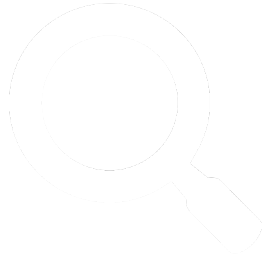


Theory of Answer Set Programming

Stable Models of Definite/Positive Programs

Objective



Objective

Compute stable models
of definite and positive
programs by hand

Syntax of Propositional Rules

We consider rules as the restricted form of formulas in which implications occur in a limited way.

- We write $F \leftarrow G$ to denote $G \rightarrow F$

A **(propositional) rule** is a formula of the form $F \leftarrow G$ where F and G are implication-free ($\perp, \top, \neg, \wedge, \vee$ are allowed in F and G)

- We often write $F \leftarrow \top$ simply as F

Example: Is each of the following a propositional rule?

- $p \leftarrow (q \vee \neg r)$
- $p \rightarrow (q \rightarrow r)$
- $(p \vee q) \wedge \neg r$

A **propositional program** is a set of propositional rules.

Representing Interpretations as Sets

| We identify an interpretation with the set of atoms that are true in it.

- **Example:** interpretations of signature $\{p, q\}$
- **Example:** for signature $\{p, q\}$, the formula $p \vee q$ has three models:



Minimal Models

Minimal Models: Definition

About a model I of a formula F , we say that it is **minimal** if no other model of F is a subset of I .

- **Example:** For signature $\{p, q\}$, the formula $p \vee q$ has three models: $\{p\}, \{q\}, \{p, q\}$.
- The minimal models are

Exercise: Find all minimal models of the program

$$\{p \leftarrow q, \quad q \vee r\}.$$

Minimal Models: A Question

- | **Statement:** If two formulas are equivalent under propositional logic, then they have the same minimal models.
- | **Question:** Is the converse true, that two formulas having the same minimal models are equivalent?



Stable Models of Definite Programs

Definite Propositional Rule: Definition

A propositional rule is **definite** if

- a) its head is an atom, and
- b) its body (if it has one) does not contain negation

Example:

$$\begin{array}{l} p \\ r \leftarrow p \wedge q \end{array}$$

$$\begin{array}{l} p, \\ q \leftarrow p \wedge r, \\ r \leftarrow p \vee t, \\ s \leftarrow r \wedge t. \end{array}$$

What is the minimal model of each program?

Definite Propositional Rule: An Algorithm

| An algorithm to find a **minimal model** of a **definite** propositional program

- $S = \emptyset$
- Repeat for each rule $h \leftarrow B$ in Π until there is no change in S
 - If $S \models B$, then $S := S \cup \{h\}$

Definite Propositional Rule: An Exercise

| Let Γ be the program

$$\{p_1 \leftarrow p_2 \wedge p_3, \quad p_2 \leftarrow p_3 \wedge p_4, \quad \dots, \quad p_8 \leftarrow p_9 \wedge p_{10}\}.$$

| For each of the following programs, describe the step-by-step process of constructing its minimal model:

(a) Γ

(b) $\Gamma \cup \{p_5\}$

(c) $\Gamma \cup \{p_5, p_6\}$

Definite Propositional Rule: A Proposition

| A propositional rule is **definite** if

- a) its head is an atom, and
- b) its body (if it has one) does not contain negation

| **Proposition:** A definite program has a unique minimal model

| Q: Find a counterexample to the proposition if “definite” is dropped from the statement

| A **stable model** of a definite program Π is **the** minimal model of Π

| Q: Is every stable model of Π a model Π ?

Informal Reading: Rationality Principle

Informally, program Π can be viewed as a specification for stable models—sets of beliefs that could be held by a rational reasoner associate with Π .

$$\begin{array}{l} p \\ r \leftarrow p \wedge q \end{array}$$

Informal Reading: Rationality Principle, cont'd

Stable models will be represented by collections of atoms. In forming such sets the reasoner must be guided by the following informal principles:

- Satisfy the rules of Π . In other words, if one believes in the body of a rule, one must also believe in its head.
- Adhere to the “the rationality principle,” which says, “Believe nothing you are not forced to believe.”

$$\begin{array}{l} p \\ r \leftarrow p \wedge q \end{array}$$



Stable Models of Positive Programs

Positive Programs

- | A rule/program is **positive** if it doesn't contain negation.
 - **Example:** $p \leftarrow q$ is positive; $p \leftarrow \neg q$ is not positive
- | Q: True or false? Every definite program is positive
- | For positive programs, stable models are defined as minimal models
- | Q: Does every positive program have a unique minimal model?
- | A **stable model** of a **definite** program Π is **the** minimal model of Π
- | A **stable model** of a **positive** program Π is **a** minimal model of Π

Lecture Wrap-Up

