

Name: _____

ID#: _____

Write your answers in the space provided.

Full marks will not be awarded if full working/explanation is not shown.

Attach the R code and Assignment as SEPARATE files. **You should NOT use any R code that may be freely available on the internet.**

You should NOT write R code or any output ...paste it from R. You can write any explanations in the spaces provided.

Upload (1) the R code script file and (2) the assignment (as a PDF) in mylearning.

Question 1:

Suppose that X_1, X_2, \dots, X_n are i.i.d. Poisson random variables whose distribution is a mixture of Poisson random variables with parameters $\lambda \wedge \mu$. We observe a Poisson random variable with mean λ with probability θ and a Poisson variable with mean μ with probability $1 - \theta$.

- (i) Derive the EM Algorithm to estimate the parameters. [10]

- (ii) Suppose the following data are observed 1, 2, 3, 8 and 12 perform one iteration (BY HAND!!) of the EM algorithm to estimate the parameters. You can use an ordinary calculator OR the calculator facilities in R. [10]

- (iii) Write R code to perform the EM Algorithm and include in the R code an appropriate stopping tolerance level. [10]

- (iv) Generate 100 values from a Poisson distribution with mean 5 and 900 values from a Poisson distribution with mean 10. Amalgamate those 1000 values into a single column. Then, run the R code you constructed to see how well your algorithm is able to estimate the parameters. [10]

Question 2:

(a) Show that the function illustrated in Table 1 represents the joint probability mass function of the two discrete random variables X and Y? Justify your answer. (2 marks)

f(x,y)		x		
		1	2	3
y	1	1/20	2/20	1/20
	2	4/20	7/20	5/20

Table 1

(b) Find $E(XY)$ by hand. Leave your answer in exact form. (3 marks)

(c) Write an appropriate algorithm to generate pairs of random variables X and Y from the joint probability mass function in Table 1. (5 marks)

State the Algorithm

(d)(i) Implement the algorithm in R and generate 100,000 pairs of random variables (X,Y) from the distribution. (PASTE R CODE) (5 marks)

(ii) Estimate $E(XY)$ from the 100,000 pairs of values you simulated (USE R!). (3 marks)

(iii) Compare the estimated mean of $E(XY)$ with the true mean you found in (b). (2 marks)