Design of spur gears

Introduction

- Bending failure of teeth

  tooth stress ≥ yield strength or bending endurance strength
- Pitting failure of teeth

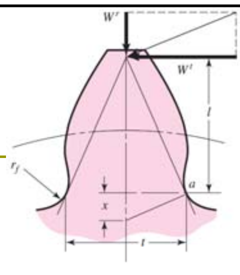
  contact stresses ≥ surface endurance strength

- General AGMA method- American Gear Manufacturer Association – AGMA 2001-C95

  AGMA 2101-C95- Metric edition
- Many charts and graphs for this course to tackle
- Simplified by taking single pressure angle and full depth teeth

The Lewis Bending Equation

- Wilfred Lewis (1892)



(b)

Assumptions of Lewis

- The full load is applied to the tip of a single tooth
- The radial component of load is negligible- conservative assumption
- The load is distributed uniformly along the full face width
- Forces which are due to tooth sliding friction are negligible
- Stress concentration in the tooth fillet is negligible

3

The Lewis Bending Equation

$$\sigma = \frac{M}{I/c} = \frac{6W^t l}{Ft^2}$$

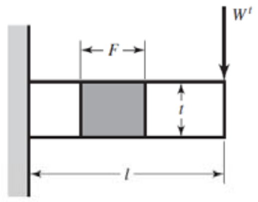
$$\frac{t/2}{x} = \frac{l}{t/2} \qquad \text{or} \qquad x = \frac{t^2}{4l} \qquad \text{or} \qquad l = \frac{t^2}{4x}$$

$$\sigma = \frac{6W^t l}{Ft^2} = \frac{W^t}{F}\frac{1}{t^2/6l} = \frac{W^t}{F}\frac{1}{t^2/4l}\frac{1}{\frac{4}{6}}$$

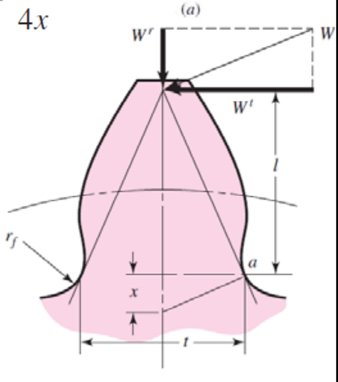
$$\sigma = \frac{W^t p}{F\left(\frac{2}{3}\right)xp}$$

$$y = 2x/(3p)$$

$$\sigma = \frac{W^t}{Fpy}$$



(a)



(b)

Lewis Equation

$$\sigma = \frac{W^t}{Fpy} \qquad (14\text{–}1)$$

y may be obtained by a graphical layout of the gear tooth or by digital computation

$$p = \pi \mathrm{m} \qquad \mathrm{Y} = \pi \mathrm{y}$$

Lewis Equation $\sigma = \dfrac{W^t}{FY\mathrm{m}}$

Lewis Form Factor $Y = \dfrac{2x}{3\mathrm{m}}$

Values of Lewis Form Factor *Y*

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

Dynamic Effects

- Effective load increases as velocity increases
- *Velocity factor* K_v accounts for this
- With pitch-line velocity V in meters per second,

$$K_v = \frac{3.05 + V}{3.05} \quad \text{(cast iron, cast profile)} \tag{14–6a}$$

$$K_v = \frac{6.1 + V}{6.1} \quad \text{(cut or milled profile)} \tag{14–6b}$$

$$K_v = \frac{3.56 + \sqrt{V}}{3.56} \quad \text{(hobbed or shaped profile)} \tag{14–6c}$$

$$K_v = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}} \quad \text{(shaved or ground profile)} \tag{14–6d}$$

Lewis Equation

- The Lewis equation including velocity factor
  - Metric version

$$\sigma = \frac{K_v W^t}{F m Y} \tag{14–8}$$

  - σ in MPa; face width F and module m in mm
  - W^t in N (Newton)
  - For spur gear, in general, $F \approx 3p - 5p$
- Acceptable for general estimation of stresses in gear teeth
- Forms basis for AGMA method, which is preferred approach

Example 14–1

A stock spur gear is available having a module of 3.0 mm, a 36 mm face, 16 teeth, and a pressure angle of 20° with full-depth teeth. The material is AISI 1020 steel in as-rolled condition. Use a design factor of $n_d = 3$ to rate the kW power output of the gear corresponding to a speed of 20 rev/s and moderate applications.

The term *moderate applications* seems to imply that the gear can be rated using the yield strength as a criterion of failure. From Table E–18, we find $S_{ut} = 380$ MPa and $S_y = 210.0$ MPa. A design factor of 3 means that the allowable bending stress is $210/3 = 70$ MPa. The pitch diameter is $m(N) = 3(16) = 48$ mm, so the pitch-line velocity is

$$V = \frac{\pi dn}{1000} = \frac{\pi(48)20}{1000} = 3.0 \text{ m/s}$$

The velocity factor from Eq. (14–5) is found to be

Example 14–1

$$K'_V = \frac{6.1 + V}{6.1} = 1.5$$

Table 14–2 gives the form factor as $Y = 0.296$ for 16 teeth. We now arrange and substitute in Eq. (14–8) as follows:

$$W' = \frac{FmY\sigma_{\text{all}}}{K'_V} = \frac{36(3)(0.296)70}{1.5} = 1492 \text{ N}$$

Power that can be transmitted is

$$\text{Power} = H = \frac{W'V}{1000} = \frac{1492(3)}{1000} = 4.475 \text{ kW}$$

It is important to emphasize that this is a rough estimate, and that this approach must not be used for important applications. The example is intended to help you understand some of the fundamentals that will be involved in the AGMA approach.

Example 14–2

Estimate the power rating of the gear in the previous example based on obtaining an infinite life in bending.

The rotating-beam endurance limit is estimated from

$S'_e = 0.506S_{ut} = 0.506(380) = 192.3$ MPa

To obtain the surface finish Marin factor k_a we refer to Table 7–5 for machined surface finding $a = 445$ and $b = -0.265$. Then Eq. (7–9) gives the surface finish Marin facto k_a as

$k_a = 4.45(380)^{-0.265} = 0.922$

The next step is to estimate the size factor k_b. From Table 13–2,

$l = 1.0m + 1.25m = 3 + 3.75 = 6.75$ mm

Example 14–2

The next step is to estimate the size factor k_b. From Table 13–2,

$l = 1.0m + 1.25m = 3 + 3.75 = 6.75$ mm

The tooth thickness t in Fig. 14–1b is given in Sec. 14–1 [Eq. (b)] as $t = (4lx)^{1/2}$ when $x = \frac{3Ym}{2}$ from Eq. (14–3). Therefore

$$x = \frac{3Ym}{2} = \frac{3(0.296) \times 3}{2} = 1.332 \text{ mm}$$

then

$t = (4lx)^{1/2} = [4(6.75)1.332]^{1/2} = 6.0$ mm

We have recognized the tooth as a cantilever beam of rectangular cross section, so the equivalent rotating-beam diameter must be obtained from Eq. (7–15):

Example 14–2

$$d_e = 0.808[36(6)]^{1/2} = 11.90 \text{ mm}$$

Then, Eq. (7–10) gives k_b as

$$k_b = \left(\frac{11.90}{7.62}\right)^{-0.107} = 0.953$$

The load factor k_c from Eq. (7–20) is unity. The temperature factor k_d is likewise unity.

Two effects are used to evaluate the miscellaneous-effects Marin factor k_e. The first of these is the effect of one-way bending. In general, a gear tooth is subjected only to one-way bending. Exceptions include idler gears and gears used in reversing mechanisms.

For one-way bending the steady and alternating stress components are $\sigma_a = \sigma_m = \sigma/2$ where σ is the largest repeatedly applied bending stress as given in Eq. (14–8). If a material exhibited a Goodman failure locus,

$$\frac{S_a}{S'_e} + \frac{S_m}{S_{ut}} = 1$$

Example 14–2

Since S_a and S_m are equal for one-way bending we substitute S_a for S_m and solve the preceding equation for S_a, giving

$$S_a = \frac{S'_e S_{ut}}{S'_e + S_{ut}}$$

Now replace S_a with $\sigma/2$, and in the denominator replace S'_e with $0.506S_{ut}$ to obtain

$$\sigma = \frac{2S'_e S_{ut}}{0.506S_{ut} + S_{ut}} = \frac{2S'_e}{0.506 + 1} = 1.33S'_e$$

Now $k_e = \sigma/S'_e = 1.33S'_e/S'_e = 1.33$. However, a Gerber fatigue locus gives mean values of

$$\frac{S_a}{S'_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

Example 14–2

Setting $S_a = S_m$ and solving the quadratic in S_a gives

$$S_a = \frac{S_{ut}^2}{2S_e'}\left(-1 + \sqrt{1 + \frac{4S_e'^2}{S_{ut}^2}}\right)$$

Setting $S_a = \sigma/2$, $S_{ut} = S_e'/0.506$ gives

$$\sigma = \frac{S_e'}{0.506^2}\left[-1 + \sqrt{1 + 4(0.506)^2}\right] = 1.65S_e'$$

and $k_e = 1.54$. Since a Gerber locus runs in and among fatigue data and Goodman does not, we will use $k_e = 1.65$.

The second effect to be accounted for in using the miscellaneous-effects Marin factor k_e is stress concentration, for which we will use our fundamentals from Chap. 7. For a 20° full-depth tooth the radius of the root fillet is denoted r_f, where

Example 14–2

$$r_f = 0.3m = 0.3(3) = 0.9 \text{ mm}$$

From Fig. E–13–6

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.9}{6.0} = 0.15$$

Since $D/d = \infty$, we approximate with $D/d = 3$, giving $K_t = 1.68$. From Eq. (7–27),

$$K_f = \frac{K_t}{1 + \dfrac{2}{\sqrt{r}}\dfrac{K_t - 1}{K_t}\sqrt{a}} = \frac{1.68}{1 + \frac{2}{\sqrt{0.9}}\frac{1.68-1}{1.68}\left(\frac{139}{380}\right)} = 1.28$$

The miscellaneous-effects Marin factor for stress concentration is

$$k_e = \frac{1}{K_f} = \frac{1}{1.28} = 0.78125$$

Example 14–2

Design

The final value of k_e is the product of the two k_e factors, that is, 1.65(0.78125) = 1.289. The Marin equation for the fully corrected endurance strength is

$$S_e = k_a k_b k_c k_d k_e S'_e$$
$$= 0.922(0.953)(1)(1)1.289(192.3) = 217.8 \text{ MPa}$$

For a design factor of $n_d = 3$ applied to the load or strength is

$$\sigma_{\text{all}} = \frac{S_e}{n_d} = \frac{217.8}{3} = 72.6$$

The transmitted load W^t is

$$W^t = \frac{FY\sigma_{\text{all}}m}{K'_V} = \frac{36(0.296)72.6(3)}{1.5} = 1547 \text{ N}$$

and the power is

$$\text{Power } H = \frac{W^t V}{1000} = \frac{1547(3)}{1000} = 4.64 \text{ kW}$$

Fatigue Stress-Concentration Factor

$$K_f = H + \left(\frac{t}{r}\right)^L \left(\frac{t}{l}\right)^M \qquad (14\text{–}9)$$

where
$$H = 0.34 - 0.458\,366\,2\phi$$
$$L = 0.316 - 0.458\,366\,2\phi$$
$$M = 0.290 + 0.458\,366\,2\phi$$
$$r = \frac{(b - r_f)^2}{(d/2) + b - r_f}$$

Surface Durability

- Wear(pitting): a surface failure due to many repetitions of high contact stress
- Scoring: surface failure due to failure of lubrication
- Abrasion: wear due to the presence of foreign material
- Hertz stress, no of cycles, the surface finish, the hardness, the degree lubrication, and the temperature- all influence the surface strength
- Mechanism of pitting: surface fatigue failure is initiated by the max shear stress (just below the contact surface) and then is propagated rapidly to the surface. The lubricant then enters the crack which is formed and under pressure eventually wedges the chip loose.

19

Contact stresses

- When two bodies having curved surfaces are pressed together, point or line contact changes to area contact, and the stresses developed in the two bodies are three dimensional.
- E.g. contact of a wheel and a rail, in automotive valve cams and tappets, in mating gear teeth, and in the action of rolling bearings.
- Typical failures are seen as cracks, pits, or flaking in the surface material.
- *Hertzian stresses.*

20

- the radius *a of the circular contact area is given by the equation*

$$a = \sqrt[3]{\frac{3F}{8}\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

- The maximum pressure occurs at the center of the contact area and is

$$p_{\max} = \frac{3F}{2\pi a^2}$$

- Equations are perfectly general and also apply to the contact of a sphere and a plane surface or of a sphere and an internal spherical surface.

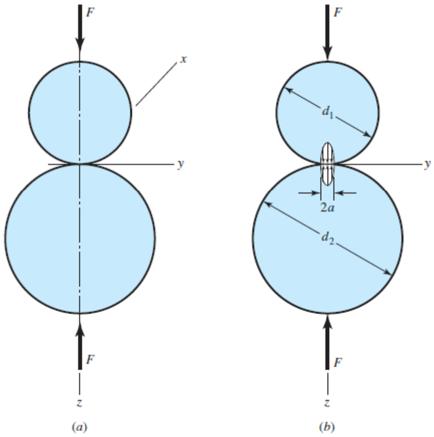


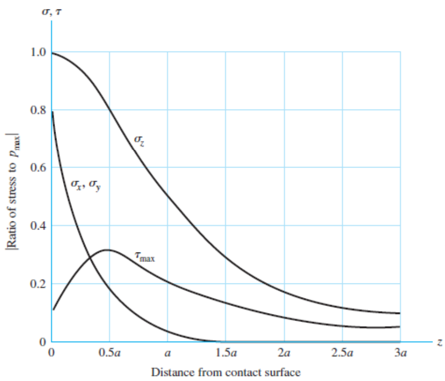
Fig: *(a) Two spheres held in* contact by force *F; (b) contact* stress has a hemispherical distribution across contact zone of diameter 2*a.*

21

- For a plane surface, use *d* =∞. *For an internal surface, the diameter is expressed as a* negative quantity.
- The maximum stresses occur on the *z axis, and these are principal stresses. Their* values are

$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y = -p_{\max}\left[\left(1 - \left|\frac{z}{a}\right|\tan^{-1}\frac{1}{|z/a|}\right)(1+\nu) - \frac{1}{2\left(1+\dfrac{z^2}{a^2}\right)}\right]$$

$$\sigma_3 = \sigma_z = \frac{-p_{\max}}{1 + \dfrac{z^2}{a^2}}$$



Note that the maximum shear stress is slightly below the surface at *z = 0.48a and is* approximately $0.3p_{max}$. *The* chart is based on a Poisson ratio of 0.30. Note that the normal stresses are all compressive stresses.

22

Cylindrical Contact

- The half-width b *is given by the equation*

$$b = \sqrt{\frac{2F}{\pi l} \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

- The maximum pressure is

$$p_{max} = \frac{2F}{\pi b l}$$

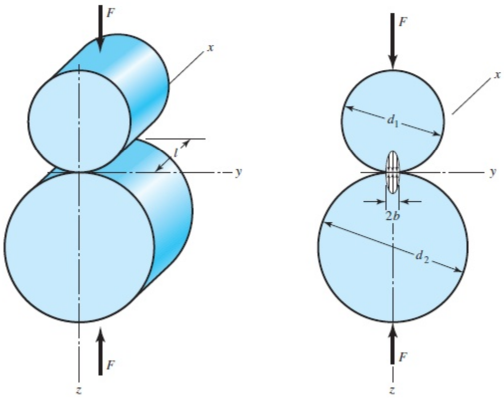


Fig: *(a) Two right circular* cylinders held in contact by forces *F uniformly distributed* along cylinder length *l.*
(b) Contact stress has an elliptical distribution across the contact zone of width 2*b.*

23

- The stress state along the *z axis is given by the equations*

$$\sigma_x = -2\nu p_{max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right)$$

$$\sigma_y = -p_{max}\left(\frac{1+2\frac{z^2}{b^2}}{\sqrt{1+\frac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right)$$

$$\sigma_3 = \sigma_z = \frac{-p_{max}}{\sqrt{1+z^2/b^2}}$$

- For $0 \le z \le 0.436b$, $\sigma_1 = \sigma_x$, and $\tau_{max} = (\sigma 1 - \sigma 3)/2$.
- *For* $z \ge 0.436b$, $\sigma_1 = \sigma_y$, *and* $\tau_{max} = (\sigma_y - \sigma_z)/2$.

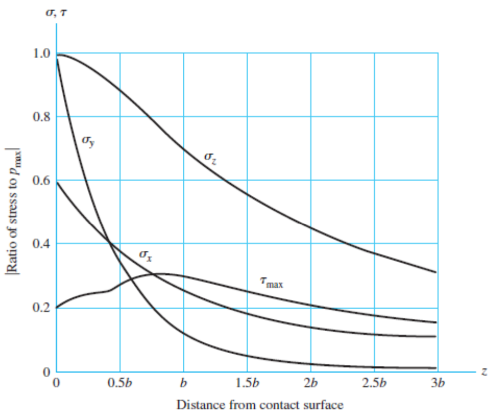


Fig: The largest value of maximum shear stress occurs at *z/b = 0.786. Its* maximum value is $0.30p_{max}$. The chart is based on a Poisson ratio of 0.30.

24

Surface Durability

- Another failure mode is wear due to contact stress.
- Modeling gear tooth mesh with contact stress between two cylinders,

$$p_{max} = \frac{2F}{\pi b l}$$

where p_{max} = largest surface pressure
F = force pressing the two cylinders together
l = length of cylinders

$$b = \left[\frac{2F}{\pi l}\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}\right]^{1/2} \qquad (14\text{–}10)$$

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- Converting to terms of gear tooth, the *surface compressive stress* (*Hertzian stress*) is found.
  $F = w^t/\cos\varphi$; $d = 2r$; $l = F$; $P_{max} = \sigma_c$

$$\sigma_C^2 = \frac{W^t}{\pi F \cos\phi}\frac{1/r_1 + 1/r_2}{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2} \qquad (14\text{–}11)$$

  where r_1 and r_2 are instantaneous values of the centre of curvature on the pinion and gear tooth profile at the point of contact
- Critical location is usually at the pitch line, where

$$r_1 = \frac{d_P \sin\phi}{2} \qquad r_2 = \frac{d_G \sin\phi}{2} \qquad (14\text{–}12)$$

- Define *elastic coefficient* (Z_E or C_p) from denominator of Eq. (14–11),

$$C_p = \left[\frac{1}{\pi\left(\dfrac{1-\nu_P^2}{E_P} + \dfrac{1-\nu_G^2}{E_G}\right)}\right]^{1/2} \qquad (14\text{–}13)$$

Surface Durability

- Incorporating elastic coefficient and velocity factor, the contact stress equation is

$$\sigma_C = -C_p \left[\frac{K_v W^t}{F \cos\phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2} \qquad (14\text{–}14)$$

(compressive stress '-' sign)

- Again, this is useful for estimating, and used as the basis for the preferred AGMA approach.

AGMA Method

- The American Gear Manufacturers Association (AGMA) provides a recommended method for gear design.
- It includes bending stress and contact stress as two failure modes.
- It incorporates modifying factors to account for various situations.
- It imbeds much of the detail in tables and figures.

AGMA Bending Stress

$$\sigma = W^t K_o K_v K_s \frac{1}{bm_t} \frac{K_H K_B}{Y_J}$$

where

W^t is the tangential transmitted load, (N)
K_o is the overload factor
K_v is the dynamic factor
K_s is the size factor
F (b) is the face width of the narrower member, in (mm)
K_m (K_H) is the load-distribution factor
K_B is the rim-thickness factor
J (Y_J) is the geometry factor for bending strength (which includes root fillet stress-concentration factor K_f)
(m_t) is the transverse metric module

AGMA Contact Stress

$$\sigma_c = Z_E \sqrt{W^t K_o K_v K_s \frac{K_H}{d_{w1} b} \frac{Z_R}{Z_I}}$$

where W^t, K_o, K_v, K_s, K_m, F, and b are the same terms as defined for Eq. (14–15). For U.S. customary units (SI units), the additional terms are

C_p (Z_E) is an elastic coefficient, $\sqrt{\text{lbf/in}^2}$ ($\sqrt{\text{N/mm}^2}$)
C_f (Z_R) is the surface condition factor
d_P (d_{w1}) is the pitch diameter of the *pinion,* in (mm)
I (Z_I) is the geometry factor for pitting resistance

30

AGMA Strengths

- AGMA uses *allowable stress numbers* rather than *strengths*.
- We will refer to them as strengths for consistency within the textbook.
- The gear strength values are only for use with the AGMA stress values, and should not be compared with other true material strengths.
- Representative values of typically available bending strengths are given in Table 14–3 for steel gears and Table 14–4 for iron and bronze gears.
- Figs. 14–2, 14–3, and 14–4 are used as indicated in the tables.
- Tables assume repeatedly applied loads at 10^7 cycles and 0.99 reliability.

Bending Strengths for Steel Gears

Table 14–3

Repeatedly Applied Bending Strength S_t at 10^7 Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Bending Stress Number S_t,[2] psi (MPa) Grade 1	Grade 2	Grade 3
Steel[3]	Through-hardened	See Fig. 14–2	See Fig. 14–2	See Fig. 14–2	—
	Flame[4] or induction hardened[4] with type A pattern[5]	See Table 8*	45 000 (310)	55 000 (380)	—
	Flame[4] or induction hardened[4] with type B pattern[5]	See Table 8*	22 000 (151)	22 000 (151)	—
	Carburized and hardened	See Table 9*	55 000 (380)	65 000 or 70 000[6] (448 or 482)	75 000 (517)
	Nitrided[4,7] (through-hardened steels)	83.5 HR15N	See Fig. 14–3	See Fig. 14–3	—
Nitralloy 135M, Nitralloy N, and 2.5% chrome (no aluminum)	Nitrided[4,7]	87.5 HR15N	See Fig. 14–4	See Fig. 14–4	See Fig. 14–4

Shigley's Mechanical Engineering Design

Bending Strengths for Iron and Bronze Gears

Table 14–4

Repeatedly Applied Bending Strength S_t for Iron and Bronze Gears at 10^7 Cycles and 0.99 Reliability

Source: ANSI/AGMA 2001-D04.

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Bending Stress Number, S_t,[3] psi (MPa)
ASTM A48 gray cast iron	Class 20	As cast	—	5000 (35)
	Class 30	As cast	174 HB	8500 (58)
	Class 40	As cast	201 HB	13 000 (90)
ASTM A536 ductile (nodular) Iron	Grade 60–40–18	Annealed	140 HB	22 000–33 000 (151–227)
	Grade 80–55–06	Quenched and tempered	179 HB	22 000–33 000 (151–227)
	Grade 100–70–03	Quenched and tempered	229 HB	27 000–40 000 (186–275)
	Grade 120–90–02	Quenched and tempered	269 HB	31 000–44 000 (213–275)
Bronze		Sand cast	Minimum tensile strength 40 000 psi	5700 (39)
	ASTM B–148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	23 600 (163)

Figure. 1.3 Allowable bending stress number σ_{FP} or AGMA bending fatigue strength S'_{fb} for through-hardened steel gears. [Source: ANSI/AGMA 2001-C95 and 2101-C95.]

The SI equations are σ_{FP} or $S'_{fb} = 0.533HB + 88.3$ MPa, grade 1,

and

σ_{FP} or $S'_{fb} = 0.703HB + 113$ MPa, grade 2.

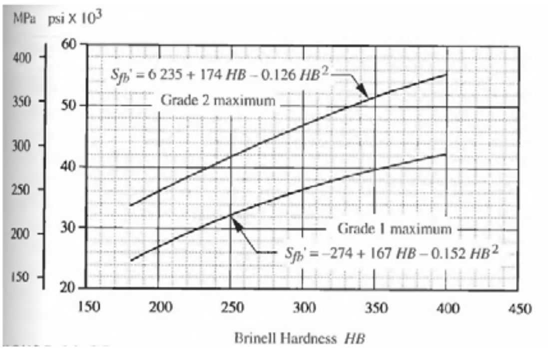


Figure. 1.4 Allowable bending stress number σ_{FP} or AGMA bending fatigue strength S'_{fb} for nitride through-hardened steel gears (I.e., AISI 4140, 4340) [Source: ANSI/AGMA 2001-C95 and 2101-C95.] .

The SI equations are σ_{FP} or $S'_{fb} = 0.568HB + 83.8$ MPa, grade 1,

and

σ_{FP} or $S'_{fb} = 0.749HB + 110$ MPa, grade 2.

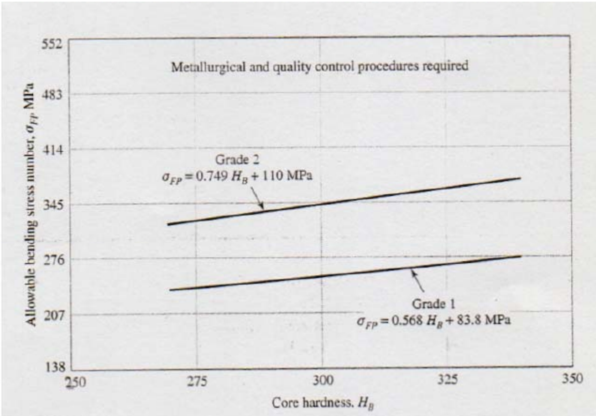
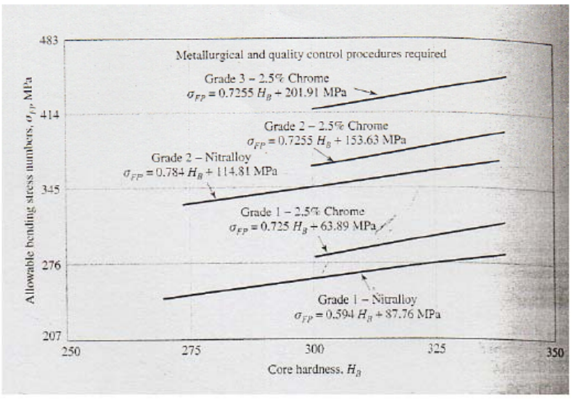


Figure 1.5. Allowable bending stress numbers σ_{FP} or AGMA bending fatigue strength S'_{fb} nitriding steel gears



The SI equations are σ_{FP} or $S'_{fb} = 0.594HB + 87.76$ MPa, grade 1- Nitralloy,

$= 0.725HB + 63.89$ MPa, grade 1- 2.5% Chrome

$= 0.784HB + 114.81$MPa,grade 2- Nitralloy

$= 0.7255HB + 153.63$ MPa, grade 2- 2.5% Chrome

$= 0.7255HB + 201.91$ MPa, grade 3- 2.5% Chrome

Table 1.15. Repeatedly Applied Bending Strength at 10^7 Cycles and 0.99 Reliability for Steel Gears; Source: ANSI/AGMA 2001-C95.

Material Designation	Heat Treatment	Minimum Surface Hardness'	Allowable Bending Stress Number σ_{FP} or S'_{fb},[2] MPa		
			Grade 1	Grade 2	Grade 3
Steel[3]	Through hardened	See Fig. 1.3	See Fig. 1.3	See Fig. 1.3	_
	Flame[4] or induction hardened[4] with type A pattern[5]		310 MPa	380 MPa	_
	Flame[4] or induction hardened[4] with type B Pattern[5]		151 MPa	151 MPa	_
	Carburized and hardened		380 MPa	448 or482 MPa	517MPa
	Nitrided[4,7] (Through-hardened steels)	83.5 HR15N	See Fig. 1-4	See Fig. 1-4	-
Nitralloy 135M, Nitralloy N, and 2.5% chrome (No aluminum)	Nitrided[4,7]	87.5 HR15N	See Fig. 1.5	See Fig. 1.5	See Fig. 1.5

Notes: See ANSI/AGMA 2001-C95 for references cited in notes 1-7.
[1] Hardness to be equivalent to that at the root diameter in the center of the tooth space and width.
[2] See table 7 through 10 of source for major metallurgical factors for each stress grade of steel gears.
[3] The steel selected must be compatible with the heat treatment process selected and hardness required.
[5] See figure 12 of source for type A and type B hardness patterns.
[6] If bainite and microcracks are limited to grade 3 levels, 482 MPa may be used.
[7] The overload capacity of nitride gears is low. Since the shape of the effective S-N curve is flat, the sensitivity to shock should be investigated before proceeding with the design.

37

Table 1.16 Repeatedly Applied bending stress numbers σ_{FP} or AGMA bending fatigue strength S'_{fb} for Iron and Bronze Gears at 10' Cycles and 0.99 reliability Source ANSI/AGMA 2001-C95.

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surfaca Hardness[2]	σ_{FP} or S'_{fb}, MPa
ASTM A48 gray	Class 20	As cast	_	35
Cost iron	Class 30	As cast	174 HB	58
	Class 40	As cast	201HB	90
ASTM A 536 ductile (nodular) Iron	Grade 60-40-18	Annealed	140HB	151-227
	Grade 80-55-06	Quenched and tempered	179HB	151-227
	Grade 100-70-03	Quenched and tempered	229HB	186-275
	Grade 120-90-02	Quenched and tempered	269 HB	213-275
Bronze		Sand cast	Minimum tensile strength 275 MPa	39
	ASTM48 Alloy 954	Heat treated	Minimum tensile strength 620 MPa	163

Notes:
[1] See ANSI/AGMA 2004-B89, Gear Materials and Heat Treatment Manual.
[2] Measured hardness to be equivalent to that which would be measured at the root diameter in the center of the tooth space and face width.
3. The lower values should be used for general design purposes. The upper vollues may be used when:
High quality material is used.
Section size and design allow maximum response to heat treatment.
Proper quality control is affected by adequate inspection.
Operating experience justifies their use.

38

Allowable Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \qquad (14\text{–}17)$$

where for U.S. customary units (SI units),

S_t is the allowable bending stress, lbf/in^2 (N/mm^2)
Y_N is the stress-cycle factor for bending stress
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_F is the AGMA factor of safety, a stress ratio

Allowable Contact Stress

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H} \dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H} \dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \qquad (14\text{–}18)$$

S_c is the allowable contact stress, lbf/in^2 (N/mm^2)
Z_N is the stress-cycle factor
C_H (Z_W) are the hardness ratio factors for pitting resistance
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_H is the AGMA factor of safety, a stress ratio

Table 1.17 Nominal Temperature Used in Nitriding and Hardnesses Obtained. Source: Darle W. Dudley, Handbook of Practical Gear Design, rev, ed. McGraw-Hill New Yark, 1984

Steel	Temperature Before nitriding, °C	Nitriding, °C	Hardness, Rockwell C Scale Case	Core
Nitralloy 135	620	524	62-65	30-35
Nitralloy 135M	620	524	62-65	32-36
Nitralloy N	537	524	62-65	40-44
AISI 4340	593	524	48-53	27-35
AISI 4140	593	524	49-54	27-35
31 Cr Mo V9	593	524	58-62	27-33

Contact Strength for Steel Gears

Table 14–6

Repeatedly Applied Contact Strength S_c at 10^7 Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Contact Stress Number,[2] S_c, psi (σ_{HP}, MPa) Grade 1	Grade 2	Grade 3
Steel[3]	Through hardened[4]	See Fig. 14–5	See Fig. 14–5	See Fig. 14–5	—
	Flame[5] or induction hardened[5]	50 HRC	170 000 (1172)	190 000 (1310)	—
		54 HRC	175 000 (1206)	195 000 (1344)	—
	Carburized and hardened[5]	See Table 9*	180 000 (1240)	225 000 (1551)	275 000 (1896)
	Nitrided[5] (through hardened steels)	83.5 HR15N	150 000 (1035)	163 000 (1123)	175 000 (1206)
		84.5 HR15N	155 000 (1068)	168 000 (1158)	180 000 (1240)
2.5% chrome (no aluminum)	Nitrided[5]	87.5 HR15N	155 000 (1068)	172 000 (1186)	189 000 (1303)
Nitralloy 135M	Nitrided[5]	90.0 HR15N	170 000 (1172)	183 000 (1261)	195 000 (1344)
Nitralloy N	Nitrided[5]	90.0 HR15N	172 000 (1186)	188 000 (1296)	205 000 (1413)
2.5% chrome (no aluminum)	Nitrided[5]	90.0 HR15N	176 000 (1213)	196 000 (1351)	216 000 (1490)

Contact Strength for Iron and Bronze Gears

Table 14–7

Repeatedly Applied Contact Strength S_c 10^7 Cycles and 0.99 Reliability for Iron and Bronze Gears

Source: ANSI/AGMA 2001-D04.

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Contact Stress Number,[3] S_c, psi (σ_{HP}, MPa)
ASTM A48 gray cast iron	Class 20	As cast	—	50 000–60 000 (344–415)
	Class 30	As cast	174 HB	65 000–75 000 (448–517)
	Class 40	As cast	201 HB	75 000–85 000 (517–586)
ASTM A536 ductile (nodular) iron	Grade 60–40–18	Annealed	140 HB	77 000–92 000 (530–634)
	Grade 80–55–06	Quenched and tempered	179 HB	77 000–92 000 (530–634)
	Grade 100–70–03	Quenched and tempered	229 HB	92 000–112 000 (634–772)
	Grade 120–90–02	Quenched and tempered	269 HB	103 000–126 000 (710–868)
Bronze	—	Sand cast	Minimum tensile strength 40 000 psi	30 000 (206)
	ASTM B-148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	65 000 (448)

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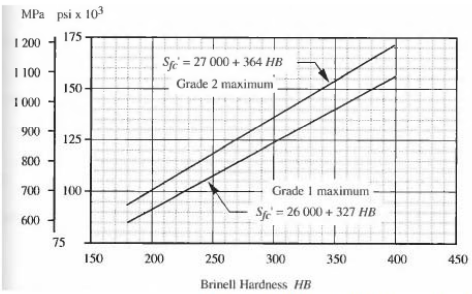
Figure 1.7: Allowable contact stress numbers σ_{HP} or AGMA surface fatigue strength S'_{fs} at 10^7 cycles and 0.99 reliability for through-hardened steel gears (Source: ANSI/AGMA 2001-C95 and 2101-C95)



The SI equations are

σ_{HP} or $S'_{fc} = 2.22HB + 200$ MPa, grade 1,

and

σ_{HP} or $S'_{fc} = 2.41HB + 237$ MPa, grade 2.

Geometry Factor J (Y_J in SI)

- Accounts for shape of tooth in bending stress equation
- Includes
  - A modification of the Lewis form factor Y
  - Fatigue stress-concentration factor K_f
  - Tooth *load-sharing ratio* m_N
- AGMA equation for geometry factor is

$$J = \frac{Y}{K_f m_N} \qquad (14\text{–}20)$$

$$m_N = \frac{p_N}{0.95Z} \qquad (14\text{–}21)$$

p_N is normal base pitch; Z: length of line of action

- Values for Y and Z are found in the AGMA standards.
- For most common case of spur gear with 20º pressure angle, J can be read directly from Fig. 14–6.

Spur-Gear Geometry Factor J Y_J

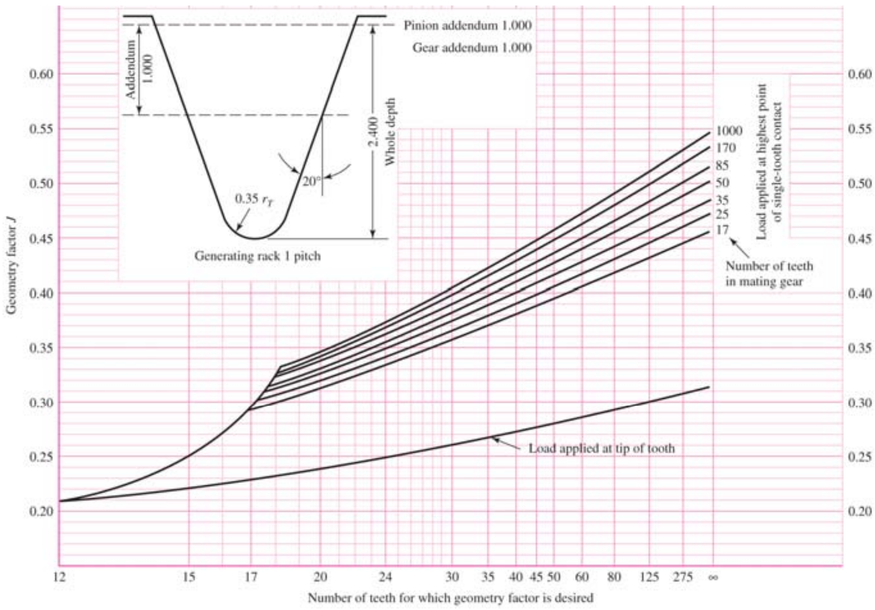


Fig. 14–6

Surface Strength Geometry Factor I (Z_I in SI)

- Called *pitting resistance geometry factor* by AGMA

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G+1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G-1} & \text{internal gears} \end{cases} \qquad (14\text{–}23)$$

$$m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P} \qquad (14\text{–}22)$$

$$m_N = \frac{p_N}{0.95Z} \quad \text{=1 for spur gear} \qquad (14\text{–}21)$$

$$p_N = p_n \cos\phi_n \qquad (14\text{–}24)$$

$$Z = \left[(r_P+a)^2 - r_{bP}^2\right]^{1/2} + \left[(r_G+a)^2 - r_{bG}^2\right]^{1/2} - (r_P+r_G)\sin\phi_t \qquad (14\text{–}25)$$

$$r_b = r\cos\phi_t \qquad (14\text{–}26)$$

Elastic Coefficient C_p (Z_E)

- Obtained from Eq. (14–13) or from Table 1.25.

$$C_p = \left[\frac{1}{\pi\left(\dfrac{1-\nu_P^2}{E_P} + \dfrac{1-\nu_G^2}{E_G}\right)}\right]^{1/2} \qquad (14\text{–}13)$$

Table 1.25: Elastic Coefficient Z_E or C_P Source : AGMA 218.01

Pinion Material	Pinion Modulus of Elasticity E_P $\sqrt{MPa}$	Gear Material and Modulus of Elasticity E_G, MPa*					
		Steel 2 x 10^5	Malleable Iron 1.7 x 10^5	Nodular Iron 1.5 x 10^5	Cast Iron 1.5 x 10^5	Aluminum Bronze 1.2 x 10^5	Tin Bronze 1.1 x 10^5
Steel	2.0X10^5	191	181	179	174	162	158
Malleable iron	1.7X10^5	181	174	172	168	158	154
Nodular iron	1.7X10^5	179	172	170	166	156	152
Cast Iron	1.5X10^5	174	168	166	163	154	149
Aluminum bronze	1.2X10^5	162	158	156	154	145	141
Tin bronze	1.1X10^5	158	154	152	149	141	137

Poisson's ratio = 0.30

*When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

Dynamic Factor K_v

- Accounts for increased forces with increased speed
- Affected by
  - manufacturing quality of gears
  - Vibration of the tooth during
  - Magnitude of pitch line velocity
  - Dynamic unbalance of the rotating members
  - Wear and permanent deformation of contacting portion
  - Gear shaft misalignment and the deflection of the shaft
  - Tooth friction
- A set of *quality numbers* Q_v define tolerances for gears manufactured to a specified accuracy.
- Quality numbers 3 to 7 include most commercial-quality gears.
- Quality numbers 8 to 12 are of precision quality.
- The AGMA *transmission accuracy-level number* A_v is basically the same as the quality number.

Dynamic Factor K_v

- Dynamic Factor equation

$$K_v = \begin{cases} \left(\dfrac{A+\sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\ \left(\dfrac{A+\sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases} \quad (14\text{–}27)$$

$$A = 50 + 56(1 - B)$$
$$B = 0.25(12 - Q_v)^{2/3} \quad (14\text{–}28)$$

- Or can obtain value directly from Fig. 14–9
- Maximum recommended velocity for a given quality number,

$$(V_t)_{\max} = \begin{cases} [A + (Q_v - 3)]^2 & \text{ft/min} \\ \dfrac{[A + (Q_v - 3)]^2}{200} & \text{m/s} \end{cases} \quad (14\text{–}29)$$

Dynamic Factor K_v

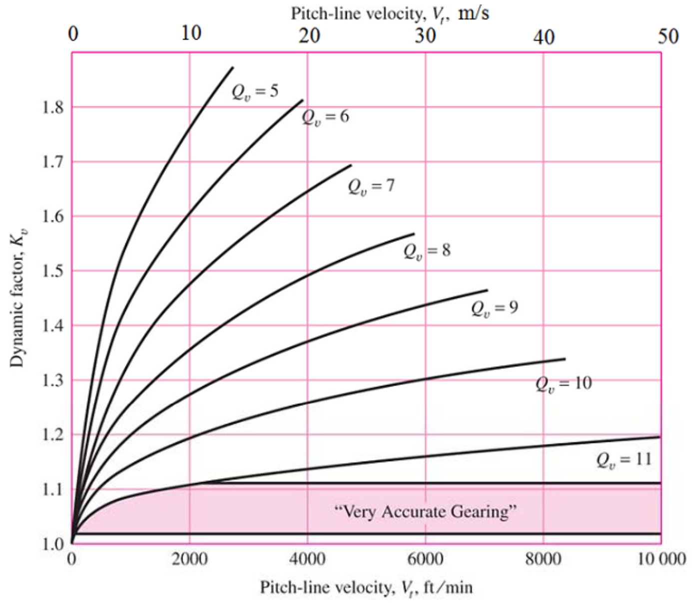


Fig. 14–9

Recommended AGMA Gear Quality Numbers for Various Applications

Application	Q_v
Cement mixer drum drive	3–5
Cement kiln	5–6
Steel mill drives	5–6
Corn picker	5–7
Cranes	5–7
Punch press	5–7
Mining conveyor	5–7
Paper-box making machine	6–8
Gas meter mechanism	7–9
Small power drill	7–9
Clothes washing machine	8–10
Printing press	9–11
Computing mechanism	10–11
Automotive transmission	10–11
Radar antenna drive	10–12
Marine propulsion drive	10–12
Aircraft engine drive	10–13
Gyroscope	12–14

Overload Factor K_O

- Variation in torque from the mean value due to firing of cylinders in an IC engine or reaction to torque variations in a piston pump
- To account for likelihood of increase in nominal tangential load due to particular application.
- Recommended values,

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Surface Condition Factor C_f (Z_R)

- Used only in the pitting resistance equation
- To account for detrimental surface finish as affected by, but not limited to cutting shaving , lapping , grinding, shot peening
- Standard surface conditions for gear tooth have not yet been established
- No values currently given by AGMA
- Use value of 1 for normal commercial gears

Size Factor K_s

- Accounts for fatigue size effect, and non-uniformity of material properties for large sizes
  - Tooth size, diameter of part, ratio of tooth size to diameter of the part, face width, area of stress pattern, ratio of case depth to tooth size, hardenability and heat treatment
- AGMA has not established size factors
- Use 1 for normal gear sizes
- Could apply fatigue size factor method from Ch. 6, where this size factor is the reciprocal of the Marin size factor k_b. Applying known geometry information for the gear tooth,

$$K_s = \frac{1}{k_b} = 0.904(bm\sqrt{Y})^{0.535}$$

- If $K_s < 1$; use $K_s = 1$

Load-Distribution Factor K_H (K_m)

- Accounts for non-uniform distribution of load across the line of contact
- Ideal to locate the gear mid span between two bearings at the zero slope place when the load is applied
- Depends on mounting and face width
- Load-distribution factor is currently only defined for
  - Face width to pinion pitch diameter ratio $b/d_p \leq 2$
  - Gears mounted between bearings
  - Face widths up to 1 meter
  - Contact across the full width of the narrowest member

Load-Distribution Factor K_m (K_H)

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \qquad (14\text{–}30)$$

- Face load-distribution factor

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \qquad (14\text{–}31)$$

$$C_{pf} = \begin{cases} \dfrac{b}{10d} - 0.025 & b \le 25 \text{ mm} \\ \dfrac{b}{10d} - 0.0375 + 4.92(10^{-4})b & 25 < b \le 425 \text{ mm} \\ \dfrac{b}{10d} - 0.1109 + 8.15(10^{-4})b - 3.53(10^{-7})b^2 & 425 < b \le 1000 \text{ mm} \end{cases} \qquad (14\text{–}32)$$

- For value of $b/10d < 0.05$ use $b/10d = 0.05$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad (14\text{–}35)$$

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \ge 0.175 \end{cases} \qquad (14\text{–}33)$$

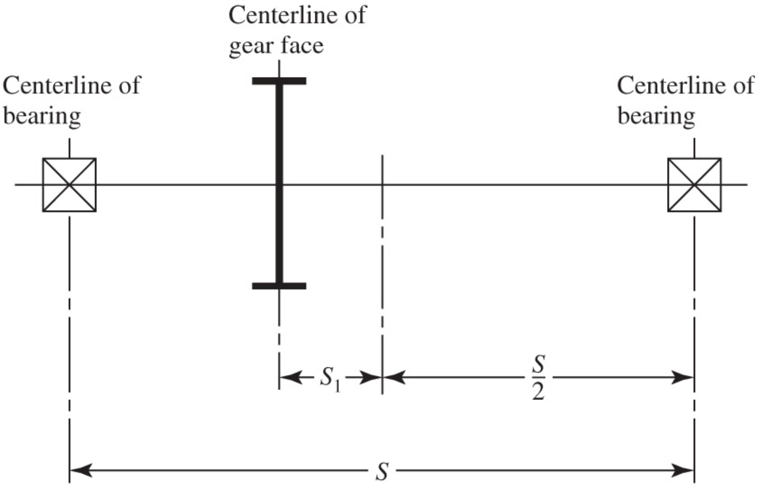


Fig. 14–10

- C_{ma} can be obtained from Eq. (14–34) with Table 14–9

$$C_{ma} = A + B\,b + C\,b^2 \qquad \text{(see Table 14–9 for values of } A, B, \text{ and } C) \qquad (14\text{–}34)$$

Table 14–9

Empirical Constants A, B, and C for Eq. (14–34), Face Width F in mm

Source: ANSI/AGMA 2001-D04.

Condition	A	B	C
		$\times 10^{-4}$	$\times 10^{-7}$
Open gearing	0.247	6.5748	-1.1858
Commercial, enclosed units	0.127	6.2205	-0.1442
Precision, enclosed units	0.0675	5.0394	-1.4353
Extraprecision enclosed gear units	0.00360	4.0157	-1.2741

- Or can read C_{ma} directly from Fig. 14–11

Load-Distribution Factor K_m (K_H)

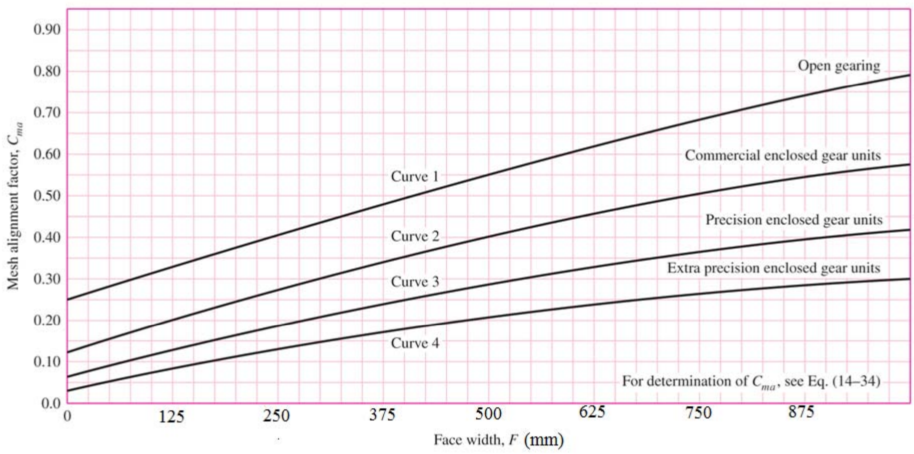


Fig. 14–11

Hardness-Ratio Factor C_H (Z_W)

- Since the pinion is subjected to more cycles than the gear, it is often hardened more than the gear.
- The hardness-ratio factor accounts for the difference in hardness of the pinion and gear.
- Z_w is only applied to the gear. That is, Z_w = 1 for the pinion.
- For the gear,

$$Z_W = 1.0 + A'(m_G - 1.0) \qquad (14\text{–}36)$$

$$A' = 8.98(10^{-3})\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29(10^{-3}) \qquad 1.2 \leq \frac{H_{BP}}{H_{BG}} \leq 1.7$$

- If $H_{BP}/H_{BG} < 1.2$, A'=0 and If $H_{BP}/H_{BG} > 1.7$, A'=0.00698
- Eq. (14–36) in graph form is given in Fig. 14–12.

Hardness-Ratio Factor

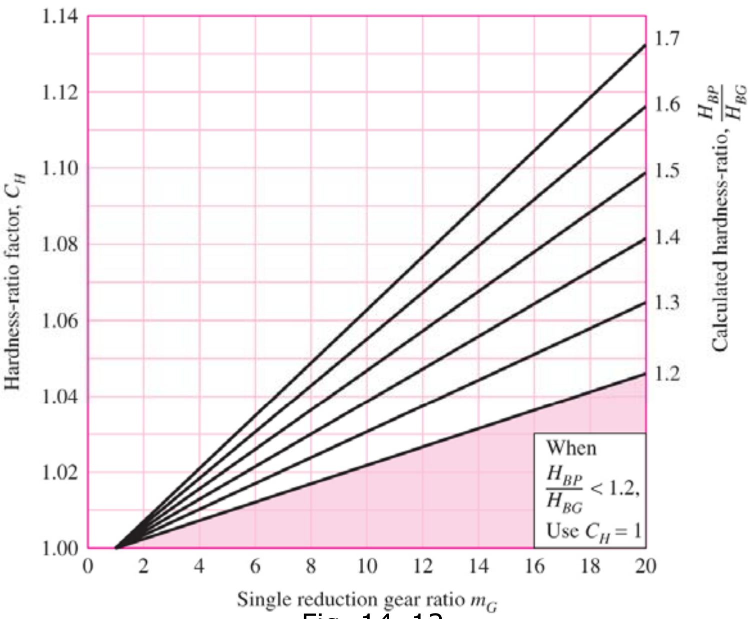


Fig. 14–12

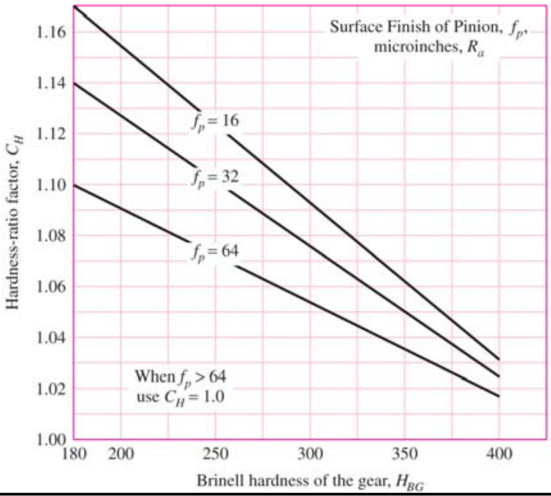
Hardness-Ratio Factor

- If the pinion is surface-hardened to 48 Rockwell C or greater, the softer gear can experience work-hardening during operation. In this case,

$$C_H = 1 + B'(450 - H_{BG})$$

$B' = 0.00075 \exp(-0.448 f_p)$

Where f_p is surface finish of the pinion expresses as Root mean squared Roughness in μm



Stress-Cycle Factors Y_N and Z_N

- AGMA strengths are for 10^7 cycles
- Stress-cycle factors account for other design cycles
- Fig. 14–14 gives Y_N for bending
- Fig. 14–15 gives Z_N for contact stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} \\ \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} \end{cases} \qquad \sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Stress-Cycle Factor Y_N

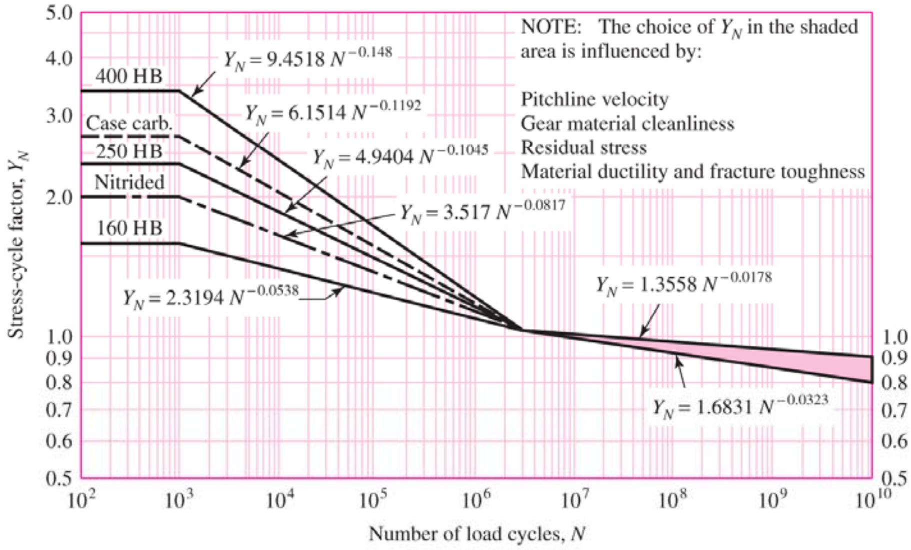


Fig. 14–14

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Stress-Cycle Factor Z_N

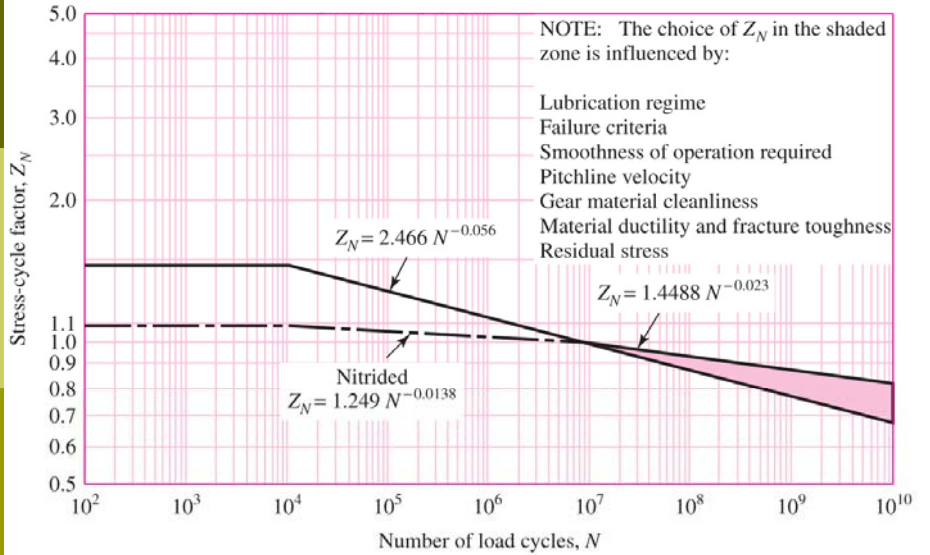


Fig. 14–15

Shigley's Mechanical Engineering Design

Reliability Factor K_R (Y_Z)

- Accounts for statistical distributions of material fatigue failures
- Does not account for load variation
- Use Table 14–10
- Since reliability is highly nonlinear, if interpolation between table values is needed, use the least-squares regression fit,

$$K_R = \begin{cases} 0.658 - 0.0759 \ln(1 - R) & 0.5 < R < 0.99 \\ 0.50 - 0.109 \ln(1 - R) & 0.99 \le R \le 0.9999 \end{cases} \quad (14\text{–}38)$$

Table 14–10

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

Temperature Factor K_T (Y_θ)

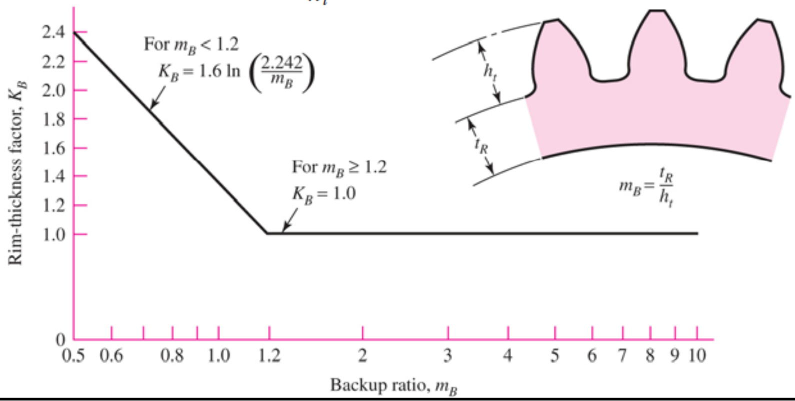
- AGMA has not established values for this factor.
- For temperatures up to 250ºF (120ºC), K_T = 1 is acceptable.

Rim-Thickness Factor K_B

- Accounts for bending of rim on a gear that is not solid

$$K_B = \begin{cases} 1.6 \ln \dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases} \quad (14\text{–}40)$$

$$m_B = \frac{t_R}{h_t} \quad (14\text{–}39)$$



Safety Factors S_F and S_H

- Included as design factors in the strength equations
- Can be solved for and used as factor of safety

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \quad (14\text{–}41)$$

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} = \frac{\text{fully corrected contact strength}}{\text{contact stress}} \quad (14\text{–}42)$$

- Or, can set equal to unity, and solve for traditional factor of safety as $n = \sigma_{all}/\sigma$

Comparison of Factors of Safety

- Bending stress is linear with transmitted load.
- Contact stress is not linear with transmitted load
- To compare the factors of safety between the different failure modes, to determine which is critical,
  - Compare S_F with S_H^2 for linear or helical contact
  - Compare S_F with S_H^3 for spherical contact

Summary for Bending of Gear Teeth

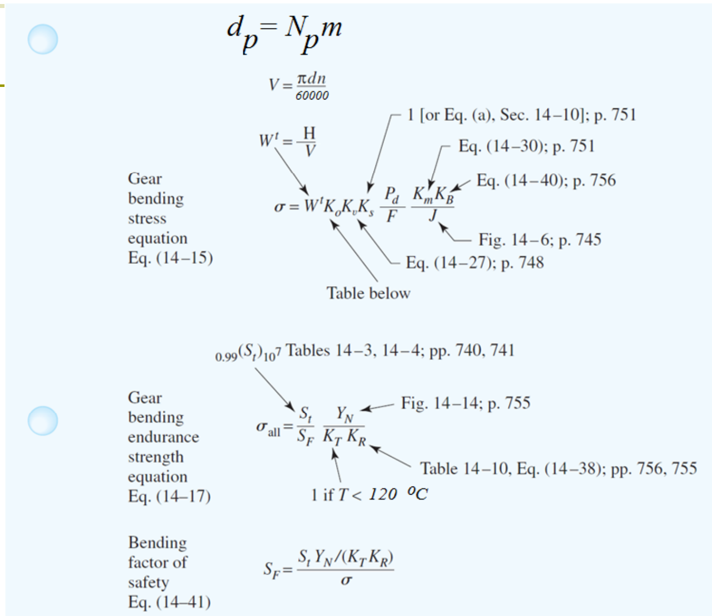


Fig. 14–17

Summary for Surface Wear of Gear Teeth

Fig. 14–18

$$d_p = N_p m$$

$$V = \frac{\pi d n}{60000}$$

$$W^t = \frac{33\,000\,H}{V}$$

Gear contact stress equation Eq. (14–16)

$$\sigma_c = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_p F}\frac{C_f}{I}\right)^{1/2}$$

- K_s: 1 [or Eq. (a), Sec. 14–10]; p. 751
- K_m: Eq. (14–30); p. 751
- C_f: 1
- I: Eq. (14–23); p. 747
- K_v: Eq. (14–27); p. 748
- K_o: Table below
- C_p: Eq. (14–13), Table 14–8; pp. 736, 749

Gear contact endurance strength Eq. (14–18)

$$\sigma_{c,\text{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

- S_c: $_{0.99}(S_c)_{10^7}$ Tables 14–6, 14–7; pp. 743, 744
- Z_N: Fig. 14–15; p. 755
- C_H: Section 14–12, gear only; pp. 753, 754
- K_R: Table 14–10, Eq. (14–38); pp. 756, 755
- K_T: 1 if $T < 120$ °C

Wear factor of safety Eq. (14–42)

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c}$$

C_H: Gear only

Example 14.4

A 17-tooth 20° pressure angle spur pinion rotates at 30 rev/s and transmits 3.0 kW to a 52-tooth disk gear. The module is 2.5, the face width 38.0 mm, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30, $(Y_J)_P = 0.30$, $(Y_J)_G = 0.40$, and Young's modulus is $2(10^5)$ MPa. The loading is smooth due to motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90, and use $Y_N = 1.3558N^{-0.0178}$, $Z_N = 1.4488N^{-0.023}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.

(a) Find the factor of safety of the gears in bending.

(b) Find the factor of safety of the gears in wear.

(c) By examining the factors of safety, identify the threat to each gear and to the mesh.

Solution

We begin with some preliminaries: $K_O = 1, d_P = 17(2.5) = 42.5$ mm, $d_G = 52(2.5) = 130$ mm, speed ratio $m_G = 52/17 = 3.06$, Lewis form factor $Y_P = 0.303$, $Y_G = 0.412$,

$$V = \frac{\pi d_P n_P}{1000} = \frac{\pi(42.5)30}{1000} = 4.0\text{m/s}$$

$$W^t = \frac{1000(3)}{4} = 750 \text{ N}$$

From Eq. (14–28)

$$B = 0.25(12-6)^{2/3} = 0.8255$$
$$A = 50 + 56(1 - 0.8255) = 59.77$$

From Eq. (14–27) velocity factor

$$K'_V = \left(\frac{A + \sqrt{200V}}{A}\right)^B = \left(\frac{59.77 + \sqrt{4(200)}}{59.77}\right)^{0.8255} = 1.377$$

The size factors (Eq. (a) Sec. 14–10) are

$$(K_s)_P = [0.904(bm\sqrt{Y_P})]^{0.0535} = 0.904\left[38(2.5)\sqrt{0.303}\right]^{0.0535} = 1.117$$

$$(K_s)_G = 0.904\left[38(2.5)\sqrt{0.412}\right]^{0.0535} = 1.126$$

From Eq. (14–30) the load distribution factor K_H is

$$K_H = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$C_{mc} = 1$, $\quad C_{pf} = 38/(10)(42.5) - 0.0375 + 4.92(10^{-4})38 = 0.0706$

$C_{pm} = 1$, for commercial enclosed gear units $C_{ma} = 0.15$ from Fig. 14–11, $C_e = 1$

$$K_H = 1 + (1)[0.0706(1) + 0.15(1)] = 1.2206$$

The load cycle factors from Fig. 14–14 are

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$
$$(Y_N)_G = 1.3558(10^8/3.06)^{-0.0178} = 0.996$$

From Eq. (14–38) the reliability factor is

$$Y_Z = 0.658 - 0.0759\ln(1 - 0.9) = 0.833$$

The temperature and surface condition factors are

$$Y_\theta = 1, \quad Z_R = 1$$

From Eq. (14–23) the geometry factor is

$$Z_I = \frac{1}{2}\cos 20^\circ \sin 20^\circ \frac{3.06}{3.06 + 1} = 0.121$$

The elastic coefficient is

$$Z_E = 191\sqrt{\text{MPa}}, \ \text{Table 14–8}$$

AGMA strengths at 10^7 cycles and 0.99 reliability:

From Fig. 14–2 the allowable bending strengths a

$$(\sigma_{FP})_P = 0.533(240) + 88.3 \text{ MPa} = 216 \text{ MPa}$$

$$(\sigma_{FP})_G = 0.533(200) + 88.3 = 195 \text{ MPa}$$

From Fig. 14–5 the allowable contact strengths are

$$(\sigma_{HP})_P = 2.22(240) + 200 = 733 \text{ MPa}$$

$$(\sigma_{HP})_G = 2.22(200) + 200 = 644 \text{ MPa}$$

From Fig. 14–5 the allowable contact strengths are

$$(\sigma_{HP})_P = 2.22(240) + 200 = 733 \text{ MPa}$$

$$(\sigma_{HP})_G = 2.22(200) + 200 = 644 \text{ MPa}$$

From Fig. 14–15 the stress cycle factors are

$$(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$$

$$(Z_N)_G = 1.4488(10^8/3.06)^{-0.023} = 0.973$$

$$A' = 8.98(10^{-3})\frac{240}{200} - 8.29(10^{-3}) = 0.002\,49$$

$$Z_W = 1 + 0.002\,49(3.06 - 1) = 1.005$$

(*a*) **Pinion tooth bending.** Substituting into Eq. (14–15) gives

$$\sigma = 750(1)(1.377)(1.117)\frac{1}{38}\frac{(1.2206)(1)}{(2.5)0.30} = 49.4 \text{ MPa}$$

Substituting into Eq. (14–17) with $S_F = 1$,

$$\sigma_{\text{all}} = \frac{216(0.977)}{(1)0.833} = 253.3 \text{ MPa}$$

$$S_F = \frac{253.3}{49.4} = 5.13$$

Gear tooth bending. Substituting into Eq. (14–15) gives the gear tooth bending stress

$$\sigma = 750(1)(1.377)\frac{1.126}{38(2.5)}\frac{1.2206(1)}{0.40} = 37.35 \text{ MPa}$$

Substituting into Eq. (14–17) with $S_F = 1$ gives

$$\sigma_{\text{all}} = \frac{195(0.996)}{(1)0.833} = 233 \text{ MPa}$$

$$S_F = \frac{233}{37.35} = 6.24$$

(*b*) **Pinion tooth wear.** Substituting into Eq. (14–16) gives

$$\sigma_c = 191\left[750(1)1.377(1.117)\frac{1.2206}{42.5(38)}\frac{1}{0.121}\right]^{1/2} = 512.8 \text{ MPa}$$

Substituting into Eq. (14–18) with $S_H = 1$ gives

$$\sigma_{c,\text{all}} = \frac{733(0.948)}{(1)0.833} = 834.2 \text{ MPa}$$

$$S_H = \frac{834.2}{512.8} = 1.627$$

Gear tooth wear. The Hertzian stress σ_C, changing only K_s, is 514.8 MPa. Substituting in Eq. (14–18) without S_H gives

$$\sigma_{c,\text{all}} = \frac{644(0.973)1.005}{(1)0.833} = 756.0 \text{ MPa}$$

$$S_H = \frac{756}{514.8} = 1.468$$

(*c*) For the pinion we compare S_F with S_H^2, or 5.13 with $1.627^2 = 2.647$, so the threat in the pinion is from wear. For the gear we compare S_F with S_H^2, or 6.24 with $1468^2 = 2.155$, so the threat in the gear is also from wear. Gear wear will probably retire the gearset.

Comparing Pinion with Gear

- Comparing the pinion with the gear can provide insight.
- Equating factors of safety from bending equations for pinion and gear, and cancelling all terms that are equivalent for the two, and solving for the gear strength, we get

$$(S_t)_G = (S_t)_P \frac{(Y_N)_P}{(Y_N)_G} \frac{J_P}{J_G}$$

- Substituting in equations for the stress-cycle factor Y_N,

$$(S_t)_G = (S_t)_P m_G^{\beta} \frac{J_P}{J_G} \qquad (14\text{–}44)$$

- Normally, $m_G > 1$, and $J_G > J_P$, so Eq. (14–44) indicates the gear can be less strong than the pinion for the same safety factor.

Comparing Pinion and Gear

- Repeating the same process for contact stress equations,

$$(S_c)_G = (S_c)_P \frac{(Z_N)_P}{(Z_N)_G} \left(\frac{1}{C_H}\right)_G = (S_C)_P m_G^{\beta} \left(\frac{1}{C_H}\right)_G$$

- Neglecting C_H which is near unity,

$$(S_c)_G = (S_c)_P m_G^{\beta} \qquad (14\text{–}45)$$

Example 14–6

In a set of spur gears, a 250-Brinell 14-tooth 16-pitch 20° full-depth pinion meshes with a 60-tooth gear. Both gear and pinion are of grade 1 through-hardened steel. Using $\beta = -0.023$, what hardness can the gear have for the same factor of safety?

For through-hardened grade 1 steel the pinion strength $(S_t)_P$ is given in Fig. 14–2:

$$(S_t)_P = 0.533(250) + 88.3 = 221.55 \text{ MPa}$$

From Fig. 14–6 the form factors are $J_P = 0.32$ and $J_G = 0.41$. Equation (14–44) gives

$$(S_t)_G = 221.55\left(\frac{60}{14}\right)^{-0.023}\frac{0.32}{0.41} = 167.23 \text{ MPa}$$

Use the equation in Fig. 14–2 again.

$$(H_B)_G = \frac{167.23 - 88.3}{77.3} = 148.26 \text{ Brinell}$$

Example 14–7

For $\beta = -0.056$ for a through-hardened steel, grade 1, continue Ex. 14–6 for wear.

From Fig. 14–5,

$$(S_c)_P = 2.22(300) + 200 = 866 \text{ MPa}$$

From Eq. (14–45),

$$(S_c)_G = (S_c)_P\left(\frac{64}{18}\right)^{-0.056} = 866\left(\frac{64}{18}\right)^{-0.056} = 807 \text{ MPa}$$

$$(H_B)_G = \frac{807 - 200}{2.22} = 273 \text{ Brinell}$$

which is slightly less than the pinion hardness of 300 Brinell.

Example 14.8

Design a 4:1 spur-gear reduction for a 75 kW, three-phase squirrel-cage induction motor running at 18.667 rev/s. The load is smooth, providing a reliability of 0.95 at 10^9 revolutions of the pinion. Gearing space is meager. Use Nitralloy 135M, grade 1 material to keep the gear size small. The gears are heat-treated first then nitrided.

Make the a priori decisions:

- Function: 75 kW, 18.667 rev/s, $R = 0.95$, $N = 10^9$ cycles, $K_O = 1$
- Design factor for unquantifiable exingencies: $n_d = 2$
- Tooth system: $\phi_n = 20°$, addendum $1.0m$, dedendum $1.25m$; $r_f = 0.300m$
- Tooth count: $N_P = 18$ teeth, $N_G = 72$ teeth (no interference)
- Quality number: $Q_V = 6$, use grade 1 material

Module: Select a trial module: $m = 6.0$ mm. The geometry factors from Fig. 14–6 are $(Y_J)_P = 0.32$, $(Y_J)_G = 0.415$. Preliminaries: $Y_P = 0.309$, $Y_G = 0.4324$, $d_P = 18(6) = 108$ mm, $d_G = 72(6) = 432$ mm.

$$V = \frac{\pi(108)(18.667)}{1000} = 6.33 \text{ m/s}$$

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

$$K_v = \left(\frac{59.77 + \sqrt{1266.7}}{59.77}\right)^{0.8255} = 1.47$$

$$W^t = \frac{75 \times 1000}{6.33} = 11848 \text{ N}$$

The reliability factor from Eq. (14–38) is

$$Y_Z = 0.658 - 0.0759 \ln(1 - 0.95) = 0.885$$

From Fig. 14–14,

$$(Y_N)_P = 1.3558(10^9)^{-0.0178} = 0.938$$

$$(Y_N)_G = 1.3558(10^9/4)^{-0.0178} = 0.961$$

From Fig. 14–15

$$(Z_N)_P = 1.4488(10^9)^{-0.023} = 0.900$$

$$(Z_N)_G = 1.4488(10^9/4)^{-0.023} = 0.929$$

Use a midrange face width $b = 4\pi m = 4\pi(6) = 75.4$ mm. Then the size factor is

$$K_s = 0.904\left(75.4(6)\sqrt{0.309}\right)^{0.0535} = 1.215$$

$$\frac{b}{10d_P} = \frac{75.4}{10(108)} = 0.0698$$

so Eq. (14–32) applies. $C_{mc} = 1$, $C_{pm} = 1$, $C_{ma} = 0.175$ from Fig. 14–11, $C_e = 1$ and

$$C_{pf} = 0.0698 - 0.0375 + 4.92(10^{-4})75.4 = 0.0694$$

$$K_H = 1 + (1)[0.0694(1) + 0.175(1)] = 1.244$$

Pinion tooth bending. With the above estimates of K_s and K_H from the trial module, we check to see if the mesh width b is controlled by bending or wear considerations. Equating Eqs. (14–15) and (14–17), substituting $n_d W^t$ for W^t, and solving for the face width $(b)_{\text{bend}}$ necessary to resist bending fatigue, we obtain

$$(b)_{\text{bend}} = \frac{n_d W^t K_O K_V' K_s}{m_t} \frac{K_H K_B}{(Y_J)_P} \frac{Y_\theta Y_Z}{\sigma_{FP} Y_N}$$

Equating Eqs. (14–16) and (14–18), substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{\text{wear}}$ necessary to resist wear fatigue, we obtain

$$(b)_{\text{wear}} = \left(\frac{Z_E Z_N}{\sigma_{HP} Y_\theta Y_Z}\right)^2 n_d W^t K_O K_V' K_s \frac{K_H Z_R}{d_P Z_I}$$

$$Z_I = \frac{1}{2}\cos 20^\circ \sin 20^\circ \frac{4}{4+1} = 0.129$$

From Table 14–5 the hardnesses of Nitralloy 135M are Rockwell C32–36 (302–33...

Brinell). Choosing a midrange hardness as attainable, using 320 Brinell. From Fig. 14–4,

$$\sigma_{FP} = 0.594(320) + 87.78 = 278 \text{ MPa}$$

Inserting the numerical value of σ_{FP} in the equation for $(b)_{\text{bend}}$ to estimate the face width gives

$$(b)_{\text{bend}} = 2(11848)(1)\frac{1.47(1.215)}{6}\frac{1.244(1)(1)0.885}{(0.32)(278)0.938} = 93 \text{ mm}$$

From Table 14–6 for Nitralloy 135M, $\sigma_{HP} = 1172$ MPa. Inserting this in the equation for $(b)_{\text{wear}}$ we find

$$(b)_{\text{wear}} = \left(\frac{191(0.900)}{1172(1)0.885}\right)^2 2(11848)1(1.47)1.215\frac{1.244(1)}{108(0.129)} = 103.8 \text{ mm}$$

Make face width 105 mm. Correct K_s and K_H:

$$K_s = 0.904\left[105(6)\sqrt{0.309}\right]^{0.0535} = 1.236$$

$$\frac{b}{10d_P} = \frac{105}{10(108)} = 0.097$$

$$C_{pf} = 0.097 - 0.0375 + 4.92(10^4)105 = 0.111$$

$$K_H = 1 + (1)[0.111(1) + 0.175(1)] = 1.286$$

The allowable stress in the pinion in bending is

$$\sigma_{\text{all}} = \frac{278(0.938)}{(1)0.885} = 294.6 \text{ MPa}$$

The bending stress induced by W^t in bending is

$$\sigma = 11848(1)1.47\frac{(1.236)}{6}\frac{1}{105}\frac{1.286(1)}{0.32} = 137.3 \text{ MPa}$$

The factor of safety in bending of the pinion is

$$(S_F)_P = \frac{\sigma_{\text{all}}}{\sigma} = \frac{294.6}{137.3} = 2.145$$

Gear tooth bending. *Decision:* Use cast gear blank because of the 432 mm pitch diameter. Use the same material, heat-treatment, and nitriding. The load-induced bending stress is in the ratio of $(Y_J)_P/(Y_J)_G$. Then

$$\sigma = 130.9\frac{0.32}{0.415} = 100.9 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{278(0.961)}{(1)0.885} = 302 \text{ MPa}$$

The factor of safety of the gear in bending is

$$(S_F)_G = \frac{\sigma_{\text{all}}}{\sigma} = \frac{302}{100.9} = 2.99$$

Pinion tooth wear. The case hardness of Nitralloy 135M is Rockwell C62–65, which is over 600 Brinell. We do not need the Brinell hardness of a gear specifically to find σ_{HP}. Use Table 14–6 to find $\sigma_{HP} = 1172$ MPa. Then

$$\sigma_c = 191\left[11848(1)1.47(1.236)\frac{1.286}{108(105)0.129}\right]^{1/2} = 841 \text{ MPa}$$

$$\sigma_{c,\text{all}} = \frac{1172(0.900)}{(1)0.885} = 1191 \text{ MPa}$$

The factor of safety based on load W^t is

$$n = (S_H)_P^2 = \left(\frac{\sigma_{c,\text{all}}}{\sigma_c}\right)^2 = \left(\frac{1191}{841}\right)^2 = 2.00$$

Gear tooth wear. The contact stress on the gear is the same as on the pinion, $\sigma_{HP} = 814.6$ MPa, as is the wear strength $\sigma_{HP} = 1172$ MPa, $Z_W = 1$ since the hardnesses are the same:

$$\sigma_{c,\text{all}} = \frac{1172(0.929)(1)}{(1)0.885} = 1230 \text{ MPa}$$

The factor of safety of the gear in bending based on load W' is

$$n = (S_H)_G^2 = \left(\frac{\sigma_{c,\text{all}}}{\sigma_c}\right)^2 = \left(\frac{1230}{814.6}\right)^2 = 2.28$$

Rim. Keep $m_B \geq 1.2$. The whole depth is h_t = addendum + dedendum = $m(1+1.25) = 2.25m = 2.25(6) = 13.5$ mm. The rim thickness t_R is

$$t_R \geq m_B h_t = 1.2(13.5) = 16.2 \text{ mm}$$

In the design of the gear blank, be sure the rim thickness exceeds 16.2 mm; if it does not, review and modify this mesh design.

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92

Questions ??

Thank You.