ECS 132 Summer 2019 - **Homework 1**

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1. Write a recursive function $pik(i, k)$ that returns t_{ik}.

```
pik <- function(i,k) {
    if(k==1){
        prob <- 0
        #If you are at position 1 or 2, you can never roll to 0 in a time.
        #As you getting closer to 0, so as i increases
        #Your chances of reaching or passing 0 is greater.
        if (i < 2) prob <- 0 # because max dice value is 6 (1 to 6)
        if (i == 2) prob <- 1/6
        if (i == 3) prob <- 2/6
        if (i == 4) prob <- 3/6
        if (i == 5) prob <- 4/6
        if (i == 6) prob <- 5/6
        if (i == 7) prob <- 1
        return (prob)
    }
    tot <- 0
    for(j in 1:6)
        if(i+j < 8) tot <- tot + 1/6 * pik(i+j,k-1)  # recursion
    return (tot)
}
```

2. Consider the ALOHA example. Suppose it is known that $X_1 \neq X_2$ the probability that there were 0, 1 or 2 collisions during those two epochs analytically and confirm via R simulation.

Epoch 1 (Both nodes start active)

Case	Node1	Status1	Node2	Status2	Collision	X_1	Probability
A	NS	Active	NS	Active	0	2	$(1-p)^2$
B	NS	Active	S	Not Active	0	1	$p(1-p)$
C	S	Not Active	NS	Active	0	1	$p(1-p)$
D	S	Active	S	Active	1	2	p^2

Epoch 2 (Given Case A)

Case	Node1	Status1	Node2	Status2	Collision	X_2	Probability
A	NS	Active	NS	Active	0	2	$(1-p)^2$
A	NS	Active	S	Not Active	0	1	$p(1-p)$
A	S	Not Active	NS	Active	0	1	$p(1-p)$
A	S	Active	S	Active	1	2	p^2

Epoch 2 (Given Case B)

Case	Node1	Status1	Node2	Status2	Collision	X_2	Probability
B	NS	Active	NS	Not Active	0	1	$(1-p)(1-q)$
B	NS	Active	NS	Active	0	2	$q(1-p)^2$
B	S	Not Active	NS	Not Active	0	0	$p(1-q)$
B	S	Not Active	NS	Active	0	1	$pq(1-p)$
B	NS	Active	S	Not Active	0	1	$p(1-p)q$
B	S	Active	S	Active	1	2	qp^2

Epoch 2 (Given Case C) is same as (Given Case B) above, because two nodes are exchangeable ideas.

Epoch 2 (Given Case D both nodes experienced collision, start active)

Case	Node1	Status1	Node2	Status2	Collision	X_2	Probability
D	NS	Active	NS	Active	1	2	$(1-p)^2$
D	NS	Active	S	Not Active	1	1	$p(1-p)$
D	S	Not Active	NS	Active	1	1	$p(1-p)$
D	S	Active	S	Active	2	2	p^2

Now we are looking for the given requirements

And we also know that P(X1 = 2, X2 = 0) = 0 because that is impossible to happen so we eliminate that option, so then we have

P(X1 $\neq$ X2)

= P(X1 = 2, X2 = 1) + P(X1 = 1, X2 = 2) + P(X1 = 1, X2 = 0)

Solve the algebra separately.

P(X1 = 2, X2 = 1)=P(X1 = 2)P(X2 = 1|X1 = 2)

= $[(1-p)^2 + p^2][2p(1-p)] = 2p(1-2p+2p^2)(1-p)$

P(X1 = 1, X2 = 2) = P(X1 = 1)P(X2 = 2|X1 =1)

= $2p(1-p) \times (q(1-p)^2 + qp^2) = 2p(1-p)(q-2pq+2p^2q)$

P(X1 = 1, X2 = 0) = P(X1 = 1)P(X2 = 0|X1 = 1)=$2p(1-p)p(1-q) = 2p^2(1-p)(1-q)$

P(X1 $\neq$ X2)= $[2p(1-2p+2p^2)(1-p)] + [2p(1-p)(q-2pq+2p^2q)] + [2p^2(1-p)(1-q)]$

$= -4p^4q - 4p^4 + 10p^3q + 6p^3 - 8p^2q - 4p^2 + 2pq + 2p$

Look for the probabilities of collisions with conditions.

- P(C=0| (X1$\neq$ X2))

  P(C=0| (X1$\neq$ X2)) $= \dfrac{P(C=0 \cap (X1 \neq X2))}{P(X1 \neq X2)}$

  $P(C=0 \cap (X1 \neq X2))$

  $= P(X1=1, X2=0) + P(X1=1, X2=2) + P(X1=2, X2=1)$

  $= P(X1=1)P(X2=0 \mid X1=1) + P(X1=1)P(X2=2 \mid X1=1) + P(X1=$

2)$P(X2 = 1 \mid X1 = 2)$
$= (2p(1-p)p(1-q)) + (2p(1-p))((1-p)^2q) + (1-p)^2(2p(1-p))$
$= -2p^4q - 2p^4 + 8p^3q + 4p^3 - 8p^2q - 4p^2 + 2pq + 2p$
Plug back in to P(C=0| (X1≠ X2))
$$= \frac{P(C = 0 \cap (X1 \neq X2))}{P(X1 \neq X2)} = \frac{-p^3q - p^3 + 4p^2q + 2p^2 - 4pq - 2p + q + 1}{-2p^3q - 2p^3 + 5p^2q + 3p^2 - 4pq - 2p + q + 1}$$

- P(C=1| (X1≠ X2))
  P(C=1| (X1≠ X2)) $= \dfrac{P(C = 1 \cap (X1 \neq X2))}{P(X1 \neq X2)}$
  $P(C = 1 \cap (X1 \neq X2))$ = Collision in Epoch1 or Epoch2, not both.
  $= P(X1 = 2, X2 = 1) + P(X1 = 1, X2 = 2)$
  $= P(X1 = 2)P(X2 = 1 \mid X1 = 2) + P(X1 = 1)P(X2 = 2 \mid X1 = 1)$
  $= p^2(2p(1-p)) + (2p(1-p))(qp^2)$
  Plug back in
  P(C=0| (X1≠ X2)) $= \dfrac{p^2(2p(1-p)) + (2p(1-p))(qp^2)}{-4p^4q - 4p^4 + 10p^3q + 6p^3 - 8p^2q - 4p^2 + 2pq + 2p}$

- P(C=2| (X1≠ X2))
  P(C=2| (X1≠ X2)) $= \dfrac{P(C = 2 \cap (X1 \neq X2))}{P(X1 \neq X2)}$
  $= \dfrac{\mathbf{P(X1 = 2, X2 = 2)}}{\mathbf{P(X1 \neq X2)}} = \mathbf{0}$
  = Since X1 = X2 = 2; Does not work

TESTING RESULTS(CALCULATE BASED ON ABOVE) :
Plug in values p=.4 and q=.8
P(X1 ≠ X2)= 0.48768
P(C=0| (X1≠ X2)) $= \dfrac{0.4368}{0.6096} = 0.71653543307$
P(C=1| (X1≠ X2)) $= \dfrac{0.13824}{0.48768} = 0.28346456692$
P(C=2| (X1≠ X2)) = 0

Here is a correct R simulation of the logic above

```
aloha <- function(p, q, nreps){
  x1_notEqual_x2 <- 0
  zero_collision <- 0
  one_collision <- 0
  two_collision <- 0
  for (i in 1:nreps) {
    # number of collisions
    temp_collision <- 0
    # Epoch 1:
```

```
# starts off active or not active
node_a <- runif(1) < p
node_b <- runif(1) < p
if(node_a + node_b == 2) {
  # 1 collision
  temp_collision <- temp_collision + 1
}
if (node_a + node_b == 1) {
    # 1 node sends
    X1 <- 1
} else {
    # neither node sends
    X1 <- 2
}
# Epoch 2:
# number of active nodes
active_node <- X1
# Try generating a new message
if (X1 == 1 && runif(1) < q) {
  active_node <- 2
}
if(active_node == 1) {
# Try sending message
    if (runif(1) < p){
        X2 <- 0
    } else {
        X2 <- 1
    }
} else {
    node_a <- runif(1) < p
    node_b <- runif(1) < p
    if(node_a + node_b == 2) {
        # 1 collision happended
        temp_collision <- temp_collision + 1
        X2 <- active_node
    } else if (node_a + node_b == 1) {
        # 1 node is sent
        X2 <- active_node - 1
    } else {
        # neither node are sent
        X2 <- active_node
    }
}
# if X1 != X2
if(X1 != X2){
```

```
      x1_notEqual_x2 <- x1_notEqual_x2 + 1
      #Count the number of collisions
      if (temp_collision == 0) {
        zero_collision <- zero_collision + 1
      }
      if (temp_collision == 1) {
        one_collision <- one_collision + 1
      }
      if(temp_collision == 2) {
        two_collision <- two_collision + 1
      }
    }
  }
  # print results:
  cat("P(C=0|(X1 != X2)) = ", zero_collision / x1_notEqual_x2,
   "\nP(C=1|(X1 != X2)) = ",one_collision / x1_notEqual_x2,
   "\nP(C=2|(X1 != X2)) = ",two_collision / x1_notEqual_x2 )
}

aloha(0.4,0.8,1000)
# one of the outputs:
# P(C=0|(X1 != X2)) =  0.7022587
# P(C=1|(X1 != X2)) =  0.2977413
# P(C=2|(X1 != X2)) =  0
```

3. Suppose we deal a 5-card hand from a regular 52-card deck. Which is larger, P(2 access) or P(3 diamonds)? Before continuing, take a moment to guess which one is more likely to happen. Please solve both analytically and use R to simulate solve and provide probabilities.
There are total $\binom{52}{5}$ ways to get a 5-card hand.
There are total $\binom{4}{2}$ ways to get 2 ace since there total 4 aces.
There are total $\binom{13}{3}$ ways to get 3 diamonds since there total 13 diamonds.
$\text{P(2 Aces)} = \frac{\binom{4}{2}\binom{48}{3}}{\binom{52}{5}} = \frac{6.17296}{2598960} = 0.03993$
$\text{P(3 Diamonds)} = \frac{\binom{13}{3}\binom{39}{2}}{\binom{52}{5}} = \frac{286.741}{2598960} = 0.08154$
Therefore chances of more likely to have a hand with 3 diamonds than a hand with 2 aces since probability for 3 diamonds are larger.

Here is a correct implementation of the R simulation of the logic above:

```
# use simulation to find P(2 aces) when deal a 5-card hand from a
# standard deck
```

```r
# think of the 52 cards as being labeled 1-52, with the 4 aces having
# numbers 1-4
cards <- function(nreps) {
   count2aces <- 0  # count of number of hands with 2 aces
   count3diamonds <- 0  # count of number of hands with 3 diamonds
   for (rep in 1:nreps) {
      # replace=FALSE means unique cards
      hand <- sample(1:52, size=5,replace=FALSE)  # deal hand
      aces <- intersect(1:4,hand)  # find which aces, if any, are in hand
      if (length(aces) == 2) count2aces <- count2aces + 1
      diamonds <- intersect(1:13,hand)  # find which diamonds, if any, are in hand
      if (length(diamonds) == 3) count3diamonds <- count3diamonds + 1
   }
   cat("The probability of 2 aces in a hand of 5 cards is",
   count2aces/nreps, "\n")
   cat("The probability of 3 diamonds in a hand of 5 cards is",
   count3diamonds/nreps, "\n")
}

cards(1000)
# output:
# The probability of 2 aces in a hand of 5 cards is 0.035
# The probability of 3 diamonds in a hand of 5 cards is 0.076
```

4. You are running a camp of 30 students, including John and Jane.

   (a) What is the total possible ways you can arrange 2 focus groups of students one group being size A(from step 1), and the other size B.
   The total number of ways to arrange the groups: $\binom{30}{A} * \binom{30-A}{B} = \binom{30}{B} * \binom{30-B}{A}$

   (b) What is the probability that John and Jane are not in the same group (so either not chosen or are chosen but not in the same group).
   We can calculate the probability that they are both together in group A or B. Then we can use 1-P(In the same group)
   P(Both In Group A) $= \frac{\binom{30-2}{A-2}\binom{30-A}{B}}{\binom{30}{A}\binom{30-A}{B}}$. P(Both In Group B) $= \frac{\binom{30-2}{B-2}\binom{30-B}{A}}{\binom{30}{A}\binom{30-A}{B}}$
   P(Same Group) $= \frac{\binom{30-2}{A-2}\binom{30-A}{B}}{\binom{30}{A}\binom{30-A}{B}} + \frac{\binom{30-2}{B-2}\binom{30-B}{A}}{\binom{30}{A}\binom{30-A}{B}}$
   P(Not In Same Group) = 1 - P(John and Jane together)
   $= 1 - [\frac{\binom{30-2}{A-2}\binom{30-A}{B}}{\binom{30}{A}\binom{30-A}{B}} + \frac{\binom{30-2}{B-2}\binom{30-B}{A}}{\binom{30}{A}\binom{30-A}{B}}]$

5. A new patient with low levels of protein X wants to know how probable is it that he has cancer. (H := Healthy and C:= Cancer)
P(High | C) = 0.95 ⇒ P(Low | C)= 1−0.95=0.05
P(High | H) = 0.1
P(C)=0.01 ⇒ P(H)=0.99
P(High) = P(High | C)*P(C) + P(High | H)*P(H)
= 0.95 * 0.01 + 0.1 * 0.01
= 0.185
⇒ P(Low) = 0.815
Therefore we can use Bayes rule to calculate P(C|Low)
$$\text{P(C| Low)} = \frac{P(Low \mid C)P(C)}{P(Low)} = \frac{0.05 * 0.01}{0.815} = 0.00613$$