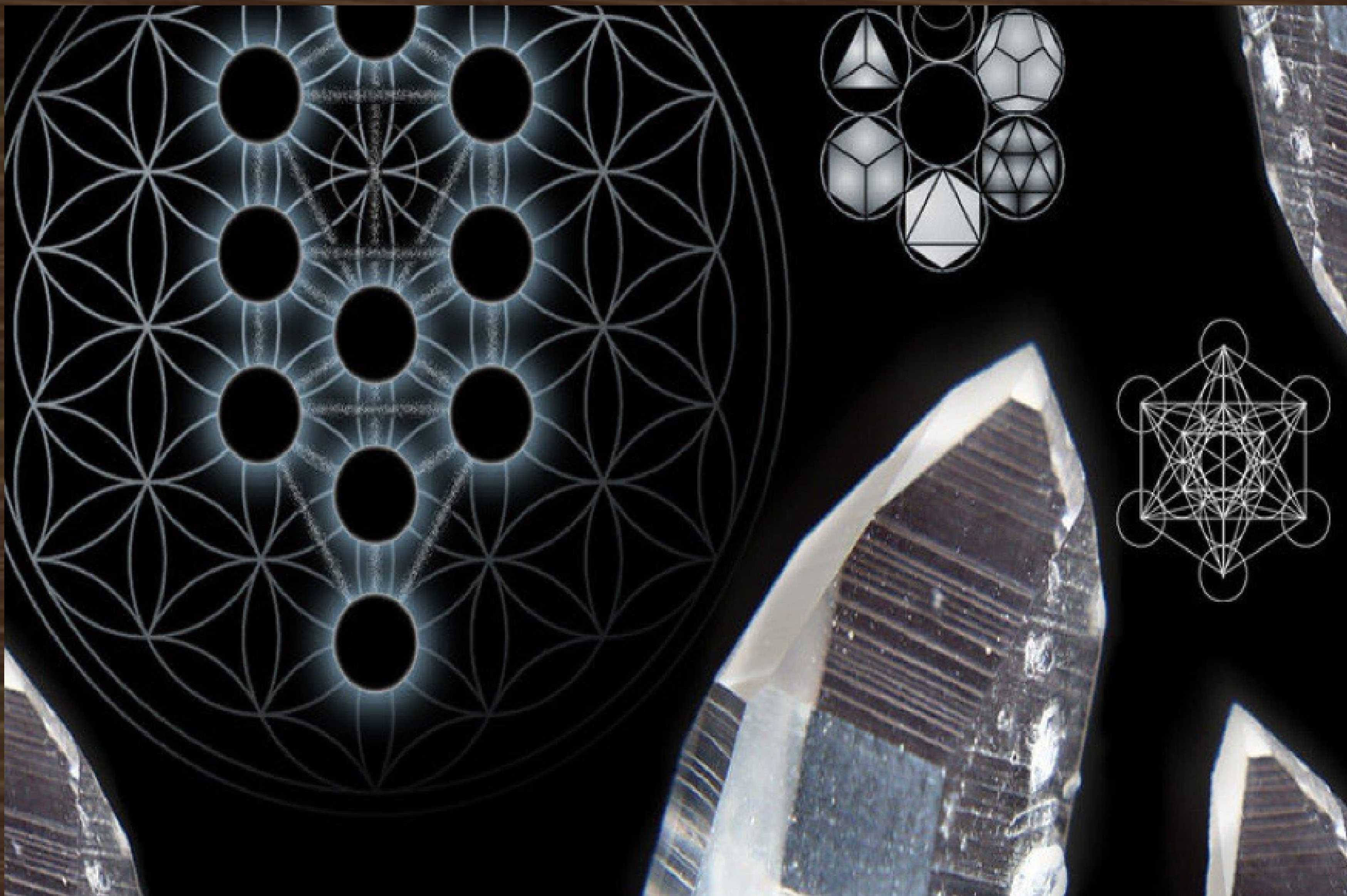

Geometry

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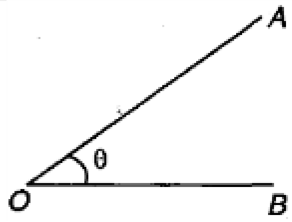
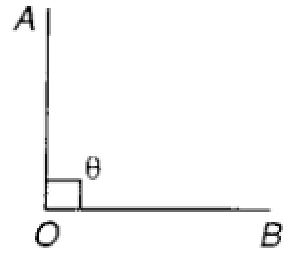
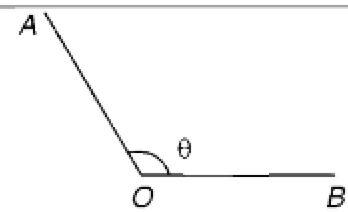
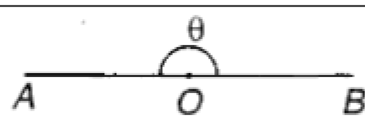
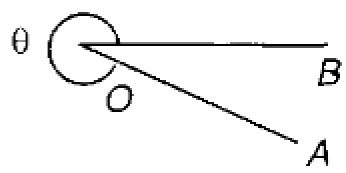
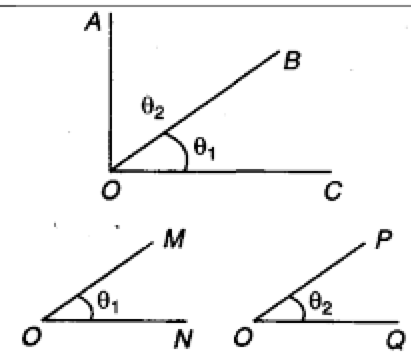
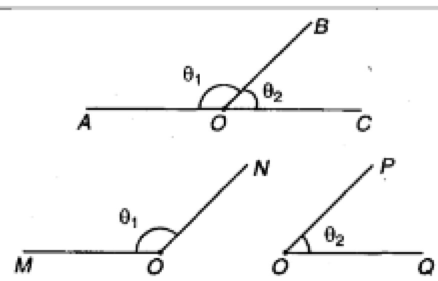
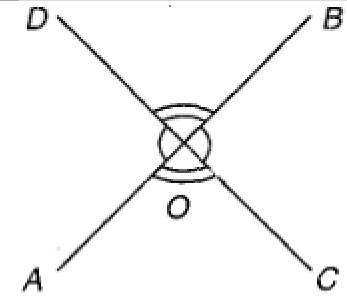
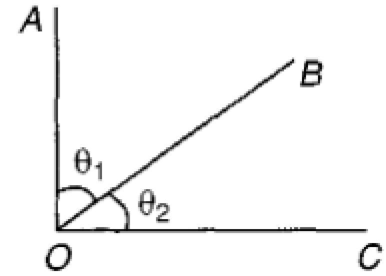
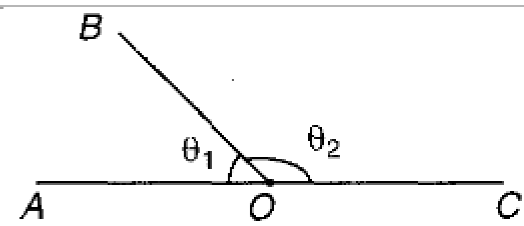
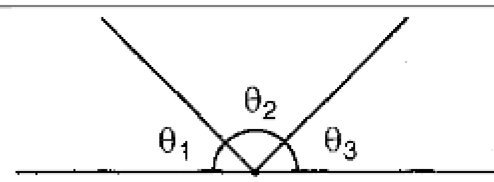
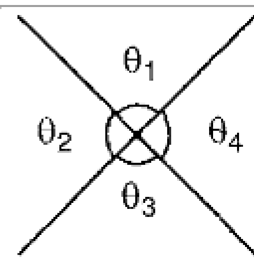
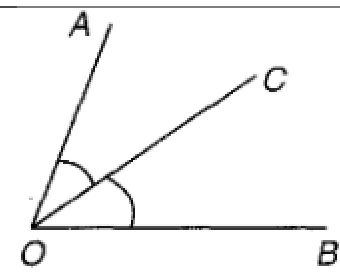
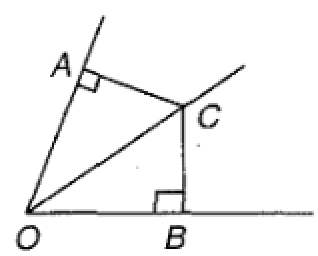
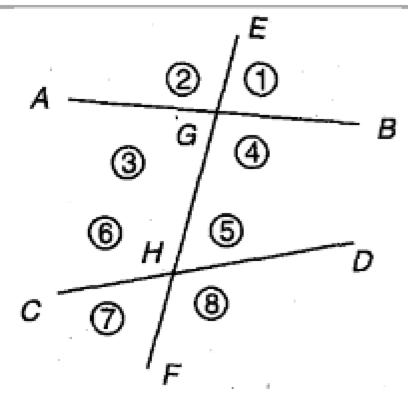
Basic of Geometry

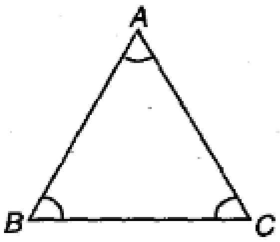
Types of Angles	Property	Diagram
Acute	$0° < \theta < 90°$	
Right	$\theta = 90°$ $(\angle AOB \; is \; a \; right \; angle)$	
Obtuse	$90° < \theta < 180°$	
Straight	$\theta = 180°$ $(\angle AOB \; is \; a \; straight \; angle)$	
Reflex	$180° < \theta < 360°$	
Complementary	$\theta_1 + \theta_2 = 90°$ Two angles whose sum is 90°, are complementary to each other	
Supplementary	$\theta_1 + \theta_2 = 180°$ Two angles whose sum is 180°, are supplementary to each other.	

Vertically opposite	$\angle DOA = \angle BOC$ and $\angle DOB = \angle AOC$	
Adjacent angles	$\angle AOB$ and $\angle BOC$ are adjacent angles Adjacent angles must have a common side. (e.g., OB)	
Linear pair	$\angle AOB$ and $\angle BOC$ are linear pair angles. One side must be common (e.g., OB) and these two angles must be supplementary	
Angles on the one side of a line	$\theta_1 + \theta_2 + \theta_3 = 180°$	
Angles round the point	$\theta_1 + \theta_2 + \theta_3 + \theta_4 = 360°$	
Angles bisector	OC is the angles bisector of $\angle AOB$. i.e., $\angle AOC = \angle BOC = \frac{1}{2}(\angle AOB)$ When a line segment divides an angle equally into two parts, then it is said to be the angle bisector (OC)	 (Angle bisector is equidistant from the two sides of the angle) i.e.,  $AC = BC$

Corresponding angles	When two lines are intersected by a transversal, then they form four pairs of corresponding angles (a) $\angle AGE, \angle CHG \Rightarrow (\angle 2, \angle 6)$ (b) $\angle AGH, \angle CHF \Rightarrow (\angle 3, \angle 7)$ (c) $\angle EGB, \angle GHD \Rightarrow (\angle 1, \angle 5)$ (d) $\angle BGH, \angle DHF \Rightarrow (\angle 4, \angle 8)$	
Exterior angles	These are following four angles (i) $\angle AGE \Rightarrow \angle 2$ (ii) $\angle CHF \Rightarrow \angle 7$ (iii) $\angle EGB \Rightarrow \angle 1$ (iv) $\angle DHF \Rightarrow \angle 8$	
Interior angles	These are following four angles (i) $\angle AGH \Rightarrow \angle 3$ (ii) $\angle GHC \Rightarrow \angle 6$ (iii) $\angle BGH \Rightarrow \angle 4$ (iv) $\angle DHG \Rightarrow \angle 5$	
Alternate angles	These are two pairs of angles as following: (i) $\angle AGH, \angle GHD\ (\angle 3, \angle 5)$ (ii) $\angle GHC, \angle BGH\ (\angle 6, \angle 4)$	

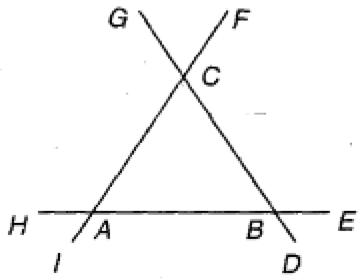
Triangles

Triangle : A three sided closed plane figure, which is formed by joining the three non-collinear points, is called as a triangle. It is denoted by the symbol Δ.



In the above Δ $(triangle)$ ABC, A, B and C are three vertices, line segments AB, BC and AC are the three sides of the triangle. $\angle A, \angle B$ and $\angle C$ are the three interior angles of a triangle ABC.

In the adjoining figure $\angle FCB, \angle CBE, \angle ABD, \angle IAB, \angle HAC, \angle GCA$ are the exterior angles of the ΔABC.

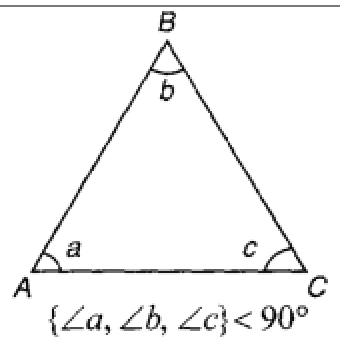
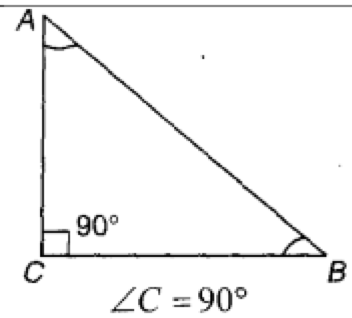


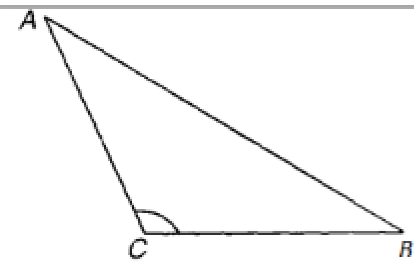
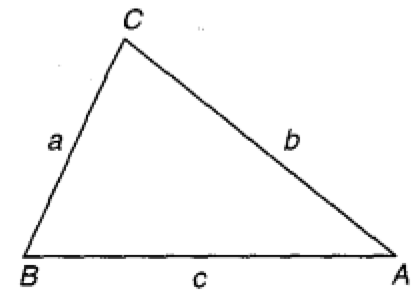
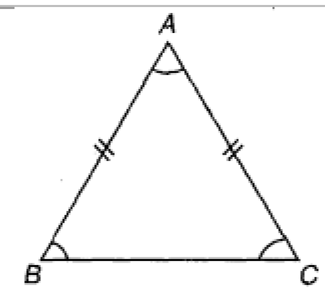
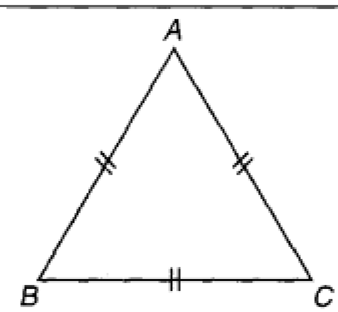
- Sum of the three interior angles of a triangle is always 180°.
- Exterior angle = Sum of two interior opposite angles e.g., $\angle CBE = \angle CAB + \angle BCA$

Perimeter of triangle is equal to sum of all the three sides i.e., $a + b + c$

Semi perimeter of a triangle is half of the perimeter i.e., $s = \frac{a+b+c}{2}$, a, b, c are the length of three sides of a triangle.

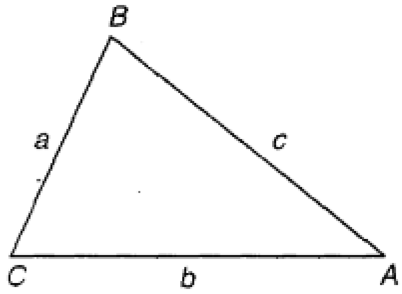
(A) According to interior angles

Types of Triangles	Property/Definition	Diagram
Acute angle triangle	Each of the angle of a triangle is less than 90° $i.e., a < 90°, b < 90°, c < 90°$	
Right angled triangle	One of the angle is equal to 90°, then it is called as right angled triangle. Rest two angles are complementary to each other.	

Obtuse angle triangle	One of the angle is obtuse (i.e., greater than 90°), then it is called as obtuse angle triangle	 $\angle C > 90°$
Scalene triangle	A triangle in which none of the three sides is equal is called a scalene triangle (all the three angles are also different).	 $a \neq b \neq c$
Isosceles triangle	A triangles in which at least two sides are equal is called as isosceles triangle. In this triangle, the angles opposite to the congruent sides are also equal.	 $AB = AC$ $\angle B = \angle C$
Equilateral triangle	A triangle in which all the three sides are equal is called as, an equilateral triangle. In this triangle each angle is congruent and equal to 60°.	 $AB = BC = AC$ $\angle A = \angle B = \angle C = 60°$

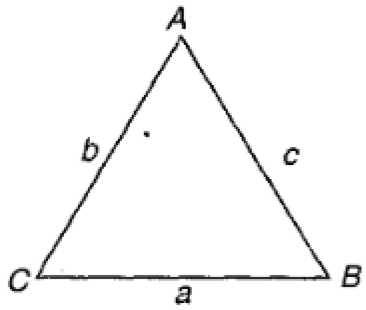
Fundamental Properties of Triangles

1. Sum of any two sides is always greater than the third side.
2. The difference of any two sides is always less than the third side.
3. Greater angle has a greater side opposite to it and smaller angle has a smaller side opposite to it *i.e.,* if two sides of a triangle are not congruent then the angle opposite to the greater side is greater.
4. Let a, b and c be the three sides of a ΔABC and c is the largest side. Then



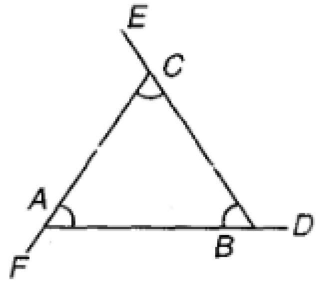
(i) If $c^2 < a^2 + b^2$, the triangle is acute angle triangle
(ii) If $c^2 = a^2 + b^2$, the triangle is right angled triangle
(iii) If $c^2 > a^2 + b^2$, the triangle is obtuse angle triangle

5. **Sine rule:** In a ΔABC, if a, b, c be the three sides opposite to the angles A, B, C respectively, then $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

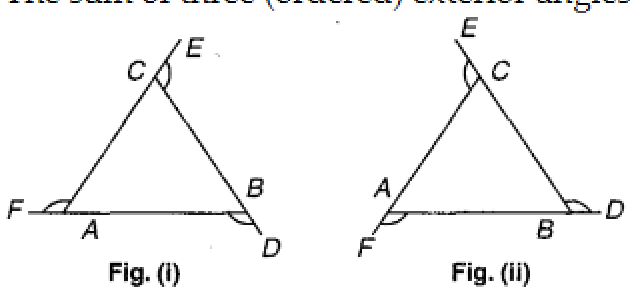


6. **Cosine rule:** In a $\Delta ABC, if\ a, b, c$ be the sides opposite to angle $A, B\ and\ C$ respectively, then
$\cos A = \frac{b^2+c^2-a^2}{2bc}$
$\cos B = \frac{c^2+a^2-b^2}{2ca}$
$\cos C = \frac{a^2+b^2-c^2}{2ab}$

7. The sum of all the three interior angles is always 180°
i.e., $\angle CAB + \angle ABC + \angle BCA = 180°$



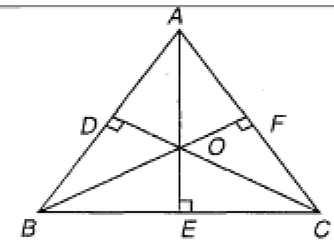
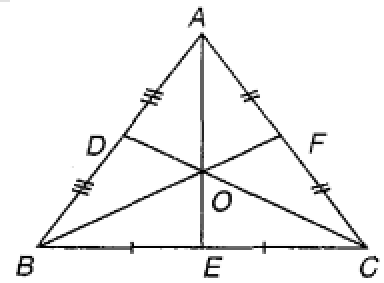
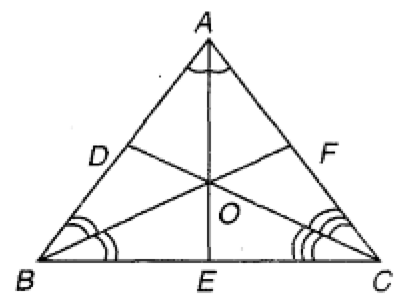
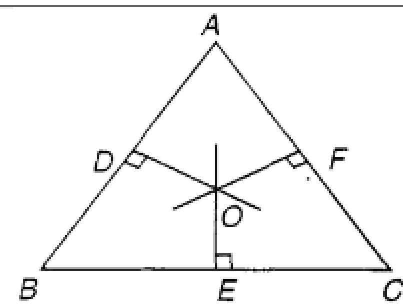
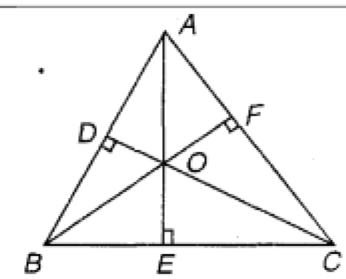
8. The sum of three (ordered) exterior angles of a triangle is 360°

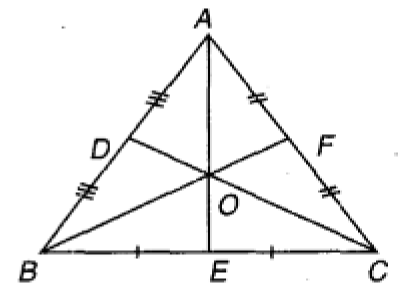
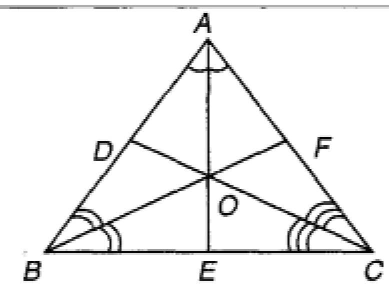
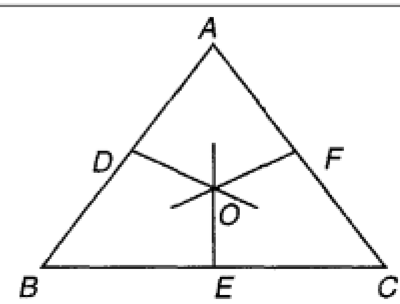


In fig. (i) : $(\angle FAC + \angle ECB + \angle DBA) = 360°$
In fig. (ii): $(\angle FAB + \angle DBC + \angle ECA) = 360°$

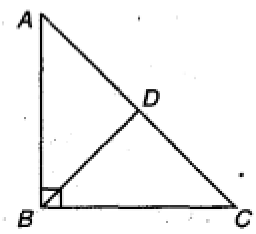
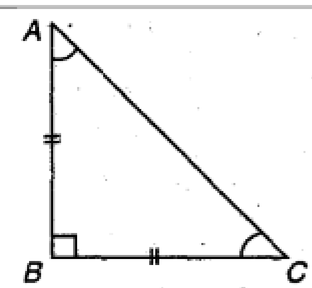
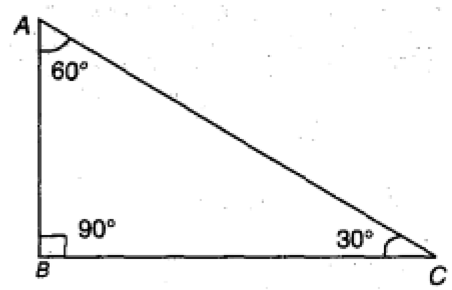
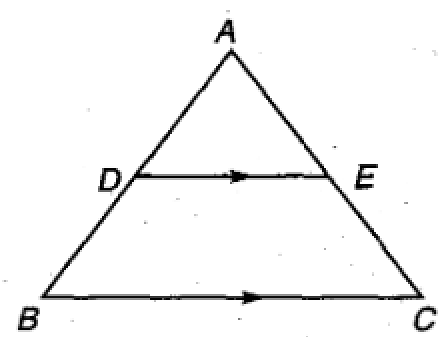
9. The sum of an interior angle and its adjacent exterior angle is 180°
10. A triangle must has at least two acute angles
11. The measure of an exterior angle of a triangle is greater than the measure of each of the opposite interior angles.

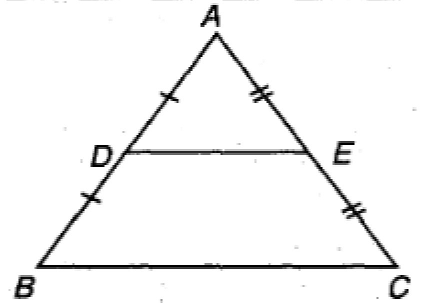
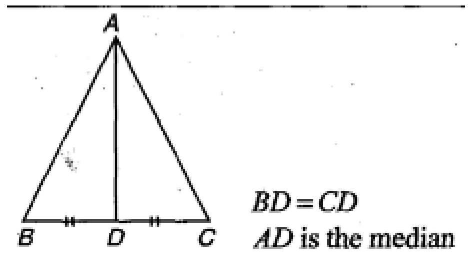
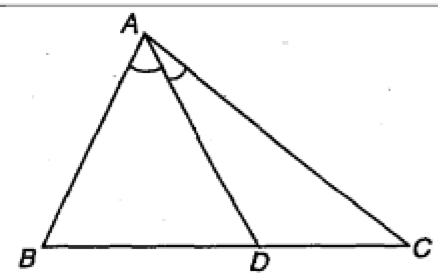
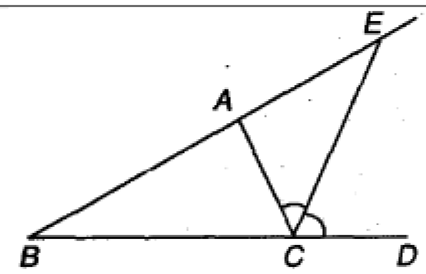
Some Important Definitions

Nomenclature	Property/Definition	Diagram
Altitude (or height)	The perpendicular drawn from the opposite vertex of a side in a triangle is called an altitude of the triangle. ➢ There are three altitudes in a triangle.	 AE, CD *and* BF are the altitudes
Median	The line segment joining the mid- point of a side to the vertex opposite to the side is called a median. ➢ There are three medians in a triangle ➢ A median bisects the area of the triangle i.e., $A(ABE) = A(AEC) = \frac{1}{2}A(\Delta ABC)$ *etc.*	 $(AE, CD$ *and* BF *are the medians*) $BE = CE, AD = BD, AF = CF$
Angle bisector	A line segment which originates from a vertex and bisects the same angle is called an angle bisector. $\left(\angle BAE = \angle CAE = \frac{1}{2}\angle BAC\right)$ *etc.*	 AE, CD *and* BF *are the angle bisectors.*
Perpendicular bisector	A line segment which bisects a side perpendicularly (i.e, at right angle) is called a perpendicular bisector of a side of triangle. ➢ All points on the perpendicular bisector of a line are equidistant from the ends of the line.	 DO, EO *and* FO are the perpendicular bisectors.
Orthocenter	The point of intersection of the three altitudes of the triangle is called as the orthocenter. $\angle BOC = 180 - \angle A$ $\angle COA = 180 - \angle B$ $\angle AOB = 180 - \angle C$	 *'O' is the orthocentre*

Types of Triangles	Property/ Definition	Diagram
Centroid	The point of intersection of the three medians of a triangle is called the centroid. A centroid divides each median in the ratio 2 : 1 (vertex : base) $\frac{AO}{OE}=\frac{CO}{OD}=\frac{BO}{OF}=\frac{2}{1}$	 'O' is the centroid.
Incentre	The point of intersection of the angle bisectors of a triangle is called the incentre. Incentre O is the always equidistant from all three sides i.e., the perpendicular distance between the sides and incentre is always same for all the three sides.	 'O' is the incentre.
Circumcentre	The point of intersection of the perpendicular bisectors of the sides of a triangle is called the Circumcentre. $OA = OB = OC = (circum\ radius)$ Circumcentre O is always equidistant from all the three vertices $A, B\ and\ C$.	 'O' is the circumcentre.

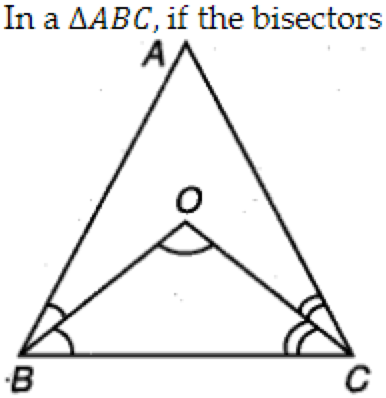
Important Theorems on Triangles

Theorem	Statement/Explanation	Diagram
Pythagoras theorem	The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides, i.e., $(AC)^2 = (AB)^2 + (BC)^2$ ➢ The converse of this theorem is also true. ➢ The numbers which satisfy this relation, are called Pythagorean triplets. e.g., (3,4,5), (5, 12,13), (7,24,25), (8,15, 17), (9,40,41), (11,60,61), (12,35,37)(16,63,65), (20,21,29), (28,45,53), (33,56,65)	 $\angle B = 90°$ $\overline{AC} \rightarrow$ Hypotenuse $AD = CD = BD$ (D is the mid-point of AC)
$45° - 45° - 90°$ Triangle theorem	If the angles of a triangle are $45°, 45°$ *and* $90°$, then the hypotenuse (i.e., longest side) is $\sqrt{2}$ times of any smaller side. ➢ Excluding hypotenuse rest two sides are equal. i.e., $AB = BC \text{ and } AC = \sqrt{2}AB = \sqrt{2}BC$	 $\angle A = 45°$ $\angle B = 90°$ $\angle C = 45°$
$30° - 60° - 90°$ triangle theorem	If the angles of triangle are $30°, 60°$ *and* $90°$, then the sides opposite to $60°$ *is* $\frac{\sqrt{3}}{2}$ times the hypotenuse. e.g., $AB = \frac{AC}{2} \text{ and } BC = \frac{\sqrt{3}}{2}AC$ $\therefore\ AB : BC : AC = 1 : \sqrt{3} : 2$	
Basic proportionality theorem (BPT) or Thales theorem	Any line parallel to one side of a triangle divides the other two sides proportionally. So if DE is drawn parallel to BC, it would divide sides AB *and* AC proportionally i.e., $\frac{AD}{DB} = \frac{AE}{EC} \text{ or } \frac{AD}{AB} = \frac{AE}{AC}$	

Mid- point theorem	If the mid-points of two adjacent sides of a triangle are joined by a line segment, then this segment is parallel to the third side. i.e., if $AD = BD$ and $AE = CE$ $then$ $DE\backslash\backslash BC$	
Apollonius theorem	In a triangle, the sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and square of half the third side. i.e., $AB^2 + AC^2 = 2(AB^2 + BD^2)$	
Interior angle bisector theorem	In a triangle the angle bisector of an angle divides the opposite side to the angle in the ratio of the remaining two sides. i.e., $\frac{BD}{CD} = \frac{AB}{AC}$ and $BD \times AC - CD \times AB = AD^2$	
Exterior angle bisector theorem	In a triangle the angle bisector of any exterior angle of a triangle divides the side opposite to the external angle in the ration of the remaining two sides i.e., $\frac{BE}{AE} = \frac{BC}{AC}$	

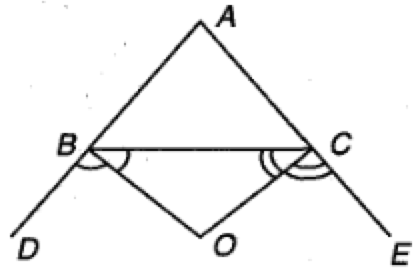
Some Useful Results

1. In a ΔABC, if the bisectors of $\angle B$ and $\angle C$ meet at O $then$



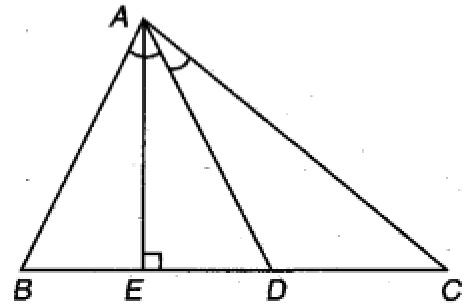
$$\angle BOC = 90^\circ + \frac{1}{2}\angle A$$

2. In a ΔABC, if sides AB and AC are produced to D and E respectively and the bisectors of $\angle DBC$ and $\angle ECB$ intersect at O, then

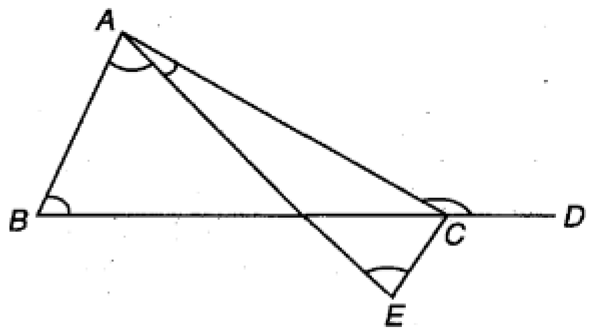


$\angle BOC = 90° - \frac{1}{2}\angle A$

3. In a ΔABC, if AD is the angle bisector of $\angle BAC$ and $\angle AE \perp BC$, the $\angle DAE = \frac{1}{2}(\angle ABC - \angle ACB)$

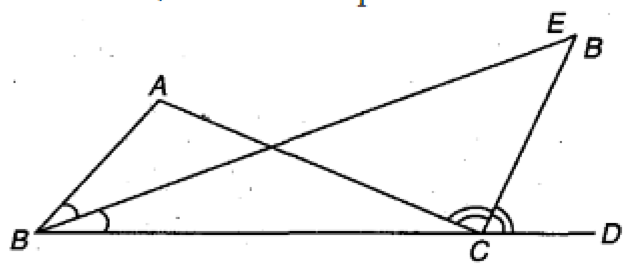


4. In a ΔABC, if BC is produced to D and AE is the angle bisector of $\angle A$, then



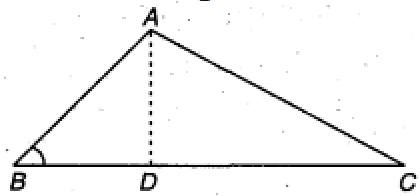
$\angle ABC + \angle ACD = 2\angle AEC.$

5. In a ΔABC, if side BC is produced to D and bisectors of $\angle ABC$ and $\angle ACD$ meet at E, then



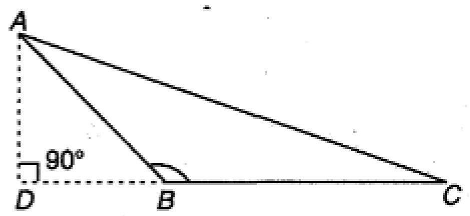
$$\angle BEC = \frac{1}{2}\angle BAC$$

6. In an acute angle ΔABC, AD is a perpendicular dropped on the opposite side of $\angle A$,then



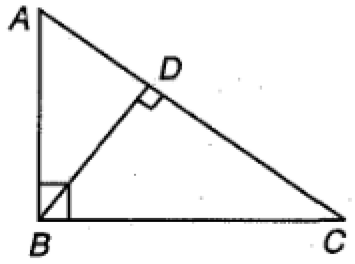
$AC^2 = AB^2 + BC^2 - 2BD \cdot BC \quad (\angle B < 90°)$

7. In an obtuse angle ΔABC, AD is perpendicular dropped on BC. BC is produced to D to meet AD, then



$AC^2 = AB^2 + BC^2 + 2\,BD \cdot BC \quad (\angle B > 90°)$

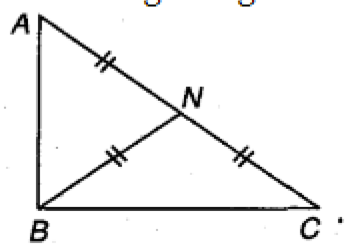
8. In a right angle ΔABC, $\angle B = 90°$ *and* AC is hypotenuse. The perpendicular BD is dropped on hypotenuse AC from right angle certex B, then



(i) $BD = \frac{AB \times BC}{AC}$ (ii) $AD = \frac{AB^2}{AC}$

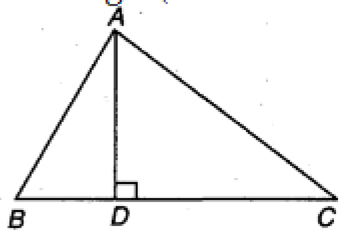
(iii) $CD = \frac{BC^2}{AC}$ (iv) $\frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$

➢ In a right angled triangle, the median to the hypotenuse $= \frac{1}{2} \times hypotenuse$



i.e., $BN = \frac{AC}{2}$ (as per the fig.)

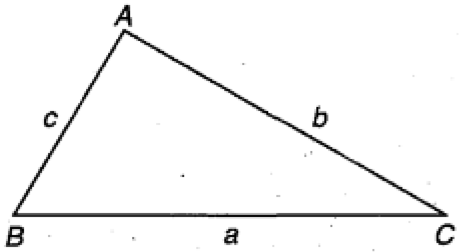
9. Area of a triangle (General formula)



$A(\Delta) = \frac{1}{2} \times \text{base} \times \text{height}$

$A(\Delta) = \frac{1}{2} \times BC \times AD$ (as per the figure.)

10. Area of scalene triangle $= \sqrt{s(s-a)(s-b)(s-c)}$



Also, $A(\Delta) = r \times s = \frac{abc}{4R}$

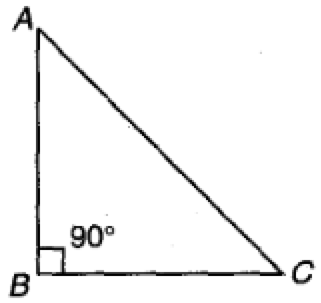
Where a, b and c are the sides of the triangle.

$s \rightarrow semiperimeter = \frac{a+b+c}{2}$

$r \rightarrow inradius$

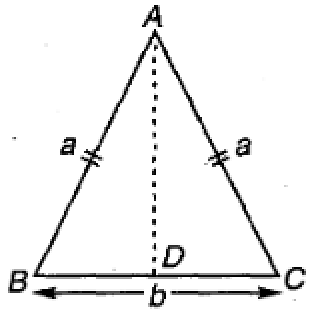
$R \rightarrow circumradius$

11. Area of right angle triangle
$= \frac{1}{2} \times base \times height$
$= \frac{1}{2} \times BC \times AB$



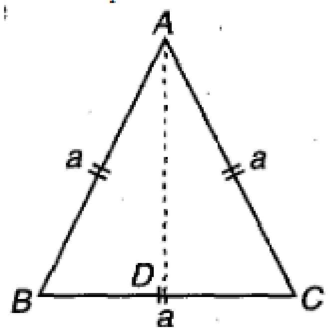
(as per the figure)

12. Area of an isosceles triangle
$= \frac{b}{4}\sqrt{4a^2 - b^2}$
$AB = AC \text{ and } \angle B = \angle C$

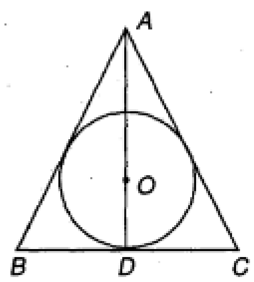


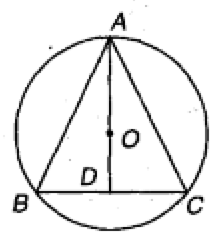
$\Delta ABD \cong \Delta ACD$
$(AD \to Angle\ bisector, median, altitude\ and\ perpendicular\ bisector)$

13. Area of an equilateral triangle
$= \frac{\sqrt{3}}{4}a^2$



$\left[A(\Delta) = \frac{1}{2}BC \times AD = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{4}a^2\right]$
$(a \to each\ side\ of\ the\ triangle)$
$AD \to Altitude, median, angle\ bisector\ and\ perpendicular\ bisector\ also.$
Inradius: $\frac{1}{3} \times height = \frac{side}{2\sqrt{3}}, OD \to Inradius$





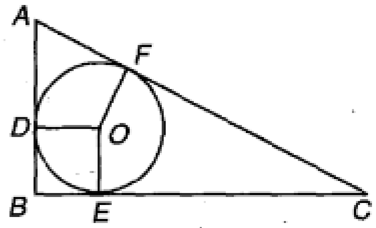
Circumradius $= \frac{2}{3} \times height = \frac{side}{\sqrt{3}}$
$OA \to Circumradius$

Note: *In equilateral triangle orthocenter centroid, incentre are Circumcentre coincide at the same point.*

- Circumradius $= 2 \times inradius$
- For the given perimeter of a triangle, the area of equilateral triangle is maximum.

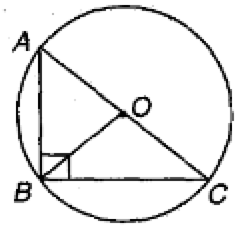
- For the given area of a triangle, the perimeter of equilateral triangle is minimum.

14. In a right angled triangle
(i) Inradius $(r) = \frac{AB+BC-AC}{2}$
(ii) Inradius $(r) = \frac{Area}{Semiperimeter}$



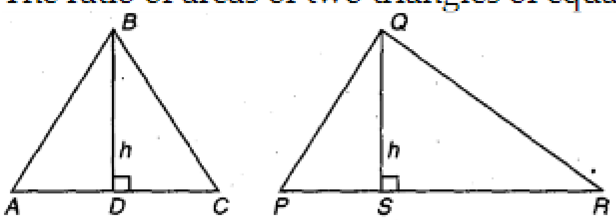
$DO = EO = FO\ (r)$

(iii) Circumradius $(R) = \frac{AC}{2} = \left(\frac{hypotenus}{2}\right)$



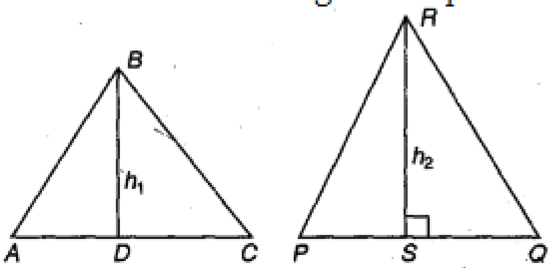
$AO = CO = BO = (R)$
AC is the diameter.

15. The ratio of areas of two triangles of equal heights is equal to the ratio of their corresponding bases. i.e.,



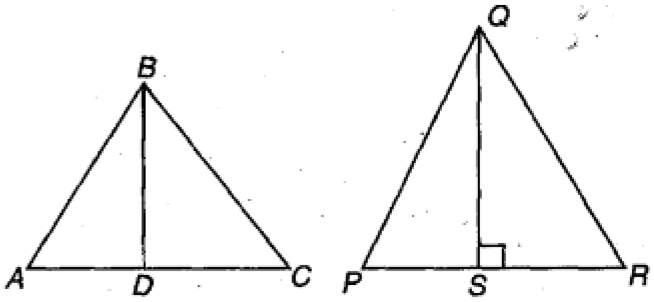
$$\frac{A\,(\Delta ABC)}{A\,(\Delta PQR)} = \frac{AC}{PR}$$

16. The ratio of areas of triangles of equal bases is equal to the ratio of their heights.



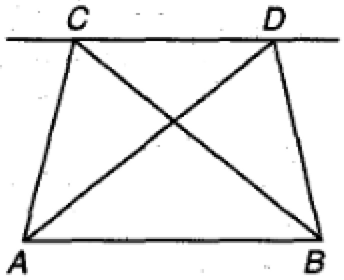
i.e., $$\frac{A\,(\Delta ABC)}{A\,(\Delta PQR)} = \frac{BD}{RS}$$

17. The ratio of the areas of two triangles is equal to the ratio of the products of base and its corresponding height i.e.,



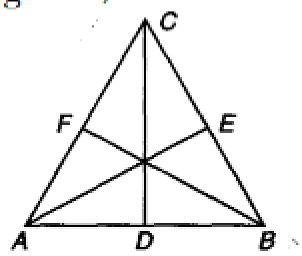
$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AC\times BD}{PR\times QS}$$

18. If the two triangles have the same base and lie between the same parallel lines (as shown in figure), then the area of two triangles will be equal.



i.e., $A(\Delta ABC) = A(\Delta ADB)$

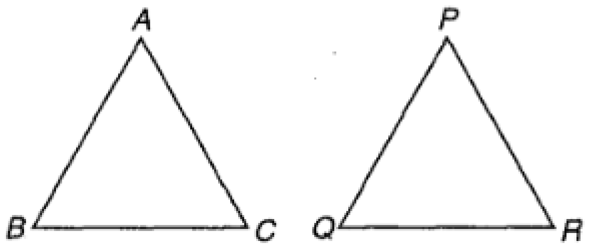
19. In a triangle AE, CD and BF are the medians then



$$3(AB^2 + BC^2 + AC^2) = 4(CD^2 + BF^2 + AE^2)$$

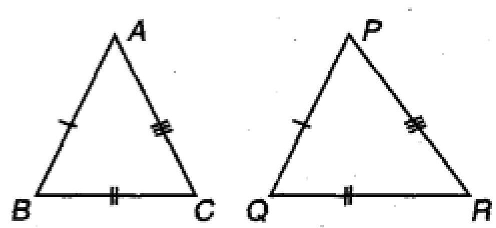
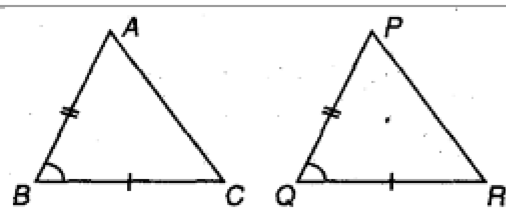
Congruency of triangles: Two triangles are said to be congruent if they are equal in all respects. i.e.,

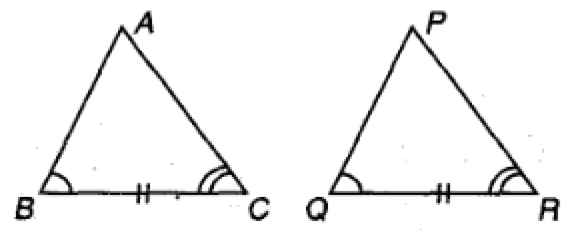
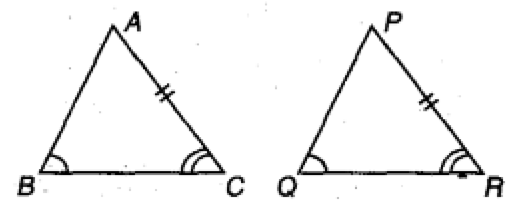
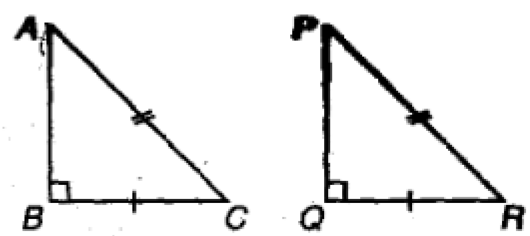
1. Each of the three sides of one triangle must be equal to the three respective sides of the other.
2. Each of the three angles of the one triangle must be equal to the three respective angles of the other.



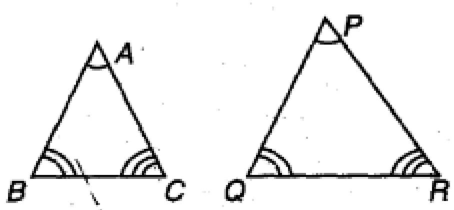
$$i.e., \quad \left.\begin{array}{l} AB = PQ \\ AC = PR \\ BC = QR \end{array}\right\} \text{ and } \left.\begin{array}{l} \angle A = \angle P \\ \angle B = \angle Q \\ \angle C = \angle R \end{array}\right\}$$

Tests of congruency: With the help of the following given tests, we can deduce without having detailed information about triangles that whether the given two triangles are congruent or not.

Test	Property	Diagram
$S-S-S$	(Side – Side – Side) If the three sides of one triangle are equal to the corresponding three sides of the other triangle, then the two triangles are congruent. $AB \cong PQ, AC \cong PR, BC \cong QR$ $\therefore\ \Delta ABC = \Delta PQR$	
$S-A-S$	$(Side - Angle - Single)$ If two sides and the angle included between them are congruent to the corresponding sides and the angle included between them, of the other triangle then the two	

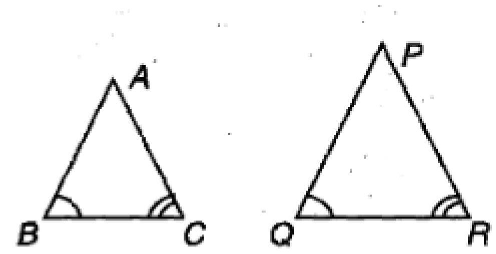
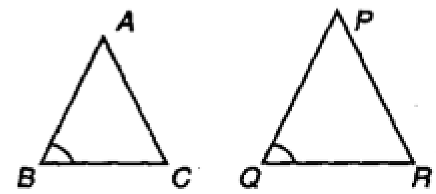
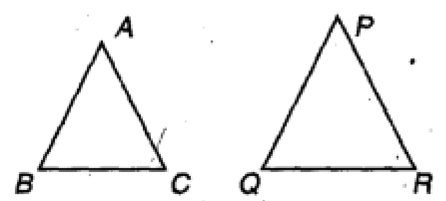
	triangles are congruent. $AB \cong PQ, \angle ABC \cong \angle PQR, BC \cong QR$ $\Delta ABC \cong \Delta PQR$	
$A-S-A$	(Angle – Side – Angle) If two angles and the included side of a triangle are congruent to the corresponding angles and the included side of the other triangle, then the two triangles are congruent. $\angle ABC \cong \angle PQR, BC \cong QR, \angle ABC \cong \angle PRQ$ $\Delta ABC \cong \Delta PQR$	
$A-A-S$	(Angle-Angle-Side) If two angles and a side other than the included side of a one triangle are congruent to the corresponding angles and a corresponding side other than the included side of the other triangle, then the two triangles are congruent. $\angle ABC \cong \angle PQR, \angle ACB \cong \angle PRQ$ And, $AC \cong PR \quad (or\ AB \cong PQ)$	
$R-H-S$	(Right angle-Hypotenuse-Side) If the hypotenuse and one side of the right angled triangle are congruent to the hypotenuse and a corresponding side of the other right angled triangle, then the two given triangles are congruent. $AC \cong PR, \angle B = \angle Q\ and\ BC \cong QR$ $\therefore\ ABC \cong \Delta PQR$	

Similarity of triangles: Two triangles are said to be similar if the corresponding angles are congruent and their corresponding sides are in proportion. The symbol for similarity is$'\sim'$.
If $\Delta ABC - \Delta PQR\ then$
$\angle ABC \cong \angle PQR, \angle BCA \cong \angle QRP, \angle BAC \cong QPR$



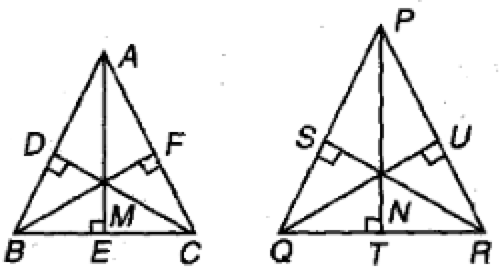
Tests for Similarity: Through the tests for similarity we can deduce the similarity of triangles with minimum required information.

Test	Property/Definition	Diagram

$A-A$	Angle-Angle If the two angles of one triangle are congruent to the corresponding two angles of the other triangle, then the two triangles are said to be similar $\angle ABC \cong \angle PQR$ $\angle ACB \cong \angle PRQ$ $\Delta ABC \sim \Delta PQR$	
$S-A-S$	Side-Angle-Side If the two sides of one triangle are proportional to the corresponding two sides of the other triangle and the angle included by them are congruent, then the two triangles are similar. i.e., $\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle ABC = \angle PQR$ $\therefore \ \Delta ABC \sim \Delta PQR$	$\frac{AB}{PQ} = \frac{BC}{QR} = K$ (K is any constant)
$S-S-S$	Side-Side-Side If the three sides of one triangle are proportional to the corresponding three sides of the other triangle, then the two triangles are similar, *i.e.,* $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ $\therefore \ \Delta ABC \sim \Delta PQR$	$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = K$

Properties of Similar Triangles

If the two triangles are similar, then for the proportional/corresponding sides we have the following results.



1. Ratio of sides = Ratio of heights (altitudes)
   = Ratio of medians
   = Ratio of angle bisectors
   = Ratio of inradii
   = Ratio of circumradii

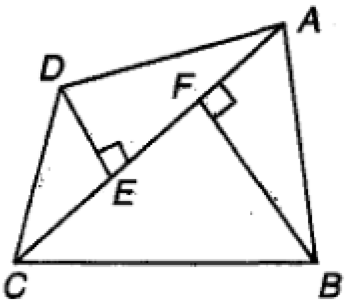
2. Ratio of areas = Ratio of squares of corresponding sides. i.e., if $\Delta ABC \sim \Delta PQR$, then
   $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$
   Note: Rule 1 can also apply with rule 2.

Quadrilaterals

A four sided closed figure is called a quadrilateral.

Properties

1. Sum of four interior angles is 360°.
2. The figure formed by joining the mid-points of a quadrilateral is a parallelogram.



3. The sum of opposite sides of a quadrilateral circumscribed, about a circle, is always equal.
4. Area of quadrilateral $= \frac{1}{2} \times$ one of the diagonals $\times$ sum of the perpendiculars drawn to the diagonals from the opposite vertices.

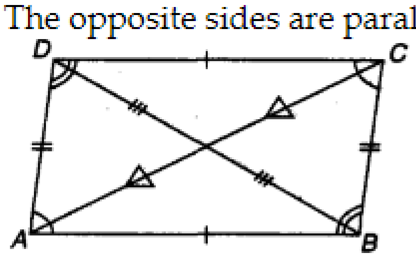
   i.e., $A(quadrilateral\ ABCD) = 1/2 \times AC \times (DE + BF)$

Different Types of Quadrilaterals are given below:

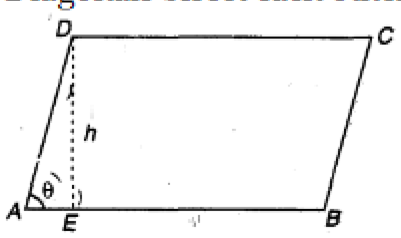
Parallelogram: A quadrilateral whose opposite sides are parallel

Properties

1. The opposite sides are parallel and equal.



2. Opposite angles are equal.
3. Sum of any two adjacent angles is 180°
4. Diagonals bisect each other



5. Diagonals need not be equal in length.
6. Diagonals need not bisect at right angle.
7. Diagonals need not bisect angles at the vertices.
8. Each diagonal divides a parallelogram into two congruent triangles.
9. Lines joining the mid-points of the adjacent sides of a **quadrilateral** form a **parallelogram.**
10. Lines j oining the mid-points of the adjacent sides of a **parallelogram is a parallelogram.**
11. The parallelogram that is inscribed in a circle is a rectangle.
12. The parallelogram that is circumscribed about a circle is a rhombus.
13. (a) Area of a parallelogram = base x height

    (b) Area of parallelogram

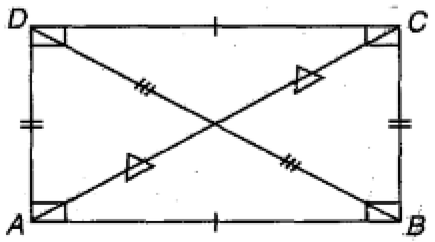
    = product of any two adjacent sides x sine of the included angles. $= AB \times AD \times \sin\theta$
14. Perimeter of a parallelogram *-2* (sum of any two adjacent sides)
15. $(AC)^2 + (BD)^2 = (AB)^2 + (BC)^2 + (CD)^2 + (AD)^2 = 2(AB^2 + BC^2)$
16. Parallelogram that lie on the same base and between the same parallel lines are equal in area.
17. Area of a triangle is half of the area of a parallelogram which lie on the same base and between the same parallel lines.

18. A parallelogram is a rectangle if its diagonals are equal.

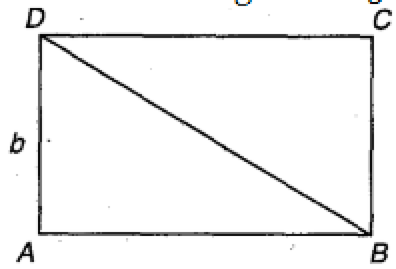
Rectangle: A parallelogram in which all the four angles at vertices are right *(i.e., 90°)*, is called a rectangle.

Properties

1. Opposite sides are parallel and equal.
2. Opposite angles are equal and of 90°.



3. Diagonals are equal and bisect each other, but not necessarily at right angles.
4. When a rectangle is inscribed in a circle, the diameter of the circle is equal to the diagonal of the rectangle.
5. For the given perimeter of rectangles, a square has maximum area.
6. The figure formed by joining the mid-points of the adjacent sides of a rectangle is a rhombus.
7. The quadrilateral formed by joining the mid-points of intersection of the angle bisectors of a parallelogram is a rectangle.
8. Every rectangle is a parallelogram.
9. Area of a rectangle = $length \times breadth \ (= l \times b)$

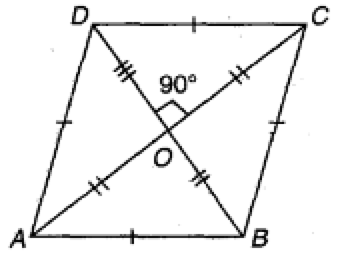


10. Diagonals of a rectangle = $\sqrt{l^2 + b^2}$
11. Perimeter of a rectangle = $2(l + b)$
 $l \rightarrow length \ and \ b \rightarrow breadth$

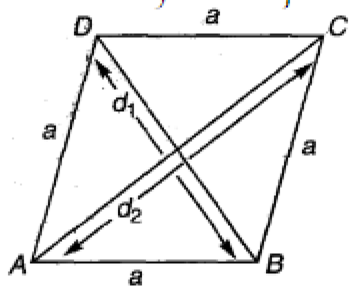
Rhombus: A parallelogram in which all sides are equal, is called a rhombus.

Properties

1. Opposite sides are parallel and equal.
2. Opposite angles are equal.
3. Diagonals bisect each other at right angle, but they are not necessarily equal.



4. Diagonals bisect the vertex angles.
5. Sum of any two adjacent angles is 180°



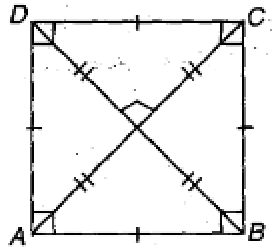
6. Figure formed by joining the mid-points of the adjacent sides of a rhombus is a rectangle.

7. A parallelogram is a rhombus if its diagonals are perpendicular to each other.
8. (a) Area of a rhombus $= \frac{1}{2} \times$ product of diagonals $= \frac{1}{2} \times d_1 \times d_2$
9. (b) Area of a rhombus = Product of adjacent sides × sine of the included angle.

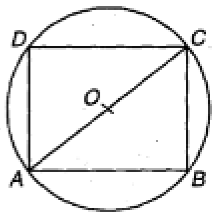
Square: A rectangle whose all sides are equal or a rhombus whose all angles are equal is called a square. Thus each rhombus is a parallelogram, a rectangle and a rhombus.

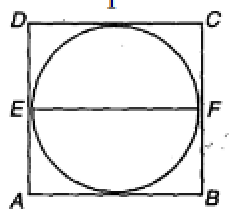
Properties

1. All side are equal and parallel.
2. All angles are right angles.
3. Diagonals are equal and bisect each other at right angle.

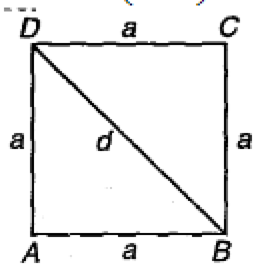


4. Diagonal of an inscribed square is equal to diameter of the inscribing circle.
5. Side of a circumscribed square is equal to the diameter of the inscribed circle.





6. The figure formed by joining the mid-points of the adjacent sides of a square is a square.
7. Area $= (side)^2$



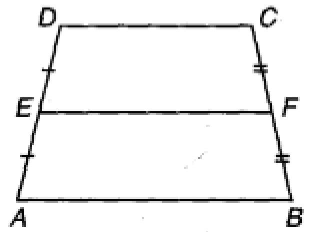
$= a^2 = \frac{(diagonal)^2}{2} = \frac{d^2}{2}$

8. Diagonal $= side\sqrt{2} = a\sqrt{2}$
9. Perimeter $= 4 \times side = 4a$

Trapezium: A quadrilateral whose only one pair of sides is parallel and other two sides are not parallel

Properties

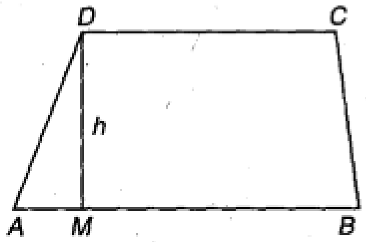
1. The line joining the mid-points of the oblique (non-parallel) sides is half the sum of the parallel sides and is called the median.



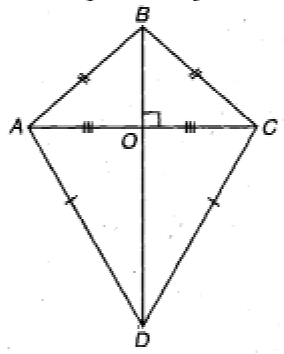
(i.e, Median $= \frac{1}{2} \times sum\ of\ parallel\ sides = \frac{1}{2} \times (AB + DC)$=EF)

2. If the non-parallel sides are equal then the diagonals will also be equal to each other.
3. Diagonals intersect each other proportionally in the ratio of lengths of parallel sides.
4. By joining the mid-points of adjacent sides of a trapezium four similar triangles are obtained.
5. If a trapezium sis inscribed in a circle, then it is an isosceles trapezium with equal oblique sides.

6. Area of trapezium $= \frac{1}{2} \times$ (sum of parallel sides $\times$ height) $= \frac{1}{2} \times (AB + CD) \times h$
7. $AC^2 + BD^2 = BC^2 + AD^2 + 2AB \cdot CD$



Kite: In a kite two pairs of adjacent sides are equal.



Properties

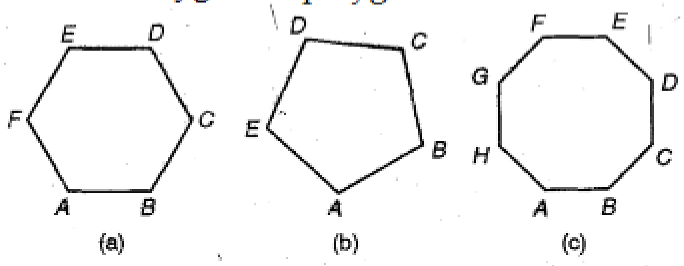
1. $AB + BC\ and\ AD = CD$
2. Diagonals intersect at right angles.
3. Shorter diagonal is bisected by the longer diagonal.
4. Area $= \frac{1}{2} \times product\ of\ diagonals$

S. No.	Property	Parallelogram	Rectangle	Rhombus	Square
1.	Opposite sides are equal	✔	✔	✔	✔
2.	All sides are equal	✗	✗	✔	✔
3.	Opposite sides are parallel	✔	✔	✔	✔
4.	Opposite angles are equal	✔	✔	✔	✔
5.	All angles are equal and right angle	✗	✔	✗	✔
6.	Diagonals bisect each other	✔	✔	✔	✔
7.	Diagonals bisect each other at right angles	✗	✗	✔	✔
8.	Diagonals bisect vertex angles	✗	✗	✔	✔
9.	Diagonals are equal	✗	✔	✗	✔
10.	Diagonals form four triangles of equal area	✔	✔	✔	✔
11.	Diagonals form four congruent triangles	✗	✗	✔	✔

Polygons

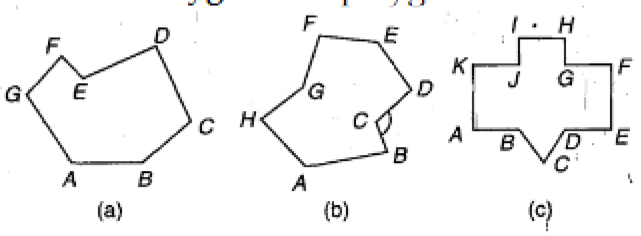
Polygon : It is a closed plane figure bounded by three or more than three straight lines.
e.g., triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon and decagon.

Convex Polygon: A polygon in which none of its interior angle is more than 180°, is known as a convex polygon.



Convex Polygons

Concave Polygon: A polygon in which atleast one interior angle is more than 180°, then it is said to be concave.



Concave Polygons

Regular Polygon : A polygon in which all the sides are equal and also all the interior angles are equal, is called a regular polygon.

Formulae: For regular polygons

1. Sum of all interior angles $= (n-2) \times 180 = (2n-4) \times 90°$
2. Each interior angle $= 180 - exterior\ angle$
3. Each exterior angle$= \left(\frac{360}{Number\ of\ sides}\right) (in\ degrees)$
4. Sum of all exterior angle $= 360°(always\ constant)$
5. Number of sides in a polygon $= \frac{360}{Exterior\ angle}$

   $= 2(x+1)\ \left(x = \frac{Interior\ angle}{Exterior\ angle}\right)$
6. Number of diagonals $= {}^nC_2 - n = \frac{n(n-1)}{2} - n$

   $= \frac{n^2-3n}{2} = \frac{n(n-3)}{2}$

   $where\ n \rightarrow$ number of sides of a polygon.
7. Star : Sum of angles of a $n\ poin\ star = (n-4)\pi$
8. Area of a polygon $= \frac{na^2}{4} \times \cot\left(\frac{180}{n}\right)$;

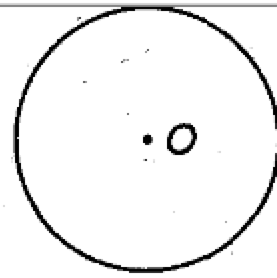
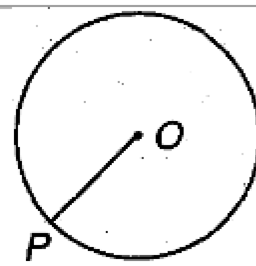
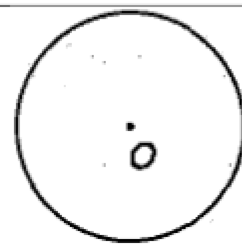
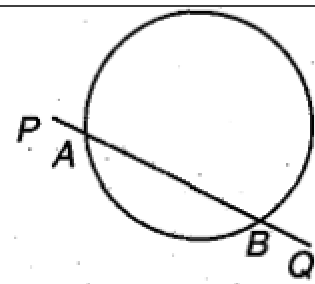
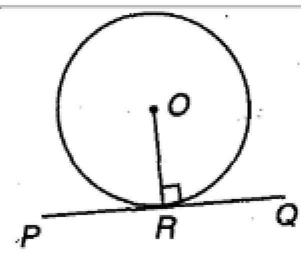
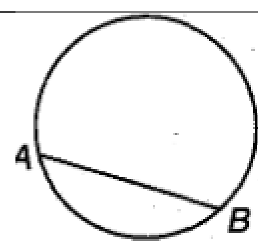
   Where $n \rightarrow number\ of\ sides$

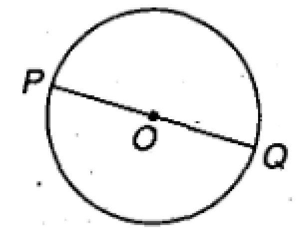
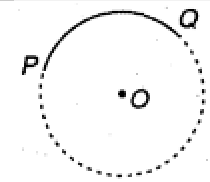
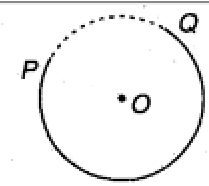
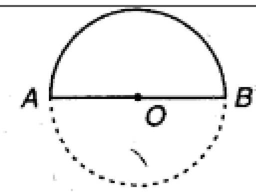
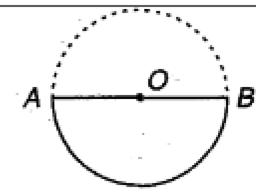
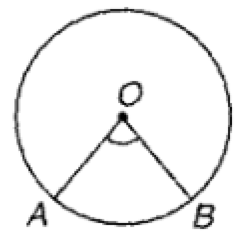
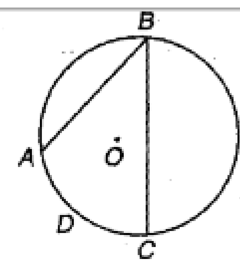
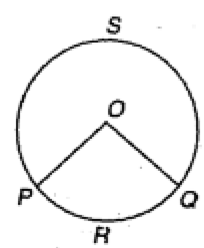
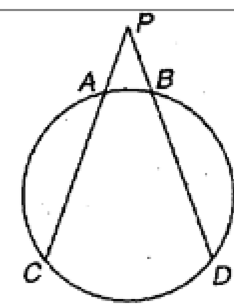
   $a \rightarrow length\ of\ sides$

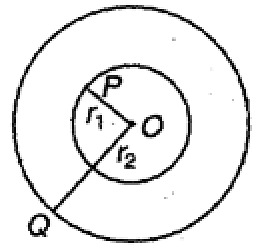
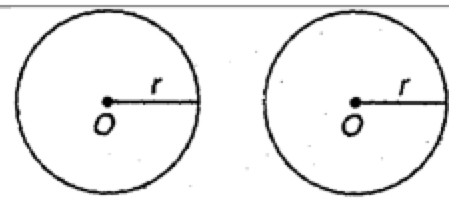
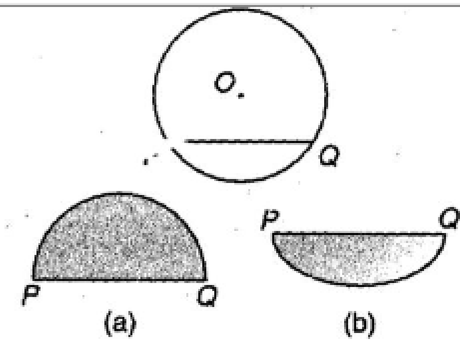
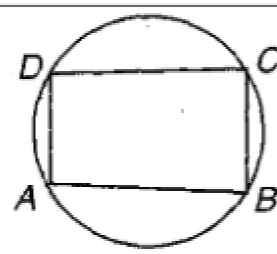
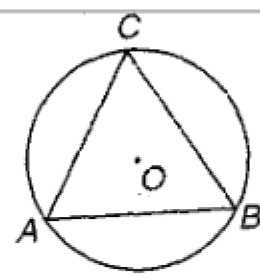
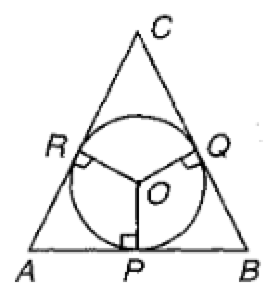
No. of sides	Polygon	Sum of all the angles	Each interior angle	Each exterior angle	No. of diagonals
3	Triangle	180°	60°	120°	0
4	Quadrilateral	360°	90°	90°	2
5	Pentagon	540°	108°	72°	5
6	Hexagon	720°	120°	60°	9
7	Heptagon	900°	$\left(128\frac{4}{7}\right)^\circ$	$\left(51\frac{3}{7}\right)^\circ$	14
8	Octagon	1080°	135°	45°	20
9	Nonagon	1260°	140°	40°	27
10	Decagon	1440°	144°	36°	35

Circle

Circle : A circle is a set of points on a plane which lies at a fixed distance from a fixed point.

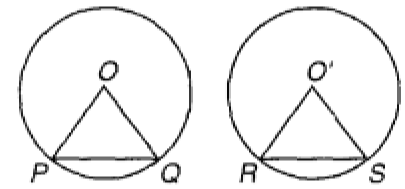
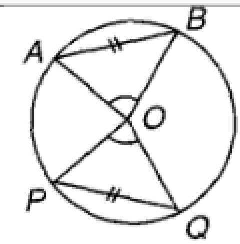
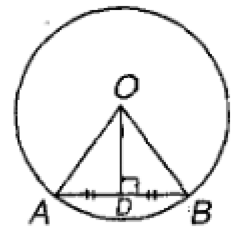
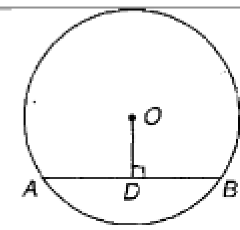
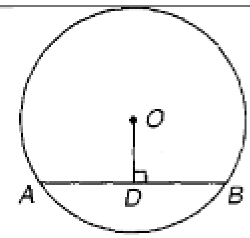
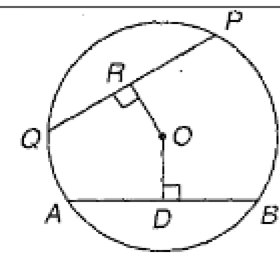
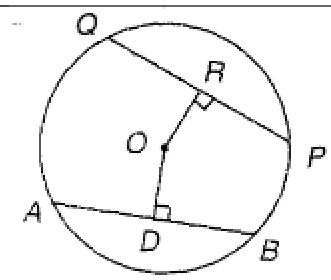
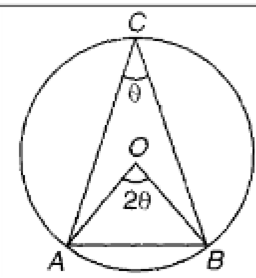
Nomenclature	Definition	Diagram
Centre	The fixed point is called the centre. In the given diagram 'O' is the centre of the circle.	
Radius	The fixed distance is called a radius. In the given diagram OP is the radius of the circle. (Point P lies on the circumference)	
Circumference	The circumference of a circle is the distance around a circle, which is equal to $2\pi r$. $(r \rightarrow \ radius\ of\ the\ circle)$	
Secant	A line segment which intersects the circle in two distinct points, is called as secant. In the given diagram secant PQ intersects circle at two points at A and B.	
Tangent	A line segment which has one common point with the circumference of a circle *i.e.,* it touches only at only one point is called as tangent of circle. The common point is called as point of contact. In the given diagram PQ is a tangent which touches the circle at a point R.	(R is the point of contact) Note: Radius is always perpendicular to tangent.
Chord	A line segment whose end points lie on the circle. In the given diagram AB is a chord.	

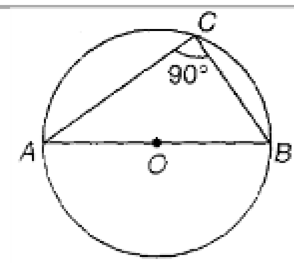
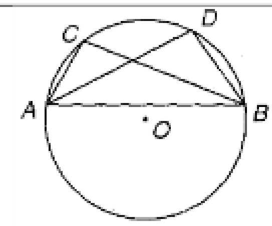
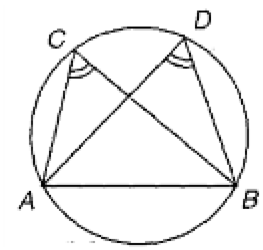
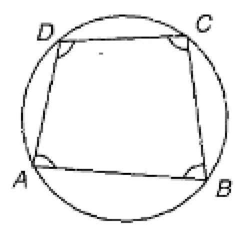
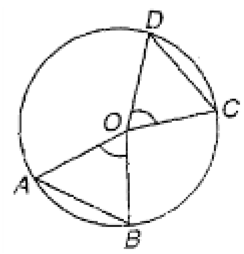
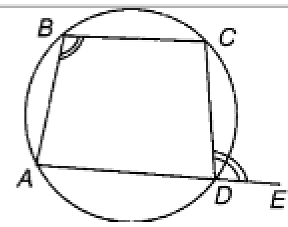
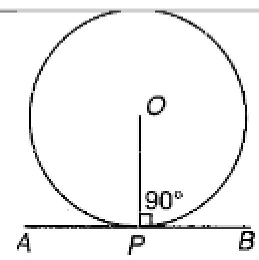
Diameter	A chord which passes through the centre of the circle is called the diameter of the circle. The length of the diameter is twice the length of the radius. In the given diagram PQ is the diameter of the circle. ($O \rightarrow$ *is the centre of the circle*)	
Arc	Any two points on the circle divides the circle into two parts the smaller part is called as minor arc and the larger part is called as major arc. It is denoted as $\frown$ In the given diagram $\widehat{PQ}$ is arc.	  $PQ \rightarrow$ *minor arc* $PQ \rightarrow$ *major arc*
Semicircle	A diameter of the circle divides the circle into two equal parts. Each part is called as semicircle.	 
Central angle	An angle formed at the centre of the circle, is called the central angle. In the given diagram $\angle AOB$ is the central angle.	
Inscribed angle	When two chords have one common end point, then the angle included between these two chords at the common point is called the inscribed angle. $\angle ABC$ is the inscribed angle by the arc $\widehat{ADC}$.	
Measure of an arc	Basically it is the central angle formed by, an arc. *e.g.,* (a) measure of a circle = 360° (b) measure of a semicircle =180° (c) measure of a minor arc = $\angle POQ$ (d) measure of a major arc = 360 - $\angle POQ$	 m (arc PRQ) = $m\,\angle POQ$ m (arc PSQ) = 360° − m (arc PRQ)
Intercepted arc	In the given diagram $\widehat{AB}$ and $\widehat{CD}$ are the two intercepted arcs, intercepted by $\angle CPD$. The end points of the arc must touch the arms of $\angle CPD$ *i.e.*, CP and DP.	

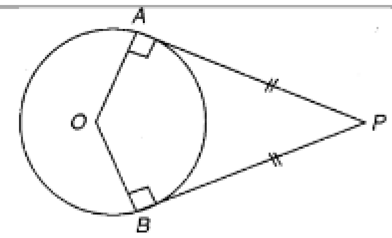
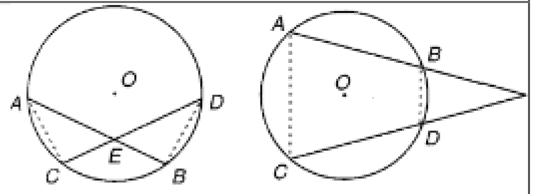
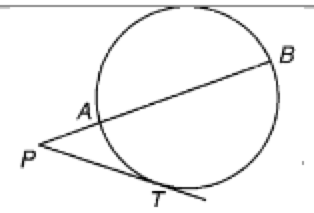
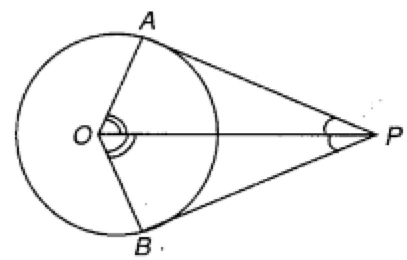
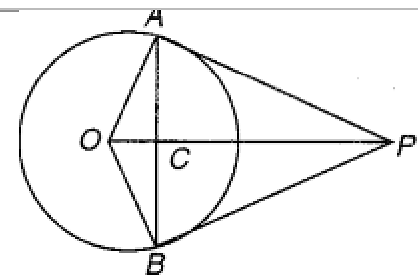
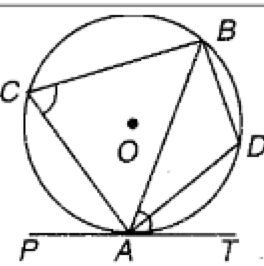
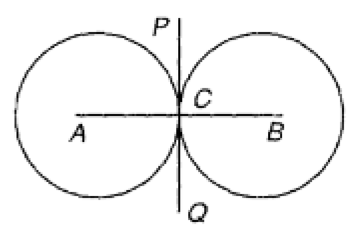
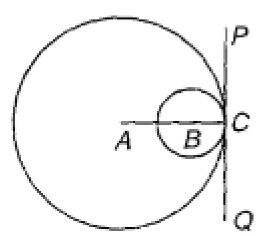
Concentric circles	Circles having the same centre at a plane are called the concentric circles. In the given diagram there are two circles with radii r_1 and r_2 having the common (or same) centre. These are called as concentric circles.	
Congruent circles	Circles with equal radii are called as congruent circles.	
Segment of a circle	A chord divides a circle into two regions. These two regions are called the segments of a circle, (a) major segment (b) minor segment	
Cyclic quadrilateral	A quadrilateral whose all the four vertices lie on the circle.	
Circum – circle	A circle which passes through all the three vertices of a triangle. Thus the circumcentre is always equidistant from the vertices of the triangle. $OA = OB = OC$ (circumradius)	
Incircle	A circle which touches all the three sides of a triangle *i.e.,* all the three sides of a triangle are tangents to the circle is called an incircle. Incircle is always equidistant from the sides of a triangle. $OP=OQ=OR$ (inradius of the circle)	

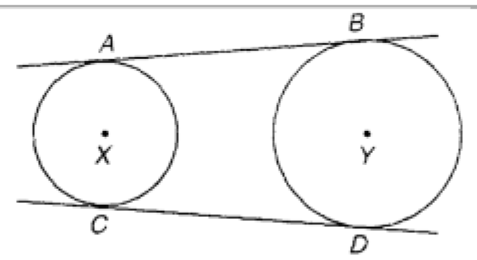
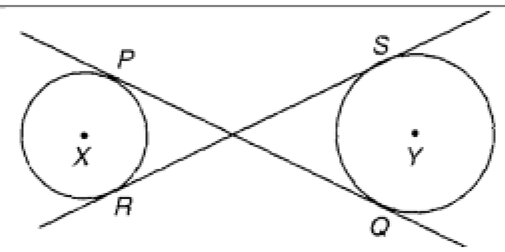
- Two arcs of a circle (or of congruent circles) arc congruent if their degree measures are equal.
- There is one and only one circle passes through three non-collinear points

S. No	Theorem	Diagram

1.	**In a circle (or in congruent circles) equal chords are made by equal arcs.** $\{OP = OQ\} = \{O'R = O'S\}$ and $\widehat{PQ} = \widehat{RS}$ $\therefore$ $PQ = RS$	
2.	Equal arcs (or chords) subtend equal angles at the centre $\widehat{PQ} = \widehat{AB}$ (or $PQ = AB$) $\therefore$ $\angle POQ = \angle AOB$	
3.	The perpendicular from the centre of a circle to a chord bisects th *i.e.*, if $OD \perp AB$ $\therefore$ $AB = 2AD = 2BD$	
4.	The line joining the centre of a circle to the mid-point of a cho perpendicular to the chord. $\because$ $AD = DB$ $\therefore$ $OD \perp AB$	
5.	Perpendicular bisector of a chord passes through the centre. i.e., $OD \perp AB$ and $AD = DB$ $\therefore$ O is the centre of the circle.	
6.	Equal chords of a circle (or of congruent circles) are equidistant from the centre. $\because$ $AB = PQ$ $\therefore$ $OD = OR$	
7.	Chords which are equidistant from the centre in a circle (or in congruent, circles) are equal. $\because$ $OD = OR$ $\therefore$ $AB = PQ$	
8.	The angle subtended by an arc (the degree measure of the arc) at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle. $M\angle AOE = 2m\angle ACB,$	

9.	Angle in a semicircle is a right angle.	
10.	Angles in the same segment of a circle are equal *i.e.*, $\angle ACB = \angle ADB$	
11.	If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, then the four points lie on the same circle. $\angle ACB = \angle ADB$ $\therefore$ Points A, C, D, B are concyclic *i.e.*, lie on the circle.	
12.	The sum of pair of opposite angles of a cyclic quadrilateral is 180° $\angle DAB + \angle BCD = 180°$ and. $\angle ABC + \angle CDA = 180°$ (Inverse of this theorem is also true)	
13.	Equal chords (or equal arcs) of a circle (or congruent circles) subtended equal angles at the centre. $AB = CD$ (or $\overset{\frown}{AB} = \overset{\frown}{CD}$) $\therefore \quad \angle AOB = \angle COD$ (Inverse of this theorem is also true).	
14.	If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle. $m\angle CDE = m\angle ABC$	
15.	A tangent at any point of a circle is perpendicular to the radius through the point of contact. (Inverse of this theorem is also true)	

16.	The lengths of two tangents drawn from an external point to a circle are equal. *i.e.,* $AP=BP$	
17.	If two chords AB and CD of a circle, intersect inside a circle (outside the circle when produced at a point E), then $AE \times BE = CE \times DE$	
18.	If PB be a secant which intersects the circle at A and B and PT be a tangent at T then $PA \cdot PB = (PT)^2$	
19.	From an external point from which the tangents are drawn to the circle with centre O, then (a) they subtend equal angles at the centre. (b) they are equally inclined to the line segment joining the centre of that point. $\angle AOP = \angle BOP$ and $\angle APO = \angle BPO$	
20.	If P is an external point from which the tangents to the circle with centre O touch it at A and B then OP is the perpendicular bisector of AB. $OP \perp AB$ and $AC = BC$	
21.	**Alternate segment theorem**: If from the point of contact of a tangent, a chord is drawn then the angles which the chord makes with the tangent line are equal respectively to the angles formed in the corresponding alternate segments. In the adjoining diagram. $\angle BAT = \angle BCA$ and $\angle BAP = \angle BDA$	
22.	The point of contact of two tangents lies on the straight line joining the two centres. (a) When two circles touch externally then the distance between their centres is equal to sum of their radii, *i.e.,* $AB = AC + BC$ (b) When two circles touch internally the distance between their centres is equal to the difference between their radii *i.e.,* $AB = AC - BC$	

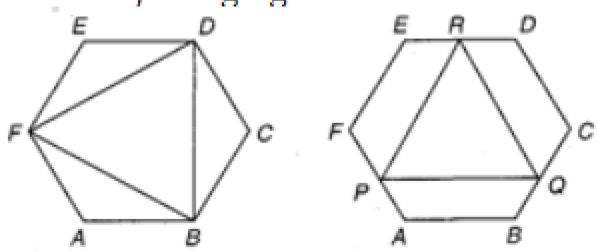
23.	For the two circles with centre X and Y and radii r_1 and r_2. AB and CD are two Direct Common Tangents (DCT), then the length of DCT $= \sqrt{(distance\ between\ centres)^2 - (r_1 - r_2)^2}$	
24.	For the two circles with centre X and Y and radii r_1 *and* r_2 PQ and RS are two transverse common tangent, then length of TCT $= \sqrt{(distance\ between\ centres)^2 - (r_1 + r_2)^2}$	

Exercise 01
Polygons

1. An n sided polygon has 'n' diagonals, then the value of n is:
(a) 4 (b) 6 (c) 7 (d) 5

2. What is the sum of all the angles of a 9 pointed star ($i.e..\ \angle 1 + \angle 2 + \angle 3 + \ldots \angle 8 + \angle 9$):
(a) 909° (b) 900° (c) 720° (d) 540°

3. In the adjoining figure there are two congruent regular hexagons each with side 6cm.

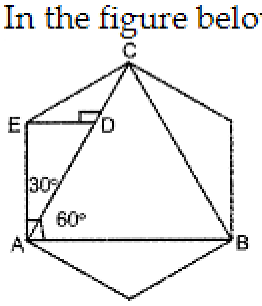


What is the ratio of area of ΔBDF and ΔPQR, if P, Q and R are the mid- points of side AF, BC and DE?
(a) $6:5$ (b) $7:6$ (c) $4:3$ (d) $1:1$

4. If each interior angle of a regular polygon is 135°, find the number of diagonals.
(a) 54 (b) 48 (c) 20 (d) None of these

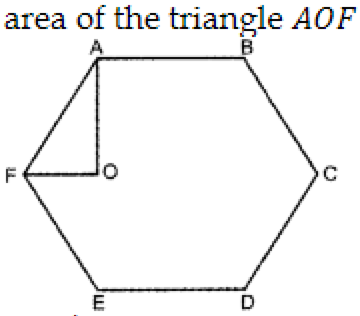
5. A square, whose side is 2 m, has its corners cut away so as to form an octagon with all sides equal. Then, the length of the each side of the octagon, in metres is:
(a) $\frac{2}{\sqrt{2}+1}$ (b) $\frac{2}{\sqrt{2}-1}$ (c) $\frac{\sqrt{2}}{\sqrt{2}-1}$ (d) $\frac{\sqrt{2}}{\sqrt{2}+1}$

6. In the figure below, if the perimeter of ∇ABC is p, then the perimeter of the regular hexagon is:



(a) $\frac{3p}{\sqrt{2}}$ (b) $\frac{\sqrt{2}p}{3}$ (c) $\frac{\sqrt{3}p}{2}$ (d) $\frac{2p}{\sqrt{3}}$

7. In the figure below, $ABCDEF$ is a regular hexagon and $AOF = 90°$. FO is parallel to ED. What is the ratio of the area of the triangle AOF of that of the hexagon $ABCDEF$?



(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{24}$ (d) $\frac{1}{18}$

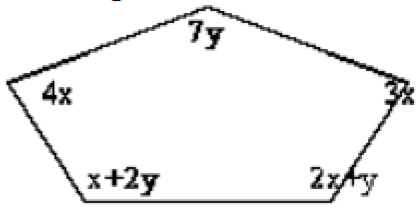
8. The sum of the interior angles of a regular polygon is 40 times the exterior, angle. Find the number of sides of the polygon.

9. A regular polygon with n sied has interior angles measuring 178°. What is the value of $\frac{180}{n}$?

10. A regular hexagon is inscribed in a circle of radius 6. What is the area of hexagon?

11. A regular hexagon has a perimeter of 12 cm. What is its area?
(a) $6\sqrt{3}$ (b) $72\sqrt{3}$ (c) $144\sqrt{3}$ (d) $216\sqrt{3}$

12. If the expressions shown are the degree measures of the angles of the pentagon, find the value of $x + y$.



13. One angle of a regular polygon measures 177°. How many sides does it have?
(a) 89 (b) 120 (c) 177 (d) 183

14. Octagon $ABCDEFGH$ is similar to octagon $JKLMNOPQ$. If $AB = 10, JK = 8, and\ m\angle A = 120°$, what is $m\angle J$ in degrees?
(a) 96° (b) 120° (c) 135° (d) 186°

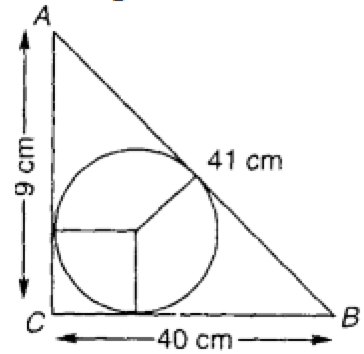
15. Find the sum of the measures of one interior and one exterior angle of a regular 40 – gon.
(a) 168° (b) 174° (c) 180° (d) 186°

Exercise 02

Triangles

1. Three sides of a triangle ABC are $a, b, c. a = 4935$ cm, $b = 4700$ cm and c = 6815 cm. The internal bisector of $\angle A$ meets BC at P, and the bisector passes through incentre 'O.
What is ratio of $PO : OA$?
(a) 3 : 2 (b) 2 : 3 (c) 2 : 5 (d) can't be determined

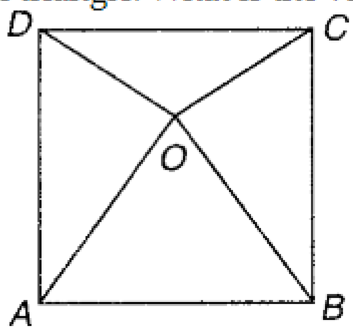
2. What is the inradius of the incircle shown in the figure?



(a) 9 cm (b) 4 cm
(c) can't be determined (d) none of the above

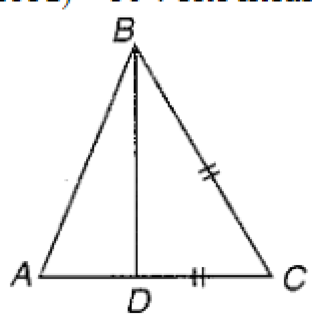
3. In a ΔPQR, points M and N are on the sides PQ and PR respectively such that $PM = 0.6.PQ$ and $NR = 0.4\ PR$. What percentage of the area of the triangle PQR does that of triangle PMN form?
(a) 60% (b) 50% (c) 36% (d) 55%

4. $ABCD$ is a square and AOB is an equilateral triangle. What is the value of ∠DOC?



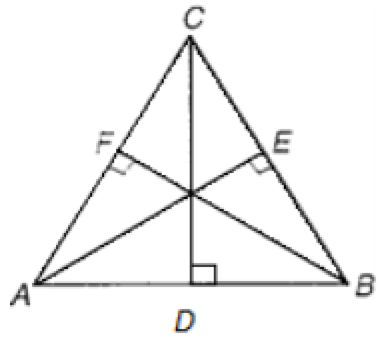
(a) 120° (b) 150° (c) 125° (d) can't be determined

5. In the triangle ABC, $BC = CD$ am $(\angle ABC - \angle BAC) = 30°$. The measure of $\angle ABD$ is :



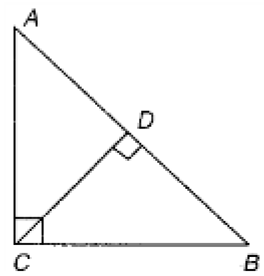
(a) 30° (b) 45° (c) 15° (d) can't be determined

6. In the given triangle ABC, CD, BF and AE are the altitudes. If the ratio of $CD: AE : BF = 2: 3: 4$, then the ratio of AB : BC :CA is :



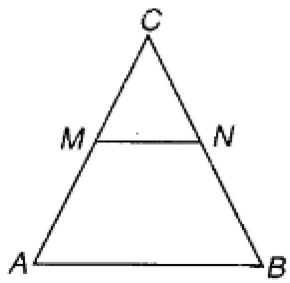
(a) $4:3:2$ (b) $2:3:4$ (c) $4:9:16$ (d) $6:4:3$

7. In a right angled triangle ABC, CD is the perpendicular on the hypotenuse AB. which of the following is correct?



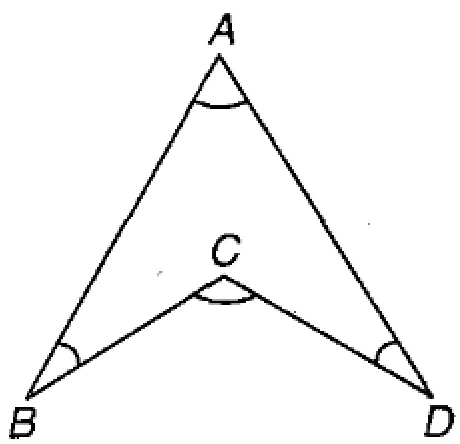
(a) $CD = \frac{AC \times BC}{AB}$ (b) $AD = \frac{AC \times AC}{AB}$ (c) $BD = \frac{BC \times BC}{AB}$ (d) All of the above

8. In the triangle ABC, MC is parallel to AB. Area of trapezium $ABNM$ is twice the area of triangle CMN. What is ratio of $CM : AM$?



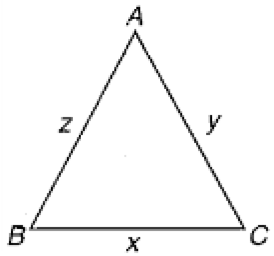
(a) $\frac{1}{\sqrt{3}+1}$ (b) $\frac{\sqrt{3}-1}{2}$ (c) $\frac{\sqrt{3}+1}{2}$ (d) none of these

9. In the adjoining figure $\angle BAD = a$, $\angle ABC = b$ and $\angle BCD =$ c and $\angle ADC = d$, find the value of $\angle ABC$ in terms of a, c and d:



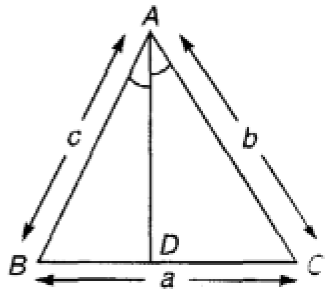
(a) $c-(a+d)$ (b) $a-(c+d)$ (c) $b-(c+d)$ (d) none of the above

10. If $x^2 + y^2 + z^2 = xy + yz + zx$, then the triangle is:



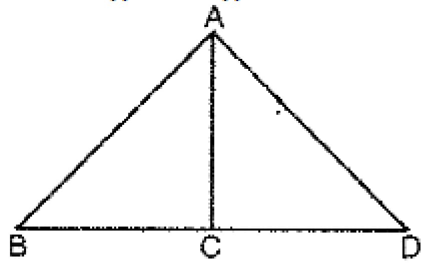
(a) isosceles (b) right angled (c) equilateral (d) scalene

11. In a triangle ABC, AD is the angle bisector of $\angle BAC$ and $\angle BAD = 60°$. What is the length of AD?



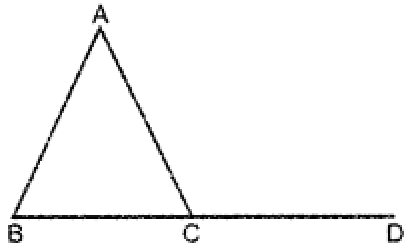
(a) $\frac{b+c}{bc}$ (b) $\frac{bc}{b+c}$ (c) $\sqrt{b^2+c^2}$ (d) $\frac{(b+c)^2}{bc}$

12. In the given figure, $AB = AD. \angle ACB = 95° + \angle BAC$ and $\angle BAD = 150°$. Find $\angle ACB$.



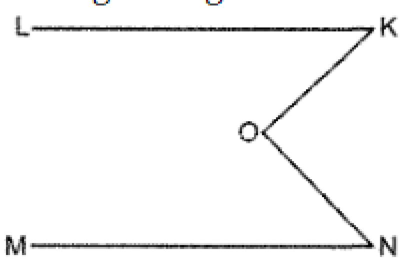
(a) 110° (b) 120° (c) 130° (d) 140°

13. In $\Delta ABC, \angle ACD = 100°$ and $\angle ABC = 40°. Find$ $\angle BAC$.



(a) 40° (b) 100° (c) 80° (d) 60°

14. In the given figure, MN and KL are parallel lines.



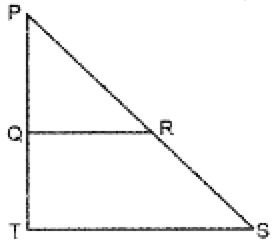
$\angle LKO = 70°, \angle KON = 100°$
Find $\angle MNO$.

(a) 20° (b) 30° (c) 40° (d) 50°

15. The supplement of an angle is five times the angle. What is the measurement of the angle?
(a) 30° (b) 65° (c) 40° (d) 45°

16. In a triangle ABC, the incentre is at O. If $\angle BOC = 100°$, find $\angle BAC$.
(a) 10° (b) 20° (c) 30° (d) 40°

17. In the given figure $PQ \times PS = PT \times PR$. If $\angle PQR = \angle PST + 30°$ and $\angle PTS = 100°$, then find $\angle PRQ$.



(a) 100° (b) 70° (c) 130° (d) 50°

18. The largest angle of a triangle of sides 7 cm, 5 cm, and 3 cm is:
(a) 45° (b) 60° (c) 90° (c) 120°

19. The sides of a triangle are in the ratio of $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. If the perimeter is 52 cm, then the length of the smallest side is:
(a) 9 cm (b) 10 cm (c) 11 cm (d) 12 cm

20. What is the area of the triangle in which two of its medians 9 cm and 12 cm long intersect at the right angles?
(a) 72 (b) 60 (c) 56 (d) 48

21. AB is the hypotenuse in the right angled triangle ABC. N is the point inside the triangle which divides the triangle in three equal parts ($\Delta ABN, \Delta CAN, \Delta BCN$). What is the distance between the circumcentre of this triangle from this point N?
(a) $\frac{AB}{4}$ (b) $\frac{AB}{6}$ (c) $\frac{AB}{3}$ (d) $\frac{2}{1+\sqrt{3}}$

22. In $\Delta ABC, P\ and\ Q$ are mid-point of sides $AB\ and\ BC$, respectively, right angled at B, then:
(a) $AQ^2 + CP^2 = AC^2$ (b) $AQ^2 + CP^2 = \frac{4}{5}AC^2$
(c) $AQ^2 + CP^2 = \frac{3}{4}AC^2$ (d) $AQ^2 + CP^2 = \frac{5}{4}AC^2$

23. Let $C_1\ and\ C_2$ be the inscribed and circumscribed circles of a triangle with sides 3cm, 4cm and 5 cm. The $\frac{area\ of C_1}{area\ of\ C_2}\ equals.$
(a) $\frac{16}{25}$ (b) $\frac{4}{25}$ (c) $\frac{9}{25}$ (d) $\frac{9}{16}$

24. In an equilateral D, 3 coins of radii 1 unit each are kept in such a way that they touch each other and also the sides of the triangle. What is the area of the triangle (in sq. units)?
(a) $4 + 5\sqrt{2}$ (b) $6 + 4\sqrt{3}$ (c) $4 + 6\sqrt{3}$ (d) $3 + 8\sqrt{3}$

25. A vertical stick 20 m long casts a shadow 10m long on the ground. At the same time, a tower casts the shadow 50 m long on the ground. Find the height of the tower.
(a) 100m (b) 120m (c) 25m (d) 200m

26. The area of similar triangles, ABC and DEF are 144 cm^2 and 81cm^2respectively. If the longest side of larger ΔABC be 36 cm, then the longest side of smaller ΔDEF is
(a) 20 cm (b) 26 cm (c) 27 cm (d) 30 cm

27. Two isosceles Δs have equal angles and their areas are in the ratio16 : 25. Find the ratio of their corresponding heights.
(a) 4/5 (b) 5/4 (c) 3/2 (d) 5/7

28. The areas of two similar Δs are respectively$9\ cm^2\ and\ 16cm^2$. Find the ratio of their corresponding sides.
(a) 3 : 4 (b) 4 : 3 (c) 2 : 3 (d) 4 : 5

29. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, find the distance between their tops.
(a) 12 m (b) 14 m (c) 13 m (d) 11 m

30. If ΔABC is a right angled triangle such that $\angle B = 90°$, $(AB + BC) - AC = 20cm$ and perimeter of $\Delta ABC = 60\ cm$, then area of the triangle (in cm^2) is.

31. The area of an isosceles triangle is 12 cm². If one of the equal sides is 5 cm long, mark the option which can give the length of the base,
(a) 4 cm (b) 6 cm (c) 10cm (d) 9cm

32. The perimetre of *a* right angled triangle measures 234 m and the hypotenuse measures 97 m. Then the other two sides of the triangle are measured as
(a) 100m and 37m (b) 72m and 65m
(c) 80m and 57m (d) None of these

33. A 25 feet long ladder is placed against the wall with its base 7 feet from the wall. The base of the ladder is drawn out so that the top comes down by half the distance that the base is drawn out. This distance is in the range:
(a) (2, 7) (b) (5, 8) (c) (9, 10) (d) None of these

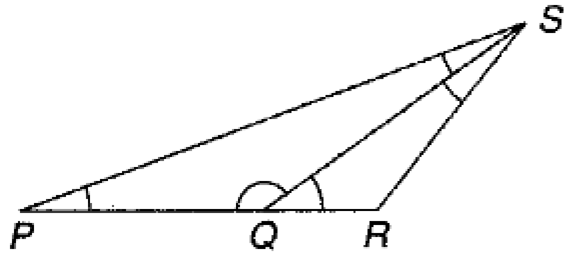
34. In an equilateral triangle *ABC*, whose length of each side is 3 cm, D is a point on *BC* such that *BD* = *CD*/2. What is the length of *AD*?
(a) $\sqrt{5}$ cm (b) $\sqrt{6}$ cm (c) $\sqrt{7}$ cm (d) $\sqrt{8}$ cm

35. In a triangle ABC, point D is on side AB and point E is on side AC, such that BCED is a trapezium. $DE: BC = 3: 5$. Calculate the ratio of the area of ΔADE and the trapezium $BCED$.
(a) 3 : 4 (b) 9 : 16 (c) 3 : 5 (d) 9 : 25

36. D, E, F are the mid-points of the sides BC, CA *and* AB respectively of a ΔABC. Determine the ratio of the area of triangles DEF *and* ABC.
(a) 1 : 4 (b) 1 : 2 (c) 2 : 3 (d) 4 : 5

37. A ladder reaches a window that is 8 m above the ground on one side of the street. Keeping its foot on the same point, the ladder is twined to the other side of the street to reach a window 12 m high. Find the width of the street if the ladder is 13 m.
(a) 15.2m (b) 14m (c) 14.6m (d) 12m

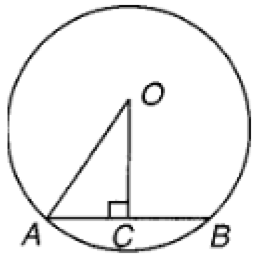
38. In the figure below, $PQ = QS, QR = RS$ and angle $SRQ = 100°$. How many degrees is angle QPS?



(a) 20° (b) 40° (c) 15° (d) 30°

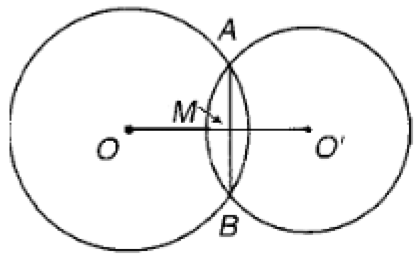
Exercise – 03
Circle

1. In the given figure, 0 is the centre of the circle. Radius of the circle is 17 cm. If OC = 8 cm, then the length of the chord AB is:



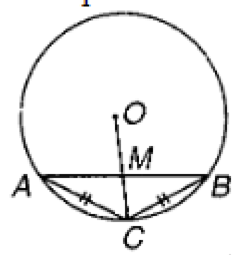
(a) 35 cm (b) 30 cm (c) 15 cm (d) 18cm

2. In the given figure, two circles with their respective centres intersect each other at A and B and AB ontersects OO′ at M, then $m\ \angle OMA$ is:



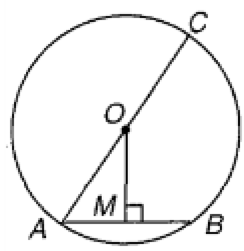
(a) 60° (b) 80° (c) 90° (d) can't be determined

3. In the given figure the two chords AC and BC are equal. The radius OC intersect AB at M, then



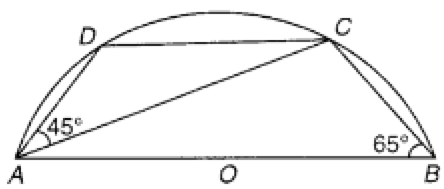
(a) 1 : 1 (b) $\sqrt{2}$: 3 (c) 3: $\sqrt{2}$ (d) none of the above

4. In the adjoining figure, O is the centre of circle and diameter AC = 26 cm. If chord AB = 10cm, then the distance between chord AB and centre O of the circle is:



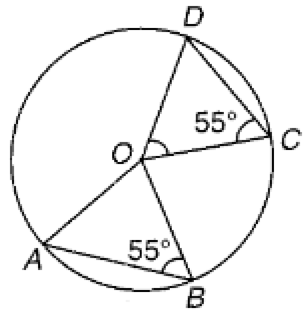
(a) 24 cm (b) 16 cm (c) 12 cm (d) none of the above

5. In the given figure, AB is diameter of the circle. C and D lie on the semicircle. $\angle ABC = 65°$ *and* $\angle CAD = 45°$. Find $m\ \angle DCA$:



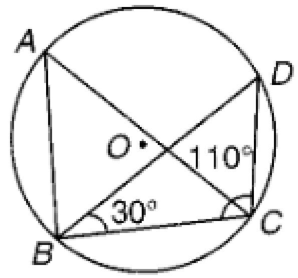
(a) 45° (b) 25° (c) 20° (d) none of these

6. In the given figure, chords AB and CD are equal. If $\angle OBA = 55°$, then $m\ \angle COD$ is:



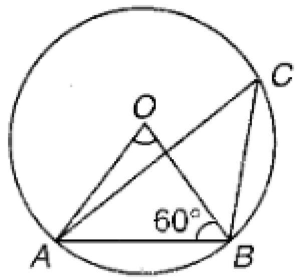
(a) 65° (b) 55° (c) 70° (d) 50°

7. In the given figure, $\angle BAC$ *and* $\angle BDC$ are the angles of same segments. $\angle DBC = 30°$ *and* $\angle BCD = 110°$. *Find m* $\angle BAC$ *is*:



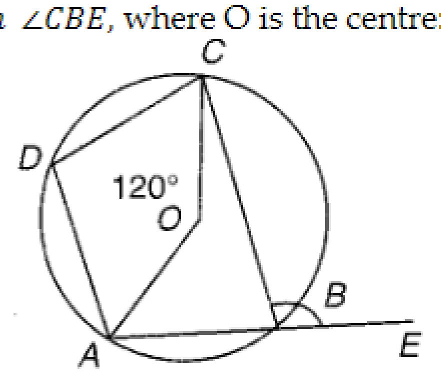
(a) 35° (b) 40° (c) 55° (d) 60°

8. In the given figure, O is the centre of the circle. $\angle ABO = 60°$. Find the value of $\angle ACB$:



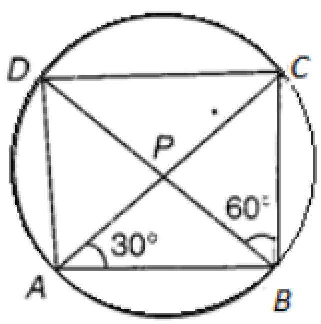
(a) 40° (b) 60° (c) 50° (d) 30°

9. In the given figure, $\angle AOC = 120°$. *FInd m* $\angle CBE$, where O is the centre:



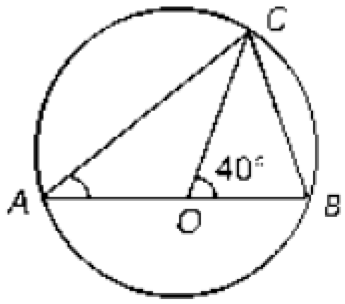
(a) 60° (b) 100° (c) 120° (d) 150°

10. In the given figure, ABCD is a cyclic quadrilateral and diagonals bisect each other at P. If $\angle DBC = 60°$ and $\angle BAC = 30°$, *then* $\angle BCD$ is:



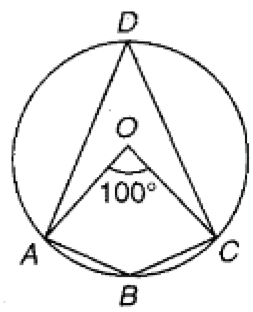
(a) 90° (b) 60° (c) 80° (d) none of the above

11. In the given figure, $\angle COB = 40°$, AB is the diameter of the circle. Find $m\,\angle CAB$:



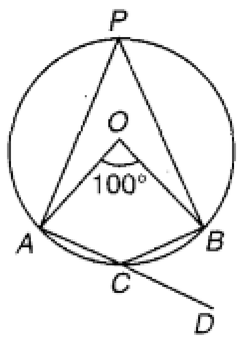
(a) 40° (b) 20° (c) 30° (d) none of the above

12. In the given figure, O is the centre of the circle and $\angle AOC = 100°$. Find the ratio of $m\,\angle ADC : m\,\angle ABC$:



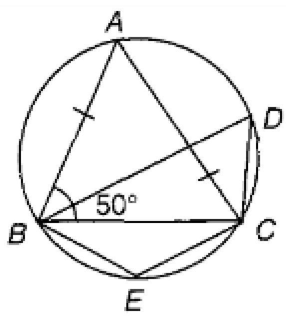
(a) 5 : 6 (b) 1 : 2 (c) 5 : 13 (d) none of the above

13. In the given figure, O is the centre of circle, $\angle AOB = 100°$. Find $m\,\angle BCD$:



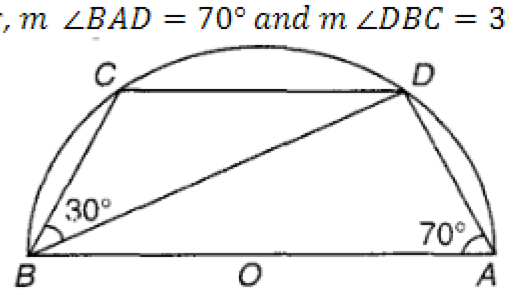
(a) 80° (b) 60° (c) 50° (d) 40°

14. In the given figure, ABC is an isosceles triangle in which AB = AC and $m\,\angle ABC = 50°$, $m\,\angle BDC$:



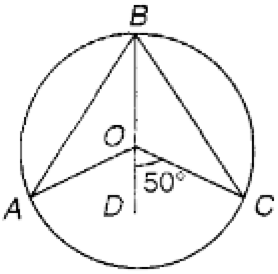
(a) 80° (b) 60° (c) 65° (d) 100°

15. In the given figure, AB is the diameter, $m\,\angle BAD = 70°$ *and* $m\,\angle DBC = 30°$. Find $m\,\angle BDC$:



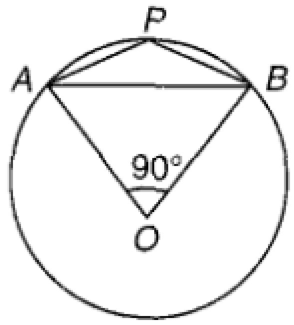
(a) 25° (b) 30° (c) 40° (d) 60°

16. 'O' is the centre of the circle, line segment BOD is the angle bisector of $\angle AOC$, $m\,\angle COD = 50°$. Find $m\,\angle ABC$:



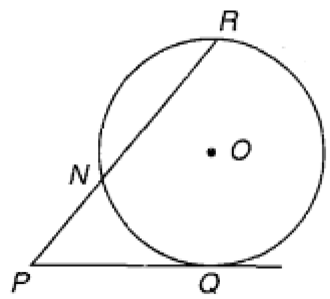
(a) 25° (b) 50° (c) 100° (d) 120°

17. In the given figure, O is the centre of the circle. $\angle AOB = 90°$. Find $m\ \angle APB$:



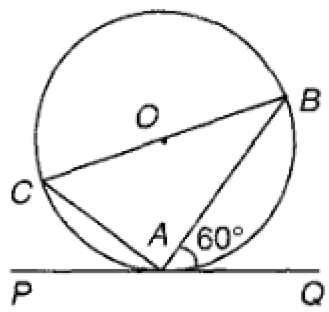
(a) 130° (b) 150° (c) 135° (d) can't be determined

18. In the given figure, PQ is the tangent of the circle. Line segment PR intersects the circle at N and R. $PQ = 15\ cm$, $PR = 25cm$, find PN:



(a) 15 cm (b) 10 cm (c) 9 cm (d) 6 cm

19. In the given figure, PAQ is the tangent. BC is the diameter of the circle. $m\ \angle BAQ = 60°$, find $m\ \angle ABC$:



(a) 25° (b) 30° (c) 45° (d) 60°

20. In a circle of radius 5 cm, AB and AC *are* the two chords such that $AB = AC = 6\ cm$. Find the length of the chord BC:
(a) 4.8cm (b) 10.8cm (c) 9.6 cm (d) none of these

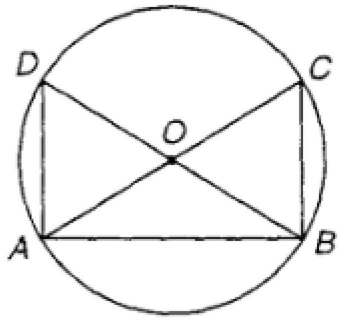
21. If two circles are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of two circles to the radius of any of the circles is :
(a) $\sqrt{3} : 2$ (b) $\sqrt{3} : 1$ (c) $\sqrt{5} : 1$ (d) none of these

22. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the other circle which is outside the inner circle, is of length :
(a) $2\sqrt{2}$ cm (b) $3\sqrt{2}$ cm (c) $2\sqrt{3}$ cm (d) $4\sqrt{2}$ cm

23. The radius of a circle is 20 cm. The radii (in cm) of three concentric circles drawn in such a manner that the whole area is divided into four equal parts, are:

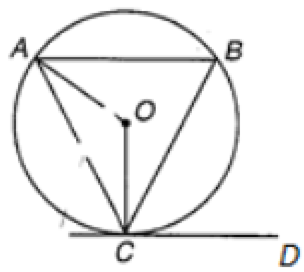
(a) $20\sqrt{2}, 20\sqrt{3}, 20$
(b) $\frac{10\sqrt{3}}{3}, \frac{10\sqrt{2}}{3}, \frac{10}{3}$
(c) $10\sqrt{3}, 10\sqrt{2}, 10$
(d) $17, 14, 9$

24. In the given figure O is the centre of the circle and $\angle BAC = 25°$, then the value of $\angle ADB$ is:



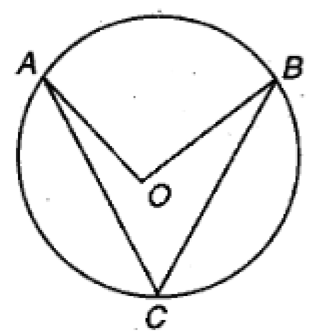
(a) 40° (b) 55° (c) 50° (d) 65°

25. In the given diagram O is the centre of the circle and CD is a tangent. $\angle CAB$ *and* $\angle ACD$ are supplementary to each other $\angle OAC = 30°$. Find the value of $\angle OCB$:



(a) 30° (b) 20° (c) 60° (d) none of the above

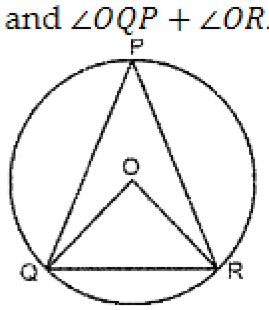
26. In the adjoining figure 'O' is the centre of circle. $\angle CAO = 25°$ and $\angle CBO = 35°$. What is the value of $\angle AOB$?



(a) 55° (b) 110° (c) 120° (d) data insufficient

28. The three sides of a triangle measure 6 cm, 8 cm and 10 cm, respectively. A rectangle equal in area ot the triangle has a length of 8 cm. The perimeter of the rectangle is:
(a) 11 cm (b) 22 cm (c) 16 cm (d) None of these

29. In the given figure, O is the centre of the circle and $\angle OQP + \angle ORP = 70°$. Find $\angle ORQ$.



(a) 20° (b) 30° (c) 40° (d) 50°

30. Three identical right angle cones with base radius r are placed on their bases so that each is touching the other two. The radius of the circle drawn through their vertices is:
(a) Smaller then r (b) Equal to r (c) Larger than r (d) Depends on the height of the cones

31. Two chords of lengths a and b of a circle subtend $60°$ and $90°$ angles at the centre, respectively. Which of the following is correct?
(a) $b = \sqrt{2}a$ (b) $b = \sqrt{2b}$ (c) $a = 2b$ (d) $b = 2a$

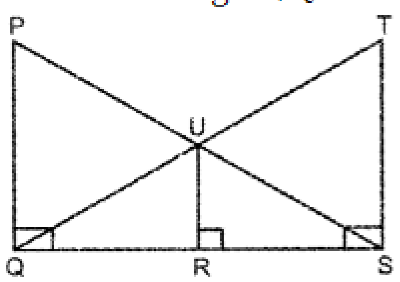
32. A chord of length 32 cm is placed inside a circle of radius 20 cm and a point whose distance from the centre of the circle is 13 cm, is marked on the chord. Calculate the lengths of the segment of the chord.
(a) 21 cm and 11 cm (b) 19 cm and 13 cm
(c) 16 cm each (d) 18 cm and 14 cm

33. Points A and B are on a circle of radius 5 and $AB = 6$. Point C is the mid – point of the minor are AB. What is the length of the line segment AC?
(a) $\sqrt{10}$ (b) $\frac{7}{2}$ (c) $\sqrt{14}$ (d) $\sqrt{15}$

34. $ABCD$ is a cyclic quadrilateral and the points A, B and C form an equilateral triangle. What is the sum of the lengths of line segments DA and DC?
(a) DB (b) $DB/2$ (c) $\sqrt{2}DB$ (d) $DB/\sqrt{2}$

35. The length of the common chord of two circles of radii 15 cm and 20 cm, whose centres are 25 cm apart, is (in cm):
(a) 24 (b) 25 (c) 15 (d) 20

36. In the above figure, $QR = 4cm$ and $RS = 12cm. TS = 8$ cm and QU is extended to T. Find PQ.



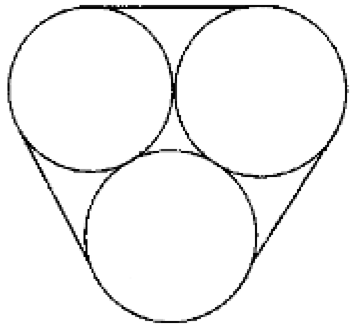
(a) 3 cm (b) 24 cm (c) 21 cm (d) None of these

39. The radius of circle is 9 cm and length of one of its chords is 14 cm. Find the distance of the chord from the centre.
(a) 5.66 cm (b) 6.3 cm (c) 4 cm (d) 7 cm

40. If the inradius of an equilateral triangle is $\sqrt{3}$ cm, then its area is:
(a) $7\sqrt{3}\ cm^2$ (b) $9\sqrt{3}\ cm^2$ (c) $10\sqrt{3}\ cm^2$ (d) $12\sqrt{3}cm^2$

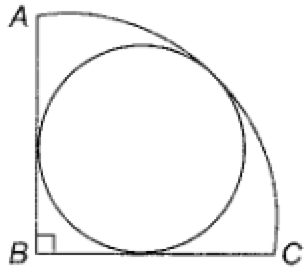
41. There are two circles C_1, and C_2 of radii 3 and 8 units respectively. The common internal tangent T, touches the circles at points P and Q respectively. The line joining the centers of the circles intersects T at X. The distance of X from the center of the smaller circle is 5 units. What is the length of the line segment PQ?
(a) ≤ 13 (b) >13 and ≤ 14 (c) > 14 and < 15 (d) > 15 and ≤ 16

42. The diagram below represents three circular garbage cans, each of diametre 2 m. The three cans are touching as shown. Find, in metres, the perimetre of the rope encompassing the three cans.



(a) $2\pi + 6$ (b) $3\pi + 4$ (c) $4\pi + 6$ (d) $6\pi + 6$

43. If ABC is a quarter circle and a circle is inscribed in it and if $AB = 1cm$, find radius of smaller circle.



(a) $\sqrt{2} - 1$ (b) $(\sqrt{2} + 1)/2$ (c) $\sqrt{2} - 1/2$ (d) $1 - 2\sqrt{2}$

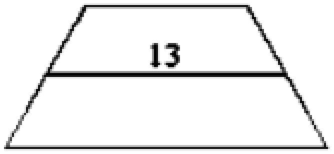
44. If a circle is provided with a measure of 19° on centre, is it possible to divide the circle into 360 equal parts?
(a) Never
(b) Possible when one more measure of 20° is given
(c) Always
(d) Possible if one more measure of 21° is given

45. A right triangle with legs measuring 12 cm and 16 cm is inscribed in a circle. What is the circumference of the circle in centimeters?
(a) 14π (b) 16π (c) 20π (d) 28π

46. A central angle measuring $M°$ intercepts an arc in a circle of radius r cm. The length of the subtended arc is 8π cm. The area of the sector formed by (and including) the angle is $48\pi\ cm^2$.
Evaluate $\left(\frac{M}{r}\right)$.
(a) 5 (b) 10 (c) 20 (d) 40

47. A circle is inscribed in a triangle with sides measuring $4cm, 6\ cm, and\ 8\ cm$. What is the area of the circle in square centimeters?
(a) $\frac{7\pi}{6}$ (b) $\frac{3\pi}{2}$ (c) $\frac{5\pi}{3}$ (d) $\frac{7\pi}{4}$

48. An isosceles trapezoid has a mid-segment measuring 13 cm and an area of 52 cm^2. If one base has length 10 cm, find the perimeter of the trapezoid in centimeters.



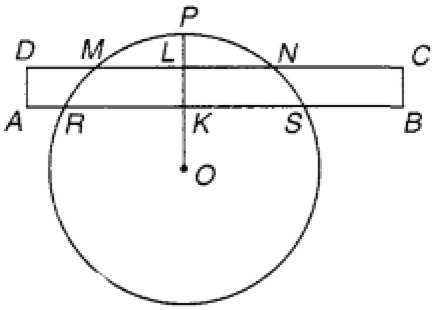
Exercise – 04

Quadrilaterals

1. In a trapezium $ABCD$, the diagonals AC and BD intersect each other at O such that $OB : OD = 3:1$ then the ratio of areas of $\Delta AOB : \Delta COD$ is:

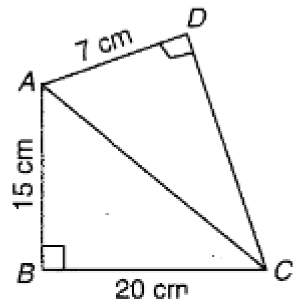
(a) 3 : 1 (b) 1 : 4 (c) 9 : 1 (d) can't be determined

2. In the adjoining figure O is the centre of the circle. The radius OP bisects a rectangle $ABCD$, at right angle. $DM = NC = 2$ cm and $AR = SB = 1cm$ and $KS = 4$ cm and $OP = 5\ cm$. What is the area of the rectangle?



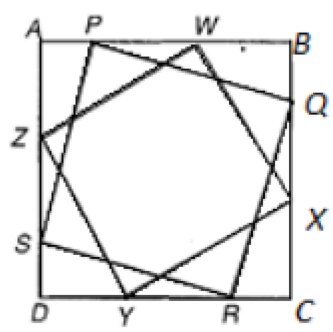
(a) $8\ cm^2$ (b) $10\ cm^2$ (c) $12cm^2$ (d) None of these

3. In the given quadrilateral $ABCD$, $AB = 15$ cm, $BC = 20$ cm and $AD = 7$ cm, $\angle ABC = \angle ADC = 90°$. Find the length of side CD:



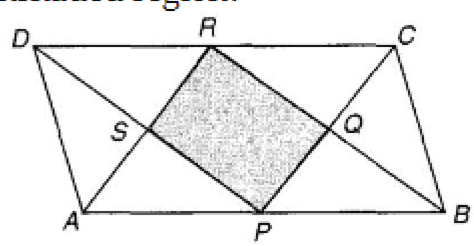
(a) 12cm (b) 18 cm (c) 24 cm (c) none of the above

4. In the adjoining figure ABCD, $PQRS$ and $WXYZ$ are three squares. Find number of triangles and quadrilaterals in the figure:



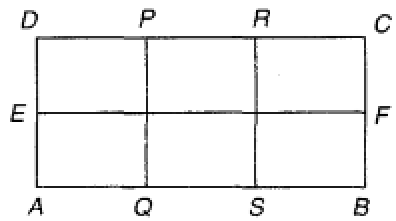
(a) 24 and 16 (b) 28 and 15 (c) 27 and 16 (d) none of the above

5. In the adjoining figure $ABCD$, P and R are the mid-points of the sides AB and CD. $ABCD$ is a parallelogram. What is the ratio of the shaded to the unshaded region?



(a) 1/2 (b) 1/3 (c) 1/4 (d) none of these

6. In the adjoining figure $ABCD$ is a rectangle. Find the maximum number of rectangles including the largest possible rectangle:



(a) 16 (b) 7 (c) 18 (d) 24

7. The three sides of a triangle measure 6 cm, 8 cm and 10 cm, respectively. A rectangle equal in area ot the triangle has a length of 8 cm. The perimeter of the rectangle is:
(a) 11 cm (b) 22 cm (c) 16 cm (d) None of these

8. A cyclic quadrilateral is such that two of its adjacent angles are divisible by 6 and 10, respectively. One of the remaining angles will necessarily be divisible by:
(a) 3 (b) 4 (c) 8 (d) None of these

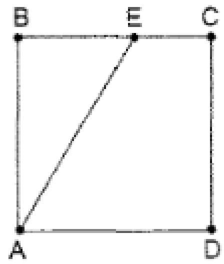
9. A square is inscribed in a semi-circle of radius 10 cm. What is the area of the inscribed square? (Given that the side of the square is along the diameter of the semicircle.)
(a) $70cm^2$ (b) $50cm^2$ (c) $25\ cm^2$ (d) $80cm^2$

10. Let A be the area of a square inscribed in a circle of radius $'r'$, and let B be the area of a hexagon inscribed in the same circle. Then, B/A equals:
(a) $2\sqrt{3}$ (b) $\frac{3\sqrt{3}}{4}$ (c) $\frac{2\sqrt{3}}{4}$ (d) None of these

11. In a trapezium, the diagonals intersect at point O. The ratio of the length of one of the diagonals from one vertex of the trapezium to the point O to its entire length is $2:5$. Find the ratio of its parallel sides (smaller side : larger side).
(a) $2:5$ (b) $2:3$ (c) $2:7$ (d) $5:7$

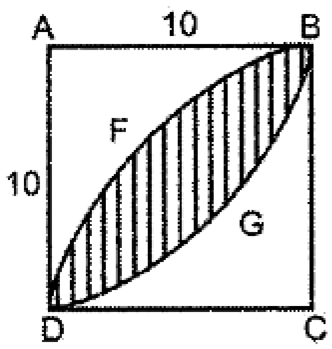
12. The sum of the lengths of the hypotenuse and one of the perpendicular sides of a right angled triangle is L. When the area of this triangle is maximum, the angle between these two sides is:
(a) 45° (b) 22.5° (c) 60° (d) None of these

13. Square $ABCD$ has side length 10. Point E is on $\overline{BC}$, and the area of ΔABE is 40. What is BE?



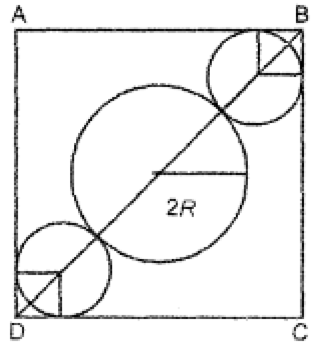
(a) 4 (b) 5 (c) 8 (d) 7

14. In the figure, $ABCD$ is a square with side 10. BFD is an arc of a circle with centre C. BGD is an arc of a circle with centre A. What is the area of the shaded region?



(a) $100\pi - 50$ (b) $100 - 25\pi$ (c) $50\pi - 100$ (d) $25\pi - 10$

15. See the figure given below:
Inside a square ABCD, three circles are drawn touching one another as shown in the figure. Radius of two smaller circles is R units and radius of the bigger circle is 2R units. Diagonal BD of the square passes through the centre of all the circles. What is the ratio of the radius of the smaller circles to the side of the square?



(a) 2 : 27 (b) 3 : 8 (c) 1 : 4 (d) None of these

16. Lines joining mid-point of a quadrilateral form *a*____
(a) square (b) parallelogram (c) rectangle (d) None of these

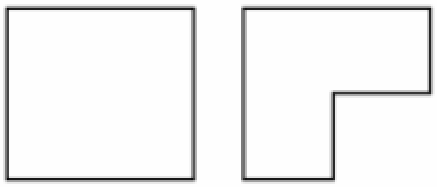
17. A rhombus has a perimeter of 52 cm and a diagonal measuring 24 cm. What is the length, in centimeters, of the other diagonal?

18. A rhombus has diagonals measuring 6 cm and 10 cm. What is its area in square centimeters?
(a) 30 (b) 32 (c) 60 (d) 64

19. One angle of a parallelogram measures $(2x + y)°$. Another angles of the same quadrilateral (but not the opposite angle) measures $(x + 2y)°$. What is $(x + y)$?
(a) 30 (b) 60 (c) 90 (d) 120

20. A square has a diagonal measuring 8 cm. When its area is expressed as 2^k square centimeters, what is K?
(a) 4 (b) 5 (c) 6 (d) 7

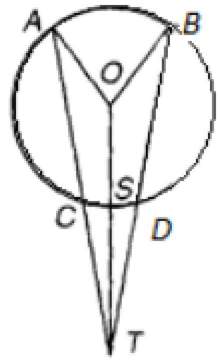
21. One – fourth of the area of a square with each side measuring $2x$ cm is sectioned off and removed. ("Before and After" pictures of the procedure appear to the right.) The area removed is itself square – shaped. What is the perimeter of the resultant figure in centimeters?



(a) $6x$ (b) $7x$ (c) $8x$ (d) $9x$

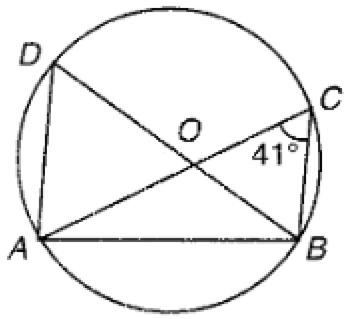
Exercise – 5
HOTS

1. In the adjoining figure 'O' is the centre of circle AC and BD are the two chords of circle which meets at T outside the circle. OT bisects CD, $OA = OB = 8cm$ and $OT = 17$ cm. What is the ratio of distance of AC and BD from the centre of the circle?



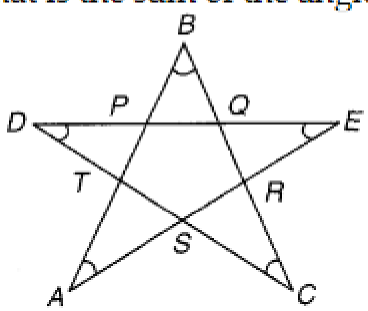
(a) 15 : 17 (b) 8 : 15 (c) 8 : 9 (d) none of the above

2. In the adjoining figure BD is the diameter of the circle and $\angle BCA = 41°$. Find $\angle ABD$:



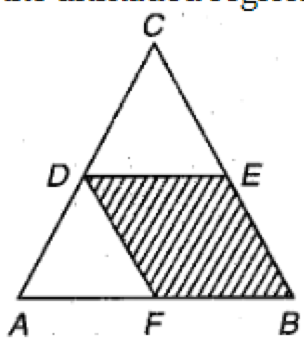
(a) 41° (b) 49° (c) 22.5° (d) 20.5°

3. In the adjoining figure, a star is shown. What is the sum of the angles *A, B, C, D* and *E?*



(a) 120° (b) 180° (c) 240° (d) can't be determined

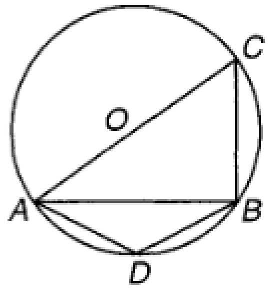
4. ABC is a triangle in which D, E and F are the mid – points of the sides AC, BC and AB respectively. What is the ratio of the area of the shaded to the unshaded region in the triangle?



(a) 1 : 1 (b) 3 : 4 (c) 4 : 5 (d) none of the above

5. In an isosceles right angled triangle ABC, $\angle B$ is right angle. Angle bisector of $\angle BAC$ is AN cut at M to the median BO.
Point 'O' lies on the hypotenuse. OM is 20 cm, then the value of AB is :
(a) 38.96cm (b) 24.18cm (c) 34.134cm (d) none of these

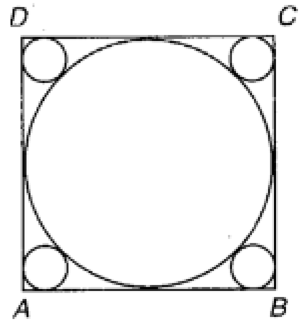
6. In the given diagram, 'O' is the centre of the circle and AC is the diameter. $\angle ADB$ is 120°. Radius of the circle is 6 cm, what is the area of the triangle ABC?



(a) $18\sqrt{3}\ cm^2$ (b) $24\sqrt{3}\ cm^2$ (c) $27\ cm^2$ (d) Data insufficient

7. A circle is circumscribed by the rhombus which in turn is made up by joining the mid-points of a rectangle whose sides 12 cm and 16 cm respectively. What is the area of the circle?
(a) $\frac{625}{26}\pi$ (b) $\frac{676}{25}\pi$ (c) $\frac{576}{25}\pi$ (d) can't be determined

8. *ABCD* is a square, in which a circle is inscribed touching all the sides of square. In the four corners of square 4 smaller circles of equal radii are drawn, containing maximum possible area. What is the ratio of the area of larger circle to that of sum of the areas of four smaller circles?



(a) $1 : (68 - 48\sqrt{2})$ (b) $1 : 17\sqrt{2}$ (c) $3 : (34 - 12\sqrt{2})$ (d) none of these

9. A trapezium *PQRS* inscribes a circle which touches the circle at M, *A, N, B,* Radius of circle is 10 cm. The length of each non-parallel side is 21 cm. What is the perimeter of trapezium?
(a) 82cm (b) 84cm (c) 85.5 cm (d) can't be determined

10. Identical spherical balls are spread on a table top so as to form an equilateral triangle. How many balls are needed so that a side of the equilateral triangle contains n balls
(a) $\frac{n(n+1)}{2}$ (b) $n^2 - 1$ (c) $n(n-1)$ (d) $n!$

11. A square is inscribed in a circle which is inscribed in an equilateral triangle. If one side of the triangle is 'a', find the area of the square.
(a) $\frac{1^2}{2}$ (b) $\frac{a^2}{6}$ (c) $\frac{3a^2}{8}$ (d) $\frac{\pi a^2}{12}$

12. There is a fan with 3 blades at 120° to each other whose central circular disc has an area of $3\pi cm^2$ and a blade is $(20 - \sqrt{3})$ cm long. If the tips of the blades are joined so as to form an equilateral triangle, what will be its area?
(a) $900cm^2$ (b) $300\sqrt{3}cm^2$ (c) $(900 + 9\pi)cm^2$ (d) $(3\pi + 300)cm^2$

13. Two circles of an equal radii are drawn, without any overlap, in a semicircle of radius 2 cm. If these are the largest possible circles that the semicircle can accommodate, what is the radius (in cm) of each of the circles?
(a) 0.414 (b) 0.828 (c) 0.172 (d) 0.586

14. A rhombus $OABC$ is drawn inside a circle, whose centre is at O, in such a way that the vertices A, B, and C of the rhombus are on the circle. If the area of the rhombus is $32\sqrt{3}m^2$, then the radius of the circle is:
(a) 64 m (b) 8 m (c) 32 m (d) 46 m

15. The ratio of the area of a square inscribed in a semicircle to that of the area of a square inscribed in the circle of the same radius is:

(a) 2 : 1 (b) 2 : 3 (c) 2 : 5 (d) 2 : 7

16. Let S_1 be a square of side a. Another square S_2 is formed by joining the mid – points of the sides of S_1. The same process is applied to S_2 to form yet another square S_3, and so on. If $A_1, A_2, A_3 \ldots$ be the areas if $P_1, P_2, P_3, \ldots$ be the perimeters of $S_1, S_2, S_3, \ldots$, respectively, then the ratio of $\frac{P_1+P_2+P_3+\cdots}{A_1+A_2+A_3+\cdots}$ $equals$.
(a) $\frac{(2+\sqrt{2})}{a}$ (b) $\frac{2(2-\sqrt{2})}{a}$ (c) $\frac{2(2+\sqrt{2})}{a}$ (d) $\frac{2(1+2\sqrt{2})}{a}$

17. The perimeter of a right – angled triangle is four times the shortest side. The ratio of the other two sides is:
(a) 5 : 6 (b) 3 : 4 (c) 4 : 5 (d) 2 : 3

18. In a triangle ABC, the medians AM and CN to the sides BC and AB, respectively, intersect at the point O. Let P be the mid – point of AC and let MP intersect CN and Q. If the area of the triangle OMQ is s square units, the area of ABC is:
(a) 16s (b) 18s (c) 12s (d) 24s

19. The sides of a triangle are given by:
$\sqrt{b^2+c^2}, \sqrt{c^2+a^2}$ and $\sqrt{a^2+b^2}$ $where\ a, b, c\ are\ positive$. Then, the area of the triangle equals.
(a) $\frac{1}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$ (b) $\frac{1}{2}\sqrt{a^4+b^4+c^4}$
(c) $\frac{\sqrt{3}}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$ (d) $\frac{\sqrt{3}}{2}(bc+ca+ab)$

20. If the number of square inches in the area of a square is equal to the number of inches in its circumference, then the diagonal of the square is equal to:
(a) 4 (b) $4\sqrt{2}$ (c) $3\sqrt{2}$ (d) $\sqrt{2}$

21. Three circles of equal radii have been drawn inside an equilateral triangle, of side a, such that each circle touches the other two circles as well as two sides of the triangles. Then, the radius of each circle is:
(a) $\frac{a}{2(\sqrt{3}+1)}$ (b) $\frac{a}{2(\sqrt{3}-1)}$ (c) $\frac{a}{\sqrt{3}-1}$ (d) $\frac{a}{\sqrt{3}-1}$

22. In a triangle $ABC, AB = 3, BC = 4$ and $CA = 5$. Point D is the midpoint of AB, point D is the midpoint of AB, point E is on the segment AC and point F is on the segment BC. IF $AE = 1.5$ and $BF = 0.5$ $then$ $\angle DEF =$
(a) 30° (b) 60° (c) 45 ° (d) 75°

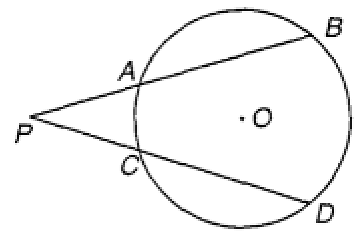
23. Two poles, of height 2 metres and 3 metres, are 5 metres apart. The height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is,
(a) 1.2 metres (b) 1.0 metres (c) 5.0 metres (d) 3.0 metres

24. A circle is inscribed inside a square. The square is inscribed inside another circle. If the area of the small circle is $\pi\ cm^2$, what is the area of the large circle, in square centimeters?
(a) $\pi\sqrt{2}$ (b) 2π (c) $2\pi\sqrt{2}$ (d) 4π

Exercise – 6

TITA/SHORT ANSWERS

1. In the given figure, $AP = 3cm, BA = 5\ cm\ and\ CP = 2cm$. Find CD:



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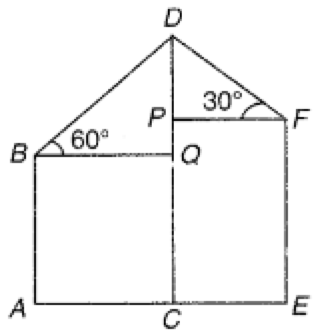
2. The distance between the centres of equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is :

____

3. In the given figure $AB, CD\ and\ EF$ are three towers. The angle of elevation of the top of the tower CD from the top of the tower AB is 60° and that from EF is 30°. BD $= 2\sqrt{3}m, CD : EF = 5 : 4\ and\ DF = 4\ m$. What is the height of the tower AB?



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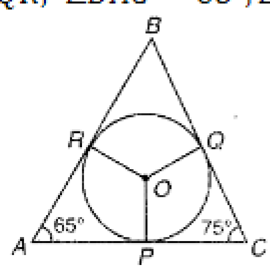
4. In a triangle ABC, O is the centre of incircle PQR, $\angle BAC = 65°, \angle BCA = 75°, find\ \angle ROQ$:



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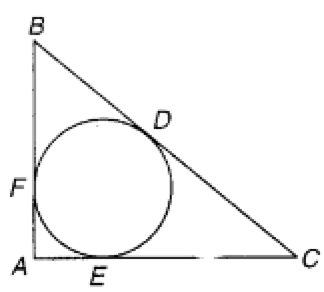
5. In the given diagram an incircle DEF is circumscribed by the right angled triangle in which $AF = 6cm$ and $EC = 15\ cm$. Find the difference CD and BD:



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6. How many distinct equilateral triangles can be formed in a regular nonagon having at two of their vertices as the vertices of nonagon?

____

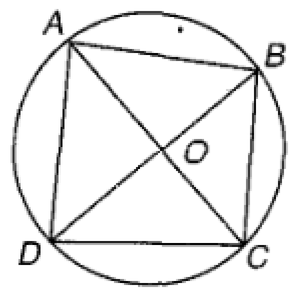
7. In a triangle all the three angles A, B, C are in integers, then the number of values that A, B and C can take :

____

8. Two trains Punjab mail and Lucknow mail starts simultaneously from Patiyala and Lakhimpur respectively towards each other with the speed of 40 km/h and 60 km/h respectively on the same track Lakhimpur is 500 km due east of Patiyala. A plane starts flying at 200 km/h at the same time from Patiyala to Jalandhar. Jalandhar is 100 knulue north of Patiyala. After travelling sometime two "trains Punjab mail and Lucknow mail collides with each other. The plane moves continuously to and fro between Patiyala to Jalandhar till the collision of the trains. How far would the plane have travelled?

____

9. In the given figure $ABCD$ is a cyclic quadrilateral DO = 8 cm and CO = 4 cm. AC is the angle bisector of $\angle BAD$. The length of AD is equal to the length of AB. DB intersects diagonal AC at O, then what is the length of the diagonal AC?



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10. There are n rectangles each with area 200 cm^2. If the dimensions of each n rectangles are in integers then the value of n is:

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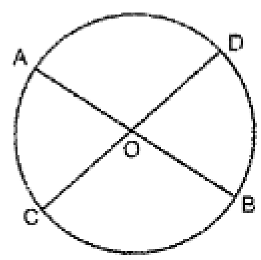
11. In the given figure, AB and CD are two chords of a circle intersecting at O. If AO = 4 cm, OB = 6 cm, and OC - 3 cm, then find OD.



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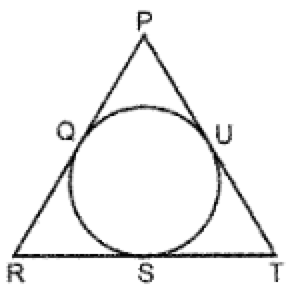
12. In the given figure PR, PT and RT are tangents to the circle at Q, U and S, respectively. $PR = (RT + 3)$ cm; $PR = (PT + 1)cm$. If the perimeter of triangle RPT is $26cm$, what is value of $QR + PT$?



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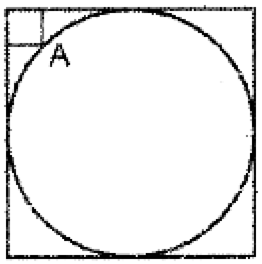
13. In the figure below, the rectangle at the corner measures 10 cm $\times$ 20 cm. The corner A of the rectangle is also a point on the circumference of the circle. What I the radius of the circle in cm?



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14. Let $s = \{(x, y): |x| + |y| = 2\}$. Then, the diameter is S is:

____

15. $ABCD$ is a square with sides of length 10 units. OCD is an isosceles triangle with base CD. OC cuts AB at point Q and OD cuts AB at point P. The area of trapezoid $PQCD$ is 80 square units. The altitude from O of the triangle OPQ is:

____

16. In a triangle ABC the length of side BC is 295. If the length of side AB is a perfect square, then the length of side AC is a power of 2, and the length of side AC is twice the length of side AB. Determine the perimetre of the triangle.

____

17. The center of a circle inside a triangle is at a distance of 625 cm, from each of the vertices of the triangle. If the diametre of the circle is 350 cm and the circle is touching only two sides of the triangle, find the area of the triangle,

____

18. Eight points lie on the circumference of a circle. The difference between the number of triangles and the number of quadrilaterals that can be formed by connecting these points is:

____

19. If the sides of a triangle measure 72, 75 and 21. What is the measure of its inradius?

____

20. If the interior angle of a regular polygon is 120°, find the number of diagonals of the polygon __.

____

21. The internal angle of a regular polygon exceeds the internal angle of another regular polygon by 18°. If the second polygon has half the number of sides as the first, then the number of sides in the first polygon is__.

____

22. An equilateral triangle T_1 has area $100\sqrt{3}\ sq.cm$. A second triangle T_2, is drawn with vertices on the midpoints of the sides of T_1. The midpoint of the sides of T_2 are the vertices of triangle T_3, and so on. What is the sum of the perimeters, in centimeters, of all the triangles, $T_1, T_2, T_3 \dots etc$?

____

23. In a $30° - 60° - 90°$ triangle, the longest side and the shortest side differ in length by 2002 units. What is the length of the longest side?

____

24. The base of an isosceles triangle is 80 cm long. If the area of the triangle cannot exceed 1680 square centimeters, what is the maximum number of centimeters in the perimeter of the triangle?

____

25. A triangle has side measuring 41 cm, 41 cm and 18 cm. A second triangle has side measuring 41 cm, 41 cm and x cm, where x is a whole number not equal to 18. If the two triangles have equal areas, what is the value of x?

PercentileClasses
Geometry Solutions

Exercise 01
Polygons
Solution

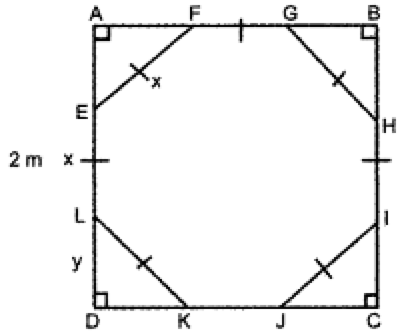
1. Ans: d
Solution
Go through options. It is pentagon
Alternatively: ${}^{n}C_2 - n = n$
$\Rightarrow \quad {}^{n}C_2 = 2n$
$\Rightarrow \quad \frac{n(n-1)}{2} = 2n$
$\Rightarrow \quad n = 5$

2. Ans: b
$9 \times 180 - 2 \times 360$
$= 180 \times 5 = 900°$ $\because$
$\left\{\begin{matrix} n \times 180 - 2 \times 360 \\ = 180(n-4) \end{matrix}\right\}$

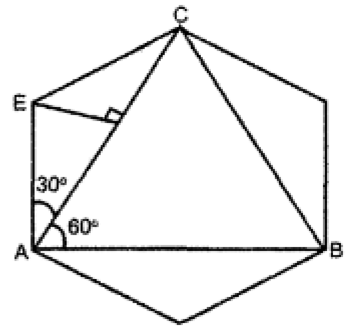
3. Ans: c
Solution
Area of hexagon $ABCDEF = 6 \times \frac{\sqrt{3}}{4} \times (6)^2$
$= 54\sqrt{3}cm^2$
$\therefore \quad Area\ od\ BDF = \frac{1}{2}(Area\ of\ hexagon)$
$= 27\sqrt{3}cm^2$

4. Ans: c
$\frac{(2n-4)\times 90}{n} = 135$
$(2n-4)2 = 3\,n$
$4n - 8 = 3n$
$n = 8$
Number of diagonals $= \frac{1}{2}n(n-3) = 20$

5. Ans: a
Solution
$x + 2y = 2$
(i)
In $\Delta AEF, x^2 = 2y^2$
(ii)
Use $(i) and\ (ii)$ to get the answer.

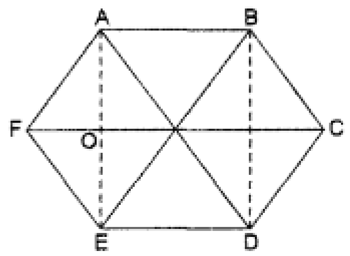


6. Ans: d
Solution
As hexagon is regular and $AD = CD$
So, ΔABC is equilateral triangle with $AB = BC\ AC = \frac{P}{3}$
$AD = \frac{P}{6}$



So,
$Cos\ 30° = \frac{\sqrt{3}}{2} = \frac{AD}{AE}$
$AE = AD \times \frac{2}{\sqrt{3}} = \frac{2p}{3\sqrt{3}}$
So, perimeter of hexagon $= 6 \times \frac{2p}{6\sqrt{3}} = \frac{2p}{\sqrt{3}}$

7. Ans: a
Solution

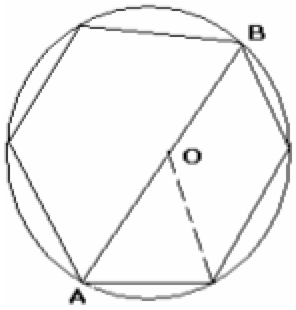


For regular hexagon. The area of all small triangles will be equal, and there are 12 small equal triangles.
Then, $\frac{Area\ of\ \Delta AOF}{Area\ of\ ABCDEF} = \frac{1}{12}$

8. Ans: 10
Solution
Let the number of sides in the polygon be n as per the question:
$(n-2)180° = 40\left(180° - \left[\frac{n-2}{n}\right]180°\right)$
$(n-2)180° = 40\left\{\frac{n-n+2}{n}\right\}180°$
$\frac{2}{n} = \frac{(n-2)}{40}$
$n = 10$

9. Ans: 1°
Solution
Exterior angle = 180 – 178 = 2°
$\Rightarrow \quad 360°/2 = 180\ so\ 180°/180 = 1°$

10. Ans: $54\sqrt{3}$
Solution
As all the sides of a regular hexagon makes an equilateral Δ with the centre of hexagon so $AB = 2a$, where a is the side of each hexagon.



$\Rightarrow \quad 2a = 2r = 2 \times 6 \Rightarrow a = 6$

So area of the hexagon $= \frac{3\sqrt{3}}{2}6^2 = 54\sqrt{3}$.

11. Ans: a

Solution

Perimeter of the hexagon = 12 cm = 6a

$\Rightarrow \quad$ a = 2 cms.

So area $= \frac{3\sqrt{3}}{2}.2^2 = 6\sqrt{3}$.

12. Ans: 54°

Solution

Sum of all the angles of a pentagon $= (5-2) \times 180° = 540°$

$\Rightarrow 10x + 10y = 540° \quad \Rightarrow$

$x + y = 54°$

13. Ans: b

Solution

Each angle of a regular polygon $= \frac{(n-2)180°}{n} = 177°$

$\Rightarrow n = 120$

14. Ans: b

Solution

Because the two figures are similar, all cones angles would be equal. Hence $\angle J = \angle A = 120°$

15. Ans: c

Solution

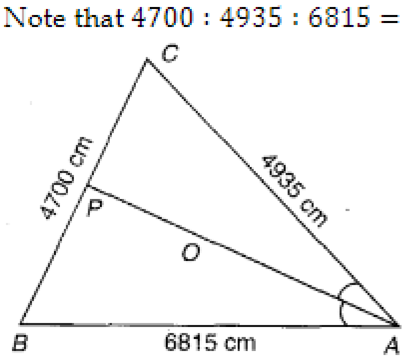
Interior $\angle + Exterior \angle = 180°$ always.

Exercise 02
Triangles
Solution

1. Ans: c

Solution

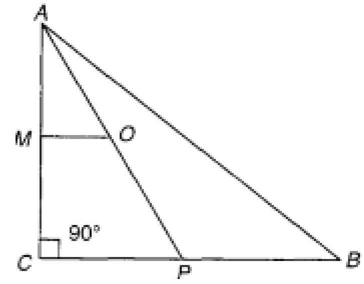
Note that 4700 : 4935 : 6815 = 20 : 21 : 29



$\therefore \quad \Delta ACB$ is right angled triangle and $\angle ACB$ is a right angle.

$\therefore \quad$ The inradius of the right angled triangle

$= \frac{20+21-29}{2} = 6\ cm$



$\therefore \quad MO = MC = 6\ cm$

$\therefore \quad AM = 21 - 6 = 15\ cm$

$\therefore \quad \frac{AM}{MC} = \frac{AO}{OP} = \frac{15}{6} = \frac{5}{2}$

$\Rightarrow \quad \frac{PO}{AO} = \frac{2}{5}$

2. Ans: b

Solution

Inradius of a right angled triangle $= \frac{AC+BC-AB}{2}$

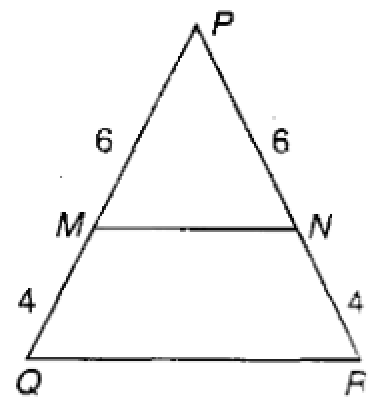
$= \frac{9+40-41}{2} = 4cm$

3. Ans: c

Solution

It is clear that

$\frac{PM}{MQ} = \frac{PN}{NR} = \frac{6}{4} = \frac{3}{2}$



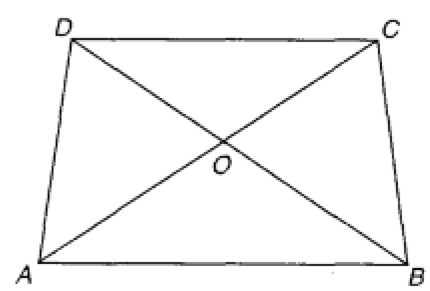
Therefore, $\Delta PMN\ and\ \Delta PQR$ are similar

Now Ratio of area of $\Delta PMN : \Delta PQR$

$= 3 \times 3 : 5 \times 5 = 9 : 25$

$\therefore \quad$ *Pertange of area of* ΔPMN over $\Delta PQR = \frac{9}{25} \times 100 = 36\%$

4.



Ans: b

Solution

$\angle AOB = 60°$

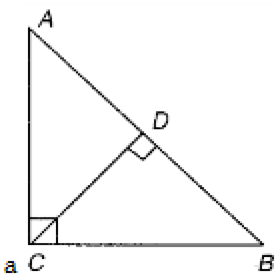
And, $\quad \angle AOD = \angle ADO = \frac{(180°-30°)}{2} = 75°$

Also, $\quad \angle BOC = 75°$

$\therefore \quad \angle COD = 360° - (75° + 60° + 75°) = 150°$

5. Ans: c
Solution
$\angle ABD = \angle ABC - \angle DBC$
$= \angle ABC - \angle BDC$
$= \angle ABC - (\angle ABD + \angle BAD)$
$\therefore$ $2(\angle ABD) = \angle ABC - \angle BAD = 30°$
$\therefore$ $\angle ABD = 15°$ $(\because \angle BAD = \angle BAC)$

6. Ans: d
Solution
$AB : BC ; CA = \frac{1}{CD} : \frac{1}{AE} : \frac{1}{BF} = \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$



7. a

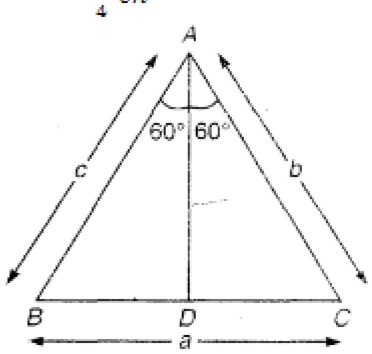
(a) $CD = \frac{AC \times BC}{AB}$ (b) $AD = \frac{AC \times AC}{AB}$
(c) $BD = \frac{BC \times BC}{AB}$
(d) All of the above
Ans: d
Go back to the basics.

8. Ans: c

9. Ans: a
Solution
Best way is to consider some values and verify the results.

10. Ans: c
Solution
$x^2 + y^2 + z^2 = xy + yz + zx$
$\Rightarrow$ $x^2 + y^2 + z^2 - xy - yz - zx = 0$
$\Rightarrow$ $2(x^2 + y^2 + z^2 - xy - yz - zx) = 0$
$\Rightarrow$ $(x^2 + y^2) + (y^2 + z^2) + (z^2 + x^2) - 2xy - 2yz - 2zx = 0$
$\Rightarrow$ $(x - y)^2 + (y - z)^2 + (z - x)^2 = 0$
$\Rightarrow$ $x = y = z$
$\therefore$ The given triangle is an equilateral triangle.

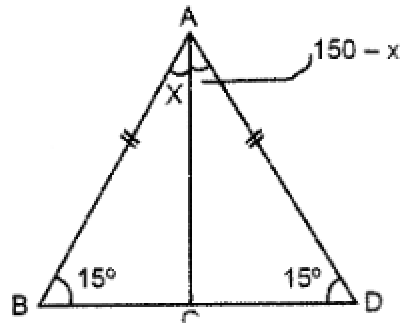
11. Ans: b
Solution
Let $AD = h(say)$
Then Area of $\Delta ABC = \frac{1}{2} bc \sin 120° = \frac{\sqrt{3}}{4} bc$
Area of ΔBAD
$= \frac{1}{2} ch \sin 60°$
$= \frac{\sqrt{3}}{4} ch$



And area of $\Delta CAD = \frac{1}{2} bh \sin 60° = \frac{\sqrt{3}}{4} bh$

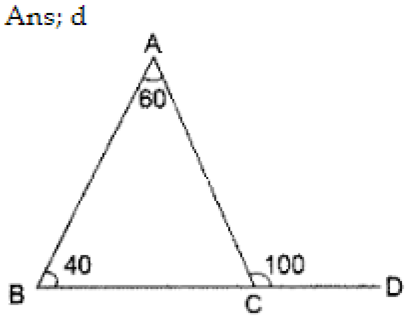
Now, $A(\Delta ABC) = A(\Delta BAD) + A(\Delta CAD)$
$\frac{\sqrt{3}}{4} bc = \frac{\sqrt{3}}{4} ch + \frac{\sqrt{3}}{4} bh$
$\Rightarrow$ $bc = h(b + c)$
$\Rightarrow$ $h = \frac{bc}{b+c}$

12. Ans: c

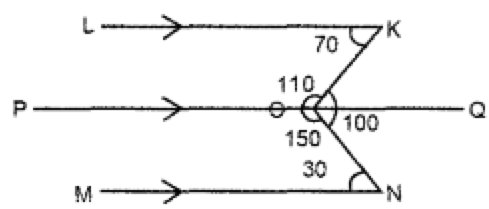


$150° - x + 15° = 95° + x$
$2x = 70°$
$x = 35°$
Therefore, $\angle ACB = 95° + 35° = 130°$

13. Ans; d



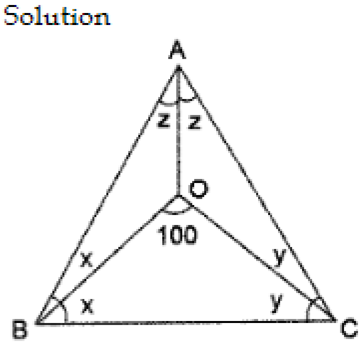
14. $\angle LKO = 70°, \angle KON = 100°$
Find $\angle MNO$.
(a) 20° (b) 30°
(c) 40° (d) 50°
Ans: b
Solution



Hence, $\angle MNO = 30°$

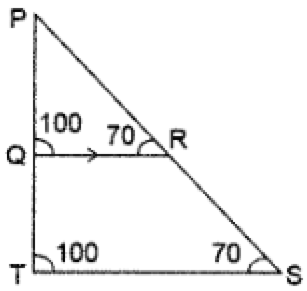
15. Ans: a
Solution
$x + 5x = 180°$
$6x = 180°$
$x = 30$

16. Ans: b
Solution



In $\angle BOC, x + y = 80$
$2x + 2y = 160°$
Also, $2x + 2y + 2z = 180°$
$2z = 20°$
$\angle BAC = 2z = 20°$

17. Ans: b

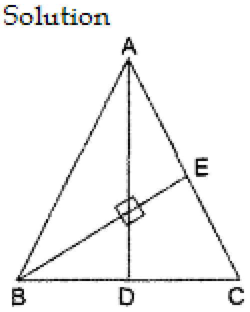


$PQ \times PS = PT \times PR$
$QR || TS$
$\angle PQR = \angle PST + 30°$
$\angle PST = 70°$
$\angle PRQ = 70°$

18. Ans: d
Solution
Clearly, triangle is obtuse. So, (d) is the correct option.

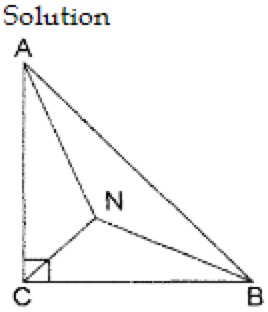
19. Ans: d
Solution
$\frac{1}{2}:\frac{1}{3}:\frac{1}{4} = 6:4:3$
$6x + 4x + 3x = 52, or, 13x = 52$ and $x = 4$
Required length = $12\ cm$

20. Ans: a
Solution



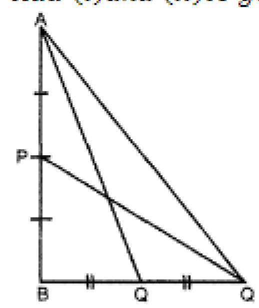
Area $\Delta AOB = \frac{1}{2} \cdot 8.6 = 24cm^2$
Area of $\Delta BOD = \frac{1}{2} \cdot 8.3 = 12cm^2$
Area of $\Delta ABD = 36cm^2$
Area of $\Delta ABC = 72cm^2$

21. Ans: b
Solution



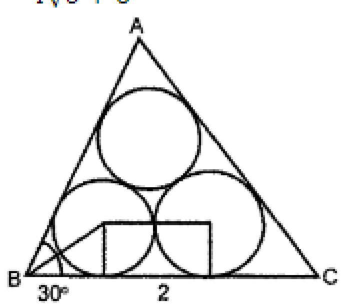
22. Ans: d
Solution
Use Pythagoras theorem, in right – angled ΔABC

$AQ^2 = AB^2 + \left(\frac{BC}{2}\right)^2$
(i)
And in right – angled ΔCBP
$CP^2 = \left(\frac{AB^2}{2}\right) + (BC)^2$
(ii)
Add (i) and (ii) to get the answer.



23. Ans: b
Solution
Since sides of Δ are: 3, 4, and 5, so this Δ will be a right angle triangle.
Then, radius of inscribed circle $C_1 = \frac{A}{s} = \frac{6}{6} = 1$
Radius of circumscribed circle $C_2 = \frac{abc}{4A} = \frac{3\times4\times5}{4\times6} = \frac{5}{6}$
Then, $\frac{Area\ of\ C_1}{Area\ of\ C_2} = \frac{\pi r_1^2\,(r_1)^2}{\pi r_2^2\,(r_2)^2} = \frac{(1)^2}{(5/2)^2} = \frac{4}{25}$

24. Ans: b
Solution
It can be seen that the side of the triangle
$= \sqrt{3} + 2 + \sqrt{3} = 2 + 2\sqrt{3}$
Then, area $= \frac{\sqrt{3}}{4}a^2$
$= \frac{\sqrt{3}}{4}(2 + 2\sqrt{3})^2$
$= 4\sqrt{3} + 6$



25. Ans: (a)
Solution
When the length of stick = 20 m, then length of shadow = 10 m i.e., in this case length = 2 ×shadow.
With the same angle of inclination of the sun, the length of tower that casts a shadow of 50 m ⇒ $2\times 50m = 100m$.

26. Ans: c
Solution
For similar triangles ⇒ $(Ratio\ of\ sides)^2 = Ratio\ of\ areas$
Then as per question $= \left(\frac{36}{x}\right)^2 = \frac{144}{81}$

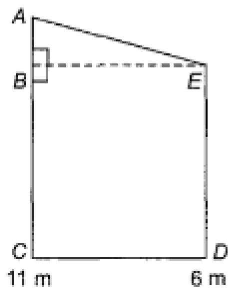
$\{Let\ the\ longest\ side\ of\ \Delta DEF = x\}$
$\Rightarrow \frac{36}{x} = \frac{12}{9} \Rightarrow x = 27\ cm$

27. Ans: a
Solution

$(Ratio\ of\ correspongind\ sides)^2 =$
$Ratio\ of\ area\ of\ similar\ triangles$
∴ Ratio of corresponding sides in this question
$= \sqrt{\frac{16}{25}} = \frac{4}{5}$

28. Ans: a
Solution
Ratio of corresponding sides $= \sqrt{\frac{9}{16}} = \frac{3}{4}$

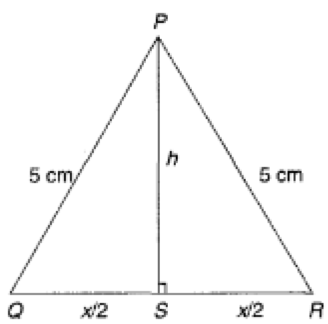
29. Ans: c
Solution



So, $BC = ED = 6m$
$AB = AC - BC = 11 - 6 = 5\ m$
$CD = BE = 12m$
Then by Pythagoras theorem:
$AE^2 = AB^2 + BE^2 \Rightarrow AE = 13m$

30. Ans: $300\ cm^2$
Solution
Inradius of $\Delta ABC = \frac{(AB+AC)-AC}{2} = \frac{20}{2} = 10cm$
(Formula for inradius)
Semiperimetre $= s = \frac{60}{2} = 30cm$
$r = \frac{Area}{s}$
Area $= r \times s$(Formula for area of any triangle using the semi – perimeter and inradius).
$= 10 \times 30 = 300\ cm^2$

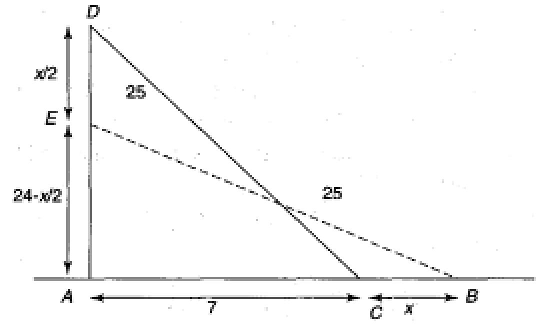
31. Ans: b
Solution



Area of $PQR = \frac{1}{2} \times x \times h = 12\ cm^2$
$xh = 24cm^2$
Also in the right angle ΔPQS,
$(5)^2 = \left(\frac{x}{2}\right)^2 + h^2$
$x^2 + 4h^2 = 100$
$x^2 + 4\left(\frac{24}{x}\right)^2 = 100$
The above equation is satisfied only for $x = 6$. So option (b) is correct.

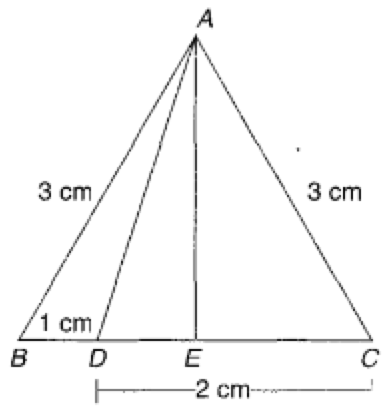
32. Ans: b
Solution
Let the other sides of the right angle triangle be $x\ and\ y$ respectively.
Then according to the question: $\sqrt{x^2 + y^2} = 97, x + 7 = 234 - 97 = 137$
Now by checking the option we can see that only option (b) satisfies both the equations.
So option (b) is correct.

33. Ans: d
Solution
Let the base of the ladder is drawn out by $x\ feet$.



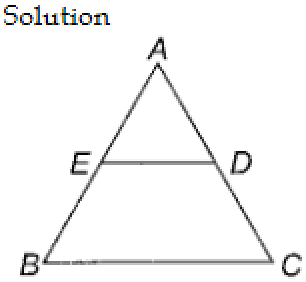
In ΔEAB : $\left(24 - \frac{x}{2}\right)^2 + (7 + x)^2 = 25^2$ (Using the Phythagoras theorem).
By solving the above quadratic equation we get $x = 0, 8$.
So option (d) is correct.

34. Ans: c
Solution



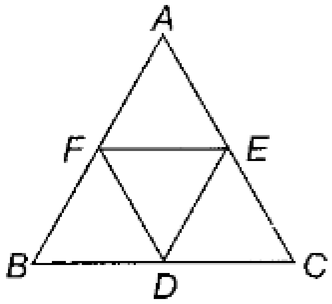
Draw $AE \perp BC$. Since $ABC\ is\ an\ equilateral\ triangle\ so\ AE\ will\ bisect\ BC$.
$DE = DC - EC = 2 - \frac{3}{2} =$ ½ cm
$AE = \sqrt{3^2 - 1.5^2} = \frac{3}{2}\sqrt{3}cm$
$AD = \sqrt{\left(\frac{3}{2}\sqrt{3}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{7}cm$

35. Ans: b
Solution



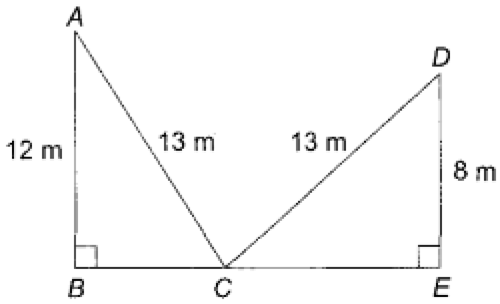
ΔADE is similar to ΔABC $(AAA\ property)$
$ED: BC = 3:5$
Area of ΔADE : $Area\ of\ \Delta ABC = 9 : 25$
Area of trapezium $= area\ of\ ABC - Area\ of\ ADE$
$= 25 - 9 = 16$
Thus,
Area of ΔADE : $Area\ of\ trapezium\ EDBC = 9 : 16$

36. Ans: a
Solution



The area of a triangle formed by joining the mid – points of the sides of another triangle is always $\frac{1}{4}$th of the area of the bigger triangle.
So, the ratio is = 1 : 4

37. Ans: a
Solution



In $\Delta ABC; BC = \sqrt{13^2 - 12^2} = 5$
In $\Delta CDE; CE = \sqrt{13^2 - 8^2} = \sqrt{105} = 10.2\ approximately$
Width of street $= BC + CE = 5 + 10.2 = 15.2\ m$

38. Ans: a
Solution
In $\Delta QRS; QR = RS$, therefore $\angle RQS = \angle RSQ$ (because angles opposite to equal sides are equal).
Thus, $\angle RQS + \angle RSQ = 180° - 100° = 80°$
∴ $\angle RQS = RSQ = 40°$
$\angle PQS = 180° - 40° = 140°$
(sum of angles on a line = 180°)
Then again $\angle QPS = \angle QSP$
($\because$ *angles opposite to equal sides are equal*)
Thus, $\angle QPS + \angle QSP = 180° - 140° = 40°$
And, $\angle QPS = \angle QSP = 20°$

Exercise – 03
Circle
Solution

1. Ans: b
Solution
$AC = \sqrt{OA^2 - OC^2} = \sqrt{(17)^2 - (8^2)} = 15cm$
∴ $AB = 2ac = 30cm$ ($\because$
$AC = BC$)

2. Ans: c
Solution
$\because$ AB is the perpendicular bisector of OO′
∴ $\angle OMA = 90°$

3. Ans: a
Solution
$\angle AOC = \angle BOC$ ($\because$
$AC + BC$)
∴ OC is the perpendicular bisector of AB.
∴ $AM = BM$

4. Ans: c
$OA = 13cm$
$AM = 5cm$
∴ $OM = \sqrt{(13)^2 - (5)^2} = 12cm$

5. Ans: c
Solution:
$\angle ADC = 180° - 65° = 115°$
∴ $\angle dca = 180° - (115° + 45°) = 20°$
(*ADCB is a cyclic quadrilateral*)

6. Ans: c
Solution:
$\angle OBA = \angle OAB = 55°$
∴ $\angle AOB = 180° - (55° + 55°) = 70°$
And $\angle AOB = \angle COD = 70°$

7. Ans: b
Solution:
$\angle BDC = 180° - (110° + 30°) = 40°$
And $\angle BAC = \angle BDC = 40°$ (angles of the same segment)

8. Ans: d
Solution:
$\angle ABO = 60° = \angle BAO (\because AO = BO, radii\ of\ the\ circle)$
∴ $\angle AOB = 180° - (60° + 60°) = 60°$
∴ $\angle ACB = \frac{1}{2}\angle AOB = 30°$

9. Ans: c
Solution:
$\angle CBA = \frac{1}{2}\angle COA = 60°$
∴ $\angle CBE = 180° - \angle CBA$
$180° - 60° = 120°$

10. Ans: a
Solution
$\angle BDC = \angle BAC = 30°$
∴ $\angle BCD + \angle BDC + \angle DBC = 180°$
∴ $\angle BCD = 180° - (30° + 60°)$
$\angle BCD = 90°$

11. Ans: b
Solution
$\angle CAB = \frac{1}{2}\angle BOC = \frac{1}{2} \times 40^\circ = 20^\circ$

12. Ans: c
Solution
$\angle ADC = \frac{100^\circ}{2} = 50^\circ$
$\therefore \quad \angle ABC = 180^\circ - 50^\circ = 130^\circ$
$\therefore \quad \frac{m\ \angle ADC}{m\ \angle ABC} = \frac{50^\circ}{130^\circ} = \frac{5}{13}$

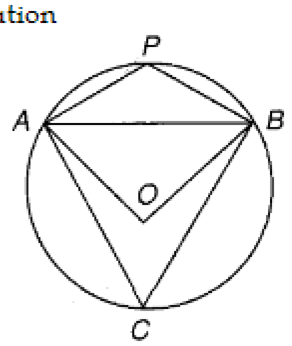
13. Ans: c
Solution
$\angle APB = 50^\circ$
$\left(\because m\ \angle APB = \frac{1}{2}\ m\ \angle AOB\right)$
$\therefore \quad \angle ACB = 180^\circ - 50^\circ = 130^\circ$
$\therefore \quad \angle BCD = 180^\circ - 130^\circ = 50^\circ$

14. Ans: a
Solution
$\angle BAC = 80^\circ \qquad (\because \angle ABC = \angle ACB = 50^\circ)$
$\therefore \quad \angle BDC = \angle BAC = 80^\circ$

15. Ans: c
Solution
$\angle ADB = 90^\circ$
$\therefore \quad \angle DBA = 20^\circ \qquad [180^\circ - (90^\circ + 70^\circ)]$
$\therefore \quad \angle CBA = 30^\circ + 20^\circ = 50^\circ$
$\therefore \quad \angle CDA = 180^\circ - 50^\circ = 130^\circ$
$\therefore \quad \angle CDB = 130^\circ - 90^\circ = 40^\circ$

16. Ans: b
Solution
$\angle AOC = 2 \times 50 = 100^\circ$
$\therefore \quad \angle ABC = \frac{100}{2} 50^\circ$

17. Ans: c
Solution



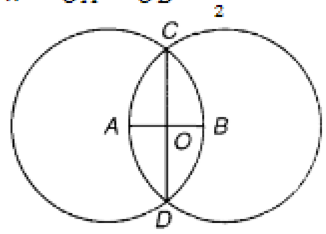
$\angle APB = 180^\circ - \frac{1}{2} \times 90^\circ = 135^\circ$
$\left(\begin{array}{l}\because \angle ACB = 45^\circ = \left(\frac{1}{2} \times 90^\circ\right) \\ and\ \angle APB + \angle ACB = 180^\circ\end{array}\right)$

18. Ans: c
Solution
$PN \times PR = PQ^2$
$PN \times 25 = (15)^2$
$PN = 9\ cm$

19. Ans: b
$\angle BAC = 90^\circ$
$\angle BCA = 60^\circ \qquad (\because \angle BCA = BAQ)$
$\therefore \quad \angle ABC = 180^\circ - (90^\circ + 60^\circ)$
$\angle ABC = 30^\circ$

20. Ans: c
$AB = AC, OB = OC$ and BC is common and OA is common
Also, $\qquad \angle BOA = \angle COA$
$\therefore \quad \Delta BOA \cong \Delta COA$
$\therefore \quad BC$ and OA are perpendicular bisector to each other
$\left[Area\ of\ \Delta = \sqrt{s(s-a)(s-b)(s-c)}\right]$
$s = \frac{a+b+c}{2} = \frac{5+5+6}{2} = 8cm$
Area of $\Delta BOA = \sqrt{8 \times 3 \times 3 \times 2} = 12cm^2$
Again area of $\Delta BOA = \frac{1}{2} OA \times BP = \frac{1}{2} \times 5 \times BP$
$12 = \frac{1}{2} \times 5 \times BP$
$\Rightarrow \quad BP = 4.8cm$
$\therefore \quad BC = 2BP = 2 \times 4.8 = 9.6cm$

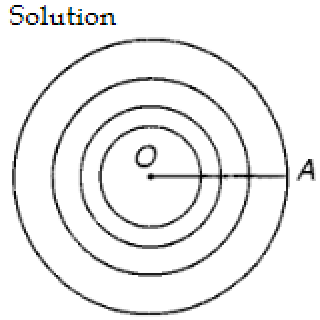
21. Ans: b
Solution
$AB = r\ (say)$
Then, $\qquad AC = BC = r, also$
$\therefore \quad OA = OB = \frac{r}{2} \qquad (CD\ is\ a\ common\ chord)$



$\therefore \quad OC = \sqrt{(AC)^2 - (OA)^2} = \sqrt{r^2 - \left(\frac{r}{2}\right)^2} = \sqrt{\frac{3r^2}{4}} = \frac{\sqrt{3}}{2} r$
$\therefore \quad CD = 2CO = 2 \times \frac{\sqrt{3}}{2} r = \sqrt{3}r$
$\therefore \quad \frac{l(CD)}{l(AC)} = \frac{\sqrt{3}r}{r} = \frac{\sqrt{3}}{1}$

22. Ans: d
Solution
Let A be the centre of larger circle and B be the centre of the smaller circle.
$OA = 3cm\ \ and\ BO = 2\ cm$
And, $AB = (OA - OB) = 1\ cm$
Also, $BP = 2\ cm$
$\therefore \quad AP = 1\ cm$
P is the point of contact of tangent MN
$\therefore \quad MP = \sqrt{AM^2 - AP^2} = \sqrt{3^2 - 1^2} = 2\sqrt{2}cm$
$\therefore \quad MN = 2MP = 4\sqrt{2}cm$

23. Ans: c
Solution



OA is the radius of the largest circle and O is the centroid of the all concentric circles.
Let
$r_1, r_2, r_3\ and\ r_4\ be\ the\ radii\ of\ the\ concentric\ circles\ in\ increasing$
Total area of given circle
$= \pi \times (20)^2 = 400\pi$

Area of each region $= \frac{1}{4} \times 400\pi = 100\pi$

$\therefore$ Area of central region $= 100\pi = \pi r_1^2$

$\Rightarrow$ $r_1 = 10cm$

Similarly, area of second region $= 100\pi = \pi(\pi_2^2 - r_1^2)$

$100\pi = \pi(r_2^2 - 100)$

$\Rightarrow$ $r_2 = 10\sqrt{2}cm$

Again, area of third region $= 100\pi = \pi(r_3^2 - r_2^2)$

$100\pi = \pi(r_3^2 - 200)$

$\Rightarrow$ $r_3^2 = 10\sqrt{3}cm$

$\therefore$ The required radii are $10, 10\sqrt{2}$ *and* $10\sqrt{3}$.

24. Ans: d

Solution

$\angle ACB = 180 - (90 + 25) = 65°$ $(\because \angle ABC = 90°)$

But since $\angle ADB = \angle ACB$

$\therefore$ $\angle ADB = 65°$

25. Ans: a

Solution

$\angle OCD = 90°$

$\angle OAC = \angle OCA = 30°$

$\angle ACD = \angle ACO + \angle OCD = 30° + 90° = 120°$

$\therefore$ $\angle BAC = 180° - 120° = 60°$

$\Rightarrow$ $\angle BCD = 60°$

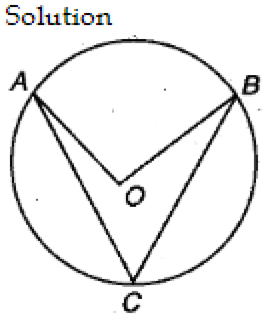
$(\angle BCD = \angle BAC)$

$\Rightarrow$ $\angle OCB = \angle OCD - \angle BCD$

$= 90° - 60° = 30°$

26. Ans: c

Solution



$\angle OCA = \angle OAC = 25°$

And, $\angle OCB = \angle OBC = 35°$

$\therefore$ $\angle ACB = \angle ACO + \angle BCO$

$= 25° + 35° = 60°$

$\therefore$ $\angle AOB = 2\angle ACB = 120°$

28. Ans: b

Solution

Clearly, triangle is a right-angled triangle.

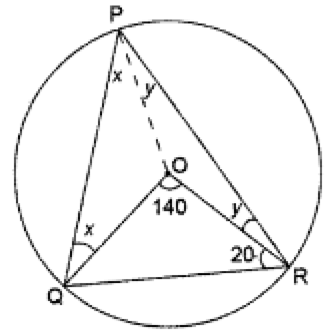
Its area $= \frac{1}{2} \times 6 \times 8 = 24cm^2$

Now, $8 \times b = 24$

$b = 3$

29. Ans: a

Solution



$\angle OQP + \angle ORP = 70°$

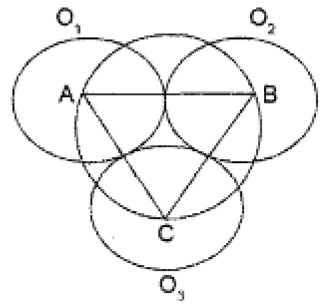
$x + y = 70°$

$\angle QPR = 70°$

$\angle ORQ = 20°$

30. Ans: c

Solution



It is given that radius of circle O_1, O_2

$AB = BC = CA = 2r$

Then, the radius of circle ABC

$= \frac{ABC}{4A} = \frac{2r \times 2r \times 2r}{4 \times \frac{1}{2} 2r \times 2r \times sin60°}$

$= \frac{8r^3}{4r^2\sqrt{3}} = \frac{2r}{\sqrt{3}}$, Which is greater than r.

31. Ans: a

Solution

In ΔCOD

$OC = OD = OB = OA = radius = r$

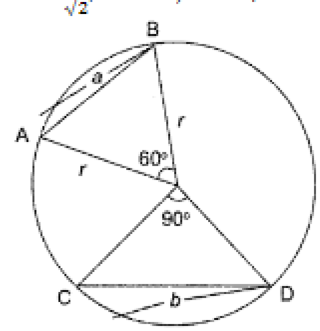
$b^2 = 2r^2$

$r = \frac{b}{\sqrt{2}}$

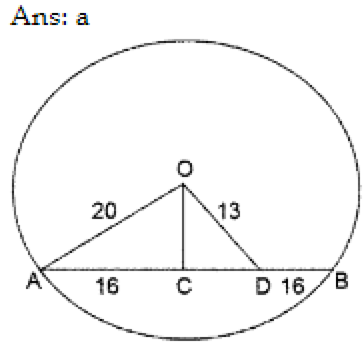
$\Delta AOB = equilateral\ triangle$

Hence, $a = r$

$r = \frac{b}{\sqrt{2}}$, hence, $b = \sqrt{2a}$



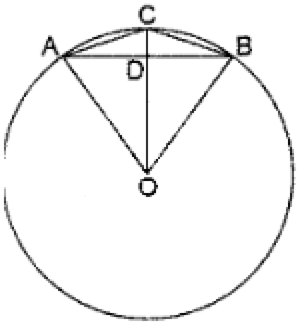
32. Ans: a



It is given that $OA = 20, AC = \frac{AB}{2} = 16$ *and* $OD = 16$

Then, lengths of segment AD and DB are 21 and 11, respectively.

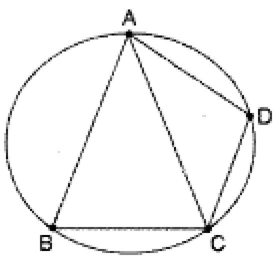
33. Ans: a
Solution
Method 1



Define D as the mid – point of line segment $\overline{AB}$, and O centre of the circle. Then, $O, C,$ and D are collinear, and since D is the mid – point of AB, m angle ODA = 90 degree, and so, $OD = \sqrt{5^2 - 3^2} = 4$. Since $OD = 4$, $CD = 5 - 4 = 1$, and so, $AC = \sqrt{3^2 + 1^2} = \sqrt{10}$. Hence option (a) is the answer.

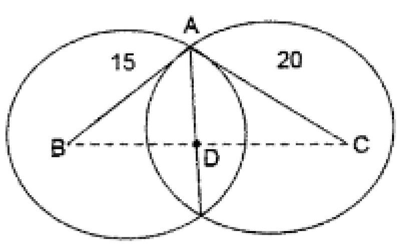
Method 2
Using Trigonometry
$\cos(\alpha/2) = \frac{\sqrt{1+\cos(\alpha)}}{2} = \sqrt{\frac{32/25}{2}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$
Hence, option (a) is the answer.

34. Ans: a
Solution



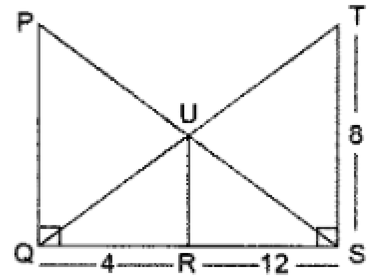
Since ΔABC is a equilateral triangle.
Let $AB = BC = CA = x$
Then, from the Ptolemy theorem
$AB \times CD + AD \times BC = AC \times BD$
$x\ CB + x\ AD = x\ BD$. Hence, $CB + AD = BD$

35. Ans: a
Solution



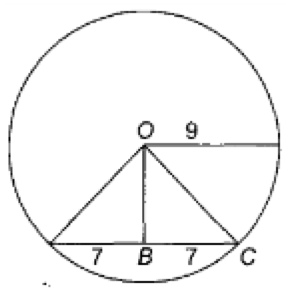
In ΔABC, the sides are $-15, 20,$ and 25.
Then, from the Pythagoras ΔABC will be a right angle triangle at $\angle A$
Then, area of $\Delta ABC = \frac{1}{2} AB \times AC = \frac{1}{2} \times 15 \times 20 = 150 cm^2$
Let height of ΔABC is AD
Then, area of $\Delta ABC = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 25 \times AD (ii)$
From equations, (i) and (ii)
$AD = 12\ cm$
Hence, length of the common chord $= 2\ AD = 24\ cm$

36. Ans: d



In
$\Delta\ QST$ and ΔQRU (ΔQST and ΔQRU are similar triangles)
.
$\frac{QR}{QS} = \frac{RU}{TS} \Rightarrow \frac{4}{16} = \frac{RU}{8} = RU = 2$
Also, $\frac{QR}{QS} = \frac{RU}{TS} \Rightarrow \frac{4}{6} = \frac{RU}{8} = RU = 2$
$\frac{1}{PQ} = \frac{1}{2} - \frac{1}{8} = \frac{4-1}{8} = \frac{3}{8}$
$PQ = \frac{8}{3} cm$

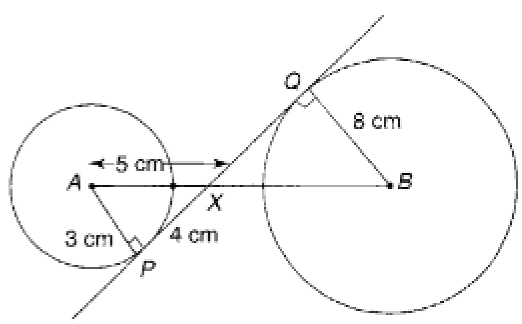
39. Ans: a
Solution



In the $\Delta OBC; BC = 7 cm$ and $OC = 9\ cm$, then using Pythagoras theorem.
$OB^2 = OC^2 - BC^2$
$OB = \sqrt{32} = 5.66\ cm\ (approx)$

40. Ans: b
Solution
Inradius of an equilateral triangle $= \frac{side}{2\sqrt{3}}$ (*Formula for inradius*)
Side $= 2\sqrt{3} \times \sqrt{3} = 6cm$
Required area $= \frac{\sqrt{3}}{4} \times 6^2 = 9\sqrt{3} cm^2$ (Using the formula that the area of an equilateral triangle of side a is given by the formula: $\frac{\sqrt{3}}{4} \times a^2$

41. Ans: c
Solution



Let A and B are centers of circles. $PX = \sqrt{5^2 - 3^2} = 4cm$
ΔAPX and ΔBQX are similar to each other as their three angles are equal.
$\frac{AP}{BQ} = \frac{PX}{QX}$
$QX = 8 \times - = 10.66\ cm$
$PQ = PX + XQ = 4 + 10.66 = 14.66\ cm$

42. Ans: a

Solution

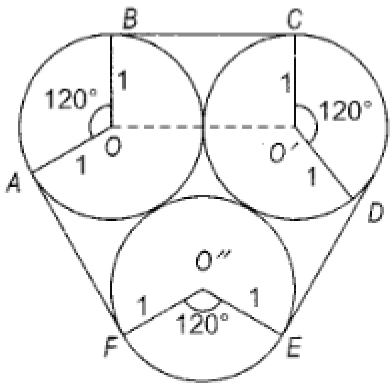
$\angle AOB = CO'D = \angle FO''E = 120°$

Distance between 2 centres $= 2\ m$

$\therefore \quad BC = DE = FA = 2\ m$

Perimetre of the figure $= BC + DE + FA +$ $circumference\ of\ sectors\ AOB, CO'B\ and\ FO''E.$

But three equal sectors of $120° = 1$ full circle of same radius.



Therefore, perimeter of surface

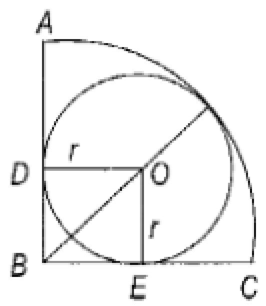
$= 2\pi r + BC + DE + FA = (2\pi + 6)m$

43. Ans: a

Solution

Assume the radius of the inner smaller circle to be 'r' and that of the outer semicircle to be R. Then, we have from the solution figure, we known that

$BO = R - r\ and\ BD = OD = r.$



$(R - r)^2 = r^2 + r^2\ (Using\ Pythagoras\ theorem)$

But we are given that $R = 1$; hence we get:

$1 - 2r + r^2 = 2r^2\ or\ r^2 + 2r - 1 = 0$

Solving this quadratic equation, we get the solution for $r = (\sqrt{2} + 1)\ or\ (\sqrt{2} - 1)cm$

However, r cannot be greater than R and hence cannot exceed a value of 1 cm. Hence, we would reject the first value of $(\sqrt{2} + 1)$ for r and select $r = (\sqrt{2} - 1)$. Option (a) is correct.

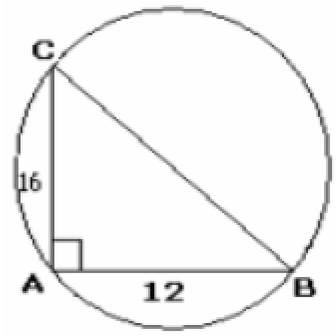
44. Ans: c

Solution

Since, we are given a measure of a 19° angle, if we use the measure 19 times, we would be able to measure 361° and hence, we can measure $361 - 360 = 1°$. Hence, it would be possible to divide the circle into 360 equal parts.

45. Ans: c

Solution



$BC = \sqrt{12^2 + 16^2} = 20cm$

$\therefore radius\ of\ circle = 10\ cm\ (\because$ $angle\ in\ a\ semi\ circle\ is\ 90°)$

$\therefore circumference = 2\pi.10 = 20\pi cm$

46. Ans: 10

Solution

Length of $AB = 2\pi \frac{M}{360°} = 8\pi$

$\Rightarrow r.\frac{M}{360°} = 4.$

Area of the sector $= \frac{\pi r^2 M}{360°} = 48\pi$

$\Rightarrow \frac{M}{360}° = \frac{1}{3} \Rightarrow M = 120°$

Hence,

$\left(\frac{M}{r}\right) = \frac{120}{12} = 10$

47. Ans: c

Solution

If r is the radius of the circle

$\Rightarrow A = r \times 5, \qquad s = \frac{4+6+8}{2} = 9\ cm., \qquad A =$ $\sqrt{9.5.3.1} = 3\sqrt{15}$

$3\sqrt{15} = r \times 9 \Rightarrow r = \sqrt{\frac{5}{3}}cms$

Area of the circle $= \pi\frac{5}{3}$

48. Ans: 36 cms

Solution

Mid-segment $13 = \frac{10+a}{2} \Rightarrow a = 16$

Area of trapeze $= \frac{1}{2}(10 + 16)h = 52$

$\Rightarrow h = 4cm.$

As $AB = EF = 10, CE = FD = \frac{(16-10)}{2} = 3cm.$

$\therefore AC = \sqrt{AE^2 + CE^2} = \sqrt{4^2 + 3^2} = 5cm$

Hence perimeter $= 10 + 5 + 16 + 5 = 36\ cms$

Exercise – 04
Quadrilaterals
Solution

1. Ans: c

Solution

Since, ΔAOB is similar to ΔCOD.

$\therefore \quad$ Ratio of areas of $\frac{\Delta AOB}{\Delta COD} = \left(\frac{3}{1}\right)^2 = \frac{9}{1}$

2. Ans: b

Solution

$(OS)^2 = (OK)^2 + (KS)^2$

$25 = OK^2 + 16 \Rightarrow OK = 3$

And, $(OS)^2 = (OL)^2 + (LN)^2$

$25 = (OL)^2 + 9$

$\Rightarrow \quad OL = 4\ cm$

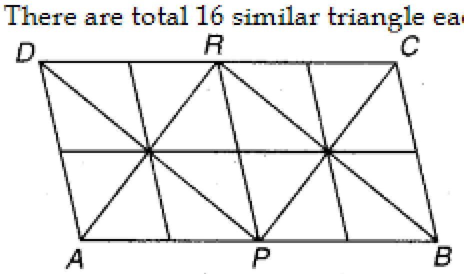
$\therefore \quad KL = OL - OK = 1\ cm$

$\therefore$ Area of rectangle $= 1 \times 10 = 10cm^2$

3. Ans: c
Solution
$AC = \sqrt{(15)^2 + (20)^2} = 25\ cm$
$CD = \sqrt{(25)^2 - (7)^2} = 24\ cm$

4. Ans: b
Solution
Calculate them physically (or manually)

5. Ans: b
Solution
There are total 16 similar triangle each with equal area.



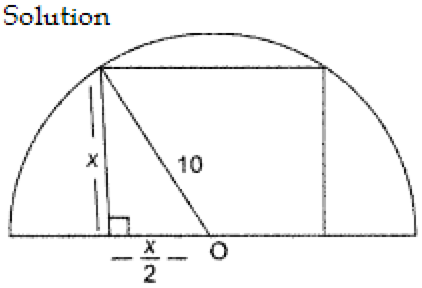
Here, 4 out of 16 triangles are taken. So the number of shaded triangles = 4 and number of unshaded triangles = 12
$\therefore$ Required ratio $= \frac{1}{3}$

6. Ans: c
Solution
Number of total rectangles ${}^4C_2 \times {}^3C_2 = 6 \times 3 = 18$

7. Ans: b
Solution
Clearly, triangle is a right-angled triangle.
Its area $= \frac{1}{2} \times 6 \times 8 = 24cm^2$
Now, $8 \times b = 24$
$b = 3$

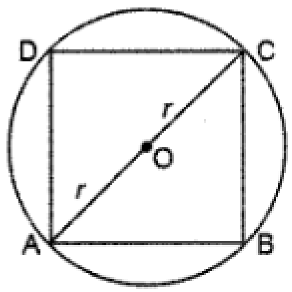
8. Ans: a
Solution
The two adjacent angles can be $30°$ and $60°$; therefore, others angle can be $150°$ and $120°$.

9. Ans: d
Solution

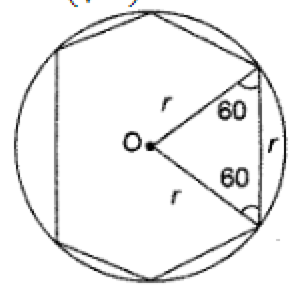


$x^2 + \frac{x^2}{4} = 100, x^2 = 80$

10. Ans: b



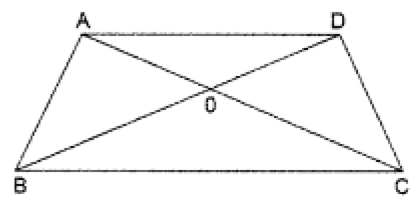
$A = (\sqrt{2}r)^2 = 2r^2$



$B = 6 \cdot \frac{\sqrt{3}}{4} r^2$

$$\frac{B}{A} = \frac{\frac{3}{2}\sqrt{3r^2}}{2r^2} = \frac{3\sqrt{3}}{4}$$

11. Ans: b
Solution

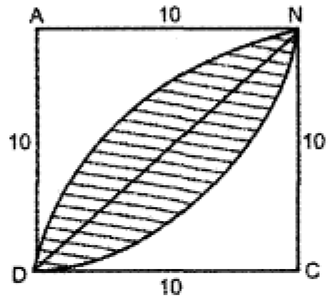


It is given that $\frac{OD}{DB} = \frac{2}{5}$
Then, $\frac{OD}{OB} = \frac{2}{3}$
We know that $\frac{AD}{BC} = \frac{OD}{OB} = \frac{OA}{OC}$
Then, $\frac{AD}{BC} = \frac{2}{3}$

12. Ans: c

13. Ans: c
Solution
We are given that the area of ΔABE is 40, and that $AB = 10$.
The area of triangle:
$A = \frac{bh}{2}$
Using AB as the height of ΔABE,
$40 = \frac{10b}{2}$ and solving for $b, b = 8$. Hence, option (c) is the answer.

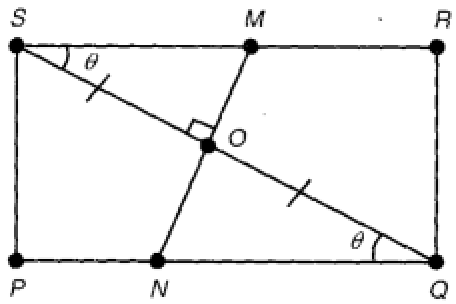
14. Ans: c
Solution
In square ABCD,
Area of shaded region = Area of sector BCD – Area of $\Delta BCD + Area\ of\ sector\ ABD - Area\ of\ \Delta ABD$
Since, all sides of a square are same,
Area of sectors and triangles will be the same having same radius (side of square) and angle 90°.
So, area of shaded region = 2 (Area of sector – Area of ΔABD)



$= 2\left(\frac{\pi}{4}(AB)^2 - \frac{1}{2} \times AD \times BA\right)$
$= (AB)^2\left(\frac{\pi}{2}\right) = 100\left(\frac{\pi}{2} - 1\right)$
$= 50\pi - 100$

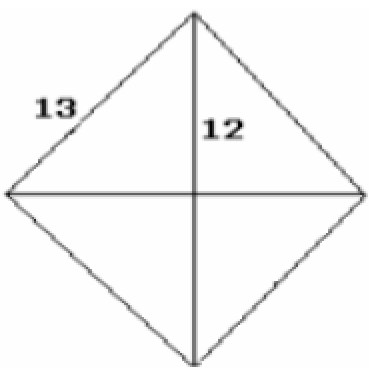
15. Ans: d

16. Ans: b
Solution



$SQ = \sqrt{3^2 + 4^2} = \sqrt{25} = 5\ cm$
$SO = OQ = 2.5\ cm$
If $\angle RSQ = \theta$
$tan\theta = \frac{MO}{SO}$ (i)
$tan\theta = RQ/SR$ (ii)
From equation (i) and (ii)
$\frac{MO}{SO} = \frac{RQ}{SR} = \frac{3}{4}$
$\frac{MO}{SO} = 0.75$

17. Ans: 10 cm
Perimeter of the rhombus $= 4a = 52$



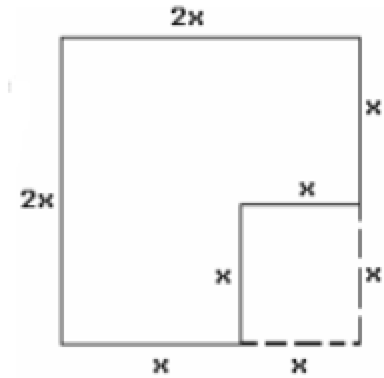
$\Rightarrow a = 13cm$. One diagonal $= 24\ cm$.
$\Rightarrow \frac{1}{2}(other\ diagonal) = \sqrt{13^2 - 12^2} = 5cm$
So length of the other diagonal $= 5 \times 2 = 10\ cm$

18. Ans: $30sqcm$
Solution
Area of rhombus $= \frac{1}{2}d_1 d_2 = \frac{1}{2}.6.10 = 30sqcm$

19. Ans: b
Solution
Sum of two consecutive angles of a $\pi gm\ is\ 180°$
Hence $2x + y + x + 2y = 180° \Rightarrow x + y = 60°$

20. Ans: b
Solution
Diagonal of a sq with side $a = \sqrt{2}$
$\therefore a\sqrt{2} = 8$ $\Rightarrow a = \frac{8}{\sqrt{2}} = 4\sqrt{2}$
Hence area $= (4\sqrt{2})^2 = 32 = 2^5$. Hence $K = 5$.

21. Ans: c
Solution



AS area of original square $= (2x)^2 = 4x^2$.
$\Rightarrow$ area of square cut out $= x^2$.
So each of side $= x\ cm$
Hence perimeter of rem. Figure $= 8x\ cm$.

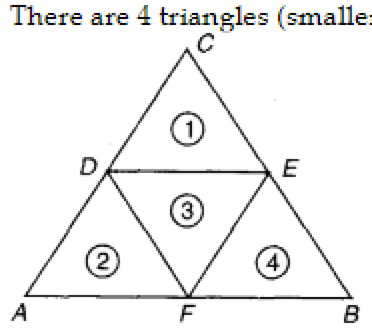
Exercise – 5
HOTS
Solution

1. Ans: d
Solution
Since $CS = SD$
$\therefore$ The two chords must be equidistant from the centre 'O'.
Thus, the required ratio is 1 : 1

2. Ans: b
Solution:
$\angle ADB = \angle ACB = 41°$
(angles of the same segment)
$\therefore \angle ABD = 180 - (\angle ADB + 90) = 49°$ $(\angle DAB$

3. Ans: b
Solution
$(\angle BPQ + \angle BQP) + (\angle EQR + \angle ERQ) + (\angle CRS + \angle CSR) + (\angle TSA + \angle ATS) + (\angle DTP + \angle DPT) = 2 \times 360$
$\angle A + \angle D + \angle B + \angle E + \angle C = 5 \times 180° - 2 \times 360° = 180°$

4. Ans: a
Solution
There are 4 triangles (smaller) are congruent.



So, out of these 4 triangles 2 triangles are taken thus the ratio of the shaded to the unshaded region is 1 : 1 (since two triangles are shaded and 2 are unshaded).

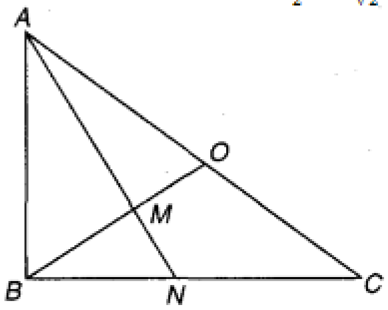
5. Ans: d

Solution

Let $AB = BC = a$

Then, $AC = \sqrt{2a}$

$\therefore \quad AO = OC = BO = \frac{\sqrt{2a}}{2} = \frac{a}{\sqrt{2}}$



Now, by angle bisector theorem

$\frac{AB}{AO} = \frac{BM}{MO} \Rightarrow \frac{BM}{MO} = \frac{a}{a/\sqrt{2}} = \frac{\sqrt{2}}{1}$

$\therefore \quad MO = 20cm$

$\therefore \quad BM = 20\sqrt{2}$

$\therefore \quad BO = 20 + 20\sqrt{2} = 20(1+\sqrt{2})cm$

Now, since $\quad BO = \frac{a}{\sqrt{2}} = \frac{AB}{\sqrt{2}}$

$\therefore \quad AB = \sqrt{2}(BO) = 1.414[20(1+1.414)]$
$= 68.2679 = 68.27\ cm$

6. Ans: a

Solution

$\angle ACB = 60°$ $\quad (\because \angle ACB + \angle ADB = 180°)$

And $\angle CAB = 30°$ $\quad (\because \angle ACB + \angle CAB = 90°)$

$AC = 2 \times 6 = 12\ cm^2$

$\frac{BC}{AC} = sin30° = \frac{1}{2}$

$\Rightarrow \quad BC = 6\ cm$

And, $\frac{BC}{AB} = tan30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad AB = 6\sqrt{3}$

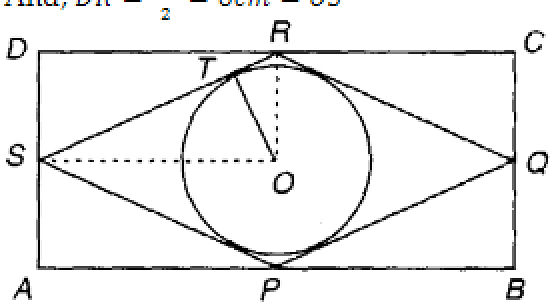
$\therefore \quad Area\ of\ \Delta ABC = \frac{1}{2} \times 6 \times 6\sqrt{3} = 18\sqrt{3}\ cm$

7. Ans: c

Solution

$DS = \frac{AD}{2} = 6cm$

And, $DR = \frac{DC}{2} = 8cm = OS$



$\therefore \quad SR = 10cm\ and\ OR = 6cm$

$\therefore \quad Area\ of\ \Delta ORS = \frac{OS \times OR}{2} = \frac{SR \times OT}{2}$

$\frac{8 \times 6}{2} = \frac{10 \times OT}{2}$

$\Rightarrow \quad OT = \frac{48}{10} cm$

$\therefore \quad$ Area of circle $= \pi r^2 = \pi \times \left(\frac{48}{10}\right)^2 = \frac{576}{25}\pi$

8. Ans: a

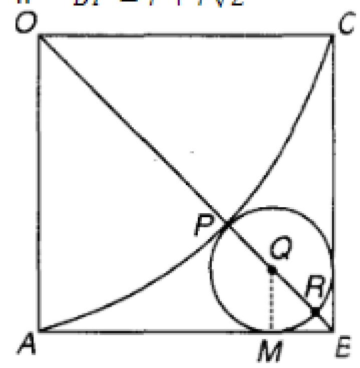
$OA = AB = BC = OC = OP$

Let $OA = R(radius\ of\ the\ larger\ circle)$then $OB = R\sqrt{2}$

Similarly $\quad PQ = MQ = QR =$
$r\ (radius\ of\ the\ smaller\ circle)$

Then, $BQ = r\sqrt{2}$

$\therefore \quad BP = r + r\sqrt{2}$



And, $BP = OB - OP = R\sqrt{2} - R$

$R\sqrt{2} - R = r + r\sqrt{2}$

$R(\sqrt{2}-1) = r(\sqrt{2}+1)$

$\Rightarrow \quad r = R\,(\sqrt{2}-1)^2$

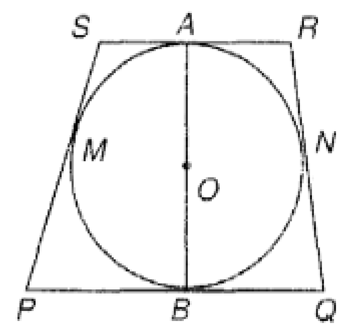
$r = R(3-2\sqrt{2})$

$\therefore \quad \frac{Area\ of\ larger\ circle}{Area\ of\ smaller\ circle} = \frac{\pi R^2}{4\pi r^2}$

$= \frac{R^2}{4(3-2\sqrt{2})^2 R^2} = \frac{1}{4(17-12\sqrt{2})}$

9. Ans: b

Solution



It can be solved using the property of tangents.

(Tangents on the circle drawn from the same points are same in length)

Points $M, A, N\ and\ B$ are the points of tangent.

$\therefore \quad PS + QR = PQ + SR = 2(21) = 42cm$

$\therefore \quad Perimeter\ of\ trapezium = 2(42) = 84cm$

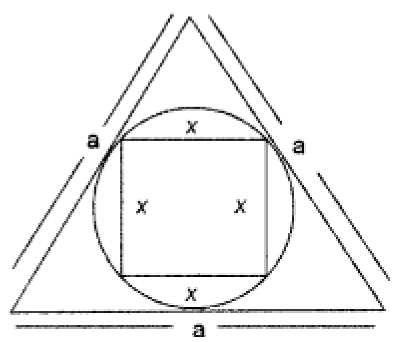
10. Ans: a

Solution

Putting the value of $n = 2, 3,$ and so on, we get option (a) as answer.

11. Ans: b

Solution



Radius of circle $R = \frac{\frac{\sqrt{3}}{4}a^2}{\frac{3a}{2}} = \frac{\sqrt{3}}{6}a$

Diagonal of square = Diameter of circle

$= 2.\frac{\sqrt{3}}{6}a$

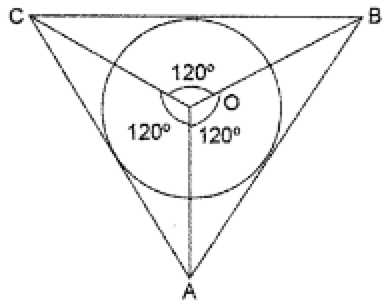
$\sqrt{2}x = \frac{\sqrt{3}}{3}a$

$x = \frac{\sqrt{3}}{3\sqrt{2}} a = \frac{\sqrt{6}}{6} a$

Area $= x^2 = \left[\frac{(\sqrt{6}a)}{6}\right]^2 = \frac{6}{36} a^2 = \frac{1}{6} a^2$

12. Ans; b

Solution

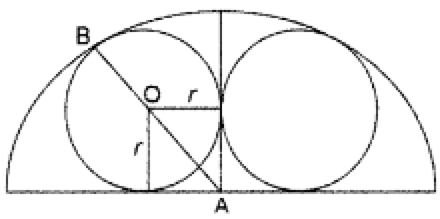


Radius of disc $= \sqrt{3}, OB = 20$

Use Sine law to find the length of $AB = BC = AC$.

13. Ans. b

Solution



Let radius of small circles

$AD = DB = r$

Then, $AC = OD = r (since\ AC = r\ (radius))$

Since, it is given that $OO_1 = 2$

$OB = OA = 2 - r$

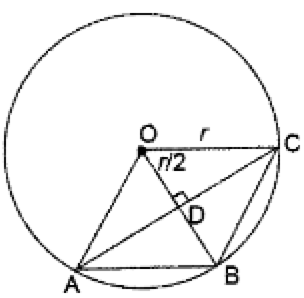
Now, in ΔADO

$AD^2 + DO^2 = OA$

$r^2 + r^2 = (2 - r)^2 = 4 + r^2 - 4r$

14. Ans: b

Solution



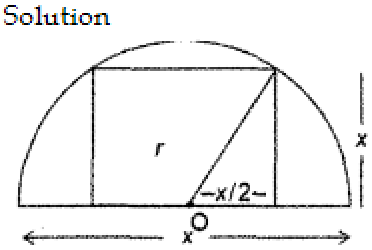
$AD = \sqrt{r^2 - \frac{r^2}{4}} = \frac{\sqrt{3}}{2} r$

$\frac{1}{2} \cdot r \cdot \sqrt{3} r = 32 \times \sqrt{3}, r^2 = 32 \times 2$

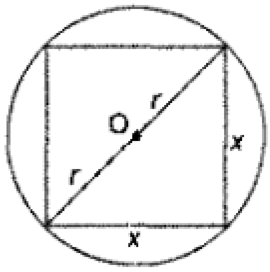
$r^2 = 64\ and\ r = 8$

15. Ans: c

Solution



Area of square $= \left(\frac{2}{\sqrt{5}} r\right)^2 = \frac{4}{5} r^2$



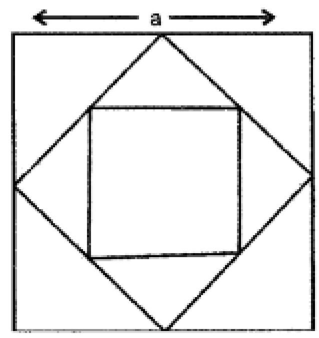
Area $= 2r^2$

Required ratio $= \frac{5}{2r^2} = 2 : 5$

16. Ans: c

Find the sides of all the squares.

Then, use geometric progression concept and get the answer.



17. Ans: c

Solution

Let the sides be $x, y\ and\ z, and\ x$ is the smallest side.

Then, $x + y + z = 4x$ [It is given]

$y + z = 3x$

$x = \frac{y+2}{3}$

Since it is right angle triangle, $x^2 + y^2 = z^2$

Put $\frac{y+2}{3} = x$ from equation (i)

Then, $\left(\frac{y+z}{3}\right)^2 + y^2 = z^2$

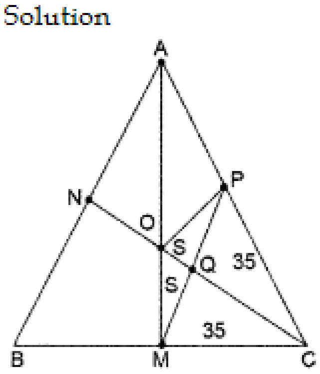
$\frac{y^2+z^2+2yz}{9} + y^2 = z^2 \rightarrow 8z^2 - 2yz - 10y^2 = 0$

$8z^2 + 8zy - 10zy + 10y^2 = 0, or, (8z - 10y)(z + y) = 0$

$8z - 10y = 0 \rightarrow 8z = 10y, then \frac{z}{y} = \frac{8}{10} = \frac{4}{5}$

18. Ans: d

Solution



We knows that medians intersect each other in the ratio of 2 : 1.

Then, $\frac{AO}{OM} = \frac{CO}{ON} = \frac{BO}{OP} = \frac{2}{1}$

Since P and M are the mid – points of AC and BC

$\therefore \frac{AP}{PC} = \frac{BM}{MC} = \frac{NQ}{QC} = \frac{1}{1}$

In the $\Delta ABC \frac{CO}{ON} = \frac{1}{2}$

$\frac{NQ}{QC} = \frac{1}{1}$

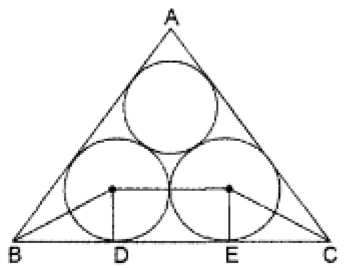
Then, $\frac{OQ}{QC} = \frac{1}{3}$

Then, in ΔOMQ and ΔQMC
Heights are same and base are in the ratio of 1 : 3.
Then, ratio of area $= s:3s$
Then, area of $\Delta OMC = 4s$
Then, area of $\Delta ABC = 6 \times 4S = 24s$

19. Ans: a

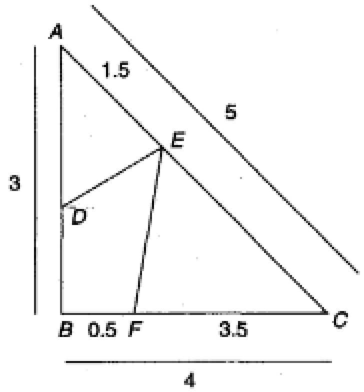
20. Ans: b
Solution
Let side of square = a, then perimeter = 4a
Area of square $= a^2$
According to the question, numerically $4a = a^2$
So, $a = 4$
Diagonal of square $= \sqrt{2a} = 4\sqrt{2}$

21. Ans: a
Solution



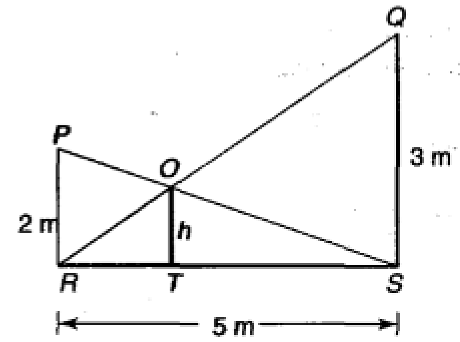
Let the radius of circles $= r$
Then, $BD = EC = r\sqrt{3}$
$DE = 2r$
Then, $BC = BD + DE + EC = 2(r + r\sqrt{3})$
$BC = 2r(1+\sqrt{3}) = a$
$[since\ BC = a]$
$r = \frac{a}{2+2\sqrt{3}} = \frac{a}{2(\sqrt{3}+1)}$

22. Ans: c
Solution



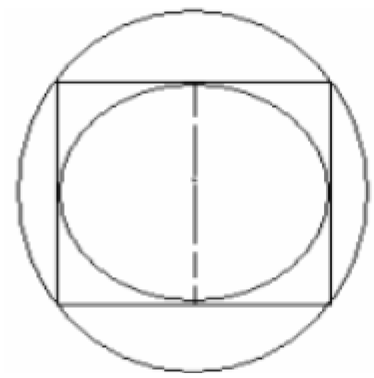
ΔABC is right angle triangle and $\angle B = 90°$
Let $\angle BAC = A$
$\Rightarrow \quad \angle BCA = 90° - A$
In $\Delta ADE: AD = AE = 1.5$
$\Rightarrow \quad \angle ADE = \angle AED = \frac{180°-A}{2} = 90° - \frac{A}{2}$
In $\Delta EFC : EC = FC = 3.5\ cm$
$\angle CEF = \angle CFE = \frac{180°-(90°-A)}{2} = 45° + \frac{A}{2}$
$\angle AED + \angle DEF + \angle FEC = 180°$
$90° - \frac{A}{2} + \angle DEF + 45° + \frac{A}{2} = 180°$
$\angle DEF = 45°$

23. Ans: a
Solution



ΔROT and ΔRQS are similar to each other:
$\frac{OT}{QS} = \frac{RT}{RS}$
$\frac{RT}{RS} = \frac{h}{3} \ldots\ldots (1)$
ΔSOT and ΔSPR are similar to each other :
$\frac{ST}{SR} = \frac{h}{2} \ldots\ldots (2)$
Adding (1) and (2), we get:
$\frac{(RT+ST)}{RS} = \frac{RS}{RS} = 1 = \frac{h}{2} + \frac{h}{3}$
$h = 1.2\ m$

24. Ans: b
Solution

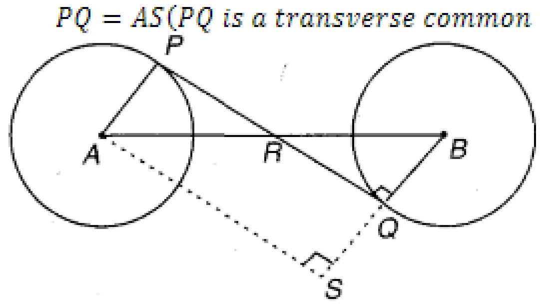


Area of smaller circle $= \pi r^2 \pi$
$\Rightarrow \quad r = 2\ cm.$
So side of square = 2 cm.
Diameter of bigger circle. = Diagonal of the square = $2\sqrt{2}$.
So are of bigger circle $= \pi\left(\frac{2\sqrt{2}}{2}\right)^2 = 2\pi cm$

Exercise – 6
TITA/SHORT ANSWERS
Solution

1. Ans: 10 cm
Solution
$PA \times PB = PC \times PD$
$3 \times (3+5) = 2 \times PD$
$\therefore \quad PD = 12cm$
$\therefore \quad CD = PD - PC$
$= 12 - 2 = 10\ cm$

2. Ans: 8 cm
Solution
$PQ = AS(PQ$ is a transverse common tangent$)$



And, $BS = BQ + SQ = BQ + AP$

$BS = 2BQ \qquad (BQ = AP = radius)$

$BS = 6\ cm$

$\therefore \quad AS = \sqrt{(AB)^2 - (BS)^2} = \sqrt{(10)^2 - (6)^2} = 8cm$

$\therefore \quad AS = 8cm$

$\therefore \quad PQ = 8cm$

Alternatively: The length of transverse common tangent

$= \sqrt{(distance\ between\ the\ centres)^2 - (r_1 + r_2)^2}$

$= \sqrt{(10)^2 - (3+3)^2} = 8\ cm$

3. Ans: 7 m

Solution

$\frac{DQ}{BD} = sin60° \Rightarrow DQ = BD \times \frac{\sqrt{3}}{2} = 3m$

And, $\qquad \frac{DP}{DF} = \sin 30° = \frac{1}{2}$

$\therefore \qquad DP = 2m$

$(\because DF = 4\ m)$

$\therefore \qquad PQ = 3 - 2 = 1\ m$

Let $\qquad AB = CQ = x\ m$

$\therefore \qquad \frac{CD}{EF} = \frac{5}{4}$

$\frac{x+3}{x+1} = \frac{5}{4} \Rightarrow x = 7\ m.$

4. Ans: 140°

Solution

$\angle ABC = 180 - (65 + 75) = 40°$

$\angle ORB = \angle OQB = 90°$

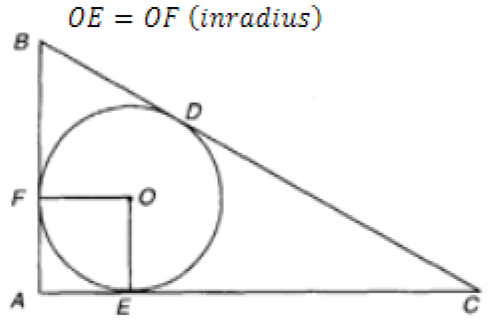
$\therefore \quad \angle ROQ = 360 - (90 + 90 - 40)$

$\therefore \quad \angle ROQ = 140°$

5. Ans: 1 cm

Solution

$OE = OF\ (inradius)$



And, $EC = DC = 15\ cm$

Let, $BF = x\ cm$

Then, $AB = (6 + x)cm$

And, $AC = 6 + 15 = 21\ cm$

$\therefore \quad (BC)^2 = (AB)^2 + (BC)^2$

$(x + 15)^2 = (6 + x)^2 + (21)^2 \qquad (\because$

$BC = BD + CD)$

$\Rightarrow \quad x = 14\ cm$

$\Rightarrow \quad BD = 14\ cm$

$\therefore \quad CD - BD = 15 - 14 = 1\ cm$

6. Ans: 66

Solution

Let $P_1, P_2, P_3, P_4, \ldots, P_9$ be the vertices of the regular nonagon then, we can select two vertices out of 9 vertices in ${}^9C_2$ ways and ${}^9C_2 = 36$ ways and number of equilateral triangles formed $= 36 \times 2 = 72$

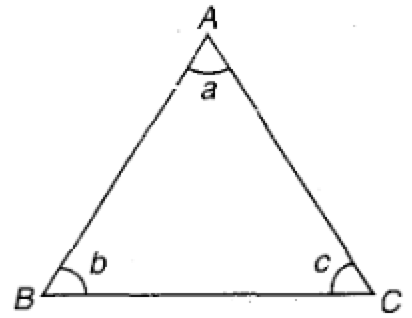
But the three triangles $\{P_1, P_4, P_7\}$, $\{P_2, P_5, P_8\}$ and $\{P_3, P_6, P_9\}$ are each counted 3 times *i.e.,* counted as 9 triangles instead by 3 triangles.

So, net number of equilateral triangles $= 72 - 6 = 66.$

7. Ans: 178

Solution

$\angle A + \angle B + \angle C = 180$



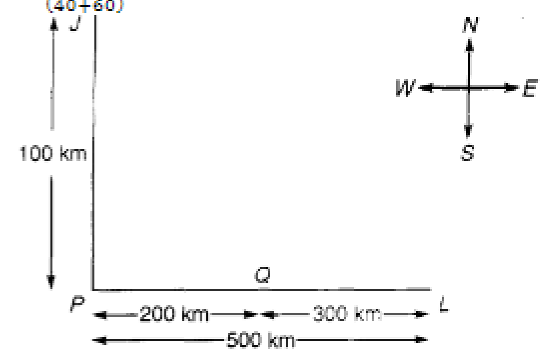
Any one of angle can posses the values from 1 to 178.

8. Ans: 1000 km

Solution

Time taken in the collision the two trains

$= \frac{500}{(40+60)} = 5h$



In 5 hours, plane will cover $5 \times 200 = 1000km\ distance$

9. Ans: 20cm

Solution

$\frac{AD}{AB} = \frac{DO}{BO} = 1$

$\Rightarrow \quad OB = OD = 8\ cm$

$\because \quad ABCD\ is\ a\ cyclic\ quadrilateral$

$\therefore \quad DO \times BO = CO \times AO$

$8 \times 8 = 4 \times AO$

$\Rightarrow \quad AO = 16\ cm$

$\therefore \quad AC = 16 + 4 = 20cm$

10. Ans: 6

Solution

$200 = 2^3 \times 5^2$

Number of total factors $= (3+1) \times (2+1) = 12$

$\therefore \quad Total\ number\ of\ required\ rectangles = \frac{12}{2} = 6$

$Area = b \times l$

$200 = 1 \times 200 = 2 \times 100$

$= 4 \times 50 = 5 \times 40$

$= 8 \times 25 = 10 \times 20$

11. Ans: 8 cm

Solution

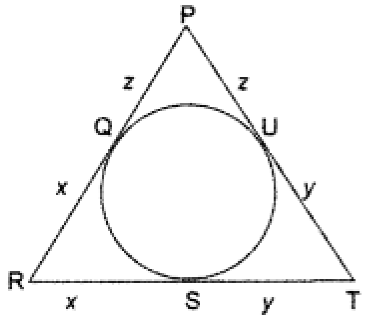
$AO \times OB = OC \times OD$

$4 \times 6 = 3 \times OD$

$OD = 8\ CM$

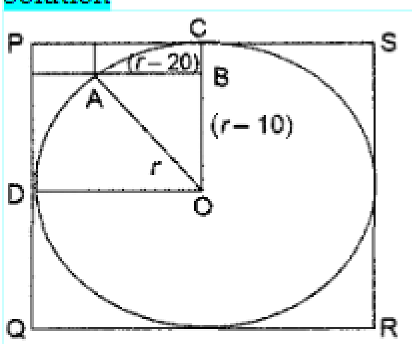
12. Ans: 13 cm

Solution



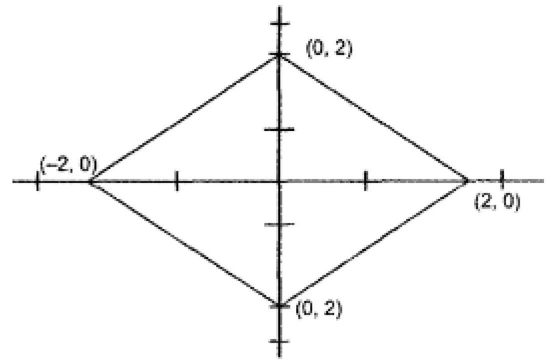
$PR = RT + 3$
$x + z = x + y + 3$
$z - y = 3$
(i)
Similarly, $z + x = z + y + 1$
$x - y = 1$
(ii)
From $(i) and (ii)$
$z - 3 = x - 1$
$z - x = 2$
Also, $2x + 2y + 2z = 26$
$x + y + z = 13$
(iii)
$3x + 1 = 13$
$3x = 12$
$x = 4$
$z = 6$
$y = 3$
$QR + PT = x + y + z$
$= 4 + 6 + 3 = 13$

13. Ans: 50 cm
Solution

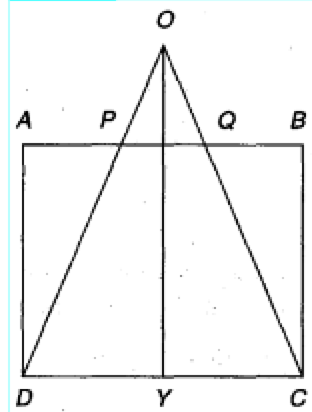


Let the centre of circle is O.
Then, $OA = OD = OC = r, or, OB = r - 10 \text{ and } AB = r - 20$
Then, in ΔABO,
$AO^2 = OB^2 + AB^2$, or $r^2 = (r-10)^2 + (r-20)^2$
Or, $r^2 = r^2 - 20r + 100 + r^2 - 40r + 400$
Or, $r^2 = 2r^2 - 60r + 500$
Or, $r^2 - 60r + 500 = 0 \Rightarrow r^2 - 50r - 10r + 500 = 0$
$\Rightarrow r(r-50) - 10(r-50) = 0$
$(r-10)(r-50) = 0$. Hence, $R = 10 \text{ or } 50$. Since 10 is not possible, hence, $r = 50$

14. Ans: 4
Solution
$x + y = 1 \text{ when } x \geq 0 \text{ and } y \geq 0$
$x + y = -2 \text{ when } x \leq 0\ y \leq 0$



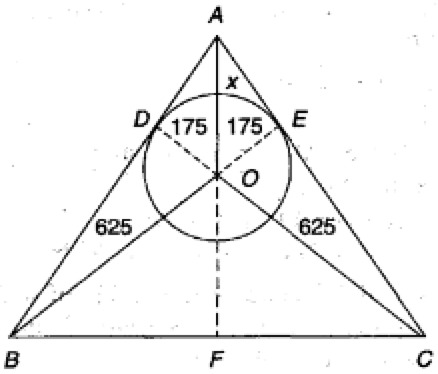
15. Ans: 15
Solution



Let the altitude from O of ΔOPQ is OX and $OY \perp DC$
Area of trapezoid $PQCD = \frac{1}{2}(PQ + DC)BC = 80 sq\ units$
$\Rightarrow \frac{1}{2}(PQ + 10)10 = 80$
$PQ = 6\ units$
$\Delta OPQ \text{ and } \Delta ODC \text{ are similar triangles}$
$\frac{OX}{OY} = \frac{PQ}{DC} = \frac{6}{10}$
$\frac{OX}{OX+10} = \frac{6}{10}$
$10\ OX = 6\ OX + 60$
$OX = \frac{60}{4} = 15\ units$

16. Ans: 1063
Solution
Let $AC = 2^a, where\ a \in N$
$AB = \frac{AC}{2} = \frac{2^a}{2} = 2^{a-1}$
According to the question AB is a perfect square.
So $a - 1$ should be even or 'a' should be odd.
$AB + AC > BC$
$3AB > 295$
$AB > 98.33$
$2^{a-1} > 98.33 \ldots (A)$
$AC - AB < BC$
$2^a - 2^{a-1} < 295 \text{ or } 2^{a-1} < 295 \ldots$ (B)
Only $a = 9$ satisfies equation A and B.
$\therefore$ $AB = 2^8 = 256,\ AC = 2^9 = 512$
$\therefore$ Perimetre of the triangle $= 256 + 512 + 295 = 1063$

17. Ans: 387072
Solution
Let ABC is the triangle and the circle touches AB, AC at D, E respectively as shown in the diagram.



$OD \perp AB$ & $OE \perp AC$
$OA = OB = OC = 625\ cm\ (Given)$
In $\Delta ODB, BD^2 + OD^2 = OB^2 (Using\ Pythagoras\ theorem)$
$BD^2 + 175^2 = 625^2$
$\Rightarrow \quad BD = 600\ cm$
Similarly, $AD = AE = EC = 600\ cm$
Hence, ΔABC is an isosceles triangle and $AB = AC = 1200\ cm$
So, $AF \perp BC$
In $\Delta AEO\ and\ \Delta AFC$
$\angle OAE = \angle CAF$
$\angle AEO = \angle AFC = 90°$
So, $\Delta AEO \sim \Delta AFC$
$\frac{AE}{AF} = \frac{OE}{CF} = \frac{OA}{AC}$
$\frac{600}{AF} = \frac{175}{CF} = \frac{625}{1200}$
So, $AF = 1200 \times \frac{600}{625} = 1152\ cm\ and\ CF = 1200 \times \frac{175}{625} = 336\ cm$
$CB = 672\ cm$
Area of $\Delta ABC = \frac{1}{2} \times 672 \times 1152 = 387072\ cm^2$

18. Ans: 14
Solution
Number of quadrilaterals that can be formed = $^8C_4 = 70$
Number of triangles that can be = $^8C_4 = 56$.
Required difference = 70 − 56 = 14

19. Ans: 9
Solution
72, 21, 75 form a Pythagorean triplet. The triangle is a right – angled triangle.
The measure of inradius of a right angle triangle

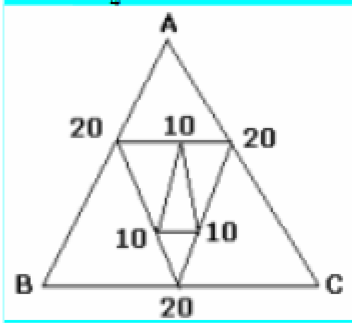
$$= \frac{Sum\ of\ legs\ of\ the\ right\ angled\ triangle - hypotenuse}{2}$$

$$\frac{72 + 21 - 75}{2} = \frac{18}{2} = 9$$

20. Ans: 9
Solution
$\frac{(2n-4)90°}{n} = 120°$ (Formula for interior angle of a regular polygon).
$\frac{n-2}{n} = \frac{120°}{180°} = 2/3$
$3n - 6 = 2n$
$n = 6$
Number of diagonals = $^6C_2 - 6 = 9$ (Number of diagonals of any n sided polygon is given by the formula $^nC_2 - n$)

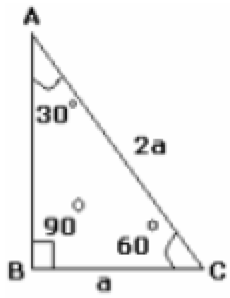
21. Ans: 20
Solution
Let the number of sides of the polygons be $2n\ and\ n$ respectively.
As per the question:
$18 = \frac{2n-2}{2n} \times 180° - \frac{n-2}{n} \times 180°$ (Formula for interior angle of a regular polygon applied to both the polygons).
$18 = \left[\frac{n-1}{n} - \frac{n-2}{n}\right] 180°$
$18 = \frac{1}{n} \times 180°$
$\Rightarrow n = 10\ or\ 2n = 20.$

22. Ans: 120
Solution
Area $= \frac{\sqrt{3}}{4}a^2 = 100\sqrt{3} \Rightarrow a^2 = 400 \Rightarrow a = 20$



The perimeters of every next triangle would be half of its previous triangle. Therefore sum of perimeters = $3 \times 20\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right) = 60 \times \frac{1}{1-\frac{1}{2}} = 60 \times 2 = 120$

23. Ans: 4004
Solution



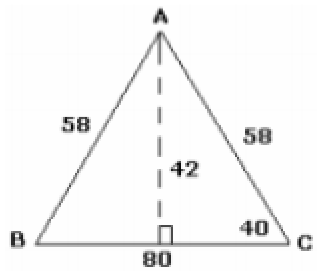
In a 30-60-90 triangle, the side opposite to 30° is half the hypotenuse.

$2a - a = 200$ $a = 2002$

Therefore longest side $= 2a = 4004$

24. Ans: 196

Solution



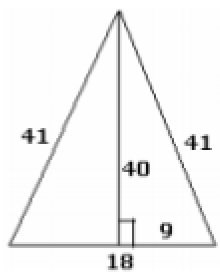
Area $= 1/2\ base\ X\ height$ $1680 = \frac{1}{2} \times 80 \times height$

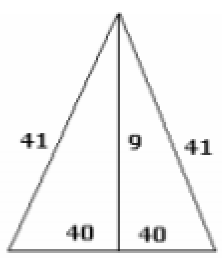
Height $= 42$. In the given figure $AB = AC = 58$

Therefore, perimeter $= 58 + 58 + 80 = 196$

25. Ans: 80

Solution





Area $= \frac{1}{2} base\ \times height = \frac{1}{2} \times 18 \times 40$

To keep the area constant we can interchange base and height as shown in the second figure. Therefore $x = 80$