B17 Two probabilistic bits

TASK 2:

$$(0.2|0\rangle + 0.8|1\rangle) \otimes (0.6|0\rangle + 0.4|1\rangle)$$

$$= 0.12|00\rangle + 0.08|01\rangle + 0.48|10\rangle + 0.32|11\rangle$$

TASK 3:

$$\begin{pmatrix} 0.2 \\ 0.8 \end{pmatrix} \otimes \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix} = \begin{pmatrix} 0.12 \\ 0.08 \\ 0.48 \\ 0.32 \end{pmatrix}$$

TASK 4:

$$|00\rangle = |0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Similarly, $|01\rangle$, $|10\rangle$ & $|11\rangle$ complete the basis, so we'll obtain --

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \& \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$ respectively from their tensor products

B18 correlation solutions

TASK 1:

Asja: $\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$, Balvis: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

composite system: $\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 0 \\ 1/2 \\ 0 \end{pmatrix}$

$= 1/2\,|00\rangle + 1/2\,|10\rangle$

TASK 2:

CNOT $\left(1/2\,(|\overset{c\ t}{00}\rangle + |\overset{c\ t}{10}\rangle) \right)$

c → control bit
t → target bit

0 so no operation

flips

$\Rightarrow \quad 1/2\,|00\rangle + 1/2\,|11\rangle$

TASK 4:

$\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$

as if $ac = 1/2$, $a \neq 0$, $c \neq 0$

↳ $bd = 1/2$, $\Rightarrow b \neq 0, d \neq 0$

TASK 7:

$1/2\,|000\rangle + 1/2\,|110\rangle \rightarrow 1/2\,|000\rangle + 1/2\,|111\rangle$

flips

TASK 8:

composite system: $(1/2\,|000\rangle + 1/2\,|111\rangle) \otimes |1\rangle \rightarrow 1/2\,|0001\rangle + 1/2\,|1111\rangle$

after CNOT application:

$\Rightarrow 1/2\,|0001\rangle + 1/2 \Rightarrow 1/2\,|0001\rangle + 1/2\,|1110\rangle$

Big Operators on Multiple bits

TASK 1:

$$M \otimes I = \begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.2 & 0 & 0.7 & 0 \\ 0 & 0.2 & 0 & 0.7 \\ 0.8 & 0 & 0.3 & 0 \\ 0 & 0.8 & 0 & 0.3 \end{pmatrix}$$

TASK 2:

$I \otimes M \otimes I$ (applied on first & third bits)

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \left(\begin{array}{cccc|cccc} 0.9 & 0 & 0.4 & 0 & & & & \\ 0 & 0.9 & 0 & 0.4 & & & & \\ 0.1 & 0 & 0.6 & 0 & & & 0 & \\ 0 & 0.1 & 0 & 0.6 & & & & \\ \hline & & & & 0.9 & 0 & 0.4 & 0 \\ & 0 & & & 0 & 0.9 & 0 & 0.4 \\ & & & & 0.1 & 0 & 0.6 & 0 \\ & & & & 0 & 0.1 & 0 & 0.6 \end{array}\right)$$

TASK 3:

$00 \xrightarrow{I} 00$

$10 \xrightarrow{I} 10$ (second bit is zero)

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

When second bit is 1:

$|01\rangle \rightarrow 0.7|01\rangle + 0.3|11\rangle$

$|11\rangle \rightarrow 0.4|01\rangle + 0.6|11\rangle$

Columns:

$$\begin{pmatrix} 0 \\ 0.7 \\ 0 \\ 0.3 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0.4 \\ 0 \\ 0.6 \end{pmatrix}$$

$\Rightarrow$ overall matrix is - -

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0.9 \\ 0 & 0 & 1 & 0 \\ 0 & 0.3 & 0 & 0.6 \end{pmatrix}$$