Web Mining Lab - 6

Aadhitya Swarnesh

[redacted]

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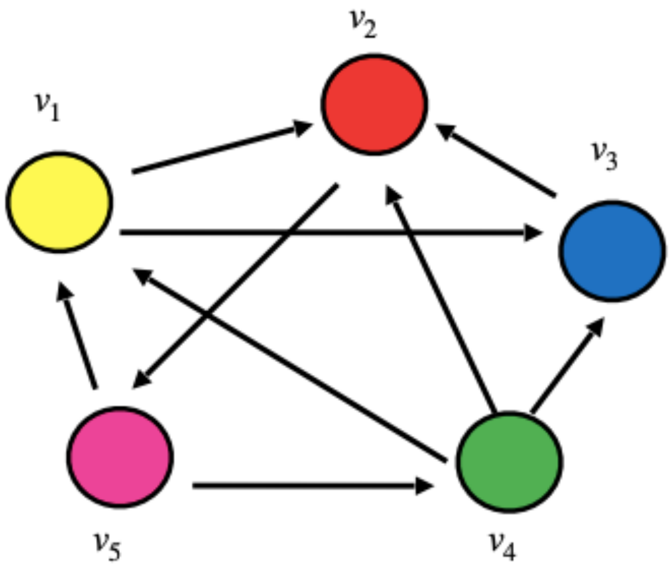
Question

Implement the basic PageRank algorithm using Python to display the rank of 5 pages.

Verify the consistency of results obtained with Random walk and inbuilt PageRank function.



Before moving on to the actual question, we need to first model the graph given to us. In order to do so, we form an adjacency matrix for the graph shown above. This is available in the code section for reference.

Now to move on to the first part of the question, we will use the networkX library to model the graph, and then use the page rank function available in this library to compute the page ranks of these vertices. We use a Directed Graph here because the like which connect one web page to another is just in one direction, and there is no need for a link to be in both ways.

The result obtained after following this method is as follows :

```
The Pages in the order of importance with the page rank scores obtained by using NetworkX are :
v2 = 0.2713164308772404
v5 = 0.26061906832422166
v1 = 0.18064505060873787
v3 = 0.14665711544131715
v4 = 0.14076233474848301
```

Now for the second part of the question, we are to validate the results obtained in the first half by implementing the actual page rank algorithm which was present inside the back box representation used above because of the networkX library. In order to do this we first find all the in-bound and the out-bound connections for all the vertices of the graph, decide on a particular error tolerance level for the convergence, and also fix the maximum number of iterations to perform in case the values do not seem to converge.

The formula to calculate the page rank for a vertex in an iteration is as follows :

$$PR(u) = \sum_{v \in B_u} \frac{PR(v)}{L(v)}$$

where B_u denotes the list of all the in-bound nodes to the node **u.**
$PR(v)$ denote the Page Rank of the node **v**.
$L(v)$ denote the number of out-bound nodes from the node **v**.

We use this formula for the page rank calculation, and the result obtained after the random walks (until the convergence condition was reached) is as follows :

```
The Pages in the order of importance with the page rank scores obtained by performing Random Walk are :
v2 = 0.2727282232687022
v5 = 0.27272760406078966
v1 = 0.1818174601173467
v3 = 0.1363637933458539
v4 = 0.13636291920730784
```

We can now conclude by the results obtained from both the methods that the first method was in fact valid and had produced the correct result. The result is the following with the page with the highest rank in the front :

$$v2 > v5 > v1 > v3 > v4$$

The **code** that has delivered these results are as follows :

Page Ranking

- Implement the basic PageRank algorithm using Python to display the rank of 5 pages.
- Verify the consistency of results obtained with Random walk and inbuilt PageRank function

![image.png]

1. Model the Query Graph

We will create an adjacency matrix that denotes the edjes between the nodes.

In [1]:

```
adjacency_matrix = [
    [0, 1, 1, 0, 0],
    [0, 0, 0, 0, 1],
    [0, 1, 0, 0, 0],
    [1, 1, 1, 0, 0],
    [1, 0, 0, 1, 0]
]
```

In [2]:

```
num_vertices = 5
```

In [3]:

```
# list of nodes

vertices_list = ['v1', 'v2', 'v3', 'v4', 'v5']
```

2. Use the Networkx Library to rank the pages

We will be using the `networkx` library to create a graph instance and then rank these nodes. The Nodes in the graph are the pages and the links between them are the hyperlinks that are available between the pages, which serve as a medium through which they are connected.

[Reference Material (https://medium.com/sicara/fraud-detection-personalized-page-rank-networkx-15bd52ba2bf6)](https://medium.com/sicara/fraud-detection-personalized-page-rank-networkx-15bd52ba2bf6)

In [4]:

```
# Import the Networkx Library for forming the graph
import networkx as nx

# Import the Pyplot library for plotting the created graph
from matplotlib import pyplot as plt
```

In [5]:

```
# Create an instance of the Graph class
# We are using a Directed graph due to the nature of our problem.

graph = nx.DiGraph()
```

In [6]:

```
# Load the nodes into the graph

graph.add_nodes_from(vertices_list)
```

In [7]:

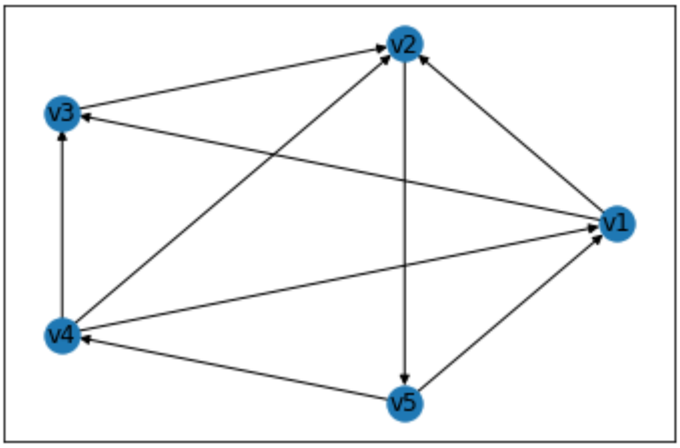
```
# Add the edges from the adjacency matrix

for i in range(num_vertices) :
    for j in range(num_vertices) :
        if adjacency_matrix[i][j] == 1 :
            graph.add_edge(vertices_list[i], vertices_list[j])
```

In [8]:

```
# Draw generated graph

nx.draw_networkx(graph, pos=nx.circular_layout(graph), arrows=True, with_labels=
True)
plt.show()
```



In [9]:

```
# Compute the page ranks

page_ranks_networkx = nx.pagerank(graph, alpha=0.85)
print("The page ranks are :\n")
page_ranks_networkx
```

```
The page ranks are :

```

Out[9]:

```
{'v1': 0.18064505060873787,
 'v2': 0.2713164308772404,
 'v3': 0.14665711544131715,
 'v4': 0.14076233474848301,
 'v5': 0.26061906832422166}
```

In [10]:

```
# Sort the pages by their ranks

page_ranks_networkx = dict(sorted(page_ranks_networkx.items(), key=lambda item:
item[1], reverse=True))
page_ranks_networkx
```

Out[10]:

```
{'v2': 0.2713164308772404,
 'v5': 0.26061906832422166,
 'v1': 0.18064505060873787,
 'v3': 0.14665711544131715,
 'v4': 0.14076233474848301}
```

In [11]:

```
# Print the pages by the order of their ranks

print("The Pages in the order of importance with the page rank scores obtained b
y using NetworkX are : ")
for k, v in page_ranks_networkx.items() :
    print(k, "=", v)
```

```
The Pages in the order of importance with the page rank scores obtai
ned by using NetworkX are :
v2 = 0.2713164308772404
v5 = 0.26061906832422166
v1 = 0.18064505060873787
v3 = 0.14665711544131715
v4 = 0.14076233474848301
```

3. Page Rank with Random Walk

We will use the Random walk of graph in order to compute the page rank of these pages.
We resort to this approach because this can serve as a validator to the above approach. We essentially perform the operations of the library systematically so as to gain a better knowledge of the inner workings of the algorithm.
[Reference Link (https://www.geeksforgeeks.org/page-rank-algorithm-implementation/)](https://www.geeksforgeeks.org/page-rank-algorithm-implementation/)

In [12]:

```
# Calculate the number of out-bound links for each vertex

out_bound_vertices_count = [0 for _ in range(num_vertices)]

for i in range(num_vertices) :
    out_bound_vertices_count[i] = sum(adjacency_matrix[i])

out_bound_vertices_count
```

Out[12]:

```
[2, 1, 1, 3, 2]
```

In [13]:

```
print("The Out-Bound vertices count for each vertex is as follows : ")
for i in range(num_vertices) :
    print(vertices_list[i], " : ", out_bound_vertices_count[i])
```

```
The Out-Bound vertices count for each vertex is as follows :
v1  :  2
v2  :  1
v3  :  1
v4  :  3
v5  :  2
```

In [14]:

```
# List and store all the in-bound vertices for a particular vertex

in_bound_vertices_list = {}

for i in range(num_vertices) :
    in_bound_vertices_list[i] = []

    for j in range(num_vertices) :
        if adjacency_matrix[j][i] == 1 :
            in_bound_vertices_list[i].append(j)

in_bound_vertices_list
```

Out[14]:

```
{0: [3, 4], 1: [0, 2, 3], 2: [0, 3], 3: [4], 4: [1]}
```

In [15]:

```
print("The In-bound vertices for each vertex is as follows : ")

for i in range(num_vertices) :
    print(vertices_list[i], " : ", end="")
    print(", ".join([vertices_list[j] for j in in_bound_vertices_list[i]]))
```

```
The In-bound vertices for each vertex is as follows :
v1  : v4, v5
v2  : v1, v3, v4
v3  : v1, v4
v4  : v5
v5  : v2
```

The actual Page rank Algorithm

In [16]:

```
def pageRank(graph, vertices_names, in_bound_vertices_list, out_bound_vertices_c
ount, tolerance=1.0e-6, max_iterations=100) :
    """
    Finds and returns the page rank for all the vertices.
    The algorithms continues for the maximum number of iterations as specfied(or
100) in case it does not converge.

    Parameters
    ----------

    graph : The adjacency matrix of the graph.
    vertices_names : The names of the vertices.
    in_bound_vertices_list : The list of all the in-bound vertices of all the ve
rtices. A dictionary matching the vertex index with the index of all the in-boun
d vertices.
    out_bound_vertices_count : The list of count of all the out-bound vertices f
rom a vertex. An array of the out-bound vertex count from a particular vertex(in
dex positioned).
    tolerance : The accuracy of the page rank that is required.
    max_iterations : The maximum iterations that the algorithm should continue i
n case of no convergence.

    Returns
    -------

    page_ranks_manual : The page ranks calculated
    """

    # Number of vertices
    num_vertices = len(vertices_names)

    # Initialize the page ranks of all the vertices to be equal (1 / num_vertice
s)
    page_rank = [(1/num_vertices) for _ in range(num_vertices)]

    # Calculate the overall convergence condition for the combination of the who
le list of vertices
    epsilon = num_vertices * tolerance

    # Start the algorithm
    converged = False
    for i in range(max_iterations) :

        # Store the old page ranks
        page_rank_old = page_rank[:]

        # Update the page ranks of all the vertices
        for j in range(num_vertices) :

            # Because of addition further ahead, we re-initialize this value
            page_rank[j] = 0

            # Add all the in-bound vertices page rank/out-bound count
            for k in in_bound_vertices_list[j] :
                page_rank[j] += page_rank_old[k] / out_bound_vertices_count[k]

        # Check for convergence condition
        error = sum([abs(page_rank[j] - page_rank_old[j]) for j in range(num_ver
tices)])
```

```
        if error < epsilon :
            converged = True
            break

    # Return the solution, only if convergence has taken place
    if converged :
        page_rank_manual = {}

        for i, pr in enumerate(page_rank) :
            page_rank_manual[vertices_names[i]] = pr

        return page_rank_manual
```

In [17]:

```
page_rank_manual = pageRank(graph, vertices_list, in_bound_vertices_list, out_bo
und_vertices_count)
page_rank_manual
```

Out[17]:

```
{'v1': 0.1818174601173467,
 'v2': 0.2727282232687022,
 'v3': 0.1363637933458539,
 'v4': 0.13636291920730784,
 'v5': 0.27272760406078966}
```

In [18]:

```
# Sort the pages by their ranks

page_rank_manual = dict(sorted(page_rank_manual.items(), key=lambda item: item[1
], reverse=True))
page_rank_manual
```

Out[18]:

```
{'v2': 0.2727282232687022,
 'v5': 0.27272760406078966,
 'v1': 0.1818174601173467,
 'v3': 0.1363637933458539,
 'v4': 0.13636291920730784}
```

In [19]:

```
# Print the pages by the order of their ranks

print("The Pages in the order of importance with the page rank scores obtained b
y performing Random Walk are : ")
for k, v in page_rank_manual.items() :
    print(k, "=", v)
```

```
The Pages in the order of importance with the page rank scores obtai
ned by performing Random Walk are :
v2 = 0.2727282232687022
v5 = 0.27272760406078966
v1 = 0.1818174601173467
v3 = 0.1363637933458539
v4 = 0.13636291920730784
```

Result

We can by this conclude that the page ranks obtained by both the algorithms are nearly the same.
And further we see that the order of the pages delivered by both the algorithms is the same.
The order of the nodes by their page ranks is : `v2 > v5 > v1 > v3 > v4`

In [ ]:

```

```