Percentile Classes

Logarithm Theory

Properties and Formulas of Logarithm

1. $log_a 1 = 0$, a > 0, a ≠ 1
2. log_a a = 1, a > 0, a ≠ 1
3. $log_a a^x = x \; \forall \; x \in R, x > 0$
4. $a^{log_a x} = x \; \forall \; x \in R, x > 0$

NOTE: $log_a a^x$ is the inverse function of a^x.

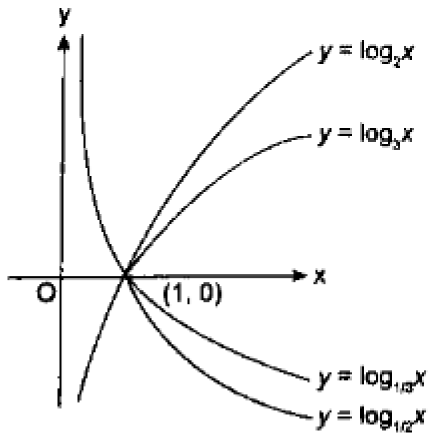
5. log_a (m. n) = log_a m + log_a n $\forall \; m, \; n > 0, a > 0, a \neq 1$
6. log_a (m / n) = log_a m - log_a n $\forall \; m, \; n > 0, a > 0, a \neq 1$
7. $log_a (m^n)$ = n log_a m $\forall \; m > 0, a > 0, a \neq 1$
8. $log_a \left(\frac{1}{m}\right)$ = - log_a m $\forall \; m > 0, a > 0, a \neq 1$
9. $log_a b = \frac{1}{log_b a} = \frac{log_c b}{log_c a}$ m ∀ a, b, c > 0 and a ≠1, b ≠1, c ≠1
10. $If \; log_a b = x$ ∀ a > 0, a ≠1, b ≠0 and $x \in$ R
    (i) $log_{(1/a)} b = - x$
    (ii) $log_a \left(\frac{1}{b}\right) = - x$
    (iii) $log_{\frac{1}{a}} \left(\frac{1}{b}\right) = x$
11. $log_{a^m} (b) = \frac{1}{m} log_a b$
12. $log_a x$ is a decreasing function, if 0 < a < 1
13. $log_a x$ is an increasing function, if a > 1

Restrictions with Logarithm of Any Number

For logarithm of any number to be defined, the number should be greater than zero and base should be positive and not equal to 1.

- ⇨ $log_a x$ to be defined as x>0 and a>0 and a ≠ 1
- ⇨ log of negative number is not defined. For example, log(-10) is not defined.
- ⇨ Log to the base of any negative number or log to the base = 1 is not defined. For example, log $_{(-5)}$x is not defined. Similarly, $log_1 x$ is not defined.

It can be seen with the help of a graph (log x is also given alongside.)



Following observation can be made from this graph:

1. Value of y can be negative for some value of x.
2. Value of x cannot be negative in any case.
3. For constant x, if base is lying in between 0 and 1, then log x becomes decreasing function. Otherwise, it is an increasing function.

Logarithmic Inequality

Case I: If base (assume to be N) > 1

(i) If x > y, then $\log_N x > \log_N y$.

(ii) Vice versa of the above rule is also true, i.e., if $\log_N x > \log_N y$ ➔ x > y

Case II: If base = N is 0 <N< 1

(i) If x > y, then $\log_N x < \log_N y$

(ii) Vice versa of the above rule is also true, i.e., if $\log_N x > \log_N y$ ➔x < y

Base change Rule

Till now all rules and theorems you have studied in Logarithms have been related to operations on logs with the same basis. However, there are a lot of situations in Logarithm problems where you have to operate on logs having different basis. The base change rule is used in such situations.

This rule states that

(i) $log_a (b) = log_c (b) / log_c (a)$

It is one of the most important rules for solving logarithms

(ii) $log_b (a) = log_c$ (a) / $log_c (b)$

A corollary of this rule is

(iii) $log_a (b) = 1/log_b$(a)

CHARACTERISTICS AND MANTISSA

Characteristics: The integral part of logarithm is known as characteristic.

Mantissa: The decimal part is known as mantissa and is always positive.

e.g., In $log_a x$, the integral part of x is called the characteristic and decimal part of x is called the mantissa.

For example: In log 3274 = 3.5150, the integral part is 3 i.e., characteristic is 3 and the decimal part is .5150 i.e., mantissa is .5150.

Exercise 01

1. What is the value of $log_{3\sqrt{3}}27$?
(a) 2 (b) 3 (c) 4 (d) 5

2. Find the logarithm of 144 to the base $2\sqrt{3}$:
(a) 8 (b) 4 (c) $2\sqrt{3}$ (d) none of these

3. Evaluate log $(36\sqrt{6})$ to the base 6.
(a) 1/2 (b) 5/2 (c) 3/2 (d) 7/2

4. Find the value of $3\log\frac{81}{80} + 5log\frac{25}{24} + 7log\frac{16}{15}$
(a) log2 (b) log3 (c) 1 (d) None of these

5. If $\log_{10}2 = 0.301$ find $\log_{10}125$.
(a) 2.097 (b) 2.301 (c) 2.10 (d) 2.087

6. $\log_{32}8$ = ?
(a) 2/5 (b) 5/3 (c) 3/5 (d) 4/5

7. $\log_a 4 + \log_a 16 + \log_a 64 + \log_a 256 = 10$. Then a = ?
(a) 4 (b) 2 (c) 8 (d) 5

8. If $\log_m n = p$, then:
(a) m=p (b) $p^n = m$ (c) $m^p = n$ (d) $n^p = m$

9. If $\log_a(mn)$ is equal to:
(a) $\log_a(m)$ (b) $\log_a m \times \log_a n$
(c) $\log_a m + \log_a n$ (d) $\log_a m - \log_a n$

10. $\log_a\left(\frac{m}{n}\right)$ is equal to?
(a) $\log_a(m-n)$ (b) $\log_a m - \log_a n$ (c) $\left(\frac{log_a m}{n}\right)$ (d) $\log_a m \div \log_a n$

11. The value of $\log_{81}27$ is:
(a) 3^7 (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

12. The value of $\log_{36}\frac{1}{216}$ is:
(a) $\frac{2}{3}$ (b) $-\frac{3}{2}$ (c) $\frac{1}{6}$ (d) none of these

13. Find the value of $\log_{10}(0.0001)$ is:
(a) $\frac{1}{1000}$ (b) -3 (c) -4 (d) none of these

14. The value of $\log_{(0.01)}(10000)$ is:
(a) 1/2 (b) -2 (c) 1/4 (d) -4

15. The value of $\log_2 \log_2 \log_3 \log_5(125)^3$ is:

(a) 0 (b) 1 (c) 2 (d) 3

16. The value of $\log_2 \log_2 \log_2 \log_2(65536)$ is:
(a) 4 (b) 2 (c) 1 (d) 0

17. The value of $\log_{10}1 + \log_{10}10 + \log_{10}100 + \ldots \log_{10}10000000000$
(a) 10 (b) 11 (c) 11111111111 (d) 55

18. The value of $\log_5 5 + \log_5 5^2 + \log_5 5^3 + \ldots + \log_5 5^n$
(a) n! (b) n^2-1 (c) $\frac{(n+1)n}{2}$ (d) none of these

19. The value of $216^{\log_6 49}$ is:
(a) 117694 (b) 117649 (c) 65631 (d) none of these

20. $\log_{625}\sqrt{5} = ?$
(a) 4 (b) 8 (c) 1/8 (d) 1/4

21. $\frac{1}{\log_{ab}abc} + \frac{1}{\log_{bc}abc} + \frac{1}{\log_{ca}abc}$ is equal to:
(a) 0 (b) 1 (c) 2 (d) 3

22. The value of $\left[\frac{1}{\log_{a/b}x} + \frac{1}{\log_{b/c}x} + \frac{1}{\log_{c/a}x}\right]$ is:
(a) 0 (b) 1 (c) abc (d) x^3

23. The value of $(\log\tan 1^0 + \log\tan 2^0 + \ldots + \log\tan 89^0)$ is:
(a) -1 (b) 0 (c) $\frac{1}{\sqrt{2}}$ (d) 1

24. If a, b, c, are in GP then $\log_{10}a, \log_{10}b, \log_{10}c$ are in;
(a) GP (b) HP (c) AP (d) none of these

25. If $\log_{10}x, \log_{10}y, \log_{10}z$ are in AP then x, y, z are in:
(a) AP (b) GP (c) HP (d) none of these

Solutions Exercise 01

1. Ans. (a)

Solution: $\log_{3\sqrt{3}} 27 = \log_{3\sqrt{3}}(3\sqrt{3})^2 = 2\log_{3\sqrt{3}} 3\sqrt{3} = 2.$
Hence option (a) is the answer.

2. Ans. (b)

Solution:

$$\log_{2\sqrt{3}}144 = \log_{2\sqrt{3}}(2\sqrt{3})^4 = 4$$

3. Ans. (b)

Solution: $\log_6 36\sqrt{6} = \log_6 6^{2.5} = 2.5(\log_6 6) = 2.5$
Hence, option (b) is the answer.

4. Ans. (a)

Solution: $3\log\frac{81}{80} = \log[\frac{81}{80}]^3$; $5\log\frac{25}{24} = \log[\frac{25}{24}]^5$; $7\log\frac{16}{15} = \log[\frac{16}{15}]^7$

So, $3\log\frac{81}{80} + 5\log\frac{25}{24} + 7\log\frac{16}{15} = \log$

$\left([\frac{81}{80}]^3 \times [\frac{25}{24}]^5 \times [\frac{16}{15}]^7\right) = \log 2$

Hence option (a) is the answer.

5. Ans. (a)

Solution: $\log_{10}125 = \log_{10}(1000/8) = \log 1000 - 3\log 2$
$= 3 - 3 \times .301 = 2.097$

6. Ans. (c)
Solution: $\log_{32}8 = \log 8/\log 32$ (By base change rule) = $3\log 2/5\log 2 = 3/5$.

7. Ans. (a)
Solution: The given expression is: $\log_a(4 \times 16 \times 64 \times 256) = 10$
i.e. $\log_a 4^{10}=10$
Thus, a = 4.

8. Ans. (c)
Solution: $\log_m n=p$
$m^p=n$

9. Ans. (c)
Solution: $\log_a(mn)$
$\log_a m + \log_a n$

10. Ans. (b)
Solution: $\log_a\left(\frac{m}{n}\right)$
$\log_a m - \log_a n$

11. Ans. (c)
Solution: $\log_{81}27\ \frac{\log_{10}27}{\log_{10}81} = \frac{\log_{10}3^3}{\log_{10}3^4} = \frac{3\log_{10}3}{4\log_{10}3} = \frac{3}{4}$

12. Ans. (b)
Solution: $\log_{36}\frac{1}{216} = \log_{36}6^{-3} = \frac{\log 6^{-3}}{\log 36}$

$= \frac{-3\log 6}{2\log 6}$

$= -\frac{3}{2}$

13. Ans. (c)
Solution: $\log_{10}(0.0001)=x$
$10^x=0.0001 = 10^{-4}$
$x=-4$

14. Ans. (b)
Solution: $\log_{(0.01)}(10000)=x$
$(0.01)^2=10000$
$(10^{-2})^x=10000$
$10^{-2x}=10^4$
$X=-2$

15. Ans. (a)
Solution: $\log_2 \log_2 \log_3 \log_5 5^9$
$= \log_2 \log_2 \log_3 9 = 0$

16. Ans. (c)
Solution: $\log_2 \log_2 \log_2 \log_2 (65536) = \log_2 \log_2 \log_2 \log_2(2^{16})$
$= \log_2 \log_2 \log_2 16 = \log_2 \log_2 4$
$= \log_2 = 1$

17. Ans. (d)
Solution: $\log_{10}1 + \log_{10}10 + \log_{10}100 + \ldots$
$\text{Log}_{10}1000000000$

$\log_{10}(1 \times 10 \times 100 \times \ldots\ldots \times 10000000000)$
$\log_{10}10^{(0+1+2+3+\ldots.10)}$
$\log_{10}10^{55}=55$

18. Ans. (c)
Solution: $\log_5 5 + \log_5 5^2 + \log_5 5^3 + \ldots. + \log_5 5^n = 1+2+3+\ldots.+n$
$= \frac{n(n+1)}{2}$

19. Ans. (b)
Solution: $216^{\log_6 49}$
$= 6^{\log 6\,(49)3}$
$= (49)^3 = 117649$

20. Ans. (c)
Solution: $\frac{1}{2}\log_{625}5 = [(1/2) \times 4]\log_5 5 = 1/8$.

21. Ans. (c)
Solution: $\frac{1}{\log_{ab}abc} + \frac{1}{\log_{bc}abc} + \frac{1}{\log_{ca}abc}$

$\log_{abc}(ab) + \log_{abc}(bc) + \log_{abc}(ca)$
$\log_{abc}(abc)^2 = 2$

22. Ans. (a)
Solution: $\frac{1}{\log_{a/b}x} + \frac{1}{\log_{b/c}x} + \frac{1}{\log_{c/a}x}$

$\log_x\frac{a}{b} + \log_x\frac{b}{c} + \log_x\frac{c}{a}$
$= \log_x\left(\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a}\right) = \log_x 1=0$

23. Ans. (b)
Solution: $\log\tan 1^0 + \log\tan 2^0 + \ldots. + \log\tan 89^0$
$= (\log\tan 1^0 + \log\tan 89^0) + (\log \tan 2^0 + \log\tan 88^0) + \ldots. + \log\tan 45^0$
$= \log(\tan 1^0.\tan 89^0) + \log(\tan 2^0.\tan 88^0) + \ldots. + \log 1$

$= \log(\tan 1^0.\cot^0) + \log(\tan 2^0.\cot 2^0) + \ldots. + \log 1$

$= \log 1 + \log 1 + \ldots..+\log 1$

$(\tan(90-\theta) = \cot\theta$ and $\tan 45^0 = 1)$

$= 0$

24. Ans. (c)

Solution: a, b, c, are in GP

$b^2 = ac$

$2\log b = \log a + \log c$

$\log_x a, \log_x b, \log_x c$, are in AP

$\log_{10} a, \log_{10} b, \log_{10} c$ are in AP

25. Ans. (b)

Solution: $\log_{10} x, \log_{10} y, \log_{10} z$ are in AP

$2\log_{10} y, = \log_{10} x + \log_{10} z$

$\log_{10} y^2, = \log_{10}(xz)$

$y^2 = xz$

x, y, z are in GP

Exercise 02

1. If a, b, c are in GP then $\frac{1}{\log_a X}, \frac{1}{\log_b X}, \frac{1}{\log_c X}$ are in:
   (a) AP (b) GP (c) HP (d) none of these

2. If $2[\log(x+y) - \log 5] = \log x + \log y$, then what is the value of x^2+y^2?
   (a) 20xy (b) 23xy (c) 25xy (d) 28xy

3. If x>1, y>1, and z>1 are three numbers in geometric progression, then $\frac{1}{1+\log x}, \frac{1}{1+\log y}, and\ \frac{1}{1+\log z}$, are in:
   (a) Arithmetic progression (b) Harmonic progression
   (c) Geometric progression (d) None of these

4. What is the value of x if $\log_3 x + \log_9 x + \log_{27} x + \log_{81} x = \frac{25}{4}$?
   (a) 9 (b) 27 (c) 81 (d) None of these

5. What is the value of x in the expression $\log_2(3-x) + \log_2(1-x) = 3$?
   (a) 1 (b) 0 (c) -1 (d) Not possible

6. If $x = \log_a(bc)$, $y = \log_b(ca)$, and $z = \log_c(ab)$, then what is the value of $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$?
   (a) 0 (b) 1 (c) xyz (d) -1

7. Solve the following equations for x: $\log_{10}(3-x) > \log_{10}(x-1)$.
   (a) x>2 (b) 1<x<2 (c) 0<x<8 (d) 1<x<3

8. If $3^{x+1} = 6^{\log_2 3}$, then x is:
   (a) 2 (b) 3 (c) $\log_3 2$ (d) $\log_2 3$

9. If $\log_4 5 = a$ and $\log_5 6 = b$, then what is the value of $\log_3 2$?
   (a) $\frac{1}{2a+1}$ (b) $\frac{1}{2b+1}$ (c) 2ab+1 (d) $\frac{1}{2ab-1}$

10. Find the value of $\frac{1}{\log_2 X} + \frac{1}{\log_3 X} + \frac{1}{\log_4 X} + \frac{1}{\log_5 X} + \frac{1}{\log_6 X} + \frac{1}{\log_7 X}$.
    (a) 1 (b) $\log_{27} X$ (c) $\log_{5040} X$ (d) $\log_x 5040$

11. If $\frac{logX}{log4} = \frac{log343}{log49} = \frac{logY}{log64}$, then what is the value of x+y?
(a) 520 (b) 740 (c) 880 (d) Cannot be determined

12. If $\log_y x=8$ and $\log_{10y}16x=4$, then find the value of y.
(a) 1 (b) 2 (c) 3 (d) 5

13. What is the value of $log_{3\sqrt{3}}27$?
(a) 2 (b) 3 (c) 4 (d) 5

14. If $3\log_{3x}{}^2 27-2\log_{3x} 9=0$, then what is the value of x?
(a) 1/243 (b) 1/7 (c) 880 (d) Cannot be determined

15. If a_1, a_2, a_3….are positive numbers in GP, then log a_n, $\log a_{n+1}$, ...and $\log a_{n+2}$ are in:
(a) AP (b) GP (c) HP (d) none of these

16. If $\log_{30}3$ = x and $\log_{30}5$ = y, then $\log_8 30$ is equal to:
(a) 3(1-x-y) (b) $\frac{1}{3(1-x-y)}$ (c) $\frac{3}{(1-x-y)}$ (d) $\frac{1-x-y}{3}$

17. What is the value of $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \ldots + \frac{1}{\log_{40} n}$?
(a) $\frac{1}{\log_{40!} n}$ (b) $\log_{(40!)} n$ (c) 1 (d) none of these

18. What is the value of $\log_3 2$, $\log_4 3$, $\log_5 4$ ….$\log_{16} 15$?
(a) 1/2 (b) 1/3 (c) 2/3 (d) ¼

19. If loga =b, find the value of 10^{3b} in terms of a.
(a) a^3 (b) 3a (c) a × 1000 (d) a × 100

20. 3 log 5 + 2 log 4 – log2 = ?
(a) 4 (b) 3 (c) 200 (d) 1000

21. $3^x=7$, then find x
(a) $1/\log_7 3$ (b) $\log_7 3$ (c) $1/\log_3 7$ (d) None of these

22. $\log \frac{12}{13} - \log \frac{7}{25} + \log \frac{91}{3} = x$
(a) 0 (b) 1 (c) 2 (d) 3

23. $\log \frac{16}{15} + 5 \log \frac{25}{24} + 3 \log \frac{81}{80} = \log x.\ x = ?$
(a) 2 (b) 3 (c) 0 (d) none of these

24. If log 3 = .4771 and log 2= .301 then number of digits in 60^{12}
(a) 25 (b) 22 (c) 23 (d) 24

25. If $log3$ = .4771 and log 2= .301 then number of digits in 72^9
(a) 17 (b) 20 (c) 18 (d) 15

Solution Exercise 02

1. Ans. (a)

Solution: a, b, c are in GP

$\log_x a, \log_x b, \log_x c$, are in AP

$\frac{1}{\log_a x}, \frac{1}{\log_b x}, \frac{1}{\log_c x}$ are in AP

2. Ans. (c)

Solution: 2[log(x+y)-log5] =logx+logy can be written as;

[log(x+y)-log5] $=\frac{1}{2}$ (logx+logy)

Or $\log\frac{x+y}{5} = \log\sqrt{xy}$

Taking antilog on both the sides, we obtain the following;

$\frac{x+y}{5} = \log\sqrt{xy}$

$x+y = 5\sqrt{xy}$

Squaring both the sides, we get $(x+y)^2=25xy$

$x^2+y^2+2xy=25xy$

$x^2+y^2=23xy$

Hence, option (b) is the answer.

3. Ans. (b)

Solution: In this question, it is better to assume values and verify the options.

Assume x=10, y=100, z=1000

$\frac{1}{1+logx} = \frac{1}{1+log10} = \frac{1}{1+1} = \frac{1}{2}$

$\frac{1}{1+logy} = \frac{1}{1+log100} = \frac{1}{1+2} = \frac{1}{3}$

$\frac{1}{1+logz} = \frac{1}{1+log1000} = \frac{1}{1+3} = \frac{1}{4}$

Now, it can be clearly seen that ½, 1/3, and ¼ are in harmonic progression, Hence, option (b) is the answer.

4. Ans. (d)

Solution: Go through the options.

5. Ans. (c)

Solution: Go through the options.

It can be seen that x=-1 satisfies the equations. Hence, option (c) is the answer.

6. Ans. (b)

Solution: $1+x= \log_a a+ \log_a(bc),= \log_a(abc)$

Similarly, $1+y= \log_b(abc)$ and $1+z= \log_c(abc)$

$\frac{1}{1+x} = \log_{abc}a$

$\frac{1}{1+y} = \log_{abc}b$

$\frac{1}{1+z} = \log_{abc}c$

$\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} = \log_{abc}a + \log_{abc}b + \log_{abc}c$

$= \log_{abc}abc = 1$

Hence option (b) is the answer.

7. Ans. (b)

Solution: For log to be defined:

3-X>0; So, x<3, and x-1>0; so, 1<x

Hence, 1<x<3

As, the base >1, therefore, 3-x>x-1

2x<4

x<2

Combining equations (i) and (ii), we get the following range of values of x;1<x<2.

Hence, option (b) is the answer.

8. Ans. (d)

Solution: $3^{x+1}=(3\times2)log_2^3$

$3^{x+1}=3^{log_2^3} \times 2^{log_2^3}$

$3^{x+1}=3^{log_2^3} \times 3$

$3^x= 3^{log_2^3}$

$x= log_2 3$

9. Ans. (d)

Solution: log_4^5 = a and log_5^6 = b

$log_4^5 \times log_5^6$ =ab

$log_5^6 = ab.$

$\frac{1}{2}\, log_5^6$ =ab

$(1 + log_2^3 = 2ab)$

$log_2^3 = 2ab - 1$

$log_2^3 = \frac{1}{2ab-1}$

10. Ans. (d)

Solution: Expression is equal to

$\log_x 2 \times 3 \times 4 \times 5 \times 6 \times 7$

$= \log_x 5040.$

11. Ans. (a)

12. Ans. (d)

Solution: $\log_y x=8$

$y^8=x$

$\log_{10y} 16x=4$

$10^4.y^4=16x$

Dividing equations (ii) by (i), we get $10^4.y^4=16$

y=5

13. Ans. (a)

Solution: $log_{3\sqrt{3}}27$

$= log_{3\sqrt{3}}(3\sqrt{3})^2$

$= 2$

14. Ans. (a)

Solution: $3\log_{3x^2}27-2\log_{3x}9=0$

$3\log_{3x^2}3^3-2\log_{3x}3^2=0$

$9\log_{3x^2}3=4\log_{3x}3$

$$\frac{9}{\log_{3x}3}=\frac{4}{3\log_{3x^2}3}$$

$9\log_3 3x=4.\log_3 3x^2$

$\log_3(3x)9=\log_3(3x^2)^4$

$3^9\times x^9=3^4\times x^8$

$x=3^{-5}$

$x = \frac{1}{243}$

15. Ans. (a)

Solution: $a_1= a_1$; $a_2= a_1r$; $a_3= a_1r^3$ are in GP with common difference 'r^2.

$\log a_1=\log a_1$

$\log a_2=\log a_1+\log r$

$\log a_3=\log a_1+2\log r$

$\log a_4=\log a_1+3\log r$

$\log a_{n+1}=\log a_1+n\log r$

$\log a_1, a_2, a_3, a_4,\ldots.. \log a_{n+2}$ are in AP with common difference 'log r'.

16. Ans. (b)

Solution: $\log_{30}3 = x$, $\log_{30}5 = y$

$x+y = \log_{30}5*3$

$x+y = \log_{30}(30/2)$

$(x+y)=1- \log_{30}2$

$\log_{30}2=1-x-y$

$3 \log_{30}2=3(1-x-y)$

$\log_{30}8=3(1-x-y)$

$\log_8 30 = \frac{1}{3(1-x-y)}$

17. Ans. (a)

Solution: $\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+\ldots..\frac{1}{\log_{40} n}$

$\log_n 2+\log_n 3+\log_n 4+\ldots\ldots\log_n 40$

$\log_n(40!)$

$\frac{1}{\log(40)!n}$

18. Ans. (d)

Solution: $\log_3 2.\log_4 3.\log_5 4\ldots.3\log_{16} 15$

$\frac{log2}{log3}.\frac{log3}{log4}.\frac{log4}{log5}\cdots\frac{log15}{log16}=\frac{log2}{log16}=\frac{log2}{log2^4}.$

$=\frac{log2}{4log2}=\frac{1}{4}$

19. Ans. (a)

Solution: $\log_{10}a = b$ ➔ $10^b = a$ ➔ By definition of logs.

Thus $10^{3b} = (10^b)^3 = a^3$.

20. Ans. (b)

Solution: 3 log 5 + 2 log 4 – log 2

= log 125 + log 16 – log 2

= log (125 x 16)/ 2 = log 1000 = 3.

21. Ans. (a)

Solution: $3^x = 7$ ➔ $\log_3 7 = x$

Hence $x = 1/\log_7 3$

22. Ans. (c)

23. Ans. (d)

Solution: $x = (16/15) \times (25^5 / 24^5) \times (81^3 / 80^3)$

None of these is correct.

24. Ans. (b)

25. Ans. (a)

Exercise 03

1. If log 3 = .4771 and log 2= .301 then number of digits in 27^{25}

(a) 38 (b) 37 (c) 36 (d) 35

2. If $\frac{logx}{b-c}=\frac{logy}{c-a}=\frac{logz}{a-b}$. mark all the correct options.

(a) $xyz = 1$ (b) $x^a y^b z^c = 1$ (c) $x^{b+c} y^{c+a} z^{a+b} = 1$ (d) all the options are correct.

3. If three positive real numbers a, b and c (c > a) are in Harmonic Progression, then log (a + c) + log (a-2b+c) is equal to:
(a) 2 log (c - b) (b) 2 log (c-1)
(c) 2 log (c-a) (d) log a + log b + log c

4. If $\log_2 x \log_{x/64} 2 = \log_{x/16} 2$ then x is
(a) 2 (b) 4 (c) 16 (d) 12

5. What is the value of $\sqrt{\frac{a}{b}}$, if $\log_4 \log_4 4^{a-b} = 2\log_4(\sqrt{a} - \sqrt{b}) + 1$
(a) -5/3 (b) 2 (c) 5/3 (d) 1

6. If $\log_{10} x - \log_{10} \sqrt[3]{x} = 6\log_x 10$ then a possible value of x is
(a) 10 (b) 30 (c) 100 (d) 1000

7. If $\log_{13} \log_{21} \{\sqrt{x+21} + \sqrt{x}\} = 0$, then the value of x is
(a) 21 (b) 13 (c) 81 (d) none of these

8. Find the value of x from the following equation: $\log_{10} 3 + \log_{10}(4x+1) = \log_{10}(x+1)+1$
(a) 2/7 (b) 7/2 (c) 9/2 (d) none of above

9. The value of the expression:
$\sum_{i=2}^{100} \frac{1}{\log_i 100!}$ is:
(a) 0.01 (b) 0.1 (c) 1 (d) 10

10. The domain of the function $f(x) = \log_7\{\log_3(\log_5(20x-x^2-91))\}$ is:
(a) (7, 13) (b) (8, 12) (c) (7, 13) (d) (12, 13)

11. If $\log_5[\log_3(\log_2 x)] = 1$, then x is:
(a) 2^{234} (b) 243 (c) 2^{243} (d) none of these

12. If $\log_e(x-1) + \log_e x + \log_e(x+1) = 0$, then
(a) x^2+e^{-1} (b) $x^3-x-1=0$ (c) $x^2+e-1=0$ (d) $x^3-x-e=0$

13. $\left(\log\frac{a^3}{bc} + \log\frac{b^3}{ac} + \log\frac{c^3}{ab}\right)$ is equal to:
(a) 1 (b) log abc (c) abc (d) none of these

14. $\frac{1}{(\log_a bc)+1} + \frac{1}{(\log_b ac)+1} + \frac{1}{(\log_c ab)+1}$ is equal to:
(a) 1 (b) 2 (c) 0 (d) abc

15. If $\log_a N = (\log_b N) \times P$, then find P in terms of a and b:
(a) b^a (b) a^b (c) $\log_a b$ (d) none of these

16. The value of $\log (ab)^2 - \log (ac) + \log (bc^4) - 3\log (bc)$ is:
(a) 0 (b) log b (c) log c (d) log a

17. If $\log_q (xy) = 3$ and $\log_q (x^2y^3) = 4$, find the value of $\log_q x$:

(a) 4 (b) 5 (c) 3 (d) 2

18. If $u=v^2=w^3=z^4$, then $\log_u(uvwz)$ is equal to:
(a) $1+\frac{1}{2}-\frac{1}{3}-\frac{1}{4}$ (b) 24 (c) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$ (d) $\frac{1}{24}$

19. If a, b, c are in GP, then $\log_a d$, $\log_b d$, $\log_c d$ are in:
(a) AP (b) HP (c) GP (d) none of these

20. Find the value of $\frac{1}{\log_3 e}, \frac{1}{\log_3 e^2}, \frac{1}{\log_3 e^4}$ +.....is:
(a) $\log_e 9$ (b) 0 (c) 1 (d) $\log_e 270$

21. Find the value of $(b^2)^{5\log_b x}$:
(a) 10^x (b) x^4 (c) 10x (d) x^{10}

22. The value of x satisfying the following relation
$\text{Log}_{1/2}x=\log_2(3x-2)$

23. If $\log_3 2$, $\log_3(2^x-5)$ and $\log_3(2^x-7/2)$ are in AP then x is equal to:
(a) 2 (b) 3 (c) 4 (d) 5

24. If $\log_x a$, $a^{x\backslash 2}$ and $\log_b x$ are in GP, then x is equal to:
(a) $\log_a(\log_b a)$
(b) $\log_a(\log_e a)$- $\log_a(\log_e b)$-
(c) - $\log_a(\log_b a)$
(d) both a and b

25. The value of $\frac{1}{\log_{100} n}+\frac{1}{\log_{99} n}+\frac{1}{\log_{98} n}+\cdots ...+\frac{1}{\log_2 n}$ is:

(a) 1 (b) $\frac{1}{\log_{100} n}$ (c) $\frac{1}{\log_{99} n}$ (d) $\frac{1}{\log_{100!} n}$

26. Find x, if $log_{2\sqrt{x}}x + log_{2x}\sqrt{x}$ =0:
(a) $1,2^{-5/6}$ (b) $1,2^{-6/5}$ (c) 4,-2 (d) none of these

27. If $a=1+\log_x yz$, $b=1+\log_y zx$, and $c=1+\log_z xy$, then ab+bc+ca is:
(a) 1 (b) 0 (c) abc (d) none of these

28. Find x if $log_{1\sqrt{2}}(1/\sqrt{8})=\log_2(4^x+1).\log_2(4^{x+1}+4)$
(a) 0 (b) 1 (c) 2 (d) none of these

29. Find the sum of 'n' terms of the series.
$\text{Log}_2\left(\frac{x}{y}\right)+log_4\left(\frac{x}{y}\right)^2+log_8\left(\frac{x}{y}\right)^3+log_{16}\left(\frac{x}{y}\right)^4+\cdots$.:

(a) $log_2\left(\frac{x}{y}\right)^{4n}$
(b) $n\left(log_2\frac{x}{y}\right)$
(c) $log_2\left(\frac{x^{n-1}}{y^{n-1}}\right)$
(d) $\frac{1}{2}log_2\left(\frac{x}{y}\right)^{n(n+1)}$

30. The set of solutions for all x satisfying the equation.
$x^{\log_3 x^2+(\log_3 x)^2-10}=\frac{1}{x^2}$
(a) (1,9) (b) (1,4,16) (c) (1,9,81) (d) $\left\{1,\frac{1}{81},9\right\}$

31. The set of all values of x satisfying $x^{\log_x |3-x|^2} = 4$

(a) $(\sqrt{3},2)$ (b) (1,5) (c) $(1,\infty)$ (d) {1,5}

32. Find all real values of x satisfying equation
$|x-1|^{(\log_3 x2 - 2\log_x 9)}$ = $(x-1)^7$

(a) (2,9) (b) 9,81 (c) 2,81 (d) none of these

Solutions Exercise 03

1.

Ans. (c)

2. Ans. (d)

Solution: $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$ = k

⇨ x = $10^{k(b-c)}$, y = $10^{k(c-a)}$, z = $10^{k(a-b)}$

∴ xyz = $10^{k(b-c+c-a+a-b)}$ = 10^0 = 1

Therefore option (a) is correct.

$x^a y^b z^c$ = $10^{k[a(b-c)+b(c-a)+c(a-b)]}$

= $10^{k(ab-ac+bc-ab+ca-bc\ (a-b))}$

= 10^{k0} = 1

Therefore option (c) is also correct.

3. Ans. (c)

Solution: b = $\frac{2ac}{a+b}$ (As a, b and c are in harmonic progression.)

$\log(a+c) + \log(a+c-2b) = \log[(a+c)(a+c-2b)]$

$= \log\left[(a+c)\left(a+c-\frac{4ac}{a+c}\right)\right]$

$= \log\left[(a+c) x \frac{(a+c)^2-4ac}{(a+c)}\right]$

$= \log[(c-a)]^2 = 2\log(c-a)$.

4. Ans. (b)

Solution: We can solve this problem just by checking the operations.

By putting x = 4 in the LHS we have

$$(2\log_2 2)\left(\log_{\frac{1}{16}} 2\right) = \frac{2\log_2 2}{(-4)\log_2 2} = -\frac{1}{2}$$

Similarly, the RHS = $\log_{\frac{1}{4}} 2 = -\frac{1}{2}$

Similarly we can check the other options as well. We find that only option (b) is correct.

5. Ans. (c)

Solution: $\log_4\log_4 4^{a-b} = \log_4(\sqrt{a} - \sqrt{b})^2 + 1$

Replacing the value of 1 in this equation with $\log_4 4$

We get: $\log_4(a-b) = \log_4 4\ (a+b-2\sqrt{ab})$

$(a-b) = 4\ (a+b-2\sqrt{ab})$ & a – b ≠ 0 or a ≠ b

(Since log 0 is not defined

$3a + 5b - 5\sqrt{ab} = 0$

Dividing throughout by b, we get: 3

$$\left(\sqrt{\frac{a}{b}}\right)^2 - 8\sqrt{\frac{a}{b}} + 5 = 0$$

Let $\sqrt{\frac{a}{b}}$ = x.

Then, $3x^2 - 8x + 5 = 0$.

Now by putting the options in place of x in the above equation we get x = 5/3, 1 satisfy the above equation but x = 1 is not possible because a ≠ b so x = $\sqrt{\frac{a}{b}}$ = 5/3. Option (c) is the correct answer.

6. Ans. (d)

Solution: $\log_{10}x - \log_{10}(x)^{1/3} = \frac{6}{\log_{10}x}$

$$\therefore \log_{10}x - \frac{1}{3}\log_{10}x = \frac{6}{\log_{10}x}$$

If we replace $\log_{10}x$ = t we get the quadratic equation:

$$t^2 - \frac{1}{3}t^2 = 6$$

by solving the above equation we get t = ± 3

When $\log_{10}x$ = -3

∴ x = 10^{-3}

When $\log_{10}x$ = 3

x = 1000.

Note: You could solve this by using options too.

7. Ans. (d)

Solution: $\log_{13}\log_{21}\left\{\sqrt{x+21} + \sqrt{x}\right\} = 0$

$\therefore \log_{21}\{\sqrt{x+21} + \sqrt{x}\} = 13^0 = 1$

$\therefore \{\sqrt{x+21} + \sqrt{x}\} = 21^1 = 21$

$\therefore \sqrt{x+21} = 21 - \sqrt{x}$

By putting the options in the given equation we find that none of the options is correct.

8. Ans. (b)
Solution: : $\log_{10}3 + \log_{10}(4x+1) = \log_{10}(x+1)+\log_{10}10$

$\log_{10}3\,(4x+1) = \log_{10}(x+1)10$
$3\,(4x+1) = (x+1)10$
By solving the above equation we get
$X = 7/2$
Note: You could solve this by using options too.

9. Ans. (c)
Solution: $\sum_{i=2}^{100}\frac{1}{\log_i 100!} = \sum_{i=2}^{100}\log_{100!}i = \log_{100!}2$
$+ \log_{100!}3 + \log_{100!}4 + \ldots + \log_{100!}100$
$\log_{100!}(2,3,4\ldots100) = \log_{100!} 100! = 1$

10. Ans. (b)
Solution: $f(x) = \log_7\{\log_3(\log_5(20x-x^2-91))\}$
⇨ $\log_3(\log_5(20x-x^2-91)) > 0$
⇨ $20x - x^2 - 91 > 5$
⇨ $X^2 - 20x + 96 < 0$
⇨ $(x-8)(x-12)<$
⇨ $x\in(8,12)$

11. Ans. (c)
Solution: $\log_5[\log_3(\log_2 x)] = 1 = \log_5 5$
$\text{Log}_3(\log_2 x) = 5 = \log_3 3^5$
$\log_2 x = 3^5 = 243$
$2^{243} = x$

12. Ans. (b)
Solution: $\log_e(x-1)+\log_e x+\log_e(x+1)=0$
$\log_e(x-1)\times x\,(x+1)=\log_e 1$
$\log_e(x^2-1)\,x = \log_e 1$
$(x^2-1)\,x = 1$
$X^3 - x - 1 = 0$

13. Ans. (b)
Solution: $\log\frac{a^3}{bc} + \log\frac{b^3}{ac} + \log\frac{c^3}{ab} = \left(\frac{a^3b^3c^3}{ab+bc+ca}\right)$
= log abc

14. Ans. (a)
Solution: $\frac{1}{(\log_a bc)+1} + \frac{1}{(\log_b ac)+1} + \frac{1}{(\log_c ab)+1}$

$= \frac{1}{\log_a bc+\log_a a} + \frac{1}{\log_b ac+\log_b b} + \frac{1}{\log_c ab+\log_c c}$

$= \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$

$= \log_{abc}a + \log_{abc}b + \log_{abc}c$
$= \log_{abc}abc = 1$

15. Ans. (c)
Solution:

$\log_a N = \log_b N\times P,$

$\frac{\log_a N}{\log_b N} = p$

$\frac{\frac{1}{\log_a N}}{\frac{1}{\log_b N}} = p$

$\frac{\log_N b}{\log_N a} = p$

$\log_a b = p$

16. Ans. (d)
Solution:

$\log(ab)^2 - \log(ac) + \log(bc^4) - 3\log(bc)$
$= \log\frac{(ab)^2}{ac} + \log\frac{(bc^4)}{(bc)^3}$

$= \log\left[\frac{(ab)^2}{ac}\times\frac{(bc^4)}{(bc)^3}\right] = \log a$

17. Ans. (b)
Solution:

$\log_q(xy) = 3$ and $\log_q(x^2y^3)=4$
$\log_q(x^2y^3)=4$
$\log_q[(xy)^2.y]$
$\log_q(xy)^2+\log_q y = 4$
$2\log_q(xy)+\log_q y = 4$
$2\text{x}3+\log_q y = 4$
$\log_q y = -2$
Again $\log_q x + \log_q y = 3$
$\log_q x + (-2) = 3$

$\log_q x = 5$

18. Ans. (c)
Solution:

$v=u^{1/2}$ $w=u^{1/3}$ $z=^{1/4}$

uvwz = $u^{\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)}$

$\log_u$ uvwz = $\log_u u^{\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)}$

$= \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right) log_u u = \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)$

19. Ans. (b)
Solution:

a,b,c are in GP

$b^2=ac$

2logb = log a + logc

Loga, logb, log c are in AP

l,e, $\log_d$ a, $\log_d$ b, $\log_d$ c are in AP

$\log_a$ d, $\log_b$ d, $\log_c$ d are in HP

20. Ans. (a)
Solution:

$\frac{1}{\log_3 e}, \frac{1}{\log_3 e^2}, \frac{1}{\log_3 e^4} + \ldots\ldots$

$= \frac{1}{\log_3 e}, \frac{1}{2\log_3 e}, \frac{1}{4\log_3 e} + \ldots\ldots$

$= \frac{1}{\log_3 e}\left(1+\frac{1}{2}+\frac{1}{4}+\cdots\right)$

$= \log_e 3\left(\frac{1}{1-1/2}\right)$.

(Using sum of infinite GP $\left(S_\infty = \frac{a}{1-r}; |r|<1\right)$

$= 2\log_e 3 = \log_e 3^2 = \log_e 9$

21. Ans. (d)
Solution:

$(b^2)^{5\log bx} = b^{10\ \log bx} = b^{\log b(x)10 = x10}$

22. Ans. (d)
Solution:

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$
(c) 3 (d) none of these

$\log_{1/2}x=\log_2(3x-2)$
$-\log_2 x=\log_2(3x-2)$
$\log_2(x^{-1})=\log_2(3x-2)$
$(x^{-1})=(3x-2)$
$3x^2-3x+x-1=0$
$3x(x-1)+1(x-1)=0$
x=1,. -1/3

but at x=-1/3, log x is not defined
The only admissible value of x is 1.

23. Ans. (b)
Solution:
Since we know that when a, b, c, are in GP, then log a, log b and c are in AP.

Therefore 2,(2^x-5) and (2^x-7/2) must be in GP

Now, going through options, we get

At x = 3 the three terms 2, (2^x-5) and log(2^x-7/2) are in AP.

Alternatively:

We have $2log_3(2^x-5) = log_3 2 + log_3(2^x-7/2)$

⇨ $log_3(2^x-5)^2 = log_3 2\,(2^x-7/2)$

⇨ $(2^x-5)^2 = 2\,(2^x-7/2)$

⇨ $(2^x)^2 * 10.\ 2^x + 25 = 2.2^x-7$

$[10.2^x+2.2^x = 12.2^x]$ $[\therefore (a-b)^2 = a^2-2ab+b^2]$

⇨ $(2^x)^2 - 12.2^x + 32 = 0$

⇨ $(2^x - 8)(2^x-4) = 0$

⇨ X = 3, x = 2

But at x = 2, log (2^x-5) is undefined Hence x = 3

24. Ans. (d)
Solution: As $\log_x$ a, $a^{x/2,}$ log $_b$x are in GP.

$\therefore (a^{x/2})^2 = \log_x a.\ \log_b x$

⇨ $a^x = \frac{log a}{log x}.\frac{log x}{log b} = \frac{\log a}{\log b} = \log_b a$

⇨ $a^x = \log_b a$

⇨ $x = \log_a(\log_b a)$

⇨ $= \log_a\left(\frac{log_e a}{log_e b}\right)$

⇨ $log_a(log_e a) - log_e(log_e b)$

25. Ans. (b)
Solution:

$$\frac{1}{\log_{100} n} + \frac{1}{\log_{99} n} + \cdots \ldots + \frac{1}{\log_2 n}$$

$$= \log_n 100 + \log_n 99 + \cdots + \log_n 2$$

$$= \log_n (100 \times \log_n 99 + \cdots + \times 2)$$

$$= \log_n 100! = \frac{1}{\log_{100!} n}$$

26. Ans. (b)

Solution: Best way is to go through options.

Alternatively: Support $\log_2 x = t$, then

$$log_{2\sqrt{x}} = \frac{log_{2\sqrt{x}}}{log_2 2x} = \frac{\frac{1}{2} log_2 x}{1 + log_{2x}} = \frac{t/2}{1+t}$$

$$log_{2\sqrt{x}} x + log_{2x}\sqrt{x} = \frac{2t}{2+t} + \frac{t}{2+2t} = 0$$

⇨ $2t(2 + 2t) + 2t + t^2 = 0$

⇨ $4t + 4t^2 + 2t + t^2 = 0$

⇨ $5t^2 + 6t = 0$

⇨ $t(5t+6) = 0$

⇨ $t = 0$ or $t = -\frac{6}{5}$

$\log^2 x = 0$ ➔ $x = 2^0 = 1$

and $\log_2 x = -\frac{6}{5}$ ➔ $x = 2^{-6/5}$

$x = 1 \quad x = 2^{-6/5}$

27. Ans. (c)

Solution:

$a = 1 + \log_x yz = \log_x x + \log_x yz = \log_x xyz$

Similarly $b = \log_y xyz$

And $c = \log_z xyz$

Now, $ab + bc + ca = abc\left[\frac{1}{c} + \frac{1}{a} + \frac{1}{b}\right]$

$= abc\left[\frac{1}{log_x xyz} + \frac{1}{log_y xyz} + \frac{1}{log_z xyz}\right]$

$= abc[\log_{xyz} x + \log_{xyz} y + \log_{xyz} z]$

$= abc[\log_{xyz} xyz] = abc$

28. Ans. (a)

Solution:

Best way is to check the options

Alternatively:

$log_{1\sqrt{2}} = (1/\sqrt{8}) = \log_2(4^x+1).\log_2(4^{x+1}+4)$

$\frac{log_2(1/\sqrt{8})}{log_2(1/\sqrt{2})} = \log_2(4^x+1).\log_2(4^{x+1}+4)$

$3 = [\log_2 4 + \log_2(4^x+1)].\log_2(4^x+1)]$

$(2+t)t = 3$ where $t - \log_2(4^x+1)$

$t = -3, 1$

If $\log_2(4^x+1) = -3$, then $4^x = -7/8$ which is not possible

If $\log_2(4^x+1) = 1$

29. Ans. (b)

Solution: $\log_2\left(\frac{x}{y}\right) + \log_4\left(\frac{x}{y}\right)^2 + \log_8\left(\frac{x}{y}\right)^3 + \ldots.$

$= \log_2\left(\frac{x}{y}\right) + \log_2\left(\frac{x}{y}\right) + \log_2\left(\frac{x}{y}\right) + \ldots.$

$= \log_2\left(\frac{x}{y} \times \frac{x}{y} \times \frac{x}{y} \times \ldots . n\ time\right)$

$\text{Log}_2\left(\frac{x}{y}\right)^n = n \log_2\left(\frac{x}{y}\right)$

30. Ans. (d)

Solution: taking log of both sides with base 3 we have,

$[\log_3 x^2 + (\log_3 x)^2 - 10] \log_3 x = -2\log_3 x$

⇨ $\log_3 x = 10$

⇨ $(\log_3 x)^2 + 2\log_3 x - 3 = 0$

⇨ $(\log_3 x + 4)(\log_3 x - 2) = 0$

⇨ $x = 1$, $\log_3 x = -4$ and $\log_3 x = 2$

⇨ $x = 1$, $x = \frac{1}{81}$ and $x = 9$

$x = 1$,

$x = \left\{1, \frac{1}{81}, 9\right\}$

31. Ans. (d)

Solution: $x^{log_x |3-x|^2} = 4$

⇨ $|3-x|^2 = 4$

⇨ $|3-x| = 2$ (-2 is inadmissible)

⇨ $(3-x) = 2$ or $-(3-x) = 2$

⇨ $x = 1$ or $x = 5$

32. Ans. (c)

Solution: $x > 0, x \neq 1$

Since exponential function assumes positive value, so we must have $(x-1)^7 > 0$ i.e., $x > 1$.

Taking logarithm on both side, we get

$(\log_3 x^2 - 2\log_x 9) \log (x-1) = 7 \log(x-1)$

Either $\log(x-1) = 0$ i.e., $x = 2$

Or $\log_3 x^2 - 2\log_x 9 = 7$

⇨ $2(\log_3 x) - 4\log_x 3 = 7$

⇨ $2t - \frac{4}{t} = 7$, $(\because t = \log_3 x)$

⇨ $2t^2 - 7t - 4 = 0$

⇨ $T = 4, -\frac{1}{2}$

$\text{Log}^3 x = 4 \rightarrow x = 81$

If $\log_3 x = -\frac{1}{2}$ then $x = 3^{-1/2} < 1$, which is not the case

hence, x = 2, 81

Exercise 04 (Short Answers/TITA)

1. The number of solutions of the equation
$\log_{x/2} x^2 + 40\log_{4x}\sqrt{x} - 14\log_{16x} x^3 = 0$

2. Find the values of x satisfying
$log_{x^2+6x+8}log_{(2x^2+2x+3)}(x^2-2x) = 0$

3. The value of x satisfying
$\log_3 4 - 2\log_3\sqrt{3x+1} = 1 - \log_3(5x-2)$

4. Find x, if $\log x^3 - \log 3x = 2\log 2 + \log 3$.

5. The number of solutions of $\log_9(2x-5) = \log_3(x-4)$ is:

6. What will be the value of x if it is given that :
$\log_x\left[\frac{1}{5} + \frac{1}{12} + \frac{1}{21} + \frac{1}{32} + \frac{1}{45} + \cdots. + \infty\ terms\right]^2 = 2$

7. For how many real values of x will the equation $\log_3\log_6(x^3-18x^2+108x) = \log_2\log_4 16$ be satisfied?

8. If $n = 12\sqrt{3}$
$\frac{1}{log_2 n} + \frac{1}{log_3 n} + \frac{1}{log_4 n} + \frac{1}{log_6 n} + \frac{1}{log_8 n} + \frac{1}{log_9 n} + \frac{1}{log_{18} n} = ?$

9. $(\log_2 x)^2 + 2\log_2 x - 8 = 0$, where x is a natural number. If $x^p = 64$, then what is the value of x + p.

10. If [N] = the greatest integer less than or equal to N, then $[\log_{10} 6730.4]$ is equal to:

11. Find x if log x = log 7.2 – log 2.4

12. Find x if log x = 2 log 5 + 3 log 2

13. log (2x-2) – log (11.66-x) = 1 + log 3

14. If log 3 = .4771 and log 2= .301, find the number of digits in 108^{10}

15. If log 2 = .301, find the number of digits in $(125)^{25}$

Exercise 04 (Solutions)

1. Solution: By changing the base to 2 the given equation becomes
$\frac{\log_2 x^2}{\log_2 x/2} + \frac{40\log_2\sqrt{x}}{\log_2 4x} - 14\frac{\log_2 x^3}{\log_2 16x} = 0$

⇨ $\frac{2\log_2 x}{\log_2 x - 1} + 20\frac{\log_2 x}{2+\log_2 x} - 42\frac{\log_2 x}{4+\log_2 x} = 0$

$Let\ t = \log_2 x$, then we have

2t (4+t) (2+t) – 42t (t-1) (t+2) + 20t (t-1) (t+4) = 0

⇨ $2t[t^2+6t+8-21t^2-21t+42 + 10t^2+30t-40] = 0$
⇨ $t [2t^2 – 3t - 2] = 0$
⇨ t = 0, t = 2, t = -1/2
⇨ x = 1, x = 4, x=1/$\sqrt{2}$ Three

2. Ans. 1
Solution: $x^2 + 6x + 8 > 0$ and $2x^2 + 2x + 3 > 0$
➔ (x+4) (x+2) > 0 and $\left(x+\frac{1}{2}\right)^2 + \frac{5}{4} > 0$
⇨ x ∈ (-∞, -4) ∪ (-2, ∞)
The given equation can be written as
$log_{(2x^2-2x+3)}(x^2-2x) = 1$
⇨ $x^2-2x = 2x^2 + 2x +3$
⇨ $x^2 + 4x + 3 = 0$
⇨ x = -1, -3
but at x = -3, $log_{(x^2+6x+8)}$is not defined
Hence, x = -1

3. Ans. 1
Solution:

$\log_3 4 - \log_3(3x+1) = \log_3 \frac{3}{(5x-2)}$

$\log_3 \frac{4}{3x+1} = \log_3 \frac{3}{(5x-2)}$

$\frac{4}{3x+1} = \frac{3}{(5x-2)}$

20x-8=9x+3

11x=11

X=1

4. Ans. 6
Solution: log x^3 – log 3x = 2 log 2 + log3
Log $\frac{x^2}{3}$ = log 12

⇨ $\frac{x^2}{3}$ = 12 ➔ x = ±6
But at x = -6, log 3x and log x^3 are not defined.
Hence x = 6 is the only correct answer.

5. Ans. 1
Solution:
log_9 (2x-5) = $log_3\ (x-4)$
➔ $\frac{1}{2}log_3(2x-5) = log_3(x-4)$
⇨ $log_3(2x-5) = log_3(x-4)^2$
⇨ $(2x-5) = (x-4)^2$
⇨ $(2x-5) = x^2+16-8x$
⇨ $x^2-10x +21 = 0$
⇨ (x-3)(x-7) = 0
⇨ x= 3 or x = 7
But at = 3, $\log_3(x-4)$ is not defined since (x-4) becomes negative
Hence x = 7 is the only possible solution.

6. Ans. (25/48)
Solution: $\log_x [\frac{1}{5}+\frac{1}{12}+\frac{1}{21}+\frac{1}{32}+\frac{1}{45}+\cdots.+\infty\ terms]^2 = 2$
$\log_x (\frac{1}{5}+\frac{1}{12}+\frac{1}{21}+\frac{1}{32}+\frac{1}{45}+\cdots.+\infty\ terms)$

Let $\frac{1}{5}+\frac{1}{12}+\frac{1}{21}+\frac{1}{32}+\frac{1}{45}+\cdots.+\infty\ terms = P$
$P = \frac{1}{4}\left[\frac{4}{1\,x\,5}+\frac{4}{2\,x\,6}+\frac{4}{3\,x\,7}+\frac{4}{4\,x\,8}+\frac{4}{5\,x\,9}+\cdots.\infty\right]$
$4P = \left[1-\frac{1}{5}+\frac{1}{2}-\frac{1}{6}+\frac{1}{3}-\frac{1}{7}+\frac{1}{4}-\frac{1}{8}+\frac{1}{5}-\frac{1}{9}\right]$
$4\ P = \left[1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right]$
$4P = \frac{25}{12}$
$P = \frac{25}{48}$
$\text{Log}_x \frac{25}{48}$ = 1 or x = 25/48

7. Ans. (1)
Solution: $\log_2 (\log_4 16) = \log_2 \log_4 4^2 = \log_2 2 = 1$
$\log_3 \log_6(x^3-18x^2+108x) = 1$
$\log_6 (x^3-18x^2+108x) = 3$
$x^3-18x^2+108x = 6^3$
$x^3-18x^2+108x – 216 = 0$
$(x-6)^3 = 0$
X= 6 is the only value for which the above question is true.

8. Ans. (4)
Solution: n = $12\sqrt{3} = 2^2 \times 3^{1.5}$
$\frac{1}{log_2 n}+\frac{1}{log_3 n}+\frac{1}{log_4 n}+\frac{1}{log_6 n}+\frac{1}{log_8 n}+\frac{1}{log_9 n}+\frac{1}{log_{18} n}$
= $\log_n 2 + \log_n 3 + \log_n 4 + \log_n 6 + \log_n 8 + \log_n 9 + \log_n 18$
= $\log_n$(2 x 3 x 4 6 x 8 x 9 x 18)
= $\log_n (2^8 \times 3^6)$
= $\log_n (2^2 \times 3^{1.5})^4$
= $\log_n (2^2 \times 3^{1.5})$
= $4log_{2^2 x 3^{1.5}} (2^2 \times 3^{1.5}) = 4$

9. Ans. (7)
Solution: $(\log_2 x)^2 + 2 \log_2 x – 8 = 0$
$(\log_2 x)^2 + 4\log_2 x – 2\log_2 x – 8 = 0$
$\log_2 x [\log_2 x+4] – 2 [\log_2 x+4] = 0$

$[\log_2 x-2]\,[\log_2 x+4] = 0$
Since, x is a natural number hence $[\log_2 x+4]$ cannot be zero. Hence, $\log_2 x-2=0$
$\log_2 x = 2$
$x = 2^2 = 4$
We are given that: $x^p = 64$. Since x is 4, this means that:
$4^p = 64$
$P= 3$
Hence, the value of (x+p) = 4+3 = 7.

10. Ans. 3
Solution: Assume that the value of $log_{10}6730.4 = z$
It can be seen that 1000 < 6730.4 < 10000.
Hence, $log_{10}(10^3) < \log_{10} 6730.4 < log_{10}(10^4)$
Taking antilog, 3 < z < 4. So, the value of z lies between 3 and 4.
Therefore, [z] = greatest integer less than or equal to z = 3.

11. Ans. 3
Solution: log x = log (7.2/2.4) = log 3 ➔ x = 3

12. Ans. 200
Solution: log x = log 25 + log 8 = log (25 x 8) = log 200.

13. Ans. 11
Solution: log (2x - 2) / (11.66 - x) = log30
⇨ (2x - 2)/(11.66 - x) = 30
2x – 2 = 350 – 30x
Hence, 32x = 352 ➔ x = 11.

14. Ans. 21
Solution: Let the number be y.
$Y = 108^{10}$
⇨ Log y = 10 log (27 x 4)
⇨ Log y = 10 [3 log3+2 log2]
⇨ Log y = 10 [1.43 + 0.602]
Hence log y = 10 [2.03] = 20.3
Thus, y has 21 digits.

15. Ans. 53
Solution: log y = 25 log 125
= 25 [log1000 – 3 log 2] = 25 x (2.097)
= 52 +
Hence 53 digits.