First Order Logic and KR

Introduction to First Order Logic

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Objectives

Objective
Explain the limitation of propositional logic

Objective
Explain the basic idea of first-order logic

Limitations of Propositional Logic and KR

Propositional Logic

- **Study of declarative sentences, statement about the world which can be given a truth value**
- **Dealt very well with sentence components like: *not*, *and*, *or*, *if…then…***
- **Propositional logic is compositional:**
  - meaning of F $\wedge$ G is derived from meaning of F and of G
- **Limitations:**
  - Cannot express individuals and relations between them
  - Cannot deal with modifiers like *there exists, all, among, only*

Representation in Propositional Logic (1 of 4)

Knowledge from a medical domain:

- A juvenile disease affects only children or teenagers
- Children and teenagers are not adults
- Juvenile arthritis is a kind of arthritis and a juvenile disease
- Arthritis affects some adults

In propositional logic:

- $JuvDisease \rightarrow AffectsChild \vee AffectsTeenager$
- $Child \vee Teenager \rightarrow \neg Adult$
- $JuvArthritis \rightarrow JuvDisease \wedge Arthritis$
- $Arthritis \rightarrow AffectsAdult$

Limitations of Propositional Logic (2 of 4)

In propositional logic:

- $JuvDisease \rightarrow AffectsChild \vee AffectsTeenager$
- $Child \vee Teenager \rightarrow \neg Adult$
- $JuvArthritis \rightarrow JuvDisease \wedge Arthritis$
- $Arthritis \rightarrow AffectsAdult$

- **Some intuitive consequences of our statements:**
  - Juvenile arthritis does not affect adults
  - Arthritis is not a juvenile disease
- **We expect the following formulas to be entailed:**
  - $JuvDisease \rightarrow \neg AffectsAdult$
  - $Arthritis \rightarrow \neg JuvDisease$
- **However, neither of them is entailed.**

Limitations of Propositional Logic (3 of 4)

In propositional logic:

- $JuvDisease \rightarrow AffectsChild \vee AffectsTeenager$
- $Child \vee Teenager \rightarrow \neg Adult$
- $JuvArthritis \rightarrow JuvDisease \wedge Arthritis$
- $Arthritis \rightarrow AffectsAdult$

- **Even worse, if we add to our initial formulas the following ones, we obtain an unsatisfiable set of formulas**
  - $JuvDisease \rightarrow \neg AffectsAdult$
  - $JuvArthritis$

Limitations of Propositional Logic (4 of 4)

What is going wrong?

- A juvenile disease affects only children or teenagers
- Children and teenagers are not adults
- Juvenile arthritis is a kind of arthritis and a juvenile disease
- Arthritis affects some adults

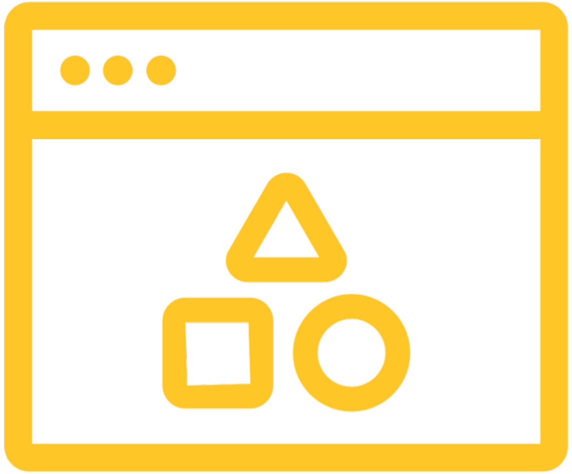
Intuitively…

- Green color represents sets of objects
- Blue color represents relationships between objects
- Red color indicates whether a statement holds for “all” or for “some” objects.

Need for a Richer Language

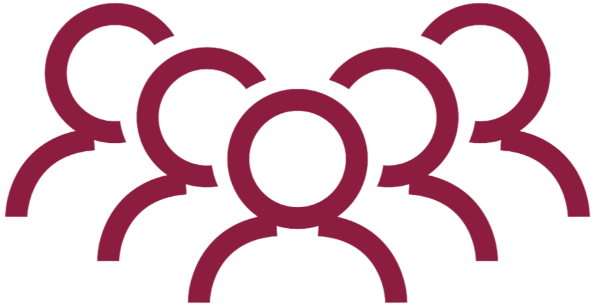
We need a language that allows us to

- Represent sets of objects
- Represent relationships between objects
- Write statements that are true for some or all objects satisfying certain conditions
- Express everything we can express in propositional logic (and, or, implies, not, …)



Need for a Richer Language, cont’d

- **Examples of conditions we want to express:**
  - For all objects c,
    - if c belongs to the set of juvenile diseases and it affects object d,
    - then d belongs to the set of children or to the set of teenagers.
- **There exist objects c, d, such that c belongs to the set of arthritis and d belongs to the set of adults and c affects d.**



Introduction to First Order Logic (FOL)

Example

- Express: **"Every student is younger than some instructor"**

- We could identify the entire phrase with the propositional symbol p

- However, the phrase has a finer logical structure. It is a statement about the following properties:
  - Being a student
  - Being an instructor
  - Being younger than somebody else

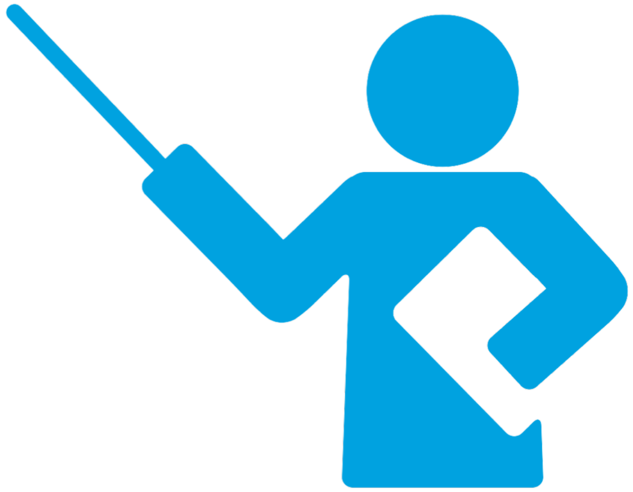
Predicates

- Individuals are expressed by object/function constants: **andy, paul, father(andy)**

- Properties are expressed by predicates. **S, I, Y** are predicates.
  - S(andy): Andy is a student.
  - I(paul): Paul is an instructor.
  - Y(andy, paul): Andy is younger than Paul.

Variables and Quantifiers

- **Variables are placeholders for concrete values.**
  - S(x): x is a student.
  - I(x): x is an instructor.
  - Y(x, y): x is younger than y.
- **Quantifiers make possible encoding the phrase:**
  - "Every student is younger than some instructor."
  - Encoding $\forall x\, (S(x) \rightarrow (\exists y\, (I(y) \land Y(x, y))))$



More Examples (1 of 3)

"No books are gaseous. Dictionaries are books.

Therefore, no dictionary is gaseous."

We denote: $B(x)$: x is a book

$G(x)$: x is gaseous

$D(x)$: x is a dictionary

More Examples (2 of 3)

"No books are gaseous. Dictionaries are books."

Therefore, no dictionary is gaseous.

We denote: $\mathrm{B(x)}$: x is a book

$\mathrm{G(x)}$: x is gaseous

$\mathrm{D(x)}$: x is a dictionary

$$\neg\exists x\big(b(x) \land G(x)\big), \forall x(D(x) \to B(x))$$

$$\vDash$$

$$\neg\exists x(D(x) \land G(x))$$

More Examples (3 of 3)

> "Every child is younger than his mother"

- We denote: $C(x)$: x is a child

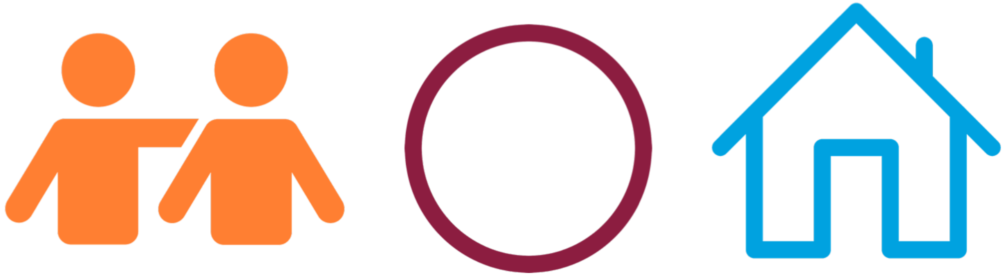
  $M(y, x)$: y is x's mother $\quad \forall x \forall y (C(x) \land M(y,x) \to Y(x,y))$

- Denote $m(x)$: mother of x $\quad \forall x \left(C(x) \to Y\left(x, m(x)\right) \right)$

First-Order Logic

Whereas propositional logic assumes world contains facts, first-order logic (like natural language) assumes the world contains

- Objects: people, houses, numbers, colors, cities, nba player, …
- Functions: father of, best friend, successor, one more than, end of, . . .
- Relations: red, round, bogus, prime, brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, . . .



Wrap-Up