UNIVERSITY OF CAMBRIDGE
Faculty of Mathematics

MATHEMATICAL TRIPOS
2025-26

GUIDE TO COURSES
IN PART IA

This booklet provides an introduction for new students, giving an outline of the first year with informal and non-technical descriptions of the courses.

This *Guide to Courses* is intended to supplement the more formal descriptions contained in the booklet *Schedules of Lecture Courses and Form of Examinations*.

These and other Faculty documents for students taking the Mathematical Tripos are available from the undergraduate pages on the Faculty's website at `https://www.maths.cam.ac.uk/undergrad/`

Revised 15 September 2025

1 Introduction

The *Mathematical Tripos* consists of Parts IA, IB and II, normally taken in consecutive years, with an optional fourth year, Part III, which can taken by students who do sufficiently well. Those who successfully complete three years are eligible to graduate with a BA honours degree, while those who go on to complete the additional fourth year graduate with both BA honours and MMath degrees.

The Mathematical Tripos is tightly structured, with no choice in the first year, some choice in the second year, and a very wide choice in the third year.

This booklet provides an introduction for new students, with an outline of Part IA and informal descriptions of the courses. The descriptions are intended to be comprehensible without much prior knowledge and to convey something of the flavour of each course (corresponding booklets are also available for Part IB and Part II).

You will also find it helpful to consult the booklet *Schedules of Lecture Courses and Form of Examinations*, available for download at `https://www.maths.cam.ac.uk/undergrad/course/schedules.pdf`, which is usually known as *the Schedules*. It describes the structure of the Tripos formally, in more technical detail, and it is the definitive reference for matters of course content and assessment, for students, lecturers and examiners. The introductory sections, in particular, should be read carefully alongside this booklet.

Your college Director of Studies will be able to provide further advice and guidance on all aspects of the Mathematical Tripos.

Changes since last year

There are no changes to the content of lecture courses in Part IA in 2025-26 compared to 2024-25.

2 The First Year in Outline

Students are admitted to study one of two options in Part IA: (a) Pure and Applied Mathematics; or (b) Mathematics with Physics. (You will have chosen one of these options when you applied.) There is no choice within each option: you are expected to follow all the courses.

- For option (a) there are eight 24-lecture courses (four in Michaelmas Term and four in Lent Term) and four 3-hour examination papers, with two courses examined on each paper.
- For option (b), the lecture courses Numbers & Sets and Dynamics & Relativity are replaced by the complete Physics course from Part IA of the Natural Sciences Tripos, which has lectures in Michaelmas, Lent and Easter Terms and assessed practical work throughout the year. Paper 4 of the Mathematics examination is replaced by the Physics paper from Natural Sciences.
- Students taking options (a) and (b) are classed together, as a single group, following the examinations at the end of the year.
- Options (a) and (b) do not extend into the second year. Those taking option (a) will (usually) continue to Part IB Mathematics; those taking option (b) may do the same, or they may change to Part IB Natural Sciences.

Option (b) is designed for students who have a strong interest in mathematics but who may wish to change to Part IB of the Natural Sciences Tripos (and take the Physics A / Physics B / Mathematics options) after the first year. It provides an excellent mathematical background for students who plan to study theoretical or experimental physics: their greater mathematical knowledge, compared with students who come to Physics through Part IA of the Natural Sciences Tripos, can be a significant benefit.

Changing from option (b) to option (a) is usually feasible if it is done early enough, but the courses cover ground rapidly; if this is something you are considering then you should discuss it with your Director of Studies as soon as possible. Changing from option (a) to option (b) is likely to be more complicated, because of the additional assessed practical work.

Michaelmas Term: Non-Examinable Mechanics Course

In Michaelmas Term there is a short non-examinable course on Mechanics. This is intended to provide catch-up material for those students who have taken only a limited amount of Mechanics at A-level (or the equivalent). You should discuss with your Director of Studies whether it might be sensible for you to attend these lectures.

Easter Term courses

A number of lecture courses are given in Easter Term, none of them examinable in Part IA.

- Lectures on Computational Projects (CATAM). It is essential to attend these in your first year, to prepare for the project work that will be submitted and assessed during your second year.
- Two courses examinable in Part IB can be attended in either your first or second year (or both): Variational Principles, and Optimisation.

One advantage of attending Part IB Easter Term courses in your first year is that material prepared well in advance can sink in much better than material prepared just before the exams. An additional benefit is that the Easter Term courses can provide helpful background for other courses in Part IB, e.g. the content of Variational Principles will connect well with topics covered in a number of other applied courses. It is common for Directors of Studies to advise their students to attend lectures in the Easter Term of their first year, work on examples sheets over the summer, and then have supervisions at the very start of their second year.

For the Computational Projects course, the Part IB CATAM manual is usually available at the end of July or the beginning of August at the end of your Part IA year. You should bear in mind that the computational work can be time-consuming and so an early start is strongly recommended. The two core projects require little mathematical knowledge from Part IB courses, so you can make substantial progress over the summer before the start of your second year.

Additional activities and lectures

Most students will find that there is enough mathematics in Part IA to keep them busy (or very busy!), and the Faculty places no expectations on students beyond keeping up with the first-year lectures, examples sheets and supervisions. There are many other educational and recreational opportunities to enjoy at university, though mathematics itself can hopefully be recreational.

For those who do want something extra or something a bit different from the mathematics in Part IA, one of the first choices could be the many excellent talks and lectures provided by the student maths societies. These provide an unparalleled opportunity to hear leading experts talking in an accessible way about some of the most important new ideas in mathematics and the mathematical sciences.

It is even possible to attend/preview some lectures in Part IB in your first year, but this must not be allowed to detract from the time and effort you devote to the IA courses and example sheets. Working to achieve a thorough understanding of the IA material, and developing more mature and subtle ways of thinking about mathematics in the process, is the key to making a successful transition from school to university.

The Faculty attempts to timetable the Part IB course Groups, Rings and Modules in the Lent Term so that it can be attended by first-year students. Hence, if you felt very comfortable with the workload in the Michaelmas Term, you might consider previewing this course (though it is generally unwise to have supervisions too - these are better left to next year when you will gain more benefit). As always, your Director of Studies will be able to provide guidance, e.g. they may advise you to concentrate on learning the Part IA courses as thoroughly as possible; that way, you would need less revision time in the Easter Term and this would allow you to take the courses provided then, i.e. Variational Principles and/or Optimisation.

3 Informal Descriptions of Courses in Part IA

Each lecture course has an official syllabus, or *schedule*, that sets out formally, and in technical terms, the material to be covered. The *schedules* are listed in the booklet *Schedules of Lecture Courses and Form of Examinations* that is available for download at `https://www.maths.cam.ac.uk/undergrad/course/schedules.pdf`.

This section, by contrast, provides an *informal* description of each lecture course given by the Faculty of Mathematics that is examinable in Part IA. Students taking option (b) Mathematics with Physics should refer to `https://www.phy.cam.ac.uk/students/teaching` for a description of the examinable material that replaces the courses Numbers & Sets and Dynamics & Relativity from option (a).

Each description below ends with a summary of the learning outcomes for the course. The full learning outcome for Part IA is that you should understand the material described in the formal syllabuses given in the *Schedules* booklet and be able to apply it to the sorts of problems that can be found on Tripos papers from earlier years.

Vectors and Matrices — 24 lectures, Michaelmas Term

Most students will be familiar with vectors and matrices to some extent; this course recaps the basics before introducing new ideas, results and techniques. The material is absolutely fundamental to nearly all areas of mathematics.

You will learn to deal with vectors in a rather general sense, allowing the description of points, lines and planes in 2 or 3 dimensions to be extended to higher, and even complex, dimensions. You will also enhance and refine your understanding of matrices: studying them as transformations or *linear maps*; using them to analyse incisively, and in full generality, sets of linear equations; and deriving new results and applications, e.g. the classification of quadric surfaces. A complete understanding of the material calls for a good balance between complementary points of view; in particular between algebra and geometry, but also between abstract ideas (e.g. what do we really mean by *dimension*?) and practical applications (e.g. how do we calculate determinants and matrix inverses?).

The course begins with a recap of complex numbers and their connection to geometry in the plane, via de Moivre's theorem, logarithms, and powers. We then move to three dimensions, discussing scalar and vector products and introducing index notation and the summation convention (due to Einstein) – this is used extensively in applied maths and theoretical physics. In parallel, key concepts applicable to all vectors such as *span*, *linear independence* and *basis* are introduced. As the course progresses, there is plenty of practice with matrix algebra, and we meet additional useful notions such as *rank*, *image* and *kernel*. Much of the last third of the course deals with the important concepts of *eigenvectors* and *eigenvalues* and discusses certain standard or *canonical forms* for matrices (how to describe a given transformation as simply as possible). Finally, there are some examples of groups of matrices, including the symmetry group of Special Relativity in two (*spacetime*) dimensions.

Learning outcomes. By the end of this course, you should:

- be able to manipulate complex numbers and be able to solve geometrical problems using complex numbers;
- be able to manipulate vectors in $\mathbb{R}^3$ (using index notation and summation convention where appropriate), and to solve geometrical problems using vectors;
- be able to manipulate matrices and determinants, and understand their relation to linear maps and systems of linear equations;
- be able to calculate eigenvectors and eigenvalues and understand their relation with diagonalisation of matrices and canonical forms.

Analysis I 24 lectures, Lent Term

Analysis involves the rigorous investigation of limits and calculus. You need to study analysis to have a firm foundation for techniques you already know, such as basic differentiation and integration. This not only allows you to understand exactly when these techniques can be used, but also allows you to generalise them to more complicated situations.

The sorts of questions that you will be asking in this course are: 'what does it mean to say that a sequence or a function tends to a limit?'; 'what is the exact definition of a derivative or an integral?'; 'which functions can be differentiated and which can be integrated?'; 'what conditions are needed for a Taylor series to be valid?'.

In later courses on analysis, differentiation and integration of functions of more than one variable are investigated.

In Analysis I, you will encounter the 'ϵ-δ' method of characterising the properties of functions. This is the basis of rigorous thought in this area of mathematics, and will repay you handsomely for all the work you put into understanding it.

Learning outcomes. By the end of this course, you should:

- be able to apply the basic techniques of rigorous analysis and be familiar with examples of 'good behaviour' and 'bad behaviour' in basic analysis;
- know the definition of a limit and be able to establish the convergence or divergence of simple real and complex sequences and series;
- understand the completeness of the real line and be able to derive the basic properties of continuous real-valued functions;
- be able to establish the rules for differentiation, and to prove and apply the mean value theorem;
- be acquainted with complex power series and be able to determine the radius of convergence in simple cases;
- know the definition of the Riemann integral, be able to test simple functions for integrability, and establish the rules for integration.

Differential Equations 24 lectures, Michaelmas Term

The main aim is to develop the skill of representing real (physical or biological) situations by means of differential (or difference) equations. The course follows smoothly from the A-level syllabus, starting with revision of differentiation and integration.

A particularly important sort of differential equation is one which is linear and has constant coefficients. These equations are unusual in that they can be solved exactly (the solutions are exponential or trigonometric functions). Many of the equations of physics are of this sort: the equations governing radioactive decay, Maxwell's equations for electromagnetism and the Schrödinger equation in quantum mechanics, for example.

In other cases, it is useful to try to represent solutions which cannot be obtained explicitly by means of phase-plane diagrams. Sometimes a particular solution describing some important situation is known although the general solution is not. In this case, it is often important to determine whether this solution is typical, or whether a small change in the conditions will lead to a very different solution. In the latter case, the solution is said to be unstable. This property is determined by linearising the original equation to obtain an equation with constant coefficients of the sort discussed above. Sometimes, the solutions are so unstable that they are called *chaotic*.

The very important idea of partial differentiation is also introduced in the course. This is the analogue of familiar differentiation to functions which depend on more than one variable. The approach is mainly geometrical and one of the applications is determining the stationary points of, for example, a function that gives height above sea-level and classifying them into maxima (mountain peaks), minima (valley bottoms) and saddle points (cols or passes).

Learning outcomes. By the end of this course, you should:

- understand the theory of, and be able to solve (in simple cases), linear differential or difference equations, and standard types of non-linear equations;
- calculate partial derivatives and use the chain rule;
- find and classify stationary points of functions of more than one variable;
- be able to investigate the stability of solutions of differential or difference equations.

Probability — 24 lectures, Lent Term

From its origin in games of chance and the analysis of experimental data, probability theory has developed into an area of mathematics with many varied applications in physics, biology and business.

This course introduces the basic ideas of probability and should be accessible to students who have no previous experience of probability or statistics. While developing the underlying theory, the course should strengthen students' general mathematical background and manipulative skills by its use of the axiomatic approach. There are links with other courses, in particular Vectors and Matrices, the elementary combinatorics of Numbers and Sets, the difference equations of Differential Equations and calculus of Vector Calculus and Analysis. Students should be left with a sense of the power of mathematics in relation to a variety of application areas.

After a discussion of basic concepts (including conditional probability, Bayes' formula, the binomial and Poisson distributions, and expectation), the course studies random walks, branching processes, geometric probability, simulation, sampling and the central limit theorem. Random walks can be used, for example, to represent the movement of a molecule of gas or the fluctuations of a share price; branching processes have applications in the modelling of chain reactions and epidemics. Through its treatment of discrete and continuous random variables, the course lays the foundation for the later study of statistical inference.

Learning outcomes. By the end of this course, you should:

- understand the basic concepts of probability theory, including independence, conditional probability, Bayes' formula, expectation, variance and generating functions;
- be familiar with the properties of commonly-used distribution functions for discrete and continuous random variables;
- understand and be able to apply the central limit theorem.
- be able to apply the above theory to 'real world' problems, including random walks and branching processes.

Groups — 24 lectures, Michaelmas Term

In university mathematics, *algebra* is the study of abstract systems of objects whose behaviour is governed by fixed rules or *axioms*. An example is the set of real numbers, governed by the rules of addition and multiplication. One of the simplest forms of abstract algebraic systems is a group, which is roughly a set of objects and a rule for multiplying them together. Groups arise all over mathematics, particularly where there is symmetry.

This course introduces groups and their properties. The emphasis is on both the general theory and a range of examples, such as groups of symmetries and groups of linear transformations.

Learning outcomes. By the end of this course, you should:

- be familiar with elementary properties of abstract groups, including the theory of mappings between groups;
- understand the group-theoretic perspective on symmetries in geometry.

Vector Calculus **24 lectures, Lent Term**

This course is about functions of more than one variable. It is an 'applied' course, meaning that you are expected to be able to apply techniques, but not necessarily to prove rigorously that they work – that will come in future analysis courses.

In the first part of the course, the idea of integration is extended from $\mathbb{R}$ to $\mathbb{R}^2$ and $\mathbb{R}^3$ (with an obvious extension to higher dimensions): integrals along the x-axis are replaced by integrals over curves, surfaces and volumes.

Then the idea of differentiation is extended to vectors (div, grad and curl), which is a basic tool in many areas of theoretical physics (such as electromagnetism and fluid dynamics).

Two important theorems are introduced, namely the divergence theorem and Stokes's theorem; in both cases, an integral over a region (in $\mathbb{R}^3$ and in $\mathbb{R}^2$, respectively) is converted to an integral over the boundary of the region.

All the previous ideas are then applied to Laplace's equation $\nabla^2\phi = 0$ and the related Poisson's equation, which are amongst the most important equations in all of mathematics and physics.

Finally, the notion of a vector is generalised to that of a *tensor*. A vector can be thought of as a 3×1 matrix that carries physical information: namely, magnitude and direction. This information is preserved when the axes are rotated only if the components change according to a certain rule. Very often, it is necessary to describe physical quantities using a 3×3 matrix (or even a $3 \times 3 \times 3 \cdots$ 'matrix'). Such a quantity is called a tensor if its components transform according to a certain rule when the axes are rotated. This rule means that the physical information embodied in the tensor is preserved.

Learning outcomes. By the end of this course, you should:

- be able to manipulate, and solve problems using, vector operators;
- be able to calculate line, surface and volume integrals in $\mathbb{R}^3$, using Stokes theorem and the divergence theorem where appropriate;
- be able to solve Laplace's equation in simple cases, and be able to prove standard uniqueness theorems for Laplace's and related equations.
- understand the notion of a tensor and the general properties of tensors in simple cases.

Numbers and Sets **24 lectures, Michaelmas Term**

This course is concerned not so much with teaching you new parts of mathematics as with explaining how the language of mathematical arguments is used. We will use simple mathematics to develop an understanding of how results are established.

Because you will be exploring a broader and more intricate range of mathematical ideas at university, you will need to develop greater skills in understanding arguments and in formulating your own. These arguments are usually constructed in a careful, logical way as proofs of propositions. We begin with clearly stated and plausible assumptions or *axioms* and then develop a more and more complex theory from them. The course, and the lecturer, will have succeeded if you finish the course able to construct valid arguments of your own and to examine critically those that are presented to you. Example sheets and supervisions will play a key role in achieving this. These skills will form the basis for the later courses, particularly those devoted to Pure Mathematics.

In order to give examples of arguments, we will take two topics: sets and numbers. Set theory provides a basic vocabulary for much of mathematics. We can use it to express in a convenient and precise shorthand the relationships between different objects. Numbers have always been a fascinating and fundamental part of Mathematics. We will use them to provide examples of proofs, algorithms and counter-examples.

Initially we will study the natural numbers $1, 2, 3, \ldots$ and especially *mathematical induction*. Then we expand to consider integers and arithmetic leading to codes like the RSA code used on the internet. Finally we move to rational, real and complex numbers where we lay the logical foundations for analysis. (Analysis is the name given to the study of, for example, the precise meaning of differentiation and integration and the sorts of functions to which these processes can be applied.)

Learning outcomes. By the end of this course, you should:

- understand the need for rigorous proof in mathematics, and be able to apply various different methods, including proof by induction and contradiction, to propositions in set theory and the theory of numbers;
- know the basic properties of the natural numbers, rational numbers and real numbers;
- understand elementary counting arguments and the properties of the binomial coefficients;
- be familiar with elementary number theory and be able to apply your knowledge to the solution of simple problems in modular arithmetic;
- understand the concept of countability and be able to identify typical countable and uncountable sets.

Dynamics and Relativity — 24 lectures, Lent Term

This course assumes knowledge from A-level mechanics (or the equivalent). If you are unsure whether you have the necessary background, then you should attend at least the first lecture of the non-examinable introductory Mechanics course in the Michaelmas Term.

This course is the first look at theoretical physics. The course is important not just for the material is contains; it is also important because it serves as a model for the mathematical treatment of all later courses in theoretical physics.

The first 17 or so lectures are on classical dynamics. The basis of the treatment is the set of laws due to Newton that govern the motion of a particle under the action of forces, and which can be extended to solid bodies. The approach relies heavily on vector methods.

One of the major topics is motion in a gravitational field. This is not only an important application of techniques from this course and the Differential Equations course, it is also of historical interest: it was in order to understand the motions of the planets that Isaac Newton developed calculus.

With the advent of Maxwell's equations in the late nineteenth century came a comfortable feeling that all was well in the world of theoretical physics. This complacency was rudely shaken by Michelson's attempt to measure the velocity of the Earth through the surrounding aether by comparing the speed of light measured in perpendicular directions. The surprising result was that it makes no difference whether one is travelling towards or away from the light source; the velocity of light is always the same. Various physicists suggested a rule of thumb (time dilation and length contraction) which would account for this phenomenon, but it was Einstein who deduced the underlying theory, special relativity, from his considerations of the Maxwell equations.

In this short introduction, the last 7 or so lectures of this course, there is time only to develop the framework in which the theory can be discussed (the amalgamation of space and time into Minkowski space-time) and tackle simple problems involving the kinematics and dynamics of particles.

Learning outcomes. By the end of this course, you should:

- appreciate the axiomatic nature of, and understand the basic concepts of, Newtonian mechanics;
- be able to apply the theory of Newtonian mechanics to simple problems including the motion of particles, systems of particles and rigid bodies, collisions of particles and rotating frames;
- be able to calculate orbits under a central force and investigate their stability;
- be able to tackle problems in rotating frames;
- be able to solve relativistic problems involving space-time kinematics and simple dynamics.

Mechanics (a non-examinable introduction) 10 lectures, Michaelmas Term

This course covers the background material in mechanics required for the Dynamics and Relativity course in the Lent Term (and for later courses in applied mathematics and theoretical physics). It is intended for students who have taken only a limited amount of mechanics at A-level (or the equivalent), e.g. students who have not taken mechanics as part of Further Maths A-level. You should attend at least the first lecture if you are unsure whether you have covered the right material. Each of the lectures will discuss an important topic, such as conservation of momentum or conservation of energy, including worked examples, and each topic will be announced beforehand, so you can decide whether you should attend. The course should not require a significant investment of time.

4 Computational Projects (CATAM)

The lectures for this course should be attended in the Easter Term of the first year.

The Computational Projects course (CATAM) consists mainly of practical projects, with an emphasis on understanding the physical and mathematical problems being modelled rather than on the details of computer programming. Projects must be written up and submitted during the second year (with deadlines just after the start of Lent and Easter Terms) and marks contribute to the total result for the Part IB examination. Lectures are given in the Easter Term of the first year to introduce some of the mathematical and practical aspects of the various projects. This allows an early start to be made on CATAM over the summer, which is strongly recommended. More details are available in the *Part IB Computational Projects Manual*, which is online at `https://www.maths.cam.ac.uk/undergrad/catam/IB`. The supported programming language is MATLAB.

Learning outcomes. By the end of this course, you should:

- be able to programme using a traditional programming language;
- understand the limitations of computers in relation to solving mathematical problems;
- be able to use a computer to solve problems in both pure and applied mathematics involving, for example, solution of ordinary differential equations and manipulation of matrices.

5 Informal Descriptions of Part IB Easter Term Courses

The following courses are lectured in the Easter Term and examined in Part IB.

Optimisation 12 lectures, Easter Term

This course may be taken in the Easter Term of either the first or the second year.

A typical problem in optimisation is to find the cheapest way of supplying a set of supermarkets from a set of warehouses: in more general terms, the problem is to find the minimum (or maximum) value of a quantity when the variables are subject to certain constraints. Many real-world problems are of this type and the theory discussed in the course are practically extremely important as well as being interesting applications of ideas introduced earlier in Numbers and Sets and Vectors and Matrices.

The theory of Lagrange multipliers, linear programming and network analysis is developed. Topics covered include the simplex algorithm, the theory of two-person games and some algorithms particularly well suited to solving the problem of minimising the cost of flow through a network.

Learning outcomes. By the end of this course, you should:

- understand the nature and importance of convex optimisation;

- be able to apply Lagrangian methods to solve problems involving constraints;
- be able to solve problems in linear programming by methods including the simplex algorithm and duality;
- be able to solve network problems by methods using, for example, the Ford–Fulkerson algorithm and min-cut max-flow theorems.

Variational Principles 12 lectures, Easter Term

This course may be taken in the Easter Term of either the first year or the second year; however it contains helpful background material for many of the other applied courses in Part IB.

The techniques developed in this course are of fundamental importance throughout physics and applied mathematics, as well as in many areas of pure and applicable mathematics.

The first part of the course considers stationary points of functions on $\mathbb{R}^n$ and extends the treatment in Part IA Differential Equations to deal with *constraints* using the method of *Lagrange multipliers*. This allows one to determine e.g. the stationary points of a function on a surface in $\mathbb{R}^3$.

The second part of the course deals with *functionals* (and functional derivatives) and enables one to find the path that minimises the distance between two points on a given surface (a *geodesic*), the path of a light ray that gives the shortest travel time (satisfying *Fermat's Principle*), or the minimum energy shape of a soap film.

Many fundamental laws of physics (in Newtonian mechanics, relativity, electromagnetism or quantum mechanics) can be expressed as variational principles in a profoundly elegant and useful way that brings underlying symmetries to the fore.

Learning outcomes. By the end of this course, you should:

- understand the concepts of a functional, and of a functional derivative;
- be able to apply constraints to variational problems;
- appreciate the relationship between variational statements, conservation laws and symmetries in physics.

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MATHEMATICAL TRIPOS

2025-26

GUIDE TO COURSES IN PART IB

This booklet contains informal and non-technical descriptions of courses to be examined in Part IB in the academic year 2025-26, as well as summaries of learning outcomes.

This *Guide to Courses* is intended to supplement the more formal descriptions contained in the booklet *Schedules of Lecture Courses and Form of Examinations*.

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1 Introduction

Each lecture course in the Mathematical Tripos has an official syllabus, or *schedule*, that sets out formally, and in technical terms, the material to be covered. The *schedules* are listed in the booklet *Schedules of Lecture Courses and Form of Examinations* that is available for download at `https://www.maths.cam.ac.uk/undergrad/course/schedules.pdf`. The *Schedules* booklet is the definitive reference for matters of course content and assessment, for students, lecturers and examiners.

The present guide, by contrast, provides an *informal* description of each lecture course in Part IB. These descriptions are intended to be comprehensible without much prior knowledge, and to convey something of the flavour of each course. Summaries of the learning outcomes for each course are also included, along with some suggestions for preparatory reading, if appropriate.

The full learning outcome for Part IB is that you should understand the material described in the formal syllabuses given in the *Schedules* booklet and be able to apply it to the sorts of problems that can be found on Tripos papers from earlier years.

Changes to lecture courses since last year

Two new courses, IB Analysis II and IB Topological Spaces, will run in 2025-26, replacing the courses IB Analysis & Topology and IB Geometry from 2024-25.

2 The Structure of Part IB

The structure of Part IB may be summarised as follows:

- There are four courses of 24 lectures, eight courses of 16 lectures, three courses of 12 lectures, and an additional Computational Projects course (CATAM).
- Five courses are lectured in Michaelmas Term, building on the core material in Part IA, while eight courses are lectured in Lent Term, allowing more specialisation in preparation for Part II.
- Two of the 12-lecture courses are given in Easter Term and may be taken in either the first or second year (*Optimisation* and *Variational Principles*).
- The examination consists of four papers, with Section I ('short') questions and Section II ('long') questions spread as evenly as possible subject to
  - each 24-lecture course having two short questions and four long questions;
  - each 16-lecture course having two short questions and three long questions;
  - each 12-lecture course having two short questions and two long questions;
  - each course having at most one question of each type (long or short) but at least one question of either type on each paper.

  The precise distribution of questions can be found in the *Schedules* booklet.
- Only four short questions and six long questions may be attempted on each paper, with
  - each short (Section I) question marked out of 10 with one beta quality mark;
  - each long (Section II) question marked out of 20 with one quality mark, alpha or beta.
- The Computational Projects course carries 160 marks and no quality marks.

3 Choice of Courses

The Faculty Board has issued the following guidance:

> *Part IB of the Mathematical Tripos provides a wide range of courses from which students should, in consultation with their Directors of Studies, make a selection based on their individual interests and preferred workload, bearing in mind that it is better to do a smaller number of courses thoroughly than to do many courses scrappily.*

So, you are certainly not expected to take all the courses in Part IB, and the informal course descriptions below are intended to help you start thinking about your choices. It is important to choose courses that you will find rewarding, and to be aware of the consequences of your choices for options in Part II; to this end the *Schedules* booklet contains a table summarising the relationships between courses in Part II and those in Part IB.

In Part IA you were expected to follow four 24-lecture courses each term, i.e. to attend two lectures per day for two terms (total 192 lectures). If you were comfortable with that, then this might be a sensible target for Part IB. Some students prefer to take slightly fewer courses and learn them more thoroughly, while other students may choose to take more. You should check the distribution of questions on the four examination papers (given in the *Schedules* booklet) before making your final choices for revision.

4 CATAM and Preparatory Work

It is important to start work as early as possible on the computational projects (CATAM). You are strongly encouraged to complete the non-examinable Introductory Project 0.1 (see the CATAM manual) over the summer; this will give you valuable practice in programming as well as in producing a coherent write-up. A model answer for this project will be available in Michaelmas Term and comparison of this with your own write-up (before having to submit write-ups for real marks) should be instructive.

You are warned that project work can take much longer than you first expect, and rushing to complete things at the last moment is not a recipe for securing good marks, as well as being a major distraction from your other work, so it is good to get ahead. If you don't know how to program in Matlab, then you should try to crack this as soon as possible.

Any mathematics that you do over the summer vacation will stand you in good stead for Part IB. It is suggested, first, that if your College expects you to have supervisions in Michaelmas Term on either of the IB Easter Term courses then make sure you are up-to-speed and ready for them, and secondly, that you might wish to do some preparatory reading for one of the Michaelmas Term courses e.g. Analysis & Topology or Quantum Mechanics. The books suggested below are intended to give an idea of the appropriate level and approach for each course. They should all be in your college library, and by browsing there you may find other sources which are just as helpful. More comprehensive reading lists are also given in the *Schedules* booklet.

5 Informal Description of Courses

Linear Algebra — Michaelmas, 24 lectures

The first-year course Vectors and Matrices includes a concrete introduction to vector spaces. Here, vector spaces are investigated from an abstract axiomatic point of view. This has two purposes: firstly to provide an introduction to abstract algebra in an already familiar context and secondly to provide a foundation for the study of infinite-dimensional vector spaces which are required for advanced courses in analysis and physics. One important application is to function spaces and differential and difference operators. A striking result is the Cayley-Hamilton theorem which says (roughly) that any square matrix satisfies the same equation as its eigenvalues (the characteristic equation).

The spaces studied for the first part of the course have nothing corresponding to length or angle. These are introduced by defining an inner product (i.e. a 'dot' product) on the vector space. This is generalised to the notion of a bilinear form ('lengths' do not have to be positive) and even further. There are direct applications to quantum mechanics and statistics.

The last part of the course covers the theory of bilinear and hermitian forms, and inner products on vector spaces. An important example is the quadratic form. The discussion of orthogonality of eigenvectors and properties of eigenvalues of Hermitian matrices has consequences in many areas of mathematics and physics, including quantum mechanics.

There are many suitable books on linear algebra: for example *Finite-dimensional Vector Spaces* by Halmos (Springer, 1974), Birkhoff and MacLane's *Algebra* (Macmillan, 1979) and Strang's *Linear Algebra*

(Academic Press, 1980).

Learning outcomes. By the end of this course, you should:

- understand the concepts of, and be able to prove results in the theory of, real and complex vector spaces;
- understand the concepts of, and be able to prove results in the theory of, linear maps between and endomorphisms of real and complex vector spaces, including the role of eigenvectors and eigenvalues and Jordan canonical form;
- understand, and be able to prove and apply, the Cayley-Hamilton theorem;
- understand, and be able to prove results in the theory of, dual vector spaces;
- understand bilinear forms and their connection with the dual space, and be able to derive their basic properties;
- know the theory of canonical forms for symmetric, alternating and hermitian forms, and be able to find them in simple cases;
- understand the theory of hermitian endomorphisms of a complex inner product space, and know and be able to apply the Gram-Schmidt orthogonalisation process.

Groups, Rings and Modules — Lent, 24 lectures

This course unites a number of useful and important algebraic and geometric ideas by developing three concepts which are fundamental in abstract algebra. Firstly there is the notion of a *group* which you met in Part IA Groups and which is found in so much of mathematics, both pure and applied. The basic concepts of group theory are recalled from the first year and then built upon, resulting in beautiful theorems that reveal much about the structure of finite groups.

Whereas a group has only one operation, a *ring* is a set that is equipped with two operations: that of addition and multiplication, such as the integers. The next third of the course develops this idea in a way that mirrors the approach to groups, as well as considering examples such as fields and the important case of a ring of polynomials in one, and in many, variables.

The last part of the course defines and deals with the notion of a *module*, which can be described as the immediate generalisation of a vector space where the scalars form a ring rather than a field. The advantage of this approach is that it allows proof of general results which can then be used to unify theorems in specific cases, as shown at the end of the course where applications to Jordan Normal Form are given, along with a proof of the classification of finitely generated abelian groups.

For an introduction to groups, J. F. Humphreys, *A course in group theory* (Oxford Science Publications) amongst others is very readable whereas B. Hartley and T. O. Hawkes, *Rings, Modules and Linear Algebra* (Chapman and Hall), although somewhat dry, contains nearly all of the rings part of the course and more than all of the material on modules.

The course also lays the foundations for most of the algebra options in Part II. In particular it is essential for Galois Theory, and highly desirable for areas such as Number Fields and Representation Theory.

Learning outcomes. By the end of this course, you should:

- have a firm understanding of the fundamental concepts of group theory and be comfortable applying these to groups of small order;
- know the definition of a ring, a field and an ideal, and be able to determine whether an ideal is principal, maximal or prime;
- be able to factorise elements in specific rings, including cases where factorisation is non-unique;
- understand the concept of a module and its application to finitely generated abelian groups.

Analysis II — Michaelmas, 24 lectures

In the Analysis I course in Part IA, you encountered for the first time the rigorous mathematical study of the concepts of limit, continuity, differentiability and integrability, applied to functions of a single real variable. This course extends that study in two different ways. First, it introduces the important notion of uniform convergence, which helps to answer the basic question of when we can ensure continuity, differentiability and integrability are preserved when passing to limits of sequences of functions. Then the fundamental ideas of Analysis I are extended from the real line $\mathbb{R}$, first to finite-dimensional Euclidean spaces $\mathbb{R}^n$ and then to still more general 'metric spaces' whose 'points' may be objects such as functions or sets.

The advantages of this more general point of view are demonstrated using Banach's contraction mapping theorem, whose striking applications include a general existence and uniqueness theorem for solutions of differential equations, and the inverse function theorem, a result of fundamental importance.

If you wish to do some vacation reading, W.A. Sutherland's Introduction to Metric and Topological Spaces (OUP, 1975) provides a good introduction to analysis on more general spaces.

Learning outcomes. By the end of this course, you should:

- understand and be able to prove basic results about uniform convergence and continuous functions in $\mathbb{R}^n$;
- understand and be able to prove basic results about differentiability of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, be able to calculate derivatives and use them to analyse the behaviour of functions;
- understand the notion of uniform continuity of functions and appreciate its significance in the theory of Riemann integration of a function of a single real variable;
- understand the basic theory of metric spaces and continuous functions on metric spaces;
- understand the statement and proof of the contraction mapping theorem and its applications to the solution of differential equations and to the inverse function theorem.

Topological Spaces — Lent, 16 lectures

Topology is the branch of mathematics that is concerned with the properties of a geometric object that are preserved under continuous deformation. In other words, it is the study of shapes defined by the continuous maps that you learned about in Analysis I and II. The first theorems of topology were proved by Euler in the 18th century, but the subject wasn't properly formalised until the beginning of the 20th century. Since then, it has emerged to play a central role in large parts of modern mathematics and physics.

This course serves as a first introduction to topology, building on the metric spaces that you studied in Analysis II. After the definition of a topological space, the key topological ideas of connectedness and compactness are introduced and their applications explained. Fresh views emerge of important results from Analysis I.

The last part of the course moves on to discuss the topology of manifolds, providing a foundation for some of the more advanced topics covered in Part II of the tripos. Plenty of examples are discussed, and some basic properties are proved. The course finishes with a detailed discussion of 2-dimensional manifolds, i.e. surfaces. You will learn how to classify them via Euler characteristic.

Munkres' book Topology (Prentice Hall, 2000) and Lee's Topological manifolds (Springer, 2000) are good sources for this course.

Learning outcomes. By the end of this course, you should:

- appreciate the definition of topological spaces and be able to distinguish between standard topological and non-topological properties;
- understand the topological notion of connectedness and its relation to path-connectedness;

- understand the topological notion of compactness, know its significance in basic analysis and be able to apply it to identify standard quotients of topological spaces;
- understand the notion of a topological manifold, and recognise several examples;
- be familiar with the classification of surfaces, including the concepts of orientability and Euler characteristic.

Variational Principles — Easter, 12 lectures

The techniques developed in this course are of fundamental importance throughout physics and applied mathematics, as well as in many areas of pure and applicable mathematics.

The first part of the course considers stationary points of functions on $\mathbb{R}^n$ and extends the treatment in Part IA Differential Equations to deal with *constraints* using the method of *Lagrange multipliers*; e.g. this allows one to determine the stationary points of a function on a surface in $\mathbb{R}^3$.

The second part of the course deals with *functionals* (and functional derivatives) and enables one to find the path that minimises the distance between two points on a given surface (a *geodesic*), the path of a light ray that gives the shortest travel time (satisfying *Fermat's Principle*), or the minimum energy shape of a soap film.

Many fundamental laws of physics (in Newtonian mechanics, relativity, electromagnetism or quantum mechanics) can be expressed as variational principles in a profoundly elegant and useful way that brings underlying symmetries to the fore.

Learning outcomes. By the end of this course, you should:

- understand the concepts of a functional, and of a functional derivative;
- be able to apply constraints to variational problems;
- appreciate the relationship between variational statements, conservation laws and symmetries in physics.

Methods — Michaelmas, 24 lectures

This course continues the development of mathematical methods which can be applied to physical systems. The material is fundamental to nearly all areas of applied mathematics and theoretical physics.

The course introduces the important class of ordinary differential equations that are self-adjoint. The equivalent in the complex domain, used in Quantum Mechanics, are Hermitian operators. Self-adjoint equations have nice properties such as having real eigenvalues and orthogonal eigenfunctions, which allow eigenfunction expansions, the prototype being Fourier series. Fourier series generalise, for non-periodic functions, to Fourier transforms which provide a useful way of solving linear differential ordinary and partial differential equations.

Much of the remainder of the course concentrates on second-order partial differential equations: classification into wave, diffusion and Laplace type equations; the fundamental solutions of the three different types; solution by separation of variable which ties in with the earlier work on self-adjoint equations.

The course also introduces the famous Dirac δ, or spike, function and the Green's function which can be regarded as the inverse operator for a differential equation: it is used to express the solution in terms of an integral. It will reappear as a basic tool in quantum field theory.

It is worthwhile to get to grips early with the major new ideas introduced here: Fourier series/transforms and Sturm-Liouville equations. Reasonably friendly accounts can be found in *Mathematical Methods in the Physical Sciences* by Boas (Wiley, 1983), *Mathematical Methods for Physicists* by Arfken (Academic Press, 1985) and *Mathematical Methods for Physicists and Engineers* by Riley, Hobson and Bence (CUP, 98). It is also worthwhile to revise thoroughly the Variational Principles course from the Easter term.

Learning outcomes. By the end of this course, you should:

- be able to apply the theory of Green's functions to ordinary differential equations;
- understand the basic properties of Sturm-Liouville equations;
- be able to apply the method of separation of variables to partial differential equations;
- be able to use standard methods to solve partial differential equations.
- be able to solve wave problems using Fourier analysis and advanced/retarded coordinates.

Complex Methods — Lent, 16 lectures

Complex variable theory was introduced briefly in Analysis I (for example, complex power series). Here, the subject is developed without the full machinery of a pure analysis course. Rigorous justification of the results used is given in the parallel course, Complex Analysis.

The course starts with the definition of analyticity and the Cauchy Riemann equations (which must be satisfied by the real and imaginary parts of a complex function in order for it to be analytic; i.e. in order for it to be expressible as a power series). There follows a brief discussion of conformal mapping with applications to Laplace's equation. Then a heuristic version of Cauchy's theorem leads, via Cauchy's integral formula, to the residue calculus. This is a remarkable technique for evaluating integrals in the complex plane, which can also be used to calculate definite integrals on the real line. It allows the calculation of integrals which one would not have a hope of calculating by other means, as well as remarkably simple and elegant derivations of standard results such as

$$\int_{-\infty}^{\infty} \exp(-x^2/2 + ikx)\, dx = \sqrt{(2\pi)}\, \exp(-k^2/2) \qquad \text{and} \qquad \int_0^{\infty} (\sin x)/x\, dx = \pi/2\,.$$

An important application is to the theory of Fourier transforms (which were introduced in the Methods course) and Laplace transforms. The transforms are used to represent, for example, a time dependent signal as a sum (in fact, an integral unless the function is periodic) over its frequency components. This is important because one often knows how a system responds to pure frequency signals rather than to an arbitrary input. In many situations, the use of a transform simplifies a physical problem by reducing a partial differential equation to an ordinary differential equation. This is a particularly important technique for numerous branches of physics, including acoustics, optics and quantum mechanics.

For a fairly applied approach, look at chapters 6 and 7 of *Mathematical Methods for Physicists* by Arfken (Academic Press, 1985). This material is also sympathetically dealt with in *Mathematical Methods in the Physical Sciences* by Boas (Wiley, 1983).

Learning outcomes. By the end of this course, you should:

- understand the concept of analyticity;
- be able to use conformal mappings to find solutions of Laplace's equations;
- be able to use the theory of contour integration, including the residue theorem, to evaluate integrals;
- understand the theory of Fourier and Laplace transforms and apply it to the solution of ordinary and partial differential equations.

Complex Analysis — Lent, 16 lectures

This course covers about 2/3 of the material in Complex Methods, from a more rigorous point of view. The main omissions are applications of conformal mappings to solutions of Laplace's equations and the theory of Fourier and Laplace transforms.

The theory of complex variable is exceptionally elegant. It is used in many branches of pure mathematics, including number theory. It also forms one of the guiding models for the modern development of geometry.

A rigorous course not only provides a firm foundation for, and makes clear the underlying structure of, this material but also allows a deeper appreciation of the links with material in other analysis courses — in particular, Metric & Topological Spaces.

An excellent book both for the course and for preliminary reading is Hilary Priestley's *Introduction to Complex Analysis* (OUP, paperback). The books by Stewart and Tall (*Complex Analysis*) and by Jameson (*A First Course in Complex Functions*) are also good.

Learning outcomes. By the end of this course, you should:

- understand the concept of analyticity;
- be able to prove rigorously the main theorems in the course;
- be able to use the theory of contour integration, including the residue theorem, to evaluate integrals.

Quantum Mechanics — Michaelmas, 16 lectures

Quantum mechanics introduces a profound different way of thinking about the physical world, formulated using precise mathematical language. It explains phenomena beyond the reach of classical physics, such as the duality of particles and waves, and the structure and behaviour of atoms, but quantum mechanics is also at work all around us in our daily uses of modern technology.

This course introduces the subject from scratch and deals mainly with the quantum mechanics of a single particle, as described by a complex-valued *wavefunction* obeying the *Schrödinger equation*. For a quantum particle there is no definite trajectory (as determined classically from Newton's Laws) and information about position and momentum must instead be extracted from the wavefunction in terms of probabilities. One consequence of this is the *Heisenberg uncertainty principle*.

The Schrödinger equation is first studied in simple but instructive cases in one dimension, before moving on to three dimensions, culminating in the solution of the Hydrogen atom. The underlying mathematics involves hermitian or self-adjoint (differential) operators whose eigenvalues give the possible outcomes of a physical measurement. Consequently, there are significant overlaps with material in Part IB Methods (and Part IA Vectors and Matrices or equivalently Part IB Linear Algebra), although the treatment in this course is essentially self-contained.

Standard introductory textbooks are *Essential Quantum Physics* by Landshoff, Metherell and Rees (CUP, 2010) and *Quantum Mechanics* by Rae (IOP Publishing, 2002), while *The Quantum Universe* by Hey and Walters (CUP, 1987) contains readable and non-mathematical accounts with lots of pictures, going well beyond the Part IB course.

Learning outcomes. By the end of this course, you should:

- understand the basic theory of quantum mechanics, including: wavefunctions, the Schrödinger equation, observables and operators—measurements, eigenvalues, and expectation values;
- be able to solve, and interpret the solution of, the Schrödinger equation in simple cases, including: 1-dimensional potential wells and steps; the harmonic oscillator; and the hydrogen atom.

Electromagnetism — Lent, 16 lectures

Maxwell's equations of electromagnetism are among the great triumphs of nineteenth century physics. These equations unify the electric and magnetic forces and provide an explanation for many natural phenomena, including the existence of light itself. The equations also hold the seed of the theory of special relativity. This course gives the first opportunity in the Tripos to study a modern physical field theory.

After a brief discussion of electric and magnetic forces, Maxwell's equations are introduced. A key idea is the use of potentials to represent the electric and magnetic fields and it is shown how Maxwell's equations imply the existence of such potential functions. The equations are solved in special cases of physical interest. First, time-independent situations are covered: for example, point charges, bar magnets, currents in wires. Next, time-varying situations are investigated: for example, induction. It is also shown how Maxwell's equations have wave-like solutions which we identify as light. The course ends with a discussion of special relativity in the context of electromagnetism. When viewed through the lens of relativity, the Maxwell equations become remarkably simple.

The course relies heavily on vector calculus. The latter part of the course also uses the theory of tensors from Part IA Vector Calculus and special relativity from Part IA Dynamics and Relativity. Electromagnetism is important for all of the theoretical physics courses in Part II, and is particularly relevant to General Relativity through its use of 4-vectors and tensors.

Learning outcomes. By the end of this course, you should:

- understand the physical significance of and be able to manipulate Maxwell's equations (including deriving the integral forms);
- solve simple problems in electrostatics including calculation of electrostatic energy, capacity and force;
- derive, and apply to simple situations, the Biot-Savart law;
- use Gauss's law and Ampère's law to calculate electric and magnetic fields in symmetrical situations;
- calculate forces using the Lorentz force;
- derive and apply Faraday's law of induction to simple circuits;
- solve Maxwell's equations to obtain plane waves.

Fluid Dynamics — Lent, 16 lectures

Fluid dynamics investigates the motion of liquids and gases, such as the motion that enables aircraft to fly. Newton's laws of motion apply – acceleration equals force per unit mass – but a subtlety arises because acceleration means the rate of change of velocity of a fluid particle. It does not mean the rate of change of the fluid velocity at a fixed point in space. A special mathematical operator, the material derivative, expresses the required rate of change using vector calculus. The forces entering Newton's laws can be external, such as gravity, or internal, arising from pressure or from viscosity (internal friction). When the viscosity is small enough to be negligible, the motion is often irrotational as well as incompressible: both the curl and divergence of velocity field vanish. In this situation, the fluid velocity can be described by a potential, and standard potential theory applies, including in some cases solutions of Laplace's equation.

The topics studied include jets, bubbles, waves, vortices, flow around aircraft wings, and flow in weather systems. Suitable introductory reading material can be found in Worster's *Understanding Fluid Flow* (CUP) or in Acheson's *Elementary Fluid Dynamics* (Oxford). For background motivation, see also the visionary discussion in the Feynman Lectures on Physics, last two chapters of Volume II (Addison-Wesley).

Learning outcomes. By the end of this course, you should:

- understand the basic principles governing the dynamics of parallel viscous flows and flows in which viscosity in negligible;
- be able to derive and deduce the consequences of the equation of conservation of mass;
- be able solve kinematic problems such as finding particle paths and streamlines;
- be able to apply Bernoulli's theorem and the momentum integral to simple problems including river flows;
- understand the concept of vorticity and the conditions in which it may be assumed to be zero;

- calculate velocity fields and forces on bodies for simple steady and unsteady flows derived from potentials;
- understand the theory of interfacial waves and be able to use it to investigate, for example, standing waves in a container;
- understand fundamental ideas relating to flows in rotating frames of reference, particularly geostrophy.

Numerical Analysis — Lent, 16 lectures

An important aspect of the application of mathematics to problems in the real world is the ability to compute answers as accurately as possible subject to the errors inherent in the data presented and the limits on the accuracy of calculation. Numerical analysis is the branch of mathematics studying such computations.

The course commences from approximation theory, focusing on the approximation of functions and data by polynomials, continues with the numerical solution of ordinary differential equations and concludes with the solution of linear algebraic systems. Although computational algorithms form a central part of the course, so do mathematical theories underlying them and investigating their behaviour: computation and approximation at their best should be done with proper mathematical justification.

An Introduction to Numerical Analysis by Suli & Mayers (CUP, 2003) and *Interpolation and Approximation* by Davis (Dover, 1975) are two excellent introductory texts.

Learning outcomes. By the end of this course, you should:

- understand the role of algorithms in numerical analysis;
- understand the role and basic theory (including orthogonal polynomials and the Peano kernel theorem) of polynomial approximation;
- understand multistep and Runge–Kutta methods for ordinary differential equations and the concepts of convergence, order and stability;
- understand the theory of algorithms such as LU and QR factorisation, and be able to apply them, for example to least squares calculations.

Statistics — Lent, 16 lectures

Statistics is the study of what can be learnt from data. We regard our data as realisations of random variables, and consider models for the (joint) distribution of these random variables. In this course, we focus entirely on *parametric* models, where the class of distributions considered can be indexed by a finite-dimensional parameter. As a simple example, the family of normal distributions can be indexed by a two-dimensional parameter, representing the mean and variance. Nonparametric models are treated in more advanced courses.

Our aim is to make inference about the unknown parameter by, for example, providing a point estimate, a confidence interval or conducting a hypothesis test. Building on Part IA Probability, this course will present basic techniques of inference, together with their theoretical justification. The final chapter will cover the ubiquitous *linear model*, with its elegant theory of orthogonal projection and application of results from linear algebra.

The most appropriate book for the course is *Statistical inference* by Casella and Berger (Duxbury, 2001).

Learning outcomes. By the end of this course, you should:

- understand the basic concepts involved in point estimation, the construction of confidence intervals and Bayesian inference;

- understand and be able to apply the ideas of hypothesis testing, including the Neyman–Pearson lemma, and generalised likelihood ratio tests, including applications to goodness of fit tests and contingency tables.
- understand and be able to apply the theory of the linear model, including examples of linear regression and one-way analysis of variance.

Markov Chains — Michaelmas, 12 lectures

A Markov process is a random process for which the future (the next step) depends only on the present state; it has no memory of how the present state was reached. A typical example is a random walk (in two dimensions, the drunkard's walk).

The course is concerned with Markov chains in discrete time, including periodicity and recurrence. For example, a random walk on a lattice of integers returns to the initial position with probability one in one or two dimensions, but in three or more dimensions the probability of recurrence in zero. Some Markov chains settle down to an equilibrium state and these are the next topic in the course.

The material in this course will be essential if you plan to take any of the applicable courses in Part II. Further introductory material and notes on the course are available from links on the Study pages on the DPMMS website.

Learning outcomes. By the end of this course, you should:

- understand the notion of a discrete-time Markov chain and be familiar with both the finite state-space case and some simple infinite state-space cases, such as random walks and birth-and-death chains;
- know how to compute for simple examples the n-step transition probabilities, hitting probabilities, expected hitting times and invariant distribution;
- understand the notions of recurrence and transience, and the stronger notion of positive recurrence;
- understand the notion of time-reversibility and the role of the detailed balance equations;
- know under what conditions a Markov chain will converge to equilibrium in long time;
- be able to calculate the long-run proportion of time spent in a given state.

Optimisation — Easter, 12 lectures

A typical problem in optimisation is to find the cheapest way of supplying a set of supermarkets from a set of warehouses: in more general terms, the problem is to find the minimum (or maximum) value of a quantity when the variables are subject to certain constraints. Many real-world problems are of this type and the theory discussed in the course are practically extremely important as well as being interesting applications of ideas introduced earlier in Numbers and Sets and in Vectors and Matrices.

The theory of Lagrange multipliers, linear programming and network analysis is developed. Topics covered include the simplex algorithm, the theory of two-person games and some algorithms particularly well suited to solving the problem of minimising the cost of flow through a network.

Whittle's *Optimisation under Constraints* (Wiley, 1971) gives a good idea of the scope and range of the subject but is a little advanced mathematically; Luenberger's *Introduction to Linear and and Non-linear Programming* (Addison-Wesley, 1973) is at the right level but provides less motivation.

Learning outcomes. By the end of this course, you should:

- understand the nature and importance of convex optimisation;
- be able to apply Lagrangian methods to solve problems involving constraints;

- be able to solve problems in linear programming by methods including the simplex algorithm and duality;
- be able to solve network problems by methods using, for example, the Ford–Fulkerson algorithm and min-cut max-flow theorems.

Computational Projects (CATAM)

This course consists mainly of practical computational projects carried out and written up for submission a week after the beginning of the Lent and Easter terms. For full credit, you do four projects. The first two are prescribed and are submitted soon after the beginning of the Lent term. The remaining two are chosen from a list of projects and are submitted soon after the beginning of the Easter term. There is also a non-examinable project that allows you to practice programming and writing up results, with a model answer provided in Michaelmas Term for comparison with your own answer.

The emphasis in the projects is on understanding the mathematical problems being modelled rather than on the details of computer programming. Some students find the projects somewhat time consuming, especially those who are not used to programming or have not completed the non-examinable project. The CATAM manual will be available over the summer and it would be extremely helpful for you to start as early as possible on the non-examinable project and, if time, the first two examinable projects.

The amount of credit available for the Computational Projects course in Part IB is 160 marks (and no quality marks), which is additional to the marks gained on examination papers. In recent years approximately 99% of Part IB students submitted projects (not necessarily complete).

Learning outcomes. By the end of this course, you should:

- be able to programme using a traditional programming language;
- understand the limitations of computers in relation to solving mathematical problems;
- be able to use a computer to solve problems in both pure and applied mathematics involving, for example, solution of ordinary differential equations and manipulation of matrices.

UNIVERSITY OF CAMBRIDGE
Faculty of Mathematics

MATHEMATICAL TRIPOS

2025-26

GUIDE TO COURSES IN PART II

This booklet contains informal and non-technical descriptions of courses to be examined in Part II in the academic year 2025-26, as well as a summary of the overall structure of Part II.

This *Guide to Courses* is intended to supplement the more formal descriptions contained in the booklet *Schedules of Lecture Courses and Form of Examinations*.

These and other Faculty documents for students taking the Mathematical Tripos are available from the undergraduate pages on the Faculty's website at `https://www.maths.cam.ac.uk/undergrad/`

Revised 15 September 2025

1 Introduction

Each lecture course in the Mathematical Tripos has an official syllabus, or *schedule*, that sets out formally, and in technical terms, the material to be covered. The *schedules* are listed in the booklet *Schedules of Lecture Courses and Form of Examinations* that is available for download at `https://www.maths.cam.ac.uk/undergrad/course/schedules.pdf`. The *Schedules* booklet is the definitive reference for matters of course content and assessment, for students, lecturers and examiners.

The present guide, by contrast, provides an *informal* description of each lecture course in Part II. These descriptions are intended to be comprehensible without much prior knowledge, and to convey something of the flavour of each course, and to suggest some preparatory reading, if appropriate.

A summary of the overall structure of Part II is also given below, including the distribution of questions on examination papers.

Changes to lecture courses since last year

The wording of the schedules of IIC Statistical Modelling, IIC Coding & Cryptography and IID Principles of Statistics have changed for 2025-26.

2 Structure of Part II

The structure of Part II may be summarised as follows:

- There are two types of lecture courses, labelled C and D. C-courses are all 24 lectures, D-courses may be 16 or 24 lectures. This year there are 10 C-courses and 27 D-courses. There is in addition a Computational Projects course (CATAM).

- C-courses are intended to be straightforward, whereas D-courses are intended to be more challenging.

- There is no restriction on the number or type of courses you may present for examination.

- The examination consists of four papers, with questions on the courses spread as evenly as possible over the four papers subject to:
  - each C-course having four Section I ('short') questions and two Section II ('long') questions;
  - each 24-lecture D-course having no Section I questions and four Section II questions;
  - each 16-lecture D-course having no Section I questions and three Section II questions.

- Only six questions from Section I may be attempted on each paper.

- Each Section I question is marked out of 10 with one beta quality mark, while each Section II question is marked out of 20 with one quality mark, alpha or beta. Thus each C-course and 24-lecture D-course carries 80 marks and a number of quality marks, while each 16-lecture D-course carries 60 marks and a number of quality marks. The Computational Projects course carries 150 marks and no quality marks.

3 Distribution of Questions on the Examination Papers

The distribution of Section II ('long') questions on the examination papers is as follows:

C-Courses	Paper 1	Paper 2	Paper 3	Paper 4
Number Theory			*	*
Topics in Analysis		*		*
Coding and Cryptography	*	*		
Automata and Formal Languages	*		*	
Statistical Modelling	*			*
Mathematical Biology			*	*
Further Complex Methods	*	*		
Classical Dynamics		*		*
Cosmology	*		*	
Quantum Information and Computation		*	*	

D-Courses	Paper 1	Paper 2	Paper 3	Paper 4
Logic and Set Theory	*	*	*	*
Graph Theory	*	*	*	*
Galois Theory	*	*	*	*
Representation Theory	*	*	*	*
Number Fields	*	*		*
Algebraic Topology	*	*	*	*
Linear Analysis	*	*	*	*
Analysis of Functions	*	*	*	*
Riemann Surfaces	*	*	*	
Algebraic Geometry	*	*	*	*
Differential Geometry	*	*	*	*
Probability and Measure	*	*	*	*
Applied Probability	*	*	*	*
Principles of Statistics	*	*	*	*
Stochastic Financial Models	*	*	*	*
Mathematics of Machine Learning	*	*		*
Asymptotic Methods		*	*	*
Dynamical Systems	*	*	*	*
Integrable Systems	*	*	*	
Principles of Quantum Mechanics	*	*	*	*
Applications of Quantum Mechanics	*	*	*	*
Statistical Physics	*	*	*	*
Electrodynamics	*		*	*
General Relativity	*	*	*	*
Waves	*	*	*	*
Fluid Dynamics	*	*	*	*
Numerical Analysis	*	*	*	*

4 Informal Description of Courses

C-Courses

Number Theory — Michaelmas, 24 lectures

Number Theory is one of the oldest subjects in mathematics and contains some of the most beautiful results. This course introduces some of these beautiful results, such as a proof of Gauss's Law of Quadratic Reciprocity, and a proof that continued fractions give rise to excellent approximations by rational numbers. The new RSA public codes familiar from Part IA Numbers and Sets have created new interest in the subject of factorisation and primality testing. This course contains results old and new on the problems.

On the whole, the methods used are developed from scratch. You can get a better idea of the flavour of the course by browsing Davenport *The Higher Arithmetic* CUP, Hardy and Wright *An introduction to the theory of numbers* (OUP, 1979) or the excellent *Elementary Number Theory* by G A and J M Jones. (Springer 1998).

Automata and Formal Languages — Michaelmas, 24 lectures

The lecture course deals with three basic ideas: the ideas of computability and decidability and how what is computable or decidable depends on the model of computation. The notion of computability is one of the fundamental concepts of modern science, independent of the concrete technology that the current generation of computers uses. It is directly related to Turing's famous limitative theorem that established that the power of computation based on any given model of computation is limited.

In the lecture course, we discuss automata and register machines as models of computation, giving rise to three different levels of computational power:

1. regular languages, recognised by finite automata,
2. context-free languages, recognised by push-down automata,
3. computably enumerable languages, recognised by register machines.

Prerequisites are in IA Numbers and Sets, including the notions of set, function, relation, product, partial order, and equivalence relation.

Coding and Cryptography — Lent, 24 lectures

When we transmit any sort of message errors will occur. Coding theory provides mathematical techniques for ensuring that the message can still be read correctly. Since World War II it has been realised that the theory is closely linked to cryptography – that is to techniques intended to keep messages secret. This course will be a gently paced introduction to these two commercially important subjects concentrating mainly on coding theory.

Discrete probability theory enters the course as a way of modelling both message sources and (noisy) communication channels. It is also used to prove the existence of good codes. In contrast the construction of explicit codes and cryptosystems relies on techniques from algebra. Some of the algebra should already be familiar – Euclid's Algorithm, modular arithmetic, polynomials and so on – but there are no essential prerequisites. IB Linear Algebra would be useful. IB Groups, Rings and Modules is very useful.

The book by Welsh recommended in the schedules (*Codes and Cryptography*, OUP), although it contains more than is in the course, is a good read.

Topics in Analysis — Lent, 24 lectures

Some students find the basic courses in Analysis in the first two years difficult and unattractive. This is a pity because there are some delightful ideas and beautiful results to be found in relatively elementary Analysis. This course represents an opportunity to learn about some of these. There are no formal prerequisites: concepts from earlier courses will be explained again in detail when and where they are needed. Those who have not hitherto enjoyed Analysis should find this course an agreeable revelation.

Statistical Modelling — Michaelmas, 24 lectures

This course is complementary to Part II Principles of Statistics, but takes a more applied perspective. By embedding statistical theory with real datasets and practical problems, students will learn to think like a statistician and gain confidence in solving real world problems with statistics.

There will be approximately 16 hours of lectures and eight hours of practical classes. The bulk of the lectures will focus on classical statistical models including linear and generalised linear models, which provide a powerful and flexible framework for the study of the relationship between a response (e.g. alcohol consumption) and one or more explanatory variables (age, sex etc.). We will also spend a few lectures on alternative perspectives of statistical modelling that focus on predictive performance or causality. In the practical classes, we will learn how to implement the techniques and ideas covered in the lectures by analysing several real data sets. We will be making extensive use of the statistical computer programming language `R`, which can be downloaded free of charge and for a variety of platforms from `https://cran.r-project.org/`. Most students will find it useful to write and execute their code in `RStudio`, an integrative development environment for `R` that can be downloaded from `https://posit.co/products/open-source/rstudio`.

This course should appeal to a broad range of students, including those considering further research in any aspect of Statistics and those considering careers in data-intensive industries (finance, manufacturing, healthcare, etc.). Those interested might like to try downloading `R` and experimenting with one of the excellent tutorials in

- `https://cran.r-project.org/doc/manuals/r-release/R-intro.html` (maintained by `R` Core Team)
- `https://rforcats.net/`
- `https://www.dpmms.cam.ac.uk/~pmea/`

Cosmology — Michaelmas, 24 lectures

This course presents a mathematically rigorous description of 13.8 billion years of history, from the Big Bang to the present day and beyond. The course starts by deriving the equations which describe an expanding universe. The need to include a number of surprising and mysterious ingredients, such as dark energy, dark matter, and inflation, will be discussed. Subsequently, the course will introduce the mathematics necessary to understand the first few minutes after the Big Bang, when the universe was very hot and the elements were forged. The course ends by explaining how small perturbations in the early universe subsequently grew into the glorious galaxies and structures that we see today. You will need to be comfortable with Newtonian dynamics, special relativity and some basic facts about quantum mechanics. No knowledge of astrophysics or general relativity is needed.

Classical Dynamics — Michaelmas, 24 lectures

This course follows on from the dynamics sections of Part IA Dynamics and Relativity and also uses the Euler–Lagrange equations from Part IB Variational Principles. The laws of motion for systems of particles and for rigid bodies are derived from a Lagrangian (giving Lagrange's equations) and from a Hamiltonian (giving Hamilton's equations) and are applied, for example, to the axisymmetric top.

One advantage of the formalism is the use of generalised coordinates; it is much easier to find the kinetic and potential energy in coordinates adapted to the problem and then use Lagrange's equations than to work out the equations of motion directly in the new coordinates. At a deeper level, the formalism gives rise to conserved quantities (generalisations of energy and angular momentum), and leads (via Poisson brackets) to a system which can be used as a basis for quantization.

The material in this course will be of interest to anyone planning to specialise in the applied courses. It is not used directly in any of the courses but an understanding of the subject is fundamental to Theoretical Physics.

Mathematical Biology — Lent, 24 lectures

The aim of the course is to explain from a mathematical point of view some underlying principles of biology, ranging from biochemistry and gene regulation to population dynamics and spread of infectious disease. In particular we examine mechanisms for feedback control, sensitivity amplification, oscillations, developmental instabilities, pattern-formation, competitive growth, and predator-pray interactions.

The material should be of interest to anyone who is fascinated by the richness of biological dynamics, but has been discouraged by too detail-oriented biological explanations. Mathematical methods include basic stochastic theory, nonlinear dynamics, differential equations, and numerical analysis. The concepts and techniques are not very difficult, and intuitive guiding principles and illustrative examples will be favoured over rigorous proofs. This is an exciting field with large unexplored territories for applied mathematicians.

While this course fits well with Dynamical Systems and there are places where understanding in one course will help the other, it is not essential and this course does not rely on any other Part II course, assuming concepts only from Part IA (Differential Equations and Probability) and IB (Methods).

Quantum Information and Computation — Lent, 24 lectures

Quantum processes can provide enormous benefits for information processing, communication and security, offering novel features beyond the possibilities of standard (classical) paradigms. These benefits include (i) new kinds of algorithms (so-called quantum algorithms) providing an exponentially faster method for some computational tasks, (ii) new modes of communication such as quantum teleportation, and (iii) the possibility of unconditionally secure communication in quantum cryptography. Most of these exciting developments have occurred in just the past few decades and they underpin striking applications of quantum technologies that are currently being developed.

This course will provide an introduction to these topics. No previous contact with the theory of computation or information will be assumed. IB Quantum Mechanics is essential, but only to provide prior exposure to basic ideas. This course rests on quantum theory in just a finite-dimensional setting, so the principal mathematical ingredients (from finite-dimensional linear algebra) will be readily accessible. We will begin by expounding the postulates of quantum mechanics in this setting (using Dirac notation) and then immediately make connections to information (quantum states viewed as information carriers, quantum teleportation) and computation (notion of qubits and quantum gates). Then we will discuss quantum cryptography (quantum key distribution), and quantum computing, culminating in an exposition of principal quantum algorithms, including the Deutsch–Jozsa algorithm, Grover's searching algorithm and an overview of Shor's quantum factoring algorithm. The course is cross-disciplinary in its conceptual ingredients and will be of interest to pure and applied mathematicians alike.

Further Complex Methods — Lent, 24 lectures

This course is a continuation in both style and content of Part IB Complex Methods, which is the only prerequisite. It will appeal to anyone who enjoyed that course. The material is classical — much of it can be found in Whittaker and Watson's 'Modern Analysis', written in 1912. The passage of time has not diminished the beauty of material, though the Faculty Board decided against naming the course 'Modern Analysis'.

The course starts with revision of Complex Methods and continues with a discussion of of the process of analytic continuation, which is at the heart of all modern treatments of complex variable theory. There follows a section on special functions, including the Gamma function (which is basically the factorial function when looked at on the real line, but on the complex plane it really blossoms) and the Riemann zeta and its connection with number theory. Then the theory of series solutions of differential questions in the complex plane is developed, and suddenly the treatment given in Part IA Differential Equations makes sense. Naturally, the messy business of actually solving specific equations by series is not in the style of the course. Particularly important are those equations that have exactly three singular points, all regular. This leads to a study of the properties of the delightful hypergeometric function, of which almost every other function you know can be thought of as a special case. This is the high point of the course, involving nearly all the theory that has preceded it.

D-Courses

Graph Theory — Michaelmas, 24 lectures

Discrete mathematics is commonplace in modern mathematics, both in theory and in practice. This course provides an introduction to working with discrete structures by concentrating on the most accessible examples, namely graphs. After a discussion of basic notions such as connectivity (Menger's theorem) and matchings (Hall's marriage theorem), the course develops in more detail the theory of extremal graphs, ideas of graph colouring, and the beautiful theorem of Ramsey. A significant feature is the introduction of probabilistic methods for tackling discrete problems, an approach which is of great importance in the modern theory.

There are no formal prerequisites but it will be helpful to recall some of the elementary definitions from the Part IA Probability course. The attractions and drawbacks of Graph Theory are similar to those of that course and of the Part IA Numbers and Sets course; whilst the notions are not conceptually difficult, the problems might on occasion require you to think a little.

The text *Modern Graph Theory* by Bollobás is an excellent source and contains more than is needed for the course. For a lighter introduction try Wilson's *Introduction to Graph Theory*, or for a little more look at Bondy and Murty's old but now online *Graph Theory with Applications*.

Representation theory — Michaelmas, 24 lectures

This course, suitable for pure and applied mathematicians, is an introduction to the basic theory of linear (matrix) actions of finite groups on vector spaces. The key notion we define is the character of a linear representation: this is a function on conjugacy classes of the group which determines the representation uniquely. Orthogonality relations between characters lead to a convenient and efficient calculus with representations, once the basic character table of the group has been computed. Later in the course 'finite' is replaced by 'compact' generalising the results with little extra effort.

The Linear Algebra course is essential and Groups, Rings and Modules is helpful.

Algebraic Topology — Michaelmas, 24 lectures

Topology is the abstract study of continuity: the basic objects of study are metric and topological spaces, and the continuous maps between them (concepts which were introduced in Part IB Analysis and Topology). One important difference between topology and algebra is that in constructing continuous maps one has vastly more freedom than in constructing algebraic homomorphisms; thus problems which involve proving the *non*-existence of continuous maps with particular properties (e.g. the problem of showing that $\mathbb{R}^m$ and $\mathbb{R}^n$ are not homeomorphic unless $m = n$) are hard to solve using purely topological methods. The technique that has proved most successful in tackling such problems is that of developing *algebraic invariants*, which assign to every topological space (in a suitable class) an algebraic structure such as a group or vector space, and to every continuous map a homomorphism of the appropriate kind. Thus questions of the non-existence of continuous maps are reduced to questions of non-existence of homomorphisms, which are easier to solve.

Two particular algebraic invariants are studied in this course: the fundamental group, and the simplicial homology groups. Of these, the former is easier to define, but hard to calculate except in a few particular cases; the latter requires the erection of a considerable amount of machinery before it can even be defined, but once this is done it becomes relatively easy to calculate. The course concludes with a classic example of the application of simplicial homology: the classification of all compact 2-manifolds up to homeomorphism.

Apart from Part IB Analysis and Topology, the only prerequisite is a modicum of geometrical intuition. Concepts and techniques of algebraic topology are used almost everywhere in mathematics where topological spaces occur; they also arise in many research areas in mathematical physics, as well as pure mathematics.

For introductory reading, browse *Basic Topology* by M.A. Armstrong (Springer-Verlag).

Galois Theory — Michaelmas, 24 lectures

The most famous application of Galois theory – discussed at the end of this course – is the proof that the general quintic equation with rational coefficients cannot be solved by radicals. Apart from this, Galois theory plays an indispensable role in algebraic number theory and several other areas of pure mathematics. It is a subject which (in favourable circumstances) allows one to handle given polynomials elegantly and with a minimum of algebraic manipulation.

Familiarity with the material concerning field extensions and the polynomial ring $K[t]$ from Part IB Groups, Rings and Modules is essential, while Part IB Linear Algebra is useful. The most closely related Part II courses are Representation Theory and Number Fields. The book *Galois Theory* by I. Stewart (Chapman and Hall, 1989) gives a very readable introduction to the subject.

Linear Analysis — Michaelmas, 24 lectures

Functional Analysis provides the framework, and a great deal of machinery, for much of modern mathematics: not only for pure mathematics (such as harmonic analysis and complex analysis) but also for the applications of mathematics, such as probability theory, the ordinary and partial differential equations met in applied mathematics, and the mathematical formulation of quantum mechanics.

The basic idea of Functional Analysis is to represent functions as points in an infinite-dimensional vector space. Since the space is infinite-dimensional, algebraic arguments are not enough, and it is necessary and appropriate to introduce the idea of convergence by a norm, which in turn defines a metric on the space.

In this course, most attention is paid to two sorts of spaces. The first consists of spaces of continuous functions: here the appropriate convergence is uniform convergence. The second is Hilbert space (particularly important in Quantum Mechanics) which provides an infinite-dimensional analogue of Euclidean space, and in which geometrical ideas and intuitions are used.

Riemann Surfaces — Lent, 16 lectures

A Riemann surface is the most general abstract surface on which one can define the notion of an analytic function, and hence study complex analysis. Roughly speaking, a surface is made into a Riemann surface when the change from one local coordinate system to another system is given by an analytic function. Not every Riemann surface has a global coordinate system; this accounts for both the interesting and the difficult parts of the theory.

The course begins with a study of the Riemann sphere (which is just the complex plane with infinity attached) and of elliptic functions (that is to say, doubly periodic analytic functions) which are the analytic functions defined on a torus. Abstract Riemann surfaces and holomorphic maps are then introduced and some of the results already studied in earlier courses on complex analysis are extended to this more general context.

Another view on Riemann surfaces comes from Riemann's original idea that the so-called 'multivalued functions' are just considered on a wrong domain: the natural domain is a surface covering the complex plane several (possibly infinitely many) times. This surface is called the Riemann surface of an analytic function and is obtained by the process of analytic continuation, extending the function (while keeping it analytic) in a maximal way from a domain in $\mathbb{C}$.

The last part of the course shows that most Riemann surfaces carry their own intrinsic non-Euclidean geometry; thus complex analysis is much more closely connected to non-Euclidean geometry than to Euclidean geometry (despite the fact that it is first studied in the Euclidean plane).

Prerequisite for this course is IB Complex Analysis (some knowledge of IB Analysis and Topology will also be useful, especially for elliptic functions). Related Part II courses include those on Algebraic Topology, Algebraic Geometry and Differential Geometry. As a preliminary reading, consider the early parts of G.A. Jones and D. Singerman, *Complex functions* CUP, 1987, and of A.F. Beardon, *A primer on Riemann surfaces* CUP, 1984.

Algebraic Geometry **Lent, 24 lectures**

Algebraic geometry is a branch of mathematics which, as the name suggests, combines techniques of abstract algebra, especially commutative algebra, with the language and the problematics of geometry. It occupies a central place in modern mathematics and has multiple conceptual connections with such diverse fields as complex analysis, topology and number theory. Initially a study of polynomial equations in many variables, the subject of algebraic geometry starts where equation solving leaves off, and it becomes at least as important to understand the totality of solutions of a system of equations, as to find some solution; this does lead into some of the deepest waters in the whole of mathematics, both conceptually and in terms of technique.

This course is an introduction to the basic ideas of algebraic geometry (affine and projective spaces, varieties), followed by a more detailed study of algebraic curves. We will develop the basic tools for understanding the properties of algebraic curves, and apply these at the end of the course to the beautiful theory of elliptic curves, which among other things played an essential part in the proof of Fermat's Last Theorem! You will find it highly advantageous to have attended the Part IB course Groups, Rings and Modules. Part II courses with which this course is related include Galois Theory, Differential Geometry, and Algebraic Topology. Students wishing to do some preliminary reading could browse the books of Reid or Kirwan noted in the schedules.

Differential Geometry **Lent, 24 lectures**

A manifold is a space that looks locally like euclidean space. The surface of a sphere or the surface of a torus are natural examples, but manifolds often arise indirectly, for example, as the space of solutions of some set of conditions, the parameter space of a family of mathematical objects, configuration spaces in mechanical systems and so on. Manifolds provide the appropriate arena on which one can explore interactions between various branches of Mathematics and Theoretical Physics.

Manifolds often come endowed with geometric structures, for example, with a way of measuring the length of a curve (Riemannian metrics) as in the case of a surface in 3-space. One can then define geodesics and curvature and study how these objects influence one another and interact with the topology of the manifold. A key illustration of this interplay, and a central result in this course, is the Gauss-Bonnet theorem, which shows that the average curvature determines the topological type of the surface.

Rather than worrying about how to define abstract manifolds (which you will see in Part III), we will study manifolds as objects already embedded in euclidean space. This will allow us to have a very short working definition of manifold and get fairly quickly into examples and the basic notions in Differential Topology (such as regular values, degree and transversality), giving a measure of how much one space folds onto another. Once we have set up the framework we will study the (Riemannian) geometry of curves and surfaces in euclidean space and we will prove at the end of the course that curvature can detect knottedness.

IB Geometry provides useful examples and an introduction to some of the ideas that we will develop and IB Analysis and Topology will be very useful when we set up the manifold framework in the first few lectures.

Analysis of Functions **Lent, 24 lectures**

The analysis of functions has its roots in the rigorous study of the equations of mathematical physics, and is now a key part of modern mathematics. This course builds on the Part II courses Linear Analysis and Probability & Measure, which are pre-requisites, applying the theory of integration and the tools of functional analysis to explore such topics as Lebesgue and Sobolev spaces, the Fourier transform and the generalised derivative.

These topics are important and interesting in themselves, but the emphasis is on their use in other areas of mathematics (for instance in the representation of functions and in partial differential equations), rather than their maximal generalisation. You can get an idea of the flavour of the course by browsing *Analysis* by Lieb & Loss (Springer).

Logic and Set Theory **Lent, 24 lectures**

The aim of this course is twofold: to provide you with an understanding of the logical underpinnings of the pure mathematics you have studied in the last two years, and to investigate to what extent, if any, the 'universe of sets' can be considered as a structure in its own right. As such, it has few formal prerequisites: some familiarity with naive set theory, as provided by the IA Numbers and Sets course, is helpful, but no previous knowledge of logic is assumed. On the other hand, the course has links to almost all of pure mathematics, and examples will be drawn from a wide range of subjects to illustrate the basic ideas.

The course falls into three main parts. One part develops the notions of validity and provability in formal logic, culminating in the Completeness Theorem, which asserts that these two notions coincide. Another part is concerned with ordinals and cardinals: these are notions that generalise the ideas of size and counting to the infinite. The final part is an introduction to formal set theory, where one makes precise the idea of a 'universe of sets', and studies its structure.

The book 'Notes on Logic and Set Theory' by P.T. Johnstone (C.U.P., 1987) covers most of the material of the course, and is suitable for preliminary reading.

Number Fields **Lent, 16 lectures**

Number theory studies properties of the integers and the rationals. The questions that arise are usually very simple to state but their solutions are often very deep and involve techniques from many branches of mathematics. It is also a subject where numerical experiments have proved useful as a guide to the sort of result one might seek to establish. Diophantine equations constitute a central theme; they are basically polynomial in form and lead to the study of integer or rational points on algebraic varieties. Two particularly famous problems here, Fermat's Last Theorem and the Catalan Conjecture, have only recently proved amenable to solution.

The course provides an introduction to algebraic number theory – it arose historically from investigations of reciprocity laws and attempts to solve the Fermat problem and it now forms one of the nicest and most fundamental topics in mathematics. Knowledge of the course on Groups, Rings and Modules is desirable.

The book *Problems in algebraic number theory* by J. Esmonde and R. Murty contains most of the material covered in the course. For a historical introduction to the subject, see also Chapter 1 of D. Cox's *Primes of the form* $x^2 + ny^2$.

Probability and Measure **Michaelmas, 24 lectures**

Measure theory is basic to some diverse branches of mathematics, from probability to partial differential equations. This course combines a systematic introduction to measure theory with an account of some of the main ideas in probability. You will be familiar with the Riemann integral from Parts IA and IB and have done some elementary probability in IA. The expectation operator of probability behaves somewhat like the integral, and in this course we see that they are both examples of some more general integral. These general integrals and the measures which underlie them have advantages over the Riemann integral, even for functions defined on the reals. In Part IA the definition and properties of expectation were only partially explored and here we do it more fully.

If you like to see how a substantial and coherent mathematical theory is put together, you will enjoy the measure theory part of this course, and this will be essential to any further work you do in analysis. It also underpins the probability which provides motivation and application throughout the course. The course ends with the Strong Law of Large Numbers and Central Limit Theorem, both of which are of real practical importance, being the mathematical basis for the whole of statistics.

A good book to read for the early part of the course is Probability with Martingales, by D. Williams (CUP, 1991).

Principles of Statistics **Michaelmas, 24 lectures**

In Part IB Statistics, an introduction to the main statistical problems and inference techniques was given, including parameter estimation, hypothesis testing and the construction of confidence sets in a variety of examples, including specific families of distributions and models. The *Principles of Statistics* course aims to give a unified perspective on these problems, and develops the main mathematical theory that underpins these basic principles of statistical inference.

The first pillar will be the inferential paradigm surrounding the likelihood function of the observations, and the associated *maximum likelihood estimator*, providing a conceptually unified and in most situations also practical solution to the problem of statistical inference. The distribution of this estimator will be shown to have a universal limiting normal distribution, permitting the use of the estimator for statistical inference. A generalisation of the Gauss–Markov theorem from the linear model can be proved for this estimator, establishing that it is in a certain sense the best among all estimators. Related to the likelihood principle, but in other respects fundamentally different, is the *Bayesian* approach to statistical inference. This approach, likewise, will be developed for general families of parametric statistical models.

The study of the notion of optimality of certain statistical procedures from a general perspective is known as *statistical decision theory*, of which the main ideas will be presented in the course. The course will also develop the main ideas of some related classical fields in statistics that are crucial in applications, such as inference methods for multivariate data, nonparametric techniques, and resampling (Monte Carlo) procedures.

Requirements: Probability IA and Statistics IB are essential.

Stochastic Financial Models **Michaelmas, 24 lectures**

This is concerned with the pricing of financial assets under uncertainty. It builds towards a presentation of the celebrated Black-Scholes formula for the price of an option on stocks. The holder of a call option on a stock has the right to purchase one unit of that stock at a specified 'strike' price within a designated time period. The holder hopes that within the period the stock price will go above the strike price whereupon the option may be exercised with the stock being bought at the strike price and sold immediately at the higher current price to yield a profit. What is the fair price to charge for such an option? In seeking an answer to this question, the course introduces some important ideas of probability theory including martingales and Brownian motion. Deciding when the holder should exercise the option leads to the techniques of dynamic programming and optimal stopping which are applicable throughout applied probability and statistics.

The main prerequisite for this material is Part IA Probability – if you liked that course then you should enjoy this one. Probability and Measure is recommended as a companion course, but it is not strictly necessary. No previous knowledge of economics or finance is necessary. It complements, but does not rely on, Markov Chains. To get a better idea of the sort of problems the course is seeking to tackle it is worth browsing in the book *Option, Futures and Other Derivative Securities* by J. Hull (Prentice-Hall, 2nd Ed. 1993).

Applied Probability **Lent, 24 lectures**

This course provides an introduction to some of the probabilistic models used to study phenomena as diverse as queueing, insurance ruin, and epidemics. The emphasis is on both the mathematical development of the models, and their application to practical problems. For example, the queueing models studied will be used to address issues that arise in the design and analysis of telecommunication networks.

The material is likely to appeal to those who enjoyed Part IA Probability; Markov Chains is useful, but the style of the course, involving a mix of theory and applications, will more closely resemble the earlier course. Probability and Measure is a loosely related Part II course.

Mathematics of Machine Learning **Lent, 16 lectures**

Suppose we are given a collection of emails labelled as either being spam or legitimate (known as ham). With this data, we aim to build an algorithm to annotate new unlabelled emails as spam or ham. This is known as a classification problem and is a central problem studied in Machine Learning, a field whose broader remit, like Statistics, is about how to learn from data. The focus of this course is the classification problem, other instances of which include predicting whether or not patients have a particular disease based on their medical history, or predicting whether a user will click an internet advertisement or not (a problem of great interest to search engines).

The first part of the course will deal with the theory of empirical risk minimization (ERM), a simple but powerful general strategy for building classification algorithms. We will work with a mathematical framework where each data instance (i.e. each email and label pair in the case of our first example) is viewed as a realisation of random elements, all sharing the same distribution. Studying ERM in this context will require us to develop important probabilistic tools, so-called concentration inequalities. Part IA Probability is a necessary prerequisite and in practice it is also advisable to have attended Part IB Statistics. The results we will derive were essentially state-of-the-art until the late 1990's and early 2000's; despite being suboptimal in some cases, studying them is a useful starting point for more sophisticated results.

The second part of the course studies computational aspects of ERM, building on parts of the IB Optimisation course (relevant material will be reviewed in the course). In particular, we will introduce stochastic gradient descent, which is now the most popular optimisation strategy for machine learning problems and currently the centre of a great deal of research. The last part of the course will illustrate some of the concepts and ideas developed in the earlier parts by studying random forests, boosting and neural networks, perhaps the three most successful methods in machine learning.

Asymptotic Methods **Michaelmas 16 lectures**

There are many instances, arising not only in mathematical physics, but also in analysis and number theory, where one needs an approximation to a function for which no usable convergent series expansion is available. Typically, the function is given as an integral or else as the solution of a differential equation. It turns out that excellent approximations can be obtained using certain series, called asymptotic expansions, which are normally non-convergent. Such an expansion might describe, for example, the behaviour of an integral depending on a parameter, as the parameter becomes large; alternatively, it might describe the behaviour of a solution of an ordinary differential equation, as the independent variable becomes large.

A certain amount of familiarity with the basics of complex-variable theory is essential, either through Part IB Complex Methods or Part IB Complex Analysis. This would be reinforced by the Part II Further Complex Methods course, which is desirable but not essential. An introduction to the course material is given in A. Erdelyi *Asymptotic Expansions* (Dover 1956).

Numerical Analysis **Michaelmas, 24 lectures**

Many mathematical problems, e.g. differential equations, can be solved generally only by computation, using discretisation methods. In other cases, e.g. large systems of linear equations, calculation of the exact solution is impractical and, again, we need to resort to numerical methods. Numerical analysis concerns itself with the design, implementation and mathematical understanding of computational algorithms. The course will address iterative techniques for linear equations, the calculation of eigenvalues and eigenvectors, and the solution of partial differential equations by finite differences (following the treatment of ordinary differential equations in Part IB). The last section of the course deals with Fourier expansions and their generalisations.

Part IB Numerical Analysis is an obvious prerequisite but Part IB Analysis courses, Complex Methods and Linear Algebra are also highly relevant. Mathematical ability is sufficient for understanding the course, while computational experience provides only a useful advantage.

Dynamical Systems — Michaelmas, 24 lectures

Contrary to the impression that you may have gained, most differential equations can not be solved explicitly. In many cases, however, a lot can still be said about the solutions. For example, for some systems of differential equations one can show that every solution converges to an equilibrium, while for others one can prove that there is a subset of solutions which are equivalent to infinite sequences of coin tosses ('chaos'). In this course, we study differential equations which can be written in the form $\dot{x} = v(x)$ with x in some (mainly two or three-dimensional) 'state' space. We take the "dynamical systems" viewpoint, concentrating on features which are invariant under coordinate change and time rescaling. We will find that two-dimensionality imposes severe restrictions though many interesting 'bifurcations' are possible: ways that the behaviour of a system $\dot{x} = v_\mu(x)$ can change as external parameters μ are varied. We shall also study nonlinear maps, which can be thought of either as difference equations or as a way of investigating the stability of periodic solutions of differential equations. We conclude with a discussion of chaotic behaviour in maps and differential equations, including a treatment of the famous logistic map. The treatment is 'applied' in flavour, with the emphasis on describing phenomena, though key theorems will be proved when needed.

If you browse P Glendinning *Stability, instability and chaos*, CUP, 1994 you will be well prepared.

The material contained in this course is relevant to any subject involving a modern treatment of differential equations. This includes most areas of Theoretical Physics, but usually not at the undergraduate level.

Electrodynamics — Michaelmas, 16 lectures

Electrodynamics is the most successful field theory in theoretical physics and it has provided a model for all later developments. This course develops from Part IB Electromagnetism. It starts by developing electromagnetism as a classical relativistic field theory, showing how the relativistic form of Maxwell's equations can be derived from a variational principle, and presenting the covariant treatment of the energy and momentum carried by the electromagnetic field. Such a treatment is essential for later use in quantum field theory. The remainder of the course shows how Maxwell's equations describe realistic phenomena. In particular, the production of electromagnetic waves by accelerated charges is discussed, and mechanisms for scattering radiation are briefly introduced. The course ends with the treatment of electromagnetism in continuous media, which allows one to understand some of the tremendous diversity in the electric and magnetic properties of real materials. The propagation of electromagnetic waves in materials is a particular focus.

The main prerequisite is familiarity with the basic ideas (especially those involving Maxwell's equations and their relativistic formulation) of Part IB Electromagnetism and knowledge of special relativity. As often for a theoretical physics course, the *Feynman Lectures* provide good introductory reading.

Principles of Quantum Mechanics — Michaelmas, 24 lectures

This course develops the principles and ideas of quantum mechanics in a way which emphasizes the essential mathematical structure, while also laying the foundations for a proper understanding of atomic and sub-atomic phenomena. In contrast to the introductory treatment given in Part IB, which is based entirely on wavefunctions and the Schrödinger equation, observables are presented as linear operators acting on vector spaces of states. This new approach has practical as well as aesthetic advantages, leading to elegant and concise algebraic solutions of problems such as the harmonic oscillator and the quantum theory of angular momentum. Some of the other key aspects of quantum behaviour that are treated include: intrinsic spin, multi-particle systems, symmetries, and their implications. Perturbation theory techniques, which are indispensable for realistic applications, are also discussed. The course ends by examining in more detail the inherently probabilistic nature of quantum mechanics, as illustrated by Bell's inequality and related ideas.

Applications of Quantum Mechanics **Lent, 24 lectures**

This course develops the ideas and methods introduced in Part IB Quantum Mechanics and Part II Principles of Quantum Mechanics and uses them to explain how we probe and understand the structure of atoms and solids. The various material objects that surround us in the everyday world exist as vast collections of particles (electrons and nuclei) making up atoms, molecules and various crystalline substances. Quantum mechanics is essential for an understanding of how this happens.

An important tool for probing the structure of matter (finding out where the particles are, how the electric charge is distributed) is the scattering of a beam of particles of appropriate energy on targets of interest. The course develops the theory of scattering in a form applicable to both atomic and crystalline targets.

There are two particularly important aspects of crystalline materials: the elastic vibrations of the atoms in the crystal matrix and the dynamics of electrons moving through the crystal. In quantum theory the elastic vibrations are understood as particle-like excitations known as phonons. In travelling through a crystal both phonons and electrons exhibit a band structure in their permitted energies. The role of phonons and electrons in condensed matter physics and the significance of this energy band structure is explained by means of simple but physically significant quantum mechanical models. Energy bands are used to understand the properties of semiconductors and some simple devices such as the *pn* junction are explained.

Some idea of the material of the course can be gained by consulting a book such as *Principles of the Theory of Solids* by J. M. Ziman, (CUP, 1972).

Statistical Physics **Lent, 24 lectures**

Statistical mechanics is the art of turning the microscopic laws of physics into a description of Nature on a macroscopic scale. This requires the development of tools to understand the properties of systems which contain a very large number of particles.

The course starts by defining new concepts such as as entropy, temperature and heat, all viewed from a microscopic perspective. These ideas are then used to understand a wide array of phenomena, from gases, to the vibrations of solids (phonons), to the behaviour of light (blackbody radiation). In each case, the laws governing these phenomena are derived from first principles. The course goes on to describe more subtle quantum effects that occur at low temperature such as Bose-Einstein condensation and the formation of Fermi surfaces. The course ends with a discussion of thermodynamics and phase transitions, in which the properties of a system change discontinuously.

Ideas from Part IB Quantum mechanics are essential and Part II Principles of Quantum Mechanics is highly desirable. Lecture notes from David Tong's course are available to download at https://www.damtp.cam.ac.uk/user/tong/statphys.html, while a shorter set of notes representing almost exactly the material presented in the classes is available at https://www.damtp.cam.ac.uk/user/us248/Lectures/lectures.html.

General Relativity **Lent, 24 lectures**

General Relativity is a relativistic theory of gravitation which supersedes the Newtonian Theory. This course shows how the theory can be built up on the foundations of Part IA Special Relativity. The necessary ideas from differential geometry will be taught ab initio, relying on the methods courses in Parts IA and IB. As an extended example, the course includes a careful treatment of the Schwarzschild spacetime and its interpretation as a black hole.

An elegant informal treatment of much of the material is contained in chapters 1,2,7 and 8 of W. Rindler *Essential Relativity* (Springer, 1977). A slightly more formal introduction is chapters 5,6,8,9,10,14-16 of R. d'Inverno *Introducing Einstein's Relativity* (Oxford, 1992).

Integrable Systems **Lent, 16 lectures**

A *soliton* was first observed in 1834 by a British experimentalist, J. Scott Russell. It was mathematically discovered in 1965 by Kruksal and Zabusky, who also introduced this name in order to emphasise the analogy with particles ('soli' for solitary and 'ton' for particles).

Solitons appear in a large number of physical circumstances, including fluid mechanics, nonlinear optics, plasma physics, elasticity, quantum field theory, relativity, biological models and nonlinear networks. This is a consequence of the fact that a soliton is the realisation of a certain physical coherence which is natural, at least asymptotically, to a variety of nonlinear phenomena. The mathematical equations modelling such phenomena are called integrable. There exist many types of integrable equations including ODEs, PDEs, singular integrodifferential equations, difference equations and cellular automata.

The mathematical structure of integrable equations is incredibly rich. Indeed soliton theory impacts on many areas of mathematics including analysis, algebraic geometry, differential geometry, group theory and topology. However, it must be emphasised that the basic concepts of the integrable theory can be introduced with only minimal mathematical tools. This course will give an introduction to soliton theory with emphasis on the occurrence of solitons in nonlinear dispersive PDEs.

Fluid Dynamics **Michaelmas, 24 lectures**

How does a hummingbird hover? How does a bumblebee fly? How, for that matter, does a Boeing 747 defy the pull of gravity? Does the bath-tub vortex really rotate anti-clockwise – or is it clockwise – in the Northern Hemisphere? How can a flow which exhibits an infinite sequence of eddies in a confined space satisfy a 'minimum dissipation' theorem? How can a flow that is strictly reversible have irreversible consequences?

Such questions lie within the domain of Fluid Dynamics, a subject that contains the seeds of chaos (and indeed provides the main stimulus for much of the current intense interest in chaos). The course will address the above questions, among others, in a progression from phenomena on very small scales ('low Reynolds number problems') to phenomena on very large scales ('large Reynolds number problems'). The course thus encompasses, at one extreme, flows that arise at the biological level (e.g. the swimming of microscopic organisms) and, at the other, flows on the scale of the Earth's atmosphere and oceans, or even larger. And, in between of course, it encompasses the bath-tub! Mathematical techniques, further to those developed in IB, will be used to determine solutions to the nonlinear, time-dependent Navier–Stokes equations.

Part II Asymptotic Methods covers some material which would be useful for this course. The course has natural links with Part II Waves and less obvious links with Dynamical Systems. A number of the Computational Projects are directly relevant. Introductory reading: *Elementary Fluid Dynamics* by D.J. Acheson, chapters 1-4.

Laboratory Demonstrations in Fluid Dynamics **Non-examinable**

A series of laboratory demonstrations and experiments is used to expose you to material covered by the Part IB and Part II Fluid Dynamics lecture courses. The emphasis is on understanding the physics behind the mathematics, along with the limitations of the simple analytical models. Attending this course will help you develop the physical insight necessary to derive and evaluate mathematical models, and to determine whether their predictions are reasonable. Specific topics covered include potential flow, surface waves, Reynolds experiment, Stokes flow, Kelvin's circulation theorem, spin-up, boundary layers and bubbles. Student participation is encouraged but not required.

There are no prerequisites for this course and the course is not examined.

Further details will be announced during the Part II Fluid Dynamics lectures.

Waves **Lent, 24 lectures**

Waves occur in almost all physical systems including continuum mechanics, electromagnetic theory and quantum mechanics. In this course examples will be drawn from fluid and solid mechanics, although much of the theory has application in other contexts. In the first part of the course sound waves in a gas are studied (after which you will understand why you can hear the lecturer). Small amplitude acoustic waves are described by the wave equation (see IB Methods); however at larger amplitudes nonlinear effects must be included. The change in the governing equations caused by nonlinearity leads to the formation of shocks, i.e. sonic booms. Applications of the underlying theory to both traffic flow and blood flow are mentioned.

Linear elastic waves, e.g. seismic waves, split into two types: the faster-travelling compressional waves (cf. sound waves) and the slower-travelling shear waves. The surface waves that cause most destruction in an earthquake are also studied.

Not all linear waves have a wavespeed that is independent of wavelength. In such systems it is important to distinguish the speed of wavecrests from the speed at which energy propagates; indeed, the wavecrests and energy can propagate in opposite directions. As a consequence, (a) if you throw a stone into a pond to generate a circular wave packet, you will see that the wavecrests propagate outward through the wave packet and disappear, and (b) atmospheric waves generated near ground level can appear to the eye as if they are propagating down from the heavens! Finally, the ray tracing equations are derived. These are used to describe, *inter alia*, why you can go surfing (i.e. why waves tend to approach a beach perpendicularly), why the wave pattern behind a ship (or a duck) subtends a half-angle of $19\frac{1}{2}^\circ$, and why sound can travel long distances at night.

The mathematical techniques assumed are those covered in the IA and IB Methods courses. While the course is otherwise self-contained, there is a small amount of complementary material in Asymptotic Methods, Electrodynamics, and Fluid Dynamics.

A good book to look at is *Wind waves: their generation and propagation on the ocean surface* by B. Kinsman (Prentice-Hall).

Computational Projects (CATAM)

This course is similar in nature to the Part IB course. There are a variety of projects to choose from; some are closely related to Part II courses and others are not. As in Part IB, the projects are listed in a CATAM manual which is available on the Faculty website at https://www.maths.cam.ac.uk/undergrad/catam/II.

The course is examined by means of work handed in close to the beginning of the Easter Term.

5 Lecture Timetable: Clashes

Each term, there are only 8 slots for lecture courses[1] This year there are a total of 37 courses to be fitted into a total of 16 slots, so there will be 11 slots with double clashes and 5 slots with triple clashes.

The general policy is to avoid triple clashing any C-courses, and to avoid clashing any applied courses, any pure courses and any applicable courses. Further, by using examination statistics to gauge the popularity of combinations of courses, the aim is to minimise the effect of the clashes (for example, by not clashing a very popular pure course with a very popular applied course). However, there are also significant constraints associated with the fact that the most popular courses in Parts II and III will fit only in the larger lecture theatres (MR2, MR3 and MR9). In addition there are constraints concerning lecturer availability. At the end of the day, it is unfortunately inevitable that some students would have wanted to attend courses that clash.

A provisional lecture timetable should be available early in September, or thereabouts, from which you will be able to see the clashes.

[1] Or 9 if 16-lecture courses were distributed asynchronously.