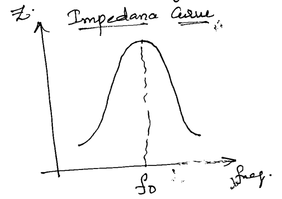
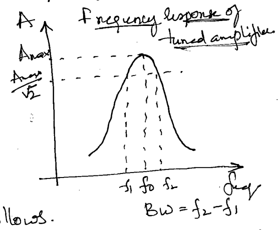
Tuned amplifier.

Syllabus:-
→ Introduction
→ classification of small signal tuned amplifiers,
→ Single tuned amplifier capacitively coupled,
→ tapped single-tuned capacitively coupled amplifier,
→ Single tuned inductively coupled amplifier,
→ Double tuned amplifier.

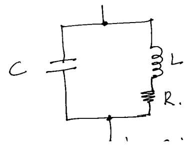
5.1 Introduction:-

→ An amplifier which amplies a selective frequency and rejects all other frequencies is called as tuned amplifier.
→ It is an amplifier with tuned or tank ckt. at the collector end called as tuned amplifier.
→ As it has narrow bandwidth hence the name Narrow band amplifier.
→ It uses a parallel tuned ckt. as its load impedance.
→ Parallel tuned ckt. has high impedance at resonance & falls sharply as we move away from resonant frequency.





→ The tuned ckt. or tank circuit is as follows.



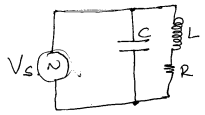
R → coil resistance (distributed parameter).

... ... or tuned amplifier will amplifiers.

→ A resonant circuit is a parallel combination of L & C for adjusting fo at variable capacitor or a variable inductor can be used.
→ In tuned amplifier, harmonic distortion is very small as it selects a single frequency & rejects all other frequencies.
→ If a parallel-tuned circuit is applied with a source then, if the frequency of the applied voltage is equal to the resonant frequency of LC circuit, then the electrical resonance occurs, impedance of the resonant ckt. will be maximum & the line current is minimum & the powerfactor is unity.

The admittance of inductive branch is

$$Y_L = \frac{1}{R+j\omega L}.$$



admittance of capacitive branch is.

$$Y_c = j\omega C.$$

Total admittance $= Y_t = Y_L + Y_c = \frac{1}{R+j\omega L} + j\omega C.$

solving by rationalizing, we get

$$Y_t = \frac{R-j\omega L}{R^2+\omega^2L^2} + j\omega C.$$

$$= \frac{R}{R^2+\omega^2L^2} + j\left(\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right)$$

As we know at resonance reactance = zero $(X=0)$
$\omega = \omega_0$.

$$\omega_0 C - \frac{\omega_0 L}{R^2+\omega_0^2L^2} = 0$$

$$\therefore \omega_0 C = \frac{\omega_0 L}{R^2+\omega_0^2L^2}$$

$$R^2C + \omega_0^2L^2C = L.$$

$$\therefore \omega_0 = \sqrt{\frac{1}{LC}} \quad (\because L \gg R)$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \qquad f_0 = \frac{1}{2\pi\sqrt{LC}}}$$ ⇒ Resonant frequency.

Quality factor (Q) of the tank circuit.

$$Q = \frac{\text{Voltage drop across } L}{\text{Voltage drop across } R} = 2\pi\left[\frac{\text{max energy stored per cycle}}{\text{max energy dissipated per cycle}}\right]$$

$$Q = \frac{V_L}{V_R} = \frac{I_L \cdot X_L}{I_L \cdot R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{1}{\omega RC}.$$

or Q at resonance.

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}.$$

Bandwidth

$$BW = \frac{f_0}{Q_0} \quad Hz.$$

$$BW = \frac{\omega_0}{Q_0} \text{ rad/sec.}$$

Selectivity. (δ)

$$\delta = \frac{\omega-\omega_0}{\omega_0} = \frac{\omega}{\omega_0} - 1.$$

Parallel resonant circuit has.
→ $Z_0 = \frac{L}{RC}$ at resonance & its high.
→ $I = \frac{V}{Z_0}$
→ $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2} \approx \frac{1}{2\pi\sqrt{LC}}.$
→ Q-factor $Q = \frac{\omega L}{R} = \frac{1}{\omega CR}.$

Classification of tuned amplifiers

→ Based on the i/p signal that has to be amplified tuned amplifiers are classified into
ⓐ Small signal tuned amplifiers
ⓑ large signal tuned amplifier.
→ As the input signal is small for small signal tuned amplifier the operating point is choosen at the centre of the load line hence used in class A mode.
→ As the i/p signal is large for large signal tuned amplifier we used the amplifier in class AB or B or C modes for large collector circuit efficiency. If design using push-pull operation further haemonic distortion will be reduced.
→ As the tuned circuit it self eliminates the harmonic distortions.
→ Small signal tuned amplifiers are further classified as.
ⓐ Single tuned amplifier
(i) capacitively coupled single-tuned amplifier.
(ii) tapped capacitively coupled single-tuned amplifier.
(iii) Inductively coupled single-tuned amplifier.
ⓑ Double tuned amplifier.
ⓒ Stagger tuned amplifier.
→ Single tuned amplifier uses one parallel tuned circuit as the load impedance in each stage and all the tuned circuits are in different stages are tuned to the same frequency (when cascaded).
→ Double tuned ckt. is the one which has a inductively coupled two tuned circuit designed for same frequencies.
→ Stagger tuned amplifier is the one which has cascaded single tuned ... tuned to slightly different frequencies.

→ Its a single tuned amplifier i.e, it has one tank circuit & the o/p of the amplifier is coupled to the next circuit through a capacitor hence the name capacitively coupled single-tuned amplifier as shown below.

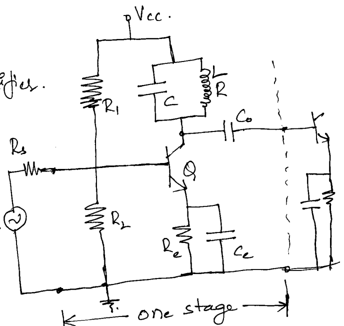
→ R_1 & R_2 are biasing elements to make the transistor to be in active mode so that it works like an amplifier.

→ R_e & C_e provides stabilization.

→ V_s is the source with its internal resistance R_s.

→ parallel LC forms a tank ckt or tuned circuit.

→ R is the internal resistance of coil L.



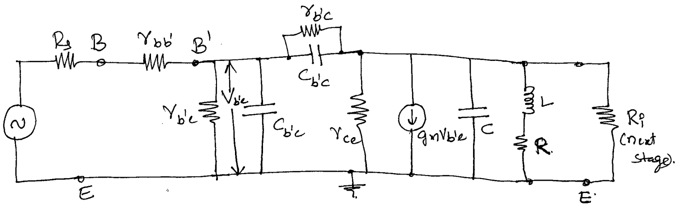
→ C_o forms a coupling capacitor to couple o/p of first stage as the input to the next stage.

→ As the input V_s is applied the transistor will amplify the signal as it is in active mode & the tuned circuit will select the i/p frequency or at the resonant frequency it offers high impedance & hence the o/p voltage & also the gain will be high for all other frequencies gain gets reduced as we move away from the resonant frequencies.

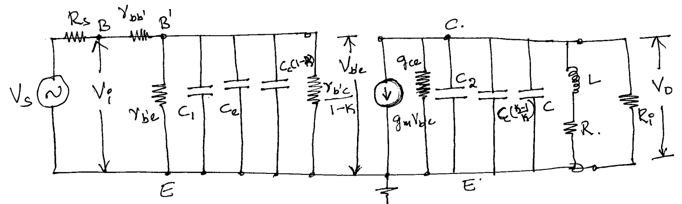
→ To analyze the above circuit we replace our transistor Q with a hybrid π model rather than h-hybrid model because the tuned amplifiers are used at radio (high) frequencies.

→ To analyze draw the a.c. equivalent ckt. & replacing the transistor with hybrid π model & applying miller's theorem to it will be as follows.

circuit, then the equivalent ckt is as follows



→ by applying miller's theorem on to $C_{b'c}$ & $r_{b'c}$ gives.

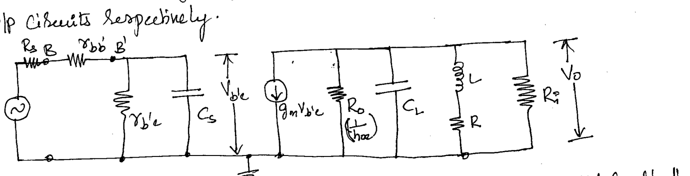


→ Replacing all i/p section capacitance with C_s & o/p section capacitance with C_L

which are. $C_s = C_1 + C_e + C_c(1-K)$

$C_L = C_2 + C_c\left(\frac{K-1}{K}\right) + C$

→ C_1 & C_2 are wiring capacitance or stray capacitances at the i/p & o/p circuits respectively.



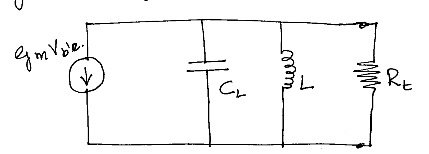
→ Converting the series combination of L & R into equivalent parallel combination

$\frac{1}{..} = \frac{R-j\omega L}{..} = \frac{R}{..} - j\frac{\omega L}{..} = \frac{1}{R_p} + \frac{1}{j\omega L_p}$

$R \approx R \quad [\because \omega L >> R]$

$L_p = \frac{R^2+\omega^2L^2}{\omega^2 L} \approx L \quad [\because \omega^2L^2 >> R^2]$

replacing & redrawing the o/p equivalent ckt. we get.



where $R_t = R_o \,\|\, R_p \,\|\, R_i$.

→ Resonant angular frequency is.

$\omega_0 = \frac{1}{\sqrt{LC_L}}$; $f_0 = \frac{1}{2\pi\sqrt{LC_L}}$

→ The effective quality factor or the circuit magnification factor of the entire output circuit at resonance is given as.

$$Q_e = \frac{\text{Susceptance of Inductor or capacitor } C_L}{\text{Conductance of shunt resistance } R_t}$$

$$= \frac{R_t}{\omega_0 L} = \omega_0 R_t C_L.$$

→ To define the gain of the amplifier we define o/p voltage V_o & is given as.

$$V_o = -g_m V_{b'e} \cdot Z$$

→ where $Z \Rightarrow$ equivalent impedance of R_t, C_L & L of the output circuit.

→ its admittance is given as $Y = \frac{1}{Z}$.

$$Y = \frac{1}{Z} = \frac{1}{R_t} + \frac{1}{j\omega L} + j\omega C_L$$

$$Y = \frac{1}{R_t}\left[1 + \frac{R_t}{j\omega L} + j\omega R_t C_L\right]$$

→ multiply & divide 2nd & 3rd terms with ω_0.

$$Y = \frac{1}{R_t}\left[1 + \frac{\omega_0 R_t}{j\omega\omega_0 L} + j\frac{\omega\,\omega_0 R_t C_L}{\omega_0}\right]$$

$$Y = \frac{1}{R_t}\left[1 - j\frac{\omega_0}{\omega} \cdot Q_e + j\frac{\omega}{\omega_0} Q_e\right]$$

$$Y = \frac{1}{R_t}\left[1 + jQ_e\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$$

$$\therefore Z = \frac{1}{Y} = \frac{R_t}{1+jQ_e\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}$$

→ indicating a fractional frequency variation with δ which is

$$\delta = \frac{\omega-\omega_0}{\omega_0} = \frac{\omega}{\omega_0} - 1$$

$$\frac{\omega}{\omega_0} = 1+\delta.$$

→ Substituting $\frac{\omega}{\omega_0}$ in Z we get.

$$Z = \frac{R_t}{1+jQ_e\left[(1+\delta) - \frac{1}{(1+\delta)}\right]} = \frac{R_t}{1+jQ_e\left[\frac{(1+\delta)^2-1}{1+\delta}\right]}$$

$$= \frac{R_t}{1+jQ_e\left[\frac{1+2\delta+\delta^2-1}{1+\delta}\right]} = \frac{R_t}{1+jQ_e\left[\frac{2\delta+\delta^2}{1+\delta}\right]}$$

$$= \frac{R_t}{1+j2\delta Q_e\left[\frac{1+\delta/2}{1+\delta}\right]} =$$

$$Z = \frac{R_t}{1+j2\delta Q_e} \quad [\text{As } \omega \to \omega_0 \;\; \delta << 1]$$

$\therefore \; Z = K_t = R_o \| R_p \| R_p$

Where $R_p = \frac{\omega_0^2 L^2}{R} = \frac{\omega_0 L}{\omega_0 CR} = \frac{L}{CR}$.

& at Resonance

$R_p = \omega_0 L Q_e = Q_e^2 R = Q_e\sqrt{\frac{L}{C}}$.

$= \omega_0 L Q_0 = Q_0^2 R = Q_0\sqrt{\frac{L}{C}}$.

$\because \; V_0 = -g_m V_{b'e} \cdot Z$.

as $V_{b'e} = \frac{r_{b'e}}{r_{bb'} + r_{b'e}} \cdot V_i$ [neglecting C_s & applying Voltage divider rule to o/p]

$\therefore \; V_0 = -g_m \frac{r_{b'e}}{r_{bb'} + r_{b'e}} \cdot Z \, V_i$

$\simeq -g_m \cdot Z \cdot V_i \quad [\because r_{bb'} + r_{b'e} \simeq r_{b'e}]$

$A_v = \frac{V_o}{V_i} = -g_m Z$

$A_v = -g_m \frac{R_t}{1 + j2\delta Q_e}$.

$|A_v| = \frac{g_m \cdot R_t}{\sqrt{1 + (2\delta Q_e)^2}}$.

at resonance $\omega = \omega_0 \; \therefore \; \delta = 0$

$\therefore \; |A_{vres}| = g_m \cdot R_t$.

$A_{res} = -g_m R_t$

gain $A_v = A = \frac{A_{res}}{1 + j2\delta Q_e}$

$\frac{A}{A_{res}} = \frac{1}{1 + j2\delta Q_e} \Rightarrow \left|\frac{A}{A_{res}}\right| = \frac{1}{\sqrt{1 + (2\delta Q_e)^2}}$

→ at half power points $\omega_1 < \omega_0 < \omega_2$

$\frac{A}{A_{res}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (2\delta Q_e)^2}}$

$\therefore \; 1 + (2\delta Q_e)^2 = 2$.

$(2\delta Q_e)^2 = 1$.

$\delta = \pm \frac{1}{2Q_e}$

at $\omega = \omega_1$,

$\delta = -\frac{1}{2Q_e}$

$\omega = \omega_2$

$\delta = +\frac{1}{2Q_e}$

Bandwidth $\Delta\omega = \omega_2 - \omega_1$

add, subtract, multiply & divide with ω_0

$= \frac{[\omega_2 - \omega_0 + \omega_0 - \omega_1]\,\omega_0}{\omega_0}$.

$= \left[\frac{\omega_2 - \omega_0}{\omega_0} + \frac{\omega_0 - \omega_1}{\omega_0}\right]\omega_0$.

$= [\delta + \delta]\,\omega_0 = 2\delta\omega_0$.

$\delta = \frac{1}{2Q_e}$.

$\therefore \; BW = \omega_2 - \omega_1 = \frac{2\omega_0}{2Q_e} = \frac{\omega_0}{Q_e}$.

Bandwidth $= \frac{\omega_0}{\omega_0 R_t C_L} = \frac{1}{R_t C_L}$ Rad/sec.

In Hertz

Bandwidth $= f_2 - f_1 = \frac{1}{2\pi R_t C_L}$ Hz.

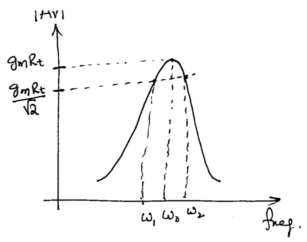
→ frequency response is as follows.

→ Its gain Bandwidth product is

$A_{vres} \times$ Bandwidth.

$GBW = A_{vres} \times B.W.$

$GBW = \frac{g_m R_t}{2\pi R_t C_L} = \frac{g_m}{2\pi C_L}$.



Tapped capacitively coupled single-tuned amplifier :-

→ This circuit is similar to that of capacitively coupled single tuned amplifier with Inductor in the tank circuit is a tapped Inductor hence the name.

→ By a tapped inductor in tuned circuit the o/p impedance of the CE amplifier & the input impedance of the next stage are made to get matched by which maximum power from output to next stage input is been transferred.

→ The ckt is as shown in the fig. ⓐ

→ R_1, R_2, R_E & C_E are Biasing & Stabilizing elements.

→ C & L constitute a tank ckt.

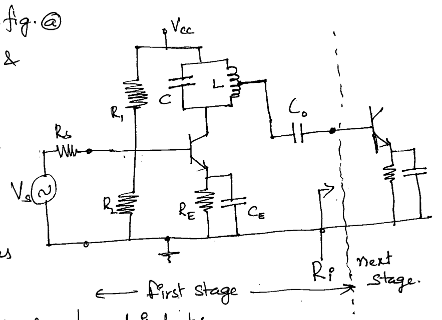
→ V_s & R_s are source & its internal resistance.

→ C_o is coupling capacitor.

→ R_i of the next stage works as load to the first stage.

→ Inductor is a Variable inductor or a tapped inductor

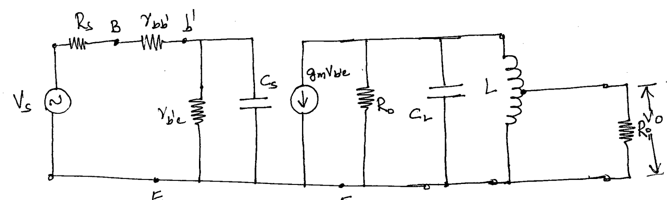
→ Position of tapping is adjusted as per the requirement. i.e, as per the output



→ As when input signal is applied as the transistor is in active mode it amplifies the signal & the tuned circuit will select the frequency & under resonant frequency the impedance offered will be maximum due to which the o/p voltage will be maximum, hence gain will also be maximum.

→ As this is a tapped circuit the tapping is adjusted for maximum Power transfer hence the power transfered will be more than to that of the previous circuit hence the gain is improved due to impedance matching by tapped inductor.

→ In order to analyse the above circuit first an ac equivalent is drawn and the transistor is replaced by a hybrid π model since these amplifiers are used at radio (High) frequencies and applying miller's theorem on $C_{b'c}$ & $r_{b'c}$ we get the simplified ckt. as below.

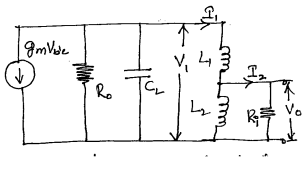


Equivalent circuit after applying millers & combining the capacitances.

$C_s = C_1 + C_e + C_c(1-k)$; $C_L = C_2 + C + C_c\left(\frac{k-1}{k}\right)$

→ Simplified output circuit can be redrawn by splitting the inductor L into L_1 & L_2 where $L_1 = nL$ & $L_2 = (1-n)L = (1-n)L = L - nL = L - L_1$

$\because \; L_1 + L_2 = L$ & the ckt. i.e.



$$V_1 = j\omega L I_1 - j\omega(L_2+M) I_2 \longrightarrow ①$$

$$0 = -j\omega(L_2+M) I_1 + (R_i + j\omega L_2) I_2 \longrightarrow ②$$

→ Where M is mutual inductance between L_1 & L_2

→ The equations are written by neglecting the coil resistance R.

Solving ① & ②

$$0 = -j\omega(L_2+M) I_1 + (R_i + j\omega L_2) I_2.$$

$$\therefore I_2 = \frac{j\omega(L_2+M)}{R_i + j\omega L_2} \cdot I_1 \rightarrow ③$$

Sub ③ in ①

$$\therefore V_1 = j\omega L I_1 - j\omega(L_2+M) \cdot \frac{j\omega(L_2+M)}{R_i+j\omega L_2} \cdot I_1$$

$$V_1 = I_1 \left(j\omega L + \frac{\omega^2(L_2+M)^2}{R_i + j\omega L_2}\right)$$

$$\therefore I_1 = \frac{V_1 \cdot (R_i + j\omega L_2)}{j\omega L(R_i+j\omega L_2) + \omega^2(L_2+M)^2}$$

→ Impedance offered by the coil along the resistance R_i can be written as.

$$Z_1 = \frac{V_1}{I_1} = \frac{j\omega L(R_i+j\omega L_2) + \omega^2(L_2+M)^2}{R_i + j\omega L_2}$$

$$= j\omega L + \frac{\omega^2(L_2+M)^2}{R_i + j\omega L_2}$$

→ At operating frequencies $\omega L_2 >> R_i$, hence neglecting ωL_2 as it is in denominator we get

$$j\omega L + \frac{\omega^2(L_2+M)^2}{}$$

Space in the coil's small in comparison with its diameter.

$$\because M = k\sqrt{L_1L_2} = k\sqrt{nL\cdot(1-n)L} = kL\sqrt{(n-n^2)}$$

as $k=1$. $M = L\sqrt{(n-n^2)}$

Substituting Z_1 we get:

$$Z_1 = j\omega L + \frac{\omega^2[(1-n)L + L\sqrt{(n-n^2)}]^2}{R_i}$$

→ from the equation it can seen that Z_1 is a combination of Inductor L in series with a resistance say R_{is}.

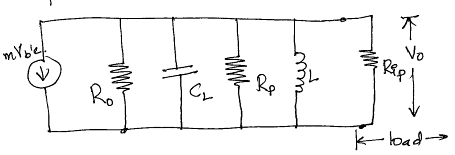
$$\text{Where } R_{is} = \frac{\omega^2L^2[(1-n)+\sqrt{(n-n^2)}]^2}{R_i}$$

$$\therefore Z_1 = j\omega L + R_{is}$$

This series combination can be resolved into equivalent parallel combination with inductance L in parallel with R_{ip}

$$\text{where } R_{ip} = \frac{\omega^2L^2}{R_{is}}$$

→ By this equivalent circuit can be drawn as.



where $R_p = \frac{\omega^2L^2}{R}$; R → Coil resistance.

→ As R_{ip} works as a load, under resonance maximum power will transfer if $R_{ip} = R_o \| R_p = R_{op}$

here $R_p = \frac{\omega_0^2L^2}{R} = \omega_0 L \cdot Q_0$

R_{is}.

Substituting R_{is} we get.

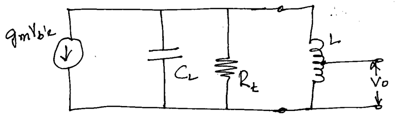
$$R_{op} = \frac{\omega_0^2L^2 R_i}{\omega_0^2L^2[(1-n)+\sqrt{(n-n^2)}]^2}$$

$$\frac{R_i}{R_{op}} = [(1-n)+\sqrt{(n-n^2)}]^2$$

$$\therefore (1-n) + \sqrt{(n-n^2)} = \pm\sqrt{\frac{R_i}{R_{op}}}$$

→ This above equation states about the tapping of ~~transformer~~ coil that has to be done as per o/p resistance & load resistance R_i for max power transfer.

→ The equivalent circuit of above figure is as follows



$$\text{where } \frac{1}{R_t} = \frac{1}{R_o} + \frac{1}{R_p} + \frac{1}{R_{ip}}$$

& effective quality factor is given as $Q_e = \frac{R_t}{W_0 L}$.

& the resonant frequency is given as $f_0 = \frac{1}{2\pi\sqrt{LC_L}}$

→ under matched condition $R_t = \frac{R_{op}}{2}$ [$\because R_t = R_{op} \| R_{ip}$ & matched condition $R_{op} = R_{ip}$]

$$\therefore Q_e = \frac{R_{op}}{2W_0L}$$

$$R_{op} = 2W_0Q_eL = \frac{R_oR_p}{R_o+R_p} \text{ (from fig.)}$$

$$\therefore 2W_0Q_eL = \frac{R_oW_0Q_0L}{R_o+W_0Q_0L}$$

→ Solving for L we get ⇒ $2Q_e = \frac{R_oQ_0}{R_o+W_0Q_0L}$

$$\frac{}{2W_0Q_0Q_e} = \frac{R_o}{W_0} \cdot \left[\frac{1}{2Q_e} - \frac{1}{Q_0}\right]$$

→ This value of L defines maximum transfer of power & also 3dB bandwidth as defined by Q_e.

→ Voltage gain can be defined from output voltage which $(1-n)$ times the voltage developed across the complete coil. which is

$$V_o = -g_m V_{b'e} \cdot Z(1-n)$$

$$V_o = -g_m V_i \frac{r_{b'e}}{r_{bb'}+r_{b'e}} \cdot Z(1-n)$$

$$A_V = \frac{V_o}{V_i} = -g_m \frac{r_{b'e}}{r_{bb'}+r_{b'e}} \cdot \frac{R_t}{1+j2\delta Q_e}(1-n)$$

$$\therefore \boxed{A_V \simeq -g_m \cdot \frac{R_t}{1+j2\delta Q_e}(1-n)} = A$$

at resonance as $\omega=\omega_0 \Rightarrow \delta=0$

$$\therefore A_{res} \simeq -g_m R_t(1-n).$$

$$\frac{A}{A_{res}} = \frac{1}{1-j2\delta Q_e}$$

$$\left|\frac{A}{A_{res}}\right| = \frac{1}{\sqrt{1+(2\delta Q_e)^2}}$$

→ Its similar to that of capacitively coupled but the only variation lie is in effective quality factor Q_e

→ at half power points $\left|\frac{A}{A_{res}}\right| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(2\delta Q_e)^2}}$

$$\therefore \sqrt{2} = \sqrt{1+(2\delta Q_e)^2}$$

$$\delta = \pm\frac{1}{2Q_e}$$

at $\omega = \omega_1$

$$\delta = -\frac{1}{2Q_e}$$

$\omega = \omega_2$

$$\delta = +\frac{1}{2Q_e}$$

$\therefore$ Bandwidth $= \omega_2 - \omega_1$.

multiply, divide, add & Subtract ω_0 to band width.

$$BW = \frac{\omega_2 - \omega_0 + \omega_0 - \omega_1}{\omega_0}\cdot\omega_0$$

$$= \left[\frac{\omega_2-\omega_0}{\omega_0} + \frac{\omega_0-\omega_1}{\omega_0}\right]\omega_0$$

$$= [\delta+\delta]\,\omega_0 = 2\delta\omega_0$$

as $\delta = \frac{1}{2Q_e}$ at cutoff.

$$BW = \omega_2 - \omega_1 = \frac{2\omega_0}{2Q_e} = \frac{\omega_0}{Q_e}$$

$$\therefore BW = \frac{\omega_0}{\omega_0 R_t C_L} = \frac{1}{R_t C_L}$$

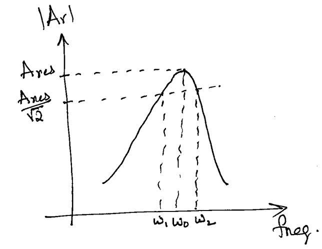
Under matched condition $R_t = \frac{R_{op}}{2}$.

$$\therefore \text{Bandwidth} = \frac{2}{R_{op}C_L}\ \text{Rad/sec}$$

Bandwidth in Hertz is given as:

$$\text{Band width} = f_2 - f_1 = \frac{2}{2\pi R_{op} C_L} = \frac{1}{\pi R_{op} C_L}\ \text{Hz}.$$

→ The frequency response is as follows.



Gain Bandwidth product

$$GBP = A_{res} \times \text{Bandwidth}$$

$$= \frac{g_m R_t (1-n)}{2\pi R_t C_L} \quad \left[\text{matched Condition } R_t = \frac{R_{op}}{2}\right]$$

$$GBP = \frac{g_m(1-n)}{2\pi C_L}$$

→ This is a single tuned amplifier whose output signal is coupled to the next stage through an Inductor hence the name.

→ It is also called as Transformer Coupled single-tuned amplifier since the tank circuit Inductor forms the primary of the transformer and the coupling inductor is called as Secondary of the transformer.

→ The Secondary of the transformer is connected to the next stage input impedance which work as a load to the first circuit and the circuit is as shown below.

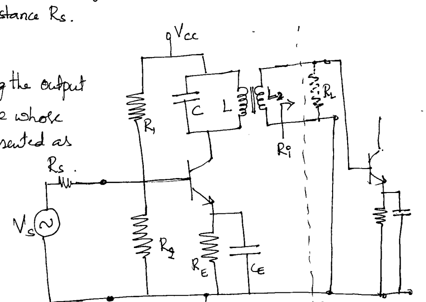
→ R_1, R_2, R_E & C_E forms the biasing & stabilizing network.

→ V_s is a Source with its internal resistance R_s.

→ L & C forms the tank circuit.

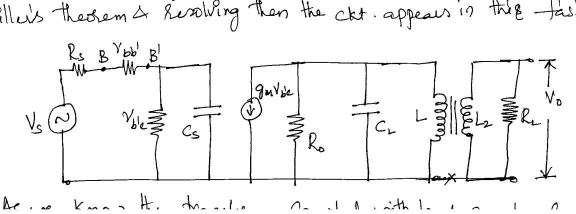
→ L & L_2 forms a transformer for coupling the output signal of first stage to the i/p of next stage whose input resistance is R_i which is represented as a load R_L shown by dotted line.

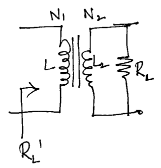
→ As per the tank circuits resonant frequency the gain is maximum at f_0 & for other frequencies the gain falls rapidly.



→ This signal is coupled through another coil L_2 & the turns ratio of transformer is chosen in such a way that the output impedances gets matched with the load impedance so that maximum power transfer occurs from output to load.

→ Analysis can be done by drawing an ac equivalent & replacing the transistor by its high frequency equivalent model i.e, hybrid π model & apply the miller's theorem & resolving then the ckt. appears in this fashion as below.



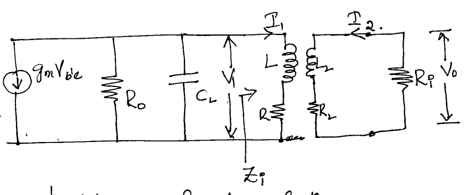


R_L' = resistance obtained from primary to the load & is given as.

$$R_L' = \frac{1}{n^2}\cdot R_L.$$

where n is N_1/N_2.

→ Redrawing the o/p circuit including the coil resistances



Applying KVL to primary & Secondary winding

$$V_1 = I_1 Z_{11} + I_2 Z_{12}.$$
$$0 = I_1 Z_{21} + I_2 Z_{22}.$$

where
$Z_{11} = R + j\omega L$
$Z_{12} = Z_{21} = j\omega M$
$Z_{22} = R_2 + R_i + j\omega L_2$.

→ Solving the equations we get

$$I_1 = \frac{V_1 \cdot Z_{22}}{Z_{11}Z_{22} - Z_{12}^2}$$

$$\therefore \text{Impedance } Z_i = \frac{V_1}{I_1} = Z_{11} - \frac{Z_{12}^2}{Z_{22}}$$

$$Z_i = (R + j\omega L) + \frac{\omega^2 M^2}{R_2 + R_i + j\omega L_2}$$

R_i is generally greater than R_2 & ωL_2

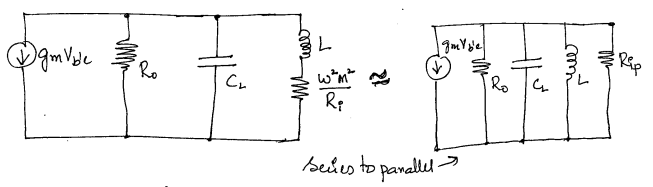
$$\therefore Z_i = R + j\omega L + \frac{\omega^2 M^2}{R_i}$$

Primary.

→ If M is large then $R \ll \frac{\omega^2 M^2}{R_i}$

$$Z_i \simeq j\omega L + \frac{\omega^2 M^2}{R_i}$$

→ Now by this Z_i which is a series combination of L & $\frac{\omega^2 M^2}{R_i}$ so the o/p circuit can be redrawn as.



$$\therefore R_{ip} = \frac{\omega^2 L^2}{\frac{\omega^2 M^2}{R_i}} = \left(\frac{L}{M}\right)^2 \cdot R_i$$

for maximum power transfer $R_o = R_{ip} = \left(\frac{L}{M}\right)^2 \cdot R_i$

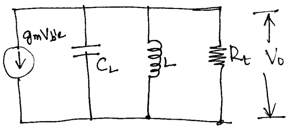
→ which says for the given load of R_i, the desired value of M for maximum transfer of power happens.

→ as we know $M = K\sqrt{L L_2}$ → sub in R_o

$$R_o = \left(\frac{L}{M}\right)^2 \cdot R_i = \frac{L}{K^2 L_2} \cdot R_i$$

→ as per R_o & R_i, L, L_2 & K are choosen.

→ The final equivalent o/p circuit can be drawn as



Where $R_t = R_o \parallel R_{ip}$

& $Q_e = \frac{R_t}{\omega_o L}$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

Under matched condition $R_t = \frac{R_o}{2}$ $[\because R_o = R_{ip}]$

$$\therefore Q_e = \frac{R_o}{2\omega_o L}.$$

$$R_o = 2 Q_e \omega_o L.$$

→ gain of the circuit can be obtained by solving KVL eq's.

$$V_1 = I_1 Z_{11} + I_2 Z_{12}$$

$$0 = I_1 Z_{21} + I_2 Z_{22}$$

by solving

$$I_2 = \frac{V_1 \cdot Z_{21}}{Z_{21}^2 - Z_{11} Z_{22}}$$

→ from the fig.

$$V_o = -I_2 R_i.$$

$$= -V_1 \cdot R_i \cdot \frac{Z_{21}}{Z_{21}^2 - Z_{11} Z_{22}}$$

$\because V_1 = -g_m V_{b'e} \cdot Z$

& $Z = \frac{R_t}{1 + j2\delta Q_e}$.

$$\therefore V_o = +g_m V_{b'e} \cdot \frac{R_t \cdot R_i}{1 + j2\delta Q_e} \cdot \frac{Z_{21}}{Z_{21}^2 - Z_{11} Z_{22}}$$

$$V_o = -g_m \frac{r_{b'e}}{r_{bb'} + r_{b'e}} \cdot V_i \cdot \frac{R_t R_i}{1 + j2\delta Q_e} \cdot \frac{Z_{21}}{Z_{11} Z_{22} - Z_{21}^2}$$

$$A = \frac{V_o}{V_i} \simeq -g_m \frac{R_t R_i}{1 + j2\delta Q_e} \cdot \frac{Z_{21}}{Z_{11} Z_{22} - Z_{21}^2}$$

$\omega = \omega_o \Rightarrow \delta = 0$

$$\therefore A_{res} \simeq -g_m \cdot R_t R_i \frac{Z_{21}}{Z_{11} Z_{22} - Z_{21}^2}$$

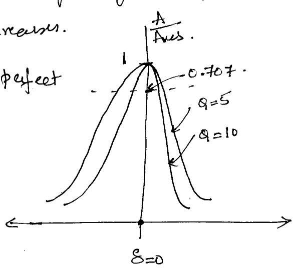
$$\frac{A}{A_{res}} = \frac{1}{1 + j2\delta Q_e} \Rightarrow \left|\frac{A}{A_{res}}\right| = \frac{1}{\sqrt{1 + (2\delta Q_e)^2}}$$

→ Its relative response can be plotted by taking δ Vs A/A_{res} as below.

→ As Q increases bandwidth decreases.

→ Gain is been improved due to perfect matching of impedance

→ 3dB band width.



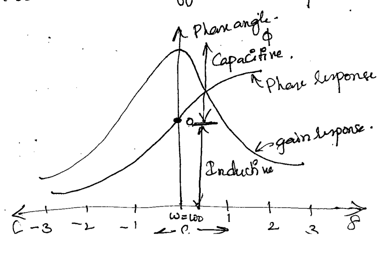
$$Q_e = \frac{\omega_o}{BW}$$

as we know $Q_e = \frac{R_t}{\omega_o L} = \frac{R_o}{2\omega_o L}$.

$$\frac{\omega_o}{BW} = \frac{R_o}{2\omega_o L}$$

$\therefore$ Bandwidth BW is directly proportional to L.

→ Reactance curve for different δ's plotted as below.



→ As this circuit is a combination of two tuned circuit hence the name double tuned amplifier.

→ It's a single-tuned amplifier with wider bandwidth.

→ both the tuned circuits are designed for same resonant frequency & the circuit is as shown below.

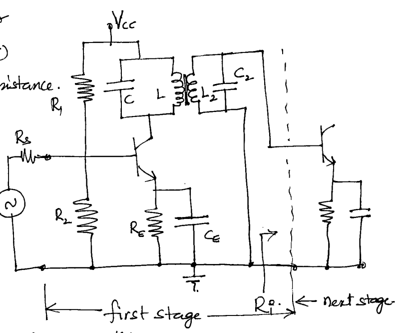
→ R_1, R_2, R_E & C_E forms a biasing & stabilizing elements for transistor (BJT)

→ V_s & R_s are source & its internal resistance.

→ C & L forms one tuned circuit.

→ L_2 & C_2 forms the 2nd tuned circuit.

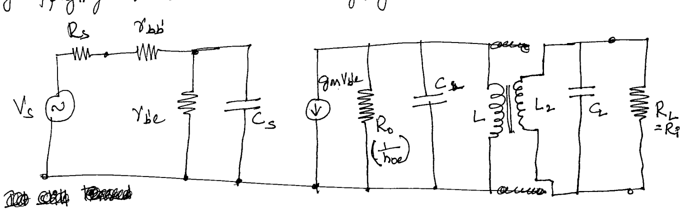
→ L & L_2 forms a transformer for power transfer from primary to the secondary i.e, on to its load which is the input resistance of next stage R_i.



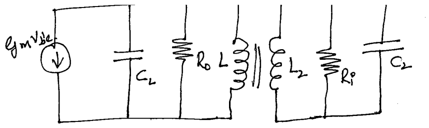
→ As the transistor is in active mode it works like an amplifier & as per the resonant frequency of tank circuit, it amplifies the resonant signal frequency with maximum gain & the gain reduces as we move away from resonant frequency

→ To analyse we go for drawn an a.c. equivalent & replace the transistor with its hybrid π model as these circuits are used for high (radio) frequencies

→ By applying miller's theorem & analysing for reduced network.

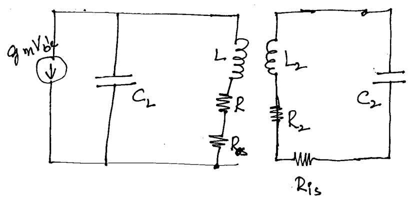


→ Redrawing the output circuit in a simplified form it is as follows

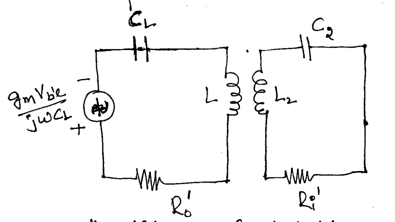


Equivalent output circuit

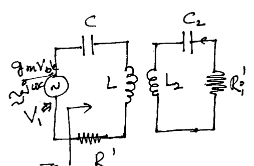
→ Convert the parallel combination of R_0 & L along with L_2 & R_i in ~~series~~ equivalent series as below.



→ R_{os} is equivalent series resistance of $R_0 \parallel L$
→ R_{is} is equivalent series resistance of $R_i \parallel L_2$
→ R & R_2 are internal resistances of L & L_2.
→ Now combining the resistances & converting a norton's source into thevinin's equivalent. we get:



Modified equivalent ckt.



final equivalent ckt.

→ $R_0' = R_{os} + R$; $R_i' = R_{is} + R_2$
→ The impedance Z_{in} looking from primary we get Z_{in} as

$$Z_{in} = \frac{-Z_{12}}{Z_{22}} = \frac{\ldots}{R_i' + j\left(\omega L_2 - \frac{1}{\omega C_2}\right)}$$

at resonance $\omega_0 L = \frac{1}{\omega_0 C}$ ⇒ $\omega_0 L_2 = \frac{1}{\omega_0 C_2}$

$$Z_{in} = \frac{\omega_0^2 M^2}{R_i'}$$

→ at resonance for maximum power transfer is at R_0' must be equal to Z_{in} ie, M is adjusted to critical value M_c, such that

$$R_0' = \frac{\omega_0^2 M_c^2}{R_i'}$$

$$\therefore R_0' R_i' = \omega_0^2 M_c^2$$

$$\sqrt{R_0' R_i'} = \omega_0 k_c \sqrt{L L_2}$$

k_c is critical coefficient of coupling.

$$\therefore k_c = \frac{\sqrt{R_0' R_i'}}{\omega_0 \sqrt{L L_2}} = \left(\frac{R_0'}{\omega_0 L}\right)^{1/2} \cdot \left(\frac{R_i'}{\omega_0 L_2}\right)^{1/2} = \frac{1}{\sqrt{Q_1 Q_2}}$$

In general $k \neq k_c$.
& then ~~k~~ let $k = b k_c$.

To define the maximum gain w.r.t. maximum current consider the KVL equations.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$0 = Z_{21} I_1 + Z_{22} I_2.$$

solving we get $I_2 = -\frac{V_1 \cdot Z_{21}}{Z_{11} Z_{22} - Z_{21}}$

$Z_{12} = Z_{21} = j\omega_0 M$ & $Z_{22} = R_i' + j\left(\omega L_2 - \frac{1}{\omega C_2}\right)$

$Z_{11} = R_0'$ & $Z_{22} = R_i'$

$Z_{12} = j\omega_0 M_c$

→ Substituting these values of Z_{11}, Z_{22} & Z_{12} in I_2 gives a relation of maximum power transfer at resonance.

$$I_{2max} = -j \frac{V_1 \omega_0 M_c}{R_0' R_i' + \omega_0^2 M_c^2}$$

Substitute ωM_c as $\sqrt{R_0' R_i'}$ we get

$$I_{2max} = -j \frac{V_1 \sqrt{R_0' R_i'}}{2 R_0' R_i'} = \frac{-jV_1}{2\sqrt{R_0' R_i'}}$$

Magnitude is

$$|I_{2max}| = \frac{V_1}{2\sqrt{R_0' R_i'}}$$

→ at $\omega = \omega_0$ we have maximum power along with this frequencies there are two other ~~freq~~ frequencies for which I_2 is maximum & can be obtained equating I_{2max} with I_2 & resolving other 2 frequencies.

$$\left|\frac{-jV_1}{2\sqrt{R_0' R_i'}}\right| = \left|\frac{-jV_1 \sqrt{R_0' R_i'}}{\left(R_0' + j\omega L + \frac{1}{j\omega C}\right)\left(R_i' + j\omega L_2 + \frac{1}{j\omega C_2}\right) + b^2 R_0' R_i'}\right|$$

by putting $k = b k_c$

$L_2 = L$
$C_2 = C_1 = C$
$R_0' = R_i' = R$.
$j\left(\omega L - \frac{1}{\omega C}\right) = jX$

$$\left|\frac{V_1}{2R}\right| = \frac{V_1 b R}{(R + jX)^2 + b^2 R^2}$$

$$|2bR^2| = |-X^2 + j2RX + R^2 + b^2R^2|$$

Squaring on both sides & solving we get.

$$4b^2R^4 = \left[R^2(1+b^2) - X^2\right]^2 + 4R^2X^2$$

& solving for X, we get.

$$X = \pm\sqrt{(b^2-1)} \cdot R.$$

$$\therefore \omega L - \frac{1}{\omega C} = \pm\sqrt{(b^2-1)} \cdot R.$$

$$\omega^2 LC - 1 = \pm\sqrt{(b^2-1)} \cdot \omega C R. \rightarrow (a).$$

In general Q is large, so that

$$\omega C R \simeq \omega_0 C R = \frac{1}{Q}.$$

& also $\omega_0^2 = \frac{1}{LC}$

∴ eq. (a) can be rewritten as

$$\frac{\omega^2}{\omega_0^2} - 1 = \pm\frac{\sqrt{b^2-1}}{Q}$$

$$\frac{\omega^2}{\omega_0^2} = 1 \pm \frac{\sqrt{b^2-1}}{Q}.$$

$$\omega \simeq \pm\omega_0 \sqrt{1 \pm \frac{\sqrt{(b^2-1)}}{Q}}.$$

→ If $b<1$, coupling coefficient is less than the critical value then from above equation we find the ω ~~is~~ becomes complex which leads to no real frequencies at which maximum power gets transferred.

$\therefore$ $b > 1$ has to be taken it results two frequencies ... maximum.

→ The 3-dB frequencies at which I_L Reduces to 0.707 of its maximum Values is lower & upper cut off frequencies Can be Obtained by.

$$\left| \frac{-jV_1 b\sqrt{R_o' R_i'}}{(R_o' + j\omega L + \frac{1}{j\omega C})(R' + j\omega L_2 + \frac{1}{j\omega C_2}) + b^2 R_o' R_i'} \right| = \frac{1}{\sqrt{2}} \left| \frac{-jV_1}{2\sqrt{R_o' R_i'}} \right|$$

by applying the Condition it Can be reduced to.

$$\left| \frac{V_1 bR}{(jX + R)^2 + b^2 R^2} \right| = \frac{1}{\sqrt{2}} \left| \frac{V_1}{2R} \right|$$

$$|2\sqrt{2}\, bR^2| = |-X^2 + j2XR + R^2 + b^2R^2|.$$

(or) $$8b^2R^4 = [R^2(1+b^2) - X^2]^2 + 4X^2R^2.$$

by solving

$$X = \pm \sqrt{(b^2 - 1 \pm 2b)}\, R.$$

$$\omega L - \frac{1}{\omega C} = \pm \sqrt{b^2 - 1 \pm 2b} \cdot R.$$

$$\omega^2 LC - 1 = \pm \sqrt{b^2 - 1 \pm 2b}\; \omega CR.$$

$\because$ $\omega_0^2 = \frac{1}{LC}$ & $\omega CR = \frac{1}{Q}$. Substituting

we get $$\frac{\omega^2}{\omega_0^2} - 1 = \pm \frac{\sqrt{b^2 - 1 \pm 2b}}{Q}.$$

Or by multiplying on both side with ω_0 w.r.t. (6) & sending ω to the left side.

we get

$$\omega\omega_0 LC - \frac{\omega_0}{\omega} = \pm \sqrt{b^2 - 1 \pm 2b}\; \omega_0 CR$$

$$\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} = \pm \frac{\sqrt{b^2 - 1 \pm 2b}}{Q_0}$$

at two 3dB frequencies ω_2 & ω_1, one corresponds to a positive value & other for Negative Sign.

→ Then $$\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2} = -\left[\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1}\right]$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

→ Taking +ve Sign & Negative & solving we get 3-dB band as

$$(\omega_2 - \omega_1) = \sqrt{(b^2 - 1) \pm 2b} \cdot \frac{\omega_0}{Q}$$

→ BW is proportional to $\frac{\omega_0}{Q}$

→ In a Single tuned amplifier $BW = \frac{\omega_0}{Q}$, in double tuned is exceeds by a factor $\sqrt{b^2 - 1 \pm 2b}$

→ If $b < 0.414$ no Real value of bandwidth. & b should be only with Positive sign.

→ If $b < 2.414$ ~~yes~~ yields four values of 3-dB freq. in practice.

→ So compromise value of b is kept in the Range of 1 to 1.7

→ If $b < 1$ leads to a Steeper sides of the Curve & b $\geq$ is increased beyond the limit the overshoots of the frequency response also increases.

—— X ——