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```
format short

syms t x
```

Problem 1

```
% Part a
fprintf("\nPart a\n");

R_x = [
    cos(x) -sin(x);
    sin(x), cos(x);
];

v = [
    5;
    -5;
]

A = double(subs(R_x, x, pi/9))
v_rotated = A*v

% Part b
fprintf("\nPart b\n");
B = double(subs(R_x, x, pi/11));
A * B
B * A

fprintf("Thus AB = BA\n");

% Part c
fprintf("\nPart c\n")
fprintf("The result from part b shows that the order of rotation doesn't
matter for rotational transformations\n")

% Part d
fprintf("\nPart d\n");
format rat
C = A * B
t = acos(C(1, 1))
t/pi
```

```
% Part e
fprintf("\nPart e\n");
format short
inv(A)
double(subs(R_x, x, -pi/9))

fprintf("Therefore R_theta = R_-theta\n")

% Part f
fprintf("\nPart f\n");
L0 = [
    1, 0;
    0, -1;
]
L_x = R_x * L0 * subs(R_x, x, -x)
L1 = double(subs(L_x, x, pi/9))

% Part g
fprintf("\nPart g\n");
L1L0 = L1 * L0
L0L1 = L0 * L1

fprintf("Thus L1L0 is not equivalent to L0L1\n");

%Part h
fprintf("\nPart h\n");
format rat

acos(L1L0(1, 1)) / pi
```

```
Part a

v =

     5
    -5


A =

    0.9397   -0.3420
    0.3420    0.9397


v_rotated =

    6.4086
   -2.9884


Part b
```

```
ans =

    0.8053   -0.5929
    0.5929    0.8053


ans =

    0.8053   -0.5929
    0.5929    0.8053
```

Thus AB = BA

Part c

The result from part b shows that the order of rotation doesn't matter for rotational transformations

Part d

```
C =

     550/683       -418/705
     418/705        550/683


t =

    1223/1927


ans =

      20/99
```

Part e

```
ans =

    0.9397    0.3420
   -0.3420    0.9397


ans =

    0.9397    0.3420
   -0.3420    0.9397
```

Therefore R_theta = R_-theta

Part f

```
L0 =
```

```
     1     0
     0    -1


L_x =

[cos(x)^2 - sin(x)^2,     2*cos(x)*sin(x)]
[    2*cos(x)*sin(x), sin(x)^2 - cos(x)^2]


L1 =

    0.7660    0.6428
    0.6428   -0.7660


Part g

L1L0 =

    0.7660   -0.6428
    0.6428    0.7660


L0L1 =

    0.7660    0.6428
   -0.6428    0.7660

Thus L1L0 is not equivalent to L0L1

Part h

ans =

       2/9
```

Problem 2

```
A = [
    3, 4, 9;
    5, 8, 5;
    6, 7, 3;
]

% Part a
fprintf("\nPart a\n");
M = [A eye(3)]
M_red = rref(M)
A_inv_1 = M_red(:, 4:6)

% Part b
```

```
fprintf("\nPart b\n");
A_inv_2 = inv(A)

fprintf("The inverse matrix from part and b are the same\n");
```

```
A =

       3              4              9
       5              8              5
       6              7              3


Part a

M =

  Columns 1 through 5

       3              4              9              1              0
       5              8              5              0              1
       6              7              3              0              0

  Column 6

       0
       0
       1


M_red =

  Columns 1 through 5

       1              0              0             11/90         -17/30
       0              1              0             -1/6            1/2
       0              0              1             13/90          -1/30

  Column 6

      26/45
      -1/3
      -2/45


A_inv_1 =

      11/90         -17/30          26/45
      -1/6            1/2           -1/3
      13/90          -1/30          -2/45


Part b
```

```
A_inv_2 =

      11/90          -17/30          26/45
      -1/6             1/2           -1/3
      13/90           -1/30          -2/45

The inverse matrix from part and b are the same
```

Problem 3

```
% Part a
fprintf("\nPart a\n");

A = [
    5, 0, 0, 0;
    13, 2, 0, 0;
    -6, 4, -1, 0;
    10, 0, 3, -2;
]

B = [
    -1, -1, 1, 1;
    2, 0, 1, 3;
    2, -1, 1, 2;
    1, 0, 3, 3;
]

A_det = det(A)
B_det = det(B)

% Part b
fprintf("\nPart b\n")

fprintf("The first row only has one non zero entry, so the cofactors computed
by the first row will be 0 except for the non-zero element. This makes the
computation of the determinant significantly easier\n");

% Part c
fprintf("\nPart c\n");

C = A*B
C_det = det(C)

% Part d
fprintf("\nPart d\n");
fprintf("Since C = AB, det(C) = det(A)*det(B)\n");
```

```
Part a

A =

       5              0              0              0
```

```
      13             2              0              0
      -6             4             -1              0
      10             0              3             -2


B =

      -1            -1              1              1
       2             0              1              3
       2            -1              1              2
       1             0              3              3


A_det =

      20


B_det =

      13


Part b
The first row only has one non zero entry, so the cofactors computed by
the first row will be 0 except for the non-zero element. This makes the
computation of the determinant significantly easier

Part c

C =

      -5            -5              5              5
      -9           -13             15             19
      12             7             -3              4
      -6           -13              7             10


C_det =

     260


Part d
Since C = AB, det(C) = det(A)*det(B)
```

Problem 4

```
A = [
    -1, 3, 9, -2;
    1, -3, -2, 0;
    0, 0, -4, -1;
    2, -8, -1, 7;
```

```
]

% Part a
fprintf("\nPart a\n");

A_det = det(A)

% Part b
fprintf("\nPart b\n");

B_det_1 = -det(A)
C_det_1 = 5*det(A)
D_det_1 = det(A)

% Part c
fprintf("\nPart c\n");

B = A;
B([2, 4], :) = B([4, 2], :)
C = A;
C(2, :) = 5*C(2, :)
D = A;
D(1, :) = 6*D(4, :) + D(1, :)

% part d
fprintf("\nPart d\n");

B_det_2 = det(B)
C_det_2 = det(C)
D_det_2 = det(D)

fprintf("The determinants found in part b are the same as the ones computed
in part d\n");
```

```
A =

    -1              3              9             -2
     1             -3             -2              0
     0              0             -4             -1
     2             -8             -1              7


Part a

A_det =

   -30


Part b

B_det_1 =
```

```
        30


C_det_1 =

      -150


D_det_1 =

       -30


Part c

B =

        -1             3             9            -2
         2            -8            -1             7
         0             0            -4            -1
         1            -3            -2             0


C =

        -1             3             9            -2
         5           -15           -10             0
         0             0            -4            -1
         2            -8            -1             7


D =

        11           -45             3            40
         1            -3            -2             0
         0             0            -4            -1
         2            -8            -1             7


Part d

B_det_2 =

        30


C_det_2 =

      -150


D_det_2 =

       -30
```

The determinants found in part b are the same as the ones computed in part d

Problem 5

```
% Part a
fprintf("\nPart a\n");

syms a b c d

A = [
    a, b;
    c, d;
]
% Part b
fprintf("\nPart b\n");

A_inv = inv(A)

% Part c
fprintf("\nPart c\n");

syms e f g h i

B = [
    a, b, c;
    d, e, f;
    g, h, i;
]

B_inv = inv(B)

% Part d
fprintf("\nPart d\n");

B_inv_simplified = B_inv * det(B)
```

```
Part a

A =

[a, b]
[c, d]


Part b

A_inv =

[ d/(a*d - b*c), -b/(a*d - b*c)]
[-c/(a*d - b*c),  a/(a*d - b*c)]
```

Part c

```
B =

[a, b, c]
[d, e, f]
[g, h, i]


B_inv =

[ (e*i - f*h)/(a*e*i - a*f*h - b*d*i + b*f*g + c*d*h - c*e*g), -(b*i - c*h)/
(a*e*i - a*f*h - b*d*i + b*f*g + c*d*h - c*e*g),  (b*f - c*e)/(a*e*i - a*f*h
- b*d*i + b*f*g + c*d*h - c*e*g)]
[-(d*i - f*g)/(a*e*i - a*f*h - b*d*i + b*f*g + c*d*h - c*e*g),  (a*i - c*g)/
(a*e*i - a*f*h - b*d*i + b*f*g + c*d*h - c*e*g), -(a*f - c*d)/(a*e*i - a*f*h
- b*d*i + b*f*g + c*d*h - c*e*g)]
[ (d*h - e*g)/(a*e*i - a*f*h - b*d*i + b*f*g + c*d*h - c*e*g), -(a*h - b*g)/
(a*e*i - a*f*h - b*d*i + b*f*g + c*d*h - c*e*g),  (a*e - b*d)/(a*e*i - a*f*h
- b*d*i + b*f*g + c*d*h - c*e*g)]
```

Part d

```
B_inv_simplified =

[e*i - f*h, c*h - b*i, b*f - c*e]
[f*g - d*i, a*i - c*g, c*d - a*f]
[d*h - e*g, b*g - a*h, a*e - b*d]
```

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