

Assignment 2

Problem 1 [Energy Minimization] (7 points)

Consider a unit mass electric vehicle with position $x(t) \in \mathbb{R}$, moving with velocity $v(t)$ and acceleration $a(t)$. The vehicle needs to be maneuvered from initial position of rest to final position $x(T)$ at time $t = T$ satisfying $x(T) \geq x_f$.

Requirements are $v \leq A$, $0 \leq v(t) \leq V$, $v(T) \leq v_f$

Let $p(t)$, $p_b(t)$, $p_d(t)$ be power delivered by battery, power lost in braking, power lost due to drag respectively and $k(t)$ be kinetic energy of vehicle which is $v(t)^2/2$. Then,

$$p(t) = k(t) + p_b(t) + p_d(t)$$

where, $p_d(t) = cv(t)^3$ and the power used for braking $p_b(t)$ can be controlled.

Take $T = nh$ and divide interval $[0, T]$ into n subintervals of length h to discretize the vehicle equations of motion,

$$x = v, v = a.$$

In discrete form we get following equations

$$x_{r+1} = x_r + h(v_r + v_{r+1})/2$$

$$v_{r+1} = v_r + ha_r$$

$$p_r = (k_{r+1} - k_r)/h + p_{b,r} + p_{d,r}$$

$$k_r = v_r^2/2$$

Total energy consumed is $E = h \sum_{r=0}^n p_r$.

1. Formulate an optimization problem for minimum energy consumption. Write all variables and constraints on them. Indicate decision variables. Comment on the nature of the problem.
2. Solve the problem by taking $x_f = 12 \text{ m}$, $v_f = 2 \text{ m/s}$, $V = 10 \text{ m/s}$, $A = 2 \text{ m/s}^2$, $c = 2$, $h = 0.1$, $T = 5 \text{ s}$. Experiment by changing the values of x_f , v_f , V , A and compare the results. Plot $x(t)$, $v(t)$, $p(t)$ v/s time on separate graphs.
3. Make a relaxation in the equality constraint $k_r = v_r^2/2$, by replacing it with constraint $k_r \geq v_r^2/2$. **Make reasonable additional assumptions and see if the problem can be converted into convex optimization problem.** Solve the problem for above values and note down your observations. Justify these observations.

Problem 2 [Variational Form of Circuit Equations] (7 points)

Consider a circuit with m branches and n nodes. Let vector $e \in \mathbb{R}^n$ denote node voltages, $v \in \mathbb{R}^m$ denote branch voltages and $i \in \mathbb{R}^m$ denote branch currents. Also, let the current-voltage characteristics of a two terminal device present in branch k be given by function $v_k = f_k(i_k)$. For example, if the device is resistor of resistance R then $v_k = i_k R$, if the device is voltage source of V_k volts with resistance r then $v_k = V_k + i_k r_k$. According to KCL, the current vector i satisfies, $Ai = 0$ where $A \in \mathbb{R}^{n \times m}$ is node-edge incidence matrix. Similarly, KVL gives $v = A^T e$. Now consider function

$$\Phi(i) = \sum_{k=1}^m \int_0^{i_k} f_k(z_k) dz_k$$

1. Write the dual of following optimization problem. Note the problem is equality constrained one. Indicate key differences w.r.t. the inequality constrained case in terms of Lagrange multipliers, constraints, etc.

minimize $\Phi(i)$

s.t. $Ai = 0$

2. Make use of KKT conditions. Comment on the equations obtained from the dual formulation.

3. Solve the following circuits by solving both primal and dual formulation. Comment on the results.

a. Circuit with nodes = $\{1,2,3,4,5\}$, branches = $\{(12,R),(23,R),(34,R),(45,R),(24,R),(35,R),(25,R),(15,V)\}$ where R indicates resistor in the branch, V indicates a voltage source in the branch. $R_{12}=10\Omega$, $R_{23}=100\Omega$, $R_{34}=10\Omega$, $R_{45}=100\Omega$, $R_{25}=10\Omega$, $R_{35}=10\Omega$, $R_{24}=10\Omega$, $V=10V$ with internal resistance of 2Ω .

b. Circuit with nodes = $\{1,2,3,4,5\}$, branches = $\{(12,R),(23,R),(34,D1, >|),(45,R),(35,D2, >|),(25,R),(15,V)\}$. $R_{12}=1000\Omega$, $R_{23}=1000\Omega$, $R_{25}=1000\Omega$, $R_{45}=100\Omega$ $D1$, $D2$ are diodes with $V_T = 30mV$ and $I_s = 0.5mA$, $V=1V$ with internal resistance of 1Ω . Take for diode $v_k = V_T \log(1 + i_k/I_s)$.

(Possible solvers that can be used are CVX for Python <https://cvxopt.org/userguide/solvers.html>, CVX for Matlab <http://cvxr.com/cvx/doc/quickstart.html>, 'fmincon' for matlab. Depending on the nature of problem you may search and choose any other solver too if you find it to be suitable. Justify your choice in your response.).