

Co – Ordinate Geometry



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Co - Ordinate Geometry

Distance Formula

The distance between two points $P(x_1, x_2)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Also, Distance of point (x_1, y_1) from origin $= \sqrt{x_1^2 + y_1^2}$

Section Formula

If $R(x, y)$ divides the line segment joined by the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m : n$ then

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

If $R(x, y)$ divides the line segment joined by the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ externally in the ratio $m : n$ then

$$x = \frac{mx_2 - nx_1}{m-n}, y = \frac{my_2 - ny_1}{m-n}$$

Area of A Triangle

The area of a triangle ABC whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is denoted by Δ .

$$\begin{aligned} \therefore \Delta &= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \\ &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \end{aligned}$$

Area of Polygon

The area of the polygon whose vertices are $(x_1, y_1), (x_2, y_2), (x_3, y_3) \dots (x_n, y_n)$ is

$$\Delta = \frac{1}{2} [(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + \dots + (x_ny_1 - x_1y_n)]$$

Condition of Collinearity of three points

The three given points are collinear i.e., lie on the same straight line if

- (i) Area of triangle ABC is zero.
- (ii) Slope of AB = Slope of BC = Slope of AC
- (iii) Distance between A and B + distance between B and C = Distance between A and C .
- (iv) Find the equation of the line passing through any two points, if third point satisfies the equation of the line then three points are collinear.

Some Important Results

- (i) If the vertices of a triangle have integral co-ordinates then the triangle cannot be equilateral.
- (ii) In order to prove that a given figure is a square, parallelogram, rectangle etc. We will prove the following points given in the table below corresponding the name of the figure.

S.no	Name of the figure	Conditions
1.	Square	Four sides are equal and the diagonals are also equal.
2.	Rhombus	Four sides are equal
3.	Rectangle	Opposite sides are equal and diagonals are also equal
4.	Parallelogram	Opposite sides are equal
5.	Parallelogram but not a rectangle	Opposite sides are equal but the diagonals are not equal.
6.	Rhombus but not a square	All side are equal but the diagonals are not equal

Some Important points in A Triangle

Centroid

If $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are the vertices of a triangle, then the coordinates of its centroid are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Incentre

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle ABC such that $BC = a$, $CA = b$ and $AB = c$, then the coordinates of its centre are $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

Note: If the equations of the sides of the triangle are given then we find the bisectors of internal angles and then their point of intersection to determine the in - centre.

Circumcentre

If 'O' is the circumcentre of a triangle ABC , then $OA = OB = OC$ and OA is called the circumradius.

To find the circumcentre of triangle ABC , we use the relation $OA = OB = OC$. This gives two simultaneous linear equation and their solution provides the coordinates of circumcentre.

Orthocentre

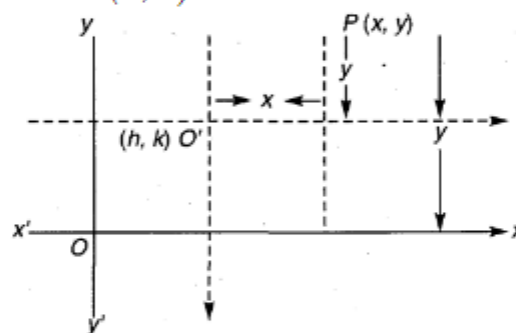
To determine the orthocentre, first we find equations of lines passing through vertices and perpendicular to the opposite sides. Solving any two of these three equations we get the coordinates of orthocentre.

Note: The circumcentre O , centroid G and orthocentre O' of a triangle ABC are collinear such that G divides $O'O$ in the ratio $2:1$ i.e., $O'G:OG = 2:1$

Shifting of Origin

Let O be the origin and $X'OX$ and $Y'OY$ be the axis of x and y respectively. Let O' and P be two points in the plane having coordinates (h, k) and (x, y) respectively referred to $X'OX$ and $Y'OY$ as coordinate axes.

Let the origin be transferred to O' and let $X'O'X$ and $Y'O'Y$ be new rectangular axes. Let the co-ordinates of P referred to new axes as the co-ordinate axes be (X, Y)



Thus if (x, y) are coordinates of a point referred to old axis and (X, Y) are the coordinates of the same point referred to new axis then.

$$x = X + h \text{ and } y = Y + k$$

If therefore the origin is shifted at a point (h, k) substitute $X + h$ and $Y + k$ for x and y respectively.

The transformation formula from new axes to old axes is

$$X = x - h, \quad Y = y - k$$

The coordinates of the old origin referred to the new axes are $(-h, -k)$

Definition of A Straight Line

A straight line is a curve such that every point on the line segment joining any two points on it lies on it.

The general form of the equation of straight line

$$ax + by = c$$

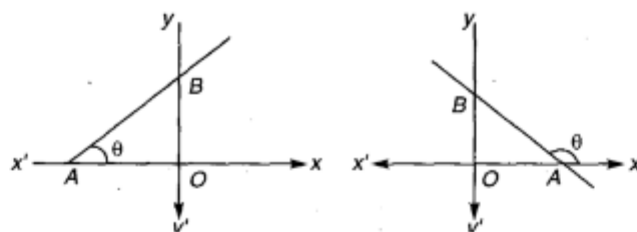
Where a, b and c are real constants and x, y are two unknowns

Slope (Gradient) of a Line

$$m = \tan\theta = \frac{a}{b}$$

$$\left\{ \because ax + by + c = 0 \Rightarrow by = -ax - c \Rightarrow y = -\frac{a}{b}x - \frac{c}{b} \right\}$$

Here m is called the slope or gradient of a line and c is the intercept on y -axis. The slope of a line is always measured in anticlockwise.



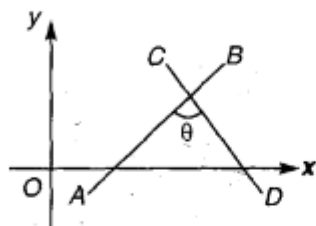
Slope of a line in terms of coordinates of any two points on it.

If (x_1, y_1) and (x_2, y_2) are coordinates of any two points on a line, then its slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Difference of ordinates}}{\text{Difference of abscissae}} = \frac{\text{rise}}{\text{run}}$$

Angle between two lines

$$\tan \theta = \pm \left(\frac{m_2 - m_1}{1 + m_1 m_2} \right)$$



Condition of Parallelism of lines

If the slopes of two lines is m_1 and m_2 and if they are parallel then,

$$m_1 = m_2$$

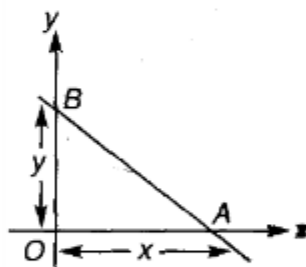
Condition of Perpendicularity of lines

If the slopes of two lines is m_1 and m_2 and if they are perpendicular to each other, then

$$m_1 \cdot m_2 = -1$$

Intercepts of a line on the axis

If a straight line cuts x-axis at A and the y-axis at B then OA and OB are known as the intercepts of the line on x-axis and y-axis respectively where O is the origin.



Exercise -01

1. The point $(-2, 3)$ lies in the quadrant :
(a) First (b) Second (c) Third (d) Fourth
2. Find the co-ordinates of the point which divides the join of the points $(2, 4)$ and $(6, 8)$ externally in the ratio 5:3:
(a) $(5, 6)$ (b) $(12, 14)$ (c) $(3, 8)$ (d) $(2, 7)$
3. If the coordinates of the mid points of the sides of a triangle are $(1, 1)$, $(2, -3)$ and $(3, 4)$. Find its centroid:
(a) $(3, \frac{2}{3})$ (b) $(2, \frac{3}{4})$ (c) $(2, \frac{2}{3})$ (d) none of these
4. $A(-2, -1)$, $B(1, 0)$, $C(4, 3)$ and $D(1, 2)$ are the four points of a quadrilateral. The quadrilateral is a:
(a) Square (b) Rhombus (c) Parallelogram (d) none of (a), (b), (c)
5. Find the slope of the line joining the points $(7, 5)$ and $(9, 7)$:
(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 3
6. Points $A(6, 6)$, $B(2, 3)$ and $C(4, 7)$ are the vertices of a triangle which is :
(a) right angled (b) acute angled (c) obtuse angled (d) none of these
7. Find the equation of the line with slope $\frac{2}{3}$ and intercept on the y -axis is 5:
(a) $y = \frac{2}{3}x + 5$ (b) $y = \frac{3}{2}x + 5$ (c) $y = 3x + 6$ (d) none of these
8. Find the equation of the line passing through the point $(2, -3)$ and having its slope $\frac{5}{4}$:
(a) $4x - 5y = 20$ (b) $3x - 2y = 5$ (c) $5x - 4y = 22$ (d) none of these
9. Find the equation of the line which cuts off intercepts 2 and 3 from the axes:
(a) $9x - 7y = 6$ (b) $3x + 2y = 6$ (c) $4x + 3y = 7$ (d) none of these
10. Find the intercepts made by the line $3x + 4y - 12 = 0$ on the axes:
(a) 2 and 3 (b) 4 and 3 (c) 3 and 5 (d) none of these
11. Find the equation of the line through the points $(-1, -2)$ and $(-5, 2)$:
(a) $2x + y = 3$ (b) $3x - 2y + 7 = 0$
(c) $x + y + 3 = 0$ (d) none of these
12. Find the equation of the straight line which passes through the point $(5, -6)$ which is parallel to the line $8x + 7y + 5 = 0$:
(a) $3x - 5y + 8 = 0$ (b) $7x + 8y + 5 = 0$
(c) $7x - 8y + 2 = 0$ (d) $8x + 7y + 2 = 0$
13. Find the distance between two parallel lines $5x + 12y - 30 = 0$ and $5x + 12y - 4 = 0$
(a) 3 (b) 7 (c) $\frac{5}{2}$ (d) 2
14. $(1, 2)$ and $(3, 8)$ are a pair of opposite vertices of square. Find the diagonals of the square passing through $(1, 2)$:
(a) $x - 2y = 1$ (b) $2x + 7y = 0$ (c) $3x + 2y + 7 = 0$ (d) $3x - y = 1$
15. Find the distance between the points $(3, 4)$ and $(8, -6)$
(a) $\sqrt{5}$ (b) $5\sqrt{5}$ (c) $2\sqrt{5}$ (d) $4\sqrt{5}$

Exercise – 02

- Find the distance between the points $(-5, 3)$ and $(3, 1)$:
(a) $2\sqrt{7}$ (b) $3\sqrt{14}$ (c) $5\sqrt{17}$ (d) $2\sqrt{17}$
- The coordinates of the vertices of a triangle are $(3, 1)$, $(2, 3)$ and $(-2, 2)$. Find the coordinates of the centroid of the triangle ABC :
(a) $(1, 2)$ (b) $(2, 3)$ (c) $(4, 5)$ (d) $(5, 6)$
- Find the co-ordinates of the incentre of the triangle whose vertices are the points $(4, -2)$, $(5, 5)$ and $(-2, 4)$:
(a) $(\frac{7}{3}, \frac{2}{3})$ (b) $(\frac{5}{2}, \frac{5}{2})$ (c) $(\frac{6}{5}, 5)$ (d) none of these
- The three vertices of a parallelogram taken in a order are $(-1, 0)$, $(3, 1)$ and $(2, 2)$ respectively. Find the coordinates of the fourth vertex:
(a) $(-1, 2)$ (b) $(-2, 1)$ (c) $(2, 3)$ (d) $(3, -2)$
- Find the area of quadrilateral formed by joining the points $(-4, 2)$, $(1, -1)$, $(4, 1)$ and $(2, 5)$:
(a) 25.4 (b) 20.5 (c) 24.5 (d) none of these
- Find (x, y) if $(3, 2)$, $(6, 3)$, (x, y) and $(6, 5)$ are the vertices of a parallelogram:
(a) $(5, 6)$ (b) $(6, 5)$ (c) $(9, 6)$ (d) $(9, 5)$
- Find the equation of the straight line making intercepts on the axes equal in magnitude but opposite in sign and passing through the point $(-5, -8)$:
(a) $x - y = 7$ (b) $2x + y = 3$ (c) $x - y = 3$ (d) none of these
- Find the equation of the line through the point of intersection of $2x - 3y + 1 = 0$ and $x + y - 2 = 0$ which is parallel to the y -axis.
(a) $x = 1$ (b) $8x = 7$ (c) $x + 3 = 0$ (d) $x = 6$
- Find the equation of the line through the intersection of the lines $3x + 4y = 7$ and $x - y + 2 = 0$ having slope 3.
(a) $4x - 3y + 7 = 0$ (b) $21x - 7y + 16 = 0$
(c) $8x + y + 8 = 0$ (d) none of these
- What will be the centroid of a triangle whose vertices are $(2, 4)$, $(6, 4)$ and $(2, 0)$?
(a) $(\frac{7}{2}, \frac{5}{2})$ (b) $(3, 5)$ (c) $(\frac{10}{3}, \frac{8}{3})$ (d) $(1, 4)$
- The distance between the lines $4x + 3y = 11$ and $8x + 6y = 15$ is
(a) 4 (b) $\frac{7}{10}$ (c) $\frac{5}{7}$ (d) 26
- Find the area of the triangle whose vertices are $(1, 3)$, $(-7, 6)$ and $(5, -1)$.
(a) 20 (b) 10 (c) 18 (d) 24
- Four vertices of a parallelogram taken in order are $(-3, -1)$, (a, b) , $(3, 3)$ and $(4, 3)$. What will be the ratio of a to b ?
(a) 4 : 1 (b) 1 : 2 (c) 1 : 3 (d) 3 : 1

14. The extremities of a diagonal of a parallelogram are the points $(3, -4)$ and $(-6, 5)$. If the third vertex is the point $(-2, 1)$, the coordinate of the fourth vertex is
 (a) $(1, 0)$ (b) $(-1, 0)$ (c) $(-1, 1)$ (d) $(1, -1)$
15. If p is the length of the perpendicular from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$, then which of the following is true?
 (a) $\frac{1}{p^2} = \frac{1}{b^2} - \frac{1}{a^2}$ (b) $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$
 (c) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (d) None of these
16. How many points on $x + y = 4$ are there that lie at a unit distance from the line $4x + 3y = 10$?
 (a) 1 (b) 2 (c) 3 (d) None of these
17. What will be the area of the rhombus $ax \pm by \pm c = 0$?
 (a) $\frac{3c^2}{ab}$ (b) $\frac{4c^2}{ab}$ (c) $\frac{2c^2}{ab}$ (d) $\frac{c^2}{ab}$
18. Find the area enclosed by the graph $y = |X + 3|$ with the coordinate axes in square units,
 (a) 9 (b) 4.5 (c) 0 (d) 12
19. Find the distance of a point $(3, 5)$ from the line $x + y = 2$, measured along a line making an angle 45° with the positive x-axis. (The anti-clockwise direction is positive angle.)
 (a) $\sqrt{2}$ unit (b) $2\sqrt{2}$ unit (c) $3\sqrt{2}$ units (d) $4\sqrt{2}$ units
20. If the sum of the distances of a point from two perpendicular lines in a plane is 1, then it will form a ____ structure.
 (a) square (b) circle (c) straight line (d) two intersecting lines

Exercise – 03

1. Let the vertices of a triangle ABC be $(4, 3)$, $(7, -1)$, $(9, 3)$ then the triangle is :
 (a) Scalene (b) Isosceles (c) Equilateral (d) none of (a), (b), (c)
2. Let the vertices of a triangle ABC be $(4, 4)$, $(3, 5)$, $(-1, -1)$, then the triangle is:
 (a) scalene (b) equilateral (c) right angled (d) none of (a), (b), (c)
3. Find the coordinates of the circumcentre of the triangle whose vertices are $(8, 6)$, $(8, -2)$ and $(2, -2)$:
 (a) $(2, 3)$ (b) $(5, 2)$ (c) $(5, 3)$ (d) $(1, 2)$
4. Find the equation of the straight line which passes through the point $(3, 4)$ and has intercepts on the axes such that their sum is 14:
 (a) $4x + 3y = 24$ (b) $x + y = 7$ (c) $3x + 7y = 43$ (d) both (a) and (b)
5. Find the third vertex of the triangle whose two vertices are $(-3, 1)$ and $(0, -2)$ and the centroid is the origin.
 (a) $(2, 3)$ (b) $(-\frac{4}{3}, \frac{14}{3})$ (c) $(3, 1)$ (d) $(6, 4)$
6. Find the area of the triangle whose vertices are $(a, b + c)$, $(a, b - c)$ and $(-a, c)$.
 (a) $2ac$ (b) $2bc$ (c) $b(a + c)$ (d) $c(a - b)$
7. What will be the reflection of the point $(4, 5)$ in the second quadrant?
 (a) $(-4, -5)$ (b) $(-4, 5)$ (c) $(4, -5)$ (d) None of these
8. What will be the reflection of the point $(4, 5)$ in the third quadrant?

- (a) $(-4, -5)$ (b) $(-4, 5)$ (c) $(4, -5)$ (d) None of these
9. If the origin gets shifted to $(2, 2)$, then what will be the new coordinates of the point $(4, -2)$
(a) $(-2, 4)$ (b) $(2, 4)$ (c) $(4, 2)$ (d) $(2, -4)$
10. Find the area of the quadrilateral the coordinates of whose angular points taken in order are $(1, 1)$, $(3, 4)$, $(5, -2)$ and $(4, -7)$.
(a) 20.5 (b) 41 (c) 82 (d) 61.5
11. Find the equation of the straight line passing through the origin and the point of intersection of the lines $x/a + y/b = 1$ and $x/b + y/a = 1$.
(a) $y = x$ (b) $y = -x$ (c) $y = 2x$ (d) $y = -2x$
12. The points $(p - 1, p + 2)$, $(p, p + 1)$, $(p + 1, p)$ are collinear for
(a) $p = 0$ (b) $p = 1$ (c) $p = -1/2$ (d) Any value of p
13. The area of a triangle is 5 square units. Two of its vertices are $(2, 1)$ and $(3, -2)$. The third vertex lies on $y = x + 3$. What will be the third vertex?
(a) $(4, -7)$ (b) $(4, 7)$ (c) $(-4, -7)$ (d) $(-4, 7)$
14. If the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear then which of the following is true?
(a) $\frac{1}{a} + \frac{1}{b} = 2$ (b) $\frac{1}{a} - \frac{1}{b} = 1$ (c) $\frac{1}{a} - \frac{1}{b} = 2$ (d) $\frac{1}{a} + \frac{1}{b} = 1$
15. The area of the figure formed by the lines $ax \pm by \pm c = 0$ is:
(a) $\frac{c^2}{ab}$ (b) $\frac{2c^2}{ab}$ (c) $\frac{c^2}{2ab}$ (d) None of these

Co-ordinate Geometry Answers Key & Solutions

Co-ordinate Geometry Solutions

Exercise - 01

- Ans: b
Solution
The point $(-2, 3)$ lies in the second quadrant.
- Ans: b
Solution
The required coordinates of the point which divides the join of $(2, 4)$ and $(6, 8)$ externally in the ratio $5 : 3$ are $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$
Here, $m : n = 5 : 3$, $(x_1, y_1) = (2, 4)$, $(x_2, y_2) = (6, 8)$
Hence the required co-ordinates = $(12, 14)$.
- Ans: c
Solution
Let $P(1, 1)$, $Q(2, -3)$, $R(3, 4)$ be the midpoints of sides AB , BC and CA respectively triangle ABC .
Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of triangle ABC .
Then, P is the mid point of AB
 $\Rightarrow \frac{x_1 + x_2}{2} = 1, \frac{y_1 + y_2}{2} = 1$
 $\Rightarrow x_1 + x_2 = 2$ and $y_1 + y_2 = 2$
...(i)
 Q is the midpoint of BC
 $\Rightarrow \frac{x_2 + x_3}{2} = 2, \frac{y_2 + y_3}{2} = -3$
 $\Rightarrow x_2 + x_3 = 4$ and $y_2 + y_3 = -6$
...(ii)
 R is the mid point of AC
 $\Rightarrow x_1 + x_3 = 6$ and $y_1 + y_3 = 8$
...(iii)
From (i), (ii) and (iii), we get
 $(x_1, y_1) \equiv (2, 8)$, $(x_2, y_2) \equiv (0, -6)$
and, $(x_3, y_3) \equiv (4, 0)$
Then the coordinates of the centroid
 $= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right) = \left(2, \frac{2}{3}\right)$
- Ans: c
Solution
Let A, B, C and D be the vertices of the quadrilateral whose coordinates are $(-2, -1)$, $(1, 0)$, $(4, 3)$ and $(1, 2)$ respectively.
Now, $AB = \sqrt{10}$, $BC = \sqrt{18}$, $DC = \sqrt{10}$, $AD = \sqrt{18}$
 $\therefore AB = CD$ and $BC = AD$ i.e., the opposite sides are equal.
Hence $ABCD$ is a parallelogram.
- Ans: a

Solution

$$\text{Slope of the line} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 5}{9 - 7} = 1$$

Here $(x_1, y_1) \equiv (7, 5)$ and $(x_2, y_2) \equiv (9, 7)$

- Ans: a

Solution

$$m_1 = \text{Slope of } AB = \frac{3 - 6}{2 - 6} = \frac{3}{4}$$

$$m_2 = \text{Slope of } BC = \frac{7 - 3}{4 - 2} = 2$$

$$\text{And } m_3 = \text{Slope of } AC = \frac{7 - 6}{4 - 6} = -1/2$$

$$\therefore m_2 \cdot m_3 = 2 \times -\frac{1}{2} = -1$$

This shows that BC is perpendicular to AC .Hence, ABC is a right-angled triangle.

- Ans: a

Solution

The equation of the line with slope $2/3$ and intercept on the y -axis 5 is $y = \frac{2}{3}x + 5$
($\because y = mx + c$)

- Ans: c

Solution

We have $m = \frac{5}{4}$ and $(x_1, y_1) = (2, 3)$

\therefore The equation of the line as point slope form is
 $y - y_1 = m(x - x_1)$

$$\text{Or } y - (-3) = \frac{5}{4}(x - 2)$$

$$\text{Or } y + 3 = \frac{5}{4}(x - 2)$$

$$\text{Or } 5x - 4y = 22$$

- Ans: b

Solution

Here $a = 2$ and $b = 3$

\therefore The required equation of the line is $\frac{x}{2} + \frac{y}{3} = 1$

$$\Rightarrow 3x + 2y = 6$$

- Ans: b

Solution

We have $3x + 4y - 12 = 10$

$$\Rightarrow 3x + 4y = 22$$

$$\Rightarrow \frac{3x}{12} + \frac{4y}{12} = 1 \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$$

- Ans: c

Solution

The equation of the line through the points $(-1, -2)$ and $(-2, 2)$ is $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Where, $(x_1, y_1) \equiv (-1, -2)$

And $(x_2, y_2) \equiv (-5, 2)$

\therefore Required equation is

$$y - (-2) = \frac{2 - (-2)}{-5 - (-1)} [x - (-1)]$$

Or $y + 2 = \frac{4}{-4}(x + 1)$

Or $x + y + 3 = 0$

12. Ans: d

Solution

The equation of any straight line parallel to the line $8x + 7y + 5 = 0$ is $8x + 7y + c = 0 \dots (1)$

Where c is an arbitrary constant.

If the line (1) passes through the point $(5, -6)$, then $8 \times 5 + 7 \times (-6) + c = 0 \Rightarrow c = 2$

Hence from (1), the required equation of the straight line is $8x + 7y + 2 = 0$

13. Ans: d

Solution

Putting $y = 0$ in $5x + 12y - 30 = 0$, we get

$$5x - 30 = 0 \text{ or } x = 6$$

$\therefore (6, 0)$ is a point on the first line $5x + 12y - 30 = 0$

Required distance between the parallel lines = Perpendicular distance of the point $(6, 0)$ from the second line $5x + 12y - 4 = 0$.

$$= \frac{5 \cdot 6 + 12 \cdot 0 - 4}{\sqrt{5^2 + 12^2}} = \frac{30 - 4}{13} = 2 \text{ units}$$

14. Ans: d

Solution

Let $ABCD$ be the square and let $(1, 2)$ and $(3, 8)$ be the coordinates of opposite vertices A and C respectively.

The equation of the diagonal AC is $y - 2 = \frac{8 - 2}{3 - 1}(x - 1)$
 $\Rightarrow 3x - y = 1$

15. Ans: b

Exercise - 2

1. Ans: d

Solution

Distance between two points =

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Let $(x_1, y_1) = (-5, 3)$ and $(x_2, y_2) = (3, 1)$

$$\therefore \text{Required distance} = \sqrt{(3 + 5)^2 + (1 - 3)^2} \\ = \sqrt{64 + 4} = \sqrt{68} = 2\sqrt{17} \text{ unit}$$

2. Ans: a

Solution

Let the co-ordinate of the centroid of $\triangle ABC$ be (x, y) then

$$(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) \\ = \left(\frac{3 + 2 + 2}{3}, \frac{1 + 3 + 2}{3} \right) = (1, 2)$$

3. Ans: b

Solution

Let $A = (4, -2)$, $B = (5, 5)$ and $C = (-2, 4)$

$$\text{Then } a = BC = \sqrt{(-2 - 5)^2 + (4 - 5)^2} = 5\sqrt{2}$$

$$b = AC = \sqrt{(4 + 2)^2 + (-2 - 4)^2} = 6\sqrt{2}$$

$$c = AB = \sqrt{(5 - 4)^2 + (5 + 2)^2} = 5\sqrt{2}$$

and $(x_1, y_1) \equiv (4, -2)$, $(x_2, y_2) \equiv (5, 5)$, $(x_3, y_3) \equiv (-2, 4)$

\therefore The coordinates of the incentre of the $\triangle ABC$

$$\text{are } \left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c} \right)$$

$$= \left(\frac{5}{2}, \frac{5}{2} \right) \{ \text{Substitute the values of } (a, b, c) \}$$

$x_1, x_2, x_3, y_1, y_2, \text{ and } y_3 \}$

4. Ans: b

Solution

Let $A(-1, 0)$, $B(3, 1)$, $C(2, 2)$ and $D(x, y)$ be the vertices of a parallelogram $ABCD$ taken in order. Since the diagonals of a parallelogram bisect each other.

\therefore Coordinates of the mid point of AC = Coordinates of the midpoint of BD

$$\Rightarrow \left(\frac{-1 + 2}{2}, \frac{0 + 2}{2} \right) = \left(\frac{3 + x}{2}, \frac{1 + y}{2} \right)$$

$$\Rightarrow \left(\frac{1}{2}, 1 \right) = \left(\frac{3 + x}{2}, \frac{1 + y}{2} \right)$$

$$\Rightarrow \frac{3 + x}{2} - \frac{1}{2} \text{ and } \frac{1 + y}{2} = 1$$

$$\Rightarrow x = -2 \text{ and } y = 1$$

5. Ans: c

Solution

Let A, B, C, D be the points $(-4, 2)$, $(1, -1)$, $(4, 1)$ and $(2, 5)$ respectively. Then the area of the quadrilateral $ABCD$

$$= \frac{1}{2} \{ -4 \times -1 - 2 \times 1 + 1 \times 1 - 4 \times -1 + 4 \times 5 - 2 \times 1 + 2 \times 2 \times -5 \times -4 \}$$

$$= \frac{1}{2} (4 - 2 + 1 + 4 + 20 - 2 + 4 + 20)$$

$$= \frac{49}{2} = 24.5 \text{ square units.}$$

6. Ans: c

Solution

Let A, B, C, D be the points $(3, 2)$, $(6, 3)$, (x, y) and $(6, 5)$ respectively.

Since $ABCD$ is a parallelogram, the diagonals AC and BD must bisect each other i.e., the mid points of AC and the mid point of BD must coincide and hence the coordinates of the two mid points are the same

$$\therefore \frac{3 + x}{2} = \frac{6 + 6}{2} \text{ and } \frac{2 + y}{2} = \frac{3 + 5}{2}$$

$$\text{Or } 3 + x = 12 \text{ and } 2 + y = 8$$

$$\text{Or } x = 9 \text{ and } y = 6$$

$$\text{Hence } (x, y) = (9, 6)$$

7. Ans: c

Solution

Let the equation of the straight line be

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \dots (1)$$

Since intercepts a, b are equal in magnitude but opposite in sign.

$$\therefore \text{From eq. (1)} \quad \frac{x}{a} + \frac{y}{(-a)} = 1$$

$$\text{Or } x - y = a \quad \dots (2)$$

Since this line passes through the point $(-5, -8)$.

$$\therefore -5 - (-8) = a$$

$$\Rightarrow a = 3$$

Hence, from (2) the required equation of the line is $x - y = 3$

8. Ans: a

Solution

The equation of the line through the point of intersection of $2x - 3y + 1 = 0$ and $x + y - 2 = 0$, is

$$(2x - 3y + 1) + k(x + y - 2) = 0$$

$$\text{i.e., } (2 + k)x + (k - 3)y + (1 - 2k) = 0$$

$$\dots (1)$$

If this line is parallel to the y -axis, then its equation must be of the form $x = h$, i.e., the coefficient of y in (1) must be zero.

$$\therefore k - 3 = 0 \text{ or } k = 3$$

Hence, from (1) the required equation of the line is $(2 + 3)x + 0 \cdot y + (1 - 2 \times 3) = 0$ [Putting $k = 3$]

$$\Rightarrow x = 1$$

9. Ans: b

Solution

The equation of any line passing through the intersection of the line $3x + 4y - 7 = 0$ and $x - y + 2 = 0$ is

$$(3x + 4y - 7) + k(x - y + 2) = 0$$

$$\text{Slope of the line} = \frac{3+k}{4-k} = 3$$

$$\Rightarrow k = \frac{15}{2}$$

Hence, from (1) the required equation of the line is

$$(3x + 4y - 7) + \frac{15}{2}(x - y + 2) = 0$$

$$\Rightarrow 21x - 7y + 16 = 0$$

10. Ans: c

11. Ans: b

12. Ans: b

13. Ans: a

14. Ans: b

15. Ans: c

Solution

Use the formula (perpendicular distance of a point from a straight line)

16. Ans: b

17. Ans: c

18. Ans: b

19. Ans: c

20. Ans: a

Exercise - 3

1. Ans: b

Solution

$$\text{Let } A \equiv (4, 3), B \equiv (7, -1), C \equiv (9, 3)$$

$$AB = \sqrt{(7-4)^2 + (-1-3)^2} = \sqrt{25} = 5$$

$$BC = \sqrt{(9-7)^2 + (3+1)^2} = \sqrt{20} = 2\sqrt{5}$$

$$CA = \sqrt{(4-9)^2 + (3-3)^2} = \sqrt{25} = 5$$

$$\therefore AB = CA = 5$$

Hence AB is an isosceles triangle.

2. Ans: c

Solution

$$\text{Let } A \equiv (4, 4), B \equiv (3, 5), C \equiv (9, 3)$$

$$AB = \sqrt{(3-4)^2 + (5-4)^2} = \sqrt{2}$$

$$BC = \sqrt{(-1-3)^2 + (-1-5)^2} = \sqrt{52}$$

$$AC = \sqrt{(-1-4)^2 + (-1-4)^2} = \sqrt{50}$$

$$AB^2 + AC^2 = BC^2$$

Hence by the Pythagoras Theorem, ABC is a right angled triangle.

3. Ans: b

Solution

Let $S(x, y)$ be the Circumcentre, then $AS = BS = CS = R$, where R is the circumradiusNow $(AS)^2 = (BS)^2$ where $S \equiv (x, y), A \equiv$

$$(8, 6), B \equiv (8, -2)$$

$$\therefore (x-8)^2 + (y-6)^2 = (x-8)^2 + (y+2)^2$$

$$\Rightarrow 36 - 12y = 4 + 4y$$

$$\Rightarrow y = 2$$

Again $(BS)^2 = (CS)^2$, where $S \equiv (x, y), B \equiv$

$$(8, -2), C \equiv (2, -2)$$

$$\therefore (x-8)^2 + (y+2)^2 = (x-2)^2 + (y+2)^2$$

$$\Rightarrow x^2 + 64 - 16x + y^2 + 4 + 4y$$

$$= x^2 + 4 - 4x + y^2 + 4 + 4y$$

$$\Rightarrow -12x + 60 = 0$$

$$\Rightarrow x = 5$$

$$\therefore \text{The Circumcentre } S \equiv (x, y) = (5, 2)$$

4. Ans: d

Solution

Let the equation of the line in the intercept form be $\frac{x}{a} + \frac{y}{b} = 1$,Where a and b are intercepts on the axes.

$$\text{Then, } a + b = 14 \text{ or } b = 14 - a$$

Since the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the point $(3, 4)$;

$$\therefore \frac{3}{a} + \frac{4}{b} = 1 \text{ or } \frac{3}{a} + \frac{4}{14-a} = 1$$

$$\text{Or } a^2 - 13a + 42 = 0$$

$$\text{Or } (a-6)(a-7) = 0$$

$$\therefore a = 6, 7$$

If $a = 6$ then $b = 8$

If $a = 7$ then $b = 7$

Hence the required equation of the line are

$$\frac{x}{6} + \frac{y}{8} = 1 \text{ and } \frac{x}{7} + \frac{y}{7} = 1$$

$$\text{Or } 4x + 3y = 24 \text{ and } x + y = 7$$

5. Ans: c

6. Ans: a

7. Ans: b

8. Ans: a

9. Ans: d

10. Ans: a
Solution

Use the area of a triangle formula for the two parts of the quadrilateral separately and then add them.

11. Ans: a
Solution

Find the point of intersection of the lines by solving the simultaneous equations and then use the two-point formula of a straight line.

Alternative: After finding out the point of intersection, use options to check.

12. Ans: d
Solution

For 3 points to be collinear,

(i) Either the slope of any two of the 3 points should be equal to the slope of any other two points.

OR

(ii) The area of the triangle formed by the three points should be equal to zero

Solving using options

13. Ans: c
Solution

First check the options to see that which of the points lie on the equation of straight line $y = x + 3$.

And then again check options, if needed, to confirm the second constraint regarding area of triangle.

14. Ans: d
Solution

Make the slope of any two points equal to the slope of any other two points.

Slope = Difference of Y coordinates / Difference of X coordinates

15. Ans: b