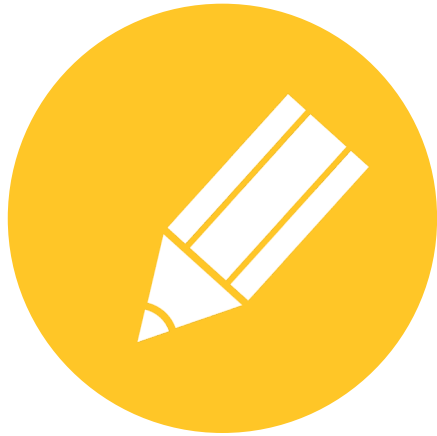


Theory of Answer Set Programming

clingo programs with negation

Objectives



Objective

Compute stable models of
programs with negation
using Clingo

Clingo Rules

Clingo rules are either

- H_1, \dots, H_m ($m \geq 1$) or
- H_1, \dots, H_m : $\neg B_1, \dots, B_n$ ($m, n \geq 0$)

where $H_1, \dots, H_m, B_1, \dots, B_n$ are atoms and comparisons,
possibly preceded with not (for \neg)

Example:

`p(a), q(b).`

`q(a) :- p(X), r(X).`

`r(X) :- p(X), not q(X).`

Recall: Propositional Image of Head and Body Expressions

Expression	Propositional Image
atom $p(t_1, \dots, t_k)$ in the head	conjunction of all formulas of the form $p(v_1, \dots, v_k)$ where v_i is a value of t_i ($i=1, \dots, k$)
atom $p(t_1, \dots, t_k)$ in the body	disjunction of all formulas of the form $p(v_1, \dots, v_k)$ where v_i is a value of t_i ($i=1, \dots, k$)
Comparison $t_1 < t_2$ in the head	\top if for every value v_1 of t_1 and every value v_2 of t_2 , $v_1 < v_2$; \perp otherwise
Comparison $t_1 < t_2$ in the body	\top if for some value v_1 of t_1 and some value v_2 of t_2 , $v_1 < v_2$; \perp otherwise

- $p(1..2)$ in the head
- $p(1..2)$ in the body
- $1..2=2..3$ in the head
- $1..2=2..3$ in the body

Recall: Propositional Image of Clingo Programs: Definition [Allowing Intervals]

The **propositional image** of a Clingo program consists of the instances of its rules rewritten as propositional formulas.

To rewrite a ground rule as a formula,

- | replace the symbol :- and all commas in the head and the body by propositional connectives as in the table shown;
- | **replace each of the expressions in the head in the body by its propositional image as in the previous slide**
- | replace the head of the rule by \perp if it is empty, and replace the body by T if it is empty.

Exercise

$p(a).$

$q(a).$

$r(X) :- p(X), \text{ not } q(X).$

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Π

$p(a)$

$q(a)$

$r(v) \leftarrow p(v) \wedge \neg q(v)$

for all $v \in \text{SUZ}$

Check if $\{p(a), q(a)\}$ is a stable model

1. Construct the propositional image

$\Pi \rightarrow p(a), q(a)$

$p(a)$

$q(a)$

$r(a) \leftarrow p(a) \wedge \neg q(a)$

$r(v) \leftarrow p(v) \wedge \neg q(v)$ for all

$v \in \text{SUZ} \setminus \{a\}$

2. Find the reduct relative to $\{p(a), q(a)\}$

3. Check if $\{p(a), q(a)\}$ is a minimal model of the reduct

Simplify $\Pi \rightarrow p(a), q(a)$

$p(a)$

$q(a)$

$r(v) \leftarrow p(v)$ for all $v \in \text{SUZ} \setminus \{a\}$

(a) Find the propositional image of

$$p(\overset{t_1}{1..3}).$$

$$q(X) :- X=2..4, \text{ not } p(X).$$

(b) Find the reduct relative to

$$\{p(1), p(2), p(3), q(4)\}$$

(c) Find the minimal model of the reduct

$$\{p(1), p(2), p(3), q(4)\}$$

$$p(1) \wedge p(2) \wedge p(3)$$

$$q(2) \leftarrow \overset{2,3}{\cancel{2}} = \overset{3,2,3,4}{\cancel{2..4}}, \text{ not } p(2)$$

$$q(3) \leftarrow \overset{1}{\cancel{3}} = \overset{1}{\cancel{2..4}}, \text{ not } p(3)$$

$$q(4) \leftarrow \overset{1}{\cancel{4}} = \overset{1}{\cancel{2..4}}, \text{ not } p(4)$$

$$\cancel{q(v) \leftarrow \overset{1}{\cancel{v}} = \overset{1}{\cancel{2..4}}, \text{ not } p(v)}$$

for all $v \in \{2,3,4\}$

$$p(1) \wedge p(2) \wedge p(3)$$

$$\cancel{q(2) \leftarrow \cancel{\neg p(2)} \perp}$$

$$\cancel{q(3) \leftarrow \cancel{\neg p(3)} \perp}$$

$$q(4) \leftarrow \neg p(4) \top$$

Definition: Prime

| N is a prime number between 1 and n if

- It is one of the numbers 2, ..., n, and
- there is no evidence that it is composite.

`composite(N) :- N=1..n, I=2..N-1, N\I=0.`

`prime(N) :- N=2..n, not composite(N).`

Lecture Wrap-Up

