

## Lecture

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Data Observations  $\rightarrow$  Machine Learning System  $\rightarrow$   $f$  (prediction functions/prediction algorithms)

Primer on Functions:

- A function takes an input value, performs a specific set of operations on it based on a defined rule, and then produces a single, corresponding output value; essentially, it establishes a relationship between an input and an output, where each input is mapped to only one unique output.
- A function from a set  $X$  to a set  $Y$  assigns to each element of  $X$  exactly one element of  $Y$ . The set  $X$  is called the domain of the function and the set  $Y$  is called the codomain of the function.
- In machine learning,  $X$  is the input set and  $Y$  is the output/label set

Examples of prediction tasks:

- Rain Prediction  
 $X: \{\text{set of all possible weather conditions}\} \ x \in X \subseteq \mathbb{R}^d$   
 $Y: \{Rain, NoRain\} = \{0, 1\}.$
- Digit Recognition  $X: \{\text{finite set of images}\} \ x \in X \subseteq \mathbb{R}^{307,200}$  (# of pixels in an image, each  $x$  has a grayscale value)  
 $Y: \{0, 1, 2, \dots, 9\}.$
- Predicting the Sale Price of a House  $X: \{\text{features of the house, zip code, square footage, age, etc.}\} \ x \in X \subseteq \mathbb{R}^d$  ( $d =$  # of features)  
 $Y: \text{price of a house in dollars} \subseteq \mathbb{R}_{>0} [10 \text{ USD} - 10,000,000 \text{ USD}]$   
 Note that we can use One-Hot Encoding to encode categorical attributes

How to find predictor function,  $f$  given domain  $D$

$D = (x_1, y_1), \dots, (x_n, y_n)$  Dataset of size  $n$ , where  $x \in X$  is an input and  $y \in Y$  is an output. And the function,  $f$  maps all  $x \in X$  to all  $y \in Y$ .

Let's suppose the following:  $\exists f^*: X \rightarrow Y$  s.t.  $f^*(x_i) = y_i, \forall i \in [n]$ . If we do find such an  $f^*$  then we are good!

We use a loss function,  $l$ , to help us understand the error between the predicted and actual values.

In supervised machine learning a loss function,  $l$ , takes the true output space (the actual target value) and the predicted output space (the model's prediction) as inputs, and calculates a numerical value representing how much the prediction deviates from the true value, essentially measuring the "error" made by the model; the goal is to minimize this loss value during training to improve the model's accuracy

$Y$  = Actual

$\hat{Y}$  = Predicted

Error =  $Y - \hat{Y}$

Depending on the task/problem, different loss functions are used to assess the quality of the model (i.e., the prediction function). We will discuss Regression and classification (binary classification and multi-classification):

Regression Loss Functions;  $Y \subseteq \mathbb{R}, \hat{Y} \subseteq \mathbb{R}$

*Absoluteloss*  $|y - \hat{y}|$

*Squaredloss*  $(y - \hat{y})^2$

Regression Loss Functions;  $Y \subseteq \{0, 1\}^m, \hat{Y} \subseteq \{0, 1\}^m$ , where  $m$  is the number of classes

*LogisticLoss*  $(y, \hat{y}) = -y * \log(\hat{y}) - (1 - y) * \log(1 - \hat{y}), \forall y \in Y, \forall \hat{y} \in \hat{Y}$

*MulticlasslogisticLoss*  $(y, \hat{y}) = \sum_{i=1}^m y_i * \log(\hat{y}_i), \forall y \in Y, \forall \hat{y} \in \hat{Y}$