Telecommunications – Session 4

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Link Equation and Link budgets

- A communications architecture will include Links that are used to carry information at a certain rate
  - Uplinks
  - Downlinks
  - Crosslinks or intersatellite links
- Links must be designed to achieve this communication with a defined maximum error rate
- The Link equation relates energy-per-bit to:
  - Transmitter power
  - Losses
  - Antenna properties
  - Noise sources
  - Bit rate
- From the Link equation construct a Link budget that shows how the required rate can be carried by the link and within some margin.

The Link Equation

Transmitted power

- Let a transmit antenna radiate with a power PL_l , where P is the power (Watts) and L_l is the line loss, with $0<L_l<1$
- We must allow for the fact that the antenna may not radiate isotropically
- We define the gain of the transmit antenna, G_t, as the ratio of power of an isotropic antenna (with the same radiated power per unit solid angle as the actual antenna's power at the centre of coverage area per unit solid angle) to the actual antenna.
  - Usually, $G_t>1$.
- Effective isotropic radiated power (EIRP):

$$EIRP = PL_lG_t$$

Transmitted power

- Consider a transmitter at the centre of a sphere radius S
- Power per unit area on the surface of the sphere (flux density) is W_t

$$W_t = \frac{P L_l G_t}{4\pi S^2} = \frac{EIRP}{4\pi S^2} \text{ W m}^{-2}$$

- This is as viewed from the direction of the centre of the beam.

Received power

- Let us calculate the flux density (Wm^{-2}) at the receiving antenna
- There will be transmission path loss due to atmosphere L_a , where $0<L_a<1$
  - E.g. gaseous components in the troposphere, water (rain, clouds, snow, ice)

$$W_r = W_t L_a$$

- Received power (W), C, is just the product of the flux density (Wm^{-2}), W_r, and an effective received antenna area, A_r (m^2):

$$C = W_r A_r = \frac{P L_l G_t L_a}{4\pi S^2} A_r$$

- The effective antenna area is the product of the physical area of the antenna and an efficiency factor (typically about 0.55 but up to 0.7)

$$A_r = \eta \frac{\pi D^2}{4}$$

Where D is the physical diameter of the receiving antenna

Received power

We can characterise the receiver by its gain, G_r, relative to an isotropic antenna (usually, G_r >1)

$$G_r = \frac{A_r}{A_{iso}} = \eta\left(\frac{\pi D}{\lambda}\right)^2 \quad where \quad A_{iso} \approx \frac{\lambda^2}{4\pi}$$

Received power:

$$C = \frac{PL_l G_t L_a}{4\pi S^2} A_r = \frac{PL_l G_t L_a}{4\pi S^2} G_r A_{iso} = PL_l G_t L_a G_r L_s$$

Where free-space loss, L_s, is the ratio of the received and transmitted powers in a link between two isotropic antennas:

$$L_s = \frac{A_{iso}}{4\pi S^2} = \left(\frac{\lambda}{4\pi S}\right)^2$$

λ = wavelength, m

Decibel representation

- Since the Received power (C) is the product of terms it can be conveniently represented in decibels (dB).
- Decibels is simply a logarithmic unit:

$$P_{dB} = 10log_{10}(P)$$

- Decibels are often used to represent power ratios (dimensionless), e.g. the output power to input power of an amplifier (it gain)

$$P_{in} = 0.3\,W,\ P_{out} = 3W, G_{absolute} = 10, G_{dB} = 10log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 10\text{dB}$$

Decibel representation

- Since when we take the log of a product we get a sum,

$$C = PL_lG_tL_aG_rL_s \quad \rightarrow \quad C_{dBW} = P + L_l + G_t + L_a + G_r + L_s$$

Where, on the right hand side the units are:

[C], [P] = dBW *

$$[L_l], [L_a], [L_s] = dB$$

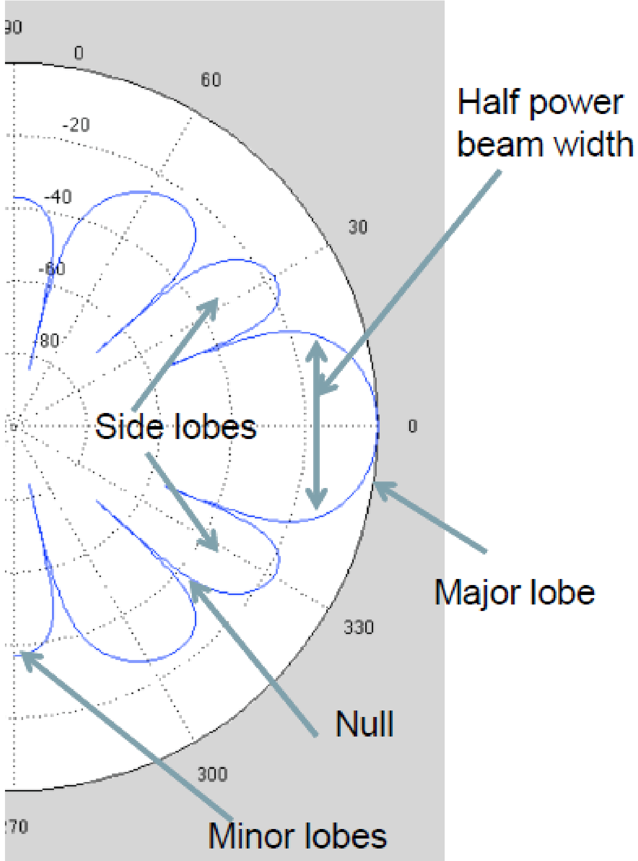
$$[G_t], [G_r] = dBi \ **$$

* When the quantity is not dimensionless, the convention is to add the unit after the dB symbol

** For antenna gains, *i* is appended to denote the gain is relative to an isotropic antenna

Antenna response

- Antenna radiation response is not isotropic
- Antenna types (e.g. patch, dipole, helix, parabolic) have different beam shapes and gains
- When selecting an antenna type the important considerations are gain and radiation pattern (beam shape)
- The full-width-half-maximum (FWHM) or half-power beam-width of the beam depend upon the frequency and the antenna type
- E.g.
  - A low gain antenna will need more input power to send the same power per unit area to a receiving antenna
  - Small half-power beam-width and poor pointing accuracy of the antenna might mean that a receiving station only sees relatively little of the radiated power



Antenna response - FWHM

- 10log(1/2) = 3.01dB ~ 3dB
- So 3dB points give the FWHM
- FWHM characterised by diffraction pattern of Airy disk:

$$\theta_{FWHM} = \theta_{3dB} \approx \frac{1.22\lambda}{D} \approx 70^{o}\frac{\lambda}{D} \cong \frac{21^{o}}{f_{GHz}D}$$

$c = f\lambda$; c = 3 x 10^{8} ms^{-1}

Antenna response - Gain

- The Gain response is approximately:

$$G(\theta) \approx G_{max} - 12\left(\frac{\theta}{\theta_{3dB}}\right)^2 \qquad \text{dBi}$$

for $0 < \theta < \theta_{\text{3dB}}/2$

Pointing errors and other losses

- Other losses can simply be added into the receiver power:

$$L = L_l L_a L_s \ldots L_x$$

Pointing errors introduce loss:

$$L_{pt} = -12\left(\frac{\theta_{pt}}{\theta_{3dB}}\right)^2 \text{ dB}\mathbf{i}$$

Receiver power is then:

$$C = P L_l G_t L_{pt,t} L_a G_r L_{pt,r} L_s$$

Bit-Rate Requirements

Energy per bit

- We have derived an expression for the received power, C, at the antenna which is a function of the power input to the transmitting antenna, P, the losses, (L_a, L_s, L_l, …) and the transmitter and receiver antenna gains (G_r, G_t):

$$C = PL_lG_tL_aG_rL_s$$

- For the digital communication channel we have a bit rate specification
- Our link has a receiver power C and will use this to communicate at a rate of R bits per second.
- Therefore, the energy per bit, E_b (J bit^{-1}), is:

$$E_b = \frac{C}{R}$$

Noise

- We must expect and allow for noise on the channel
- Assume 'White noise', i.e. the noise power density, N_0(W Hz^{-1} = J), is constant over the frequency band
- We define a system noise temperature, T_s, such that $N_0 = k_B T_s$,

  Where $k_B = 1.381 \times 10^{-23}$ J K^{-1} is the Boltzmann's constant
- System noise temperatures are typically around several hundred K

Link Equation

The Link Equation is the ratio of the energy per bit to noise power spectral density:

$$\frac{E_b}{N_0} = \frac{P L_l G_t L_a G_r L_s}{k_B T_s R} \qquad \text{bit}^{-1}$$

- Think of it as a signal to noise ratio
- In terms of dB a value of 5-10 (i.e. x 3-10) is sufficient but the actual requirement depends upon the modulation/coding chosen.

Link Equation – in dB

$$\frac{E_B}{N_0} = P + L_l + G_t + L_a + G_r + L_s - 10log_{10}(k_B) - 10log_{10}(T_s) - 10log_{10}(R)$$

Link Equation – in dB

$$\frac{E_b}{N_0} = P + L_l + G_t + L_a + G_r + L_s - 10log_{10}(k_B) - 10log_{10}(T_s) - 10log_{10}(R)$$

In calculating the space loss, Ls, be careful with squares (sometimes applied as a factor of two)

$$L_s = 10log_{10}\left[\left(\frac{\lambda}{4\pi S}\right)^2\right] = 20log_{10}\left(\frac{\lambda}{4\pi S}\right)$$

In terms of frequency (f = $\frac{c}{\lambda}$)

$$L_s = 20log_{10}\left(\frac{c}{4\pi f S}\right)$$

Quantity	Value and/or units
P	dBW
L_l	dB
G_t	dBi
L_a	dB
G_r	dBi
L_s	dB
$10log_{10}(k_B)$	-228.60 dBW Hz^{-1} K^{-1}
$10log_{10}(T_s)$	dBK
R	Bits s^{-1}

Link Design and Link Budgets

Link Budget presentation (example)

Transmitter power	**P**	**13.06 dBW**
Transmitter line loss	L_l	-1.96 dB
Transmit antenna peak gain	G_t	14.20 dBi
Pointing loss	$L_{pt,t}$	-8.67 dB
Transmit antenna net gain	G_{tnet}	3.57 dBi
Equivalent Isotropic Radiated Power	EIRP	16.63 dBW
Space Loss	L_s	-168.33 dB
Propagation and polarisation loss	L_a	-0.3 dB
Implementation loss	L_{imp}	-2.0 dB
Receiver antenna peak gain	G_{rp}	+39.1 dBi
Receiver antenna pointing loss	$L_{pt,r}$	-0.1 dB
Receiver antenna net gain	G_r	+39.0 dBi
System noise temperature	T_s	21.3 dbK (135K)
Boltzmann's constant	k_B	-228.60 dBW Hz^{-1} K^{-1}
Data rate	R	79.34 dB
Required Eb/N0 for BER of 10-5	E_b/N_0	10 dB
Margin		3 dB

- Balance terms of link budget
- Link budget has entries for each term of the equation
- Add appropriate margins
- Through the link budget it can be shown that the link can support the required data rate
- Trade parameters to balance the budget, e.g. antenna power for antenna gain for the same EIRP

$G_{tnet} = L_l + G_p + L_{pt,t}$; $EIRP = P + G_{tnet}$

$G_r = G_{rp} + L_{pt,r}$

E_b/N_0 expected (not shown)

$= EIRP + L_s + L_{imp} + G_r - T_s - k_B - R$

=13 in this case (= Required + Margin = balanced)

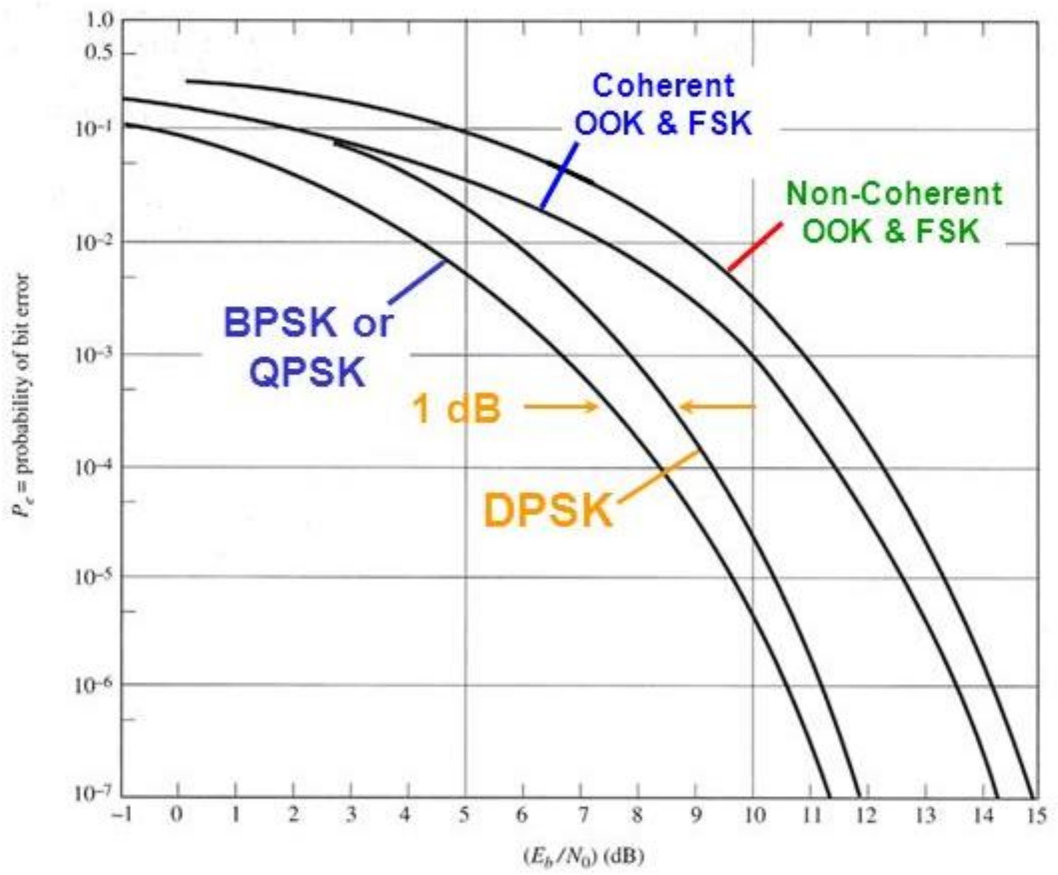
Error-Correcting Codes

- We can Code the signal to help error detection and recovery (Forward Error Correction)
  - Simple schemes are check sums and parity bits but these do not allow us to identify where an error has been introduced or to recover from it. They are vulnerable to multiple errors
- By repeating the number of transmissions of each bit (e.g. 3 times) and by taking the most frequent occurance (001 = 0; 011=1 etc) some errors can be identified
- More sophisticated schemes exist that are more effective and less wasteful, e.g. convolution codes, Reed-Solomon, Turbo
- Whatever the approach you will have:

$$\text{a bit error rate (BER)} = \frac{number\ of\ bit\ errors}{number\ of\ bits\ transmitted}\ s^{-1}$$

Bit error rate

- BER depends upon
  - Type of modulation (e.g. BPSK)
  - Type of coding (see before)
  - Energy per bit to noise density ratio
- Practically,
  - start with the BER,
  - select the modulation and coding
  - thus determine the required E_b/N_0
  - Check the Link Budget



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See e.g. Space Mission Analysis and Design or any of the other texts

Example Link Budget calculation

- In this example we seek to find the transmitter antenna diameter and transmitter power.
- Step 1 – Determine the required EIRP given:
  - Spacecraft distance
  - Ground station characteristics
  - Typical values for losses
  - Required bit rate, error rate and transmission frequency
- Step 2 – Trade diameter and power to find optimum values
  - Assuming pointing accuracy for spacecraft

Step 1 – find EIRP

- How you design the link budget depends upon the application and the constraints
- In this example we need to transmit data at 86 Mbit/s with BER of 10^{-5} at 2.2 GHz (S band) over a path length of 2831 km
  - Data rate – 86×10^6 bits/s
  - BER – 10^{-5}
  - Frequency – 2.2 GHz
  - Path length – 2831 km
- We know the ground station details which are fixed for this exercise

Transmitter power	**P**	
Transmitter line loss	L_l	
Transmit antenna peak gain	G_t	
Pointing loss	$L_{pt,t}$	
Transmit antenna net gain	G_{tnet}	$= L_l+G_t+L_{pt}$
Equivalent Isotropic Radiated Power	EIRP	$= P + G_{tnet}$
Space Loss	L_s	
Propagation and polarisation loss	L_a	
Implementation loss	L_{imp}	
Receiver antenna peak gain	G_{rp}	
Receiver antenna pointing loss	$L_{pt,r}$	
Receiver antenna net gain	G_r	$=G_{rp}+L_{pt,r}$
System noise temperature	T_s	
Boltzmann's constant	k_B	
Data rate	R	
Required E_b/N_0 for BER of 10^{-5}	E_b/N_0	
Margin		

Predetermined values

- Space loss

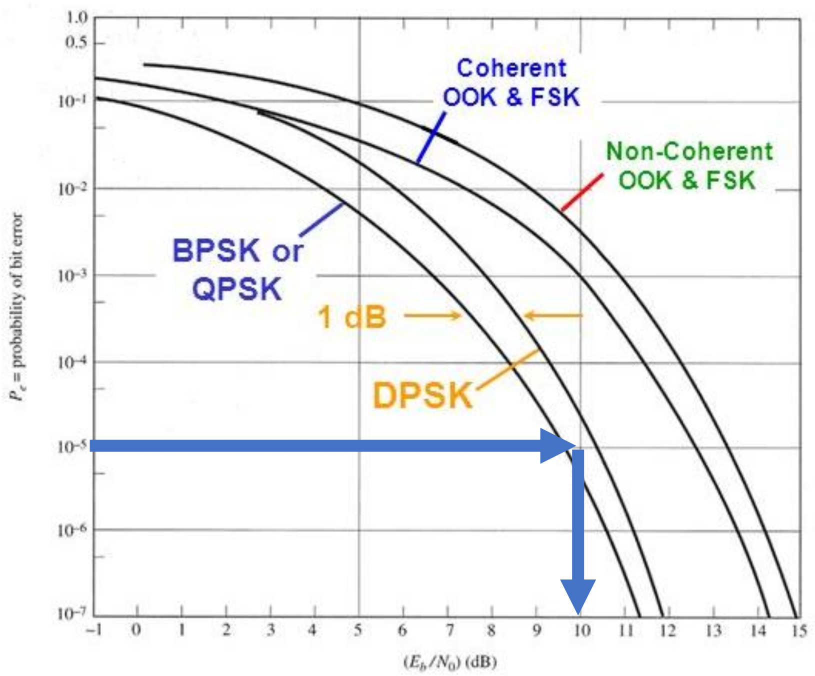
$$L_s = 20log_{10}\left(\frac{c}{4\pi Sf}\right)$$

$$= -168.33 \text{ dB}$$

Transmitter power	**P**	
Transmitter line loss	L_l	
Transmit antenna peak gain	G_t	
Pointing loss	$L_{pt,t}$	
Transmit antenna net gain	G_{tnet}	
Equivalent Isotropic Radiated Power	EIRP	
Space Loss	L_s	**-168.33 dB**
Propagation and polarisation loss	L_a	**-0.3 dB**
Implementation loss	L_{imp}	**-2 dB**
Receiver antenna peak gain	G_{rp}	+39.1 dBi
Receiver antenna pointing loss	$L_{pt,r}$	-0.1 dB
Receiver antenna net gain	G_r	**+39.0 dBi**
System noise temperature	T_s	
Boltzmann's constant	k_B	**-228.60 dBW Hz^{-1} K^{-1}**
Data rate	R	**79.34 dB**
Required E_b/N_0 for BER of 10^{-5}	E_b/N_0	
Margin		

Modulation and coding

- Selecting BPSK modulation and no coding for BER of 10^{-5} we obtain a required E_b/N_0 of 10 dB.
- Assume system noise temperature of 135 K
- Assume a margin of 3 dB



Transmitter power	**P**	
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Pointing loss	$L_{pt,t}$	
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Equivalent Isotropic Radiated Power	EIRP	
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Data rate	R	**79.34 dB**
Required E_b/N_0 for BER of 10^{-5}	E_b/N_0	**10 dB**
Margin		**3 dB**

Design transmitter

- Calculate EIRP required
- The challenge is to find a P and G_{tnet} to meet the budget.
- G_{tnet} depends on antenna diameter due to pointing errors.

Transmitter power	**P**	
Transmitter line loss	L_l	
Transmit antenna peak gain	G_t	
Pointing loss	$L_{pt,t}$	
Transmit antenna net gain	G_{tnet}	
Equivalent Isotropic Radiated Power	**EIRP**	**16.67 dBW**
Space Loss	L_s	**-168.33 dB**
Propagation and polarisation loss	L_a	**-0.3 dB**
Implementation loss	L_{imp}	**-2 dB**
Receiver antenna peak gain	G_{rp}	+39.1 dBi
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Receiver antenna net gain	G_r	**+39.0 dBi**
System noise temperature	T_s	**21.30 dB K (135K)**
Boltzmann's constant	k_B	**-228.60 dBW Hz^{-1} K^{-1}**
Data rate	R	**79.34 dB**
Required E_b/N_0 for BER of 10^{-5}	E_b/N_0	**10 dB**
Margin		**3 dB**

Step 2: P and D trade given EIRP

- Trade transmitter power, antenna diameter and pointing loss to get required EIRP (16.67 dBW)
- Choose a parabolic antenna: the antenna diameter sets the peak gain (efficiency = 55%)

$$G_t = \frac{A_r}{A_{iso}} = \eta \left(\frac{\pi D}{\lambda}\right)^2 \rightarrow G_t \cong 17.8 + 20log_{10}(D) + 20log_{10}(f_{GHz}) \text{ dBi}$$

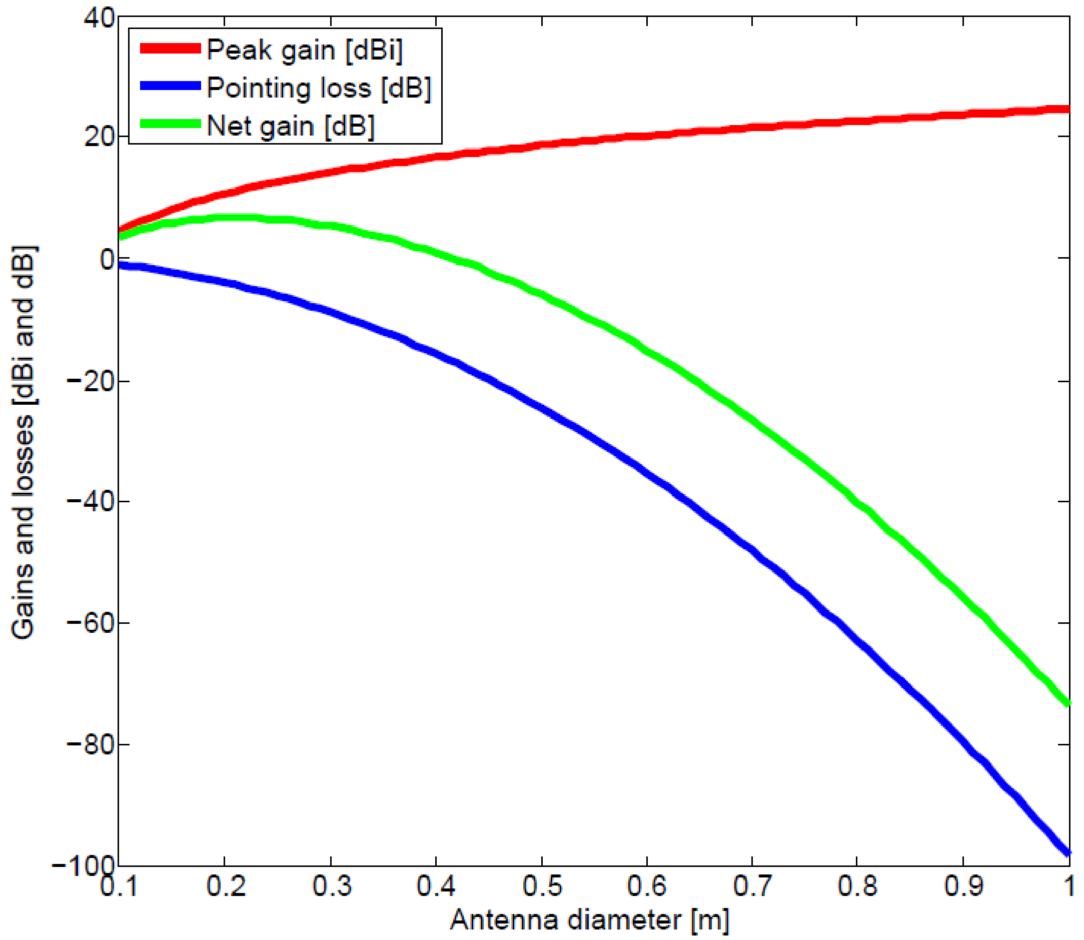
EIRP trade-off

- The antenna diameter also determines the half-power beam width and therefore the pointing loss and then the net gain.

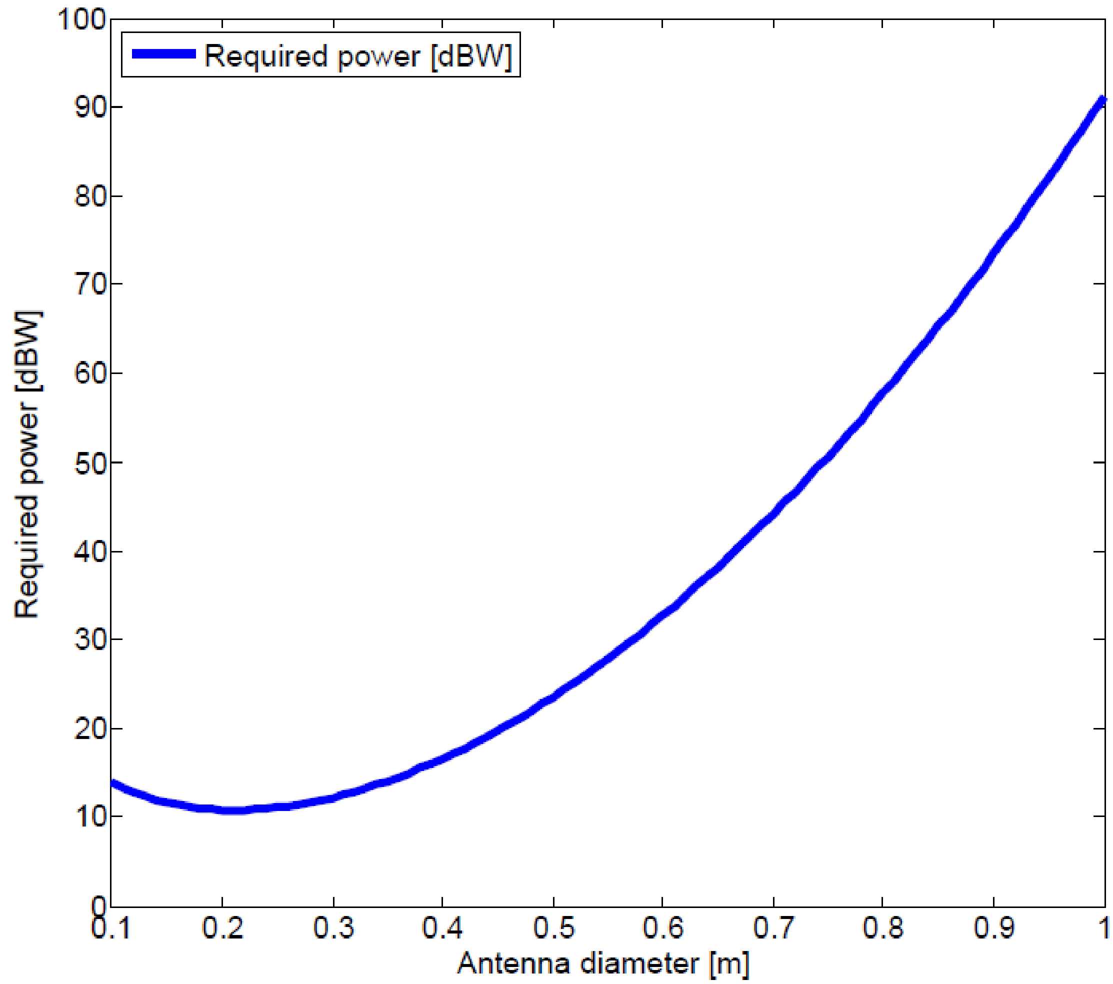
$$\theta_{3dB} \approx \frac{1.22\lambda}{D}\ rad \cong \frac{21}{f_{GHz}D}\ \text{deg} \qquad L_{pt,t} = -12\left(\frac{\theta_{pt}}{\theta_{3dB}}\right)^2$$

- Assume 27 degree pointing error
- Trade-off antenna diameter D and required P

Transmission antenna gain/pointing loss



Required power to meet EIRP



Final link budget

- While the minimum is nearer 20cm say transmitter antenna size 30 cm to allow some margin and respect that it may be used for uplink for instance
- Thus the link budget is complete with a 3 dB margin

Transmitter power	**P**	**13.26 dBW**
Transmitter line loss	L_l	-1.96 dB
Transmit antenna peak gain	G_t	14.20 dBi
Pointing loss	$L_{pt,t}$	-8.83 dB
Transmit antenna net gain	G_{tnet}	3.41 dBi
Equivalent Isotropic Radiated Power	**EIRP**	**16.67 dBW**
Space Loss	L_s	**-168.33 dB**
Propagation and polarisation loss	L_a	**-0.3 dB**
Implementation loss	L_{imp}	**-2 dB**
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Margin		**3 dB**