Stability

By Adarsh (B49)

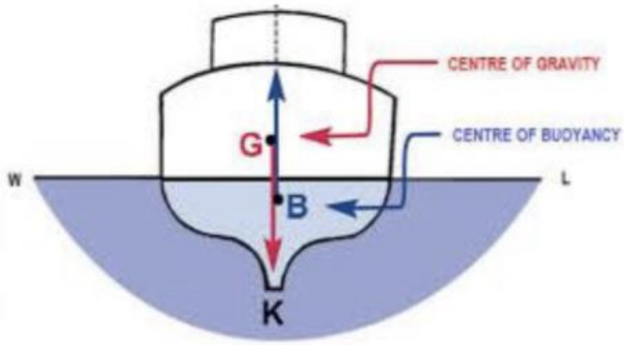
CUSAT

Conditions for static equilibrium of a floating body

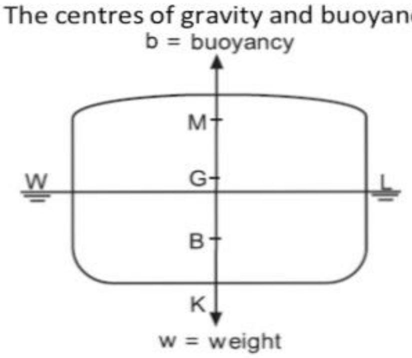
14. **Stable and Unstable Equilibrium of Ships.**

-Equilibrium of a rigid body. A rigid body is considered to be in a state of equilibrium when the resultant of all forces and moments acting on the body are zero.

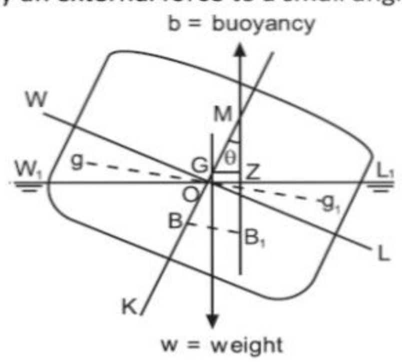
-Equilibrium of a static floating body. The ship is a complex structure and not a rigid body in a mathematical sense. However, for academic purpose, a ship floating in calm water (static floating body) is regarded as a rigid body. To assess its stability, we start with the initial position of equilibrium, with the body floating upright and at rest in a still liquid. In this case, the resultant of all gravity forces i.e. weights(W) assumed to be acting downward through its Centre of Gravity(G), and the resultant of the buoyancy forces (F_B), assumed to be acting upward on the body through its Centre of Buoyancy(B), are of equal magnitude and in the same vertical line.



-Concept of Metacentre. Consider a ship floating upright in still water at waterline WL as shown in the figure below. The centres of gravity and buoyancy are at G and B respectively.



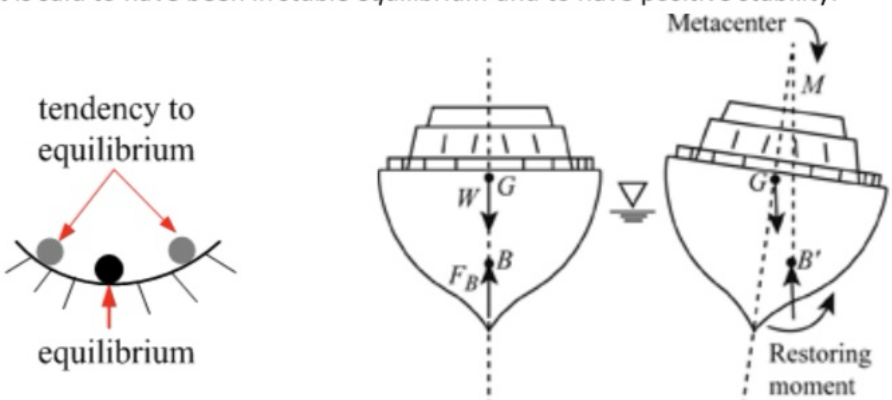
Now let the ship be inclined by an external force to a small angle θ. The new waterline is W_1L_1



Since there has been no change in the distribution of weights, the centre of gravity will remain at G and the weight of the ship (w) can be considered to act vertically downwards through this point. The centre of buoyancy, being the centre of gravity of the underwater volume, must shift from B to the new position B_1. In upright position, the line of action of the buoyancy force, is vertical through the centre of buoyancy (B) and it coincides with the ship's centreline. In the inclined position, the line of action of the buoyancy force, is the vertical through the new centre of buoyancy(B_1), which will intersect with the vertical through B in upright position (i.e. the ship's centreline). This point of intersection between the verticals through centre of buoyancy at two consecutive angles of inclination is called the Metacentre(M). The distance between G and M is referred to as the Metacentric Height (GM). For angles of heel up to about 10°, the Metacentre is at a fixed point called the Initial Metacentre and corresponding GM is called Initial GM . If M is above G, the ship is said to have positive metacentric height, and if M is below G the ship is said to have negative metacentric height. When M coincides with G the GM=0

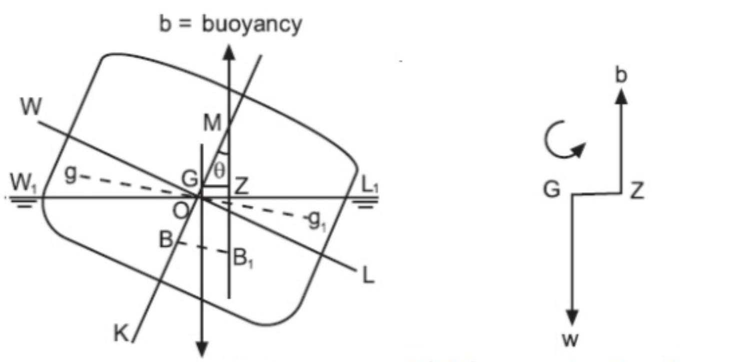
-<u>States of Equilibrium of a static floating body</u>. If a static floating body is subjected to a small disturbance (force / moment) at its initial position of equilibrium, it will move to a new angular attitude. Following the disturbance, the type of response of the body defines the state of equilibrium of the body. The three states of equilibrium of a static floating body are as follows:-

-<u>Stable equilibrium</u>. If a static floating body, initially at equilibrium, is disturbed by a force / moment, it will attain a new angular attitude or inclination. In the inclined position, if GM is positive, there is a restoring or righting moment. Therefore, upon removal of the disturbance, the body will return to its initial equilibrium position, and it is said to have been in *stable equilibrium* and to have positive stability.



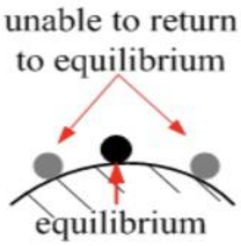
Rigid Body

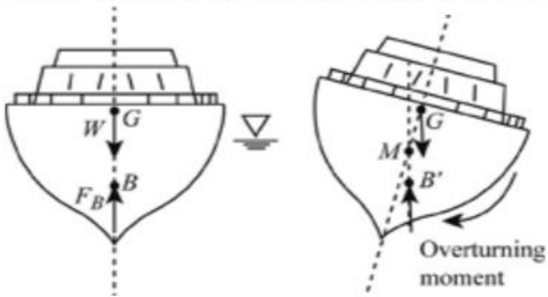
Ship (Static Floating Body)



Righting couple where b=w

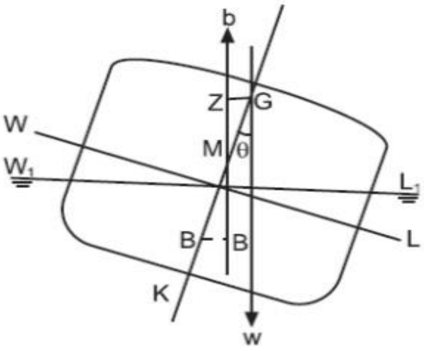
-<u>Unstable equilibrium</u>. In the inclined position, if GM is negative, there is a capsizing moment. Therefore, the body continues to move in the same direction of disturbance, it is said to have been in *unstable equilibrium*, and to have negative stability.

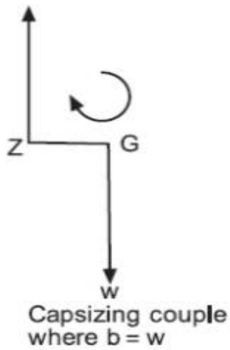




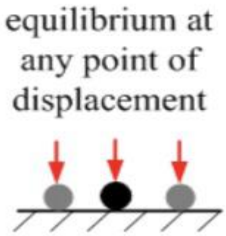
Rigid Body

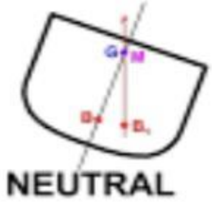
Ship (Static Floating Body)





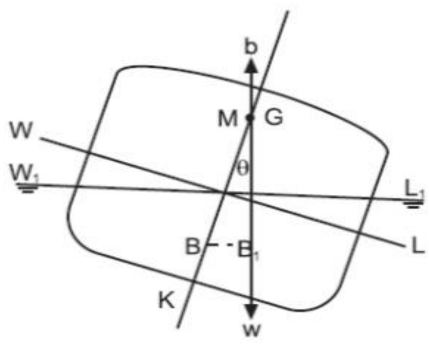
-<u>Neutral equilibrium</u>. In the inclined position, if GM is zero, there is neither a restoring / righting moment nor a capsizing moment. Therefore, upon removal of the disturbance, the body will continue to remain in the new inclined position, and it is said to have been in *neutral equilibrium* and to have neutral stability. For e.g. a cylindrical homogeneous log floating on its side is in neutral equilibrium.

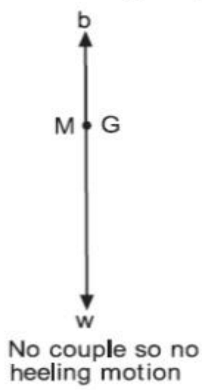




Rigid Body

Ship (Static Floating Body)





Archimedes Principle

This principle laid the groundwork for the development of hydrostatics and buoyancy theory. Even over two millennia later, it remains pivotal in disciplines ranging from shipbuilding and submarine design to fluid dynamics and meteorology. Statement of Archimedes' Principle:

"Any object, wholly or partially submerged in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

Mathematical Formulation

Let's consider an object of volume V, density ρ_{object}, and mass m, immersed in a fluid with density ρ_{fluid}. The **buoyant force** F_b acting on the object is given by:

$$F_b = \rho_{\text{fluid}} \cdot V_{\text{displaced}} \cdot g$$

Hydrostatic pressure distribution on floating cuboids and bodies of arbitrary shape

Simple Geometrical Form

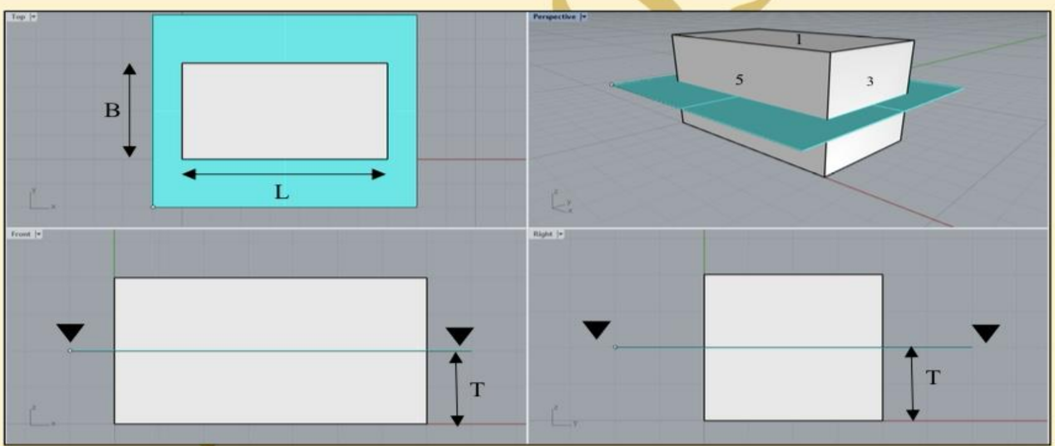
Pressure at any depth z is given by: -

$$P_{absolute} = P_{atm} + P_{gauge}$$

$$P_{gauge} = \gamma z$$

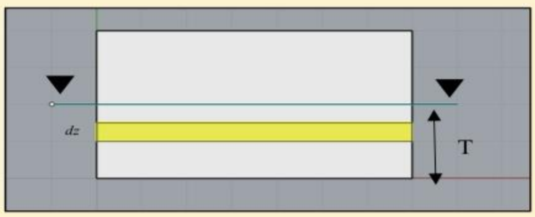
where, γ is the specific gravity of the fluid and z is the depth.

Let us find the hydrostatic forces on a cuboid (parallelepiped), as shown in fig. below. We consider the body immersed to the draft T. Let us call the top face 1, the bottom face 2, and number the vertical faces with 3 to 6. To obtain the absolute pressure we must add the force due to the atmospheric pressure p_0.



Let us consider a thin cuboidal volume of thickness dz at a depth of z below the water surface.

'z' will vary from 0 →T.



Let us calculate the hydrostatic forces on the side F_5

$$Force = Pressure\ x\ Area$$

We have discussed above, the absolute pressure acting the area ($z\ dz$) on the infinitesimally small volume at a depth z is as follows:-

$$F_5 = \underbrace{L\int_0^T \gamma z dz}_{P_{gauge}} + \underbrace{p_0 LT}_{P_{atm}}$$

As per our coordinate system, which we discussed in class, the hydrostatic pressure acting on the face F_5 is from –ve to +ve direction. Hence if we include the direction part of the **F** (F being a vector): -

$$\boldsymbol{F_5} = -L\int_0^T \gamma z dz - p_0 LT$$

$$\boldsymbol{F_5} = -\frac{1}{2}\gamma LT - p_0 LT$$

Similarly, $\mathbf{F_6}$ is given by

$$\boldsymbol{F_6} = L\int_0^T \gamma z dz + p_0 LT$$

$$\boldsymbol{F_6} = \frac{1}{2}\gamma LT + p_0 LT$$

Applying similar logic for $\mathbf{F_3}$ and $\mathbf{F_4}$.

We can see that $|\boldsymbol{F_6}| = -|\boldsymbol{F_5}|$ and $|\boldsymbol{F_4}| = -|\boldsymbol{F_3}|$, the forces on the sides cancel outs.

Now calculating the forces of $\mathbf{F_1}$ and $\mathbf{F_2}$,

$$\boldsymbol{F_1} = p_0 LB$$

$$\boldsymbol{F_2} = -p_0 LB - \gamma TLB$$

Thus, the total resultant hydrostatic forces acting on the cuboid is

$$\boldsymbol{F_H} = -\gamma T * LB$$

The product LBT is actually the volume of the immersed body.

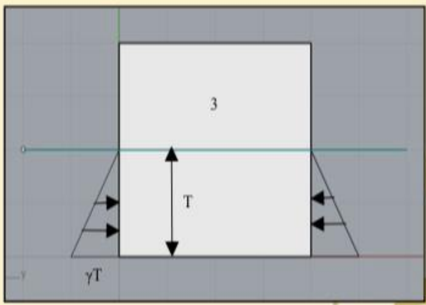
$$\boldsymbol{F_H} = -\gamma V_{submerged}$$

$$\boldsymbol{F_H} = -\rho\, g\, V_{submerged}$$

$$\boldsymbol{F_H} = -W_{submerged}$$

Thus $\mathbf{F_H}$, the net hydrostatic forces acting on the body is the weight of the water displaced, which is the Archimedes' Principle.

The same can be calculated geometrically



Thus the force acting on the side 5 is as follows:

$$\boldsymbol{F_5} = -\,Area\ of\ the\ pressure\ triangle * Length\ of\ the\ side - P_{atm} * Area\ of\ side$$

$$\boldsymbol{F_5} = -\frac{1}{2}\gamma LT - p_0 LT \text{ , which is same as that above.}$$

We can see from the above derivation that the P_{atm} does not play a role in derivation of Archimedes' Principle.

General Case

A small mathematics class before we derive the Archimedes' Principle for a general case.

We know any 3D shape (in an *x*, *y* and *z* coordinates) can be written in a mathematical form. For e.g. the mathematical equation for a sphere with center (*a, b, c*) and radius *r* is:

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

We can isolate the *z* in the above equation and write the above equation like:

$$z = f(x,y)$$

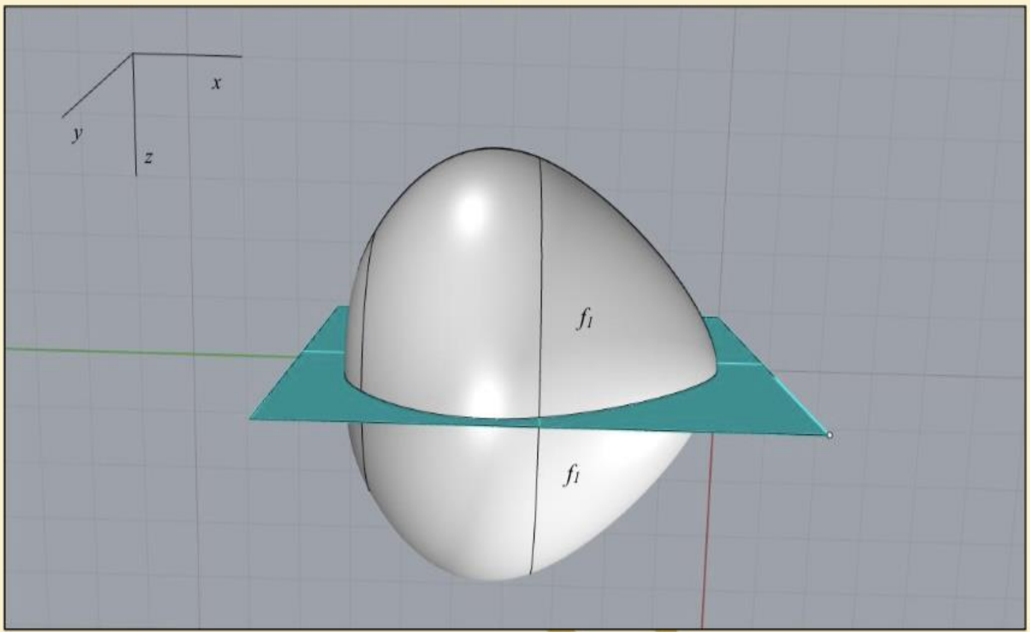
The above equation means that for a well-defined sphere, the z coordinate can be written as function of *x* and *y*.

Keeping that in mind, let us calculate the hydrostatic pressure on a very well defined 3D structure submerged in a fluid of specific density γ, as in the figure below. Let the surface be defined as *S*, let *P* be a plane which randomly divides the structure in two surfaces S_1 and S_2. Let the *z* coordinate of S_1 is given by

$$z = f_1(x, y)$$

and for S_2 is given by

$$z = f_2(x, y)$$



We know,

$$\boldsymbol{F} = -\int_S p \, . \, d\boldsymbol{A}$$

where **A** is the area vector and $\boldsymbol{A} = A \, . \, \boldsymbol{n}$ (n is the unit normal vector)

The **vertical component of hydrostatic force** on an element d**A** of S_1 is p.d**A**. This force is directed along the normal, **n**, to S_1 in the element of area. If the cosine of the angle between n and the vertical axis is cos (n, z), the vertical component of the pressure force on d**A** equals γf_1(x, y) cos (**n**, z)d**A**. As cos (**n**, z)d**A** is the projection of d**A** on a horizontal plane, that is dx dy, we conclude that the vertical hydrostatic force on S_1 is

$$\boldsymbol{F_V} = \gamma \iint_{S_1} f_1(x, y) \, dxdy$$

Similarly, on S_2 the vertical force is given by

$$\boldsymbol{F_V} = -\gamma \iint_{S_2} f_2(x, y) \, dxdy$$

Thus the total **F** on *S* is as follows:

$$\boldsymbol{F_V} = \gamma \iint_{S_1} f_1(x, y) \, dxdy - \gamma \iint_{S_2} f_2(x, y) \, dxdy$$

$$F_V = \gamma \iint_S \{f_1(x,y) - f_2(x,y)\}\, dxdy$$

We can assimilate that the value in the {} is the difference in z coordinate at a particular (x, y). In simpler terms, at a particular (x, y) the top most and bottom most point of a fully submerged body. Therefore the integral yields the volume of the submerged body.

We have considered two distinct equation while calculating the vertical forces as, the pressure varies with different *z*s.

However considering a particular d**A** at a depth *z* there would be a d**A** on the opposite side such that, the **horizontal component of the hydrostatic forces** is

$$F_H = p\cos(\boldsymbol{n}, x)dA = -\gamma z dxdy$$

The sum of both forces is **zero**. As the whole surface *S* consists of such "opposed" pairs d**A**, the horizontal component in the x-direction is zero. By a similar reasoning we conclude that the horizontal component in the y-direction is zero too. This is also the result predicted by intuition. In fact, if the resultant of the horizontal components would not be zero we would obtain a "free" propulsion force.

Thus the total force on any submerged body is given by:

$$F = F_V + F_H = \gamma \iint_S \{f_1(x,y) - f_2(x,y)\}\, dxdy$$

which is the weight of the liquid displaced by the submerged body. This is a derivation of the Archimedes' Principle for a general body.

The point at which the **F (buoyancy Force or simply buoyancy)** acts is to be determined. As discussed in class, we will just find the *x* and *y* coordinate and the force can be acting at any point in the *z* direction. The x coordinate is got by adding the net moments about *XOZ* plane and dividing by the total force.

$$x = \frac{\gamma \iint_S x\{f_1(x,y) - f_2(x,y)\}\, dxdy}{F}$$

And similarly, the x coordinate is got by adding the net moments about *XOZ* plane and dividing by the total force.

$$y = \frac{\gamma \iint_S y\{f_1(x,y) - f_2(x,y)\}\, dxdy}{F}$$

These are simply the *x* and *y* coordinates of the center of the submerged volume. We conclude that the buoyancy force passes through the center of the submerged volume, B (center of the displaced volume of liquid).

Stevin's Law

The first condition for equilibrium, that is the one regarding the sum of forces, is expressed as Archimedes' principle. The second condition,regarding the sum of moments, is stated as Stevin's law. The second condition of equilibrium of a floating body: the sum of the moments of all forces acting on it must be zero. This condition is fulfilled in Figure 2.7a where the centre of gravity, G, and the centre of buoyancy, B, of the floating body are on the same vertical line. The weight of the body and the buoyancy force are equal-that is opposed, and act along the same line. The sum of their moments about any reference is zero.

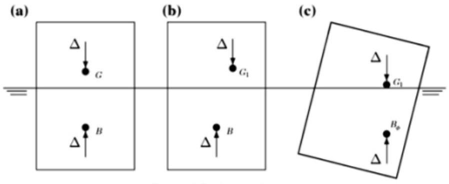


Figure 2.7 Stevin's Law, 1

Stevin's Law

The gravitational and buoyancy forces are equal and opposite. If the CoG is moved, a couple acts on the ship. Then, the CoB moves to make the couple zero. The lines of action of G and B are the same in 3D space when the ship is in equilibrium.

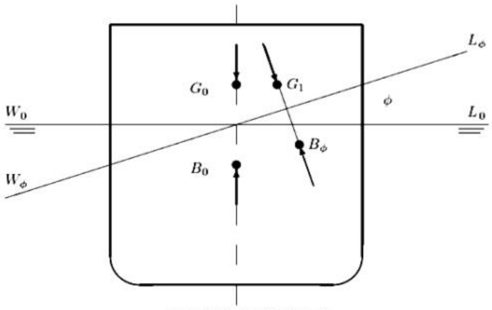


Figure 2.8 Stevin's Law, 2

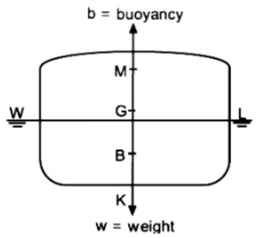
Three types of equilibrium, Small vertical change in the position, small inclination

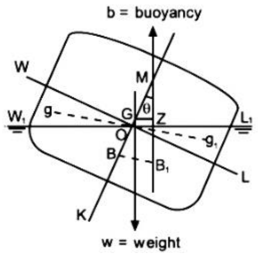
- **Neutral Equilibrium** :- A ship cannot be in neutral equilibrium with respect to vertical displacement. If the draft increases, the buoyancy force exceeds the gravitational force and there is a restoring force that causes it to move towards the equilibrium position. It is in neutral equilibrium with respect to horizontal displacement as there are no restoring forces when a gust of wind acts on it.

- **Unstable Equilibrium** :- If a floating body has zero freeboard and there are openings above the waterline it is in unstable vertical equilibrium. If it is displaced from its original position (draft) it will sink. If it has a very small freeboard, it is stable for "small" vertical displacements but unstable for larger displacements.
- **Stable equilibrium** :- If a floating body, initially at equilibrium, is disturbed by an external moment, there will be a change in its angular attitude. If upon removal of the external moment, the body returns to its original position, it is said to have been in stable equilibrium and to have positive stability.

Bouguer's Metacentre

Consider a ship floating upright in still water at waterline WL as shown in the figure below. The centres of gravity and buoyancy are at G and B respectively.





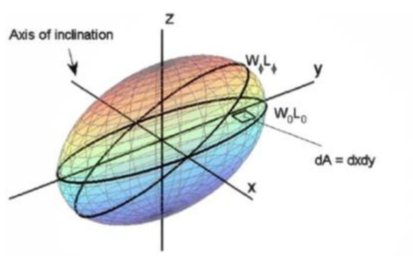
Now let the ship be inclined by an external force to a small angle theta. The new waterline is W1L1. at G and the weight of the ship (w) can be considered to act vertically downwards through this point. The centre of buoyancy, Because a volume submerges at starboard, and an equal volume emerges at port, the centre of buoyancy moves to starboard,must shift from B to the new position B1. In upright position, the line of action of the buoyancy force, is vertical through the centre of buoyancy (B) and it coincides with the ship's centreline. In the inclined position, the line of action of the buoyancy force, is the vertical through the new centre of buoyancy(B1), which will intersect with the vertical through B in upright position (i.e. the ship's centreline). This point of intersection between the verticals through centre of buoyancy at two consecutive angles of inclination is called the Metacentre(M). The distance between G and M is referred to as the Metacentric Height (GM). For angles of heel up to about 10°, the Metacentre is at a fixed point called the Initial Metacentre and corresponding GM is called Initial GM . If M is above G, the ship is said to have positive metacentric height, and if M is below G the ship is said to have negative metacentric height. When M coincides with G the GM=0. Bouguer called this point metacentre. **Remember, this definition holds for small heel angles only.**

Euler's Theorem and the axis of inclination

Axis of Equivolume Inclination :-

Let us assume that the initial waterplane of the body shown in Figure 2.11 is W_0L_0. Next we consider the same body inclined by a *small* angle ϕ, such that the new waterplane is $W_\phi L_\phi$. The weight of the body does not change; therefore, also the submerged volume does not change. If so, the volume of the "wedge" that submerges at right, between the two planes

V_0L_0 and $W_\phi L_\phi$, equals the volume of the wedge that emerges at left, between the same tw lanes. Let us express this mathematically. We take the intersection of the two planes as the -axis. This is the *axis of inclination*.



As shown in Figure 2.12, an element of volume situated at a distance y from the axis of inclination has the height y tan . If the base of this element of volume is d A = dxdy, the volume is y tan dxdy. Let the area of the waterplane W_oL_o at the right of the axis of intersection be Sy; that at the left, S2. Then, the volume that submerges is

At this point in the derivation, we do not know where the axis of intersection lies. It is to be found.

$$V_1 = \int \int_{\mathbf{S_1}} y \tan\phi \, dx dy \tag{2.24}$$

and the volume that emerges,

$$V_2 = -\int \int_{\mathbf{S_2}} y \tan\phi \, dx dy \tag{2.25}$$

By assuming a small heel angle, ϕ, we can consider the submerging and emerging volumes as wall sided and write Eqs. (2.24) and (2.25) as we did.

The condition for constant volume is

$$V_1 = V_2$$

The condition for constant volume is

$$V_1 = V_2$$

Combining this with Eqs. (2.24) and (2.25) yields

$$\int \int_{\mathbf{S_1}} y \tan\phi \, dx dy = -\int \int_{\mathbf{S_2}} y \tan\phi \, dx dy \tag{2.26}$$

and, finally

$$\int \int_{\mathbf{S}} y \, dx dy = 0 \tag{2.27}$$

where $\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}$ is the whole waterplane. In words, the first moment of the waterplane area, with respect with the axis of inclination, is zero. This happens only if the axis of inclination passes through the *centroid* of the waterplane area. We remind the reader that the coordinates of the centroid of an area A are defined by

$$x_C = \frac{\int \int_A x \, dx dy}{\int \int_A dx dy}, \quad y_C = \frac{\int \int_A y \, dx dy}{\int \int_A dx dy}$$

Centre of flotation

The centroid of the waterplane area is known as centre of flotation. Let the initial waterplane of a floating body be WoLo. After an inclination, at constant volume of displacement, with an angle , the new waterplane is W1L1. The intersection of the two waterplanes is the axis of inclination. If the angle of inclination tends to zero, the axis of inclination tends to a straight line passing through the centroid of the waterplane area. In practice, this property holds if the angle of inclination is sufficiently small. For heeling of a vessel, this can mean a few degrees, 5° for some forms, even 15° for others. If the inclination is the trimming of an intact vessel, the angles are usually small enough and this property always holds. The property also holds for larger heel angles if the floating body is wall sided.This is the name given to floating bodies whose surface includes a cylinder (in the broader geometrical sense), with generators perpendicular to the initial waterplane. The term used in some languages, such as French, Italian, or Spanish, for an axis passing through the centroid of an area is barycentric axis. This term is economic and we shall use it whenever it will help us to express ideas more concisely.

A consequence of Euler's Theorem :-

Thus the line of intersection of two equivolume waterlines, the angle between them being infinitesimally small, must pass through the centroids of both waterlines. This is Euler's theorem.We have proved it for heel but it can be proved for an equivolume inclination about any axis lying in the waterplane.Let us formulate a consequence of Euler's theorem which is its generalisation. If a floating ship is turned through an infinitesimal angle about any axis passing through the centroid of the waterplane, the volume of the immersed portion of the ship is unchanged.

Change in density

Tonnes per centimeter :- The TPC for any given draught is the weight that must be loaded or discharged to change the ship's mean draught by one centimetre.

Since: Mass = Volume × Density

then:

Mass of additional slice of water = Volume of the additional slice of water × Density.

If the WPA is assumed to not significantly change between the two waterlines, then:

Volume of the slice = WPA (m^2) × 1 cm;

Fig. 3.3

We cannot multiply m^2 by cms, therefore:

$$\text{Volume of slice} = \text{WPA}(m^2) \times \frac{1}{100}(m);$$

$$\therefore \text{Added displacement (t)} = \text{WPA}(m^2) \times \frac{1}{100}(m) \times \text{density } (t/m^3);$$

Therefore, the formula for TPC is:

$$\boxed{TPC = \frac{WPA}{100} \times P}$$

Let us suppose that we know the displacement, (delta) corresponding to a given draught, T, and we want to find by how many tonnes that displacement will change if the draught changes by,(delta)T centimetres. Let the waterplane area be Aw m^2, and the water density, Rho (p) t m-3. The problem posed can be inverted: find the change in draught, (delta)T, corresponding to a change of displacement. The obvious answer is

$$\delta T = \frac{\delta\Delta}{TPC}$$

Change in density :-

When the ship moves to water with different density (Ie, what happens if the weight of the ship remains the same and the density of the water in which it is floating is changed)what will happen, Let us examine…….a body of displacement "∇" floating in water of density W1 which passes into water of lower density W2.It will sink deeper because the water is less buoyant. The weight and buoyancy have not changed because nothing has been added to the body or taken away.Then if Δ = Delta = Displacement in tonne…..

$$\therefore\ \Delta = \nabla_1 w_1 = \nabla_2 w_2$$
$$\frac{\nabla_1}{\nabla_2} = \frac{w_2}{w_1}$$

Fresh Water Allowance (FWA)

- The Fresh Water Allowance is the amount (the number of millimetres) by which the mean draft changes when a ship passes from salt water to fresh water, or vice versa, whilst floating at the loaded draft.
- Densities of fresh water and seawater are **1000 kg/m3** and **1025 kg/m3** respectively.
- Density of fresh water is maximum at 4 degree celsius. For fixed salinity, density decreases when temperature increases.

Now derive an expression for FWA,

Consider the ship shown in Figure to be floating at the load Summer draft in salt water at the waterline WL. Let V be the volume of salt water displaced at this draft.Now let Will be the waterline for the ship when displacing the san ass of fresh water. Also, let 'v' be the extra volume of water displaced fresh water.

The total volume of fresh water displaced is then $V + v$.

$$\text{Mass} = \text{Volume} \times \text{density}$$

$$\therefore \text{Mass of SW displaced} = 1025\,V$$

$$\text{and mass of FW displaced} = 1000\,(V + v)$$

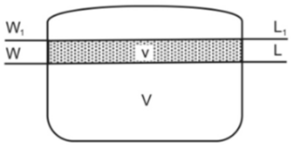
$$\text{but mass of FW displaced} = \text{mass of SW displaced}$$

$$\therefore 1000\,(V + v) = 1025\,V$$

$$1000\,V + 1000\,v = 1025\,V$$

$$1000\,v = 25\,V$$

$$v = V/40$$



Now let w be the mass of salt water in volume v, in tonnes and let W be the mass of salt water in volume V, in tonnes.

$$\therefore w = W/40$$

$$\text{but } w = \frac{FWA}{10} \times TPC$$

$$\frac{FWA}{10} \times TPC = W/40$$

or

$$FWA = \frac{W}{4 \times TPC}\ \text{mm}$$

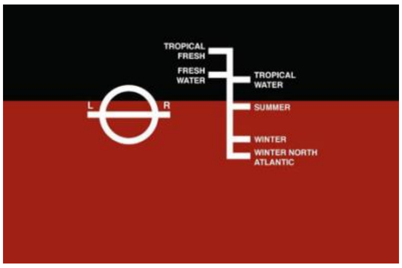
Dock Water Allowance

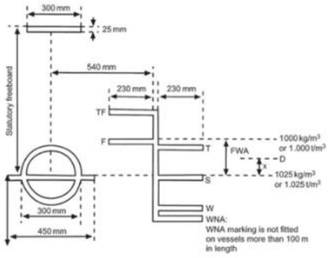
Dock Water Allowance is similar to FWA and is the amount in millimetres by which the ships mean draft changes when a vessel moves between a salt water and dock water. Dock water is the water whose density is neither that of fresh water or salt water but in-between the two. The Relative Density (RD) of dock water is between 1.0 and 1.025. DWA depends on the density of the dock water.

$$\text{Dock Water Allowance} = \frac{FWA(1025 - \rho_{DW})}{25}$$

Plimsoll line

A Plimsoll line, or load line, is a reference mark on a ship's hull that indicates the maximum safe draft a vessel can be immersed when loaded with cargo. The centre of the disk is at a distance below the deck line equal to the ship's statutory freeboard. Then 540 mm forward of the disk is a vertical line 25 mm thick, with horizontal lines measuring 230 × 25 mm on each side of it. The upper edge of the one marked **'S'** is in line with the horizontal line through the disk and indicates the draft to which the ship may be loaded when floating in salt water in a Summer Zone. Above this line and pointing aft is another line marked **'F'**, the upper edge of which indicates the draft to which the ship may be loaded when floating in fresh water in a Summer Zone. If loaded to this draft in fresh water the ship will automatically rise to 'S' when she passes into salt water. The perpendicular distance in millimetres between the upper edges of these two lines is therefore the ship's Fresh Water Allowance.

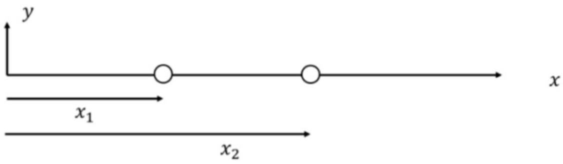




Change in the centre of gravity (Movement of mass on board)

Consider masses, $m_1, m_2, m_3, \dots m_N$, with CoGs at $\bar{x}_i, i = 1, 2, 3, \dots N$.

The CoG is at $\bar{x}_G = \frac{\sum_{i=1}^{N} m_i \bar{x}_i}{\sum_{i=1}^{N} m_i}$



- The mass m_i is at (x_i, y_i, z_i), $i = 1, 2, 3, \dots N$. The mass m_1 is moved to $(x_1 + \Delta x_1, y_1 + \Delta y_1, z_1 + \Delta z_1)$. Find the original CoG, the new CoG, and the change in the CoG.
- The original CoG is at (x_G, y_G, z_G). $x_G = \frac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i}$. $y_G = \frac{\sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i}$. $z_G = \frac{\sum_{i=1}^{N} m_i z_i}{\sum_{i=1}^{N} m_i}$.
- The new CoG is at $(x_G + \Delta x_G, y_G + \Delta y_G, z_G + \Delta z_G)$
- $x_G + \Delta x_G = \frac{m_1 \Delta x_1 + \sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i}$. $y_G + \Delta y_G = \frac{m_1 \Delta y_1 + \sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i}$. $z_G + \Delta z_G = \frac{m_1 \Delta z_1 + \sum_{i=1}^{N} m_i z_i}{\sum_{i=1}^{N} m_i}$
- The change in the CoG is $\Delta x_G = \frac{m_1 \Delta x_1}{\sum_{i=1}^{N} m_i}$. $\Delta y_G = \frac{m_1 \Delta y_1}{\sum_{i=1}^{N} m_i}$. $\Delta z_G = \frac{m_1 \Delta z_1}{\sum_{i=1}^{N} m_i}$

Heel, Trim, and CoB

- A mass is on the CoF. If it is moved transversely, the ship will heel but not trim. If it is moved longitudinally, the ship will trim but not heel. In the general case, both heel and trim will occur. **Heel and trim take place about the centre of flotation.** The axis of inclination passes through the CoF. For every attitude of a ship, there is ONE corresponding CoB.
- In general, for pure equivolume heel of a ship, the CoB will move both transversely and longitudinally because the CoB of the submerged and emerged volumes are not on the same transverse plane for a non-wall-sided ship.

Surfaces of centre of buoyancy

- For each axis about which a floating body inclines, there corresponds one Metacentre. Example: transverse and longitudinal metacenters. To each position of a floating body correspond one centre of buoyancy and on. metacentre. Each position of the floating body is defined by three parameters, for instance the triple {displacement, angle of heel, angle of trim}; we call them the **parameters of the floating condition. If we keep two parameters**

constant and let one vary, the centre of buoyancy travels along a curve and the metacentre along another. If only one parameter is kept constant and two vary, the centre of buoyancy and the metacentre generate two surfaces. In this chapter we shall briefly show what happens when the displacement is constant. The discussion of the case in which only one angle (that is, either heel or trim) varies leads to the **concept of metacentric evolute.**

- So far we have considered infinitesimal equivolume inclinations of the ship and the associated shifts of the centre of buoyancy. We shall now touch upon some problems associated with the shift of the centre of buoyancy when equivolume inclinations are not restricted to small angles. Considering all possible equivolume inclinations of the ship with no limitation imposed on the direction of the axis of inclination or the magnitude of the angle of inclination we come to the conclusion that all centres of buoyancy corresponding to all possible positions of the ship, with the constant volume of the immersed portion, lie on a closed surface which is the locus of centres of buoyancy for a given equivolume inclination. This surface is known in the ship theory as the **surface of centres of buoyancy, or the C surface.** This is for a fixed displacement When the ship is inclined in a particular plane the centre of buoyancy is shifted not only over the C surface but along a particular curve lying on this surface. This curve is termed the **trajectory of centres of buoyancy**, or the C trajectory. The C trajectory is in general a curve of double curvature. As it does not lie in one plane The projection of the C trajectory on the plane of inclination is termed the curve of C. . This curve is obviously plane.
- The CoB and the M are shown for various angles of heel. Each BM line is perpendicular to the associated waterline. Learn to draw this figure.

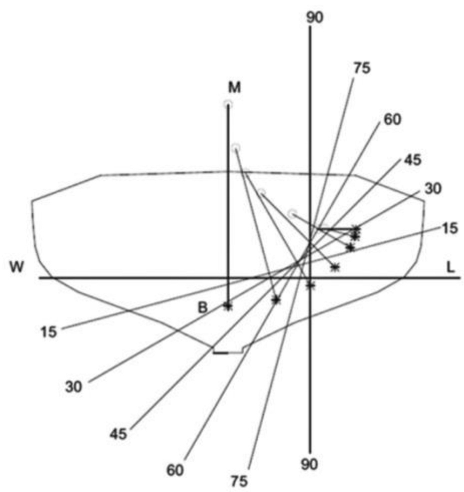


Figure 2.23 B and M curves of vessel *Lido 9*

Change in the metacentre

The transverse metacenter is shown for 2 special cases. The longitudinal metacenter is at a different point. Bouguer was sometimes described as 'the father of naval architecture' . It must be emphasised here that the definition of the metacentre is not connected at all with the form of a ship. Therefore, the fact that in the above figures the meta-centre is the intersection of the new line of action of the buoyancy force and the centreline is true only for symmetrical hulls heeled from the upright condition.

Metacentric Radius = BM = I/V

The Transverse BM is the height of the transverse metacentre above the centre of buoyancy and is found by using the formula:

$$BM = \frac{I}{V}$$

where

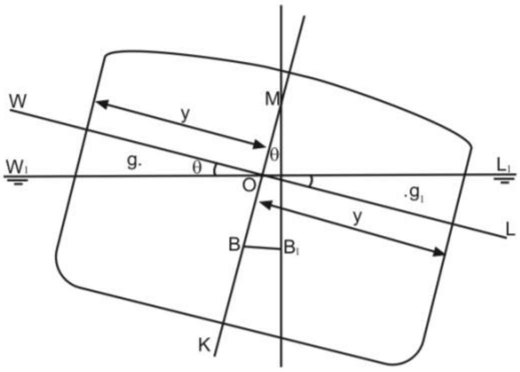
I = The second moment of the water-plane area about the centre line,

and

V = The ship's volume of displacement

The derivation of this formula is as follows:

Consider a ship inclined to a small angle (θ) as shown in Figure 12.3(a) Let 'y' be the half-breadth.

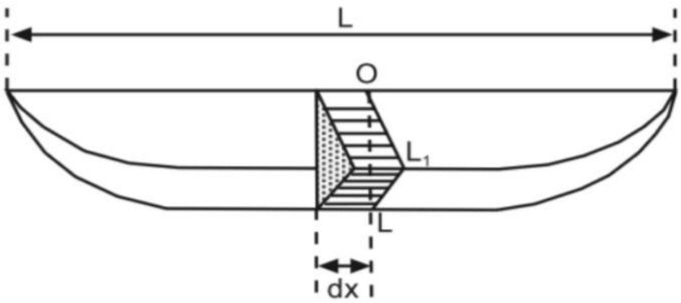


$$\text{Since } \theta \text{ is a small angle then arc } WW_1 = \text{arc } LL_1$$
$$= \theta\, y$$

Also:

$$\text{Area of wedge } WOW_1 = \text{Area of wedge } LOL_1$$
$$= \tfrac{1}{2}\,\theta\, y^2$$

Consider an elementary wedge of longitudinal length dx as in Figure 12.3(b).



$$\text{The volume of this wedge} = \tfrac{1}{2}\,\theta\, y^2\, dx$$

$$\text{The moment of the wedge about the centre line} = \tfrac{1}{2}\,\theta\, y^2\, dx \times \tfrac{2}{3}\, y$$
$$= \tfrac{1}{3}\,\theta\, y^3\, dx$$

$$\text{The total moment of both wedges about the centre line} = \tfrac{2}{3}\,\theta\, y^3\, dx$$

$$\text{The sum of the moments of all such wedges} = \int_O^L \tfrac{2}{3}\,\theta\, y^3\, dx$$

$$= \theta \int_O^L \tfrac{2}{3}\, y^3\, dx$$

But

$$\int_O^L \tfrac{2}{3}\, y^3\, dx = \left.\begin{array}{l}\text{The second moment}\\ \text{of the water-plane}\\ \text{area about the ship's centre line}\end{array}\right\} = I$$

$$\begin{array}{l}\therefore \text{The sum of the moments}\\ \text{of the wedges}\end{array} = I \times \theta$$

$$\begin{array}{l}\text{But the sum of}\\ \text{the moments}\end{array} = v \times gg_1$$

where v is the volume of the immersed or emerged wedge.

$$\therefore\ I \times \theta = v \times gg_1$$

or

$$I = \frac{v \times gg_1}{\theta} \qquad (I)$$

Now:

$$BB_1 = \frac{v \times gg_1}{V}$$

and

$$BB_1 = BM \times \theta$$

$$\therefore\ BM \times \theta = \frac{v \times gg_1}{V}$$

or

$$BM \times V = \frac{v \times gg_1}{\theta}$$

Substituting in (I) above:

$$BM \times V = I$$

$$\therefore\ BM = \frac{I}{V}$$

Module 2

Initial Transverse Stability of ships:

Righting lever arm – GZ

The lever GZ is referred to as the righting lever and is the perpendicular distance between the centre of gravity and the vertical through the centre of buoyancy. Before we look at an intro to stability at large angles, recall the figures for stability at low angles. Are 2.9a, 2.9b and 2.9c stable ? 2.9a and 2.9b are stable. 2.9c is unstable.

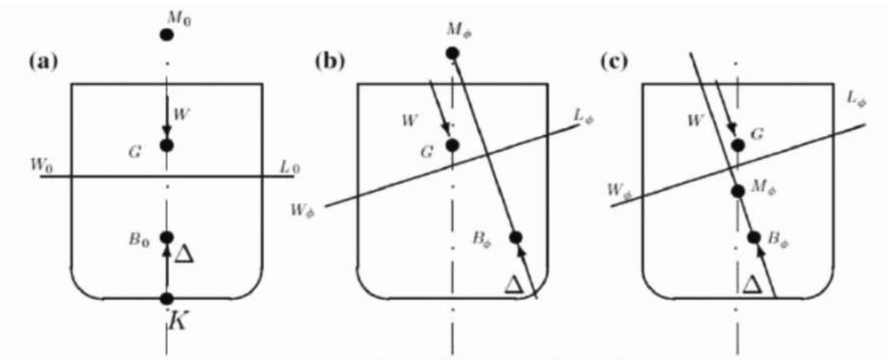


Figure 2.9 The condition of initial stability

Wall-Sided Ships

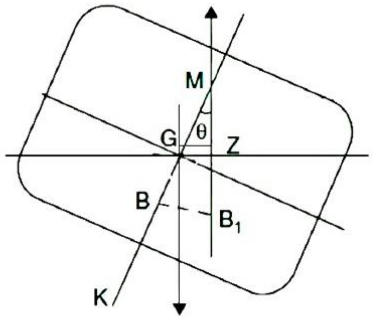
Wall-sided ships" is a term mainly used in naval architecture and ship stability studies. It refers to a theoretical or simplified type of ship form where the sides of the ship are assumed to be vertical (like walls) . At small angles of heel, most real ships behave approximately like wall-sided ships.

Lever arm of stability of weight

When we talk about stability of a ship, we usually deal with two lever arms: **Righting arm (GZ) and Heeling (or upsetting) lever arm of weight (W) .** the lever arm of stability of weight is just the horizontal distance (GZ) by which the weight force and buoyancy force separate when the ship heels.

GZ = GM sin(θ)

The lever GZ is referred to as the righting lever and is the perpendicular distance between the centre of gravity and the vertical through the centre of buoyancy.From triangle GZM, angle GMZ = θ..Taking sine θ, **GZ = GM Sin(θ)**



This equation is valid only for small angles….. Since there is a couple formed between line of action of buoyant force and line of action of gravitational force, there is a restoring moment formed. If **W** is the restoring force acting on the ship at the trimmed condition then restoring moment is…

RM = Restoring force * Righting lever = W * (GM Sin (θ))

Change in the CoB

- The CoB of the original volume depends on only the lines-plan of the underwater volume. It does not depend on the density of water .
- Take the moments about the CoB of the original volume. Take moments of the original volume and the added volume:

$$\nabla_1 0 + (\nabla_2 - \nabla_1)\overline{Bb} = \nabla_2 \overline{BB^1}$$

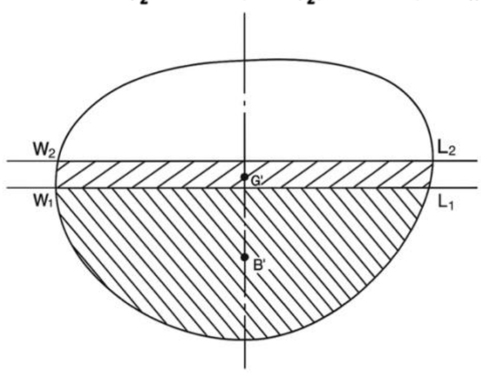
- Taking first moments of the volume about B the original centre of buoyancy ,

$$\overline{BB^1} = \frac{(\nabla_2 - \nabla_1)\overline{Bb}}{\nabla_2} = \left(1 - \frac{w_2}{w_1}\right)\overline{Bb}$$

***B* is the original CoB**
***B^1* is the new CoB**
***b* is the CoB of the layer**

- Rearranging yields the change in the CoB. To find the change in the CoB, it is necessary to know the original CoB

$$\overline{BB^1} = \frac{(\nabla_2 - \nabla_1)\overline{Bb}}{\nabla_2} = \left(1 - \frac{\nabla_1}{\nabla_2}\right)\overline{Bb} = \left(1 - \frac{w_2}{w_1}\right)\overline{Bb}$$



Atwood's formula for GZ

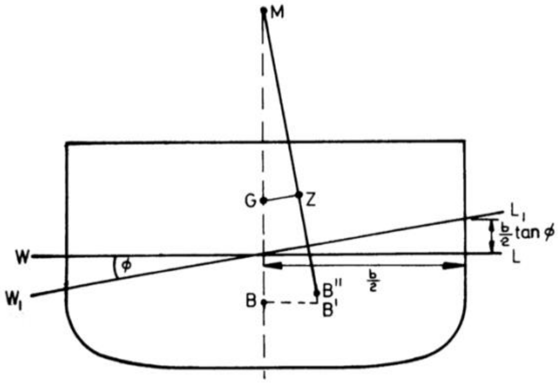
At a large angle of heel the force of buoyancy can no longer be considered to act vertically upwards through the initial metacenter (M). When the ship is heeled to an angle of more than 15°. The centre of buoyancy has moved further out to the low side, and the vertical through B, no longer passes through (M), the initial metacenter. The righting lever (GZ) is once again the perpendicular distance between the vertical through G and the vertical through B,, and the moment of statical stability is equal to W x GZ.

However, GZ is no longer equal to GM sin θ. Up to the angle at which the deck edge is immersed, it may be found by using a formula known as the **wall-sided formula**, i.e

$$GZ = (GM + \frac{1}{2} BM \tan^2 \theta) \sin \theta$$

Ship Stability at Moderate Heel Angles Wall Sided Formula

- Wall-sided ship if the area of wedge emerged is same as the wedge immersed.No practical ships are truly wall-sided, but many may be regarded as such for small angles of Fig. 4.7 Wall-sided formula inclination - perhaps up to about 10-15 degrees.



- Initial waterline WL, inclined waterline W1L1 when vessel heeled by a small angle Ø.a new waterline W1L1 by being heeled through a small angle 0. Since the vessel Since the vessel is wall-sided, WL and W1L1 must intersect on the centreline. is wall-sided, WL and W1L1 must intersect on the centre line.Referring to Fig. 4.7,

The volume transferred in an elemental wedge of length δL where the beam is b is

$$\delta L\left(\frac{1}{2} \times \frac{b}{2} \times \frac{b}{2} \tan \phi\right) = \left(\frac{b^2}{8} \tan \phi\right) \delta L$$

Moment of transfer of volume for this wedge in a direction parallel to WL

$$= \frac{b^2}{8} \tan \phi \frac{2b}{3} \delta L$$

Hence, for the whole ship, moment of transfer of volume is

$$\int_0^L \frac{b^3}{12} \tan \phi \, \mathrm{d}L$$

- Hence the horizontal component of shift of B, is given by

$$\overline{BB'} = \frac{1}{\nabla}\int_0^L \frac{b^3}{12}\tan\phi \, dL$$

$$= \frac{I}{\nabla}\tan\phi = \overline{BM}\tan\phi$$

Similarly, vertical shift $\overline{B'B''}$ is given by

$$\overline{B'B''} = \frac{1}{\nabla}\int_0^L \frac{b^2}{8}\tan\phi \frac{1}{3} b\tan\phi \, dL$$

$$= \frac{I}{2\nabla}\tan^2\phi = \frac{\overline{BM}}{2}\tan^2\phi$$

By projection on to a plane parallel to W_1L_1

$$\overline{GZ} = \overline{BB'}\cos\phi + \overline{B'B''}\sin\phi - \overline{BG}\sin\phi$$

$$= \overline{BM}\left[\sin\phi + \frac{\tan^2\phi}{2}\sin\phi\right] - \overline{BG}\sin\phi$$

$$= \sin\phi\left[\overline{BM} - \overline{BG} + \frac{\overline{BM}}{2}\tan^2\phi\right]$$

i.e.

$$\overline{GZ} = \sin\phi\left[\overline{GM} + \frac{\overline{BM}}{2}\tan^2\phi\right]$$

For a given ship if $\overline{GM}$ and $\overline{BM}$ are known, or can be calculated, $\overline{GZ}$ can readily be calculated using this formula.

- For θ up to 25° we can use this formula

NOTE :- This formula may be used to obtain the GZ at any angle of heel so long as the ship's side at WW1, is parallel to LL1, but for small angles of heel (0 up to 5°), the term **(½BM tan²(θ))** may be omitted.

Question 1 :- A ship of 6000 tonnes displacement has KB = 3 m, KM = 6 m, and KG = 5.5 m. Find the moment of statical stability at 25° heel ?

Ans –

$$GZ = (GM + \frac{1}{2}BM\tan^2\theta)\sin\theta$$

$$= (0.5 + \frac{1}{2} \times 3 \times \tan^2 25^\circ)\sin 25^\circ$$

$$= 0.8262 \sin 25^\circ$$

$$GZ = 0.35 \text{ m}$$

$$\text{Moment of statical stability} = W \times GZ$$

$$= 6000 \times 0.35$$

Ans. Moment of statical stability = 2100 tonnes m.

Question 2 :- A box-shaped vessel 65 m x 12 m x 8 m depth has KG = 4 m, and is floating in salt water upright on an even keel at 4 m draft. Calculate the moments of statical stability at

(a) 5° and (b) 25° heel

Finding M using curves of CoB and BM

In the study of initial stability we have considered small inclinations of a ship. A reference to small inclinations implies such inclinations at which the metacentre does not move in the system of axes OXYZ fixed in relation to the ship. The absence of a shift of the metacentre is a consequence of equality of moments of inertia of areas of equivolume waterplanes calculated about the axis of inclination. This assumption underlies the derivation of the metacentric formulas of stability. For ships with port-starboard symmetry, the metacentre lies on the centre-line for zero heel angle. For non-zero heel angle

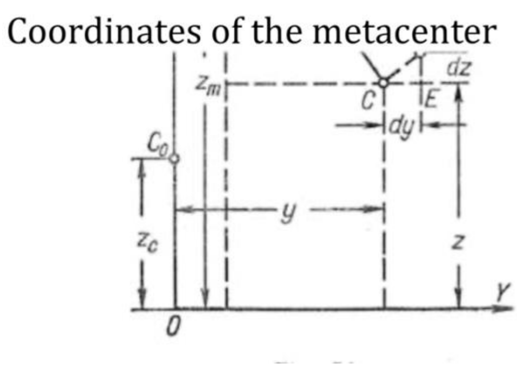
- the metacentre does not lie on the centre-line.
- For very small angles of heel, the metacentre lies very close to the centre-line but not on the centre line

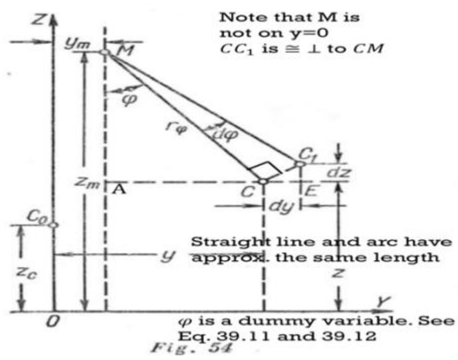
Inclination in only the transverse plane (axis is longitudinal and is the fwd-aft centreline)

If the metacentre is slightly shifted at the inclination under consideration, all the derivations of the previous chapter may be applied without fear of obtaining serious errors in final results.In cases where this condition is not fulfilled there is a risk entailed in using the relations of the previous chapter.Most relations developed in this chapter may be used to solve various problems associated with inclination in the transverse plane but in the absence of inclination in the longitudinal plane.so when **θ (Heel angle) not equal to 0 and Φ (Trim angle) not equal to 0** . It is this case that will be discussed in detail in the present chapter since it is most frequently encountered. The fact is that the angles Φ which in practice may occur along with the occurrence of the angle f are so small that they may be neglected in most cases without introducing any error. There are several possible ways of deriving the relations **to find M** of this chapter.

1. In the first instance it is possible, taking the meta-centric formula of stability as a basis, to obtain the expression for the correction to it to take account of the shift of the metacentre in the co-ordinate system OXYZ. **(Add correction terms to the expression valid for small angles.)**
2. Another way demands that the general expressions be found for the co-ordinates of the centre of buoyancy. By the use of these expressions it is possible to determine the lever of statical stability which, when multiplied by the weight, gives the righting moment. **(First find CoB curve.Then M and finally GZ.)**
3. third way implies the direct derivation of the expressions for two moments: the first one due to the immersed and emerged wedges and the second one due to the shift of the centre of gravity in relation to the vertical through the initial position of the centre of buoyancy. The righting moment is then obtained as the sum of these moments. **(Directly find GZ from fundamentals.)**

It is to be noted that whatever be the way of deriving formulas we necessarily obtain identical results since the underlying relation in the three cases is the relation between the moments of inertia of equivolume waterplanes about the longitudinal centroidal axes and the angle of heel e. This relation cannot be expressed in analytic form as the lines of the ship are not representable by mathematical expressions. It is commonly given in the form of a diagram or a table. In addition to those noted above, we can imagine various other ways of deriving the relation between the righting moment and the angle of heel but these ways cannot give anything new in principle because they will invariably be based on the use of one and the same underlying relation. Let us first derive the expressions for infinitesimal displacements of the centre of buoyancy dy and dz corresponding to any values of the angle Φ.





Directly from Fig. 54 we can obtain the expressions for the co-ordinates of the metacentre y_m and Z_m.

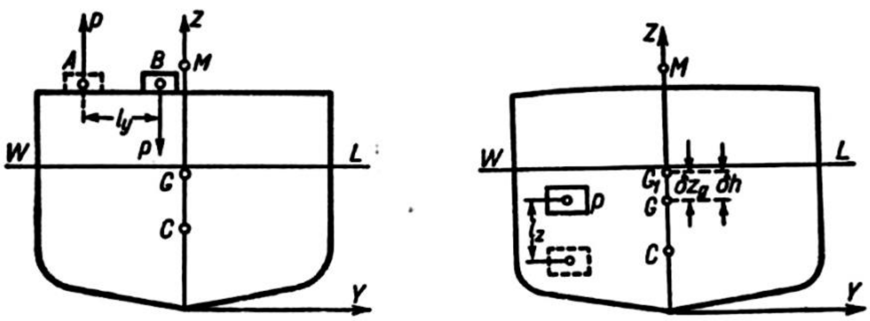
$$y_m = y_\theta - r_\theta \sin\theta;$$
$$z_m = z_\theta + r_\theta \cos\theta.$$

Expressions define the co-ordinates x,y and z of the trajectory centre of buoyancy Corresponding to inclination in the transverse plane.The xo ordinates y and z define the curve of centres of buoyancy which represents the projection of the trajectory on the plane of inclination. **(For more clarification in derivation refer Semyonov).**

Shifting of weight's

As a measure of stability it is possible also to take the product of the weight of the ship and the transverse metacentric height, Dh_o, which is called the **coefficient of stability**. The metacentric height h, is a generally accepted measure of stability. It is convenient as its values encountered in ships of widely different types lie within a relatively narrow range and depend slightly on the dimensions of the ship. On the other hand, the metacentric height does not determine the ability of the ship to resist the moments of external forces applied to her and in this respect is a measure of stability in the particular sense rather than a measure of stability in the broad sense. The stability of the ship is determined by the magnitude of the righting moment which is proportional to the magnitude of the righting moment which is proportional to the coefficient of stability at small inclinations. Hence the product Dh_o gives a more complete idea of the actual properties of the ship but its values for various ships lie within a very wide range.

Lateral shift of weight :-



A weight p which will be considered as small has been transferred on a ship. The weight has been transferred so that its centre of gravity has moved from a point with the co-ordinates X_o, Y_o and Z_o to a point with the co-ordinates ,X_1 , Y_1 and Z_1. It is apparent that there is no change in weight of the ship due to the transference of weight.The shift of the centre of gravity of the weight from the point (X_o, Y_o, Z_o) to the point (X_1, Y_1, Z_1) may be resolved into three mutually perpendicular shifts parallel to the three co-ordinate axes OX, OY and OZ. Thus we have

longitudinal shift of weight

$$l_x = x_1 - x_0;$$

transverse shift of weight

$$l_y = y_1 - y_0;$$

vertical shift of weight

$$l_z = z_1 - z_0.$$

Consider first separately the effect of each of the three shifts of the weight on the position and stability of the ship. Here and henceforth a reference to a change in position of the ship will imply a set of changes in draught, heel and trim.The shift of the weight p in the athwartship direction through the distance l, may be thought of as the removal of the weight p from the point A and the addition of the weight p at the point B. The addition and removal of the weights may be replaced by two equal and opposite forces p and p acting vertically. forces form a couple whose moment heels the ship and is expressed as

$$M_h = pl_y.$$

The heeling moment will be balanced by the righting moment M_h,=M_r, with M_r, expressed by the metacentric formula of stability.So we can say

$$\boxed{\sin(\theta) = \frac{pl_y}{Dh_0} = \frac{pl_y}{D\overline{GM}}}$$

Following the same line of reasoning, we can write for the shift in the fore-and-aft direction

$$\sin(\psi) = \frac{pl_x}{D\ GM_L}$$

Thus the shifts in the longitudinal and transverse directions cause a change in position of the ship leaving her stability un-changed. The shift in the vertical direction, as we shall see below, causes a change only in stability leaving the position of the ship unchanged.

Summary for lateral shift of weight :-

- Thus the shifts in the longitudinal and transverse directions cause a change in the position of the ship leaving her stability unchanged.
- "position" = attitude. Heel or trim or both will change
- "stability unchanged" => coefficient of stability, D GM, is approximately unchanged.Actually, D does not change, zCoG does not change, but BM increases by a small factor of [$1/\cos^3 \Phi$].

<u>Vertical shift of Weight</u> :-

- A vertical shift of a weight changes the KG and the stability of the ship.
- Transverse or longitudinal shift of weight causes a heel or trim but no change in KG (stability).
- If the CoG moves up, the GM reduces.
- At $\Phi = 0$, GZ remains zero. But at all other angles, both positive and negative, GZ decreases.

<u>Hanging Loads :-</u>

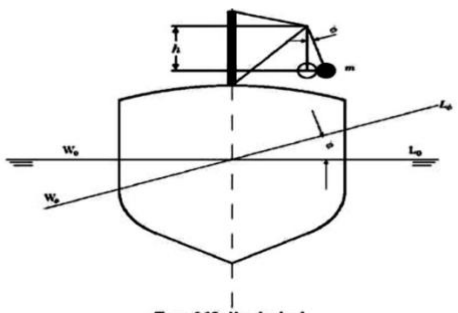


Figure 6.10 Hanging load

In Figure 6.10 we consider a mass m suspended at the end of a rope of length h. When an external moment causes the ship to heel by an angle Φ. the hanging mass moves transversely a dist. **h tan Φ** and the ship centre of gravity moves in the same direction a distance

$$\overline{GG_1} = \frac{hm}{\Delta} \tan\phi$$

As Righting arm is reduced to GZ_{eff} effect is the same as if the centre of gravity, G, moved to a higher position, Gv, given by

$$\overline{GG_v} = \frac{\overline{GG_1}}{\tan\phi} = \frac{hm}{\Delta}$$

As a result, we use for initial-stability calculations a corrected, or effective metacentric height

$$\overline{GM_{eff}} = \overline{GM} - \frac{hm}{\Delta}$$

Free-surface effect

The free-surface effect is an important concept in naval architecture and ship stability. It occurs when a liquid moves freely within a partially filled tank (or compartment) on a ship. When a ship heels), the liquid in the tank also shifts to the lower side.This shift of liquid creates a movement of the centre of gravity of the ship towards the heeled side.As a result, the ship's stability reduces because the metacentric height (GM) decreases. Liquids with free surfaces are a very common kind of moving load. Any engine-propelled vessel needs fuel and lubricating-oil tanks. Tanks are needed for carrying fresh water. The cargo can be liquid; then tanks occupy a large part of the vessel. Tanks cannot be filled to the top. Liquids can have large thermal expansion coefficients and space must be provided to accommodate for their expansion, otherwise unbearable pressure forces may develop. In conclusion. almost all vessels carry liquids that can move to a certain extent endangering thus the ship stability. **A partially-filled tank is known as a slack tank**. Effect does not depend on the position of the tank" because only the change in the CoG is important.

- Present-day computer programmes can calculate exactly and quickly the position of the centre of gravity for any heel angle. For example, one can describe the tank form as a hull surface and run the option for cross-curves calculations.
- The free-surface effect can endanger the ship, or even lead to a negative metacentric height. Therefore, it is necessary to reduce the free-surface effect. The usual way to do this is to subdivide tanks by longitudinal bulkheads. If the left-hand figure would refer to a parallelepiped hull, the moment of inertia of the liquid surface in each tank would be $1/2^3$ = 1/8 that of the undivided tank. Having two tanks, the total moment of inertia, and the corresponding free-surface effect, are reduced in the ratio 1/4.
- The reduction of stability caused by the liquids in slack tanks is known as free-surface effect.
- For equations and numerical questions based on this topic, refer ship stability for masters and mates.

Stability of grounded or docked ships

Grounding of a ship means the unintended contact of a ship's bottom (hull or keel) with the seabed, riverbed, or underwater objects. It is different from a collision (ship hits another vessel) or an allision (ship hits a stationary object above water). Grounding happens below the waterline. Figure 6.15 shows a ship grounded on the whole length of the keel. If local tide lowers the sea level, at a certain draught the ship will loose stability and capsize. To plan the necessary actions, the ship master must know how much time remains until reaching the critical draught.A similar situation occurs when a ship is laid in a floating dock. While ballast water is pumped out of the dock, the draught of the ship decreases. Props must be fully in place before the critical draught is reached. In Figure 6.15 we consider that the draught, T, descended below the value To corresponding to the ship displacement mass. A. Then, the ship weight is supported partly by the buoyancy force **ρVg** and partly by the reaction, R:

$$g\rho\nabla_T + R = g\Delta$$

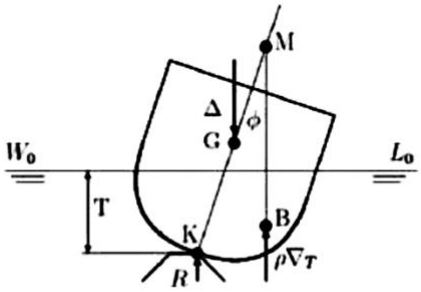


Figure 6.15 Ship grounded on the whole keel length

where **V** is the submerged volume at the actual draught, T. The ship heels and for a small angle, . the condition of stability is

$$g\rho\nabla_T \overline{KM} \sin\phi > g\Delta\overline{KG} \sin\phi$$

On simplifying

$$\overline{KM} > \frac{\nabla}{\nabla_T} \cdot \overline{KG}$$

Stiff and tender ships

Before understanding this concept lets recall some familiar topics

Single degree of freedom (SDOF) :-

A single degree-of-freedom SDOF mechanical system is the simplest dynamic model: a mass m connected to a linear spring (stiffness k) and a linear viscous damper (damping coefficient C) .The mass can move in one direction (coordinate x(t)). External time-varying force F(t) may act on the mass. Write Newton's second law (sum of forces = mass × acceleration). Take positive x to the right :-

$$\boxed{m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t)}$$

On solving the above equation we can obtain the solution

Natural (undamped) angular frequency

$$\omega_n = \sqrt{\frac{k}{m}} \quad (\text{rad/s})$$

Also we will be able to obtain the parameters for an undamped system…..

The natural angular frequency is

$$\omega_n = \sqrt{\frac{k}{m}}$$

So the time period is

$$\boxed{T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{k}}}$$

Roll period

We describe the dynamics of heeling by Newton's equation for rotational motion

$$J\frac{d^2\phi}{dt^2} + g\Delta\overline{GZ} = M_H$$

where **J** is the mass moment of inertia of the ship, **Δ** the mass displacement and $\mathbf{M_H}$, a heeling moment. The mass moment of inertia is calculated as the sum of the products of masses by the square of their distance from the axis of roll

$$J = \sum_{i=1}^{n} (y_i^2 + z_i^2) m_i$$

where j_i, is the transverse and Z_i, is the height coordinate of the mass i. In the SI system we measure J in m^2.we neglected damping and added mass. We also neglect the coupling of heeling with other ship motions. For small angles of heel, and assuming $\mathbf{M_H = 0}$,

$$J\frac{d^2\phi}{dt^2} + g\Delta\overline{GM}\phi = 0$$

We say that this equation describes unresisted roll. We define the mass radius of gyration. I_m. by

$$J = i_m^2\Delta$$

Substituting the above expression and rearranging yields

$$\frac{d^2\phi}{dt^2} + \frac{g\overline{GM}}{i_m^2}\phi = 0$$

Where we can also obtain the relation

$$\omega_0 = \sqrt{\frac{g\,GM}{i_m^2}}$$

the steady-state solution of this equation is of the form = sin ($w_o f$ + C), where w_o is the natural angular frequency of roll, and C, the phase. The natural period of roll is the inverse of the roll frequency, f_o, defined by

$$\omega_0 = 2\pi f_0$$

$$\boxed{T_0 = 2\pi\frac{i_m}{\sqrt{gGM}}.}$$

where the result is in seconds. We conclude that the larger the metacentric height. GM, the shorter the roll period. T_o. If the roll period is too short, the oscillations may become unpleasant for crew and passengers and can induce large forces in the transported cargo. Tangential forces developed in rolling are proportional to the angular acceleration, that is to

$$\frac{d^2\phi}{dt^2} = -\Phi\omega_0^2 \sin(\omega_0 t + \epsilon)$$

a quantity directly proportional to GM. Thus, while a large metacentric height is good for stability, it may be necessary to impose certain limits on it. IMO (2009), for example, referring to ships carrying timber on deck. recommends to limit the metacentric height to maximum 3% of the ship breadth (Part B, paragraph 3.7.5). Operational experience indicates that excessive initial stability should be avoided because it results in large accelerations in rolling and can cause huge stresses in lashings.

Norby (1962) quotes researches carried on by **Kempf,** in Germany, in the 1930s, **Kempf** defined a non-dimensional rolling factor, $[T(g)^{(1/2)}/ B]$, and on the basis of extensive statistics found that:

- for values of Kempf's factor under 8 the ship motions are **stiff.**
- for values between 8 and 14 the roll is **comfortable.**
- for factor values above 14 the motions are **tender.**
- In other way,we can explain stiff ships as A ship with large GM (metacentric height).Hence these ships Comes back upright very quickly when heeled, Rolls fast and jerky → uncomfortable for crew and passengers and a Tender ship is a ship with small GM.Hence these ships Returns to upright slowly, Rolls slow and gentle → more comfortable. But if GM is too small, the ship may be unsafe (risk of capsizing).

Module 3

Stability of Ships at Large angles:

The central notion in this chapter is the righting arm. We shall show how to calculate and represent the righting arm in a set of curves known as cross curves of stability. Another topic is the plot of the righting arm as function of the heel angle, for a given displacement volume and a given height of the centre of gravity. This plot is called curve of statical stability and it is used to assess the ship stability.

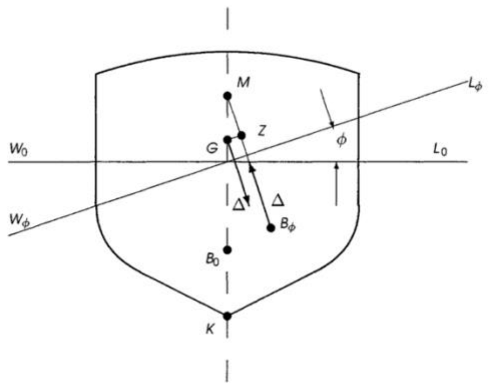


Figure 5.2 Righting arm, $\overline{GZ}$, at small angles of heel

Lets recall why GZ = GM Sin(θ) is valid for only small angles. The answer is given by the Figure 5.2. **Equation holds true as long as the metacentre, M, does not move visibly from its initial position.** Thus, for many ships an angle equal to 5° is small, while for a few others even 15° may be a small angle. The value depends on both ship forms and loading condition.

Curve of statical stability. GZ(Φ)

The plot of the righting arm, GZ, calculated from Eq. (5.2), as function of the heel angle, Φ, at constant V and KG values is called curve of statical stability. Such diagrams are used to evaluate the stability of the ship in a given loading condition. For a full appreciation, it is necessary to compare the righting arm with the various heeling arms that can endanger stability. Now let us check different properties of curve of statical stability and an example of curve of statical stability. **One important value is the maximum GZ value and the heel angle where this value occurs.** For example, in Figure 5.4 the maximum righting arm value is 1.009 m and the corresponding heel angle is 50°.

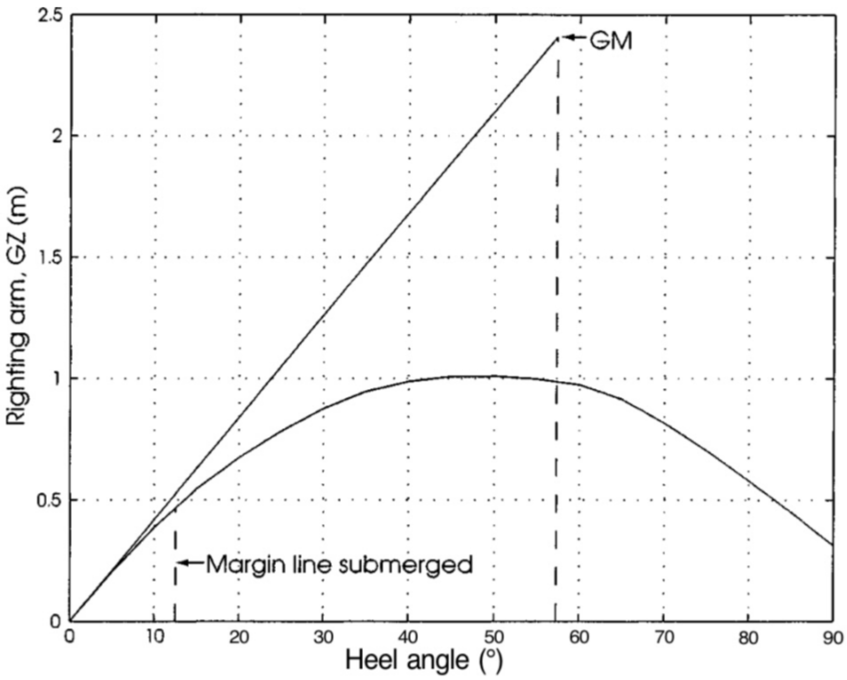


Figure 5.4 Statical-stability curve

Another important point is that in which the GZ curve crosses zero. The corresponding Φ value is called **angle of vanishing stability**. In our example, the righting-arm curve crosses zero at an angle greater than 90°, in a region outside the plot frame. The angle of vanishing stability can often occur at less than 90°.Also GZ curve passes through the origin. There is no righting lever when the ship is upright. **A very useful property refers to the tangent in the origin of the righting arm curve**. The slope of this tangent is given by (a small error here, it is *d(GZ)/dΦ* instead of d(GM)/dΦ).

$$\begin{aligned} \tan\alpha|_{\phi=0} &\approx \left|\frac{\mathrm{d}(\overline{GM})}{\mathrm{d}\phi}\right|_{\phi=0} \\ &= \frac{\mathrm{d}\overline{GM}}{\mathrm{d}\phi}\sin 0 + \overline{GM}_0\cos 0 = \overline{GM}_0 \end{aligned}$$

This Equation yields a simple rule for drawing the tangent : A knowledge of the initial GM can be used to determine the slope of the origin of the GZ curve. In the curve of statical stability, at the heel angle 1 rad (approximately 57.3°) draw a vertical and measure on it a length equal to that of GM. Draw a line from the origin of coordinates to the end of the measured segment. This line is tangent to the GZ curve. From the triangle formed by the heel-angle axis, the vertical at 1 rad, and the tangent in origin, we find the slope of the line denned as above; it is equal to GM/1, that is the same as yielded by above Eq. .The tangent in the origin of the righting-arm curve should always appear in the curve of statical stability; it gives an immediate, visual indication of the GM magnitude, and it is a check of the correctness of the curve.**Also the work done by the righting lever can be obtained by the area under GZ curve.**We can also obtain the **range of stability** from GZ curve. The range of positive stability: Is the range in degrees between the upright equilibrium angle and angle of vanishing stability. It must be emphasized that only the early part of the curve up to say 40° heel can be regarded as giving a reasonable representation of the actual GZ value, as in practice at very large angles of heel it is probable that:

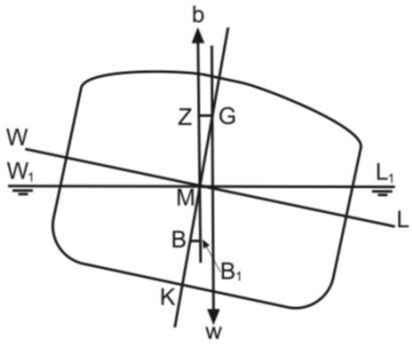
- Cargo will have shifted
- Equipment will have broken loose

- Water will have entered the vessel

Thus making invalid the assumption that G does not shift.

Angle of loll

When a ship with negative initial metacentric height is inclined to a small angle, the righting lever is negative, resulting in a capsizing moment. This effect is shown below and it can be seen that the ship will tend to heel still further.



At a large angle of heel the centre of buoyancy will have moved further out the low side and the force of buoyancy can no longer be considered to act vertically upwards though M, the initial metacentre. If, by heeling still further, the centre of buoyancy can move out far enough to lie vertically under G the centre of gravity, the **righting lever and thus the righting moment, will be zero.** The angle of heel at which this occurs is referred to as the **angle of loll**.It may be defined as **the angle to which a ship with negative initial metacentric height will lie at rest in still water**.

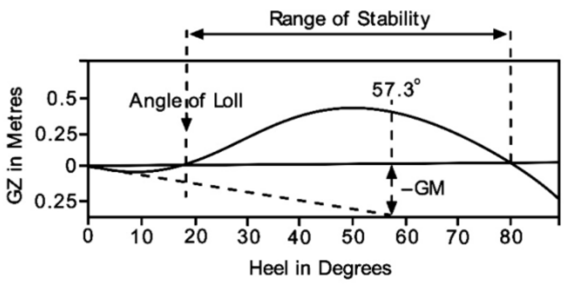


Fig. 24.2

The curve of statical stability for a ship in this condition of loading is illustrated in Figure 24.2. Note from the figure that the **GZ at the angle of loll is zero**. At angles of heel less than the angle of loll the righting levers are negative, whilst beyond the angle of loll the righting levers are positive up to the angle of vanishing stability.

To calculate the angle of loll :-

When the vessel is `wall-sided' between the upright and inclined waterlines, the GZ may be found using the formula

$$GZ = \sin\theta(GM + \tfrac{1}{2}\,BM\tan^2\theta)$$

At the angle of loll, **GZ = 0** ,

either $\sin\theta = 0$ **OR**

$$(GM + \tfrac{1}{2}\,BM\tan^2\theta) = 0$$

If

$$\sin\theta = 0$$

then

$$\theta = 0$$

But then angle of loll cannot be zero, therefore:

$$(GM + \tfrac{1}{2}\,BM\,\tan^2\theta) = 0$$

$$\tfrac{1}{2}\,BM\,\tan^2\theta = -GM$$

$$BM\,\tan^2\theta = -2\,GM$$

$$\tan^2\theta = \frac{-2\,GM}{BM}$$

$$\tan\theta = \sqrt{\frac{-2\,GM}{BM}}$$

The angle of loll is caused by a negative GM, therefore:

$$\tan\theta = \sqrt{\frac{-2(-GM)}{BM}}$$

or

$$\tan\theta = \sqrt{\frac{2\,GM}{BM}}$$

Where

θ = the angle of loll,

GM = a negative initial metacentric height, and

BM = the BM when upright.

Question :- Will a homogeneous log 6 m 3 m 3 m and relative density 0.4 foat in fresh water with a side perpendicular to the waterline? If not, what will be the angle of loll?

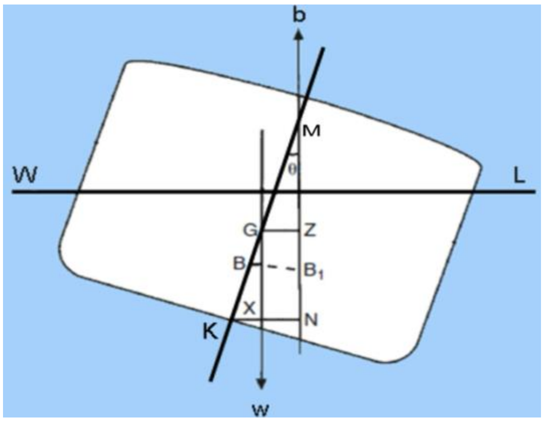
Cross-curves of stability

At large angles of vessel heel, M changes, Waterplane area, M.I, etc changes. So, small And moderate angle assumptions fail. Cross-curves of stability are hydrostatic curves that show the distance KN (from the keel, K, to the line of action of buoyancy, N) for a ship at different angles of heel and for various displacements. They are sometimes called **KN curves**. **These curves are generated based on the geometry of the hull and are independent of the ship's center of gravity (G)**. When a ship heels The center of buoyancy (B) shifts because the shape of the underwater volume changes and this shift creates a righting lever (GZ) that tries to bring the ship back upright. The problem is GZ depends both on hull geometry and the vertical position of G (KG). Instead of recalculating everything each time, naval architects prepare cross-curves once for the hull. With cross-curves, GZ can be quickly determined for any displacement and KG.

How Are Cross-Curves Obtained :-

- Take the ship's hull form and calculate hydrostatics for various displacements.
- At each chosen displacement, heel the ship through angles (0°, 10°, 20°, …, 90°).
- Find the lever KN for each angle.

- Plot KN (y-axis) vs. heel angle (x-axis) for each displacement.
- KN values for a given displacement → Draw a vertical line and read the values where this line crosses the curves.
- The result: a family of curves (cross-curves).



- K → Keel point – Intersection of ship's centreline and baseline
- θ → Heel angle
- B → CoB at θ =0
- B_1→ CoB at θ not equal to zero
- G→ CoG (Assumed to remain there even it heels)
- M – metacentre (changes at large heel angles
- KN → Horizontal separation between the keel point and inclined CoB.

From the figure we can say,

KN =KX +XN = KX +GZ (Since GZ = XN). → GZ = KN – KX

From triangle KGX,. KX = KG Sin θ

Which finally gives

$$GZ = KN - KG \cdot \sin\theta$$

KN = cross-curve ordinate at angle θ and displacement

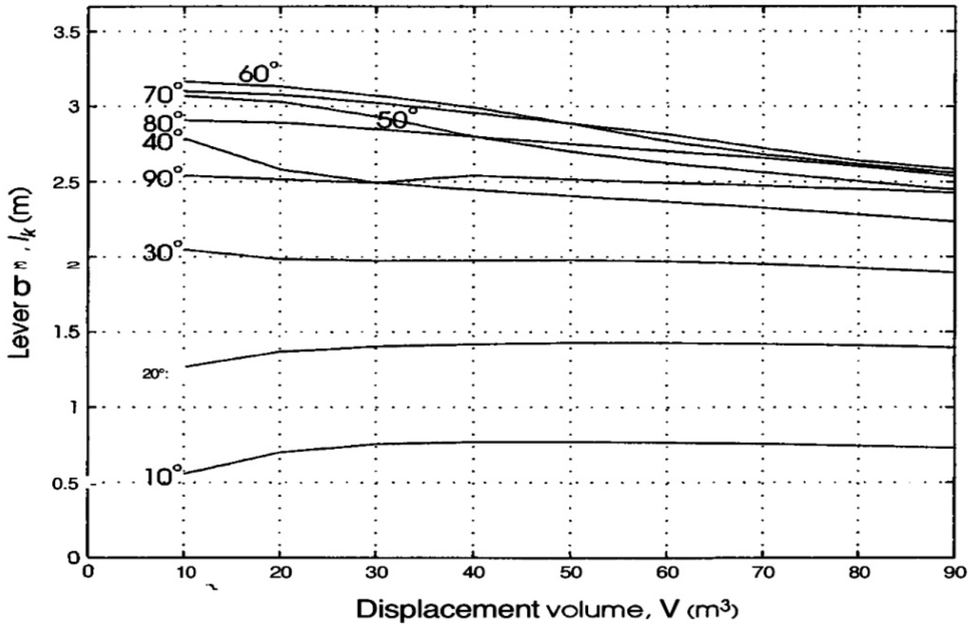


Figure 5.3 Cross-curves of stability of Ship *Lido 9*

Use of Cross Curve :-

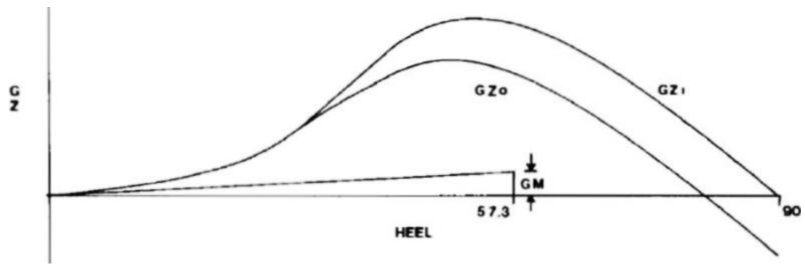
- Our interest is in knowing GM and GZ values, which are measure of ship's stability.
- These values depends on VCG (KG), which varies with loading conditions and also during ship transit.
- So we assume that CoG is initially at a geometry point of ship. Here it is taken as intersection point of keel and ship centreline (K).

- We determine the perpendicular distance from K to the inclined buoyancy line (ie; KN) for a particular displacement (draft) and for different heel angles.
- The above is repeated for different displacement (draft) of the ship.
- These KN values for different heel angles at a particular displacement are marked in a graph paper and is repeated for different displacements. The points for a particular heel angle over different displacements are joined by a curve. Same is repeated for different heel angles. This gives a set of curves known as ship stability cross curves.
- Once the KG values of ship at a particular loading condition is known, we can estimate the GZ curve for that loading condition.
- The stability parameters are then obtained from the GZ curves and are checked against the IMO or other requirements.

Effect of ship dimensions on GZ

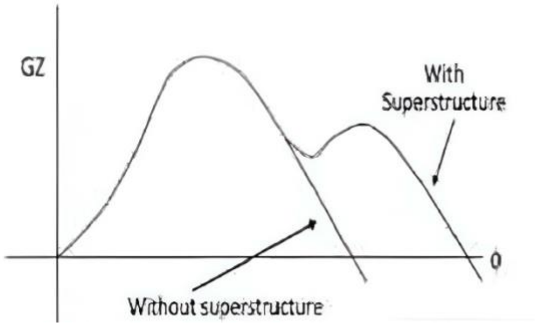
i. Change in freeboard

Suppose a vessel has centre of gravity at G_o and freeboard f_o, can have additional freeboard added to give freeboard f_1, with the draft remaining constant and the centre of gravity remaining at G_o. (G can remain fixed if weight is redistributed.). Then if the vessel is heeled by an external force the initial shape of the curve will be unchanged. However, the angle of deck edge immersion will be delayed for the vessel with high freeboard. **Thus the curve will continue to rise, until the larger angle of deck edge immersion. There is a considerable increase in max. GZ, the range of stability is increased and at large angles of heel the dynamical stability in increased**. The improved stability at very large angles can be accounted for by considering the increased width of waterplane. There is no difference between the two GZ curves upto the angle of deck edge immersion. Changing the freeboard has no effect on GM if the values of all the other parameters are the same .



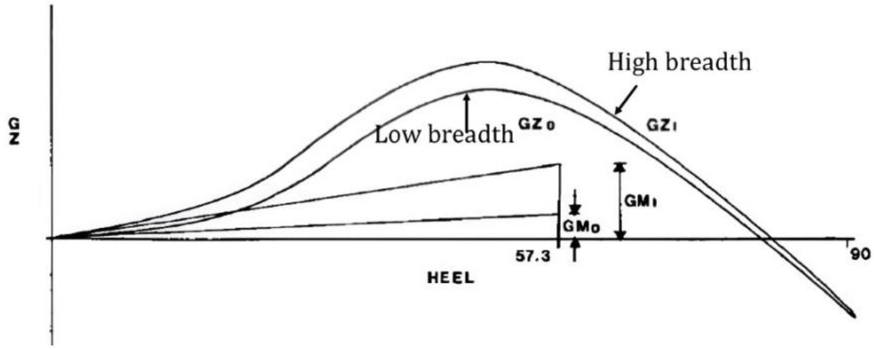
ii. Effect of the superstructure on the GZ curve

Watertight superstructure has a similar effect to increased freeboard, provided the superstructure is distributed equally about the centre of buoyancy, i.e. forecastle and poop (The name originates from the French word for stern, *la poupe*) or uniformly stowed timber deck cargo, etc. However, if the superstructure is not uniform, i.e. offshore supply vessels, there will be a considerable shift of the centre of buoyancy when the superstructure enters the water causing the vessel to trim. As in the figure the trim will be towards the part of the vessel with the lower freeboard. This change of trim due to shift buoyancy as a vessel is heeled is called free trim. All vessels will trim to some extent as they are heeled, in most cases the effect is ignored in presenting GZ curves.When the ship heels because of an external moment, the hatched forward superstructure area provides additional buoyancy. B moves forward and a restorin, noment acts to correct the trim and heel. Superstructures add weight above the main deck. This increases,KG (height of the center of gravity from the keel).From the formula GZ = KN - KG Sin θ, Higher KG → larger subtraction from KN → smaller GZ.



III . Effect of the breadth on the GZ curve

Suppose a vessel with centre of gravity at G_o and beam B_o has its beam Howesed displacemeht case the increase. Since the eidin of Che waterplane is increased, the inertia of the waterplane must be increased, hence the metacentre will rise to M_1. The initial slope of the GZ curve will be increased. However the angles of deck edge immersion will be carlier, and thereafter the slope of the curve will be reduced.At large angles the waterplane is not greatly changed, there is little change in stability at these angles, the curves coincide at some very large angle. In the special case of a box shape the curves intersect at 90°; for most other vessels the point of intersection will be at some angle less than 90°.



IV. Effect of Length on GZ Curve

Length can be increased without altering the position of G and there will be no effect on the value of KM for box shapes and little effect for ship shapes. There will be no change in the angle of deck edge immersed, the shape of the GZ curve will be little changed. However, displacement must be increased thus increasing both righting moment and dynamical stability.

Question :- Explain why a change in the length will have no effect on the value of KM for box shapes and little effect for ship shapes.

Refers other books to understand the effects of shifting weights on the GZ Curve, change in KG on GZ, Horizontal Shift of Weight on the GZ curve, shift of center of gravity of ship on GZ curve and more .

Intact stability regulations

At this point we may ask what is satisfactory stability, or, in simpler terms, how much stable a ship must be. Analyzing the data of vessels that behaved well, and especially the data of vessels that did not survive storms or other adverse conditions, various researchers and regulatory bodies prescribed criteria for deciding if the stability is satisfactory. In this chapter, we present examples of such criteria. To use picturesque language, in this chapter we present man-made laws. Man-made laws, in our case stability regulations, have another meaning. **Stability regulations prescribe criteria for approving ship designs, accepting new buildings, or allowing ships to sail out of harbour. If a certain ship fulfils the requirements of given regulations, it does not mean that the ship can survive all challenges, but her chances of survival are good because stability regulations are based on considerable experience and reasonable theoretic models.** Conversely, if a certain ship does not fulfill certain regulations, she must not necessarily capsize, only the risks are higher and the owner has the right to reject the design or the authority in charge has the right to prevent the ship from sailing out of harbour. Stability regulations are, in fact, codes of practice that provide reasonable safety margins. The purpose of IMO is the inter-governmental cooperation in the development of regulations regarding shipping, maritime safety, navigation, and the prevention of marine pollution from ships. IMO is an agency of the United Nations and has 161 members. The regulations described in this section were issued by IMO in 1995, and are valid 'for all types of ships covered by IMO instruments'.For Passenger and cargo ships, regulations are….

- The area under the righting-arm curve should not be less than 0.055 m rad up to 30°, and not less than 0.09 mrad up to 40°.
- The area under the righting-arm curve between 30° and 40°, or between 30° and the angle of flooding, should not be less than 0.03 m rad.
- The maximum righting arm should occur at an angle of heel preferably exceeding 30°, but not less than 25°.
- The initial metacentric height,GM_0, should not be less than 0.15 m.

Large change in attitude

An **equi-voluminal inclination** is caused by a (first) moment about the axis of floatation. It can be caused by an external force (not through CoF) or a movement of a mass on-board. When there is an inclination, the CoB moves. The new CoB is determined by using the old CoB and the moment that caused the inclination. It can also be determined by considering the underwater volume. When the inclination is due to an external moment, there is a restoring couple, $\boldsymbol{\Delta}\ \boldsymbol{GZ}$, where $\boldsymbol{\Delta}$ is the displacement and $\boldsymbol{GZ}$ is the righting arm. The couple is also equal to the first moment of mass in the newly emerged and submerged volumes. This is used to find the change in the CoB. Therefore, the first moment of volume is of interest.

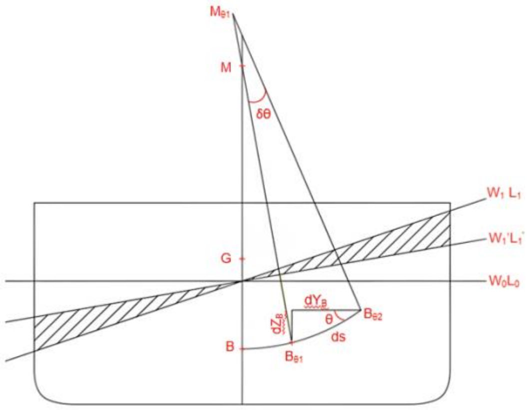
Large change in attitude: Outline of procedure :-

1. Start with **T** (draft), $\boldsymbol{\theta}$ (heel angle), and $\boldsymbol{\psi}$ (trim angle) as independent variables. Values of other parameters such as underwater volume are found using the independent variables.
2. Derive expressions for the change in volume and moments (about global coord axes) due to $\boldsymbol{\delta}$, $\boldsymbol{\delta\theta}$, and $\boldsymbol{\delta\psi}$. The expressions are in terms of the first and second moments of the waterplane area about the global axes.
3. Express changes in volume and moments as total differentials
4. Use 2 and 3 to find partial derivatives of a) volume and b) first moments of volume in terms of first and second moments of the waterplane area about the global axes.
5. Consider equivolume inclinations. Reduce the number of independent variables to two: $\boldsymbol{\delta\theta}$ and $\boldsymbol{\delta\psi}$.
6. Express the partial derivatives of moments in terms of second moments about floatation axes.

7. Find analytic expressions for change in CoB and the metacenter
8. Special case. Trim = 0.
9. Special case. Initial stability. $\boldsymbol{\theta} = \boldsymbol{\phi} = \mathbf{0}$.
10. Find the CoB and BM for zero Trim.

Krylov methods for calculating the GZ curve

A great number of various methods of calculating stability at large angles of heel may be found in the literature. We shall consider here the methods of calculation which are employed in naval architecture in the U.S.S.R. The development of these methods is due to A. N. Krylov. In order to calculate the lever of statical stability by formula (10.1) it is necessary to know, as has been noted above, the relation r($\boldsymbol{\theta}$) for equivolume inclinations. Thus it is necessary first to draw equivolume waterlines at equal angular intervals. W hall consider here two methods of drawing equivolume water lines proposed by A. N. Krylov. Both methods are based on the drawing of an auxiliary waterline cutting off an approximately constant volume and on the subsequent determination of the distance between the auxiliary and equivolume waterlines. In Krylov's method a layer correction is introduced to inclined waterline for maintaining constant volume displacement (∇ = constant) In below Fig., Location of centre of buoyancy for different small inclination angles ($\delta\theta$) are represented by, B,$B_{(\theta 1)}$, $B_{(\theta 2)}$.



Let Y_B and Z_B be transverse and vertical coordinates of B_θ (CoB at inclination angle, θ). Once ship is again inclined by $\delta\theta$, say CoB shifts from $B_{(\theta 1)}$ to $B_{(\theta 2)}$. Let the increments in coordinates of $B_{(\theta 2)}$ be dZ_B and dY_B. For **small inclination angles ($\delta\theta$), ds can be approximated as a straight line.** These distances are measured from upright CoB.Slope of B_θ Curve at any point at θ heel is

$$\frac{dZ_B}{ds} = sin\theta \qquad dZ_B = ds * sin\theta \qquad \frac{dY_B}{ds} = cos\theta \qquad dY_B = ds * cos\theta$$

Also we know $\boldsymbol{\theta} =$ **Arc / length** .So we get

$$ds \ = B_{\theta 1}M_{\theta 1} * \delta\theta$$

$$dZ_B = \ ds * sin\theta = \ B_{\theta 1}M_{\theta 1} * sin\theta * \delta\theta$$

$$dY_B = \ ds * cos\theta = \ B_{\theta 1}M_{\theta 1} * cos\theta * \delta\theta$$

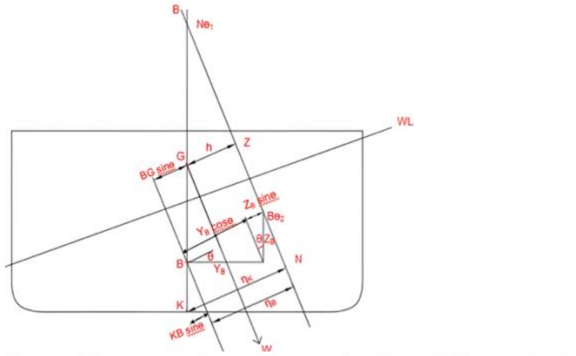
Then

Then on integrating above equations

$$Z_B = \int_0^{\theta} B_{\theta 1} M_{\theta 1} * \sin\theta * \delta\theta = \int_0^{\theta} \frac{I_{T\theta}}{\nabla} * \sin\theta * \delta\theta$$

$$Y_B = \int_0^{\theta} B_{\theta 1} M_{\theta 1} * \cos\theta * \delta\theta = \int_0^{\theta} \frac{I_{T\theta}}{\nabla} * \cos\theta * \delta\theta$$

Now from the following figure we can say that…



$$GZ = h = Y_B * \cos\theta + Z_B * \sin\theta - BG * \sin\theta$$

$$GZ = h = Y_B * \cos\theta + (Z_B - BG) * \sin\theta$$

Since the position of centre of gravity (G) depends on loading condition, it is preferable to have righting arms (GZ) without considering G. Later we can add BG or KG with corresponding $\sin\theta$ (For different loading case and inclination angle).

Righting arm in relation with "B" (COB)

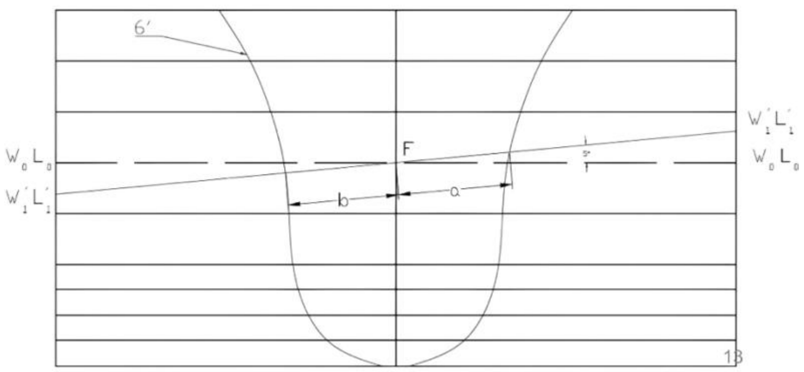
$$\eta_B = Y_B * \cos\theta + Z_B * \sin\theta$$

Righting arm in relation with "K"(Keel)

$$KN = \eta_K = Y_B * \cos\theta + (Z_B + KB) * \sin\theta$$

In Krylov's method, volume displacement is kept constant for each inclination (∇ = constant). We will consider angular intervals 0^o,5^o,………,90^o for successive inclinations. In Figure 1 , W_0L_0 is upright waterline (Zero inclination). W_0L_0 is also called **operating Waterline** $W'_1L'_1$ is drawn through centroid of previous waterline, W_0L_0 .$W'_1L'_1$ is called **auxiliary waterline**. A layer correction is introduced to take account of the change in displacement that needs to be considered.

- Let V_i = volume of immersed wedge)
- V_e = Volume of emerged wedge
- d = Layer Correction Thickness
- $A_{W'1L'1}$ = Area of auxiliary waterplane.

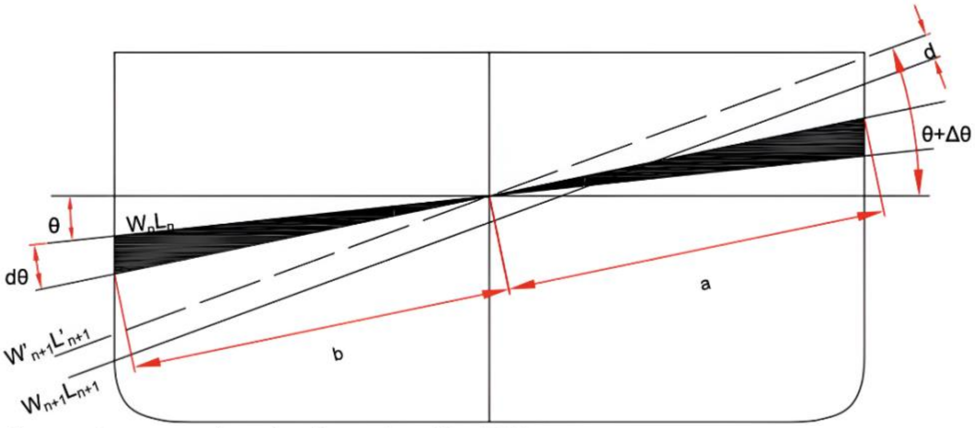


Then

$$d = \frac{V_i - V_e}{A_{W_1'L_1'}}$$

Volume of immersed wedge, $dV_i = \frac{1}{2} * a * a * \delta\theta \;=\; \frac{1}{2} * a^2 * \delta\theta$

Volume of immersed wedge over ship length, $V_i = \int_0^L \frac{1}{2} * a^2 * \delta\theta * dx$



Volume of immersed wedge from θ to $(\theta + \delta\theta)$ & over ship length,

$$V_i = \int_\theta^{\theta+\delta\theta} \left(\int_0^L \frac{1}{2} * a^2 * dx \right) \delta\theta$$

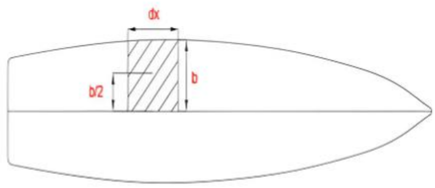
Similarly,

$$V_e = \int_\theta^{\theta+\delta\theta} \left(\int_0^L \frac{1}{2} * b^2 * dx \right) \delta\theta$$

Therefore,

$$d = \frac{1}{A_{W_1'L_1'}} * \int_\theta^{\theta+\delta\theta} \left(\int_0^L \frac{(a^2 - b^2)}{2} * dx \right) \delta\theta \quad = \frac{1}{A'_{W_1L_1}} \int_\theta^{\theta+\delta\theta} M * d\theta$$

Waterplane area moment about C.L



$$M = M_i - M_e$$

$$M_i = \int_0^L \frac{a^2}{2} dx \qquad M_e = \int_0^L \frac{b^2}{2} dx$$

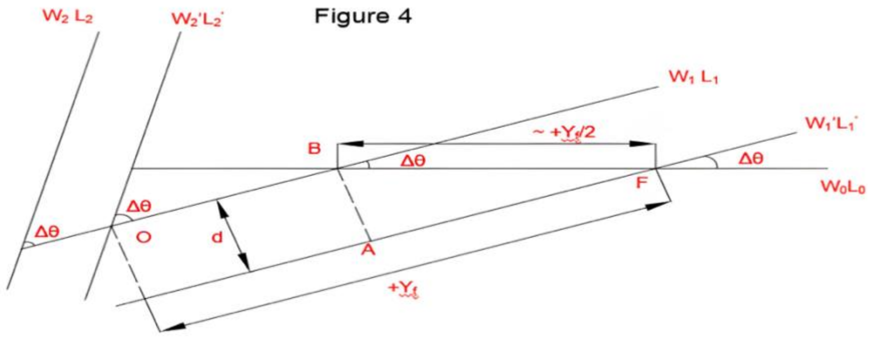
$$M = \int_0^L \frac{(a^2 - b^2)}{2} * dx$$

15

$$Y_f = \frac{Moment}{Area}$$

$$Y_f = \frac{\frac{1}{2}\int_0^L (a^2 - b^2) dx}{\int_0^L (a + b) dx}$$

Figure 4



$$d = \frac{1}{A'_{W_1L_1}} \int_{\theta}^{\theta+\delta\theta} M * d\theta = \frac{1}{A'_{W_1L_1}} * \frac{M_\theta + M'_{\theta+\delta\theta}}{2} \delta\theta$$ Using Trapezoidal rule

$$d = \frac{M'_{\theta+\delta\theta}}{A'_{W_1L_1}} * \frac{\delta\theta}{2}$$ as M_θ is moment of waterplane ($W_1'L_1'$) about CoF = 0

Therefore, $d = \frac{Y_f}{2}\delta\theta$ as $Y_f = \frac{M'_{\theta+\delta\theta}}{A'_{W_1L_1}}$

Y_f is the distance between centroid of new waterline (aux. WL) and centroid of the operating waterline. i.e. the point of intersection of the new WL with the previous WL. Approximate determination of "d" while drawing.

$$d = AB = FB * sin\delta\theta$$

$\delta\theta$ very small, hence
$FB \approx -\frac{Y_f}{2}$ and
$sin\delta\theta \approx \delta\theta$
Therefore, $d = \frac{Y_f}{2}\delta\theta$

Now steps involved are

1. Draw $W_1'L_1'$ through "F" at any angle $\delta\theta$ from W_0L_0
2. Plot $\frac{Y_f}{2}$ along W_0L_0 from the point "F" & mark point "B"
3. Draw a line parallel to $W_1'L_1'$ through point "B"
4. Perpendicular distance from point "B" to $W_1'L_1'$ is equal to "d"
5. Now use W_1L_1 as the new operating waterline and repeat steps 1 to 4

Moment of inertia, $I'_{T\theta}$ related to the intersection of W_0L_0 and $W_1'L_1'$, (about point F)

$$I'_{T\theta} = \frac{1}{3}\int_0^L b^3 dx + \frac{1}{3}\int_0^L a^3 dx = \frac{1}{3}\int_0^L (b^3 + a^3) dx$$

Statical Moment $$M'_{\theta+\delta\theta} = \int_0^L \frac{(a^2 - b^2)}{2} dx$$

Waterplane area, $$A'_{W_1L_1} = \int_0^L (b + a) dx$$

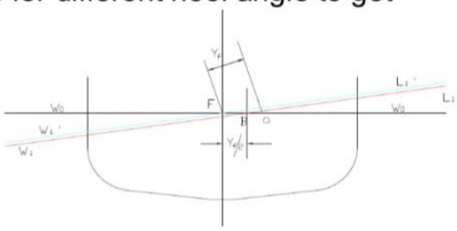
Correction for $I'_{T\theta}$, $I_{corr} = A'_{W_1L_1} * Y_f^2$ $\quad I_{T\theta} = I'_{T\theta} - I_{corr}$ $\quad BM_\theta = \frac{I_{T\theta}}{\nabla}$

Where ∇ is the volume displacement (From Hydrostatics)

- KN values are found by using the Tchebycheff's integration rules in the measurements taken from Tchebycheff's double body plan.
- Instead of calculating the immersed volume and its centroid, this method uses the water plane parameters to arrive at the centre of buoyancy (Centroid of underwater volume) and the corresponding lever.
- Waterlines are adjusted after every inclination in such a manner that the immersed volume displacement is same as the upright volume displacement.

Procedure :-

1. Tchebycheff's Double Body Plan
2. Draw initial operating waterline, W_0L_0
3. Calculate BM_0
4. Rotate waterline by 5 degree to get auxiliary waterline
5. Calculate $BM_{5.deg}$
6. Calculate Y_f and draw new operating waterline.
7. Repeat previous steps for different heel angle to get BM_θ.
8. Estimate KN_θ



Summary :-

Various rules of integration are available for finding the area enclosed by a curve. In the Krylov's II Method used in ship calculations, KN values (horizontal separation between the keel point, K and the centre of buoyancy) are found by using the Tchebycheff's integration rules on the measurements taken from Tchebycheffs double body plan, adjusting the waterlines after every inclination in such a manner that the immersed volume displacement is same as the upright volume. Accuracy of the Tchebycheff's rules can be checked by comparing the Volume Displacement and Vertical Centre of Buoyancy (KB) at the upright condition with that in Hydrostatics, obtained by using the Simpson's rules. Accuracy of the equi-volume inclination method can be ascertained by comparing the immersed volume at 90-degree heel with that at the upright condition and comparing the KN at 90-degree heel with the KB at that angle.

The Cross-curves of stability for ships are the plots of curves of righting levers (GZ) against displacement, for various angles of heel. GZ's for the same angle of heel make a curve. Righting lever, for any displacement and angle of heel, is the horizontal separation between the centre of gravity and the centre of buoyancy. Since centre of gravity varies with loading even for the same displacement, and it is not possible to calculate, in advance, the righting levers for all the possible positions of the centre of gravity, instead of the righting lever, the horizontal distance between the centre of buoyancy and an assumed point on the centreline (for instance, the keel point, K) is found out and plotted in the cross curves. The values measured from these curves are then corrected for the actual position of centre of gravity. When the keel point is used for calculations, the lever is termed as KN. Once the actual KG for a loading condition is determined, GZ for any angle of heel can be calculated using the formula, GZ = KN - KG Sin $\boldsymbol{\theta}$. Cross curves of stability are calculated by various methods.

One method is to find the centroid of the immersed volume (a displacement) for a known angle of heel and then find its horizontal distance from an assumed centreline point. Krylov's methods are also used to find these levers, which use a different approach. Instead of calculating the immersed volume and its centroid, it uses the water plane parameters to arrive at the centre of buoyancy (centroid of volume) and the corresponding lever. In the Krylov's II Method, the method of equi-volume inclinations is employed. After every inclination the water planes are adjusted so as to make the immersed volume same as that of the upright volume displacement of the ship. It is normally done graphically. If the theory is sound and the procedures are followed without error, the immersed volumes after every inclination should be the same.

Prohaska's Method for calculating GZ curve

Prohaska (1947) introduced the concept of residuary stability whereby the righting arm could be expressed as the sum of two independent terms, one loading dominated and one hull form dominated. He suggested that

$$GZ = GM \sin \Phi + MS$$

where ***MS*** represents the residuary stability of the hull form. It can be seen in Figure 5, which shows the stability of a ship at large angles of heel, that the movement of the pro-metacenter is the source of this additional term

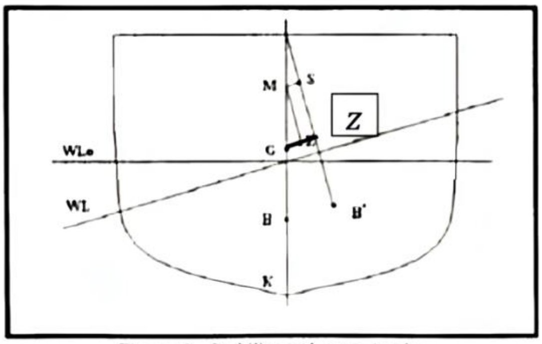


Figure 5. Stability at Large Angles

To facilitate non-dimensional plotting, he also introduced the residuary stability coefficient, C_{RS}, whereby

$$C_{RS} = \frac{MS}{BM_T}$$

Figure 6 shows curves of C_{RS} for various sized tankers and warships, calculated using General HydroStatics (GHS) software, for angles of heel up to 90 degrees. It can be readily seen that the positive C_{RS} at lower angles is what causes the GZ, curve to have a steeper slope than the sine curve used in Equation (4). At higher angles, C_{RS} quickly becomes negative, drawing the GZ curve down toward the unstable region. The transition point between the lower and higher angles, although not easily discerned, corresponds to he point where the deck edge is immersed. In general, GL can be thought of as getting its height from GM and its shape from C_{RS}.In all the cases, C is zero at zero heel angle and negative at large heel angle. From Fig. 7, C is independent of KG. Residuary stability however, is not merely hull form "dominant;"(it will be weakly dependent on KG) it is entirely independent of the location of the centre of gravity. This is illustrated in Figure 7, which shows the CRS curves for a fine-lined hull with varying heights of centre of gravity (KG).While this is true, it can not be said that residuary stability is independent of loading all together. The extent of loading or total weight will determine the displacement, and hence the draft, of the vessel which greatly influences the residuary stability. **Prohaska's further research into residuary stability and the effects of hull form on transverse stability resulted in the broad conclusion that the ratios of beam to draft (B/T) and depth of hull (B/D) have a comparatively greater influence on transverse stability than do the coefficients of form** (i.e. fullness parameters). His work included the analysis of C_{RS} for a series of 42 systematically varied hull forms, covering the range of fullness coefficients seen in the merchant ship fleet of the day. (Prohaska, 1951).

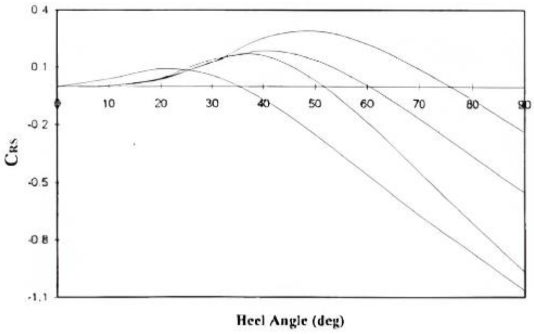


Figure 6. Residuary Stability Coefficient Curves

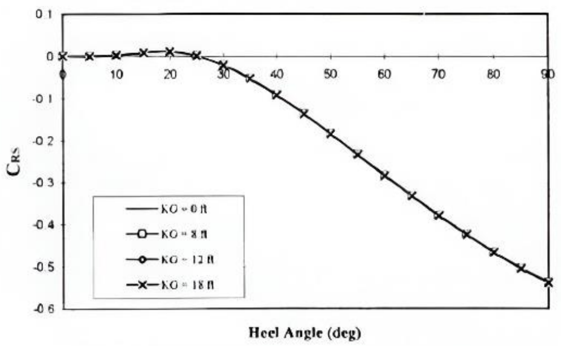


Figure 7. Residuary Stablitity Coefficient for Various KG

The only parameters besides the heel p are the draft ratio T/B and the depth ratio D,/ B here depth corrected for sheer thus:

$$D_1 = D + \frac{S_A + S_F}{6}.$$

In spite of the very simple construction this diagram gave relatively excellent results so long as the ships examined were ordinary merchant ship types having not too fine lines, i.e. block coefficient ought to be larger than say 0.65.

The following are needed to find $C_{RS}(\varphi, \frac{T}{B}, \frac{D_{11}}{B}, \delta, \beta)$

- GM = metacentric height
- BM = metacentric radius
- δ = Block coefficient = $\nabla/(LBT)$ = 0.5 in Fig. 5
- β = midship coefficient = $A_x/(BT)$ = 0.75 in Fig. 5
- T/B = draft / breadth. Fig. 5 x-axis.
- D_{11}/B = depth corrected for shear and erections/ breadth. Fig. 5. y-axis.
- Find $C_{RS} = MS/BM$ using Fig. 5. Use BM and find MS.
- $GZ = GM\ \sin(\theta) + MS$

Prohaska developed **contour plots** (nomograms) of the Coefficient of Residual Stability (C_{RS}) as a function of two key non-dimensional parameters [B/D] and [GM/B] where D is the ship's moulded depth.These contour plots allow naval architects to estimate C_{RS} directly without integrating GZ curves every time. So, the axes are:

- X axis :- B/D
- Y axis :- GM/B
- Contours:- C_{RS}

Fig. 5 is for the special case of δ = Block coefficient = $\nabla/(LBT)$ = 0.5 and β = midship coefficient = $A_x/(BT)$= 0.75. The heel angle is shown in big numbers. Contour lines of C are shown in Fig. 5a.

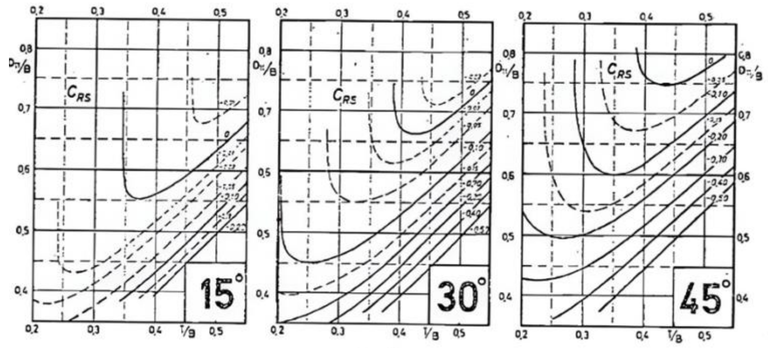


Fig. 5a.

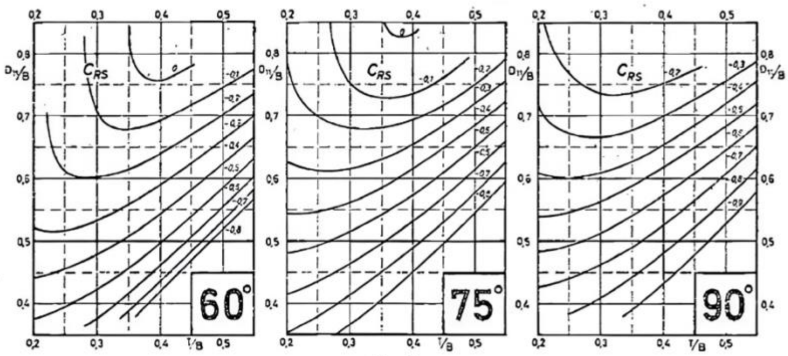


Fig. 5b.

- Read more about Prohaskas method from other books…..

Dynamic stability

Until now we assumed that the heeling moments are applied gradually and that inertial moments can be neglected. Shortly, we studied statical stability. Heeling moments, however, can appear, or increase suddenly. For example, wind speed is usually not constant, but fluctuates. Occasionally, sudden bursts of high intensity can occur; they are called **gusts.** As another example, loosing a weight on one side of a ship can cause a sudden heeling moment that sends down the other side. In the latter cases we are interested in dynamical stability.So **Heeling moments can be caused by** wind, by the centrifugal force in turning, by crowding of passengers on one side, by towing, or by the tension in the cable that links two vessels during operations at sea. Dividing a heeling moment by the displacement force we obtain a heeling arm.*It is no more sufficient to compare righting with heeling arms; we must compare the energy of the heeling moment with the work done by the opposing righting moment.* It can be easily shown that the energy of the heeling moment is proportional to the area under the heeling-arm curve, and the *work done by the righting moment is proportional to the area under the righting-arm curve* .

Proof :-

let us remember that the work done by a force, F, which produces a motion from X_1 to X_2 is equal to

$$W = \int_{x_1}^{x_2} F \, \mathrm{d}x$$

If the path of the force F is an arc of circle of radius r, the length of the arc that subtends an angle dΦ is dx = r dΦ.Substituting into above gives

$$W = \int_{\phi_1}^{\phi_2} F r \, \mathrm{d}\phi = \int_{\phi_1}^{\phi_2} M \, \mathrm{d}\phi$$

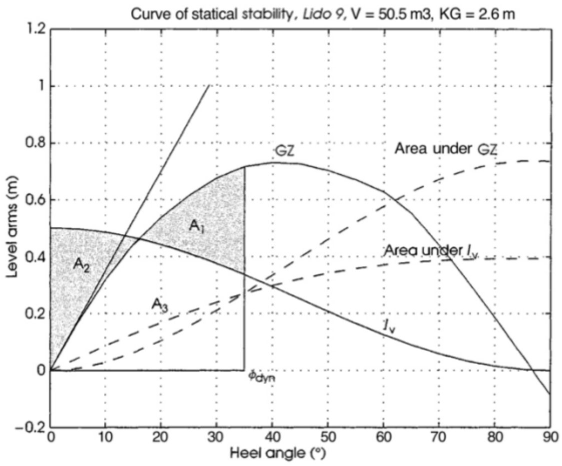
where **M** is a moment. A ship subjected to a sudden heeling moment M_h, applied when the roll angle is Φ_1, will reach for an instant an angle **fa** up to which the energy of the heeling moment equals the work done by the righting moment, so that

$$W = \int_{\phi_1}^{\phi_2} \frac{M_h}{g} \, d\phi = \int_{\phi_1}^{\phi_2} \overline{\Delta GZ} \, d\phi$$

or

$$\int_{\phi_1}^{\phi_2} \frac{M_h}{g\Delta} \, d\phi = \int_{\phi_1}^{\phi_2} \overline{GZ} \, d\phi$$

Hence work done by the righting moment is proportional to the area under the righting-arm curve . This condition is fulfilled in below Figure where the area under the heeling-arm curve is $A_2 + A_3$, and the area under the righting-arm curve is $A_1 + A_3$. As A3 is common to both areas, the condition is reduced to $A_1 = A_2$.



In the above fig. we assumed that the gust of wind appeared when the ship was in an upright condition, i.e. $\Phi_1 = 0$. As shown in Figure 6.5, the situation is less severe if **fa > 0**, and more dangerous if **fa < 0**. In both graphs the maximum dynamical angle is found by plotting the curve and looking for the point where it crosses zero.

$$\int_{\phi_1}^{\phi} \overline{GZ} \, d\phi - \int_{\phi_1}^{\phi} l_v \, d\phi$$

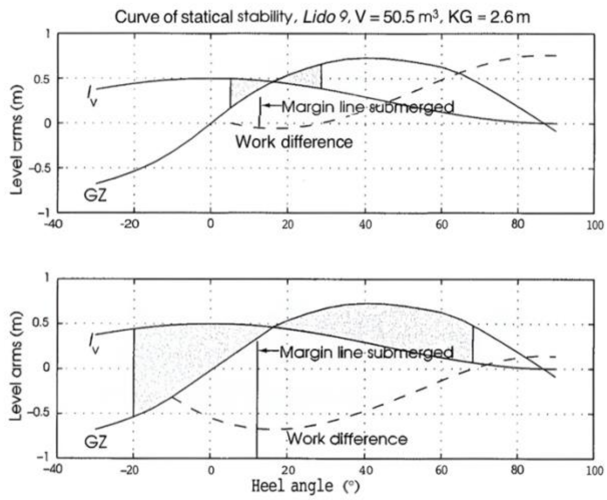


Figure 6.5 The influence of the roll angle on dynamical stability

In the healing-arm and the righting-arm graph understand what will happen if **$A_1 > A_2$** , if **$A_1 < A_2$** and if **$A_1 = A_2$**. We understood the response of a single degree of freedom system to an impulse in previous section.Similarly ***Response of a ship to a gust of wind***

- When a large wind force acts for a very short duration, or a load is dropped, an impulse acts on the ship. The oscillates and slowly comes to rest. Damping is provided by the sea and the energy that is input to the ship is dissipated. It is of interest to find the maximum angle of heel when the ship oscillates.
- When there is no wind and then a gust of wind blows for several seconds, the ship heels.

Most ships are not symmetric about a transverse plane (notable exceptions are Viking ships and some ferries). Therefore, during heeling the centre of buoyancy travels in the longitudinal direction causing trim changes. According to the German regulations this effect must be considered in the calculation of cross- curves.

The IMO Weather criterion

Ships covered by the code should meet a weather criterion that considers the effect of a beam wind applied when the vessel is heeled windwards. We explain this criterion with the help of Figure 8.2.

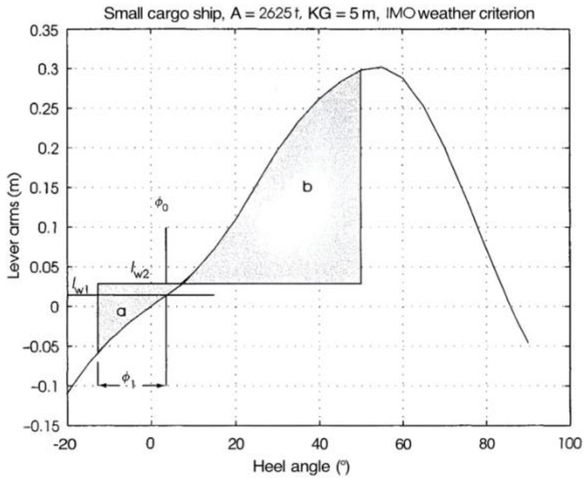


Figure 8.2 The IMO weather criterion

The code assumes that the ship is subjected to a constant wind heeling arm calculated as

$$\ell_{w1} = \frac{PAZ}{1000g\Delta}$$

where P = 504 Nm^2, A is the projected lateral area of the ship and deck cargo above the waterline, in m^2, Z is the vertical distance from the centroid of A to the centre of the underwater lateral area, or approximately to half-draught, in m, A is the displacement mass, in t, and g = 9.81 mS^2.

•For more read the part *Intact stability regulations by A.B. Biran* .

Rolling test

In Naval Architecture, the Roll Test (also known as the Heel Test) is a practical test conducted to determine a ship's rolling period and, consequently, its metacentric height (GM), a key indicator of stability. When a ship is slightly heeled and released, it oscillates (rolls) about its equilibrium position due to the restoring moment created by buoyancy and gravity.

This oscillatory motion is simple harmonic for small angles of heel.Time period of rolling is given by

$$\boxed{T_0 = 2\pi \frac{i_m}{\sqrt{gGM}}.}$$

The relationship between the metacentric height, GM_o ,and the roll period, T, is given as

$$\overline{GM}_0 = \left(\frac{fB}{T}\right)^2$$

where ***B is the ship breadth***. An interesting part of the Annex refers to the plot of heel-angle tangents against heeling moments; it explains the causes of deviations from a straight line, such as free surfaces of liquids, restrictions of movements, steady wind or wind gust.

The inclining experiment

The purpose of this procedure is to ***achieve a satisfactory accuracy in the determination of the lightship weight and of the coordinates of its centre of gravity***. This general procedure is a recommendation. Alternative requirements which are considered to be equivalent to those specified by the following items may be accepted. Acceptance of such equivalents rests with the Society and, where the inclining test is performed to satisfy a statutory requirement, such equivalents also may be subject to the acceptance of the Flag Administration. Where a surveyor of the Society is requested to attend the inclining test, the surveyor should verify that the test is conducted according to accepted procedures and that all basic measurements and data are correctly taken and recorded.

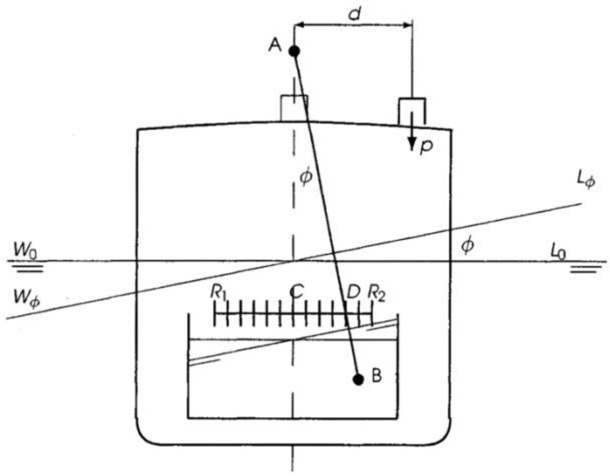
General Preparation for the Test :-

1. **Information that should be submitted** (The Instruction, containing the information of date and location of the test, responsible person, stability, inclining weight, schemes of inclining weight positions etc., should be presented to the Classification Society before the inclining test.)
2. **The inclining test condition** (The ship should be as near to completion as possible. Equipment used by the yard on board should be limited to the utmost extent possible. Prior to the inclining test, lists of all items which are to be added, removed, or relocated should be prepared. These weights and their locations should be accurately recorded. All objects should be secured in their regular positions. The ship should be cleared of residues of cargo, tools, debris, scaffolding and snow. All bilge water and other extraneous standing liquids should be removed. All spaces should be safe for inspection.)
3. **Tank contents** (Preferably, all tanks should be either full or empty. The number of tanks containing liquids should be kept to a minimum.)
4. **Mooring Arrangements and Environmental Conditions** (Mooring is the arrangement of lines, anchors, chains, or other devices used to hold a ship or floating structure at a desired location. Mooring lines should be free of any tension in the transverse direction of the ship during the reading after each weight shift. No external moments should be brought upon the ship. If possible, the ship should be located in a calm, protected area free from external forces. The ship may be moored by means of other special arrangement approved by the Society.)

Common set-up for the inclining experiment :-

A plumb line with a bob B is hung at A. The bob is immersed in a water tank that serves as an oscillation damper. A mass p is displaced transversely a distance d. The resulting heel angle, assumed small, is given by

$$\tan\theta = \frac{pd}{\Delta \overline{GM}}$$



The deflection of the plumb line is measured on a graduated batten R_1R_2 and is used to calculate

$$\tan\theta = \frac{\overline{CD}}{\overline{AC}}$$

A recommended practice is to displace the mass once to starboard and measure tan (θ_s) then to port and measure tanθ_P. The value to be substituted into first equation (a error here, it is tan θ)

$$\tan 0 = \frac{\tan\theta_S + \tan Op}{2}$$

It is recommended to repeat the inclining experiment setup at three stations along the ship. The masses should be selected so that the heel angles remain within the valid range of the governing equation. According to Hansen (1985), the plumb line should be long enough to give a batten reading of 150–200 mm — longer pendulums suit stiff ships, shorter ones suit tender ships. Kastner (1989) found that while longer plumb lines improve reading resolution, they also increase motion sensitivity. He concluded that a plumb line length of about 1.15 m provides good accuracy and stability. Today, traditional setups can be replaced by electronic devices like inclinometers or gyroscopic platforms that directly send heel angle data to an onboard computer. The accuracy of results is checked by plotting the tangent of heel angles against heeling moments — the plot should ideally form a straight line. When analyzing the results of the inclining experiment, we get….

$$\Delta\overline{GM} = \frac{pd}{\tan\theta}$$

To interpret inclining experiment results, the ship's displacement (Δ) and KM must be known. For small trim, these values can be taken from hydrostatic curves using the mean draught (Tm). According to Hansen (1985), trim should not exceed 0.67% of the ship's length for naval ships and 1% for commercial ones. If the trim is large, use Bonjean curves or hydrostatic computer programs after correcting draught marks, as they may not align with the forward and aft perpendiculars. The hull may bend due to uneven loading or temperature changes, and this deflection is determined by comparing the average of forward and aft draughts with the midship draught (Tm).

$$d = T_M - \frac{T_F - T_A}{2}$$

With modern computers and calculators, accurate hydrostatic data can be easily obtained. Assuming the equivalent draught gives a good estimate of displacement, the metacentric height (KM) can also be determined more precisely.For a deflected hull, hydrostatic data can be found using Bonjean curves by drawing a waterline through the forward, midship, and aft draughts. Though the exact waterline shape is unknown, small deflections make it close to a smooth curve. The Bonjean curves are then used to read the required data. If a computer program is available, it can perform hydrostatic calculations in wave conditions. The wave length is set to twice the ship's waterline length, and the wave height to twice the hull deflection.

- If $T_m > (T_f + T_a)/2$, the ship is ***sagging*** (wave crest at midship).
- If $T_m < (T_f + T_a)/2$ the ship is ***hogging*** (wave trough at midship).

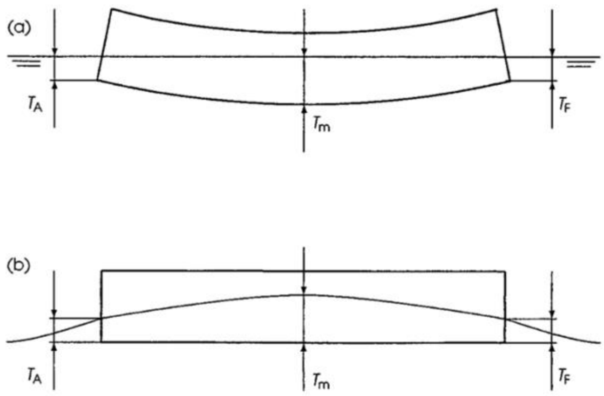


Figure 7.5 Deflected hull - sagging: (a) actual condition (b) computer input

Example 1 shows how the inclining experiment results are analysed to find the product GM, which represents the ship's stability for the test loading. The displacement (A) is obtained from hydrostatic curves, Bonjean curves, or a computer program. From this, the metacentric height (GM) is determined. Similarly, the metacentric height above the baseline (KM) is found using the same methods. Finally, the height of the centre of gravity above the baseline (KG) is calculated using

$$\overline{KG} = \overline{KM} - \overline{GM}$$

Summary :-

Because of uncertainties in the calculation of masses and centres of gravity, it is necessary to validate them experimentally. This is done in the inclining experiment, an operation to be carried out for new buildings and for ships that underwent substantial changes. The ship is brought in sheltered waters and when no wind is blowing. A known mass, p, is displaced transversely a known distance, d, and the tangent of the resulting heel angle, tan $\boldsymbol{\theta}$, is measured. The statistical analysis of several inclining tests yields the product

$$\Delta\overline{GM} = \frac{pd}{\tan\theta}$$

The displacement, A, is found as a function of the draughts measured during the experiment. If a hull deflection is measured it must be taken into account. The vertical centre of gravity is calculated as

$$\overline{KG} = \overline{KM} - \overline{GM}$$

If the trim is large the hydrostatic curves cannot be used. The Bonjean curves are helpful here, as is a computer programme. Both Bonjean curves and computer programmes can be used to calculate the effect of hull deflection.

Module 4

Longitudinal metacentre

In the theory of moments of inertia the two axes for which we obtain the extreme values of moments of inertia are called ***principal axes*** and the corresponding moments, ***principal moments of inertia***. When the waterplane area has an axis of symmetry, this axis is one of the principal axes; the other one is perpendicular to the first. The waterplane area of ships in upright condition has an axis of symmetry: the intersection of the waterplane and the centreline plane. The moment of inertia about this axis is the smallest one; it is used to calculate the ***transverse metacentric radius***. The moment of inertia about the axis perpendicular in F to the centreline is the largest; it enters in the calculation of the ***longitudinal metacentric radius***. To give an idea of the relative orders of magnitude of the transverse and longitudinal metacentric radii, let us consider a parallelepiped barge whose length is L, breadth. B, and draught, T. The volume of displacement equals V = LBT. The transverse metacentric radius results from

$$\overline{BM} = \frac{LB^3/12}{LBT} = \frac{B^2}{12T}$$

The longitudinal metacentric radius is given by

$$\overline{BM_L} = \frac{BL^3/12}{LBT} = \frac{L^2}{12T}$$

The ratio of the two metacentric radii is

$$\frac{\overline{BM_L}}{\overline{BM}} = \left(\frac{L}{B}\right)^2$$

The length-breadth ratio ranges from, for some motor boats, to 10.5, for fast cruisers. Correspondingly, the ratio of the longitudinal to the transverse metacentric radius varies roughly between 10 and 110. As a rule of thumb, the longitudinal metacentric radius is of the same order of magnitude as the ship length.

Trim and MCT$_{(1cm)}$

Trim may be considered as the longitudinal equivalent of list. Trim is also known as `longitudinal stability'. It is in effect transverse stability turned through 90 . Instead of trim being measured in degrees it is measured as the difference between the drafts forward and aft. If difference is zero then the ship is on even keel. If forward draft is greater than aft draft, the vessel is trimming by the bow. If aft draft is greater than the forward draft, the vessel is trimming by the stern. A vessel with a rectangular water-plane has its centre of flotation on the centre line amidships but, on a ship, it may be a little forward or abaft amidships, depending upon the shape of the water-plane. In trim problems, unless stated otherwise, it is to be assumed that the centre of flotation is situated amidships. Trimming moments are taken about the centre of flotation since this is the point about which rotation takes place. The longitudinal metacentre (M_L) is the point of intersection between the verticals through the longitudinal positions of the centres of buoyancy. The vertical distance between the centre of gravity and the longitudinal metacentre (GM_L) is called the longitudinal metacentric height.

The Moment to Change Trim one centimetre

It represents the trimming moment required to change the ship's trim by 1 cm. Derivation of this formula for MCT :-

Consider a ship floating on an even keel as shown in Figure 15.3(a). The ship is in equilibrium. Now shift the weight `w' forward through a distance of `d' metres. The ship's centre of gravity will shift from G to G_1, causing a trimming moment of W*GG_1, as shown in Figure 15.3(b). The ship will trim to bring the centres of buoyancy and gravity into the same vertical line as shown in Figure 15.3(c). The ship is again in equilibrium. Let the ship's length be L metres and let the tipping centre (F) be l metres from aft. The longitudinal metacentre (M_L) is the point of intersection between the verticals through the centre of buoyancy when on an even keel and when trimmed.

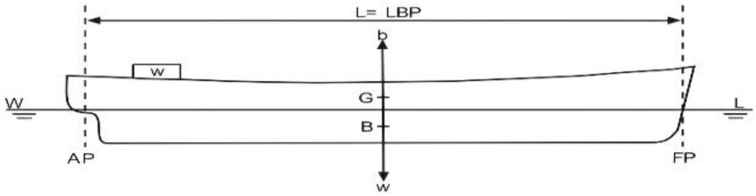


Fig. 15.3(a)

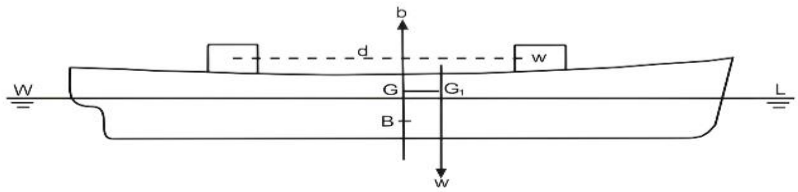


Fig. 15.3(b)

$$GG_1 = \frac{w \times d}{W} \text{ and } GG_1 = GM_L \tan\theta$$

$$\therefore \tan\theta = \frac{w \times d}{W \times GM_L}$$

but

$$\tan\theta = \frac{t}{L} \text{ (See Figure 15.4(b))}$$

Let the change of trim due to shifting the weight be 1 cm. Then w × d is the moment to change trim 1 cm.

$$\therefore \tan\theta = \frac{1}{100L}$$

but

$$\tan\theta = \frac{w \times d}{W \times GM_L}$$

$$\therefore \tan\theta = \frac{\text{MCT 1 cm}}{W \times GM_L}$$

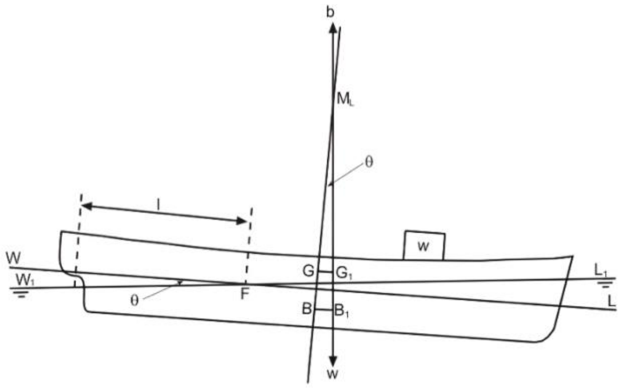


Fig. 15.3(c)

or

$$\frac{\text{MCT 1 cm}}{W \times GM_L} = \frac{1}{100L}$$

and

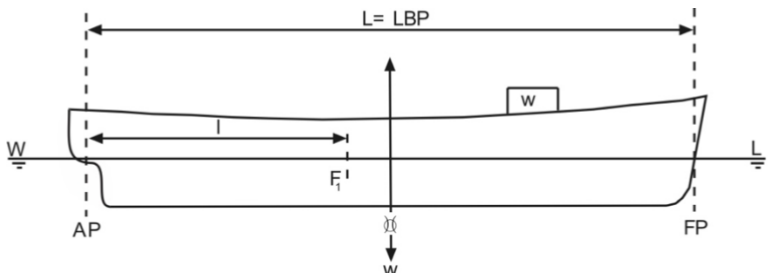
$$\text{MCT 1 cm} = \frac{W \times GM_L}{100L} \text{ tonnes m/cm.}$$

Longitudinal centre of flotation

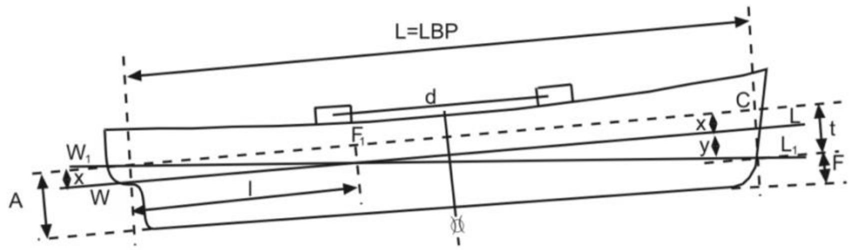
The Centre of Flotation (CF) is defined as the geometric centre of the ship's waterplane area at a given draft. It is the centroid of the plane formed by the intersection of the ship's hull with the still water surface. In simpler terms, if the waterplane were a flat plate of uniform thickness and density, the centre of flotation would be its balance point. ***The Longitudinal Centre of Flotation (LCF) refers to the longitudinal position of the centre of flotation measured from a reference point, usually the midship or the aft perpendicular.*** The LCF indicates where the buoyant forces are effectively concentrated in the longitudinal direction at the current waterplane. When a vessel experiences a change in trim, that is, when its fore and aft drafts change due to the movement of weights on board or external forces, the ship rotates about the LCF. This occurs because the ***LCF is the pivot point for trimming motion***. During such rotation, the total buoyant force remains unchanged, but its distribution along the length of the ship alters, resulting in one end immersing more deeply and the other rising. Mathematically, the LCF plays a key role in the calculation of Moment to Change Trim (MCT) and Trim by the Stern or Bow, as it determines the lever arms through which buoyant and weight moments act. Its position is not fixed for all drafts; it varies with the waterplane shape, which changes as the vessel immerses due to loading conditions. ***Typically, for most merchant ships, the LCF lies slightly abaft the midship position*** at the design waterline, though this can differ depending on hull form.

Change of trim and draft due to addition, removal, and movement of load

When a ship changes trim it will obviously cause a change in the drafts forward and aft. One of these will be increased and the other decreased. A formula must now be found which will give the change in drafts due to change of trim. Consider a ship floating upright as shown in Figure….



F_1 represents the position of the centre of flotation which is l metres from aft. The ship's length is L metres and a weight `w' is on deck forward. Let this weight now be shifted aft a distance of `d' metres. The ship will trim about F_1 and change the trim `t' cms by the stern as shown in below figure. W_1C is a line drawn parallel to the keel.



`A' represents the new draft aft and `F' the new draft forward. The trim is therefore equal to A F and, since the original trim was zero, this must also be equal to the change of trim. Let `x' represent the change of draft aft due to the change of trim and let `y' represent the change forward. In the triangles WW_1F_1 and W_1L_1C, using the property of similar triangles:

$$\frac{x\text{ cm}}{l\text{ m}} = \frac{t\text{ cm}}{L\text{ m}}$$

or

$$x\text{ cm} = \frac{l\text{ m} \times t\text{ cm}}{L\text{ m}}$$

$$\therefore \text{ Change of draft aft in cm} = \frac{l}{L} \times \text{Change of trim in cm}$$

where

l = the distance of centre of flotation from aft in metres, and

L = the ship's length in metres

It will also be noticed that $x + y = t$

$$\therefore \text{ Change of draft F in cm} = \text{Change of trim} - \text{Change of draft A.}$$

The effect of shifting weights already on board

Question:- A ship 126 m long is floating at drafts of 5.5 m F and 6.5 m A. The centre of flotation is 3 m aft of amidships. MCT 1 cm 240 tonnes m. Displacement = 6000 tonnes. Find the new drafts if a weight of 120 tonnes already on board is shifted forward a distance of 45 metres.

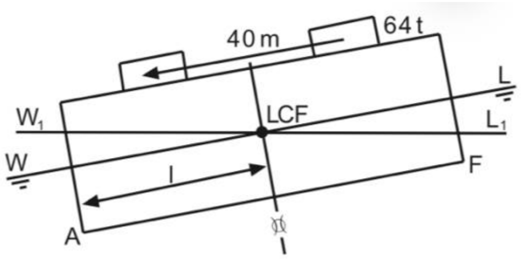
Ans :-

$$\begin{aligned}
\text{Trimming moment} &= w \times d \\
&= 120 \times 45 \\
&= 5400 \text{ tonnes m by the head} \\
\text{Change of trim} &= \frac{\text{Trimming moment}}{\text{MCT 1 cm}} \\
&= \frac{5400}{240} \\
&= 22.5 \text{ cm by the head} \\
\text{Change of draft aft} &= \frac{l}{L} \times \text{Change of trim} \\
&= \frac{60}{126} \times 22.5 \\
&= 10.7 \text{ cm} \\
\text{Change of draft forward} &= \frac{66}{126} \times 22.5 \\
&= 11.8 \text{ cm}
\end{aligned}$$

Original drafts	6.500 m A	5.500 m F
Change due to trim	−0.107 m	+0.118 m
Ans. New drafts	6.393 m A	5.618 m F

Question 2 :- A box-shaped vessel 90 m 10 m 6 m ¯oats in salt water on an even keel at 3 m draft F and A. Find the new drafts if a weight of 64 tonnes already on board is shifted a distance of 40 metres aft.

Ans :-



$$BM_L = \frac{L^2}{12d} = \frac{90 \times 90}{12 \times 3}$$

$$BM_L = 225 \text{ m}$$

$$W = L \times B \times d \times 1.025$$
$$= 90 \times 10 \times 3 \times 1.025$$
$$W = 2767.5 \text{ tonnes}$$

$$MCT\ 1\text{ cm} = \frac{W \times GM_L}{100L}$$

Since BG is small compared with GM_L:

$$MCT\ 1\text{ cm} \simeq \frac{W \times BM_L}{100L} = \frac{2767.5 \times 225}{100 \times 90}$$

$$MCT\ 1\text{ cm} = 69.19 \text{ tonnes m/cm}$$

$$\text{Change of trim} = \frac{w \times d}{MCT\ 1\text{ cm}} = \frac{64 \times 40}{69.19}$$

$$\text{Change of trim} = 37 \text{ cm by the stern}$$

$$\text{Change of draft aft} = \frac{l}{L} \times \text{Change of trim} = \tfrac{1}{2} \times 37 \text{ cm}$$

$$\text{Change of draft aft} = 18.5 \text{ cm}$$

$$\text{Change of draft forward} = 18.5 \text{ cm.}$$

Original drafts	3.000 m A	3.000 m F
Change due to trim	+0.185 m	−0.185 m
Ans. New drafts	3.185 m A	2.815 m F

The effect of loading and/or discharging weights

When a weight is loaded at the centre of flotation it will produce no trimming moment, but the ship's drafts will increase uniformly so that the ship displaces an extra weight of water equal to the weight loaded. *If the weight is now shifted forward or aft away from the centre of flotation, it will cause a change of trim.* From this it can be seen that when a weight is loaded away from the centre of flotation, it will cause both a bodily sinkage and a change of trim. Similarly, when a weight is being discharged, if the weight is first shifted to the centre of flotation it will produce a change of trim, and if it is then discharged from the centre of flotation the ship will rise bodily. Thus, both a change of trim and bodily rise must be considered when a weight is being discharged away from the centre of flotation.

Question:- A ship 90 m long is floating at drafts 4.5 m F and 5.0 m A. The centre of flotation is 1.5 m aft of amidships. TPC 10 tonnes. MCT 1 cm. 120 tonnes m. Find the new drafts if a total weight of 450 tonnes is loaded in a position 14 m forward of amidships.

Ans :-

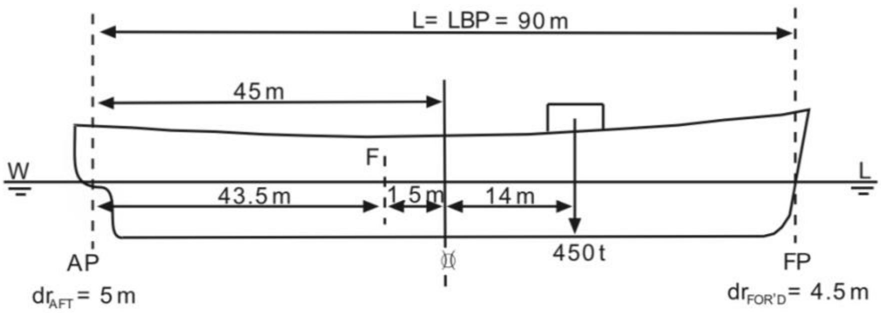


Fig. 15.6

$$\text{Bodily sinkage} = \frac{w}{\text{TPC}} = \frac{450}{10}$$

$$\text{Bodily sinkage} = 45 \text{ cm}$$

$$\text{Change of trim} = \frac{\text{Trim moment}}{\text{MCT 1 cm}} = \frac{450 \times 15.5}{120}$$

$$\text{Change of trim} = 58.12 \text{ cm by the head}$$

$$\text{Change of draft aft} = \frac{l}{L} \times \text{Change of trim} = \frac{43.5}{90} \times 58.12$$

$$\text{Change of draft aft} = 28.09 \text{ cm}$$

$$\text{Change of draft forward} = \frac{46.5}{90} \times 58.12$$

$$\text{Change of draft forward} = 30.03 \text{ cm}$$

Original drafts	5.000 m A	4.500 m F
Bodily sinkage	+0.450 m	+0.450 m
	5.450 m	4.950 m
Change due trim	−0.281 m	+0.300 m
Ans. New drafts	5.169 m A	5.250 m F

Note :- In the event of more than one weight being loaded or discharged, the net weight loaded or discharged is used to find the net bodily increase or decrease M in draft, and the resultant trimming moment is used to find the change of trim. Also, when the net weight loaded or discharged is large, it may be necessary to use the TPC and MCT 1 cm at the original draft to find the approximate new drafts, and then rework the problem using the TPC and MCT 1 cm for the mean of the old and the new drafts to find a more accurate result.

Question:- A box-shaped vessel 40 m 6 m 3 m is floating in salt water on an even keel at 2 m draft F and A. Find the new drafts if a weight of 35 tonnes is discharged from a position 6 m from forward. MCT 1 cm 8.4 tonnes m.

Ans :-

$$\text{TPC} = \frac{\text{WPA}}{97.56} = \frac{40 \times 6}{97.56}$$

$$\text{TPC} = 2.46 \text{ tonnes}$$

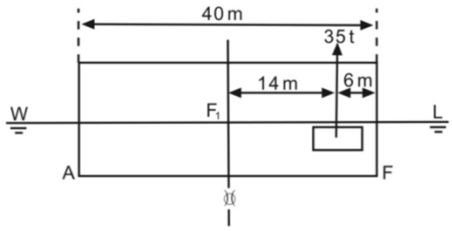


Fig. 15.7

$$\text{Bodily rise} = \frac{w}{\text{TPC}} = \frac{35}{2.46}$$

$$\text{Bodily rise} = 14.2\,\text{cm}$$

$$\text{Change of trim} = \frac{w \times d}{\text{MCT 1 cm}} = \frac{35 \times 14}{8.4}$$

$$\text{Change of trim} = 58.3\,\text{cm by the stern}$$

$$\text{Change of draft aft} = \frac{l}{L} \times \text{Change of trim} = \tfrac{1}{2} \times 58.3\,\text{cm}$$

$$\text{Change of draft aft} = 29.15\,\text{cm}$$

$$\text{Change of draft forward} = \tfrac{1}{2} \times 58.3$$

$$\text{Change of draft forward} = 29.15\,\text{cm}$$

Original drafts	2.000 m A	2.000 m F
Bodily rise	−0.140 m	−0.140 m
	1.860 m	1.860 m
Change due trim	+0.290 m	−0.290 m
Ans. New drafts	2.150 m A	1.570 m F

How Loads displaced transversely affects the ship stability

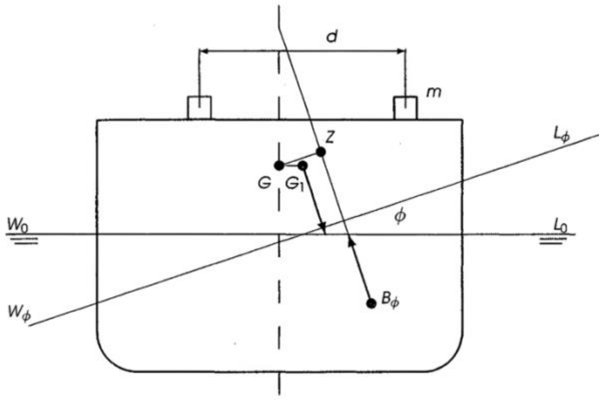
In below figure, we consider that a mass m, belonging to the ship displacement A, is moved transversely a distance d. A heeling moment appears and its value, for any heeling angle Φ is m*dcos Φ. As a result, the ship centre of gravity G moves to a new position, G1, the distance GG1 being equal to

$$\overline{GG_1} = \frac{dm}{\Delta}$$

and the righting arm is reduced to an effective value

$$\overline{GZ_{\text{eff}}} = \overline{GZ} - \frac{dm}{\Delta} \cos \phi$$

We invite the reader to check that the above reduction occurs when the vessel is inclined towards the side to which the mass m was moved, while the righting arm increases if the ship is inclined towards the other side.



Effect of density on trim and draft

1. Effect of change of density when the displacement is constant

When a ship moves from water of one density to water of another density, without there being a change in her mass, the draft will change. This will happen because the ship must displace the same mass of water in each case. Since the density of the water has changed, the volume of water displaced must also change. If the density of the water increases, then the volume of water displaced must decrease to keep the mass of water displaced constant, and vice versa.

The effect on box-shaped vessels

$$\text{New mass of water displaced} = \text{Old mass of water displaced}$$
$$\therefore \text{New volume} \times \text{new density} = \text{Old volume} \times \text{Old density}$$
$$\frac{\text{New volume}}{\text{Old volume}} = \frac{\text{Old density}}{\text{New density}}$$
$$\text{But volume} = \text{L} \times \text{B} \times \text{draft}$$
$$\therefore \frac{\text{L} \times \text{B} \times \text{New draft}}{\text{L} \times \text{B} \times \text{Old draft}} = \frac{\text{Old density}}{\text{New density}}$$

or

$$\frac{\text{New draft}}{\text{Old draft}} = \frac{\text{Old density}}{\text{New density}}$$

2. Effect of density on displacement when the draft is constant

Should the density of the water in which a ship ¯oats be changed without the ship altering her draft, then the mass of water displaced must have changed. The change in the mass of water displaced may have been brought about by bunkers and stores being loaded or consumed during a sea passage, or by cargo being loaded or discharged. In all cases :

$$\text{New volume of water displaced} = \text{Old volume of water displaced}$$

or

$$\frac{\text{New displacement}}{\text{New density}} = \frac{\text{Old displacement}}{\text{Old density}}$$

or

$$\frac{\text{New displacement}}{\text{Old displacement}} = \frac{\text{New density}}{\text{Old density}}$$

Question 1 :- A box-shaped vessel ¯oats at a mean draft of 2.1 metres, in dock water of density 1020 kg per cu. m. Find the mean draft for the same mass displacement in salt water of density 1025 kg per cubic metre.

Ans :-

$$\frac{\text{New draft}}{\text{Old draft}} = \frac{\text{Old density}}{\text{New density}}$$

$$\text{New draft} = \frac{\text{Old density}}{\text{New density}} \times \text{Old draft}$$

$$= \frac{1020}{1025} \times 2.1\,\text{m}$$

$$= 2.09\,\text{m}$$

Ans. New draft = 2.09 m

Question 2 :- A box-shaped vessel ¯oats upright on an even keel as shown in fresh water of density 1000 kg per cu. m, and the centre of buoyancy is 0.50 m above the keel. Find the height of the centre of buoyancy above the keel when the vessel is floating in salt water of density 1025 kg per cubic metre.

Note. The centre of buoyancy is the geometric centre of the underwater volume and for a box-shaped vessel must be at half draft, i.e. KB= ½ draft.

Ans :-

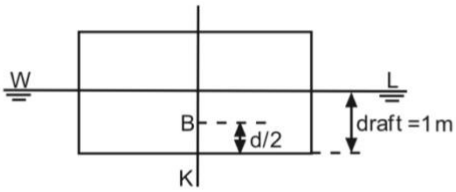


Fig. 5.1

In Fresh Water

$$\text{KB} = 0.5\,\text{m, and since KB} = \tfrac{1}{2}\text{ draft, then draft} = 1\,\text{m}$$

In Salt Water

$$\frac{\text{New draft}}{\text{Old draft}} = \frac{\text{Old density}}{\text{New density}}$$

$$\text{New draft} = \text{Old draft} \times \frac{\text{Old density}}{\text{New density}}$$

$$= 1 \times \frac{1000}{1025}$$

$$\text{New draft} = 0.976\,\text{m}$$

$$\text{New KB} = \tfrac{1}{2}\text{ new draft}$$

Ans. New KB = 0.488 m, say 0.49 m.

Question 3 :- A ship is loading in the water of density 1010 kg per cu. m .If the FWA = 150 mm ,find the change in draft on entering sea water.

Ans :-

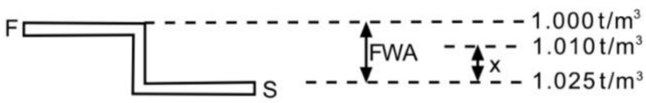


Fig. 5.4

$$\text{Let } x = \text{The change in draft in millimetres}$$

$$\text{Then } \frac{x}{FWA} = \frac{1025 - 1010}{25}$$

$$x = 150 \times \frac{15}{25}$$

$$x = 90\text{ mm}$$

Ans. Draft will decrease by 90 mm, i.e. 9 cm

- For ***rules on draft and trim and Trim and stability booklet freeboard,*** refer A. Biran and other books.

Stability in waves

In this section, we explain why the metacentric height (GM) varies when a wave travels along the ship's length. The discussion is illustrated with data calculated for a 29-metre fast patrol boat, where the influence of waves on stability is particularly significant. The still-water waterline, corresponding to the above draught, is shown as a solid line in Figures 9.1 and 9.2.

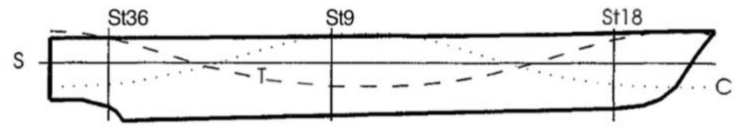


Figure 9.1 Wave profiles on a fast patrol boat outline - S = still water, T = wave trough, C = wave crest

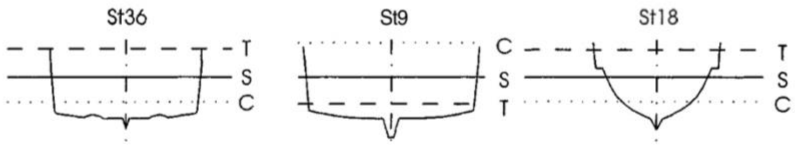


Figure 9.2 Wave profiles on FPB transverse sections - S = still water, T = wave trough, C = wave crest

➢ Ship on a wave crest

Calculations and experimental observations show that *the maximum influence of longitudinal waves on ship stability occurs when the wave length is approximately equal to the ship's waterline length.* The dot–dot lines in Figures 9.1 and 9.2 represent the waterline when the wave crest lies in the midship-section plane — this condition is referred to as the ship being on a wave crest. From Figure 9.2, it can be observed that:

- In the midship section, the waterline rises above the still-water line, while the breadth of the waterline remains nearly unchanged.
- In sections 36 and 18, the waterline descends below the still-water position.
- In section 18, the breadth decreases, a phenomenon extending through a large part of the forebody.

Since the metacentric radius (BM) depends on the cube of the waterline breadth (for constant displacement), this *reduction in breadth results in a decrease in BM, and consequently, a reduction in metacentric height when the ship is on the wave crest.*

➢ Ship in a Wave Trough

The dash–dash lines in Figures 9.1 and 9.2 correspond to the condition where the wave has advanced by half a wavelength, placing the wave trough at the midship section — this is referred to as the ship being in a wave trough.(A wave trough is the lowest point of a wave, located between two successive wave crests). As seen in Figure 9.2:

- The breadth of the waterline increases significantly in the plane of station 18.
- It decreases slightly at the midship section, and increases marginally in the plane of station 36.

The combined effect of these changes is an overall increase in the metacentric radius (BM), leading to an increase in metacentric height when the ship is in a wave trough.

- **NOTE :-** In naval architecture, *trochoidal wave theory* is an important concept used to represent real ocean waves more accurately than the simple sinusoidal (Airy) wave theory. It describes the shape and motion of finite-amplitude waves those with noticeable height and curvature which have a trochoidal profile instead of a pure sine curve. A trochoidal wave is a periodic surface wave whose profile is a trochoid the path traced by a point on a circle as it rolls along a straight line.In simpler terms, the wave crests are sharper and the troughs are flatter, closely resembling the real sea surface. Trochoidal wave theory is considered realistic in naval architecture because it captures the nonlinear, finite-amplitude, and asymmetric nature of real sea waves, leading to more accurate predictions of ship behaviour, stability variation, and wave-induced loads — particularly for smaller or faster vessels operating in steep seas.

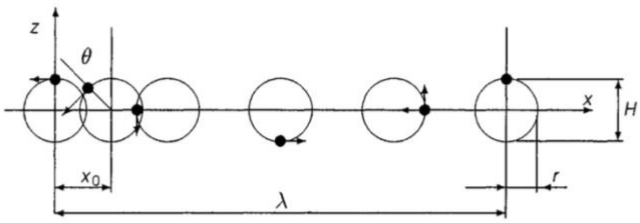


Figure 10.1 The generation of the trochoidal wave

Second-generation intact ship stability criteria

- The first-generation intact stability criteria, which are still in the IMO Code (such as those based on the static GZ curve), were developed decades ago.They focus mainly on static stability — that is, the ship's ability to resist heeling moments in calm or slowly varying conditions. *However, real ships at sea face dynamic conditions, such as*: ***Parametric rolling, Pure loss of stability in waves, Surf-riding and broaching, Dead ship condition and excessive acceleration***. These involve dynamic phenomena that cannot be captured by static GZ curve limits alone.
- The Second-Generation Intact Ship Stability Criteria (SGISC) offer significant advantages by providing a more realistic and technically accurate approach to assessing ship stability. Unlike traditional static methods, the SGISC consider the effects of irregular and unsteady sea conditions, capturing how a vessel behaves dynamically in real ocean waves. By integrating both ship dynamics and hydrodynamics, these criteria account for the complex interactions between the hull, waves, and restoring forces. This framework also promotes design flexibility, allowing naval architects to explore innovative hull forms and operational profiles while still ensuring safety. Furthermore, the SGISC support a performance-based design philosophy, emphasising actual ship behaviour and physics-based analysis rather than relying solely on fixed prescriptive limits.
- The new criteria evaluate five specific stability failure modes:

Mode of failure	
Pure loss of stability	Temporary loss of right moment, due to wave crust.(Reduces water, plane area and GM)
Parametric rolling	Periodic variation in the GM (or restoring Stiffness) in the head or following Seas causes roll amplitude to grow(Mathieu-type instability).
Surf riding/ Broaching	In following sea ship, speed will be equal to wave celerity. Ie loss of Steering control or large yaw-roll coupling
Dead ship condition (DSC)	Stability in beam seas when ship is dead in the water (no propulsion or steering or in other way loss of propulsion and steering in waves and wind.
Excessive Acceleration	High accelerations on passengers or cargo due to large roll motion (Can cause passenger injuries).

Summary:- The Second Generation Intact Stability Criteria represent a modern, physics-based framework that captures the dynamic nature of ship stability in real seas, promoting both safety and design efficiency. They mark a shift from purely prescriptive rules to a performance-based methodology aligned with contemporary naval architecture and simulation technology.

Module 5

Bilging

The bilge is the lowest internal part of the hull, where water and other fluids naturally collect. When the hull is holed, cracked, or indented in this area due to grounding, collision, or impact with submerged objects, it is called bilging. When this occurs, water enters the hull, causing flooding and a loss of buoyancy and stability. In intact and damaged stability calculations, the “effect of bilging a compartment” is often studied to assess how much stability the ship loses if that space becomes flooded. This is part of damage stability analysis required by IMO regulations.

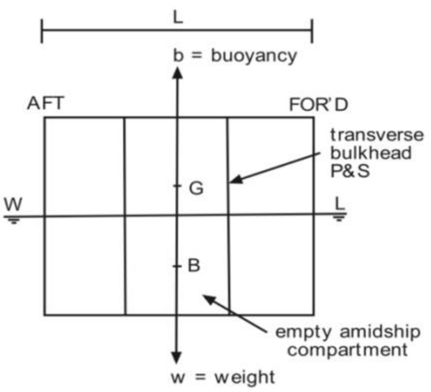


Fig. 21.1 (a)

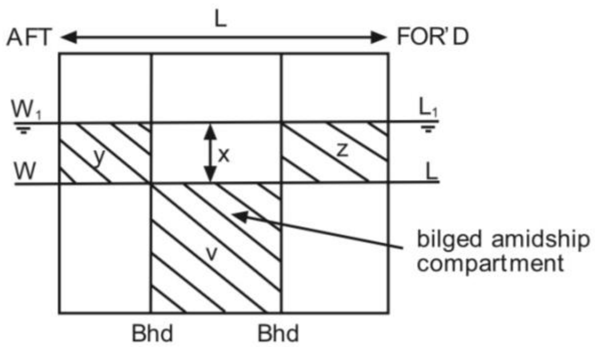


Fig. 21.1 (b)

- • When a vessel ¯oats in still water it displaces its own weight of water. Figure 21.1(a) shows a box-shaped vessel floating at the waterline WL. The weight of the vessel (W) is considered to act downwards through G, the centre of gravity. The force of buoyancy is also equal to W and acts upwards through B, the centre of buoyancy. b=W. Now let an empty compartment amidships be holed below the waterline to such an extent that the water may flow freely into and out of the compartment. A vessel holed in this way is said to be `bilged'. Figure 21.1(b) shows the vessel in the bilged condition. The buoyancy provided by the bilged compartment is lost. The draft has increased and the vessel now floats at the waterline W1L1, where it is again displacing its own weight of water. `X' represents the increase in draft due to bilging. The volume of lost buoyancy (v) is made good by the volumes `y' and `z'.

$$\therefore\ v = y + z$$

Let 'A' be the area of the water-plane before bilging, and let 'a' be the area of the bilged compartment. Then:

$$y + z = Ax - ax$$

or

$$v = x(A - a)$$

$$\text{Increase in draft} = x = \frac{v}{A - a}$$

i.e.

$$\text{Increase in draft} = \frac{\text{Volume of lost buoyancy}}{\text{Area of intact waterplane}}$$

Note. Since the distribution of weight within the vessel has not been altered the KG after bilging will be the same as the KG before bilging.

Question 1 :- A box-shaped vessel is 50 metres long and is floating on an even keel at 4 metres draft. A compartment amidships is 10 metres long and is empty. Find the increase in draft if this compartment is bilged.

Ans :-

$$x = \frac{v}{A - a} = \frac{l \times B \times B}{(L - l)B}$$

let

$$B = \text{Breadth of the vessel}$$

then

$$x = \frac{10 \times B \times 4}{50 \times B - 10 \times B}$$

$$= \frac{40B}{40B}$$

$$\underline{\text{Increase in draft} = 1 \text{ metre}}$$

Question 2 :- A box-shaped vessel is 150 metres long 24 metres wide 12 metres deep and is floating on an even keel at 5 metres draft. GM 0.9 metres. A compartment amidships is 20 metres long and is empty. Find the new GM if this compartment is bilged.

Ans :-

$$\text{Old KB} = \tfrac{1}{2}\ \text{Old draft}$$

$$= 2.5\,\text{m}$$

$$\text{Old BM} = B^2/12d$$

$$= \frac{24 \times 24}{12 \times 5}$$

$$= 9.6\,\text{m}$$

$$\text{Old KB} = +2.5\,\text{m}$$

$$\text{Old KM} = 12.1\,\text{m}$$

$$\text{Old GM} = -0.9\,\text{m}$$

$$\text{KG} = 11.2\,\text{m}$$

This KG will not change after bilging has taken place.

$$x = \frac{v}{A - a}$$

$$= \frac{20 \times 24 \times 5}{150 \times 24 - 20 \times 24}$$

$$= \frac{2400}{130 \times 24}$$

$$\text{Increase in draft} = 0.77 \text{ m}$$

$$\text{Old draft} = 5.00 \text{ m}$$

$$\text{New draft} = \underline{\overline{5.77 \text{ m}}} = \text{say draft } d_2$$

Permeability (μ)

permeability (μ) is a crucial concept used to describe how much of a ship's compartment or space can be occupied by water if it becomes flooded or bilged. When an empty compartment is bilged, the whole of the buoyancy provided by that compartment is lost.For an empty compartment permeability, μ= 100%. Consequently, the higher the value of the permeability for a bilged compartment, the greater will be a ship's loss of buoyancy when the ship is bilged. The permeability of a compartment can be found from the formula:

$$\mu = \text{Permeability} = \frac{\text{Broken Stowage}}{\text{Stowage Factor}} \times 100 \text{ per cent}$$

The broken stowage to be used in this formula is the broken stowage per tonne of stow.

- When a bilged compartment contains cargo, the formula for finding the increase in draft must be amended to allow for the permeability. If 'μ' represents the permeability, expressed as a traction, then the volume of lost buoyancy will be 'μv' and the area of the intact waterplane will be 'A — μv' square metres. The formula then reads:

$$x = \frac{\mu v}{A - \mu a}$$

Question 1 :- A box-shaped vessel is 64 metres long and is floating on an even keel at 3 metres draft. A compartment amidships is 12 m long and contains cargo having a permeability of 25 per cent. Calculate the increase in the draft if this compartment be bilged ?

Ans :-

$$x = \frac{\mu v}{A - \mu a}$$

$$= \frac{\frac{1}{4} \times 12 \times B \times 3}{64 \times B - \frac{1}{4} \times 12 \times B}$$

$$= \frac{9B}{61B}$$

Ans. Increase in draft = 0.15 m

Bilging end compartments

When the bilged compartment is situated in a position away from amidships, the vessel's mean draft will increase to make good the lost buoyancy but the trim will also change. Consider the box-shaped vessel shown in Figure 21.3(a). The vessel is floating upright on an even keel, WL representing the waterline. The centre of buoyancy (B) is at the centre of the displaced water and the vessel's centre of gravity (G) is vertically above B. There is no trimming moment.

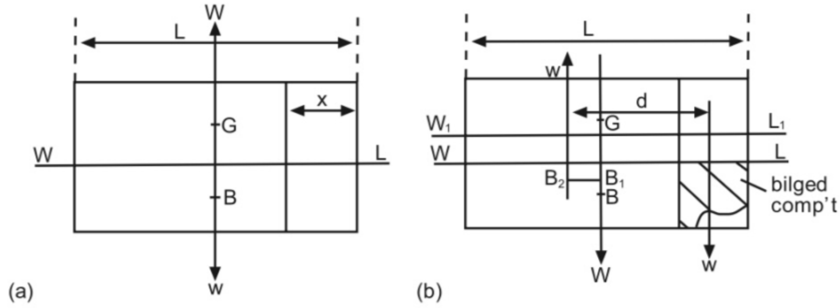


Fig. 21.3

Now let the forward compartment which is X metres long be bilged. To make good the loss in buoyancy, the vessel's mean draft will increase as shown in Figure 21.3(b), where W1L1 represents the new waterline. Since there has been no change in the distribution of mass within the vessel, the centre of gravity will remain at G. It has already been shown that the effect on mean draft will be similar to that of loading a mass in the compartment equal to the mass of water entering the bilged space to the original waterline. The vertical component of the shift of the centre of buoyancy (B to B1) is due to the increase in the mean draft. KB1 is equal to half of the new draft. The horizontal component of the shift of the centre of buoyancy (B1B2) is equal to X/2. A trimming moment of W B1B2 by the head is produced and the vessel will trim about the centre of flotation (F), which is the centre of gravity of the new water-plane area.

$$B_1B_2 = \frac{w \times d}{W}$$

or

$$W \times B_1B_2 = w \times d$$

but

$$W \times B_1B_2 = \text{Trimming moment,}$$

$$\therefore\ w \times d = \text{Trimming moment}$$

It can therefore be seen that the effect on trim is similar to that which would be produced if a mass equal to the lost buoyancy were loaded in the bilged compartment.

Question 2 :- A box-shaped vessel 75 metres long 10 metres wide 6 metres deep is floating in salt water on an even keel at a draft of 4.5 metres. Find the new drafts if a forward compartment 5 metres long is bilged.

Ans :- New drafts [Aft = 3.788 m] and [Fwd = 6.002 m]. Refer Ship stability for masters and mates.

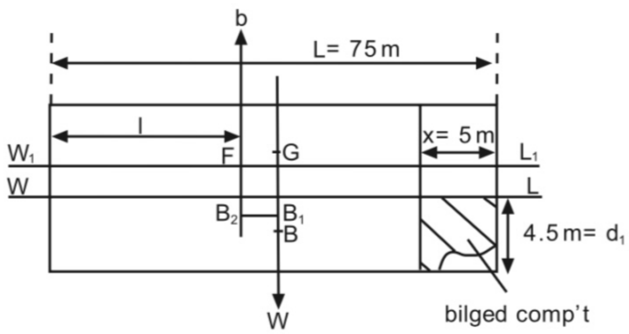


Fig. 21.4

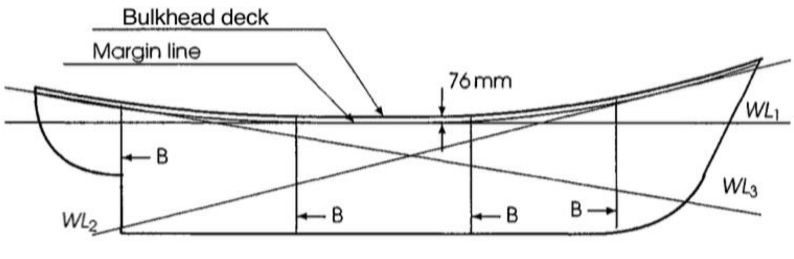
Effect of bilging on stability :- It has already been shown that when a compartment in a ship is bilged the mean draft is increased. The change in mean draft causes a change in the positions of the centre of buoyancy and the initial metacentre. Hence KM is changed and, since KG is constant, the GM will be changed.

Floodable length

- The hull is subdivided into compartments by means of **watertight bulkheads**.
- After flooding of a prescribed number of compartments the ship shall not submerge beyond a line situated at least 76 mm (3 in) below the deck at side. The said line is called in English **margin line**.
- *The floodable length at a given point of the ship length is the maximum length, with the centre at that point, that can be flooded without submerging the ship beyond the margin line*.
- In Figure 11.1, we see the sketch of a ship subdivided by four bulkheads. The three waterlines WL_1, WL_2 and WL_3 are tangent to the margin line. They are examples of limit lines beyond which no further submergence of the damaged ship is admissible. If the bulkhead deck is not continuous, a continuous margin line can be assumed such as having no point at a distance less than 76 mm below the deck at side. Let us suppose that calculating the volume of a compartment starting from its dimensions we obtain the value v. There is almost no case in which this volume can be fully flooded because almost always there are some objects in the compartment. Even in an empty tank there are usually structural members – such as frames, floors and deck beams - sounding instruments and stairs for entering the tank and inspecting it. If we deduct the volumes of such objects from the volume v, we obtain the volume of the water that can flood the compartment; let it be v_f. The ratio

$$\mu = \frac{v_F}{v}$$

Is actually permeability.



B - Watertight bulkhead

Two methods for finding the ship condition after flooding

There are two ways of calculating the effect of flooding. One way is known as the method of lost buoyancy, the other as the method of added weight.

I. The **method of lost buoyancy** assumes that when a compartment is flooded, it no longer provides buoyancy to the vessel. This reflects real conditions: when seawater freely enters a compartment, the pressure inside the flooded space equals the external water pressure, cancelling the buoyant force that compartment would normally contribute. In this method, the flooded volume is no longer considered part of the vessel's effective hull form, while the structural weight of that compartment remains part of the ship's total displacement. As a result, the ship's displacement and centre of gravity (G) remain unchanged. The vessel will then adjust its position (heel and trim) until both force and moment equilibrium are restored. Since the flooding water is treated as external to the ship, it does not produce any free-surface effect.This method is also known as ***method of constant displacement***.

II. he **method of added weight** assumes that the water entering a damaged compartment becomes part of the ship's mass. Therefore, the mass of the flooding water is added to the ship's displacement, giving rise to the term "added weight." Although modern analysis is performed using mass units, the term "weight" is retained for convention and to avoid confusion with the unrelated concept of added mass used in hydrodynamic studies. In the method of added weight the displacement of the flooded vessel is calculated as the sum of the intact displacement and the mass of the flooding water. The new centre of gravity (G_1) is determined by combining the moments of the intact ship and the flooding water about a chosen reference point. Since the flooding water is considered part of the ship, it introduces a free-surface effect, which must be evaluated and included in the stability calculations.

III. refer other textbooks for numerical related to this topic.

The curve of floodable length

Before the advent of digital computers it was not possible to check in a few seconds what happens when certain compartment combinations are flooded. To improve efficiency, Naval Architects devised ingenious, very elegant methods; one of them produces the curve of floodable lengths.

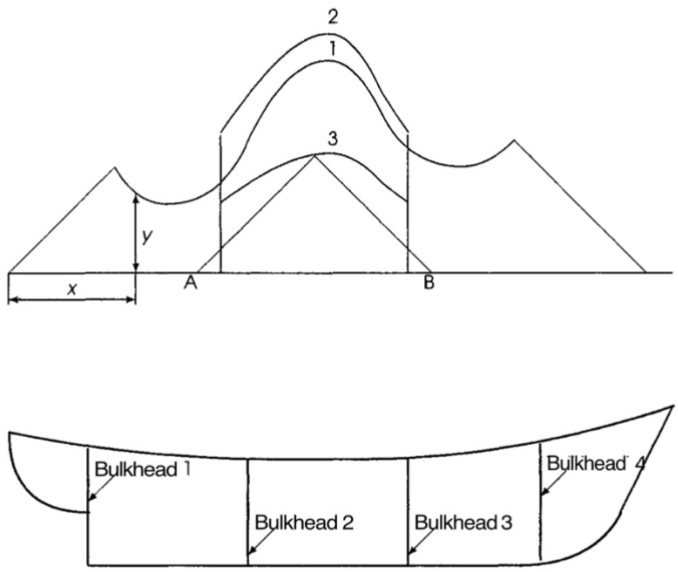


Figure 11.7 The curve of floodable lengths

In Figure 11.7, we can identify the properties common to all curves of floodable lengths and give more insight into the flooding process.

I. At the extremities, the curve turns into straight-line segments inclined 45° with respect to the horizontal. Let us choose any point of the curve in that region. Drawing from it lines at 45°, that is descending along the first or the last curve segment, we reach the extremities of the ship. These are indeed the limits of the floodable compartments at the ship extremities because there is no vessel beyond them.

II. The straight lines at the ship extremities rise up to local maxima. Then the curve descends until it reaches local minima. Usually the ship breadth decreases toward the ship extremities and frequently the keel line turns up. Thus, compartment volumes per unit length decrease toward the extremities. Therefore, floodable lengths in that region can be larger and this causes the local maxima.
III. As we go towards the midship the compartment volumes per unit length increase, while still being remote from the midship. Flooding of such compartments can submerge the margin line by trimming the vessel. Therefore, they must be kept short and this explains the local minima.
IV. The curve has an absolute maximum close to the midship. Flooding in that region does not cause appreciable trim; therefore, floodable lengths can be larger.