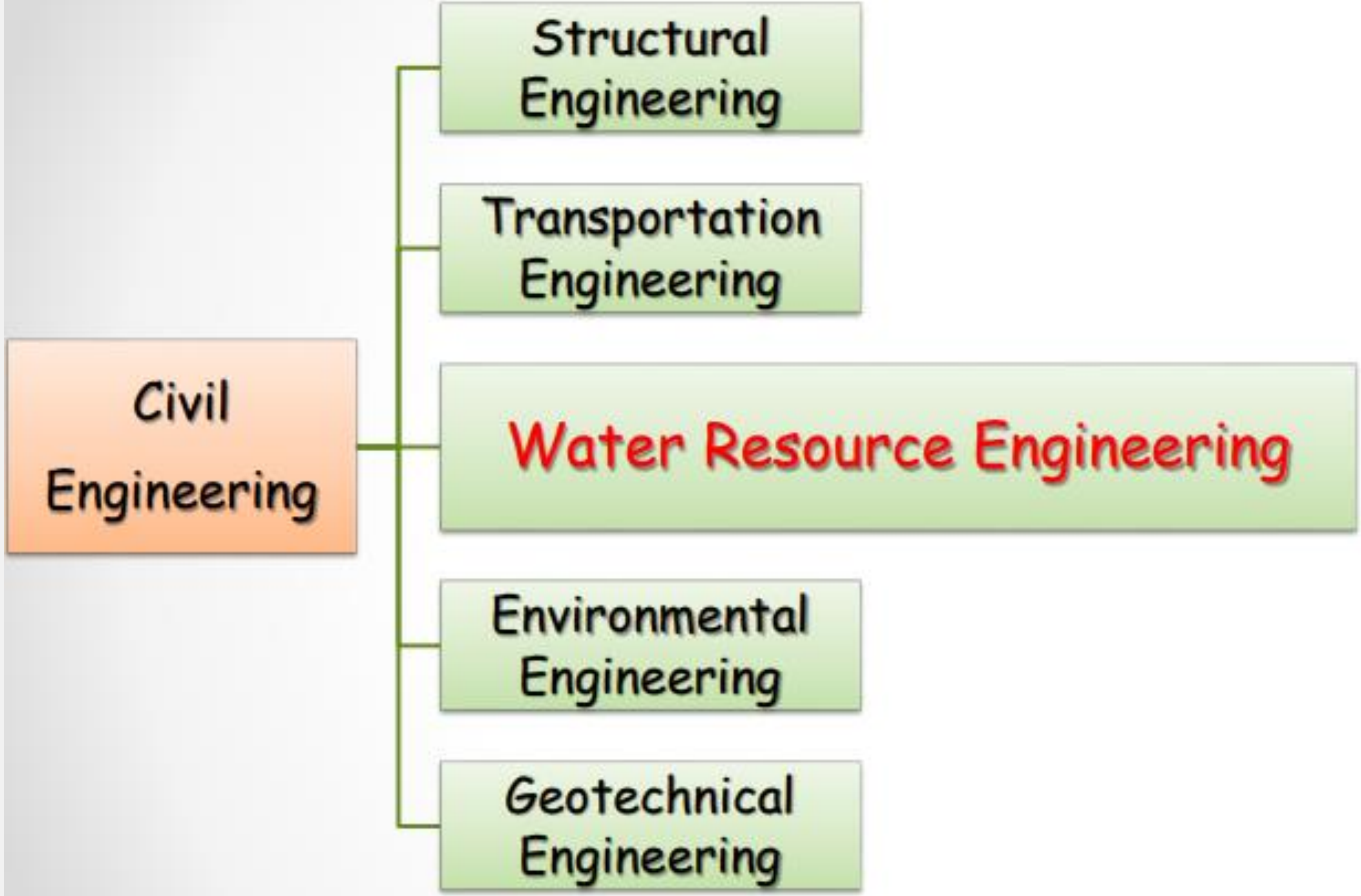
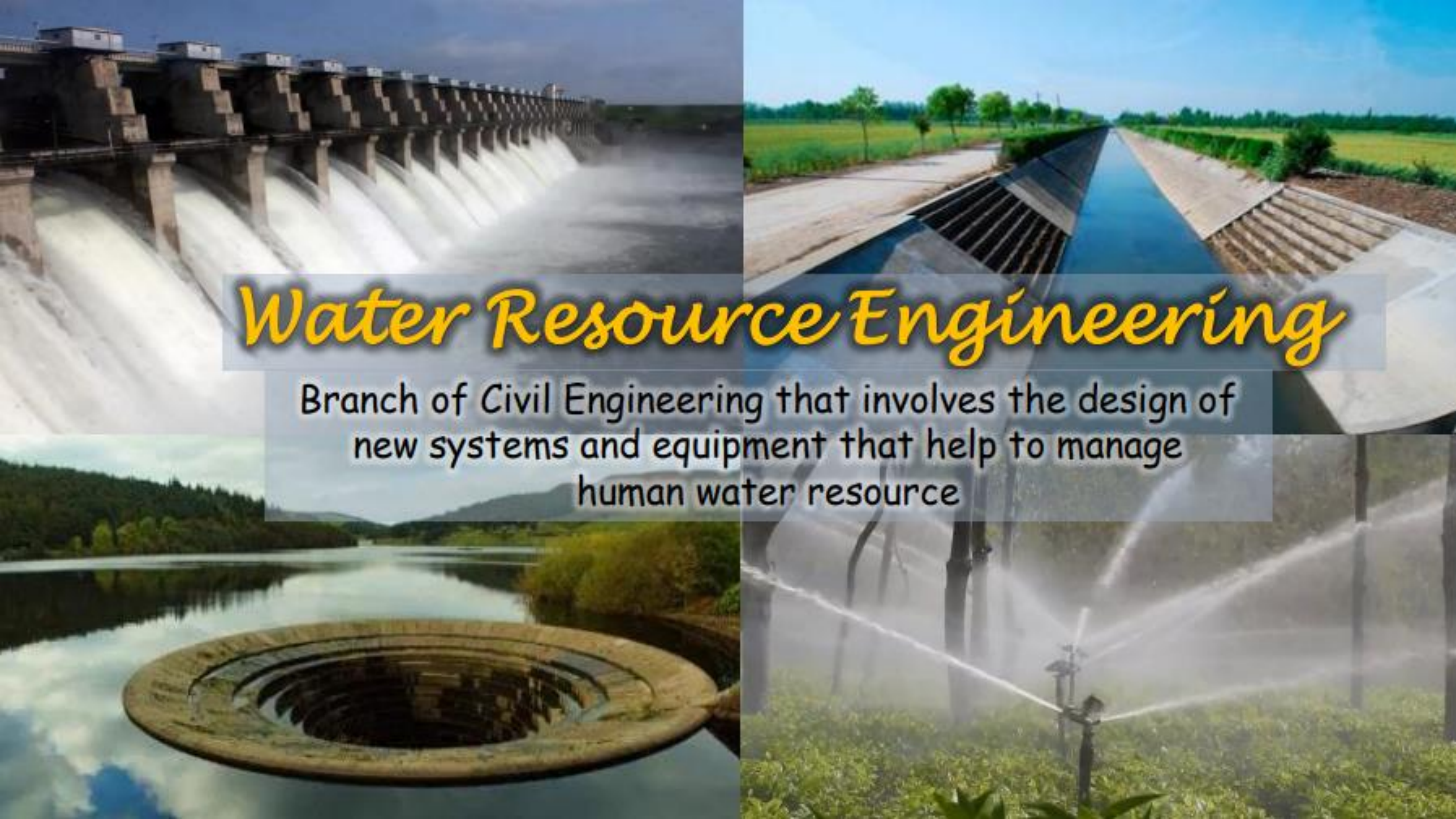

HYDRAULICS



<u>In Diploma Curriculum</u>

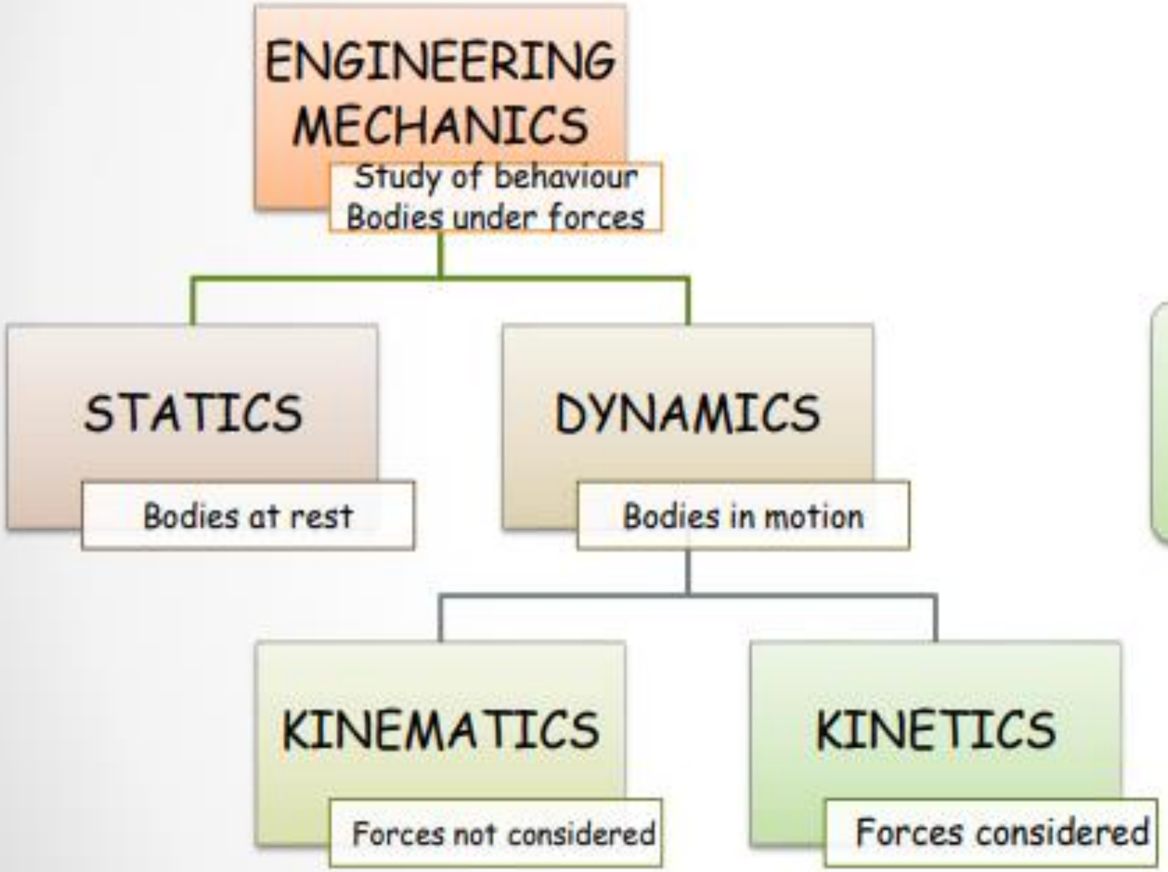
1. Hydraulics
2. Irrigation Engineering
3. Irrigation Drawing

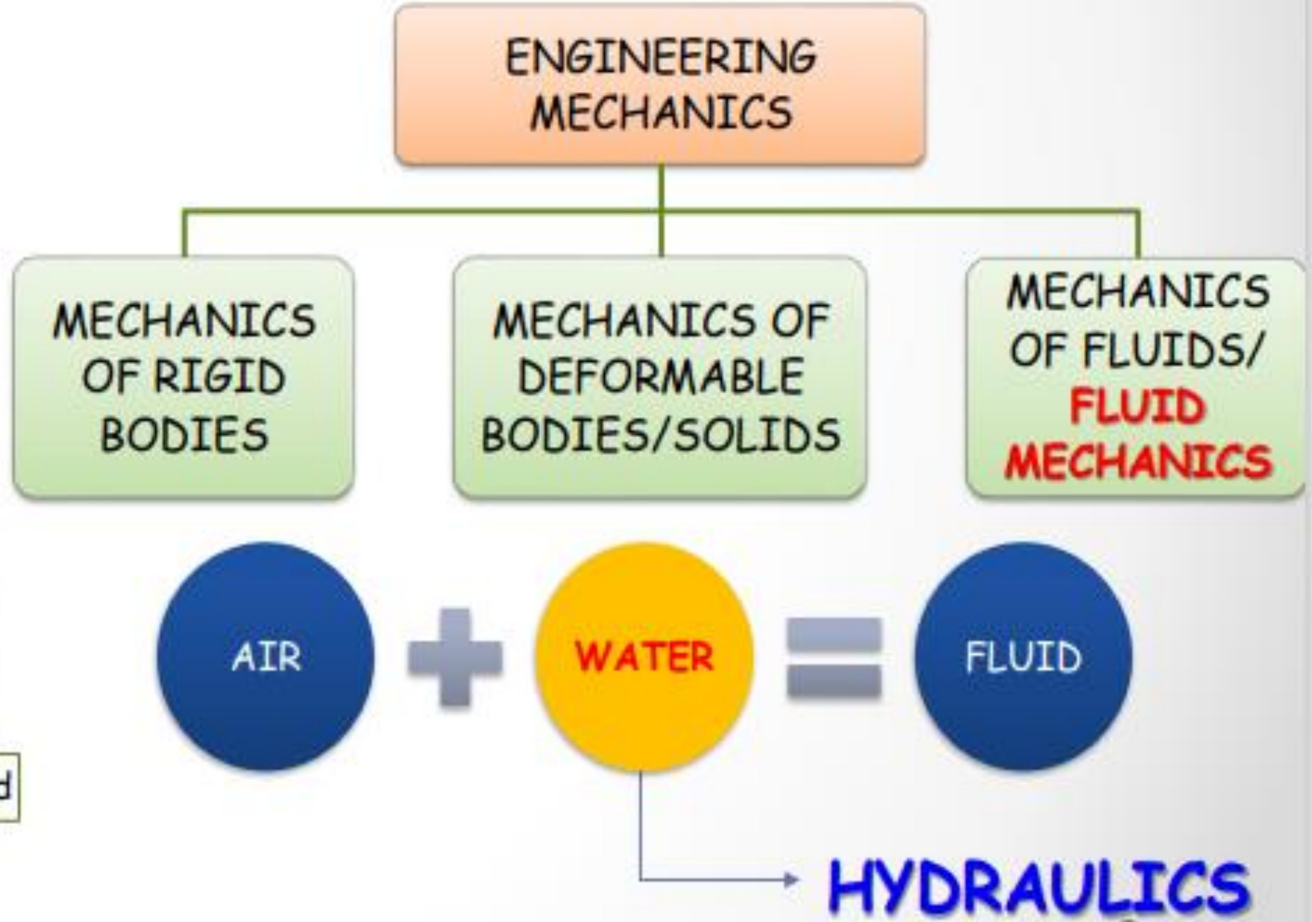

Water Resource Engineering
Branch of Civil Engineering that involves the design of new systems and equipment that help to manage human water resource

HYDRAULICS

Hydraulics is the branch of engineering science that deals with **behaviour of water** under the conditions of rest and motion.

RECALL!!!





OVERVIEW OF THE SYLLABUS

PROPERTIES OF LIQUIDS

- Know the general properties of liquids
- Understand liquid pressure and its measurement

FLOW OF LIQUIDS

- Apply general principles of flow of liquids
- Understand flow through orifices and mouthpieces
- Understand flow over notches and weirs
- Understand flow through pipes and open channels

HYDRAULIC MACHINERY

- Understand working of water turbines
- Understand working of common pumps
- Know general layout of hydro electric plants

MODULE 1

PROPERTIES OF LIQUID

LIQUID PRESSURE AND ITS MEASUREMENT

BERNOULLI'S PRINCIPLE AND ITS APPLICATIONS

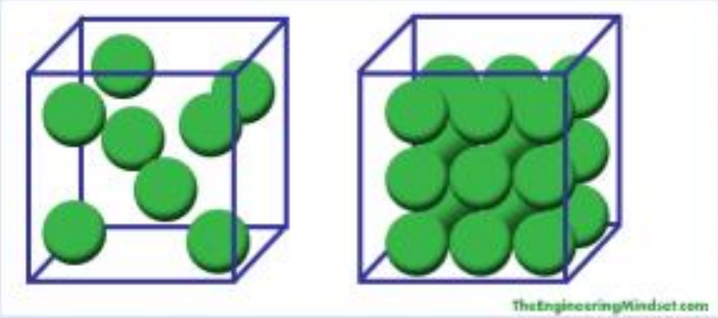
PROPERTIES OF LIQUIDS

1. DENSITY or MASS DENSITY (ρ)

- Density or mass density of a liquid is the ratio of the mass of a liquid to its volume.

$$\rho = \frac{Mass\ of\ liquid}{Volume\ of\ liquid}$$ [SI Unit kg/m^3]

Density of water : 1 gm/cm^3 or 1000 kg/m^3



2. SPECIFIC WEIGHT or WEIGHT DENSITY (w)

- Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume.

$$w = \frac{Weight\ of\ liquid}{Volume\ of\ liquid}$$ [SI Unit N/m^3]

Weight = mass x acceleration due to gravity (g)

$$w = \frac{\text{Mass of liquid}}{\text{Volume of liquid}} * g$$

$$w = \rho * g$$

Weight density of water : 9.81 kN/m^3 *(Approx. 10 kN/m^3)*

3. SPECIFIC GRAVITY OF LIQUIDS (S)

- Specific Gravity is the ratio of weight density (or density) of a liquid to weight density (or density) of water.

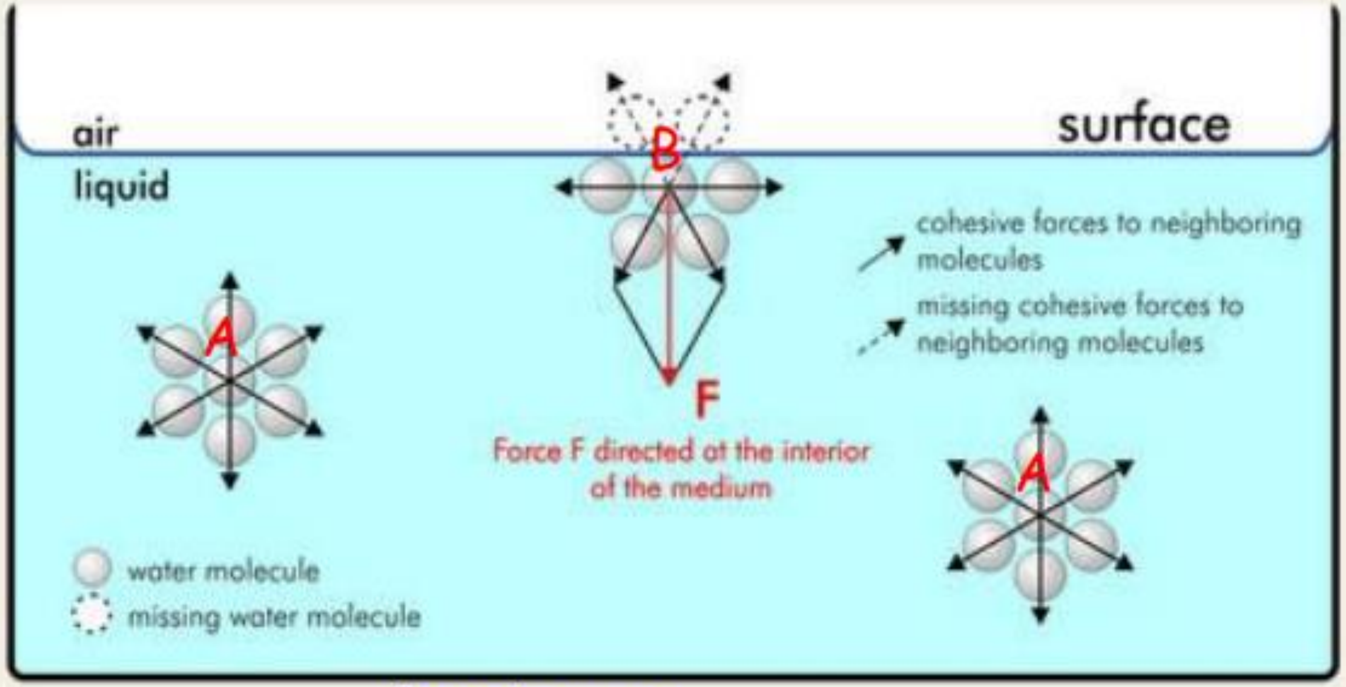
$$S = \frac{Weight\ density/density\ of\ liquid}{Weight\ density/density\ of\ water} = \frac{w_{liquid}}{w_{water}} = \frac{\rho_{liquid}}{\rho_{water}}$$

SPECIFIC GRAVITIES OF STANDARD LIQUIDS	
Water : 1	Mercury : 13.6

Thus, If you know specific gravity of a liquid, you can find its density by just remembering the density of water !!!!

Try This !!!! If specific gravity of some oil is 0.8. Find out its weight density in kN/m^3

4 SURFACE TENSION



Source : sita-process.com

MOLECULE A : IN LIQUID MEDIUM

- Molecule is surrounded by molecules from around.
- Cohesive forces from all around. (net force is zero)

MOLECULE B : AT FREE SURFACE

- No attractive forces in upward direction.
- So there is net downward force.

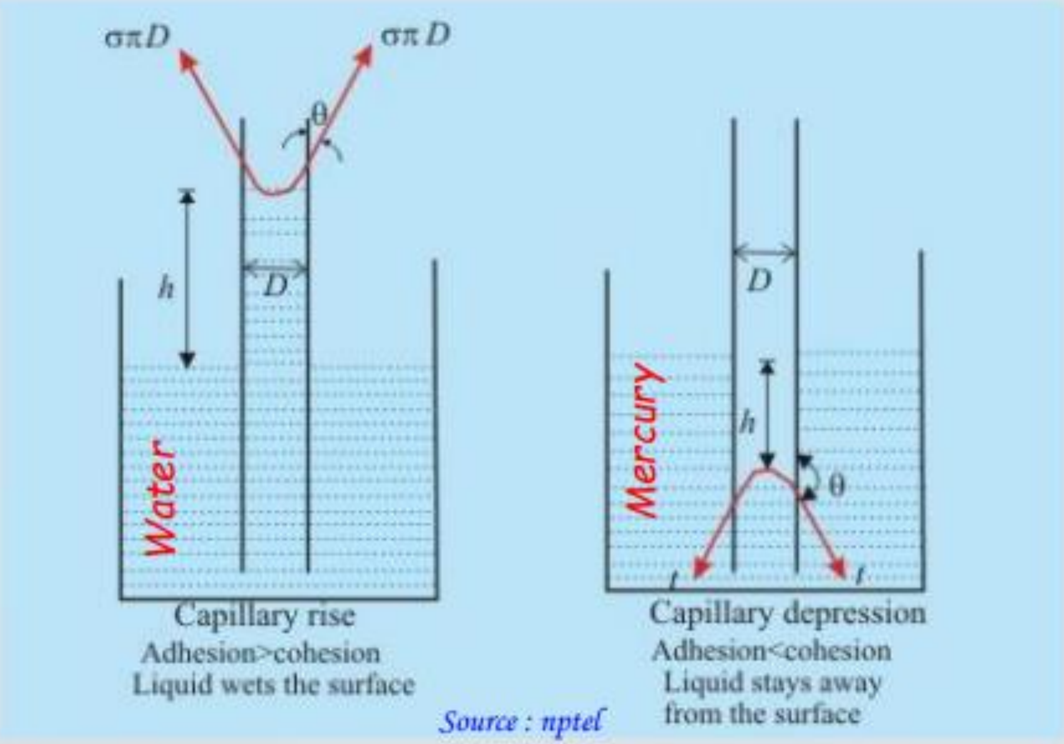
- Surface tension is defined as the tensile force acting on the surface of a liquid in contact with air or on the surface between two immiscible liquids such that the contact behaves as a membrane under tension.

[SI Unit N/m]

5. CAPPILARITY

- Capillarity is defined as a phenomenon of rise or fall of a liquid in a tube relative to the adjacent level of liquid when the tube is held vertically in liquid.

Rise of liquid surface : Capillary rise
Fall of liquid surface : Capillary fall



$$Cappillary\ rise/fall, h = \frac{4\sigma\cos\theta}{\rho g d}$$

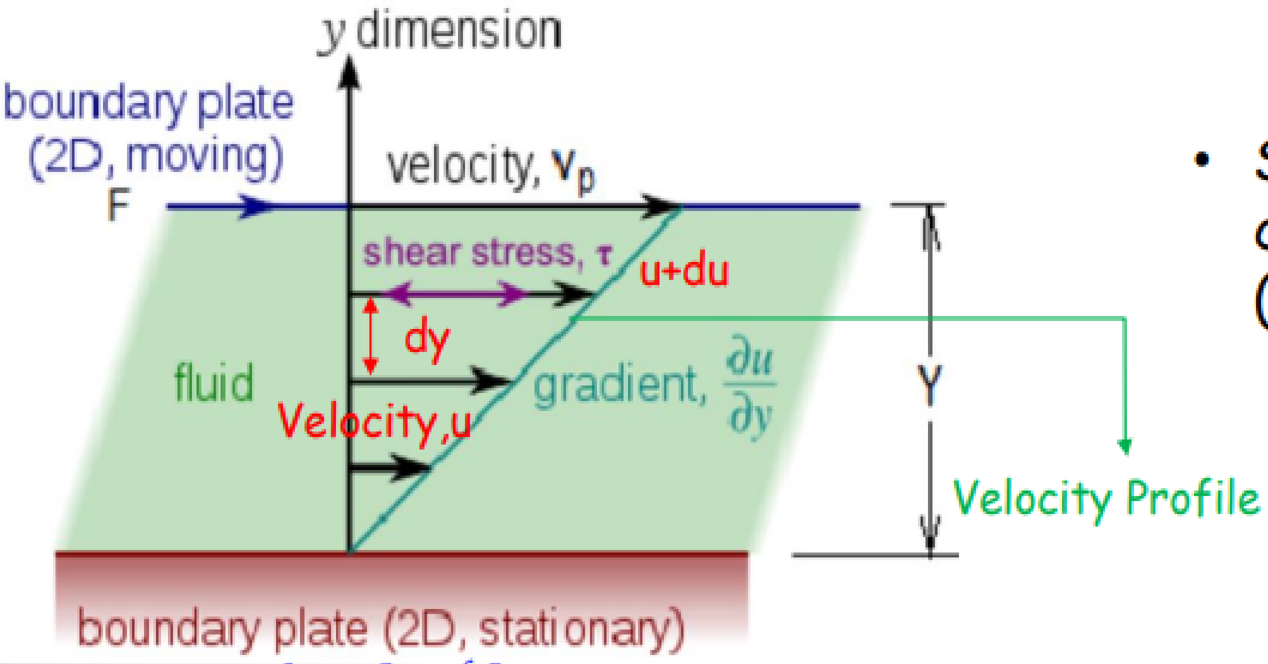
σ	:	Surface tension
θ	:	Angle between liquid surface and glass tube
ρ	:	Density of liquid
g	:	Acceleration due to gravity
d	:	Diameter of the tube

[Unit mm or cm]

6 VISCOSITY

- Property of fluid which offers resistance to the movement of one layer of fluid over adjacent layer of the fluids.
- Flow of different fluids is different as their viscosities are different

Source : wikipedia



Source : Research Gate

NEWTON'S LAW OF VISCOSITY

- Shear stress (τ) is proportional to the change in velocity with respect to y (velocity gradient).

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

COEFFICIENT OF DYNAMIC VISCOSITY

DYNAMIC VISCOSITY (μ)

- Ratio of shear stress to velocity gradient.
- Mathematically,

$$\mu = \frac{\tau}{du/dy}$$

SI Unit : $\frac{N}{m^2}s$ or Pa.s

CGS Unit : Poise or $\frac{dyne}{cm^2}s$

[1 poise = 0.1 Pa.s]

KINEMATIC VISCOSITY (υ)

- Ratio of dynamic viscosity(μ) and density of fluid (ρ)
- Mathematically,

$$\upsilon = \frac{\mu}{\rho}$$

SI Unit : $\frac{m^2}{s}$

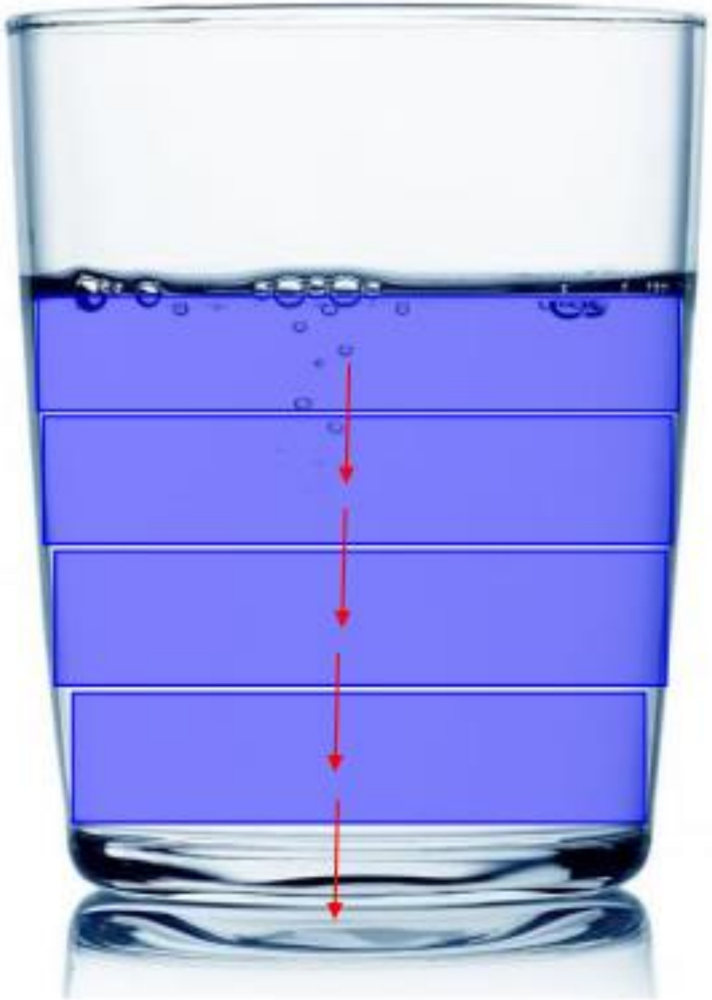
CGS Unit : stokes

[1 stoke = 1 $\frac{cm^2}{s}$]

PRESSURE

Force acting per unit area

LIQUID PRESSURE



Pressure is exerted by each layer of the liquid on the other layer due to the gravity force (weight of each layer)

PRESSURE AT A POINT ON LIQUID

Consider a small point of area dA in a large liquid mass.

The surrounding water exerts force dF on this area.

Intensity of pressure at that point, $p = \frac{dF}{dA}$.

If this force 'F' is uniformly distributed over area 'A' ,

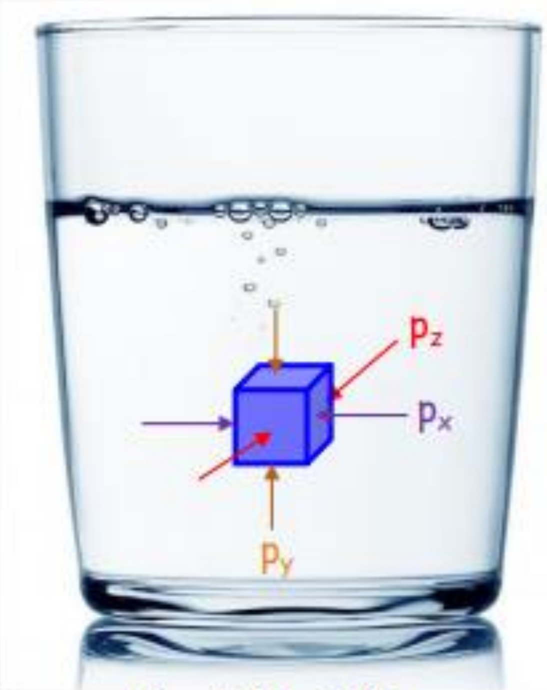
$$\boxed{\text{Pressure intensity, } p = \frac{F}{A}}$$

[SI Unit Pa or N/m²]

PASCAL'S LAW

Blaise Pascal

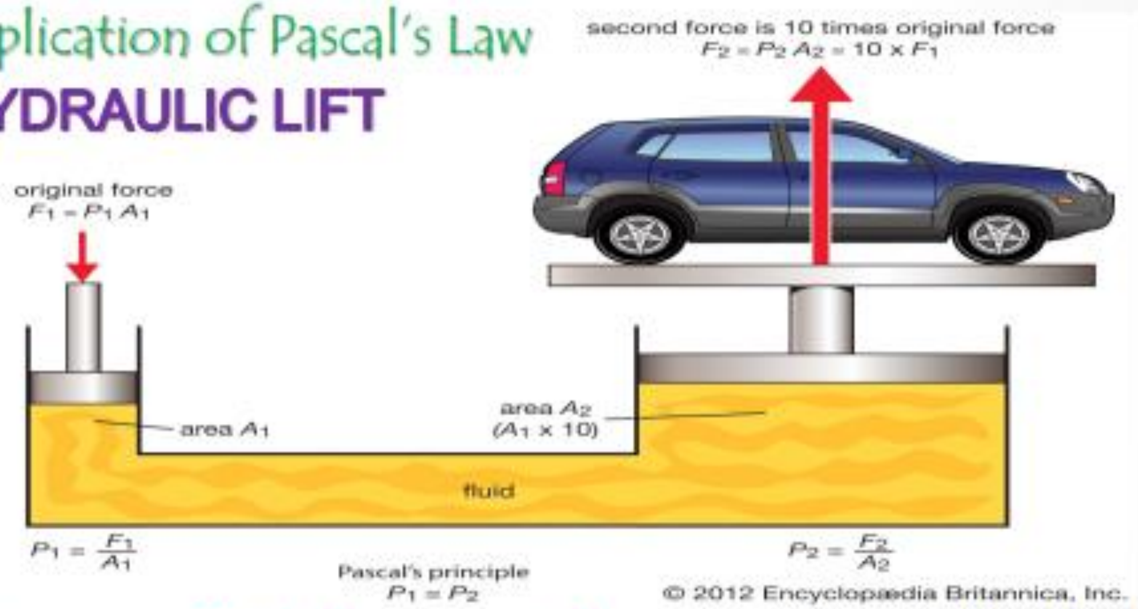
Pascal's law states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.



$p_x = p_y = p_z$

Application of Pascal's Law

HYDRAULIC LIFT



Pressure applied at one point in a confined liquid will be transmitted equally in all the directions

PRESSURE HEAD AT A POINT

Consider a container having liquid filled to height 'h'.

Let 'z' be the distance of any point from the surface of the liquid.

The fluid surrounding the point exerts force on all points at this depth z.

Force exerted by the liquid, F = Weight of the liquid

Weight = Unit weight (w) x Volume (V)

So force, $F = w \times V = w \times A \times z$

Now, pressure of liquid at a depth 'z' from the top,

Pressure intensity, $p = \frac{F}{A} = \frac{w \times A \times z}{A} = wz$

$$\boxed{p = wz = \rho g z}$$

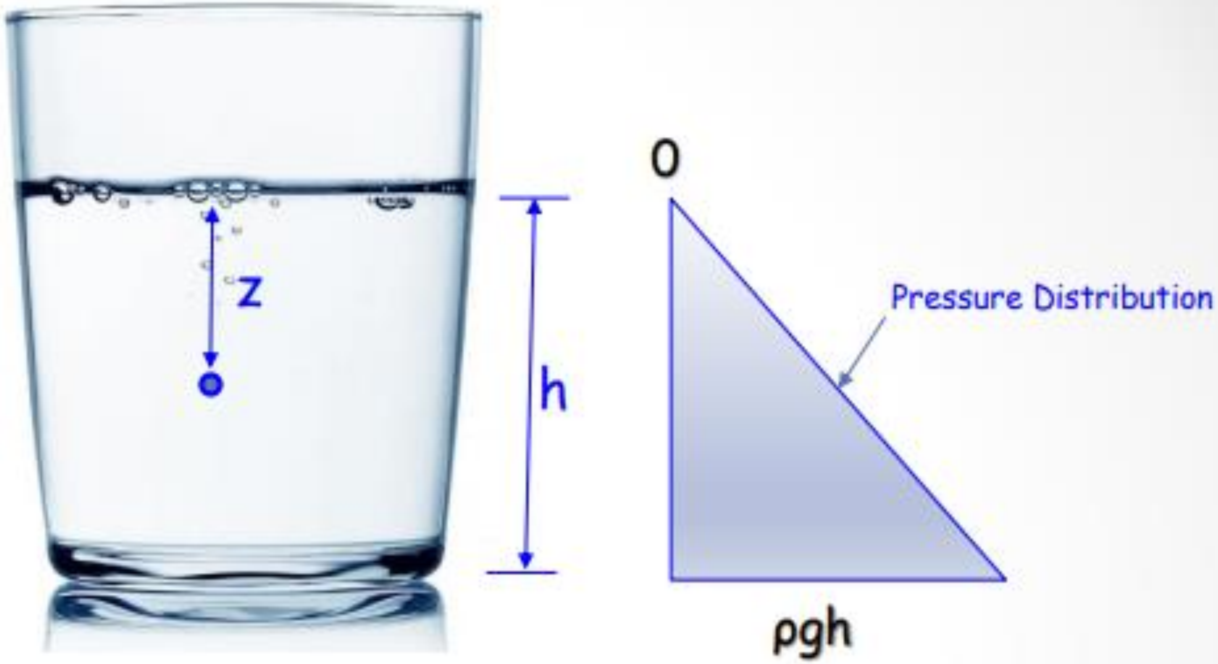
ρ	:	*Density of liquid*
g	:	*Acceleration due to gravity*
w	:	*Unit weight of liquid ($w=\rho g$)*
z	:	*Depth of liquid layer from the top of surface*

$$\boxed{p = wz = \rho g z}$$

When z=0, $p = 0$

When z=h, $p = \rho g h$

So pressure of liquid increases with the depth from free surface



Pressure in a particular liquid is dependent only on the depth from free surface.

The pressure represented in terms in terms of depth from free surface of liquid is known as pressure head

$$\boxed{\text{Pressure head}, z = \frac{p}{w} = \frac{p}{\rho g}}$$

If you give value of density/weight density of any liquid you can express pressure as pressure head in terms of that liquid !!!

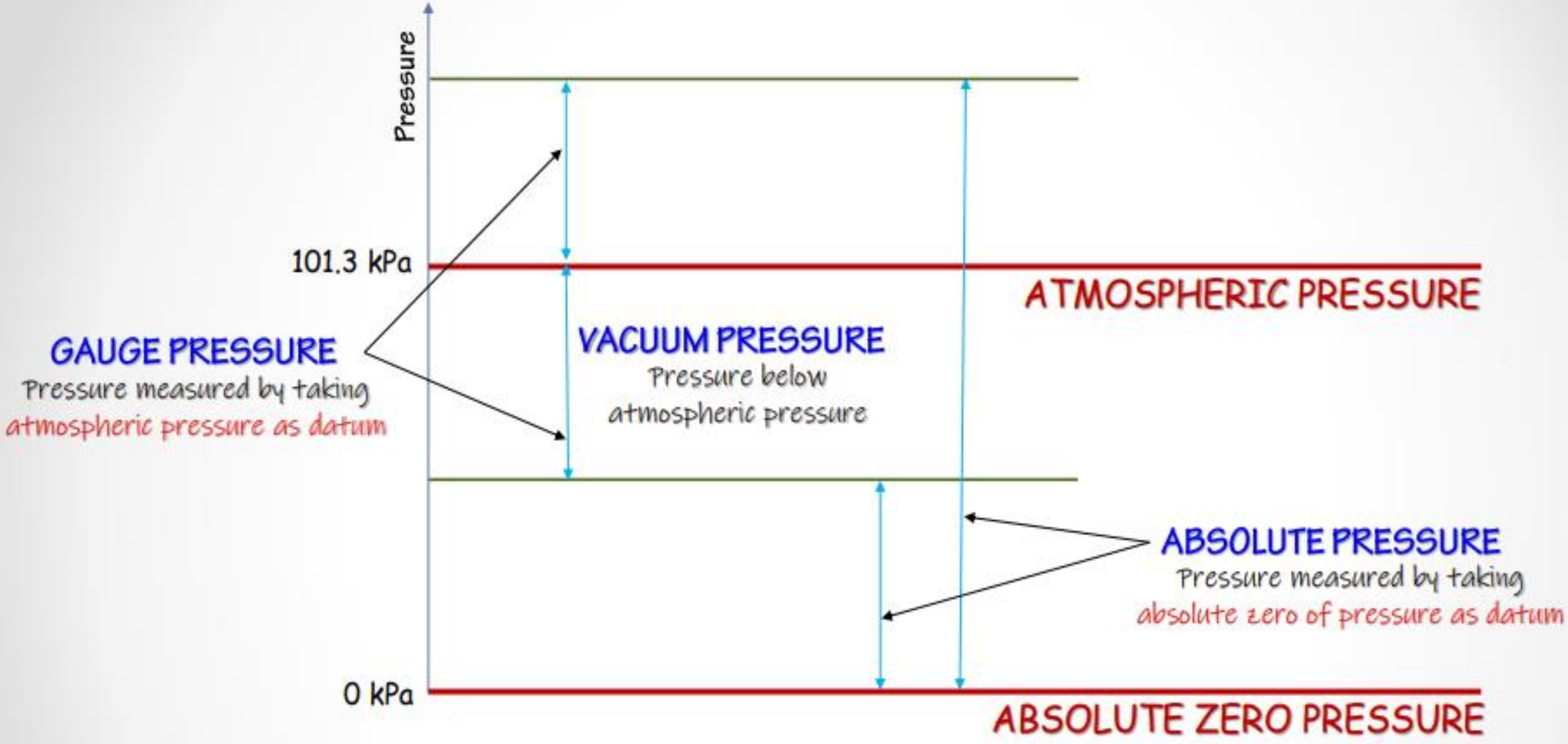
ATMOSPHERIC PRESSURE

- The normal pressure exerted by atmospheric air on all surfaces which is in contact with it is the atmospheric pressure.
- Also known as the barometric pressure.
- Standard atmospheric pressure is taken at 15°C at mean sea level

STANDARD ATMOSPHERIC PRESSURE

1 bar (10^5 N/m^2 approx.)	101.3 kPa	760 mm of Mercury	10.33 mm of water

Source : Fisher Scientific



Absolute Pressure = Atmospheric Pressure + Gauge Pressure

Vacuum Pressure = Atmospheric Pressure − Absolute Pressure

GAUGE PRESSURE

- Pressure measured with the help of pressure measuring devices in which atmospheric pressure is taken as datum (Atmospheric pressure is taken as zero.)

$$\boxed{P = \rho g z}$$

VACUUM PRESSURE

- If the measured pressure is below the atmospheric pressure then it is known as vacuum pressure. (value of pressure intensity will be negative)

ABSOLUTE PRESSURE

- Any pressure measured above absolute zero of pressure is termed as absolute pressure.
- Independent of atmospheric pressure.
- Absolute zero pressure exists only at complete vacuum.

- PRESSURE MEASURING DEVICES

  (Gauge Pressure)

  - SIMPLE MANOMETERS
    - PIEZOMETERS
    - U-TUBE MANOMETERS
    - SINGLE COLUMN MANOMETERS
      - VERTICAL SINGLE COLUMN MANOMETERS
      - INCLINED SINGLE COLUMN MANOMETERS
  - DIFFERENTIAL MANOMETERS
    - U-TUBE DIFFERENTIAL MANOMETERS
    - INVERTED U-TUBE DIFFERENTIAL MANOMETERS
  - MECHANICAL GAUGES
    - DIAPHRAGM PRESSURE GAUGES
    - BOURDON TUBE PRESSURE GAUGES
    - DEAD WEIGHT PRESSURE GAUGES
    - BELLOWS PRESSURE GAUGES

1. SIMPLE MANOMETERS

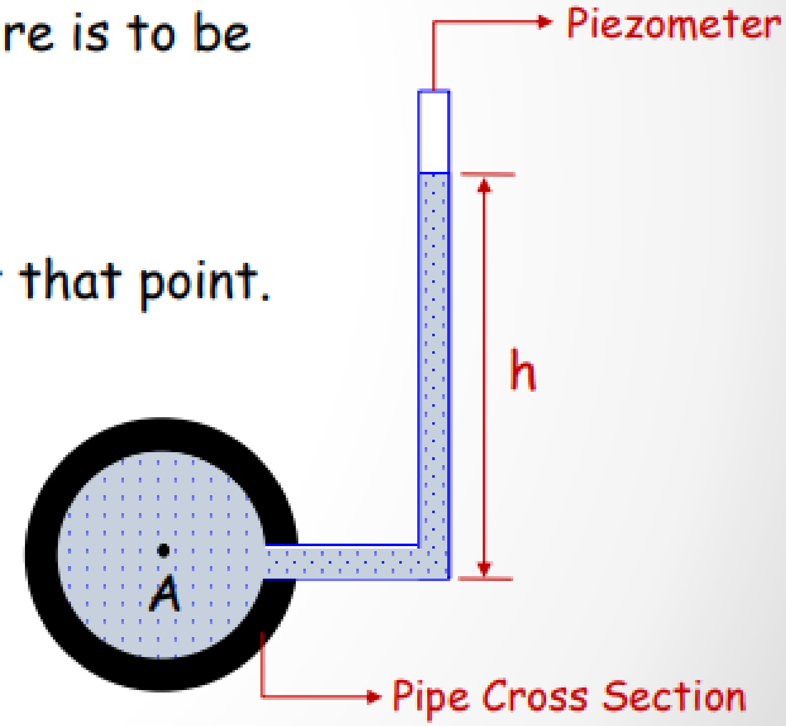
Manometers which consists of a glass tube having one of its ends connected to a point where pressure is to be measured and the other end is open to atmosphere.

PIEZOMETER

- A simple glass tube is connected to a point where the pressure is to be measured.
- The liquid rises in the tube connected.
- The rise of the water in the tube 'h', is the pressure head at that point.
- Pressure in SI unit at the point is,

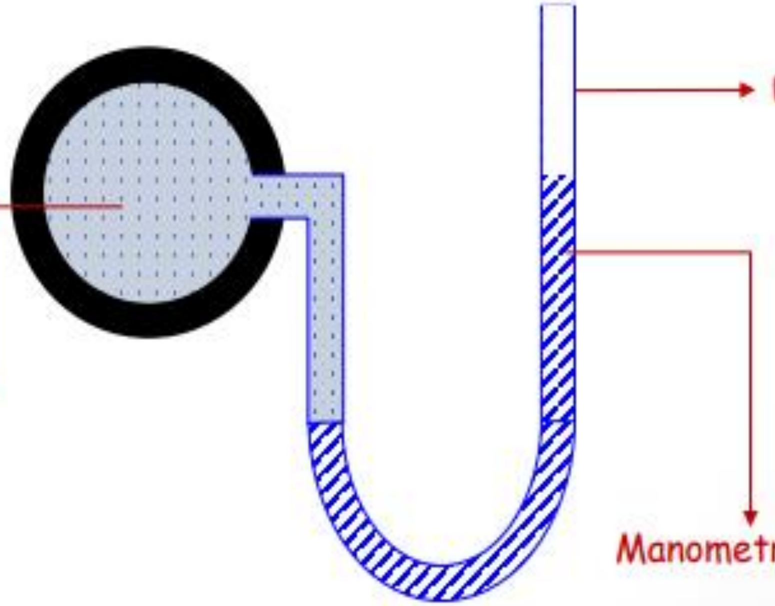
$$\boxed{P = \rho g h}$$

ρ	:	*Density of liquid in the pipe*
g	:	*Acceleration due to gravity*
h	:	*Rise of water in the piezometer*

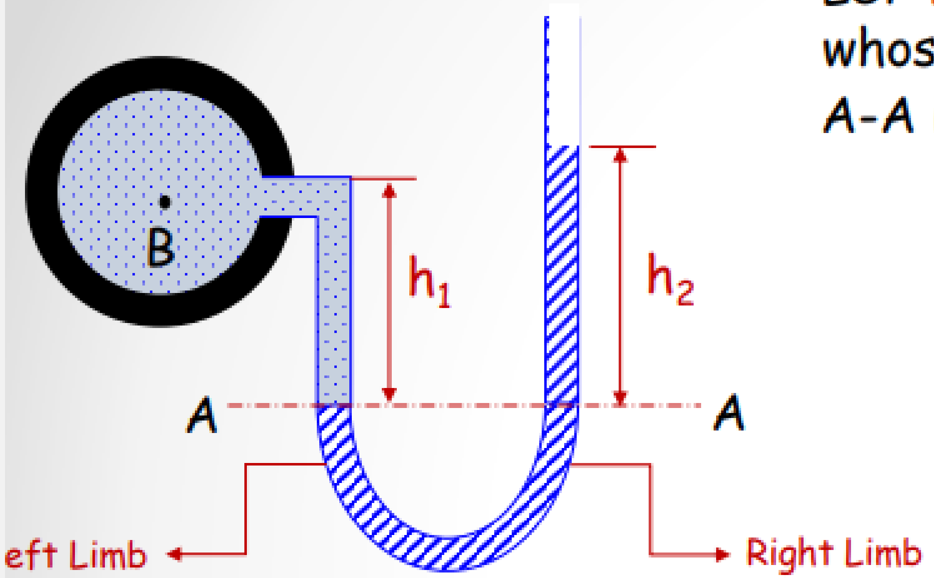


U-TUBE MANOMETER

- U-tube manometers are used when large pressures are to be measured.
- Consists of a glass tube bent in U-shape, one end of which is connected to the point where pressure is to be measured and other end is open to atmosphere.
- U tube is filled with a liquid whose specific gravity is higher than that of the liquid whose pressure is to be measures.



1. MEASUREMENT OF GAUGE PRESSURE/POSITIVE PRESSURE



Let B be the point where pressure is to be measured whose value is p.

A-A is the datum line.

h_1	:	*Height of the light liquid above the datum line*
h_2	:	*Height of the heavy liquid above the datum line*
S_1	:	*Specific Gravity of light liquid*
S_2	:	*Specific Gravity of heavy liquid*
ρ_1	:	*Density of light liquid = 1000*S_1*
ρ_2	:	*Density of heavy liquid = 1000*S_2*

Pressure is same for a horizontal surface

Therefor,

Pressure above datum in left limb = Pressure above datum in right limb

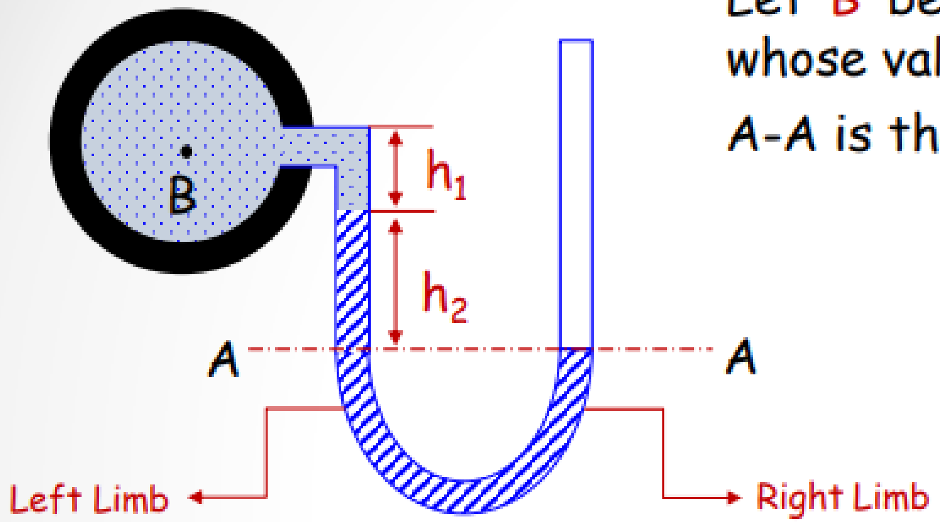
Pressure above datum in left limb = $p + \rho_1 g h_1$ .... (1)

Pressure above datum in right limb = $\rho_2 g h_2$ .... (2)

(1)=(2) , Hence, $p + \rho_1 g h_1 = \rho_2 g h_2$

Pressure at point B, $\boxed{p = \rho_2 g h_2 - \rho_1 g h_1}$

2. MEASUREMENT OF VACUUM PRESSURE/NEGATIVE PRESSURE



Let B be the point where pressure is to be measured whose value is p.

A-A is the datum line.

h_1	:	*Height of the light liquid above the datum line*
h_2	:	*Height of the heavy liquid above the datum line*
S_1	:	*Specific Gravity of light liquid*
S_2	:	*Specific Gravity of heavy liquid*
ρ_1	:	*Density of light liquid = 1000*S_1*
ρ_2	:	*Density of heavy liquid = 1000*S_2*

Pressure above datum in left limb = $p + \rho_1 g h_1 + \rho_2 g h_2$ .... (1)

Pressure above datum in right limb = 0 .... (2)

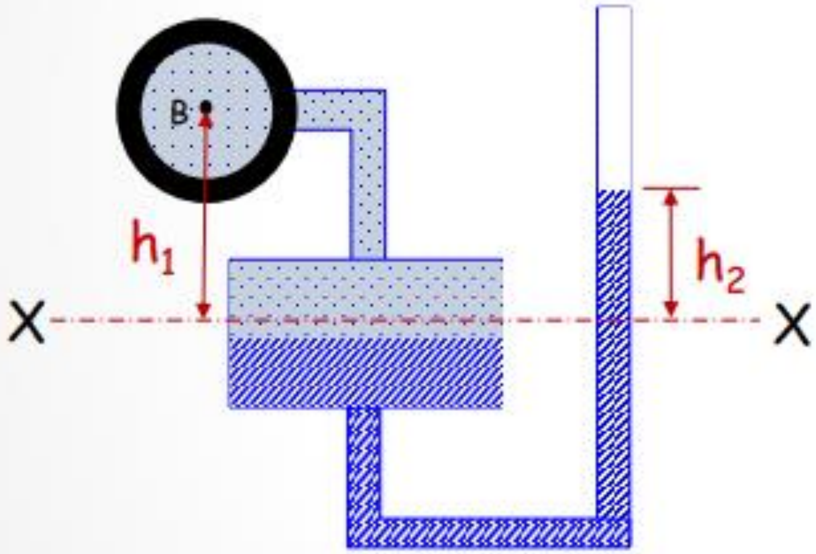
(1)=(2) , Hence, $p + \rho_1 g h_1 + \rho_2 g h_2 = 0$

Pressure at point B, $\boxed{p = -(\rho_1 g h_1 + \rho_2 g h_2)}$

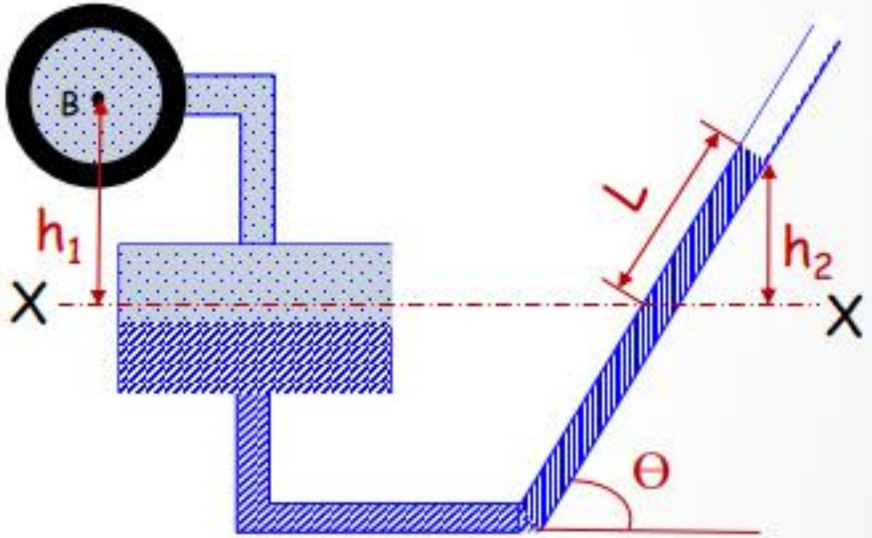
SINGLE COLUMN MANOMETER

- U- tube manometer modified with a reservoir
- Reservoir diameter is 100 times than that of the pipe.
- Higher the cross sectional area lesser will be the change in level of fluid in pipe.
- Useful in measuring large pressure.

1. VERTICAL SINGLE COLUMN MANOMETER



2. INCLINED SINGLE COLUMN MANOMETER



X-X is the datum line in the reservoir before connecting the pipe

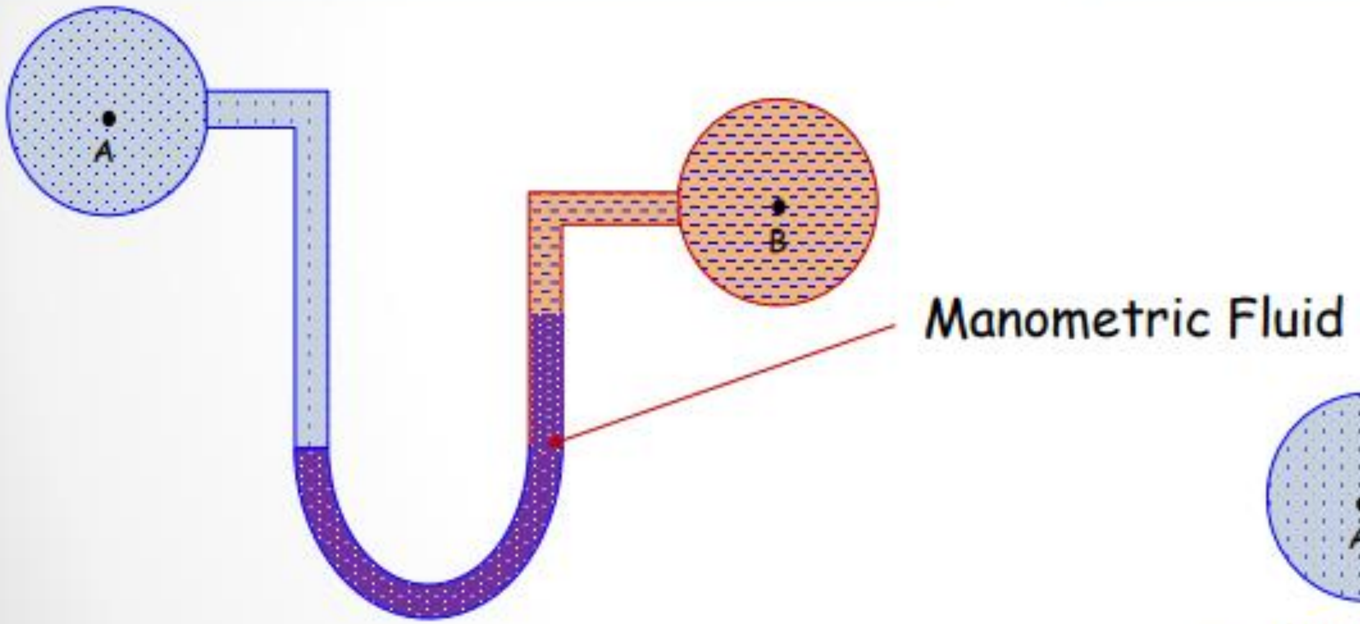
$$p = \rho_2 g h_2 - \rho_1 g h_1$$

$$p = \rho_2 g\, L(\sin\Theta) - \rho_1 g h_1$$

2. DIFFERENTIAL MANOMETERS

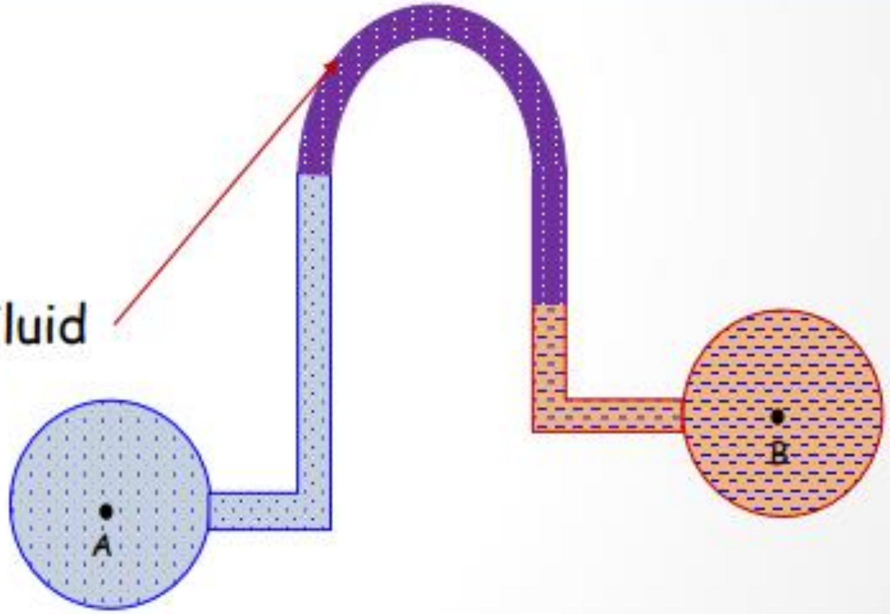
- Used to measure pressure between two points in a pipe or two different pipes.
- Differential manometers consists of a U-tube containing liquid, whose two ends are connected to the points whose difference of pressure is to be measured.

1. U-TUBE DIFFERENTIAL MANOMETER



- Contains heavy liquid in U-Tube.

2. INVERTED U-TUBE DIFFERENTIAL MANOMETER



- Inverted U-Tube contains lighter liquid.
- Measures difference in low pressures.

ASSIGNMENT 1

QUE 1 – BOURDON TUBE PRESSURE GAUGE – DESCRIPTION WITH NEAT SKETCH

HYDROSTATIC FORCES ON SURFACES

Source : Pinterest

TOTAL PRESSURE

- **Force exerted by a static fluid on a surface** when a fluid comes in contact with the surface is called total pressure.

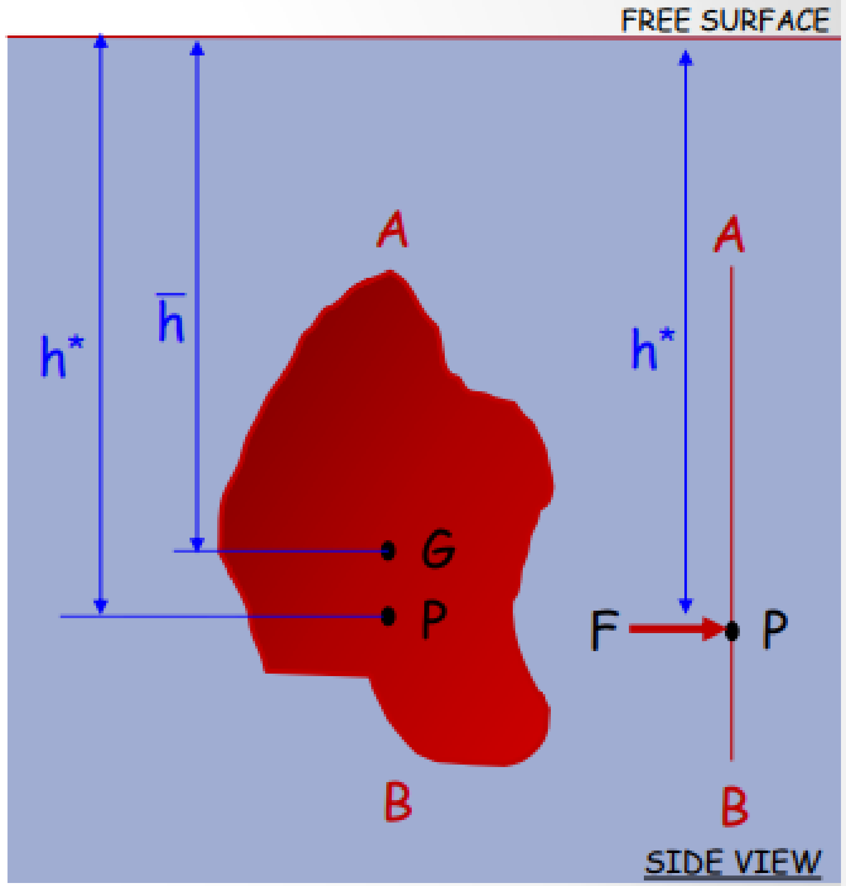
CENTRE OF PRESSURE

- Centre of pressure is defined as the **point of application of total pressure** on the surface.
- Force will always be normal to the surface

VERTICAL PLANE SURFACE IMMERSED IN A LIQUID

Consider a plane vertical surface of any arbitrary shape immersed in a liquid.

A	:	Area of the surface
G	:	Centre of gravity of plane surface
$\bar{h}$	:	Depth of C.G. of area from the free surface of liquid
P	:	Centre of Pressure
h^*	:	Depth of centre of pressure from free surface of liquid
ρ	:	Density of liquid



TOTAL PRESSURE (F)

Consider a small strip of thickness dh and width b at a depth of h from the free surface.

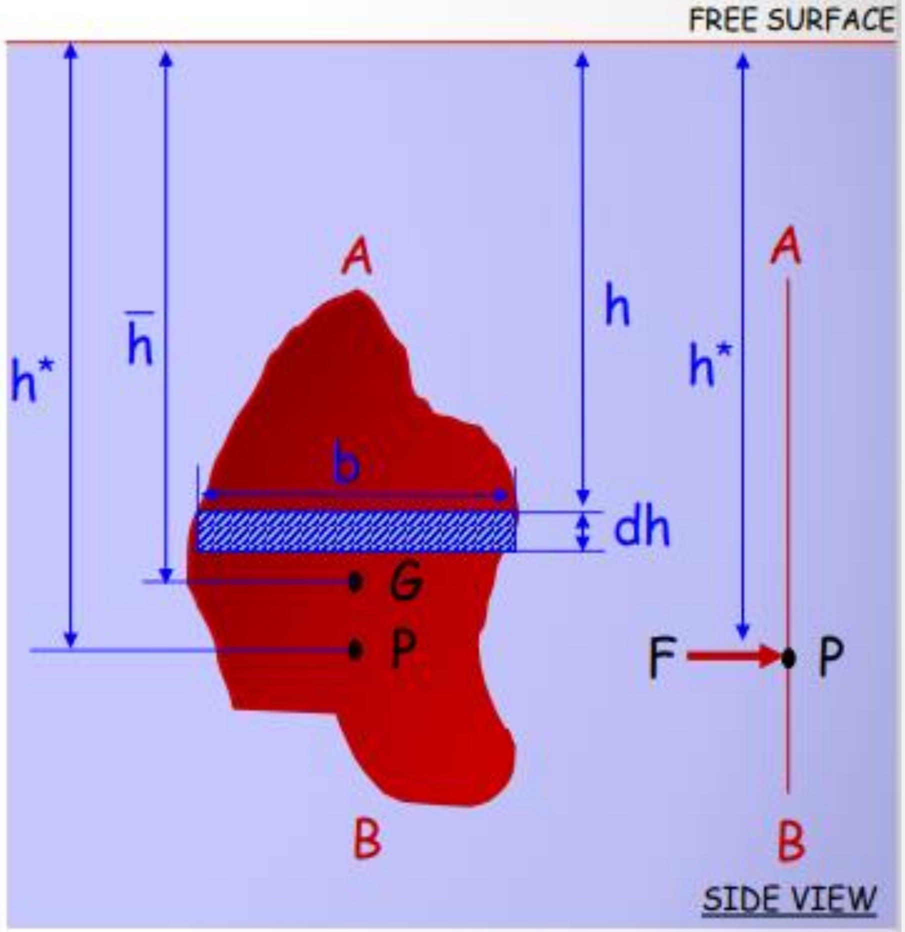
Pressure intensity on the strip, $p = \rho g h$

Area of the strip , $dA = b * dh$

Total pressure force on the strip, $dF = P * dA = \rho g h * b * dh$

Total pressure force on the surface,
$$F = \int dF$$
$$= \int \rho g h * b * dh$$
$$= \rho g \int h * (b * dh)$$
$$= \rho g \int h * dA$$

Moment of Area = $A\bar{h}$



Hence, total pressure force on the surface,

$$\boxed{F = \rho g A \bar{h}}$$

CENTRE OF PRESSURE (h^*)

Recalling Principle of moments, We have,

Moment due to resultant forces about an axis = Sum of moments due to component forces about the same axis

Moment due to resultant force about the free surface $= F \times h^* = \rho g A\bar{h} \times h^*$ ..(1)

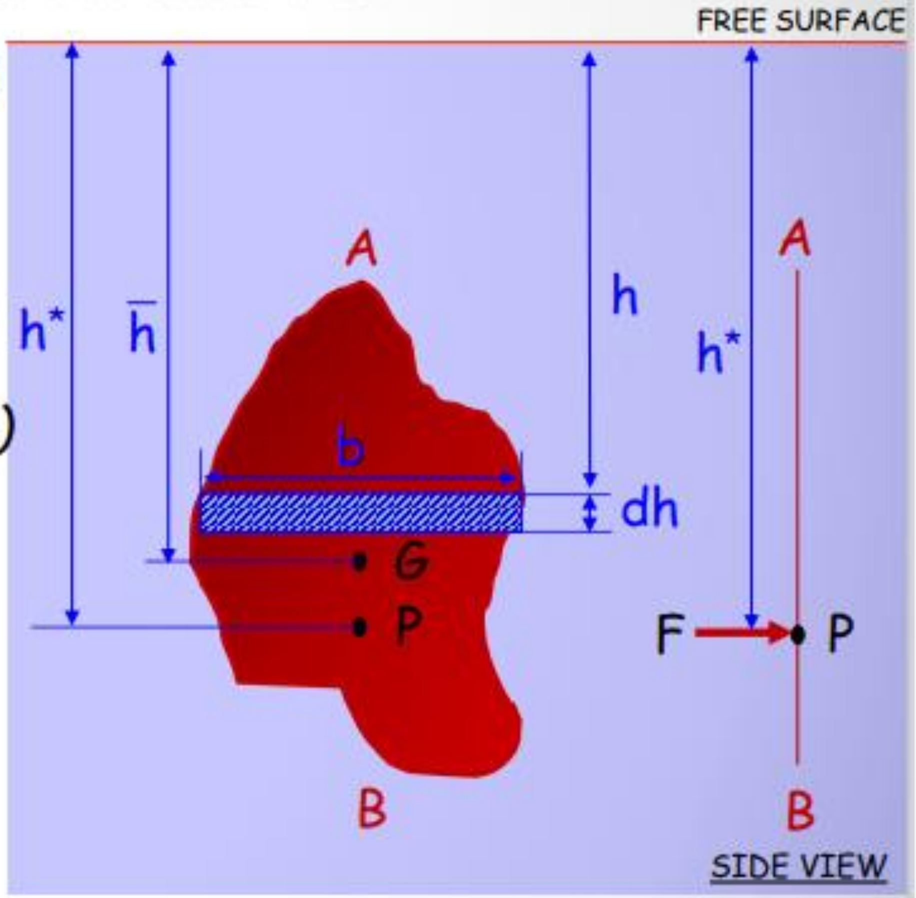
Moment due to a single strip about the free surface $= (\rho g h \times dA) \times h = \rho g h^2 dA$

Moment due to all such strips about the free surface $= \int \rho g h^2 dA = \rho g \int h^2 dA = \rho g I_0$ ..(2)

Second moment of Area/ Moment of Inertia = I_0

(1)=(2) , Hence, $\cancel{\rho g} A\bar{h} \times h^* = \cancel{\rho g} I_0$

Centre of pressure, $h^* = \frac{I_0}{A\bar{h}}$



Centre of pressure, $h^* = \frac{I_0}{A\bar{h}}$ → *Moment of inertia of the plane figure about the free surface*

From parallel axis theorem,

$$I_0 = I_G + A\bar{h}^2$$

A : *Area of the plane figure*

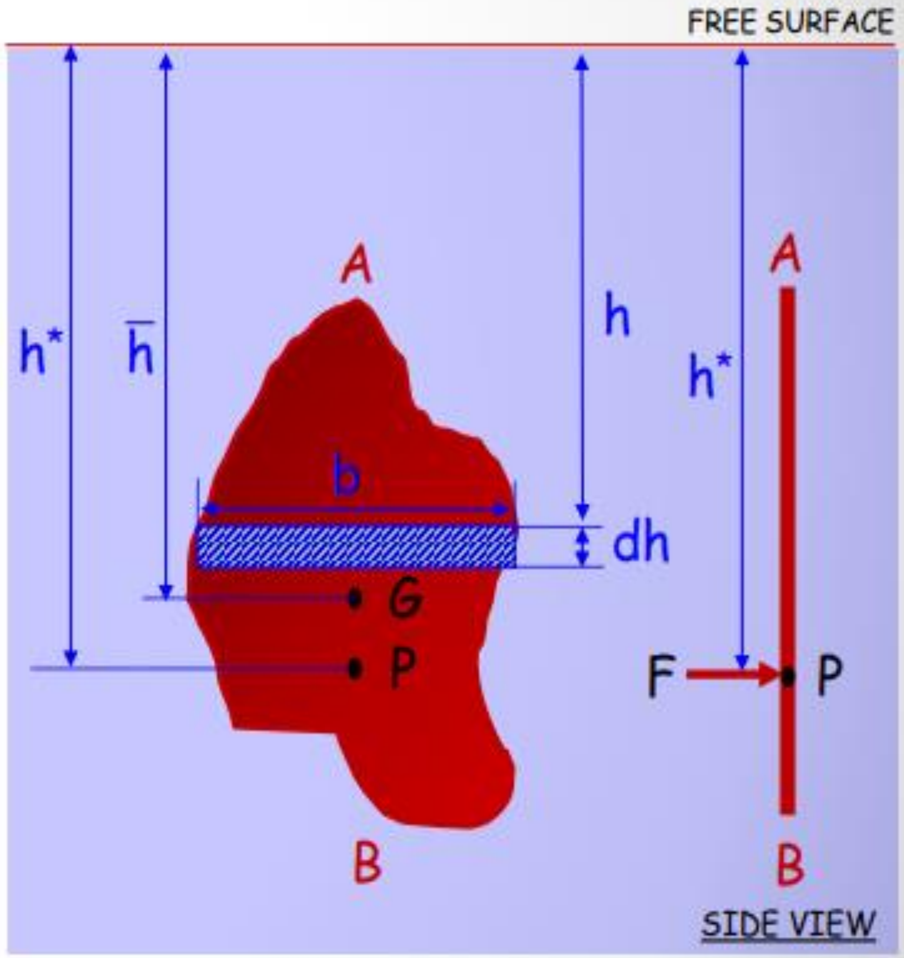
I_G : *Moment of inertia about centroidal axis*

$\bar{h}$: *Distance between C.G. and Free surface*

Substituting for I_0 in the equation for centre of pressure,

$$\text{Centre of pressure, } h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \frac{\cancel{A}\bar{h}^{\cancel{2}}}{\cancel{A}\cancel{\bar{h}}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$\boxed{\text{Centre of pressure, } h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$$



HORIZONTAL PLANE SURFACE IMMERSED IN A LIQUID

- Every point on the surface lies at same depth from the free surface of liquid.
- Hence the pressure intensity (p) will be equal on the entire surface.

$$p = \rho g h$$

h : Depth of plane surface from free surface of liquid

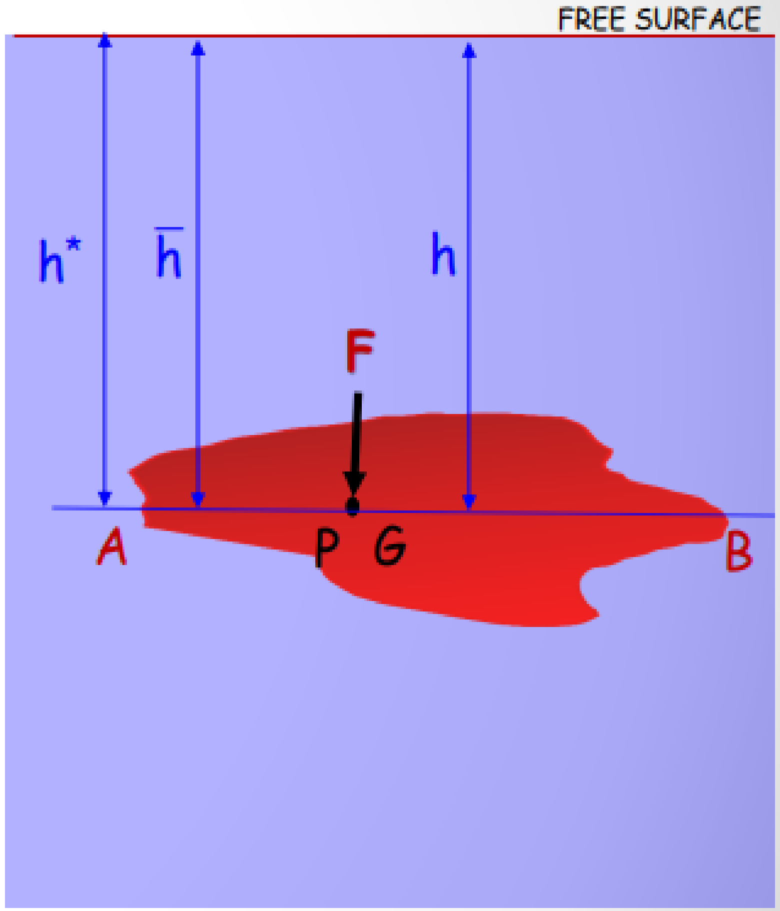
If A is the total surface area of the plane figure,

Total pressure force on the surface, $F = \rho g h^* A = \rho g \bar{h} A$

$$\boxed{F = \rho g A \bar{h}}$$

$\bar{h}$: Depth of C.G of plane surface from free surface of liquid

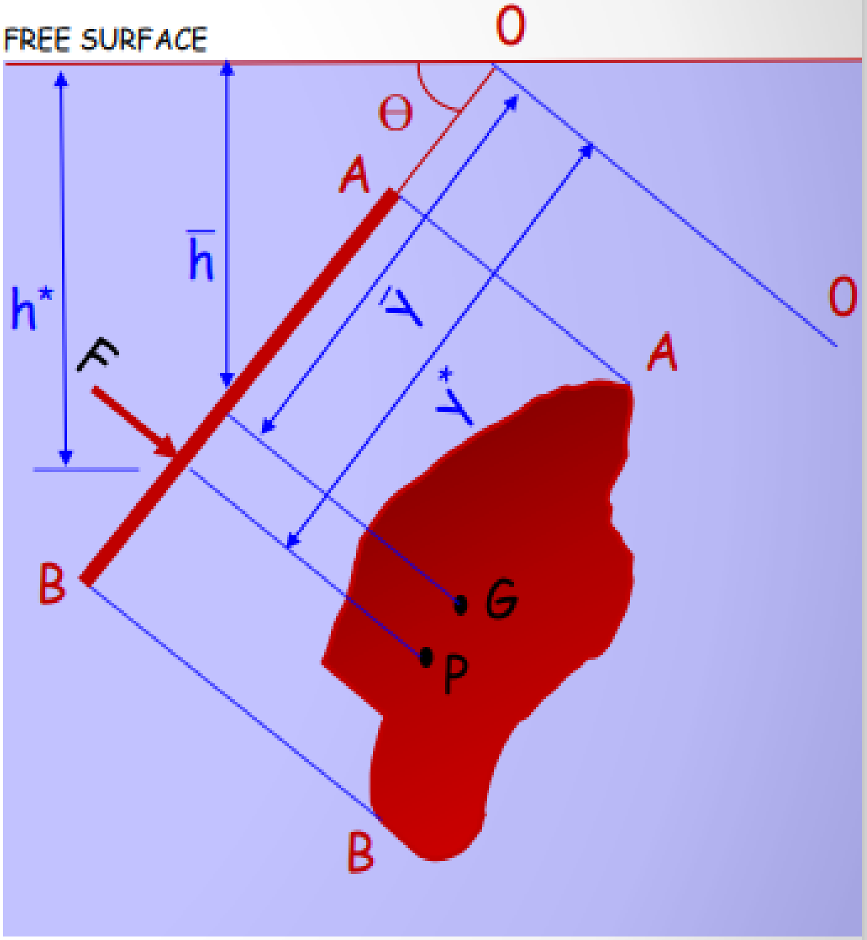
$$\boxed{\text{Centre of pressure, } h^* = \bar{h} = h}$$



INCLINED PLANE SURFACE IMMERSED IN A LIQUID

Consider a plane inclined surface of any arbitrary shape immersed in a liquid.

A	:	Area of the surface
θ	:	Angle made by plane surface with the free liquid surface
G	:	Centre of gravity of plane surface
$\bar{h}$	:	Depth of C.G. of area from the free surface of liquid
P	:	Centre of Pressure
h^*	:	Depth of centre of pressure from free surface of liquid
ρ	:	Density of liquid



Let O be the point of projection of the plane surface on the free surface of liquid.

$\bar{y}$	:	Distance of C.G of inclined surface from O.
y^*	:	Distance of centre of pressure from O

TOTAL PRESSURE

Consider a small strip of area dA at a depth h from the free surface and at a distance y from the axis O-O.

Pressure intensity on the strip, $p = \rho g h$

Total pressure force on the strip, $dF = p * dA = \rho g h * dA$

Total pressure force on the surface, $F = \int dF = \int \rho g h * dA$ ..(1)

From the figure,

$$\text{Sin}\,\Theta = \frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} \longrightarrow \begin{array}{l} h = y\sin\Theta \quad ..(2) \\ \bar{h} = \bar{y}\sin\Theta \quad ..(3) \end{array}$$

Substituting (2) in (1),

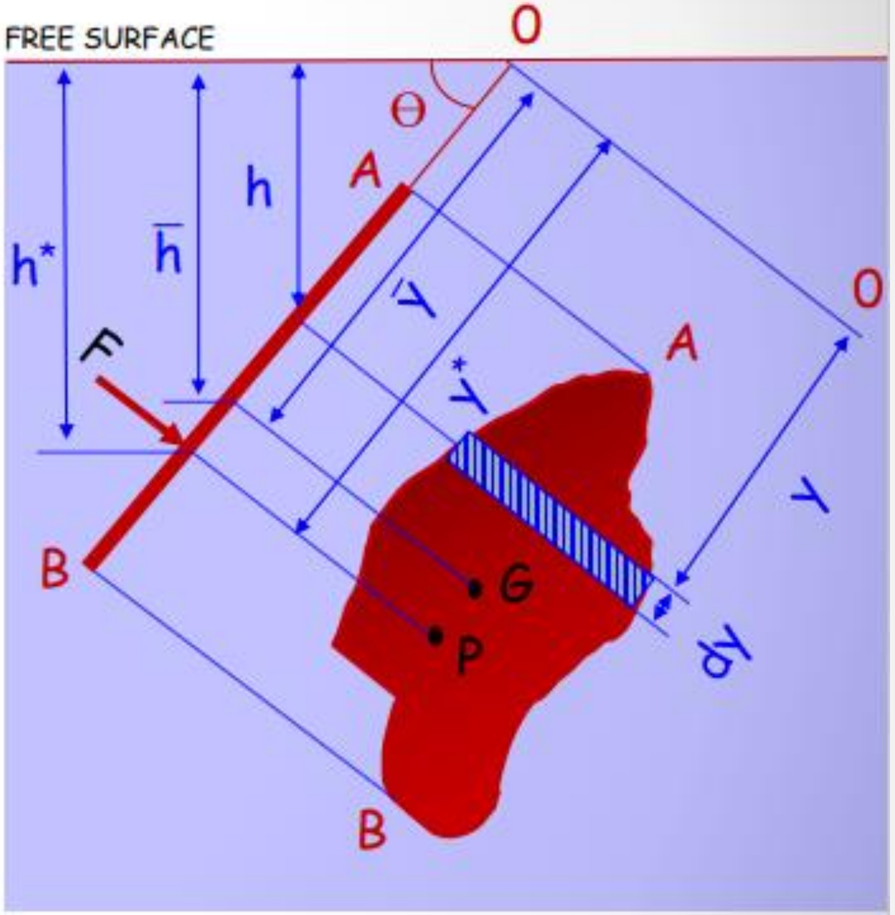
$$F = \int \rho g y \sin\theta * dA = \rho g \sin\theta \int y * dA$$

Moment of Area = $A\bar{y}$

So, $F = \rho g \sin\theta\, A\bar{y} = \rho g A(\bar{y}\sin\theta)$

Substituting for $\bar{y}\sin\theta$ from *(3)*,

$$\boxed{\text{Total pressure force, } F = \rho g A \bar{h}}$$



CENTRE OF PRESSURE (h^*)

Total pressure force on the strip, $dF = \rho gh * dA = \rho g (y \sin\Theta) * dA$ (As $h = y \sin\Theta$)

Moment of force dF about the axis O-O $= dF*y$

$= \rho gy \sin\Theta * dA * y$

$= \rho gy^2 \sin\Theta * dA$

Sum of moments of all forces about O-O $= \int \rho gy^2 \sin\Theta * dA$

$= \rho g \sin\Theta \int y^2 * dA$

Second moment of Area/Moment of Inertia $= I_0$

So,

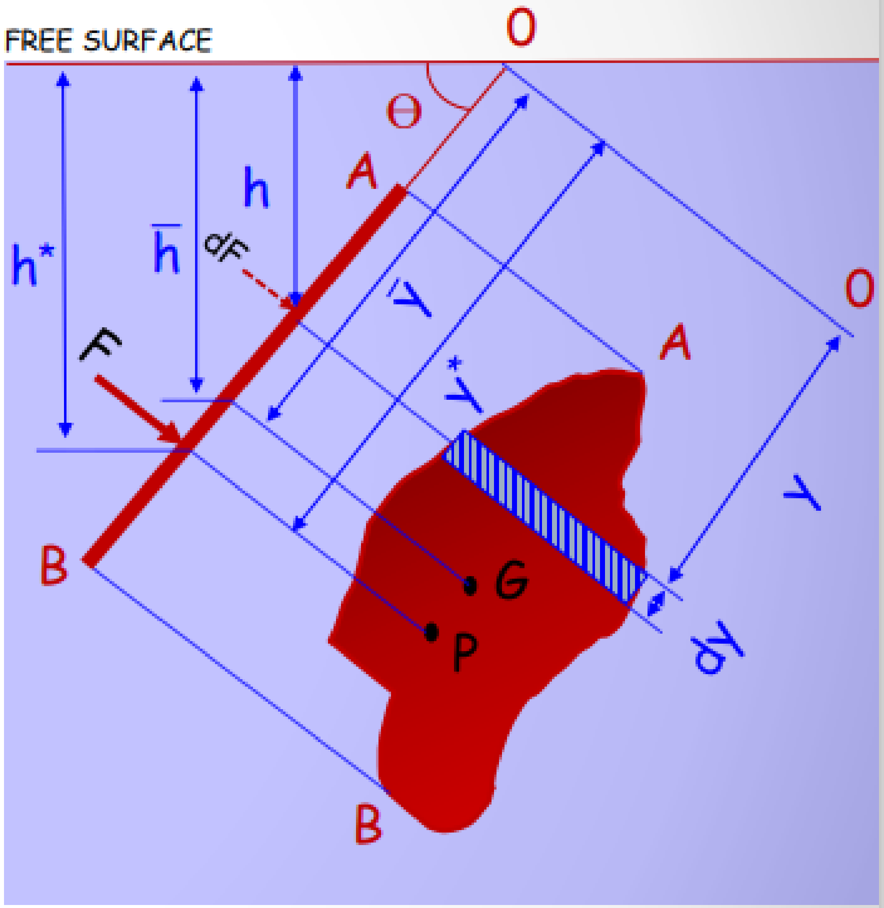
Sum of moments of all forces about O-O $= \rho g \sin\Theta I_0$ ..(1)

Moment due to resultant force about O-O $= F \times y^* = \rho g A\bar{h}\, y^*$ ..(2)

From Principle of moments, We have,

(1)=(2), $\cancel{\rho g} \sin\Theta I_0 = \cancel{\rho g} A\bar{h}\, y^*$

$$y^* = \frac{I_0 \sin\Theta}{A\bar{h}}$$



$$y^* = \frac{I_0 \sin\Theta}{A\bar{h}} \quad ..(3)$$

From the figure,

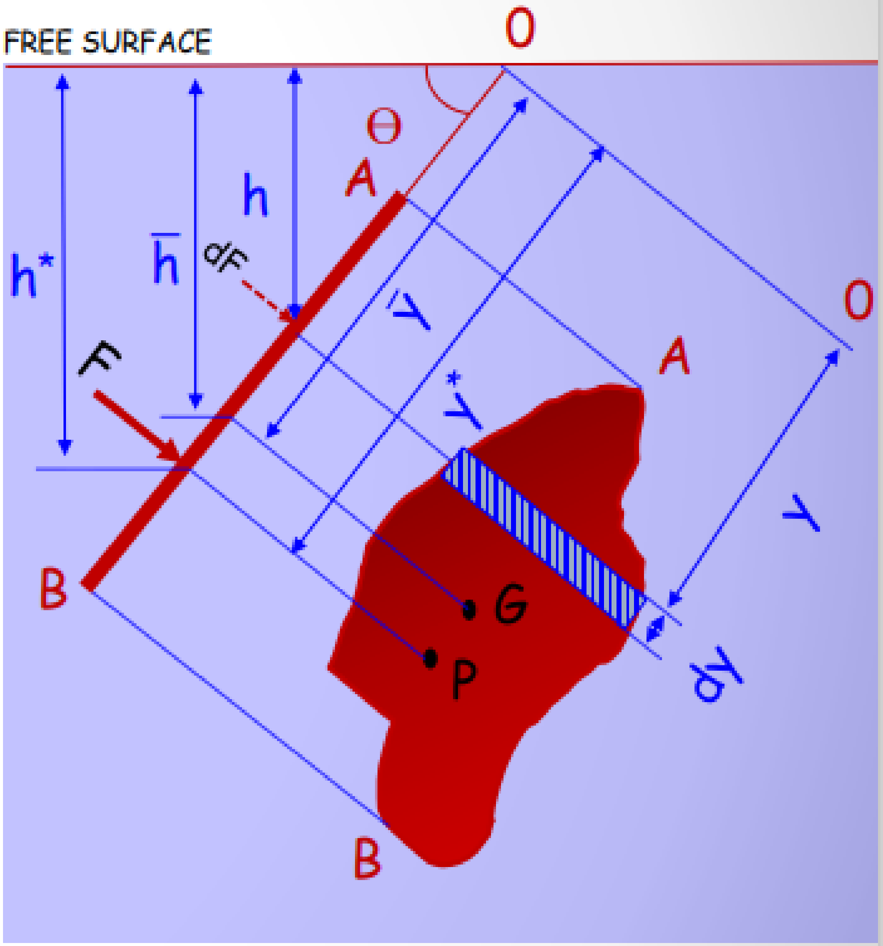
$$\sin\Theta = \frac{h^*}{y^*} \longrightarrow y^* = \frac{h^*}{\sin\Theta} \quad ..(4)$$

Substituting (4) in (3),

$$\frac{h^*}{\sin\Theta} = \frac{I_0 \sin\Theta}{A\bar{h}} \longrightarrow h^* = \frac{I_0 \sin^2\Theta}{A\bar{h}} \quad ..(5)$$

Now, $I_0 = I_G + A\bar{y}^2$ *(Parallel axis theorem)*

$$I_0 = I_G + A\left(\frac{\bar{h}}{\sin\Theta}\right)^2 \quad \left(As,\ \bar{y} = \frac{\bar{h}}{\sin\Theta}\right)$$



Substituting I_0 for in (5),

$$h^* = \left(I_G + A\left(\frac{\bar{h}}{\sin\Theta}\right)^2\right)\frac{\sin^2\Theta}{A\bar{h}}$$

$$= \left(I_G \frac{\sin^2\Theta}{A\bar{h}}\right) + \left(\frac{\cancel{A}\bar{h}^{\cancel{2}}}{\cancel{\sin^2\Theta}} * \frac{\cancel{\sin^2\Theta}}{\cancel{A\bar{h}}}\right)$$

$$\boxed{\text{Centre of pressure, } h^* = \frac{I_G}{A\bar{h}}\sin^2\Theta + \bar{h}}$$

SUMMARY

- For VERTICAL/HORIZONTAL/INCLINED PLANE SURFACE immersed in liquid,

$$\boxed{\text{Total pressure force, } F = \rho g A\bar{h}}$$

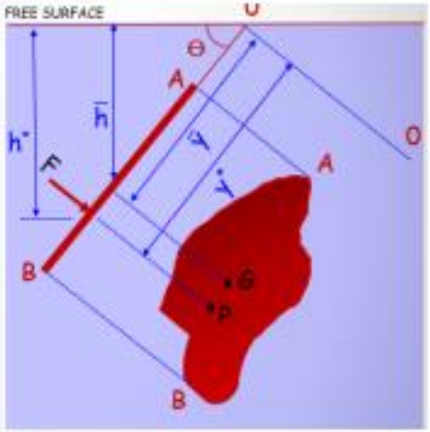
- For INCLINED PLANE SURFACE immersed in liquid,

$$\boxed{\text{Centre of pressure, } h^* = \frac{I_G}{A\bar{h}}\sin^2\Theta + \bar{h}}$$

When $\Theta = 0°$, HORIZONTAL PLANE SURFACE

$$\boxed{\text{Centre of pressure, } h^* = \bar{h} = h}$$

h : Depth of plane surface from free surface of liquid

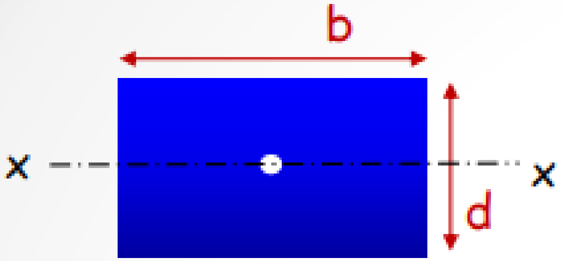


When $\Theta = 90°$, VERTICAL PLANE SURFACE

$$\boxed{\text{Centre of pressure, } h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$$

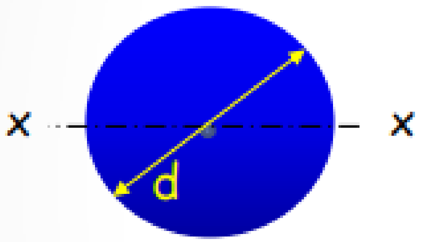
A	:	Area of the surface
Θ	:	Angle made by plane surface with the free liquid surface
$\bar{h}$	:	Depth of C.G. of area from the free surface of liquid
I_G	:	*Moment of inertia about centroidal axis*
ρ	:	Density of liquid
g	:	Acceleration due to gravity

RECALL



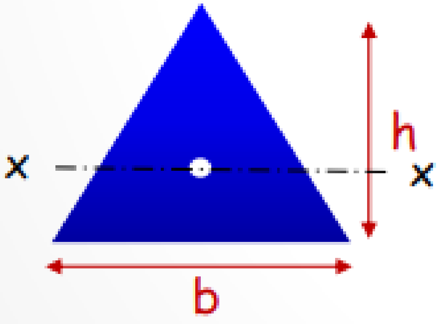
CG@ d/2

$$I_G = \frac{bd^3}{12}$$



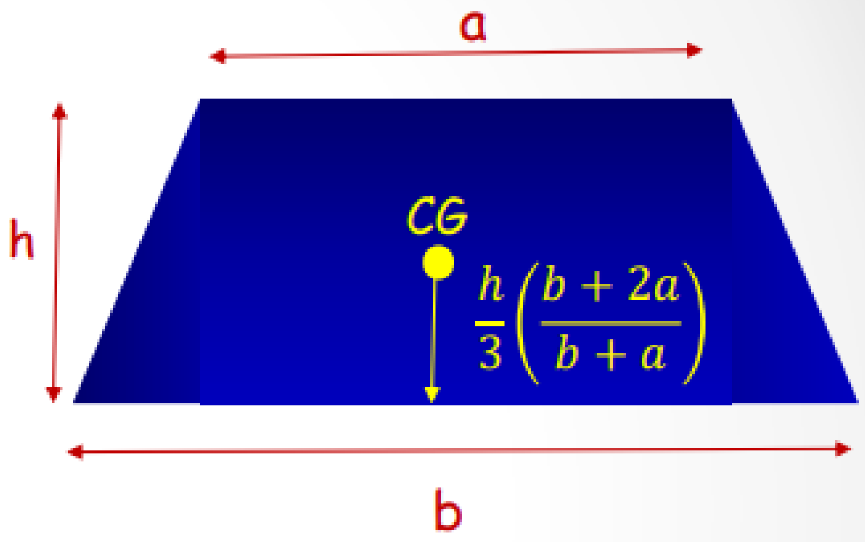
CG@ d/2

$$I_G = \frac{\pi d^4}{64}$$



CG@ h/3 from base

$$I_G = \frac{bh^3}{36}$$



$$\text{Area} = \left(\frac{a+b}{2}\right) \times h$$

$$I_G = \left[\frac{a^2+4ab+b^2}{36(a+b)}\right] \times h^3$$

1. A rectangular plane surface is 2 m wide and 3 m deep. It lies in a vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and a) coincides with water surface, b) 2.5 m below the free water surface.

SOLUTION

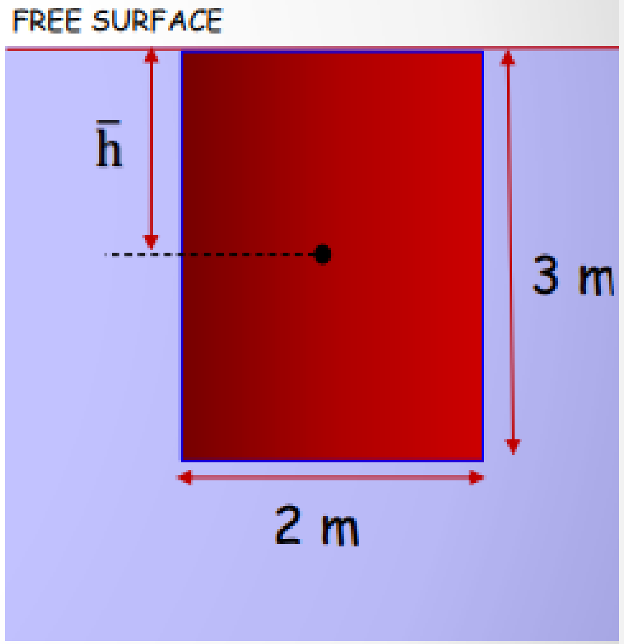
CASE 1 : UPPER EDGE COINCIDES WITH WATER SURFACE

Given, $b = 2$ m $d = 3$ m Density of liquid, $\rho = 1000\ kg/m^3$

Area of the plane figure, $A = b*d = 2*3 = 6\ m^2$

Depth of CG from free surface, $\bar{h} = d/2 = 3/2 = 1.5$ m

Moment of inertia about C.G axis, $I_G = bd^3/12 = 2*3^3/12 = 4.5\ m^4$



Now,

Total pressure force, $F = \rho g A\bar{h}$ $= 1000*9.81*6*1.5$
$= 88.29x10^3$ N $=$ **88.29 kN**

Position of centre of pressure, $h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{4.5}{6*1.5} + 1.5 = \mathbf{2\ m}$

CASE 2 : UPPER EDGE IS 2.5 m BELOW WATER SURFACE

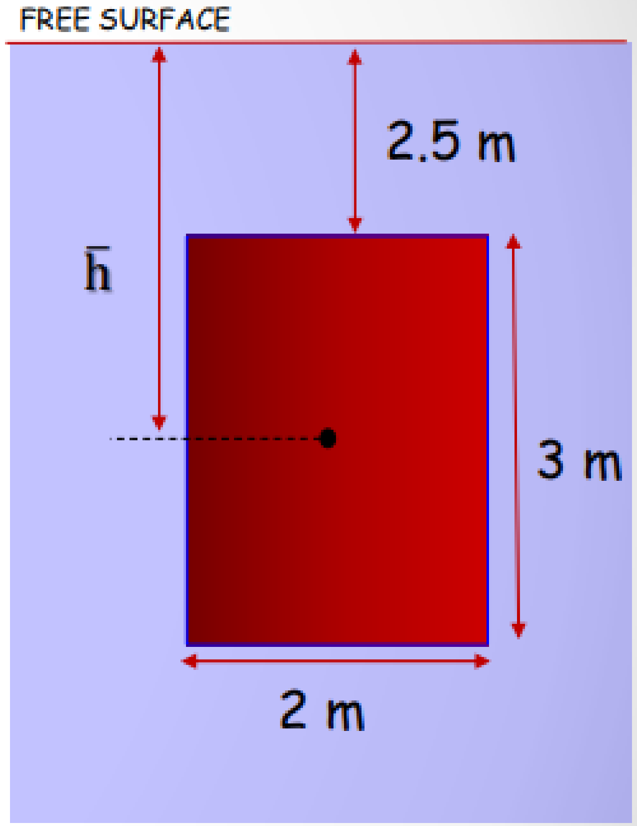
Given, b = 2 m d = 3 m Density of liquid, ρ = 1000 kg/m^3

Area of the plane figure, A = b*d = 2*3 = 6 m^2

Depth of CG from free surface, $\bar{h}$ = 2.5 + (d/2)

= 2.5 + (3/2) = 4 m

Moment of inertia about C.G axis, I_G = bd^3/12 = 2*3^3/12 = 4.5 m^4



Now,

Total pressure force, **F** = $\rho g A\bar{h}$ = 1000*9.81*6*4

= 235.44x10^3 N

= **235.44 kN**

Position of centre of pressure, **h*** = $\frac{I_G}{A\bar{h}} + \bar{h}$

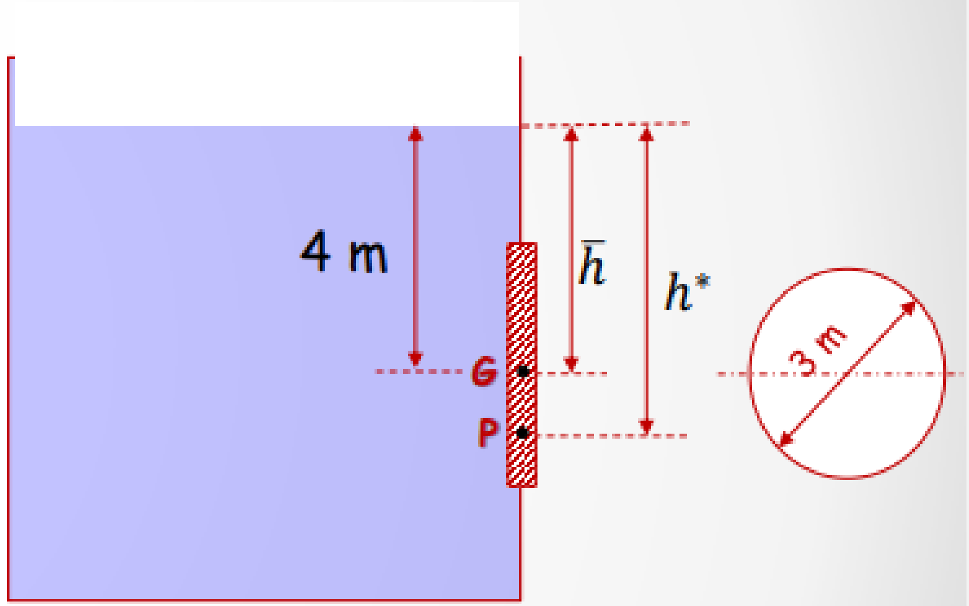
$= \frac{4.5}{6*4} + 4$ = **4.1875 m**

2. A circular opening, 3 m diameter, in a vertical side of a water tank is closed by a disc of 3 m diameter which can rotate about a horizontal diameter,

i) the force on the disc, and

ii) the torque required to maintain disc in equilibrium in the vertical position when the head of the water above horizontal diameter is 4 m.

SOLUTION



Here the circular disc is like a circular plane figure immersed in liquid.

i) FORCE ON DISC

The force on the disc is the total pressure on the circular disc.

Given, $d = 3$ m, Density of liquid, $\rho = 1000$ kg/m^3

Area of the plane figure, $A = \pi d^2/4 = \pi * 3^2/4 = 7.069$ m^2

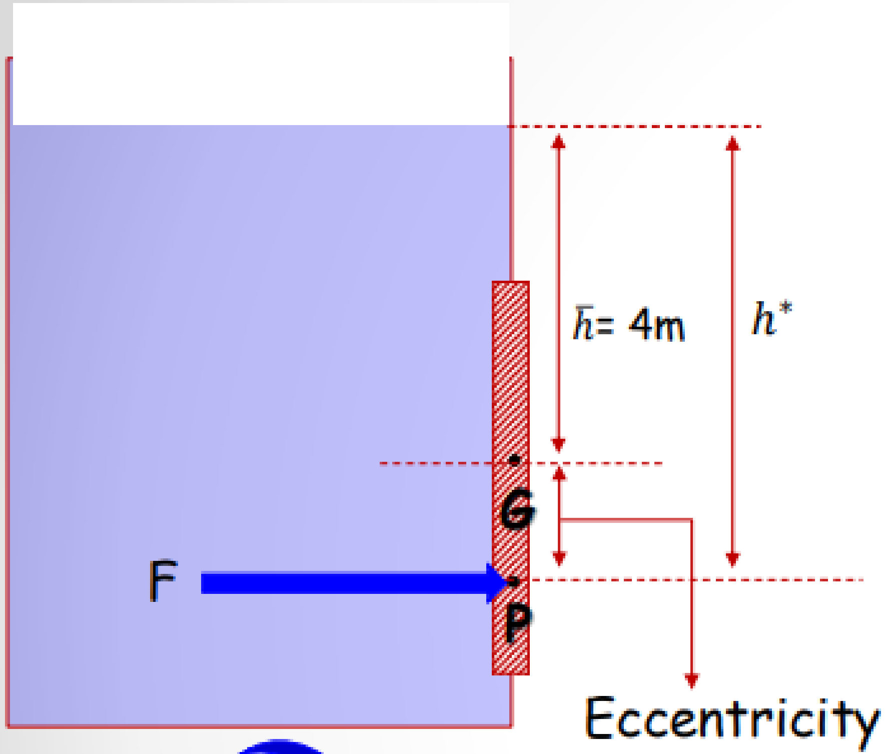
Moment of inertia about C.G axis, $I_G = \pi d^4/64 = \pi * 3^4/64 = 3.98$ m^4

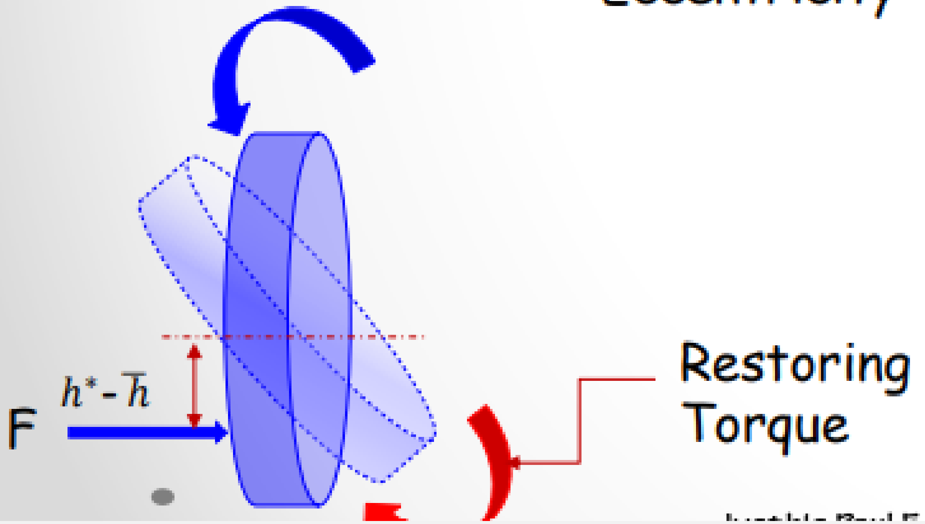
Depth of CG from free surface, $\bar{h} = 4$ m

Total pressure force, $F = \rho g A \bar{h} = 1000*9.81*7.069*4$

$= 277.39 \times 10^3$ N $=$ **277.39 kN**

ii) TORQUE REQUIRED TO MAINTAIN EQUILIBRIUM





The total force acts at an eccentricity .

So total force causes moment about the centroidal axis of the disc.

$$M = F * (h^* - \bar{h})$$

A restoring torque is required to maintain the equilibrium and numerically value is equal to **M.**

$\bar{h}$ = 4 m I_G= 3.98 m^4 A = 7.069 m^2

Position of centre of pressure, $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

$$= \frac{3.98}{7.069*4} + 4 = 4.14 \text{ m}$$

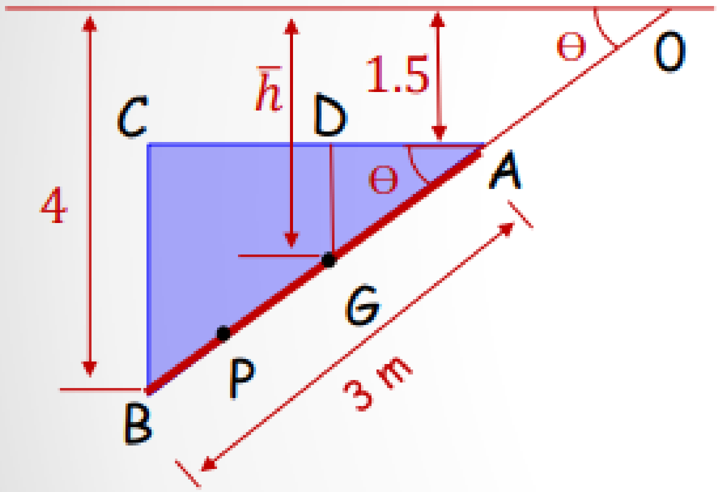
Now, torque required to maintain equilibrium,

$$M = F * (h^* - \bar{h})$$

$$= 277.39 *(4.14-4) = 38.83 \text{ kN-m}$$

3. A circular plate 3.0 m diameter is immersed in water in such a way that its greatest and least depth below the free surface are 4 m and 1.5 m respectively. Determine the total pressure on one face of the plate and position of the centre of pressure.

SOLUTION



From ΔABC,

$$sin\Theta = \frac{BC}{AB} = \frac{4-1.5}{3} = 0.833$$

From ΔADG,

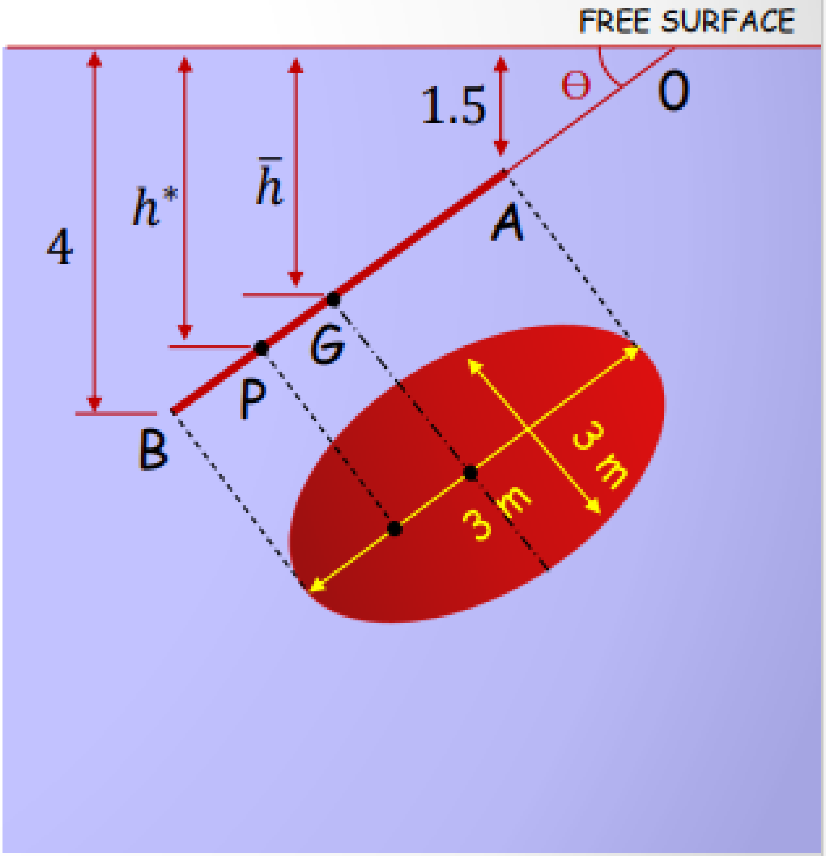
$$sin\Theta = \frac{DG}{AG} = \frac{DG}{(^3/_2)}$$

$$DG = 1.5 \sin\Theta = 1.5*0.833 = 1.25 \text{ m}$$

Hence, Distance of C.G. from free surface, $\bar{h}$ = 1.5 + DG

= 1.5 + 1.25

= 2.75 m



So, We have,

$\bar{h}$ = 2.75 m $\sin\theta = 0.833$ Density of liquid, ρ = 1000 kg/m^3

Area of the plane figure, $A = \pi d^2/4 = \pi * 3^2/4 = 7.069$ m^2

Moment of inertia about C.G axis, $I_G = \pi d^4/64 = \pi * 3^4/64 = 3.98$ m^4

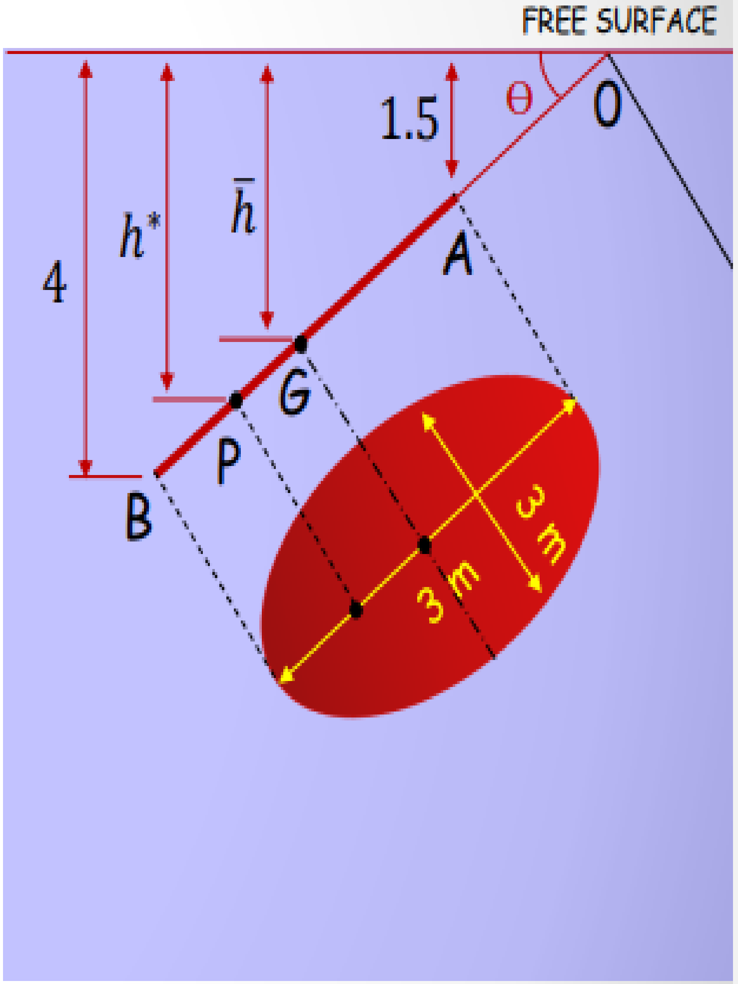
Total pressure force, $F = \rho g A\bar{h} = 1000*9.81*7.069*2.75$

$= 190.7\times10^3$ N $=$ **190.7 kN**

Position of centre of pressure, $h^* = \frac{I_G}{A\bar{h}}\sin^2\theta + \bar{h}$

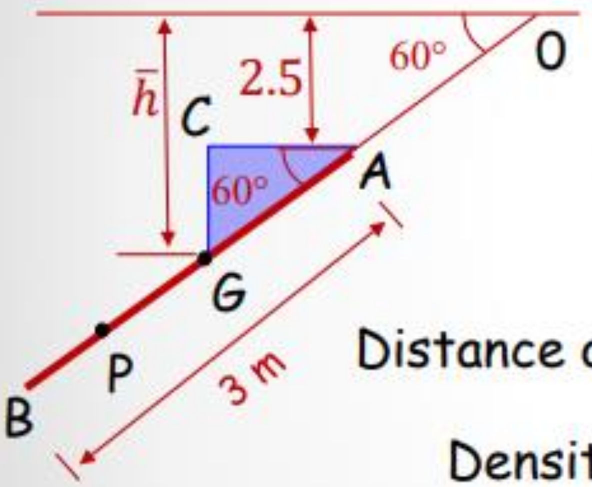
$$= \frac{3.98}{7.069*2.75} * 0.833^2 + 2.75$$

= 2.89 m



4. Find the total pressure and position of centre of pressure on a triangular plate of base 2m and height 3m which is immersed in water in such a way that the plane of the plate makes an angle of 60° with the free surface of the water. The base of the plate is parallel to water surface and at a depth of 2.5 m from water surface.

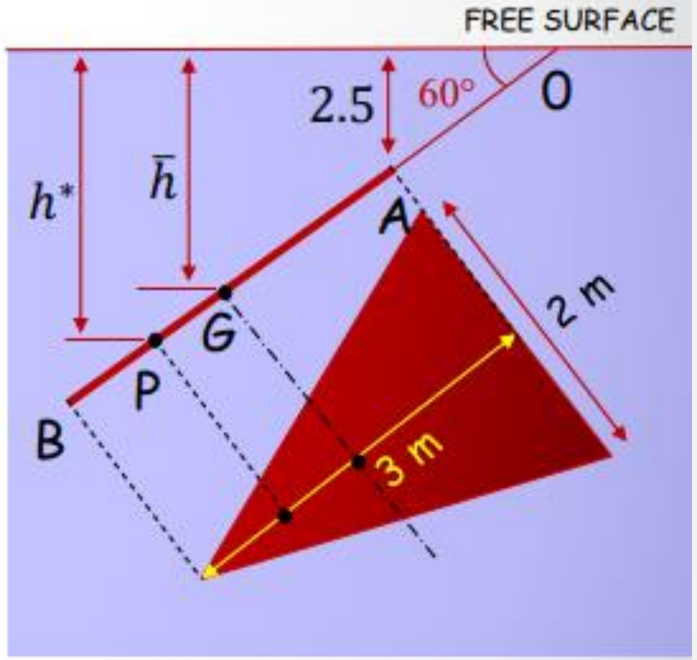
SOLUTION



$\sin\theta = \sin 60 = 0.866$

From ΔAGC,

$$\sin\theta = \frac{GC}{AG} = \frac{GC}{(3/3)} \rightarrow GC = 1*0.866 = 0.866$$



Distance of C.G. from free surface, $\bar{h}$ = 2.5 + GC = 3.37 m

Density of liquid, $\rho = 1000\ kg/m^3$

Area of the plane figure, A = bh/2 = 2*3/2 = 3 m^2

Moment of inertia about C.G axis, I_G = $bh^3/36 = 2*3^3/36$ = 1.5 m^4

Total pressure force, **F** = $\rho g A\bar{h}$ = 1000*9.81*3*3.37 = **99.18 kN**

Position of centre of pressure, $\mathbf{h^*} = \frac{I_G}{A\bar{h}}\sin^2\Theta + \bar{h} = \frac{1.5}{3*3.37} * 0.866^2 + 3.37$ = **3.48 m**

OPEN BOOK TUITORIAL

I. Find the total pressure on a triangular area vertically dipped in water so that its vertex is in the water surface and the base is parallel to the water surface. The base of the triangle is 1.8 m and its height is 0.9m . Find the depth and centre of pressure

II. A circular plate is 2m in dia is immersed in water so that its plane makes an angle of 30 deg to horizontal water surface. The highest point of the plate is 1.6m below the surface. Calculate the total water pressure and centre of pressure

TYPES OF FLOW

1. STEADY AND UNSTEADY FLOW

STEADY FLOW

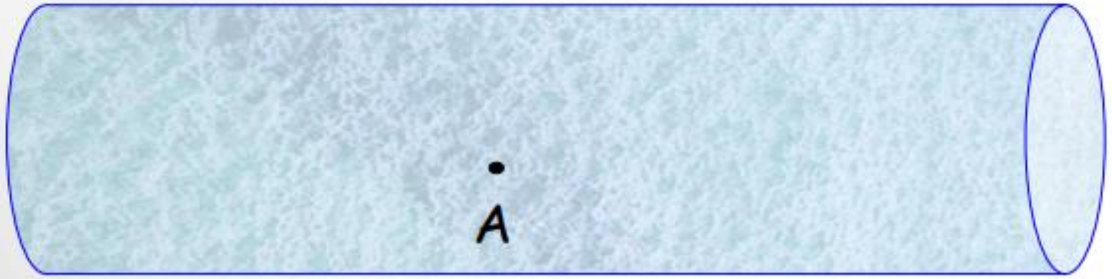
- Type of flow in which fluid characteristics like velocity, pressure, and density at a point **do not** change with **time.**

$$\frac{\partial v}{\partial t} = 0 \quad \& \quad \frac{\partial P}{\partial t} = 0 \quad \& \quad \frac{\partial \rho}{\partial t} = 0$$

UNSTEADY FLOW

- Type of flow in which fluid characteristics like velocity, pressure or density at a point **change** with **time.**

$$\frac{\partial v}{\partial t} \neq 0 \quad \text{or} \quad \frac{\partial P}{\partial t} \neq 0 \quad \text{or} \quad \frac{\partial \rho}{\partial t} \neq 0$$



Properties at different time at same point

Same → STEADY FLOW

Different → UNSTEADY FLOW

2. UNIFORM AND NON-UNIFORM FLOW

UNIFORM FLOW

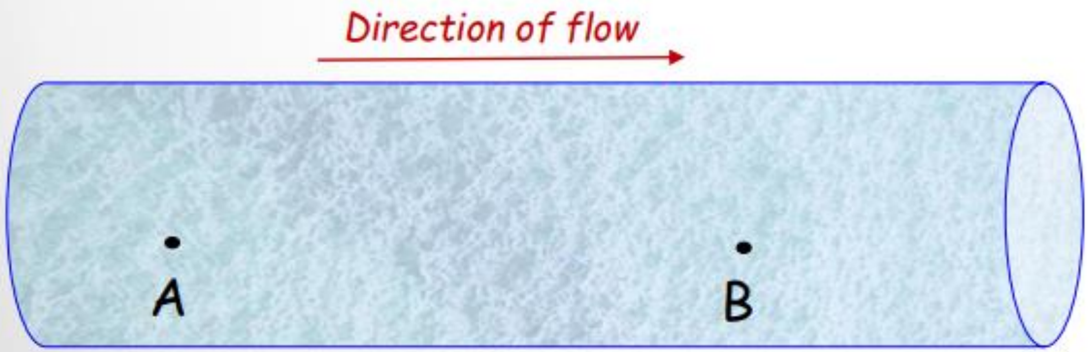
- Type of flow in which **velocity** at any given time **does not** change **along the length** of pipe in the direction of flow.

$$\frac{\partial v}{\partial s} = 0 \quad \textit{(@ Constant time)}$$

NON-UNIFORM FLOW

- Type of flow in which **velocity** at any given time **changes along the length** of pipe in the direction of flow.

$$\frac{\partial v}{\partial s} \neq 0 \quad \textit{(@ Constant time)}$$



Velocity at different points at the same time

Same → UNIFORM FLOW

Different → NON-UNIFORM FLOW

3. COMPRESSIBLE AND INCOMPRESSIBLE FLOW

COMPRESSIBLE FLOW

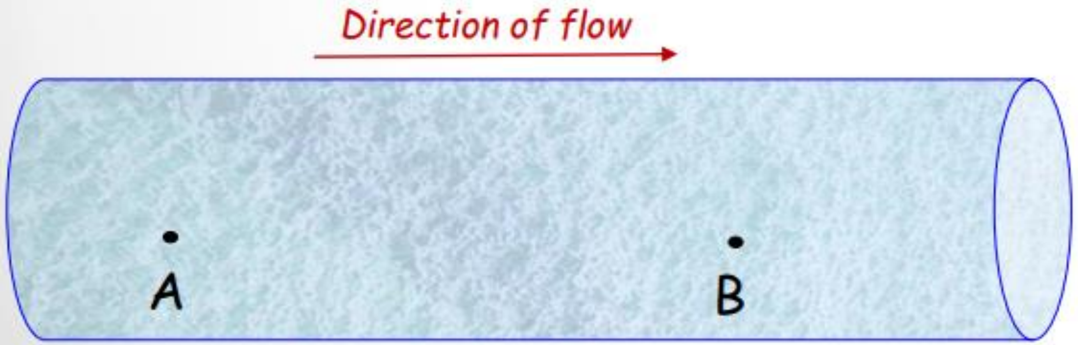
- Type of flow in which density of liquid changes from one point to another.

$$\frac{\partial \rho}{\partial s} \neq 0$$

INCOMPRESSIBLE FLOW

- Type of flow in which density of liquid does not change from one point to another.
- Generally, all liquids are incompressible.

$$\frac{\partial \rho}{\partial s} = 0$$



Density at different points

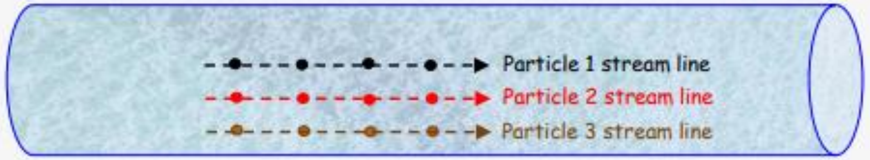
Varies → COMPRESSIBLE FLOW

Constant → INCOMPRESSIBLE FLOW

4. LAMINAR AND TURBULENT FLOW

LAMINAR FLOW

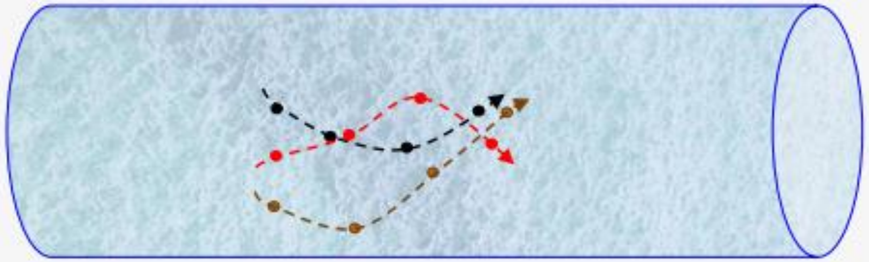
- The flow in which fluid particles moves along well defined flow paths (stream lines).



- The stream lines are straight and parallel.
- The fluid layers moves in layers (as laminas) sliding smoothly over adjacent layers.
- Also known as stream line flow or viscous flow.

TURBULENT FLOW

- The flow in which fluid particles moves along zig-zag flow paths.



- Flow paths are not straight and parallel and hence they cross each other.
- Results in loss of energy during flow of liquid.

REYNOLD'S NUMBER

$$\boxed{R_e = \frac{\rho V D}{\mu}}$$

ρ : *Density of liquid*

V : *Velocity of flow*

D : *Diameter of the pipe*

μ : *Dynamic Viscosity of the liquid*

Dimensionless quantity used to designate type of flow

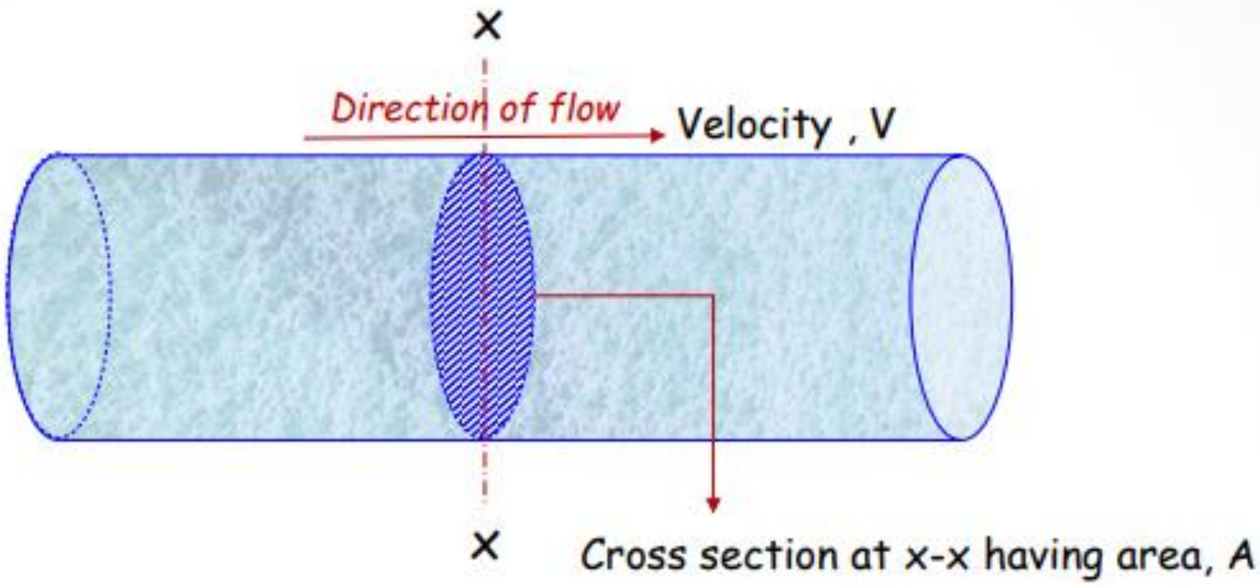
FOR PIPE FLOW $\Rightarrow$

$R_e < 2000$ $\Rightarrow$ Flow is **laminar**

$R_e > 4000$ $\Rightarrow$ Flow is **turbulent**

$2000 < R_e > 4000$ $\Rightarrow$ Flow is in **transition state**

RATE OF FLOW OR DISCHARGE



Quantity of fluid flowing per second through a section of a pipe or channel is discharge or rate of flow of the liquid (Q).

$$\text{Discharge}, Q = \frac{\text{Volume of liquid flowing through section}}{\text{Time}} = AV$$

A : Area of cross section

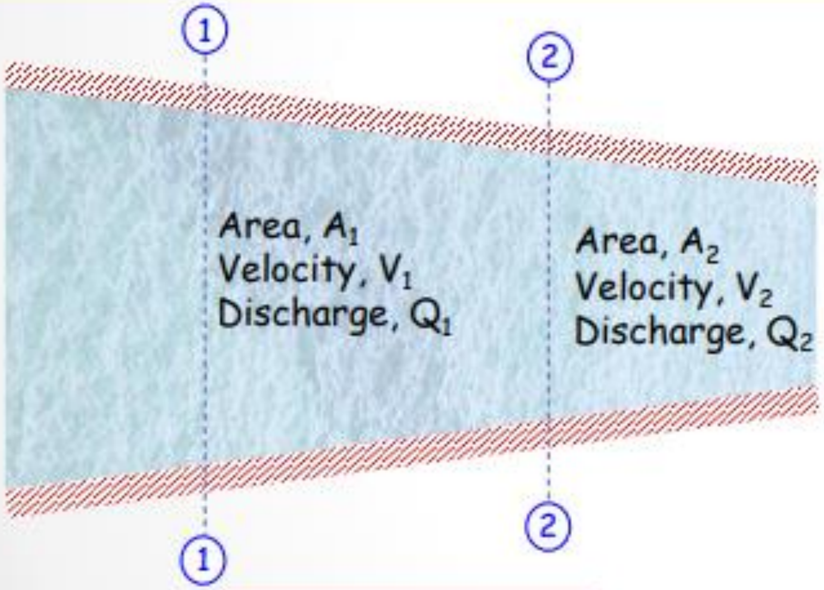
V : Average velocity of flow

(Unit m^3/s)

CONTINUITY EQUATION

STATEMENT

- Continuity equation states that for a fluid flowing through a pipe or channel, **at all cross sections** quantity of fluid flowing per second **(Q) is constant.**

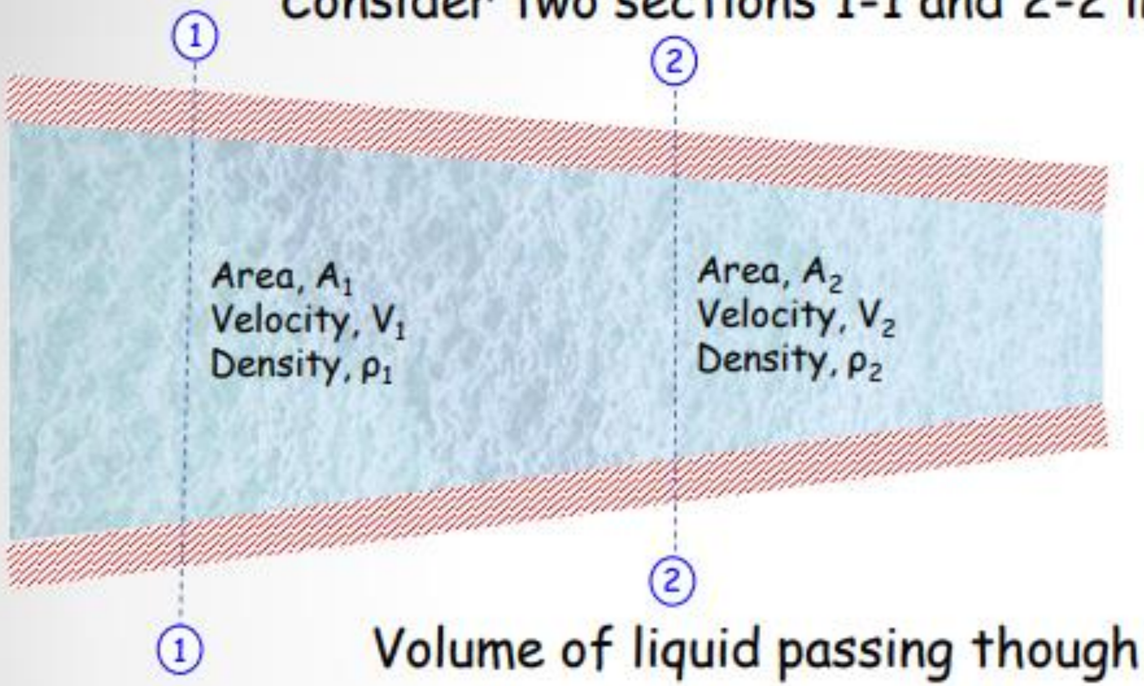


$$\boxed{\begin{aligned} Q_1 &= Q_2 \\ A_1V_1 &= A_2V_2 \end{aligned}}$$

Area of flow decreases and **hence velocity increases** for maintaining constant discharge

DERIVATION

Consider two sections 1-1 and 2-2 in channel as shown in the figure.



A_1	:	Area of cross section @ section 1-1
V_1	:	Average velocity of flow @ section 1-1
ρ_1	:	Density of liquid flowing through section 1-1
A_2	:	Area of cross section @ section 2-2
V_2	:	Average velocity of flow @ section 2-2
ρ_2	:	Density of liquid flowing through section 2-2

Volume of liquid passing though 1-1 at unit time , $Q_1 = A_1V_1$

Volume of liquid passing though 2-2 at unit time , $Q_2 = A_2V_2$

Mass of liquid passing though 1-1 at unit time , $M_1 = Q_1\rho_1 = A_1V_1\rho_1$

Mass of liquid passing though 2-2 at unit time , $M_2 = Q_2\rho_2 = A_2V_2\rho_2$

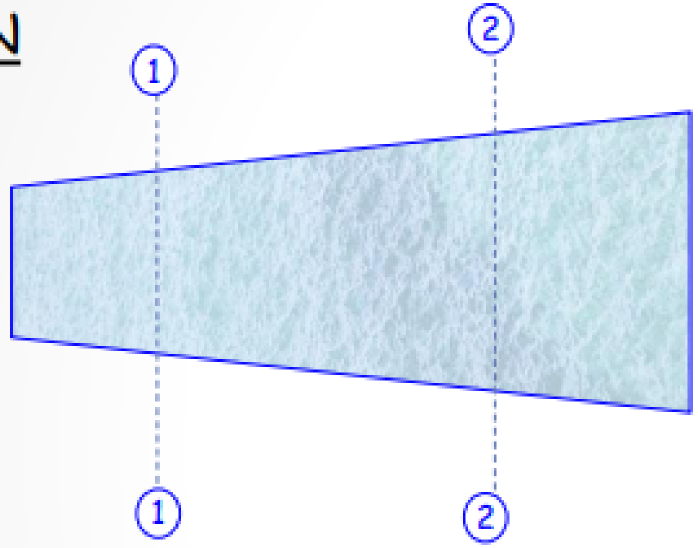
As per principle of conservation of mass, $M_1 = M_2$

Hence the general continuity equation, $\boxed{A_1V_1\rho_1 = A_2V_2\rho_2}$

And for incompressible fluids, $\boxed{A_1V_1 = A_2V_2}$ [As for incompressible fluids, $\rho_1 = \rho_2$

1. The diameter of a pipe at two sections 1-1 and 2-2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through pipe at section 1-1 is 5 m/s. Also find the velocity at section 2-2.

SOLUTION



$d_1 = 10$ cm $\quad A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} * 10^2 = 78.53\ cm^2$

$d_2 = 15$ cm $\quad A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} * 15^2 = 176.72\ cm^2$

$V_1 = 5$ m/s

$V_2 = ?$

Discharge/ Rate of flow at section 1-1, $Q_1 = A_1V_1 = 78.53*10^{-4}*5$

$= \mathbf{0.0393\ m^3/s}$

As per continuity equation, $A_1V_1 = A_2V_2 = 0.0393\ m^3/s$

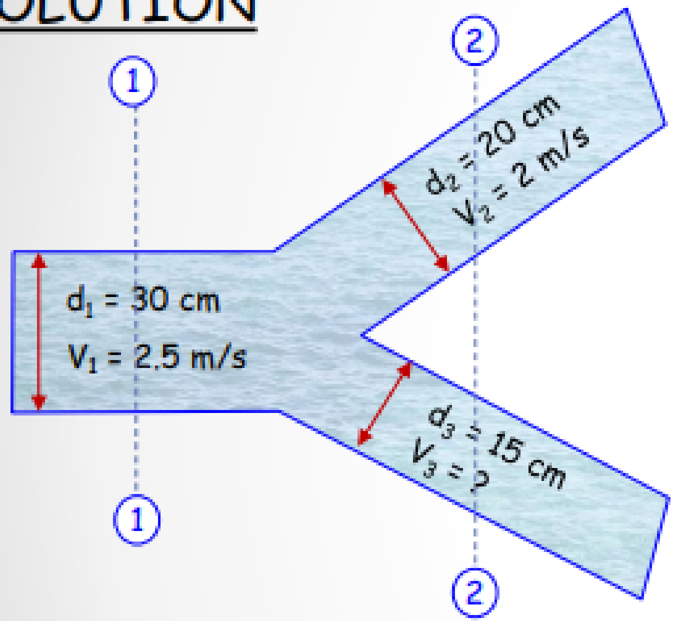
Therefore velocity @ 2-2 , $V_2 = 0.0393/A_2$

$= 0.0393/(176.72*10^{-4})$

$= \mathbf{2.22\ m/s}$

2. A 30 cm pipe conveying water branches into two pipes of diameters 20cm and 15cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s find the discharge in these pipes. Also determine the velocity in 15 cm pipe if average velocity is 20 cm diameter pipe is 2 m/s.

SOLUTION



$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} * 30^2 = 706.86\ cm^2$$

$$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} * 20^2 = 314.16\ cm^2$$

$$A_3 = \frac{\pi}{4} d_3^2 = \frac{\pi}{4} * 15^2 = 176.72\ cm^2$$

Discharge 30 cm diameter pipe, **Q_1** = A_1V_1 = $706.86*10^{-4}*2.5$

= 0.1767 m^3/s

Discharge 20 cm diameter pipe, **Q_2** = A_2V_2 = $314.16*10^{-4}*2$

= 0.0628 m^3/s

As per continuity equation,

Discharge @ section 1-1 = Discharge @ section 2-2

$Q_1 = Q_2+Q_3$

Discharge 15 cm diameter pipe, **Q_3** = $Q_1 - Q_2$ = 0.1767-0.0628

= 0.1139 m^3/s

Now, A_3V_3 = 0.1139 m^3/s

Therefore velocity @ 15 cm pipe , **V_3** = $0.1139/A_3$

$$= \frac{0.1139}{176.72*10^{-4}} = \mathbf{6.44\ m/s}$$

ENERGY OF FLOWING LIQUID

PRESSURE ENERGY

Pressure energy is the energy possessed by a liquid by virtue of its existing pressure.

$$\text{Pressure head},\ h = \frac{p}{w} = \frac{p}{\rho g} \quad (m)$$

$$\text{Pressure Energy per unit mass} = \frac{p}{\rho} \quad (Nm/kg)$$

ρ	:	Density of liquid
g	:	Acceleration due to gravity
w	:	Unit weight of liquid (w=ρg)
p	:	Pressure intensity

KINETIC ENERGY OR VELOCITY ENERGY

Energy processed by liquid in motion due to its velocity is called kinetic energy or velocity energy.

$$\text{Kinetic Energy} = \frac{1}{2} mV^2 \quad (N\text{-}m)$$

$$\text{Kinetic Energy per unit mass} = \frac{V^2}{2} \quad (Nm/kg)$$

$$\text{Kinetic head} = \frac{V^2}{2g} \quad (m)$$

m	:	Mass of liquid
V	:	Velocity of flow
g	:	Acceleration due to gravity

POTENTIAL ENERGY

Potential Energy is the energy due to the position of a fluid particle above or below any datum line or plane.

$$\text{Potential Energy} = mgz \quad (N\text{-}m)$$

$$\text{Potential Energy per unit mass} = gz \quad (Nm/kg)$$

$$\text{Potential head} = z \; (m)$$

m	:	Mass of liquid
z	:	Distance of point from the datum
g	:	Acceleration due to gravity

TOTAL ENERGY

PRESSURE ENERGY + KINETIC ENERGY + POTENTIAL ENERGY

$$\text{Total Energy} = \frac{p}{\rho} + \frac{V^2}{2} + gz \quad (Nm/kg \text{ of liquid})$$

TOTAL HEAD

PRESSURE HEAD + KINETIC HEAD + POTENTIAL HEAD

$$\text{Total Head} = \frac{p}{\rho g} + \frac{V^2}{2g} + z \quad (m)$$

BERNOULLI'S EQUATION/THEOREM

Daniel Bernoulli

For the steady flow of an ideal incompressible liquid, the **total energy is a constant.**

$$\boxed{\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{Constant}}$$

In terms of energy

$$\boxed{\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{Constant}}$$

In terms of head

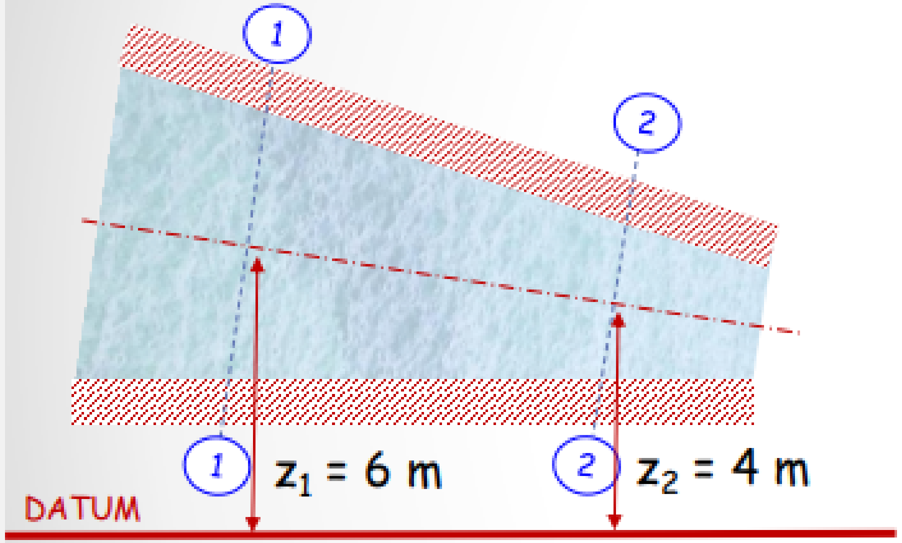
P	:	Pressure intensity
ρ	:	Density of liquid
g	:	Acceleration due to gravity
V	:	Velocity of flow
$\frac{P}{\rho g}$	:	Pressure head
$\frac{V^2}{2g}$	:	Velocity head/Kinetic Head
z	:	Datum Head

ASSUMPTIONS IN BERNOULLI'S THEOREM

1. The liquid is ideal (zero viscous) and incompressible.
2. The flow is steady and continuous.
3. The flow is along streamline and is one dimensional.
4. The velocity is uniform over the section and is equal to the mean velocity.
5. Only forces acting on the liquid are gravity forces and pressure forces.

1. The water flowing through a pipe having diameter 20 cm and 10 cm at section 1 and 2 respectively. The rate of flow through the pipe is 35 litres/second. The section 1 is 6 m above datum and section 2 is 4 m above datum. If pressure at section 1 is 39.24 N/cm². Find the intensity of pressure at section 2.

SOLUTION



d_1 = 20 cm $\quad\quad$ d_2 = 10 cm

$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} * 20^2 = 314.16\ cm^2$

$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} * 10^2 = 78.53\ cm^2$

P_1 = 39.24 N/cm² = 39.24x10⁴ N/m²

P_2 = ?

Rate of flow/ discharge, Q = 35 lit/second = $35\text{x}10^{-3}\ m^3/s$

[1 m³ = 1000 litres]

From continuity equation, $Q = A_1V_1 = A_2V_2$

Velocity @ 1-1, $V_1 = Q/A_1 = \frac{35*10^{-3}}{314.16*10^{-4}}$ = 1.114 m/s

Velocity @ 2-2, $V_2 = Q/A_2 = \frac{35*10^{-3}}{78.53*10^{-4}}$ = 4.46 m/s

Applying Bernoulli's Equation at 1-1 & 2-2,

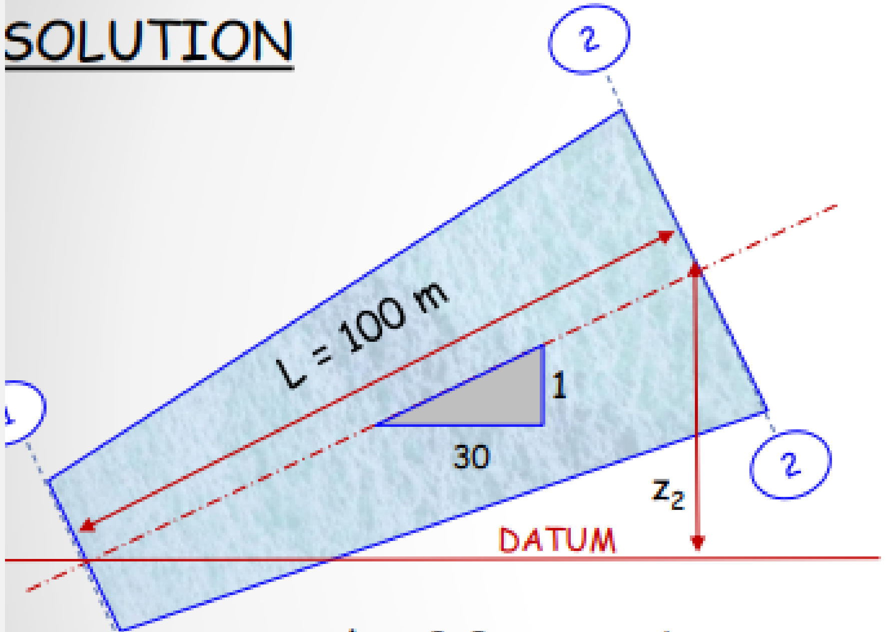
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{39.24*10^4}{1000*9.81} + \frac{1.114^2}{2*9.81} + 6 = \frac{P_2}{1000*9.81} + \frac{4.46^2}{2*9.81} + 4$$

Pressure at section 2, P_2 = $40.27\text{x}10^4$ N/m² = **40.27 N/cm²**

2. The water flowing through a pipe of length 100 m having diameter 600 mm at the upper end and 300 mm at the lower end at the rate of 50 litres/second. The pipe has a slope of 1 in 30. Find the pressure at lower end if the pressure at the higher level is 19.62 N/cm^2.

SOLUTION



$d_1 = 0.3$ m $\quad d_2 = 0.6$ cm

$A_1 = \frac{\pi}{4} * 0.3^2 = 70.68 * 10^{-3}\ m^2$

$A_2 = \frac{\pi}{4} * 0.6^2 = 282.74 * 10^{-3}\ m^2$

$P_2 = 19.62$ N/cm^2 = 19.62×10^4 N/m^2

$P_1 = ?$

Discharge, Q = 50 lit/second = 50×10^{-3} m^3/s

Velocity @ 1-1, $V_1 = Q/A_1 = \frac{50*10^{-3}}{70.68*10^{-3}} = 0.707$ m/s

Velocity @ 2-2, $V_2 = Q/A_2 = \frac{50*10^{-3}}{282.74*10^{-3}} = 0.177$ m/s

Taking starting of datum line at centre line of pipe @ lower end,

$z_1 = 0$ m

Slope 1 in 30 means ⟶ For 1 vertical 30 horizontal

So, $z_2 = (1/30)*100 = 3.33$ m

Applying Bernoulli's Equation at 1-1 & 2-2,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{P_1}{1000*9.81} + \frac{0.707^2}{2*9.81} + 0 = \frac{19.62*10^4}{1000*9.81} + \frac{0.177^2}{2*9.81} + 3.33$$

Pressure at section 1, $P_1 = 228.57 \times 10^3$ N/m^2 = **22.86 N/cm^2**

Try yourself !!!!

1. Water is flowing through a uniform pipe of 5 cm diameter under a pressure of 29.43 N/cm^2 and with mean velocity of 2.0 m/s. Find the total head at a cross section 5 m above datum.

2. Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end of pipe respectively. The intensity of pressure at upper end is 9.81 N/cm^2 and at lower end is 24.525 N/cm^2. Determine the difference in datum head if the rate of flow through the pipe is 40 liters/second.

Thank you ……