**Homework-4 Solutions**

# Question 1

You are given the following color image, where the pixel values are in sRGB. The value of each pixel is given as the triplet $(r, g, b)$.

| | | | |
|---|---|---|---|
| (0,0,0) | (0,0,0) | (0, 0, 0) | (0, 0, 0) |
| (255, 0, 0) | (255, 0, 0) | (255, 0, 0) | (255, 0, 0) |
| (100, 100, 100) | (100, 100, 100) | (100, 100, 100) | (100, 100, 100) |
| (0, 100, 100) | (0, 100, 100) | (0, 100, 100) | (0, 100, 100) |

**A**

Convert the above image to nonlinear RGB.

| | | | |
|---|---|---|---|
| (0, 0, 0) | (0, 0, 0) | (0, 0, 0) | (0,0,0) |
| (1, 0, 0) | (1, 0, 0) | (1, 0, 0) | (1, 0, 0) |
| (0.392, 0.392, 0.392) | (0.392, 0.392, 0.392) | (0.392, 0.392, 0.392) | (0.392, 0.392, 0.392) |
| (0, 0.392, 0.392) | (0, 0.392, 0.392) | (0, 0.392, 0.392) | (0, 0.392, 0.392) |

**B**

Convert the above image to linear RGB.

| | | | |
|---|---|---|---|
| (0, 0, 0) | (0, 0, 0) | (0, 0, 0) | (0,0,0) |
| (1, 0, 0) | (1, 0, 0) | (1, 0, 0) | (1, 0, 0) |
| (0.127,0.127,0.127) | (0.127,0.127,0.127) | (0.127,0.127,0.127) | (0.127,0.127,0.127) |
| (0,0.127,0.127) | (0,0.127,0.127) | (0,0.127,0.127) | (0,0.127,0.127) |

**C**

Convert the above image to XYZ.

| | | | |
|---|---|---|---|
| (0, 0, 0) | (0, 0, 0) | (0, 0, 0) | (0,0,0) |
| (0.412, 0.212, 0.019) | (0.412, 0.212, 0.019) | (0.412, 0.212, 0.019) | (0.412, 0.212, 0.019) |
| (0.121,0.127,0.138) | (0.121,0.127,0.138) | (0.121,0.127,0.138) | (0.121,0.127,0.138) |
| (0.07,0.1,0.136) | (0.07,0.1,0.136) | (0.07,0.1,0.136) | (0.07,0.1,0.136) |

**D**

Convert the above image to xyY.

| | | | |
|---|---|---|---|
| (0, 0, 0) | (0, 0, 0) | (0, 0, 0) | (0,0,0) |
| (0.64, 0.33, 0.212) | (0.64, 0.33, 0.212) | (0.64, 0.33, 0.212) | (0.64, 0.33, 0.212) |
| (0.3127,0.3129,0.127) | (0.3127,0.3129,0.127) | (0.3127,0.3129,0.127) | (0.3127,0.3129,0.127) |
| (0.2247,0.3288,0.1) | (0.2247,0.3288,0.1) | (0.2247,0.3288,0.1) | (0.2247,0.3288,0.1) |

**E**

Convert the above image to Luv.

| | | | |
|---|---|---|---|
| (0, 0, 0) | (0, 0, 0) | (0, 0, 0) | (0,0,0) |
| (53.2, 175.0, 37,7) | (53.2, 175.0, 37,7) | (53.2, 175.0, 37,7) | (53.2, 175.0, 37,7) |
| (42.37, 0.0, 0.0) | (42.37, 0.0, 0.0) | (42.37, 0.0, 0.0) | (42.37, 0.0, 0.0) |
| (37.9, -29.29, -6.339) | (37.9, -29.29, -6.339) | (37.9, -29.29, -6.339) | (37.9, -29.29, -6.339) |

**F**

Compute linear illumination stretching in the Luv domain, and convert the result back to sRGB.

The minimum value of $L$ is 0, and the maximum value is 53.2 Multiply each $L$ value by 100/53.2. This gives the following image in $Luv$ (only first column is shown):

| (0,0, 0) |
|---|
| (100, 175.0, 37.7) |
| (79.64, 0, 0) |
| (71.2, -29.29, -6.339) |

Converting to $XYZ$:

| (0,0, 0) |
|---|
| (1.5, 1, 0.53) |
| (0.533, 0.56, 0.61) |
| (0.344, 0.425, 0.523) |

Converting to linear $RGB$:

| (0,0, 0) |
|---|
| (1, 0.69, 0.69) |
| (0.56, 0.56, 0.56) |
| (0.2, 0.485, 0.485) |

Converting to nonlinear $RGB$ in bytes:

| (0,0, 0) |
|---|
| (255, 177, 177) |
| (197, 197, 197) |
| (124, 185, 185) |

# Question 2

$$f_1 \otimes g = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 3 & 3 & 3 & 1 & 1 \\ 0 & 2 & 4 & 7 & 10 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 4 & 6 & 6 & 6 & 2 & 2 \end{bmatrix}$$

$$f_1 * g = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 & 3 & 3 & 2 & 2 \\ 0 & 1 & 2 & 5 & 8 & 6 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 6 & 6 & 6 & 4 & 4 \end{bmatrix}$$

$$f_2 \otimes g_1 = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \otimes \begin{bmatrix} 2 & 2 & 3 & 3 & 3 & 1 & 1 \\ 0 & 2 & 4 & 7 & 10 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 4 & 6 & 6 & 6 & 2 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 & -3 & -3 & -3 & -1 & -1 & 0 \\ 2 & 2 & 1 & -1 & -4 & 1 & -2 & 1 \\ 0 & 2 & 6 & 11 & 17 & 13 & 7 & 4 \\ -4 & -4 & -6 & -6 & -6 & -2 & -2 & 0 \\ 4 & 8 & 10 & 12 & 12 & 8 & 4 & 2 \end{bmatrix}$$

**Q2**

$$
\begin{aligned}
g_2 &= f_2 \otimes g_1 \\
&= f_2 \otimes (f_1 \otimes g) \\
&= \boxed{\begin{matrix} 1 & 1 \\ 0 & -1 \end{matrix}} \otimes \Big( \boxed{\begin{matrix} 1 & 0 & 2 \end{matrix}} \otimes g \Big) \\
&= \boxed{\begin{matrix} -1 & 0 \\ 1 & 1 \end{matrix}} * \Big( \boxed{\begin{matrix} 2 & 0 & 1 \end{matrix}} * g \Big) \\
&= \left( \boxed{\begin{matrix} -1 & 0 \\ 1 & 1 \end{matrix}} * \boxed{\begin{matrix} 2 & 0 & 1 \end{matrix}} \right) * g \\
&= \left( \boxed{\begin{matrix} 1 & 1 \\ 0 & -1 \end{matrix}} \otimes \boxed{\begin{matrix} 2 & 0 & 1 \end{matrix}} \right) * g \\
&= \boxed{\begin{matrix} -2 & 0 & -1 & 0 \\ 2 & 2 & 1 & 1 \end{matrix}} * g \\
&= \boxed{\begin{matrix} 1 & 1 & 2 & 2 \\ 0 & -1 & 0 & -2 \end{matrix}} \otimes g
\end{aligned}
$$