + \frac{1}{x} = 1 \Rightarrow x^2 - x + 1 = 0$

Now, as $x \neq -1 \Rightarrow (x+1)(x^2 - x + 1) = 0$

$\Rightarrow x^3 + 1 = 0$

$\therefore x^3 = -1 \Rightarrow x^{4000} = (x^3)^{1333}.\, x = -x$

$\therefore P = x^{4000} + \frac{1}{x^{4000}}$

$= -x - \frac{1}{x} = -1 \Rightarrow P = -1$

Now, let $n = 2$

$\therefore$ $p =$ unit digit of 17, that is, 7. So, $p + q = 7 - 1 = 6$

13. Ans: c

Let α, β be the roots of the equation, then

$$\alpha + \beta = \frac{1}{2}\alpha\beta$$

$$-\frac{b}{a} = \frac{1}{2} \cdot \frac{c}{a}$$

$\Rightarrow$ $$-b = \frac{c}{2}$$

$\Rightarrow$ $$(k+6) = \frac{2(2k-1)}{2}$$

$\Rightarrow$ $$k = 7$$

14. Ans: b

Let $$x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots \infty}}}}$$

$\therefore$ $$x = \sqrt{2 + x}$$

$\Rightarrow$ $$x^2 = 2 + x$$

$\Rightarrow$ $$x^2 - x - 2 = 0$$

$\Rightarrow$ $$x = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2} = 2, -1$$

$\because$ $$x > 0, \ \therefore x = 2$$

15. Ans: b

$2|x|^2 - 5|x| + 2 = 0$

$\Rightarrow (2|x| - 1)(|x| - 2) = 0$

$\therefore \ |x| = \frac{1}{2}, 2$

$\therefore \ x = \pm\frac{1}{2}, \pm 2$

16. Ans: a

$xy = 2(x+y) \Rightarrow y(x-2) = 2x$

$\therefore \ y = \frac{2x}{x-2}$ *but x, y* $\in N$ *by trial, we get* $x = 3, 4, 6$

$\therefore \ y = 6, 4, 3$ but $x \leq y$

$\therefore \quad x = 3, 4 \; and \; y = 6, 4$
Thus two solutions are possible

17. Ans: d

$$x = 7 + 4\sqrt{3}$$

$$\therefore \quad y = \frac{1}{7 + 4\sqrt{3}} = 7 - 4\sqrt{3}$$

$$\therefore \quad \frac{1}{x^2} + \frac{1}{y^2} = \frac{x^2 + y^2}{(xy)^2}$$

$$= \frac{(7 + 4\sqrt{3})^2 + (7 - 4\sqrt{3})^2}{[(7 + 4\sqrt{3})(7 - 4\sqrt{3})]^2}$$

$$= \frac{2(49 + 48)}{1} = 194$$

18. Ans: c

Putting $x = \frac{1}{y}, we\ get$

$$27y^3 + 54y^3 + cy - 10 = 0$$

This above eq. (i) must be in AP.
Let the roots of equation in y be

$$a - \beta, a, a + \beta \qquad (\because roots\ are\ in\ AP)$$

$$\therefore \sum a = a - \beta + a + a + \beta = 3a$$

$$\Rightarrow 3a = \frac{-54}{27} \Rightarrow a = \frac{-2}{3}$$

Now $a = \frac{-2}{3}$ $will\ satisfy\ the\ eq.(i) we\ get$

$$27 \times \frac{-8}{27} + 54 \times \frac{4}{9} - \frac{2c}{3} - 10 = 0$$

$$\Rightarrow c = 9$$

19. Ans: c

$$\log_{100} |x + y| = \frac{1}{2} \Rightarrow (100)^{1/2} = |x + y|$$

$$\Rightarrow \quad |x + y| = 10 \qquad \ldots(1)$$

Again, $\log_{10} y - \log_{10} |x| = \log_{100} 4$

$$\log_{10} y - \log_{10} |x| = \log_{10} 2$$

$$\Rightarrow \quad \log_{10} \frac{y}{|x|} = \log_{10} 2$$

$$\Rightarrow \quad y = 2|x| \qquad \ldots(2)$$

From eq. (2) we can conclude that y is always positive.
Now, when $x > 0$ and $y > 0$ (always)

$$|x + y| = 10 \Rightarrow |x + 2|x|| = 10$$

$$\Rightarrow \quad x + 2|x| = 10 \qquad (\because x > 0)$$

$$\Rightarrow \quad x + 2x = 10$$

$$\Rightarrow \quad x = \frac{10}{3}$$

$$\therefore \quad y = \frac{20}{3}$$

Again, $x < 0$ and $y > 0$ (always positive)

$$|-x + 2|-x|| = 10$$

$$\Rightarrow \quad |-x + 2x| = 10$$

$$\Rightarrow \quad |x| = 10$$

$$\Rightarrow \quad x = -10 \qquad (\because x < 0)$$

$$\therefore \quad y = 20$$

Hence, $x = -10, \; y = 20$ and $x = \frac{10}{3}$ and $y = \frac{20}{3}$.

20. Ans: c

The given equation is $|x-2|^2+|x-2|-2=0$.

Let us assume $|x-2|=m$

Then $m^2+m-2=0$

$(m-1)(m+2)=0$

Only admissible value is

$m=1 \quad (\because m \neq -2 as\ m \geq 0)$

$\therefore \ |x-2|=1$

$\Rightarrow \ x-2=1 \Rightarrow x=3$

Or $-(x-2)=1 \Rightarrow x=1$

Hence, $x=1,3$

$\therefore$ Sum of the roots of equation $=1+3=4$.

21. Ans: b

Best way is to go through options.

Consider option (b)

$|3^4-1|^{log_3{}^{(3^4)^2}-2log_{81}{}^9}=(3^4-1)^7$

$|80|^{log_3{}^{3^8-log_{81}81}}=(80)^7$

$\Rightarrow \ log_3 3^8-log_{81}81=7$

$\Rightarrow \ 8-1=7$

$7=7$

Hence option (b) is correct.

22. Ans : b

23. Ans: d

24. Ans: d

25. Ans: b