Homework-8 Solutions

Question 1

You are given a picture with 5 point at the following (x, y) coordinates:

(1,3), (2,1), (2,5), (3,3), (3,5)

Apply the Hough Transform algorithm to search for circles in the parametric representation

$$(x - x_0)^2 + (y - y_0)^2 = r^2.$$

Quantize r^2 into three values: $2, 3, 4$.
Quantize x_0 into four values: $-1, 1, 3, 5$.
Quantize y_0 into four values: $-1, 1, 3, 5$.
Follow these steps:

Initialization: Prepare and initialize to 0 the three dimensional accumulator space. You can visualize it (and write it in your notebook) as 3 two dimensional arrays. The first for $r^2 = 2$, the second for $r^2 = 3$, and the third for $r^2 = 4$.

Voting: for each point (x, y) of the five picture points
for each possible value of x_0
for each possible value of y_0
{
compute r^2 from the equation $r^2 = (x - x_0)^2 + (y - y_0)^2$
If r^2 is in the range 2–4 vote by incrementing the corresponding cell
}
(Notice that this requires calculating r^2 80 times.)

Choose a winner determine the cell with max number of votes.

a. What are the values of the accumulator space after the voting phase?

$r^2 = 4$:

	$x_0 = -1$	$x_0 = 1$	$x_0 = 3$	$x_0 = 5$
$y_0 = -1$	0	0	0	0
$y_0 = 1$	0	1	1	0
$y_0 = 3$	1	1	2	1
$y_0 = 5$	0	2	1	1

b. What is the most likely circle?

Two solutions:

$$(x - 3)^2 + (y - 3)^2 = 4, \qquad (x - 1)^2 + (y - 5)^2 = 4,$$

Question 2

You are given the following values of the camera calibration parameters:

$$f = 2, \quad u_0 = 0, \quad v_0 = 1$$

Compute the image location of the following 3D point:

$$X = 7, \quad Y = 13, \quad Z = 2$$

Answer:

$$u = u_0 + fX/Z = 0 + 2 \cdot 7/2 = 7, \quad v = v_0 + fY/Z = 1 + 2 \cdot 13/2 = 14$$

Question 3

A point at the coordinates (u, v) in the picture is a projection of a 3D point X, Y, Z. Given that the camera calibration parameters $f, u_0, v_0 f$, and that the 3D point X, Y, Z is on the plane

$$Z = aX + bY + c,$$

prove that:

$$X = \frac{c(u - u_0)}{f - a(u - u_0) - b(v - v_0)} = \frac{cx}{f - ax - by}, \quad Y = \frac{c(v - v_0)}{f - a(u - u_0) - b(v - v_0)} = \frac{cy}{f - ax - by}.$$

Compute Z as as an explicit function of $u, v, a, b, c, f, u_0, v_0$.

Answer

$$u = u_0 + f\frac{X}{Z} = u_0 + f\frac{X}{aX + bY + c} \quad \rightarrow \quad (ax - f)X + bxY = -cx$$

$$v = v_0 + f\frac{Y}{Z} = v_0 + f\frac{Y}{aX + bY + c} \quad \rightarrow \quad ayX + (by - f)Y = -cy$$

This can be viewed as two equations with in the two unknowns X, Y. The solution gives the above relations.

Substituting these values of X, Y into the plane equation and simplifying gives:

$$Z = \frac{cf}{f - ax - by}.$$