Theory of Answer Set Programming

clingo programs with negation

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Objectives

Objective

Compute stable models of programs with negation using Clingo

Clingo Rules

Clingo rules are either

- $\underline{H_1, \ldots, H_m}$ $(m \geq 1)$ or
- $\underline{H_1, \ldots, H_m} :- \underline{B_1, \ldots, B_n}$ $(m, n \geq 0)$

where $H_1, \ldots, H_m, B_1, \ldots, B_n$ are atoms and comparisons,

possibly preceded with not (for $\neg$)

Example:

```
p(a),q(b).

q(a) :- p(X), r(X).

r(X) :- p(X), not q(X).
```

Recall: Propositional Image of Head and Body Expressions

Expression	Propositional Image
atom $p(t_1,\ldots,t_k)$ in the head	conjunction of all formulas of the form $p(v_1,\ldots,v_k)$ where v_i is a value of t_i ($i=1,\ldots,k$)
atom $p(t_1,\ldots,t_k)$ in the body	disjunction of all formulas of the form $p(v_1,\ldots,v_k)$ where v_i is a value of t_i ($i=1,\ldots,k$)
Comparison $t_1 \prec t_2$ in the head	$\top$ if for every value v_1 of t_1 and every value v_2 of t_2, $v_1 \prec v_2$; $\bot$ otherwise
Comparison $t_1 \prec t_2$ in the body	$\top$ if for some value v_1 of t_1 and some value v_2 of t_2, $v_1 \prec v_2$; $\bot$ otherwise

- p(1..2) in the head
- p(1..2) in the body
- 1..2=2..3 in the head
- 1..2=2..3 in the body

Recall: Propositional Image of Clingo Programs: Definition [Allowing Intervals]

The propositional image of a Clingo program consists of the instances of its rules rewritten as propositional formulas.

To rewrite a ground rule as a formula,

- replace the symbol :- and all commas in the head and the body by propositional connectives as in the table shown;
- replace each of the expressions in the head in the body by its propositional image as in the previous slide
- replace the head of the rule by ⊥ if it is empty, and replace the body by ⊤ if it is empty.

Exercise

```
p(a).
q(a).
r(X) :- p(X), not q(X).
```

Π

$p(a)$

$q(a)$

$r(v) \leftarrow p(v) \wedge \neg q(v)$ for all $v \in S \cup Z$

Check if Z = {p(a), q(a)} is a stable model

1. Construct the propositional image
2. Find the reduct relative to {p(a),q(a)}
3. Check if {p(a),q(a)} is a minimal model of the reduct

$\Pi^{\{p(a), q(a)\}}$

$p(a)$

$q(a)$

$r(a) \leftarrow p(a) \wedge \cancel{\neg q(a)}$ $\bot$

$r(v) \leftarrow p(v) \wedge \cancel{\neg q(v)}$ $\top$ for all $v \in S \cup Z \setminus \{a\}$

Simplify $\Pi^{\{p(a), q(a)\}}$

$p(a)$ $q(a)$

$r(v) \leftarrow p(v)$ for all $v \in S \cup Z \setminus \{a\}$

EXERCISE

(a) Find the propositional image of

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p(1..3).
q(X) :- X=2..4, not p(X).
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(b) Find the reduct relative to

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{p(1),p(2),p(3),q(4)}
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(c) Find the minimal model of the reduct

$\{p(1), p(2), p(3), q(4)\}$

$p(1) \wedge p(2) \wedge p(3)$

$q(2) \leftarrow \cancel{2 = 2..4}^{\top}, \text{not } p(2)$

$q(3) \leftarrow \cancel{3 = 2..4}^{\top}, \text{not } p(3)$

$q(4) \leftarrow \cancel{4 = 2..4}^{\top}, \text{not } p(4)$

$\cancel{q(v) \leftarrow v = 2..4, \text{not } p(v)}$ $\bot$ for all $v \in S \cup Z \setminus \{2,3,4\}$

$p(1) \wedge p(2) \wedge p(3)$

$\cancel{q(2) \leftarrow \neg p(2)}$ $\bot$

$\cancel{q(3) \leftarrow \neg p(3)}$ $\bot$

$q(4) \leftarrow \cancel{\neg p(4)}$ $\top$

Definition: Prime

N is a prime number between 1 and n if

- It is one of the numbers 2, …, n, and
- there is no evidence that it is composite.

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composite(N) :- N=1..n, I=2..N-1, N\I=0.

prime(N) :- N=2..n, not composite(N).
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Lecture Wrap-Up