# Introduction to KRR

## Propositional Logic: Semantics

Ira A. Fulton Schools of Engineering
Arizona State University

# Objective

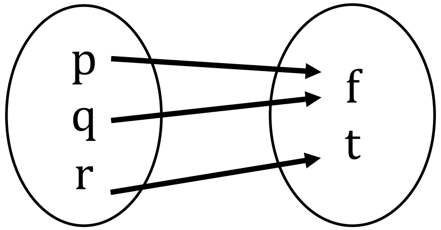
## Objective

Explain the semantics of propositional logic

# Interpretation

- A **propositional signature** is a set of symbols called atoms, such as p, q, r
- The symbols f and t are called **truth values**
- An **interpretation** of a propositional signature $\sigma$ is a function from $\sigma$ into $\{f, t\}$
- If $\sigma$ is finite, an interpretation can be defined by the truth table

| p | q | r |
|---|---|---|
| f | f | t |




Q: How many interpretations for {p, q, r}?

| p | q | r |
|---|---|---|
| f | f | f |
| f | f | t |
| f | t | f |
| f | t | t |
| t | f | f |
| t | f | t |
| t | t | f |
| t | t | t |

# Interpretation, cont'd

## Tables associated with the propositional connectives

| $x$ | Neg $\neg(x)$ |
|---|---|
| f | t |
| t | f |

| $x$ | $y$ | $\wedge(x,y)$ | $\vee(x,y)$ | $\rightarrow(x,y)$ | $\leftrightarrow(x,y)$ |
|---|---|---|---|---|---|
| f | f | f | f | t | t |
| f | t | f | t | t | f |
| t | f | f | t | f | f |
| t | t | t | t | t | t |

# Evaluation of a Formula and Satisfaction

- For any formula F and any interpretation I, the truth value $F^I$ that is assigned to F by I is defined recursively, as follows:
  - For any atom F, $F^I = I(F)$ $\quad p^I = I(p) = f$
  - $\bot^I = f$, $\top^I = t$
  - $(\neg F)^I = \underline{\overset{Neg}{\neg}}(F^I)$ $\quad (\neg p)^I = Neg(p^I) = Neg(f) = t$
  - $(F \odot G)^I = \odot(F^I, G^I)$ for every binary connective $\odot$

$$(p \wedge r)^I = \wedge(p^I, r^I) = \underline{\wedge}(f, t) = f$$

- If $F^I = t$ then we say that the interpretation I **satisfies** F (symbolically **$I \vDash F$**)

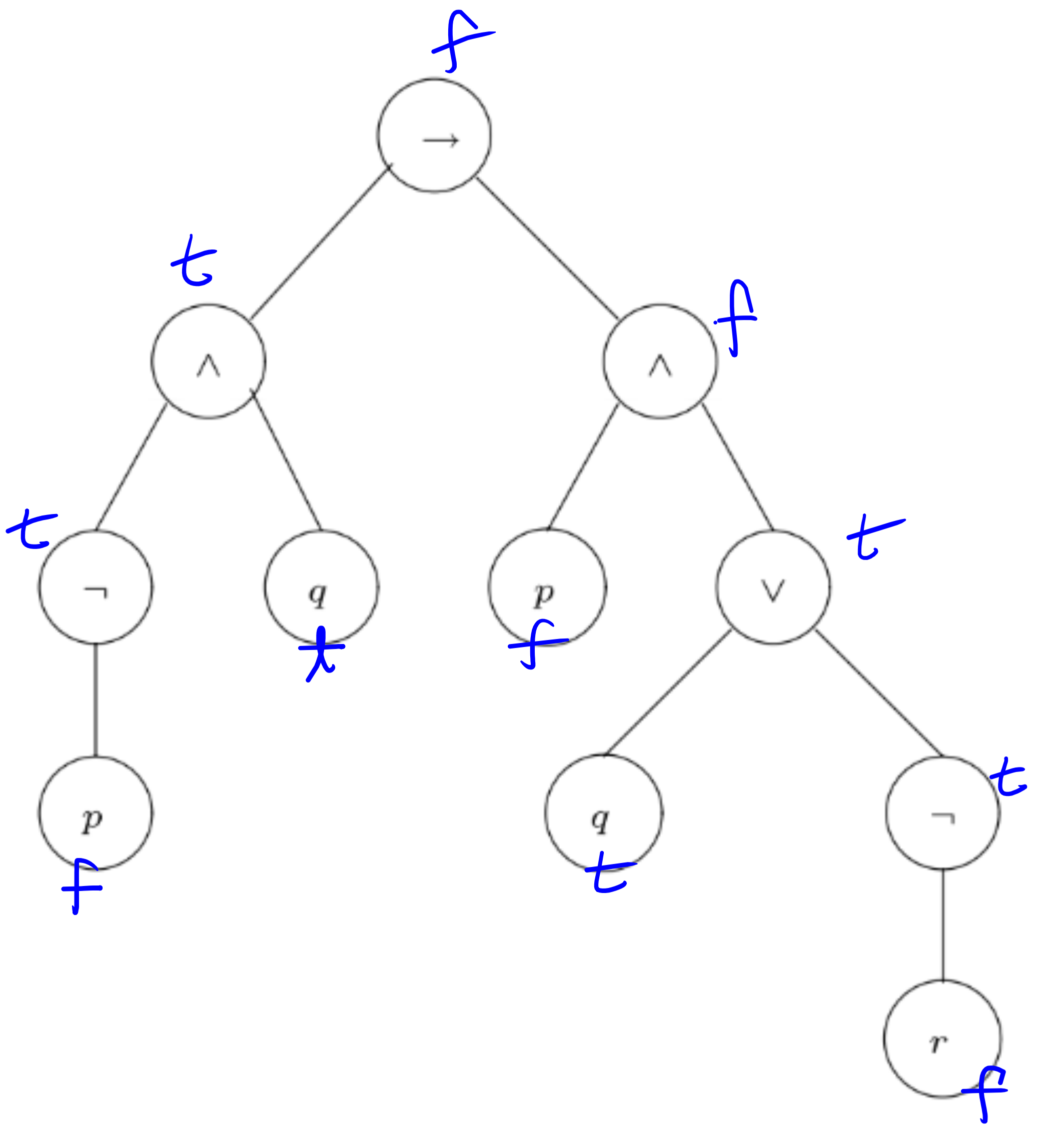
# Example

**Formula:** $((\neg p \wedge q) \rightarrow (p \wedge (q \vee \neg r)))$

**Q: The truth value of the formula**

1. if I(p)=t, I(q)=t, I(r)=t? t

2. if I(p)=f, I(q)=t, I(r)=f? f



# Wrap-Up