Theory of Answer Set Programming

Stable Models of Definite/Positive Programs

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Objective

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Compute stable models of definite and positive programs by hand

Syntax of Propositional Rules

- **We consider rules as the restricted form of formulas in which implications occur in a limited way.**
  - We write $F \leftarrow G$ to denote $G \rightarrow F$

- **A (propositional) rule is a formula of the form $F \leftarrow G$ where F and G are implication-free ($\bot, \top, \neg, \wedge, \vee$ are allowed in F and G)**
  - We often write $F \leftarrow \top$ simply as F

- **Example: Is each of the following a propositional rule?**
  - $p \leftarrow (q \vee \neg r)$
  - $p \rightarrow (q \rightarrow r)$
  - $(p \vee q) \wedge \neg r$

- **A propositional program is a set of propositional rules.**

Representing Interpretations as Sets

We identify an interpretation with the set of atoms that are true in it.

- **Example:** interpretations of signature $\{p, q\}$

- **Example:** for signature $\{p, q\}$, the formula $p \vee q$ has three models:

Minimal Models

Minimal Models: Definition

About a model I of a formula F, we say that is is **minimal** if no other model of F is a subset of I.

- **Example:** For signature $\{p, q\}$, the formula $p \vee q$ has three models: $\{p\}, \{q\}, \{p, q\}$.
- The minimal models are

Exercise: Find all minimal models of the program

$$\{p \leftarrow q, \qquad q \vee r\}.$$

Minimal Models: A Question

Statement: If two formulas are equivalent under propositional logic, then they have the same minimal models.

Question: Is the converse true, that two formulas having the same minimal models are equivalent?

Stable Models of Definite Programs

Definite Propositional Rule: Definition

A propositional rule is definite if

a) its head is an atom, and

b) its body (if it has one) does not contain negation

Example:

$$p$$
$$r \leftarrow p \wedge q$$

$$p,$$
$$q \leftarrow p \wedge r,$$
$$r \leftarrow p \vee t,$$
$$s \leftarrow r \wedge t.$$

What is the minimal model of each program?

Definite Propositional Rule: An Algorithm

- **An algorithm to find a minimal model of a definite propositional program**
  - $S = \emptyset$
  - Repeat for each rule $h \leftarrow B$ in Π until there is no change in S
    - If $S \vDash B$, then $S := S \cup \{h\}$

Definite Propositional Rule: An Exercise

- **Let Γ be the program**

$$\{p_1 \leftarrow p_2 \land p_3, \quad p_2 \leftarrow p_3 \land p_4, \quad \ldots \quad , p_8 \leftarrow p_9 \land p_{10}\}.$$

- **For each of the following programs, describe the step-by-step process of constructing its minimal model:**

(a) Γ

(b) $\Gamma \cup \{p_5\}$

(c) $\Gamma \cup \{p_5, p_6\}$

Definite Propositional Rule: A Proposition

- **A propositional rule is definite if**
  a) its head is an atom, and
  b) its body (if it has one) does not contain negation

- **Proposition:** A definite program has a unique minimal model

- Q: Find a counterexample to the proposition if “definite” is dropped from the statement

- A stable model of a definite program Π is the minimal model of Π

- Q: Is every stable model of Π a model Π?

Informal Reading: Rationality Principle

- Informally, program Π can be viewed as a specification for stable models—sets of beliefs that could be held by a rational reasoner associate with Π.

$$p$$
$$r \leftarrow p \wedge q$$

Informal Reading: Rationality Principle, cont'd

Stable models will be represented by collections of atoms. In forming such sets the reasoner must be guided by the following informal principles:

- Satisfy the rules of Π. In other words, if one believes in the body of a rule, one must also believe in its head.
- Adhere to the "the rationality principle," which says, "Believe nothing you are not forced to believe.

$$p$$

$$r \leftarrow p \wedge q$$

Stable Models of Positive Programs

Positive Programs

- A rule/program is **positive** if it doesn't contain negation.
  - **Example:** $p \leftarrow q$ is positive; $p \leftarrow \neg q$ is not positive
- **Q: True or false? Every definite program is positive**
- **For positive programs, stable models are defined as minimal models**
- Q: Does every positive program have a unique minimal model?
- **A stable model of a definite program Π is the minimal model of Π**
- **A stable model of a positive program Π is a minimal model of Π**

Lecture Wrap-Up