Alan Adame
CSE202: Theory Problems
17JAN17

Theory Problem 1: Sum of squares of Fibonacci numbers

Claim $\sum_{i=1}^{n} F_i^2 = F_i\, F_{i+1}$

Since we know F_1 = 1 and F_2 =1, it is possible to verify n=1 as a base case.

$\sum_{i=1}^{1} F_i^2 = F_1\, F_2 \rightarrow 1^2 = 1 \cdot 1$

The inductive hypothesis is for n + 1 when n$\geq$1:

$$\sum_{i=1}^{n+1} F_{n+1}^2 = F_{n+1}\, F_{n+1+1}$$

Step 1. $F_1\, F_1 + \ldots + F_n\, F_n \qquad = F_n\, F_{n+1}$

Step 2. $F_1\, F_1 + \ldots + F_n\, F_n + F_{n+1}\, F_{n+1} \qquad = F_{n+1}\, F_{n+1+1}$

We can substitute $F_n\, F_{n+1}$ from Step 1.

Now we have: $F_n\, F_{n+1} + F_{n+1}\, F_{n+1} = F_{n+1}\, F_{n+1+1}$

After simplification, we are left with the definition of Fibonacci sequence: $F_n + F_{n+1} = F_{n+1+1}$.