Bicycle State Estimation with Extended Kalman Filter, GNSS and Wheel Speed Sensor Fusion

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I. INTRODUCTION

II. RELATED WORK

III. PROBLEM STATEMENT

IV. APPROACH

1. Dynamics Model

$$
\begin{aligned}
\vec{X} &= [x, y, z, \theta, v_x, v_y, \omega_w, b_u]^T \\
\vec{u} &= [\psi, \tau]^T
\end{aligned}
$$

$$
\begin{aligned}
\dot{x} &= v_x \\
\dot{y} &= v_y \\
\dot{\theta} &= V \tan\psi \\
\dot{v_x} &= \dot{v}\cos\theta - v\sin\theta\dot{\theta} \\
\dot{v_y} &= \dot{v}\sin\theta + v\cos\theta\dot{\theta} \\
\dot{\omega_w} &= \frac{1}{I_w}(\tau - \dot{v}mr_w) \\
\dot{b_u} &= 0
\end{aligned}
$$

$$
\begin{aligned}
v &= \sqrt{v_x^2 + v_y^2} \\
s &= \frac{r\omega_w - v}{v} \\
\mu &= \mu(s) \\
\dot{v} &= \frac{\mu g l_1}{l_1 + l_2}
\end{aligned}
$$

$$
\begin{aligned}
\psi &\sim \mathcal{N}\left(0, \left(\frac{\alpha}{v}\right)^2\right) \\
\tau &\sim \mathcal{N}\left(0, (\sigma_\tau)^2\right)
\end{aligned}
$$

$$
\begin{aligned}
\vec{f}(x_k) &= \vec{X}_k + \Delta t \dot{\vec{X}}|_{x_k, u_k = 0} \\
F_k &= \left.\frac{\partial \vec{f}}{\partial \vec{X}}\right|_{\vec{X}_k} = I + \Delta t \left.\frac{\partial \dot{\vec{X}}}{\partial \vec{X}}\right|_{\vec{X}_k} \\
B_k &= \left.\frac{\partial \vec{f}}{\partial \vec{u}}\right|_{\vec{X}_k} = \Delta t \left.\frac{\partial \dot{\vec{X}}}{\partial \vec{u}}\right|_{\vec{X}_k}
\end{aligned}
$$

$$
\begin{aligned}
&h_\omega(\vec{X}) = \omega \\
&h_\rho^i(\vec{X}) = \sqrt{(x^i - x)^2 + (y^i - y)^2 + (z^i - z)^2} + b_u - B^i \\
&h_{\dot{\rho}}^i(\vec{X}) = (\vec{v}^i - \vec{v})^T \vec{1}^i + \dot{b}_u - \dot{B}^i
\end{aligned}
$$

$$
Q_k = Q_{process} + B_k Q_u B_k^T
$$

$$
\begin{aligned}
\hat{\vec{X}}_{k+1} &= \vec{f}(X_k) \\
\hat{P}_{k+1} &= F_k P_k F_k^T + Q_k
\end{aligned}
$$

$$
\begin{aligned}
\vec{\nu}_k &= \vec{z}_k - h(\hat{\vec{X}}_k) \\
S_k &= H_k \hat{P}_{k+1} H_k^T + R_k \\
K_k &= \hat{P}_{k+1} H_k^T S_k^{-1} \\
\vec{X}_{k+1} &= \hat{\vec{X}}_{k+1} + K_k \vec{\nu} \\
P_{k+1} &= (I - K_k H_k) \hat{P}_{k+1}
\end{aligned}
$$

V. RESULTS

VI. CONCLUSION

VII. FUTURE DIRECTIONS