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1

Department of Ship Technology
Cochin University of Science & Technology

Dec24-Apr25

20 Dec 2024

Stability of Ships

B. Tech. NA&SB. 2023-27. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

Dr. D. D. Ebenezer
Adjunct Faculty
9446577239
ebenezer.cusat@gmail.com

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2

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
   Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
   Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
   Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
   Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
   Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
   Intact stability rules, Heel/ Load test.
   Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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3

Stability of Ships. Course Content

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4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

73

   removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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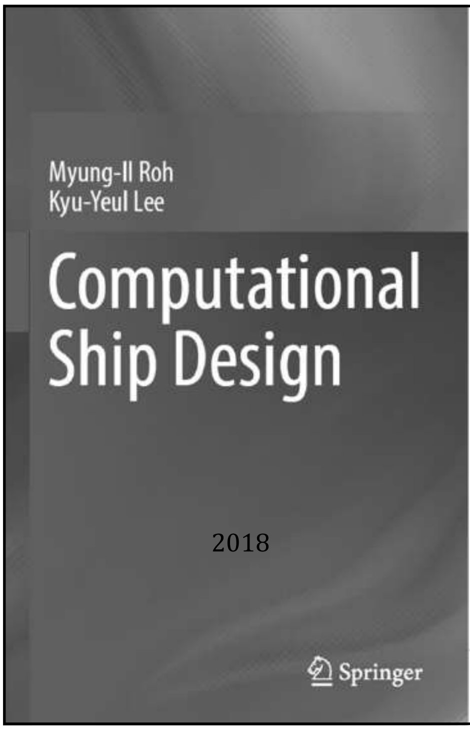

2018

SHIP STRUCTURAL ANALYSIS AND DESIGN

by Owen F. Hughes and Jeom Kee Paik
with Dominique Béghin, John B. Caldwell, Hans G. Payer and Thomas E. Schellin

2010

Published by
The Society of Naval Architects and Marine Engineers
601 Pavonia Avenue
Jersey City, New Jersey 07306

Ship Design for Efficiency and Economy

Second edition

H. Schneekluth and V. Bertram

1998

BUTTERWORTH HEINEMANN

OXFORD BOSTON JOHANNESBURG MELBOURNE NEW DELHI SINGAPORE

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5

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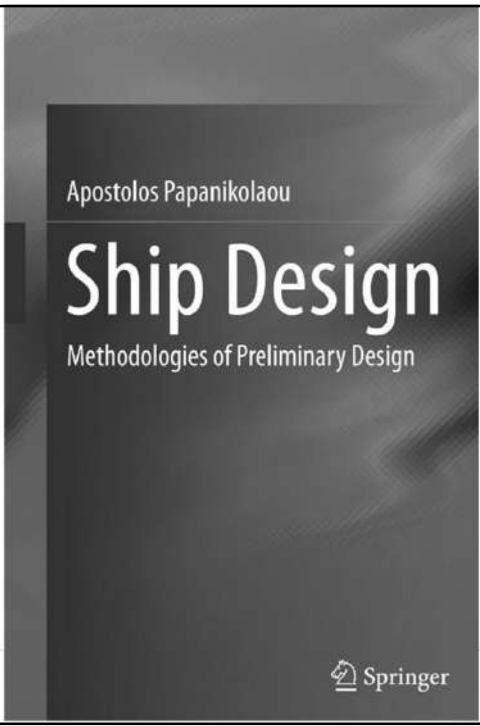

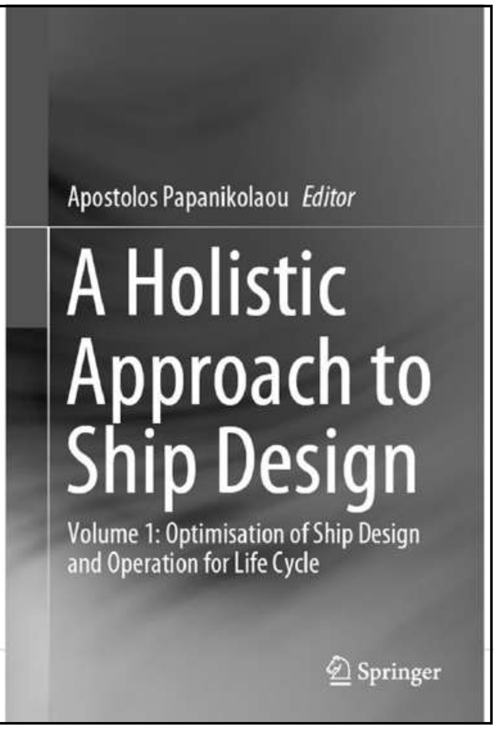

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6

Technical Report No. 72

The Stability of a Freely Floating Ship

Final report

Maciej Pawłowski

2nd edition, revised and updated

Gdansk, January 2016

Some stability issues of ships

J. Pinkster

Report 1266-P 2001

Published in Schip en Werf de Zee, Marine Technology, 11th edition, January 2001

TU Delft

The Principles of Naval Architecture Series

Intact Stability

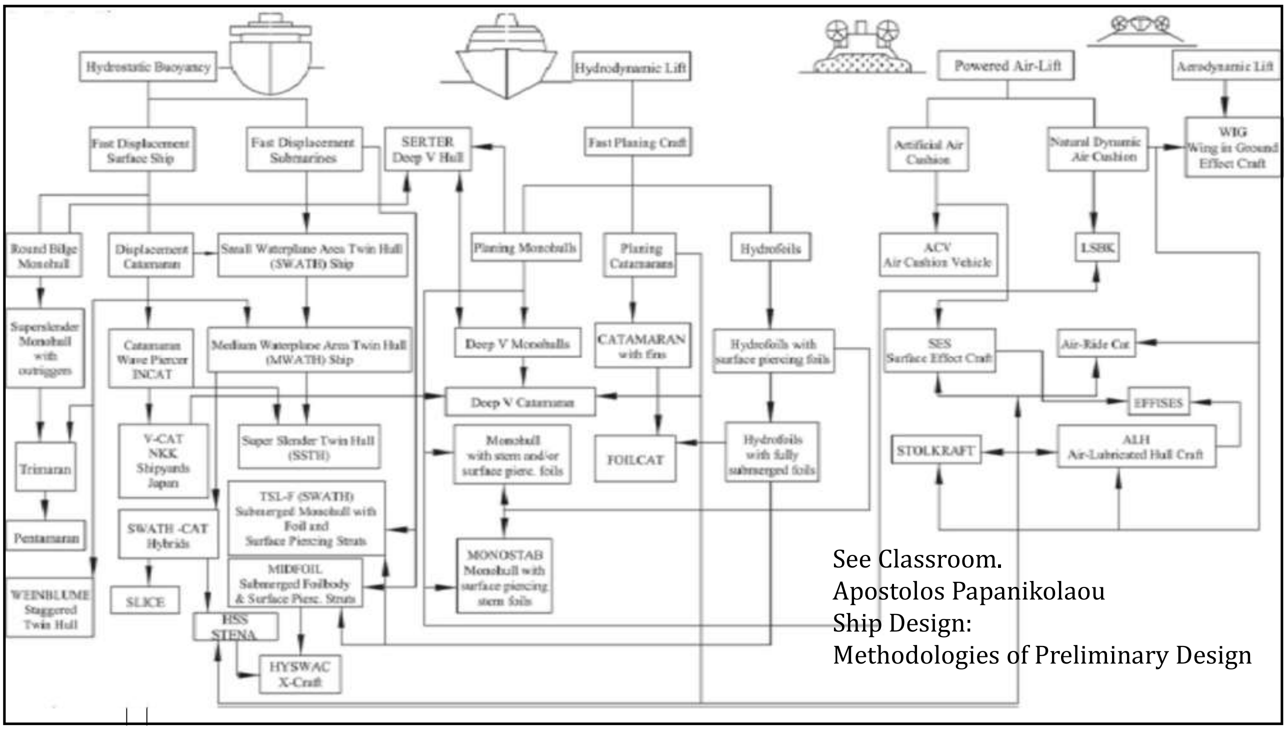
Colin S. Moore

J. Randolph Paulling, Editor

2010

Published by
The Society of Naval Architects and Marine Engineers
601 Pavonia Avenue
Jersey City, New Jersey 07306

Apr25



See Classroom.
Apostolos Papanikolaou
Ship Design:
Methodologies of Preliminary Design

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8

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MODULE 2. Initial Transverse Stability

Completed

Module 1. 06 lectures.

Multiple choice Google forms test on 08 Jan 2025.

Today

Module 2

2.1 $\overline{GM_0}$

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9

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What is Naval Architecture?

- "Naval Architecture is that branch of engineering which embraces all aspects of design, research, developments, construction, trials, and effectiveness of all forms of <u>man-made vehicles</u> which operate either at or below the surface of any body of water."

A. Blyth, FRINA

In

Ship Hydrostatics and Stability, Second Edition (2014)

Adrian B. Biran, Technion, Faculty of Mechanical Engineering, and

Rubén López-Pulido, Former Representative of Spain to the IMO

With contributions by Javier de Juana Gamo

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10

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Ship Design Cycle

- Taggart. Ship Design and Construction.
- A design is <u>the</u> solution to a constrained optimization problem

2 SHIP DESIGN AND CONSTRUCTION

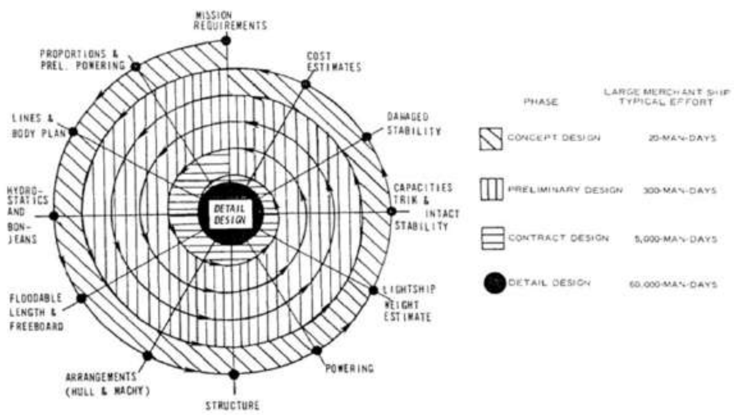

Fig. 1 Basic design spiral

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11

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Why is a Cycle Needed?

Wilson. Basic Naval Architecture.

It is typical of a design calculation that it is necessary to know the result of the calculation before the calculation can be carried out.

Consider the structural design problem:

- The structure must be strong enough to sustain the loads applied to it.
- A substantial part of the loading is associated with structural weight.
- The structural weight depends on the size of the structural components.
- The size of the components depends on the strength required

Thus to estimate the required structural strength you need to know the structural weight which cannot be found unless you already know how strong the structure needs to be.

Likewise consider the power problem:

- The power required to propel the ship depends on its total weight and other factors
- Major items of weight are propulsion machinery and fuel.
- These items of weight cannot be estimated until the power to propel the ship is known.

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12

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Technical Design of Ships

- Wilson. Basic Naval Architecture.
- In order to proceed with the technical design process in this sort of circumstance the starting point has to be a good first guess at the answer. A typical procedure would be (do this for your final year ship design project)

  1. Guess (estimate) total weight for ship and contents.
  2. Carry out a power calculation, select the size and type of main engine, estimate machinery and fuel weights.
  3. Carry out a structural calculation, select the sizes of structural components and estimate structural weights.
  4. Estimate the total weight including structure, machinery, fuel, cargo, equipment and stores.
  5. Compare new total weight with the starting guess and repeat the calculation if the discrepancy is too large.

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Apostolos Papanikolaou
Ship Design: Methodologies of Preliminary Design

1.3 Introduction to Ship Design 17

Apostle: One sent on a mission

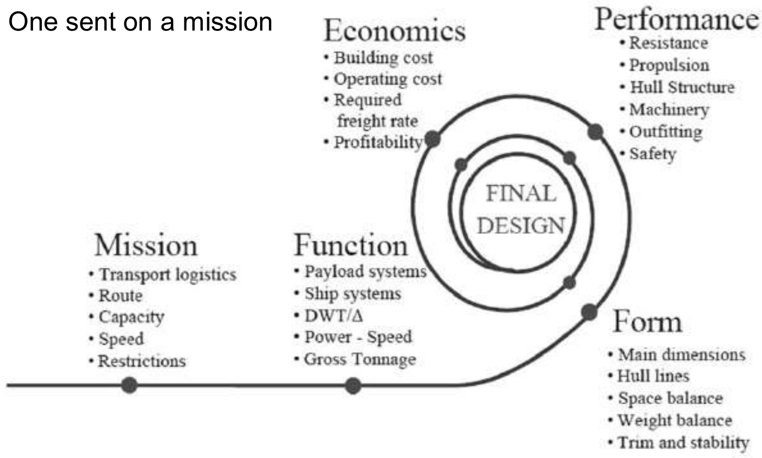

Fig. 1.13 Ship design procedure according to K. Levander (2009) and Papanikolaou et al. (2009d; coordinator)

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14

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2.1 $\overline{GM_0}$

- $\overline{GM_0}$ is the metacentric height for heeling
- To find $\overline{GM_0}$, we should know (estimate) the coordinates of G and M_0
- Several methods are used to find the CoG. Some are approximate and they are used in the preliminary design and stages. Some are nearly exact and used when the ship is loaded. These methods are presented next.
- Recall. The transverse metacenter is often simply known as the metacenter

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15

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Biran. 7.2 Weight Calculations

7.2.1 Weight Groups

- A vessel is composed of hundreds, sometimes thousands of mass items. To systematize calculations, it is necessary to organize them into weight groups. The first subdivision is into two main sets: lightship and deadweight. The lightship (less frequently known as lightweight) is the mass of the empty ship; it is composed of the hull, the outfit, and the machinery masses, including the liquids in the machinery and various systems, but not those in tanks or storage spaces. The deadweight is the sum of the masses of crew, cargo, and passengers, fuel, lubricating oil, provisions, water, stores, and spare parts. The usual acronym for deadweight is DWT. In simpler terms, the deadweight is the weight that the ship "carries."

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Biran. 7.2 Weight Calculations

- In the first stages of ship design, known as preliminary design, the lightship masses and their centres of gravity are estimated by empirical equations, based on statistics of similar ships, or are derived from the masses of a given parent ship.
- For merchant ships, the lightship groups are the hull, the outfit, and the machinery. Fitting out, or outfitting, is the process in shipbuilding that follows the float-out or launching of a vessel and precedes sea trials. It is the period when all the remaining construction of the ship is completed and readied for delivery to her owners.
- The classification of warship weight groups may be somewhat different. Thus, the classification system of the US Navy, SWBS, distinguishes the following main weight groups: hull structure, propulsion plant, electric plant, command and surveillance, auxiliary systems, outfit and furnishings, armament.

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17

Biran. 7.2 Weight Calculations

- As the design progresses by successive iterations, the weight estimations are refined by subdividing the weight groups into subgroups, the subgroups into lower-level subgroups, and so on. Thus, the hull mass is subdivided into hull and superstructure, then the hull into bottom, sides, decks, bulkheads, etc. The machinery components are first subdivided into main, or propulsion machinery, and auxiliary machinery. In the final stages it is possible to calculate the masses and centres of gravity of individual items from detailed drawings or from data provided by equipment suppliers.
- The procedure described above requires a classification of the various weight groups, subgroups, and so on that <u>ensures that no item is forgotten</u> and that <u>no item belongs to two groups</u>.

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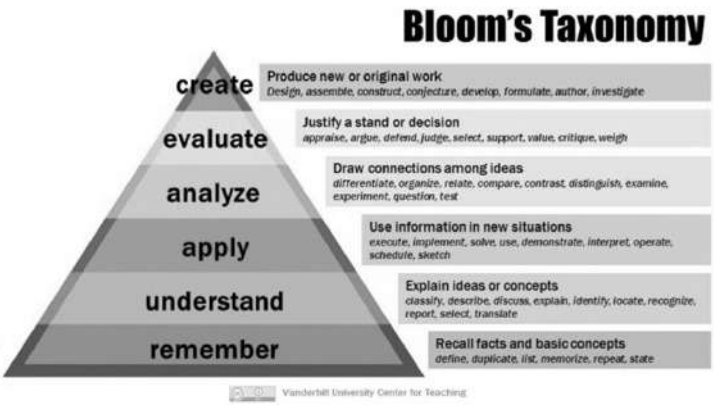
18

Taxonomy

- Taxonomy is a scheme for classification.
- Bloom’s taxonomy is used in education.
- The total weight of a ship is subdivided to include the whole and not have any duplicate entries
- Think set-theory



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Find the Center of Gravity

- Lester. Merchant Ship Stability. See the figure on the next slide.
- Distances x_0, x_1, x_2, … are used to find moments about the YY and ZZ axes

In *Figure 3.1*, W is the resultant of forces w_0, w_1, w_2 acting at distances x_0, x_1, x_2 from axis YY. Then if the resultant W is a distance x from YY and:

$$W = w_0 + w_1 + w_2$$

$$Wx = w_0x_0 + w_1x_1 + w_2x_2$$

$$x = \frac{w_0x_0 + w_1x_1 + w_2x_2}{W}$$

$$x = \frac{\text{moment of forces about YY}}{\text{weight}}$$

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59

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20

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Find the Center of Gravity

- Lester. Merchant Ship Stability

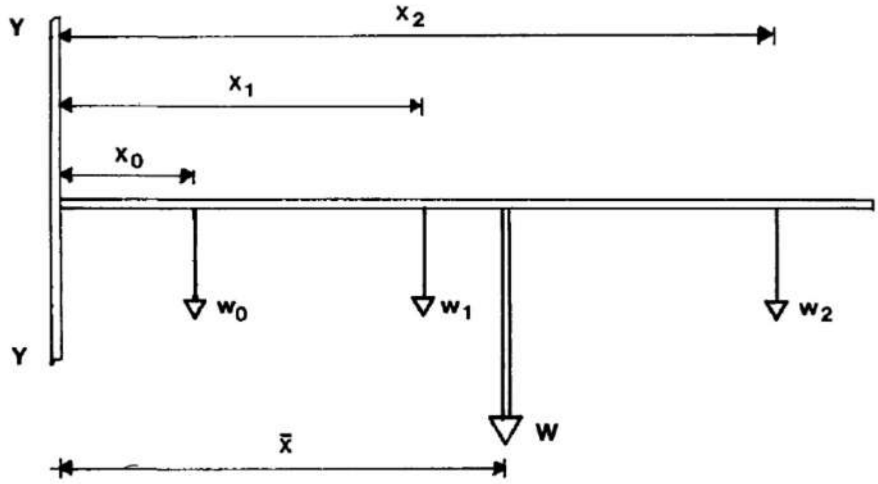

Figure 3.1 Position of centre of gravity

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21

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Find the $\overline{KG}$ (vertical CoG)

- In this figure, the ship is on its side to show that gravity is acting downwards. Use it to find $\overline{KG}$.

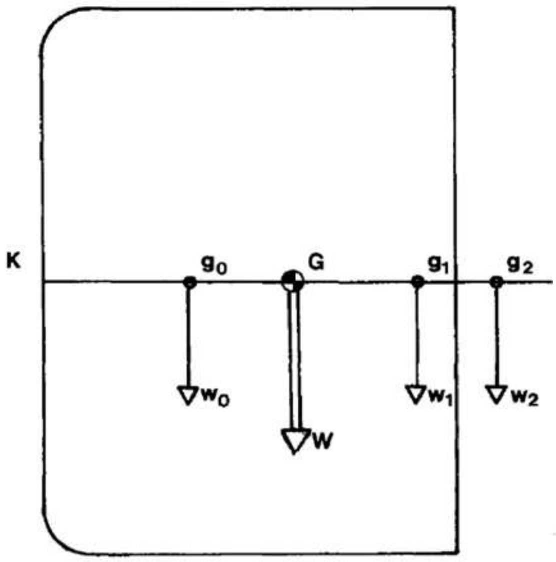

Figure 3.2 KG

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Find the Transverse Center of Gravity

- Take moments about the centreline
- Lever arms can be positive or negative

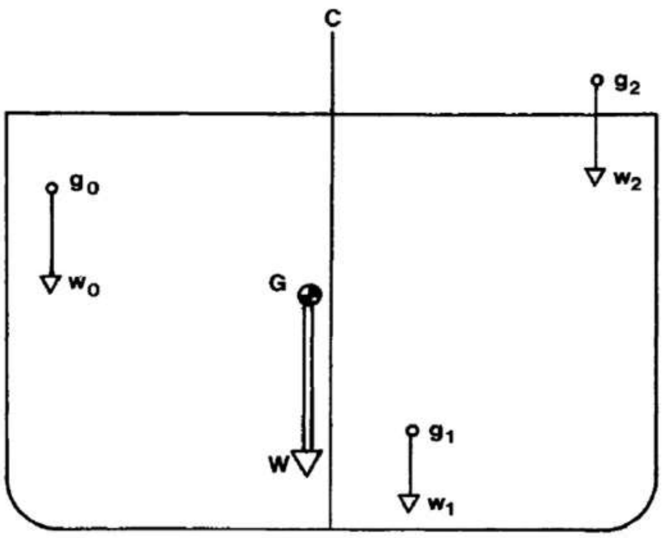

Figure 3.3 G from centreline

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23

Biran. 7.2 Weight Calculations

	A	B	C	D	E	F
1	Small cargo ship, Homogeneous cargo, departure.					
2						
3	**Weight item**	**Mass**	**vcg**	**z-Moment**	**lcg**	**x-Moment**
4		t	m	tm	m	tm
5	Lightship	1247.66	5.93	7398.62	32.04	39975.03
6	Crew and effects	3.60	9.60	34.56	11.00	39.60
7	Provisons	5.00	7.30	36.50	3.50	17.50
8	Fuel oil	177.21	1.56	276.45	30.88	5472.24
9	Lubricating oil	4.50	4.65	20.93	8.45	38.03
10	Freshwater	103.09	4.61	475.24	27.19	2803.02
11	Ballast water			0.00		0.00
12	Cargo in hold	993.94	4.35	4323.64	42.62	42361.72
13	Cargo on deck			0.00		0.00
14	Fruit cargo	90.00	6.08	547.20	38.66	3479.40
15	**Full load**	2625.00	5.00	13113.14	35.88	94186.54
16						
17	Mean draught, m	4.32				
18	KM, m	5.16				
19	KG, m	5.00				
20	GM, m	0.16				
21	FS effect, m	0.04				
22	Effective GM, m	0.12				

Biran. Figure 7.1

- Once the ship is built and in service, the lightship displacement and its centre of gravity are taken in calculations as constants. For each possible loading case, that is for each combination of cargo and other deadweight items, the masses of those items and their moments are added to those of the lightship. The calculations yield the displacement and the coordinates of the centre of gravity of the loading case under consideration. To give an example, we return to the data of the small cargo ship considered in Chapter 6. Figure 7.1 shows the calculations corresponding to the load case homogeneous cargo, departure. By departure condition we mean the ship leaving the port, with all the fuel, lubricating oil and provisions.

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Actual Weight and CoG

- Detailed computer drawings of the ship are used to find the CoG of the ship
- Present-day Naval-Architectural computer programs use mainly B-splines and Non-uniform rational basis spline (NURBS).
- The CoG is a vector quantity with 3 components
- Each item that goes on-board a ship comes with data: mass and CoG
- The CoG of the ship is determined by using moments

$$\bar{\boldsymbol{x}} = \sum_{i=1}^{N} \boldsymbol{W_i}\, \bar{\boldsymbol{x}}_{\boldsymbol{i}}$$

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Gross Tonnage

- The gross tonnage calculation is defined in Regulation 3 of Annex 1 of The International Convention on Tonnage Measurement of Ships, 1969.
- $V =$ the ship's total internal (enclosed) volume in cubic metres
- $K = 0.2 + 0.02 \times \log_{10}(V)$
- Gross Tonnage $= VK = V$ times K. The multiplication sign is omitted.
- (Gross) Tonnage is the basis for calculating registration fees and port dues. One of the convention's goals was to ensure that the new calculated tonnages "did not differ too greatly" from the traditional gross and net register tonnages.

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Preliminary Estimation of Ship Weight

- Dan Obreja and Ana-Maria Chiroşcă, “Preliminary Estimation of Ship Weight,” Annals of “Dunarea de Jos” University of Galati, Romania, 2015.
- Merchant vessels. Displacement = Lightship wt + Deadweight.
  $\Delta = \Delta_g + \boldsymbol{D_w}$
- Union of Δ_g and $\boldsymbol{D_w}$ is Δ. Intersection of Δ_g and $\boldsymbol{D_w}$ is the null set.
  - Designers often make the mistake of including the same items in both the sets!
- The light ship weight $\Delta_g = \boldsymbol{M_c} + \boldsymbol{M_m} + \boldsymbol{M_{ai}} + \boldsymbol{M_\Delta}$

  $\boldsymbol{M_c}$ is the structural weight,

  $\boldsymbol{M_m}$ is the propulsion system weight

  $\boldsymbol{M_{ai}}$ is the outfit and hull engineering weight

  $\boldsymbol{M_\Delta}$ is the displacement reserve.

  Intersection of any two sub-sets of Δ_g is the null set.

Total Weight of Ship
Dead Weight
Lightship Weight

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Lightship Weight

- The lightship weight includes the weight of completely outfitted vessel with inventory according to the List of Inventory, spare parts according to the Class Society requirements and with liquids in engine room systems. In particular lightship weight does not include:
- A. Loose container lashing equipment.
- B. Spare parts in excess of rule requirements.
- C. Provision stores, crew and effects.
- D. Fuel oil, diesel oil, lubricating oil, fresh water, ballast water in tanks.

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28

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Lightship Weight - 2

For the purpose of estimate, generally the lightship weight is considered to be the sum of three main components:

$$W_{LS} = W_S + W_E + W_M$$

in which:

W_S - Weight of the structural steel of the hull, the superstructure and of the outfit steel (machinery foundations, supports, masts, ladders, handrails, etc).

$$W_S = W_H + W_{SPS}$$

W_E - Weight of the equipment, outfit, deck machinery, etc.
W_M - Weight of all the machinery located in the engine room

M.Ventura Estimation Methods 37

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Weight Estimates

A reasonable structure for a generic expression to compute the weights of the ship can be as follows

$$W = k.V^a.\Delta^b$$

W can be W_S or W_E or W_M

See the next slide

in which:

- k - constant obtained from similar ships
- V - service speed
- Δ - displacement
- a, b - constants depending from the type of weight under consideration, obtained from statistical regressions

M.Ventura Estimation Methods 38

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30

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Weight Estimate

Hull Weight

$$W_H = k \cdot V^{0.5} \cdot \Delta$$

See the next slide for a better expression that can be used only if the principal dimensions are known

Equipment Weight

$$W_E = k \cdot V^{0.9} \cdot \Delta^{3/4}$$

Machinery Weight

$$W_M = k \cdot V^3 \cdot \Delta^{2/3}$$

Machinery weight is very sensitive to the service speed. If V increases by a factor of 2, it will increase by a factor of 8. The Hull Weight will increase by only 1.414.

M.Ventura Estimation Methods 39

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31

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Hull Weight

- From statistical analysis regression (d'Almeida, 2009):

$$W_H = k1 \cdot L_S^{k2} \cdot B^{k3} \cdot D^{k4}$$

	k1	k2	k3	k4
Oil Tankers	0.0361	1.600	1.000	0.220
Bulk Carriers	0.0328	1.600	1.000	0.220
Container Carriers	0.0293	1.760	0.712	0.374
General Cargo	0.0313	1.675	0.850	0.280

M.Ventura Estimation Methods 45

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32

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Lightship Characteristics of Merchant Ships

- CSK Prabu, N. Vishwanath, Om Prakash Sha, "Study on the lightship characteristics of merchant ships," Brodogradnja/Shipbuilding Volume 71 Number 3, 2020
- The authors are at IIT Kharagpur.
- See the next slide. Replace MT with mt = metric tonne.

1. Which type of ship has the largest displacement?
2. For a fixed displacement, which type of ship has the highest lightship wt?
3. Note the scatter in the plots. Not all points lie on the same curve.
4. Read the discussion in the paper reg Fig. 5. See Study Material.

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CSK Prabu, N. Vishwanath, Om Prakash Sha
IIT Kharagpur

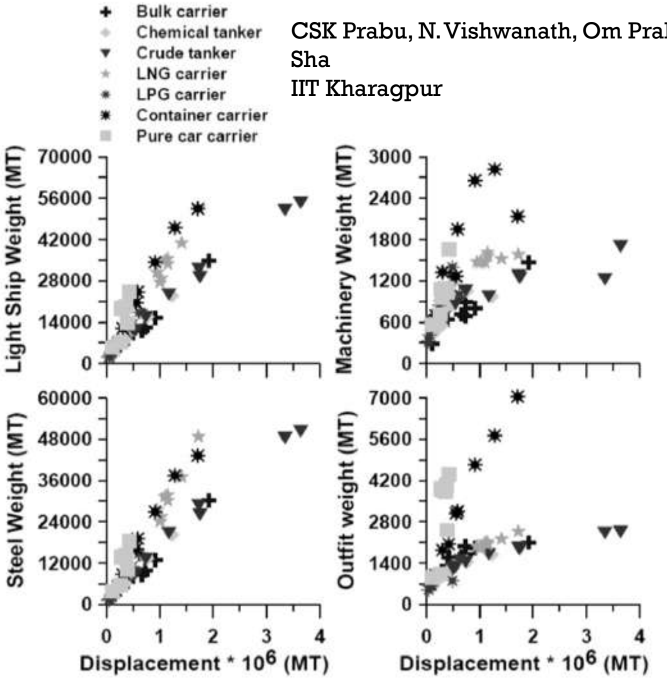

Fig. 5 Variation of different components of lightship weight with displacement for different ship types.

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34

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2.1 $\overline{GM_0}$

- The CoG is found using the procedure just described. KG = zCoG
- $\overline{GM_0}$ = transverse GM at even keel is of interest
- What is the next step?
- GM = KM – KG (any GM)
- = KB + BM – KG
- Find or estimate BM=I/V
- Find or estimate KB

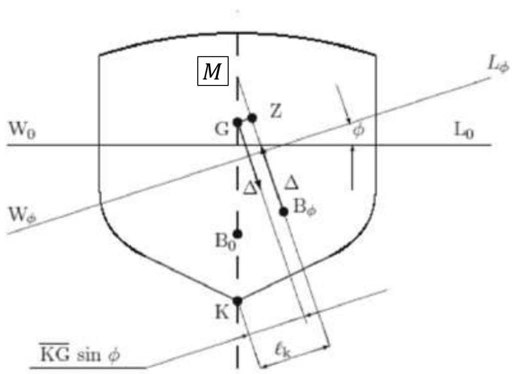

Figure 5.1 Definition of righting arm

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35

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Students are advised to read books and magazines.

Our library has a good collection of both.

Internal Assessment Marks will depend on your interaction in class, punctuality, being attentive in class, sleeping/awake, etc.

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1

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01 Jan 2025

Stability of Ships

B. Tech. NA&SB. 2023-27. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

Dr. D. D. Ebenezer
Adjunct Faculty
9446577239
ebenezer.cusat@gmail.com

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2

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Stability of Ships. Course Content.
Exam question paper will be based on this.

Course Content:

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   Intact stability rules, Heel/ Load test.
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3

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

73

removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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MODULE 2. Initial Transverse Stability

Completed

Module 1. 06 lectures.

Multiple choice Google forms test on 08 Jan 2025.

Module 2

2.1 $\overline{GM_0}$

Today

2.2 CoB

2.3 Wall-sided Ships

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5

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2.1 $\overline{GM_0}$

- The CoG is found using the procedure described in L07. KG = zCoG
- $\overline{GM_0}$ = transverse GM at even keel is of interest
- G,B_0, and and M_0 are on the centreline
- $\overline{GM_0} = \overline{KM_0} - \overline{KG}$
- $= \overline{KB_0} + \overline{B_0M_0} - \overline{KG}$
- $\overline{BM}$ =I/V. To find I see L08S31

Today

- Find or estimate V and KB
- See Wigley Hull Task where V is found

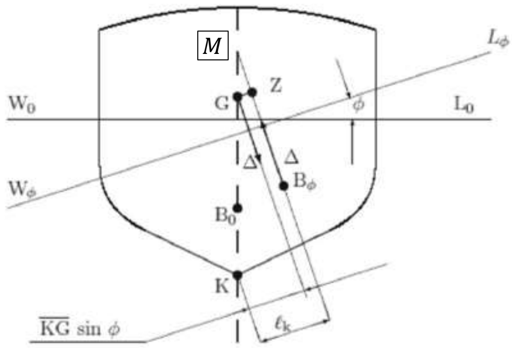

Figure 5.1 Definition of righting arm

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6

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Hydrostatic Data

- Biran. Chap. 4.1
- Numerical integration is used for calculation of areas, centroids, moments of inertia of areas, volumes, and centres of volume. We call these properties hydrostatic data and show how to plot them, as functions of draught, in curves that allow further calculations.
- Another set of plots consists of Bonjean curves; they enable the user to calculate the displacement and the centres of buoyancy for a given waterline, in an upright condition. The waterline can be not only a straight line, as is the case in still water, but also a curve. The latter case can arise when the hull is deflected because of a longitudinal bending moment or thermal expansion, or when the vessel floats in waves. The vessel is said to be in a hogging condition if the keel is concave downwards, and in a sagging condition if the keel is concave upwards.
- All the properties mentioned above are represented as functions of draught. Certain functional relationships exist between some of those curves.

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7

Two common ways to find the Underwater Volume

1. Use the Water-Plane areas
2. Use the Bonjeans (Cross-Sectional areas)
3. Use both the methods. See if they agree. A difference of 0.1% is good. This is to check for errors.
4. See Wigley Hull Task where Volume is found using both the method and the difference is 0%. Why is the difference 0%?

The Wigley Hull is defined by a function that is quadratic in both x and z. The 1/3 Simpson rule has zero error for integration of polynomials up to fourth degree. The error in $y = \int_a^b f(x)dx$ is proportional to Δx^4.

https://math.libretexts.org/Courses/Mount_Royal_University/MATH_2200%3A_Calculus_for_Scientists_II/2%3A_Techniques_of_Integration/2.5%3A_Numerical_Integration_-_Midpoint%2C_Trapezoid%2C_Simpson's_rule

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Simpson's Rule

- Let $f(x)$ be a continuous function over [a,b] and n be a positive even integer and $\Delta x = (b-a)/n$. Let [a,b] be sub-divided into n sub intervals, each of length Δx with end points at P $= \{x_0 = a, x_1, x_2, x_3, \dots . x_n = b\}$. Set

$$S_n = \frac{\Delta x}{3}\left[f(x_0) + f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots 2f\left(x_{(n-2)}\right) + 4f\left(x_{(n-1)}\right) + f(x_n)\right]$$

- Then, $\lim_{n\to\infty} S_n = \int_a^b f(x)\mathrm{d}x$
- Note that n is an even number and that $f(x_0), f(x_1), f(x_2), \dots f(x_n)$ is used.

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Error Bound for Simpson's Rule

- Let $f(x)$ be a continuous function over [a,b] having a fourth derivative, $f^{(4)}(x)$, over this interval. The error in using the Simpson's rule is proportional to the maximum value of $f^{(4)}(x)$ in the interval. If this max value is zero, the error is zero. The max value is zero for fourth degree polynomials. So, Simpson's rule is exact for polynomials of degree 4 or less.
- If M is the maximum value of $\left|f^{(4)}(x)\right|$ over [a,b], then the upper bound for the error in using S_n to estimate $\int_a^b f(x)dx$ is given by
  - Error in $S_n \leq M\frac{(b-a)^5}{180n^4} = M\frac{\Delta x^4(b-a)}{180}$ where $\Delta x = (b-a)/n$

- https://math.libretexts.org/Courses/Mount_Royal_University/MATH_2200%3A_Calculus_for_Scientists_II/2%3A_Techniques_of_Integration/2.5%3A_Numerical_Integration_-_Midpoint%2C_Trapezoid%2C_Simpson's_rule

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Simpson's Rule. Example.

- Use $f(x) = x^5$ and find $\int_0^{10} f(x)dx$ using (a) analytical integration and (b) Simpson's 1/3 rule using $f(0), f(1), f(2), \ldots f(10)$. Find the error in the Simpson's integral. Compare it with the error bound for the method.
- Ans. n = 10. Analytical area = 1e6/6 = 1.66666666e5. Simpson's area = 1.667000000000e+05. Error = Exact – Approx = 0.00033333e5 = 33.333.
- $f^{(4)}(x) = 5.4.3.2\ x = 120\ x$. max $f^{(4)}(x) = 120$ times 10 = 1200.
- M is the maximum value of $\left|f^{(4)}(x)\right|$ over [a,b] = 1200.
- Error $\leq M\frac{(b-a)^5}{180n^4} = 1200\ \frac{1e5}{180\ 10^4} = 66.66$. The actual error is 33.33.
- Error % = Error*100/Exact value = 0.04%

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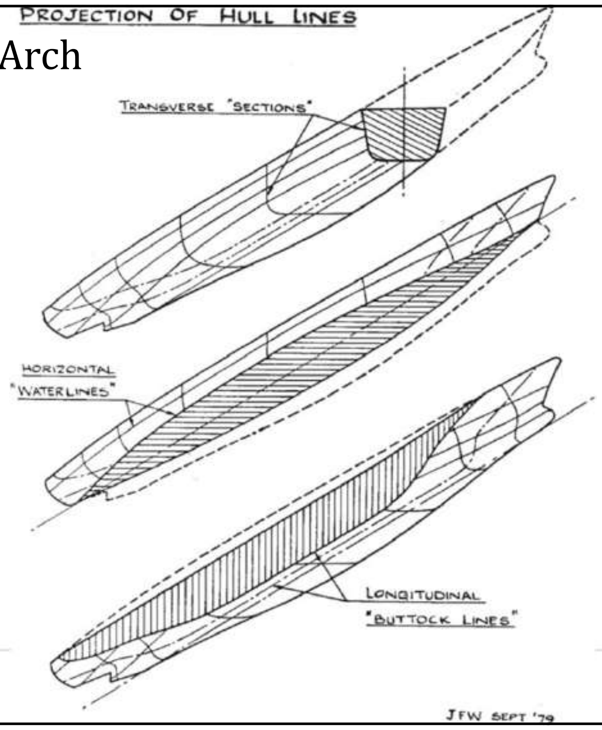
Wilson. Basic Naval Arch

- Fig. 2 Projection of hull lines
- "Adjust your eyes" to see isometric views correctly. The forward end of the ship is in the top right corner and further away than the aft.
- The view is from above, aft, and starboard of the ship.



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Bonjean Curves

- Biran. Fig. 4.4 is on the next slide.

4.4 Bonjean Curves and their Use

Figure 4.4 shows the midship section of the ship *Lido 9* in solid, thick line. Its equation is of the form

$$z = f(y)$$

y is the half-breadth of the ship

The **Bonjean curves** are defined by the equations

A = half-sectional area

$$A = \int_{keel}^{T} y dz \tag{4.16}$$

$$M = \int_{keel}^{T} zy dz \tag{4.17}$$

The first integral yields the **sectional area** as function of draught, while the second integral is the moment of the sectional area about the baseline, also as function of draught.

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Bonjean Curves

- Biran

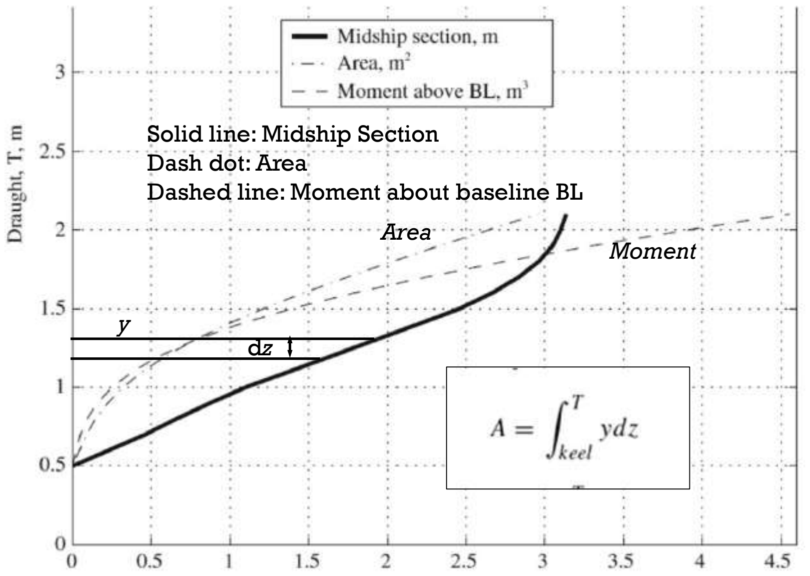

Figure 4.4 The meaning of Bonjean curves

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Bonjean Curves

- Curves of areas and moments of sections versus draught, plotted on the sheer plan.
- The diagram of Bonjean curves was first proposed at the beginning of the nineteenth century by Bonjean, a French naval engineer, for the purpose of readily obtaining, for any given waterline, the areas of the immersed portion of each transverse section throughout the ship's length.
- Only one scale is shown for the Y axis in Biran, Fig. 4.5.
- It can be used to find the draft, area, and moment.
- Area/2 and Moment/5 are shown in the figure.
- Find the area and the moment at amidships when the draft is 3 m. Use the larger figure on the next slide.

110 Chapter 4

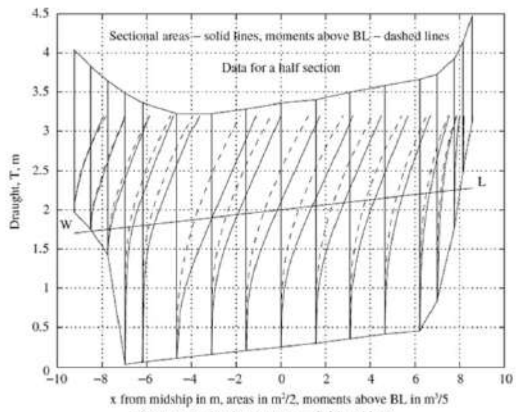

Figure 4.5 Bonjean curves of ship Lido 9

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Bonjeans

110 Chapter 4

- What is the length of Lido9?
  - Approx 18 m.
- Data is shown at N stations. What is N?
  - 17
- What is the distance along the x direction (forward aft) between the midship and the first section forward of it?
- At the first station forward of midship, what is the half-area at draft = 2.5 m?
- Approx 4 m^2

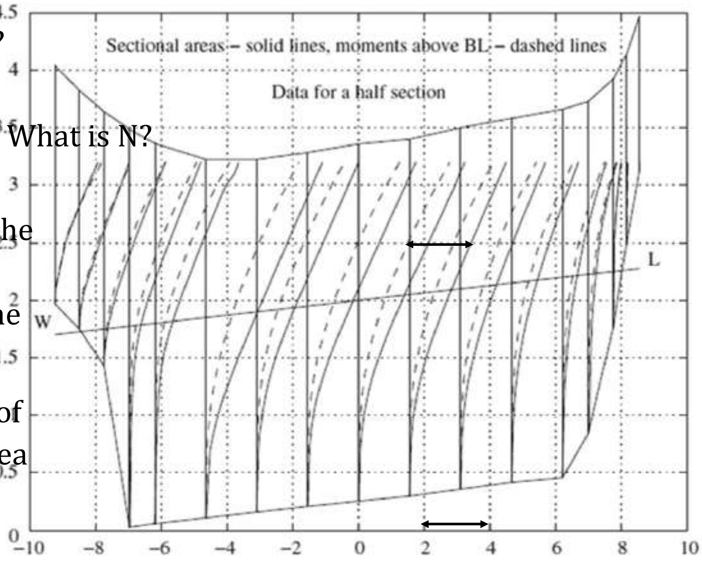

Figure 4.5 Bonjean curves of ship *Lido 9*

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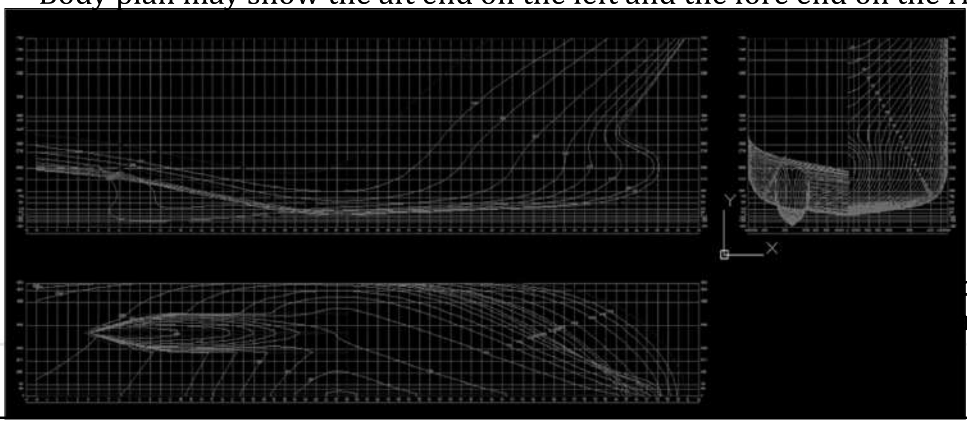
Lines Plan

- Lines plans show the hull form of a vessel. A lines plan consists of 3 views:
- The plan view (a view from above the vessel shows the waterlines)
- The profile view (a view from starboard of the ship shows buttock lines)
- The body plan view (a view from the aft of the vessel shows station lines)
- Body plan may show the aft end on the left and the fore end on the right.



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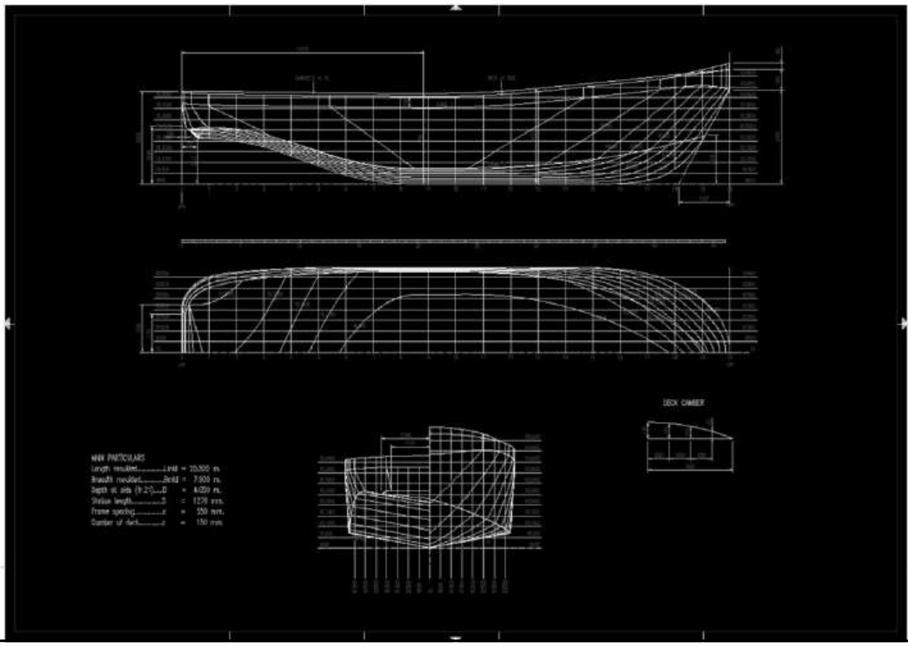
Lines Plan of a Tug



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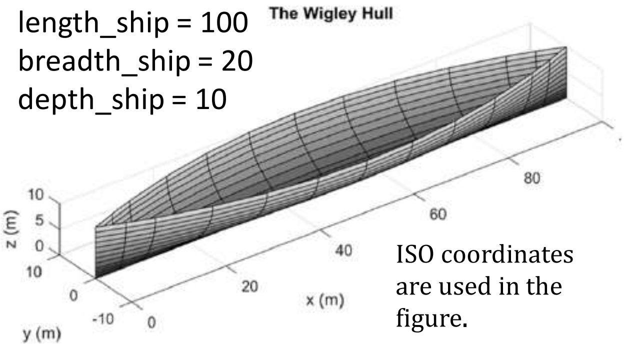
The Wigley Hull

length_ship = 100
breadth_ship = 20
depth_ship = 10



ISO coordinates are used in the figure.

- The origin is on the centerline, at midship, and on the sea surface.

$$y = \pm\frac{B}{2}\left[1-\left(\frac{2x}{L}\right)^2\right]\left[1-\left(\frac{z}{T}\right)^2\right]$$

$$-\frac{L}{2} \le x \le \frac{L}{2}, -T \le z \le 0$$

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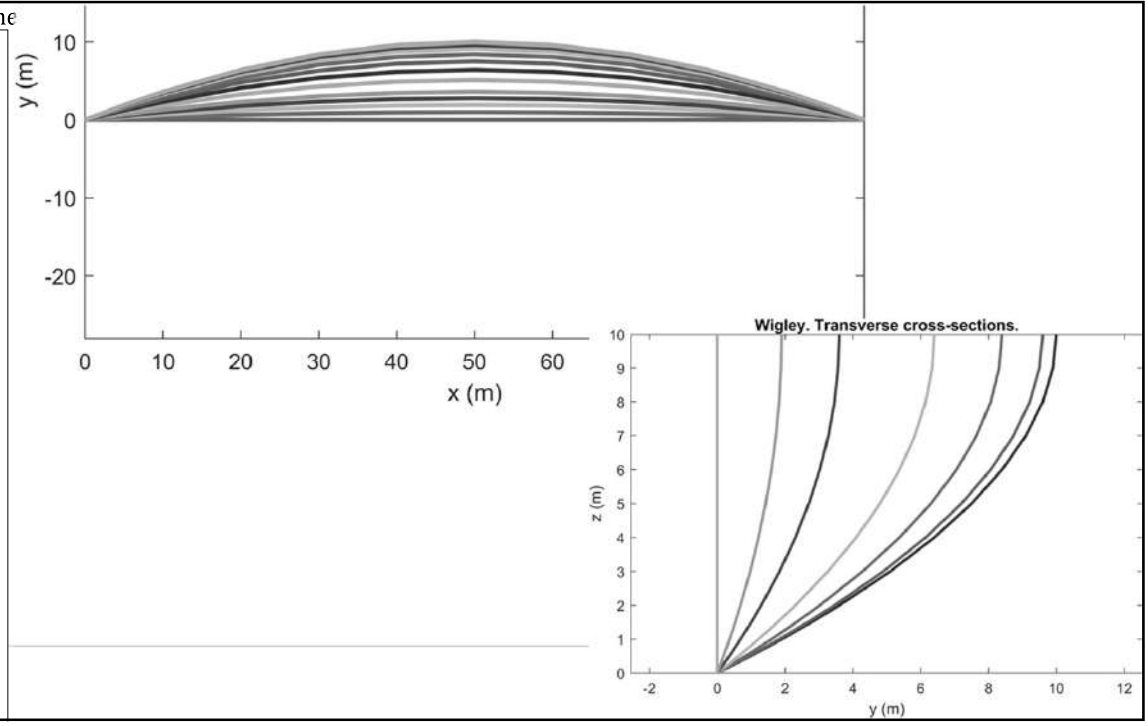

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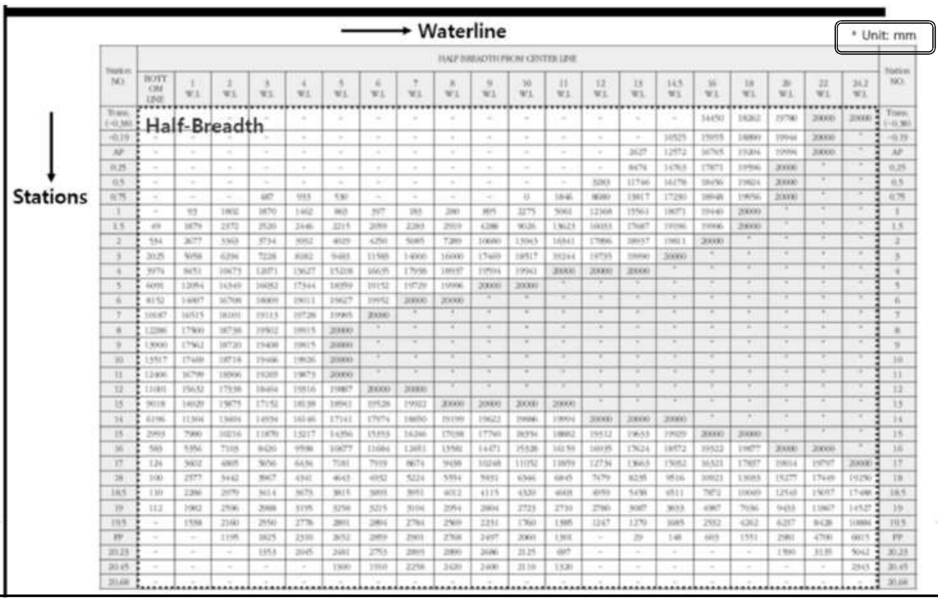
Zoom and see Offset Table posted under Study Material in Classroom



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A Portion of the Table of Offsets

In one figure, sketch the 2 m and 3 m waterlines

Station No	Waterline								
	Bottom	1 m	2 m	3 m	4 m	5 m	6 m	7 m	8 m
Transom									
-0.19									
AP									
0.25									
0.5									
0.75				487	933	530			
1		93	1802	1870	1462	863	397	183	
1.5	49	1879	2372	2520	2446	2215	2059	2283	
2	534	2677	3363	3734	3932	4029	4250		
3	2025	5058	6294	7228	8182	9483	11583		
4	3974	8451	10473	12071	13627				
5	6091	12054	14349	16032	17344				

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Find Displacement by integrating over a volume

- Semyonov. Method 1. Integrate over Water-Plane Areas. No heel or trim.
- Use this figure to understand both the methods. Note S and ω

22 *Buoyancy* Ch. II

6. DISPLACEMENT AND CO-ORDINATES OF CENTRE OF BUOYANCY

The formulas developed in this and the following sections apply to a ship floating without heel and trim.

In order to calculate the volume V of the immersed portion of the ship it is necessary to cut the ship by horizontal planes parallel to the plane XOY (Fig. 4), i.e., by waterplanes. The areas S

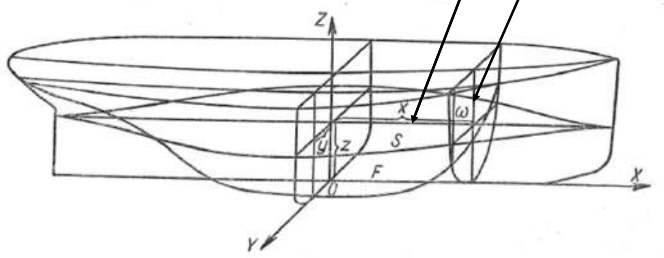

Fig. 4

- S is the waterplane area
- ω is the cross-sectional area
- Draw this isometric view of the boat in your notebook. Use the ISO coordinate system.

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1. Integrate over Water-Plane Areas

- y is the half-breadth.
- That is why Eq. (6.2) has a 2.

of waterplanes bounded by the closed lines of the ship will be called the waterplane areas. The expression for the volume V may obviously be written in the form of the following definite integral

$$V=\int_0^T S\,dz, \tag{6.1}$$

where T is the draught.

The waterplane area S may be represented also by the definite integral

$$S=2\int_{-\frac{L}{2}}^{+\frac{L}{2}} y\,dx, \tag{6.2}$$

where L is the length, $y=y\,(x)_{\text{at }z=const}$ is the equation of the waterline.

Substituting (6.2) in (6.1) we obtain the volume in the form of the double integral

$$V=2\int_0^T\int_{x_0(z)}^{x(z)} y\,dx\,dz, \tag{6.3}$$

where $x\,(z)$ and $x_0\,(z)$ are the abscissas of the extreme points in the waterline length.

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2. Integrate over Cross Sectional Area

- Compare the numerical values obtained using the two methods
- For the Wigley hull In L08S18, the uw vol for draft = 10 m is 8888.9 m^3 by using (a) analytical integration (b) waterplane areas (c) cross-sectional areas. Simpson's 1/3 rule is used.

Sec. 6 *Displacement and Centre of Buoyancy* 23

Let the ship be cut by transverse planes parallel to the plane YOZ (Fig. 4), i.e., by section planes. The areas ω of these planes located within the closed lines of the ship will be called the sectional areas. Then, obviously, the volume V may be represented in the alternative form

ω is found using Bonjeans

$$V=\int_{-\frac{L}{2}}^{+\frac{L}{2}} \omega\,dx. \tag{6.4}$$

The sectional area may be represented by the following definite integral

$$\omega=2\int_0^T y\,dz, \tag{6.5}$$

where $y=y\,(z)_{\text{at }x=const}$ is the equation of the section.

Substituting (6.5) in (6.4) we obtain

$$V=2\int_{-\frac{L}{2}}^{+\frac{L}{2}}\int_{z_0(x)}^{T} y\,dz\,dx, \tag{6.6}$$

where $z_0\,(x)$ is the height of the point of intersection of the section contour and the diametral plane.

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KB using Bonjeans
Cross Sectional Area(x,z)
Barrass & Derrett

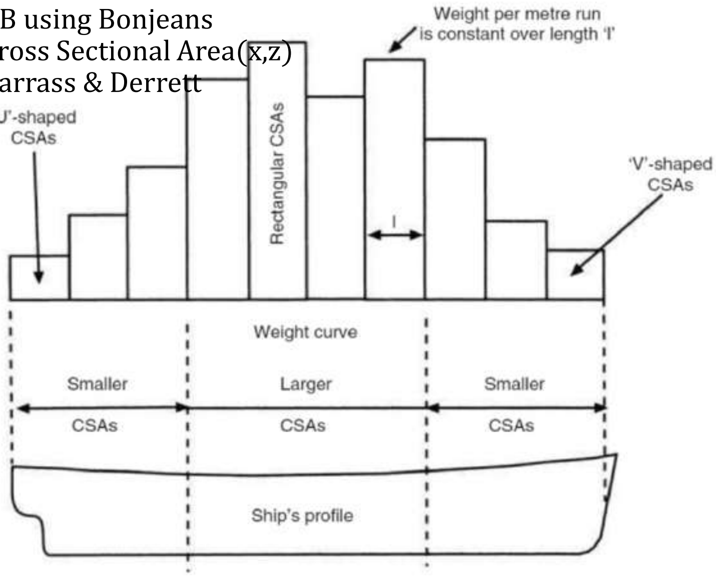

Fig. 50.10 This figure shows the ship divided into 10 elemental strips along its length LOA. In practice the Naval Architect may split the ship into 40 elemental strips in order to obtain greater accuracy of prediction for the weight distribution.

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NPOL's Steel Boat

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NPOL's Steel Boat

Home Work

See the photo and sketch the lines plan

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See L02S22

- Force acts at (x, y, z)
- Moment is a vector. It has 3 components. It is about a point. $\vec{r}$ is the vector from the point to the point of application of the force.
- The components of the moment are along the $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$ directions
- The moment about the x axis is also the component of the moment along the x axis.
- Using vector notation or by using the direction in which a screw will travel when F_z acts at y, yF_z is along the $+x$ axis and zF_y is along the $-x$ axis.

distance is z. Using the right-hand-screw rule for ascertaining the sense of each of the moments, we can say:

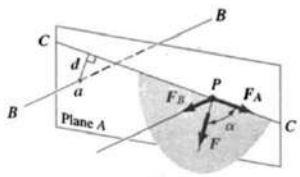
$$\text{moment about } x \text{ axis} = (yF_z - zF_y) \quad (3.10)$$

Were we to take moments of $\boldsymbol{F}$ about the origin O, we would get (see Eq. 3.8)

$$\boldsymbol{M} = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k} = \boldsymbol{r} \times \boldsymbol{F}$$
$$= (yF_z - zF_y)\boldsymbol{i} + (zF_x - xF_z)\boldsymbol{j} + (xF_y - yF_x)\boldsymbol{k} \quad (3.11)$$

Cross Product

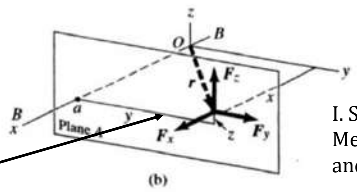
$$\begin{matrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{matrix}$$



(a)

F_A is ⊥ to the B axis

F_B is parallel to the B axis



$y \times F_z$ is a moment along the $+x$ axis

(b)

Figure 3.16. Moment about an axis.

I. Shames. Engineering Mechanics - Statics and Dynamics. 4th Ed.

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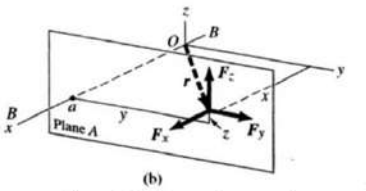
29

Center of Buoyancy

- Semyono. CoB has three coordinates. Find them. For CoB, all the "forces" are along the –z direction in Semyonov's axes. CoB is the centroid of the underwater volume.
- Force is along –z and the lever arm is x. See fig. When the moment acts, the screw will advance along the $+y$ axis. Semyonov calls it M_{yz} = the moment about the yz plane. x is the normal distance between the point of application of the force and the yz plane.
- It is better to use vector notation



(b)

Figure 3.16. Moment about an axis.

In order to find the co-ordinates of the centre of buoyancy x_c, y_c and z_c it is necessary to set up the expressions of the static moments of the volume V with respect to the co-ordinate planes YOZ, XOZ and XOY. Denoting the variable co-ordinates of the section, buttock and waterline planes by x, y and z respectively (Fig. 4), we obtain

x is the distance between ω and the yz plane

$$M_{yz} = \int_{-\frac{L}{2}}^{+\frac{L}{2}} \omega x \, dx; \qquad (6.11)$$

$$M_{xy} = \int_0^T S z \, dz. \qquad (6.12)$$

By symmetry of the lines of the ship with respect to the plane XOZ

$$M_{xz} = 0.$$

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30

Center of Buoyancy

- M_{yz} is the moment about the yz plane. $V = \int_0^T S dz$ is the underwater volume.
- The distributed buoyancy force is equivalent to a concentrated force and a moment about a point. About the CoB, the moment of the distributed buoyancy force is zero.

Sec. 6 *Displacement and Centre of Buoyancy* 25

The co-ordinates of the centre of buoyancy are

$$x_c = \frac{M_{yz}}{V} = \frac{\int_{-\frac{L}{2}}^{+\frac{L}{2}} \omega x \, dx}{\int_{-\frac{L}{2}}^{+\frac{L}{2}} \omega \, dx}; \qquad (6.13)$$

$$z_c = \frac{M_{xy}}{V} = \frac{\int_0^T S z \, dz}{\int_0^T S \, dz}; \qquad (6.14)$$

$$y_c = 0.$$

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$\overline{BM} = \mathrm{I/V}$

- For transverse stability, find $\overline{BM}$ using I about the centreline
- $y(x)$ is the half-breadth. η = eta
- $I = 2\int_0^L \int_0^{y(x)} \eta^2 d\eta \, \mathrm{d}x$
- $I = (2/3)\int_0^L y(x)^3 \mathrm{d}x$
- Draw a waterplane area and show that the above expression for I is correct.
- $V = \int_0^T S\mathrm{d}z$ is the underwater volume.

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2.1 $\overline{GM_0}$

- In the previous class we found KG
- $\overline{GM_0}$ = transverse GM at even keel is of interest
- G,B_0, and and M_0 are on the centreline
- $\overline{GM_0} = \overline{KM_0} - \overline{KG}$
- $\quad = \overline{KB_0} + \overline{B_0M_0} - \overline{KG}$
- $\overline{BM}$ =I/V. To find I see L08S31
- We have found V, $\overline{KB_0}$, and $\overline{B_0M_0}$ in this class.

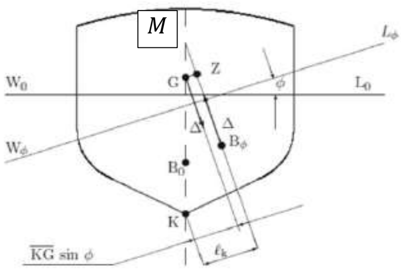

Figure 5.1 Definition of righting arm

Therefore, we know $\overline{GM_0}$.

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CoB with trim

- Semyono. The traces of a surface (hull) are the cross-sections created when the surface intersects a plane parallel to one of the coordinate planes.

15. BONJEAN SCALE **See Slide #11. Fig. 18 is on the next slide.**

A set of integral curves $\omega(z)$ for sections represents the so-called Bonjean scale. Fig. 18 illustrates the Bonjean scale in the most widely used form. To a horizontal straight line representing the trace of the base plane perpendiculars are drawn representing the traces of cross-sections. At each perpendicular, as at an axis, the curve of total area of a given section is plotted which differs from the curve of ω, described in Sec. 14, only in that its ordinates represent the total sectional areas rather than the half-areas. The ordinates of these curves are expressed as

Read the description of the Bonjeans to see if the half or full sectional area is shown.

$$\omega = 2\int_0^z y\,dz. \qquad (15.1)$$

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34

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KB using Bonjeans. Trim.

- HW. A barge with a uniform rectangular cross section has L = 10 m and B = 4 m. The displacement in fresh water is 40 t. (a) Draw the Bonjeans. (b) The trim is 0.1+n/1000 m where n = last two digits of your registration number. Find the fwd and aft drafts. (c) Use the calculated drafts. Find the CoB.

52 *Buoyancy* *Ch. II*

From the Bonjean scale it is a simple matter to calculate the displacement V and the abscissa of the centre of buoyancy x_c for a ship floating with trim. In this case the position of the waterline in relation to the ship is determined by the draught forward and the

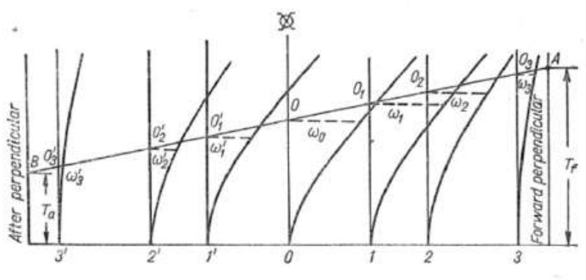

Fig. 18

draught aft. On the plan of the Bonjean scale the forward and after perpendiculars are drawn in conformity with the scale adopted for the length of the ship. The draught forward T_f and the draught aft T_a are laid off on these perpendiculars to the scale adopted for draught (Fig. 18). Joining the points so derived by a straight

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35

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KB using Bonjeans

- Draft at Section 9
- Draft Aft

line we obtain the trace of the waterline on the diametral plane. The points of intersection of this trace and the perpendiculars corresponding to the positions of sections give the immersion of each section. The ordinates of the curves of ω drawn from these points give the areas of the immersed portions of sections to the scale adopted. After the ordinates have been taken the displacement and the abscissa of the centre of buoyancy can be calculated by formulas (6.4) and (6.13).

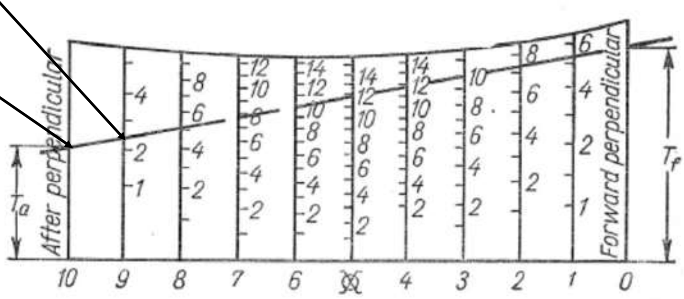

Fig. 19

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36

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Biran. Bonjeans and CoB

- It is good to read many books on the same subject

4.2 The Calculation of Hydrostatic Data

4.2.1 Waterline Properties

In this section, we refer to Figure 4.1 and assume that all waterlines are symmetric about the centreline. This assumption is true for almost all ships in upright condition.

We calculate the **waterplane area**, of a given waterline, as

$$A_W = 2\int_a^b y dx \approx 2\left(\sum_{i=n_1}^{n_n} \alpha_i y_i\right)\delta L \qquad (4.1)$$

where the waterline begins at station n_1, with $x = a$, and ends at station n_n, with $x = b$.

The **moment of the waterplane area about a transverse axis** passing through the origin of coordinates is

$$M_x = 2\int_a^b xy dx \approx 2\left(\sum_{i=n_1}^{n_n} \alpha_i x_i y_i\right)\delta L = 2\left(\sum_{i=n_1}^{n_n} \alpha_i j_i y_i\right)\delta L^2 \qquad (4.2)$$

Leaving the indexes n_1, n_n we write the x-**coordinate of the centre of flotation** of the given line as

$$x_F = \frac{M_x}{A_W} = \frac{2\left(\sum \alpha_i j_i y_i\right)\delta L^2}{2\left(\sum \alpha_i y_i\right)\delta L} = \frac{\left(\sum \alpha_i j_i y_i\right)}{\left(\sum \alpha_i y_i\right)}\delta L \qquad (4.3)$$

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37

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Biran. Bonjeans and CoB

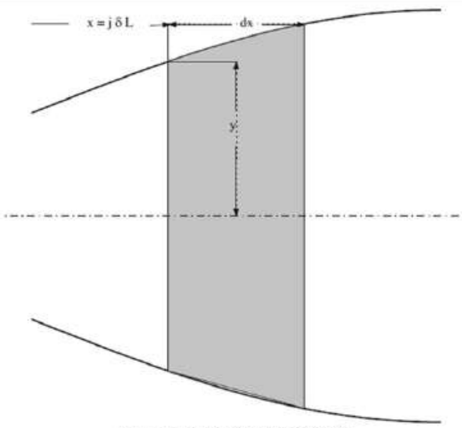

Figure 4.1 An element of waterline area

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38

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The notation x_F corresponds to the DIN 81209 standard. The notation used in English-language texts is **LCF**, an acronym for **longitudinal centre of flotation**. The corresponding curve is shown in Figure 4.2. To calculate the transverse **moment of inertia of the waterplane area**, i.e., the moment of inertia about the centreline, we first write the moment of inertia of the elemental area shown in grey in Figure 4.1:

$$dI_T = \frac{(2y)^3 dx}{12} = \frac{2}{3} y^3 dx \tag{4.4}$$

Then, the moment of inertia of the whole waterplane equals

$$I_T = \int_a^b \frac{2}{3} y^3 dx \approx \frac{2}{3} \left(\sum_{i=n_1}^{n_n} \alpha_i y_i^3 \right) \delta L \tag{4.5}$$

The **moment of inertia of the waterplane area about a transverse axis** passing through the origin of coordinates is calculated as

$$I_y = 2 \int_a^b x^2 y dx \approx 2 \left(\sum_{i=n_1}^{n_n} \alpha_i x_i^2 y_i \right) \delta L = 2 \left(\sum_{i=n_1}^{n_n} \alpha_i j_i^2 y_i \right) \delta L^3 \tag{4.6}$$

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39

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Korea Very Large Crude Carrier 2. LBP = 320 m. Benchmark Model. Offset Table is Public.

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40

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Korea Very Large Crude Carrier 2

		Ship	Model
Scale ratio		1.0	1/58
Design speed	$V[m/s]$	7.9739	1.047
Froude number	Fn	0.142	
Reynolds number	Rn	2.1×10^9	4.6×10^6
Length between perpendiculars	$Lpp[m]$	320	5.5172
Breadth	$B[m]$	58	1.0
Depth	$D[m]$	30	0.5172
Draft	$T[m]$	20.8	0.3586
WSA w/o rudder	$S_W[m^2]$	27,194	8.0838
WSA of rudder	$S_R[m^2]$	273.3	0.0812
Displacement	$\nabla[m^3]$	312,622	1.6023
Block Coeff.	C_B	0.8098	
Longitudinal C_B from midship	$LCB[m]$	0.0348Lpp	
Vertical CG from keel	$KG[m]$	18.6	0.3207
Metacentric height	$GM[m]$	5.71	0.0984
Pitch radius of gyration	K_{yy}	0.25Lpp	

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41

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2.3 Wall-Sided Ships

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42

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2.3 GZ at small angles of inclination. Wall-sided Ships.

- Biran. Example 2.5 (A parallelepipedic barge).

- Let us consider a parallelepipedic barge; it has a constant, rectangular transverse section as shown in Figure 2.17. (Next slide). L = length. B = breadth. H = depth. T = draught.
- For this simple body form we can calculate analytically the positions of the center of buoyancy and of the metacenter. We shall do this in two ways:
  - Starting from known principles of mechanics and elementary results of differential geometry;
  - Using the theorems developed in this chapter.

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43

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2.3 GZ at small angles of inclination. Wall-sided Ships.

- Transverse Center of Buoyancy (TCB) of barge as a function of heel angle, ϕ.

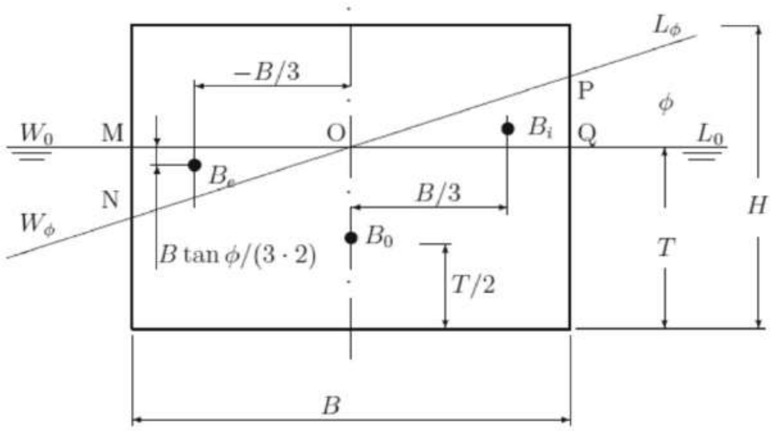

Figure 2.17 A barge with simple geometrical form

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44

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2.3 GZ at small angles of inclination. Wall-sided Ships.

- TCB. Note that M_ϕ is not on the center-line. It is the transverse metacentre when the ship has heeled by ϕ . Find it's coordinates.

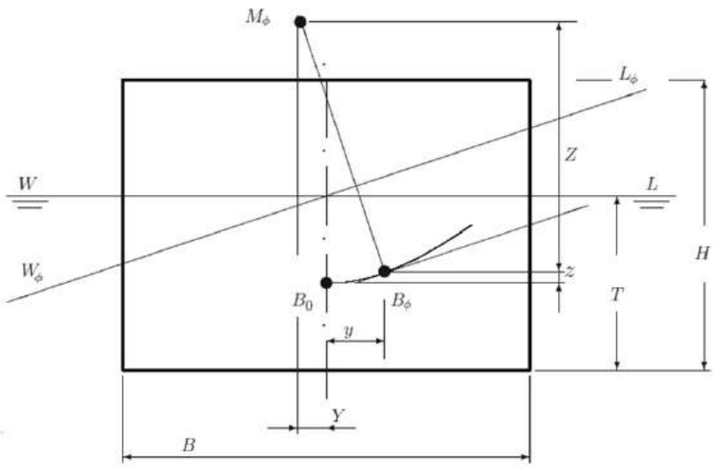

Figure 2.18 Centre of buoyancy and metacentre of simple barge

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45

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2.3 GZ at small angles of inclination. Wall-sided Ships.

- Barge. TCB as a function of heel angle.

Table 2.2 Calculating the transverse centre of buoyancy of the heeled barge

Solid	Volume	tcb	Moment
1	2	3	4 = 2 x 3
Initial	LBT	0	0
Submerged wedge	$LB^2 \tan\phi/8$	$2B/(3 \cdot 2)$	$LB^3 \tan\phi/(3 \cdot 8)$
Emerged wedge	$-LB^2 \tan\phi/8$	$-2B/(3 \cdot 2)$	$LB^3 \tan\phi(3 \cdot 8)$
Total	LBT	$B^2 \tan\phi/(12T)$	$LB^3 \tan\phi/12$

- The origin of the coordinate system is on the Center Line of the barge.
- The original CoG, the CoB, and the axis of floatation of the barge are on the Center Line
- Moment is taken about xOz plane to find yCoB = new TCB
- Last row. Final TCB = Moment/Vol

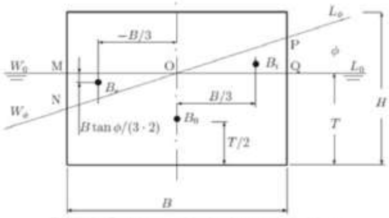

Figure 2.17 A barge with simple geometrical form

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46

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2.3 GZ at small angles of inclination. Wall-sided Ships.

- Barge. TCB. See Table 2.2 in prev slide

This is the place to stop for a short digression on this *tabular form of calculations*. Let us refer to Table 2.2. Column 2 contains the volumes of the initial hull, of the submerged wedge and of the emerged wedge. Column 3 contains the y-coordinates of the volumes entered in column 2. As said, these coordinates are measured from the centreline plane; we call them tcb, an acronym for *transverse centre of buoyancy*. We use lower-case letters and reserve the upper-case notation, TCB, for the y-coordinate of the whole body. Column 4 contains the moments of the initial body and of the wedges, about the centreline plane. These moments are calculated as products of the terms in column 2, by those in column 3. The procedure is described symbolically by the expression $4 = 2 \times 3$ written in the subheading of column 4.

The sum of the terms in column 2 equals the total volume of the heeled barge; it is written in the cell identified by the entries *Volume* and *Total*. Similarly, the sum of the partial moments in column 4 is the moment of the heeled barge about the centreline plane; it appears in the cell corresponding to the entries *Moment* and *Total*. Dividing the moment of the heeled barge by its volume yields the y-coordinate of the heeled barge:

$$TCB = \frac{LB^3 \tan\phi/12}{LBT} = B^2 \tan\phi/(12T)$$

This result is written in the cell identified by the entries *tcb* and *Total*.

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47

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2.3 GZ at small angles of inclination. Wall-sided Ships.

- Barge. Vertical Center of Buoyancy (VCB). Original VCB

Table 2.3 Calculating the vertical centre of buoyancy of the heeled barge

Solid	Volume	vcb	Moment Change
1	2	3	4 = 2 x 3
Initial	LBT	0	0
Submerged wedge	$LB^2 \tan\phi/8$	$B\tan\phi/(3\cdot 2)$	$LB^3 \tan^2\phi/(8\cdot 3\cdot 2)$
Emerged wedge	$-LB^2 \tan\phi/8$	$-B\tan\phi/(3\cdot 2)$	$LB^3 \tan^2\phi/(8\cdot 3\cdot 2)$
Total	LBT	$B^2 \tan^2\phi/(24T)$	$LB^3 \tan^2\phi/(3\cdot 8)$

Tables 2.2 and 2.3 yield the **parametric equations** of the curve of centres of buoyancy:

TCB = y
VCB = z = change in VCB
In Eq. 2.66, they are used as coords of a curve

$$y = \frac{1}{12}\cdot\frac{B^2}{T}\tan\phi$$
$$z = \frac{1}{24}\cdot\frac{B^2}{T}\tan^2\phi \qquad (2.65)$$

The x axis is the axis of floatation. Vertical moments are taken about the xOy plane.

We call the curve of centres of buoyancy **B curve**. From Eqs. (2.65) we can derive

Square 2.65a and substitute in 2.65b to get 2.66

$$\cancel{z = \frac{6T}{B^2}x^2} \qquad z = \frac{6T}{B^2}y^2 \qquad (2.66)$$

This is the equation of a parabola.

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48

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2.3 GZ at small angles of inclination. Wall-sided Ships.

- See the previous slide
- To find the change in the VCB, zCoB, it is necessary to take moment about any horizontal plane such as xOy.
- In this case, moment is taken about a horizontal plane that contains the Center of Floatation
- To find the actual VCG take moments about the origin of the coordinate system
- Before heeling, the VCB is at T/2. After heeling, it is at T/2 + zCoB.

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49

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2.3 GZ at small angles of inclination. Wall-sided Ships.

- Biran. Barge. The slope of the CoB curve dz/dy is $\tan(\phi)$. $\phi =$ heel angle.

The slope of the curves of centres of buoyancy is given by

$z = \frac{6T}{B^2}y^2$;

$$\frac{dz}{dy} = \frac{dz/d\phi}{dy/d\phi} = \tan\phi \qquad (2.67)$$

Use the parametric eqs. 2.65 in Slide #11 to differentiate z wrt y

where $\frac{dz}{dy} = \frac{12T}{B^2}y$; does not depend on the origin. In y=mx+c, c depends on the origin but m does not

$$\frac{dy}{d\phi} = \frac{B^2}{12T}\cdot\frac{1}{\cos^2\phi}\cancel{d\phi} \qquad (2.68)$$

and

$$\frac{dz}{d\phi} = \frac{B^2}{12T}\cdot\frac{\tan\phi}{\cos^2\phi}\cancel{d\phi} \qquad (2.69)$$

Equation (2.67) shows that the tangent in B_ϕ has the slope ϕ, meaning that it is parallel to the corresponding waterline.

To find the radius of curvature of the B curve we calculate

$$\frac{d^2z}{dy^2} = \frac{1}{\cos^2\phi}\frac{d\phi}{dy} = \frac{12T}{B^2} \qquad (2.70)$$

and use a formula that can be found in many books on calculus or classic differential geometry (see, for example, Stoker, 1969, p. 26; Taillé, 1975, p. 73; Gray, 1993, p. 11):

$$R = \frac{(1 + (dz/dy)^2)^{3/2}}{d^2z/dy^2} = \frac{B^2}{12T}\cdot\frac{1}{\cos^3\phi}$$

$R = \overline{BM}$

- $(1 + y'^2)^{\ /2} = [1 + \tan^2(\phi)]^{3/2} = [\sec^2(\phi)]^{3/2} = \sec^3(\phi)$

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50

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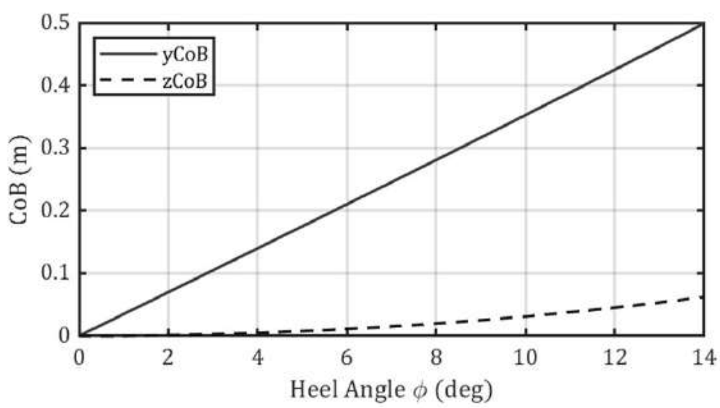
2.3 GZ at small angles of inclination. Wall-sided Ships.

```
• phi_deg = 0:14; Breadth = 6; Draft = 1.5;
• yCoB = Breadth^2 * tand(phi_deg)/(12*Draft);
• zCoB = Breadth^2 * tand(phi_deg).^2 / (24*Draft);
```



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51

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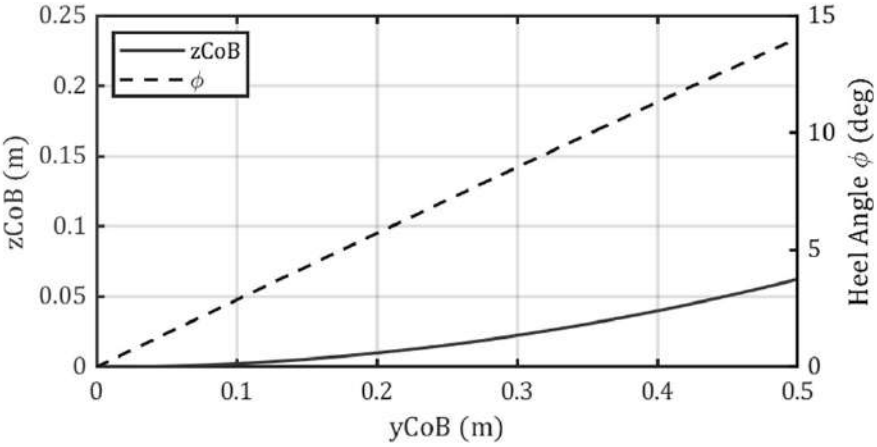
2.3 GZ at small angles of inclination. Wall-sided Ships.

- `yyaxis left; plot(yCoB,zCoB,'r-','linewidth',1.5)`
- `yyaxis right; plot(yCoB,phi_deg,'b--','linewidth',1.5)`



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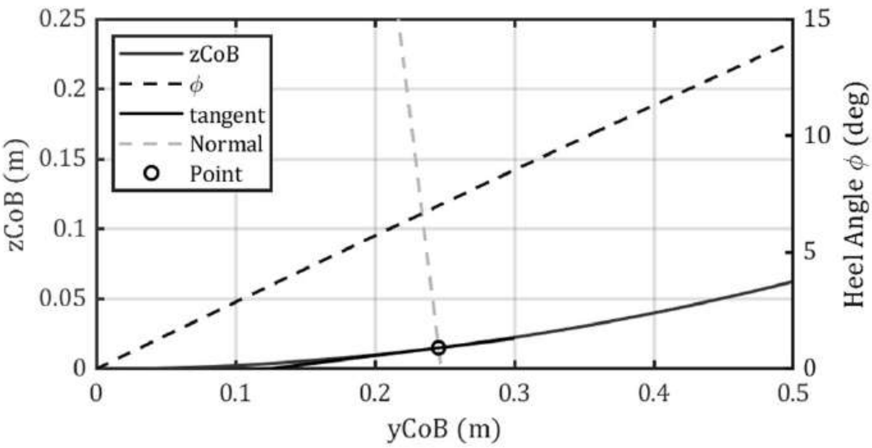
Assignment

- CoB, BM=Rad of curvature, and the slope of the BM line are known. Find the coordinates of the metacenter, M. The angle between the normal to the CoB curve and the vertical is also ϕ. Draw a right angle triangle with B and M at the vertices. Use simple trigonometry to find the coords of M. What is the horizontal distance between M and the centerline?



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1

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03 Jan 2025

Stability of Ships

B. Tech. NA&SB. 2023-27. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

Dr. D. D. Ebenezer
Adjunct Faculty
9446577239
ebenezer.cusat@gmail.com

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Dr. D. D. Ebenezer

2

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
   Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
   Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
   Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
   Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
   Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
   Intact stability rules, Heel/ Load test.
   Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

73

removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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MODULE 2. Initial Transverse Stability

Completed

Module 1. 06 lectures.

Multiple choice Google forms test on 08 Jan 2025.

Module 2

2.1 $\overline{GM_0}$

2.2 CoB

2.3 Wall-sided ship

Today

2.4 Shift of weight. Effect on $\overline{GM_0}$.

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5

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2.4 Effect of shift of weight on the $\overline{KG}$, $\overline{GM_0}$, and stability. Coefficient of Stability

- Semyonov's $h_0 = \overline{GM_0}$ = initial metacentric height. It lies within a narrow range for ships of various types and sizes. The righting lever arm is $\overline{GM_0}\ \sin\phi$.

As a measure of stability it is possible also to take the product of the weight of the ship and the transverse metacentric height, Dh_0, which is called *the coefficient of stability*. The metacentric height h_0 is a generally accepted measure of stability. It is convenient as its values encountered in ships of widely different types lie within a relatively narrow range and depend slightly on the dimensions of the ship. On the other hand, the metacentric height does not determine the ability of the ship to resist the moments of external forces applied to her and in this respect is a measure of stability in the particular sense rather than a measure of stability in the broad sense. The stability of the ship is determined by the magnitude of the righting moment which is proportional to the coefficient of stability at small inclinations. Hence the product Dh_0 gives a more complete idea of the actual properties of the ship but its values for various ships lie within a very wide range.

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6

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- Fig. 34 is on the next slide.

Since the ship is inclined at an infinitesimal angle $d\theta$ the lever $\overline{GK}$ is also an infinitesimal quantity which will be denoted by dl. With reference to Fig. 34 we can write the expression for the lever

$$dl = \overline{MG}\, d\theta.$$

The inclination through an infinitesimal angle can bring into action only an infinitesimal righting moment

$$dM_r = D\, dl = D\overline{MG}\, d\theta.$$

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7

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Notation and Coefficient of Stability

The distance $\overline{MG}$ represents the height of the transverse metacentre M above the centre of gravity G and is termed *the transverse* or *small metacentric height*. Denote it by h_0, the height of the centre of gravity G above the centre of buoyancy C_0 by a and the height of

$h_0 = \overline{GM}$
$C_0 = \overline{CoB}$
$a = \overline{GB}$
$\overline{GK} = \overline{GM}\sin\theta$
$\sin\theta \cong \theta, \theta \cong 0$
$\sin\theta \cong \theta - \frac{\theta^3}{3!}, \theta \ll 1$

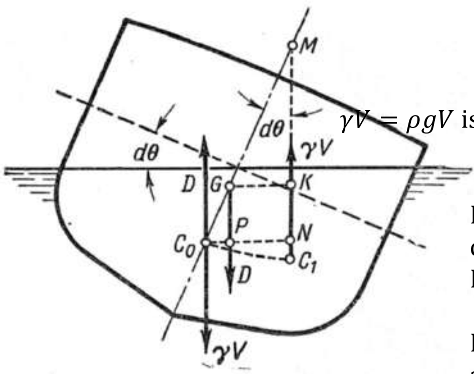

Fig. 34

$\gamma V = \rho g V$ is the buoyancy force

Different books use different notations. Be alert.

In your docs, define all symbols.

the transverse metacentre M above the base plane by z_m. We can

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8

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Sec. 24 *Metacentric Formulas of Stability* 89

Notation and Coefficient of Stability

The formula for dM_r can now be written as

$$dM_r = Dh_0\,d\theta. \qquad (24.5)$$

This is *the metacentric formula of transverse stability* represented in differential form. This formula has been derived for an infinitesimal inclination but we shall apply it for finite though small angles. So we write it in the alternative form

$$M_r = Dh_0\theta. \qquad (24.6)$$

Here the angle θ is expressed in radians. In most cases this formula may be applied for angles of heel not exceeding 15-20° without fear of obtaining insufficiently accurate results.

The metacentric formula (24.6) may be represented in still another form. Referring to Fig. 34 and assuming that the angle of heel is θ, we can write the following expression for the righting lever:

$$\overline{GK} = l = h_0 \sin\theta$$

and the metacentric formula is then

$$\boxed{M_r = Dl = Dh_0 \sin\theta.} \qquad (24.7)$$

For angles of heel not exceeding 15-20° we may put

$$\sin\theta \approx \theta$$

and formula (24.7) is converted into formula (24.6).

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9

Shifting Weights. General Case.

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- Semyonov

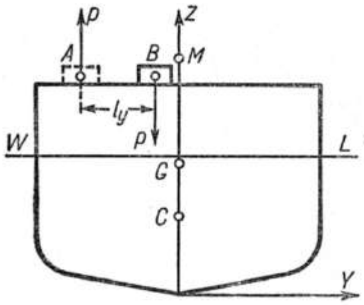

Fig. 39

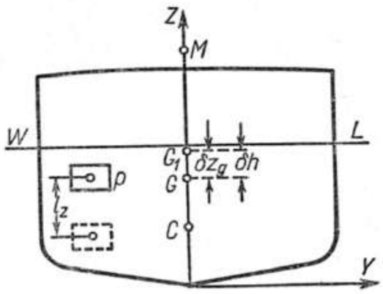

Fig. 40

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10

2.4 Shifting Weights. General Case.

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30. ALTERATION IN POSITION AND STABILITY DUE TO A TRANSFERENCE OF WEIGHT

A weight p which will be considered as small has been transferred on a ship. The weight has been transferred so that its centre of gravity has moved from a point with the co-ordinates x_0, y_0 and z_0 to a point with the co-ordinates x_1, y_1 and z_1. It is apparent that there is no change in weight of the ship due to the transference of weight.

The shift of the centre of gravity of the weight from the point (x_0, y_0, z_0) to the point (x_1, y_1, z_1) may be resolved into three mutually perpendicular shifts parallel to the three co-ordinate axes OX, OY and OZ. Thus we have

longitudinal shift of weight

$$l_x = x_1 - x_0;$$

Semyonov

transverse shift of weight

$$l_y = y_1 - y_0;$$

vertical shift of weight

$$l_z = z_1 - z_0.$$

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11

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Consider first separately the effect of each of the three shifts of the weight on the position and stability of the ship. Here and henceforth a reference to a change in position of the ship will imply a set of changes in draught, heel and trim. position = attitude

The shift of the weight p in the athwartship direction through the distance l_y may be thought of as the removal of the weight p from the point A (Fig. 39) and the addition of the weight p at the point B. The addition and removal of the weights may be replaced by two equal and opposite forces p and p acting vertically. These

forces form a couple whose moment heels the ship and is expressed as

$$M_h = pl_y.$$

The heeling moment will be balanced by the righting moment $M_h = M_r$, with M_r expressed by the metacentric formula of stability (24.7)

$$pl_y = Dh_0 \sin(\theta)$$

$pl_y = D\,GZ = D\,GM \sin(\theta)$

Solving this equation for θ, we obtain

Use an exact equation if you can. Avoid approximations.

$$\sin(\theta) = \frac{pl_y}{Dh_0} = \frac{pl_y}{D\overline{GM}} \tag{30.1}$$

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12

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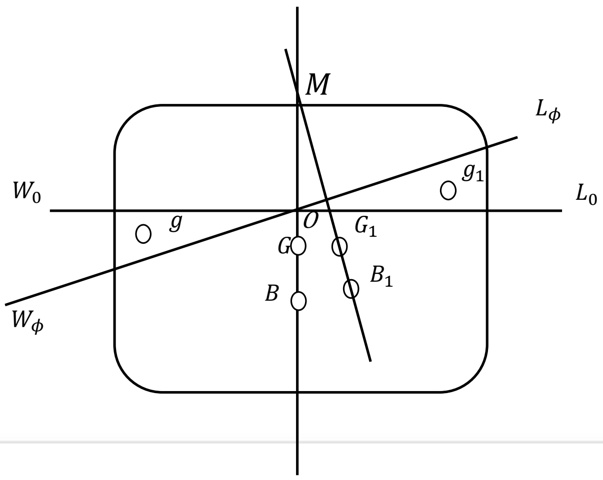
Example. Shift of Weight. Find the CoG and heel angle.

- A mass on-board is moved transversely. Find the heel angle. Angle $BMB_1 = \phi$.



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13

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Example. Shift of Weight. Find the CoG and heel angle.

- A barge with a rectangular cross-section has a displacement of 40 tons in fresh water and is on even keel. The zCoB = 0.5 m = $\overline{KB}$. $\overline{KG}$ = 0.7 m. The L/B = 2.5. One ton is moved to starboard by 1 m = δy. Find the new CoG and the heel angle. See the previous slide.
- Let m_1 = 39 t and m_2 = 1 t. It is not necessary to know the CoG of the 1 t mass. Only the change in the CoG is needed. yCoG after shifting = $m_2 \delta y/\ (m_1 + m_2)$ = 1/40 m = 2.5 cm. There is no change in the xCoG and zCoG.
- $\overline{KB}$ = 0.5 m. So, T= 1 m. Displacement = 40 tons. So, LBT = 40. L/B = 2.5. So, $2.5B^2T = 40$. B^2 = 16. B = 4 m. For heeling, $I = LB^3/12 = 10\ 4^3/12$ = 53.33 m^4. UW Vol = 40 m^3. $\overline{BM} = I/V$ = 53.33/40 = 1.333 m.
- Heeling moment = 1 t x 1 m = 1 tm.
- $\overline{KM} = \overline{KB} + \overline{BM}$ = 0.5 + 1.3333 = 1.83 m = $\overline{KG} + \overline{GM}$ = 0.7 + $\overline{GM}$. So, $\overline{GM}$ = 1.1333 m. Heel angle = ϕ. Righting Moment = Disp × $\overline{GM}$ sin ϕ.
- Heeling moment = Righting Moment. 1 tm = 40 × 1.1333 × sin ϕ. sin ϕ = 0.0221. ϕ = 0.0221 rad = 1.27 deg.
- Method 2. $\overline{GG_1}$ is parallel to the line joining the initial and final positions of the moved weight. It is parallel to the Y axis. G_1 lies on the line joining B and M. So, tan $\phi = \overline{GG_1}\ /\overline{GM}$ = 0.025/1.1333. ϕ = 0.0221 rad = 1.27 deg. See L04S31.

Dec24-Apr25

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14

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Example. Shift of Weight. Find the new CoB and $\overline{BM}$.

- Using the formulas for a wall-sided ship, the change in the CoB is …
- See L08S45. yCoB = $B^2 \tan\phi/(12T)$ = 16 tan ϕ /12 = 2.95 cm
- See L08S47. zCoB = $B^2 \tan^2\phi/(24T)$ = 0.33 mm.
- What is the new $\overline{BM}$?
- $I_\phi = LB^3/(12\cos^3\phi)$ = 53.3725 m^4.
- $\overline{BM}_\phi = I_\phi$/V = 53.3725/40 = 1.3343 m
- Home Work. Use the coordinates of the new CoG and CoB. Show that $\overline{GB}$ is perpendicular to the waterline.

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15

Shifting Weights. General Case.

Following the same line of reasoning, we can write for the shift in the fore-and-aft direction $\sin(\psi) = \frac{p l_x}{D\ GM_L}$ (30.2)

Thus the shifts in the longitudinal and transverse directions cause a change in position of the ship leaving her stability unchanged. The shift in the vertical direction, as we shall see below,

Semyonov says that the stability is not changed. See the next slide.

Position = attitude

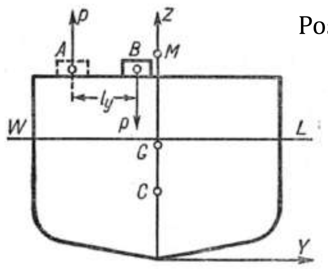

Fig. 39

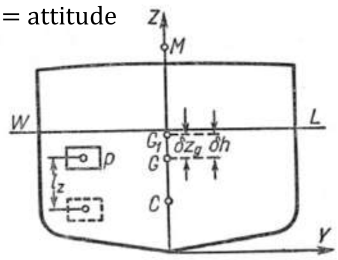

Fig. 40

causes a change only in stability leaving the position of the ship unchanged.

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16

Summary for lateral shift of weight

- Semyonov. See the previous slide.
- "Thus the shifts in the longitudinal and transverse directions cause a change in the position of the ship leaving her stability unchanged."
  - "position" = attitude. Heel or trim or both will change
  - "stability unchanged" => coefficient of stability, $D\ \overline{GM}$, is approximately unchanged. Actually, D does not change, zCoG does not change, but $\overline{BM}$ increases by a small factor $\frac{1}{\cos^3 \phi}$ (see L09S14)
- Consider a barge with a uniform rectangular cross-section. Assume a mass shifts in the transverse direction. The ship heels. G, B, and $\overline{BM}$ change. Find the new $\overline{GM}$ in the heeled condition. Do not use any approximations if they can be avoided easily.

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Vertical shift of Weights

- See the next slide for the definitions of symbols

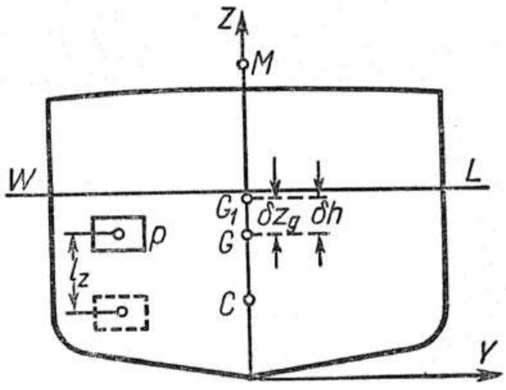

Fig. 40

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18

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The shift of the weight p in the vertical direction through the distance l_z (Fig. 40) leads to the shift of the centre of gravity of the ship, according to the theorem of moments to which reference was made in Sec. 20, through the distance

No change in $\overline{KB}$ and $\overline{BM}$. $\delta z_g = \frac{p}{D} l_z$. No change in D

The position of the metacentre is not affected by the transference of the weight. Hence on the basis of formula (24.2) we may write

δKM=0. So,

$$\delta GM = -\delta KG = \delta h = -\delta z_g = -\frac{p}{D} l_z. \quad (30.3)$$

No change in M

On the basis of formula (24.12) it is possible to obtain a similar expression for the change in longitudinal metacentric height

$$\delta GM_L = -\delta KG = \delta H = -\delta z_g = -\frac{p}{D} l_z.$$

GM decreases if the body moves up.

Thus the changes in transverse and longitudinal metacentric heights due to the transference of the weight p in the vertical direction through the distance l_z are the same. In ships with the common ratio of length to breadth, however, δH is negligibly small as compared with H_0 and may therefore be disregarded.

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19

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2.4 Shifting Weights. General Case.

If all the 3 coordinates of a weight are changed, first find the new KG and GM. Then, find the trim and heel.

If the transference of a weight is effected in the three directions, it is necessary first to calculate δh by formula (30.3), then to determine

$$h_1 = h_0 + \delta h$$

and, substituting this in formula (30.1) for h_0, to calculate θ. The angle of trim ψ can be calculated directly from formula (30.2).

Using relation (26.4), it is possible to obtain from (30.2) the formula for calculating the trim ΔT in the form

See S15 for Eq. (30.2)

$\sin(\psi) = \frac{\Delta T}{L}$

$$\Delta T = \frac{p}{D} \frac{L}{H_0} l_x. \quad (30.4)$$

where $\boldsymbol{H_0}$ is the longitudinal metacentric height.

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20

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2.4 Shifting Weights. General Case.

- The ship will heel and trim about the center of floatation, CoF $= (x_f, y_f)$.

The changes in draughts at stem and stern may be obtained on the basis of Euler's theorem (see Sec. 19), i. e., by drawing the inclined waterline at the angle of trim ψ through the point F, the centroid of the upright waterplane (Fig. 41). Referring to Fig. 41 and denoting the draughts forward and aft by T_f and T_a, we may write

- Distance between the forward perpendicular and the CoF is $L/2 - x_f$

$$\delta T_f = \left(\frac{L}{2} - x_f\right)\psi; \quad (30.5)$$

$$\delta T_a = -\left(\frac{L}{2} + x_f\right)\psi. \quad (30.6)$$

In formulas (30.5) and (30.6) the positive values of δT_f and δT_a correspond to an increase in draughts T_f and T_a while the

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21

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- In Semyonov coord sys, the distance between the forward perpendicular and the CoF is $L/2 - x_f$
- In the ISO coordinate system, the distance between the FP and the CoF is FP $- x_f = L - x_f$.
- Numerically, the distances are the same

negative values correspond to their decrease. The trim ΔT may be obtained as

$$\Delta T = \delta T_f - \delta T_a. \quad (30.7)$$

The positive value of ΔT corresponds to a trim by the head and the negative value to a trim by the stern.

In conclusion it is to be noted that formula (30.3) which gives a reduction in stability ($\delta h < 0$) due to a transference of weight upward ($l_z > 0$) and an increase in stability ($\delta h > 0$) due to a

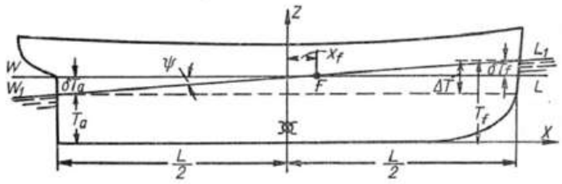

Fig. 41

transference of weight downward ($l_z < 0$) may be applied to a transference of any weight with no limitation on its amount whereas formulas (30.1) and (30.2) are valid only for small values of the heeling and trimming moments since they have been derived from the metacentric formulas of stability. Small values of the heeling and trimming moments may be obtained in this case by either transferring a small weight through a finite distance or transferring a finite weight through a small distance.

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22

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2.4 Shifting Weights. Summary

- A vertical shift of a weight changes the KG and the stability of the ship.
- Transverse or longitudinal shift of weight causes a heel or trim but no change in KG (stability).

Next Lecture

- The Inclining Experiment is very important and must be done with great care. An error can be very costly!
- Don't forget to download and read the Inclining Experiments documents.

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08 Jan 2025

Stability of Ships

B. Tech. NA&SB. 2023-27. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

Dr. D. D. Ebenezer
Adjunct Faculty
9446577239
ebenezer.cusat@gmail.com

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
   Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
   Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
   Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
   Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
   Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
   Intact stability rules, Heel/ Load test.
   Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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3

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

73

removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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4

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MODULE 2. Initial Transverse Stability

Completed

Module 1. 06 lectures.

Multiple choice Google forms test on 08 Jan 2025.

Module 2

2.1 $\overline{GM_0}$

2.2 CoB

2.3 Wall-sided ship

2.4 Shift of weight. Effect on $\overline{GM_0}$.

Today

2.5 The Inclining Test

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5

2.5 Inclining Test and Shifting weights

- Earlier, we studied equivolume displacements.
- A change in the attitude of a ship can occur because there is a constant external force acting on it.
- It can also happen because of a shift or movement of a weight that is already onboard.
- When a weight shifts, the displacement does not change. However, the CoG will change. As a result, the CoB will change (Stevin's Law).
- The angle of heel or trim will change. The ship will incline about the axis of floatation. The centroid of the waterplane area lies on the axis of floatation. (Euler's Theorem)

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6

2.5 The Inclining Test

IACS 2023. The purpose of this procedure is to achieve a satisfactory accuracy in the determination of the lightship weight and of the coordinates of its centre of gravity.

Download from Classroom and study the following

1. IACS 2023. Recommended procedure for inclining test
2. ITTC 2021 Inclining Experiment Recommended Procedure 75-02-07-047
3. M. D. Woodward et al. "Uncertainty analysis procedure for the ship inclining experiment," Ocean Engineering Vol. 114 pp 79-86 (2016).

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7

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Inclining test unified procedure
IACS ToC

1. Introduction
2. General Preparation for the Test
    1. Information to be submitted
    2. The inclining test condition
    3. Tank contents
    4. Mooring Arrangements and Environmental Conditions
    5. Inclining Weights
    6. Pendulums and Instruments
    7. Trim and Stability

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8

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Inclining test unified procedure

3. Inclining Test and Record of Data
    1. Person in Charge
    2. Accuracy of Data
    3. Draught and Water Density Measurements
    4. Weight shifts and Inclination Measurements
    5. Other Relevant Data
4. Postponement of the Test
5. Test Report and Analysis of Lightship Data

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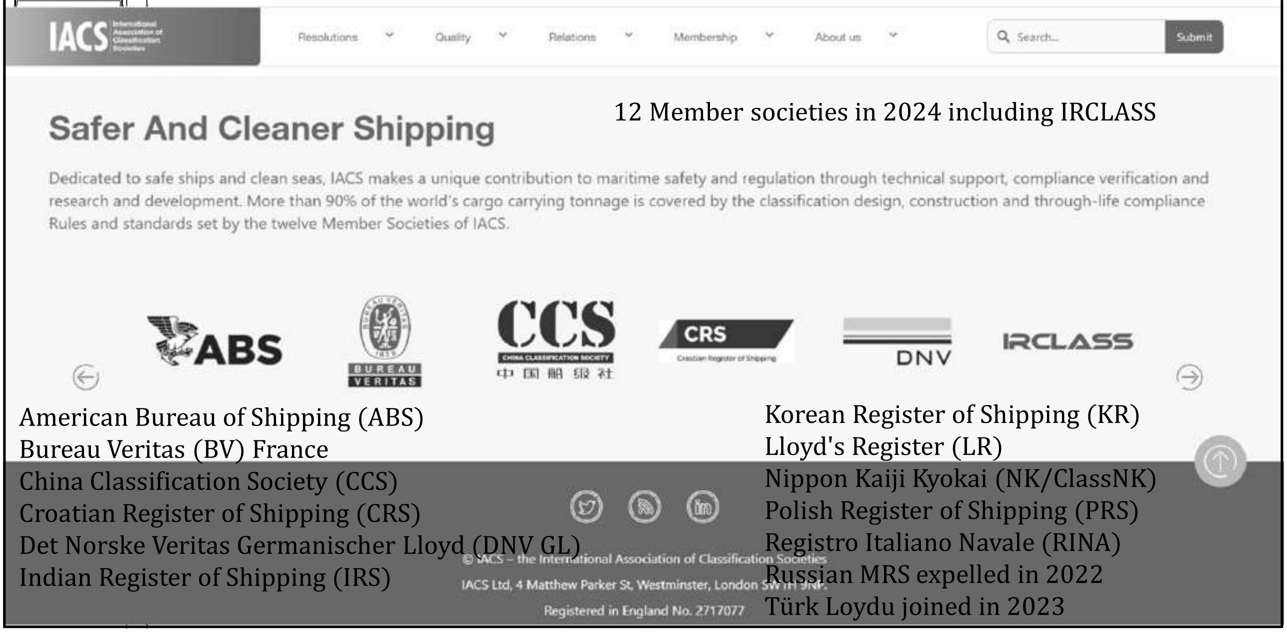
9

IACS website
International Association of Classification Societies



12 Member societies in 2024 including IRCLASS

American Bureau of Shipping (ABS)
Bureau Veritas (BV) France
China Classification Society (CCS)
Croatian Register of Shipping (CRS)
Det Norske Veritas Germanischer Lloyd (DNV GL)
Indian Register of Shipping (IRS)

Korean Register of Shipping (KR)
Lloyd's Register (LR)
Nippon Kaiji Kyokai (NK/ClassNK)
Polish Register of Shipping (PRS)
Registro Italiano Navale (RINA)
Russian MRS expelled in 2022
Türk Loydu joined in 2023

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10

IACS: Underwater Noise

- A lot of research is done on Underwater Radiated Noise (URN) for defense applications under the guise of marine mammal protection

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11

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IACS 2023: Recommended procedure for inclining test

1. Introduction

The purpose of this procedure is to achieve a satisfactory accuracy in the determination of the lightship weight and of the coordinates of its centre of gravity.

This general procedure is a recommendation. Alternative requirements which are considered to be equivalent to those specified by the following items may be accepted. Acceptance of such equivalents rests with the Society and, where the inclining test is performed to satisfy a statutory requirement, such equivalents also may be subject to the acceptance of the Flag Administration.

Where a surveyor of the Society is requested to attend the inclining test, the surveyor should verify that the test is conducted according to accepted procedures and that all basic measurements and data are correctly taken and recorded.

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Flag and Port States

- The role of the flag state is to conduct regular inspections of each of their ships to ensure the safety of their cargo and crew members. The state that the vessel registers with is also responsible for collecting taxes from that vessel and regulating the pollution levels of ships under their flag. In turn, ships must follow any policies enforced by the country that they register with.
- Owners of ships can choose which country they want to register their vessel with, and they are not obligated to register with the country in which they live. Also called flags of convenience, some states will even offer tax, port access and regulation incentives to encourage ships to register with them. About 73% of ships sail under a flag of convenience, and the top three flag states are Panama, Liberia and the Marshall Islands. A nation does not have to be a coastal state to be a flag state.
- A port state is a nation that allows Port State Control (PSC) at its ports. The PSC is responsible for inspecting ships that dock in their harbor to ensure they are up to international codes. Port State Control Officers can check any ship at their port, regardless of whether they fly the flag for their dock country.

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13

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Dec24-Apr25

IACS 2023. 2. General Preparation for the Test

2.1 Information that should be submitted

The Instruction, containing the information of date and location of the test, responsible person, stability, inclining weight, schemes of inclining weight positions etc., should be presented to the Classification Society before the inclining test.

The following information should be available at the time of the inclining test as necessary:

– General arrangement drawing;

– Tank capacity plan;

– Hydrostatic curves;

– Draft marks locations.

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14

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IACS. 2.2 Inclining Test Condition

2.2 The inclining test condition

2.2.1 The ship should be as near to completion as possible. Equipment used by the yard on board should be limited to the utmost extent possible. Prior to the inclining test, lists of all items which are to be added, removed, or relocated should be prepared. These weights and their locations should be accurately recorded.

Normally, the total value of missing weights should not exceed 2 percent and surplus weights, excluding liquid ballast, not exceed 4 percent of the light ship displacement. For smaller vessels, higher percentages may be allowed.

2.2.2 All objects should be secured in their regular positions. All weights which may swing or shift should be secured in their known position. If more than one sea stowage position is possible, the actual stowage position used during the test should be recorded.

2.2.3 The ship should be cleared of residues of cargo, tools, debris, scaffolding and snow. Icing of the inner and outer surfaces, the underwater hull included, should not be permitted.

2.2.4 All bilge water and other extraneous standing liquids should be removed. When draining individual tanks is impracticable, allowances for such liquids should be at the discretion of the Society.

2.2.5 All service tanks and machinery plant pipings should be filled as for the working condition.

2.2.6 In general, only the people participating in the inclining test should stay on board the ship.

2.2.7 All spaces should be safe for inspection.

2.3 Tank contents

2.3.1 Preferably, all tanks should be either full or empty. The number of tanks containing liquids should be kept to a minimum.

2.3.2 Soundings and density of liquids in tanks should be taken. Shapes of tanks which are partly filled should be known in order to determine the free liquid surface effect.

2.3.3 Adequate measures should be taken to preclude air pockets in completely full tanks. All connections between tanks should be closed and all empty tanks should be adequately dried.

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17

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2.4 Mooring Arrangements and Environmental Conditions

2.4.1 Mooring lines should be free of any tension in the transverse direction of the ship during the reading after each weight shift. No external moments should be brought upon the ship (from mooring lines, quay, etc.). If possible, the ship should be located in a calm, protected area free from external forces.

2.4.2 The depth of water under the hull should be sufficient to ensure that the hull will be entirely free of the bottom. Prior to the test the depth of water should be measured in as many locations as necessary to positively satisfy this requirement, taking into account tide differences, if applicable.

2.4.3 An ideal mooring arrangement would involve bow and stern lines on both sides of the ship attached at or near the centre-line. Longitudinal mooring lines should be as long as practicable. More commonly, a ship may be moored by bow and stern lines on one side only and supplemented by spring lines. Where a single bow or stern line is proposed, the surveyor should be assured that the ship's freedom of movement does not adversely effect the conduct of the experiment.

2.4.4 The ship may be moored by means of other special arrangement approved by the Society.

2.4.5 When tidal currents are present the experiment should normally be conducted at or around slack tide.

2.4.6 The ship's gangway should be in the stowed position and any shore gangway removed during the inclining test. As few cables, hoses, etc., as possible should be connected to shore. Those which are needed should be slack.

2.4.7 The test should not be conducted under adverse wind, wave and current conditions where the accuracy of the results cannot be assured.

Dec24-
Apr25

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18

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2.5 Inclining Weights

2.5.1 For the inclining test, solid inclining weights normally should be used.

2.5.2 Use of water ballast transfer to incline the vessel may be permitted only in cases where it is impractical to incline the vessel using solid weights. If the transfer of water ballast is to be used, a detailed procedure, including calculation procedure, should be submitted to the society for approval prior to the experiment.

2.5.3 The total weight used should be sufficient to provide a minimum inclination of one degree and a maximum of four degrees of heel to each side of the initial position. However, in those cases where it is absolutely impractical to reach a minimum angle of 1 degree by use of solid weights or water ballast a lesser inclination angle may be accepted, provided that the requirements on pendulum deflection or U-tube difference in height in 2.6.1 should be complied with.

2.5.4 Each weight should be compact, impervious to water and shaped such that its centre of gravity may be accurately determined. It is recommended that not fewer than four weights (or sets of weights) be used, each approximately equal in mass, and that the inclining weights (or sets of weights) be positioned as symmetrically as possible and parallel to the centre line in places convenient for the shifting of weights and measurement of the arms.

2.5.5 Each inclining weight should be marked with an identification number. The inclining weights should have been weighed with a calibrated instrument to the satisfaction of the Surveyor.

Dec24-
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19

2.6 Pendulums and Instruments

2.6.1 The use of three measuring devices is recommended to determine the vessel's inclination after each weight shift, however, a minimum of two devices should be used, one of which should be a pendulum or U-tube arrangement. The length and arrangement of pendulum/U-tube should be such as to ensure the accuracy of the readings of deflection/difference. The minimum deflection/difference, to each side of the initial position, corresponding to the total weight shift, should be 15 cm.

2.6.2 The use of a stabilograph may also be acceptable provided the calibration of the instrument has been verified to the Surveyor's satisfaction prior to the experiment. A trace of the recorded heel pattern should be included in the test report.

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Dec24-Apr25

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20

2.7 Trim and Stability

2.7.1 The vessel should be upright prior to the inclining. However, an initial list of the ship not exceeding 0.5° is permissible.

2.7.2 Excessive trim should be avoided for certain hull forms where changes in waterplane shape would occur in the region of the waterline when the ship is heeled. Such features should be taken into account to select a suitable draught and trim for the test.

2.7.3 The persons conducting the test should be satisfied that the vessel has adequate, positive stability and acceptable stress levels during the test. The estimated initial metacentric height should be at least 0.20 m.

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3 Inclining Test and Record of Data

3.1 Person in Charge

A competent person should be designated in charge of the preparation and execution of the inclining test.

3.2 Accuracy of Data

Measurement of Inclining Test data should be as accurate as possible and to the satisfaction of the attending Surveyor.

3.3 Draught and Water Density Measurements

3.3.1 Draught/freeboard should be measured immediately before and verified after the test, to ensure that no significant changes in vessel's condition have occurred during the test.

3.3.2 Draughts/freeboards should be measured at fore and aft and midship draught marks at both sides. If the freeboards are not measured from the upper edge of deck line at side of freeboard deck or at the same frame locations as the draught marks, the locations and vertical datum should be stated.

3.3.3 A suitable boat with low freeboard should be available for the draught measurements.

3.3.4 To control the correctness of draught measurements, it is recommended to plot two waterlines by draught readings and by measured values of the freeboard when the latter is available. With correct measurements, both waterlines should coincide. In case of non-coincidence of separate points, additional measurements should be taken.

3.3.5 Sufficient water samples should be taken at suitable locations and depths to enable and accurate assessment of water density to be made.

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3.4 Weight shifts and Inclination Measurements

3.4.1 Two recommended procedures of shifting weights are shown in table 1.

Table 1

No. of Weights or Weight Groups

Weight Shifts	Four		Six	
	PS	SB	PS	SB
No. 0	2, 4	1, 3	2, 4, 6	1, 3, 5
No. 1	4	1, 2, 3	4, 6	1, 2, 3, 5
No. 2		1, 2, 3, 4		1, 2, 3, 4, 5,
No. 3	1	2, 3, 4	6	1, 2, 3, 4, 5
No. 4	1, 3	2, 4	2, 4, 6	1, 3, 5
No. 5	1, 2, 3	4	1, 2, 3, 4, 6	5
No. 6	1, 2, 3, 4		1, 2, 3, 4, 5,	
No. 7	2, 3, 4	1	1, 2, 4, 6	3, 5
No. 8	2, 4	1, 3	2, 4, 6	1, 3, 5

PS and SB denotes port and starboard sides of ship respectively.

The underlined numbers indicate the last weights or weight groups shifted.

23

3.4.2 The inclining weight positions should be marked on the deck to ensure that consistency in placement is achieved. The transverse shift distance should be as great as practicable and appreciable changes in longitudinal or vertical position when moving port to starboard and vice versa should be avoided.

3.4.3 The pendulum length should be measured from its point of suspension to the recording batten on which deflections are read.

3.4.4 Pendulum, or U-tube reading on the recording batten or scale may be registered by either of the following ways:

a) on the final stable position of the pendulum or liquid column after stopping of ship motions due to shifting of the inclining weight;

b) by marking the mean value within the range of residual oscillation.

3.4.5 When using other devices, angles of inclination should be recorded according to instructions supplied with each device.

3.4.6 Checks should be made in the process of the inclining test for each measuring device. These should, generally, be a progressive plot of angles of heel against heeling moments which should give a series of points lying about a straight line passing through (or close to) the origin.

If there is a deviation of points, either between the points for a particular weight movement, or from the straight line, the deflections and moments should be checked and corrected prior to the next weight movement.

3.4.7 Personnel should be instructed to remain on their assigned positions while inclination readings are being taken and a check should be made that all mooring lines, etc., remain slack following each weight shift until all deflections have been taken and recorded.

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24

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3.5 Other Relevant Data

3.5.1 In the case where the inclinations are carried out by means of transfer of water, it should be possible to evaluate accurately the weight and the centre of the shifted liquid in relation to the ship's heel and trim.

3.5.2 The weather conditions, i.e., wind speed and direction relative to the vessel, sea state, air and water temperatures, etc., during the test should be recorded.

4 Postponement of the Test

If during the course of an inclining test circumstances arise such that the aforesaid recommendations are not adhered to,the attending Surveyor should advise the Person in Charge that the results may not be accepted.

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25

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5. Test Report and Analysis of Lightship Data

5.1 The Builder/Owner should incorporate the data gathered during the test into a comprehensive test report, which may be combined with the analysis of the lightship data. Test readings not used in the final analysis should still be recorded in the report.

5.2 The Surveyor should ensure that the data given in the report is consistent with that gathered during the test and to sign the report.

5.3 The inclining test report and analysis, combined with the report or separately, should be submitted to the Society for review and acceptance of results as the basis for approval of the stability information of the ship.

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26

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IACS 2023 Recommended procedure for inclining test

- Pay special attention to numbers in the document. Examples …
  - The total weight used should be sufficient to provide a minimum inclination of one degree and a maximum of four degrees of heel to each side of the initial position.
  - The minimum deflection/difference, to each side of the initial position, corresponding to the total weight shift, should be 15 cm.
  - However, an initial list of the ship not exceeding 0.5° is permissible
  - The estimated initial metacentric height should be at least 0.20 m.

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27

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Purpose: Has it been achieved?

- IACS 2023. The purpose of this procedure is to achieve a satisfactory accuracy in the determination of the lightship weight and of the coordinates of its centre of gravity.

- What is meant by satisfactory accuracy? The experimentally determined quantities will be used in subsequent calculations. For example, the GM is to be found. If the error in CoG is large, the error in the GM will also be large.

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28

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Standard Active | Last Updated: May 27, 2021 Track Document

ASTM F1321-21

Standard Guide for Conducting a Stability Test (Lightweight Survey and Inclining Experiment) to Determine the Light Ship Displacement and Centers of Gravity of a Vessel

Significance and Use

4.1 From the light ship characteristics one is able to calculate the stability characteristics of the vessel for all conditions of loading and thereby determine whether the vessel satisfies the applicable stability criteria. Accurate results from a stability test may in some cases determine the future survival of the

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29

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2.5 The Inclining Test

When all is ready and the ship is upright, a weight is shifted across the deck transversely, causing the ship to list. A little time is allowed for the ship to settle and then the deflection of the plumb line along the batten is noted. If the weight is now returned to its original position the ship will return to the upright. She may now be listed in the opposite direction. From the deflections the GM is obtained as follows:

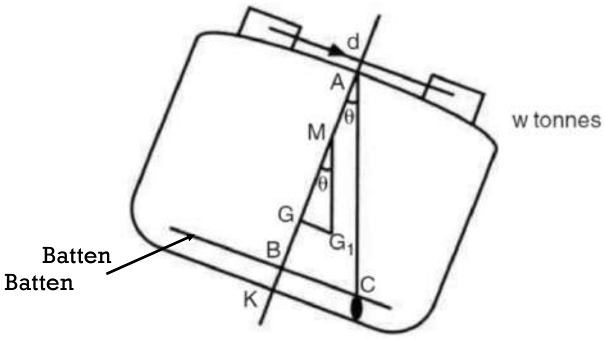

Fig. 33.1

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30

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2.5 The Inclining Test

- **The batten is parallel to the waterline when on even keel**
- **MG_1 is perpendicular to the water surface**
- **AC is also perpendicular to the water surface**
- **Therefore, MG_1 and AC are parallel and angle BAC = θ**

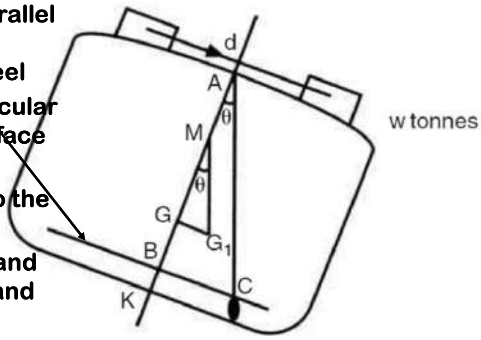


Fig. 33.1

In Figure 33.1 let a mass of 'w' tonnes be shifted across the deck through a distance 'd' metres. This will cause the centre of gravity of the ship to move from G to G_1 parallel to the shift of the centre of gravity of the weight. The ship will then list to bring G_1 vertically under M, i.e. to θ degrees list. The plumb line will thus be deflected along the batten from B to C. Since AC is the new vertical, angle BAC must also be θ degrees.

2.5 The Inclining Test

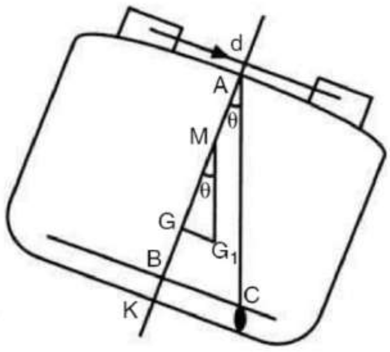
- $\tan(\boldsymbol{\theta}) = \mathbf{GG_1/GM} = \mathbf{BC/AB}$.
- $\mathbf{GM} = \frac{\boldsymbol{wd}}{\boldsymbol{W}\tan(\boldsymbol{\theta})}$

In this formula AB, the length of the plumb line and BC, the deflection along the batten can be measured. 'w' the mass shifted, 'd' the distance through which it was shifted, and 'W' the ship's displacement, will all be known. The GM can therefore be calculated using the formula.

The naval architects will already have calculated the KM for this draft and hence the present KG is found. By taking moments about the keel, allowance can now be made for weights which must be loaded or discharged to bring the ship to the light condition. In this way the light KG is found.

2.5 The Inclining Test

- $\tan(\boldsymbol{\theta}) = \mathbf{GG_1/GM} = \mathbf{BC/AB}$. Measure **AB** and **BC**. Calculate $\tan\boldsymbol{\theta}$.
- $\boldsymbol{w}$ = weight of shifted object
- $\boldsymbol{d}$ = distance of shift
- $\boldsymbol{W}$ = displacement of ship. Find it by 1) using the linesplan 2) adding all the weights.
- Find $\mathbf{GG_1} = \boldsymbol{wd/W}$
- Find **GM** using $\tan(\boldsymbol{\theta}) = \mathbf{GG_1/GM} = \boldsymbol{wd/W}/\mathbf{GM}$
- $\mathbf{GM} = \frac{\boldsymbol{wd}}{\boldsymbol{W}\tan(\boldsymbol{\theta})}$
- Find the **KB** from the hydrostatics curves
- Find $\mathbf{BM} = \mathbf{I/V}$
- Find K**M** = **KB** + **BM** = **KG** + **GM** and check



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1

15 Jan 2025

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Stability of Ships

B. Tech. NA&SB. 2023-27. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

Dr. D. D. Ebenezer
Adjunct Faculty
9446577239
ebenezer.cusat@gmail.com

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2

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
   Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
   Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
   Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
   Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
   Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
   Intact stability rules, Heel/ Load test.
   Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

Dec24-Apr25

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3

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

   73

   removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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4

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MODULE 2. Initial Transverse Stability

Completed

Module 1. 06 lectures.

Multiple choice Google forms test on 10 Jan 2025.

Module 2

2.1 $\overline{GM_0}$

2.2 CoB

2.3 Wall-sided ship

2.4 Shift of weight. Effect on $\overline{GM_0}$.

2.5 The Inclining Test

Today

2.5 The Inclining Test. Example.

2.6 Loads that affect transverse stability

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5

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2.5 The Inclining Test

- **The batten is parallel to the waterline when on even keel**
- **MG_1 is perpendicular to the water surface**
- **AC is also perpendicular to the water surface**
- **Therefore, MG_1 and AC are parallel and angle BAC = θ**

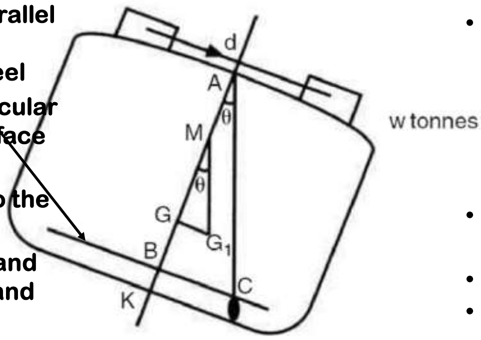

Fig. 33.1

- After the mass is moved, the heel angle changes and there is a new equilibrium position.
- There is no righting moment.
- $\tan\theta = \overline{GG_1}/\overline{GM}$
- $\Delta\,\overline{GG_1} = pd$
- $\Delta\overline{GM} = pd/\tan\theta$

Every time a mass is moved, we can find the $\Delta\overline{GM}$ and average all the values. This is a bad idea.

Measure $\boldsymbol{pd}$ and $\boldsymbol{tan\,\theta}$. Find $\Delta\overline{\boldsymbol{GM}}$.

In Figure 33.1 let a mass of 'w' tonnes be shifted across the deck through a distance 'd' metres. This will cause the centre of gravity of the ship to move from G to G_1 parallel to the shift of the centre of gravity of the weight. The ship will then list to bring G_1 vertically under M, i.e. to θ degrees list. The plumb line will thus be deflected along the batten from B to C. Since AC is the new vertical, angle BAC must also be θ degrees.

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6

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Example from Biran

- All experiments have errors. Efforts should be taken to reduce the errors.
- Plots of moments, pd, vs $\tan\theta$ (not θ) are prepared as the slope $= \Delta\overline{GM}$

- See L11S11

A common way of checking the accuracy of the results consists in plotting the tangents of heel angles, ~~arc~~tan θ, against the heeling moments, pd. Equation (7.8) shows that the ideal plot should be a straight line. Years ago Naval Architects fitted by eye a straight line passing through the plotted points. Nowadays computers and many hand calculators yield easily a **least-squares fit**. Example 7.1 shows how to do it.

When analysing the results of the inclining experiment, Eq. (7.8) is rewritten as

$$\Delta\overline{GM} = \frac{pd}{\tan\theta}$$

The interpretation of the results of inclining experiments requires the knowledge of the displacement, Δ, and of the height of the metacentre above the baseline, $\overline{KM}$. If the trim is

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7

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Example from Biran

- See the data on the next slide and the figure in L11S11. Finding the value of $\Delta\overline{GM} = \frac{pd}{\tan\theta}$ for each measurement and then finding the average is a bad idea. $\frac{1}{N}\sum_{n=1}^{N}(\frac{pd}{\tan}) =$ 53679. This is a poor estimate for the slope. See the slope on the next slide obtained by minimising the root mean square error.

7.6 Examples

Example 7.1 (Least-squares fit of the results of an inclining experiment). The results of the inclining experiment presented here are taken from an example in Hansen (1985), but are converted into SI units. The data are plotted as points in Figure 7.6. At a first glance it seems reasonable to fit a straight line whose slope equals the mean of $pd/\tan\theta$ values. In this example some trials performed with very small pd values produced zero heel-angle tangents. Those cases must be discarded when averaging because they yield $pd/\tan\theta = \infty$. After eliminating the pairs corresponding to zero heel-angle tangents we calculate the mean slope and obtain 53679.638. The reader can easily verify that the line having this slope is far from being satisfactory. Available programmes for linear least-squares interpolation cannot be used

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8

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Example from Biran

Table 7.1 Results of inclining experiment

Inclining Moment (tm) pd	Heel Angle Tangent $\tan\theta$	Inclining Moment (tm) pd	Heel Angle Tangent $\tan\theta$
1156.9	0.0187	−1136.5	−0.0179
1156.9	0.0185	−1136.5	−0.0180
1156.9	0.0179	−1136.5	−0.0185
771.5	0.0126	−757.5	−0.0119
771.5	0.0126	−757.5	−0.0120
771.5	0.0121	−757.5	−0.0124
386.3	0.0065	−379.4	−0.0057
386.3	0.0064	−379.4	−0.0060
386.3	0.0062	−379.4	−0.0065
1.1	0.0004	−0.2	0.0000
1.1	0.0005	−0.2	0.0000
1.1	0.0000	−0.2	0.0006

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9

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Example from Biran

- $M = \text{slope} = \Delta\overline{GM}$. Find M. Then use Δ to find $\overline{GM}$. Find $\overline{KB}$ and $\overline{BM}$ from hydrostatic data. Find $\overline{KM} = \overline{KB} + \overline{BM}$. Find $\overline{KG} = \overline{KM} - \overline{GM}$.

because, in general, they fit a line having an equation of the form

$$y = c_1 x + c_2 \qquad x = \tan\theta, \quad y = pd = \text{Moment}$$

Obviously, in our case the line must pass through the origin, that is $c_2 = 0$. Therefore, let us derive by ourselves a suitable procedure.

To simplify notations let x_i be the tangents of the measured heeling angles, and y_i the corresponding inclining moments. As said, we want to fit to the measured data a straight line passing through the origin

$$y = Mx \tag{7.11}$$

The error of the fitted point to the ith measured point is

$$y_i - Mx_i \tag{7.12}$$

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10

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Example from Biran

- $M = \text{slope} = \Delta\overline{GM}$. Find M. Then use Δ to find $\overline{GM}$. Find $\overline{KB}$ and $\overline{BM}$ from hydrostatic data. Find $\overline{KM} = \overline{KB} + \overline{BM}$. Find $\overline{KG} = \overline{KM} - \overline{GM}$.

We want to minimize the sum of the squares of errors

$$e = \sum (y_i - Mx_i)^2 \tag{7.13}$$

To do this we differentiate e with respect to M and equal the derivative to zero

$$\sum x_i (y_i - Mx_i) = 0 \tag{7.14}$$

The solution is

$$M = \frac{\sum x_i y_i}{\sum x_i^2} \qquad M = \frac{\sum \tan\theta \times \text{Moment}}{\sum \tan^2\theta} \tag{7.15}$$

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11

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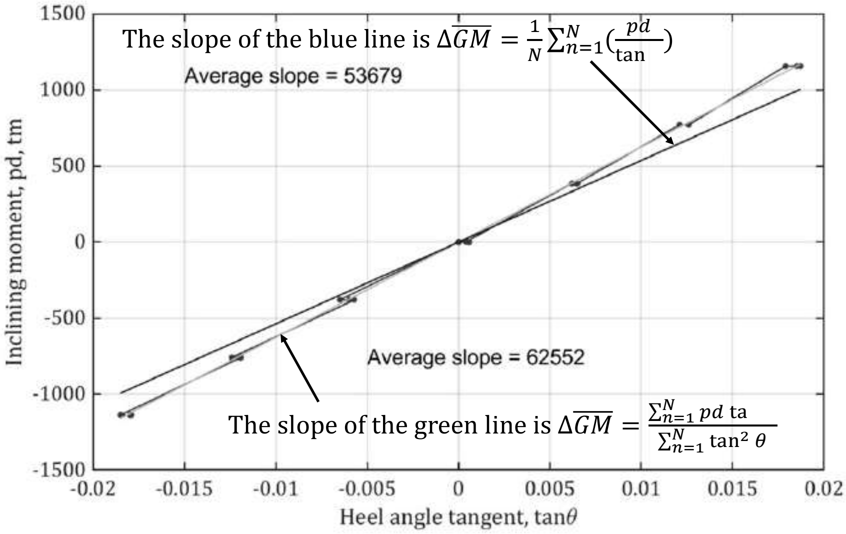
Example from Biran

- Biran. Figure 7.6 A plot of the results of an inclining test. “Root mean square” slope = 62552



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12

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Line fitting using Least Square Error

- https://www.youtube.com/watch?v=PaFPbb66DxQ
- The Main Ideas of Fitting a Line to Data (The Main Ideas of Least Squares and Linear Regression.)

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13

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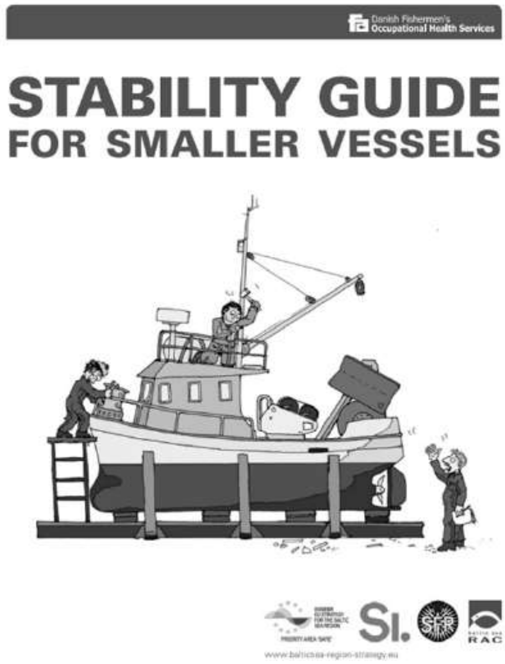


16 | Chapter 2

14

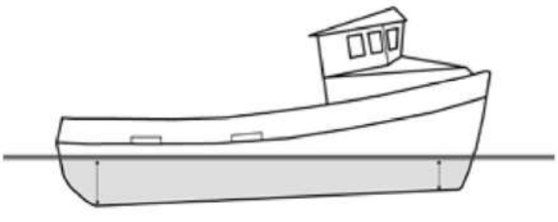
Find Δ

Calculation of the vessel's lightweight and centre of gravity

After the heeling test, the marine engineer calculates the vessel's stability back at his office.

The vessel's lightweight is calculated first.

Based on the line drawing or survey of the hull, a computer model of the vessel's hull is generated. This model is used for all subsequent calculations.



Measuring the draught forward and aft

The actual water line is determined with the data measured fore and aft under the heeling test. Based on the computer model of the hull, the vessel's displacement is calculated.

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15

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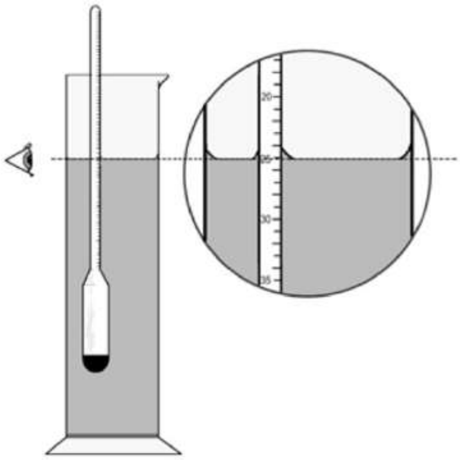
Find the Displacement

If for example the calculated displacement of the vessel is 20 m^3 of water, the vessel's weight can also be determined using the density of water measured under the heeling test. In fresh water, which weighs 1.00 t/m3, the ship's weight would equal 20 tonnes, but as the sea water contains salt, it is slightly heavier. For example, on the Danish west coast, seawater commonly weighs approx. 1.025 t/m^3.

So the weight of the vessel under the heeling test can be determined as 20 m3 * 1.025 t/m^3 = 20.50 tonnes.



The water density in the harbour is determined with a hydrometer. The higher the hydrometer floats in the water, the more salt there is in the water and the higher is the density of the water.

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16

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Find $\overline{GM}$

- In this simple example, the $\overline{GM}$ is found using one inclining or heel test
- Find $\Delta\overline{GM}$ by finding the slope of the line. Find Δ by using the drafts and the offsets table. Find $\overline{GM}$.

Determining a vessel's stability

Under the heeling test the following data was measured:

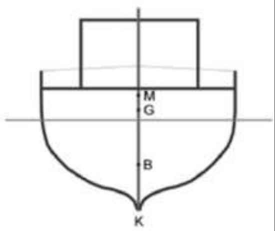
a	=	Distance of the heel weight moved transversely, measured in metres
w	=	Weight of the heel weight, measured in tonnes
W	=	The vessel's weight in tonnes
L	=	Length of the 2 lines including the pendulums
V	=	Transverse movement of the pendulum

This data is used to calculate the Metacentric height (GM) of the vessel. This is the first step in calculating the vessel's centre of gravity G.

$$GM = \frac{a * w * L}{W * V}$$

Example:

a	=	The heel weight moved 3 metres transversely
w	=	The heel weight weighs 200 kg
W	=	The vessel's weight was 20.50 tonnes under the heel test
L	=	The length of the lines including pendulum is 2,200 mm
V	=	The transverse movement of the pendulum is 80 mm



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17

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$$GM = \frac{3{,}00\ m * 0{,}20\ t * 2200\ mm}{20{,}50\ t * 80\ mm} = 0{,}80\ m$$

In some countries, the comma is used instead of the decimal point. On this page, both are used!

The Metacenter M for a given water line is determined by the shape of the hull below the waterline. The marine engineer can calculate the Metacentre M from the model of the hull.

The value is read from a hydrostatic print such as KMT:

Version of Hydrostatic Particulars: krpro

TRIM ON BASE LINE	-0.247 m	SPECIFIC GRAVITY	1.025 t/m^3
CODE OF ORIGIN	AP	KEEL THICKNESS	240 mm
TRIM UNDER KEEL	0.670 m	RISE OF KEEL	0.917 m

HYDROSTATIC PARTICULARS

D/BASE (m)	DEXT (m)	DISPLT (t)	TPCM (t)	MCT/CM (t.m)	LCB (m)	LCF (m)	KMT (m)	KML (m)	KB (m)
1.410	1.650	20.50	0.28	0.149	4.640	4.743	2.170	8.42	0.936

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18

Find the CoG

18 Chapter 2

The vertical centre of gravity KG at the heeling test can now be calculated using the KMT, derived from the table and the GM calculated at the heeling test as follows:

$$KG = KMT - GM = 2{,}17 - 0{,}80 = 1{,}37\ m$$

Recall that the purpose of the inclining test is to find, with satisfactory accuracy, the lightship weight and the CoG of the ship.

To find the centre of gravity of the empty ship - also called lightship - the calculation has to be corrected for the weights and centres of gravity which at the heeling test were noted as excess weights and lack of weights.

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19

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2.6 Loads that affect transverse stability

Change in Effective $\overline{GM}$

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20

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$\overline{GZ} = \overline{GM}\ \sin\phi$ is the righting arm

- A wind blows. The ship heels. The CoG does not move. The CoB moves to B_ϕ.
- A righting couple acts
- The righting lever arm is $\overline{GZ} = \overline{GM}\ \sin\phi$

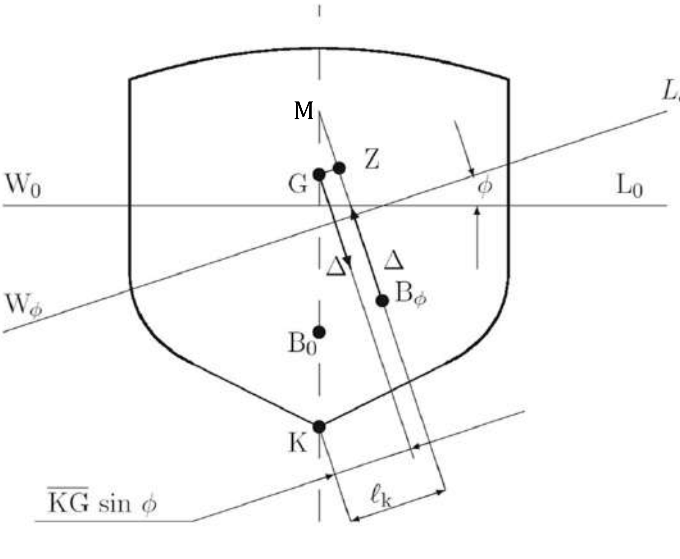

Figure 5.1 Definition of righting arm

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21

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2.6 Loads that affect transverse stability

- Biran. Adverse is an antonym of favourable.

6.9 Loads that Adversely Affect Stability

6.9.1 Loads Displaced Transversely

In Figure 6.8 we consider that a mass, m, belonging to the ship displacement, Δ, is moved transversely a distance d. A heeling moment appears and its value, for any heeling angle, ϕ, is $dm\cos\phi$. As a result, the ship centre of gravity, G, moves to a new position, G_1, the distance $\overline{GG_1}$ being equal to

$$\overline{GG_1} = \frac{dm}{\Delta} \tag{6.30}$$

and the righting arm is reduced to an **effective** value

$$\overline{GZ_{eff}} = \overline{GZ} - \frac{dm}{\Delta}\cos\phi \tag{6.31}$$

The effect is the same as if the centre of gravity, G, moved to a higher position, G_{eff}. During roll the ship inclines also to the other side, as in Figure 6.9. Then the **effective righting arm**, $\overline{GZ_{eff}}$, increases and the ship behaves as if the centre of gravity moved to a lower position, G_{eff}.

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Loads that affect transverse stability

Ship is upright. Mass is moved and ship lists.

Mass moved to stbd. Ship heels to starboard.

$$\overline{GZ_{eff}} = \overline{GZ} - \frac{dm}{\Delta}\cos\phi \tag{6.31}$$

The effect is the same as if the centre of gravity, G, moved to a higher position, G_{eff}. During roll the ship inclines also to the other side, as in Figure 6.9. Then the **effective righting arm**, $\overline{GZ_{eff}}$, increases and the ship behaves as if the centre of gravity moved to a lower position, G_{eff}.

Before the mass is shifted, $\overline{GZ} = \overline{GM}\sin\phi$.
After G shifts to G_1, it is $\overline{G_1Z_{eff}}$
$\overline{GG_{eff}}\sin\phi$ = reduction in $\overline{GZ}$ =
$(\overline{GG_1}/\tan\phi)\sin\phi = \frac{dm}{\Delta}\cos\phi$.

$\overline{G_1Z_{eff}} < \overline{GZ}$ when the ship rolls to the same side to which the mass is moved

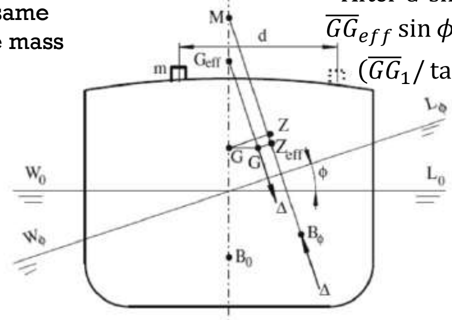

Figure 6.8 The destabilizing effect of a mass moved transversely

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Effective $\overline{GM}$ when ship rolls to the side to which mass is moved

- In the upright condition, before the mass, m, shifts, $\overline{GZ} = \overline{GM}\sin\phi$.
- After m shifts to starboard by d and G shifts to G_1, when the ship heels to starboard by ϕ, using $\overline{GG_1} = \frac{md}{\Delta}$,

$$\overline{G_1 Z_{eff}} = \overline{GZ} - \frac{md}{\Delta}\cos\phi = \overline{GM}\sin\phi - \frac{md}{\Delta}\cos\phi = \overline{G_{eff}M}\sin\phi.$$

- Therefore, $\overline{G_{eff}M} = \frac{\overline{GM}\sin\phi - \frac{md}{\Delta}\cos\phi}{\sin\phi} = \overline{GM} - \frac{md}{\Delta\tan\phi}$
- Reduction in $\overline{GM} = \frac{md}{\Delta\tan\phi} = \frac{\overline{GG_1}}{\tan}$.
- Note that reduction in $\overline{GM} \gg \overline{GG_1}$.
  A small transverse movement of the CoG is equivalent to a large reduction in $\overline{GM}$.

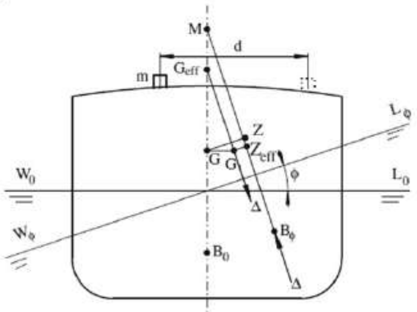

Figure 6.8 The destabilizing effect of a mass moved transversely

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- Mass moved to starboard. Ship heels to port by ϕ. Increase in effective $\overline{GM}$
- $\overline{G_{eff}M} = \frac{\overline{GM}\sin\phi + \frac{md}{\Delta}\cos\phi}{\text{si}} = \overline{GM} + \frac{md}{\Delta\tan\phi}$. Increase in $\overline{GM} = \frac{\overline{GG_1}}{\tan\phi}$.
-

Simple Models of Stability 143

$\overline{GZ_{eff}}$ increases when the ship rolls to the opposite side to which the mass is moved

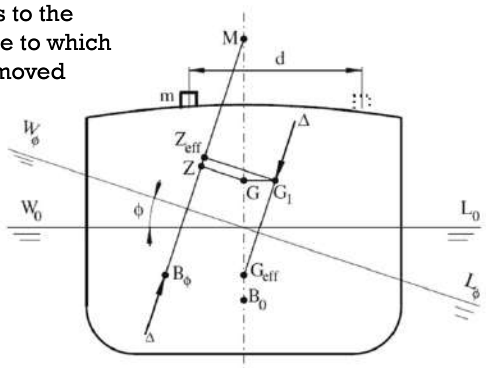

Figure 6.9 The effect of a mass moved transversely when the ship rolls to the other side

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Effect of vertical shift in CoG

- If the CoG moves up, the $\overline{GM}$ reduces.
- $\overline{GZ} = \overline{GM} \sin \phi$
- At $\phi = 0$, $\overline{GZ}$ remains zero. But at all other angles, both positive and negative, $\overline{GZ}$ decreases. See the next slide.

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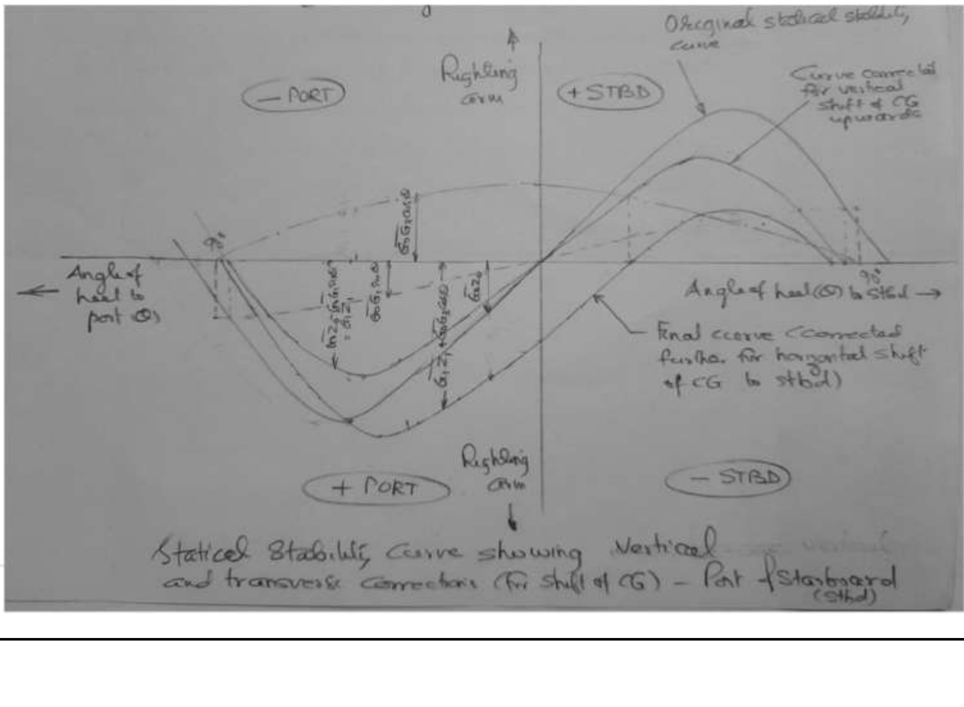
Effect of vertical and horizontal shift in CG



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6.9.2 Hanging Loads

In Figure 6.10 we consider a mass m suspended at the end of a rope of length h. When an external moment causes the ship to heel by an angle ϕ, the hanging mass moves transversely a distance $h \tan\phi$, and the ship centre of gravity moves in the same direction a distance

$$\overline{GG_1} = \frac{hm}{\Delta}\tan\phi \tag{6.32}$$

In Figure 6.11 we see that the righting arm is reduced from $\overline{GZ}$ to $\overline{G_1Z_1} = \overline{GZ_{eff}}$. The effect is the same as if the centre of gravity, G, moved to a higher position, G_V, given by

$$\overline{GG_v} = \frac{\overline{GG_1}}{\tan\phi} = \frac{hm}{\Delta} \tag{6.33}$$

As a result, we use for initial-stability calculations a corrected, or **effective metacentric height**

$$\overline{GM_{eff}} = \overline{GM} - \frac{hm}{\Delta} \tag{6.34}$$

The destabilizing effect appears immediately after raising the load sufficiently to let it move freely. Looking at Eq. (6.34) we see that the metacentric height is reduced by the same amount that would result from raising the load by a distance h. In other words, we can consider that the mass acts in the hanging point.

Hanging Loads

- In this analysis, the vertical movement of the hanging load is neglected. The exact transverse movement of the load is $h\sin\phi$. Righting lever without hanging load $= \overline{GZ}$. With hanging load $= \overline{G_1Z_1}$

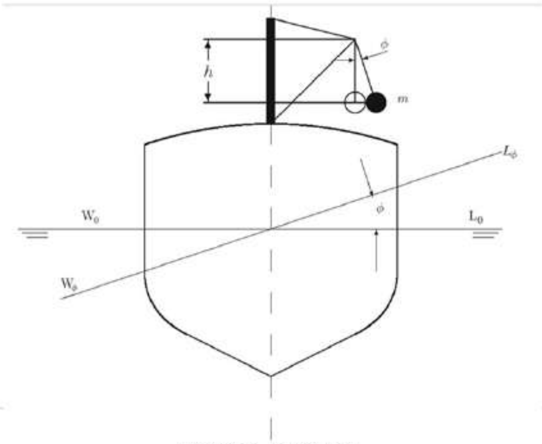
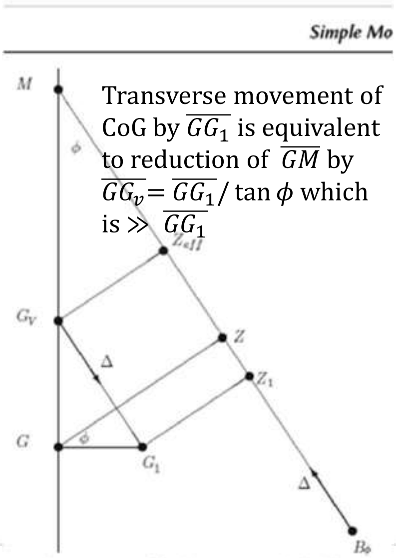


Figure 6.10 Hanging load



Transverse movement of CoG by $\overline{GG_1}$ is equivalent to reduction of $\overline{GM}$ by $\overline{GG_v} = \overline{GG_1}/\tan\phi$ which is $\gg$ $\overline{GG_1}$

Figure 6.11 Effective metacentric height

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29

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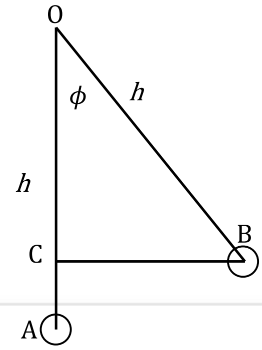
Hanging Load

- $\overline{OA} = \overline{OB} = h$; $\overline{BC} = h\sin\phi$ is the transverse displacement of the hanging load when the ship heels by ϕ.
- $\overline{GG_1} = \frac{mh\sin\phi}{\Delta}$
- Decrease in $\overline{GM} = \overline{GG_v} = \frac{\overline{GG_1}}{\tan\phi} = \frac{mh\sin\phi}{\Delta\tan\phi} = \frac{mh\cos\phi}{\Delta} \cong \frac{mh}{\Delta}$
- Note that decrease in $\overline{GM} \gg \overline{GG_1}$
- For small angles, $\sin\phi \cong \tan\phi$. This approximation is used to get a simple expression for the decrease in $\overline{GM}$ that is similar to the expression when a mass is moved horizontally.
- Decrease in $\overline{GM} = \overline{GG_v} = \frac{\overline{GG_1}}{\tan\phi} = \frac{mh\tan\phi}{\Delta\tan\phi} \cong \frac{mh}{\Delta}$
- Decrease in $\overline{GM}$ = what will happen if the mass is moved to it's suspension point



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30

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6.9.3 Free Surfaces of Liquids

Liquids with free surfaces are a very common kind of moving load. Any engine-propelled vessel needs fuel and lubricating-oil tanks. Tanks are needed for carrying fresh water. The cargo can be liquid; then tanks occupy a large part of the vessel. Tanks cannot be filled to

the top. Liquids can have large thermal expansion coefficients and space must be provided to accommodate for their expansion, otherwise unbearable pressure forces may develop. In conclusion, almost all vessels carry liquids that can move to a certain extent endangering thus the ship stability. A partially-filled tank is known as a **slack tank**.

Figure 6.12a shows a tank containing a liquid whose surface is free to move within a large range of heeling angles without touching the tank top or bottom. Let us consider that the liquid volume behaves like a ship hull and consider the free surface a waterplane. Then, the centre of gravity of the liquid is the buoyancy centre of the liquid hull. Therefore, we use for it the notation b_0. While the ship heels, the centre of gravity of the liquid moves along the curve of the centre of the buoyancy, "around" the metacentre, m. The horizontal distance between the initial position, b_0, and the inclined position, b_ϕ, is

Recall that change in the tcb for a wall-sided ship is $B^2\tan\phi/(12T) = \overline{BM}\tan\phi$

$$\overline{b_0 m}\tan\phi$$

If v is the volume occupied by the liquid, i_B the moment of inertia of the liquid surface with respect to the barycentric axis (of the free surface) parallel to the axis of heeling, and ρ the

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31

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Free-Surface

- Heeling moment ≅ Mass of liquid in the tank × transverse distance its CoG has moved
- Transverse distance CoG of entire liquid in tank has moved = $B^2 \tan\phi\, /(12T) = \overline{b_0 m_0} \tan\phi$
- $\overline{b_0 m_0} = i_B/v$. See the next slide for notation.
- When the ship heels, the CoG of the tank moves. So, the CoG of the ship moves.

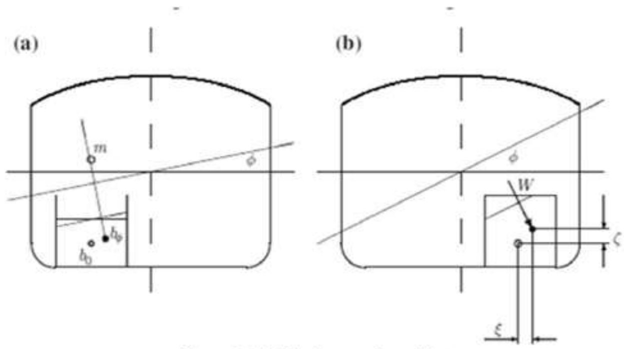

Figure 6.12 The free-surface effect

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32

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liquid density, the heeling moment produced by the inclination of the liquid surface is

$$M_l = \rho v \frac{i_B}{v} \tan\phi = \rho i_B \tan\phi$$

where M_l has the dimensions of mass times length

As a result, the ship centre of gravity moves transversely a distance equal to

$$\overline{GG_1} = \frac{\rho \cdot i_B}{\Delta} \tan\phi \qquad (6.35)$$

By comparison with the preceding subsection we conclude that the **effective metacentric height** is

$$\overline{GM_{eff}} = \overline{GM} - \frac{\rho \cdot i_B}{\Delta} \qquad \text{See L11S23} \qquad (6.36)$$

and the **effective righting arm**,

$$\overline{GZ_{eff}} = \overline{GZ} - \boxed{\frac{\rho \cdot i_B}{\Delta} \sin\phi} \qquad (6.37)$$

$= \overline{GG_1} \cos\phi$

Instead of considering the free-surface effect as a virtual reduction of the metacentric height and of the righting lever, we can take it into account as the heeling lever of free movable liquids. Its value is

$$\ell_F = \frac{\rho \cdot i_B}{\Delta} \qquad (6.38)$$

Free surface effect can be thought of as reduction in GM , reduction in righting lever, or a heeling lever

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Free-Surface

- "Effect does not depend on the position of the tank" because only the change in the CoG is important.

and the respective curve is proportional to $\sin\phi$. The latter approach is that adopted in the stability regulations of the German Navy.

The reduction of stability caused by the liquids in slack tanks is known as **free-surface effect**. Two of its features must be emphasized:

- the mass of the liquid plays no role, only the moment of inertia of the free surface appears in equations;
- the effect does not depend on the position of the tank.

In general, ships have more than one tank, and different tanks can contain different liquids. The destabilizing effects of all tanks must be summed up when calculating the effective metacentric height

$$\overline{GM_{eff}} = \overline{GM} - \frac{\sum_{k=1}^{n} \rho_k \cdot i_{Bk}}{\Delta} \tag{6.39}$$

and the effective righting arm,

$$\overline{GZ_{eff}} = \overline{GZ} - \frac{\sum_{k=1}^{n} \rho_k \cdot i_{Bk}}{\Delta} \sin\phi \tag{6.40}$$

where n is the total number of tanks.

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34

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Free-Surface

- Self-study. Not for exam. Derive Eq. (6.41).

Often the liquid surface is not free to behave as in Figure 6.12a and its shape changes when it reaches the tank top or bottom. Then, we cannot use the equations shown above. The same happens when the heeling angle is large and the forms of the tank such that the shape of the free surface changes in a way that cannot be neglected. In such cases the exact trajectory of the

Simple Models of Stability 147

centre of gravity must be calculated. As shown in Figure 6.12b, the resulting heeling moment is

$$M_\ell = W(\xi \cos\phi + \zeta \sin\phi) \tag{6.41}$$

where W is the liquid mass, ξ the horizontal distance and ζ the vertical distance travelled by the centre of gravity.

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Free-Surface

Some books and articles on Naval Architecture contain tables and curves that allow the calculation of the free-surface effect for various tank proportions. Present-day computer programmes can calculate exactly and quickly the position of the centre of gravity for any heel angle. For example, one can describe the tank form as a hull surface and run the option for cross-curves calculations. Therefore, correction tables and curves are not included in this book.

The free-surface effect can endanger the ship, or even lead to a negative metacentric height. Therefore, it is necessary to reduce the free-surface effect. The usual way to do this is to subdivide tanks by **longitudinal bulkheads**, such as shown in Figure 6.13. If the left-hand figure would refer to a parallelepipedic hull, the moment of inertia of the liquid surface in each tank would be $1/2^3 = 1/8$ that of the undivided tank. Having two tanks, the total moment of inertia, and the corresponding free-surface effect, are reduced in the ratio 1/4. An usual arrangement in tankers is shown in Figure 6.13b.

Some materials that are not really liquid can behave like liquids. Writes Price (1980), "Whole fish when carried in bulk in a vessel's hold behave like liquid," and should be considered as such in stability calculations.

We end this section by noting that transverse bulkheads do not reduce the free-surface effect of slack tanks.

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36

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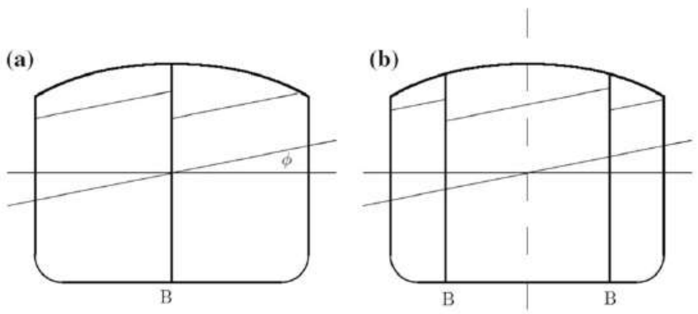
Free-Surface

- Assume that Fig. 6.13b shows tanks with breadth 0.2B 0.6B, 0.2B. What is the reduction in the total I? B is the breadth of the ship.

We end this section by noting that transverse bulkheads do not reduce the free-surface effect of slack tanks.



Longitudinal bulkheads are marked by 'B'

Figure 6.13 Reducing the free-surface effect

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37

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Read books, magazines, and journals

and become outstanding Naval Architects

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1

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17 Feb 2024

Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

Dr. D. D. Ebenezer
Adjunct Faculty
9446577239
ebenezer.cusat@gmail.com

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2

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
Intact stability rules, Heel/ Load test.
Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

73

   removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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4

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MODULE 2. Initial Transverse Stability

Completed

Module 1. 06 lectures.

Multiple choice Google forms test on 10 Jan 2025.

Module 2

2.1 $\overline{GM_0}$

2.2 CoB

2.3 Wall-sided ship

2.4 Shift of weight. Effect on $\overline{GM_0}$.

2.5 The Inclining Test

2.6 Loads that affect transverse stability

Today

2.6 Free Surface Effect. Example.

2.7 Stability during Docking and Grounding

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5

2.6 Example. Free-Surface Effect

- Barrass and Darrett

Example 1

A ship of 8153.75 tonnes displacement has KM = 8 m, KG = 7. 5 m, and has a double bottom tank 15 m × 10 m × 2 m which is full of salt water ballast. Find the new GM if this tank is now pumped out till half empty.

206 Ship Stability for Masters and Mates

Note. The mass of the water pumped out will cause an actual rise in the position of the ship's centre of gravity and the free surface created will cause a virtual loss in GM. There are therefore two shifts in the position of the centre of gravity to consider.

In Figure 21.2 the shaded portion represents the water to be pumped out with its centre of gravity at position g. The original position of the ship's centre of gravity is at G.

Δ is uppercase Delta

∇ is Del or Nabla. It is used for volume. Note that it like V with a line joining the 2 ends of the V.

- To find the CoG after the tank is half-emptied, let the disp of the ship with full tank be W. Mass of removed water = w. KG = original KG. g= CoG of removed water. KG1 = KG after tank is half-emptied. Take moments about the keel to find KG1. W KG – w Kg = (W − w) KG1. KG1 – KG = increase in CoG = (W KG – w kg)/ (W-w) – KG = w(KG-kg)/ (W-w).
- Exercise: take moments about G and find the change in the CoG.

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6

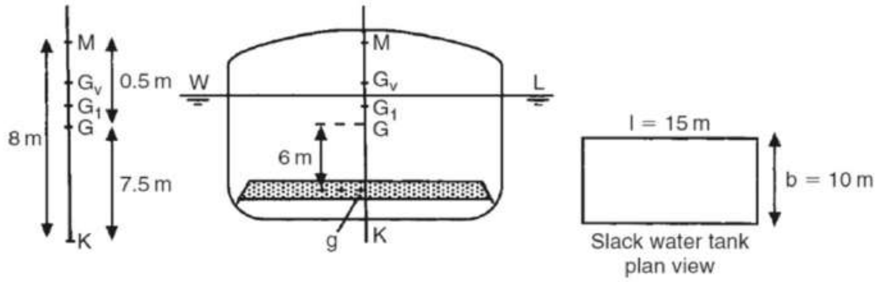

Fig. 21.2

Let GG_1 represent the actual rise of G due to the mass discharged.

$$\text{The mass of water discharged (w)} = 15 \times 10 \times 1 \times 1.025 \text{ tonnes}$$

$$w = 153.75 \text{ tonnes}$$

$$W_2 = W_1 - w = 8153.75 - 153.75$$

$$= 8000 \text{ tonnes}$$

$$GG_1 = \frac{w \times d}{W_2}$$

$$= \frac{153.75 \times 6}{8000}$$

$$\underline{GG_1 = 0.115 \text{ m}}$$

- Take moments about the original CoG to find the change in the CoG
- Find decrease in the moment due to removal of mass. Divide by the remaining mass.

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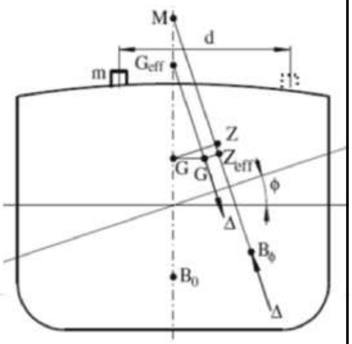
7

Effect of the free surface when the ship inclines

- The surface of the liquid in the tank is parallel to the water-surface outside the ship
- The CoG of the liquid in the tank moves transversely (port-stbd) and vertically
- From wall-sided ship analysis, liquid in tank has $\text{yCoG}_\text{t} = B_t^2 \tan\phi/(12T_t)$. Subscript t is for tank. $\text{yCoG}_\text{t} = BM_t \tan\phi = I_t \tan\phi\,/V_t$ (1)
- After heeling, yCoG of ship moves from G_1 to G_2. $\overline{G_1G_2} = \text{yCoG}_\text{t}\,\Delta_t/\Delta = \text{yCoG}_\text{t} V_t\rho_t/\Delta$ (2)
- Using (1) and (2) $\overline{G_1G_2} = I_t \tan\phi\,\rho_t/\Delta$ (3)
- See Fig. on L12S08.
- $\overline{G_1G_2}\cos\phi = \overline{G_1G_v}\sin\phi$ = reduction in $\overline{GZ}$. $\overline{G_1G_2}/\tan\phi = \overline{G_1G_v}$ (4)
- Transverse movement of G_1 to G_2 is equivalent to reduction in GM of $\overline{G_1G_v}$. (4)
- Using (3) and (4) Effective reduction in GM is $\overline{G_1G_v} = I_t\rho_t/\Delta$ (5)
- Barrass and Darrett neglect the vertical movement of the CoG of the liquid



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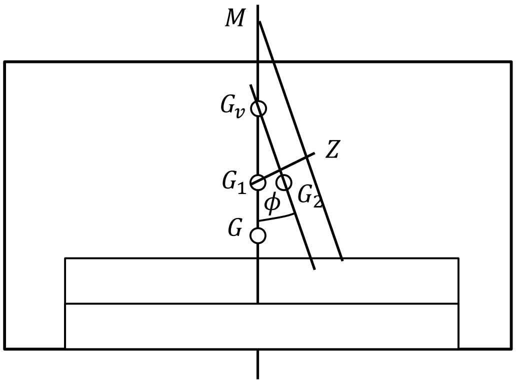
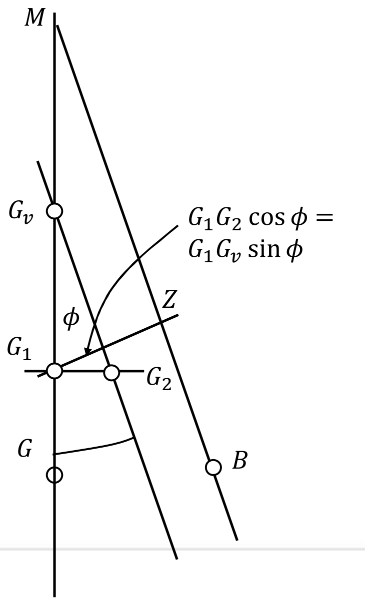
8

G_v = Effective or virtual CoG because of the free surface effect





- G = Original CoG
- G_1 = CoG after half the tank is emptied w/o considering the free surface effect
- G_2 = CoG after the ship heels
- G_v = Effective or virtual CoG because of the free surface effect

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9

2.6 Example. Free-Surface Effect

Let G_1G_v represent the virtual loss of GM due to free surface or rise in G_1. Then:

i = Moment of inertia of the WPA of the tank

W = mass of the ship

$$G_1G_v = \frac{i}{W} \times \rho_1 \times \frac{1}{n^2}$$

as per equation (I) at the beginning of this chapter:

n is the number of equal longitudinal compartments into which the tank is divided. See Fig. 6.13.

$$n = 1$$

$$\therefore \quad G_1G_v = \frac{i}{W_2} \times \rho_{SW}$$

G_1G_v = distance by which CoG shifts virtually. New virtual CoG is G_v

or

$$G_1G_v = \frac{1b^3}{12} \times \frac{\rho_{SW}}{W_2} \times \frac{1}{n^2}$$

$$\text{Loss in GM} = \text{FSE}$$

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10

2.6 Example. Free-Surface Effect

The rise in G due to these two effects can, in some cases, cause a very significant decrease in $\overline{GM}$ and reduce the stability to an unsafe level

The virtual rise in the CoG is independent of the volume of water removed. It is present as soon as there is a free surface in the tank.

or

$$G_1G_v = \frac{15 \times 10^3 \times 1.025}{12 \times 8000}$$

$$\underline{G_1G_v = 0.160\ \text{m}} \uparrow$$

$$\text{Old KM} = \underline{8.000\ \text{m}}$$

$$\text{Old KG} = \underline{7.500\ \text{m}}$$

$$\text{Old GM} = 0.500\ \text{m}$$

$$\text{Actual rise of G} = 0.115\ \text{m}$$

$$0.385\ \text{m} = \text{GM}_{solid}$$

$$G_1G_v = \text{Virtual rise of G} = 0.160\ \text{m} \uparrow$$

Ans. New GM = 0.225 m = GM_{fluid}

Hence G_1 has risen due to the discharge of the ballast water (loading change) and has also risen due to free surface effects.

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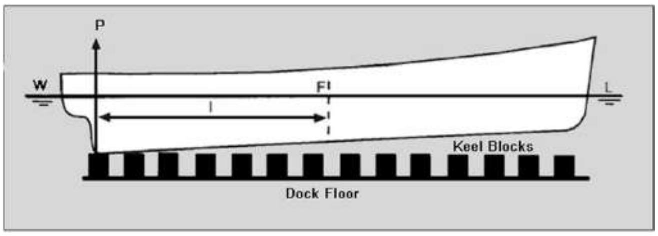
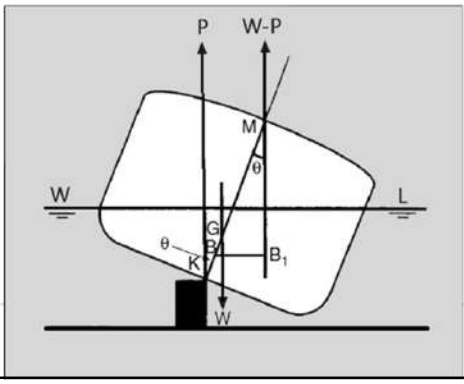
2.6 Example. Free-Surface Effect

- It is not necessary to neglect the change in the zCoG of the liquid. Find the change in the KG of the ship due to the upward movement of the liquid in the tank. This is equivalent to a reduction in the GM.
- From wall-sided ship analysis, liquid in tank has $\text{zCoG}_t = B_t^2 \tan^2 \phi/(24T_t) \cong B_t^2\phi^2/(24T_t)$. Subscript t for tank. $I_t = L_t B_t^3/12$; $I_t/V_t = B_t^2/(12T_t)$
- After heeling, $\text{zCoG}_t = I_t \tan^2 \phi/(2V_t)$ (6)
- After heeling, zCoG of ship moves to G_4. $\overline{G_2G_4} = \text{zCoG}_t\, \Delta_t/\Delta = \text{zCoG}_t V_t\rho_t/\Delta = I_t \tan^2 \phi\, \rho_t/(2\Delta)$ (7) where $\Delta =$ displacement of the ship
- The reduction in GM due to the vertical shift of the CoG of the tank is much less than the reduction in GM due to the horizontal shift of the CoG of the tank.

2.7 Stability during Docking and Grounding





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13

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2.7 Stability during Docking and Grounding

6.10 The Stability of Grounded or Docked Ships

6.10.1 Grounding on the Whole Length of the Keel

Figure 6.15 shows a ship grounded on the whole length of the keel. If local tide lowers the sea level, at a certain draught the ship will loose stability and capsize. To plan the necessary actions, the ship master must know how much time remains until reaching the critical draught. A similar situation occurs when a ship is laid in a floating dock. While ballast water is pumped out of the dock, the draught of the ship decreases. Props must be fully in place before the critical draught is reached.

- M is the point at which the line that passes through B and is perpendicular to the waterline intersects the centre-line of the ship. It is approximately the metacentre which does not lie exactly on the centre-line.

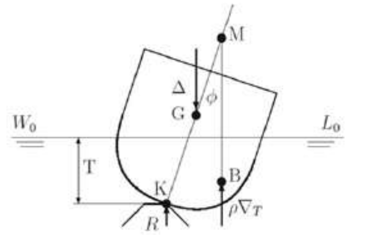

Figure 6.15 Ship grounded on the whole keel length

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14

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Biran. 2nd Ed. Sec. 2.10. "As the tangent to the B-curve is parallel to the corresponding waterline, it follows that the buoyancy force is normal to the B-curve." Therefore, … see box.

In Figure 6.15 we consider that the draught, T, descended below the value T_0 corresponding to the ship displacement mass, Δ. Then, the ship weight is supported partly by the buoyancy force $g\rho\nabla_T$ and partly by the reaction, R:

$$g\rho\nabla_T + R = g\Delta \tag{6.49}$$

where ∇_T is the submerged volume at the actual draught, T. The ship heels and for a small angle, ϕ, the condition of stability is

$$g\rho\nabla_T\overline{KM}\sin\phi > g\Delta\overline{KG}\sin\phi \tag{6.50}$$

or

$$\overline{KM} > \frac{\Delta\overline{KG}}{\rho}\cdot\frac{1}{\nabla_T} \tag{6.51}$$

The exact metacentre lies on the normal to the CoB curve which is also a normal to the waterline. E.g., wall-sided barge.

Simplifying we obtain

$$\overline{KM} > \frac{\nabla}{\nabla_T}\cdot\overline{KG} \tag{6.52}$$

where ∇ is the displacement volume corresponding to the ship mass, Δ. As an example, Figure 6.16 shows the curves $\overline{KM}$ and $\nabla\overline{KG}/\nabla_T$ as functions of draught, that is local depth, T, for the ship *Lido 9*. The critical draught in this case is 1.53 m.

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15

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Biran

152 *Chapter 6*

For draft > critical draft, KM>$\nabla KG/\nabla_T$ and the ship is stable

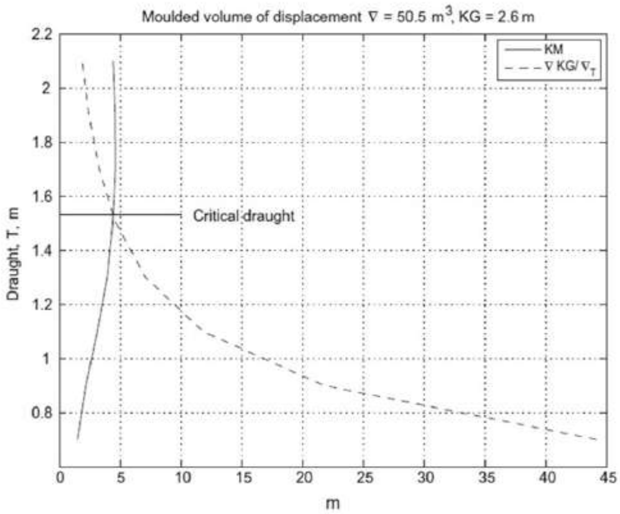

Figure 6.16 Finding the critical draught of a ship grounded on the whole keel length

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16

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Students are advised to read books and magazines.

Our library has a good collection of both.

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1

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22 Jan 2025

Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

Dr. D. D. Ebenezer
Adjunct Faculty
9446577239
ebenezer.cusat@gmail.com

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2

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Stability of Ships. Course Content.
Exam question paper will be based on this.

Course Content:

1. **Module I**
Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
Intact stability rules, Heel/ Load test.
Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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3

Stability of Ships. Course Content

4. **Module IV**
Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

73

removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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4

MODULE 2. Initial Transverse Stability

Completed

Module 1. 06 lectures.

Multiple choice Google forms test on 10 Jan 2025.

Module 2

2.1 $\overline{GM_0}$

2.2 CoB

2.3 Wall-sided ship

2.4 Shift of weight. Effect on $\overline{GM_0}$.

2.5 The Inclining Test

2.6 Loads that affect transverse stability

2.7 Stability during Docking and Grounding

Today

2.8 Roll period

2.9 Atwood's formula for $\overline{GZ}$

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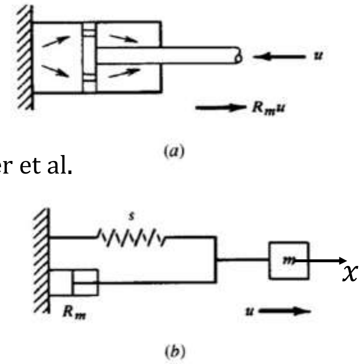
Free Vibration of damped SDOF systems

- A Single Degree of Freedom system that comprises a mass, a spring, and a damper
- When the mass is displaced to the right (+ve x) the spring and damper exert forces towards the left. The dynamic equilibrium equation is $m\ddot{x} = -sx - R_m\dot{x}$
- The solution to $m\ddot{x} + R_m\dot{x} + sx = 0$ is obtained after normalizing the equation.
- Let $\omega_0^2 = s/m$ and $\beta = R_m/(2m)$. Then, $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$;



Kinsler et al.

Figure 1.6.1 (*a*) Representative sketch of a dashpot with mechanical resistance R_m.
(*b*) Schematic representation of a damped, free oscillator consisting of a mass m attached to a spring of spring constant s and a dashpot with mechanical resistance R_m.

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6

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Governing Equation and Solution

- Special case: $\beta = 0$
- The solutions to $\ddot{x} + \omega_0^2 x = 0$ (1) can be written in many equivalent forms
- $x = A_1 \cos(\omega_0 t) + A_2 \sin(\omega_0 t)$ is a solution. Check this.
- $x = A \sin(\omega_0 t + \epsilon)$ is another form of the solution. See L13S10
- The second order ordinary differential equation has 2 solutions and 2 independent coefficients
- The solution shows that the mass vibrates sinusoidally when it is disturbed from its equilibrium position
- Recall that $\omega_0^2 = s/m$. $\omega_0 = \sqrt{s/m}$ is the natural angular frequency
- $f_0 = \omega_0/(2\pi)$ is the natural frequency
- $T_0 = 1/f_0 = \frac{2\pi}{\sqrt{s/m}}$ is the natural period

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7

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2.8 Roll Period. Stiff and Tender Ships.

The heeling moment, M_H, is caused by wind and waves.

If the wind blows the ship undergoes forced roll.

When the wind or excitation stops, $M_H = 0$.

Then, the ship undergoes free roll.

6.7 Stability Conditions—A More Rigorous Derivation

We describe the dynamics of heeling by Newton's equation for rotational motion

Does every term in this equation have the same dimensions? Check.

The righting moment is $g\Delta\overline{GZ}$

$$J\frac{d^2\phi}{dt^2} + g\Delta\overline{GZ} = M_H \tag{6.12}$$

where J is the **mass moment of inertia** of the ship, Δ, the mass displacement, and M_H, a heeling moment. The mass moment of inertia is calculated as the sum of the products of

138 Chapter 6

masses by the square of their distance from the axis of roll

$$J = \sum_{i=1}^{n} \left(y_i^2 + z_i^2\right) m_i \tag{6.13}$$

where y_i is the transverse and z_i is the height coordinate of the mass i. In the SI system we measure J in m^2 t. In Eq. (6.12) we neglected damping and added mass, terms briefly introduced in Section 6.13 and used in Chapter 12. We also neglect the **coupling** of heeling with other ship motions.

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8

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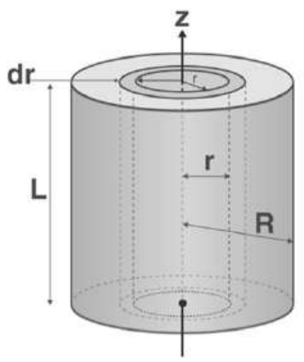
Moment of Inertia of a Solid Cylinder

- Consider a solid cylinder that has a mass, M, a radius, R, and a length, L. The moment of inertia, I, about the z axis of the cylinder is of interest. The solid cylinder can be divided into infinitesimally thin hollow cylinders. Each hollow cylinder has a wall thickness, dr, and a length of L. The moments of these infinitesimally thin hollow cylinders are added. ρ is the density. V is the volume.
- The elemental moment of inertia is $\mathrm{d}I = r^2\mathrm{d}m$ where $\mathrm{d}m = \rho\mathrm{d}V$, $\mathrm{d}V = L\mathrm{d}A$, and $\mathrm{d}A = \pi[(r+\mathrm{d}r)^2 - r^2]$ is the cross-sectional area of the hollow cylinder. $\mathrm{d}A = 2\pi r\mathrm{d}r$. $M = \rho\pi R^2 L$
- $I = \int_0^R dI = \rho\int_0^R r^2 2\pi r L\mathrm{d}r = \rho 2\pi L\left.\frac{r^4}{4}\right|_0^R = \rho\pi L\frac{R^4}{2} = \frac{1}{2}MR^2$. If all the mass is concentrated at $r = R/\sqrt{2}$, the moment of inertia will be the same



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9

2.8 Roll Period. Stiff and Tender Ships.

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- Free unresisted (no damping) rolling with no external force.

6.8 Roll Period

In Eq. 6.12, use $\overline{GZ} \cong \overline{GM}\sin\phi \cong \overline{GM}\phi$

For small angles of heel, and assuming $M_H = 0$, we rewrite Eq. (6.12) as

$$J\frac{d^2\phi}{dt^2} + g\Delta\overline{GM}\phi = 0 \tag{6.23}$$

We say that this equation describes **unresisted roll**. We define the **mass radius of gyration**, i_m, by

$$J = i_m^2\Delta \tag{6.24}$$

Substituting the above expression into Eq. (6.23) and rearranging yields

$$\frac{d^2\phi}{dt^2} + \frac{g\overline{GM}}{i_m^2}\phi = 0 \tag{6.25}$$

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10

2.8 Roll Period. Stiff and Tender Ships.

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With the notation

$$\omega_0 = \sqrt{\frac{g\overline{GM}}{i_m^2}} \tag{6.26}$$

the steady-state solution of this equation is of the form $\phi = \Phi\sin(\omega_0 t + \epsilon)$, where ω_0 is the **natural angular frequency** of roll, and ϵ, the **phase**. The **natural period of roll** is the inverse of the roll frequency, f_0, defined by

$$\omega_0 = 2\pi f_0$$

Using algebra, we obtain

$$\boxed{T_0 = 2\pi\frac{i_m}{\sqrt{g\overline{GM}}},} \tag{6.27}$$

where the result is in seconds.

We conclude that the larger the metacentric height, $\overline{GM}$, the shorter the roll period, T_0. If the roll period is too short, the oscillations may become unpleasant for crew and passengers and can induce large forces in the transported cargo. Tangential forces developed in rolling are proportional to the angular acceleration, that is to

$$\frac{d^2\phi}{dt^2} = -\Phi\omega_0^2\sin(\omega_0 t + \epsilon)$$

a quantity directly proportional to $\overline{GM}$.

11

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2.8 Roll Period. Stiff and Tender Ships.

Thus, while a large metacentric height is good for stability, it may be necessary to impose certain limits on it. IMO (2009), for example, referring to ships carrying timber on deck, recommends to limit the metacentric height to maximum 3% of the ship breadth (Part B, paragraph 3.7.5). Operational experience indicates that excessive initial stability should be avoided because it results in large accelerations in rolling and can cause huge stresses in lashings. Norby (1962) quotes researches carried on by Kempf, in Germany, in the 1930s. Kempf defined a non-dimensional rolling factor, $T\sqrt{g/B}$, and, on the basis of extensive statistics found that:

- for values of Kempf's factor under 8 the ship motions are **stiff**;
- for values between 8 and 14 the roll is **comfortable**;
- for factor values above 14 the motions are **tender**.

When the motions become too tender the ship master will worry because the metacentric height may be too low.

The exact value of the radius of gyration, i_m, can be calculated from Eq. (6.24) and requires the knowledge of all masses and their positions. This knowledge is not always available, certainly not in the first phases of ship design. Therefore, it is usual to assume that the radius of gyration, i_m, is proportional to the ship breadth, B,

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12

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Roll Period. Stiff and Tender Ships.

The exact value of the radius of gyration, i_m, can be calculated from Eq. (6.24) and requires the knowledge of all masses and their positions. This knowledge is not always available, certainly not in the first phases of ship design. Therefore, it is usual to assume that the radius of gyration, i_m, is proportional to the ship breadth, B,

$$i_m = aB$$

Let us define

$$c = 2a = \frac{2i_m}{B}$$

Substituting into Eq. (6.27) we obtain

$$T_0 = \frac{\pi c B}{\sqrt{gGM}} \tag{6.28}$$

As $\pi \approx \sqrt{g}$, we can rewrite Eq. (6.28) as

$$T_0 \approx \frac{cB}{\sqrt{GM}} \tag{6.29}$$

Rose (1952) quotes the following c values: large cargo and passenger vessels, 0.85; small cargo and passenger vessels, 0.77; loaded ore carriers, 0.81; tugs, 0.76; wide barges, 0.79. These values are based on old-type vessels. More recently, Costaguta (1981) recommends to

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13

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Roll Period

Use the recommendation by Costaguta and find the roll period of a merchant ship with B = 15 m and $\overline{GM}$ = 0.2 m

Rose (1952) quotes the following c values: large cargo and passenger vessels, 0.85; small cargo and passenger vessels, 0.77; loaded ore carriers, 0.81; tugs, 0.76; wide barges, 0.79. These values are based on old-type vessels. More recently, Costaguta (1981) recommends to take $i_m = B/3$ for merchant ships, and $c = 0.8$–0.9 for round-bilge, motor yachts. Some shipyards use $i_m = 0.35B$.

For actual ships, i_m can be obtained experimentally by measuring the roll period. When i_m is known, Eq. (6.27) can be used to control the metacentric height by measuring the roll period.

142 Chapter 6

This can be done automatically and online with the help of modern technology. Wendel (1960b) describes an instrument that did the job many years ago. The use of the roll period as a stability indicator is discussed, for example, by Norby (1962) and Jons (1987).

Normally, the roll period is measured in the still water of a harbour, and the ship is tied by the stern and by the aft to minimize other motions than roll. When measuring the roll period in a seaway it is necessary to distinguish between the ships own period and the period of encounter with the waves (see Jons, 1987, and Chapter 9).

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14

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2.9 Atwood's Formula. Wall-sided ship. Large angle of heel.

- Expressions for the yCoB and zCoB of a wall-sided ship from Biran are in L08
- Here, using the same assumptions, an expression for $\overline{GZ}$ is derived.

The Moment of Statical Stability at a Large Angle of Heel

At a large angle of heel the force of buoyancy can no longer be considered to act vertically upwards through the initial metacenter (M). This is shown in Figure 17.3, where the ship is heeled to an angle of more than 15°. The center of buoyancy has moved further out to the low side, and the vertical through B_1 no longer passes through (M), the initial metacenter. The righting lever (GZ) is once again the perpendicular distance between the vertical through G and the vertical through B_1, and the moment of statical stability is equal to W × GZ.

However, GZ is no longer equal to GM sin $\theta°$. Up to the angle at which the deck edge is immersed, it may be found by using a formula known as the *wall-sided formula*, i.e.

$$GZ = (GM + \frac{1}{2}BM\tan^2\theta)\sin\theta$$

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15

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2.9 Atwood's Formula. Wall-sided ship. Large angle of heel.

- In the figure, the ship is wall-sided for small angles of heel and the waterline is WL.
- However, when the waterline is W_1L_1, it is not wall-sided. Draw a figure showing a ship that is wall sided at both the waterlines. Hint: decrease the roll angle.

$$GZ = (GM + \frac{1}{2} BM \tan^2 \theta) \sin \theta$$

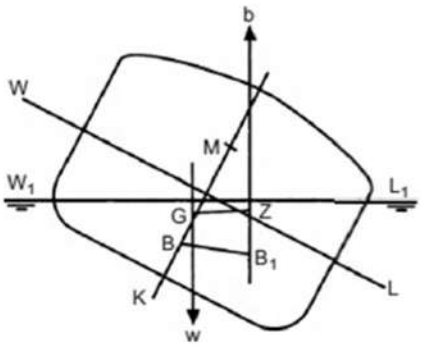

Figure 17.3

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16

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Fig. 17.4 is on the next slide

160 Chapter 17

The derivation of this formula is as follows:

Refer to the ship shown in Figure 17.4. When inclined the wedge WOW_1 is transferred to LOL_1 such that its center of gravity shifts from g to g_1. This causes the center of buoyancy to shift from B to B_1. The horizontal components of these shifts are hh_1 and BB_2 respectively, the vertical components being $(gh + g_1h_1)$ and B_1B_2 respectively.

Let BB_2 be 'a' units and let B_1B_2 be 'b' units (see Figure 17.5(a)). Now consider the wedge LOL_1:

$$\text{Area} = \frac{1}{2} y^2 \tan \theta$$

Consider an elementary strip longitudinally of length dx as in Figure 17.5(b):

$$\text{Volume} = \frac{1}{2} y^2 \tan \theta \, dx$$

The total horizontal shift of the wedge (hh_1) is 2/3 × 2y or 4/3 × y.

$$\therefore \text{Moment of shifting this wedge} = \frac{4}{3} y \times \frac{1}{2} y^2 \tan \theta \, dx$$
$$= \frac{2}{3} y^3 \tan \theta \, dx$$

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17

2.9 Atwood's Formula. Wall-sided ship. Large angle of heel.

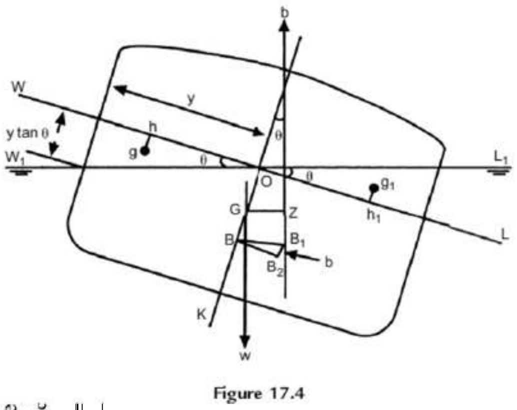

Figure 17.4

- $\angle BB_2B_1$ is 90 deg

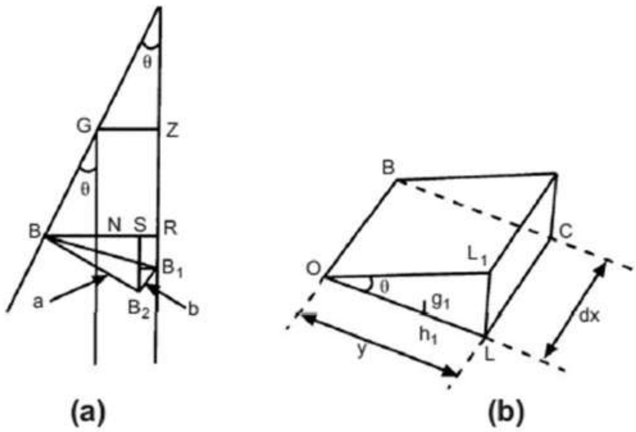

Figure 17.5

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18

- See the proof for $\overline{BM} = I/V$. The same fundamental ideas are used here also.

$$\text{The sum of the moment of all such wedges} = \int_O \frac{2}{3} y^3 \tan\theta \, dx$$

$$= \tan\theta \int_O^L \frac{2}{3} y^3 \, dx$$

$$\text{But the second moment of the waterplane area about the centerline (I)} = \int_O^L \frac{2}{3} y^3 \, dx$$

$$\therefore \text{Sum of the moment of all such wedges} = I \tan\theta$$

$$BB_2 = \frac{v \times hh_1}{V}$$

or

$$V \times BB_2 = v \times hh_1$$

$$\text{But the sum of the moments of the wedges} = v \times hh_1$$

$$\therefore V \times BB_2 = I \tan\theta$$

$$BB_2 = \frac{I}{V} \tan\theta$$

$$BB_2 = BM \tan\theta \qquad \text{(a)}$$

$$\text{The vertical shift of the wedge} = gh + g_1h_1$$

$$= 2gh$$

$$\therefore \text{The vertical moment of the shift} = v \times 2gh$$

$$= 2vgh$$

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19

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2.9 Atwood's Formula. Wall-sided ship. Large angle of heel.

In Figure 17.5(b):

$$OL = y \quad \text{and} \quad Oh_1 = \frac{2}{3}y$$

But

$$LL_1 = y\tan\theta$$

$$\therefore g_1h_1 = \frac{1}{3}y\tan\theta$$

$$\text{The volume of the wedge} = \frac{1}{2}y^2\tan\theta\,dx$$

First moment

$$\text{The moment of the vertical shift} = \frac{1}{2}y^2\tan\theta\,dx \times \frac{2}{3}y\tan\theta$$

$$= \frac{1}{3}y^3\tan^2\theta\,dx$$

$$\text{The vertical moment of all such wedges} = \int_O^L \frac{1}{3}y^3\tan^2\theta\,dx$$

$$= \frac{1}{2}I\tan^2\theta$$

$$\therefore \text{The moment of the vertical shift} = \frac{1}{2}I\tan^2\theta$$

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20

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Dec24-Apr25

2.9 Atwood's Formula. Wall-sided ship. Large angle of heel.

Also:

Vertical shift in the CoB of the ship = $b = B_1B_2 = \frac{v \times 2gh}{V}$

or

$$V \times b = 2vgh$$

but

$$2vgh = \text{The vertical moment of the shift}$$

$$\therefore V \times b = \frac{1}{2}I\tan^2\theta$$

or

$$b = \frac{I}{V} \times \frac{\tan^2\theta}{2}$$

$$B_1B_2 = \frac{BM\tan^2\theta}{2} \qquad \text{(b)}$$

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21

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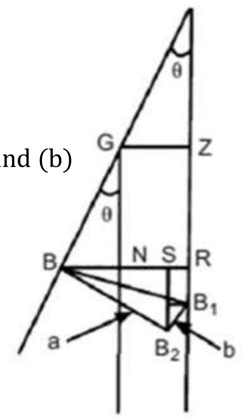
2.9 Atwood's Formula. Wall-sided ship. Large angle of heel.

Referring to Figure 17.5(a),

$\overline{BB_1} = a$
$\overline{B_1B_2} = b$

$$
\begin{aligned}
GZ &= NR \\
&= BR - BN \\
&= (BS + SR) - BN \\
&= a \cos\theta + b \sin\theta - BG \sin\theta \quad \text{Then, using Eqs. (a) and (b)} \\
&= BM \tan\theta \cos\theta + \frac{1}{2} BM \tan^2\theta \sin\theta - BG \sin\theta \\
&= BM \sin\theta + \frac{1}{2} BM \tan^2\theta \sin\theta - BG \sin\theta \\
&= \sin\theta \left(BM + \frac{1}{2} BM \tan^2\theta - BG\right) \\
GZ &= \sin\theta \left(GM + \frac{1}{2} BM \tan^2\theta\right) \quad (\text{for } \theta \text{ up to } 25°)
\end{aligned}
$$



(a)

This is the *wall-sided formula*.

Note. This formula may be used to obtain the GZ at any angle of heel so long as the ship's side at WW_1 is parallel to LL_1, but for small angles of heel (θ up to 5°), the term ½BM $\tan^2\theta$ may be omitted.

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22

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2.9 Example

Example 1

A ship of 6000 tonnes displacement has KB = 3 m, KM = 6 m, and KG = 5.5 m. Find the moment of statical stability at 25° heel.

$$
\begin{aligned}
GZ &= \left(GM + \frac{1}{2} BM \tan^2\theta\right)\sin\theta \\
&= \left(0.5 + \frac{1}{2} \times 3 \times \tan^2 25°\right)\sin 25° \\
&= 0.8262 \sin 25° \\
GZ &= 0.35 \text{ m} \\
\text{Moment of statical stability} &= W \times GZ \\
&= 6000 \times 0.35
\end{aligned}
$$

Ans. Moment of statical stability = 2100 tonnes m.

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23

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2.9 Example

164 Chapter 17

Example 2

A box-shaped vessel 65 m × 12 m × 8 m depth has KG = 4 m, and is floating in salt water upright on an even keel at 4 m draft. Calculate the moments of statical stability at (a) 5° and (b) 25° heel.

- At both the angles, calculate GZ using
  (a) $\overline{GZ} = \overline{GM} \sin\phi$ and (b) $\overline{GZ} = [\overline{GM} + 0.5\overline{BM}\tan^2\phi]\sin\phi$
  Find the difference between the results. Comment on the difference.
- Ans. 5 deg. (a) 278.9 tonnes m. 25 deg. (b) 1790.9 tonnes m.

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24

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Module 2 is completed

A Google Forms test will be held on 29 Jan 2025

Dec24-Apr25

Fresh Water Allowance

- A ship at sea is loaded up to the maximum waterline. The density is 1025 kg/m^3. It moves to fresh water with density 1000 kg/m^3. Find the increase in the draft. The increase is the FWA.

- A ship is loaded at dock with water density 1000 kg/m^3 to the maximum waterline + FWA. When it goes to sea, with water density 1025 kg/m^3, the waterline reduces to the permitted level.

- Derive an expression for the FWA using either of the above definitions.

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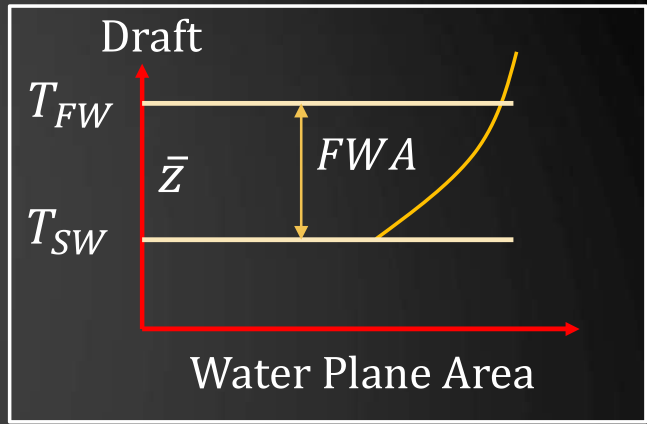
To find the FWA …



- T_{FW} = draft in fresh water. T_{SW} = draft in sea water.
- The exact change in the volume is $v = \int_{T_{SW}}^{T_{FW}} \text{WPA(z)}dz$

  where FWA = $T_{FW} - T_{SW}$ is the exact fresh water allowance
- For most ships, WPA(z) increases when z increases and WPA(T_{FW})> WPA(T_{SW}).
- So, $\text{WPA}_{FW}\text{FWA} > v$ (1) and $v > \text{WPA}_{SW}\text{FWA}$. (2)
- Further, $v = \text{WPA}_{\bar{z}}\text{FWA}$ (3) where $\bar{z}$ lies between T_{FW} and T_{SW} but its value is not known.
- When moving from sea to fresh water, the displacement is constant. The density decreases from 1025 kg/m^3 to 1000 kg/m^3. The underwater volume at sea = V_{SW}.
- It increases to $V_{\text{FW}} = V_{\text{SW}} + v = V_{\text{SW}}\ 1025/1000 = V_{SW}\left(1 + \frac{1}{40}\right)$. $v = V_{SW}\ /40$ is known.
- Using this in Eq. (1), FWA > v / WPA_{FW}. So, FWA ≅ v / WPA_{FW} is a conservative estimate (under-estimate) of the FWA.
- Using Eq. (2), v/WPA_{SW} > FWA. So, FWA ≅ v/WPA_{SW} is an over-estimate of the FWA
- The difference between WPA_{SW} and WPA_{FW} is small and books neglect the difference.

ASSIGNMENT 4. DUE AT 9 PM ON 11 JAN 2025.

Stability of Ships
B. Tech. NA&SB. 2023-27. 20-215-0406. 3 credits. IV Semester. Dec24-Apr25.
Department of Ship Technology
CUSAT, Kochi

Dr. D. D. Ebenezer
Adjunct Faculty
Mobile and WhatsApp 9446577239
ebenezer.cusat@gmail.com

Instructions.

1. Upload your answers as one pdf file named RegNo_Name_HW04.pdf. Eg. 01_Abhiram_HW04.pdf.
2. Use MATLAB wherever possible.
3. Do not copy. Upload whatever YOU did.

Question. Let the last 2 digits of your Registration number be n.

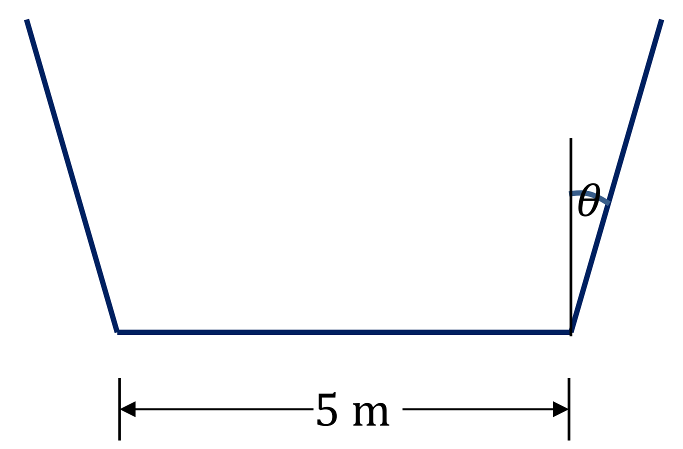


Fig. 1. The cross-section of a barge.

1. The cross-section of a barge with port-starboard symmetry is shown in Fig. 1. It is in fresh water of density 1 t/m^3. The displacement is 300 + n tons. The length is 40 – n/50 m. The angle θ is 5+n/10 degrees. Find the draft and the CoB.

2. A mass of 30 tons is added to the barge at the centre of flotation. Find the new draft and the new CoB using lectures notes (a) 1.5.1 Infinitesimally small mass (b) 1.5.2 Moderately small mass and (c) 1.5.3 Arbitrary mass. What do the three answers tell you about errors due to approximations?

Assignments that are submitted after the due date and time will not be considered for marks.

ASSIGNMENT 5. DUE AT 9 PM ON 25 JAN 2025.

Stability of Ships
B. Tech. NA&SB. 2023-27. 20-215-0406. 3 credits. IV Semester. Dec24-Apr25.
Department of Ship Technology
CUSAT, Kochi

Dr. D. D. Ebenezer
Adjunct Faculty
Mobile and WhatsApp 9446577239
ebenezer.cusat@gmail.com

Instructions.

1. Upload your answers as one pdf file named RollNo_Name_HW04.pdf. Eg. 01_Abhiram_HW04.pdf.
2. Use MATLAB wherever possible.
3. Do not copy. Upload whatever YOU did.

Question.

1. A cuboidal barge is floating on even keel in fresh water. The length, L, breadth, B, and draft, T, are 50 m, 6 m, and 2 m, respectively. The KG is 1 m. A mass of two tonnes on board is moved towards the starboard by 1.5 m. Find the heel angle.

2. A cuboidal barge is floating on even keel in fresh water. The length, L, breadth, B, and draft, T, are 50 m, 6 m, and 2 m, respectively. It is uniformly loaded with cargo to a depth of H=4 m. Neglect the mass of the barge. The moment of inertia of the barge about the x axis of flotation is

   $\Delta\left(\frac{B^2}{12}+\frac{(H)^2}{12}+(\frac{H}{2}-T)^2\right)$. Find the roll period and the Kempf factor.

Use MATLAB and upload your m file and the output.

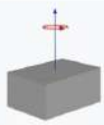
Assignments that are submitted after the due date and time will not be considered for marks.

Moment of Inertia

- See https://en.wikipedia.org/wiki/List_of_moments_of_inertia
- https://en.wikipedia.org/wiki/Parallel_axis_theorem

Solid rectangular cuboid of height h, width w, and depth d, and mass m.[7]

For a similarly oriented cube with sides of length s, $I_{CM} = \frac{1}{6}ms^2$



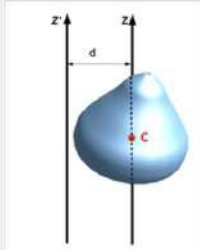
$$I_h = \frac{1}{12}m\left(w^2+d^2\right)$$
$$I_w = \frac{1}{12}m\left(d^2+h^2\right)$$
$$I_d = \frac{1}{12}m\left(w^2+h^2\right)$$

Mass moment of inertia [edit]

Suppose a body of mass m is rotated about an axis z passing through the body's center of mass. The body has a moment of inertia I_{cm} with respect to this axis. The parallel axis theorem states that if the body is made to rotate instead about a new axis z', which is parallel to the first axis and displaced from it by a distance d, then the moment of inertia I with respect to the new axis is related to I_{cm} by

$$I = I_{cm} + md^2.$$

Explicitly, d is the perpendicular distance between the axes z and z'.



The mass moment of inertia of a body around an axis can be determined from the mass moment of inertia around a parallel axis through the center of mass.

Moment of Inertia about the Axis of Flotation

- A rectangular cuboid has sides L, B, H. The moment of inertia (second moment) about the x axis which passes through its centre of mass is

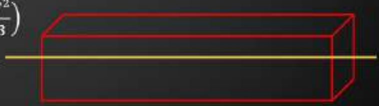
$$I_x = \int_{-H/2}^{H/2}\int_{-B/2}^{B/2}\int_{-L/2}^{L/2}(y^2+z^2)\,\rho dxdydz = L\int_{-H/2}^{H/2}\int_{-B/2}^{B/2}(y^2+z^2)\,\rho dydz$$

$$I_x = L\int_{-H/2}^{H/2}(y^3/3+z^2y)\Big|_{-B/2}^{B/2}\rho dydz = L\int_{-H/2}^{H/2}(B^3/12+Bz^2)\,\rho dz$$

$$I_x = L(HB^3/12+BH^3/12)\rho = LBH\rho\left(\frac{B^2}{12}+\frac{H^2}{12}\right) = M\left(\frac{B^2}{12}+\frac{H^2}{12}\right)$$

The moment of inertia of a cuboidal barge about its axis of flotation is obtained by using the parallel axis theorem. The distance between the two axes is H/2T where T is the draft

$I_f = M\left(\frac{B^2}{12}+\frac{H^2}{12}\right) + M(H/2-T)^2$. For H = T, $I_f = M\left(\frac{B^2}{12}+\frac{T^2}{3}\right)$



Radius of gyration $= i_m = \sqrt{\frac{I_f}{M}} = \sqrt{\left(\frac{B^2}{12}+\frac{H^2}{12}\right)+(H/2-T)^2}$

No. 31

(1990)
Corr.1997
(Corr.Aug. 1998)
(Rev.1 June 2000)
(Rev.2 June 2002)
(Corr.1 Jan 2004)
(Rev.3 Apr 2023)

Recommended procedure for inclining test

Inclining test unified procedure

1. Introduction

The purpose of this procedure is to achieve a satisfactory accuracy in the determination of the lightship weight and of the coordinates of its centre of gravity.

This general procedure is a recommendation. Alternative requirements which are considered to be equivalent to those specified by the following items may be accepted. Acceptance of such equivalents rests with the Society and, where the inclining test is performed to satisfy a statutory requirement, such equivalents also may be subject to the acceptance of the Flag Administration[1].

Where a surveyor of the Society is requested to attend the inclining test, the surveyor should verify that the test is conducted according to accepted procedures and that all basic measurements and data are correctly taken and recorded.

2. General Preparation for the Test

2.1 Information that should be submitted

The Instruction, containing the information of date and location of the test, responsible person, stability, inclining weight, schemes of inclining weight positions etc., should be presented to the Classification Society before the inclining test.

The following information should be available at the time of the inclining test as necessary:

– General arrangement drawing;
– Tank capacity plan;
– Hydrostatic curves;
– Draft marks locations.

2.2 The inclining test condition

2.2.1 The ship should be as near to completion as possible. Equipment used by the yard on board should be limited to the utmost extent possible. Prior to the inclining test, lists of all items which are to be added, removed, or relocated should be prepared. These weights and their locations should be accurately recorded.

Normally, the total value of missing weights should not exceed 2 percent and surplus weights, excluding liquid ballast, not exceed 4 percent of the light ship displacement. For smaller vessels, higher percentages may be allowed.

2.2.2 All objects should be secured in their regular positions. All weights which may swing or shift should be secured in their known position. If more than one sea stowage position is possible, the actual stowage position used during the test should be recorded.

[1] The inclining test procedure described in the DETAILED GUIDANCE FOR THE CONDUCT OF AN INCLINING TEST in Annex 1 to 2008 IS Code is considered as equivalent alternative.

2.2.3 The ship should be cleared of residues of cargo, tools, debris, scaffolding and snow. Icing of the inner and outer surfaces, the underwater hull included, should not be permitted.

2.2.4 All bilge water and other extraneous standing liquids should be removed. When draining individual tanks is impracticable, allowances for such liquids should be at the discretion of the Society.

2.2.5 All service tanks and machinery plant pipings should be filled as for the working condition.

2.2.6 In general, only the people participating in the inclining test should stay on board the ship.

2.2.7 All spaces should be safe for inspection.

2.3 Tank contents

2.3.1 Preferably, all tanks should be either full or empty. The number of tanks containing liquids should be kept to a minimum.

2.3.2 Soundings and density of liquids in tanks should be taken. Shapes of tanks which are partly filled should be known in order to determine the free liquid surface effect.

2.3.3 Adequate measures should be taken to preclude air pockets in completely full tanks. All connections between tanks should be closed and all empty tanks should be adequately dried.

2.4 Mooring Arrangements and Environmental Conditions

2.4.1 Mooring lines should be free of any tension in the transverse direction of the ship during the reading after each weight shift. No external moments should be brought upon the ship (from mooring lines, quay, etc.). If possible, the ship should be located in a calm, protected area free from external forces.

2.4.2 The depth of water under the hull should be sufficient to ensure that the hull will be entirely free of the bottom. Prior to the test the depth of water should be measured in as many locations as necessary to positively satisfy this requirement, taking into account tide differences, if applicable.

2.4.3 An ideal mooring arrangement would involve bow and stern lines on both sides of the ship attached at or near the centre-line. Longitudinal mooring lines should be as long as practicable. More commonly, a ship may be moored by bow and stern lines on one side only and supplemented by spring lines. Where a single bow or stern line is proposed, the surveyor should be assured that the ship's freedom of movement does not adversely effect the conduct of the experiment.

2.4.4 The ship may be moored by means of other special arrangement approved by the Society.

2.4.5 When tidal currents are present the experiment should normally be conducted at or around slack tide.

2.4.6 The ship's gangway should be in the stowed position and any shore gangway removed during the inclining test. As few cables, hoses, etc., as possible should be connected to shore. Those which are needed should be slack.

2.4.7 The test should not be conducted under adverse wind, wave and current conditions where the accuracy of the results cannot be assured.

2.5 Inclining Weights

2.5.1 For the inclining test, solid inclining weights normally should be used.

2.5.2 Use of water ballast transfer to incline the vessel may be permitted only in cases where it is impractical to incline the vessel using solid weights. If the transfer of water ballast is to be used, a detailed procedure, including calculation procedure, should be submitted to the society for approval prior to the experiment.

2.5.3 The total weight used should be sufficient to provide a minimum inclination of one degree and a maximum of four degrees of heel to each side of the initial position. However, in those cases where it is absolutely impractical to reach a minimum angle of 1 degree by use of solid weights or water ballast a lesser inclination angle may be accepted, provided that the requirements on pendulum deflection or U- tube difference in height in 2.6.1 should be complied with.

2.5.4 Each weight should be compact, impervious to water and shaped such that its centre of gravity may be accurately determined. It is recommended that not fewer than four weights (or sets of weights) be used, each approximately equal in mass, and that the inclining weights (or sets of weights) be positioned as symmetrically as possible and parallel to the centre line in places convenient for the shifting of weights and measurement of the arms.

2.5.5 Each inclining weight should be marked with an identification number. The inclining weights should have been weighed with a calibrated instrument to the satisfaction of the Surveyor.

2.6 Pendulums and Instruments

2.6.1 The use of three measuring devices is recommended to determine the vessel's inclination after each weight shift, however, a minimum of two devices should be used, one of which should be a pendulum or U-tube arrangement. The length and arrangement of pendulum/U-tube should be such as to ensure the accuracy of the readings of deflection/difference. The minimum deflection/difference, to each side of the initial position, corresponding to the total weight shift, should be 15 cm.

2.6.2 The use of a stabilograph may also be acceptable provided the calibration of the instrument has been verified to the Surveyor's satisfaction prior to the experiment. A trace of the recorded heel pattern should be included in the test report.

2.7 Trim and Stability

2.7.1 The vessel should be upright prior to the inclining. However, an initial list of the ship not exceeding 0.5° is permissible.

2.7.2 Excessive trim should be avoided for certain hull forms where changes in waterplane shape would occur in the region of the waterline when the ship is heeled. Such features should be taken into account to select a suitable draught and trim for the test.

2.7.3 The persons conducting the test should be satisfied that the vessel has adequate, positive stability and acceptable stress levels during the test. The estimated initial metacentric height should be at least 0.20 m.

No. 31 (cont)

3 Inclining Test and Record of Data

3.1 Person in Charge
A competent person should be designated in charge of the preparation and execution of the inclining test.

3.2 Accuracy of Data
Measurement of Inclining Test data should be as accurate as possible and to the satisfaction of the attending Surveyor.

3.3 Draught and Water Density Measurements

3.3.1 Draught/freeboard should be measured immediately before and verified after the test, to ensure that no significant changes in vessel's condition have occurred during the test.

3.3.2 Draughts/freeboards should be measured at fore and aft and midship draught marks at both sides. If the freeboards are not measured from the upper edge of deck line at side of freeboard deck or at the same frame locations as the draught marks, the locations and vertical datum should be stated.

3.3.3 A suitable boat with low freeboard should be available for the draught measurements.

3.3.4 To control the correctness of draught measurements, it is recommended to plot two waterlines by draught readings and by measured values of the freeboard when the latter is available. With correct measurements, both waterlines should coincide. In case of non-coincidence of separate points, additional measurements should be taken.

3.3.5 Sufficient water samples should be taken at suitable locations and depths to enable and accurate assessment of water density to be made.

3.4 Weight shifts and Inclination Measurements.

3.4.1 Two recommended procedures of shifting weights are shown in table 1.

Table 1

No. of Weights or Weight Groups

Weight Shifts	Four		Six	
	PS	SB	PS	SB
No. 0	2, 4	1, 3	2, 4, 6	1, 3, 5
No. 1	4	1, 2, 3	4, 6	1, 2, 3, 5
No. 2		1, 2, 3, 4		1, 2, 3, 4, 5,
No. 3	1	2, 3, 4	6	1, 2, 3, 4, 5
No. 4	1, 3	2, 4	2, 4, 6	1, 3, 5
No. 5	1, 2, 3	4	1, 2, 3, 4, 6	5
No. 6	1, 2, 3, 4		1, 2, 3, 4, 5,	
No. 7	2, 3, 4	1	1, 2, 4, 6	3, 5
No. 8	2, 4	1, 3	2, 4, 6	1, 3, 5

PS and SB denotes port and starboard sides of ship respectively.

The underlined numbers indicate the last weights or weight groups shifted.

3.4.2 The inclining weight positions should be marked on the deck to ensure that consistency in placement is achieved. The transverse shift distance should be as great as practicable and appreciable changes in longitudinal or vertical position when moving port to starboard and vice versa should be avoided.

3.4.3 The pendulum length should be measured from its point of suspension to the recording batten on which deflections are read.

3.4.4 Pendulum, or U-tube reading on the recording batten or scale may be registered by either of the following ways:

a) on the final stable position of the pendulum or liquid column after stopping of ship motions due to shifting of the inclining weight;

b) by marking the mean value within the range of residual oscillation.

3.4.5 When using other devices, angles of inclination should be recorded according to instructions supplied with each device.

3.4.6 Checks should be made in the process of the inclining test for each measuring device. These should, generally, be a progressive plot of angles of heel against heeling moments which should give a series of points lying about a straight line passing through (or close to) the origin.

If there is a deviation of points, either between the points for a particular weight movement, or from the straight line, the deflections and moments should be checked and corrected prior to the next weight movement.

3.4.7 Personnel should be instructed to remain on their assigned positions while inclination readings are being taken and a check should be made that all mooring lines, etc., remain slack following each weight shift until all deflections have been taken and recorded.

3.5 Other Relevant Data

3.5.1 In the case where the inclinations are carried out by means of transfer of water, it should be possible to evaluate accurately the weight and the centre of the shifted liquid in relation to the ship's heel and trim.

3.5.2 The weather conditions, i.e., wind speed and direction relative to the vessel, sea state, air and water temperatures, etc., during the test should be recorded.

4 Postponement of the Test

If during the course of an inclining test circumstances arise such that the aforesaid recommendations are not adhered to,the attending Surveyor should advise the Person in Charge that the results may not be accepted.

5. Test Report and Analysis of Lightship Data

5.1 The Builder/Owner should incorporate the data gathered during the test into a comprehensive test report, which may be combined with the analysis of the lightship data. Test readings not used in the final analysis should still be recorded in the report.

No. 31 (cont)

5.2 The Surveyor should ensure that the data given in the report is consistent with that gathered during the test and to sign the report.

5.3 The inclining test report and analysis, combined with the report or separately, should be submitted to the Society for review and acceptance of results as the basis for approval of the stability information of the ship.

End of Document

ITTC Quality System Manual

Recommended Procedures and Guidelines

Procedure

Inclining Tests

7.5 Process Control

7.5-02 Testing and Extrapolation Methods

7.5-02-07 Loads and Responses

7.5-02-07-04.7 Inclining Tests guideline

Disclaimer

Updated / Edited by	Approved
Stability in Waves Committee of the 29th ITTC	29th ITTC 2021
Date: 03/2020	Date: 06/2021

Table of Contents

1. PURPOSE OF PROCEDURE..............3
  1.1 Historical background.......................3
  1.2 Objectives...3
  1.3 Inclining tests.....................................3
2. MANDATORY CONDITIONS 3
  2.1 Environmental conditions.................3
    2.1.1 Wave effects.................................4
    2.1.2 Wind effects.................................4
    2.1.3 Current effects..............................4
    2.1.4 Sea water density..........................4
    2.1.5 Other effects.................................4
  2.2 Ship conditions..................................4
    2.2.1 Tanks..5
    2.2.2 Machinery.....................................5
  2.3 Numerical model and numerical tools...5
3. PROCEDURES 6
  3.1 Preliminary 6
  3.2 Survey 6
  3.3 Displacement measurements 6
  3.4 Inclining tests.....................................7
  3.5 Post-treatments..................................7
4. UNCERTAINTY 8
5. FINAL CHECK.....................................9
6. REPORT 9
7. RECOMMENDATIONS 10
8. REFERENCES 10

Inclining Tests

1. PURPOSE OF PROCEDURE

1.1 Historical background.

The historical background is essential to understand inclining tests procedure. It is very well explained by Nowacki and Ferreiro (2003). They show that the first theoretical written base came from Archimedes (ca. 287-212 B.C.) who explain stability for a homogeneous floating solid made of simple geometrical shapes (in those case the centre of gravity of the immersed part of the body is also the centre of volume). It was necessary to wait until the 17th century to found mention of procedures to estimate loads (which was needed to estimates taxes) and then displacement of the ship, by draught measurement and waterplane estimation (Anthony Deane in UK or Johannes Hudde in Netherlands). It must be noticed that those measurements were made at full scale and not using drawing plans. Notable theoretical improvement from Archimedes theory came simultaneously and independently from Bouguer and Euler, the first introduce the metacentre and the second the restoring moment, both around 1746-1749. Only few years later, in 1748, the first inclining test (referenced by Nowacki and Ferreiro) was performed in Brest (France) by Clairin des Lauriers on a new-built 74-gun ship *Intrépide* specifically made in order to test the new theory.

1.2 Objectives

The objectives of these procedures are to determine the effective displacement and position of centre of gravity of a ship in the situation during the experiments. Effective, means that the vertical position of the centre of gravity obtained, take into account the effect of free surface in tank, necessary for the stability evaluation. In particular, that means that no extrapolation at another displacement is considered (except for inclining weight and gear). As for the ATM standard guide (2014), this procedure is not applicable to vessels such as tension-leg platforms, semisubmersibles, rigid hull inflatable boats and so on.

1.3 Inclining tests

In this procedure, inclining tests is decomposed in four parts:

- Survey of the ship,
- The determination of displacements,
- The inclining experiment itself, which is based on a transverse shift of weight,
- Post-treatments.

The three first parts can be done in any order but must be done in the smallest delays between them in order to reduce change in mass and position. It must be specified in the report where the test have been performed, time and duration of the test.

2. MANDATORY CONDITIONS

2.1 Environmental conditions

During test, environmental conditions must be as favourable as possible and very well documented in the report of the experiments. It must be clearly specified in the report of the experiment:

- Wave conditions during draft measurements (should be less than 5 cm high but can be depend of the technical solution used),

- Wind conditions (should be less than 10 knots in gust but the limit can depend of the ship) including relative heading (temporal wind measurement is encouraged),
- Current if any, including tide,
- Other meteorological phenomenon, as rain (potential water accumulation on deck) or environmental phenomenon (wake of others ships) which can have an effect on the results have to be mentioned,
- For estimation of displacement by draft or freeboard measurements, seawater density must be measured.

2.1.1 Wave effects

In order to evaluate waves effects it is recommended to record temporal trace of measurements and to compare it to natural roll period of the ship. Effect of a relatively large roll resonance motion can be limited by computation of the average of the signal during a large period but it must be checked that other frequencies do not disturbed too much the motions.

2.1.2 Wind effects

In order to evaluate the effects of wind it is suggested to determine the heel angle due to the wind. For this purpose, the wind surface and others ship parameters can be the ones chosen in the stability booklet for the verification of the IMO's meteorological criteria with a raw approximated value for the vertical position of the centre of gravity. More data can be found in Blendermann (1996) for the draft coefficient. The wind velocity is the gust value (average over 5 seconds of duration). If the gust velocity is unknown, it can be estimated from the nominal wind velocity (average over 10 minutes at 10 meters high) usely given by measurements or hindcast. A gust factor of $\sqrt{2}$ has to be used from the nominal wind velocity and gust velocity. A relative reduction of the velocity can be obtained by taking into account the relative heading. The heel angle due to this gust wind velocity should be very small and in all case much smaller than the first heel inclination observed during the inclining experiments.

2.1.3 Current effects

Because effects of current is difficult to estimate it is recommended to avoid this situation whenever it is possible for example by conducting the experiment around slack tide (if any).

2.1.4 Sea water density

If necessary seawater density must be evaluated in one or more places around the ship depending of the conditions (for instance: suspicion of non-uniform density due to mixing of seawater and freshwater after rain or near a river). The sample of water used should be taken at a depth representative to the draft of the ship. In some cases, depending on density evaluation techniques used, temperature of the water has to be measured too.

2.1.5 Other effects

In order to avoid any external perturbation a continuous visual observation outside the ship is mandatory and must be reported.

2.2 Ship conditions

The ship must be as less linked as possible to the quay or other part, floatable or not, non-include in the ship definition. Mass modification and mass transfer should be avoid during the whole tests. Precautions should be taken to prevent both deliberate and accidental liquid transfer (Moore 2010).

It must be verified that there is enough water under the keel in order to be sure that the ship is entirely free in all experiment situation and during the entire duration of the experiment.

All gear should be secure in order to not shift during the experiment.

Draft at which abrupt changes in the water plane are to be avoid as possible (Moore, 2010). In order to check it, it is suggested to draught the metacentre movement with heel angle on a figure as the figure 1c of Dunworth (2014) or figure 2 of Karolius (2016). The non-linearity in the heeling moment vs. angle of heel can imply different solutions for postreatment.

2.2.1 Tanks

The results of those tests will gave the effective position of centre of gravity. That means that the effect of liquid in tanks shifting with heel will be included in the results. To avoid this phenomenon, it is preferable to fully empty as many tanks as possible and exclude the cases of reservoir or decks containing relatively small amount of liquid that could cause disturbances difficult to quantify (corner effects in particular). Excluding almost full tank is also preferable to avoid air pocket and venting problems.

Empty tank is the preferable situation. Slack tank can contain small quantity of fluid inducing a large free surface effect, non-expected linearity with heel angle and hysteresis phenomenon. Full tank can induce non-predictable free surface effect. In pressed tank, it can be observed air trap dependant to location of events which also induce inaccuracy in the results.

In the case of non-empty tank, free surface effect have to be include. If this effect can be remain constant during the experiment, usual correction obtain from surface inertia o the free surface in the tank can be used. If not, the shift of centre of gravity of the fluid have to be calculated for each inclinations and considered for the evaluation of the heeling moment.

Anti-roll tanks using liquid have, by definition, a large free surface effect. Those tanks have to be fully empty.

Pipe between tanks should be preferably closed.

2.2.2 Machinery

All unnecessary machinery should be shut down or isolated to prevent fluid tranfert. Fluid consumption are to be minimised and be drawn preferably by centreline tanks. Estimation of transfert of fluid during the experiments have to be reported (MAP, 2010).

2.3 Numerical model and numerical tools

The hull geometry must be very precisely described in order to estimate the level of uncertainty obtained. Using numerical model of the ship and modern numerical tools is necessary and have not only to be used through hydrostatic pre-calculated tables. The following elements must be specified:

- The sign convention for roll, heel and moment must be fixed,
- The reference of the numerical hull file used as input data, and the name of the numerical tools used,
- The uncertainty expected of the numerical hull definition (2D or 3D representation).
- The representation of the numerical hull file (out frame size or the overall size of the hull) and keel thickness if available,
- The list of appendices (including bow thruster, added keel etc…) taken into account and those, which are not.

3. PROCEDURES

3.1 Preliminary

The motion of the ship must be simulated before the inclining test with estimated value of the displacement and the centre of gravity (from sister ships results or of from the design office) in order to check:

- If the expected weight and their location are acceptable,
- The possible security problems due to excessive heel angle,
- The possible excessive change in the surface of flotation (it can induce the used of different numerical tools post-treatments),
- The influence of wind to estimate the maximum wind velocity acceptable for experiments.

3.2 Survey

Survey of the ships is important in order to check if everything is in an acceptable situation for the experiment (inventory, and if necessary limit, any weight, including liquid, with possible shift) and to be able to describe precisely the conditions of the ship during the experiments (conditions for light operational or full displacement). All tanks have to be verified (sounding, filling rate, density) and adequate measures have to be taken to preclude air pockets in about full tanks (IACS, 2004). It is also preferable to check all compartments and voids.

3.3 Displacement measurements

Displacement estimation can be done by many different ways depending on the size of ship, the knowledge of the geometry of the ship and the incertitude wanted. Because, at an early stage, this procedure is more focused on the determination of the centre of gravity where displacement is only an input data, this part will not be developed so much. Only a list of solutions with advantages and disadvantages is proposed for the moment.

Even if it should be the more accurate solution, direct mass measurement is possible only for small ship, then the basic theory used is to estimate the volume of water displaced. By water density multiplication, the displacement is directly accessible.

The usual solution to determine the displacement of a ship at sea is to measure drafts or freeboard (at least one, preferably four and usually six). Draft measurement on official draft mark are preferable for a better accuracy of the results. Depending of number of drafts measurements, it can be necessary to also measure heel and trim. The usual combinations is six draft measurements and one density measurement. The six draft measurements (at front, middle, and fore end for both side) can allow to take into account a potential hull deformation. For hull deformation, different approximations can be used and have to be well referenced in the final report of the experiment. The default one can be the one described in Principle of Naval Architecture (1988), and theoretically strictly justified for a rectangular barge with homogeneous weight repartition. In this case, parabolic deformation is expected and cannot be discarded because there is always a parabolic lines passing through three points. A more rigorous methodology could be obtained with modern tools using, for example, the mechanical inertia of the main section in order to find a more realistic hull deformation.

Those drafts measurements, eventually completed by angles, determine the exact position of the theoretical geometry of the ship beside the water surface considered as flat (water surface can be lightly deformed in order to simulate hull deformation because it can be easier than deforming the hull geometry). Then it is necessary to use a hull geometry to obtain volume of displaced water. Using the numerical model to obtain the best fit between all measurements if preferable than using only equivalent draft and hydrostatic tank table.

With adequate post treatments, it could be possible to estimate the volume of the water displaced, the mass of the ship, the position of the centre of gravity (longitudinal, transversal and vertical) located somewhere along the vertical from the centre of buoyancy which is determined by numerical tools.

3.4 Inclining tests

The methodology is to shift mass in the ship and measure precisely the induce motions of the ship. Any kind of mass shift can be used (even water transfer in ballast) but it must be kept in mind the objective of uncertainty because some of the solutions can be less precise than others. For example, a car carrying the weights and rolling in transverse rails gives excellent results because little rolling of the ship it induces and the movement of the weights can be and measured accurately (Moore 200) and adjusted. Calibrated mass transversely shifted on horizontal deck nearest the middle of the ship should be preferable. In any case, the shifted mass should be shaped so that its centre of gravity may be accurately determined (IACS, 2004) and already inboard for the draft measurement. In case of using water ballast heel and trim have to be taken into account. In case of outside location during storage and/or experiments impervious to water is needed (MAP, 2010).

For all shift mass (including the reference situation), the motion of the ship must be recorded. Measurement techniques is free but must be well documented in order to estimate the uncertainty of the motions measurements.

The final results should avoid effect of resonance roll usually observed after shifting weight.

The zero point is the reference attitude of the ships where final position of centre of gravity will be determined by the inclining test. Heeling points must be symmetrical (in number and in values) from this reference point.

Numerous tests are needed; the more they are, the more accurate will be the final results. At least one (the zero point) should be done twice.

During experiments, it must be report any observation of change in the surface of flotation due to heel from the zero point as reference (transom, bulbous, bilge keel, ..).

3.5 Post-treatments

It is strongly recommended to reproduce all the experiments with adequate numerical tools and not only using traditional formulas and hydrostatic table in order to take into account at all angles:

- The real displacement of the metacentre,
- The real location of the centre of buoyancy,
- The real free surface effect of tanks,
- The real initial attitude of the ship (trim and heel),

Those more accurate methodologies where re-demonstrated recently by Wilezynski (2015), Dunworth (2013, 2014 and 2015) Smith et al (2016) and Karolius et al (2018). Those methods allow more attitudes of the ship even with drastic change of the waterplane area with heel than traditional methods as mentioned by ASTM F1321-14 (2014).

At the end, results came usually from the slope of a graph (it is not mandatory that the regression line pass through the origin). The evaluation of this value should preferably use likelihood method in order to not minimize the incertitude along one axe from the other, as linear regression do. For each point the uncertainty of heeling moment and heel angle can be evaluate using following table and taken into account.

This graph (tangent of the heel angle vs heeling moment) should be draught during the test in order to found potential error before the end of the experiments (Moore 2010). Different typical error visible by examination of the slope of the graph is commented, for example, in IMO IS 2008.

4. UNCERTAINTY

Uncertainty study can be based on Whitrow (2003) and Hansen (1985) work. Whitrow include results of a questionnaire sent to a sample of naval architects, surveyors and Royal Navy personnel. The first step must be the evaluation of input errors. Whitrow propose a summary of input errors estimation, used in (MAP, 2010), and resumed and lightly completed in the following table.

Table 1: Inputs errors, mainly from Whitrow (1985)

Input parameter	**Error applied**	**Source**
visual draught reading (depending of the weather)	0.005 m	ASTM / MAP
draught mark vertical position	0.006 m	Whitrow
seawater density	0.00045 t/m^3	Withrow
tank content dip tape reading	0.003 m	Whitrow / ASTM
tank content gauge readings	3.2 %	Whitrow
density of liquids in tanks	0.00045 t/m^3	Whitrow
free surface moment of inertia of tanks	1.50 %	Hansen
solid deadweight estimates	1 %	MAP
KG of deadweight estimates	0.150 m	Hansen
weight of personnel	5.0 %	Hansen
KG of personnel	0.150 m	Hansen
inclining gear weight	4.15%	Hansen
KG of inclining gear	0.050 m	Hansen
longitudinal distance between forward marks and aft marks	0.100 m	Hansen
longitudinal distance between aft marks and midships marks	0.100 m	Hansen
longitudinal distance between aft marks and midships marks	0.100 m	Hansen
hull defelection parameter	10.0 %	Hansen
calculated volume	0.1 %	Hansen
volume due to appendages	1.0 %	-
difference of centre of buoyancy due to appendages	1.0 %	-
difference between design and build dimensions	0.06 m / 100 m	Hansen
vertical moment of displaced volume	0.05 %	Hansen
water line moment of inertia	0.09 %	Hansen
visual pendulum defection	0.002 m	-
hell angle measurement	0.01°	ASTM
KM Metacentre postion	1.0 %	-

pendulum lengths and reading	0.002 m	

Most of the items in the previous table depend on measuring device (for example, draught measurement can be improved by adequate device as draft tube) and numerical tools (using numerical tools allow to not use the hypothesis of a fixe metacentre which improve the final results).

It must be noticed that using a single weight for all inclinations imply that and error in the mass of this weight will be present in all points and will not be visually detectable on the slope of the graph, tangent of the heel angle vs heeling moment (Moore 2010).

5. FINAL CHECK

In order to check the quality of the experiments and the results it can be reported those questions:

- Were the mooring lines checked?
- Was the meteorological condition good enough?
- Was the wind speed measured?
- Was water under the hull measured/evaluated?
- Were redundant heel measurements used?
- If any, have all visual measurements been done properly?
- Are all measurements systems properly calibrated?
- Are weights properly calibrated?
- How many natural periods were used in the heel angle inclination method?
- Were metrological tools used?
- Were adequate systems used for draught or freeboard measurements?
- Is the Metacentre assumed fixe or not?
- Were numerical tools accurate?
- Are there enough measuring points? Are they symmetric?
- Personnel on board is minimized, limited to the crew, and informed of the requirements of an inclining test in order to not disturb the measurements?
- Is there more than one measurement for one inclination situation? At least two measurements for the initial zero angle, preferably three.

6. REPORT

Report must be consistent in form and in content in order to give the possibility to redo the calculation. Report have to use ITTC symbols and ISO units. Photographs of draft marks weight and location of measurements are strongly recommended.

Drawing of reading position of the draft/freeboard measurements must be given in the report. If the draft mark was used, the draft mark plan should be at least referenced.

In addition, in order to estimate the uncertainty, some measurements details must be specified:

- Technical solutions used to measure draft (visual, pressure, …) and the number of independent catch,
- Number positions and technical solutions used to estimate the water density (and if necessary the water temperature),
- How was determine the zero point (on board value, reference place in the ship, …),
- Details of numerical output used for the post-treatment (hydrostatic tables, equilibrium at each point, …), and the numerical tools used (including reference of the input data),
- Calibration certificate of all measurements systems used.

A list of all tank with levels and density is require. Additional information as position of centre of gravity, free surface effect and variation of this effect with heel and tank level have to be also given. The list must show how the filling level have been measured (by hand, electric gauges, …).

A list of all compartments and voids showing those which have been checked for the experiments.

Comprehensive description of the weight situation of the ship during the experiments (reference of the capacity plan used is needed).

Estimation of transfer of fluid during the experiments have to be reported.

The report must contain basic data, such as the weight of each inclining weight, the distance it was moved and the lengths and deflections of each pendulum, rather than only the moments and tangents, in order to permit further checking in case any data appear later to be questionable (Moore, 2010).

A chapter with evaluation of the uncertainty must be include in the report. For example, regression coefficient of the slope of the graph if used, or average value and variance of results from all inclinations.

The report must contain a conclusion from the personnel in charge of the experiments and some comparisons against similar ships results.

7. RECOMMENDATIONS

Final recommendations are:

- Use of modern tools for hydrostatic calculations is preferable,
- Use numerical model as precise as possible,
- Environmental condition should be as favourable as possible,
- Ship conditions should be the nearest to the loading conditions expected for the rest of trials in order to reduce extrapolations,
- Use modern and adequate metrological instrumentation,
- Roll period measurement is recommended during the tests in order to follow the change after the tests as suggested by IMO, using the link between the roll period and the metacentric high.

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Uncertainty analysis procedure for the ship inclining experiment

Article *in* Ocean Engineering · March 2016

DOI: 10.1016/j.oceaneng.2016.01.017

Martijn van Rijsbergen

Maritime Research Institute Netherlands

Ocean Engineering 114 (2016) 79–86

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Uncertainty analysis procedure for the ship inclining experiment

Michael D. Woodward [a,*], Martijn van Rijsbergen [b], Keith W. Hutchinson [c], Andrew Scott [d]

[a] *Australian Maritime College, University of Tasmania, Launceston, Australia*
[b] *MARIN, Wageningen, The Netherlands*
[c] *Babcock International Group Centre for Advanced Industry, Newcastle-upon-Tyne, UK*
[d] *Maritime & Coastguard Agency, Newcastle-upon-Tyne, UK*

ARTICLE INFO

Article history:
Received 31 July 2015
Accepted 16 January 2016

Keywords:
Inclining experiment
Uncertainty analysis
Ship stability
KG
GM

ABSTRACT

The inclining experiment is typically performed for all new-build ships and after any major refit. The purpose of the inclining experiment is to establish the vertical distance of the centre-of-mass of the ship above its keel in the lightship condition. This value is then taken as the point of reference when loading the ship, for establishing the 'in-service' stability, throughout the life of the ship. Experimental uncertainty analysis is commonly utilised in hydrodynamic testing to establish the uncertainty in a result as a function of the input variables. This can in turn be utilised to establish an interval about the result that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurement. This paper provides a methodology for calculating a confidence interval for the location of the centre-of-mass of a ship from an inclining experiment; and ultimately, in any load condition.

The uncertainty compared to an assumed metacentric height of 0.15 m is provided for four classes of ship: buoy tender 0.15 ± 0.15 m ($\pm 100\%$); super yacht 0.150 ± 0.033 m ($\pm 22.0\%$); supply ship 0.150 ± 0.047 m ($\pm 31.3\%$), container ship 0.150 ± 0.029 m ($\pm 19.3\%$), ropax 0.150 ± 0.077 m ($\pm 100\%$).

1. Aims and objectives

The aim is to establish procedures for identifying the experimental uncertainty in the estimate of the centre-of-mass height above the keel (referred to as $\overline{KG}$) by method of an inclining experiment (IE).

The first objective is to give procedures for performing a pre-test analysis that can be employed to identify the best course of action for reducing the experimental uncertainty. The second objective is to give procedures for performing a post-test analysis that can be employed to identify a confidence interval for the resulting estimate of $\overline{KG}$.

2. Background

The IE is a required procedure [unless exceptions apply; see IMO, 2008] for all new-build ships and after any major refit. The purpose of the IE is to establish $\overline{KG}$, in the lightship condition. This value is then taken as the point of reference when loading the ship, for establishing the 'in-service' $\overline{KG}$, throughout the life of the ship. An accurate estimate of the limiting $\overline{KG}$ is absolutely necessary for the safe operation of the ship, so as to ensure adequate stability. Clearly, this is dependent on an accurate estimate of the lightship $\overline{KG}$ obtained from the IE.

While typically all attempts are made to conduct the IE in a manner that minimises the introduction of error, many potential sources of error exist. For example, all attempts are made to remove the influence of fluid free-surface effects, by emptying or pressing-full all tanks. Any suspended loads are secured or removed and anything that may move is removed or made secure. Similarly, all attempts are made to conduct the IE in calm conditions, when the effect of wind, waves, current and the wash from passing ships is minimised.

Notwithstanding all attempts to minimise errors, sources of uncertainty will always be present – uncertainty being different from error. Due to the stochastic nature of the world, all input variable measurements are only known with limited accuracy. The uncertainty in the results (in this case the estimate of $\overline{KG}$) is dependent on the magnitude of the uncertainties of each input variable and on the particular sensitivity of the results to each input, which is dependent on the form of the data reduction equations.

* Corresponding author. Tel.: +44 191 222 6750; fax: +44 191 222 5491.
E-mail addresses: michael.woodward@utas.edu.au (M.D. Woodward), m.x.v.rijsbergen@marin.nl (M.v. Rijsbergen), keith.w.hutchinson@babcockinternational.com (K.W. Hutchinson), andrew.scott@mcga.gov.uk (A. Scott).

http://dx.doi.org/10.1016/j.oceaneng.2016.01.017

2.1. Overview of the inclining experiment

Explanations of the procedure for an IE exist in many texts, with the fundamental description given by (IMO, 2008). In brief, an IE is conducted by forcibly inclining the ship by moving a known weight a known transverse distance across the ship. The inclination is measured from the movement of a plumb-line relative to a mark-board, that is horizontal when the ship is upright. Typically, two or three plumb-lines are employed (forward-amidships-aft) to account for any torsional deformation of the ship. Then, the metacentric height $\overline{GM}$ is obtained according to,

$$\overline{GM} = \frac{wd}{\rho\nabla \tan\theta} \tag{1}$$

where w is the mass of the weight moved, d is the distance the weight is moved, ρ is the water density, ∇ the displaced volume of the ship and θ is the induced heel-angle. Eq. (2) calculates the height of the metacentre above the centre-of-buoyancy as a function-of-form for the given draught.

$$\overline{BM} = \frac{I}{\nabla} \tag{2}$$

In Eq. (2), I is the transverse second moment of area of the water-plane at that draught. The height of the centre-of-buoyancy above the keel $\overline{KB}$, (the centroid of volume at that draught) being a geometric property, is readily calculated from the hydrostatic particulars. The height of the mass-centroid (centre of gravity) above the keel $\overline{KG}$, is then given by Eq. (3).

$$\overline{KG} = \overline{KB} + \overline{BM} - \overline{GM} \tag{3}$$

2.2. Overview of experimental uncertainty analysis

The expression of experimental uncertainty is generally dealt with by National Metrology Institutions. However, for the application of specific procedures, scientific committees or societies more often take responsibility. Considering hydrodynamic testing, the International Towing Tank Conference (ITTC) provides Procedures and Guidelines for many aspects of ship related testing. Though the IE is not within its scope; one procedure (ITTC, 2008) does have relevant information, as it describes the application of uncertainty to hydrodynamic testing. Also, the development of all new procedures and guidelines should be expressed in line with the International Organisation for Standards (ISO), Guide to the Expression of Uncertainty in Measurement (ISO/IEC, 1995).

In accordance with ISO uncertainties can be categorised into Type-A and Type-B. Type-A uncertainties are components obtained utilising a method based on statistical analysis of a series of observations. Type-B uncertainties are components obtained by means other than repeated observations. For the IE most measurements are Type-B; or at least must be treated as such due to the nature of the measurement methods applied. In many respects however, the distinction is arbitrary as, for onward calculations, Type-A and Type-B uncertainties are treated in the same way. In its most simple form, the combined uncertainty in a result $u_c(y)$ is the root-sum-square of the standard uncertainties $u(x_i)$ for each ith input variable multiplied by a corresponding sensitivity coefficient c_i for each variable, given by Eq. (4).

$$u_c^2(y) = \sum_{i=1}^{N} c_i^2 u^2(x_i) \tag{4}$$

Of course, this is a somewhat simplified form, neglecting the possibility of correlation between various variables. Such correlation will be dealt with later in the paper, but for the immediate discussion this simplified form is sufficient. The sensitivity coefficient c_i is the partial derivative of the results with respect to any given input variable x_i; given by Eq. (5).

$$c_i = \frac{\partial y}{\partial x_i} \tag{5}$$

The standard uncertainty of any given variable is relatively easy to obtain. If a sufficiently large number of samples of measurement data are available, the Type-A standard uncertainty for a single sample is equal to the sample standard deviation. If there is no recent measurement data available, the limits of the uncertainty need to be estimated or e.g. taken from a specification of a measurement device. With these limits and an assumed probability distribution, the Type-B standard uncertainty can be derived (for application guidance see (ISO/IEC, 1995) Section 4.3).

3. Derivation of sensitivity coefficients

By assuming linearity, for small changes in draught T, for the variables $\overline{KB}$, I and ∇, the sensitivity coefficients can be obtained directly. Going to the hydrostatic tables for the ship, the tangent to the curves at the lightship 'as inclined' draught are utilised to obtain the coefficient α_n and constant terms β_n shown in Eq. (6).

$$\begin{aligned} \overline{KB} &= \alpha_1 T + \beta_1 \\ I &= \alpha_2 T + \beta_2 \\ \nabla &= \alpha_3 T + \beta_3 \end{aligned} \tag{6}$$

Eq. (7) is obtained by substituting Eqs. (1), (2) and (6) back into Eq. (3).

$$KG^{(\alpha_1 T + \beta_1) + \left(\frac{\alpha_2 T + \beta_2}{\alpha_3 T + \beta_3}\right) - \left[\frac{wd}{\rho(\alpha_3 T + \beta_3)\tan\theta}\right]} \tag{7}$$

Simplifying as much as possible, the relevant sensitivity coefficients are then given by Eqs. (8)–(12), for the ith heel-angle measurement induced by weight shift. In Eq. (12) the gradient terms α_n are replaced with the specific differential terms, as they are perhaps more meaningful.

$$c_{1i} = \frac{\partial\overline{KG}}{\partial\theta_i} = \frac{wd}{\rho\nabla \sin^2\theta_i} \tag{8}$$

$$c_{2i} = \frac{\partial\overline{KG}}{\partial\rho} = \frac{wd}{\rho^2\nabla \tan\theta_i} \tag{9}$$

$$c_{3i} = \frac{\partial\overline{KG}}{\partial w} = -\frac{d}{\rho\nabla \tan\theta_i} \tag{10}$$

$$c_{4i} = \frac{\partial\overline{KG}}{\partial d} = -\frac{w}{\rho\nabla \tan\theta_i} \tag{11}$$

$$c_{5i} = \frac{\partial\overline{KG}}{\partial T} = \frac{\partial\overline{KB}}{\partial T} + \frac{1}{\nabla}\left(\frac{\partial I}{\partial T} - \frac{\partial\nabla}{\partial T}\overline{BM} + \frac{\partial\nabla}{\partial T}\frac{wd}{\rho\nabla \tan\theta_i}\right) \tag{12}$$

The uncertainty in the ship geometry is an important consideration in comparison to the drawings. This takes into account the uncertainty in the position of the centre-of-buoyance and the metacentre, from which all other calculations are taken. Taking the partial derivatives of Eq. (3) (with Eqs. (1) and (2) substituted accordingly) the sensitivity coefficients given by Eqs. (13)–(15) are obtained.

$$c_6 = \frac{\partial\overline{KG}}{\partial\nabla} = \frac{1}{\nabla^2}\left(\frac{wd}{\rho \tan\theta_i} - I\right) \tag{13}$$

$$c_7 = \frac{\partial\overline{KG}}{\partial I} = \frac{1}{\nabla} \tag{14}$$

$$c_8 = \frac{\partial \overline{KG}}{\partial \overline{KB}} = 1 \tag{15}$$

4. Identification of the variable uncertainties

With various types of calculation involved in an analysis, a description of uncertainty in 'levels' is more practical. That is to say, use the sensitivity coefficient and standard uncertainty at one level to output the combined uncertainty. Then use this as the input standard uncertainty at the next level. An example of such an approach is implemented within this methodology, utilising the output combined uncertainty for the heel angle measurement as input standard uncertainty for the next calculations. The next section will look at the necessary variables and provides practical methods for obtaining the required values.

4.1. Uncertainty in the heel-angle by plumb-line measurement, $\boldsymbol{u}(\boldsymbol{\theta})$

Taking the length of the plumb-line to be l, and the horizontal measured plumb-line displacement to be η, then the heel angle θ, is given by Eq. (16).

$$\theta = \tan^{-1}\left(\frac{\eta}{l}\right) \tag{16}$$

The combined uncertainty for the measured heel angle is dependent both on the standard uncertainty in l and in η; as given by Eq. (17).

$$u^2(\theta) = \left(\frac{\partial\theta}{\partial l}\right)^2 u^2(l) + \left(\frac{\partial\theta}{\partial\eta}\right)^2 u^2(\eta) \tag{17}$$

Typically, the plumb-line will be swinging back-and-forth in an approximately sinusoidal oscillation. The value for η is typically obtained by trying to estimate the middle of the plumb-line swing. Ideally the estimate of the uncertainty would be obtained as the sample standard deviation of the signal, over a sufficiently large number of cycles. In the case of the IE however, the time history of the plumb-line displacement is typically not recorded. Taking the extremes of the swing would somewhat overestimate the uncertainty. A reasonable estimate for uncertainty in the plumb-line displacement measurement can nevertheless be obtained in terms of the approximate maximum and minimum observed values. The standard deviation of a sinusoidal signal σ_S, of amplitude ζ can be shown to be as given in Eq. (18); with proof provided in Appendix A.

$$\sigma_S = \frac{\zeta}{\sqrt{2}} \tag{18}$$

Assuming that the swinging plumb-line motion is a pure sinusoid, then the signal height is the maximum observed value minus the minimum observed value. The amplitude is by definition half the signal height; given by Eq. (19),

$$\zeta = \frac{\left(s^{max} - s^{min}\right)}{2} \tag{19}$$

where s^{max} is the maximum observed swing of the plumb-line and s^{min} the minimum. Considering that the plumb-line will be oscillating about both the reference position and then later about the measurement position, the uncertainty related to both situations needs to be taken into account. If the magnitude of the oscillations is not far different in either case, the uncertainties in the amplitudes are correlated. Then the standard uncertainty in η is equal to $2\sigma_S$. Substituting Eq. (19) back into Eq. (18), and multiplying by two, the uncertainty in the estimated plumb-line displacement, as giving in Eq. (20), is obtained.

$$u(\eta) = \frac{\left(s^{max} - s^{min}\right)}{\sqrt{2}} \tag{20}$$

If the induced heel-angle is given by Eq. (16), then the sensitivity is the partial derivative of θ with respect to η, given by Eq. (21).

$$\frac{\partial}{\partial\eta}\left[\tan^{-1}\left(\frac{\eta}{l}\right)\right] = \frac{l}{\eta^2 + l^2} \tag{21}$$

In a similar way, the sensitivity with respect to the plumb-line length is given by Eq. (22).

$$\frac{\partial}{\partial l}\left[\tan^{-1}\left(\frac{\eta}{l}\right)\right] = \frac{-\eta}{\eta^2 + l^2} \tag{22}$$

It is important to remember that although several plumb-line measurements are taken at various locations, these are not independent measurements of the same thing. In actual fact, these are discrete measurements each contributing to a part of a data reduction equation. In this case the data reduction equation is rather simplistic, being simply the mean value for N plumb-lines. From this, the sensitivity coefficient for each measurement can be shown to be equal to $\frac{1}{N}$. Bringing together Eqs. (20)–(22) into the form given in Eq. (17), the uncertainty in the heel-angle induced by the ith moment (induced by weight shift) is obtained as given in Eq. (23). Here, the standard uncertainty of the jth plumb-line length $u(l_j)$ is the combination of two uncertainties. The first is the best measurement capability of the measuring equipment utilised to measure it, including components such as calibration uncertainty and resolution. The second is the uncertainty in the measuring process with contributions such as alignment, repeatability.

$$u^2(\theta_i) = \sum_{j=1}^{N}\left(\frac{1}{N}\right)^2\left\{\left[\frac{l_j}{\left(\eta_{ji}^2 + l_j^2\right)}\right]^2\left[\frac{\left(s_{ji}^{max} - s_{ji}^{min}\right)}{\sqrt{2}}\right]^2 + \left[\frac{-\eta_{ji}}{\left(\eta_{ji}^2 + l_j^2\right)}\right]^2 u^2(l_j)\right\} \tag{23}$$

4.2. Uncertainties related to the water density, $\boldsymbol{u}(\boldsymbol{\rho})$

Typically, the water density around the ship will be sampled at several locations and at more than one depth. The average water density is then taken as the basis for subsequent calculations. Utilising this method there are two main areas to be considered. Firstly, there is uncertainty related to the best measurement capability of the device employed to measure the water density. Secondly, there is the uncertainty due to the measuring process.

If for example, the water density is determined by measuring the specific gravity, then the best measurement capability is the combined uncertainty of the calibration uncertainty as provided by the calibration certificate and the resolution (smallest scale division on the gauge), $u(\rho_{bmc})$. The second source of uncertainty to be considered is the uncertainty in the measuring process. The main contribution to this uncertainty is the process of sampling. Since the samples can be assumed to be independent, the standard uncertainty of the mean value can be calculated by dividing the sample standard deviation by the square root of the number of samples, $u(\rho_\sigma)$.

The uncertainty for any necessary temperature correction associated with the hydrometer reading can also be taken into consideration by applying ITTC (2011). However, based on the findings of the case studies (in Section 8), such finesse may be superfluous. The total uncertainty associated with the water density $u(\rho)$, is then given by the root-sum-square of the component uncertainties; given by Eq. (24).

$$u^2(\rho) = u^2(\rho_{bmc}) + u^2(\rho_\sigma) \tag{24}$$

4.3. Uncertainty in the weight of objects moved, $\boldsymbol{u}(\boldsymbol{w_i})$

In an ideal situation, a quayside crane will be employed to move the inclining weights. However, more typically, a forklift truck will be employed to move the inclining weights and then return itself to a known position. Similarly, the staff involved in conducting the IE must also return to known positions before the necessary measurement readings are made. The uncertainty related to items such as the forklift, the personnel and any other equipment are covered in Section 4.7.

The uncertainty of the mass of each inclining weight is assumed to be equal to the calibration uncertainty of the measuring device utilised to weigh it. If a given weight is made up of multiple smaller weights, each having been weighed separately on the same device, then their uncertainties in mass are correlated. This results in a simple addition of the individual uncertainties instead of a root-sum-square calculation. Eq. (25) gives the uncertainty for each ith inclining weight, where N is the number of component weights making up each inclining weight.

$$u(w_i) = \sum_{j=1}^{N} u(w_j) \tag{25}$$

4.4. Uncertainty in the distance objects are moved, $\boldsymbol{u}(\boldsymbol{d})$

When considering the placement of inclining weights, two sources of uncertainty must be taken into account. Specifically, the uncertainty in the location of the marks made for positioning the weights and the uncertainty of the placement of the weights with respect to those marks.

If for example a measurement mark were made on a piece of white paper with a fine pencil and a steel rule calibrated in millimetres, then it would be fair to say that the uncertainty was plus-or-minus a millimetre. Conversely, just because a tape measure calibrated in millimetres is utilised to mark the placement of the inclining weights, to assume such accuracy would be spurious. Stretching a tape-measure across a, perhaps uneven, deck and marking with chalk or sticky-tape, or some such similar crude marking, could be more realistically considered as plus-or-minus a centimetre. Of course, a more sophisticated method might be employed such as a laser measurement, to improve accuracy. Notwithstanding, the task at hand is to make a realistic judgment of the accuracy that can be assumed with the tools utilised. When taking multiple measurements to calculate the total distance the total measurement uncertainty is taken as the root-sum-square of the contributing measurement uncertainties (or simply the sum if the individual measurements are correlated e.g. taken with the same device). Then, the measurement of the mark d_{Mi} relating to the ith inclining weight has an uncertainty $u(d_{Mi})$.

As with the above, when trying to line up an inclining weight (itself on a forklift truck pallet) with a mark made with sticky-tape, then to assume millimetre accuracy would be spurious. As above, the task at hand is to make a realistic judgment of the accuracy that can be assumed with the tools utilised. Then, alignment with respect to the mark d_{Ai} for the ith inclining weight has an uncertainty $u(d_{Ai})$.

For each ith inclining weight moved, the total uncertainty is the root-sum-square of the uncertainty related to the marks and the uncertainty related to the position with respect to the marks. Then, Eq. (26) gives the uncertainty of the distance the ith inclining weight is moved.

$$u^2(d_i) = u^2(d_{Mi}) + u^2(d_{Ai}) \tag{26}$$

4.5. Uncertainties related to the draught marks, $\boldsymbol{u}(\boldsymbol{T})$

The estimate of the draught marks has two sources of uncertainty. The uncertainty related to the position of the draught marks and the uncertainty of the water-level with respect to those marks. For the first of these, the draught mark represents a distance above the keel. The flat bottom of the ship however has itself some variation. Realistically, adjudging the 'flatness' of the keel to be, say plus-or-minus 10 mm, then the uncertainty of the draught marks must be at least this. Depending on the construction methods and the quality of build, the task is to make a realistic judgment on the likely building tolerance; here represented by $u(\epsilon_M)$.

In addition to this, the effect of surface tension causes an uncertainty in the exact position of the water level due to the curved meniscus; here represented by $u(\gamma)$. The magnitude of this depends on the roughness of the surface that the fluid is in contact with. A typical value would be in the order of 3 mm and should be added (as a root-sum-square) to the other draught related sources of uncertainty.

As the water surface is invariable moving and, to some extent, the ships itself, then the measurement is problematic. This can be improved upon by the use of a glass tube to damp out the wave action; but some oscillation will always be present. For comparison with the above, typical amplitudes could be in the order of 50 mm. For simplicity, a reasonable estimate of the uncertainty may be obtained by multiplying the oscillation amplitude by the standard deviation of a sinusoidal signal; described in Section 4.1 and Appendix A. Letting the maximum local observed jth draught mark be τ_j^{max} and the minimum be τ_j^{min}, then Eq. (27) gives the combined uncertainty for the draught measurement as,

$$u_c^2(T) = \sum_{j=1}^{3} c_{5j}^2 \left[\left(\frac{\tau_j^{max} - \tau_j^{min}}{2\sqrt{2}} \right)^2 + u^2(\gamma) + u^2(\epsilon_M) \right] \tag{27}$$

where $j=1$ corresponds to the forward draught measurement, 2 the measurement amidships and 3 the aft measurement. Taking into consideration the hog/sag correction and the layer correction, the draught at the longitudinal centre of flotation is given in Eq. (28) (which is typically the reference point in tables describing the ship hydrostatic characteristics),

$$T_{LCF} = \frac{1}{6}(T_1 + 4T_2 + T_3) + LCF\,\frac{(T_3 - T_1)}{L_{bm}} \tag{28}$$

where LCF is the position of the longitudinal centre of flotation with respect to amidships and L_{bm} is the length between draught marks. The corresponding sensitivity coefficients c_{5j} are given by Eqs. (29)–(31).

$$c_{51} = \frac{\partial T_{LCF}}{\partial T_1} = \frac{1}{6} - \frac{LCF}{L_{bm}} \tag{29}$$

$$c_{52} = \frac{\partial T_{LCF}}{\partial T_2} = \frac{4}{6} \tag{30}$$

$$c_{53} = \frac{\partial T_{LCF}}{\partial T_3} = \frac{1}{6} + \frac{LCF}{L_{bm}} \tag{31}$$

By taking an average from N draught measurements and assuming that their uncertainties are independent, the uncertainty of the average draught is given by Eq. (32).

$$u^2(\overline{T}) = \sum_{i=1}^{N} \left(\frac{1}{N} \right)^2 u^2(T_i) \tag{32}$$

4.6. Uncertainties related to hull-form tolerances, $\boldsymbol{u}(\nabla)$, $\boldsymbol{u}(\boldsymbol{I})$ and $\boldsymbol{u}(\overline{\boldsymbol{KB}})$

Taking the usual definition of volume to be $\nabla = LBTC_B$ and taking logarithms, Eq. (33) is obtained.

$$\log \nabla = \log L + \log B + \log T + \log C_B \tag{33}$$

Recognising that if $y = \log x$ then $\frac{dy}{dx = \frac{1}{x}}$ so $dy = \frac{dx}{x}$, Eq. (34) is obtained.

$$\frac{\partial \nabla}{\nabla} = \frac{\partial L}{L} + \frac{\partial B}{B} + \frac{\partial T}{T} + \frac{\partial C_B}{C_B} \tag{34}$$

Considering the change in any given parameter to be the manufacturing tolerance in that given dimension (denoted ϵ), then Eq. (34) can be rewritten. To assign a tolerance to the block coefficient an assumption is made that any horizontal transverse measurement from the centre-line has the same tolerance as that of the breadth. This leads to a simplification (factor of 2 on breadth tolerance) where Eq. (35) gives the uncertainty in displaced volume.

$$u(\nabla) = \nabla\left(\frac{\epsilon_L}{L} + 2\frac{\epsilon_B}{B} + \frac{\epsilon_T}{T}\right) \tag{35}$$

In a similar way, assuming that the water-plane area can be approximated by a rectangle, the second moment of area is given by $I = \frac{LB^3}{12}$. Again taking logarithms and with the same process as above, Eq. (36) gives the uncertainty in the transverse second moment of water-plane area.

$$u(I) = I\left(\frac{\epsilon_L}{L} + 3\frac{\epsilon_B}{B}\right) \tag{36}$$

From a similar analogy, Eq. (37) gives the uncertainty in the height of the centre of buoyancy.

$$u(\overline{KB}) = \overline{KB}\left(\frac{\epsilon_T}{T}\right) \tag{37}$$

4.7. Uncertainties related to the removal or addition of weights $\boldsymbol{u}(\boldsymbol{\delta G})$

It is necessary to remove the inclining weights and other equipment from the ship after the IE is finished. The estimate of $\overline{KG}$ must then be amended accordingly. In addition, though not ideal, the ship may well have weights on-board that will be removed or still to be added. Eq. (38) gives a change in the position of the ships centre-of-gravity due to the addition on removal of an ith weight of vertical distance h_i from the original centre-of-gravity (w_i will be a negative value for the removal of a weight).

$$\delta G_i = \frac{h_i w_i}{\Delta + w_i} \tag{38}$$

The sensitivity coefficients for a shift in the centre-of-gravity, due to the addition or removal of an ith weight are given in Eqs. (39)–(41).

$$c_{9i} = \frac{\partial(\delta G_i)}{\partial w_i} = \frac{h_i \Delta}{(\Delta + w_i)^2} \tag{39}$$

$$c_{10i} = \frac{\partial(\delta G_i)}{\partial h_i} = \frac{w_i}{\Delta + w_i} \tag{40}$$

$$c_{11i} = \frac{\partial(\delta G_i)}{\partial \Delta} = \frac{-w_i h_i}{(\Delta + w_i)^2} \tag{41}$$

The standard uncertainty of the mass of the ith weight $u(w_i)$ and the height of the ith weight $u(h_i)$ should be taken as the combined uncertainty of the calibration uncertainty of the devices utilised to measure them (or a realistic estimate) and the uncertainty in the measurement. The standard uncertainty for the displacement $u(\Delta)$ can be obtained from the density and volume uncertainties (given in Eqs. (24) and (35) respectively) by Eq. (42).

$$u(\Delta) = \Delta\left[\frac{u(\nabla)}{\nabla} + \frac{u(\rho)}{\rho}\right] \tag{42}$$

4.8. Uncertainties related to free-surface corrections $\boldsymbol{u}(\boldsymbol{FSC})$

After the IE is conducted a correction to $\overline{KG}$ may be required-if there are any free-surfaces aboard the ship during the test. Assuming tanks to be approximately rectangular, the free-surface correction is given by Eq. (43). In the equation ϱ_i is the density of the fluid in the ith tank and a_i and b_i are the length and breadth of that tank respectively.

$$FSC = \frac{\varrho_i}{\rho}\frac{a_i b_i^3}{12\nabla} \tag{43}$$

The sensitivity coefficients for the free-surface correction are given in Eqs. (44)–(48).

$$c_{12i} = \frac{\partial FSC}{\partial \varrho_i} = \frac{1}{\rho}\frac{a_i b_i^3}{12\nabla} \tag{44}$$

$$c_{13i} = \frac{\partial FSC}{\partial \rho} = -\frac{\varrho_i}{\rho^2}\frac{a_i b_i^3}{12\nabla} \tag{45}$$

$$c_{14i} = \frac{\partial FSC}{\partial a} = \frac{\varrho_i}{\rho}\frac{b_i^3}{12\nabla} \tag{46}$$

$$c_{15i} = \frac{\partial FSC}{\partial b} = \frac{\varrho_i}{\rho}\frac{a_i b_i^2}{4\nabla} \tag{47}$$

$$c_{16i} = \frac{\partial FSC}{\partial \nabla} = -\frac{\varrho_i}{\rho}\frac{a_i b_i^2}{12\nabla^2} \tag{48}$$

The standard uncertainty for the density of fluid in the ith tank $u(\varrho_i)$ is obtained in a similar way as the uncertainty for the seawater density $u(\rho)$; see Section 4.2. The standard uncertainties for the length a_i and breadth b_i of each tank are taken as the calibration uncertainty of the device utilised to measure them, and the uncertainty in ship displaced volume $u(\nabla)$ as given in Eq. (34).

4.9. Other sources of uncertainty

4.9.1. Uncertainty of the position of the inclining weight centroid

While methods do exist for finding the centroid of a mass by direct measurement, they are unlikely to be undertaken. Provided the inclining weights are not rotated when moved, the position of the centroid is not important. That is to say, the distance moved by the centroid will be the same as the distance moved by any point of reference. Therefore, careful attention to the procedure can remove this source of uncertainty.

4.9.2. Uncertainty of the marks made on deck for longitudinal placement

The difficulty with the longitudinal marks is more one of finding a suitable point of reference. If a hatch combing or accommodation block bullhead is utilised for reference, then the uncertainty in their placement must be considered. Sighting transversely across the deck, at right angles to the parallel-mid-body, is again not without difficulties. Considering this, an uncertainty of approximately 10 cm is reasonable. While this may sound alarmingly large, remember this value will be multiplied by a sensitivity coefficient. This then considers the sensitivity coefficient for the change in I and ∇ with respect to a small change in trim. These terms will be negligibly small provided the trim is minimal.

4.9.3. Uncertainties when utilising ballast tanks as inclining weights

In some cases the general arrangements of the ship prohibit the use of mobile inclining weights. In such cases, the ballast tanks are employed as an alternative. For example, a port side tank may be filled. Then, when ready, the tank will be emptied and an equivalent tank on the opposite side filled. In such cases, the uncertainty is related to the relative positions of the centroid of each tank, the volume of each tank and the density of the fluid used to fill them. Taking the root-sum-square for these items then the sensitivity can be taken with respect to the induced moment. Also, the uncertainty in any free-surface correction must be taken into account.

5. Combined uncertainty

It is not uncommon in an IE to take multiple measurements by additional or repeated weight movements. Estimates of the random uncertainty from the standard deviation of the mean are possible, if multiple truly independent measurements are made. This however provides only the uncertainty in the estimate of $\overline{GM}$ and not $\overline{KG}$. The estimates of $\overline{KB}$ and $\overline{BM}$, both necessary for the estimate of $\overline{KG}$, are dependent on parameters also measured as part of the IE; and must be dealt with appropriately. Notwithstanding, more likely the individual measurements are not truly independent. For example, the second induced angle may include the moment from both the first and second inclining weights. Similarly, a third weight move may be achieved by returning the first weight to its original position. The uncertainties should thus be assumed to be fully correlated and combined accordingly. As, in this case, the data reduction equation is a simple average then the uncertainty for fully correlated variables is also a simple average, given by Eq. (49).

$$u_c\left(\overline{KG}\right) = \frac{1}{N}\sum_{i=1}^{N} u\left(\overline{KG}_i\right) \tag{49}$$

For items that are to be removed (as described in Sections 4.7 and 4.8), the corresponding uncertainties should be included after the samples of $u\left(\overline{KG}_i\right)$ are combined utilising Eq. (49).

6. Expanded uncertainty (*U*)

The combined uncertainty is defined as equivalent to one standard deviation. This corresponds to a confidence interval of approximately 68% if the uncertainty can be assumed to be normally distributed. In engineering applications a higher confidence interval when expressing the uncertainty is more practical. This can simply be achieved by multiplying the combined uncertainty u_c by a coverage factor k, which gives the expanded uncertainty U. For example, assuming a normal distribution, $k=2$ gives a U_{95} with a 95% confidence interval.

7. Method

An experimental uncertainty analysis may be performed prior to the IE, as a process of experimental design, or post analysis to establish a confidence interval in the result. The main difference is that, prior to the test being conducted, the limits of some parameters must be estimated. In either case the calculations are relatively straightforward and can be performed easily with a typical spread-sheet application. Also in either case, the process is predominantly the same and can be structured into seven key steps as described in Fig. 1.

Utilising the methods outlined for Step 6 (Sections 4.7 and 4.8), the uncertainty in $\overline{GM}$ for any load condition can readily be obtained.

8. Case studies

To establish the fitness-for-purposes of the procedure and to meet with the objectives of the paper, the procedure is applied to five case-study ship inclining experiments. In line with the objectives of the paper, the results are utilised to find the uncertainty in the estimated $\overline{KG}$ and, explore the origins of contributing uncertainties to help target improvements in the experimental procedures.

As the data is historic, not all of the necessary parameters specified by this procedure are available. Nevertheless, the data serves perfectly well to perform a typical pre-test analysis. This has in fact some advantages in that environmental inputs are made the same for all five ships, making the results more directly

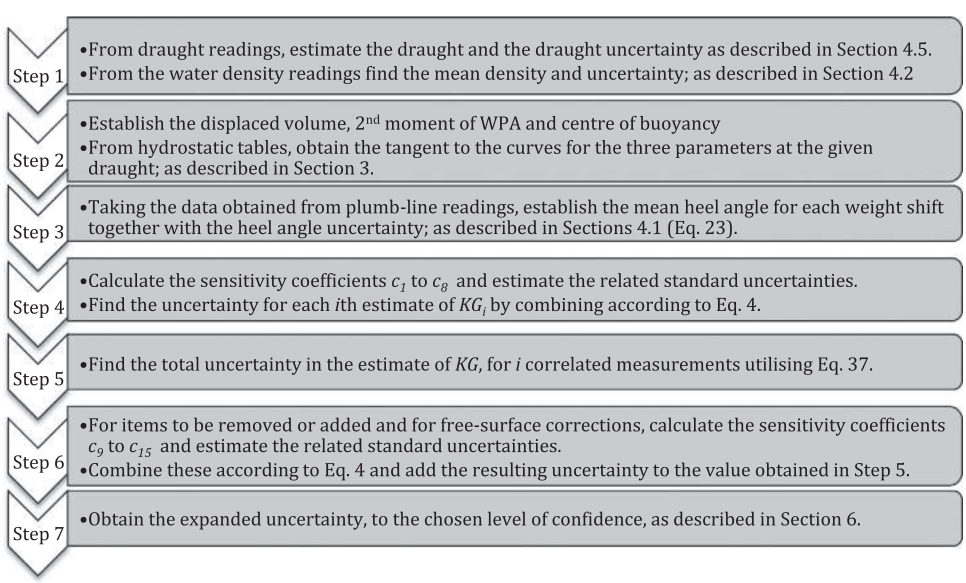

Fig. 1. Step-by-step flow chart for the implementation of the uncertainty analysis procedure for an inclining experiment, either pre- or post-analysis.

Table 1
Results from case study ships.

Parameter (units)	L_{bp} (m)	Δ_{Design} (tonne)	$\overline{KG}$ (m)	$u_c\left(\overline{KG}\right)$ (m)	U_{95} (m)	$U_{95}\left(\overline{GM}\right)$ (%)[a]
Buoy tender	37	453	3.580	0.075	0.15	100
Super yacht	50	698	4.340	0.016	0.033	22.0
Supply ship	51	904	4.173	0.024	0.047	31.3
Container	124	15,718	10.245	0.014	0.029	19.3
Ropax	204	23,370	16.620	0.077	0.15	100

[a] The expanded uncertainty is given as a % of an assumed metacentric height of 0.15 m.

comparable. For example, in all cases wave amplitudes of 5 cm are assumed for the water surface when taking draught readings. Similarly, plumb-line readings are all assumed to be oscillating with amplitude of 1mm and the water density is assumed to have a 5 kg/m^3 standard uncertainty in all cases. In all cases the manufacturing tolerances are assumed to be ± 10 mm in the length and ± 3 mm in all other dimensions.

For commercial considerations, the full details of the particular ships are not published. Table 1 contains however all of the pertinent values necessary to form a judgment. For reference, the table gives the length between perpendiculars and the design displacement for each ship, together with a descriptive ship-type title. In each case, the estimated $\overline{KG}$ is given together with the combined uncertainty and the expanded uncertainty for a 95% confidence interval. This contains only the uncertainty associated with the light ship estimate and not the uncertainty of all other items (cargo, fuel, water, ballast etc.) on-board the ship in its loaded condition. The uncertainty of the light-ship $\overline{KG}$ is the minimum possible values and the implication of this for the operation of the ship is certainly worth considering. Comparing the uncertainty in $\overline{KG}$ to the value of $\overline{KG}$ is not particularly meaningful in this case as the magnitude of $\overline{KG}$ is somewhat arbitrary, and will change as the ship is loaded. Strictly speaking, the uncertainty in the position of the centre of mass (G) is important and not its magnitude with respect to an arbitrary reference point such as the keel (K). Consideration of the magnitude of the expanded uncertainty for a typical $\overline{GM}$ limitation is perhaps more meaningful. Considering the basic IMO requirement for $\overline{GM}$ to be greater than 0.15 m, the given values of expanded uncertainty can simply be added on to find the necessary $\overline{GM}$ that would have a 95% confidence of achieving the given criterion. For comparison, the percentage of expanded uncertainty with respect to an assumed $\overline{GM}$ of 0.15 m is given in the last column of Table 1.

For the three smaller ships, if the confidence interval encompasses a potentially negative $\overline{GM}$, this does not necessarily present a problem, as they would not normally be loaded to this limit (or be required to do so). In the case of the container ship however $\overline{GM}$ would typically be close to this limit; to prevent high roll accelerations that might otherwise cause damage to the container stacks. In this case the ship would have to be loaded to a $\overline{GM}$ value of nearly 0.18 m to ensure a 95% confidence of compliance. Similarly, the Ropax would typically load close to the limiting $\overline{GM}$ to reduce the risk of high acceleration causing a shift of cargo. In this case the ship would have to be loaded to a $\overline{GM}$ value of nearly 0.33 m; more than double the criterion limit. Note, this estimate does not accounting for uncertainties in the loading of the ship; that could be much larger.

It is clear from the results that the magnitude of estimated uncertainty varies widely for the ships considered; with at least one, the Buoy tender, showing a markedly high value. To better explore the origins of the uncertainties, the contributions from various inputs are examined. Fig. 2 gives the uncertainties for

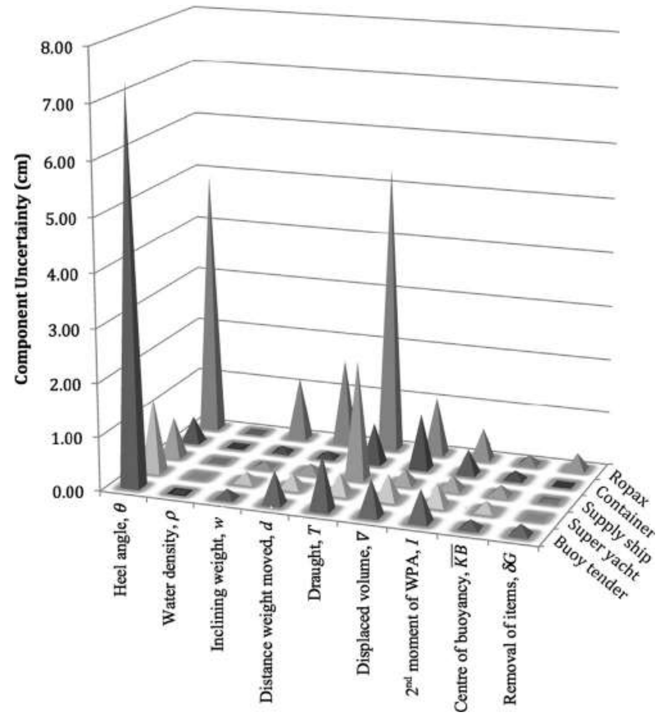

Fig. 2. Component uncertainty contribution in the vertical location of the centre of mass for various inclining experiment parameters for the five case study ships.

various inputs for each ship. On examination the importance of heel angle is clear; and notably so for both the Buoy tender and the Ropax. The Buoy tender does have the smallest average induced heel angle ($\approx 0.6°$). The Ropax however has an average induced heel angle in line with and sometimes greater that the other ships examined. This is a function of the sensitivity of the results to the heel angle that depends on the relationship between various parameters (heel angle; plumb-line length; plumb-line oscillation; $\overline{GM}$). This perhaps exemplifies well the value of performing a pre-test uncertainty analysis, to avoid such situations. Notwithstanding, the two parameters here that may be readily controlled are the induce heel angle (which should be appropriately large) and the plumb-line length (which should be as long as possible).

The second most influential parameter appears to be the draught measurement. In actual fact, the 5 cm wave amplitude is most likely very optimistic, and could be much larger. Nevertheless, repeating the draught measurement more than once quickly reduces the uncertainty in this parameter. Establishing the minimum number of necessary draught measurements needed for any particular wave condition is a relatively easy process using this procedure.

The next most important parameters appear to be the estimate of displaced volume, followed by the estimate of the second moment of water plane area. These are dependent on the manufacturing tolerances, and the estimate thereof. Of course, this can vary depending on the shipyard. More sophisticated ways of measuring the 'as-built' form/dimensions may be considered if this parameter is identified as significant.

It is worth also considering the inferred relationships from the sensitivity coefficients. Assuming that the ship is simply a box with the same length, breadth and draught but with a block coefficient tending to unity, then Eq. (12) can be substantially simplified. The centre of buoyancy of a box is always at half the draught, so $\frac{\partial \overline{KB}}{\partial T} = 0.5$. Also, the second moment of water plane area does not change with draught, so $\frac{\partial I}{\partial T} = 0$. Substituting also Eq. (1) and recognising that $\overline{GM} - \overline{BM} = \overline{BG}$, Eq. (12) can be reduced to the simplified form given in Eq. (50).

$$\frac{\partial \overline{KG}}{\partial T} = 0.5 - \frac{\partial \nabla}{\partial T}\frac{\overline{BG}}{\nabla} \tag{50}$$

This indicates that, to reduce sensitivity, $\overline{BG}$ must be as high as possible. As the height of the centre-of-buoyancy at a particular draught is fixed by the geometry of the ship, a more generally inference can be made in that the centre of gravity must be as high as possible. Also Eq. (50) indicates that ∇ must be as small as possible. Inspection of Eq. (1) shows that both situation result in increased induced heel angles. Some caution should be exercised however as, while large heel angles may reduce uncertainty, they will at the same time increase error due to changes in the position of the metacentre. Nevertheless, heel angles in excess of 7° would be needed before metacentric theory is seriously compromised; far in excess of those needed for a successful IE.

9. Conclusions and recommendations

The aim of this study was to establish procedures for identifying the experimental uncertainty in the estimate of $\overline{KG}$, obtained by IE. The objective were to give procedures for performing a pre-test analysis to help reducing the experimental uncertainty and post-test analysis to identify a confidence interval for the resulting estimate of $\overline{KG}$.

A procedure is provided together with case studies, demonstrating how the uncertainty in an IE can be utilised to improve the design of the experiment. No one parameter can be identified in all cases as problematic from the case studies. There is however a strong indication that the uncertainty in the heel-angle measurement is important but this may be a function of other factors such as $\overline{GM}$. Nevertheless, the longest possible plumb-line (or perhaps an electronic alternative) with sufficiently large induce heel angles should help to reduce uncertainty. The draught measurement uncertainty was also seen to be important, but can be substantially improved with increased sample size. Also, the knowledge of the 'as-built' condition in terms of manufacturing tolerances was identified as important. If this were identified as critical for any particular ship, alternative methods could be employed to establish the as build dimensions more accurately.

A procedure is provided for estimating a confidence interval for $\overline{KG}$ and argued to be more usefully considered as a confidence interval for $\overline{GM}$. The case studies show that, for some ships, a substantial increase in the minimum $\overline{GM}$ may be necessary to ensure safe operation.

In addition to the original objectives, the methods outlined for the addition or removal of weights and for free-surface correction, provide a full and complete procedure for establishing the uncertainty in $\overline{GM}$ for any load condition.

Appendix A. Standard deviation of a sinusoidal signal

Taking the definition of standard deviation to be given by Eq. (A1.1), where x_i is the ith sample amplitude, μ is the mean value of all samples and N is the number of samples.

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2} \tag{A1.1}$$

The mean value μ, of a sinusoidal signal, between the limits of zero and $\frac{2\pi}{\omega}$, will be by definition zero. Then, replacing x_i with $\zeta \sin \omega t$ [where ζ is the amplitude, ω is the frequency and t is time] we get Eq. (A1.2).

$$\sigma_S = \sqrt{\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \zeta^2 \sin^2 \omega t dt} \tag{A1.2}$$

Integrating between the given limits, gives:

$$\zeta^2 \int_0^{\frac{2\pi}{\omega}} \sin^2 \omega t dt = \zeta^2 \left[\frac{t}{2} - \frac{\sin(2\omega t)}{4\omega}\right]_0^{\frac{2\pi}{\omega}}$$

which, by substituting in the values for the limits, can be seen to equal $\frac{\zeta^2 \pi}{\omega}$. Substituting this back into Eq. (A1.2), we obtain:

$$\sigma_S = \sqrt{\frac{\omega}{2\pi}\frac{\zeta^2 \pi}{\omega}}$$

Cancelling out, the standard deviation of a sinusoidal signal for any number of whole cycles, is by definition thus given by Eq. (A1.3).

$$\sigma_S = \frac{\zeta}{\sqrt{2}} \tag{A1.3}$$

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