Theory of Answer Set Programming

Definite/Positive Programs in the Language of clingo Allowing Intervals

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Objective

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Represent definite and positive programs allowing intervals in the language of clingo

Positive Programs in the Language of Clingo (Allowing Intervals)

Terms in Clingo Language [Allowing Intervals]

The input language of Clingo allows the notion of terms

Terms in Clingo language are

- Integers, symbolic constants (representing specific objects), and variables
  - 1, 2, 3, a, b, X
- $t_1 \circ t_2$ where $\circ$ is an arithmetic operation, and t_1, t_2 are terms
  - 3 * 4, 3+4, 3**X
- $|t|$ where t is a term
  - |-3|
- $t_1..t_2$ where t_1, t_2 are terms
  - 1..10

Terms that do not contain variable are called ground.

Values of Ground Terms [Allowing Intervals]

The set of values of a ground term is defined recursively.

- If t is an integer or a symbolic constant then the only value of t is t itself.
- If t is $t_1 \circ t_2$, where $\circ$ is an arithmetic operation, then the values of t are integers $n_1 \circ n_2$, where the integer n_1 is a value of t_1, and the integer n_2 is a value of t_2.
- If t is $|t_1|$ then the values of t are integers of the form $|n_1|$, where the integer n_1 is a value of t_1.
- If t is $t_1 \,..\, t_2$ then the values of t are the integers n for which there exist integers n_1 and n_2 such that
  - n_1 is a value of t_1 and n_2 is a value of t_2,
  - $n_1 \leq n \leq n_2$.

$1..10$ $\{1, 2, 3 ... 10\}$

$1..10 + 1$ $\{2, 3 .... 11\}$

Example

Term	Values
2*2	{4}
2/0	∅
2*a	∅
$\underbrace{(2..4)}_{\{2,3,4\}}*\underbrace{(2..4)}_{\{2,3,4\}}$	{4, 6, 8, 9, 12, 16}
6..5	∅
a..(a+1)	∅
2**(-2)	{0}

Q: Find a ground term with values 1, 3, 9. 3**(0..2)

Propositional Image of Head and Body Expressions

Expression	Propositional Image
atom $p(t_1,\ldots,t_k)$ in the head	conjunction of all formulas of the form $p(v_1,\ldots,v_k)$ where v_i is a value of t_i $(i=1,\ldots,k)$
atom $p(t_1,\ldots,t_k)$ in the body	disjunction of all formulas of the form $p(v_1,\ldots,v_k)$ where v_i is a value of t_i $(i=1,\ldots,k)$
Comparison $t_1 \prec t_2$ in the head	$\top$ if for every value v_1 of t_1 and every value v_2 of t_2, $v_1 \prec v_2$; $\bot$ otherwise
Comparison $t_1 \prec t_2$ in the body	$\top$ if for some value v_1 of t_1 and some value v_2 of t_2, $v_1 \prec v_2$; $\bot$ otherwise

$1 < 2 \quad \top$

- p($\boxed{1..2}$) in the head — t_1, $\{1,2\}$ — $p(1) \wedge p(2)$
- p(1..2) in the body — $p(1) \vee p(2)$
- 1..2=2..3 in the head — $\{1,2\}$ $\{2,3\}$ — $1=2 \quad 2=2 \quad 1=3 \quad 2=3 \rightarrow \bot$
- 1..2=2..3 in the body $\rightarrow \top$

Propositional Image of Clingo Programs: Definition [Allowing Intervals]

The propositional image of a Clingo program consists of the instances of its rules rewritten as propositional formulas.

To rewrite a ground rule as a formula,

- replace the symbol :- and all commas in the head and the body by propositional connectives as in the table shown;
- replace each of the expressions in the head in the body by its propositional image as in the previous slide
- replace the head of the rule by ⊥ if it is empty, and replace the body by ⊤ if it is empty.

EXERCISES

Propositional image of

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square(1..2, 1..2)
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$\{1, 2\}$, $\{1, 2\}$

$sq(1,1) \wedge sq(1,2) \wedge sq(2,1) \wedge sq(2,2)$

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p(1..3).

X = 1  :-  p(X).
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$p(1) \wedge p(2) \wedge p(3)$

$\top \leftarrow p(1)$ ✓

$\bot \leftarrow p(v)$ for all $v \in S \cup Z \setminus \{1\}$

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p(1), p(2), p(3).

:- p(X),  X>2.
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$p(1) \vee p(2) \vee p(3)$

$\bot \leftarrow p(v) \wedge \top$ $v > 2$ and $v \in S \cup Z$

$\bot \leftarrow p(v) \wedge \bot$ for all other v

Lecture Wrap-Up