BT SH-I(R/S)-11-23-310 Reg. No. [ ][ ][ ][ ][ ][ ][ ][ ]

A

B.Tech. Degree I Semester Regular/Supplementary Examination in Naval Architecture and Ship Building November 2023

20-215-0102 MATHEMATICS - I
(*2020 Scheme*)

Time: 3 Hours Maximum Marks: 100

Course Outcome
On successful completion of the course, the students will be able to:

CO1: Learn the properties of hyperbolic functions.
CO2: Compute Taylor and Maclaurin Series of different functions and learning Leibnitz rule of differentiation.
CO3: Familiarize with important curves in engineering practice and learn about curvature.
CO4: Method of finding Envelopes and Evolutes of curves.
CO5: Compute partial derivatives of functions of two variables and applications.

Bloom's Taxonomy Levels (BL): L1 – Remember, L2 – Understand, L3 – Apply, L4 –Analyze, L5 – Evaluate, L6 – Create
PO – Programme Outcome

PART A
(Answer *ALL* questions)

		(5 × 4 = 20)	Marks	BL	CO	PO
I.	(a)	If x is real show that $\cosh^{-1}x = \log\left(x+\sqrt{x^2-1}\right)$.	4	L2	1	1(2),2(1), 6(2),7(1),12(2)
	(b)	Find the n^{th} derivative of $y = e^x\cos 2x\cos x$.	4	L1	2	1(3),2(1)
	(c)	Find the centre, foci, directrices and axes of the conic $9x^2 + 16y^2 - 18x + 32y - 119 = 0$.	4	L2	3	1(2),2(2), 4(2),5(2)
	(d)	If $z = e^{ax+by}f(ax-by)$, prove that $b\dfrac{\partial z}{\partial x}+a\dfrac{\partial z}{\partial y} = 2abz$.	4	L3	4	1(2),2(2), 3(1),4(1),12(1)
	(e)	Find the Envelope of the family of lines. $y = mx + \sqrt{9m^2-4}$	4	L2	5	1(3),2(3),6(2), 7(2),12(1)

PART B

		(5 × 16 = 80)	Marks	BL	CO	PO
II.	(a)	Prove that $\sinh 3x = 3\sinh x + 4\sinh^3 x$.	3	L1	1	1(2),2(1), 6(2),7(1),12(2)
	(b)	If $u = \log\tan\left(\dfrac{\pi}{4}+\dfrac{\theta}{2}\right)$ prove that $\tanh\dfrac{u}{2} = \tan\dfrac{\theta}{2}$.	5	L1	1	1(2),2(1), 6(2),7(1),12(2)
	(c)	If $\cos(x + Iy) = \cos\theta + i\sin\theta$ prove that: (i) $\sin^2 x = \pm\sin\theta$ (ii) $\cos 2x + \cosh 2y = 2$.	8	L2	1	1(2),2(1), 6(2),7(1),12(2)
		OR				
III.	(a)	Separate $\tan^{-1}(x+iy)$ in to real and imaginary parts.	8	L1	1	1(2),2(1), 6(2),7(1),12(2)
	(b)	If $\sin(A + IB) = x + iy$, show that: (i) $\dfrac{x^2}{\cosh^2 B}+\dfrac{y^2}{\sinh^2 B}=1$ (ii) $\dfrac{x^2}{\sin^2 A}-\dfrac{y^2}{\cos^2 A}=1$	8	L2	1	1(2),2(1), 6(2),7(1),12(2)

BT SH-I(R/S)-11-23-310

		Marks	BL	CO	PO
IV.	(a) If $y = e^{a\sin^{-1}x}$ prove that: $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$	8	L2	2	1(3),2(1)
	(b) Using Maclaurin's series show that: $e^{\sin x}=1+x+\frac{x^2}{2}-\frac{x^4}{8}+\ldots$	8	L3	2	1(3),2(1)
	OR				
V.	(a) If $y = \sin(m\sin^{-1}x)$ prove that: $(1-x^2)y_{n+2}-2(n+1)xy_{n+1}+(m^2-n^2)y_n=0$	8	L2	2	1(3),2(1)
	(b) Find the Maclaurin's series expansion of $\tan x$ up to the terms containing x^5.	8	L3	2	1(3),2(1)
VI.	(a) Define orthoptic locus of a conic. Also find the orthoptic locus of the parabola $y^2 = 4ax$.	8	L2	3	1(2),2(2), 4(2),5(2)
	(b) Find the condition that $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.	8	L1	3	1(2),2(2), 4(2),5(2)
	OR				
VII.	(a) If the chord joining two points whose eccentric angles are α and β on $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the major axis at a distance d from the centre, prove that $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{d-a}{d+a}$.	8	L1	3	1(2),2(2), 4(2),5(2)
	(b) Find the radius of curvature at any point θ on $x = a\cos^3\theta$, $y = a\sin^3\theta$	8	L2	3	1(2),2(2), 4(2),5(2)
VIII.	(a) Find the Evolute of the parabola $y^2 = 4ax$.	8	L2	4	1(2),2(2), 3(1),4(1),12(1)
	(b) Find the Envelope of the family of lines $\frac{x}{a}+\frac{y}{b}=1$ where $a^n + b^n = c^n$.	8	L1	4	1(2),2(2), 3(1),4(1),12(1)
	OR				
IX.	(a) Show that the Evolute of the Cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, is another cycloid $x = a(\theta + \sin\theta)$, $y = -a(1 - \cos\theta)$.	8	L2	4	1(2),2(2), 3(1),4(1),12(1)
	(b) Find the Envelope of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a+b=c$.	8	L1	4	1(2),2(2), 3(1),4(1),12(1)

(Continued)

X.	(a)	If $V=\left(x^2+y^2+z^2\right)^{\frac{-1}{2}}$, prove that: $$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0$$	5	L3	5	1(3),2(3), 6(2),7(2),12(1)
	(b)	If $U=F(x-y,y-z,z-x)$ prove that: $$\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}+\frac{\partial U}{\partial z}=0.$$	5	L2	5	1(3),2(3), 6(2),7(2),12(1)
	(c)	If $U=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial U}{\partial x}+y\frac{\partial U}{\partial y}=\tan U$.	6	L3	5	1(3),2(3), 6(2),7(2),12(1)
		OR				
XI.	(a)	If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ prove that: $$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\cos 2u\sin u.$$	10	L3	5	1(3),2(3), 6(2),7(2),12(1)
	(b)	If $u=\frac{yz}{x},\ v=\frac{xz}{y},\ w=\frac{xy}{z}$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	6	L2	5	1(3),2(3), 6(2),7(2),12(1)

Bloom's Taxonomy Levels
L1 = 28%, L2 = 50%, L3 = 22%.

***