Distributed Rumor Blocking With Multiple Positive Cascades

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***Abstract*— Misinformation and rumor can spread rapidly and widely through online social networks and therefore rumor controlling has become a critical issue. It is assumed in the existing works that there is a single authority whose goal is to minimize the spread of rumor by generating a positive cascade. In this paper, we study a more realistic scenario when there is multiple positive cascades generated by different agents. For the multiple-cascade diffusion, we propose the peer-to-peer independent cascade model for private social communications. The main contribution of this paper is an analysis of the rumor blocking effect (i.e., the number of the users activated by rumor) when the agents noncooperatively generate the positive cascades. We show that the rumor blocking effect provided by the Nash equilibrium will not be arbitrarily worse even if the positive cascades are generated noncooperatively. In addition, we give a discussion on how the cascade priority and activation order affect the rumor blocking problem. We experimentally examine the Nash equilibrium of the proposed games by simulations done on real social network structures.**

***Index Terms*— Game theory, rumor blocking, social network.**

I. Introduction

WITH the recent advancements of information technologies, social networks have significantly changed the world by allowing efficient interchange of ideas and innovations. Especially, in online social networks of which there are a drastic increase of usage in the past decade, the hitting news may break out even before officially announced [1]. However, misinformation or rumor also spreads through the network [2], which may lead to serious public panic or economic consequence. Therefore, rumor control has become one of the important issues in social networks research.

The topics regarding rumor control are closely related to the study of the influence diffusion in social networks. In a social network, it is assumed that information spreads in the fashion of influence cascades. Under the classic models, a cascade starts to spread from a set of seed users and then propagates from active users to inactive users. Rumor is taken as a certain cascade spreading along with other cascades, and the cascades holding opposite opinions may compete against each other. In particular, a user who has received the genuine news will not accept the rumor. Conversely, when rumor comes first, the undesirable effect can be caused immediately, and therefore, the true fact arriving later is futile. For example, when affected by the misinformation of swine flu on Twitter, people might have taken mistake vaccines before receiving the clarification from WHO. In order to prevent people from being misled by rumor, a natural method is to introduce a positive cascade that is able to reach users before the arrival of rumor. Once the rumor is detected, the network manager can generate a competing positive cascade by selecting appropriate seed users such that the number of rumor-activated users can be minimized. Motivated by this framework, several works (e.g., [3]–[6]) have studied the rumor blocking problem under the competitive diffusion models.

The feasibility of the existing work is limited by the following aspects. On one hand, due to the great magnitude of a social network, the whole network cannot be efficiently controlled by a single manager. In a more realistic scenario, there are usually more than one positive cascades generated by different users or institutes, which we call the *agents*. Although all fight against rumor, when designing rumor containment strategies, such agents do not cooperate with each other. In this case, the rumor blocking task is distributed to the agents. Specifically, each agent makes their own choice according to the actions of other agents such that their own utility can be maximized, which forms a game between the agents. Under this setting, the social objective is to minimize the number of rumor-activated users while the private utility of each agent varies under different games. In this paper, we study such a noncooperative rumor blocking game and investigate the problem that *how bad can the equilibrium of the game be in the worst case compared with the optimal seeding strategy, with respect to the number of nonrumor-activated nodes*.

By extending the classic independent cascade (IC) model, we herein develop the peer-to-peer IC (PIC) model supporting the multiple-cascade diffusion. Unlike the existing models, the PIC model assumes that an active user can only activate one inactive neighbor at each time step. The PIC model represents the private social communication where the content is not open to all the users in the network. One example is the mobile social network where the communication is established by mobile phones in a person-to-person manner. Based on the PIC model, we formulate the rumor blocking game with one cascade of rumor and k agents where each agent generates one positive cascade. In such a game, the social utility is the number of rumor-activated nodes, which is a function over the strategy space of the agents. In this paper, we first show that under the PIC model the social utility is a *set*

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function of the union of the seed sets of the positive cascades, and furthermore, it is monotone increasing and submodular. For the private utility, we consider two games, the rumor-aware game and the rumor-oblivious game, depending on whether or not the agents are able to distinguish the rumor from the genuine news. For the proposed games, we provide an analysis of the equilibrium under the best-response assumption and the approximate-response assumption, respectively. Under the former, the agents are able to make optimal decisions and the equilibrium of the game provides a 2-approximation with respect to the social utility. In another issue, we consider that case that the agents cannot obtain the optimal strategy in polynomial time due to the NP-hard nature of the problem. As shown later, the private utility is submodular and it is well known that the submodular maximization problem admits an efficient $(1 - 1/e)$-approximation. Assuming that the agents adopt such an approximation strategy, we prove that the equilibrium of the rumor blocking game provides a $(2\ e - 1/e - 1)$-approximation. We simulate the rumor blocking game on graphs extracted from real-world social networks and record the number of nodes influenced by rumor. The experimental results have shown that the effect of noncooperative rumor blocking game is comparable to that of the single-positive-cascade case when the seed nodes are selected by the state-of-the-art algorithms.

In addition to the game-theoretic analysis, we further discuss the property of the competitive diffusion models. When developing such kind of models, there are two critical settings. One is to determine which cascade should a user u select when multiple cascades reach u at the same time. Another one is the order of activation. That is, when a node becomes active, which of its neighbor will be first selected for activation. The activation order of the neighbors plays an important role in the diffusion of multiple cascades.[1] As discussed later in Section V, such issues become tricky and complicated when there are more than two cascades. For example, under certain reasonable settings, when more positive cascades appear in the network, the rumor may paradoxically spread more widely. In this paper, we will discuss such issues and provide several interesting observations on the property of competitive diffusion model.

A. Contribution

The contribution of this paper is summarized as follows.

1) We propose a new competitive cascade model that represents the private peer-to-peer communication in social systems.
2) We formulate the rumor blocking game and provide the analysis of the equilibrium regarding the effect of rumor blocking. The main result is that the rumor blocking effect can be guaranteed with a provable ratio even if the agents work noncooperatively.
3) We discuss the property of the competitive cascade model under different settings of cascade priority and activation order.

[1]As discussed in prior works, e.g., [4], [7], such issues become less important for the classic IC model, because the influence spreads from one active node to all of their neighbors simultaneously.

B. Organization

The rest of this paper is organized as follows. In Section II, we survey the related work. In Section III, we provide the preliminaries and formulate the PIC model. In Section IV, a game-theoretic analysis is given. In Section V, we discuss the property of the competitive cascade model under different settings. In Section VI, the experimental results are shown. In Section VII, conclusions are drawn. Most of the proofs are given in the Appendix.

II. Related Work

Rumor control has drawn significant attention from both academia and industry. In what follows, we briefly introduce the prior works related to this topic.

Rumor detection aims to distinguish rumor from genuine news. Leskovec *et al.* [8] develop a framework for tracking the spread of misinformation and observe a set of persistent temporal patterns in the news cycle. Ratkiewicz *et al.* [9] built a machine learning framework to detect the early stages of viral spreading of political misinformation. Qazvinian *et al.* [10] address this problem by exploring the effectiveness of three categories of features: content-based, network-based, and microblog-specific memes. Takahashi and Igata [11] later study the characteristics of rumor and design a system to detect the rumor on Twitter. He *et al.* [12] consider the model for propagation dynamics of rumors and develop corresponding countermeasures.

Rumor source detection is another important problem for rumor control. The prior works primarily focus on the classic susceptible-infected-recovered model where the nodes can be infected by rumor and may recover later. Shah and Zaman [13] provide a systematic study and design a rumor source estimator based on the concept of rumor centrality. Wang *et al.* [14] later study this problem with the consideration of multiple observations.

The rumor blocking problem is mainly considered under the influence propagation models. The study of influence diffusion can be tracked back to Domingos and Richardson [15]. Later in the seminal work of Kempe *et al.* [16], two basic operational models, IC model and linear threshold (LT) model, are proposed. Based on those models, advanced models supporting multiple cascades are then developed, and the competitive influence diffusion problem has been studied in such models. Bharathi *et al.* [17] show a $(1 - 1/e)$-approximation algorithm for the best response to an opponent's strategy. Borodin *et al.* [18] study several competitive diffusion models by extending the classic LT model and show that the original greedy approach proposed in [16] may not be applicable to such settings. The rumor blocking problem is similar but not identical to the competitive influence maximization problem. The goal of the competitive influence maximization problem is to maximize the spread of a certain cascade while rumor blocking aims to minimize the spread of rumor (i.e., minimize the number of rumor-activated nodes). For the rumor blocking problem, He *et al.* [5] show a $(1 - 1/e)$-approximation algorithm for the competitive LT model, and, Fan *et al.* [3] study this problem under the opportunistic one-activate-one and deterministic one-activate-many models. From another perspective, Nguyen *et al.* [1] propose the β_T^I-node protector problem that limits the spread of misinformation by blocking the high-influential nodes. He *et al.* [19] study the rumor blocking problem in the mobile social networks. The above-mentioned all works aim to design the seeding algorithms, which is essentially different from the topic of this paper.

We are not the first who study the influence diffusion via game-theoretical approaches. Kostka *et al.* [20] formulate the seeding process as a game and study the best-response strategy under a new model, which is more restricted than the IC and LT models. Different from that paper, we do not design response strategies and instead our analysis focuses on the equilibrium of the game where there is one rumor cascade and multiple positive cascades. In another issue, Jiang *et al.* [21], [22] propose an evolutionary game-theoretic framework to model the dynamic information diffusion process in social networks.

III. Model

In this section, we introduce the system model and provide the preliminaries. The notations that are frequently used in this paper are listed in Table I.

A. Influence Diffusion

1) Single Cascade: A social network is given by a directed graph $G = (V, E)$ where V and E denote the users and social ties, respectively. Let $N_u = \{v|(u, v) \in E\}$ be the set of the out-neighbors of node u and define $d_u = |N_u|$ as the number of the out-neighbors of u. We will use terms *user* and *node* interchangeably. We speak of each user as being *active* and *inactive*. To trigger the spread of influence, some users are first activated as seed users who will later attempt to activate their out-neighbors. Under the independent cascade model, associated with each edge (u, v) there is a propagation probability $p^G_{(u,v)} \in [0, 1]$, which is the probability that u successfully activates v. For each pair of nodes u and v, u has only one chance to activate v. By the fashions of influence propagation, the independent cascade model can be classified into the following two categories.

Broadcast Independent Cascade Model[2]*:* Under this model, when node u becomes active at time $t - 1$, it attempt to activates all of its out-neighbors simultaneously at time step t.

PIC Model: Under this model, an active node u can only attempt to activate one of its out-neighbors at a time step.

Example 1: An illustrative example of the above-mentioned two models is shown in Fig. 1. Suppose the propagation probability of each edge is 1 and node v_3 is selected as the seed node. As shown in the figure, after the first step, all the neighbors of v_3 are activated under the BIC model, while only one neighbor of v_3 is activated under the PIC model.

The BIC model represents the open social communication namely, Facebook or Twitter. For example, a public post on Facebook is simultaneously available to all the user's friends. The PIC model represents the private social communication such as personal online message or e-mail, where a user has to take an action to pass the message to their friends. Note that, for the rumor blocking problem, there is a significant difference between these two models. One can see that the PIC model tends to slow the spread of influence, and when multiple cascades exist whether a node will be rumor-activated depends on the first cascade reaching it. In this paper, we focus on the PIC model, which has not been studied in the literature.

2) Multiple Cascade: Suppose there are multiple cascades each of which is generated by its own seed set. We denote by C_r the cascade generated by rumor with a fixed seed set a_r. The basic definitions are shown as follows.

Definition 1: For a certain cascade C, we call a node C*-active* (resp. $\overline{C}$*-active*) if it is activated (resp. not activated) by cascade C.

[2]This is the model that has been considered in most of the prior works [16], [23]–[25].

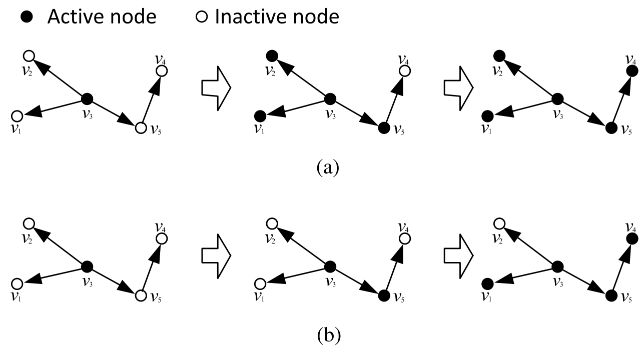

Fig. 1. Illustrative example of (a) BIC and (b) PIC models.

TABLE I
NOTATIONS

Symbol	Definition
$G = (V, E)$	a PIC network.
N_u	the set of out-neighbors of node u.
$p^G_{(u,v)}$	propagation probability of edge (u, v).
C_r	the cascade of rumor.
a_r	the seed set of rumor.
b_u^t	the neighbors of u that can be activated by u at time step t.
$\Pr[g]$	the probability that the realization g can be generated.
$\mathcal{G}$	the set of all possible realizations.
k	the number of agents.
C_i	the positive cascade generated by the i-th agent.
a_i	the seed set of cascade C_i.
B_i	the budget of the seed set of cascade C_i.
$\overline{\gamma}()$	social utility of the game.
$\overline{\delta}_i()$	private utility of the i-th agent.
$\overline{\sigma}_i(S)$	the expected number of C_i-active nodes under strategy S.
$t_A^g(u)$	the activation time of node u in g under the full-action A.

Definition 2 (Cascade Priority): Each cascade is assigned a distinct priority and we assume that the rumor always has the highest priority. We denote by Priority(C) the priority of cascade C and, for two cascades C_1 and C_2, Priority$(C_1) <$ Priority(C_2) if and only if cascade C_2 has a higher priority than that of cascade C_1.

Definition 3 (Activation Order): Let b_u^t be the set of the node v such that $v \in N_u$ and u has not tried to activate v before time t. At time step t, an active node u will uniformly at random select a node in b_u^t to activate.[3]

Recall that the PIC model represents the private communication and consequently a user cannot know whether the other users have been activated or not. Therefore, one user may attempt to activate another user who has already been activated by others.

3) Diffusion Process: Given a PIC network G and the seed sets of the cascades, the diffusion process unfolds in discrete, as described in the following.

1) Time step 0. Each cascade C activates its seed nodes. If one node is selected by more than one cascade, it will be activated by the cascade with the highest priority.
2) Time step $t > 0$. Each active node u randomly select one node v in b_u^t and activates v with a success probability of $p_{(u,v)}^G$, where each node in b_u^t has the same probability to be selected by u. If u is C-active and u successfully activates v then v becomes C-active. If a node is successfully activated by two or more neighbors pertaining to different cascades, it will be activated by the cascade with the highest priority.

The PIC model is a probabilistic model where the randomness comes from that: 1) at each step who to select to activate and 2) whether the activation succeeds. The following definition shows a derandomization of the diffusion process under the PIC model.

Algorithm 1 Realization Generation

Input: A PIC network $G = (V, E)$.
Output: A realization $g = (V_g, E_g)$ together with α_u^g for each node u and p_e^g for each e.
$V_g \leftarrow V$ and $E_g \leftarrow E$;
for each edge $e \in E$ **do**
  $rand \leftarrow$ a random number from 0 to 1 generated in uniform;
  if $rand \leq p_e^G$ **then**
    $p_e^g \leftarrow 1$;
  else
    $p_e^g \leftarrow 0$;
for each node $u \in V$ **do**
  $\alpha_u^g \leftarrow$ a permutation of N_u generated uniformly at random;
Return g, p_e^g and α_u^g;

Definition 4 (Realization): A *realization* [26] $g = (V_g, E_g)$ of a PIC network $G = (V, E)$ is a special PIC network randomly constructed by Algorithm 1. First, $V_g = V$ and $E_g = E$. The propagation probability p_e^g of each edge e in g is either 0 or 1 determined in random. In particular, for each edge e, the probability that $p_e^g = 1$ (resp. $p_e^g = 0$) is p_e^G (resp. $1 - p_e^G$). Each node u randomly decides a permutation α_u^g (i.e., an order) of all its out-neighbors N_u in G where each possible permutation of N_u has the same probability to be selected by u. We take a permutation α_u^g as an one-to-one mapping from N_u to $\{1, \ldots, |N_u|\}$. In g, the activation order of the out-neighbors of u is determined by the permutation α_u^g. That is, when u becomes active, u selects its neighbor to activate one by one according to the order given by α_u^g. The cascade priority in g remains the same as that in G. Furthermore, we assign a weight of each edge in g. Suppose u has d_u out-neighbors $v_1, \ldots, v_{d_u}$ in G. For $1 \leq i \leq d_u$, the weight $w^g(u, v_i)$ of edge (u, v_i) is j in g if $\alpha_u^g(v_i) = j$. For two nodes u and v, let $\text{dis}^g(u, v)$ be the length of the shortest path from u to v in g. For a node set V' and a node v, define that $\text{dis}^g(V', v) = \min_{u \in V'} \text{dis}^g(u, v)$. For a certain realization g, let $\Pr[g]$ be the probability that g can be generated by Algorithm 1. Let $\mathcal{G}$ be the set of all possible realizations.

One can see that each realization g corresponds to a basic event of the PIC model. If an edge (u, v) has a probability of 1 in g, then it means u can successfully activate v. The weight $w^g(u, v)$ of an edge (u, v) implies that if u is activated at time t then it will try to activate v at time $t + w^g(u, v)$. The following theorem shows the relationship between a PIC network and its realizations.

Theorem 1: Given the seed set of each cascade, the following two diffusion processes are equivalent to each other, with respect to the distribution of the spreading results.

1) Execute the stochastic diffusion process on the PIC network G.
2) Randomly generate a realization g of G according to Algorithm 1, and execute the deterministic diffusion process on g.

Proof: See Appendix A. □

In Section IV, we will discuss the property of the rumor blocking game where Theorem 1 plays an important role.

IV. Game-Theoretical Analysis

We assume each cascade is generated by an *agent* who decides the seed set of that cascade. For example, an agent can be a company that posts an advertisement for its product. In the traditional rumor blocking problem, it is assumed that there is an authority who generates a single positive cascade. However, the real social networks are extremely large and such an authority is not efficient and sometimes even unfeasible. In this section, we consider the scenario that there are multiple positive cascades generated by different agents and each agent aims to limit the spread of rumor by itself, which forms a game between the agents.

A. Some Notations

Suppose there are k positive cascades $\{C_1, \ldots, C_k\}$ generated by k agents, respectively. Together with the rumor C_r there are totally $k + 1$ cascades in the network.

[3] Other kinds of activation orders will be discussed later in Section V.

Definition 5 (Action Space): Associated with each agent, there is an action space which is a collection of seed sets that they can select. We denote by $\mathcal{A}_i$ the action space of the ith agent.

$\mathcal{A}_i$ is usually not equal to 2^V. For example, a company can only convince the users who like the product to be the seed users. The most considered constraint is the budget constraint where each agent can select at most a certain number of seed nodes.

Definition 6 (Full-Action): A *full-action* $A = (a_1, \ldots, a_k) \in \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_k$ specifies the seed sets selected by the agents.

Instead of taking a single action from the action space, an agent may decide an action according to a distribution s over all of their actions. We call such a distribution s as a strategy and denote by $\mathcal{S}_i$ the set of all strategies of the ith agent.

Definition 7 (Strategy Space): The *strategy space* $\mathcal{S}_i$ of the ith agent is a set of the distributions over the actions in $\mathcal{A}_i$. For each $s \in \mathcal{S}_i$ and $a \in \mathcal{A}_i$, we use $\Pr[a|s]$ to denote the probability that action a is taken under the strategy s. We denote by $\emptyset$ the empty strategy where $\Pr[a|\emptyset] = 0$ for each action a.

In analogy with Definition 6, we have the following term for strategies.

Definition 8 (Full-Strategy): A *full-strategy* $S = (s_1, \ldots, s_k) \in \mathcal{S}_1 \times \mathcal{S}_2 \times \cdots \times \mathcal{S}_k$ specifies the strategy adopted by each agent, where s_i is the strategy adopted by the ith agent. For a full-strategy S and a full-action A, let $\Pr[A|S]$ be the probability that A is implemented under S.

B. Social Utility

For the rumor blocking game, the social utility is the number of the users that are not activated by rumor.

Definition 9 (Social Utility): For a full-strategy S of the agents, we use $\overline{\gamma}(S)$ to denote the expected number of $\overline{C}_r$-active nodes.

We are particularly interested in the marginal return of $\overline{\gamma}(S)$ resulted by adding more agents to game. For the purpose of analysis, we introduce the following notations.

Definition 10: For a full-strategy $S = (s_1, \ldots, s_k)$, a full-action $A = (a_1, \ldots, a_k)$ and an integer $i \leq k$, let $S^i = (s_1, \ldots, s_i, \emptyset, \ldots, \emptyset)$ and $A^i = (a_1, \ldots, a_i, \emptyset, \ldots, \emptyset)$. For a full-action $A = (a_1, \ldots, a_k)$, we denote by $A(i, a_i')$ the full-action where the ith agent replaces its action a_i in A by a_i'. Similarly, we have the notation $S(i, s_i')$ for a full-strategy S and a strategy s_i' of the ith agent.

Intuitively, for $i \leq j$, $\overline{\gamma}(S^i(j, s)) - \overline{\gamma}(S^i)$ denotes the marginal return when the jth agent join the game with a strategy s. The following result indicates that the social utility of our rumor blocking game has the property of diminishing marginal return.

Theorem 2: $\overline{\gamma}(S^{i_1}(i_3, s^*)) - \overline{\gamma}(S^{i_1}) \geq \overline{\gamma}(S^{i_2}(i_3, s^*)) - \overline{\gamma}(S^{i_2})$, for $1 \leq i_1 \leq i_2 \leq i_3 \leq k$ and any strategy $s^* \in \mathcal{S}_{i_3}$.

Proof of Theorem 2: In the rest of this section, we provide a sketch of the proof of Theorem 2. The details can be found in the Appendix.

For a full-action $A = (a_1, \ldots, a_k)$, let $\gamma(A)$ be the expected number of $\overline{C}_r$-active nodes under A. For a full-action $A = (a_1, \ldots, a_k)$ and a realization g, let $\gamma^g(A)$ be the number of $\overline{C}_r$-active nodes in g under A. By Theorem 1

$$\gamma(A) = \sum_{g \in \mathcal{G}} \Pr[g] \cdot \gamma^g(A)$$

The key to proving Theorem 2 is that $\gamma(A)$ only depends on the union of the actions in A, shown as follows.

Lemma 1: For a full-action $A = (a_1, \ldots, a_k)$, let $A^* = a_1 \cup a_2 \cup \ldots \cup a_k$ be the union of the seed sets of the agents. $\gamma(A)$ is a set function on A^*. That is, for any two full-actions A_1 and A_2, $\gamma(A_1) = \gamma(A_2)$ if $A_1^* = A_2^*$.

Proof: See Appendix B. □

It is worthy to note that in some other models $\gamma()$ may not be a set function of the union of positive seed sets, as discussed in Section V. Since $\gamma()$ is a set function, for any $X \subseteq V$, let $\gamma^g(X)$ be the number of $\overline{C}_r$-active nodes in g when the union of the seed sets of positive cascades is X.

A set function $f : 2^V \to \mathbb{R}$ is called monotone increasing if $f(X) \leq f(Y)$ for any $X \subseteq Y$. A set function $f : 2^V \to \mathbb{R}$ is called *submodular* if $f(X) + f(Y) \geq f(X \cap Y) + f(X \cup Y)$ for any X and $Y \in 2^V$.

Lemma 2: $\gamma()$ is monotone increasing and submodular.

Proof: See Appendix C. □

By Lemma 2, $\gamma(A^{i_1}(i_3, a)) - \gamma(A^{i_1}) \geq \gamma(A^{i_2}(i_3, a)) - \gamma(A^{i_2})$, for any full-action A, $1 \leq i_i \leq i_2 \leq i_3 \leq k$ and $a \in \mathcal{A}_{i_3}$. Since $\overline{\gamma}(S) = \sum_A \Pr[A|S] \cdot \gamma(A)$, Theorem 2 follows immediately.

C. Private Utility and the Nash Equilibrium

Now let us consider the private utility of the games. We consider two games depending on whether or not the agents are able to distinguish rumor from other positive cascades.

1) Rumor-Aware Game: In a social network, the agents are able to identify the rumor when the content of rumor is completely different from the facts. Assuming the agents are aware of the rumor, the private utility $\overline{\delta}_i()$ of the ith agent is

$$\overline{\delta}_i(S) = \overline{\gamma}(S) - \overline{\gamma}(S(i, \emptyset)) \tag{1}$$

which is the effort made by the ith agent to limit the spread of rumor. For a full-action A, we use $\delta_i(A) = \gamma(A) - \gamma(A(i, \emptyset))$ to denote the private utility of the ith agent under A. We term this game as the *rumor-aware game*.

Since the agents aim to maximize $\overline{\delta}_i()$ and the rumor has the highest priority, we can assume $a_i \cap a_r = \emptyset$ for any $a_i \in \mathcal{A}_i$ without loss of generality.

An agent may change their strategy to gain more private utilities according to the strategies of other agents. For a full-strategy S, it reaches the Nash equilibrium if no player can gain more utility by changing their own strategy. That is

$$\overline{\delta}_i(S) \geq \overline{\delta}_i(S(i, s_i'))$$

for each i, $1 \leq i \leq k$, and each s_i' in $\mathcal{S}_i$. Due to Nash [27], the finite k-agent noncooperative game always has at least one Nash equilibrium. In the following, we will show that any Nash equilibrium of the rumor-aware game guarantees the social utility with a provable ratio compared to the optimal strategy.

A game is a *valid utility system* if, under any full-strategy S: 1) the private utility is not less than the marginal social utility and 2) the total private utility is not larger than the social utility. That is

$$\overline{\delta}_i(S) \geq \overline{\gamma}(S) - \overline{\gamma}(S(i, \emptyset)) \tag{2}$$

for each i, and

$$\sum_{i=1}^{k} \overline{\delta}_i(S) \leq \overline{\gamma}(S) \tag{3}$$

Theorem 3: The rumor-aware game is a valid utility system under the PIC model.

Proof: See Appendix E. □

According to Vetta [28], if the social utility is a submodular set function, the Nash equilibrium of the game guarantees the social utility by a factor of 2, and therefore, we have the following result immediately.

Corollary 1: Suppose a full-strategy S forms a Nash equilibrium of the rumor-aware game, and let Ω be the full-strategy such that $\overline{\gamma}(\Omega)$ is maximized. Then, $\overline{\gamma}(S) \geq (1/2) \cdot \overline{\gamma}(\Omega)$.

Recall that in the rumor-aware game the social utility is the expected number of $\overline{C}_r$-active nodes while the private utility is the effort made by each agent to limit the spread of rumor. Due to the nature of noncooperation, each agent only concerns their marginal contribution. Nevertheless, Corollary 1 shows that the social utility will not be arbitrarily far from the optimal and in fact it guarantees a 2-approximation. Intuitively speaking, even if there is no powerful authority dealing with rumor in a social network, the rumor can be efficiently blocked by the users who participate and propagate positive information.

A *pure* full-strategy is a special full-strategy where each of the ith agent decides to carry out one specific action. When a full-strategy $S = (s_1, \ldots, s_k)$ is pure, each s_i is a vector of 0 and 1. In other words, the pure full-strategy reduces to the full-action. The Nash equilibrium formed by a pure full-strategy called pure Nash equilibrium. Note that there always exists an optimal strategy Ω, which is pure. For general games, the Nash equilibrium may not be pure. However, the agents usually make pure strategies instead of making decisions according to a distribution. The following result shows that the pure Nash equilibrium always exists in the rumor-aware game.

Theorem 4: For the rumor-aware game, there exists a full-action Φ such that $\delta_i(\Phi) \geq \delta_i(\Phi(i, a_i'))$ for each $1 \leq i \leq k$ and each action a_i' of the ith agent.

Proof: Let Φ_0 be an arbitrary full-action and consider the following process to generate a series of full-actions $\Phi_0, \ldots, \Phi_m$. For the full-action Φ_j, if for some i there exists an action $a_i \in \mathcal{A}_i$ such that $\delta_i(\Phi_j) < \delta_i(\Phi_j(i, a_i))$, we denote $\Phi(i, a_i)$ as Φ_{j+1}. By such process, we finally obtain a sequence of full-actions Φ_j. According to the construction, for any Φ_j and Φ_{j+1}, there exists some i such that $\delta_i(\Phi_j) < \delta_i(\Phi_{j+1})$, which implies $\gamma(\Phi_j) < \gamma(\Phi_{j+1})$. Therefore, any two full-actions in the sequence cannot be identical. Since the action space is finite, the sequence Φ_j must be finite and the last full-action reaches the pure Nash equilibrium. □

Algorithm 2 Simple Game

$A \leftarrow (\emptyset, \ldots, \emptyset)$;
sign $\leftarrow true$;
while $A' \neq A$ **do**
  $A \leftarrow A'$;
  for $i = 1 : k$ **do**
    $v \leftarrow \arg\max_{v \in V} \ \delta_i(A(i, \{v\}))$;
    $A' \leftarrow A(i, \{v\})$;
Return A;

The proof of Theorem 4 implies that we can build a Nash equilibrium starting from any full-action by increasing the private utility of some agent.

Simple Game: In the prior works, the goal is to block the rumor by introducing a competing cascade. Due to the expense of activating seed nodes, there is a budget of the seed nodes. In our rumor blocking game, such a budget is distributed to multiple agents. Suppose the budget is distributed to k agents where each agent has one budget, the agents make decisions in turn, and, the agents always make pure decisions. The evolution of the Nash equilibrium is shown in Algorithm 2. We denote the game under such a setting as *Simple Game*.

Under the Budget Constraint: The result shown in Corollary 1 requires that each agent follows the best-response policy. However, the agent in real case may not be able to efficiently[4] find such an optimal action that maximizes the private utility. Under the budget constraint, each agent can select at most a certain number of seed nodes. In this case, finding the best response is NP-hard [4] so the polynomial approximation response is the best that each agent can adopt. According to Lemma 2, given the seed sets of other agents, $\delta_i()$ is also monotone increasing and submodular, and therefore, the ith agent can easily obtain an action a_i such that $\gamma(A(i, a_i)) \geq (1 - e^{-1}) \cdot \gamma(A(i, a_i^*)$, where a_i^* is the best response [29]. If all the agents adopt the approximation action, the game finally reaches an approximate Nash equilibrium. The next result shows that such an equilibrium guarantees the social utility within a factor of $(2e + 1/e + 1)$.

Lemma 3: Let $A = (a_1, \ldots, a_k)$ be a pure Nash equilibrium under the approximate response. Then, $\gamma(A^* \cup \Omega^*) \leq (2e + 1/e + 1) \cdot \gamma(A^*)$ where Ω is the optimal pure full-action that maximizes $\gamma()$.

Proof: See Appendix D. □

Theorem 5: If each agent adopts the $(1 - e^{-1})$-approximate response, the Nash equilibrium guarantees an $(2 \cdot e - 1/e - 1)$-approximation with respect to the expected number of $\overline{C}_r$-active nodes.

Proof: Since $\gamma()$ is a set function and $\gamma(\Omega^*) \leq \gamma(A^* \cup \Omega^*)$, the theorem directly follows from Lemma 3. □

2) Rumor-Oblivious Game: In another issue, the rumor may be well disguised such that they cannot be distinguished from the genuine news. In this case, the best that an agent can do is to maximize the spread of its own cascade. Therefore,

[4]Ideally decisions should be made in polynomial time.

the private utility $\overline{\delta}_i()$ of the ith agent is $\overline{\delta}_i(S) = \overline{\sigma}_i(S)$, where

$$\overline{\sigma}_i(S) = \sum_A \Pr[A|S] \cdot \sigma_i(A) \tag{4}$$

is the expected number of C_i-active nodes under S and $\sigma_i(A)$ is the expected number of C_i-active nodes under A. Such a game is called rumor-oblivious game. In the following, we will show that the rumor-oblivious game also forms a valid utility system. However, the proof slightly differs from that of the rumor-aware game.

Lemma 4: The rumor-oblivious game is a valid utility system under the PIC model.

Proof: See Appendix F. □

We have the following result due to Vetta [28]

Corollary 2: For any Nash equilibrium S of the rumor-oblivious game, $\overline{\gamma}(S) \geq (1/2) \cdot \overline{\gamma}(\Omega)$, where Ω is the full-strategy maximizing $\overline{\gamma}()$.

As discussed in the prior works, e.g., [17], given the actions of other agents, $\sigma_i(A)$ is also monotone and submodular with respect to the seed set of the ith agent. Therefore, similar to the analysis in Section IV-C, the agents in the rumor-oblivious game are also able to make the $(1-1/e)$-approximate pure response. However, unlike the rumor-aware game, there may not be a $(2 \cdot e - 1/e - 1)$-approximation equilibrium for the rumor-oblivious game. This is because the pure Nash equilibrium may not exist in the rumor-oblivious game.

V. Discussion on Cascade Priority and Activation Order

In this section, we provide several observations concerning the competitive diffusion model. The discussion herein may help us to further understand the scenario when more than two cascades exist. We introduce the following two types of cascade priority.

Definition 11: (Homogeneous and Heterogeneous Cascade Priority): The cascade priority is homogeneous if the priority of the cascades is the same for each user. Otherwise, it is called heterogeneous cascade priority.

We list some observations, as follows.

Fact 1: Under the homogeneous cascade priority, if rumor does not have the highest priority, $\gamma(A)$ is not a set function anymore.

Example: Consider the network shown in Fig. 2(a) and (b), where $p_e^G = 1$ for each edge e. Suppose $a_r = \{v_2\}$ and there are two agents with cascades C_1 and C_2, respectively. Suppose that Priority(C_1) $\leq$ Priority(C_r) $\leq$ Priority(C_2). One can easily check that action $A_1 = (\{v_1\}, \{v_3\})$ and $A_2 = (\{v_3\}, \{v_1\})$ result different values of social utility. Under action A_1, v_4 will be activated by the rumor seed v_2 and therefore many nodes will be later activated by rumor spreading from v_4. However, under A_2, only the node v_2 will be activated by rumor as v_4 will be activated by cascade C_1. Thus, $\gamma(A_1) \neq \gamma(A_2)$ even if $A_1^* = A_2^*$. □

Fact 2: Under the heterogeneous cascade priority, the social utility $\gamma(A)$ is not monotone increasing.

Example: The heterogeneous cascade priority setting has been adopted in prior works, e.g., [18]. We observe that under this setting, the social utility may decrease when more agents join in game. Consider the illustrative example shown in Fig. 2(c), where each $p_e^G = 1$ for each edge e and $a_r = \{v_3\}$. Suppose there are two agents C_1 and C_2, and for v_4 and v_5, the priority of the cascades is

$$\text{Priority}(C_2) < \text{Priority}(C_1) < \text{Priority}(C_r)$$

and

$$\text{Priority}(C_1) \leq \text{Priority}(C_r) \leq \text{Priority}(C_2)$$

respectively. Consider two actions A_1 and A_2 where $A_1 = (\{v_1\}, \emptyset)$ and $A_2 = (\{v_1\}, \{v_2\})$. Under A_1 there is only one agent in the game and v_5 will be activated by C_1 because C_1 has the higher priority than C_r at v_5. However, when another agent joins the game as shown by A_2, v_5 will become C_r-active because v_4 will be activated by C_2 and Priority(C_r) $\leq$ Priority(C_2) at v_4. Therefore, under this setting, $\gamma(A)$ may not be monotone increasing with respect to A^*. □

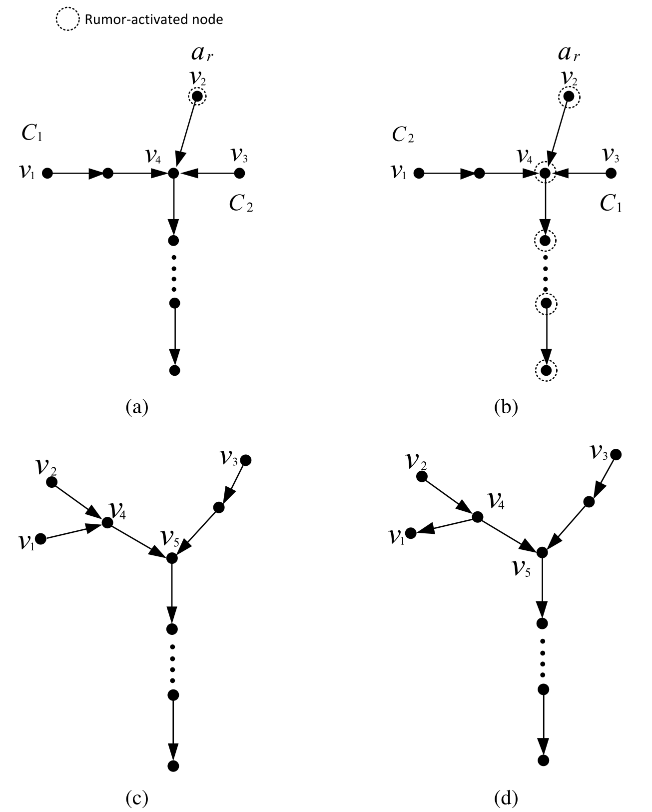

Fig. 2. Examples. (a) Action A_1. (b) Action A_2. (c) Second example. (d) Third example.

Fact 3: Under the homogeneous cascade priority, if an active user only attempts to activate inactive neighbors, the social utility $\gamma(A)$ is not monotone increasing.

Example: Note that we in this paper assume that a node may try to activate the neighbor that has been active. It is worthy to note that if each node only attempts to activate inactive neighbors, then $\gamma(A)$ is not monotone increasing under the PIC model. An example is shown in Fig. 2(d). Again, we assume that each edge has the probability of 1, $a_r = \{v_2\}$ and there are two agents. Suppose each node only selects inactive node to activate and v_4 will activate v_1 and v_5 in order after becoming active. Consider the two actions $A_1 = (\{v_3\}, \emptyset)$ and $A_2 = (\{v_3\}, \{v_1\})$. One can see that v_5 will be activated by C_1 under A_1 because at the second step v_4 will activate v_1 after rumored by v_2. Nevertheless, if another agent participates and selects v_1 as the seed node, as shown in A_2, then v_5 will be rumor-activated by v_4, because at the second step v_4 will not try to activate v_1 as v_1 has been activated in the first step. Thus, when more agents come to limit the spread of rumor, the rumor may surprisingly spread more widely. □

As shown earlier, under certain settings the model does not have good properties anymore and consequently, the rumor blocking problem becomes more complicated in such scenarios.

VI. Experiment

In this section, we experimentally evaluate the rumor blocking effect in the equilibrium of the proposed game by comparing it with the traditional rumor blocking framework where there is only one positive cascade. We first simulate the behavior of the agents to obtain the seed nodes, and then simulate the diffusion process to see how many users will be influenced by rumor.

A. Setup

Our experiments are performed on a server with 16-GB ram and a 3.6-GHz quadcore processor running 64-bit JAVA VM 1.6.

1) Data Set: We adopt the network structure of the following data sets. The first data set, denoted by **Facebook**, is collected from the Facebook social platform, provided by the Stanford Network Analysis Project [30]. The Facebook data set contains 4039 nodes with 88 234 edges and it has been widely used in prior works [31]–[33]. Another real-world social network is an academic collaboration from co-authorships in physics, denoted by **Hep**. This data set is compiled from the "High Energy Physics-Theory" section of the e-print arXiv,[5] including about 15 000 nodes and 58 000 edges. Hep data set has been studied in [16] and [34]–[36].

2) Propagation Probability: In the experiments, we consider three settings of the probability on the edges. In the first and second settings, the probability of each edge is uniformly set as 0.1 and 0.01, respectively. The third setting follows the classic weighted cascade model [16] where $p^G_{(u,v)} = 1/d_v$ and d_v is the number of out-neighbors of node v.

3) Seeds of Rumor: The seed nodes of rumor are selected from the nodes with the highest degree. The number of the rumor seeds will be discussed later.

4) Game: Given a social network and budget k, we deploy k agents each of which generates one positive cascade with one seed node. The seed nodes are obtained by simulating the Simple Game developed in Section IV. The diffusion result of the Simple Game is labeled as *Game*.

[5]http://www.arXiv.org

5) Single-Positive-Cascade Case: For the single-positive-cascade case, we set the budget of the seed set as k and select the seed nodes of the positive cascade according to the following methods.

1) *Greedy*: This is the state-of-the-art rumor blocking algorithm. Given a budget k, we assume there is one positive cascade with k seed nodes in which the nodes are decided by successively adding the node that can maximize the social value. Such a method provides a $(1 - 1/e)$-approximation due to the submodularity and it has been widely used in the prior works [3]–[5].
2) *MaxDegree:* Assuming there is one positive cascade, MaxDegree selects the k users in $V \setminus a_r$ with the highest degree.
3) *Random:* Assuming there is one positive cascade, Random selects k seed nodes at random.
4) *NoBlocking:* This is the case when there is no positive cascade.

Another popular heuristic rumor blocking algorithm, called Proximity, which selects the neighbor of rumor seed nodes as positive seed nodes, is not included in our experiments, because its performance is worse than that of Greedy as shown in [3] and [5]. Due to space limitation, we will not discuss all combinations of the above-mentioned settings. For a given full-action (i.e., the seed sets of each cascade) and a specified network, $\gamma(A)$ is calculated by taking the average of 10 000 simulations.

B. Results

We perform three series of experiments. The experimental results are discussed as follows.

1) Experiment I: In the first experiment, the number of seed nodes of the rumor is set from 1 to 30 and the budget k is equal to the number of rumor seed nodes. The results of this experiment on Facebook under the three propagation probability settings are shown in Fig. 3(a)–(c). As shown in the figures, when the propagation probability is 0.01, the effectiveness of Game is slightly worse than that of Greedy. Under the other two settings of propagation probability, the equilibrium of the game has the same degree of effect as Greedy does in limiting the spread of the rumor. The result of this experiment done on Hep is shown in Fig. 3(d). In this case, Game provides the best performance among all the considered methods. In general, both the Game and Greedy are effective for rumor containment. However, under different settings and network structures, the patterns of the curves are diverse. The first observation is that the spread of rumor may have a saturation point with respect to the number of seed nodes. For the cases shown in Fig. 3(a) and (b), the number of C_r-active nodes under NoBlocking will not notably increase when k is larger than 5. Nevertheless, in Fig. 3(d), the number of C_r-active nodes continuously increases with the increase of k. Another observation is that when k increases by one, the number of C_r-active nodes does not necessarily increase. As shown in Fig. 3(c), when one rumor seed and one agent are added at $k = 15$, the number of C_r-active nodes decreases by about 500 under Game. Such a case suggests that the marginal effect

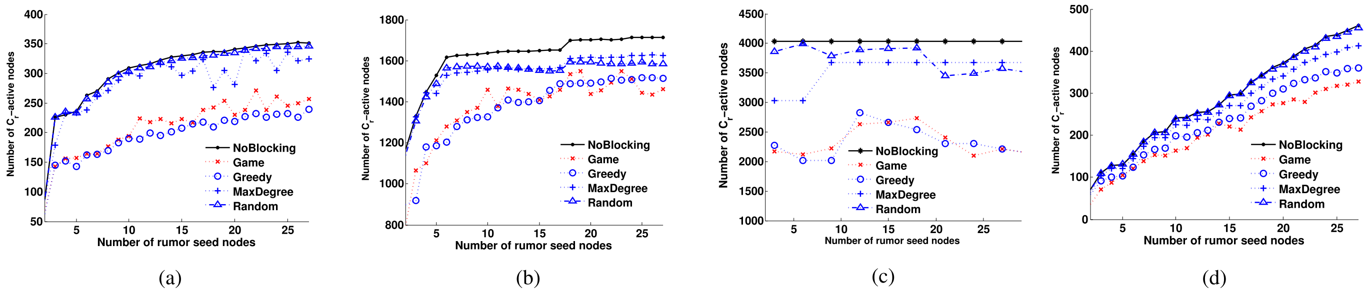

Fig. 3. Results of the first experiment. The y- and x-axes denote the expected number of C_r-active nodes and the number of rumor seed nodes, respectively. Each graph gives five curves plotting the number of C_r-active nodes under NoBlocking, Game, Greedy, MaxDegree, and Random, respectively. (a) Facebook with $p^G_{(u,v)} = 0.01$. (b) Facebook with $p^G_{(u,v)} = 0.1$. (c) Facebook under weighted cascade setting. (d) Hep with $p^G_{(u,v)} = 0.1$.

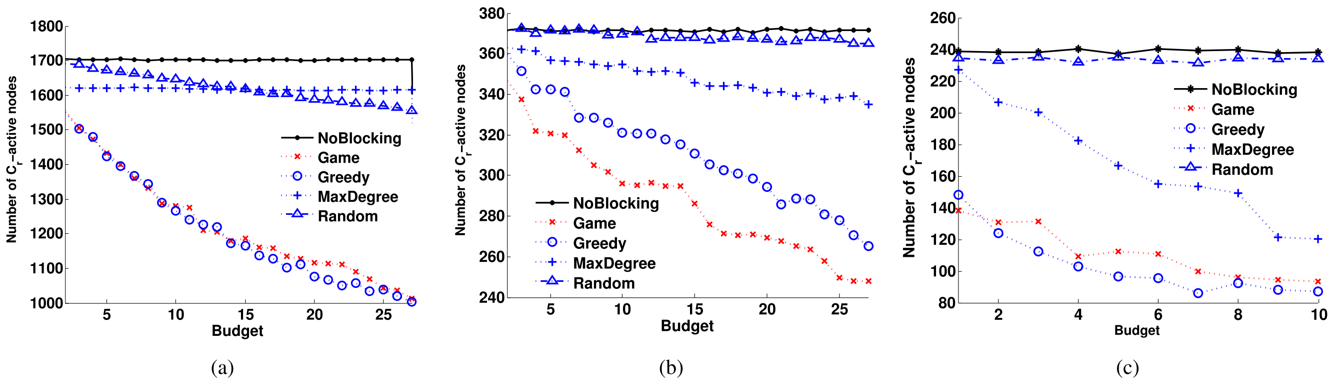

Fig. 4. Results of the second experiment. The y- and x-axes denote the expected number of C_r-active nodes and the budget, respectively. Each graph gives five curves plotting the number of C_r-active nodes under NoBlocking, Game, Greedy, MaxDegree, and Random, respectively. (a) Facebook with $p^G_{(u,v)} = 0.1$ and $|a_r| = 20$. (b) Hep with $p^G_{(u,v)} = 0.1$ and $|a_r| = 20$. (c) Facebook with $p^G_{(u,v)} = 0.01$ and $|a_r| = 5$.

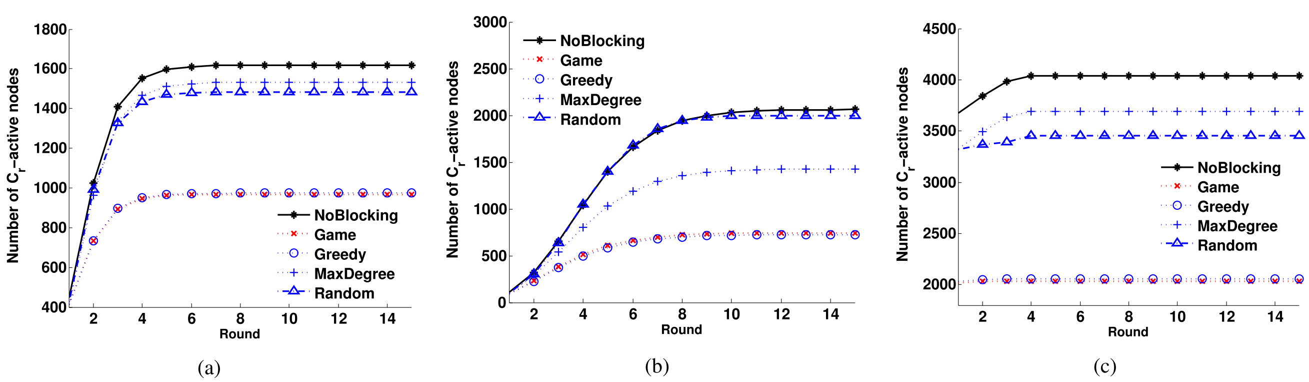

Fig. 5. Results of the third experiment. The y- and x-axes denote the expected number of C_r-active nodes and the index of spread round, respectively. Each graph gives five curves plotting the number of C_r-active nodes under NoBlocking, Game, Greedy, MaxDegree, and Random, respectively. (a) Facebook with $p^G_{(u,v)} = 0.1$, $|a_r| = 10$ and $k = 15$. (b) Hep under weighted cascade setting, $|a_r| = 10$ and $k = 25$. (c) Facebook under weighted cascade setting, $|a_r| = 10$ and $k = 10$.

of adding one seed node not only depends on the selection of the seed nodes but also on the network structure.

2) Experiment II: In the second experiment, we fix the number of rumor seed nodes and see how the number of C_r-active nodes varies with the increase of the budget k. The results of the experiments under three different settings are shown in Fig. 4(a)–(c). One can see that when more budgets is added the number of C_r-active nodes become less and less. On the Facebook network, when $p^G_{(u,v)}$ is equal to 0.1 and $|a_r|$ is set as 20, as shown in Fig. 4(a), the number of C_r-active nodes decreases significantly under Game and Greedy but hardly changes under MaxDegree and Random, which implies that the agent should not arbitrarily select seed nodes or use simple heuristics. For the case shown in Fig. 4(c), one can see that adding the first positive seed node can reduce the number of C_r-active nodes by a half. Such a scenario exemplifies the submodularity nature of the rumor blocking problem and indicates that the first several actions of the agents are important.

3) Experiment III: In the third experiment, we fix both the number of rumor seeds and budget k, and record the number of C_r-active nodes round by round. That is, we take the snapshots of the first two experiments and examine how fast the rumor spread under different cases. The results are shown in Fig. 5(a)–(c). As indicated by the figures, the equilibrium of the game formulated in this paper can effectively limit the spread of the rumor. Furthermore, under appropriate strategies, the rumor can be blocked at an early stage. For example, in Fig. 5(b), the number of C_r-active nodes stops increasing at about the eighth round under Game. However, it increases until the eleventh round under MaxDegree.

VII. Conclusion and Future Work

In this paper, we study the rumor blocking problem when there are multiple positive cascades. By formulating the rumor-aware game and the rumor-oblivious game, we have shown that under the best-response and the approximate response, the equilibrium and the game provide a 2-approximation and $(2e + 1/e + 1)$-approximation, respectively, with respect to the social utility, i.e., the number of nodes that is not influenced by rumor. The theoretical results herein are well supported by the experiments done on real-world networks.

As shown in this paper, the rumor containment in a distributed mode is effective for rumor blocking. Therefore, it is interesting to design rumor blocking strategies for multiple positive cascades with the concern of the cascade priority. Another direction of the future work is to study the pure Nash equilibrium of the rumor-oblivious game. In particular, it is interesting to study the circumstance under which the pure Nash equilibrium exists. Finally, as discussed in Section V, the competitive cascade model becomes evasive under certain settings. To the best our knowledge, none of the prior works has considered the rumor blocking problem in such models. We leave this part for the future work.

Appendix
Proofs

A. Proof of Theorem 1

Proof: For each cascade C, we denote by C^t be the set of C-active nodes after time step t. Now, let us consider the spreading result after time step $t+1$. To prove the theorem, it is sufficient to show that given the C^t, for each cascade C, and b_u^t for each u at time step t, the distributions of the spread result after time step $t+1$ are the same under the two spread processes. In particular, because the cascade priorities are the same under the both process, it suffices to prove that, for any inactive node u^* and any active node v^*, the probability that v^* successfully actives u^* at time step $t+1$ under the first spreading process is the same as that under the second one.

Process a: An active node v^* successfully activates the inactive node u^* at time step $t+1$ if and only if $u^* \in b_{v^*}^t$, v^* select u^* to activate and the activation is successful. By Definition 3, this probability is $(p_{(v^*,u^*)}/|b_{v^*}^t|)$.

Process b: According to Definition 4, v^* will try to activate u^* at time step $t+1$ if and only if $\alpha_{v^*}^g(u^*) = d_{v^*} - |b_{v^*}^t| + 1$ and $p_{(v^*,u^*)}^g = 1$. Since each permutation has the same probability to be generated, for each node $u \in b_{v^*}^t$, $\alpha_{v^*}^g(u) = d_{v^*} - |b_{v^*}^t| + 1$ happens with the same probability. Therefore, for the node u^*, with the probability of $(1/|b_{v^*}^t|)$, v^* will try to activate u^* at time step $t+1$. Thus, under this process, that probability that v^* successfully activates u^* at time step $t+1$ is still $(p_{(v^*,u^*)}/|b_{v^*}^t|)$. □

B. Proof of Lemma 1

We introduce some preliminaries before proving Lemma 1. For a fixed realization g and a full-action $A = (a_1, \ldots, a_k)$, the outcome of the influence diffusion in g under A is determined. Let $t_A^g(u)$ be the time that u becomes active in g under A.

Lemma 5: For two nodes u_1 and u_2, and any simple path $P = (v_1 = u_1, \ldots, v_i, \ldots, v_m = u_2)$ from u_1 to u_2 in g, $t_A^g(u_2) \leq t_A^g(u_1) + |P|$ where $|P|$ is the sum of the weights of the edges in P.

Proof: For any two successive nodes v_i and v_{i+1} in the path, v_i will attempt to activate v_{i+1} at $t_A^g(v_i) + w^g(v_i, v_{i+1})$. Therefore, $t_A^g(v_{i+1}) \leq t_A^g(v_i) + w^g(v_i, v_{i+1})$. The property follows inductively from u_1 to u_2 along the given path. □

Corollary 3: Given a realization g and a full-action A, for any node u and v where v is a seed selected by one or more cascades, $t_A^g(u) \leq \mathrm{dis}^g(v, u)$.

Proof: Since v is a seed node, $t_A^g(u) = 0$. The corollary directly follows from Lemma 5. □

The next lemma provides the condition for a node u to be rumor-activated in a realization g under a full-action A.

Lemma 6: Given a full-action $A = (a_1, \ldots, a_k)$ and a realization g, a node u^* will be activated by rumor C_r in g under process b defined in Theorem 1 $\Leftrightarrow$ $\mathrm{dis}^g(a_r, u^*) \leq \mathrm{dis}^g(a_i, u^*)$, for each i and $\mathrm{dis}^g(a_r, u^*) \neq +\infty$.[6]

Proof: Let v_i be the node in a_i such that $\mathrm{dis}^g(v_i, u^*) = \mathrm{dis}^g(a_i, u^*)$, for $1 \leq i \leq k$.

$\Rightarrow$: Clearly, $\mathrm{dis}^g(a_r, u^*) \neq +\infty$. Since u^* is activated by rumor, there is path P from a certain node $v_r \in a_r$ to u^* such that all the nodes in this path are activated by rumor and $t_A^g(u^*) = |P|$. By definition, $\mathrm{dis}^g(a_r, u^*) \leq |P|$. By Corollary 3, $t_A^g(u^*) \leq \mathrm{dis}^g(v_i, u^*)$ for each i, which implies $\mathrm{dis}^g(a_r, u^*) \leq |P| = t_A^g(u^*) \leq \mathrm{dis}^g(v_i, u^*)$.

$\Leftarrow$: Now suppose $\mathrm{dis}^g(a_r, u^*) \leq \mathrm{dis}^g(a_i, u^*)$ for each i and $\mathrm{dis}^g(a_r, u^*) \neq +\infty$. Let v_r be the node in a_r such that $\mathrm{dis}^g(v_r, u^*) = \mathrm{dis}^g(a_r, u^*)$ and P be the shortest path from v_r to u^* in g. Suppose the nodes in P are $u_1, \ldots, u_l$ where $u_1 = v_r$ and $u_l = u^*$. *It suffices to show that every node u_j in P will be activated by rumor at time* $\mathrm{dis}^g(u_1, u_j)$. We prove this inductively. Clearly, $t_A^g(u_1) = \mathrm{dis}^g(u_1, u_1) = 0$ and u_1 is activated by rumor. Suppose this is true for the first j nodes in P and u_{j+1} is activated by its certain in-neighbor v^*. There are two cases shown as follows.

Case 1 ($v^* = u_j$): By the inductive hypothesis, u_j is activated by rumor at time $\mathrm{dis}^g(u_1, u_j)$. Therefore, v_{j+1} is

[6]We define that $\mathrm{dis}^g(u_1, u_2) = \mathrm{dis}^g(u_3, u_4)$ if $\mathrm{dis}^g(u_1, u_2) = +\infty$ and $\mathrm{dis}^g(u_3, u_4) = +\infty$, for any four nodes u_1, u_2, u_3, and u_4.

also activated by rumor and

$$\begin{aligned} t_A^g(u_{j+1}) &= t_A^g(u_j) + w^g(u_j, u_{j+1}) \\ &= \text{dis}^g(u_1, u_j) + w^g(u_j, u_{j+1}) = \text{dis}^g(u_1, u_{j+1}). \end{aligned}$$

Case 2 *($v^* \neq u_j$):* Suppose v^* is activated at $t_A^g(v^*)$ via a certain path P' from a certain seed node v' to v^*. Then, u_{j+1} is activated at $|P|' + w^g(v^*, u_{j+1})$. By Corollary 3

$$|P|' + w^g(v^*, u_{j+1}) = t_A^g(u_{j+1}) \leq \text{dis}^g(u_1, u_{j+1}). \tag{5}$$

Furthermore, since $\text{dis}^g(u_1, u^*) = \text{dis}^g(a_r, u^*) \leq \text{dis}^g(a_i, u^*)$, $\text{dis}^g(u_1, u_{j+1}) \leq \text{dis}^g(v_i, u_{j+1})$ for each i. Since P' together with (v^*, u_{j+1}) is a path from v' to u_{j+1}, $\text{dis}^g(u_1, u_{j+1}) \leq |P'| + w^g(v^*, u_{j+1})$. Combining (5)

$$\text{dis}^g(u_1, u_{j+1}) = |P'| + w^g(v^*, v_{i+1}) = t_A^g(v_{j+1}).$$

By the inductive hypothesis, u_j is C_r-active at $\text{dis}^g(v_1, u_j)$ and it will attempt to activate u_{j+1} at $\text{dis}^g(v_1, u_j) + w^g(u_j, u_{j+1}) = \text{dis}^g(u_1, u_{j+1})$, which means u_j and v^* activate u_{j+1} at the same time. Since the rumor has the highest priority, u_{j+1} will be activated by rumor at time step $\text{dis}^g(u_1, u_{j+1})$. By the above-mentioned induction, all the nodes in path P, including u^*, are C_r-active. □

One can see that the minimum of $\text{dis}^g(a_i, u^*)$ only depends on the union of the positive seed sets, and therefore, the social utility is a set function, shown as follows.

Proof of Lemma 1: It suffices to show that, for any realization g and two full-actions A_1 and A_2, $\gamma^g(A_1) = \gamma^g(A_2)$ if $A_1^* = A_2^*$. Let u be an arbitrary $\overline{C}_r$-active node in g under $A_1 = (a_1, \ldots, a_k)$. By Lemma 6, $\text{dis}^g(a_r, u) > \text{dis}^g(a_i, u)$ for some i or $\text{dis}^g(a_r, u) = \infty$. If $\text{dis}^g(a_r, u) = \infty$, then there is no path from any rumor seed to u and therefore u cannot be activated by rumor in g under A_2. Now suppose $\text{dis}^g(a_r, u) \neq \infty$ and $\text{dis}^g(a_r, u) > \text{dis}^g(v_i, u)$ for some i and some $v_i \in a_i$. Since $A_1^* = A_2^*$, v_i must be a seed node of some agent i^* in A_2 and therefore $\text{dis}^g(a_r, u) < \text{dis}^g(a_{i^*}, u)$, which means u is also $\overline{C}_r$-active in g under A_2. By the above-mentioned analysis, $\gamma^g(A_1) \leq \gamma^g(A_2)$. It can be easily seen that $\gamma^g(A_2) \leq \gamma^g(A_1)$ can be proved in the similar manner, and therefore $\gamma^g(A_1) = \gamma^g(A_2)$. □

C. Proof of Lemma 2

By Theorem 1, we only need to show that $\gamma^g()$ is monotone increasing and submodular for each realization g. Due to Lemma 6, $\gamma^g()$ is clearly monotone increasing. To prove the submodularity, it suffices to show that, for each realization g

$$\gamma^g(X \cup \{v\}) - \gamma^g(X) \geq \gamma^g(Y \cup \{v\}) - \gamma^g(Y) \tag{6}$$

where $X \subseteq Y \subseteq V$ and $v \in V \setminus Y$. Since $\gamma^g()$ is monotone increasing, $\gamma^g(Y \cup \{v\}) - \gamma^g(Y)$ is the number of nodes that is C_r-active under Y but $\overline{C}_r$-active under $Y \cup \{v\}$. Now let us consider such a node u. Since u is C_r-active under $Y \cup \{v\}$, $\text{dis}^g(a_r, u) \neq +\infty$. By Lemma 6, $\text{dis}^g(a_r, u) \leq \text{dis}^g(Y, u)$ and $\text{dis}^g(a_r, u) > \text{dis}^g(Y \cup \{v\}, u)$, which means $\text{dis}^g(a_r, u) > \text{dis}^g(\{v\}, u)$. Because X is a subset of Y, $\text{dis}^g(X, u) \geq \text{dis}^g(Y, u) \geq \text{dis}^g(a_r, u)$ and thus u will be C_r-active in g under X. Meanwhile, u cannot be C_r-active in g under $X \cup \{v\}$ because $\text{dis}^g(X \cup \{v\}, u) \leq \text{dis}^g(\{v\}, u) < \text{dis}^g(a_r, u)$. Therefore, each node that contributes 1 to the right-hand side of (6) must contribute 1 to the left-hand side. Equation (6) thus follows.

D. Proof of Lemma 3

Proof: By the definition of $\overline{\delta}_i(\cdot)$, (2) directly follows. Let $\delta_i^g(A) = \gamma^g(A) - \gamma^g(A(i, \emptyset))$. To prove (3), it suffices to show that $\sum_{i=1}^k \delta_i^g(A) \leq \gamma^g(A)$ holds for each full-action A and g. Note that

$$\begin{aligned} \sum_{i=1}^k \delta_i^g(A) &= \sum_{i=1}^k \gamma^g(A) - \gamma^g(A(i, \emptyset)) \\ &= \sum_{i=1}^k \gamma^g(A^k) - \gamma^g(A^k(i, \emptyset)) \\ &\quad \{\text{By Corollary 2}\} \\ &\leq \sum_{i=1}^k \gamma^g(A^i) - \gamma^g(A^i(i, \emptyset)) \\ &= \gamma^g(A^k) = \gamma^g(A). \end{aligned}$$

Thus, proved. □

E. Proof of Theorem 3

Proof: Suppose $\Omega = (b_1, \ldots, b_k)$. For two node sets V_1 and V_2, we denote by $V_1^{V_2}$ the set of the nodes in V_1 but not in V_2, i.e., $V_1^{V_2} = V_1 - V_2$. Under such notation, $A^* \cup \Omega^* = A^* \cup b_1^{A^*} \cup, \ldots, \cup b_k^{A^*}$. Let $B_i = \{b_1^{A^*}, \ldots, b_i^{A^*}\}$ and $B_0 = \{\emptyset\}$. Then

$$\begin{aligned} &\gamma(A^* \cup \Omega^*) - \gamma(A^*) \\ &\quad = \sum_{i=1}^k \gamma\left(A^* \cup B_i^*\right) - \gamma\left(A^* \cup B_{i-1}^*\right). \end{aligned}$$

Due to submodularity, for each $1 \leq i \leq k$

$$\begin{aligned} &\gamma\left(A^* \cup B_i^*\right) - \gamma\left(A^* \cup B_{i-1}^*\right) \\ &\quad \leq \gamma\left(A^* \cup b_i^{A^*}\right) - \gamma(A^*) \\ &\quad \leq \gamma\left(A^* - a_i \cup b_i^{A^*}\right) - \gamma(A^* - a_i). \end{aligned}$$

According to the monotonicity of $\gamma()$

$$\gamma\left(A^* - a_i \cup b_i^{A^*}\right) \leq \gamma(A^* - a_i \cup b_i).$$

Therefore

$$\begin{aligned} &\gamma(A^* \cup \Omega^*) - \gamma(A^*) \\ &\quad \leq \sum_{i=1}^k \gamma\left(A^* - a_i \cup b_i^{A^*}\right) - \gamma(A^* - a_i) \\ &\quad \leq \sum_{i=1}^k \gamma\left(A^* - a_i \cup b_i\right) - \gamma(A^* - a_i) \qquad (7) \\ &\{\text{by the approximate response}\} \\ &\quad \leq \sum_{i=1}^k (1 - e^{-1})^{-1} \cdot \delta_i(A^*) \\ &\{\text{by (3)}\} \\ &\quad \leq (1 - e^{-1})^{-1} \cdot \gamma(A^*). \qquad (8) \end{aligned}$$

Thus, proved. □

F. Proof of Lemma 4

Let $\sigma_i^g(A)$ be the number of C_i-active nodes in g, By Theorem 1

$$\sigma_i(A) = \sum_{g \in \mathcal{G}} \Pr[g] \cdot \sigma_i^g(A)$$

and therefore, in order to prove (2), it suffices to show that $\sigma_i^g(A) \geq \gamma^g(A) - \gamma^g(A(i, \emptyset))$ holds for each full-action A and each realization g. Note that $\gamma^g(A)$ is a set function on A^* and it is monotonically increasing. Therefore, the right-hand side is a number of nodes that are C_r-active under A, but $\overline{C}_r$-active under $A(i, \emptyset)$. Let u be such a node that contribute 1 to the right-hand side. According to Lemma 6, $\mathrm{dis}^g(a_i, u) < \mathrm{dis}^g(a_r, u) \leq \mathrm{dis}^g(a_j, u)$ for $j \neq i$. Thus, u must be C_i-active under A in g and therefore also contributes 1 to the left-hand side. Equation (2) thus proved.

Now to prove (3), it suffices to show that $\sum_{i=1}^{k} \delta_i^g(A) \leq \gamma^g(A)$ holds for each A and g. Note that $\gamma^g(A)$ is the number of $\overline{C}_r$-active nodes, i.e., the nodes activated by the positive cascades together with the nodes that are not activated by any cascade. Therefore, (3) follows directly.

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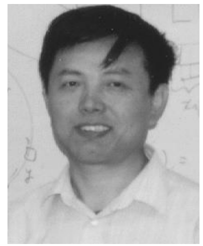

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