ECE 2k1 Final

Impedance

$$X_R = R \tag{1a}$$
$$X_C = \frac{-j}{\omega C} = \frac{1}{j\omega C} \tag{1b}$$
$$X_L = j\omega L \tag{1c}$$
$$Z = \frac{V}{I} \tag{1d}$$
$$Y = \frac{1}{Z} = \frac{I}{V} \tag{1e}$$

Phase Changes

Resistance does not change phase of current or voltage
Capacitor makes voltage lag behind current
Inductor makes current lag behind voltage

Impedance, Admittance, Susceptance, Conductance

Admittance is the reciprocal of impedance [$Y = \frac{1}{Z}$]
Susceptance is the imaginary part of admittance[$B = Im(Y)$]
Conductance is the real part of admittance[$G = Re(Y)$]

Combination of Elements [Series]

$$R_{eq} = \Sigma_0^n R_n \tag{2a}$$
$$\frac{1}{C_{eq}} = \Sigma_0^n \frac{1}{C_n} \tag{2b}$$
$$L_{eq} = \Sigma_0^n L_n \tag{2c}$$

Combination of Elements [Parallel]

$$\frac{1}{R_{eq}} = \Sigma_0^n \frac{1}{R_n} \tag{3a}$$
$$C_{eq} = \Sigma_0^n C_n \tag{3b}$$
$$\frac{1}{L_{eq}} = \Sigma_0^n \frac{1}{L_n} \tag{3c}$$
$$\tag{3d}$$

Current Division Formula

$$I_n = I_0 \frac{R_{eq}}{R_n} = I_0 \frac{Z_{eq}}{Z_n} \tag{4a}$$

Capacitance and Inductance in terms of I, V

$$i_C(t) = C\frac{dv_C(t)}{dt} \tag{5a}$$
$$v_L(t) = L\frac{di_L(t)}{dt} \tag{5b}$$
$$v_C(t) = \frac{1}{C}\int_{t_0}^{t}(i_C(t)dt) + v_C(0) \tag{5c}$$
$$i_L(t) = \frac{1}{L}\int_{t_0}^{t}(v_L(t)dt) + i_L(0) \tag{5d}$$

Energy Stored / Dissipated over time T

$$w(t) = \frac{1}{R}\int_0^T (v_R^2(t)dt) = R\int_0^T (i_R^2(t)dt) \tag{6a}$$
$$w(t) = \frac{1}{2}C(v_c(t))^2 \tag{6b}$$
$$w(t) = \frac{1}{2}L(i_c(t))^2 \tag{6c}$$

Unit Function $u(t)$

$$u(t) = \left\{ \begin{array}{ll} 1 & \text{if } t > 0 \\ 0 & \text{if } t < 0 \end{array} \right\} \tag{7a}$$
$$u(-t) = \left\{ \begin{array}{ll} 1 & \text{if } t < 0 \\ 0 & \text{if } t > 0 \end{array} \right\} \tag{7b}$$
$$u(t-k) = \left\{ \begin{array}{ll} 1 & \text{if } t > k \\ 0 & \text{if } t < k \end{array} \right\} \tag{7c}$$
$$u(k-t) = \left\{ \begin{array}{ll} 1 & \text{if } t < k \\ 0 & \text{if } t > k \end{array} \right\} \tag{7d}$$

Capacitor Voltage and Inductor Current in RC, RL Circuits

$$v_C(t) = v_C(\infty) + (v_C(t_0) - v_C(\infty))e^{\frac{-(t-t_0)}{RC}} \tag{8a}$$
$$i_L(t) = i_L(\infty) + (i_L(t_0) - i_L(\infty))e^{\frac{-R(t-t_0)}{L}} \tag{8b}$$

Time Constants for RC, RL Circuits

$$\tau = RC \tag{9a}$$
$$\tau = \frac{L}{R} \tag{9b}$$

AC Circuit Math

$$y(t) = B\sin(\omega t + \beta) = B\cos\big(\omega t + \beta - 90^\circ\big) \tag{10a}$$
$$f = \frac{1}{T}(Hz) \tag{10b}$$
$$\omega = 2\pi f(rad)(s^{-1}) \tag{10c}$$

Imaginary Numbers

$$Z = a + bj \tag{11a}$$
$$Re(Z) = a, Im(Z) = b \tag{11b}$$
$$|Z| = \sqrt{a^2 + b^2} \tag{11c}$$
$$\theta = \tan^{-1}\left(\frac{b}{a}\right) \tag{11d}$$
$$\frac{1}{j} = -j = 1\angle -90^\circ \tag{11e}$$
$$j = 1\angle 90^\circ \tag{11f}$$
$$Z^* = a - bj = |Z|\angle -\tan^{-1}\left(\frac{b}{a}\right) \tag{11g}$$

Power

Amplitude

$$P = Re(\frac{1}{2}V_L I_L^*) \tag{12a}$$
$$P = \frac{1}{2}R_L|I_L|^2 \tag{12b}$$
$$P = \frac{1}{2}|V_L|^2 \frac{R_{load}}{R_{load}^2 + X_{load}^2} \tag{12c}$$

RMS

$$P = Re(V_L I_L^*) \tag{13a}$$
$$P = R_L|I_L|^2 \tag{13b}$$
$$P = |V_L|^2 \frac{R_{load}}{R_{load}^2 + X_{load}^2} \tag{13c}$$

Max Power Transfer

$$Z_{load} = Z_{eq}^* \tag{14a}$$
$$P_{max} = \frac{|V_{th}|^2}{8R_{eq}} \tag{14b}$$

Aadi Rave

RMS Formula

$$K_{rms} = \sqrt{\frac{1}{T}\int_0^T (K(t))^2 dt} \tag{15a}$$

Mutual Inductance

$$k = \frac{L_{12}}{\sqrt{L_1 L_2}} \tag{16a}$$

$$E = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm L_{12} i_1 i_2 \tag{16b}$$

$$\tag{16c}$$

Note that $L_{12}i_1i_2$ term is positive if transformer dots are opposite to each other; otherwise, it is negative.

Ideal Transformer

$$k = 1 \tag{17a}$$

$$\frac{V_2}{V_1} = \frac{n_2}{n_1} \tag{17b}$$

$$\frac{I_2}{I_1} = \frac{n_1}{n_2} \tag{17c}$$

$$\frac{Z_2}{Z_1} = \left(\frac{n_2}{n_1}\right)^2 \tag{17d}$$

Currents in Semiconductors

Diffusion evens out charge distribution in semiconductor

$$f = \text{particle flux density} \tag{20a}$$

$$n = \text{particle concentration (n or p)} \tag{20b}$$

$$D = \text{diffusion coefficient} \tag{20c}$$

$$f = -D\frac{d\eta}{dx} \tag{20d}$$

$$\tag{20e}$$

Drift Current moves charges according to electric fields

$$J_N = q\mu_n n\epsilon_x \tag{21a}$$

$$J_P = q\mu_p p\epsilon_x \tag{21b}$$

$$J = \text{current density} \tag{21c}$$

$$\epsilon = \text{electric field} \tag{21d}$$

$$q = \text{particle charge} \tag{21e}$$

$$\mu = \text{electron mobility} \tag{21f}$$

Total Equations

$$J_P = J_{P,drift} + J_{P,diff} = q\mu_p p\epsilon_x - qD_p\frac{d\eta}{dx} \tag{22a}$$

$$J_N = J_{N,drift} + J_{N,diff} = q\mu_n n\epsilon_x + qD_n\frac{d\eta}{dx} \tag{22b}$$

Einstein Relation

$$k = 8.617 \cdot 10^{-5} eVK^{-1} = \text{Boltzmann constant} \tag{23a}$$

$$\frac{D}{\mu} = \frac{kT}{q} = 0.026 \tag{23b}$$

Quantum Principles

Aufbau Principle Orbitals are filled from lowest to highest energy
Pauli Exclusion Principle No two electrons can have the same quantum numbers $\implies$ two electrons per orbital, with different spin
Hund's Rule Equal energy orbitals get one electron each, before a second is filled
Bonding Orbitals There are two orbitals involved in a bond—bonding and anti-bonding. Bonding orbital $\implies$ strong stable bond, anti-bonding $\implies$ unstable bond.

pn Junction

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) \tag{24a}$$

$$\frac{x_n}{x_p} = \frac{N_A}{N_D} \tag{24b}$$

$$x_p = \left[\frac{2\epsilon_r\epsilon_0}{q}\left(\frac{N_D}{N_A(N_A+N_D)}(V_{bi}-V_A)\right)\right]^{\frac{1}{2}} \tag{24c}$$

$$x_n = \left[\frac{2\epsilon_r\epsilon_0}{q}\left(\frac{N_A}{N_D(N_A+N_D)}(V_{bi}-V_A)\right)\right]^{\frac{1}{2}} \tag{24d}$$

$$W = \left[\frac{2\epsilon_r\epsilon_0}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)(V_{bi}-V_A)\right]^{\frac{1}{2}} \tag{24e}$$

$$I = I_0(e^{\frac{qV_A}{kT}} - 1) \tag{24f}$$

$$Q = q\left(\frac{N_A N_D}{N_A+N_D}\right)WA \text{ [A = Area]} \tag{24g}$$

$$\epsilon_0 = 8.85 \cdot 10^{-14} Fcm^{-1} \tag{24h}$$

$$q_e = -1.6 \cdot 10^{-19} C \tag{24i}$$

I_0 is the reverse-bias current for the pn-junction

Semiconductors

Conducting Band Anti-bonding orbitals of Si-Si bonds; electrons are free to move
Valence Band Bonding orbitals; electrons are bound to atoms
Band Gap Energy it takes to go from the valence to the conducting band. $E_{ins} > E_{sem} > E_{con}$

In semiconductors, mobile holes or electrons are needed for charge transfer. Electrons move in conducting band, holes move in valence band. $E_{Si} = 1.12eV$, but doping changes this.

$$n_i = \text{intrinsic carrier density} \tag{18a}$$

$$n = \text{electron concentration} \tag{18b}$$

$$p = \text{hole concentration} \tag{18c}$$

$$np = n_i^2 \tag{18d}$$

Doping
Donors: Group 5, n doping
Acceptors: Group 3, p doping

$$N_D = \text{donor doping concentration} \tag{19a}$$

$$N_A = \text{acceptor doping concentration} \tag{19b}$$

$$p \approx N_A \text{ (for p doped)} \tag{19c}$$

$$n \approx \frac{n_i^2}{N_A} \tag{19d}$$

$$n \approx N_D \text{ (for n doped)} \tag{19e}$$

$$p \approx \frac{n_i^2}{N_D} \tag{19f}$$

Diodes

$$V_A = V_D + I_0(e^{\frac{qV_D}{kT}} - 1)R_D \tag{25a}$$

$$V_D = \frac{kT}{q}\ln\left(\frac{I_D}{I_0} + 1\right) \tag{25b}$$

Transistors

$$I_D = \mu_n C_{ox} \left(\frac{W}{L}\right) \left[(V_{GS} - V_T)V_{DS} - \frac{1}{2}V_{DS}^2\right] \tag{26a}$$

$$V_{DS} \leq V_{GS} - V_T \text{ [Triode]} \tag{26b}$$

$$I_D = \mu_n C_{ox} \left(\frac{W}{L}\right) \left[\frac{1}{2}(V_{GS} - V_T)^2\right] \tag{26c}$$

$$V_{DS} \geq V_{GS} - V_T \text{ [Saturation]} \tag{26d}$$

$$I_D = \mu_n C_{ox} \left(\frac{W}{L}\right) \left[\frac{1}{2}V_{DS}^2\right] \tag{26e}$$

$$V_{GS} = V_T \text{ [Edge]} \tag{26f}$$

$$P = V_{DS} I_D \tag{26g}$$

$$k = \mu_n C_{ox} \left(\frac{W}{L}\right) \tag{26h}$$

$$V_{triode} = V_T + \frac{\sqrt{2kR_D V_{DD}} - 1}{kR_D} \tag{26i}$$

Transistor Amplifier

$$A = -R_D k(V_G - V_T) \text{ [Gain]} \tag{27a}$$

$$D = \frac{\hat{V_G}}{2(V_G - V_T)} \text{ [Distortion with small signal condition]} \tag{27b}$$