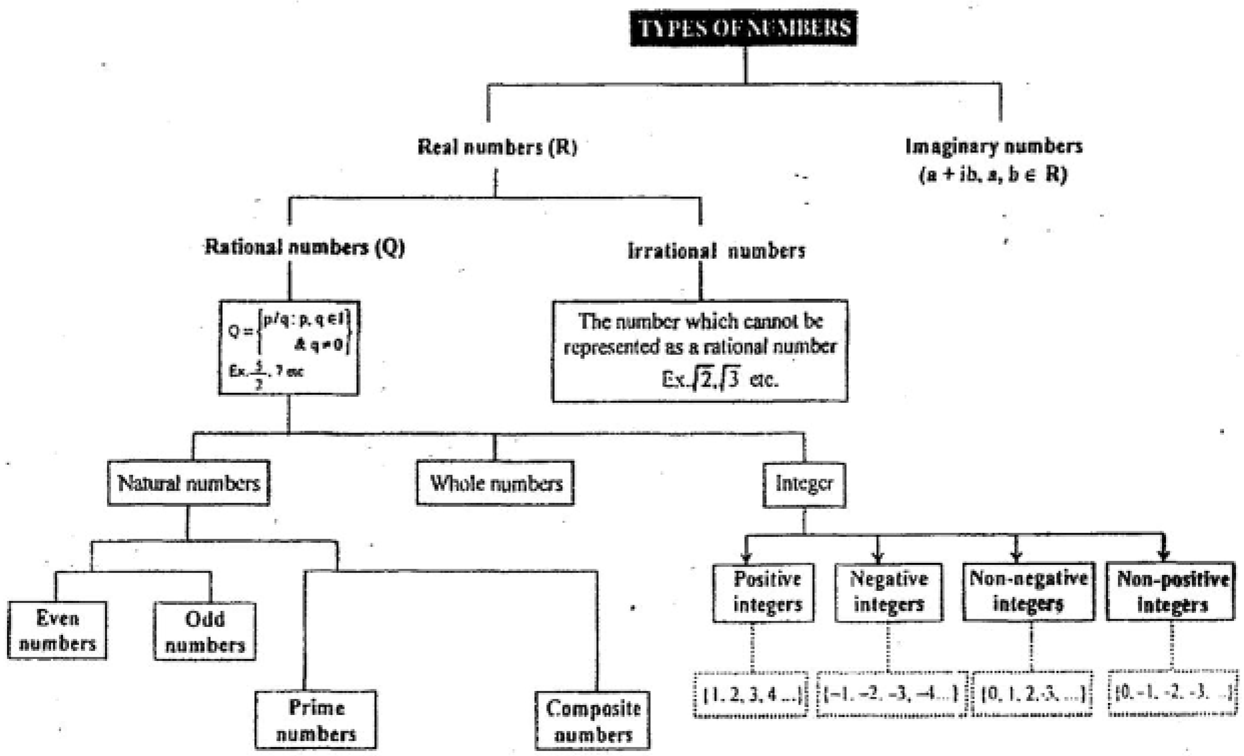
Number System

The ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called digits, which can represent any number.

Number System

It is a mathematical structure which includes natural numbers, whole numbers, rational numbers, irrational numbers, etc.



Remember

- *1 is neither prime nor composite*
- *1 is an odd integer*
- *0 is neither positive nor negative.*
- *0 is an even integer.*
- *2 is prime & even both.*
- *All prime numbers (except 2) are odd.*

Natural Numbers:

These are the number (1, 2, 3, etc.) that are used for counting.
It is denoted by N.
There are infinite natural numbers and the smallest natural number is one (1).

Even Numbers:

Natural numbers which are divisible by 2 are even numbers.
It is denoted by E.
$E = 2, 4, 6, 8, \ldots$
Smallest even number is 2. There is no largest even number.

Odd numbers:

Natural numbers which are not divisible/by late odd numbers.
It is denoted by O.
O = 1, 3, 5, 7,... Smallest odd number is 1.
There is no largest odd number.

✓ Based on divisibility, there could be two types of natural numbers: Prime and Composite.

Prime Numbers:
Natural numbers which have exactly two factors, i.e., I and the number itself are called prime numbers.
The lowest prime number is 2.
2 is also the only even prime number.

Composite Numbers:
It is a natural number that has atleast one divisor different from unity and itself.
Every composite number can be factorised into its prime factors.
For Example: $24 = 2 \times 2 \times 2 \times 3$. Hence, 24 is a composite number.
The smallest composite number is 4.

Whole Numbers:
The natural numbers along with zero (0), form the system of whole numbers.
It is denoted by W.
There is no largest whole number and;
The smallest whole number is 0.

Integers:
The number system consisting of natural numbers, their negative and zero is called integers.
It is denoted by Z or I.
The smallest and the largest integers cannot be determined.

The number line:
The number line is a straight line between negative infinity on the left to positive infinity on the right.

Real Numbers:
All numbers that can be represented on the number line are called real numbers. It is denoted by R.
R^+: Positive real numbers and
R^-: Negative real numbers.
Real numbers = Rational numbers + Irrational numbers.
(A) Rational numbers: Any number that can be put in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called rational number.
It is denoted by Q.
Every integer is a rational number.
Zero (0) is also rational number. The smallest and largest rational numbers cannot be determined. Every fraction (and decimal fraction) is a rational number.
$Q = \frac{p}{q} \frac{(Numerator)}{Denominator}$

- If x and y are two rational numbers, then $\frac{x+y}{2}$ is also a rational number and its value lies between the given two rational numbers x and y.
- An infinite number of rational numbers can be determined between any two rational numbers.

✓ *Irrational numbers:* The numbers which are not rational or which cannot be put in the form of $\frac{p}{q}$, where p and q are intergers and $q \neq 0$, is called irrational number.
It is denoted by Q′ or Q^c
$\sqrt{2}, \sqrt{3}, \sqrt{5}, 2+\sqrt{3}, 3-\sqrt{5}, 3\sqrt{3}$ are irrational numbers

✓ Every real number is either rational or irrational

Fraction: A fraction is a quantity which expresses a part of the whole.

$$Fraction = \frac{Numerator}{Denominator}$$

Types of Fractions:

1. *Proper fraction:* If numerator is less than its denominator, then it is a proper fraction.
   For example: $\frac{2}{5}, \frac{6}{18}$
2. *Improper fraction:* If numerator is greater than or equal to its denominator, then it is a improper fraction.
   $For\ example: \frac{5}{2}, \frac{18}{7}, \frac{13}{13}$

Division Algorithm:

Dividend = (Divisor $\times$ Quotient) + Remainder where, Dividend = The number which is being divided Divisor = The number which performs the division process Quotient = Greatest possible integer as a result of division Remainder= Rest part of dividend which cannot be further divided by the divisor.

Complete remainder: A complete remainder is the remainder obtained fay a number by the method of successive-division.
Complete remainder = [I divisor $\times$ II remainder] + I remainder
$C.R. = d_1r_2 + r_1$
$C.R. = d_1d_2r_3 + d_1r_2 + r_1$

✓ Two different numbers x and y when divided by a certain divisor D leave remainder r_1 and r_2 respectively. When the sum of them is divided by the same divisor, the remainder is r_3. Then,
$divisor\ D = r_1 + r_2 - r_3$

1. The product of n consecutive natural numbers is always divisible by n!, where n! = 1 x 2 x 3 x 4 x 5 ……x n
   Sum of first n natural numbers = $\frac{n(n+1)}{2}$
   Sum of squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$
   Sum of cubes of first n natural numbers = $\left(\frac{n(n+1)}{2}\right)^2$
   Number of single – digit natural numbers: 1-9 = 9
   Number of two digit natural numbers: 10-99 = 90
   Number of three digit natural numbers: 100-999 = 900
   Square of every natural number can be written in the form 3n or 3n+1.
   Square of every natural number can be written in the form 4n or 4n +1.
   Square of a natural number can only end in 00, 1, 4, 5, 6, and 9. No perfect square can end in 2, 3, 7, 8
   Or a single 0.
   The tens digit of every perfect square is even unless the square is ending in 6 in which case the tens digit is odd.

2. To find whether a number N is prime or not
   Find the root R (approximate) of the number N, i.e. R = $\sqrt{N}$. Divide N by every prime number less than or equal to R. If N is divisible by at least one of those prime numbers it is not a prime number. If N is not divisible by any of those prime numbers, it is a prime number.

Remember! *Odd + Odd = Even* *Even + Even = Even* *Odd + Even = Odd* $(Odd)^{Even} = Odd$	$(Even)^{Odd} = Even$ *Even × Odd = Even* *Even × Even = Even* *Odd × Odd = Odd* $(Odd)^{Even} \times (Even)^{Odd} = Even$ $(Odd)^{Even} + (Even)^{Odd} = Odd$

3. Suppose the numbers N_1, N_2, N_3… give quotients Q_1, Q_2, Q_3… and remainders R_1, R_2, R_3..., respectively, when divided by a common divisor D.
   Therefore
   $N_1 = D \times Q_1 + R_1$,
   $N_2 = D \times Q_2 + R_2$,
   $N_3 = D \times Q_3 + R_3$.. and so on.

Let P be the product of N_1, N_2, N_3…
Therefore, $P = N_1N_2N_3.. = (D \times Q_1 + R_1)(D \times Q_2 + R_2)(D \times Q_3 + R_3)..$
$= D \times K + R_1R_2R_3...$ where K is some number ---- (1)

In the above equation, since only the product $R_1R_2R_3$… is free of D, therefore the remainder when P is divided by D is the remainder when the product $R_1R_2R_3$… is divided by D.
Let S be the sum of N_1, N_2, N_3…
Therefore, $S = (N_1) + (N_2) + (N_3) +...$
$= (D \times Q_1 + R_1) + (D \times Q_2 + R_2) + (D \times Q_3 + R_3)..$
$= D \times K + R_1 + R_2 + R_3...$ where K is some number--- (2)

Hence the remainder when S is divided by D is the remainder when $R_1 + R_2 + R_3$ is divided by D.

4. What is the remainder when the product 1998 × 1999 × 2000 is divided by 7?

Answer: the remainders when 1998, 1999, and 2000 are divided by 7 are 3, 4, and 5 respectively.
Hence the final remainder is the remainder when the product 3 × 4 × 5 = 60 is divided by 7. Therefore, remainder = 4

5. What is the remainder when 2^{2004} is divided by 7?
Answer: 2^{2004} is again a product (2 × 2 × 2... (2004 times)). Since 2 is a number less than 7 we try to convert the product into product of numbers higher than 7. Notice that 8 = 2 × 2 × 2. Therefore we convert the product in the following manner- $2^{2004} = 8^{668}$ = 8 × 8 × 8... (668 times).
The remainder when 8 is divided by 7 is 1. Hence the remainder when 8^{668} is divided by 7 is the remainder obtained when the product 1 × 1 × 1... is divided by 7. Therefore, remainder = 1

6. What is the remainder when 2^{2006} is divided by 7?
Answer: This problem is like the previous one, except that 2006 is not an exact multiple of 3 so we cannot convert it completely into the form 8x
. We will write it in following manner- $2^{2006} = 8^{668} \times 4$.
Now, 8^{668} gives the remainder 1 when divided by 7 as we have seen in the previous problem. And 4 gives a remainder of 4 only when divided by 7. Hence the remainder when 2^{2006} is divided by 7 is the remainder when the product 1 × 4 is divided by 7. Therefore, remainder = 4

7. What is the remainder when 25^{25} is divided by 9?
Answer: Again $25^{25} = (18 + 7)^{25} = (18 + 7)(18 + 7)...25 \text{ times} = 18K + 7^{25}$
Hence remainder when 2525 is divided by 9 is the remainder when 7^{25} is divided by 9.
Now $7^{25} = 7^3 \times 7^3 \times 7^3$
.. (8 times) × 7 = 343 × 343 × 343... (8 times) × 7.

The remainder when 343 is divided by 9 is 1 and the remainder when 7 is divided by 9 is 7.
Hence the remainder when 7^{25} is divided by 9 is the remainder we obtain when the product 1 × 1 × 1... (8 times) × 7 is divided by 9. The remainder is 7 in this case. Hence the remainder when 2525 is divided by 9 is 7.

8. What is the remainder when $32^{32^{32}}$ is divided by 7?
Let me put up the steps for finding remainder when X^{Y^z} is divided by D.
1. Divide X by D. let the remainder be r. therefore, you have to find the Remainder when R^{Y^z} is divided by D. 32 gives a remainder 4 when divided by 7. Therefore, you are trying to find the remainder when $4^{32^{32}}$ is divided by 7.
2. Find a power of R that gives a remainder of + 1 or -1 with D, if you find a power that gives a remainder –1, twice of that power will give a remainder of +1. Now I know that $4^3 = 64$ gives a remainder 1 when divided by 7.
3. Find the remainder when Y^z is divided by that power. Here, find the remainder when 32^{32} is divded by 3. The remainder is 1. Therefore, 32^{32} can be written as 3k +1 and $4^{32^{32}}$ can be written as 4^{3k+1} or $(4^3)^k \times 4$.

Let P be the product of N_1, N_2, N_3…
Therefore, $P = N_1N_2N_3.. = (D \times Q_1 + R_1)(D \times Q_2 + R_2)(D \times Q_3 + R_3)..$
$= D \times K + R_1R_2R_3...$ where K is some number ---- (1)

In the above equation, since only the product $R_1R_2R_3$… is free of D, therefore the remainder when P is divided by D is the remainder when the product $R_1R_2R_3$… is divided by D.
Let S be the sum of N_1, N_2, N_3…
Therefore, $S = (N_1) + (N_2) + (N_3) +...$
$= (D \times Q_1 + R_1) + (D \times Q_2 + R_2) + (D \times Q_3 + R_3)..$
$= D \times K + R_1 + R_2 + R_3$… where K is some number--- (2)

Hence the remainder when S is divided by D is the remainder when $R_1 + R_2 + R_3$ is divided by D.

4. What is the remainder when the product $1998 \times 1999 \times 2000$ is divided by 7?

Answer: the remainders when 1998, 1999, and 2000 are divided by 7 are 3, 4, and 5 respectively.
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The remainder when 8 is divided by 7 is 1. Hence the remainder when 8^{668} is divided by 7 is the remainder obtained when the product $1 \times 1 \times 1$... is divided by 7. Therefore, remainder = 1

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7. What is the remainder when 25^{25} is divided by 9?
Answer: Again $25^{25} = (18 + 7)^{25} = (18 + 7)(18 + 7)$...25 times $= 18K + 7^{25}$
Hence remainder when 2525 is divided by 9 is the remainder when 7^{25} is divided by 9.
Now $7^{25} = 7^3 \times 7^3 \times 7^3$
.. (8 times) $\times 7 = 343 \times 343 \times 343$... (8 times) $\times 7$.

The remainder when 343 is divided by 9 is 1 and the remainder when 7 is divided by 9 is 7.
Hence the remainder when 7^{25} is divided by 9 is the remainder we obtain when the product $1 \times 1 \times 1$... (8 times) $\times 7$ is divided by 9. The remainder is 7 in this case. Hence the remainder when 2525 is divided by 9 is 7.

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   2. Find a power of R that gives a remainder of + 1 or -1 with D, if you find a power that gives a remainder – 1, twice of that power will give a remainder of +1. Now I know that $4^3 = 64$ gives a remainder 1 when divided by 7.
   3. Find the remainder when Y^z is divided by that power. Here, find the remainder when 32^{32} is divded by 3. The remainder is 1. Therefore, 32^{32} can be written as 3k +1 and $4^{32^{32}}$ can be written as 4^{3k+1} or $(4^3)^k \times 4$.

4. Now (4^3) gives a remainder 1 when divided by 7. Therefore, we need to find the remainder when 4 is divided by7. Therefore, the remainder is 4.

SOME SPECIAL CASES:

When both the dividend and the divisor have a factor in common.

Let N be a number and Q and R be the quotient and the remainder when N is divided by the divisor D.
Hence, $N = Q \times D + R$.
Let $N = k \times A$ and $D = k \times B$ where k is the HCF of N and D and $k > 1$. Hence $kA = Q \times kB + R$.
Let Q1 and R1 be the quotient and the remainder when A is divided by B. Hence $A = B \times Q_1 + R_1$.
Putting the value of A in the previous equation and comparing we get

$k(B \times Q_1 + R_1) = Q \times kB + R \rightarrow R = kR_1$.

Hence to find the remainder when both the dividend and the divisor have a factor in common,

- Take out the common factor (i.e. divide the numbers by the common factor)
- Divide the resulting dividend (A) by resulting divisor (B) and find the remainder (R_1).
- The real remainder R is this remainder R1 multiplied by the common factor (k).

What the remainder when 2^{96} is divided by 96?

The common factor between 2^{96} and 96 is $32 = 2^5$

.

Removing 32 from the dividend and the divisor we get the numbers 2^{91} and 3 respectively.
The remainder when 2^{91} is divided by 3 is 2.
Hence the real remainder will be 2 multiplied by common factor 32.
Remainder = 64

The concept of negative remainder

$15 = 16 \times 0 + 15$ or $15 = 16 \times 1 - 1$.
The remainder when 15 is divided by 16 is 15 the first case and −1 in the second case. Hence, the remainder when 15 is divided by 16 is 15 or −1.

When a number $N < D$ gives a remainder R (= N) when divided by D, it gives a negative remainder of $R - D$.
For example, when a number gives a remainder of −2 with 23, it means that the number gives a remainder of $23 - 2 = 21$ with 23.

9. Find the remainder when 752 is divided by 2402.
Answer: $7^{52} = (7^4)^{13} = (2401)^{13} = (2402 - 1)^{13} = 2402K + (-1)^{13} = 2402K-1$.

Hence, the remainder when 7^{52} is divided by 2402 is equal to −1 or $2402 - 1 = 2401$.
Remainder = 2401.
When dividend is of the form $a^n + b^n$ or $a^n - b^n$

Theorem1: $a^n + b^n$ is divisible by $a + b$ when n is **ODD**.
Theorem 2: $a^n - b^n$ is divisible by $a + b$ when n is **EVEN**.
Theorem 3: $a^n - b^n$ is ALWAYS divisible by $a - b$.

10. What is the remainder when $3^{444} + 4^{333}$ is divided by 5?
Answer: The dividend is in the form $a^x + b^y$. We need to change it into the form $a^n + b^n$.
$3^{444} + 4^{333} = (3^4)^{111} + (4^3)^{111}$. Now $(3^4)^{111} + (4^3)^{111}$ will be divisible by $3^4 + 4^3 = 81 + 64 = 145$. Since the number is divisible by 145 it will certainly be divisible by 5. Hence, the remainder is 0.

11. What is the remainder when $(5555)^{2222} + (2222)^{5555}$ is divided by 7?

Answer: The remainders when 5555 and 2222 are divided by 7 are 4 and 3 respectively. Hence, the problem reduces to finding the remainder when $(4)^{2222} + (3)^{5555}$ is divided by 7.

Now $(4)^{2222} + (3)^{5555} = (4^2)^{1111} + (3^5)^{1111} = (16)^{1111} + (243)^{1111}$.

Now $(16)^{1111} + (243)^{1111}$ is divisible by 16 + 243 or it is divisible by 259, which is a multiple of 7. Hence the remainder when $(5555)^{2222} + (2222)^{5555}$ is divided by 7 is zero.

12. $20^{2004} + 16^{2004} - 3^{2004} - 1$ is divisible by:
(a) 317 (b) 323 (c) 253 (d) 91

Answer: $20^{2004} + 16^{2004} - 3^{2004} - 1 = (20^{2004} - 3^{2004}) + (16^{2004} - 1^{2004})$. Now $20^{2004} - 3^{2004}$ is divisible by 17 (Theorem 3) and $16^{2004} - 1^{2004}$ is divisible by 17 (Theorem 2). Hence the complete expression is divisible by 17.

$20^{2004} + 16^{2004} - 3^{2004} - 1 = (20^{2004} - 1^{2004}) + (16^{2004} - 3^{2004})$. Now $20^{2004} - 1^{2004}$ is divisible by 19 (Theorem 3) and $16^{2004} - 3^{2004}$ is divisible by 19 (Theorem 2). Hence the complete expression is also divisible by 19.

Hence the complete expression is divisible by $17 \times 19 = 323$.

13. When $f(x) = a + bx + cx^2 + dx^3 +...$ is divided by $x - a$
The remainder when $f(x) = a + bx + cx^2 + dx^3 +..$ is divided by $x - a$ is $f(a)$. Therefore, If $f(a) = 0$, $(x- a)$ is a factor of $f(x)$.

What is the remainder when $x^3 + 2x^2 + 5x + 3$ is divided by $x + 1$?

Answer: The remainder when the expression is divided by $(x - (-1))$ will be $f(-1)$. Remainder $= (-1)^3 + 2(-1)^2 + 5(-1) + 3 = -1$

14. If $2x^3 -3x^2 + 4x + c$ is divisible by $x - 1$, find the value of c.
Answer: Since the expression is divisible by $x - 1$, the remainder $f(1)$ should be equal to zero $\Rightarrow 2 - 3 + 4 + c = 0$, or $c = -3$.

The remainders when F(x) is divided by x - 99 and x - 19 are 19 and 99, respectively. What is the remainder when F(x) is divided by (x - 19) (x - 99)?

Answer: $F(x) = (x - 99)a + 19$ --- (1) and $F(x) = (x - 19)b + 99$ --- (2). Multiplying (1) by $(x - 19)$ and (2) by $(x - 99)$ we get
$(x - 19)F(x) = (x - 99)(x - 19)a + 19(x - 19)$ and
$(x - 99)F(x) = (x - 99)(x - 19)b + 99(x - 99)$

Subtracting, we obtain:

$80F(x) = (x - 99)(x - 19)k - 80x + 9440 \Rightarrow F(x) = \frac{(x-19)(x-99)k}{80} - x + 118$. We know from (1) and (2) that F(x) does not have fractional coefficients. Therefore, k will be divisible by 80. Therefore, the remainder when F(x) is divided by $(x - 99)(x - 19)$ is $118 - x$.

Euler's Theorem

If M and N are two numbers coprime to each other, i.e. HCF(M,N) 1 and $N = a^p b^q c^r$..., Remainder $\left[\frac{M^{\Phi(N)}}{N}\right] = 1$, where $\Phi(N) = N\,(1-\frac{1}{a})\,(1-\frac{1}{b})\,(1-\frac{1}{c})$ … and is knows as Euler's Totient function.. $\Phi(N)$ is also the number of numbers less than abc and prime

15. Find the remainder when 5^{37} is divided by 63.
Answer: 5 and 63 are coprime to each other, therefore we can apply Euler's theorem here.
$63 = 32 \times 7 \Rightarrow \Phi(63) = (63-\frac{1}{3})\,(1-\frac{1}{7}) = 36$

Therefore, Remainder $[\frac{5^{37}}{63}]$ Remainder $[\frac{5^{36} x 5}{63}] = 5$

16. Find the last three digits of 57^{802}.

Answer: Many a times (not always), the quicker way to calculate the last three digits is to calculate the remainder by 1 000. We can see that 57 and 1 000 are coprime to each other. Therefore, we can use Euler's theorem here if it's useful.

$1000 = 2^3 \times 5^3 \Rightarrow \varphi\ (1000)\ 1000(1 - \frac{1}{2})(1 - \frac{1}{4}) = 400$

Therefore,

Remainder $[\frac{57^{400}}{1000}] = 1 \rightarrow$ Remainder $[\frac{57^{400} x 57^{400}}{100}] =$ Remainder $[\frac{57^{800}}{1000}] = 1$

Remainder $[\frac{57^{802}}{1000}]$ Remainder $[\frac{57^{800} x 57^{2}}{1000}] = 249$

Hence, the last two digits of 57802 are 249.

Fermat's Theorem

If p is a prime number and N is prime to p, then $N^p - N$ is divisible by p.

17. What is the remainder when $n^7 - n$ is divided by 42?

Answer: Since 7 is prime, $n^7 - n$ is divisible by 7. $n^7 - n = n(n^6 - 1) = n\ (n + 1)(n - 1)(n^4 + n^2 + 1)$.

Now $(n - 1)(n)(n + 1)$ is divisible by $3! = 6$. Hence $n^7 - n$ is divisible by $6 \times 7 = 42$. Hence the remainder is 0.

Fermat's Little Theorem

If N in the above Euler's theorem is a prime number, then $\Phi\ (N) = N(1 - \frac{1}{N}) = N - 1$. Therefore, if M and N are coprime to each other and N is a prime number, Remainder $[\frac{M^{N-1}}{N}] = 1$

18. Find the remainder when 5^{260} is divided by 31.

Answer: 31 is a prime number therefore $\varphi(N) = 30$. 52 and 31 are prime to each other. Therefore, by Fermat's theorem:

Remainder $[\frac{52^{30}}{31}] = 1$ Remainder $[\frac{52^{60}}{31}] = 1$

Wilson's Theorem

If P is a prime number then $[\frac{(P-1)!+1}{P}] = 0$. In other words, $(P - 1)! + 1$ is divisible by P if P is a prime number. It also means that the remainder when $(P - 1)!$ Is divided by P is $P - 1$ when P is prime.

19. Find the remainder when 40! is divided by 41.

Answer: By Wilson's theorem, we can see that $40! + 1$ is divisible by 41 $\Rightarrow$ Remainder $[\frac{40!}{41}] = 41 - 1 = 40$

20. Find the remainder when 39! is divided by 41.

Answer: In the above example, we saw that the remainder when 40! is divided by 41 is 40.

$\Rightarrow 40! = 41k + 40 \Rightarrow 40 \times 39! = 41k + 40.$

The R.H.S. gives remainder 40 with 41 therefore L.H.S. should also give remainder 40 with 41. L.H.S. = $40 \times 39!$ where 40 gives remainder 40 with 41.

Therefore, 39! should give remainder 1 with 41.

Chinese Remainder Theorem

This is a very useful result. It might take a little time to understand and master Chinese remainder theorem completely but once understood, it is an asset.

If a number N = a × b, where a and b are prime to each other, i.e., hcf(a, b) = 1, and M is a number such that MM M Remainder$[\frac{M}{a}]$ = r_1 and Remainder$[\frac{M}{b}]$ = r_2 then Remainder$[\frac{M}{N}]$ = ar_2 x br_1 y, where ax + by = 1

Confused?

Following example will make it clear.

21. Find the remainder when 3^{101} is divided by 77.

Answer: 77 = 11 × 7.

By Fermat's little theorem, $\frac{3^{101}}{7}$ = 1 AND Remainder $\frac{3^{10}}{11}$ = 1

Remainder$\frac{3^{101}}{7}$ = Remainder $[\frac{3^{96}\ x\ 3^5}{7}]$ = Remainder$[\frac{(3^6)^{16}\ x\ 3^5}{7}]$ = Remainder $\frac{1\ x\ 3^5}{7}$ = 5 = r_1

Remainder$\frac{3^{101}}{11}$ = Remainder $[\frac{3^{100}\ x\ 3}{11}]$ = Remainder$[\frac{(3^{10})^{10}\ x\ 3}{11}]$ = Remainder $\frac{1\ x\ 3}{11}$ = 3 = r_2

Now we will find x and y such that 7x + 11y = 1. By observation we can find out, x = −3 and y = 2.

Now we can say that Remainder $[\frac{3^{101}}{77}]$ = 7 x 3 x -3 + 11 x 5 x 2 = 47

Some Special Problems:

22. Find the remainder when 123456789101112……..40 is divided by 36.

Answer: 36 = 9 × 4. Therefore, we first find the remainders when this number is divided by 9 and 4.
The remainder by 9 would be the remainder when the sum of digits is divided by 9. Sum of digits = 4 × (1 + 2 + 3 + 4 + … + 9) + 10 × (1 + 2 + 3) + 4 = 180 + 60 + 4 = 244 ⇒ remainder by 9 = 1.
The remainder by 4 would be the remainder when the last two digits are divided by 4 ⇒ remainder by 4= 0.
Therefore, to find the remainder we need to find the smallest multiple of 4 that gives remainder 1 with 9. The smallest such number = 28. Therefore, remainder = 28.

23. Find the remainder when 112123123412345…12345678 is divided by 36.

Answer: 36 = 9 × 4. Therefore, we first find the remainders when this number is divided by 9 and 4.
The remainder by 9 would be the remainder when the sum of digits is divided by 9. Sum of digits = 1 × 8 + 2 × 7 + 3 × 6 + … + 8 × 1 = 120 ⇒ remainder by 9 = 3.

The remainder by 4 would be the remainder when the last two digits are divided by 4 ⇒ remainder by 4 = 2.
The overall remainder would be the smallest number that gives remainder 3 with 9 and remainder 2 with 4. Therefore, the number would satisfy the equation 9a + 3 = 4b + 2 ⇒ 4b − 9a = 1 ⇒ (a, b) = (3, 7) and the number = 30. Therefore, remainder = 30.

Let n! = 1 × 2 × 3 × … × n for integer n ≥ 1. If p = 1! + (2 × 2!) + (3 × 3!) + … + (10 × 10!), then p+2 when divided by 11! leaves a remainder of (CAT 2005)

1. 10 2. 0 3. 7 4. 1

Answer: Nth term of the series = n × n! = (n + 1 − 1) × n! = (n + 1)! − n!

Therefore, p = 2! − 1! + 3! − 2! + 4! − 3! + … + 11! − 10! = 11! − 1! ⇒ p + 2 = 11! + 1 ⇒ remainder by 11! = 1
Find the remainder when 1 × 2 + 2 × 3 + 3 × 4 + … + 98 × 99 + 99 × 100 is divided by 101.

Answer: Nth term of the series = n × (n + 1) = n^2 + n.

Therefore, sum of the series $\sum(n^2+n) = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6} = \frac{99\ x\ 100\ x\ 101}{6}$ → remainder by 101 = 0.

24. A number when divided by 8 leaves remainder 3 and quotient Q. The number when divided by 5 leaves remainder 2 and quotient Q + 8. What is the number?

Answer: Let the number be N $\Rightarrow$ N = 8Q + 3 and N = 5(Q + 8) + 2 = 5Q + 42 8Q + 3 = 5Q + 42 $\Rightarrow$ Q = 13 $\Rightarrow$ N = 107

25. Find the largest natural number that divides 364, 414, and 539 and leaves the same remainder in each case.

Answer: Let the divisor be D and the remainder be R. Therefore, 364 = Da + R, 414 = Db + R, 539 = Dc + R
Subtracting first equation from the second and the second equation from the third we get

50 = D (b − a) and 125 = D (c − b). As D is the common factor in RHS of both the equation, it should be the common factor on the LHS of both the equation. The HCF of 50 and 125 is 25. Therefore, the highest number can be 25.

What is the remainder when $\underbrace{11111\ldots11111}_{243\ times}$ is divided by 243 ?

Answer: It can be proved that a number formed by writing any single digit 3^n times will be divisible by 3^n. This is left to students to check it out.

How many numbers between 1 and 1000 are there such that $n^2 + 3n + 5$ is divisible by 121?

Answer: 0 values. $n^2 + 3n + 5 = (n - 4)(n + 7) + 33$. Now, 33 is divisible by 11 but not 121. n + 7 and n - 4 are two numbers with a difference of 11, therefore either both are divisible by 11 or both are not divisible by 11. If both are divisible by 11, their product is divisible by 121 but 33 is divisible only by 11 therefore the expression is not divisible by 121. If both are not divisible by 11, the expression is again not divisible by 121.

26. Find the remainder when $1^{39} + 2^{39} + 3^{39} + 4^{39} + \ldots + 12^{39}$ is divided by 39.

Answer: $1^p + 2^p + 3^p + \ldots n^p$ is divisible by $1 + 2 + 3 + \ldots + n$ if p is odd. Therefore, remainder = 0 as $1 + 2 + 3 + \ldots + 12 = 78$ which is a factor of 13.

Divisors of a Number

Divisors: For a natural number N, all the numbers, including 1 and N itself, which divide N completely are called divisors of N.

Example: The number 24 is divisible by 1, 2, 3, 4, 6, 8, 12, and 24. Hence all these numbers are divisors of 24.

How to find the number of divisors of a number:
Let us the find the number of divisors of 60.
$60 = 2^2 \times 3 \times 5$.

Any divisors of 60 will have powers of 2 equal to either 2^0 or 2^1 or 2^2.
Similarly, any divisor of 60 will have powers of 3 equal to either 3^0 or 3^1, and powers of 5 equal to either 5^0 or 5^1

To make a divisor of 60, we will have to choose a power of 2, a power of 3 and a power of 5. A power of 2 can be chosen in 3 ways out of 2^0 or 2^1, or 2^2. Similarly, a power of 3 can be chosen in 2 ways and a power of 5 can be chosen in 2 ways.

Therefore, the number of divisors = 3 × 2 × 2 = 12.

Notice that we have added 1 each to powers of 2, 3 and 5 and multiplied.

Now for the formula:
Let N be a composite number such that $N = (x)^a(y)^b(z)^c$.. where x, y, z.. are prime factors. Then, the number of divisors of N = (a + 1)(b + 1)(c + 1)..

27. If N is Natural number, N has 4 factors, and summation of factors excluding N is 31, how many values for N are possible?

Answer:

Let N be a composite number such that N = $(2)^a (y)^b(z)^c$.. where y, z.. are prime factors. Then, the number of even divisors of N = (a)(b + 1)(c + 1) and number of odd divisors of N = (b + 1)(c + 1)

28. How many divisors of 21600 are perfect squares?

Answer: In a perfect square, all the prime factors have even powers. For example, $2^5 \times 6^8$ will not be a perfect square as the power of 2 is odd whereas $2^4 \times 6^8$ will be a perfect square because all the prime factors have even powers. 21600 = $2^5 \times 3^3 \times 5^2$ therefore, all the divisors made by even powers of 2, 3 and 5 will be perfect squares.

The even powers of 2 are 2^0, 2^2, 2^4, even powers of 3 are 3^0 and 3^2, and even powers of 5 are 5^0 and 5^2 We can select an even power of 2 in 3 ways, even power of 3 in 2 ways, and even power of 5 in 2 ways. Therefore, the number of combinations = 3 × 2 × 2 = 12.

Let N be a composite number such that N = $(x)^a(y)^b(z)^c$.. where x, y, z.. are prime factors. Then, the sum of divisors of N = $\frac{x^{a+1}}{x-1} \times \frac{y^{b+1}}{y-1} \times \frac{z^{c+1}}{z-1}$.....

29. What is the sum of divisors of 60?

Answer: 60 = $2^2 \times 3 \times 5 \Rightarrow$ Sum of the divisors = $\frac{2^3-1}{2-1} \times \frac{3^2-1}{3-1} \times \frac{5^2-}{5-1}$ = 168

30. Find the sum of even divisors of $2^5 \times 3^5 \times 5^4$

Answer: All the even divisors of the number will have powers of 2 equal to one of 2, 2^2, 2^3, 2^4, or 2^5.

Therefore, sum of even divisors = $(2 + 2^2 + 2^3 + 2^4 + 2^5) \times (1 + 3 + 3^2 + 3^3 + 3^4 + 3^5) \times (1 + 5 + 5^2 + 5^3 + 5^4)$

= $\frac{2(2^5-1)}{2-1} \times \frac{3^6-1}{3-1} \times \frac{5^5-1}{5-1}$ = 17625608

31. A positive integer is bold if it has 8 positive divisors that sum up to 3240. For example, 2006 is bold because its 8 positive divisors, 1, 2, 17, 34, 59, 118, 1003 and 2006, sum up to 3240. Find the smallest positive bold number.

Answer: If a number has 8 divisors it can be of the forms a^7, ab^3, or abc, where a, b and c are prime numbers. The sum of the divisors in each case is given below:

Type of Number	*a^7*	*ab^3*	*abc*
Sum = 3240	$\frac{a^8-1}{a-1}$	$\frac{a^2-1}{a-1} \times \frac{b^4-1}{b-1}$ = (a+1) $(b^3 + b^2 + 1)$	$\frac{a^2-1}{a-1} \times \frac{b^2-1}{b-1}$ = (a+1) (b + 1) (c + 1)
Examples	None	None	1614 = (2+1) (3+1)(269+1), 1790 = (2+1)(5+1)(179+1), 1958 = (2+1)(11+1)(89+1)

Therefore, 1614 is the smallest **bold** number.

Let N be a composite number such that N = $(x)^a(y)^b(z)^c$.. where x, y, z.. are prime factors. Then, the product of divisors of N = $(n)^{\frac{(a+1)(b+1)(c+1)}{2}} = (x^a y^b z^c)^{\frac{(a+1)(b+1)(c+1)}{2}}$

What is the product of divisors of 60?

Answer: 60 = $2^2 \times 3 \times 5 \Rightarrow$ product of divisors of 60 = $(60)^{\frac{3 \times 2 \times 2}{2}} = 60^6 = 2^{12} \times 3^6 \times 5^6$

32. Let A = set of all divisors of 8100 and B = set of all divisors of 21600. What is the product of the elements of AUB?

Answer: 8100 = $2^2 \times 3^4 \times 5^2$ and 21600 = $2^5 \times 3^3 \times 5^2$. AUB will have all the divisors of 8100 and 21600 with the common divisors written only once. Therefore, these common divisors will be multiplied only once. The common divisors will come from $2^2 \times 3^3 \times 5^2$ and are 36 in number. Their product will be $(2^2 \times 3^3 \times 5^2)^{18} = 2^{36} \times 3^{54} \times 5^{36}$

Required product=

$$\frac{product\ of\ divisors\ of\ 8100\ product\ of\ divisors\ of\ 21600}{\text{product of common divisors 36 54 36 235}} = \frac{(2^2\ x\ 3^4\ x\ 5^2)^{\frac{45}{2}}\ x\ (2^5\ x\ 3^3\ x\ 5^2)^{36}}{2^{36}\ x\ 3^{54}\ x\ 5^{36}} = 2^{189} \text{ x } 3^{144} \text{ x } 5^{81}$$

Let N be a composite number such that N = $(x)^a(y)^b(z)^c$.. where x, y, z.. are prime factors.
If N is not a perfect square, then, the number of ways N can be written as a product of two numbers $= \frac{(a+1)(b+1)(c+1)}{2} = \frac{Number\ of\ divisors}{2}$

If N is a perfect square, then, the number of ways N can be written as a product of two numbers $= \frac{(a+1)(b+1)(c+1)+1}{2} = \frac{Number\ of\ divisors+1}{2}$

REMEMBER! A perfect square has odd number of factors. In other words, any number which has odd number of factors is a perfect square.

For example, the divisors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60. Now,
60 = 1 × 60 = 2 × 30 = 3 × 20 = 4 × 15 = 5 × 12 = 6 × 10. Therefore, divisors occur in pairs for numbers which are not perfect squares.

The divisors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.
36 = 1 × 36 = 2 × 18 = 3 × 12 = 4 × 9 = 6 × 6. Therefore, divisors occur in pairs except for the square root for numbers which are perfect squares.

N is a composite number with an even number of factors. Consider the following statement
I: N has a factor lying between 1 and $\sqrt{N}$
II: N has a factor lying between $\sqrt{N}$ and N
Which of the following options is true?

A. Both I and II are true
B. I is true but II is false
C. I is false but II is true
D. Both I and II are false

Answer: Since N is a composite number, it has more than two factors. Since N has even number of factors, it is NOT a perfect square and therefore it has at least one factor lying between 1 and $\sqrt{N}$ and one factor lying between N and$\sqrt{N}$. Therefore, option [A].

How many ordered pairs of integers, (x, y) satisfy the equation xy = 110?
Answer: 110 = 2 × 5 × 11. Hence, the number of divisors of 110 is = 2 × 2 × 2 = 8. Hence, the number of positive ordered pairs of x and y = 8 (as (2, 55) is not same as (55, 2)). Also, since we are asked for integers, the pair consisting of two negative integers will also suffice. Hence the total number of ordered pairs = 2 × 8 = 16.

The number of ways in which a composite number can be resolved into two factors which are prime to each other = 2n – 1, where n is the number of different prime factors of the number.

For example, let the number N = $2^{10} \times 3^7 \times 5^6 \times 7^4$. We have to assign these prime factors and their powers to one of the two factors. As the two factors will be prime to each other, we will have to assign a prime factor with its power (for example 210) completely to one of the factors. For every prime factor, we have two ways of assigning it. Therefore, the total number of ways = 2 × 2 × 2 × 2 = 16. As we are not looking for ordered pairs, the required number of ways = $\frac{16}{2}$ = 8.

Number of numbers less than or prime to a given number:

If N is a natural number such that N = $a^p \times b^q \times c^r$, where a, b, c are different prime factors and p, q, r are positive integers, then the number of positive integers less than and prime to N = $\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$. Therefore,

$N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$ number have no factor in common with N.

Unit's Digit

33. To find the units digit of x^y we only consider the units digits of the number x.
To calculate units digit of 237234 we only consider the units digit of 237. Hence, we find the units digit of 7^{234}.
To find the units digit of a × b, we only consider the units digits of the numbers a and b.
To calculate units digit of 233 × 254, we only consider the units digit of 233 and 254 i.e. 3 and 4, respectively. Hence, we find the units digit of 3 x 4, respectively.

To calculate units digit of x^y where x is a single digit number

To calculate units digit of numbers in the form x^y such 7^{253}, 8^{93}, 3^{74} etc.
Case 1: When y is NOT a multiple of 4
We find the remainder when y is divided by 4. Let y = 4q + r where r is the remainder when y is divided by 4, and 0 < r < 4. The units digit of x^y is the units digit of x^r

Case 2: When y is a multiple of 4
We observe the following conditions:
Even numbers 2, 4, 6, 8 when raised to powers which are multiple of 4 give the units digit as 6.
Odd numbers 3, 7, and 9 when raised to powers which are multiple of 4 give the units digit as 1.

34. Find the units digit of 733.
Answer: The remainder when 33 is divided by 4 is 1. Hence the units digit of 7^{33} is the unit digit of 7^1 =7

35. Find the units digit of 43^{47}.
Answer: The units digit of 43^{47} can be found by finding the units digit of 3^{47}. 47 gives a remainder of 3 when divided by 4. Hence units digit = units digit of 3^3 = 7

36. Find the units digit of $28^{28} - 24^{24}$.
Answer: We have to find the units digit of $8^{28} - 4^{24}$. Since 28 and 24 are both multiples of 4, the units digits of both 8^{28} and 4^{24} will be 6. Hence the units digit of the difference will be 0.

37. Find the units digit of $43^{43} - 22^{22}$.
Answer: Units digit of 43^{43} is 7 and units digit of 22^{22} is 4. Hence the units digit of the expression will be 7 – 4 = 3.

38. Find the units digit of 3^{3^3}
Answer: Again, we find the remainder when the power is divided by 4. Therefore, we find the remainder when 3^3 is divided by 4. Now, 3^3 = 27, remainder by 4 = 3.
Therefore, units digit of 3^{3^3} = units digit of 3^3 = 7.

39. Find the units digit of $7^{11^{13^{17}}}$.
Answer: Again, we find the remainder when the power is divided by 4. Therefore, we find the remainder when $11^{13^{17}}$ is divided by 4. Now 11 = 12 − 1 ⇒ Remainder $[11^{\text{Odd}}]$ = Remainder$[(-1)^{\text{Odd}}] = -1 = 3$.
Therefore, units digit of $7^{11^{13^{17}}}$ = units digit of 7^3 = 3.

40. Find the units digit of $1^3 + 2^3 + 3^3 + \ldots + 98^3 + 99^3$
Answer: Unit digit of $1^3, 2^3, 3^3, 4^3, 5^3, 6^3, 7^3, 8^3, 9^3$ are 1, 8, 7, 4, 5, 6, 3, 2, and 9, respectively. The sum of these units digits gives a unit digit of 5. Now these units digit will repeat 10 times each. Therefore, units digit of the sum = 5 × 10 = 0.

Last Two Digits

Before we start, let me mention binomial theorem in brief as we will need it for our calculations.

$$(x+a)^n = {}^nC_0a^n + {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2 + \cdots where\ {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

Last two digits of numbers ending in 1
Let's start with an example.

What are the last two digit of 31^{786}?
Solution: $31^{786} = (30+1)^{786} = {}^{786}C_0 \times 1^{786} + {}^{786}C_1 \times 1^{785} \times (30) + {}^{786}C_2 \times 1^{784} \times 30^2 + \cdots$, Note that all the terms after the second term will end in two or more zeroes. The first two terms are ${}^{786}C_0 \times 1^{786}$ and ${}^{786}C_1 \times 1^{785} \times (30)$. Now, the second term will end with one zero and the tens digit of the second term will be the product of 786 and 3 i.e., 8. Therefore, the last two digit of the second term will be 80. The last digit of the first term is 1. So the last two digits of 31^{786} are 81.

Now, here is the shortcut:

Multiply the tens digit of the number (3 here) with the last digit of the exponent (6 here) to get the tens digit. The units digit is equal to one.

Here are some more examples:

Find the last two digits of 41^{2789}
In no time at all you can calculate the answer to be $61(4 \times 9 = 36$. Therefore, 6 will be the tens digit and one will be the units digit)

Find the last two digit of 71^{56747}
Last two digits will be 91 $(7 \times 7\ gives\ 9\ and\ 1\ as\ units\ digit)$

Now try to get the answer to this question within 10s:

Find the last two digits of $51^{456} \times 61^{567}$
The last two digits of 51^{456} will be 01 and the last two digits of 61^{567} will be 21. Therefore, the last two digits of $51^{456} \times 61^{567}$ will be the last two digits of $01 \times 21 = 21$

Last two digits of numbers ending in 3, 7 or 9

Find the last two digits of 19^{266}.
$19^{266} = (19^2)^{133}$. Now, 19^2 ends in $61(19^2 = 361)$ therefore, we need to find the last two digits of $(61)^{133}$.
Once the number is ending in 1 we can straight away get the last two digits with the help of the previous method. The last two digits are $81(6 \times 3 = 18,\ so\ the\ tens\ digit\ will\ be\ 8\ and\ last\ digit\ will\ be\ 1)$

Find the last two digits of 33^{288}.
$33^{288} = (33^4)^{72}$. Now 33^4 ends in $21(33^4 = 33^2 \times 33^2 = 1089 \times 1089 =$ xxxxx21) therefore, we need to find the last two digits of 21^{72}. By the previous method, the last two digits of $21^{72} = 41$ $(tens\ digit = 2 \times 2 = 4, unit\ digit = 1)$

So here's the rule for finding the last two digits of numbers ending in 3, 7 and 9:

Convert the number till the number gives 1 as the last digit and then find the last two digits according to the previous method.

Now try the method with a number ending in 7:

Find the last two digits of 87^{474}.

$87^{474} = 87^{472} \times 87^2 = (87^4)^{118} \times 87^2 = (69 \times 69)^{118} \times 69$ (The last two digits of 87^2 are 69) $= 61^{118} \times 69 = 81 \times 69 = 89$

If you understood the method then try your hands on these questions:

Find the last two digits of:
1. 27^{456}
2. 79^{83}
3. 583^{512}

Last two digits of number ending in 2, 4, 6 or 8

There is only one even two-digit number which always ends in itself (last two digits) – 76 i.e. 76 raised to any power gives the last two digits as 76. Therefore, our purpose is to get 76 as last two digits for even numbers. We know that 24^2 ends in 76 and 2^{10} ends in 24. Also, 24 raised to an even power always ends with 76 and 24 raised to an odd power always ends with 24. Therefore, 24^{34} will end in 76 and 24^{53} will end in 24.

Let's apply this funda:

Find the last two digits of 2^{543}.
$2^{543} = (2^{10})^{54} \times 2^3 = (24)^{54}\ (24\ \textit{raised to an even power}) \times 2^3 = 76 \times 8 = 80$

(Note: Here if you need to multiply 76 with 2^n, then you can straightaway write the last two digits of 2^n because when 76 is multiplied with 2^n the last two digits remain the same as the last two digits of 2^n. Therefore, the last two digits of 76×2^7 will be the last two digits of $2^7 = 28$. Note that this fund works only for powers of 2 ≥ 2)

Same method we can use for any number which is of the form 2^n. Here is an example:

Find the last two digits of 64^{236}
$64^{236} = (2^6)^{236} = 2^{1416} = (2^{10})^{141} \times 2^6 = 24^{141}$ (24 raised to odd power) $\times 64 = 24 \times 64 = 36$

Now those numbers which are not in the form of 2n can be broken down into the form $2n \times \textit{odd number}$. We can find the last two digits of both the parts separately.

Here are some examples:

Find the last two digits of 62^{586}.
$62^{586} = (2 \times 31)^{586} = 2^{586} \times 3^{586} = (2^{10})^{58} \times 2^6 \times 31^{586} = 76 \times 64 \times 81 = 84$

Find the last two digits of 54^{380}.
$54^{380} = (2 \times 3^3)^{380} = 2^{380} \times 3^{1140} = (2^{10})^{38} \times (3^4)^{285} = 76 \times 81^{285} = 76 \times 01 = 76.$

Find the last two digits of 56^{283}.
$56^{283} = (2^3 \times 7)^{283} = 2^{849} \times 7^{283} = (2^{10})^{84} \times 2^9 \times (7^4)^{70} \times 7^3 = 76 \times 12 \times (01)^{70} \times 43 = 16$

Find the last two digits of 78^{379}.
$78^{379} = (2 \times 39)^{379} = 2^{379} \times 39^{379} = (2^{10})^{37} \times 2^9 \times (39^2)^{189} \times 39 = 24 \times 12 \times 81 \times 39 = 92$

Power of a number contained in a factorial

Highest power of prime number p in $n! = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \left[\frac{n}{p^4}\right] + \cdots$ *where* $[x]$ denotes the greatest integer less than or equal to x.

Find the highest power of 2 in 50!
The highest power of 2 in 50! $= \left[\frac{50}{2}\right] + \left[\frac{50}{4}\right] + \left[\frac{50}{8}\right] + \left[\frac{50}{16}\right] + \left[\frac{50}{32}\right] = 25 + 12 + 6 + 3 + 1 = 47$

Find the highest power of 30! In 50!
$30 = 2 \times 3 \times 5$. Now 5 is the largest prime factor of 30, therefore, the powers of 5 in 50! Will be less than those of 2 and 3. Therefore, there cannot be more 30s than there are 5 in 50! So we find the highest power of 5 in 50! The highest power of 5 in 50! $= \left[\frac{50}{5}\right] - \left[\frac{50}{25}\right] = 10 + 2 = 12$. Hence the highest power of 30 in 50! = 12

Find the number of zeroes present at the end of 100!
We get a zero at the end of a number when we multiply that number by 10. So, to calculate the number of zeroes at the end of 100!, we have to find the highest power of 10 present in the number. Since $10 = 2 \times 5$, we have to find the highest power of 5 in 100! The highest power of 5 in 100! $= \left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] = 20 + 4 = 24$
Therefore, the number of zeroes at the end of 100! = 24

What is the rightmost non-zero digit in 15!?
Answer: We saw that $15! = 2^{11} \times 3^6 \times 5^3 \times 7^2 \times 11 \times 13$. Now $2^3 \times 5^3$ will give 10^3 *or* 3 zeroes at the end. Removing $2^3 \times 5^3$, we will be left with $2^8 \times 3^6 \times 7^2 \times 11 \times 13$. Calculating units digit of each prime factor separately, the units digit of the product $2^8 \times 3^6 \times 7^2 \times 11 \times 13$ = units digit of $6 \times 9 \times 9 \times 1 \times 3 = 8$.
Therefore, rightmost non-zero digit = 8

To find the powers of p^a in n! where p is a prime number and a is a natural number.
Highest power of prime number p^a in n! $= \left[\frac{\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\left[\frac{n}{p^4}\right]+\cdots}{a}\right]$ where p is a prime number, a is a natural number and [x] denotes the greatest integer less than or equal to x.

Find the highest power of 72 in 100!
$72 = 8 \times 9$. Therefore, we need to find the highest power of 8 and 9 in 72!.
$8 = 2^3 \Rightarrow$ highest power of 8 in 100! $= \left[\frac{\left[\frac{100}{2}\right]+\left[\frac{100}{4}\right]+\left[\frac{100}{8}\right]+\left[\frac{100}{16}\right]+\left[\frac{100}{32}\right]+\left[\frac{100}{64}\right]}{3}\right] = 32$
$9 = 3^2 \Rightarrow$ highest power 9 in 100! $= \left[\frac{\left[\frac{100}{3}\right]+\left[\frac{100}{9}\right]+\left[\frac{100}{27}\right]+\left[\frac{100}{81}\right]}{2}\right] = 24$
As power of 9 are less, powers of 72 in 100! = 24

Divisibility

Divisibility by 2, 4, 8, 16, 32..
A number is divisible by 2, 4, 8, 16, 32,.. 2^n when the number formed by the last one, two, three, four, five… n digits is divisible by 2, 4, 8, 16, 32,.. 2^n respectively.
Example: 1246384 is divisible by 8 because the number formed by the last three digits i.e., 384 is divisible by 8. The number 89764 is divisible by 4 because the number formed by the last two digits, 64 is divisible by 4.

A 101 digit number is formed by writing first 55 natural numbers next to each other. Find the remainder when the number is divided by 16.
Answer: to find remainder by 16 we only divide the number formed by the last 4 digits by 16. The last 4 digits would be 5455⇒ remainder by 16 = 15.

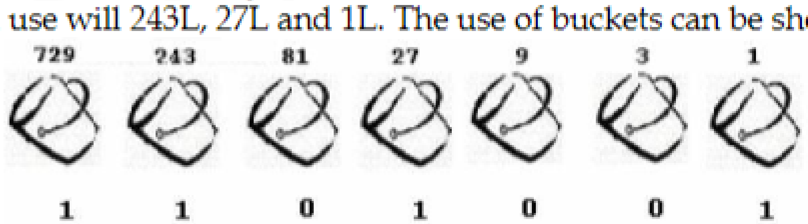
Divisibility by 3 and 9
A number is divisible by 3 or 9 when the sum of the digits of the number is divisible by 3 or 9 respectively.
Example: 313644 is divisible by 3 because the sum of the digits:- $3 + 1 + 3 + 6 + 4 + 4 = 21$ is divisible by 3.
The number 212364 is divisible by 9 because the sum of the digit:- $2 + 1 + 2 + 3 + 6 + 4 = 18$ is divisible by 9.

The six – digit number 73A998 is divisible by 6. How many values of A are possible?
Answer: Since the number is ending in an even digit, the number is divisible by 2. To find divisibility by 3, we need to consider sum of the digits of the number. The sum of the digits $= 7 + 3 + A + 9 + 9 + 8 = 36 + A$.
For the number to be divisible by 3, the sum of the digits should be divisible by 3. Hence A can take values equal to 0, 3, 6, and 9. Therefore, number of values possible = 4

Divisibily by 6, 12, 14, 15, 18..
Whenever we have to check the divisibility of number N by a composite number C, the number N should be divisible by all the prime factors (highest power of every prime factor) present in C.
Divisibility by 6: the number should be divisible by both 2 and 3.
Divisibility by 12: the number should be divisible by both 3 and 4.
Divisibility by 14: the number should be divisible by both 2 and 7.
Divisibility by 15: the number should be divisible by both 3 and 5.
Divisibility by 18: the number should be divisible by both 2 and 9.

Base System
Suppose you have a 1000 L tank to be filled with water. The buckets that are available to you all have sizes that are powers of 3, i.e., 1, 3, 9, 27, 81, 243, and 729 L. Which buckets do you use to fill the tank in the minimum possible time?

You will certainly tell me that the first bucket you will use is of 729L. That will leave 271 L of the tank still empty. The next few buckets you will use will 243L, 27L and 1L. The use of buckets can be shown as below.



We can say that $1000 = 729 + 243 + 27 + 1$
$= 1 \times 3^6 + 1 \times 3^5 + 0 \times 3^4 + 1 \times 3^3 + 0 \times 3^2 + 0 \times 3^1 + 0 \times 3^0$.

The number 1000 has been written in increasing powers of 3. Therefore, 3 is known as the 'base' in which we are expressing 1000.
For example, The number 7368 can be written as $8 + 6 \times 10 + 3 \times (10)^2 + 7 \times (10)^3$.
The number 10 is called the 'base' in which this number was written.

Let a number abcde be written in base p, where a, b, c, d and e are single digits less than p. The value of the number abcde $= e + d \times p + c \times p^2 + b \times p^3 + a \times p^4$

For example, if the number 7368 is written in base 9,
The value of $(7368)_9 = 8 + 6 \times 9 + 3 \times 9^2 + 7 \times 9^3 = 5408$ (this value is in base 10).

There are two kinds of operations associated with conversion of bases:

Conversion from any base to base ten
The number $(pqrstu)_b$ is converted to base 10 by finding the value of the number. i.e., $(pqrstu)_b = u + tb + sb^2 + rb^3 + qb^4 + pb^5$.

Convert $(21344)_5$ to base 10.
Answer: $(21344)_5 = 4 + 4 \times 5 + 3 \times 25 + 1 \times 125 + 2 \times 625 = 1474$

Conversion from base 10 to any base
A number written in base 10 can be converted to any base 'b' by first dividing the number by 'b', and then successively dividing the quotients by 'b'. The remainders, written in reverse order, give the equivalent number in base 'b'.

Write the number 25 in base 4.

$$\begin{array}{r|ll} 4 & 25 & \\ \hline & 6 & 1 \\ \hline & 1 & 2 \\ \hline & 0 & 1 \end{array}$$

Writing the remainders in reverse order the number 25 in base 10 is the number 121 in base 4.

Addition, subtraction and multiplication in bases:
Add the numbers $(4235)_7$ *and* $(2354)_7$
Answers: The numbers are written as

$$\begin{array}{cccc} 4 & 2 & 3 & 5 \\ 2 & 3 & 5 & 4 \end{array}$$

The addition of 5 and 4 (at the units place) is 9, which being more than 7 would be written as $9 = 7 \times 1 + 2$. The Quotient is 1 and written is 2. The Remainder is placed at the units place of the answer and the Quotient gets carried over to the ten's place. We obtain

$$\begin{array}{cccc} & +1 & +1 & \\ 4 & 2 & 3 & 5 \\ \underline{2} & \underline{3} & \underline{5} & \underline{4} \\ \underline{6} & \underline{6} & \underline{2} & \underline{2} \end{array}$$

At the tens place: 3 + 5 + 1 (carry) = 9
Similar procedure is to be followed when multiply numbers in the same base

Multiply $(43)_8 \times (67)_8$
Answer:
$7 \times 3 = 21 = 8 \times 2 + 5 \Rightarrow$ we write 5 and carry $2 \times base$ (8)
$7 \times 4 + 2\ (carry) = 30 = 8 \times 3 + 6$ we write 6 and carry $3 \times base$ (8)
$6 \times 3 = 18 = 8 \times 2 + 2 \Rightarrow$ we write 2 and carry $2 \times base(8)$
$6 \times 4 + 2(carry) = 26 = 8 \times 3 + 2 \Rightarrow$ we write 2 and carry $3 \times base$ (8)

$$\begin{array}{l} (4 \quad 3)_8 \\ \underline{(6 \quad 7)_8} \\ \quad 365 \\ \underline{322} \\ \underline{3605} \end{array}$$

For subtraction the procedure is same for any ordinary subtraction in base 10 except for the fact that whenever we need to carry to the right we carry the value equal to the base.

Subtract 45026 from 51231 in base 7.
Answer:

$$\begin{array}{rrrrrr} & 5 & 1 & 2 & 3 & 1 \\ - & 4 & 5 & 0 & 2 & 6 \\ \hline & & 3 & 2 & 0 & 2 \\ \hline \end{array}$$

In the units column since 1 is smaller than 6, we carry the value equal to the base from the number on the left. Since the base is 7 we carry 7. Now, $1 + 7 = 8$ *and* $8 - 6 = 2$. Hence we write 2 in the units column. We proceed the same way in the rest of the columns.

Important rules about bases
A number in base N is divisible by N – 1 when the sum of the digits of the number in base N is divisible by N – 1.

When the digits of a k – digit number N_1, written in base N are rearranged in any order to form a new k – digit number N_2, the difference $N_1 - N_2$ is divisible by N – 1.

If a number has even number of digits in base N, the number is divisible by base $N + 1$ if the digits equidistant from each end are the same, i.e., the number is a palindrome.

The number 35A246772 is in base 9. This number is divisible by 8. Find the value of digit A.
Answer: The number will be divisible by 8 when the sum of the digits is divisible by 8.
Sum of digits = $3 + 5 + A + 2 + 4 + 6 + 7 + 7 + 2 = 36 + A$. The sum will be divisible by 8 when A = 4.

A four – digit number N_1 is written in base 13. A new four – digit number N_2 is formed by rearranging the digit of N_1 in any order. Then the difference $N_1 - N_2$ is divisible by

(a) 9 (b) 10 (c) 12 (d) 13

Answer: The difference is divisible by $13 - 1 = 12$.

In what base is the equation $53 \times 15 = 732$ *valid*?
Answer: Let the base be b. $(53)_b = 3 + 5b, (15)_b = b + 5, (732)_b = 7b^2 + 3b + 2$
$\Rightarrow (3 + 5b) \times (b + 5) = 7b^2 + 3b + 2 \Rightarrow b = 13.$

A positive whole number M less than 100 is represented in base 2 notation, base 3 notation, and base 5 notation. It is found that in all three cases the last digit is 1, while in exactly two out of the three cases the leading digit is 1. Then M equals.
Answer: Whenever we change a number from base 10 to any other base, the units digit is the first remainder when the number is divided by that base. Therefore, M when divided by 2, 3 and 5 gives remainder 1 in each case. LCM of 2, 3 and 5 is 30. Therefore, $M = 30k + 1 = 31, 61$ *and* 91. Out of these 3 numbers, only the number 91 satisfies the second criterion of leading digit (last remainder).

A palindromic number reads the same forward and backward. A 10 – digit palindromic number in base 16 will always be divisible by
Answer: If a number has even number of digits in base N, the number is divisible by base N + 1 if the digits equidistant from each end are the same. Therefore, the number will be divisible by 16 +1 = 17.

If n is a natural number, find the possible terminating digits of $n^2 + n$ in base 5.
Answer: For a natural number n, $n^2 + n$ will always end in either 0, or 2 or 6. Therefore, when written in base 5, the first remainders will be 0, 1 or 2. Therefore, the units digit of $n^2 + n$ written in base 5 will be 0, 1 or 2.

Solve in base 7, the pair of equations $2x - 4y = 33$ *and* $3x + y = 31$, where x, y and the coefficients are in base 7.
Answer: Working in base 7:
$2x - 4y = 33 - - - (1)$
$3x + y = 31 - - - (2)$
Multiply (2) by 4, nothing that $3 \times 4 = 15$ in base 7 and $31 \times 4 = 154$ in base 7. We obtain

$15x + 4y = 154 - - - (3)$
Adding (1) and (3) we get $20x = 220 \Rightarrow x = 11 \Rightarrow y = -2$.

My ABN AMRO ATM Pin is a four-digit number. My HDFC BANK ATM Pin is also a four-digit number using the same digits, in a different order, as those in my ABN AMRO Pin. When I subtract the two number, I get a four-digit number whose first three digits are 2, 3 and 9. What is the unit digit of the difference?
Answer: When the digits of a k-digit number N_1, written in base N are rearranged in any order to form a new k-digit number N_2, the difference $N_1 - N_2$ is divisible by $N - 1$. Therefore, the difference of the two numbers would be divisible by 9. Hence, the sum of the digits of the difference should be divisible by 9.
Therefore, unit digit = 4.

A number N written in base b is represented as a two - digit number A2, where A = b - 2. What would N be represented as when written in base b - 1?
Answer: $N = (A2)_b = 2 + A \times b = 2 + (b-2)b = b^2 - 2b + 2 = (b-1)^2 + 1 = (101)_{b-1.}$

HCF and LCM

What is highest common factor (HCF) and least common multiple (LCM)? How do you calculate HCF and LCM of two or more numbers? Are you looking for problems on HCF and LCM? This chapter will answer all these questions.

Highest Common Factor (HCF)
The largest number that divides two or more given numbers is called the highest common factor (HCF) of those numbers. There are two methods to find HCF of the given numbers:

Prime Factorization Method:- When a number is written as the product of prime numbers, the factorization is called the prime factorization of that number. For example, $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

To find the HCF of given numbers by this method, we perform the prime factorization of all the numbers and then check for the common prime factors. For every prime factor common to all the numbers, we choose the least index of that prime factor among the given number. The HCF is product of all such prime factors with their respective least indices.

Find the HCF of 72, 288, and 1080
Answer: $72 = 2^3 \times 3^2, 288 = 2^5 \times 3^2, 1080 = 2^3 \times 3^3 \times 5$
The prime factors common to all the numbers are 2 and 3. The lowest indices of 2 and 3 in the given numbers are 3 and 2 respectively.
Hence, HCF = $2^3 \times 3^2 = 72$.

Find the HCF of $36x^2y^2$ *and* $24x^4y$.
Answer: $36x^3y^2 = 2^2 \cdot 3^2 \cdot x^3 \cdot y^2, 24x^4y = 2^3 \cdot 3 \cdot x^4 \cdot y$. The least index of 2, 3, x and y in the numbers are 2, 1, 3 and 1 respectively. Hence the HCF = $2^2 \cdot 3 \cdot x^2 \cdot y = 12x^2y$.

Division method:- To find HCF of two numbers by division method, we divide the higher number by the lower number. Then we divide the lower number by the first remainder, the first remainder by the second remainder… and so on, till the remainder is 0. The last divisor is the required HCF.

Find the HCF of 288 and 1080 by the division method.
Answer:

```
288|1080|3
     864
    216|288|1
        216
         72|216|3
            216
              0
```

Hence, the last divisor 72 is the HCF of 288 and 1080.

Exercise – 01
Unit Digit

1. If x is an even number, then x^{4n}, where n is a positive integer, will always have
(a) zero in the unit's place (b) 6 in the unit's place
(c) either 0 or 6 in the unit's place (d) None of these

2. Find the units digit of the expression $25^{6251} + 36^{528} + 73^{54}$.
(a) 4 (b) 0 (c) 6 (d) 5

3. Find *the unit digit of*
$13^{24} \times 68^{57} + 24^{13} \times 57^{68} + 1234 + 5678$.

4. The last digit of the following expression is:
$(1!)^1 + (2!)^2 + (3!)^3 + (4\,!)^4 + \ldots + (10!)^{10}$
(a) 4 (b) 5 (c) 6 (d) 7

5. Find the unit digit of $3^6 \times 4^7 \times 6^3 \times 7^4 \times 8^2 \times 9^5$.

6. Find the unit digit of $(12)^{78}$.

7. Find the unit digit of the product, of all the odd prime numbers.

8. In the product $459 \times 46 \times 28x \times 484$, the digit in the unit place is 8. The digit to come in place of x is
(a) 3 (b) 5 (c) 7 (d) None of these

9. The digit in the unit's place of $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - 16^4 + 259]$ is
(a) 1 (b) 4 (c) 5 (d) 6

10. Find *the unit digit of the product of all the elements of the set which consists all the prime numbers greater than 2 but less than 222.*

11. Find the last digit of the expression :
$1^2 + 2^2 + 3^2 + 4^2 + \cdots + 100^2$.

12. Find the unit digit of $123 + 345 + 780 + 65 + 44$.?

13. The last digit in the decimal representation of $\left(\frac{1}{5}\right)^{2000}$ is
(a) 2 (b) 4 (c) 5 (d) 6

14. $432 \times 532 + 532 \times 974 + 537 \times 531 + 947 \times 997$
(a) 5 (b) 6 (c) 9 (d) 8

15. $82^{43} \times 83^{44} \times 84^{97} \times 86^{98} \times 87^{105} \times 88^{94}$
(a) 2 (b) 6 (c) 4 (d) 8

16. The last digit of the number obtained by multiplying the numbers $81 \times 82 \times 83 \times 84 \times 85 \times 86 \times 87 \times 88 \times 89$ will be
(a) 0 (b) 9 (c) 7 (d) 2

17. The last 5 digits of the following expression will be
$(1!)^5 + (2!)^4 + (3!)^3 + (4!)^2 + (5!)^1 + (10!)^5 + (100!)^4 + (1000!)^3 + (10000!)^2 + (100000!)$
(a) 45939 (b) 00929 (c) 20929 (d) can't be determined

18. $67 \times 37 \times 43 \times 91 \times 42 \times 33 \times 42$
(a) 2 (b) 6 (c) 8 (d) 4

19. $12^{55}/3^{11} + 8^{48}/16^{18}$ will give the digit at-units place as
(a) 4 (b) 6 (c) 8 (d) 0

20. Unit's digit in $(784)^{126} + (784)^{127}$ is
(a) 0 (b) 4 (c) 6 (d) 8

21. The digit in the unit's place of the product $(2464)^{1793} \times (615)^{317} \times (131)^{491}$ is
(a) 0 (b) 2 (c) 3 (d) 5

22. Find the last digit of the number $1^3 + 2^3 + 3^3 + 4^3 \ldots + 99^3$
(a) 0 (b) 1 (c) 2 (d) 5

23. Find the units digit of the expression $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$.
(a) 1 (b) 9 (c) 7 (d) 0

24. Find the unit of $676 \times 543 \times 19$.

25. Find the unit digit of $135 \times 361 \times 970$.

26. Find the unit digit of 2^{35}.

27. Find the unit digit of $(33)^{123}$.

28. Find the unit digit of $3^{47} + 7^{52}$.

29. Find *the unit digit of* 111! (factorial 111).

30. Find the unit digit of the product of all the prime number between 1 and $(11)^{11}$.

31. Find *the last* digit of $222^{888} + 888^{222}$.

32. Find the unit digit of $1^1 + 2^2 + 3^3 + \ldots 10^{10}$.

33. Find the unit digit of the expression
$888^{9235!} + 222^{9235!} + 666^{2359!} + 999^{9999!}$.

34. $67 \times 35 \times 43 \times 91 \times 47 \times 33 \times 49$
(a) 1 (b) 9 (c) 5 (d) 6

35. $67 \times 35 \times 45 + 91 \times 42 \times 33 \times 82$
(a) 8 (b) 7 (c) 0 (d) 5

36. $(52)^{97} \times (43)^{72}$
(a) 2 (b) 6 (c) 8 (d) 4

37. $(55)^{75} \times (93)^{175} \times (107)^{275}$
(a) 7 (b) 3 (c) 5 (d) 0

38. $(173)^{45} \times (152)^{77} \times (777)^{999}$
(a) 2 (b) 4 (c) 8 (d) 6

39. Find the last digit of the number $1^2 + 2^2 + ... + 99^2$.
(a) 0 (b) 1 (c) 2 (d) 3

40. Find the units digit of the expression $55^{725} + 73^{5810} + 22^{853}$.
(a) 4 (b) 0 (c) 6 (d) 5

41. Find the units digit of the expression $11'.12^2.13^3.\ 14^4.15^5.16^6$.
(a) 4 (b) 3 (c) 7 (d) 0

42. The unit digit in the expression $36^{234}*33^{512}*39^{180} - 54^{29}*25^{123}*31^{512}$ will be
(a) 8 (b) 0 (c) 6 (d) 5

43. The unit's digit of 13^{2003} is
(a) 1 (b) 3 (c) 7 (d) 9

44. The digit in the unit's place of the number 123" is
(a) 1 (b) 4 (c) 7 (d) 8

45. The digit in the unit's place of the number $(67)^{25} - 1$ must be
(a) 0 (b) 6 (c) 8 (d) None of these

46. The unit's digit in the product $274 \times 318 \times 577 \times 313$ is
(a) 2 (b) 3 (c) 4 (d) 5

47. The digit in the unit place of the number represented by $(7^{95} - 3^{58})$ is
(a) 0 (b) 4 (c) 6 (d) 7

48. If m and n are positive integers, then the digit in the unit's place of $5^n + 6^m$ is always
(a) 1 (b) 5 (c) 6 (d) $n + m$

Exercise – 01
Unit Digit
Solutions

Solutions

1. (c)
2. (b) The units digit would be given by $5 + 6 + 9$ (numbers ending in 5 and 6 would always end in 5 and 6 irrespective of the power and 3^{54} will give a units digit equivalent to 3^{4n+2} which would give us a unit digit of 3^2 i.e.9).

3. The unit digit of 3^{24} is 1
The unit digit of 8^{57} is 8
The unit digit of 4^{13} is 4
The unit digit of 7^{68} is 1
So the resultant value of the unit digits
$= 1 \times 8 + 4 \times 1 + 4 + 8$
$= 8 + 4 + 4 + 8 = 24$
Thus the unit digit of the whole expression is 4.

4. The unit digit of the given expression will be equal to the unit digit of the sum of the unit digits of every term of the expression.
Now,
The unit digit of $(1!)^2 = 1$
The unit digit of $(2!)^2 = 4$
The unit digit of $(3!)^3 = 6$
The unit digit of $(4!)^4 = 6$
The unit digit of $(5!)^5 = 0$
The unit digit of $(6!)^6 = 0$

Thus the last digit of the $(7!)^7$, $(8!)^8$, $(9!)^9$, $(10\,!)^{10}$ will be zero. So, the unit digit of the given expression = 7
$(\because 1 + 4 + 6 + 6 + 0 + 0 + 0 + 0 + 0 + 0 = 17)$

5. The unit digit of 3^6 is 9
The unit digit of 4^7 is 4
The unit digit of 6^3 is 6
The unit digit of 7^4 is 1
The unit digit of 8^2 is 4
The unit digit of 9^5 is 9
Therefore the unit digit of the given expression is 6, (since $9 \times 4 \times 6 \times 1 \times 4 \times 9 = 7776$

6. The unit digit of $(12)^{78}$ will be same as $(2)^{78}$. Now since we know that the cyclic period of unit digit of 2 is 4. The remainder when 78 is divided by 4 is 2. Hence the unit digit of 2^{78} will be same as 2^2 which is 4. Thus the unit digit of 12^{78} is 4.

7. The prime numbers are $3, 5, 7, 11, 13, 17, 19$...etc.
Now we know that if 5 is multiplied by any odd number it always gives the last digit 5. So the required unit digit will be 5.
8. (a)
9. (b)

10. The set of required prime number $= \{3, 5, 7, 11, \ldots\}$
Since there is no any even number in the set so when 5 will multiply with any odd number, it will always give 5 as the last digit. Hence the unit digit will be 5.

11. Solution.................. 0 (zero).

12. We can find the unit digit just by adding the unit digits 3, 5, 0, 5, 4 as
$3 + 5 + 0 + 5 + 4 = 17$
So the unit digit (or the last digit) of the resultant value of the expression $123 + 345 + 780 + 65 + 44$ will be 7. (you can verify it by doing the whole sum)

13. $\left(\frac{1}{5}\right)^{2000} = (0.2)^{2000}$.
Last digit of $(0.2)^{2000}$ = Last digit of $(0.2)^4 = 6$.

14. (d)
15. (b)
16. (a) The units digit in this case would be '0' because the given expression has a pair of 2 and 5 in it's prime factors.

17. (b) The last digit of $(1!)^5 = 1$
The last digit of $(2!)^4 = 16$
The last digit of $(3!)^3 = 216$
The last digit of $(4!)^2 = 576$
The last digit of $(5!)^1 = 120$
The last 5 digit of $(10!)^5 = 00000$
The last 5 digit of $(100!)^4 = 00000$
$(1000!)^3 = 00000$
$(10000!)^2 = 00000$
$(100000!)^1 = 00000$
Thus the last 5 digits of the given expression = 00929
$[\because 1 + 16 + 216 + 576 + 120 + 00000 + 00000 + 00000 + 00000 + 00000 = 00929]$

18. (d) The unit digit would be given by the units digit of the multiplication of $4 \times 6 \times 6 \times 6 = 4$
19. (d) $\frac{12^{55}}{3^{11}} = 3^{44}.4^{55} \rightarrow 4$ as units place.
Similarly, $8^{48}/16^{18} = 2^{72} \rightarrow 6$ as the unit place.
Hence, 0 is the answer.
20. (a)
21. (a)
22. (a)
23. (b) The respective units digits for the six parts of the expression would be:
$1 + 4 + 7 + 6 + 5 + 6 + = 29 \rightarrow$ required answer is 9. Option (b) is correct.

26. We can find the unit digit of the product of the given expression just by multiplying the unit digits (6, 3, 9) instead of doing the whole sum.
Thus $6 \times 3 \times 9 = 162$
Hence, the unit digit of the product of the given expression will be 2, (you can verify it by doing the complete sum)

27. The unit digit can be obtained by multiplying the unit digits 5, 1, 0. then $5 \times 1 \times 0 = 0$ thus the unit digit will be zero.

28. Answer.......... is 8.

29. Since we know that the unit digit of $(33)^{123}$ will be same as $(3)^{123}$. Now the unit-digit of 3^{123} will be 7 since it will be equal to the unit digit of 3^3.
Thus the unit digit of $(3)^{123}$ is 7.

30. The unit digit of the given expression will be equal to the unit digit of the sum of the unit digits of both the terms individually.
Now, unit digit of 3^{47} is 7 (since it will be equal to 3^3) and the unit digit of 7^{52} is 1 (since it will be equal to 7^4)
Thus the unit digit of $3^{47} + 7^{52}$ is $7 + 1 = 8$.

31. $111! == 1 \times 2 \times 3 \times 4 \times 5 \times ... \times 110 \times 111$
Since there is a product of 5 and 2 hence it will give zero as the unit digit.
Hence the unit digit of 111! is 0 (zero).

32. The set of prime number $S = \{2, 3, 5, 7, 11, 13, \ldots\}$
Since there is one 5 and one 2 which gives 10 after multiplying mutually, it means the unit digit will be zero.

33. The last digit of the expression will be same as the last digit of $2^{888} + 8^{222}$.
Now the last digit of 2^{888} is 6 and the last digit of the 8^{222} is 4.
Thus the last digit of $2^{888} + 8^{222}$ is 0 (zero), since $6 + 4 = 10$.

34. The unit digit of $1^1 = 1$
The unit digit of $2^2 = 4$
The unit digit of $3^3 = 7$
The unit digit of $4^4 = 6$
The unit digit of $5^5 = 5$
The unit digit of $6^6 = 6$
The unit digit of $7^7 = 3$
The unit digit of $8^8 = 6$
The unit digit of $9^9 = 9$
The unit digit of $10^{10} = 0$
Thus the unit digit of the given expression will be 7: ($\because 1 + 4 + 7 + 6 + 5 + 6 + 3 + 6 + 9 = 47$)

35. Answer : 9
First of all we find the unit digit individually of all the four terms,
So, the unit digit of $888^{9235!}$ is equal to the unit digit of $8^{9235!}$
Now, the unit digit of $8^{9235!}$ is equal to the unit of 8^4 (since 9235! is divisible by 4), which is 6.

36. (c)
37. (b)
38. (a)
39. (c)
40. (c)
41. (a)
42. (c) The respective units digit for the three parts of the expression would be:
$5 + 9 + 2 = 16 \rightarrow$ required answer is 6. Option (c) is correct.

43. (d) The respective unit digits for the six parts of the expression would be:
$1 \times 4 \times 7 \times 6 \times 5 \times 6 \rightarrow$ required answer is 0. Option (d) is correct.

44. (c) It can be seen that the first expression is larger than the second one. Hence, the required answer would be given by the (units digits of the first expression – units digit of the second expression) = 6 – 0 = 6. Option (c) is correct.

45. (c)
46. (c)
47. (b)
48. (a)
49. (b)
50. (a)

Exercise – 02
Last Two Digit – 01

1. Find the last two digits of 41^{2789}?

   ....

2. Find the last two digits of 71^{56747}?

   ....

3. Find the last two digits of $51^{456} \times 61^{567}$?

   ....

4. Find the last two digits of 19^{266}.

   ....

5. Find the last two digits of 33^{288}.

   ....

6. Find the last two digits of 87^{474}.?

   ....

7. Find the last two digits of 64^{236}.

   ....

8. Find the last two digits of 56^{283}.

   ....

9. Find the last two digits of $(51)^{2008}$

   ....

10. Find the last two digits of $(61)^{223}$

    ....

11. Find the last two digits of 7^{7214}

    ....

12. Find the last two digits of 7^{1215}

    ....

13. Find the last two digits of 2^{100}

    ....

14. Find the last two digits of 2^{110}

....

15. Find the last two digits of 2^{121}

....

16. Find the last two digits of 2^{317}

....

17. Find the last two digits of $(37)^{56}$

....

18. Find the last two digits of $7^{(81!)}$

....

19. Find the last two digits of $(99)^{200}$

....

20. Find the last two digits of the following numbers
$65 \times 29 \times 37 \times 63 \times 71 \times 87 \times 85$
(a) 25 (b) 35 (c) 75 (d) 85

21. Find the last two digits of the following numbers
$75 \times 35 \times 47 \times 63 \times 71 \times 87 \times 82$
(a) 50 (b) 70 (c) 30 (d) 90

22. Find the last two digits of $9! + 99! + 999! + 9999! + 99999!$

....

23. Find the last two digits of $5! + 55! + (555)! + (5555)!$

....

24. Find the last two digits of 5^{65}

....

Exercise – 02
Last Two Digit – 01
Solutions
Hints

1. In no time at all you can calculate the answer to be $61(4 \times 9 = 36$. Therefore, 6 will be the tens digit and one will be the units digit)

2. Last two digits will be $91(7 \times 7 \text{ gives } 9 \text{ and } 1 \text{ as units digit})$

3. The last two digits of 51^{546} will be 01 and the last two digits of 61^{567} will be 21. Therefore, the last two digits of $51^{456} \times 61^{567}$ will be the last two digits of $01 \times 21 = 21$.

4. $19^{266} = (19^2)^{133}$. Now, 19^2 ends in 61 ($19^2 = 361$) therefore, we need to find the last two digits of $(61)^{133}$.
Once the number is ending in 1 we can straight away get the last two digits with the help of the previous method. The last two digits are $81 (6 \times 3 = 18,$ so the tens digit will be 8 and last digit will be 1).

5. $33^{288} = (33^4)^{72}$. Now 33^4 ends in 21 ($33^4 = 33^2 \times 33^2 = 1089 \times 1089 = \times\times\times\times\times 21$) therefore, we need to find the last two digits of 21^{72}. By the previous method, the last two digits of $21^{72} = 41$ (tens digit $= 2 \times 2 = 4$, unit digit $= 1$)
So here's the rule for finding the last two digits of numbers ending in 3, 7 and 9:

6. $87^{474} = 87^{472} \times 87^2 = (87^4)^{118} \times 87^2 = (69 \times 69)^{118} \times 69$ (The last two digits of 87^2 are $69) = 61^{118} \times 69 = 81 \times 69 = 89$.

7. $64^{236} = (2^6)^{236} = 2^{1416} = (2^{10})^{141} \times 2^6 = 24^{141}$ (24 raised to odd power) $\times 64 = 24 \times 64 = 36$
Now those numbers which are not in the form of 2n can be broken down into the form $2n \times$ odd number. We can find the last two digits of both the parts separately.

8. $56^{283} = (2^3 \times 7)^{283} = 2^{849} \times 7^{283} = (2^{10})^{84} \times 2^9 \times (7^4)^{70} \times 7^3 = 76 \times 12 \times (01)^{70} \times 43 = 16$
9. 01
10. 81
11. 49
12. 43
13. 76
14. 24
15. 52
16. 72
17. 41
18. 01
19. 01
20. (c)
21. (a)
22. 80
23. 20
24. 25

Exercise – 03

Last Two Digit – 02

1. What are the last two digits of 31^{786}?

……………………………

2. Find the last two digits of 2^{543}.

……………………………

3. Find the last two digit of 54^{380}.

……………………………

4. Find the last two digits of 7^{11}

……………………………

5. Find the last two digits of 7^{216}

....

6. The last two digits in the multiplication of $35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 \cdot 30 \cdot 29 \cdot 28 \cdot 27 \cdot 26$ is
(a) 00 (b) 40 (c) 30 (d) 10

7. Find the last two digits of the following numbers
$65 \times 29 \times 37 \times 63 \times 71 \times 87$
(a) 05 (b) 95 (c) 15 (d) 25

8. Find the last two digits of the following numbers
$65 \times 29 \times 37 \times 63 \times 71 \times 87 \times 62$
(a) 70 (b) 30 (c) 10 (d) 90

9. Find the last two digits of the following numbers
$(201 \times 202 \times 203 \times 204 \times 246 \times 247 \times 248 \times 249)^2$
(a) 36 (b) 56 (c) 76 (d) 16

10. Find the last two digits of $1! + 11! + 111! + (1111)! + (11111)!$

....

11. Find the last two digits of $(15)^{25}$

....

12. Find the last two digits of 78^{379}.

....

13. Find the last two digits of $(81)^{111}$

....

14. Find the last two digits of $(91)^{41}$

....

15. Find the last two digits of $(43)^{183}$

....

16. Find the last two digits of $(39)^{44}$

....

17. Find the last two digits of $(63)^{24}$

....

18. Find the last two digits of the following numbers
$101 \times 102 \times 103 \times 197 \times 198 \times 199$
(a) 54 (b) 74 (c) 64 (d) 84

19. Find the last two digits of $(71)^{64}$

....

20. Find the last two digits of 62^{586}.

....

21. Find the last two digits of 2^{126}

....

22. Find the last two digits of 5^{766}

....

23. Find the last two digits of $(45)^{55}$

....

24. Find the last two digits of $(55)^{66}$

....

25. Find the last two digits of $(1!)^1 + (2!)^2 + (3!)^3 + (4!)^4 + (5!)^5 + (6!)^6 + (7!)^7 + (8!)^8 + (9!)^9 + (10!)^{10}$

....

Exercise – 03
Last Two Digit – 02
Solutions

1. Solution: $31^{786} = (30+1)^{786} = {}^{786}C_0 \times 1^{786} + {}^{786}C_1 \times 1^{785} \times (30) + {}^{786}C_2 \times 1^{784} \times 30^2 + \cdots$, Note that all terms after the second term will end in two or more zeroes. The first two terms are ${}^{786}C_0 \times 1^{786}$ and ${}^{786}C_1 \times 1^{785} \times (30)$. Now, the second term will end with one zero and the tens digit of the second term will be the product of 786 and 3 i.e. 8. Therefore, the last two digits of the second term will be 80. The last digit of the first term is 1. So the last two digits of 31^{786} are 81.
2. $2^{543} = (2^{10})^{54} \times 2^3 = (24)^{54}$ (24 raised to an even power) $\times 2^3 = 76 \times 8 = 08$
3. $54^{380} = (2 \times 3^3)^{380} = 2^{380} \times 3^{1140} = (2^{10})^{38} \times (3^4)^{285} = 76 \times 81^{285} = 76 \times 01 = 76$.
6. (a)
7. (b)
8. (d)
9. (c)
10. 01
11. 75
12. $78^{379} = (2 \times 39)^{379} = 2^{379} \times 39^{379} = (2^{10})^{84} \times 2^9 \times (39^2)^{189} \times 39 = 24 \times 12 \times 81 \times 39 = 92$.
13. 81
14. 91
15. 07
16. 41
17. 61
18. (c)
19. 81
20. $62^{586} = (2 \times 31)^{586} = 2^{586} \times 3^{586} = (2^{10})^{58} \times 2^6 \times 31^{586} = 76 \times 64 \times 81 = 84$.
21. 64
22. 25
23. 25
24. 25
25. 97

Exercise – 04

H.C.F. & L.C.M. – 01

Directions: *Mark (√) against the correct answer:*

1. Find the factors of 330.
(a) $2 \times 4 \times 5 \times 11$ (b) $2 \times 3 \times 7 \times 13$
(c) $2 \times 3 \times 5 \times 13$ (d) $2 \times 3 \times 5 \times 11$

2. Find the factors of 1122.
(a) $3 \times 9 \times 17 \times 2$ (b) $3 \times 11 \times 17 \times 2$
(c) $9 \times 9 \times 17 \times 2$ (d) $3 \times 11 \times 17 \times 3$

3. 252 can be expressed as a product of primes as
(a) $2 \times 2 \times 3 \times 3 \times 7$ (b) $2 \times 2 \times 2 \times 3 \times 7$
(c) $3 \times 3 \times 3 \times 3 \times 7$ (d) $2 \times 3 \times 3 \times 3 \times 7$

4. Which of the following has most number of divisors?
(a) 99 (b) 101 (c) 176 (d) 182

5. Reduce $\frac{128352}{238368}$ to its lowest terms.
(a) $\frac{3}{4}$ (b) $\frac{5}{13}$ (c) $\frac{7}{13}$ (d) $\frac{9}{13}$

6. The simplest reduction to the lowest terms of $\frac{116,690,151}{427,863,887}$ is
(a) $\frac{3}{11}$ (b) $\frac{7}{11}$ (c) $\frac{11}{3}$ (d) None of these

7. The highest common factor of 0 and 6 is
(a) 0 (b) 3 (c) 6 (d) Undefined

8. The H.C.F. of $2^2 \times 3^3 \times 5^5$, $2^3 \times 3^2 \times 5^2 \times 7$ and $2^4 \times 3^4 \times 5 \times 7^2 \times 11$ is
(a) $2^2 \times 3^2 \times 5$ (b) $2^2 \times 3^2 \times 5 \times 7 \times 11$
(c) $2^4 \times 3^4 \times 5^5$ (d) $2^4 \times 3^4 \times 5^5 \times 7 \times 11$

9. The H.C.F. of $2^4 \times 3^2 \times 5^3 \times 7$, $2^3 \times 3^3 \times 5^2 \times 7^2$ and $3 \times 5 \times 7 \times 11$ is
(a) 105 (b) 1155 (c) 2310 (d) 27720

10. H.C.F. of $4 \times 27 \times 3125$, $8 \times 9 \times 25 \times 7$ & $16 \times 81 \times 5 \times 11 \times 49$ is
(a) 180 (b) 360 (c) 540 (d) 1260

11. Find the highest common factor of 36 and 84.
(a) 4 (b) 6 (c) 12 (d) 18

12. Even numbers are formed by taking at least two at a time from the numbers 0, 4, 8, 9. Their H.C.F. is
(a) 2 (b) 4 (c) 10 (d) None of these

13. Which of the following is a pair of co-primes?
(a) (16, 62) (b) (18, 25) (c) (21, 35) (d) (23, 92)

14. The H.C.F. of 2923 and 3239 is
(a) 37 (b) 47 (c) 73 (d) 79

15. The L.C.M. of $2^3 \text{ x } 3^2 \text{ x } 5 \text{ x } 11$, $2^4 \text{ x } 3^4 \text{ x } 5^2 \text{ x } 7$ and $2^5 \text{ x } 3^3 \text{ x } 5^3 \text{ x } 7^2 \text{ x } 11$ is
(a) $2^3 \times 3^2 \times 5$ (b) $2^5 \times 3^4 \times 5^3$
(c) $2^3 \times 3^2 \times 5 \times 7 \times 11$ (d) $2^5 \times 3^4 \times 5^3 \times 7^2 \times 11$

16. Find the lowest common multiple of 24, 36 and 40.

(a) 120 (b) 240 (c) 360 (d) 480

17. The L.C.M. of 22, 54, 108, 135 and 198 is
(a) 330 (b) 1980 (c) 5940 (d) 11880

18. The L.C.M. of 148 and 185 is
(a) 680 (b) 740 (c) 2960 (d) 3700

19. The H.C.F. of $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$ is equal to
(a) $\frac{L.C.M. of\ a,c,e}{H.C.F\ of\ b,d,f}$ (b) $\frac{H.C.F. of\ a,c,e}{L.C.M. of\ b,d,f}$
(c) $\frac{H.C.F. of\ a,c,e}{H.C.F. of\ b,d,f}$ (d) $\frac{ace}{bdf}$

20. The H.C.F. of $\frac{2}{3}, \frac{8}{9}, \frac{64}{81}$ and $\frac{10}{27}$ is :
(a) $\frac{2}{3}$ (b) $\frac{2}{81}$ (c) $\frac{160}{3}$ (d) $\frac{160}{81}$

21. The H.C.F. of $\frac{9}{10}, \frac{12}{25}, \frac{18}{35}$ and $\frac{21}{40}$ is
(a) $\frac{3}{5}$ (b) $\frac{252}{5}$ (c) $\frac{3}{1400}$ (d) $\frac{63}{700}$

22. The L.C.M. of $\frac{1}{3}, \frac{5}{6}, \frac{2}{9}, \frac{4}{27}$ is
(a) $\frac{1}{54}$ (b) $\frac{10}{27}$ (c) $\frac{20}{3}$ (d) None of these

23. The L.C.M. of $\frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{9}{13}$ is
(a) 36 (b) $\frac{1}{36}$ (c) $\frac{1}{1365}$ (d) $\frac{12}{455}$

24. The G.C.D. of 1.08, 0.36 and 0.9 is
(a) 0.03 (b) 0.9 (c) 0.18 (d) 0.108

25. H.C.F. of 3240, 3600 and a third number is 36 and their L.C.M. is $2^4 \times 3^5 \times 5^2 \times 7^2$. The third number is
(a) $2^2 \times 3^5 \times 7^2$ (b) $2^2 \times 5^3 \times 7^2$ (c) $2^5 \times 5^2 \times 7^2$ (d) $2^3 \times 3^5 \times 7^2$

Exercise – 04

H.C.F. & L.C.M. – 01

Answers Key

1. d	*2. b*	*3. a*	*4. c*	*5. c*
6. a	*7. d*	*8. a*	*9. a*	*10. a*
11. c	*12. a*	*13. b*	*14. d*	*15. d*
16. c	*17. c*	*18. b*	*19. b*	*20. b*
21. c	*22. c*	*23. a*	*24. c*	*25. a*

7. Since division by 0 is undefined, so 0 cannot be a factor of any natural number. Hence, H.C.F. of 0 and 6 is undefined.

8. H.C.F = Product of lowest powers of common factors $= 2^2 \times 3^2 \times 5$.
10. $4 \times 27 \times 3125 = 2^2 \times 3^2 \times 5^5$;
$8 \times 9 \times 25 \times 7 = 2^3 \times 3^2 \times 5^2 \times 7$;
$16 \times 81 \times 5 \times 11 \times 49 = 2^4 \times 3^4 \times 5 \times 7^2 \times 11$
$\therefore \; H.C.F. = 2^2 \times 3^2 \times 5 = 180.$

20. Required H.C.F. $= \frac{H.C.F. of\ 2,8,64,10}{L.C.M.\ of\ 3,9,81,27} = \frac{2}{81}$.

21. Required H.C.F. $= \frac{H.C.F\ of\ 9,12,18,21}{L.C.M\ of\ 10,25,35,40} = \frac{3}{1400}$.

24. Given numbers are 1.08, 0.36 and 0.90 H.C.F. of 108, 36 and 90 is 18.
$\therefore$ H.C.F. of given numbers = 0.18.

Exercise – 05

H.C.F. & L.C.M. – 02

1. Three numbers are in the ratio 1: 2: 3 and their H.C.F. is 12. The numbers are
(a) 4, 8, 12 (b) 5, 10, 15 (c) 10, 20, 30 (d) 12, 24, 36

2. The ratio of two numbers is 3: 4 and their H.C.F. is 4. Their L.C.M. is
(a) 12 (b) 16 (c) 24 (d) 48

3. The sum of two numbers is 216 and their H.C.F. is 27. The numbers are
(a) 27, 189 (b) 81, 189 (c) 108, 108 (d) 154, 162

4. The sum of two numbers is 528 and their H.C.F. is 33. The number of pairs of numbers satisfying the above conditions is
(a) 4 (b) 6 (c) 8 (d) 12

5. The number of number-pairs lying between 40 and 100 with their H.C.F. as 15 is
(a) 3 (b) 4 (c) 5 (d) 6

6. The H.C.F. of two numbers is 12 and their difference is 12. The numbers are
(a) 66, 78 (b) 70, 82 (c) 94, 106 (d) 84, 96

7. The product of two numbers is 2028 and their H.CF. is 13. The number of such pairs is
(a) 1 (b) 2 (c) 3 (d) 4

8. Three numbers which are co-prime to each other are such that the product of the first two is 551 and that of the last two is 1073. The sum of the three numbers is
(a) 75 (b) 81 (c) 85 (d) 89

9. The ratio of two numbers is 13: 15 and their L.C.M. is 39780. The numbers are (P.C.S., 2009)
(a) 884, 1020 (b) 884, 1040 (c) 670, 1340 (d) 2652, 3060

10. The L.C.M. and ratio of four numbers are 630 and 2: 3: 5: 7 respectively. The difference between the greatest and least numbers is
(a) 6 (b) 14 (c) 15 (d) 21

11. The H.C.F. and L.C.M. of two numbers are 12 and 336 respectively. If one of the numbers is 84, the other is
(a) 36 (b) 48 (c) 72 (d) 96

12. If the product of two numbers is 324 and their H.C.F. is 3, then their L.C.M. will be
(a) 972 (b) 327 (c) 321 (d) 108

13. If H.C.F. of p and q is x and $q = xy$, then the L.C.M. of p and q is

(a) pq (b) qy (c) xy (d) py

14. The H.C.F. and L.C.M. of two numbers are 21 and 84 respectively. If the ratio of the two numbers is 1: 4, then the larger of the two numbers is
(a) 12 (b) 48 (c) 84 (d) 108

15. If the sum of two numbers is 36 and their H.C.F. and L.C.M. are 3 and 105 respectively, the sum of the reciprocals of the two numbers is
(a) $\frac{2}{35}$ (b) $\frac{3}{35}$ (c) $\frac{4}{35}$ (d) None of these

16. The L.C.M. of two numbers is 12 times their H.C.F. The sum of H.C.F. and L.C.M. is 403. If one number is 93, find the other.
(a) 124 (b) 128 (c) 134 (d) None of these

17. The L.C.M. of three different numbers is 120. Which of the following cannot be their H.C.F.?
(a) 8 (b) 12 (c) 24 (d) 35

18. The H.C.F. and L.C.M. of two numbers are 21 and 4641 respectively. If one of the numbers lies between 200 and 300, the two numbers are
(a) 273, 357 (b) 273, 359 (c) 273, 361 (d) 273, 363

19. 21 mango trees, 42 apple trees and 56 orange trees have to be planted in rows such that each row contains the same number of trees of one variety only. Minimum number of rows in which the trees may be planted is:
(a) 3 (b) 15 (c) 17 (d) 20

20. The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is:
(a) 15 cm (b) 25 cm (c) 35 cm (d) 42 cm

21. The capacity of two pots is 120 litres and 56 litres respectively. Find the capacity of a container which can exactly measure the contents of the two pots.
(a) 7500 cc (b) 7850 cc (c) 8000 cc (d) 9500 cc

22. A daily wage labourer was engaged for a certain number of days for `5750, but being absent on some of those days he was paid only `5000. What was his maximum possible daily wage?
(a) `125 (b) `250 (c) `375 (d) `500

23. A person has to completely put each of three liquids: 403 litres of petrol, 465 litres of diesel and 496 litres of Mobil Oil in bottles of equal size without mixing any of the above three types of liquids such that bottle is completely filled. What is the least possible number of bottles required?
(a) 34 (b) 44 (c) 46 (d) None of these

24. The least number of square tiles required to pave the ceiling of a room 15 m 17 cm long and 9 m 2 cm broad is
(a) 656 (b) 738 (c) 814 (d) 902

25. Three sets of English, Mathematics and Science books containing 336, 240 and 96 books respectively have to be stacked in such a way that all the books are stored subject wise and the height of each stack is the same. Total number of stacks will be
(a) 14 (b) 21 (c) 22 (d) 48

Exercise – 05

H.C.F. & L.C.M. – 02

Answers Key

1. d	2. d	3. a	4. a	5. b

6. d	*7. b*	*8. c*	*9. d*	*10. c*
11. b	*12. d*	*13. d*	*14. c*	*15. c*
16. a	*17. d*	*18. a*	*19. c*	*20. c*
21. c	*22. b*	*23. b*	*24. c*	*25. a*

1. Let the required numbers be $x, 2x$ and $3x$. Then, their H.C.F. $= x$. So, $x = 12$.
   ∴ The number are 12, 24 and 36.

2. Let the number be $3x$ and $4x$. Then, their H.C.F. $= x$. So, $x = 4$.
   So, the numbers are 12 and 16
   L.C.M. of 12 and 16 = 48.

8. Since the numbers are co-prime, they contain only 1 as the common factor.
   Also, the given two products have the middle number in common
   So, middle number = H.C.F. of 551 and 1073 = 29;
   First number $= \left(\frac{551}{29}\right) = 19;$
   Third number $= \left(\frac{1073}{29}\right) = 37.$
   ∴ $Required\ sum = (19 + 29 + 37) = 85$

12. $L.C.M. = \frac{324}{3} = 108.$

14. Let the numbers be x and $4x$.
   Then, $x \times 4x = 84 \times 21 \Leftrightarrow x^2 = \left(\frac{84\times21}{4}\right) \Leftrightarrow x = 21.$
   Hence, larger number $= 4x = 84.$

Exercise – 06

H.C.F. & L.C.M. – 03

1. Four metal rods of lengths 78 cm, 104 cm, 117 cm and 169 cm are to be cut into parts of equal length. Each part must be as long as possible. What is the minimum number of pieces that can be cut?
   (a) 27 (b) 36 (c) 43 (d) 480

2. Let N be the greatest number that will divide 1305, 4665 and 6905, leaving the same remainder in each case. Then sum of the digits in N is
   (a) 4 (b) 5 (c) 6 (d) 8

3. A milkman has 3 jars containing 57 litres, 129 litres and 177 litres of pure milk respectively. A measuring can, after a different number of exact measurements of milk in each jar, leaves the same amount of milk unmeasured in each jar. What is the volume of the largest such can?
   (a) 12 litres (b) 16 litres (c) 24 litres (d) None of these

4. Which greatest number will divide 3026 and 5053 leaving remainders 11 and 13 respectively?
   (a) 15 (b) 30 (c) 45 (d) 60

5. Find the greatest number that will divide 964, 1238 and 1400 leaving remainders 41, 31 and 51 respectively.
   (a) 61 (b) 71 (c) 73 (d) 81

6. What is the least natural number which leaves no remainder when divided by all the digits from 1 to 9?
(a) 1800 (b) 1920 (c) 2520 (d) 5040

7. The sum of two numbers is 45. Their difference is $\frac{1}{9}$ of their sum. Their L.C.M. is:
(a) 100 (b) 150 (c) 200 (d) 250

8. The number between 4000 and 5000 which is divisible by 12, 18, 21 and 32 is
(a) 4023 (b) 4032 (c) 4203 (d) 4302

9. The number nearest to 43582 divisible by each of 25, 50 and 75 is
(a) 43500 (b) 43550 (c) 43600 (d) 43650

10. The greatest number which when subtracted from 5834, gives a number exactly divisible by each of 20, 28, 32 and 35 is
(a) 1120 (b) 4714 (c) 5200 (d) 5600

11. A number less than 500, when divided by 4, 5, 6, 7 leaves remainder 1 in each case. The number is
(a) 211 (b) 420 (c) 421 (d) 441

12. What is the greatest number of 3 digits which when divided by 6, 9 and 12 leaves a remainder of 3 in each case?
(a) 903 (b) 939 (c) 975 (d) 996

13. Let the least number of six digits, which when divided by 4, 6, 10 and 15, leaves in each case the same remainder of 2, be N. The sum of the digits in N is
(a) 3 (b) 4 (c) 5 (d) 6

14. Find the least number which when divided by 12 leaves a remainder of 7; when divided by 15, leave-a remainder of 10 and when divided by 16, leaves a remainder of 11.
(a) 115 (b) 235 (c) 247 (d) 475

15. The least number, which when divided by 48, 60, 72, 108 and 140 leaves 38, 50, 62, 98 and 130 as remainders respectively is
(a) 11115 (b) 15110 (c) 15120 (d) 15210

16. What is the third term in a sequence of numbers that leave remainders of 1, 2 and 3 when divided by 2, 3 and 4 respectively?
(a) 11 (b) 17 (c) 19 (d) 35

17. Find the greatest number of 4 digits which when divided by 4, 5, 6, 7 and 8 leaves 1, 2, 3, 4 and 5 as remainders.
(a) 9237 (b) 9240 (c) 9840 (d) 9999

18. The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but -when divided by 9 leaves no remainder, is
(a) 1677 (b) 1683 (c) 2523 (d) 3363

19. A gardener has to plant trees in rows containing equal number of trees. If he plants in rows of 6, 8, 10 or 12, then five trees are left unplanted. But if he plants in rows of 13 trees each, then no tree is left. What is the number of trees that the gardener plants?
(a) 485 (b) 725 (c) 845 (d) None of these

20. When Seeta made necklaces of either 16 beads, 20 beads or 36 beads, not a single bead was left over. What could be the least number of beads Seeta had?
(a) 700 (b) 720 (c) 750 (d) 780

21. An electronic device makes a beep after every 60 sec. Another device makes a beep after every 62 sec. They beeped together at 10 a.m. The next time, when they would beep together at the earliest is
(a) 10.30 a.m. (b) 10.31 a.m. (c) 10.59 a.m. (d) 11 a.m.

22. Six bells commence tolling together and toll at intervals of 2,4, 6, 8,10 and 12 seconds respectively. In 30 minutes, how many times do they toll together?

(a) 4 (b) 10 (c) 15 (d) 16

23. Four bells begin to toll together and toll respectively at intervals of 6, 7, 8 and 9 seconds. In 1.54 hours, how many times do they toll together and in what interval (seconds)?
(a) 14, 504 (b) 14, 480 (c) 12, 504 (d) 16, 580

24. Three girls start jogging from the same point around a circular track and each one completes one round in 24 seconds, 36 seconds and 48 seconds respectively. After how much time will they meet at one point?
(a) 2 minutes 20 seconds (b) 2 minutes 24 seconds
(c) 3 minutes 36 seconds (d) 4 minutes 12 seconds

25. Find the HCF of 132, 204 and 228.
(a) 12 (b) 18 (c) 6 (d) 21

Exercise – 06

H.C.F. & L.C.M. – 03

1. b	*2. a*	*3. c*	*4. c*	*5. b*
6. c	*7. a*	*8. b*	*9. d*	*10. b*
11. c	*12. c*	*13. c*	*14. b*	*15. b*
16. d	*17. a*	*18. b*	*19. c*	*20. b*
21. b	*22. d*	*23. c*	*24. b*	*25. a*

3. Required volume
$= [H.C.F.of\ (129-57),(177-129)\ and\ (177-57)]litres$
$= (H.C.F.of\ 72,48\ and\ 120)\ litres = 24\ litres.$

4. Required Number
$= H.C.F.of\ (3026-11)\ and\ (5053-13)\ = H.C.F.of\ 3015\ and\ 50540 = 45$

11. L.C.M OF 4, 5, 6, 7 = 420. $\therefore$ Required number $= 420 + 1 = 421.$

14. Here $(12-7) = 5, (15-10) = 5\ and\ (16-11) = 5.$
$\therefore$ Required number $= (L.C.M\ of\ 12, 15, 16) - 5 = 240 - 5 = 235.$

18. L.C.M. of 5,6,7,8 $= 840.$ $\therefore$ Required number is of the form $840k + 3$
Least value of k for which $(840k + 3)$ is divisible by 9 is $k = 2.$
$\therefore$ Required number $= (840 \times 25 + 3) = 1683.$

21. Interval of change $= (L.C.M.of\ 60\ and\ 62)$ sec $= 1860\ sec = 31min.$
So, the devices would beep together 31 min after 10 a.m., i.e., at 10.31 a.m.

Exercise – 07
H.C.F. & L.C.M. – 04

1. If three numbers are 2a, 5a and 7a, what will be their LCM?
(a) $70a$ (b) $65a$ (c) $75a$ (d) $70a^3$

2. The product of two whole numbers is 1500 and their HCF is 10. Find the LCM.
(a) 15000 (b) 150 (c) 140 (d) 15

3. A number x is divided by 7. When this number is divided by 8, 12 and 16. It leaves a remainder 3 in each case. The least value of x is:
(a) 148 (b) 149 (c) 150 (d) 147

4. The number of pair of positive integers whose sum is 99 and HCF is 9 is
(a) 5 (b) 4 (c) 3 (d) 2

5. The ratio of two numbers is 3 : 4 and their LCM is 120. The sum of numbers is
(a) 70 (b) 140 (c) 35 (d) 105

6. The greatest four digit number which is exactly divisible by each one of the numbers 12,18, 21 and 28
(a) 9288 (b) 9882 (c) 9828 (d) 9928

7. The traffic lights at three different signal points change after every 45 seconds, 75 seconds and 90 seconds respectively. If all change simultaneously at 7 : 20 : 15 hours, then they will change again simultaneously at
(a) 7 : 28 : 00 hours (b) 7 : 27 : 45 hours (c) 7 : 27 : 30 hours (d) 7 : 27 : 50 hours

8. The least number which is a perfect square and is divisible by each of the numbers 16, 20 and 24, is
(a) 1600 (b) 3600 (c) 6400 (d) 14400

9. The smallest number which when diminished by 7, is divisible by 12, 16, 18, 21 and 28 is
(a) 1008 (b) 1015 (c) 1022 (d) 1032

10. The least number which when increased by 5 is divisible by each one of 24, 32, 36 and 54 is
(a) 427 (b) 859 (c) 869 (d) 4320

11. The least number, which when divided by 12, 15, 20 and 54 leaves in each case a remainder of 8 is
(a) 504 (b) 536 (c) 544 (d) 548

12. Find the least multiple of 23, which when divided by 18, 21 and 24 leaves remainders 7, 10 and 13 respectively.
(a) 3002 (b) 3013 (c) 3024 (d) 3036

13. Find the least number which when divided by 16, 18, 20 and 25 leaves 4 as remainder in each case, but when divided by 7 leaves no remainder.
(a) 17004 (b) 18000 (c) 18002 (d) 18004

14. Four different electronic devices make *a* beep after every 30 minutes, 1 hour, $1\frac{1}{2}$ hour and 1 hour 45 minutes respectively. All the devices beeped together at 12 noon. They will again beep together at
(a) 12 midnight (b) 3 a.m. (c) 6 a.m. (d) 9 a.m.

15. Three persons walking around a circular track complete their respective single revolutions in $15\frac{1}{6}$ seconds, $16\frac{1}{4}$ seconds and $18\frac{2}{3}$ seconds respectively.
They will be again together at the common starting point after an hour and
(a) 10 seconds (b) 20 seconds (c) 30 seconds (d) 40 seconds

16. A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and C in 198 seconds, all starting at the same point. After what time will they meet again at the starting point?
(a) 26 minutes 18 seconds (b) 42 minutes 36 seconds
(c) 45 minutes (d) 46 minutes 12 seconds

17. Three wheels can complete 40,24 and 16 revolutions per minute respectively. There is a red spot on each wheel that touches the ground at time zero. After how much time, all these spots will simultaneously touch the ground again?
(a) 7.5 sec (b) 18 sec (c) 7 min (d) 18 min

18. A pendulum strikes 5 times in 3 seconds and another pendulum strikes 7 times in 4 seconds. If both pendulums start striking at the same time, how many clear strikes can be listened in 1 minute?
(a) 195 (b) 199 (c) 200 (d) 205

19. The LCM of two numbers is 936. If their HCF is 4 and one of the numbers is 72, the other is:
(a) 42 (b) 52 (c) 62 (d) None of these

20. Two alarm clocks ring their alarms at regular intervals of 50 seconds and 48 seconds. If they first beep together at 12 noon, at what time will they beep again for the first time?
(a) 12:10 P.M. (b) 12:12 P.M. (c) 12:11 P.M. (d) None of these

21. 4 Bells toll together at 9:00 A.M. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours?
(a) 3 (b) 4 (c) 5 (d) 6

22. On Ashok Marg three consecutive traffic lights change after 36, 42 and 72 seconds, respectively. If the lights are first switched on at 9:00 A.M. sharp, at what time will they change simultaneously?
(a) 9 : 08 : 04 (b) 9 : 08 : 24 (c) 9 : 08 : 44 (d) None of these

23. Two equilateral triangles have the sides of lengths 34 and 85, respectively.
(a) The greatest length of tape that can measure both of them exactly is:
(b) How many such equal parts can be measured?

24. A forester wants to plant 44 apple trees, 66 banana trees and 110 mango trees in equal rows (in terms of number of trees). Also, he wants to make distinct rows of trees (i.e. only one type of tree in one row). The number of rows (minimum) that are required are:
(a) 2 (b) 3 (c) 10 (d) 11

25. Three runners running around a circular track can complete one revolution in 2, 4 and 5.5 hours, respectively. When will they meet at the starting point?
(a) 22 (b) 33 (c) 11 (d) 44

Exercise – 07

H.C.F. & L.C.M. – 04

1. a	*2. b*	*3. d*	*4. a*	*5. a*
6. c	*7. b*	*8. b*	*9. b*	*10. b*
11. d	*12. b*	*13. d*	*14. d*	*15. d*
16. d	*17. a*	*18. b*	*19. b*	*20. d*

21. c	*22. b*	*23. a. 17, b, 21*	*24. c*	*25. d*

1. The given three numbers are $2a, 5a$ *and* $7a$. $LCM\ of\ 2a, 5a\ and\ 7a = 2 \times 5 \times 7 \times a = 70a$

2. Product of two numbers $= 1500, HCF = 10$
$LCM = \frac{Product\ of\ two\ numbers}{Their\ HCF}$
$= \frac{1500}{10} = 150$
Required LCM is 150.

6. LCM of $12, 18, 21, 28 = 252$
Largest divisible number $= 252 \times 39 = 9828$.

13. $L.C.M.\ of\ 16, 18, 20, 25 = 3600$.
Required number is of the form $3600k + 4$
Least value of k for which $(3600k + 4)$ is divisible by 7 is $k = 5$.
∴ Required number $= (3600 \times 5 + 4) = 18004$.

21. [The LCM of 7, 8, 11 and 12 is 1848. Hence, the answer will be got by the quotient of the ratio (10800)/(1848) → 5.]

23. (a) HCF of 34 and 84 is 17.
(b) $[\frac{34}{17} \times 3 + \frac{85}{17} \times 3 = 2 \times 3 + 5 \times 3 = 21]$

Exercise – 08

H.C.F. & L.C.M. – 05

1. The HCF and LCM of two numbers are 33 and 264, respectively. When the first number is divided by 2, the quotient is 33. The other number is?
(a) 66 (b) 132 (c) 198 (d) 99

2. Which of the following represents the largest 4 digit number which can be added to 7249 in order to make the derived number divisible by each of 12, 14, 21, 33, and 54.
(a) 9123 (b) 9383 (c) 8727 (d) None of these
[Answer: The LCM of the numbers 12, 14, 21, 33 and 54 is 8316. Hence, in order for the condition to be satisfied we need to get the number as: $7249 + n = 8316 \times 2\ Hence, n = 9383$.]

3. Find the greatest number of four digits which when divided by 10, 11, 15 and 22 leaves 3, 4, 8 and 15 as remainders, respectively.
(a) 9907 (b) 9903 (c) 9893 (d) None of these

4. Find the HCF of $(3^{125} - 1)$ and $(3^{35} - 1)$.
[Answer: The solution of this question is based on the rule that:
The HCF of $(a^m - 1)$ and $(a^n - 1)$ is given by $(a^{HCF\ of\ m,n} - 1)$
Thus, in this question the answer is: $(3^5 - 1)$. Since 5 is the HCF of 35 and 125.]

5. What will be the least possible number of the planks, if three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length?
(a) 7 (b) 8 (c) 22 (d) None of these

The least possible number of planks would occur when we divide each plank into a length equal to the HCF of 42, 49 and 63. The HCF of these numbers is clearly 7- and this should be the *size* of each plank. Number of planks in this case would be: 42/7 + 49/7 + 63/7 = 6 + 7 + 9 = 22 planks. Hence, option (c) is correct.

6. Find the greatest number, which will divide 215, 167 and 135 so as to leave the same remainder in each case.
(a) 64 (b) 32 (c) 24 (d) 16
Trial and error would give us that the number 16 would leave the same remainder of 7 in all the three cases. Hence, option (d) is correct.

7. Find the L.C.M of 2.5, 0.5 and 0.175.
(a) 2.5 (b) 5 (c) 7.5 (d) 17.5
The numbers are 5/2, ½ and 175/1000 = 7/40. The LCM of three fractions is given by the formula: LCM of numerators/HCF of denominators = (LCM of 5, 1 and 7)/(HCF of 2 and 40) = 35/2 = 17.5

8. The L.C.M of 4.5; 0.009; and 0.18 = ?
(a) 4.5 (b) 45 (c) 0.225 (d) 2.25

9. What is the greatest possible rate at which a man can walk 51 km and 85 km in an exact number of minutes?
(a) 11 km/min (b) 13 km/min (c) 17 km/min (d) None of these
The answer would be given by the HCF of 51 and 85 – which is 17. Hence, option (c) is correct.

10. The LCM of 5, 8,12, 20 will not be a multiple of
(a) 3 (b) 9 (c) 8 (d) 5

11. Find the number of divisors of 720 (including 1 am 720).
(a) 25 (b) 28 (c) 29 (d) 30

12. The LCM of $(16 - x^2)$ and $(x^2 + x - 6)$ is
(a) $(x-3)(x+3)(4-x^2)$ (b) $4(4 - x^2)(x + 3)$
(c) $(4-x^2)(x-3)$ (d) None of these

13. GCD of $x^2 - 4$ and $x^2 + x - 6$ is
(a) $x + 2$ (b) $x - 2$ (c) $x^2 - 2$ (d) $x^2 + 2$

14. Find the GCD of the polynomials $(x+3)^2(x-2)(x+1)^2$ *and* $(x+1)^3(x+3)(x+4)$.
(a) $(x+3)^3(x+1)^2(x-2)(x+4)$
(b) $(x+3)(x-2)(x+1)(x+4)$
(c) $(x+3)(x+1)^2$
(d) $(x+1)(x+3)^2$

15. Find the LCM of $(x+3)(6x^2+5x+4)$and$(2x^2+7x+3)(x+3)$
(a) $(2x+1)(x+3)(3x+4)$ (b) $(4x^2-1)(x+3)^2(3x+4)$
(c) $2(x+3)^2(6x^2+5x+4)(x+1/2)$ (d) $(2x-1)(x+3)(3x+4)$

16. The least multiple of 7 which leaves a remainder of 4 when divided by 6, 9, 15 and 18 is
(a) 94 (b) 184 (c) 364 (d) 74

17. What is the least 3 digit number that when divided by 2, 3, 4, 5 or 6 leaves a remainder of 1?
(a) 131 (b) 161 (c) 121 (d) None of these

18. The highest common factor of 70 and 245 is equal to
(a) 35 (b) 45 (c) 55 (d) 65

19. The product of two numbers is 7168 and their HCF is 16. How many pairs of numbers are possible such that the above conditions are satisfied?
(a) 2 (b) 3 (c) 4 (d) 6

20. HCF of $3^{15} - 1$ & $3^{25} - 1$ is

…………………… .

21. The HCF of two natural numbers a, b is 10 & LCM of these numbers is 45. If a = 15 then b = ?
(a) 210 (b) 220 (c) 310 (d) Can't be Determined

22. LCM and HCF of 10! and 15! are respectively
(a) 5! & 25! (b) 5! & 30! (c) 10! & 30! (d) 15! & 10!

23. Find the HCF of
(a) 420 and 1782 (b) 36 and 48 (c) 54, 72, 198 (d) 62, 186 and 279

24. Find the LCM of
(a) 13, 23 and 48 (b) 24, 36, 44 and 62 (c) 22, 33, 45 and 72 (d) 13, 17, 21 and 33

25. The L.C.M of two numbers is 1890 and their H.C.F is 30. If one of them is 270, the other will be
(a) 210 (b) 220 (c) 310 (d) 320

Exercise – 08

H.C.F. & L.C.M. – 05

Answers Key

1. b	*2. b*	*3. c*	*4.* $(3^5 - 1)$	*5. c*
6. d	*7. d*	*8. a*	*9. c*	*10. b*
11. d	*12. d*	*13. b*	*14. c*	*15. c*
16. c	*17. c*	*18. a*	*19. a*	*20. 242*
21. d	*22. d*	*23. -*	*24. -*	*25. a*

5. The least possible number of planks would occur when we divide each plank into a length equal to the HCF of 42, 49 and 63. The HCF of these numbers is clearly 7 and this should be the size of each plank. Number of planks in this case would be: $42/7 + 49/7 + 63/7 = 6 + 7 + 9 = 22$ planks. Hence, option (c) is correct.

6. Trial and error would give us that the number 16 would leave the same remainder of 7 in all the three cases. Hence, option (d) is correct.

8. Use the same process as for question 23 for the numbers 9/2; 9/1000 & 9/50.
$(LCM\ of\ 9, 9, 9)/(HCF\ of\ 2, 1000\ \&\ 50) = 9/2 = 4.5$

9. The answer would be given by the HCF of 51 and 85 – which is 17. Hence, option (c) is correct.

11. $720 = 2^4 \times 3^2 \times 5^1$. Number of factors $= 5 \times 3 \times 2 = 30$. Option (d) is correct

13. $x^2 - 4 = (x-2)(x+2)\ and\ x^2 + x - 6 = (x+3)(x-2)$ GCD or HCF of these expressions $= (x-2)$
Option (b) is correct.

15. For the LCM of polynomials write down the highest powers of all available factors of all the polynomials .
The correct answer could be $2(x+3)^2(6x^2+5x+4)\left(x+\frac{1}{2}\right)$

Exercise – 09

1. If $A381$ is divisible by 11, find the value of the smallest natural number A.
(a) 5 (b) 6 (c) 7 (d) 9

2. If 381A is divisible by 9, find the value of smallest natural number A.
(a) 5 (b) 5 (c) 7 (d) 6

3. Find the number of zeroes at the end of 1090!
(a) 270 (b) 268 (c) 269 (d) 271

4. If 146! Is divisible by 5. Then find the maximum value of n.
(a) 34 (b) 35 (c) 36 (d) 37

5. Find the number of divisors of 1420.
(a) 14 (b) 15 (c) 13 (d) 12

6. Find the remainder when the number 9^{100} is divided by 8.
(a) 1 (b) 2 (c) 0 (d) 4

7. Find the remainder of 2^{100} when divided by 3
(a) 1 (b) 2 (c) 4 (d) 6

8. Find the number of divisors of 10800.
(a) 57 (b) 60 (c) 72 (d) 64

9. A hundred and twenty digit number is formed by writing the first x natural numbers in front of each other as 12345678910111213….find the remainder when this number is divided by 8.
(a) 6 (b) 7 (c) 2 (d) 0

10. Find the number of zeros at the end of 100!
(a) 20 (b) 23 (c) 24 (d) 25

11. Find the number of zeroes at the end of 122!
(a) 20 (b) 23 (c) 24 (d) 28

12. Find the number of zeroes at the end of 380!
(a) 90 (b) 91 (c) 94 (d) 95

13. The highest power of 3 that completely divides 40! Is
(a) 18 (b) 15 (c) 16 (d) 17

14. The highest power of 7 that completely divides 80! Is:
(a) 12 (b) 13 (c) 14 (d) 15

15. XYZ is a 3 digit number such that when we calculate the difference between the two three digit numbers XYZ – YXZ the difference is exactly 90. How many possible values exist for the digits X and Y?
(a) 9 (b) 8 (c) 7 (d) 6

16. The least number which can be subtracted from 763 so that it is completely divisible by 57 is.
(a) 35 (b) 22 (c) 15 (d) 25

17. The least number which can be added to 8441 so that it is completely divisible by 57 is.
(a) 42 (b) 15 (c) 5 (d) 52

18. Find the least number of 5 digits that is exactly divisible by 79.
(a) 10003 (b) 10033 (c) 10043 (d) none of these

19. A number when divided by 84 leaves a remainder of 57. What is the remainder when the same number is divided by 12?
(a) 7 (b) 8 (c) 9 (d) cannot be determined

20. 511 and 667 when divided by the same number, leave the same remainder, how many numbers can be used as the divisor in order to make this occur?
(a) 14 (b) 12 (c) 10 (d) 8

21. The number of ways of expressing 72 as a product of 2 factors is

22. If x be a rational number and y be an irrational number, then:
(a) both x + y and xy are necessarily irrational
(b) both x + y and xy are necessarily rational
(c) both is necessarily irrational but x + y can be either rational or irrational
(d) x + y necessarily irrational, but xy can be either rational or irrational.

23. If the sum of two numbers added to the sum of their squares is 42 and the product of these numbers is 15, then the numbers are:
(a) 15, 1 (b) $\frac{15}{6}$, 6 (c) $2\frac{1}{2}$, 6 (d) 5, 3

24. If $x + \frac{1}{x} = 2$, then the value of $x^2 + \frac{1}{x^2}$ is?
(a) 6 (b) 4 (c) 2 (d) 0

25. Which one of the following sets of surds is in correct sequence of ascending order of their values?
(a) $\sqrt[4]{10}, \sqrt[8]{6}, \sqrt{3}$ (b) $\sqrt{3}, \sqrt[4]{10}, \sqrt[8]{6}$
(c) $\sqrt{3}, \sqrt[8]{6}, \sqrt[4]{10}$, (d) $\sqrt[4]{10}, \sqrt{3}, \sqrt[8]{6}$

26. The expression $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)\ldots\ldots\left(1+\frac{1}{n}\right)$ *simplifies* to:
(a) $\frac{n+1}{3}$ (b) $\frac{n}{n+1}$ (c) $\frac{3}{n}$ (d) $1+\frac{1}{3}\frac{1}{4}\frac{1}{5}\ldots\ldots\frac{1}{n}$

27. The remainder when $(20)^{23}$ is divided by 17 is:
(a) 11 (b) 3 (c) 6 (d) can't be determine

28. If p be a prime number, then $p^2 + 1$ can not have its unit digit is:
(a) 3 (b) 9 (c) 7 (d) all of these

29. A six digit number abcabc such that a,b,c ϵ N, then which is the most correct statement is:
(a) it is divisible by 91 (b) it can be divided by 143
(c) it is divisible by 6 (d) only a and b are correct

30. Which is not a prime number?
(a) 97 (b) 1001 (c) 127 (d) 101

31. When we divide 15192327 by 99 the remainder will be:
(a) 98 (b) 84 (c) 30 (d) none of these

32. The sum of 100 terms of the series
1-3+5-7+9-11+13-15+….is?
(a) 100 (b) 50 (c) 200 (d) none of these

33. The value of $(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{4})(1-\frac{1}{5})\ldots(1-\frac{1}{n})$ is :
(a) 1 (b) $(1-\frac{1}{n})^n$ (c) $\frac{1}{n}$ (d) can't be determined

34. What is the remainder of $\frac{6^{36}}{215}$?
(a) 0 (b) 1 (c) 2 (d) none of these

35. Which of the following is the smallest?
(a) $5^{1/2}$ (b) $6^{1/3}$ (c) $8^{1/4}$ (d) $12^{1/6}$

36. The number N is divisible by 6 but not divisible by 4. Which of the following will not be an integer?
(a) N/3 (b) N/2 (c) N/6 (d) N/12

37. If n^2 is a perfect cube, then which of the following statements is always true?
(a) n is odd (b) n is even
(c) n^3 is a perfect square (d) n is a perfect cube

38. For what values of x is $(25^x + 1)$ divisible by 13?
(a) All real values of x
(b) Odd natural values of x
(c) Even values of x
(d) All the integral values of x

39. How many prime numbers are there between 80 and 105?
(a) 3 (b) 4 (c) 5 (d) 8

40. What is the remainder when 5^{79} is divided by 7?
(a) 1 (b) 0 (c) 5 (d) 4

41. How many zeroes will be there eat the end of 25 x 35 x 40 x 50 x 60 65?
(a) 6 (b) 8 (c) 5 (d) 7

42. How many divisors will be there of the number 1020?
(a) 12 (b) 20 (c) 24 (d) 36

43. How many zeroes will be there at the end of the expression N – 2 x 4 x 6 x 8 x…….x 100?
(a) 10 (b) 12 (c) 14 (d) none of these

44. How many zeroes will be there at the end of the expression N = 10 x 20 x 30 ….x 1000?
(a) 1280 (b) 1300 (c) 1320 (d) none of these

45. The number 444,444…(999 times) is definitely divisible by:
(a) 22 (b) 44 (c) 222 (d) all of these

46. What is the remainder when (1! + 2! + 3! + …..1000!) is divided by 5?
(a) 1 (b) 2 (c) 3 (d) 4

47. Two friends Harry and Jayesh were discussing about two numbers. The found the two numbers to be such that one was twice the other. However, both had the same number of prime factors, while the larger one had 4 more factors than the smaller one. What are the numbers?
(a) 40, 80 (b) 20, 40 (c) 30, 60 (d) 50, 100

48. How many zeroes will be there at the end of N = 18! + 19!?

(a) 3 (b) 4 (c) 5 (d) cannot be determined

49. Manish was dividing two numbers by a certain divisor and obtained remainders as 437 and 298, respectively. When he divides the sum of the two numbers by the same divisor, the remainder is 236. What is the divisor?
(a) 499 (b) 735 (c) 971 (d) none of these

50. The product of three consecutive odd numbers is 531117. What is the sum of the three numbers?
A. 183 B. 213 C. 243 D. 273

51. N is a number such that $200 < N < 300$ and it has exactly 6 positive divisors. How many different values of N are possible?
A. 12 B. 13 C. 14 D. 15

52. In 1936, my age was equal to the last two digits of my birth year. My grandfather said that it was true for him also. Then, the sum of my age and my grandfather's age in 1936 was
A. 84 B. 86 C. 90 D. 94

53. If n is a natural number such that $10^{12} < n < 10^{13}$ and the sum of the digits of n is 2, then the number of values n can take is
A. 13 B. 12 C. 11 D. 10

54. The squares of the natural numbers are written in a straight line 149162536… to form a 200-digits number. What is the 100th digit from the left?
A. 2 B. 5 C. 6 D. 9

55. $18^{2000} + 12^{2000} - 5^{2000} - 1$ is divisible by
A. 323 B. 221 C. 299 D. 237

56. How many natural numbers between 1 and 900 are NOT multiples of any of the numbers 2, 3, or 5?
A. 240 B. 250 C. 270 D. 300

57. The numbers 123 456 789 and 999 999 999 are multiplied. How many times does digit '9' come in the product?
A. 0 B. 1 C. 2 D. 3

58. All the divisors of 72 are multiplied. The product can be written in the form $2^a * 3^b$. Then the value of a + b is
A. 28 B. 30 C. 34 D. 40

59. In the nineteenth century a person was X years old in the year X^2. How old was he in 1884?
A. 43 B. 58 C. 68 D. 78

60. How many positive integers less than or equal to 120 are relatively prime to 120?
A. 24 B. 32 C. 36 D. 40

61. If $3^{27^x} = 27^{3^x}$, then x is equal to
A. −1 B. ½ C. 1 D. 2

62. If $S = \frac{1}{\sqrt{2}-\sqrt{1}} - \frac{1}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{4}-\sqrt{3}} - \ldots\ldots - \frac{1}{\sqrt{121}-\sqrt{120}}$, then the value of S is
A. 12 B. 11 C. 10 D. −10

63. The difference between the cubes of two consecutive positive integers is 1027. Then the product of these integers is
A. 552 B. 342 C. 306 D. 132

64. Which of the following numbers can be written as the sum of the squares of three odd natural numbers?

A. 5021 B. 4445 C. 3339 D. 1233

65. The value of x + y such that $x^2 - y^2 = 343$, is
A. 343 B. 49 C. 7 D. A or B

66. The total number of integer pairs (x, y) satisfying the equation x + y = xy is
A. 0 B. 1 C. 2 D. none of the above

Exercise – 09

Solutions

1. Ans. (c)
Solution: for the number A381 to be divisible by 11, the sum of the even placed digits and the odds placed digits should be either 0 or a multiple of 11. This means that (A+B) – (3+1) should be multiple of 11-as it is possible to make it zero. Thus, the smallest value that A can take (and in fact the only value it can take) is 7. Option (c) is correct.

2. Ans. (d)
Solution: for 381A to be divisible by 9, the sum of the digits 3+8+1+A should be divisible by 9. For that in happen A should be 6. Option (d) is correct.

3. Ans. (a)
Solution: The number of zeros would be given by adding the quotients when we successively divide 1090 by 5. 1090/5 + 218/5 +43/5 + 8/5 = 218 + 43 + 8 + 1 = 270. Option (a) is correct.

4. Ans. (b)
Solution: The number of 5's in 146! Can be got by [146/5]+ [29/5] + [5/5] = 29+5+1 = 35

5. Ans. (d)
Solution: 1420 = 142 x 10 = $2^2 \times 71^1 \times 5x$.
Thus, the number of factors of the number would be (2+1)(1+1)(1+1) = 3 x 2 x 2 = 12.

6. Ans. (a)
Solution: $9^{100}/8 = (8 + 1)^{100}/8 \rightarrow$ since this is of the form $(a + 1)^n/a$, the remainder = 1 option (a) is correct.

7. Ans. (a)
Solution: $2^{1000}/3$ is of the form $(a)^{\text{EVEN POWER}} / (a + 1)$ the remainder = 1 in this case as the power is even, option (a) is correct

8. Ans. (b)
Solution: 10800 = 108 x 100 = $3^3 \times 2^4 \times 5^2$
The number of divisors would be : (3 + 1) (4 + 1) (2 + 1) = 4 x 5 x 3 = 60 divisors. Option (b) is correct.

9. Ans. (a)
Solution: The last 3 digits of the number would determine the remainder when it is divided by 8. The number upto the 12th digit would be 1234567891011..646 646 divided by 8 gives us a remainder of 6. the answer is (a)

10. Ans. (c)
Solution: The answer would be given by the quotients of 100/5 + 100/25 = 20 + 4 = 24 option (c) is correct.

11. Ans. (d)
Solution: 24 + 4 = 28 option (d) is correct.

12. Ans. (c)
Solution: 76 + 15 + 3 = 94 option (c) is correct.

13. Ans. (a)
Solution: 13 + 4 + 1 = 18. Option (a) is correct.

14. Ans. (a)
Solution: 11 + 1 = 12 = option (a) is correct.

15. Ans. (b)
Solution: This occurs for values such as 213 – 123; 324 – 234 etc where it can be seen that the value of X is 1 more than Y. the possible pairs of X and Y are: 2,1;3,2….9.8 – a total of eight pairs of values option (b) is correct.

16. Ans. (b)
Solution: Since, 763/57 leaves a remainder of 22, we would need to subtract 22 from 763 in order to get a number divisible by 57. Option (b) is correct.

17. Ans. (d)
Solution: 8441/57 leaves remainder of 5. Thus, if we were to add 52 to this number the number we obtain would be completely divisible by 57. Option (d) is correct.

18. Ans. (b)
Solution: 10000 divided by 79 leaves a remainder of 46. Hence, if we were to add 33 to 10000 we would get a number divisible by 79. The correct answer is 10033. Option (b) is correct.

19. Ans. (c)
Solution: Since 12 is a divisor of 84, the required remainder would be got by dividing 57 by 12. The required answer is 9. Option (c) is correct.

20. Ans. (b)
Solution: The numbers that can do so are going to be factors of the difference between 511 and 667 i.e. 156. the factors 156 are 1,2,3,4,6,12,13,26,39,52,78,156. There are 12 such numbers. Option (b) is correct.

21. Solution: 6
$72 = 2^3 \times 3^2$
Total number of factors of 72 = (3 + 1) (2 + 1) = 12
Total number of ways of expressing 72 as a product. Of two factors = $\frac{12}{2} = 6$

22. Ans. (a)
Solution: Go back to the basics.

23. Ans. (d)
Solution: Check through options: As
$(5+3) + (5^2 + 3^2) = 42$
And $5 \times 3 = 15$

24. Ans. (c)
Solution: $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2\left(x - \frac{1}{x}\right)$
⇨ $(2)^2 = x^2 + \frac{1}{x^2} + 2(1)$
(Substituting the value of $x + \frac{1}{x}$)
⇨ $4 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow X^2 + \frac{1}{x^2} = 2$

Hence (c) is the right choice.

25. Ans. (b)

Solution: $\sqrt[4]{10} = (10)^{1/4} = (10)^{3/12} = (1000)^{1/12}$

$\sqrt[3]{6} = (6)^{1/3} = (6)^{4/12} = (1296)^{1/12}$

$\sqrt{3} = (3)^{1/2} = (3)^{6/12} = (729)^{1/12}$

$\sqrt{3} < \sqrt[4]{10} < \sqrt[3]{6}$ is the correct order and hence (b) is correct.

26. Ans. (a)

Solution: $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right).....\left(1+\frac{1}{n}\right)$

$= \frac{4}{3} \times \frac{5}{4} \times \frac{6}{5} \times \ldots \frac{n}{(n+1)} \times \frac{n+1}{n} = \frac{n+1}{3}$

Hence (a) is the correct option.

27. Ans. (a)

Solution: $\frac{(20)^{23}}{17} \rightarrow \frac{(3)^{23}}{17} \rightarrow \frac{(3^3)^7 \times 3^2}{17} \rightarrow \frac{27^7 \times 9}{17} \rightarrow \frac{(10)^7 \times 9}{17}$

$\rightarrow \frac{(10^2)^3 \times 90}{17} \rightarrow \frac{(15)^3 \times 5}{17} \rightarrow \frac{225 \times 75}{17} \rightarrow \frac{4 \times 7}{17} \rightarrow 11$

Hence the required remainder is 11.

28. Ans. (d)

Solution: Since the unit digit of primes number = 1, 2, 3, 5, 7, 9

So, the unit digit of the squares of prime number = 1, 4, 9, 5, 9, 1

Thus 1 + 1 = 2, 4+1=5, 9+1=10, 5+1=6, 9+1=10 and 1+1=2

Hence the possible unit digits are 2, 5, 6, 0

Thus, 3 , 7 or 9 are not possible hence (d) is correct.

29. Ans. (d)

Solution: Any number of the form abcabc must be divisible by 1001 or its factors i.e. 1001 = 7 x 13 x 11

Hence (a) and (b) are correct, thus (d) is the most appropriate statement.

30. Ans. (b)

Solution: 1001 = 7 x 11 x 13

Hence (b)

31. Ans. (b)

Solution: Go through options: consider option (b)

15192327 – 84 = 15192243

Now, (3+2+9+5) - (4+2+1+1) = 11

And 1+5+1+9+2+2+4+3 = 27

Thus 15192243 is divisible by both 11 and 9 hence by 99.

Thus the presumed remainder 84, is correct.

32. Ans. (d)

Solution: 1-3 + 5 – 7 + 9 – 11 + …+197- 199

= (-2) + (-2) + (-2) + …..+(-2) [50 times]

= 50 x (-2) = -100. Hence (d)

33. Ans. (c)

Solution: $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right).....\left(1-\frac{1}{n}\right)$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \ldots.. \times \frac{(n-2)}{(n-1)} \times \frac{(n-1)}{n} = \frac{1}{n}$

34. Ans. (b)
Solution: $\frac{6^{36}}{215} \rightarrow \frac{(6^3)^{12}}{215} \rightarrow \frac{(216)^{12}}{215} \rightarrow \frac{(1)^{12}}{215} \rightarrow$ 1 is the remainder.

35. Ans. (d)
Solution: Number are $5^{1/2}$, $6^{1/3}$, $8^{1/4}$, and $12^{1/6}$
To solve such questions, we raise each number to a common ower so that the powers of the numbers are natural numbers.
In this case, raise each number to the power 12 (LCM of 2, 3, 4 and 6).
Therefore numbers obtained = $(5^{1/2})^{12}$, $(6^{1/3})^{12}$, $(8^{1/4})^{12}$ and $(12^{1/6})^{12}$ = 5^6, 6^4, 8^3, and 12^2

36. Ans. (d)
Solution: $\frac{N}{12} = \frac{N}{3} \times \frac{N}{4}$.
However it is given that $\frac{N}{4}$ is not an integer and there fore, $\frac{N}{12}$ will not be an integer.

37. Ans. (d)
Solution: If n is a perfect cube, then n^2 will also be a perfect cube, therefore, the answer is option (d).

38. Ans. (b)
Solution: $\frac{25^x+1}{13} = \frac{(-1)^x}{13} \frac{1}{13}$
Hence for odd natural values of x, $25^x + 1$ will be divisible.

39. Ans. (c)
Solution: Count the number by actual counting method. The numbers are 83, 89, 87, 101, 103.

40. Ans. (c)
Solution: Using Fermat's Theorem

$\frac{5^{79}}{7} = \frac{5^{78} \, x \, 5}{7} = \frac{5^{6|8|} \, x \, 5}{7} = \frac{15}{7} = 5$

41. Ans. (a)
Solution: 25 x 35 x 40 x 50 60 x 65 = $(5)^2$ x (5 x 7) x (5 x 8) x (5^2 x 2) x (5 x 12) x (5 x 13)
= 5^8 x 2^6 x 3 x 7 x 13.
Therefore are eight 5s and six 2s.
Number of zeroes = umber of sets of 2 and 5 = minimum of (number of 2s and number of 5s) = 6

42. Ans. (c)
Solution: Factors of 1020 will divide 1020 properly therefor factors of 1020 = 2^2 x 3 x 5 x 7
= (2+1) (1+1) (1+1) (1+1) = 24

43. Ans. (b)
Solution: N = 2 x 4 x 6 x 8 x ….100
Count the number of 5s in N, which is 12. Therefore, number of zeroes are 12.

44. Ans. (d)
Solution: N = 10 x 20 x 30 ….x 1000
There is one 5 in the multiple of 10.
There are two 5 in the multiple of 25 and three 5s in the multiple of 125.
Now, count the multiple of 5s in the expression which are (100 + 20 + 4) = 124

45. Ans. (c)
Solution: Since there are 999 terms in the number, then it is divisible by 222.

Because every term will be divisible by 222, and therefore, all 999 terms will also be divisible by 222

46. Ans. (c)
Solution: We know that 5! Or greater than 5! Will be divisible by 5. Therefore, when (1! + 2! + 3!…….1000!) is divisible by 5 equals to when (1!+2!+3!+4!) is divided by 5, we get the remainder as
$\frac{1!+2!+3!+4!}{5} = \frac{33}{5} = \frac{3}{5}$
Hence remainder obtained = 3

47. Ans. (c)
Solution: Go through the options.
The answer is option (c).
Because the number of factors of 30 $(2 \times 3 \times 5) = 8$
And the number of factors of 60 $(2^2 \times 3 \times 5) = 12$

48. Ans. (b)
Solution: The number of 5s in 18! = $\frac{18}{5}$ = 3 and in 19! = $\frac{19}{5}$ = 3
Therefore number zeroes in 18! Is 3 and in 19! Is 3.
Hence number of zeroes in 18! + 19! = 3 zeroes

49. Ans. (a)
Solution: let the number be x.
It is given that if we divide the sum of two numbers, then the remainder is 236.
Hence it means when we divide (437 + 298) by x then the remainder is 236.
From here, the number x should be 499.

50. Ans. (C)
Solution: The unit digits of three consecutive odd numbers would be (1, 3, 5), (3, 5, 7), (5, 7, 9), (7, 9, 1) and (9, 1, 3). As the unit digit of the product is 7, only the last triplet of units digits, i.e. (9, 1, 3) will qualify. Therefore, we need to find three consecutive odd numbers ending in 9, 1 and 3 such that there product is 531117. We can see that 80^3 = 512000. Therefore, the numbers would be lying around 80. The numbers are 79, 81 and 83 and the sum is 243.

51. Ans. (B)
Solution: The number which has 6 divisors will be of the form ab^2 or c^5, where a, b and c are prime numbers.
The numbers are 2×11^2, 5×7^2, 11×5^2, 23×3^2, 29×3^2, 31×3^2, 53×2^2, 59×2^2, 61×2^2, 67×2^2, 71×2^2, 73×2^2 and 3^5

52. Ans. (B)
Solution: The ages will be lying on either side of 1900 for us to have two solutions to same situation. Let the birth year of grandfather be 18ab. Therefore, his age in 1936 = 1936 − 18ab = 36 + 1900 − 18ab = 36 + 100 − ab. This should be equal to ab ⇒ ab = 36 + 100 − ab ⇒ ab = 68. Similarly, the birth year for grandson is 1918. The sum of ages = 68 + 18 = 86

53. Ans. (A)
Solution: 10^{12} = 1 000 000 000 000. Since the number is greater than 10^{12} and the sum of the digits is 2, one of the zeroes in 1 000 000 000 000 will be replaced by 1. Therefore, there will be 12 numbers generated this way. Apart from this, the number 2 000 000 000 000 also satisfies the criterion. Hence, 13 numbers are possible.

54. Ans. (D)
Solution: Single digit square = 3 (1, 3, 9) Digits written = 3
Two-digit squares = 6 (16, 25, … 81) Digits written = 2 × 6 = 12.
Three-digit squares = 22 (100, 121, … 961) Digits written = 3 × 22 = 66
Total digits written so far = 3 + 12 + 66 = 81. Digits left = 100 − 81 = 19.
After 961, we will start writing four-digit squares – 1024, 1089… with every square we cover four digits. Since we need to cover 19 digits, we will have to write 4 four-digit squares and then we will see the 3rd digit of the fifth four-digit square. The fifth four-digit square = 36^2 = 1296 ⇒ 3rd digit =9. Therefore, 100th digit = 9.

55. Ans. (B)
Solution: $a^n - b^n$ is divisible by both a + b and a − b when n is even.
$18^{2000} + 12^{2000} - 5^{2000} - 1 = 18^{2000} - 5^{2000} + 12^{2000} - 1^{2000} \Rightarrow 18^{2000} - 5^{2000}$ is divisible by 13 and 23.
Similarly, $12^{2000} - 1^{2000}$ is divisible by 11 and 13. As 13 is the common factor, the whole expression is divisible by 13.
$18^{2000} + 12^{2000} - 5^{2000} - 1 = 18^{2000} - 1 + 12^{2000} - 5^{2000} \Rightarrow 18^{2000} - 1$ is divisible by 17 and 19.
Similarly, $12^{2000} - 5^{2000}$ is divisible by 7 and 17. As 17 is the common factor, the whole expression is divisible by 17.
As the expression is divisible by both 13 and 17, it is divisible by 221.

56. Ans. (A)
Solution: The number of numbers prime to and less than 900 = $900(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{5}) = 240$

57. Ans. (A)
Solution: 123456789 × 999999999 = 123456789 × (1000000000 − 1) = 123456789000000000 − 123456789
= 123456788876543211

58. Ans. (B)
Solution: When the divisors of N $X^aY^bZ^c$ = are multiplied the product is equal to $(N)^{\frac{(a+1)(b+1)(c+1)}{2}}$. When the divisors of 72 = $2^3 \times 3^2$ are multiplied, the product is equal to $(2^3 \times 3^2)^{\frac{(2+1)(3+1)}{2}}$ - $2^{18} \times 3^{12}$. Therefore, a+ b =30

59. Ans. (D)
Solution: There is only one perfect square in the nineteenth century 432 = 1849. Therefore, the man was 43 years old in 1849. Therefore, he was 78 years old in 1884.

60. Ans. (B)
Solution: The number of numbers less than and prime to 120 = $120\ (1-\frac{1}{2})\ (1-\frac{1}{3})\ (1-\frac{1}{5}) = 32$

61. Ans. (B)
Solution: $3^{27^x} = 3^{(3)^{3x}} = 3^{3^{3x}}$
$27^{3^x} = (3^3)^{3x} = 3^{3x3^x} = 3^{3^{x+1}}$
⇨ $3x = x + 1 \rightarrow\ = ½.$

62. Ans. (D)
Solution: $= \frac{1}{\sqrt{2}-\sqrt{1}} = = \frac{1}{\sqrt{2}-\sqrt{1}} x = \frac{\sqrt{2}+\sqrt{1}}{\sqrt{2}+\sqrt{1}} = \sqrt{2}+\sqrt{1}$
$\frac{1}{\sqrt{2}-\sqrt{1}} - \frac{1}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{4}-\sqrt{3}} - \ldots\ldots \frac{1}{\sqrt{121}-\sqrt{120}}\ \sqrt{1}+\sqrt{2}-\sqrt{2}-\sqrt{3}+\sqrt{3}+\sqrt{4}-\sqrt{4}-\sqrt{5}+\ldots\sqrt{121}-\sqrt{120} = -11+1 = -10$

63. Ans. (B)
Solution: $(n + 1)^3 - n^3 = 1027 \Rightarrow 3n^2 + 3n - 1026 = 0 \Rightarrow n^2 + n - 342 = 0 \Rightarrow n = 18.$

64. Ans. (C)
Solution: The sum of the squares of three odd numbers will be in the form 4k + 3. Only C satisfies the criterion.

65. Ans. (D)
Solution: (x + y)(x − y) = 343

66. Ans (B) ; one pair only (0,0)

Exercise – 10

1. $(17^{21} + 19^{21})$ is not divisible by

(a) 36 (b) 8 (c) 9 (d) 18

2. What is the units digit of $2^{3^{4^5}}$?

(a) 2 (b) 4 (c) 8 (d) 6

3. How many zeroes will be there at the end of $36!^{36!}$?

(a) 7 x 6! (b) 8 x 6! (c) 7 x 36! (d) 8 x 36!

4. What is the units digit of $7^{11^{22^{33}}}$?

(a) 1 (b) 3 (c) 7 (d) 9

5. I purchased a ticket for the football match between France and Italy in the world cup. The number on the ticket was a 5-digit perfect square such that the first and the last digit were the same and the second and fourth digit were the same. If the third digit was 3 then what was the ticket number?

(a) 24, 342 (b) 12, 321 (c) 21, 312 (d) none of these

6. The following is known about three real numbers, x, y and z.
$-4 \le x \le 4$, $-8 \le y \le 2$ and $-8 \le z \le 2$. Then the range of values that $M = xz/y$ can take is best represented by:

(a) $-\infty < x < \infty$ (b) $-16 \le x \le 8$ (c) $-8 \le x \le 8$ (d) $-16 \le x \le 16$

7. How many numbers between 200 and 400 are divisible by 13?

(a) 14 (b) 15 (c) 16 (d) 17

8. Given that $7x + y$ is a prime number of natural numbers x and y, then what is the minimum value of $(x+y)^2$

9. If $N^2 = 1234567654321$, then X = ?

10. X is a number formed by writing 9 for 99 times. What will be the remainder of this number when divided by 7?

11. Find the remainder of $\frac{2^{41}}{41}$.

12. Find the remainder when 40! Is divided by 41.

13. Find the remainder when $x^4 + 3x^3 + 4$ is divided by $x + 3$.

14. How many numbers in the form of $2^n - 1$ which are less than 500 are prime?

15. Find the value of x in $\sqrt{x + 2\sqrt{x + 2\sqrt{x + 2\sqrt{3x}}}} = x$.

(a) 1 (b) 3 (c) 6 (d) 12 (e) 9

16. The mean of 1, 2, 2^2 ……2^{31} lies in between

(a) 2^{24} to 2^{25} (b) 2^{25} to 2^{26} (c) 2^{26} to 2^{27} (d) 2^{29} to 2^{30}

17. $16^5 + 2^{15}$ is divisible by
(a) 31 (b) 13 (c) 27 (d) 33

18. If $|x-4| + |y-4| = 4$, then how many integer values can the set (x, y) have?
(a) infinite (b) 5 (c) 16 (d) 9

19. The highest power on 990 that will exactly divide 1090! Is
(a) 101 (b) 100 (c) 108 (d) 109

20. The expression $333^{555} + 555^{333}$ is divisible by
(a) 2 (b) 3 (c) 37 (d) all of these

21. Find the number of zeros in the product $1^1 \times 2^2 \times 3^3 \times 4^4 \times \ldots 98^{98} \times 99^{99} \times 100^{100}$
(a) 1200 (b) 1300 (c) 1050 (d) 1225

22. The highest power of 45 that will exactly divide 123! Is
(a) 28 (b) 30 (c) 31 (d) 59

23. Find the 28383rd term of the series:
123456789101112….
(a) 3 (b) 4 (c) 9 (d) 7

24. The series of number (1,1/2, 1/3, ¼……..1/1972) is taken. Now two numbers are taken from this series (the first two) say x, y. the n the operation x + y + x. y is performed to get a consolidated number. The process is repeated. When will be the value of the set after all the numbers are consolidated into one number?
(a) 1970 (b) 1971 (c) 1972 (d) none of these

25. What is the remainder when $9 + 9^2 + 9^3 + \ldots 9^{2n+1}$ is divided by 6?
(a) 1 (b) 2 (c) 3 (d) 4

26. The remainder when the number 123456789101112……..484950 is divided by 16 is ?
(a) 3 (b) 4 (c) 5 (d) 6

27. What is the total number of divisors of the number $12^{33} \times 34^{23} \times 2^{70}$?
(a) 4658 (b) 9316 (c) 2744 (d) none of these

28. What is the remainder when $(1!)^3 + (2!)^3 + (3!)^3 + (4!)^3 + \ldots(1152!)3$ is divided by 1152?
(a) 125 (b) 225 (c) 325 (d) 205

29. What will be the value of remainder when (111111111……64terms) * (22222222…….55terms) is divided by 18?
(a) 0 (b) 1 (c) 2 (d) 17

30. Find the number of solutions of the equation $x^2 - y^2 = 777314$:

31. Find the lower of the two successive natural numbers if the square of the sum of those numbers exceeds the sum of their squares by 112.

(a) 6 (b) 7 (c) 8 (d) 9

32. When the natural numbers 1, 2, 3….,500 a written, then the digit 3 is sued n times in this way. The value of n is:
(a) 100 (b) 200 (c) 300 (d) 280

33. If a and b are two odd distinct prime numbers and if a>b then $a^2 - b^2$ can never be divided by:
(a) 13 (b) 11 (c) 17 (d) none of these

34. The remainder when $75^{75^{75}}$ is divided by 37:
(a) 0 (b) 1 (c) 5 (d) 7

35. If 4 is added to the numerator of a fraction, it becomes 1/3 and if 3 is added to the denominator of the same fraction it becomes 1/6 then the sum of the numerator and denominator is:
(a) 32 (b) 7 (c) 4 (d) 3

36. If $a^2 + b^2 + c^2 + d^2 = 1$, then the maximum value of abcd is:
(a) 1 (b) 2 (c) $\frac{1}{16}$ (d) data insufficient

37. $8^6 - 5^6$ is divisible by individually:
(a) 91 (b) 49 (c) 129 (d) all of these

38. The sum of all the factors of 45000 which are exactly the multiples of 10 is:
(a) 152295 (b) 14196 (c) 600 (d) none of these

39. A stairway of 20 ft height is such that each step account for half a foot upward and one foot forward. What distance will an ant travel if it starts from ground level to reach the top of the stairway?
(a) 59 (b) 60 (c) 58 (d) none of these

40. Paltry and Sundry, the two bird hunters went to woods paltry fires 5 shots when sundry fires 7 shots but paltry kills 2 out of 5 while sundry kills 3 out of 7. When sundry has missed 32 shots. Then how many birds has paltry killed?
(a) 25 (b) 24 (c) 16 (d) 12

41. What is the sum of the following series?

$$\frac{1}{1 x 2} + \frac{1}{2 x 3} + \frac{1}{3 x 4} + \cdots ...+ \frac{1}{100 x 101}$$

(a) $\frac{100}{101}$ (b) $\frac{1}{101}$ (c) $\frac{101}{100}$ (d) 101!

42. Which one of the following is greatest one?
(a) 3^{3322} (b) 33^{322} (c) 333^{32} (d) 22^{333}

43. If a, b, c, d, e, f are sequentially the terms of an A.P. belong to set (1, 2, 3,….9) where all the terms a, b, c, ….are in increasing order, then the last digit of $a^b \times c^d \times e^f$ is:
(a) 5 (b) 2 (c) 7 (d) either of (a) and (b)

44. $1^2 - 2^2 + 3^2 - 4^2 + ….-199^2$:
(a) 19900 (b) 12321 (c) 19998 (d) none of these

45. If the number $23^{32} - 9$ is divided by 16, then the remainder is:

(a) 8 (b) 0 (c) 6 (d) none of these

46. If x + y + z = 21, then the maximum value of (x+6) (y+7) (z-4) is:
(a) 343 (b) 216 (c) 125 (d) none of these

47. N is the smallest number such that N/2 is a perfect square and N/3 is a perfect cube. Then, the number of divisors of N is
A. 12 B. 16 C. 20 D. 24

48. Let n be the smallest positive number such that the number $S = (8^n)(5^{600})$ has 604 digits. Then the sum of the digits of S is
A. 19 B. 8 C. 10 D. 11

49. How many natural numbers between 1 and 100 have exactly four factors?
A. 30 B. 32 C. 300 D. 33

50. What is the value of n such that n! = 3! × 5! × 7!
A. 10 B. 11 C. 8 D. 9

Exercise – 10

Solutions

1. Ans. (b)
Solution: We know that $(a^n + b^n)$ is divisible by (a+b) if n is an odd number. It means $(17^{21} + 19^{21})$ is divisible by 36 and all the factors of 36.
Therefore the answer is 8 because 8 is not a factor of 36.

2. Ans. (a)
Solution: We know that $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$ and $2^5 = 32$
Therefore cycle of 2 is 4.
now, $\frac{3^{4^5}}{4} = \frac{(-1)^{4^5}}{4} = \frac{1}{4}$
Therefore remainder is 1, so units digit = $2^1 = 2$

3. Ans. (d)
Solution: Number of 5s in 36! = $\left[\frac{36}{5}\right] + \left[\frac{36}{25}\right] = 7 + 1 = 8$
Therefore zeros in $(36!)^{36!} = 8 \times 36!$

4. Ans. (c)
Solution: We know that $7^1 = 7$, $7^2 = 9$, $7^3 = 3$ and $7^4 = 1$, therefore, the cycle of 7 is four.
Now divide $11^{22^{33}}$ by 4.
$\frac{11^{22^{33}}}{44} = \frac{(-1)^{22^{33}}}{4} = \frac{1}{4}$

5. Ans. (b)
Solution: For being a perfect square, the last digit of the number should be 1, 4, 5, 6 and 9. And the digital sum of the number should be 1, 4, 9 and 7.

6. Ans. (a)
Solution: Since the value of y varies between -8 to 2. It is evident that if we take a very small value for y. say 0.000000000000000000000000, and we take normal integral values for x and z, the expression xz/y would become

either positive or negative infinity (depending on how you manage the signs of the number x, y and z). the answer is (a)

7. Ans. (b)
Solution: The multiple of 13 between 200 and 400 would be represented by the series:
208, 221, 234, 247, 260, 273, 286, 299, 312, 325, 338, 351, 364, 377 and 390 There are a total of 15 numbers in the above series. Option (b) is correct.
Note: the above series is an arithmetic progression the process of finding the number of terms in an arithmetic progression are defined in the chapter on progression.

8. Solution: 5
For $x = 1$, $y = 4$, $7x + y = 11$, which is a prime number, $x + y = 1 + 4 = 5$.

9. Solution: 1111111
$11^2 = 121$
$111^2 = 12321$
$1111^2 = 1234321$
$11111^2 = 123454321$
So $1234567654321 = (1111111)^2$

10. Solution: 5
999999 is divisible by 7. It mean 999999…….99 (6 x 16) times is divisible by 7.
So the required remainder = Remainder of $\frac{999}{7} = 5$

11. Solution: 2
We know that $A^{p-1} + p$ leave a remainder of 1, when p is a prime number
Here 41 is a prime number. Hence, $\frac{2^{40}}{41}$ leaves a remainder 1 thus the remainder of $2^{41} + 41$ would be equal to the remainder of $2^1 + 41 \rightarrow 2$ (required remainder).

12. Solution: 40
According to the Wilson theorem if p is a prime number then $(p - 1)! + 1$ is a multiple of p. here 41 is a prime number so $40! + 1$ is completely divisible by 41. This means that 40! Leaves a remainder – 1 when we divide it by 41 or it leaves a remainder $41 - 1 = 40$.

13. $X + 3 = 0$ for $x = -3$

14. Solution: 4
The numbers we need to check for are 1, 3, 7, 15, 31, 63, 127, 255, 511, 1023, 2047 and 4095. Out of these, the prime numbers are 3, 7, 31 and 127. Hence 4 number.

15. Ans. (b)
Solution: the value of x should be such that the left hand side after completely removing the square root sings should be an integer for this to happen, first of all the square root of 3x should be an integer. Only 3 and 12 from the option satisfy this requirement. If we try to put x as 12, we get the square root of 3x as 6. Then the next point at which we need to remove the square root sing would be $12 + 2(6) = 24$ whose square root would be an irrational number. This leaves us with only 1 possibly value ($x = 3$) checking for this value of x we can see that the expression is satisfied as LHS = RHS.

16. Ans. (c)
Solution: $1 + 2 + 2^2 + \ldots.. + 2^n = 2^{32} - 1$
Hence the average will be; $\frac{2^{32}-1}{32} = 2^{27} - 1/2^5$
Which lies between 2^{26} and 2^{27}.

17. Ans. (d)
Solution: $165 + 2^{15} = 2^{20} + 2^{15} = 2^{15}(2^5+1)$ → Hence is divisible by 33.

18. Ans. (c)
Solution: The expression would have solution based on a structure of:
4 + 0; 3+1; 2+2; 1+3 or 0+4
There will be 2 x 1 = 2 solutions for 4+0 as in this case x can take the values of 8 and 0. While y can take a value of 4; Similarly there would be 2 x 2 = 4 solutions for 3 + 1 as in this case x can take the values of 7 or 1, while y can take a value of 5 or 3.
Thus, the total number of solutions can be visualized as:
2 (for 4 +0) + 4 (for 3 + 1) + 4 (for 2 +2) + 4 (for 1+3) +2(for 0+2) = 16 solutions for the set (x,y) where both x and y are integers.

19. Ans. (c)
Solution: $990 = 11 \times 3^2 \times 2 \times 5$. The highest power of 9990 which would divide 1090! Would be the power of 11 available in 1090. This is given by [1090/11] + [1090/121] = 99+9 = 108

20. Ans. (d)
Solution: Both 333^{555} and 555^{333} are divisible by 3,37 and 111. Further, the sum of the two would be an even number and hence divisible by 2. Thus all the four option divide the given number.

21. Ans. (b)
Solution: The number of zeroes would be given by counting the number of 5s the relevant number of counting the number of 5s' in the product would be given by:
55; 10^{10}; 15^{15}; 20^{20}; 25^{25}......and so on till 100^{100}
The number of 5s' in these values would be given by:
(5+10+15+20+5030+35+40+45+100+55+60+65+70+150+80+85+90+95+200)
This can also be written as:
(5+10+15+20+25+30+35+40+45+50+55+60+65+70+75+80+8590+95+100) + (25+50+75+100)
= 1050 + 250 = 1300

22. Ans. (a)
Solution: $45 = 3^3 \times 5$. Hence we need to count the number of 3^2's and 5's that can be made out of 123!
Number of 3's = 41+13+4+1 = 59 → number of 3^2's = 29
Number of 5's = 24 + 4 = 28
The Required answer is the lower of the two (viz. 28 and 29) hence option (a) 28 is correct.

23. Ans. (a)
Solution: There will be 9 single digit numbers using 9 digits, 90 two digit numbers using 180 digits, 900 three digit numbers using 2700 digits. Thus, when the number 999 would be written, a total of 2889 digits would have been used up thus, we would need to look for the 25494th digit when we write all 4 digit numbers. Since 25494/4 = 6373.5 we can conclude that the first 6373 four digit numbers would be used up for writing the first 25492 digits the second digit of the 6374th four digit number would be the required answer since the 6374th four digit number is 7373, the required digit is 3.

24. Ans. (c)
Solution: It can be seen that for only 2 number (1 and ½) the consolidated number would be 1+½ + ½ = 2
For 3 numbers, (1, ½, 1/3) the number would be 3. Thus for the given series the consolidated number would be 1972.

25. Ans. (c)
Solution: The remainder of each power of 9 when divided by 6 would be 3. Thus for (2n+1) powers of 9, there would be an odd number of 3's hence the remainder would be 3.

26. Ans. (d)

Solution: The remainder when a number is divided by 16 is given by the remainder of the last 4 digits divided by 16 because 10000 is a multiple of 16. This principle is very similar in logic to why we look at last 2 digits for divisibility by 2 and the last 3 digits for divisibility by 8. Thus the required answer would be the remainder of 4950/16 which is 6.

27. Ans. (d)
Solution: $12^{33} \times 34^{23} \times 2^{70} = 2^{159} \times 3^{33} \times 17^{23}$.
The number of factors would be 160 x 34 x 24 = 130560. Thus, option (d) is correct.

28. Ans. (b)
Solution: $1152 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2\ 3 \times 3 = 2^7 \times 3^2$
Essentially every number starting from $4!^3$ would be divisible perfectly by 1152 since each number after that would have at least 7 twos and 2 threes.
Thus, the required remainder is got by the first three terms;
(1+8+216)/1152 = 225/1152 gives us 225 as the required remainder.

29. Ans. (c)
Solution: Remainder of

$$\frac{[111111111.......64\ terms] \times [22222222.....55\ terms]}{18}$$

= 2 x remainder of

$$\frac{[(111111111......64\ terms) \times (111111......55 terms)]}{9}$$

$\rightarrow$ Remainder = 1
So, required remainder = 2 x 1 =2

30. Solution: 0
We can see from the expression that (x-y) (x+y) is an even number both x – y and x + y are even is an even number both (x – y)(x+y) must be divisible by 4. But 777314 is not divisible by 4. So it it has no solution. The correct answer would be 0.

31. Ans. (b)
Solution: Again, spotting this with options is quite easy as we can see that $7^2 + 8^2 = 113$ and that is 112 less than the value of $(7+8)^2 = 225$, without option here you can think of $a^2 + b^2 + 112 = (a+b)^2 \rightarrow 2ab = 112$ or ab = 56. Since, the numbers are consecutive, sifting through the factor pairs of 56 we can see the numbers as 7 and 8, respectively.

32. Ans. (b)
Solution: 5 x (3, 13, 23, 33, 43, 53, 63, 73, 83, 93) + 5 x (30, 31, 32, 33, 34, 35, 36, 37, 38, 39) + (301, 302, 303, 304, ….., 399)
= 5 x 10 + 5 x 10 + 100 = 200
Explanations: In every set of 100 numbers there are 10 numbers whose unit digit is 3. Similarly in every set of 100 number there are 10 numbers whose tens digit is 3 and there are total 100 numbers whose hundreds digit is 3.

33. Ans. (d)
Solution: $a^2 – b^2 = (a+b)(a-b)$
Let us consider two odd prime number as 3 and 29
Then $(29)^2 - 3^2 = (29 + 3)\ (29-3)$
= 32 x 26.
Which is divisible by 13.
Again consider 7 and 29.
Then $(29)^2 - 7^2 = (29 + 7)\ (29-7)$
= 36 x 22.
Which is divisible by 11.
Further consider 3 and 37. Then $(37)^2 – (3)^2 = 40 \times 34$ which is divisible by 17.
Hence (d) is correct.

34. Ans. (b)

Solution: $\frac{75^{75^{75}}}{37} = \frac{1^{75^{75}}}{37}$ → remainder is 1.

35. Ans. (a)

Solution: Let $\frac{x}{y}$ be the fraction, then

$\frac{x+4}{y} = \frac{1}{3}$

And $\frac{x}{y+3} = \frac{1}{6}$

⇨ 3x + 12 = y

⇨ 6x = y + 3

⇨ X = 5 and y = 27

Thus, x + y = 32

36. Ans. (c)

Solution: Given that $a^2 + b^2 + c^2 + d^2 = 1$

Now, the maximum value of a,b,c,d will be only when

a= b = c=d = k

then $a^2 + b^2 + c^2 + d^2 = 4k^2 = 1$

⇨ $k = \pm\frac{1}{2}$

Now a,b,c,d = $k^4 = (\pm\frac{1}{2})^4 = \frac{1}{16}$

Hence (c) is correct.

37. Ans. (d)

Solution: 86 – 56 = $(8^2)^3 - (5^2)^3$ = (64)+ – $(25)^3$

= (64-25) $(64^2 + 25^2 + 64 \times 25)$ = 39 x 6321

= 13 x 3 x 3 x 7 x 7 x 43

So it can be divided by 13 x 7 = 91, 7 x 7 = 49, 3 x 43 = 129

Hence all of these are possible.

38. Ans. (b)

Solution: since 45000 = = 10 x 4500

So, the sum of all the factors of 45000 which are the multiples of 10 will be same as sum of all the factors of 4500 Multiple by 10.

Now, $4500 = 2^2 \times 3^2 \times 5^3$

Therefore the sum of all the factors of

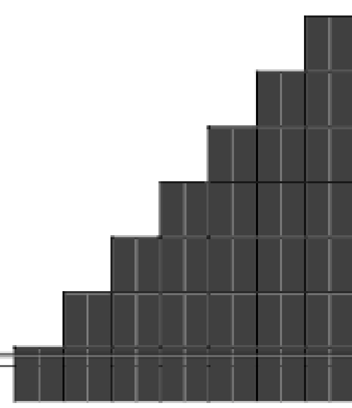
$4500 = \frac{(2^3-1)x(3^3-1)(5^4-1)}{(2-1)(3-1)(5-1)}$

$= \frac{7 x 26 x 624}{1 x 2 x4}$ = 14196

Hence the sum of all the products of 45000 which are the multiples of 10 = 10 x 14196 = 141960.

39. Ans. (a)

Solution: Total distance to be covered = total horizontal movement + total upward movement



= (20x 2- 1) + (20)

= 39 + 20 = 59 feet

40. Ans. (c)

Solution:

	Paltry	Sundry
Fire shots	5	7
Hit shots	2	3
Missed shots	3	4

When Sundry missed 32 shots it means paltry missed 24 shots when paltry missed 24 shots it means paltry hit 16 shots.

41. Ans. (a)

Solution: $\frac{1}{1 x 2}+\frac{1}{2 x 3}+\frac{1}{3 x 4}+\cdots ...+\frac{1}{100 x 101}$

$=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\cdots ...+ (\frac{1}{100}-\frac{1}{101})$

$= 1-\frac{1}{101}=\frac{100}{101}$

42. Ans. (a)

Solution: 3^{3322} and 33^{332}

⇨ $(3^4)^{830} \times 3^2$ and 33^{332}

Thus $(81)^{830} \times 3^2 > 33^{332}$

Again 3^{3322} and 333^{22}

⇨ $(3^6)^{553} \times 3^4$ and $(333)^{22}$

Thus $(729)^{553} \times 3^4 > (333)^{22}$

Again 3^{3322} and 22^{333}

⇨ $(3^3)1107 \times 3$ and 22^{333}

Thus $(27)^{1107} \times 3 > 22^{333}$

Hence (a)

43. Ans. (d)

Solution: if a, b, c, d, e, f all the six forms of an A.P. can have only 9 values hence the common difference of the A.P. must be1. (Since common difference cannot be greater than 1.)

If a = 1, then

$1^2 \times 3^4 \times 5+$ → Unit digit is 5

If a = 2, then

$2^3 \times 4^5 \times 6^7$ → unit digit is 2

If a = 3 then

$3^4 \times 5^6 \times 7^8$ → Unit digit is 5

If a = 4, then

$4^5 \times 6^7 \times 8^9$ → unit digit is 2

44. Ans. (a)

Solution: $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots..-198^2 + 199^2$

$= (1^2-2^2) + (3^2-4^2) + (5^2-6^2)+\ldots\ldots..+(197^2 - 198^2) + 199^2$

$= (-3) + (-7) + (-11) + (-15) + \ldots\ldots.+ (-395) + (199)^2$

$=\left[-\left(\frac{3+395}{2}\right) x\ 99\right] + 39601$

$= (-199 \times 99) + 39601$

$= (-19701) + 39601 = 19900$

45. Ans. (a)

Solution: $\frac{23^{32}-9}{16} \rightarrow (\frac{23^{32}}{16} - \frac{9}{16}) \rightarrow (\frac{7^{32}}{16}-\frac{9}{16}) \rightarrow [\frac{(7^5)^{16}}{16} - \frac{9}{16}] \rightarrow \frac{1^{16}}{16} - \frac{9}{16}$

$\rightarrow \frac{1-9}{16} \rightarrow \frac{-8}{16} \rightarrow \frac{16-8}{16} \rightarrow \frac{8}{16} \rightarrow 8$

46. Ans. (b)
Solution: (x - 6) (y+7) (z-4) will be maximum only when
(x-6) = (y+7) = (z-4) = k (say) for a given value of
x = k + 6
y = k -7
z = k +4
x + y + z = 21
⇨ (k+6) + (k-7) + (k + 4) = 21
3k + 3 = 21
K = 6
Therefore, (x-6), (y+7) (z-4) = $k^3 = (6)^3 = 216$

47. Ans. (C)
Solution: $\frac{N}{2}$ is a perfect square $\Rightarrow \frac{N}{2} = x^2$, $\frac{N}{3}$ is a perfect cube $\Rightarrow \frac{N}{3} = y^3$. $\frac{N}{2} \times \frac{N}{3} = x^2y^3 = N = \sqrt{6x^2y^3} = xy\sqrt{6y}$ For N to be a natural number, 6y has to be a perfect square ⇒ y = 6 ⇒ N = $2^3 \times 3^4$ ⇒ number of divisors = 20.

48. Ans. (A)
Solution: $2^{600} \times 5^{600}$ will have 601 digits (1 followed by 600 zeroes). To get 604 digits we will have to increase the power of 2 such that we get a four digit number followed by 600 zeroes. The power should also be a multiple of 3 as $8 = 2^3$. The smallest such power is $2^{12} = 8^4$. Therefore, S = $8^{204} \times 5^{600}$ = 4096000000… (604 digits). The sum of the digits of S = 19.

49. Ans. (B)
Solution: The number will be of the form a × b or c^3 , where a, b and c are prime factors. Count all the numbers below 100 of the above form.

50. Ans. (D)
Solution: Since R.H.S. has 7! Multiplied by a positive quantity, L.H.S will be greater than 7!. As R.H.S. does not have the prime factor of 11, L.H.S. will be less than 11!. As R.H.S. has only one power of 5 (in 5!), L.H.S. will be less than 10!. Therefore, only possibilities are 8! And 9!. Considering powers of 3 on both sides, n = 9.

Exercise – 11

1. How many integers N in the set of integers (1, 2, 3,……100) are there such that $N^2 + N^3$ is a perfect square?
(a) 5 (b) 7 (c) 9 (d) 11

2. A boy was carrying a basket of eggs. He fell down and some of the eggs were broken. The boy has 10 eggs left with him. When asked by his mother, how many eggs were broken, the boy could not recall. However, he recalled that when 1 egg was left, he counted the total number of eggs as 3. When counted 4 at a time, 1 egg was left and when counted 5 at a time, no egg was left. How many eggs were broken?
(a) 15 (b) 25 (c) 30 (d) 35

3. How many prime numbers are there which when divided by another prime number, gives a quotient which is same as the remainder?
(a) 0 (b) 1 (c) 2 (d) more than 2

4. What is the remainder when 7 + 77 + 777 + 7777 + …. (till 100 terms) is divided by 8?
(a) 0 (b) 2 (c) 4 (d) 6

5. N^2 = 12345678987654321. Find N.
(a) 101010101 (b) 11111 (c) 111111111 (d) 1000000001

6. How many number of zeroes will be there eat the end of 12! Expressed in base 6?
(a) 4 (b) 5 (c) 6 (d) 7

7. How many divisors of 10^5 end with a single zero?
(a) 1 (b) 3 (c) 9 (d) 16

8. When a certain two digit number is added to another two digit number having the same digits in reverse order, the sum is a perfect square. How many such two digit numbers are there?
(a) 4 (b) 6 (c) 8 (d) 10

9. If p, (p+2), and (p+4) are prime numbers, then the number of possible solutions for p is:
(a) 0 (b) 1 (c) 2 (d) none of these

10. P is a natural number, 2P has 28 divisors and 3P has 30 divisors. How many divisors of 6P will be there?
(a) 35 (b) 40 (c) 45 (d) 48

11. pqr is a three digit natural number such that pqr = $p^3 + q^3 + r^3$ what is the vaue of r?
(a) 0 (b) 1 (c) 3 (d) cannot be determined

12. When asked about his date of birth in 1996, Mayank replied that last two digits of my birth year stands for my age. When siddharth was asked about his age, he also replied the same, however, siddharth is older to mayank. What is the difference in their age?
(a) 46 (b) 50 (c) 0 (d) cannot be determined

13. What is the value of N in the following expression $345_6 + 632_7 + 487_9 = (N)_5$?
(a) 11,412 (b) 11,214 (c) 10,412 (d) 21,412

14. Find the sum of all three digit numbers that give a remainder of 4 when they are divided by 5.
(a) 98, 270 (b) 99,270 (c) 1,02,090 (d) 90,270

15. Find the sum of all odd three digit numbers that are divisible by 5.
(a) 50,500 (b) 50,250 (c) 50,000 (d) 49,500

16. How many pairs of natural numbers are there the difference of whose squares is 45?
(a) 1 (b) 2 (c) 3 (d) 4

17. In a four digit number, the sum of the digits in the thousands and tens is equal to 4, the sum of the digits in the hundreds and the units is 15, and the digit of the units exceeds by 7 the digit of the thousands, among all the numbers satisfying these conditions, find the number the sum of the product of whose digit of the thousands by the digit of the units and the product of the digit of the hundreds by that of the tense assumes the least value.

(a) 4708 (b) 1738 (c) 2629 (d) 1812

18. If we divide a two digit number by a number consisting of the same digits written in the reverse order, we get 4 as a quotient and 15 as a remainder. If we subtract 1 from the given number, we get the sum of the squares of the digits constituting that number. Find the number.
(a) 71 (b) 83 (c) 99 (d) none of these

19. The remainder obtained when $43^{101} + 23^{101}$ is divided by 66 is
(a) 2 (b) 10 (c) 5 (d) 0

20. When $2222^{5555} + 5555^{2222}$ is divided by 7 the remainder is
(a) 0 (b) 2 (c) 4 (d) 5

21. If x is a number of five digits which when divided by 8, 12, 15 and 20 leaves respectively 5, 9, 12 and 17 as remainders then find x such that it is the lowest such number.
(a) 10017 (b) 10057 (c) 10097 (d) 10077

22. $\frac{32^{32^{32}}}{9}$ will leave a remainder
(a) 4 (b) 7 (c) 1 (d) 2

23. Find the remainder of 2^{100} when divided by 3.
(a) 3 (b) 0 (c) 1 (d) 2

24. Find the remainder when the number 3^{1989} is divided by 7.
(a) 1 (b) 5 (d) 6 (d) 4

25. The remainder when $10^{10} + 10^{100} + 10^{1000} + \ldots\ldots + 10^{10000000000}$ is divided by 7 is
(a) 0 (b) 1 (c) 2 (d) 5

26. Find the remainder when $50^{51^{52}}$ is divided by 11.
(a) 4 (b) 6 (c) 7 (d) 3

27. Find the remainder when $30^{72^{87}}$ is divided by 11.
(a) 5 (b) 9 (c) 6 (d) 3

28. Find the remainder when $50^{56^{62}}$ *is* divided by 11.
(a) 7 (b) 5 (d) 9 (d) 10

29. Find the remainder when $33^{34^{35}}$ is divided by 7.
(a) 5 (b) 4 (c) 6 (d) 2

30. How many integer values of x and y satisfy the expression $4x + 7y = 3$ where $|x| < 1000$ and $|y| < 1000$?
(a) 284 (b) 285 (c) 286 (d) none of these

31. How many factors of 19! are there, whose unit digit is 5?

32. A four digit number wxyz is such that x + y = 2w & y + 6z = 2(w+x) & w+ 5z = 2y find the sum of such four digit numbers which satisfy the given conditions.

33. Amongst all the four digit natural numbers divisible by 24, how many have the number 24 in them?

34. A nine digit number abcdefghi is such that a is divisible by 1. Ab is divisible by2, abc is divisible by 3 and abcd is divisible by 4 and so on where none of a, b, c, d…. is same and every digit is a non zero digit such a number is:
(a) 123456789 (b) 381654729 (c) 126453789 (d) 826435791

35. If A and B are divided by D, then the remainder obtained are 13 and 31 respectively where A, B, D are natural numbers. Further A+ B is divided by the same divisor D the remainder is 4, then the divisor D is:
(a) 14 (b) 23 (c) 40 (d) 18

36. Which of the following is correct if A = $3^{3^{3^3}}$, B = 3^{33^3}, C = $3^{3^{33}}$, and D = 3^{333}?
(a) A > B = C > D (b) C > A > B > D
(c) A > C > D > B (d) C > B > D > A

37. A number 'p' is such that it is divisible by 7 but not b 2. Another number 'q' is divisible by 6 but not by 5, then the following expression which necessarily be an integer is:
(a) $\frac{7p+6q}{42}$ (b) $\frac{5p+6q}{71}$ (c) $\frac{6p+7q}{42}$ (d) none of these

38. The remainder when $(888!)^{9999}$ is divided by 77 is:
(a) 1 (b) 2 (c) 3 (d) none of these

39. There are six locks exactly with one key for each lock. All the keys are mixed to each other. The maximum number of attempts needed to get the correct combination is:
(a) 21 (b) 15 (c) 6 (d) can't be determined

40. One day very early morning Ravishankar went to temple to offer some flowers as a part of Puja. He purchased some flowers but the seller offered him that if the would give him all his Rs. 2 he could get all the remaining 6 flowers and thus could gain 60 paise per dozen, if each time the transaction is possible only in rupees then how many flowers did Ravishankar purchase initially?
(a) 6 (b) 3 (c) 4 (d) 12

41. The sum of :
$(2^2 + 4^2 + 6^2 + \ldots.+100^2) - (1^2 + 3^2 + 5^2 + \ldots.+99^2)$ is:
(a) 5555 (b) 5050 (c) 888 (d) 222

42. The remainder when $4^0 + 4^1 + 4^2 + 4^3 + \ldots.+4^{40}$ is divided by 17 is:
(a) 0 (b) 16 (c) 4 (d) none of these

43. Capt. Mano Pandey once decided to distribute 180 bullets among his 36 soldiers. But he gave n bullets to a soldier of nth row and there were same number of soldiers in each row. Thus he distributed all his 180 bullets among his soldiers. The number of soldiers in (n-1) the row was:
(a) 3 (b) 8 (c) 9 (d) none of these

44. All the page numbers from a book are added, beginning at page 1. However, one page number was added twice by mistake. The sum obtained was 1 000. Which page number was added twice? (CAT 2001)
A. 44 B. 45 C. 10 D. 12

45. If $x = \sqrt{2(1+\sqrt{2})}$ then the value of $x^3 + x^2 - 2x - 2$ is
A. 0
B. $6\sqrt{1+\sqrt{2}} + 3(1+\sqrt{2})$
C. $2\sqrt{2} + 4\sqrt{1+\sqrt{2}}$
D. $1+\sqrt{2}$

46. For how many integers n is $\frac{5n+23}{n-7}$ also an integer?
A. 2 B. 4 C. 6 D. 8

47. In the value of the number 30!, all the zeroes at the end are erased. Then, the unit digit of the number that is left is
A. 2 B. 4 C. 6 D. 8

48. Let $S = (3 + 3^2 + 3^3 + \ldots + 3^{400}) - (7 + 7^2 + 7^3 + \ldots + 7^{201})$. The last two digits of S are
A. 00 B. 07 C. 43 D. 93

49. How many positive integers less than or equal to 120 are relatively prime to 120?
A. 24 B. 32 C. 36 D. 40

50. Given that x and y are integers and $5x^2 + 2y^2 = 5922$, what can be the unit digit of y?
A. 3 B. 5 C. 7 D. 9

51. What is the remainder when the number $123\underbrace{123123\ldots\ldots123123}_{300\text{digits}}$ is divided by 99?
A. 18 B. 27 C. 33 D. 36

52. If $x = (16^3 + 17^3 + 18^3 + 19^3)$, then x divided by 70 leaves a remainder of (CAT 2005)
A. 69 B. 35 C. 0 D. 1

53. What is the remainder when 4^{96} is divided by 6? (CAT 2003)
A. 0 B. 3 C. 2 D. 4

54. What is the remainder when $(17)^{36} + (19)^{36}$ is divided by 111?
A. 0 B. 2 C. 36 D. 110

55. How many zeroes are present at the end of 25! + 26! + 27! + 28! + 30!?
A. 5 B. 6 C. 7 D. 8

56. Suppose you write the numbers 0, 1, 2, 3,…, 20 on the board. You perform following operation 20 times- You pick up two numbers at random (call them a and b), erase them, and write down □a -b□. Then, the final number left on the board is

A. odd B. even C. more than 10 D. less than 10

Exercise – 11

Solutions

1. Ans. (c)

Solution: $N^2 + N^3 = N^2 (N+1)$

For $(N^2 + N^3)$ to be a perfect square, $(N+1)$ should be a perfect square.

Further we know that there are 10 perfect squares till 100. However we cannot take $(N+1) = (1 \rightarrow N) = 0'$

Therefore, there are 9 number for which $N^2 (N+1)$ will be a perfect square.

2. Ans. (a)

Solution: Let us first find the number that is divided by 3, 4 and 5. Which gives remainder 1, 1 and 0. Respectively. It is equal to 25.

It is given that only 10 eggs are left now. It means 25 – 10 = 15 eggs has been broken.

3. Ans. (b)

Solution: There is only one set of prime number that satisfies the given condition, and the set of prime number is (2,3)

4. Ans. (d)

Solution: $\frac{7+7+777......(till\ 100\ terms)}{8} = \frac{7}{8} + \frac{77}{8} + \frac{777}{8}$ ….(till 100 terms)

$\frac{7+5+1+1+1.....(till\ 100\ terms)}{8} = \frac{7+5+98}{8} = \frac{6}{8}$

Hence, remainder = 6

5. Ans. (c)

Solution: Following patterns can be observed:

$(11)^2 = 121$

$(111)^2 = 12,321$

$(1111)^2 = 12,34,321$

$(111111111)^2 = 12,345,678,987,654,321$

6. Ans. (b)

Solution: In the case of decimal system we obtain 10 by multiplying 5 and 2. And then, to find the number of zeroes we search the exponents of 5. In the case of base 6, 10 will be obtained by multiplying 3 and 2 therefore, here, we will check for the exponents of 3 to know about the number of zeroes, obviously it is 5[12/3+12/9]

7. Ans. (c)

Solution: $10^5 = 2^5 \times 5^5$

Now, all the factors of 10^5 that will end in one zero will be one power of 2 and (1-5) powers of 5 and vice versa

This will be equal to 9.

8. Ans. (c)

Solution: Let the number is AB

For perfect square = AB + Ba = (10A + B) + (10B + A) = 11(A+B)

For being a perfect square, (A+B) should be equal to 11.

Then(A+B) = 11 now find the sets of values of A and B.

9. Ans. (b)

Solution: There is only one set possible where p = 3, (p+2) = 5. And (p+4) = 7
In every other set, one number will be divisible by 3 and hence that number will not be a prime number.

10. Ans. (a)
Solution: 2P is having 28 (7 x 4) divisors but 3P is not having a total divisor, which is divisible by 7. Therefore, the first part of the number P will be 2^5.
Similarly 3P is having 30 (3 x 10) divisors but 2P does not have a total divisor, which is divisible by 3 therefore, second part o the number P will be 3^3. Therefor, $P = 2^5 \times 3^3$

11. Ans. (d)
Solution: pqr can be 370 or 371. Therefore, it is not possible to arrive at a unique answer.

12. Ans. (b)
Solution: Mayank DOB = 1948 and Siddharth DOB = 1898

13. Ans. (a)
Solution: $(N)_5 = 345_6 = 632_7 + 487_9 = 137_{10} + 317_{10} + 403_{10} = 857_{10} = 11{,}412$

14. Ans. (b)
Solution: the required numbers would be numbers in the arithmetic progression 104, 109,114,119,……999 the sum of this series would be given as n x average = $180 \times \frac{1103}{2} = 99270$

15. Ans. (d)
Solution: the required numbers would be numbers in the arithmetic progression 105, 115, 125,…..995. the sum of this series would be given as n x average = $90 \times \frac{1100}{2} = 49500$

16. Ans. (c)
Solution: (x + y) (x + y) = 45 working through factor pairs of 45, we get 15 x 3; 45 x 1 and 9 x 5 as the three factor pairs here. The numbers are 9 & 6; 22& 23; 7&2.

17. Ans. (b)
Solution: Since this question has close ended options, it is fine if you were to solve it by just checking the options, however, I would encourage you to solve this in a no option scenario too.

18. Ans. (d)
Solution: The number would obviously need to have 1 as its unit digit (as otherwise the quotient of 4 would not be possible to achieve). Hence the only relevant numbers to check would be 71, 81 and 91 for a quotient of 4. Out of these, the number 91 also meets the remainder of 15 requirements. Hence. The correct answer is 91.

19. Ans. (d)
Solution: $43^{101} + 23^{101}$ is of the form $a^n + b^n$ with n odd. Such a number can be written to be a multiple of (a+b). thus the given expression is a multiple of (43+23) = 66. Hence the required remainder would be 0.

20. Ans. (a)
Solution: $2222^{5555} \div 7 \rightarrow 3^{5555} \div 7 \rightarrow 3^5 \div 7 \rightarrow$ Remainder = 5;
$5555^{2222} \div 7 \rightarrow 4^{2222} \div 7 \rightarrow 4^2 \div 7 \rightarrow$ remainder = 2.
Hence the required remainder would be $(5+2) \div 7 = 0$.

21. Ans. (d)
Solution: Since the LCM of 20, 15, 12 and 8 is 120 we need the smallest 120n – 3 number is 5 digits 120 x 84 = 10080. Thus the required number is 10080 – 3 = 10077.

22. Ans. (a)
Solution: $32^{32^{32}} \div 9 \rightarrow 5^{32^{32}} \div 9 = 5^{6n+x} \div 9$.

We write this in the form of 5^{6n+x} because 5^6 leaves a remainder of 1 when divided by 9. When we try to see 32^{32} as 6n + x, we can find the value of x as the remainder of 2^{32} when divided by 6. The following thought process would help us find this value;
$2^{32} \div 6 = 2^{31} \div 3 \rightarrow$ remainder = 2 (by the $a^n \div (a+1)$rule) thus, $2^{32} \div 6$ would have a remainder of 2 x 2 = 4
Hence the required remainder would be $5^4 \div 9$, which is 4.

23. Ans. (c)
Solution: Since the power on 2 is even, the remainder would be 1.

24. Ans. (c)
Solution: $3^6 \div 7$ leaves a remainder of 1. If we look at the power 1989 as 6n +x, we will get x as 3. Hence the remainder of $3^{1989} \div 7$ would be the same as the remainder of $3^3 \div 7$, i.e. 6.

25. Ans. (d)
Solution: The remainder of $(10^{10} + 10^{100}\ 10^{1000} + \ldots\ldots + 10^{10000000000}) \div 7 \rightarrow (3^{10} + 3^{100} + 3^{1000} + \ldots\ldots + 3^{10000000000}) \div 7 \rightarrow$ $(3^4+3^4+3^4+3^4+3^4+3^4+3^4+3^4+3^4+3^4) \div 7$ = remainder of $40 \div 7 \rightarrow 5$.

26. Ans. (b)
Solution: $\frac{50^{51^{52}}}{11} \rightarrow \frac{6^{51^{52}}}{11} = \frac{6^{10x}\ x\ 6^1}{11} \rightarrow$ remainder = 6.

27. Ans. (a)
Solution: $\frac{30^{72^{87}}}{11} \rightarrow \frac{8^{72^{87}}}{11} = \frac{8^{10x} + 8^{2^{87}}}{11} \rightarrow \frac{1 X 8^{2^{87}}}{11} \rightarrow \frac{8^{10x} + 8^{2^8}}{11} \rightarrow$ Remainder = 5

28. Ans. (b)
Solution: $\frac{50^{56^{62}}}{11} \rightarrow \frac{6^{56^{62}}}{11} \rightarrow \frac{6^{10x} + 6^6}{11} \rightarrow$ remainder = 5

29. Ans. (d)
Solution: $\frac{33^{34^{35}}}{7} \rightarrow \frac{5^{34^{35}}}{7} = \frac{5^{6x}\ X\ 5^4}{7} \rightarrow$ remainder = 2.

30. Ans. (b)
Solution: The first solution easily visibly here would be at x = -1 and y = 1 in such equations, we should know that the value of x would change with the coefficient of y, while the value of y would change with the coefficient of x (& the two values would move in the middle). Thus the series of values of x from its highest positive value below 1000 to the lower limit of being just above – 1000 would be 993, 986,.....13,6-1,-8,-15,...-995. The number of terms in this series = $\frac{1988}{7} + 1 = 285$.

31. Solution: 1296
In order to solve this question, you would need to find the odd factors of 19! That are also multiples of 5.
$19! = 2^{16} \times 3^8 \times 5^3 \times 7^2 \times 11^1 \times 13^1 \times 17^1 \times 19^1$.
The required answer would be 1 x 9 x 3 x 3 x 2 x 2 x 2 x 2 = 81 x 16 = 1296.

32. Solution: 9723
X + y = 2w 1
Y + 6z = 2 (w+x)......2
W + 5z = 2y.......3
From equation 2 – equation 1 we get: 6z – x = 2x or x = 2z
Substituting x = 2z in equation 1, we get : 2w – y = 2z 4
Solving equation 3 and 4 we get w = 3x and y = 4z z:y: x = 1 : 4 : 2 :3
So 3241 and 6482 are two possible values of wxyz so the required sum = 3241 + 6482 = 9723.
Alternately,
you can think of values and try to fit in the conditions of the other equations. z and x can take only
4 feasible values: viz 1,2; 2,4;3,6 & 4,8

This gives us fur possibilities for the numbers. 2 1; 4 2; 6_3; 8 4. The fourth of these, with z = 4 can be eliminated by looking at the third equation (w+5z= 2y) as it would need y to be greater than 10

For z = 3 and x = 6; we get w = 1 and y = 8 or w = 3 and y = 9 from the third equation, both these values do not match the second equation.
For z = 2 and x = 4 we get w = 2 and y = 6 or w = 4 and y = 7 or w = 6 and y = 8 or w = 8 and y = 9 checking for the second equation, only the values of w = 6 and y = 8 matches hence we get the number 6482.
Likewise, when you check for z = 1 and x = 2 you would be able to find the number 3241.

33. Solution: 25
If the 4- digit number is abcd then three cases are possible for the number to have 24 in it.
Ab24
Ab24 = 100ab + 24
24 is divisible by 24 and 100 x ab must be divisible by 24, 100 is divisible so 4 then ab must be divisible by 6. Possible values of ab = 12, 18, 24 …..,96 so case b; if bc = 24 then the number are of the form a24b.
A24b must be divisible by 3 and 8 if a24b is divisible by 8 then 24b is divisible by 8. Possible values of b = 0, 8
Similarly a + 2 +4 +b = (a+b)+ 6 must be divisible by 3.
When b = 0 then a = 3, 6, 9 (3 possible cases)
When b = 8 then a = 1, 4, 7 (3 possible cases)
So there are total 6 possible cases.
Case C ab = 24 so the number should be of the form 24cdb.
24ab = 2400 + cd
2400 is divisible by 24, cd divisible by 24 when cd = 00, 24, 48, 72, 96 (5 possible cases)
However the number 2424 occurred in cases a and b both so the total possible numbers = 5 + 6 + 15 – 1 = 25.

34. Ans. (b)
Solution: No explanations available

35. Ans. (c)
Solution: If you go through option, then you will find option (c) is correct. Since divisor is always greater than remainder.
Alternatively : $\frac{A}{D}$ → Remainder 13
$\frac{B}{D}$ → Remainder 31
$\frac{A+B}{D}$ → remainder should be 44
But it is given that the remainder is 4.
Hence the divisor must be 40 (=44 - 4)

36. Ans. (b)
Solution: $A = 3^{3^{3^{3}}} = 3^{3^{27}}$
$C = 3^{3^{33}} = 3^{3^{33}}$

Hence C > A.
Hence either (b) or (d) option is correct.
Now $A = 3^{3^{3^{3}}} = 3^{3^{27}}$
And $D = 3^{333}$ (Since 3^{27}>333)
Thus the correct relation is C > A > B > d/
Hence option (b) is correct.

37. Ans. (c)
Solution: Go through option
$\frac{6p+7q}{42} \rightarrow \frac{6p}{42} + \frac{7q}{42}$

Since p is divisible by 7 and q is divisible by 6 thus

$\frac{6p}{42} + \frac{7q}{42} \rightarrow \frac{6 \, x \, 7m + 7 \, x \, 6n}{42}$

$= \frac{42(m+n)}{42} = (m + n)$

38. Ans. (d)

Solution: $(888!)^{999} = (1 \times 2 \times 3 \times 4 \times \ldots\ldots \times 76 \times 77 \times 78 \times \ldots \times 888)^{999}$

Since it has a factor as 77. Thus it has no remainder.

39. Ans. (b)

Solution: Let there be six locks A, B, C, D, E and F.

Then we need maximum 5 attempts to know the right 'A'. again we need maximum 4 attempts to know the right key for 'B'. similarly we need 3, 2 and 1 attempts to know the proper key for the locks C, D and E. Now 1 key is automatically left for the lock 'F'. Thus the total number attempts required = 5 + 4 + 3 + 2 + 1 = 15

40. Ans. (c)

Solution: Initially Ravi Shankar has only R.s 2 with him. Since we know that the Ravi Shankar can pay the sum in rupees only it means he cannot pay in paise. Therefore he must have purchased initially the flowers of exactly Re. 1.

Now from the options consider option (c)

The number of flowers he has purchased for Re 1 = 4.

Later on he could purchased the number of flowers for toal Rs. 2 = 4 + 6 = 10.

Thus the initial cost of one dozen flowers = Rs. 3

And the changed cost of one dozen flowers = Rs. 2.40

Thus he could gain 60 paise per dozen. Hence the presumed option (c) is correct.

41. Ans. (b)

Solution: $(2^2 + 4^2 + 6^2 + 8^2 + \ldots + 100^2) - (1^2 + 3^2 + 5^2 + \ldots + 99^2)$

$= (2^2 + 4^2 + 6^2 + \ldots\ldots + 100^2) = [(1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + 99^2 + 100^2)]$

$= 2\,(2^2 + 4^2 + \ldots\ldots 100^2) - (1^2 + 2^2 + 3^2 + \ldots\ldots + 100^2)$

$= 8\,(1^2 + 2^2 + 3^2 + \ldots\ldots + 50^2) - (1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots 100^2)$

$= 8\left(\frac{50 \, x \, 51 \, x \, 101}{6}\right) - \left(\frac{100 \, x \, 101 \, x \, 201}{6}\right)$

$= \left(\frac{101 \, x \, 100}{6}\right)(51 \times 4 - 201) = \left(\frac{10100}{6}\right) \times (3) = 5050,$

Hence (b).

Alternatively:

$(2^2 + 4^2 + 6^2 + 8^2 + \ldots + 100^2) - (1^2 + 3^2 + 5^2 + \ldots + 99^2)$

$= [(4 + 16 + 36 + 64 + \ldots\ldots + 10000) - (1 + 9 + 25 + \ldots + 9801)]$

$= 3 + 7 + 11 + 15 + \ldots + 199 = \left(\frac{3+199}{2}\right) \times 50$

$= \left(\frac{202 \, x \, 50}{2}\right) = 101 \times 50 = 5050$

42. Ans. (d)

Solution: Let the sum of the expression be S_n then

$S_n = 40 + 4^1 + 4^2 + 4^3 + \ldots + 4^{40}$

$\Rightarrow S_n = (1 + 4 + 16 + 64) + 4^4\,(1+4+16+64) + \ldots + 4^{36}\,(1+4+16+64) + 4^{40}$

Since (1+4+16+64) = 85 is divisible by 17

hence except 4^{40} remaining expression is divisible by 17.

$\left(\frac{4^{40}}{17}\right) \rightarrow \left(\frac{(4^{40})^{10}}{17}\right) \rightarrow \left(\frac{1^{10}}{17}\right)$

Hence the required remainder is 1.

43. Ans. (d)

Solution: let there be n rows, then the number of soldier in each row $= \frac{36}{n}$

(since the number of soldier in each row is same) now the number of bullets he gave to the first row = 1 bullet x number of soldiers
And the number of bullets he gave to the second row = 2 bullets x number of soldiers
And the number of bullets he gave to the third row = 3 bullet x number of soldiers
Hence total number of bullets

$$= 1 \times \frac{36}{n} + 2 \times \frac{36}{n} + 3 \times \frac{36}{n} + \ldots + n \times \frac{36}{n}$$

$$180 = \frac{36}{n}(1+2+3+\ldots.+n)$$

Or $180 = \frac{36}{n} \times \frac{n(n+1)}{2}$

Or $180 = \frac{36(n+1)}{2}$

⇨ $(n+1) = 10$

⇨ $N = 9$

Thus the number of soldiers in each row $= \frac{36}{9} = 4$

Hence, there are 4 soldiers in 8^{th} row.

44. Ans. (C)

Solution: The actual sum would be less then 1000. Let's find out the nearest value of the sum $\underline{n(n+1)} = 1000$ $\Rightarrow \frac{n(n+1)}{2}$)= 2000. We take the roots both sides to arrive at x as n and n+1 are two consecutive numbers and their product = x^2 =2000 ⇒ n = 44. For n=44, sum = 44X45 = 990. Therefore, page number 10 was added twice.

45. Ans. (C)

Solution: $x = \sqrt{2(1+\sqrt{2})}$ $X^2 = 2+2\sqrt{2} = (x^2 - 2) = 2\sqrt{2}$

$X^3 + x^2 - 2x - 2 = x(x^2 - 2) + x^2 - 2 = (x^2 - 2)(x+1)$

$= 2\sqrt{2}(\sqrt{2(1+\sqrt{2})} + 1$

$= 2\sqrt{2} + 4\sqrt{1+\sqrt{2}}$

46. Ans. (D)

Solution: $\frac{5n+23}{n-7} = \frac{5(n+7)+58}{x-7}$

$5 + \frac{55}{x-7}$. For this to be integer $\frac{58}{z-7}$ should be an integer ⇒ 58 is divisible by x-7 ⇒ x-7 = -58, -29, -2, -1, 1, 2, 29, 58 ⇒ We have corresponding 8 value for n.

47. Ans. (D)

Solution: Finding the powers of all the prime factors in 30! We obtain, $30! = 2^{26} \times 3^{14} \times 5^7 \times 7^4 \times 11^2 \times 13^2 \times 17 \times 19 \times 23 \times 29$. Removing 27×57 for the number of zeroes, we obtain $2^{19} \times 3^{14} \times 7^4 \times 11^2 \times 132 \times 17 \times 19 \times 23 \times 29$. We need to find the units digit of this product to find the rightmost non-zero digit. The units digit of the product = units digit of $8 \times 9 \times 1 \times 1 \times 9 \times 7 \times 9 \times 3 \times 9 = 8$.

48. Ans. (D)

Solution: Let's see the last two digits of summation in the groups of 4.

$3 + 3^2 + 3^3 + 3^4 = 03 + 09 + 27 + 81 = 20$

$3^5 + 3^6 + 3^7 + 3^8 = 43 + 29 + 87 + 61 = 20$

$3^9 + 3^{10} + 3^{11} + 3^{12} = 83 + 49 + 47 + 41 = 20$

Similarly, for 7,

$7 + 7^2 + 7^3 + 7^4 = 07 + 49 + 43 + 01 = 00$

$7^5 + 7^6 + 7^7 + 7^8 = 07 + 49 + 43 + 01 = 00$

$(3 + 3^2 + 3^3 + 3^4) + (3^5 + 3^6 + 3^7 + 3^8) + \ldots + 3^{400} = (20) + (20) + \ldots + 20 = 00$

$(7 + 7^2 + 7^3 + 7^4) + (7^5 + 7^6 + 7^7 + 7^8) + \ldots + 7^{200}) + 7201 = 00 + 00 + 00 + \ldots + 00 + 07 = 07$

Therefore, difference = 00 − 07 = 93.

49. Ans. (B)
Solution: The number of numbers less than and prime to 120 = $120\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = 32$

50. Ans. (D)
Solution: $5x^2 + 2y^2 = 5922$, the R.H.S. is even and therefore, L.H.S. should also be even $2y^2$ is even therefore, $5x^2$ should also be even, therefore x will be even $\Rightarrow 5x^2$ will end in 0. As the unit digit of R.H.S. is 2, the unit digit of $2y^2$ should also be 2 $\Rightarrow$ unit digit of y^2 should be 1 $\Rightarrow$ unit digit of y can be 9.

51. Ans. (C)
Solution: $\underbrace{123123123\ldots\ldots123123}_{300\text{digitseas}} = 123\underbrace{1001001\ldots\ldots1001}_{298\text{ digits}}$ $123(10^{297} + 10^{294} + 10^{291} + \ldots 10^3 + 1)$. gives a remainder of 10 with whereas 10^{even} gives a remainder of 1 with 99. Therefore in the expression $10^{297} + 10^{294} + 10^{291} \ldots + 10^3 + 1$, we will get 50 10 therefore remainder = 24 (50x 10 +50) = = remainder when 24 x 550 is divided by 99 = 33.

52. Ans. (C)
Solution $16^3 + 19^3$ is divisible by 16 + 19 = 35, ($a^n + b^n$) is divisible by a + b is n is **ODD**). Similarly, $17^3 + 18^3$ is divisible by 35. Also, the expression contains two even and two odd terms. Therefore, the sum is even. Therefore, the whole expression is also divisible by 2 $\Rightarrow$ it is divisible by 70:

53. Ans. (D)
Solution: Solution Not Available

54. Ans. (B)
Solution: 111 = 37 × 3. Therefore we find the remainder when the expression is divided by both 3 and 37. Now we know that if x is prime to p, where p is a prime number, $(x)^{p-1} - 1$ is divisible by p. In other words, $(x)^{p-1}$ gives remainder 1 when divided by p. Therefore, both 1736 and 1936 will give remainder 1 with 37. Therefore, total remainder with 37 = 1 + 1 = 2. Also, $17^{36} = (17^{18})\ 2 \Rightarrow$ will give remainder 1 with 3. Therefore, both 17^{36} and 19^{36} will give remainder 1 with 3. Therefore, total remainder with 37 = 1 + 1 = 2. Therefore, remainder with 111 = 2.

55. Ans. (B)
Solution: 25! + 26! + 27! + 28! + 30! = 25! (1 + 26 + 27 × 26 + 28 × 27 × 26) + 30! = 25! × a number ending in 5 + 30! $\Rightarrow$ 6 zeroes.

56. Ans. (B)
Solution: The concept can be understood easily in this way :- There are 10 odd numbers and 11 even numbers. The 10 odd numbers can be reduced to an even number only and the 11 even numbers can also be reduced to an even number. The final reduced number would be even.

Exercise – 12

1. X! is completely divisible by 11^{51} but not by 11^{52}. What is the sum of digits of largest such number X?

2. How many times would I be used while writing all the natural numbers from 8 to 127 in the Binary number system?
   (a) 212 (b) 218 (c) 424 (d) 436

3. $(3132!)_{10} = (x)_{34}$, then what will be the number of consecutive zeros at the end of 'x'?

(a) 124 (b) 167 (c) 194 (d) none of these

4. Which is larger $A = 99^{11} + 100^{11}$ or $B = 101^{11}$?
A. A B. B C. Both are equal D. Cannot be determined

5. The number 523abc is divisible by 7, 8 and 9. Then the value of $a \times b \times c$
A. is 10 B. is 60 C. is 180 D. cannot be determined

6. The number of integer solutions of the equation $x^2 + 12 = y^4$ is
A. 2 B. 4 C. 6 D. 8

7. Let $N = \underbrace{111.....111}_{73 \text{ times}}$ When N is divided by 259, the remainder is R1 and when N is divided by 32, the remainder is R2. Then R1 + R2 is equal to
A. 6 B. 8 C. 253 D. none of these

8. For how many integers m is $m^3 - 8m^2 + 20m - 13$ a prime number?
A. 1 B. 2 C. 3 D. more than 3

9. What is the unit digit of $7^{15^{16^{17}}}$?
A. 1 B. 3 C. 7 D. 9

10. In how many zeroes does $\frac{2002!}{(1001!)^2}$ end in?
A. 0 B. 1 C. 2 D. 3

11. How many zeroes will be at the end of $27!^{27!}$?
A. 36 B. 627! C. $6 \times 27!$ D. $6! \times 27!$

12. M and N are two natural numbers such that M + N = 949. LCM of M and N is 2628. What is the HCF of M and N?
A. 23 B. 73 C. 69 D. none of these

13. If {x} denotes fractional part of x then $\{\frac{5^{200}}{8}\}$ is
A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{3}{8}$ D. $\frac{5}{8}$

14. The highest power of 12 that can divide $5^{36} - 1$ is
A. 1 B. 2 C. 3 D. 4

15. The number 6162 in base 10 is written as $(222)_b$. Then, the base b is equal to
A. 60 B. 55 C. 45 D. 42

16. If in some base x, $(563)_x + (544)_x + (433)_x = (2203)_x$ then x is equal to
A. 6 B. 7 C. 9 D. 11

17. The number 2006! is written in base 22. How many zeroes are there at the end?

A. 500 B. 450 C. 200 D. 199

18. A three-digit number abc is divisible by 7 if
 A. 3a + b + c is divisible by 7
 B. a + 2b + c is divisible by 7
 C. 2a + 3b + c is divisible by 7
 D. 2a + 2b + c is divisible by 7

19. The digits 1, 2, 3, 4, and 5 are each used once to compose a five-digit number *abcde* such that the three-digit number *abc* is divisible by 4, *bcd* is divisble by 5, and *cde* is divisble by 3. Find the digit a.
 A. 1 B. 2 C. 3 D. 4

20. How many factors of 10^{10} end with a zero?
 A. 21 B. 90 C. 100 D. 121

21. A number has exactly 1024 factors. What can be the maximum number of prime factors of this number?
 A. 10 B. 8 C. 6 D. 5

22. If A = 10! + 12! + 14! + 16! +………..100!, then the highest power of 2 in A is
 A. 7 B. 8 C. 9 D. 10

23. The smallest natural number n such that $\frac{16!}{N}$ is a perfect square is

24. Find the largest positive integer n such that $n^3 + 100$ is divisible by (n + 10)

25. Find the smallest number n such that n! is divisible by 990.

26. Let P be a prime number greater than 3. Then, when $P^2 + 17$ is divided by 12, the remainder is

27. A 101 digit number is formed by writing first 55 natural numbers next to each other. Find the remainder when the number is divided by 16.

28. N is a natural number. How many values of N exist, such that $N^2 + 24N + 21$ has exactly three factors?

29. What should be the values of a and b so that 30a0b03 is divisible by 13?

Exercise – 12

Solutions

1. Solution: 14

X! is completely divisible by 11^{51} so the value of X should be less than 11 x 51 = 561

Highest power of 11 in 561! $\left[\frac{561}{11}\right] + \left[\frac{561}{11^2}\right] + \ldots = 51 + 4 = 55$

If we subtract 11 x 3 = 33 from 561 we get 561 – 33 = 528 highest power of 11 in 528! Is $\left[\frac{528}{11}\right] + \left[\frac{528}{11^2}\right] = 52$

Highest power of 1 in 528 – 1 = 527! Is = $\left[\frac{527}{11}\right] + \left[\frac{527}{11^2}\right] = 47 + 4 = 51$

So the required number is 527.

Sum of the digits = 5 + 2 + 7 = 14.

2. Ans. (d)
Solution: we need to look at writing the binary number system from $8 = (1000)_2$ to $127 = (1111111)_2$
There are 64 7-digit numbers in binary system are from 1000000 to 1111111. There are six digits after the leftmost 1. Each of these 6 digits can be filled by either 0 or 1 and both are equally probable in any position so the number of 1 from 1000000 to
$1111111 = 64 + 64 \times \frac{1}{2} \times 5 = 32 + 80 = 112.$
Similarly, from 10000 to 11111 there are $16 + \frac{16}{2} \times 4 = 16 + 32 = 48.$
Similarly, from 1000 to 1111 there are $8 + 8 \times \frac{1}{2} \times 3 = 20$
So total 1s = 20 + 48 + 112 + 256 = 436.

3. Ans. (c)
Solution: in base 34, 10 means 34, in base 10, 10 is obtained by multiplying 2 and 5 in base 34, it is obtained by multiplying 2 and 17. Number of consecutive zeroes in base 34 at the end of the number is same as the number of 2's and 17 in 3132! Since the number of 2's is much more than number of 17's so we count number of 17's in 3132!
Maximum power of 17 in 3132! = $\left[\frac{3132}{17}\right] + \left[\frac{3132}{17^2}\right] = 184 + 10 = 194.$

4. Ans. ()
Solution: $(101)^{11} - 99^{11} = (100 + 1)^{11} - (100-1)^{11}$
$= (100^{11} + 100^{10} \times 11 + 55\times100^9 + \dots +1) - (100^{11} - 11\times100^{10} + 55\times100^9 \dots 1)$
$= 2(11\times100^{10} + 165\times100^9 + \dots +1)$
$\frac{101^{11} - 99^{11}}{100^{11}} = \frac{22}{100} + \frac{330}{1000} + \dots\dots + \dots + \frac{1}{100^{11}} < 1$ (we can check that rest of the 1st term will be negligible.)

$\Rightarrow\ 101^{11} < 100^{11} + 99^{11}$

5. Ans. (A % C)
Solution: LCM of 7, 8, and 9 is 504. 523 abc when divided by 504 gives 19 abc= 19000 + abc. The remainder when 19000 is divided by 504 is 352 $\Rightarrow$ 352 + abc is divided by 504 is 352.
$\Rightarrow 352 + abc = 504$ or 504×2
$\Rightarrow abc = 152$ or 656
$\Rightarrow a \times b \times c$
= 10 or 180

6. Ans. (A)
Solution: . $y^4 - x^2 = 12$
$\Rightarrow (y^2+x)(y^2-x)$
= 12.
y^2-x and y^2+x are two numbers at a difference of 2x, i.e an even number. Therefore we need to break 12 into product of two number the difference between then being even
$\Rightarrow (y^2 + x)(y^2 - x) = 2\times6 \Rightarrow x = +2 \Rightarrow y = \pm 2$

7. Ans. (D)
Solution: For a prime number P not equal to 2, 3 or 5 single digit ($\neq$P) written P-1 times is divisible by P. 259 = 7×37
$\underbrace{111\dots\dots111}_{72\text{times}}$ is divisible by both 7 and 37 → $\underbrace{111\dots\dots111}_{72\text{times}}$ leaves remainder 1 with both 7 and 37 remainder by259=1

8. Ans. (C)
Solution: . $m^3 - 8m^2 + 20m-13 = (m-1)(m^2 - 7m+13)$. If this product has to yield a prime number, one of the multiplicands has to be equal to 1 and the other one has to be equal to a prime number. $m-1 = 1 \Rightarrow m=2 \Rightarrow m^2 - 7m+13 = 3$. Therefore m=2 $(m^2 - 7m+13) = 1 \Rightarrow (m-3)(m-4)=0 \Rightarrow m = 3$ or $4 \Rightarrow m-1 = 2$ or 3
∴ therefore m = 2, 3, or 4

9. Ans. (C)
Solution: Remainder of 15, 16, 17 with 4=1
Unit digit = 7

10. Ans. (A)
Solution: 2002! Ends in 499 zeroes. $(1001!)^2$ ends in 498 zeroes $\frac{2002!}{(1001!)^2}$ Ends in a single zero.

11. Ans. (C)
Solution: $6 \times 27!$

12. Ans. (B)
Solution: $949 = 73 \times 13$, $2628 = 73 \times 9 \times 4$
$\therefore$ M and can be 73×9 , and 73×4 .
$\Rightarrow$ HCF of M and N = 73

13. Ans. ()
Solution: $\frac{5^{200}}{8} = \frac{(5^2)^{100}}{8}$ = remainder $\rightarrow$ 1

14. Ans. (B)
Solution: $5^{36} - 1 = (4 + 1)^{36} - 1 = 4^{36} + 36 \times 4^{35} + \ldots + 36 \times 4 + 1 - 1 = 4^2(4^{34} + 9 \times 4^{34} + \ldots + 9)$.
Therefore, the highest power of 4 is 2.
$5^{36} - 1 = (6 - 1)^{36} - 1 = 6^{36} - 36 \times 6^{35} + \ldots - 36 \times 6 + 1 - 1 = 6^3(633 - 6^{34} + \ldots - 1)$. Therefore, the highest power of 3 is 3. Therefore, the highest power of 12 is 2.

15. Ans. (B)
Solution: $(222)_n = 6162 \rightarrow 2b^2 + 2b + 2 = 6162 \rightarrow b = 55$

16. Ans. (B)
Solution: Adding the unit digit of the numbers on L.H.S., we get 3 + 4 + 3 = 10. But the unit digit is 3. Therefore, base x is 3 less than 10 (the sum of units digit) or x = 7.

17. Ans. (D)
Solution: When we write 2006! In base 22, we successively divide 2006! by 22 and keep writing down the remainders. The first remainder will become the units digit, the second remainder will become the tens digit, the third remainder will become the hundreds digit and so on. Therefore, the number of zeroes that 2006! written in base 22 will have will be equal to the number of times 22 divides 2006! completely. The number of times 22 divides 2006! completely is equal to the highest power of 22 in 2006! or equal to highest power of 11 in 2006!.
The highest power of 11 in 2006! = $\left[\frac{2006}{11}\right] + \left[\frac{2006}{11}\right] + \left[\frac{2006}{1331}\right] = 182 + 16 + 1 = 199$

18. Ans. (C)
Solution: a b c = 100a + 10b + c = 98a + 2a + 7b + 3b + c = 7k + 2a + 3b + c.

19. Ans. (A)
Solution: . **bcd** is divisible by 5 $\Rightarrow$ d = 5. **abc** is divisible y 4 $\Rightarrow$ bc = 12, 32, 52 or 24. As b cannot be equal to 5, only options are 12, 32, or 24 $\Rightarrow$ c can be 2 or 4. cde is divisible by 3 $\Rightarrow$ c5e is divisible by 3 $\Rightarrow$ (c, e) = (1, 3), (3, 1), (3, 4) or (4, 3). Only the last value satisfies the condition that c can be 2 or 4. Therefore, c = 4 and e = 3. Also, this gives b = 2. Therefore, a = 1

20. Ans. (C)
Solution: $10^{10} = 2^{10} \times 5^{10}$ A factor will end in a zero if it has both 2 and 5 in it. Therefore we cannot take 2^0 or 5^0 in fornins the factors. Therefore, total number of factors having both 2 and 5 in it = 10×10 = 100

21. Ans. (A)
Solution: . Since 1024= 2× 2× 2× 2× 2× 2× 2× 2× 2× 2
So this no. can be, N= $n_1 \times n_2 \times n_3 \times n_4 \times n_5 \times n_6 \times n_7 \times n_8 \times n_9 \times n_{10}$
Where n_1, n_2, n_3,----- n_{10} are prime no.
So, this number N can have 10 prime factors.

22. Ans. (B)
Solution: A= (10! + 12! + 14!+ 16!+ …..100!)
=10! [1 + e_1 +e_2 +e_3 +….. e_{99}]
= 10! × k
Since e1, e_2, e_3,……e_{99} each of them is even
∴ 1+ e_1+ e_2+ e_3+….e_{99} = k = odd
The highest power of 2 is A in same as highest power of 2 in 10!
10 = 5, $\frac{10}{2}$
∴ highest power of 2 in 10! In = 5+2+1= 8

23. Solution: 16! = $2^{15} \times 3^6 \times 5^3 \times 7^2 \times 11^1 \times 13^1$
Since $\frac{16!}{N}$ has to be a perfect square, which means all prime factors should have even power.
∴ n = 2× 5×11×13
= 1430

24. Solution: We know that (n^3 + 10^3) which is nothing but (n^3 + 1000) in divisible by (n+10) now, (n3 + 1000) = (n^3 + 100 + 900) Since (n^3 + 100 + 900) is divisible by (n +10) for any value of n which means. (n^3 +100) & 900 should be divisible by (n+10) now the largest possible value of (n+10) which can satisfy this in 900
∴ largest possible value of n = 900-10 = 890

25. Solution: 990= $2 \times 3^2 \times 5 \times 11$
We have to select n! in such a way that it should consist of 11, 5, 3^2 , and 2
∴ n= 11
And 11! = $2^8 \times 3^4 \times 5^2 \times 7^1 \times 11$
∴ the smallest possible value of n = 11

26. Solution: We know that all prime number greater than 3 can be expressed as 6n = 1
$\frac{P2+17}{12} = \frac{(6n+1)^2 17}{12} = \frac{36n^2+1+12n+17}{12}$
$36n^2$ in divisible by 12
+ 12n in divisible by 12

Remainder for the above exp = Rem $\frac{1+17}{12}$
= Rem $\frac{18}{12}$
= Rem 6

27. Solution: the last 4 digit of the given sequence in 5455 the remainder of the given sequence would be same as he remainders when we divide 5455 by 16
Remainder = Rem $\frac{5455}{10}$
= 15

28. Solution: Only squares of prime numbers have exactly three factors
Let $N^2 + 24N + 21 = p^2 \Rightarrow (N + 12)^2 - p^2 = 123$
⇒ (N+ 12 + p) (N + 12 – p) = 123 123 = 1 × 123 or 3 × 41
⇒ p = 61 or 19 ⇒ two values of N are possible.

29. Solution: Using the divisibility test of 13. 30a0b03 i.e. ['b03' + 3] – ['0a0'] should be divisible 13.

⇒ [100b + 0 + 3 + 3] – [0 + 10a +0]

= 100b – 10a + 6 should be divisible by 13.

⇒ the ordered pairs (a, b) that satisfy it are :- (2, 3), (3, 7), (5, 2), (6, 6), (8, 1), (9, 5)