Introduction to KRR

Propositional Logic: Syntax

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Objective

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Explain the intuition behind the propositional logic

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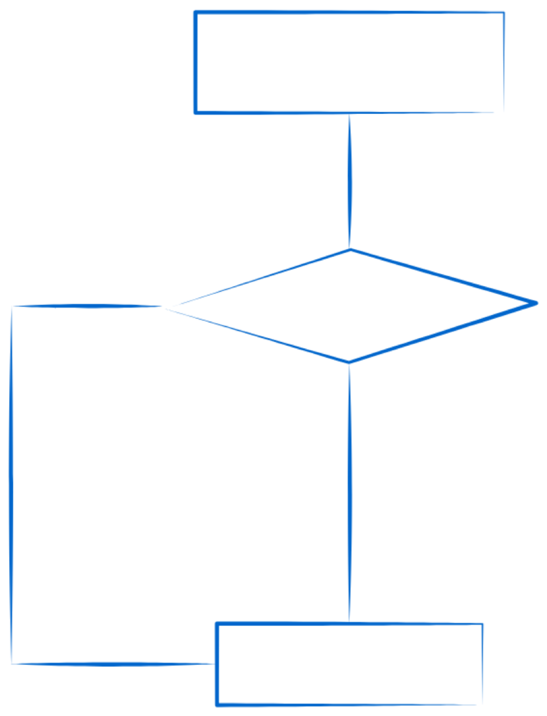
Explain the syntax of propositional logic

Propositional Logic and KR

Propositional logic is the simplest mathematical logic

- Study of declarative sentences, statements about the world which can be given a truth value
- Dealt very well with sentence components like not, and, or, if… then…
- Propositional logic is compositional:
  - Meaning of F $\land$ G is derived from meaning of F and of G



Yet satisfiability in propositional logic is NP-complete

Many efficient SAT solvers developed and actively used in many real-world problems

Rough Idea of Propositional Logic

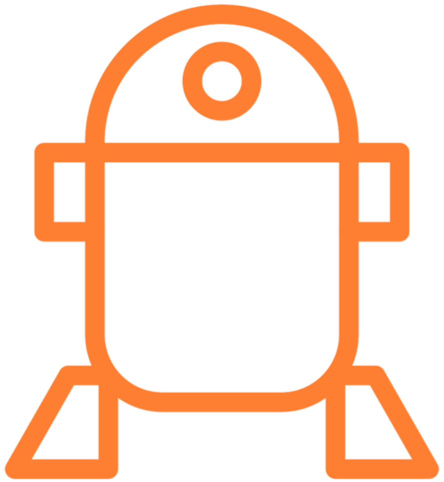
- **In propositional logic, an atom represents a proposition, which is either true or false.**
  - The sum of the numbers 3 and 5 equals 8
  - Jane reacted violently to Jack's accusations
  - Every even number greater than two is the sum of two prime numbers
  - Could you please pass the salt
  - Ready, steady, go
  - May the force be with you
- **Propositional connectives are used to compose the meaning**
  - If a number is divisible by 4, it is divislble by 2

$4 = 2+2$

$6 = 3+3$

$8 = 3+5$

$10 = 5+5$



Reasoning: Example 1

If the train arrives late (TrainLate) and there are no taxis at the station then John is late for his meeting (JohnLate).	H: $TrainLate \wedge \neg Taxi \rightarrow JohnLate$
John is not late for his meeting.	$\neg JohnLate$
The train did arrive late.	$TrainLate$
True or False? There was a taxi (Taxi) at the station.	T: $Taxi$

Reasoning: Example 2

- If it is raining and Jane does not have her umbrella with her, then she will get wet.
- Jane is not wet
- It is raining
- True or False? Jane has her umbrella with her.

Comparing the Two Examples

- Two examples have the same structure.

	Example 1	Example 2
p	The train is late	It is raining
q	There are taxis at the station	Jane has her umbrella with her
r	John is late for his meeting	Jane gets wet

- If p and not q, then r
- not r
- p.
- Therefore, q
- It does not matter what p,q,r stand for.

Alphabet of Propositional Logic

- A **propositional signature** is a set of symbols called **atoms**
  - $TrainLate, TaxiLate, JohnLate, p, q, r$
- **Propositional connectives**
  - 2-place (binary): $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (implication)
  - 1-place (unary): $\neg$ (negation)
  - 0-place: $\bot$ (bottom) and $\top$ (top)
- The **alphabet** of propositional logic consists of
  - the atoms from the signature,
  - the propositional connectives, and
  - parentheses

Definition of Propositional Formulas

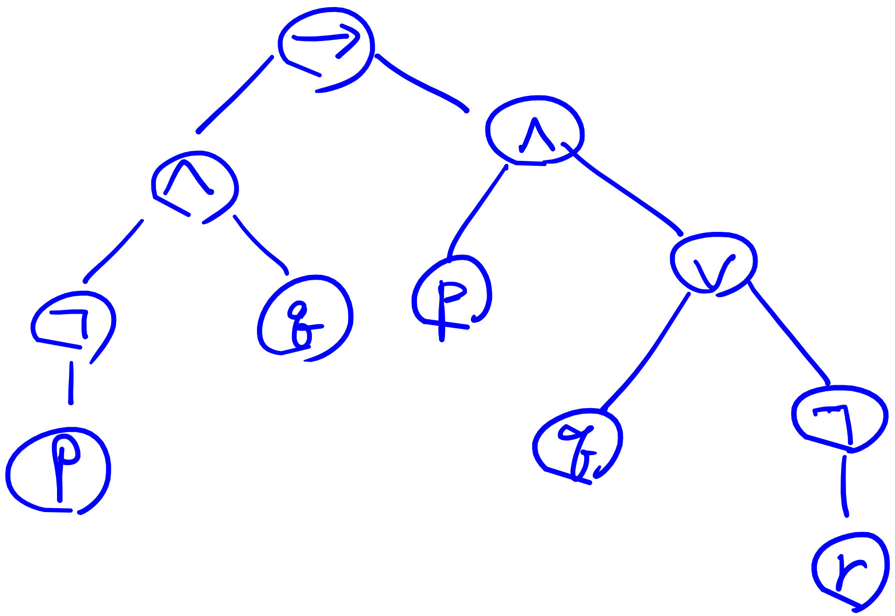
$(\neg\bot \to \top) \vee (\top \to \bot)$

$((\neg p \wedge q) \to (p \wedge (q \vee \neg r)))$

$((\neg p \wedge q) \to (p \wedge (q \vee \neg r)))$

A **propositional formula** of signature σ is defined recursively as follows:

- Every atom is a formula
- Both 0-place connectives are $(\bot, \top)$ formulas
- If F is a formula then $\neg\, \mathrm{F}$ is a formula
- For any binary connective $\odot$, if F and G are formulas then $(\mathrm{F} \odot \mathrm{G})$ is a formula



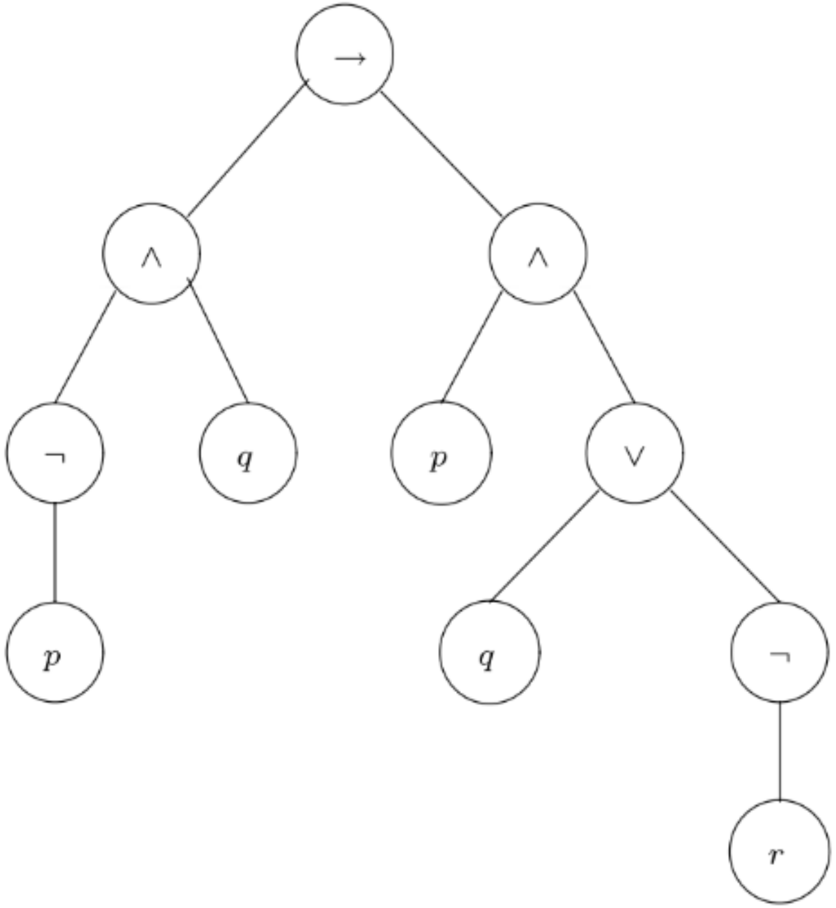
Subformulas

$$((\neg p \land q) \to (p \land (q \lor \neg r)))$$

Subformulas of ϕ are the formulas corresponding to the subtrees of the parse tree of ϕ

Q: Is the formula a subformula of itself? yes

Q: How may subformulas are there? 9



p
q
r
$\neg p$
$(\neg p \land q)$
$\neg r$
$(q \lor \neg r)$
$(p \land (q \lor \neg r))$
$((\neg p \land q) \to (p \land (q \lor \neg r)))$

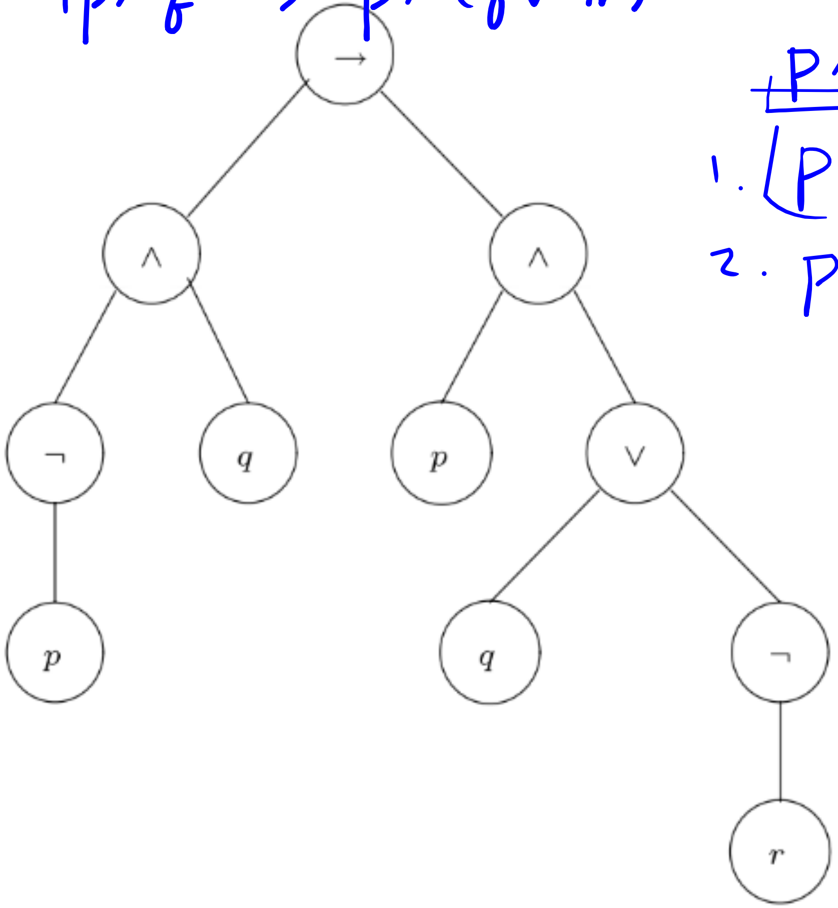
Binding Precedence

$$((\neg p \land q) \to (p \land (q \lor \neg r)))$$

Binding precedence allows us to avoid many parentheses:

$$\neg \;{\color{red}>}\; \land, \lor \;{\color{red}>}\; \to, \leftrightarrow$$

$\neg p \land q \to p \land (q \lor \neg r)$



$\underline{p \land q \lor \neg r}$

1. $(p \land q) \lor \neg r$
2. $p \land (q \lor \neg r)$

Wrap-Up