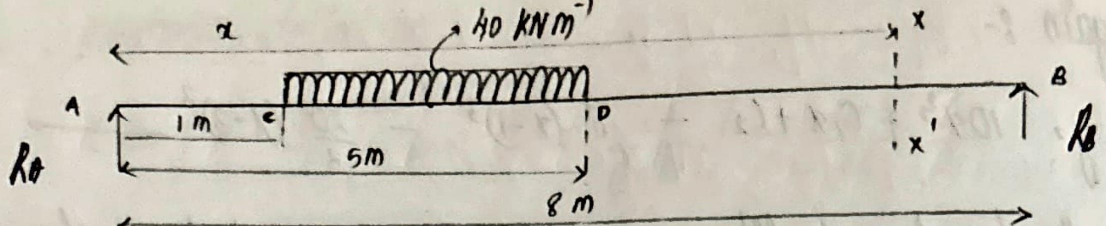
Qn 8 -



$R_A = 100$
$R_B = 60$

Ans 8- $R_A + R_B = 40 \times 4$ & $\sum M_A = 0 \Rightarrow 8R_B = 40 \times 3.5 \times 4 \Rightarrow R_B = \frac{160}{8} \times 4$

ie $\frac{160 \times 3}{8} = R_B = \underline{60\ KN}$ $\quad R_A = 160 - 60 = \underline{100\ KN}$

$M_x = 100 . x - 40 \times 4 (x - 3)$

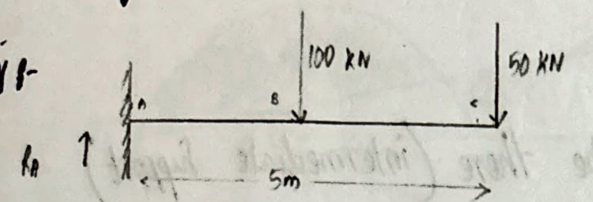
MOHR'S THEOREM

Theorem 1: change in the slope b/w 2 points under flexure is equal to the area of "M/EI" diagram b/w those 2 points..

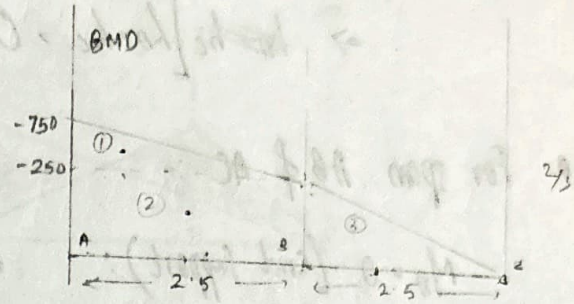
Theorem 2: Change in deflection b/w 2 points under flexure is equal to the moment of area of diagram of M/EI

Q08 A cantilever of length 5 m carries a point load of 50 KN at free end, another load of 100 KN at the centre. The material is mild steel and $I = 10^8\ mm^4$. Determine the slope & deflection at the free end using moment-area method. ($E = 2.1 \times 10^5$ N/mm²)

Ans :-



$R_A = 150$ KN



Area under BMD = slope → A = ① + ② + ③ = $\frac{1}{2} \times 2.5 \times 500 + 2.5 \times 250 + \frac{1}{2} \times 2.5 \times 250$ / EI

• Area of BMD = slope → Slope, A = ① + ② + ③ = $-\left(\frac{1}{2} \times 2.5 \times 250 + 250 \times 2.5 + \frac{500 \times 2.5}{2}\right)/EI$

$\theta_A = 0$ $\left[\ \theta_{A-C}\ \right]$ & $\theta_B - \theta_A = \theta_B$

$= -(312.5 + 625 + 625)/EI$

$= -(1562.5)/EI$

$EI = 2.1 \times 10^{13}$

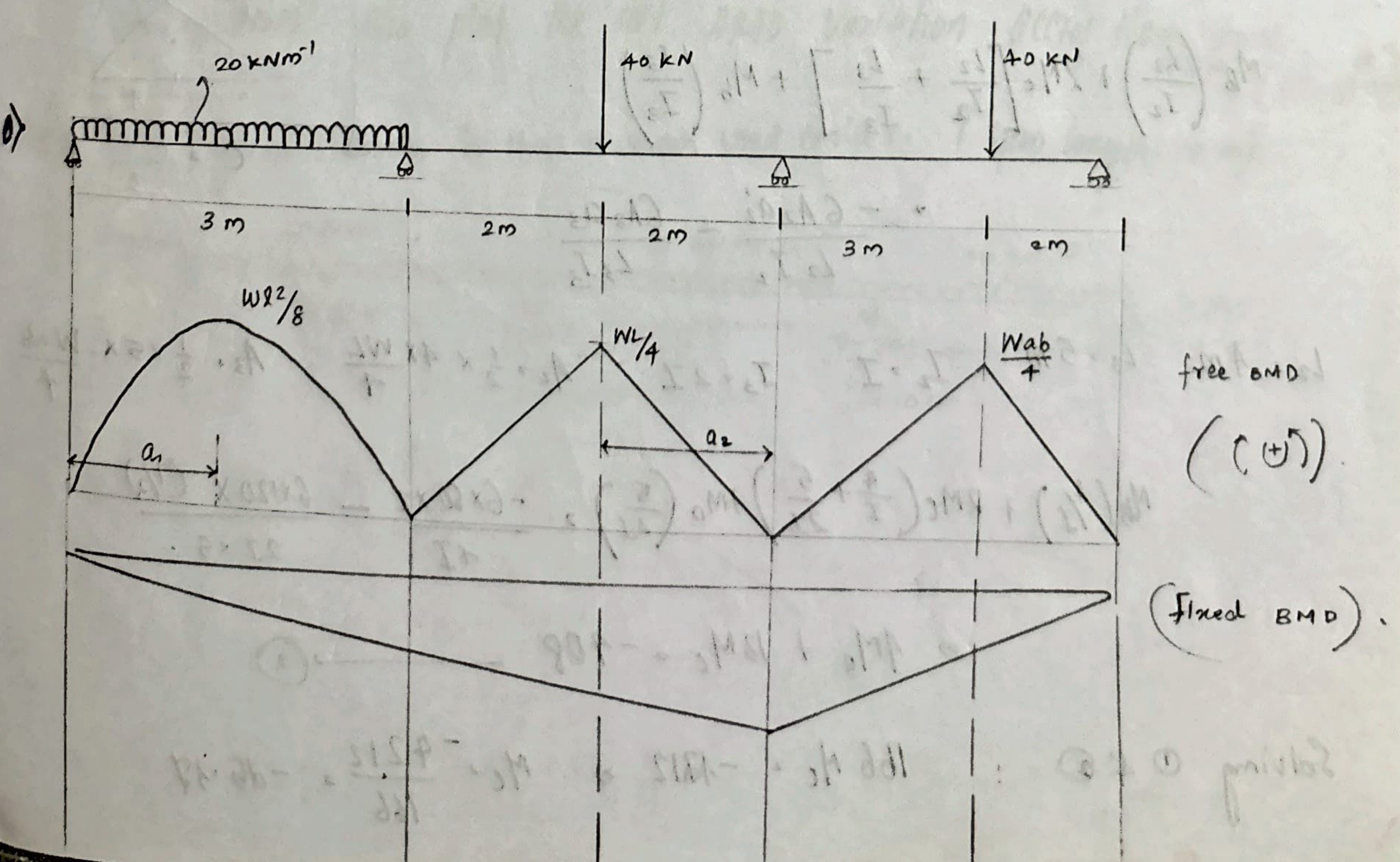
deflection = moment about C = $\frac{A\bar{X}}{EI}$

$(\delta_C - \delta_A = \delta_C)$, $\delta_A = 0$

$\frac{-1526.5}{2.1 \times 10^{9} \times 10^{-4}} = 727 \times 10^{-5}$

deflection ⇒ $\frac{A\bar{X}}{EI} = \frac{312.5 \times (.834 + 2.5) + 625 \times 3.75 + 625 \times 4.167}{EI} = \frac{5989.38}{2.1 \times 10^5} = 2852.3 \times 10^{-5}$

b)



BME :

$$M_A\left[\frac{L_1}{I_1}\right] + 2M_B\left[\frac{L_1}{I_1}+\frac{L_2}{I_2}\right] + M_C\cdot\left[\frac{L_2}{I_2}\right] = \frac{-6A_1a_1}{L_1I_1} - \frac{6A_2a_2}{L_2I_2} + \cancel{\frac{6E\delta_A}{L_1}} + \cancel{\frac{6E\delta_C}{L_2}}$$

$\Rightarrow$ ~~h_A~~ h_C [$h_A = h_C = 0$, all are at same horizontal level).

- For span AB & AC :-

$M_A = 0$ (end support) : M_B & M_C will be there (intermediate support)

$L_1 = 3m \qquad L_2 = 4m \qquad I_1 = I_2 = I$

$A_1 = \frac{2}{3}bh = \frac{2}{3}\times 3\cdot\frac{wl^2}{8} \qquad A_2 = \frac{1}{2}bh = \frac{1}{2}\times 4\times\frac{WL}{4}$

$a_1 = 1.5m \qquad a_2 = 2m.$

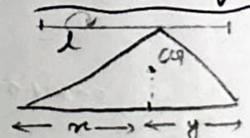
Subt. in BME,

$$\frac{3}{I}M_A + 2M_B\left(\frac{3}{I}+\frac{4}{I}\right) + M_C\cdot\frac{4}{I} = -\left(\frac{6\times 45\times 1.5}{3I}\right) - \left(\frac{6\times 80\times 2}{4I}\right)$$

$$14M_B + 4M_C = 375 \longrightarrow (1)$$

- For span BC & CD, $\underline{M_D = 0}$, M_B & M_C unknown.

Centroid for unsym. triangle



$C = \frac{l+y}{3}$

$$M_B\left(\frac{L_2}{I_2}\right) + 2M_C\left[\frac{L_2}{I_2}+\frac{L_3}{I_3}\right] + M_D\left(\frac{L_3}{I_3}\right)$$

$$= \frac{-6A_2a_2'}{L_2I_2} - \frac{6A_3a_3}{L_3I_3}$$

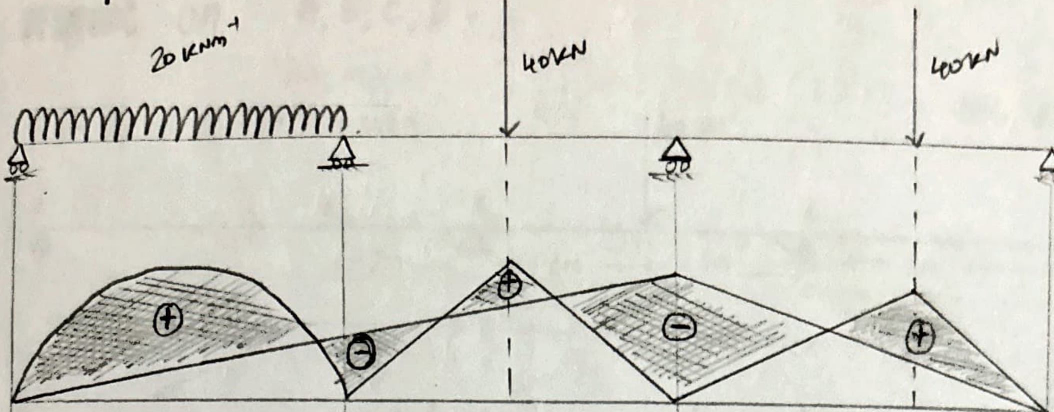
$L_2 = 4m \qquad L_3 = 5m \qquad I_2 = I \qquad I_3 = 2I \qquad A_2 = \frac{1}{2}\times 4\times\frac{WL}{4} \qquad A_3 = \frac{1}{2}\times 5\times\frac{Wab}{4}$

$$M_B(4/I) + 2M_C\left(\frac{4}{I}+\frac{5}{2I}\right) + M_D\left(\frac{5}{2I}\right) = \frac{-6\times 80\times 2}{4I} - \frac{6\times 120\times (7/3)}{2I\times 5}$$

$$\Rightarrow 4M_B + 13M_C = -408 \longrightarrow (2)$$

Solving (1) & (2) : $\quad 166M_C = -4212 \Rightarrow M_C = \frac{-4212}{166} = -25.37$

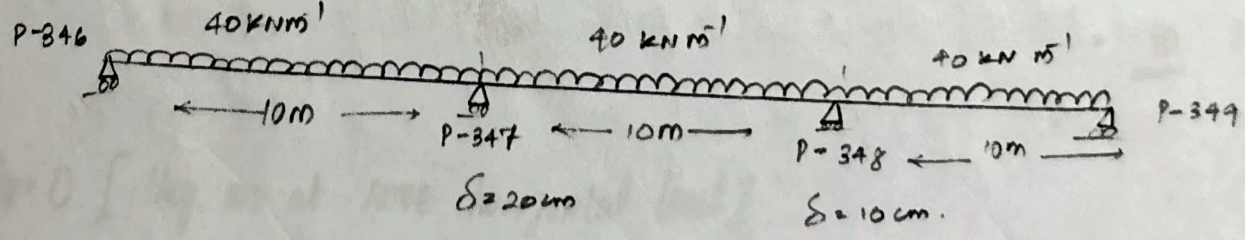
$$M_B = \frac{-273.51}{14} = \underline{\underline{-19.53}}$$



Qn: Kochin metro pillers, piller no 347 and piller no 348 @ pathadipalam misaligned due to settlement of piller foundation, by 20 cm & 10 cm respectively, relative to the piller from the either side (P-346 & P-349). Assuming these portions of the metro bridge to be considerd as a contin. beam pin jointed at all these 4 supports, find out the moments that is induced @ the intermedi-ate pilers (347 & 348). use clapyrons theorem of 3 moments and take dead wt including self wt, wt of rails ets throughout the whole spans as 40 KNm⁻¹. Also plot the net BMD variation accros these spans.

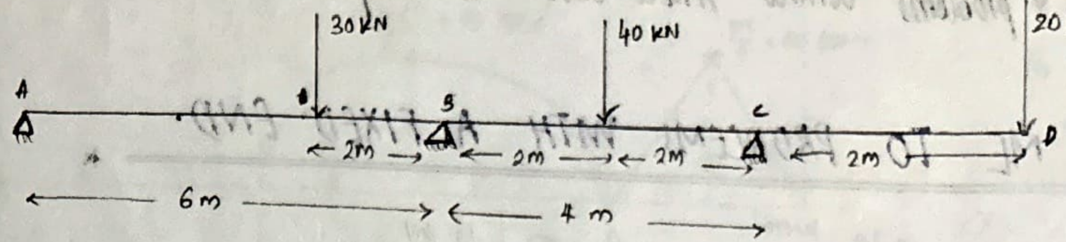
$(EI = 10^{15}\ Nm^2)$

Ans 8- Clyprons theorem is the theorem used earlier. (span length = 10 m).



Qn: If the support "C" in the figure settle down by 5 mm, find the support moment on A, B, C, D.

Ans:

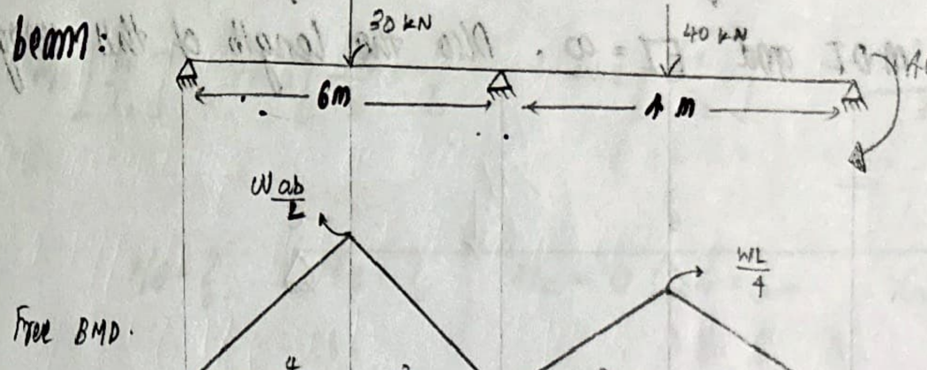


We know: 3 ME : $M_A\left[\frac{L_1}{I_1}\right] + 2M_B\left[\frac{L_1}{I_1} + \frac{L_2}{I_2}\right] + M_C\left[\frac{L_2}{I_2}\right] =$

(centroydal dist)

$$\frac{-6A_1\bar{a}_1}{L_1 I_1} - \frac{6A_2 a_2}{L_2 I_2} + \frac{6E\delta_A}{L_1} + \frac{6E\delta_C}{L_2}$$

equivalent beam:



$E = 15\ kN/mm^2$

$I =$ Const $= 5\times10^9\ m^4$

$A_1 = \frac{1}{2}\times6\times\frac{30\times4\times2}{6}$

$= 120$

$A_2 = \frac{1}{2}\times4\times\frac{40\times2}{4}$

$= \underline{\underline{80}}$

$a_1 = \frac{6+4}{3}$ $\boxed{\left(\frac{l+x}{3}\right)}$

$= 10/3\ m$

$a_2 = 4/2 = 2m$

* $\delta_A = 0$ [they are at same horizontal level]

$\delta_C = -5$ (settle down)

$M_A = 0$ $\quad M_B = ?$ $\quad M_C = -40$ kNm (Hogging).

eqn I)

$$0 + 2M_B\left[\frac{6}{1} + \frac{4}{1}\right] + (-40)[4] = \frac{-6\times120\times[10/3]}{6} - \frac{6\times80\times2}{4} + 0 + \frac{6\times EI\times-5}{4}$$

$$= -400 - 240 - \left(75\times10^6\times\frac{3}{2}\times5\times10^{-3}\right)$$

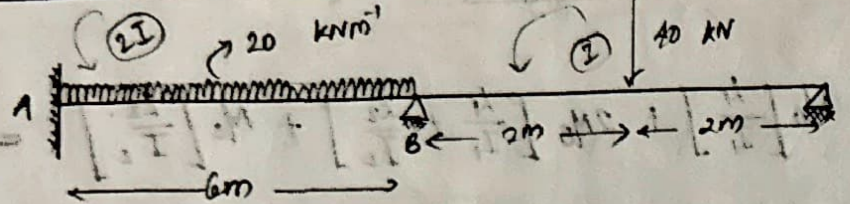
$$= -640 - 562.5\times10^6$$

ie $M_B = -52.125\ KNm^{-1}$.

Qn: 3ME to problems with a fixed end is important:

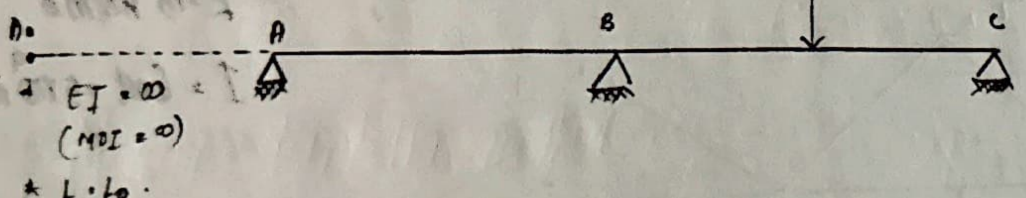
3 ME TO PROBLEMS WITH A FIXED END.

Qn:



2 unknowns.

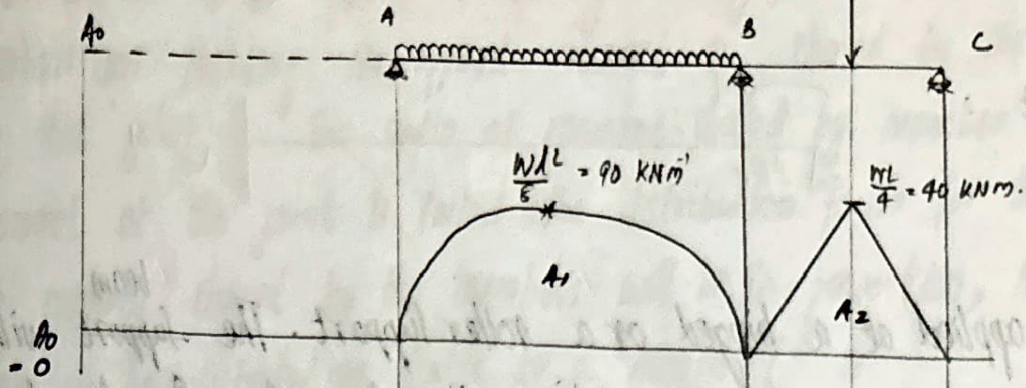
In order to solve the above qn, we are considering an imaginary beam, AA_0, of infinite MOI and $EI = \infty$. Also the length of this imaginary span $L = L_0 = 0$.



If the structural rigidity is infinity, it means the beam is very stiff such that the rotation at the support A for the span AA_0 is equal to zero. Thus owing to the continuity requirements, the rotation at support A for the span AB is also equal to zero. Thus ideally, our assumption of infinite structural rigidity matches with that of a fixed support.

ie $$M_0\left[\frac{L_0}{I_0}\right] + 2M_A\left[\frac{L_0}{I_0} + \frac{L_1}{I_1}\right] + M_B\left[\frac{L_1}{I_1}\right] = \frac{-6A_0 a_0}{I_0 L_0} - \frac{6A_1 a_1}{I_1 L_1} \longrightarrow \text{eqn } ①$$

$M_0 = 0$ $M_A = ?$ $L_0 = 0$ $I_0 = \infty$ $L_1 = 6m$ $I_1 = I2$



$A_0 = 0$

$A_1 = \frac{2}{3} \times 6 \times 90 = 360$

$a_0 = 0$

$a_1 = 3m.$

eqⁿ ① → $2M_A\left[0 + \frac{6}{2I}\right] + M_B\left[\frac{6}{2I}\right] = 0 - \frac{6 \times 360 \times 3}{2I \cdot 6}$ } for span A_0AB.

$2M_A + M_B = -180$ ⟶ ①

eqⁿ ② → $M_A\left[\frac{L_1}{I_1}\right] + 2M_B\left[\frac{L_2}{I_2} + \frac{L_1}{I_1}\right] + M_C\left[\frac{L_2}{I_2}\right] = -\frac{6A_1a_1}{I_1L_1} - \frac{6A_2a_2}{I_2L_2}$

→ for span ABC

$M_A = ?$ $M_B = ?$ $M_C = 0$ $L_1 = 6m$ $L_2 = 4m.$ $I_1 = 2I$

$M_A\left(\frac{6}{2I}\right) + 2M_B\left(\frac{6}{2I} + \frac{4}{I}\right) + 0 = \frac{-6 \cdot 360 \times 3}{2I \cdot 6} - \frac{6 \cdot 80 \times 2}{I \cdot 4}$

$\Rightarrow 3M_A + 14M_B = -780$ ⟶ ②

$6M_A + 3M_B = -540$

$6M_A + 28M_B = -1560$

$25M_B = 1020$

$M_B = \frac{1020}{25}$

$= 40.8$

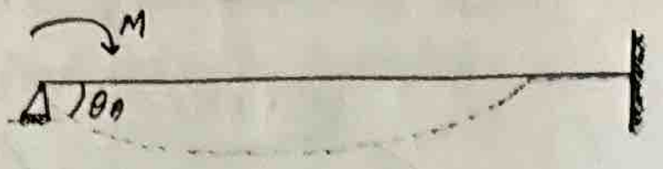
Solving ① & ② ; $M_B = 40.8$ KNm $M_A = -69.6$ KNm.

MOMENT DISTRIBUTION METHOD

It is a itrative distribution procedure. The method is ideally suited for fairely higher degree of indeterminate structures.

Terminology.

(i) Carryover moment :



When a moment is applied at a hinged or a roller support. The ~~support~~ beam will rotate by a specific angle. Due to this rotation the far end will also have a tendency to rotate and if the far end is fixed that rotation will be restricted. This induced moment at the far end is known as the carryover moment. The dirn of carryover moment is such that it will resist the possible rotation.

(ii) Carryover factor :- Ratio of $M'/M = \dfrac{\text{Carryover moment}}{\text{Applied moment}} = CF$. $\boxed{M' = \frac{1}{2}M}$

Moment distribution method :-

* Carryover factor is $\text{CO. Factor} = \dfrac{M_1}{M} = 1/2$ [for a beam fixed at one end]

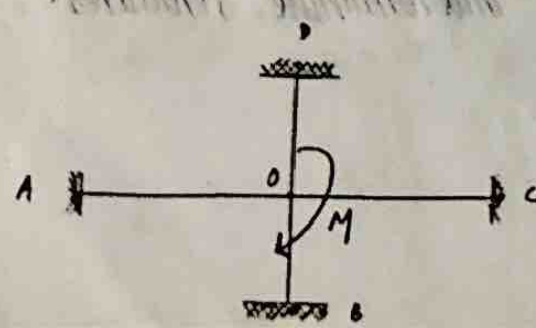
(iii) Stiffness :- Moment required to rotate an end by unit term (1 radian)

Mathematically, stiffness (k) is

* $\boxed{k = \dfrac{M}{\theta_a}}$

The stiffness for a beam with far end which is given by $k = \dfrac{4EI}{L}$

(iv) Distribution factor :-



$M_{OA} = \dfrac{M_{OA}}{M}$.

When a moment is applied to an rigid-body joint, where a number of members are meeting the applied moment is shared by the members meeting at that joint. The ratio of moment shared by member to the applied moment at the joint is called the distribution factor for that member. The moment shared by the members will be in proportion, to their stiffness values. The distribution factor for a member "i" is

$$d_i = \frac{M_i}{M} = \frac{k_i}{\Sigma k}$$

METHODOLOGY IN MDM

(I) Find Fixed End Moments (FEM) by assuming all the ends are fixed.

(II) Calculate distribution factors (DF) for all the members.

(III) Find out the unbalanced moment at each support and then distribute this unbalanced moments to the adjacent member in proportion to their distribution factors.

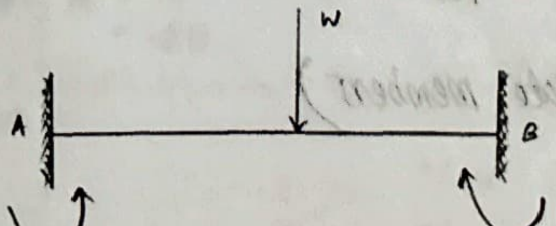
(IV) Find out the carriover moments due to the distributed moment calculated in step ③.

(V) This carriover moment again disturbs the balance of the joints which need to be balanced in the subliquent steps.

(VI) Continue this procedure until the carryover moment becomes negligible.

- STD. FORMULAS FOR CALCULATING FIXED END MOMENTS.

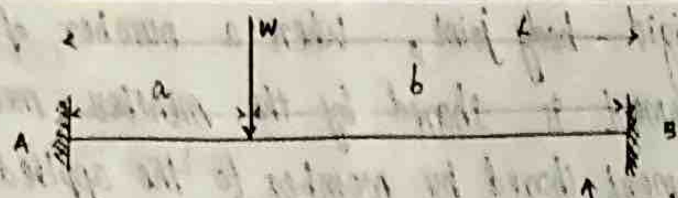
①



$M_{FAB} = \frac{-WL}{8}$ $M_{FBA} = \frac{WL}{8}$

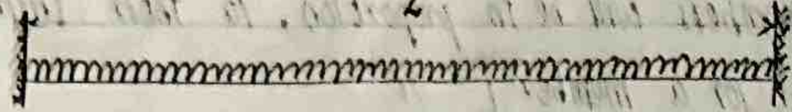
Anticlock wise = −ve
Clockwise = +ve

(ii)



$M_{FAB} = \frac{-Wab^2}{L^2}$ $\qquad$ $M_{FBA} = \frac{Wa^2b}{L^2}$

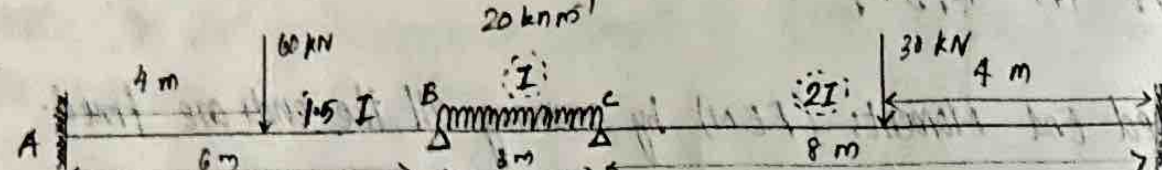
(iii)



$M_{FAB} = \frac{-WL^2}{12}$ $\qquad$ $M_{FBA} = \frac{WL^2}{12}$

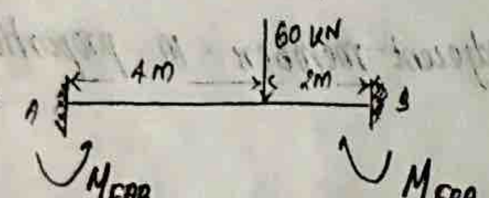
CONTINUOUS BEAM WITH FIXED ENDS.

Qn:



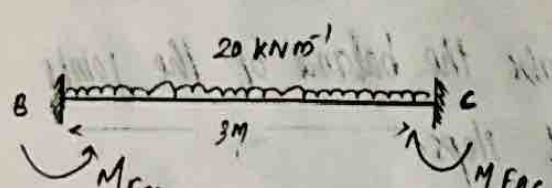
Analyse the given beam using MDM.

Ans: (S1) :



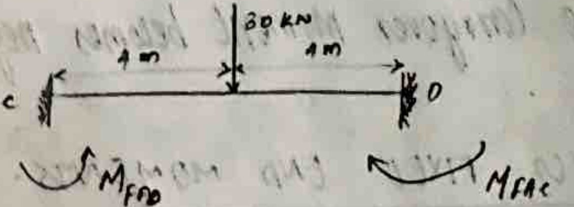
$M_{FAB} = \frac{-\cancel{60}^{10} \times 4 \times \cancel{4}^{2}}{\cancel{36}\,\cancel{6}_{3}} = \frac{-80}{3} = 26.67$

$M_{FBA} = \frac{\cancel{60}^{10} \times 16 \times \cancel{2}}{\cancel{6} \times \cancel{6}_{3}} = \frac{160}{3} = \underline{\underline{53.33}}$



$M_{FBC} = \frac{-\cancel{20}^{5} \times \cancel{3} \times 9}{\cancel{4} \times \cancel{3}} = -45$

$M_{FCB} = 45.$



$M_{FCD} = \frac{-WL}{8} = -30$

$M_{FDC} = 30.$

(S2) : DF calculation (For intermediate members)

Joints	Members	k	Σk	DF's
B	BA BC	$k_{BA} = \frac{4EI}{L} = \frac{4 \cdot E \cdot 3I}{2 \cdot 6} = EI$ $k_{BC} = \frac{4EI}{3} = 1.34 EI$	$\Sigma k_B = EI + 1.34EI = 2.34EI$	$d_{BA} = \frac{k}{\Sigma k} = \frac{EI}{2.34EI} = .429$ $d_{BC} = \frac{1.34EI}{2.34EI} = .572$
C	CB CD	$k_{CB} = \frac{4EI}{3} = 1.34 EI$ $k_{CD} = \frac{4E \cdot 2I}{8} = EI$	$\Sigma k_C = 2.34EI$	$d_{CB} = .571$ $d_{CD} = .429$

24.15

-27.338

Moment distribution table :-

Joints	A	B		C		D
DFs.		.429	.571	.571	.429	
FEMs	-26.27	58.33	-15	15	-30	30
Balancing mom.		-38.33		15 kNm		
Balancing		-38.33 × .429 -16.44	-38.33 × .571 -21.89	8.57	6.43	
COM	$\frac{1}{2}$ × -16.44 = -8.22		$\frac{1}{2}$ × 8.57 = 4.29	$\frac{1}{2}$ × -21.89 = 10.94		½ × 6.43 = 3.22
Balancing Moment (By × with DF)		-1.84	-2.45	6.25	4.7	
Carryover Moms	½ · (-1.84) = -0.93		3.13	½ (-2.45) = -1.23		2.35
Balancing mom.		$\boxed{-3.13}$		$\boxed{1.23}$		
		-1.34	-1.793	.70	.528	
Carryover mom.	-.67 (½×1.34)		.35	-.8965		.264

JOINTS	A	B		C		D
Balancing moments ↪		$\boxed{-.35}$ -.15	-.20	.50	$\boxed{+.89}$ .39	
Carryover moments	-(0.075) → (-1/2 × .15) and similarly all		.25	-.10		.195
Balancing mom.		$\boxed{-.25}$ -.10	-.15	.057	$\boxed{.10}$ .043	
Carryover mom.	-.05		.0285	-.075		.0215
Now adding all the distributed moments on the partic. Column		⇒ 33.46 [53.33 - 16.44 - 1.84 ...]	⇒ -33.46	+17.91	-17.91	36.04

$M_{AD} = -26.67 - 6.27 - .92 = -36.56.$

PLASTIC THEORY.

* In plastic theory of analysis of beams, we study behaviour of structures beyond the elastic limit, ie when material yields and forms plastic hinges. Consider the Scenario :- Bending stress distribution in a symmetric simply supported beam.

(a) Before yielding :- From elastic theory $\sigma = \frac{My}{I}$

stress distribution is triangular and material behaves linearly. Yield stress not reached at most points.

(b) At Yield moments :- when bending moment reaches the yield value of moment M_y, outermost fiber reach yield stress f_y, and inner fibers are still in elastic range. stress taper linearly towards neutral axis.

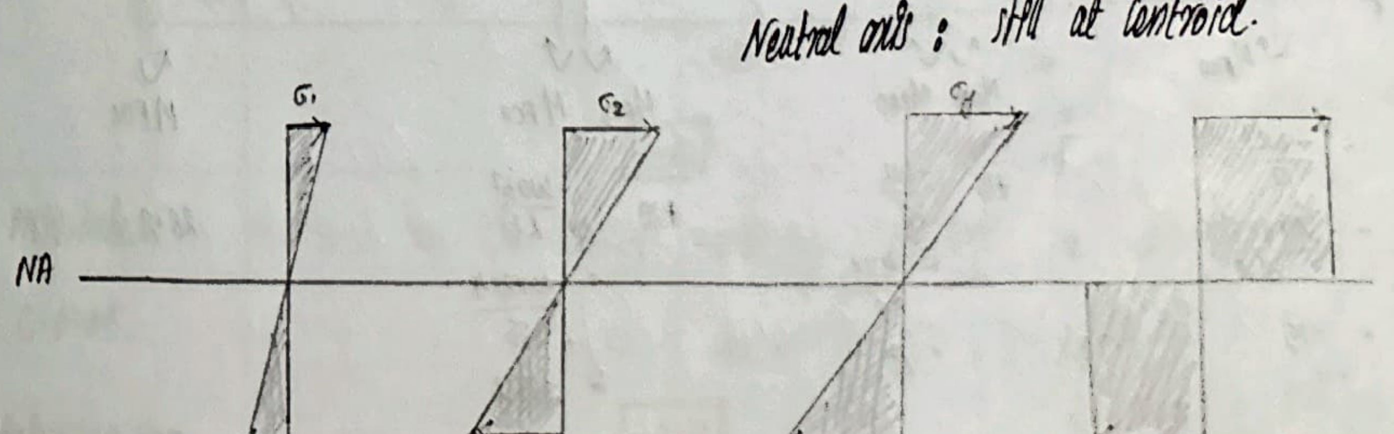
(c) Fully plastic stress distribution (M_p = plastic moment) :- As moment increase further, yield progress inward and eventually entire cross section yields.

$$\sigma = \begin{cases} +f_y & \text{, above neutral axis} \\ -f_y & \text{, below neutral axis.} \end{cases}$$

Here stress block is rectangular and ⇒ Top half : Const. Compression f_y

Bottom half : Const. tension $-f_y$

Neutral axis : still at centroid.



● To show how much conservative plastic theory than elastic theory :-

Shape factor = SF = $\frac{\text{Plastic section modulous}}{\text{Elastic section modulous}} = \frac{Z_p}{Z_e}$ $\left(\frac{I/y_{centroid}}{A/2\,(\bar{y}_1 + \bar{y}_2)} \right)$ centroidal dist.

CONTINUOUS BEAM WITH SIMPLY SUPPORTED ENDS. ⊙.

Steps :-

① Fixed end moments are calculated at simply supported ends also,

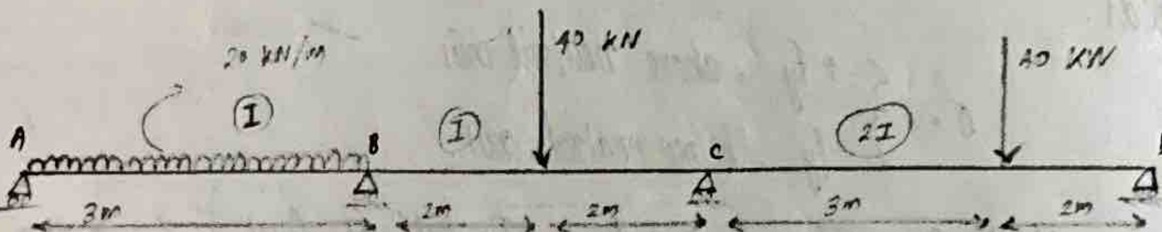
② Joint balancing is done for these simply supported joints by taking distribution factor as one or DF = 1 [only one member exists at that point]

③ Carryover moment is calculated at the adjecent ends.

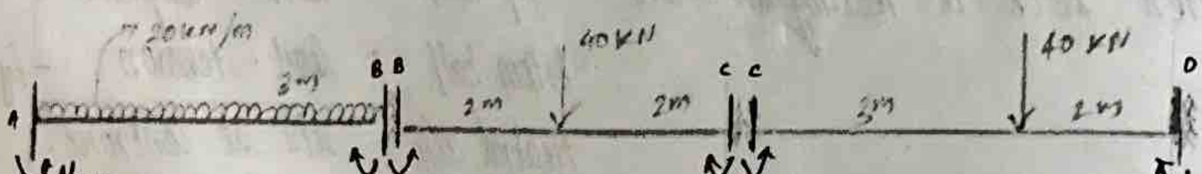
④ For the further calculations, use carriover factor = 0 and the stiffness, $k = \frac{3EI}{L}$.

$K = 3EI/L$, $COF = \frac{M'}{M} = \frac{0}{M} = 0$ $K = \frac{4EI}{L}$, $COF = \frac{M'}{M} = \frac{1}{2}$

Qn :-



Ans :-



M_{FAB} M_{FBA} M_{FBC} M_{FCB} M_{FCD} M_{FDC}

$M_{FAB} = \frac{-wL^2}{12} = -\frac{20 \times 3 \times 3}{12} = -15$

$M_{FBA} = +15$

$M_{FBC} = \frac{-WL}{8} = \frac{-40 \times 4}{8} = -20$

$M_{FCB} = +20$

$M_{FCD} = \frac{-Wab^2}{L^2} = \frac{-40 \times 3 \times 4}{25} = -19.2$

$M_{FDC} = 28.2$

DISTRIBUTION FACTOR TABLE :-

Joints	Members	k	Σk	DF's
B	BA	$k_{BA} = 3EI/L = 3\times EI/3 = EI$	2EI	.5
	BC	$k_{BC} = 4EI/L = 4EI/4 = EI$		.5
C	CB	$k_{CB} = 4EI/L = 4EI/4 = EI$	1.6EI	.625
	CD	$k_{CD} = 3EI/L = 3EI/5 = .6EI$		.375
A				1 (Because pinned)
D				1 (")

Moment distribution table :-

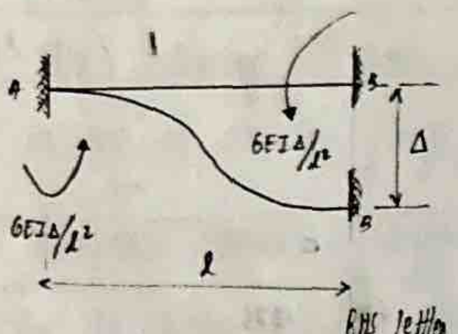
Joints	A	B		C		D
DF's	1	.5	.5	.625	.375	1
F.E.M	-15	15	-20	20	-19.2	28.2
	+15					-28.8
		7.5			-14.4	
Balancing moments		-2.5		$\boxed{+13.6}$		
Distribution		-1.25	-1.25	8.5	5.1	
C.O.M			4.25	-.63		
Balancing mom.		$\boxed{-4.25}$		$\boxed{+.63}$		
Distribution mom : $(-4.25\times.5)$ f $(.63\times.625)$		-2.13	-2.12	0.39	0.24	
			.20	-1.06		
		$\boxed{-.20}$		$\boxed{1.06}$		
Distribution		-.1	-0.1	.66	.40	
C.O.M			.33	-0.05		
Balancing mom.		$\boxed{-0.33}$		$\boxed{.05}$		

$M_A = 0$ $M_{BA} = 19.02$ $M_{BC} = 27.86$ M

$M_{AB} = -19.02$ $M_{CB} = -27.86$

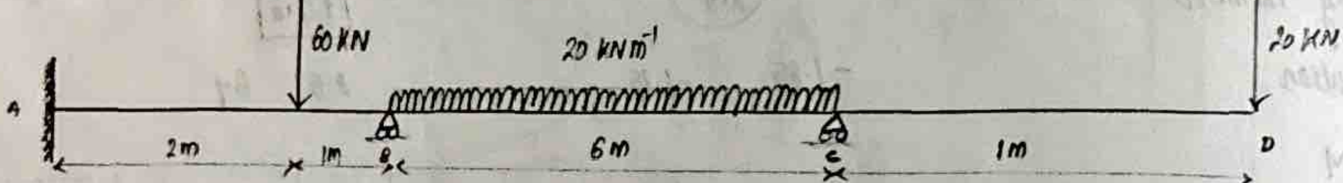
· **Note :-**

In M.D.M, f.e.m's due to the settlement of supports, are added to the f.e.m's due to the loadings. Rest of the analysis remains the same. If RHS settles $6EI\Delta/l^2$ anticlockwise moment comes additionally. If LHS settles $6EI\Delta/l^2$ clockwise moment comes additionally.

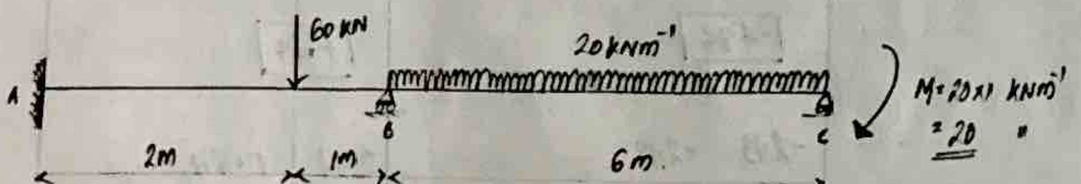


In the given condition, RHS settles. (Anticlockwise)

Q08:- If the support B of the continu. beam settles by 9 mm, find all the support moments. $EI = const = 10^{12}\ Nmm^2$

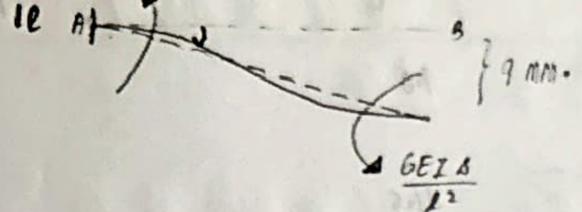


Ans:-



Ans :- Fixed end moments –

Since there is a settlement 9 mm in support B, we have to include the induced anticlockwise moment (RHS settles).



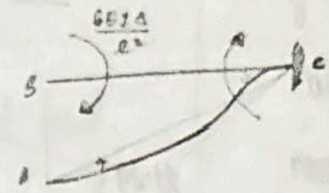
$$M_{FAB} = \frac{-Wab^2}{l^2} - \frac{6EI\Delta}{l^2} = \frac{-60\times2\times1}{9} - \frac{6EI9}{9}$$

$$M_{FBA} = \frac{Wa^2b}{l^2} + \frac{-6EI\Delta}{l^2} = \frac{60\times2\times2\times1}{9} - \frac{6EI\cdot9}{9}$$

$M_{FAB} = -13.334 - 6EI$ $\quad$ $M_{FBA} = 26.667 - 6EI$

$$M_{FBC} = \underbrace{\frac{-Wl^2}{12}}_{\text{udl}} + \underbrace{6EI\Delta/l^2}_{\text{induced moment}} - \underbrace{20}_{\text{overhang}} = \frac{-20\times\cancel{36}^{3}}{\cancel{12}} - 20 - \frac{6EI\,9}{9}$$

$$= \underline{-80 - 6EI}$$



$$M_{FCB} = \frac{Wl^2}{12} - 20 + \frac{6EI\Delta}{l^2} = 60 - 20 + \frac{6EI9}{9} = 40 + 6EI$$

$EI = 10^{12}\ Nmm^2$

$= 10^{12}\,10^{-6}\ NM$

All the EI here should be in metre....

$M_{FAB} = -19.33$ $\quad$ $M_{FBA} = 20.67$ $\quad$ $M_{FBC} = -58.6$ $\quad$ $M_{FCB} = 41.5$

Distrib. table :-

JOINTS	Member	K	ΣK	D.F
B	BA	$4EI/l = 1.33\,EI$	$1.83\,EI$	.727
	BC	$3EI/l = .5EI$		.273
C	CB	$4EI/l = .66EI$	$.66\,EI$	1
	CD			

Moment distribution table :-

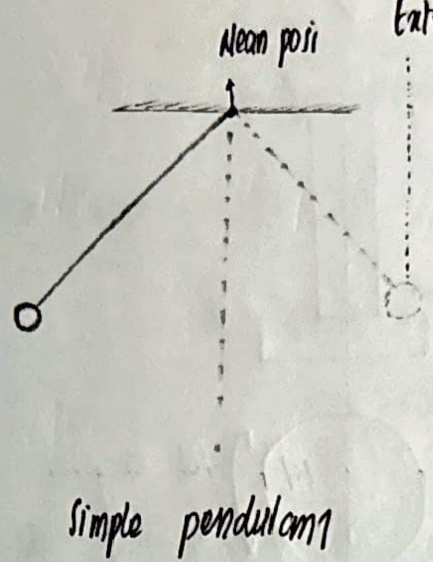
JOINTS	A	B		C	D
D.F's		.727	.273	1.0	
Fixed end moments	-19.33	20.67	-58.5 20.75	41.5	
Balancing mom:		$\boxed{+58.58}$			
Distrib.		42.58	15.99		
Carryover	21.29.				
	1.96	+ve ← 68.25 → -ve		+20	-20

a pinned support can not any moment. Hence here no. distribution from B→C

$\underbrace{+20 \quad -20}_{\text{external}}$ Cantilever beam like mom.

MODULE :- 4

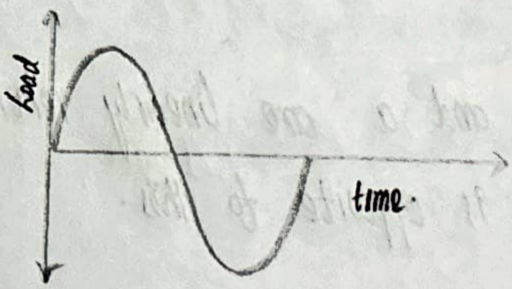
STRUCTURAL DYNAMICS



Cause of vibration :-

* Seismic load
* wind load
* wave load
* Blast load
* impact load
* Machine load

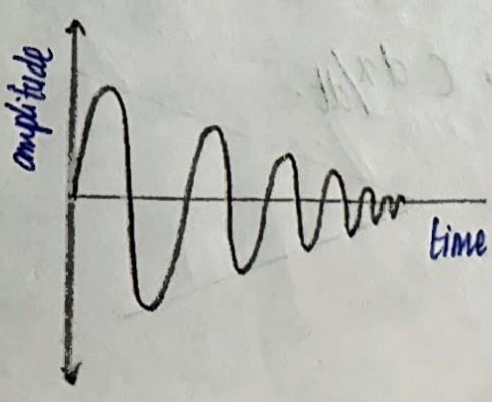
- Load, Mangnitud and direction changes with time.
- For dynamic analysis ; Inertia force will comes to effect. (Dynamic eqn. equation).
- Dynamic load :-



- Types of vibration :- ⓐ free vibration and forced vibration

  free vibration : Contin. without external force after an initial displacement

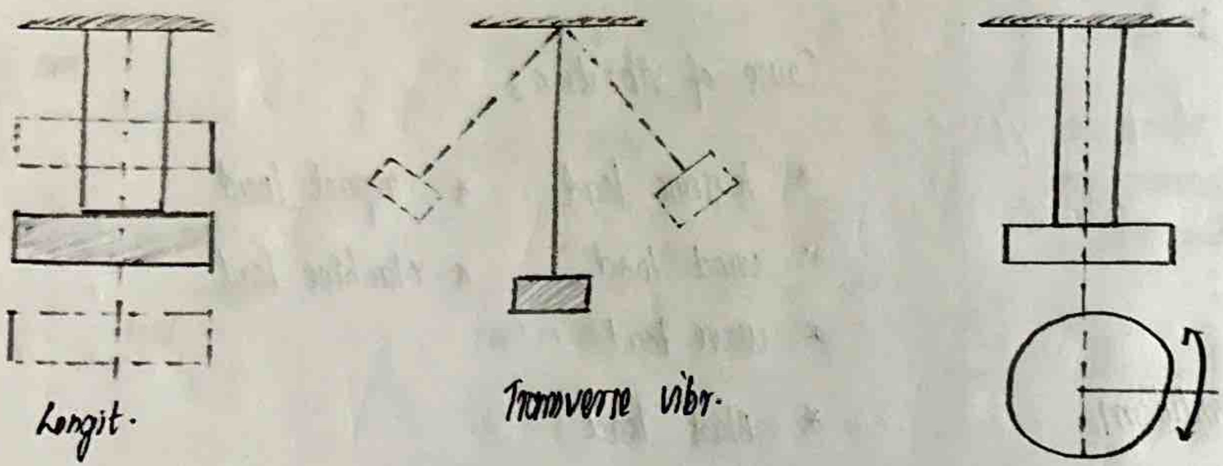
  forced vibration : Continues due to an external exciting force.



ⓑ Deterministic vibration : All charact. like [v, a, s ---] can be calculated

Non-deterministic vibration : Random nature, difficult to calculate

ⓒ Longitudinal, Transverse and torsional vibration :- Based on movment of mass



ⓓ Damped and undamped vibration :-

⇒ Damped : Energy dissipates, vibration dies dow

⇒ Undamped : Non energy dissipation and vibration Continues.

ⓔ Linear vibration :- s, v. to and a are linearly related to external exciting force, And non-linear is opposite to this.

DAMPING

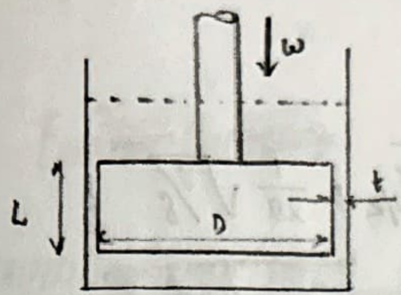
Resistance encountered during vibration reduces amplitude over time. Gradual reduction in amplitude over time. Damping force prop. to velocity ~~of~~ magnitude opposite to dirⁿ. of motion.

$$F_d \propto \dot{x} \Rightarrow F_d = cV = c\,dx/dt.$$

c = Damping force Constant.

- Common in System vibrating in a fluid.

Eg:- * shock absorbers in Cars.

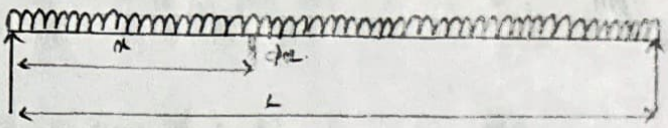


* when the weight w pushes the piston downwards, the piston tries to move inside the fluid filled cylinder.
* faster motion creates more resistance, so piston slows

• Solid or structural damping :- * Caused by internal molecular friction.

* Stress - strain diagram forms a hysteresis loop (Loop area represents the damping capacity or energy dissipated).

⊙ Static deflection on a simply supported beam with udl :-



We know : $y = \frac{w}{24EI}\left[x^4 - 2Lx^3 + L^3x\right]$

strain energy of entire beam :- $\int_0^L \frac{1}{2}(w\,dx)\,y = \frac{1}{2}w\int_0^L y\,dx$

• Whirling speed :-

if short bearing : pinned type simply supp. means take as fixed beam

Qn :- A shaft 20 mm diameter and 500 mm between the long bearing at it's ends, carries a wheel weighing 100 N midway between the bearings. Neglecting the increase of the stiffness due to the attachement of the ~~weight~~ wheel to the shaft, find the critical speed of rotation, and the maximum bending stress when the shaft is rotating at 4/5 of this speed, if the centre of gravity of the wheel is 0.4 mm from the centre of the shaft. Take $E = 2\times10^5\ Nmm^{-2}$.

Ans: shaft supported by long bearings $\Rightarrow$ effect of fixed beam.

Central deflection for such a beam $= \delta = \frac{WL^3}{192 EI}$

ie Critical speed, $N_c = \frac{1}{2\pi}\sqrt{\frac{kg}{W}} = \frac{1}{2\pi}\sqrt{g/_{W/k}} = \frac{1}{2\pi}\sqrt{g/\delta}$

ie $N_c = \frac{1}{2\pi}\sqrt{\frac{192 EI \cdot g}{WL^3}}$

I for a cylindrical shaft : $I_o = \frac{\pi}{64} d^4 = \frac{\pi}{64}(20)^4 = .785 \times 10^4 \, mm^2$

ie $N_c = \frac{1}{2\pi}\sqrt{\frac{192 \times (2\times10^5)\times(.785\times10^4)\times 9810}{100 \times [500]^3}}$ rps (multip. with 60 gives rpm).

$= \underline{4645}$ rpm

static deflection :- $y = \frac{e\omega^2}{\omega_c^2 - \omega^2}$ (given $\omega = 4/5\, \omega_c$) $e = .4$ mm

then $y = \frac{.4\times(4/5)^2\omega_c^2}{\omega_c^2 - (4/5\,\omega_c)^2} = \frac{.4 \times .64}{1 - .64} = \underline{\underline{.71 \text{ mm}}}.$

Central Centrifugal bending force : $ky = k\cdot[.71]$

Here bending is caused by Centri. force & which is equal to stiffness.

$M = \frac{WL}{8} = \frac{L}{8}(k\cdot y) = \frac{L}{8}(.71 k) = \frac{.71\times L}{8}\cdot\frac{192\,EI}{\cancel{W}L^3}$

ie $\sigma = \frac{M}{Z}$ $\Rightarrow$ $\sigma = \frac{M}{Z} = 136.3\ Nm^{-2}$ //.

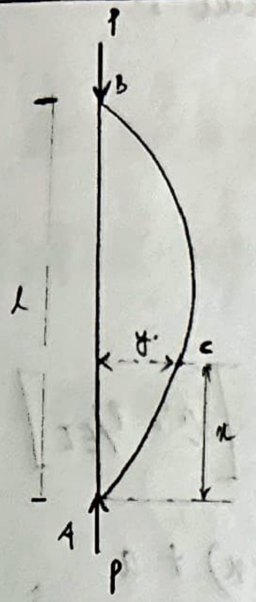
$Z_o = \frac{\pi d^4}{64\cdot(d/2)}$

section mod.

MODULE : 3

① Eulers theorem :-

CRIPPLING LOAD WHEN BOTH ENDS OF THE COLUMN ARE HINGED.



Concave deflection ⇒ signe -ve.

Moment = Load × ⊥ʳ dist = $-P \cdot y$

Also moment = $EI \frac{d^2y}{dx^2}$

ie $-Py = EI \frac{d^2y}{dx^2}$ ⇒ $EI \frac{d^2y}{dx^2} + Py = 0$

ie we get : $\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = 0$

similar to SHM derivation : $\frac{d^2y}{dx^2} + w^2x = 0$, Here

$\alpha^2 = P/EI$ and $\alpha = \sqrt{\frac{P}{EI}}$

ie Solution $y = C_1 \cos(\alpha x) + C_2 \sin(\alpha x) = C_1 \cos\left(\sqrt{\frac{P}{EI}}\, x\right) + C_2 \sin\left(\sqrt{\frac{P}{EI}}\, x\right)$.

Substituting boundary Condⁿ : @ $x = 0$, $y = 0$
$x = l$, $y = 0$

ie $0 = C_1 + 0$ ⇒ $C_1 = 0$

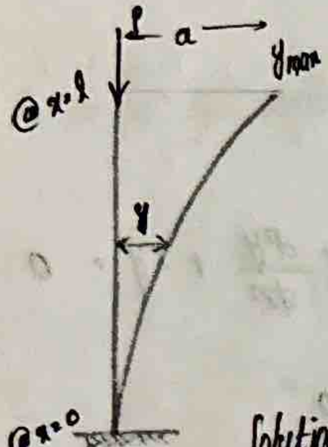
$0 = C_1 \cos\left[\sqrt{P/EI}\, l\right] + C_2 \sin\left(\sqrt{P/EI}\, l\right]$ ⇒ we already have $C_1 = 0$

ie $C_2 \sin\left(\sqrt{P/EI}\, l\right) = 0$ < $C_2 = 0$ OR $\sin\left(\sqrt{P/EI}\, l\right) = 0$

ie we say $\pi = \sqrt{\frac{P}{EI}}\, l$, $\pi^2 = \frac{P l^2}{EI}$ $\therefore$ ~~$P = \frac{l^2}{\pi^2}\frac{E}{I}$~~

$$\Rightarrow \boxed{P = \frac{\pi^2 EI}{l^2}}$$

*** One end fixed and other end free : find buckling load :-**



$M = EI \cdot \frac{d^2y}{dx^2}$ | $M = P(a-y)$ $y = 0$

$M = EI\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} + \frac{-Pa}{Ey} + \frac{Py}{EI} = 0$ @ $x = l$ $y = y_{max} = \frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} + \frac{Py}{EI}y = \frac{P}{EI}\cdot a$ $\left[\alpha^2 = P/EI\right]$

ie

$C_1\cos\left(x\sqrt{P/EI}\right) + C_2\sin\left(x\sqrt{P/EI}\right)$

Solution to this DE $\rightarrow$ $y = C_1\cos(\alpha x) + C_2\sin(\alpha x) + a$

$C_1 = 0$ & $y = C_1\cos\left[\sqrt{P/EI}\,x\right] + C_2\sin\left[\sqrt{P/EI}\,x\right] + a$

$\rightarrow \frac{dy}{dx} = C_2\sin\sqrt{P/EI}\cdot x$ $\rightarrow$ $\int_0^l k\cdot x\,dx$

Boundary Conditions :- @ $x = 0$ $\frac{dy}{dx} = 0$ & @ $x = l$ $y = a$ & @ $x = 0$ $y = 0$

$= C_2\sin\sqrt{P/EI}\left(\frac{x^2}{2}\right)_0$ $= C_2\sin\frac{\sqrt{P/EI}}{2}$

(fixed support does not have slope)

$\rightarrow$ ie $y = \sin$

$\Rightarrow$ $0 = a + C_1 \Rightarrow$ $\underline{C_1 = -a}$ (from $x = 0$, $y = 0$)

$\Rightarrow$ $\frac{dy}{dx} = 0 \rightarrow$ @ $x = 0$: $\frac{dy}{dx} = C_2\cos\left(\sqrt{P/EI}\,x\right)\cdot\sqrt{P/EI} - C_1\sin\left(\sqrt{P/EI}\,x\right)\sqrt{P/EI}$

ie we get $a\sin\left(\sqrt{P/EI}\cdot 0\right) = C_2\cos\left(\sqrt{P/EI}\,0\right)$

$C_2 = 0$

$\Rightarrow$ $a = -a\cos\left[\sqrt{P/EI}\times L\right] + a \rightarrow$ $\cos\left[\sqrt{P/EI}\,l\right] = +0$

ie we have $\cos\theta = 0$ @ $0, \pi/2, 3\pi/2 \ldots$, applying minimum. Condⁿ

$$\pi/2 = l\sqrt{P/EI} \Rightarrow \pi^2/4 = \frac{l^2 P}{EI}$$

$$P = \frac{\pi^2 EI}{4l^2}$$

⇒ Both end hinged : $\pi^2 EI/l^2 = P$ ⇒ both end fixed ⇒ $P = 4\pi^2 EI/l^2$

⇒ one end fixed & other end hinged : $P = 2\pi^2 EI/l^2$

- Beam column :- A member subjected to both axial & transverse force. In case of bending axial stress [direct stress] is ignored.