Homework-6 Solutions

Question 1

The result of detecting the template | 1 | 2 | 1 | in the image

			4	4	4				
			1	2	1				

a. with non-normalized cross correlation matching. Cross correlation of the template with the image gives: Therefore, the best match is found for cross correlation value of 16, at the coordinate $x = 4$, $y = 1$.

		4	12	16	12	4			
		1	4	6	4	1			

b. with normalized cross correlations matching. First square the original image values

			16	16	16				
			1	4	1				

Next compute the cross correlation with | 1 | 1 | 1 |

		16	32	48	32	16			
		1	5	6	5	1			

Next divide the values found in Part a by the square root of the above values giving:

		1	$3/\sqrt{2}$	$4/\sqrt{3}$	$3/\sqrt{2}$	1			
		1	$4/\sqrt{5}$	$6/\sqrt{6}$	$4/\sqrt{5}$	1			

The best match is found at coordinate $x = 4$, $y = 2$.

If desired the results can be normalize to a (0,1) range by dividing each element by $\sqrt{1^2 + 2^2 + 1^2}$

		$1/\sqrt{6}$	$\sqrt{3}/2$	$2\sqrt{2}/3$	$\sqrt{3}/2$	$1/\sqrt{6}$			
		$1/\sqrt{6}$	$4/\sqrt{30}$	1	$4/\sqrt{30}$	$1/\sqrt{6}$			

1	2
7	3

A

2	2
2	2

B

8	9
11	10

C

11	20
40	30

D

100	100
100	100

E

Question 2

A,B,C,D,E above are 5 small windows in an image. Our goal is to use template matching to detect the following pattern:

1	2
4	3

I

If the technique of non-normalized cross correlation is used, which pattern gives the best match value?

Answer: E.

	A	B	C	D	E
match value	42	20	100	291	1000

II

If the technique of normalized cross correlation is used, which pattern gives the best match value?

Answer: D.

	A	B	C	D	E
match value	$42/\sqrt{63} = 5.291$	$20/\sqrt{16} = 5$	$100/\sqrt{366} = 5.227$	$301/\sqrt{3021} = 5.476$	$1000/\sqrt{40000} = 5$

Question 3

Consider the following image:

	$x=0$	$x=1$	$x=2$	$x=3$	$x=4$
$y=0$	1	0	0	0	0
$y=1$	0	0	0	0	0
$y=2$	0	1	1	1	0
$y=3$	0	2	2	2	0
$y=4$	0	0	0	0	0
$y=5$	0	0	0	0	0

1.

Compute its integral image.

Answer:

Integral image on each row:

	$x=0$	$x=1$	$x=2$	$x=3$	$x=4$
$y=0$	1	1	1	1	1
$y=1$	0	0	0	0	0
$y=2$	0	1	2	3	3
$y=3$	0	2	4	6	6
$y=4$	0	0	0	0	0
$y=5$	0	0	0	0	0

Integral image on each column:

	$x=0$	$x=1$	$x=2$	$x=3$	$x=4$
$y=0$	1	1	1	1	1
$y=1$	1	1	1	1	1
$y=2$	1	2	3	4	4
$y=3$	1	4	7	10	10
$y=4$	1	4	7	10	10
$y=5$	1	4	7	10	10

2.

Use the integral image calculated in 1 to compute the sum of pixels in the following rectangles:

Rectangle 1: $x_1 = 1, y_1 = 1, x_2 = 4, y_2 = 2$.

Rectangle 2: $x_1 = 1, y_1 = 1, x_2 = 2, y_2 = 2$.

Rectangle 3: $x_1 = 3, y_1 = 1, x_2 = 4, y_2 = 2$.

Show your computations.

Answer:

Rectangle 1: $x_1 = 1, y_1 = 1, x_2 = 4, y_2 = 2$. $\text{sum} = 4 - 1 - 1 + 1 = 3$.

Rectangle 2: $x_1 = 1, y_1 = 1, x_2 = 2, y_2 = 2$. $\text{sum} = 3 - 1 - 1 + 1 = 2$.

Rectangle 3: $x_1 = 3, y_1 = 1, x_2 = 4, y_2 = 2$. $\text{sum} = 4 - 1 - 3 + 1 = 1$.

3.

How many rectangles inside the image have even width?

Answer:

126 rectangles.

height 1: $6 * 6$
height 2: $5 * 6$
height 3: $4 * 6$
height 4: $3 * 6$
height 5: $2 * 6$
height 6: $1 * 6$
Total: $6 * 6 * (1 + 6)/2 = 126$

4.

Suppose the given image is the only positive example in a learning task, and there is only one negative example, an image with all zero pixels. Consider the Haar feature computed over the Rectangle 1 above, where the left values (Rectangle 2) are all -1, and the right values (Rectangle 3) are all 1. What is the threshold associated with this feature?

Answer:

Applying the Haar feature to the given postive example will give the value -1. For the negative example, applying the Haar feature gives 0. Therefore, we can use a threshold of -0.5.