Average

Averages

Basically the average is the arithmetic mean of the given data. For example if the $x_1 + x_2 + x_3 + x_4 \ldots x_n$ be any 'n' quantities (i.e., data), then the average (or arithmetic mean) of these 'n' quantities.

$$= \frac{x_1 + x_2 + x_3 \ldots x_n}{n}$$

Properties of Averages

1. The average of any two or more quantities (or data) necessarily lies between the lowest and highest values of the given data, i.e., if x_1 and x_h, be the lowest and highest (or greatest) values of the given data $(x_1, x_2, \ldots x_l, \ldots x_h, \ldots x_n)$ then $x_l < Average < x_h; x_l \neq x_h$

    i.e., $x_l < \frac{(x_1 + x_2 + x_3 + x_l \ldots + x_h \ldots + x_n)}{n} < x_h$

2. If each quantity is increased by a certain value 'K' then the new average is increased by K.
3. If each quantity is decreased by a certain value K, then the new average is also decreased by K.
4. If each quantity is multiplied by a certain value K, then the new average is the product of old average with K.
5. If each quantity is divided by a certain quantity 'K', then the new average becomes $\frac{1}{K}$ times of the initial average, where $K \neq 0$.
6. If 'A' be the average of $x_1, x_2, x_m, \ldots y_1, y_2, \ldots y_n$. Where $x_1, x_2, \ldots, x_m$ be the below A and $y_1, y_2, y_3, \ldots, y_n$ be the above A, then

    $(A - x_1) + (A - x_2) + \cdots (A - x_m)$
    $$= (y_1 - A) + (y_2 - A) + \cdots (y_n - A)$$

    i.e., the surplus above the average is always equal the net deficit below average.

General Formula: $Average\ speed = \frac{Total\ distance}{Total\ time}$

Proportion Method: If the half of the distance is covered at u km/hr and the rest half of the journey is covered at v km/hr then the average speed can be found as follows:

Step 1: Divide the difference of u and v in the ratio of $u : v$ (where $u < v$)

Step 2: $u + \frac{(u-v) \times u}{(u+v)}$

Or

$v - \frac{(u-v)v}{u+v}$

Case 2: When the distance travelled at different speeds are different then we calculate the average speed with the help of general formula of average speed. e.g., A person first goes x_1 km at the speed of $u\ km/hr$ and x_2 km at the speed of $v\ km/hr$ and $x_3\ km$ at the speed of $w\ km/hr$ and so on, then the

$Average\ speed = \frac{Total\ distance}{Total\ time}$
$= \frac{x_1 + x_2 + x_3 + \cdots}{t_1 + t_2 + t_3 + \cdots}$
$= \frac{x_1 + x_2 + x_3 + \cdots}{\frac{x_1}{u} + \frac{x_2}{v} + \frac{x_3}{w} + \cdots}$

7. **Some Important Concepts**
    1. $a + b = k$ and $b + c = l$ and $a + b + c = m$ then $[(a + b) + (b + c)] - (a + b + c) = (k + l) - m$
        Or $\qquad\qquad b = k + l - m$
    2. $a + b = k, d + e = l$ and $a + b + c + d + e = m$ then $c = (a + b + c + d + e) - [(a + b) + (d + e)] = m - [k + l]$

8. **Average of Some Important Series of Numbers**:
    (i) Average of first $'n'$ natural numbers $= \frac{n+1}{2}$
    (ii) Average of first $'n'$ even numbers $= (n + 1)$
    (iii) Average of first $'n'$ odd numbers $= n$
    (iv) If there are $(p + q)$ elements in a set of group but the average of p elements is r and the average of q elements is s, then the average of all the elements of the set (or group) is $\frac{(pr+sq)}{(p+q)}$

9. **Problems based on income/salary**

    Income = Expenditure + Saving

10. **Problems based on speed, time and distance:**

    Case 1: When the distance travelled in different time slots or parts is same *i.e.,* if a person or vehicle moves x km at a speed of $u\frac{km}{hr}$ and further he goes or comes back the same distance x km at a speed of $v\frac{km}{hr}$. Then the average speed $= \frac{2uv}{(u+v)}$

    If there are 3 parts of distance x km travelled with 3 different speeds *i.e.,* if a person goes first x km @ *speed* of u km/hr and next x km @ v km/hr and the last x km @ w km/hr.

    Then the average speed $= \frac{3uvw}{(uv+vw+wu)}$.

11. **Weighted Average**: When the average of groups or sets, instead of individuals, having different number of elements is being calculated, then it is called the weighted average. Since in this case the number of elements if different for the different sets thus they carry different weightage. If the number of elements in n different groups be $K_1, K_2, K_3, K_4, ..., K_n$ and the averages of the respective groups be $A_1, A_2, A_3, A_4 ... A_n$ then the weighted average $= \frac{K_1 A_1 + K_2 A_2 + K_3 A_3 + \cdots K_n A_n}{K_1 + K_2 + K_3 + \cdots K_n}$.

Exercise – 1

1. The average of a, 11, 23, 17 is 15 and the average of a, b, 12, 25 is 16. The value of a/b is:
    (a) 1/3   (b) 1/2   (c) 2/3   (d) 3/4

2. The average of 7 consecutive odd numbers if the smallest of those numbers is denoted by k:
    (a) k +4   (b) k + 7   (c) k + 6   (d) 7k

3. The average of first 100 natural number is :
    (a) 100   (b) 50   (c) 50.50   (d) 55

4. The average of first 50 odd natural numbers is
    (a) 50   (b) 55   (c) 51   (d) 101

5. The average of first 99 even numbers is
    (a) 9999   (b) 100   (c) 9801   (d) 9009

6. Mr. Anand Roy, the renowned author, recently got his new novel launched. To his utter dismay. He found that for the 1007 pages on an average there were two mistakes every page. While in the first 612 pages there were only 434 mistake they seemed to increase for the latter pages find the average number of mistakes per page for the remaining pages.
    (a) 6   (b) 4   (c) 2   (d) None of these

7. If the average of m number is a, and on adding x to those numbers, the average of the 1+m numbers is b. then find the value of x.
    (a) m(b-a)+b   (b) m(b+a)+a   (c) m(a-b)+a   (d) None of these

8. The average of n numbers in 32. If ¾ of the numbers are increased by 4 each and the remaining are decreased by 6 each, then what is the new average?
    (a) 32   (b) 32.5   (c) 33.5   (d) 34.5

9. Three years ago, the average age of a family of five member members was 17 years. In spite of the birth of a child in the family, the present average age of the family remains the same. The present age of the child is
    (a) 3 years   (b) 1 years   (c) 2 years   (d) 1.5 years

10. The average of 10 two-digit positive integers is Z. However, one number AB is taken as BA, then the average increases to Z+2.7 what is the value of |B-A|?
    (a) 1   (b) 2   (c) 3   (d) 4

11. Milawati Sharma, the local grocer mixed three qualities of tea T1, T2 and T3 priced at Rs. 74 per kg, Rs. 68 per kg and Rs. 63 per kg in the ratio of 1:2:4. He sold this new mixture for Rs. 84 per/kg thereby making a profit of 20%. How much of T1 did the mix with the mixture.
    (a) 4 kg   (b) 6 kg   (c) 8 kg   (d) None of these

12. The average age of 24 student and the principle is 15 years. When the principal's age is excluded, the average age decreases by 1 year. What is the age of the principal?
    (a) 38   (b) 40   (c) 39   (d) 37

13. The mean temperature of Monday to Wednesday was 37^0C and of Tuesday to Thursday was 34^0C. If the temperature on Thursday 4/5 that of Monday, the temperature on Thursday was
    (a) 38^0C (b) 36^0C (c) 40^0C (d) 39^0C

14. The average of the first five multiples of 7 is
    (a) 20   (b) 21   (c) 28   (d) 30

15.   The marks obtained by Here Rama in Mathematics, English and Biology are respectively 93 out of 100, 78 out of 150 and 177 out of 200. Find his average score in percent.
       (a) 87.83   (b) 86.83   (c) 76.33   (d) 77.33

16.   The average age of a family of 6 members is 22 years. If the age of the youngest member is 7 years. What was the average age of the family at the birth of the youngest member?
       (a) 15   (b) 18   (c) 21   (d) 12

17.   The average of 3 numbers is 17 and that of the first two is 16. Find the third number.
       (a) 15   (b) 16   (c) 17   (d) 19

18.   Find the average of the first 97 natural numbers,
       (a) 47   (b) 37   (c) 48   (d) 49

19.   Find the average of all prime numbers between 30 and 50.
       (a) 39.8   (b) 38.8   (c) 37.8   (d) 41.8

20.   The average of 5 consecutive numbers is n. If the next two numbers are also included, the average will,
       (a) increase by 1   (b) remain the same   (c) increase by 1.4   (d) increase by 2

21.   The average of 13 papers is 40. The average of the first 7 papers is 42 and of the last seven papers is 35. Find the marks obtained in the 7th paper.
       (a) 23   (b) 38   (c) 19   (d) 39

22.   Out of three numbers, the first is twice the second and three times the third. The average of the three numbers is 88. The smallest number is
       (a) 72   (b) 36   (c) 42   (d) 48

23.   If a, b, c, d and e are five consecutive odd numbers, then their average is
       (a) (a +b)/5   (b) (abcde)/5   (c) 5(a+b+c+d+e)   (d) None of these

24.   The average of the first ten natural numbers is
       (a) 5   (b) 5.5   (c) 6.5   (d) 6

25.   The average of the first ten whole numbers is
       (a) 4.5   (b) 5   (c) 5.5   (d) 4

Exercise – 02

1. A train normally covers a certain distance at a speed of 60 km/hr. however, if it were to halt for a fixed time interval in each hour its average speed reduced to 50 km/hr. what is the time interval for which the train halt in each hour?
   (a) 10 Minutes   (b) 20 Minutes   (c) 6 Minutes   (d) 12 Minutes

2. The average of 4 distinct prime number a, b, c, d is 35, where a<b<c<d. a and d are equidistant from 36 and b and c are equidistant from 34 and a,b are equidistant from 30 and c and d are equidistant from 40 the difference between a and d is:
   (a) 30   (b) 14   (b) 21   (d) can't determined

3. The speed of the train in going from Nagpur to Allahabad is 100 km/hr while when coming back from Allahabad to Nagpur, its speed is 150 km/hr. Find the average speed during the whole journey,
   (a) 125   (b) 75   (c) 135   (d) 120

4. The age of Shaurya and Kauravki is in the ratio 2:6. After 5 years, the ratio of their ages will becomes 6:8. Find the average of their ages after 10 years.
   (a) 12   (b) 13   (c) 17   (d) 24

5. The average earning of Shambhu Nath Pandey for the initial three months of the calendar year 2002 is ₹1200. If his average earning for the second and third month is ₹1300 find his earning in the first month?
   (a) 900   (b) 1100   (c) 1000   (d) 1200

6. The average age of a group of men is increased by 5 years when a person aged 18 years is replaced by a new person of aged 38 years. How many men are there in the group?
   (a) 3   (b) 4   (c) 5   (d) 6

7. The average score of a class of 40 students is 52. What will be the average score of the rest of the students if the average score of 10 of the students is 61.
   (a) 50   (b) 47   (c) 48   (d) 49

8. The average of the first ten even numbers is
   (a) 18   (b) 22   (c) 9   (d) 11

9. The average of the first ten odd numbers is
   (a) 11   (b) 10   (c) 17   (d) 9

10. The average of the first ten prime numbers is
    (a) 15.5   (b) 12.5   (c) 10   (d) 12.9

11. The average of the first ten composite numbers is
    (a) 12.9   (b) 11   (c) 11.2   (d) 10

12. The average of the first ten prime numbers, which are odd is
    (a) 12.9   (b) 13.8   (c) 17   (d) 15.8

13. The average temperature of 1st, 2nd and 3rd December was 24.4 °C. The average temperature of the first two days was 24 °C. The temperature on the 3rd of December was
    (a) 20°C   (b) 25°C   (c) 25.2°C   (d) None of these

14. The average of 6 students is 11 years. If 2 more students of age 14 and 16 years join, their average will become
    (a) 12 years   (b) 13 years   (c) 21 years   (d) 19 years

15. Shyam bought 2 articles for Rs.5.50 each, and 3 articles for Rs.3.50 each, and 3 articles for Rs.5.50 each and 5 articles for Rs.1.50 each. The average price for one article is
    (a) Rs.3   (b) Rs.3.10   (c) Rs.3.50   (d) Rs.2 84.

16. The average of temperatures at noon time from Monday to Friday is 50; the lowest one is 45. What is the possible maximum range of the temperatures?

17. The average of 30 numbers is zero. Of them, at the most, how many may be greater than zero?

18. The average of 50 numbers is 38. If two numbers namely 45 and 55 are discarded, the average of the remaining numbers is:

19. The average of nine numbers is 50. The average of the first five numbers is 54 and that of last three numbers is 52. Then the 6th number is:

20. What is the average of the first six natural number multiples of 7?

21. In the month of July of a certain year, the average daily expenditure of an organisation was Rs.68. For the first 15 days of the month, the average daily expenditure was Rs.85 and for the last 17 days, Rs.51. Find the amount spent by the organisation on the 15th of the month,
    (a) ₹42   (b) ₹36   (c) ₹34   (d) ₹52

22. A train travels with a speed of 20 m/s in the first 10 minutes, goes 8.5 km in the next 10 minutes, 11 km in the next 10, 8.5 km in the next 10 and 6 km in the next 10 minutes. What is the average speed of the train in km per hour for the journey described?
    (a) 42kmph   (b) 35.8 kmph   (c) 55.2 kmph   (d) 46 kmph

23. One-fourth of a certain journey is covered at the rate of 25 km/h, one-third at the rate of 30 km/h and the rest at 50 km/h. Find the average speed for the whole journey.
    (a) 600/53 km/h   (b) 1200/53 km/h   (c) 1800/53 km/h   (d) 1600/53

24. Find the average increase in the population in the population in the first year is 30% and that in the second year is 40%.
    (a) 41   (b) 56   (c) 40   (d) 38

25. In hotel Jaysarmin, the rooms are numbered from 101 to 130 on the first floor, 221 to 260 on the second floor and 306 to 345 on the third floor. In the month of June 2012, the room occupancy was 60% on the first floor, 40% on the second floor and 75% on the third floor. If it is also known that the room charges are Rs.200, Rs.100 and Rs.150 on each of the floors-then find the average income per room for the month of June 2012.
    (a) Rs.151.5   (b) Rs.88.18   (c) Rs.78.3   (d) Rs.65.7

26. A batsman makes a score of 270 runs in the 87th inning and thus increases his average by a certain number of runs that is a whole number. Find the possible values of the new average,
    (a) 98   (b) 184   (c) 12   (d) All of these

27. There were 42 students in a hostel. Due to the admission of 13 new students, the expenses of the mess increase by Rs.31 per day while the average expenditure per head diminished by Rs.3. What was the original expenditure of the mess?
    (a) Rs.633.23   (b) Rs.583.3   (c) Rs.623.3   (d) Rs.632

28. The average weight of 47 balls is 4 gm. If the weight of the bag (in which the balls are kept) be included, the calculated average weight per ball increases by 0.3 gm. What is the weight of the bag?
    (a) 14.8gm   (b) 15.0 gm   (c) 18.6 gm   (d) None of these

29. The average age of a group of 14 persons is 27 years and 9 months. Two persons, each 42 years old, left the group. What will be the average age of the remaining persons in the group?
    (a) 26.875 years   (b) 26.25 years   (c) 25.375 years   (d) 25 years

Exercise – 03

1. In a family of 8 males and a few ladies, the average monthly consumption of grain per head is 10.8 kg. If the average monthly consumption per head be 15 kg in the case of males and 6 kg in the case of females, find the number of females in the family,
   (a) 8   (b) 7   (c) 9   (d) 15

2. A bus goes to Ranchi from Patna at the rate of 60 km per hour. Another bus leaves Ranchi for Patna at the same time as the first bus at the rate of 70 km per hour. Find the average speed for the journeys of the two buses combined if it is known that the distance from Ranchi to Patna is 420 kms.
   (a) 64.615 kmph   (b) 64.5 kmph   (c) 63.823 kmph   (d) 64.82 kmph

3. The average weight of 6 persons is increased by 2.5 kg when one of them whose weight is 50 kg is replaced by a new man. The weight of the new man is
   (a) 65kg   (b) 75kg   (c) 76kg   (d) 60kg

4. A ship sails out to a mark at the rate of 15 km per hour and sails back at the rate of 20 km/h. What is its average rate of sailing?
   (a) 16.85km   (b) 17.14km   (c) 17.85km   (d) 18km

5. In a bag, there are 150 coins of Re. 1, 50 p and 25 p denominations. If the total value of coins is Rs.150, then find how many rupees can be constituted by 50 p coins.
   (a) 16   (b) 20   (c) 28   (d) None of these

7. A person travels three equal distances at a speed of x km/h, y km/h and z km/h respectively. What will be the average speed during the whole journey?
   (a) $xyz/(xy + yz + zx)$ (b) $(xy + yz + zx)/xyz$
   (c) $3xyz/(xy + yz + xz)$ (d) None of these

8. Find the average of f(x), g(x), h(x), d(x) at x = 10. F(x) is equal to $x^2 + 2$, $g(x) = 5x^2 - 3$, $h(x) = \log x^2$ and $d(x) = (4/5)x^2$
   (a) 170   (b) 170.25   (c) 70.25   (d) 70

9. Find the average of *f(x) - g(x), g(x) - h(x), h(x) - d(x), d(x) –f(x)*
   (a) 0   (b) -2.25   (c) 4.5   (d) 2.25

10. $\sum_{r=1}^{n}(n + 1)\,r$ where r = n
    (a) $\dfrac{(n-1)(n)(n+1)}{2}$ (b) $\dfrac{n(n+1)^2}{2}$ (c) $\dfrac{n(n-1)^2}{2}$ (d) $\dfrac{n^2}{2}$

11. There are two houses in parliament. One is Lok Sabha and the other one is Rajya Sabha and the member of Parliaments (MPs) in both the houses is 300 and 200 respectively. The average age of the members of Lok Sabha and Rajya Sabha is 40 years and 50 years respectively. A member of the Rajya Sabha when elected for the Lok Sabha also, he left the Rajya Sabha and becomes the member of the Lok Sabha. Thus the average age of both the houses increases. Which one of the following statements is true?
    (a) the age of this member is greater than 50 years
    (b) the age of this member is less than 40 years
    (c) the age of this member is greater than 40 but less than 50 years
    (d) none of these

12. Eleven years earlier the average age of a family of 4 members was 28 years. Now the age of the same family with six members is yet the same, even when 2 children were born in this period. If they belong to the same parents and the age of the first child at the time of the birth of the younger child was same as there were total

family members just after the birth of the youngest members of this family, then the present age of the youngest member of the family is:
(a) 3 years   (b) 5 years   (c) 6 years   (d) none of these

13. In a set of prime and composite numbers, the composite numbers are twice the number of prime numbers and the average of all the numbers of the set is 9. If the number of prime numbers and composite numbers are exchanged then the average of the set of numbers is increased by 2. If during the exchange of the numbers the average of the prime numbers and composite numbers individually remained constant, then the ratio of the average of composite numbers to the average of prime numbers (initially) was:
(a) 7/13   (b) 13/7   (c) 9/11   (d) none of these

14. The average earning of a group of persons is Rs. 50 per day. The difference between the highest earning and lowest earning of any two persons of the group is Rs. 45. If these two people are excluded the average earning of the group decreases by Re. 1. If the minimum earning of the person in the group lies between 42 and 47 and the number of persons initially in the group was equal to a prime number, with both its digits prime. The number of persons in the group initially was
(a) 29   (b) 53   (c) 31   (d) none of these

15. The average age of Donald, his wife and their two children is 23 years. His wife is just 4 year younger than Donald himself and his wife was 24 years old when his daughter was born. He was 32 years old when his son was born. The average age of Donald and his daughter is:
(a) 25 years   (c) 26 years   (b) 22.5 years   (d) can't be determined

16. There are 6 consecutive odd numbers in increasing order. The difference between the average of the squares of the first 4 numbers and the last four numbers is 64. If the sum of the squares of the first and the last element (i. e., odd numbers) is 178, then the average of all the six numbers is average
(a) 7   (b) 8   (c) 9   (d) 10

17. Mr. Tyagi while going from Meerut to Saharanpur covered half the distance by train at the speed of 96 km/hr then he covered half of the rest distance by his scooter at the speed of 60 km/hr and finally he covered the rest distance at the speed of 40 km/hr by car. The average speed at which Mr. Tyagi completed his journey
(a) 64 km/hr   (b) 56 km/hr   (c) 60 km/hr   (d) 36 km/hr

18. A man makes 60 articles in the first hour. His efficiency decreases by 25 % in the second hour, increases by 40% in the third hour, decreases by 33% in the fourth hour and increases by 50 % in the fifth hour. If the has to work for more than one hours, then in which hour the average number of articles produced per hour then would be minimum ?
(a) second hour   (b) after fifth hour   (c) third hour   (d) none of these

19. In a school, there are two sections for class X – section A and section B. the number of students in section A and section B are 60 and 70, respectively, and their respective average weight is 35 kg and 38 kg if one of the students from section A is shifted to section B then the average weight of both the sections decreases. What can be said about the weight of that student?
(a) His weight is less than 35 kg
(b) His weight is more than 38 kg.
(c) His weight is more than 35 kg and less than 38 kg
(d) Cannot be determined

Exercise – 04 (TITA/SA)

1. Progressive express left for New Delhi, increasing its speed in each hour. It started its journey from Lucknow, but after four hours of its journey it met with accident. Its speed in the fourth hour was 7/5 times that of the third hour and the speed in the third hour was 10/7 times that of the second hour and in the second hour it was 7/5 times that of the first hour. If it would have been travelled with the half of the speed that of the third hour, then it would have gone 160 km less in the same time (i.e. in four hours). The average speed of the train during the journey of 4 hours was :

    _____

2. Ravi went to Kanpur from Lucknow by his four wheeler. During the journey he had to use the spare wheel (i. e., stepony). Thus he finished his 160 km journey. The average distance covered by the wheels of his car is :

    _____

3. Mr. Manmohan calculated the average of 10 three digit numbers'. But due to mistake he reversed the digits of a number and thus his average increased by 19.8. The difference between the unit digit and hundreds digit of that number is:

    _____

4. The average expenditure of the hotel when there are 10 guests is Rs. 60 per guests and the average expenditure is Rs. 40 when there are 20 guest. It is known that there the average is Rs. 40 when there are 20 guests. If it is known that there are same fixed expenses irrespective of the number guests then the average expenditure per guest when there are 40 guest in the hotel"

    _____

5. In a management entrance test, a student scores 2 marks for every correct answer and loses 0.5 marks for every wrong answer. A student attempts all the 100 questions and scores 120 marks. The number of questions he answered correctly was

    _____

6. 30 oranges and 75 apples were purchased for Rs.510. If the price per apple was Rs.2, then the average price of oranges was

    _____

7. A batsman made an average of 40 runs in 4 innings, but in the fifth inning, he was out on zero. What is the average after fifth inning?

    _____

8. A man covers half of his journey by train at 60 km/h, half of the remainder by bus at 30 km/h and the rest by cycle at 10 km/h. Find his average speed during the entire journey.

    _____

9. Find the average weight of four containers. If it is known that the weight of the first container is 100 kg and the total of the second, third and fourth containers' weight is defined by $f(x) = x^2 - 3/4 (x^2)$ where $x = 100$.

    _____

10. The average salary of the entire staff in an office is Rs.3200 per month. The average salary of officers is Rs.6800 and that of non-officers is Rs.2000. If the number of officers is 5, then find the number of non-officers in the office?

    _____

11. If the product of n positive integers is n^n, then what is the minimum value of their average for n=6?

    _____

12.	The average of 7 consecutive numbers is P. if the next three numbers are also added , the average increases by
_____

13.	If the average of 22, 24, x, 27, 29 lies in between 30, 35 including both. Find the number of possible values of x.
_____

14.	Ashish a great swing bowler has 12 boxes with him, which have an average of 20 balls per box. If each box has at least 9 balls and no two boxes have an equal number of balls, then what is the maximum possible number of balls in any box?
_____

15.	If the average of four different numbers is 5, and if all the numbers are positive integers then what is the largest possible value of the average of the 2 biggest numbers?
_____

Average
Answers Key & Solutions

SOLUTIONS

Exercise 01

1.	Ans. (b)
Solution: Total value of a+11+23+17 = 60
    $\Rightarrow$ a=9
Again a+b+12+25= 64
    $\Rightarrow$ b = 18
        Therefore $\frac{a}{b} = \frac{1}{2}$

2.	Ans. (c)
Solution: $\frac{k+k+2+k+4+k+6+k+8+k+10+k+12}{7}$ = k + 6

3.	Ans. (c)
Solution: $\left(\frac{1+2+3+\cdots.100}{100}\right) = \frac{\left(\frac{100+101}{2}\right)}{100}$=50.50

4.	Ans. (a)
Solution: $\left(\frac{1+3+5+7+\cdots.99}{50}\right)=\left(\frac{50 \times 50}{50}\right)$=50

5.	Ans. (b)
Solution: $\left(\frac{2+4+6+\cdots.198}{99}\right)=\left(\frac{99 \times 100}{99}\right)$= 100

6.	Ans. (b)
Solution: total mistakes = 1007 x 2 = 2014

Let x be average emistake per page for the remaining pages.
434+395x = 2014
395x = 1580
x=4

7.	Ans. (a)
Solution: $\frac{ma+x}{1+m}$ = b ➔ ma+x=b+bm
X=b+m (b-a)

8.	Ans. (c)
Solution: $\frac{32 \times n+\frac{3}{4}n.4-\frac{1}{4}n.6}{n} = \frac{70n-3n}{2n}$ = 33.5

9.	Ans. (c)
Solution: Total age of family 3 years ago = 17 x 5 = 85 years.
Total age of family now = 17 x 6 = 102 years
Total age of family excluding the child now = (85+15)= 100 years
Age of child = 2 years

10.	Ans. (c)
Solution: 10 Z – (10.A+B) + (10B+A) = 10 (Z+2.7)
0-10A-B+10B+A = 10Z + 27

$9B-9A = 27$
$B-A = 3$
$|B-A| = 3$

11.   Ans. (d)
Solution: Type of T_1 tea in 4 kg mixture = $\frac{1}{7}$ x 4kg = $\frac{4}{7}$ kg

Type of T_2 tea in 4 kg mixture = $\frac{2}{7}$ 4kg $\frac{8}{7}$ kg

Type of T_3 tea in 4 kg mixture = $\frac{4}{7}$ x 4 = $\frac{16}{7}$ kg

CP of 4 kg tea mixture = Rs. ($\frac{296 \times 504 \times 1008}{7}$)

= Rs. $\frac{1808}{7}$

SP of new mixture = Rs. 84

CP of new mixure = Rs. $(84-84.\frac{1}{4})$

=Rs. 63 Per kg

Suppose x kg of T_1 tea is added to 4 kg mixure. CP of $(4+x)$ kg tea mixture = $(4+x)$ 63 = Rs. $(252 +63x)$
Also CP of $(4+x)$ kg tea mixture

= Rs. $(\frac{1808}{7} + \frac{x}{7}.74)$

= Rs. $(\frac{1808+74x}{7})$

Therefore, $252 + 63x = \frac{1808+74x}{7}$

$1764 + 441x = 1808 +74x$

$367x = 47$

$x = \frac{47}{367}$

12.   Ans: (c)
Solution: $P = 25 \times 15 - 24 \times 14 = 375 - 336 = 39$

13.   Ans: (b)
Solution: Monday + Tuesday + Wednesday = 3×37 = 111;

Tuesday + Wednesday + Thursday = $3 \times 34 = 102$
Thus, Monday – Thursday = 9 and
Thursday = $4 \times$ Monday/5 → Thursday = 36 and Monday = 45.

14.   Ans: (b)
Solution: $7 \times 3 = 21$

15.   Ans: (d)
Solution: His total score is 93 + 78 + 177 = 348 out of 450% score = 77.33

16.   Ans: (b)
Solution: Today's total age = $6 \times 22 = 132$ years. Total age of the family excluding the youngest member (for the remaining 5 people) = 132 – 7 = 125. Average age of the other 5 people in the family = 25 years
7 years ago their average age = 25- 7 = 18 years

17.   Ans: (d)

Solution: $3 \times 17 – 2 \times 16 = 51 – 32 = 19$

18.   Ans: (d)
Solution: The average would be given by the average of the first and last numbers (since the series 1,2,3,4....97 is an arithmetic progression)
Hence the average = (1+97/2 = 49)

19.   Ans: (a)
Solution: We need the average of the numbers 31, 37, 41, 43 and 47

20.   Ans: (a)
Solution: If the numbers are a+1, a+2, a+3, a+4 and a+5 the average would be a+3. If we take 7 numbers as: a+1, a+2, a+3, a+4, a+5, a+6 and a+7 their average would be a+4 hence. The average increases by 1.

21.   Ans: (c)
Solution: Let the number of marks in the 7th paper be M. then the total of the firs seven papers = 7×42 while the total of the last 7 (i.e. 7th to 13th papers) would be 7×35.

Total of 1st 7 total of 7th to 13th = total of all 13+ marks in the 7th paper → $7 \times 42 + 7 \times 35 = 13 \times 40 + M$
$539 = 520 + M$ → M = 19
(**Note:- We write this equation since marks in the seventh paper is counted in both the first 7 and the last 7**)

22.   Ans: (d)
Solution: If we take the first number as n the second number would be 3n and the third would be 2n sum of the three numbers = 6n + 3n + 2n = 11n = 88×3 → n = 24.
The smallest number would be 2n = 48.

23.   Ans: (d)
Solution: Five consecutive odd numbers would always be in an arithmetic progression and their average would be the middle number. The average would be c in this case.

24.   Ans: (b)
Solution: Required average = (1+2+3…..10)/10 = 55/10 = 5.5.
Alternately you could use the formula for sum of the first n natural numbers as n(n+1)/2 with n as 10. Then average – Sum/10 = $10 \times 11/2 \times 10 = 5.5$

25.   Ans: (a)
Solution: Required average = (0+1+2+….+9)/10 = 45/10 = 4.5

Exercise – 02

1.   Ans. (a):
Solution: for this type of questions take the LCM of speeds and assume the LCM as the distance
Then the time taken @ speed of 60 km/hr = $\frac{300}{60}$ = 5 hrs
Again the time taken @ speed of 50 km/hr = $\frac{300}{50}$ = 6 hrs
Thus we see that is place of 5 hrs train takes 6 hours.it means the train takes 1 hours extra and this one hour is stopping period in the total time of 6 hours thus in 6 hour train halts for 1 hours. So in 1 hour train will stop for $\frac{1}{6}$ hours or 10 minutes
Alternatively : short cut
Halting or stopping time = $\left(1 - \frac{slower\ speed}{fastest\ speed}\right)$ hours

$$= \left(1 - \frac{50}{60}\right) = \frac{10}{60} = \frac{1}{6} \text{ hours}$$

= 10 Minutes
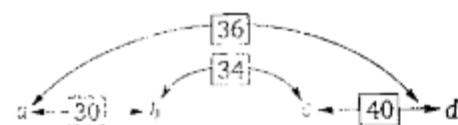(:- 1 hours = 60 minutes)

2.   Ans. (b):
Solution: given that a<b<c<d

The only possible prime number between 30 and 34 is 31
Hence b=31 therefore a=29
      Similarly c= 37 and d=43
      Therefore d-a = 43 -29=14

3.   Ans: (d)
Solution: The average speed can be calculated by assuming a distance of 6300 km LCM of 100 and 150 then time taken @ 100 kmph = 3 hours and time taken @ 150 kmph = 2 hours, average speed = total distance / total time = 600/5 = 120 kmph

4.   Ans: (a)
Solution: Let their current ages be x and 3x (ratio of 2:6). Then their ages after 5 years would be x + 5 and 3x + 5 now it is given that (x+5) /(3x + 5) = ¼ → x=1
And hence their current ages are 1 years and 3 years respectively. After 10 years their average age would be 12 years.

5.   Ans: (c)
Solution: 1200 × 3 – 1300 × 2 = 1000

6.   Ans: (b)
Solution: When a person aged 18 years is replaced by a person aged 38 years, the total age of the group goes up by 20 years since this leads to an increase in the average by 5 years, it means that there are 20/5 = 4 persons in the group.

7.   Ans: (d)
Solution: 10 × 61 + 30 × A = 40 × 52 → A = (2080 - 610)/ 30 = 1470/30 = 49.

8.   Ans: (d)
Solution: Required   average   = (2+4+6+8+10+12+14+16+18+20)/10 = 110/10= 11
Alternately you could use the formula for sum of the first n even natural numbers as n (n+1) with n as 10. Then average = sum/10 = 10× 11/10 = 11

9.   Ans: (b)
Solution: The sum of the first n odd numbers = n^2 in this case n = 10 → Sum = 10^2 = 100. Required average = 100/10 = 10

10.   Ans: (d)
Solution: Required   average   = (2+3+5+7+11+13+17+19+23+29)/10=12.9

11.   Ans: (c)
Solution: Required   average   = (4+6+8+9+10+12+14+15+16+18)/10=112/10=11.2

12.   Ans: (d)
Solution: Required   average   = (3+5+7+1+13+17+19+23+29+31)/10 = 158/10=15.8

13.   Ans: (c)
Solution: Temperature on 3rd December = 24.4 × 3 – 24 × 2 = 73.2 – 48= 25.2

14.   Ans: (a)
Solution: Required average = (6 × 11 + 14 + 16)/8 = 96/4 = 12.

15.   Ans: (c)
Solution: Required Average = (2 × 5.5 + 3 × 3.5 + 3 × 5.5 + 5 × 1.5)/ 13 = 45.5/13 = 3.5

16.   Ans: (70)
Solution: M + T + W+ Th + Fr = 50 × 5 = 250
If four of these were equal to the lowest possible, temperature for the fifth day is 250 – 45

17.   Ans: (29)

Solution: Average of 30 number is 0 then at most 29 of them greater than 0 and 30th number is equal to -1 times the sum of 29 numbers.

18. Ans: (37.5)
Solution: Required average = $\frac{50 \times 38 - (45+55)}{48}$ = 37.5

19. Ans: (24)
Solution: 6th Number = 50 × -9- (54× 5) – (52 × 3) = 24

20. Ans: (24.5)
Solution: Required average = $\frac{7+14+21+28+35+42}{6}$ 24.5
Alternately, since you can see that this is an arithmetic progression, the average of the six numbers is simply the average of the first and the last numbers in the series i.e., the average of 7 and 42 which is 24.5

21. Ans: (c)
Solution: standard question requiring good calculation speed. Obviously the 15th days deficit in last 17 days = 255 – 289 → Net deficit of 34. This means tha the average is reducing by 34 due to the double counting of the 15th day. This can only means that the 15th day's expenditure is Rs 68 – 34 = 34
(Lengthy calculations would have yielded the following calculations: 85 × 15 + 51 × 17 – 68 × 31 = 34)

22. Ans: (c)
Solution: Find the total distance covered in each segment of 10 minutes. You will get total distance = 46 kilometer in 50 mints

23. Ans: (c)
Solution: Assume that the distance is 120 km. hence 30 km is covered @ 25 kmph 40 @ 30 kmph and so on. Then average speed is 120/total time.

24. Ans: (a)
Solution: 100 → 130 → 182. Hence 82/2 = 41

25. Ans: (a)
Solution: The number of rooms in 18 + 16+30 on the three floors respectively.
Total revenue are 18 × 200+ 16 × 100 + 30 × 150 = 9700 required average = 9700/110 = 88.18
Note: here that if you could visualize here that since the number of rooms is 110 the decimal values cannot be 0.3 or 0.7 which effectively means tha options 3 and 4 are rejected.

26. Ans: (d)
Solution: Part of the runs scored in the 87th innings will go towards increasing the average of the first 86 innings to the new average and the remaining part of the runs will go towards maintaining the new average for the 87th innings. The only constraint in this problem is that there is an increase in the average by a whole number of runs. This is possible for all three options.

27. Ans: (a)
Solution: 42 A + 31 = 55 (A-3 →, 13 A = 196 → A = 196/13 = 15.07.
Total expenditure original = 15.07 x 42 = 633.23)

28. Ans: (d)
Solution: The average weight per ball is asked. Hence the bag does not have to be counted as the 48th item.

29. Ans: (c)
Solution: (14 * 333-2-504)/22

Exercise – 03

1. Ans: (b)
Solution: Let the number of ladies be n. then we have 8 × 15 + n × 6 = (8+n) × (10.8) → 120 + 6n = 86.4 + 10.8n → 4.8n = 33.6 → n=7

2. Ans: (a)
Solution: Total distance by total time = 840/13 = 64.615.

3. Ans: (a)
Solution: The total weight of the six people goes up by 15 kgs (When the average for 6 person goes up by 2.5 kg)

thus. The new person must be 15 kgs more than the person who he replaces. Hence the new person weight = 5 + 15 = 65 kg

4. Ans: (b)
Solution: Assume a distance of 60 km in such a case, the required average = total distance / total time = (60 +60)/ (4+3)= 120/7= 17.14

5. Ans: (d)
Solution: For 150 coins to be of a value of Rs. 150, using only 25 paise, 50 paise and 1 Re coins, we cannot have

any coins lower than the value of 1 Rs. Thus the number of 50 paise coins would be 0. Option (d) is correct

7.   Ans: (c)
Solution: Let the equal distances be 'd' each.
Then $3d/(d/x + d/y + d/z) = 3xyz /(x+y+z)$.

8.   Ans: (b)
Solution: Put $x = 10$ in the given equations and find the average of the resultant values.

9.   Ans: (a)
Solution: Put $x = 10$ in the given equations and find the average of the resultant values.

10.   Ans: (b)
Solution: Solve through options.

11.   Ans. (c):
Solution:

	Lok Sabha	Rajya Sabha
No. of MPs	300	200
Ave. Age	40	50

Since when a member of rajya Sabha joins the Lok Sabha and the average age of both the houses increases, it means the average age of this member must lie between 40 and 50 when the age of this member is greater than 40, then the average age of the Lok Sabha increases.

H	W	D	S
28	24	0	x
32	28	4	0

Again when the age of this member is less than 50 then after leaving it the average age of the Rajya Sabha increases.

12.   Ans. (a):
Solution:

	No. of family members	Average	Total
Eleven years earlier	4	28	112
	If 4	39	156
Presently	6	28	168

Since it is obvious that just after the birth of the youngest member was 6 family members in the family. Therefore at the time of the birth of the youngest child the elder child age was 6 years.
Now the sum of their ages = $x + (x+6) = 12 = (168 – 156)$
= $x=3$ and $(x+3) = 9$

13.   Ans. (a):
Solution: let the average of prime numbers be P and average of composite numbers be C.
Again the number of prime numbers be x, then the number of composite numbers be $2x$.

Then $\dfrac{Px+2Cx}{3x} = 9$ → $P+2C = 27$ ......(1)
And $\dfrac{2Px+Cx}{3x} = 11$

⇨$2P + C = 33$ ......(2)

On adding eq. (1) and (2)
We get $P+C = 20$
And on subtracting eq. (1) and (2)
We get $P-C = 6$
Therefore $P=13$ and $C=7$

Thus $\dfrac{C}{P} = \dfrac{7}{13}$

14.   Ans. (d):
Solution: Let there be n people initially in the group, then the total earning of the group = $n \times 50$
$(n-2) \times 49 + (2x + 45)$
⇨$N = 2x-53$;
Where x is the lowest earning of any person.
Now since $42<x<47$ and $n \in$ prime numbers
Then the only possible value of
n = 37 x = 45

15.   Ans. (a):
Solution: Let Donald be denoted by H (Husband)
His wife be denoted by W (wife)
His daughter be denoted by d (Daughter)
His son be denoted by S (Son)
The average age of 4 person = $\dfrac{H+W+D+S}{4}$ = 23
⇨$H + W + D + S = 92$
Again   H= W+4
So,

At the time when daughter is born
At the time when son is born

So at the time of birth of his son, total age of his family = 64 years (32+28+4+0=60)
And presently the total age of his family = 92 years
It means total increase in age of the whole family = 28 years
Thus the average increase in age = $\dfrac{28}{4} = 7$ years
It means the age of Donald = 39 years
And age of his daughter = 11 years
Therefore the average age of Donald and his daughter is 25 years.

16.   Ans. (b):
Solution: let the numbers be (a-5), (a-3), (a-1), (a+3), (a+5)
Then their average = $\dfrac{(a-5)+ (a-3)+(a-1)+ (a+3)+ (a+5)}{6} = a$

Again the value of a can be found by using the last statement
i.e,
$(a-5)^2 + (a+5)^2 = 178$
$a^2 = 64$ → a=8

17.   Ans. (a):
Solution: Average speed of the later half journey
$$= \frac{2 \times 40 \times 60}{100} = 48 \text{ km/hr}$$

Now the average speed of the whole journey

$$= \frac{2 \times 48 \times 96}{144} = 64 \text{ km/hr}$$

18.   Ans. (d)
Solution: Number of articles made in first hour = 60
Number of articles made in second hour = 45

Number of articles made in third hour = 63
Number of articles made in f fourth hour = 42
Number of articles made in fifth hour = 63
So, obviously articles made in fourth hour is minimum.

19.   Ans. (c)
Solution: Since a boy is increased in section B and the average is decreased ➔ hence, his weight is less than 38 kg. again, the average weight is also decreased in both the sections. Hence, his weight is more than 35 kg. Hence, option (c) is the answer.

Exercise – 04
TITA/SA

1.   Ans.
    Solution: Let the speed for the first hour be x km/hr
    Then the speed for the second hour be $\frac{7}{5} x$ km/hr
    Then the speed for the third hour be $\frac{10}{7} x \frac{7}{5} x = 2x$ km/hr
    Then the speed for the fourth hour be $2x \times \frac{7}{5} = \frac{14x}{5}$ km/hr
    Therefore total distance in four hours $= x + \frac{7}{5}x + 2x + \frac{14x}{5} = \frac{36x}{5}$ km
    Average speed $= \frac{total\ distance}{total\ time} = \frac{\frac{36x}{5}}{4} = \frac{9x}{5}$ km/hr

2.   Ans.
Solution: total distance covered by all the wheels = 4 x 160
Number of wheels used = 5
Therefore average distance covered by each wheel = $\frac{4\ x\ 160}{5}$ = 128 km

3.   Ans.
Solution: Remember $\frac{-cba}{99\ (a-c)}$
    Where abc and cba are the three digit numbers and $(a,c) \neq 0$
Again since the difference in average = 19.8
Therefore the difference in total = 19.8 × 10 = 198

⇨ 99 × (a-c) = 198
⇨ (a-c) = 2

4.   Ans.

Solution: let the fixed expenditure of the hotel be Rs. x and the variable expenditure (which is dependent on the guest) is Rs. y, then
        x+10y = 600   ..(1)
        x+20y = 800   ….(2)
        10y = 200
        Y=Rs. 20 and x = 400
Hence the total expenditure when there are 40 guests = 400 + 40 x 20 = 1200
Therefore average expenditure = $\frac{1200}{400}$ = Rs. 30

5.
Solution: If the number of questions correct is N, then the number of wrong answer is 100-N. this we get
N × 2 – (100 - N) × 0.5 = 120 ➔ 2.5 N = 170 ➔ N = 68.

6.
Solution: 30 × P + 75 × 2 = 510 ➔ P = (510-150)30 = 12.

7.
Solution: Required average = total runs / total innings = (40 × 4 + 0)/5 =160/5 =32

8.
Solution: Assume that the distance is 120 km. hence 30 km is covered @ 25 kmph 40 @ 30 kmph and so on. Then average speed is 120/total time. (same process like this explanations)

9.
Solution: Put x = 100 to get the weight of the containers. Use these weights to find average weight as 2600/4 = 650

10.
Solution: Use allegation to solve 20------32------68. Thus. 5 corresponds to 12, hence for 36 the answer will be 15.

11.   Ans: (6)

Solution: Let the numbers be $n_1, n_2, n_3, n_4 \ldots \ldots n_n$

$$\frac{n_1 + n_2 + n_3 + \cdots + n_n}{n} \geq (n_1 . n_2 . n_3 \cdots n_n)^{1/n}$$

$$\geq (n)^{1/n}$$

$$\frac{n_1 + n_2 + n_3 + \cdots + n_n}{n} \geq n$$

12. Ans: (1.5)
Solution: If the numbers are 1, 2, 3, 4, 5, 6, 7 and we add 8, 9, 10
Initial average = (1+2+3+4+5+6+7)/7 = 4
Final average = (1+2+3+4....10)/10 = 5.5
Therefore, the average increases by 1.5

13. Ans: (26)
Solution: $30 \leq \dfrac{22+24+x+27+29}{5} \leq 35$
$150 \leq 102 + x \leq 175$

$48 \leq x \leq 73$
Number of possible values of x= 26

14. Ans: (86)
Solution: Total number of balls in all 12 – boxes = 20 x 12 = 240.
Minimum possible number of balls in 11 of these 12 boxes = 9+10+11+12+13+14+15+16+17+18+19 = 154.
Maximum possible number of balls in any box = 240-154 = 86.

15. Ans: (8.5)
Solution: Sum of these four numbers = 5 x 4 = 20.
Maximum average of the largest two numbers would occur when the two smaller numbers are as small as possible (i.e. when they are 1 and 2 respectively)

➔ Max. Average = $\dfrac{20-(1+2)}{2} = \dfrac{17}{2} = 8.5$