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Department of Ship Technology
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21 Feb 2025

Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406
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3 credits

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
Intact stability rules, Heel/ Load test.
Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Stability of Ships. Course Content

4. **Module IV**
Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

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removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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Course Content

Earlier

Module 1. Stability Terms. 06 Lectures.

Module 2. Initial (Transverse) Stability. 07 lectures.

Module 3. Large Angle Transverse Stability. 07 lectures.

Today

- 4. 1 Trim and Pitch

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Longitudinal Stability

Today

- 4.1 Trim and Pitch

Later

- 4.2 Longitudinal Metacenter
- 4.3 Longitudinal Center of Floatation
- 4.4 Moment to Change Trim (by 1 cm)
- 4.5 Trim of a loaded ship
- 4.6 Trim and draft due to change in weight
- 4.7 Rules on draft and trim

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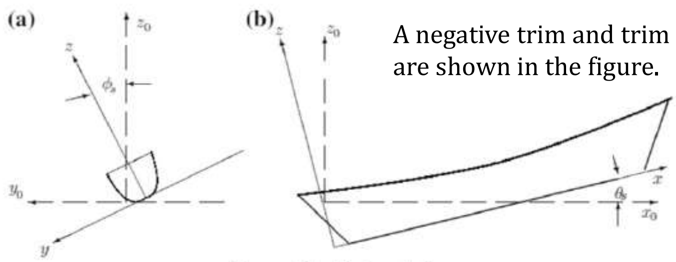
4.1 Trim and Pitch

Definitions, Principal Dimensions 11



A negative trim and trim angle are shown in the figure.

Figure 1.7 Heel and trim

When the ship-fixed x-axis is parallel to the space-fixed x_0-axis, we say that the ship floats on **even keel**. A static inclination of the ship-fixed system around an axis parallel to the ship-fixed y-axis is called **trim**. If the inclination is dynamic, that is a function of time resulting from ship motions, it is called **pitch**. A graphic explanation of the term trim is given in Figure 1.7. The trim is measured as the difference between the forward and the aft draught. Thus, the trim is positive if the ship is **trimmed by the head**. As defined here the trim is measured in metres.

- Linesplan is in the ship-fixed sys
- Trim angle is easily measured in the global coord sys.
- In the ship-fixed system, trim is the diff in the fwd and aft water levels

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Two Convenient Coordinate Systems

and the z-axis positive upwards. For dynamic applications taking the origin in the centre of gravity simplifies the equations. However, it should be clear that to each loading condition corresponds one centre of gravity, while a point like the intersection of the aft perpendicular with the baseline is independent of the ship loading. The system of coordinates used for the hull surface can be also employed for the location of weights. By its very nature, the system in which the hull is defined is fixed in the ship and moves with her. To define the various **floating conditions**, that is the positions that the vessel can assume, we use another system, fixed in space, that is defined in ISO 7463 as x_0, y_0, z_0. Let this system initially coincide with the system x, y, z. A vertical translation of the system x, y, z with respect to the space-fixed system x_0, y_0, z_0 produces a draught change.

in the space-fixed coordinate system,

If the ship-fixed z-axis is vertical we say that the ship floats in an *upright condition*.
A rotation of the ship-fixed system around an axis parallel to the x-axis is called **heel** (Figure 1.7) if it is temporary, and **list** if it is permanent. The heel can be produced by lateral wind, by the centrifugal force developed in turning, or by the temporary, transverse displacement of weights. The list can result from incorrect loading or from flooding. If the transverse inclination is the result of ship motions, it is time-varying and we call it **roll**.

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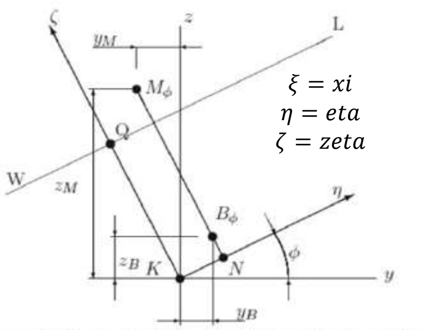
4.1 CoB in the Ship-Fixed Sys

- Biran. Chap 2.
- Global coord sys. (x, y, z)
- Ship fixed sys (ξ, η, ζ)
- See L15S16 for (y_B, z_B) in the ship-fixed coordinate system. Fig. 56.



$\xi = xi$
$\eta = eta$
$\zeta = zeta$

Figure 2.22 The coordinates of the points *B* and *M*

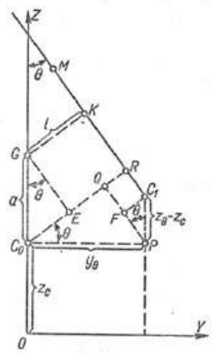


Fig. 56

To each position of a floating body correspond one centre of buoyancy ~~and one metacentre~~. Each position of the floating body is defined by three parameters, for instance the triple {*displacement*, *angle of heel*, *angle of trim*}; we call them the **parameters of the floating condition**. If we keep two parameters constant and let one vary, the centre of buoyancy travels along a curve and the metacentre along another. If only one parameter is kept constant and two vary, the centre of buoyancy and the metacentre generate two surfaces. In this chapter we shall

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4.1 Trim

Example 2.6 (B and M curves of *Lido 9*). Table 2.4 contains hydrostatic data of the vessel *Lido 9* for a volume of displacement equal to 44.16 m^3 and the heel angles 0°, 15°, 30°, . . . , 90°. As shown in Figure 2.22, all the data are measured in a system of coordinates ξ, η, ζ. In this example, the axes $K\eta$ and $K\zeta$ rotate with an angle ϕ with respect to the axes Ky, Kz in which the hull surface is defined. The angle ϕ is the *heel angle*. The draught, T, is measured perpendicularly to the waterline; in our figure it is $T = \overline{KQ}$. As we see, $\overline{KN}$ is parallel to the waterline. The centre of buoyancy corresponding to the heel angle ϕ is marked B_ϕ and the respective metacentre, M_ϕ. In the table we dropped the subscripts ϕ. The height of the centre of buoyancy, $\overline{NB_\phi}$, is measured perpendicularly to the waterline and so is the height of the metacentre, $\overline{NM_\phi}$.

The draught is measured on the ⊥ to the waterline. See Fig. 2.23.

Note that LCB and $\overline{NM}_L$ change

Table 2.4 Data of vessel *Lido 9* at 44.16 m^3 volume of displacement

Heel Angle (°)	Draught (m)	$\overline{KN}$ (m)	$\overline{NB}$ (m)	$\overline{NM}$ (m)	LCB (m)	$\overline{NM_L}$ (m)
0	1.729	0.000	1.272	4.596	−1.735	23.371
15	1.575	1.122	1.121	3.711	−1.799	23.730
30	1.163	1.979	0.711	2.857	−1.932	23.154
45	0.600	2.595	0.107	1.830	−2.047	23.133
60	−0.012	2.945	−0.625	0.479	−2.072	17.473
75	−0.693	2.874	−1.393	−0.869	−2.008	14.298
90	−1.354	2.539	−2.108	−13.314	−1.970	12.792

- Observe that the ship has heeled wrt a coordinate system that is fixed in space

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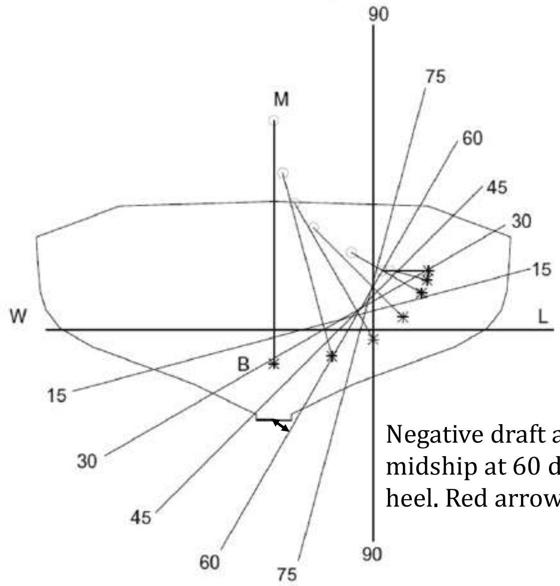
Lido 9. Negative draft at amidships.

- At 60 deg. angle of heel, is any part of the center-line underwater?
- What is the draft at amidships when heel = 60 deg?
- Draft at amidships = distance at amidships between the keel and the waterline along the perpendicular to the waterline
- See Table 2.4 on S09. Find the mid-ship draft at 60 deg. heel. It is negative.



Negative draft at midship at 60 deg heel. Red arrow.

Figure 2.23 B and M curves of vessel *Lido 9*

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LCB

- Read this carefully and understand the implications. When heel angle changes LCB and trim also change because there is no forward-aft symmetry.

Table 2.4 contains a column that we did not use until now: the LCB values. We included these data to show that at finite angles of heel the centre of buoyancy can leave its initial transverse plane and move along the ship. This is the case of ships that do not have a fore-to-aft symmetry. Then, when the heel changes, the trim also changes until centre of gravity and centre of buoyancy lie again on the same vertical (Stevin's law).

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Transformation between Coordinate Systems

- Ship-fixed and Space-fixed coordinate systems

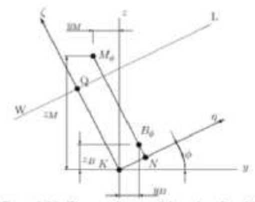

Figure 2.22 The coordinates of the points B and

Let x_B, y_B be the coordinates of the centre of buoyancy, B, and x_M, y_M those of the metacentre, M. With the help of Figure 2.22 we can write

In the η, ζ coord sys, $\overline{KB} = \overline{NB}$ = zCoB and $\overline{KM} = \overline{NM}$

$$\begin{aligned} y_B &= \overline{KN}\cos\phi - \overline{KB}\sin\phi \\ z_B &= \overline{KN}\sin\phi + \overline{KB}\cos\phi \end{aligned} \tag{2.75}$$

$\xi = xi$
$\eta = eta$
$\zeta = zeta$

and

$$\begin{aligned} y_M &= \overline{KN}\cos\phi - \overline{KM}\sin\phi \\ z_M &= \overline{KN}\sin\phi + \overline{KM}\cos\phi \end{aligned} \tag{2.76}$$

See S08 and S13

Equation (2.75) can be rewritten in matrix form as

$$\begin{bmatrix} y_B \\ z_B \end{bmatrix} \cancel{\begin{bmatrix} x_B \\ y_B \end{bmatrix}} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \overline{KN} \\ \overline{KB} \end{bmatrix} \tag{2.77}$$

Similarly, Eq. (2.76) can be written in matrix form as

$$\begin{bmatrix} y_M \\ z_M \end{bmatrix} \cancel{\begin{bmatrix} x_M \\ y_M \end{bmatrix}} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \overline{KN} \\ \overline{KM} \end{bmatrix} \tag{2.78}$$

The **transformation matrix**

$$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \tag{2.79}$$

performs counterclockwise rotation, around the origin, with the angle ϕ. In this example we

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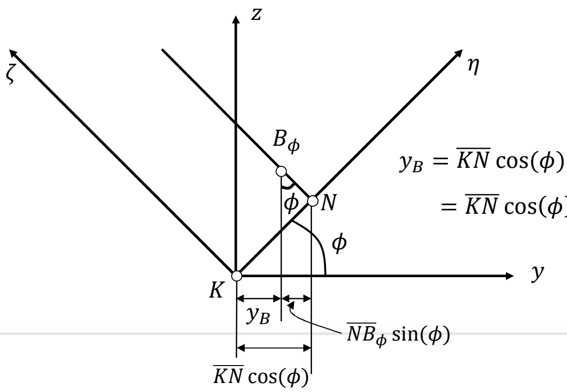
4.1 Trim

- Transformation of Coordinates
  - See the previous slide. The coordinates of B_ϕ in the ship-fixed (η, ζ) plane are known to be $(\overline{KN}, \overline{NB}_\phi)$. Find the coords in the earth-fixed (y, z) coordinate system.



$$y_B = \overline{KN}\cos(\phi) - \overline{NB}_\phi \sin(\phi)$$
$$= \overline{KN}\cos(\phi) - \overline{KB}_\phi \sin(\phi)$$

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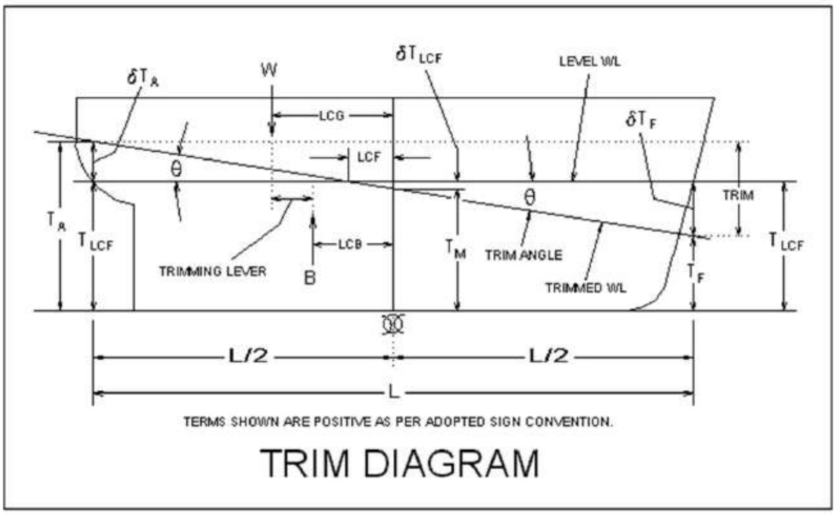
Trim $= \boldsymbol{T_{fwd}} - \boldsymbol{T_{aft}}$

- This ship has negative Trim



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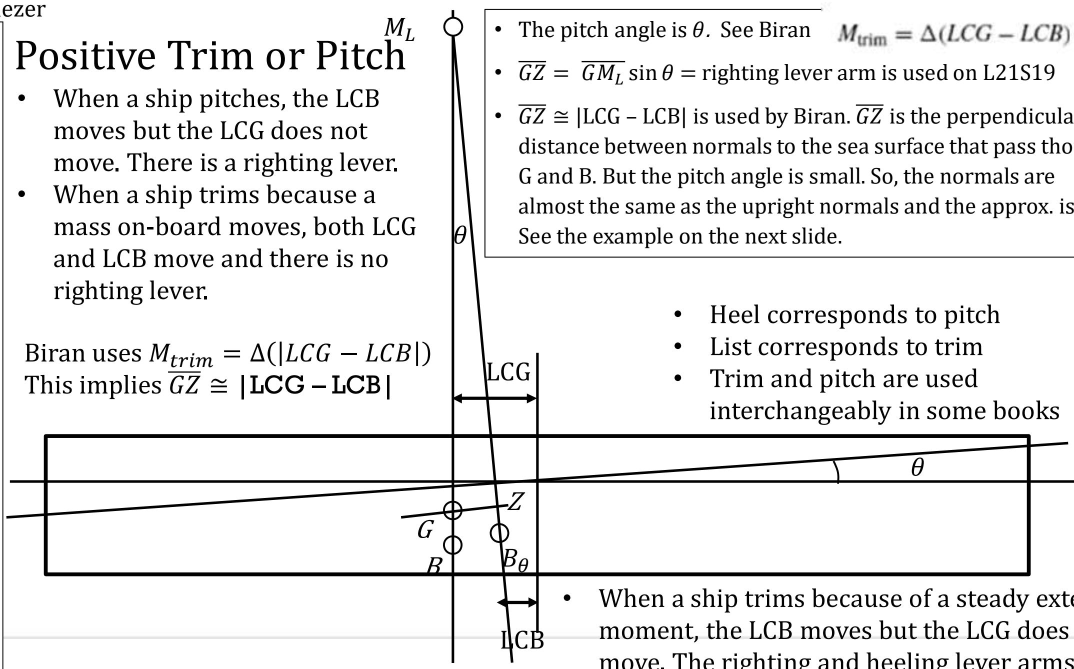
Positive Trim or Pitch

- When a ship pitches, the LCB moves but the LCG does not move. There is a righting lever.
- When a ship trims because a mass on-board moves, both LCG and LCB move and there is no righting lever.

Biran uses $M_{trim} = \Delta(|LCG - LCB|)$
This implies $\overline{GZ} \cong$ **|LCG – LCB|**

- The pitch angle is θ. See Biran $M_{\text{trim}} = \Delta(LCG - LCB)$ (7.1)
- $\overline{GZ} = \overline{GM_L} \sin\theta$ = righting lever arm is used on L21S19
- $\overline{GZ} \cong$ |LCG – LCB| is used by Biran. $\overline{GZ}$ is the perpendicular distance between normals to the sea surface that pass though G and B. But the pitch angle is small. So, the normals are almost the same as the upright normals and the approx. is ok. See the example on the next slide.

- Heel corresponds to pitch
- List corresponds to trim
- Trim and pitch are used interchangeably in some books



- When a ship trims because of a steady external moment, the LCB moves but the LCG does not move. The righting and heeling lever arms are equal at the new attitude.

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$\overline{GZ}$ $\cong$ LCB – LCG for small pitch and trim angles

- A cuboidal barge of length, L, breadth, B, and draft, T, has zCoG = KG. It is on even keel. Then, it pitches (or trims due to a steady external moment) by 1 deg. Find the exact and approximate values of the righting lever arm.
- KM = KB + BM = $\frac{T}{2} + \frac{L^2}{12T}$. $\overline{GM} = \overline{KM}$ - $\overline{KG}$.
- Exact $\overline{GZ} = \overline{GM}\sin\theta = \left[\frac{T}{2} + \frac{L^2}{12T} - \overline{KG}\right]\sin\theta$.
- For small θ and $\frac{L^2}{12T} \gg \left(\frac{T}{2} - \overline{KG}\right)$, $\overline{GZ} \cong \frac{L^2}{12T}\theta$. More generally, $\overline{GZ} = \overline{GM}\theta \cong \overline{BM}\theta$
- xCoG is at midship. After pitching fwd, xCoB = $\frac{L^2}{12T}\tan\theta$ fwd of midship. Using $\overline{GZ} \cong$ LCB – LCG, $\overline{GZ} \cong \frac{L^2}{12T}\theta$. Note the sign difference between this and Biran Eq. (7.1).
- For small pitch angles, it has been shown that the righting lever arm of a ship that pitches is approx. LCB – LCG. Here, LCB is the xCoB after pitching.

$$M_{\text{trim}} = \Delta(LCG - LCB)$$

Biran Eq. (7.1)

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Numerical Example

- On the previous slide, use L = 24 m, B = 4 m, T = 2 m, KG = 1.4 m.
- Then, KM = KB + BM = $\frac{T}{2} + \frac{L^2}{12T}$ = 1 +24 = 25 m. KG = 1.4 m.
- GM = KM – KG = 25-1.4 = 23.6 m. $\overline{GZ} = \overline{GM}\sin\theta = 0.412$ m
- xCoB = $\frac{L^2}{12T}\tan\theta = 24\tan\theta$ = 0.419 m = LCB.
- Use LCG = 0 and $\overline{GZ}$ ≅ LCB – LCG ≅ 0.419 m.
- The error percentage is (0.419-0.412)*100/0.412 = 1.7%

Use the exact equation whenever you can

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Read books, magazines, and journals

and become outstanding Naval Architects

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4.2 Longitudinal Metacenter

- $\overline{BM_L} = I/V$

https://www.youtube.com/watch?v=r-rQKU8jOGE

In the theory of moments of inertia the two axes for which we obtain the extreme values of moments of inertia are called **principal axes** and the corresponding moments, **principal moments of inertia**. When the waterplane area has an axis of symmetry, this axis is one of the principal axes; the other one is perpendicular to the first. The waterplane area of ships in upright condition has an axis of symmetry: the intersection of the waterplane and the centreline plane. The moment of inertia about this axis is the smallest one; it is used to calculate the **transverse metacentric radius**. The moment of inertia about the axis perpendicular in F to the centreline is the largest; it enters in the calculation of the **longitudinal metacentric radius**.

To give an idea of the relative orders of magnitude of the transverse and longitudinal metacentric radii, let us consider a parallelepipedic barge whose length is L, breadth, B, and draught, T. The volume of displacement equals $\nabla = LBT$. The transverse metacentric radius results from

$$\overline{BM} = \frac{LB^3/12}{LBT} = \frac{B^2}{12T}$$

The longitudinal metacentric radius is given by

$$\overline{BM_L} = \frac{BL^3/12}{LBT} = \frac{L^2}{12T}$$

The ratio of the two metacentric radii is

$$\frac{\overline{BM_L}}{\overline{BM}} = \left(\frac{L}{B}\right)^2$$

The length-breadth ratio ranges from 3.1, for some motor boats, to 10.5, for fast cruisers. Correspondingly, the ratio of the longitudinal to the transverse metacentric radius varies roughly between 10 and 110. As a rule of thumb, the longitudinal metacentric radius is of the same order of magnitude as the ship length.

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4.3 Longitudinal Center of Flotation

- Centre of flotation – The geometric center of the waterplane on which a vessel floats.
- LCF The longitudinal coordinate of the center of floatation
- A vessel pitches, or rotates (about a transverse axis) through this point, when moved by an external force or a mass on-board is moved longitudinally

- The LCG of a ship is known. A steady external moment acts and the ship trims. LCG does not change. The new LCB is known. Find the trim.
- $\overline{GZ} \cong$ |LCG – LCB| See S15
- To find the trim of a loaded ship, find the LCG and the upright LCB. Then use Eq. (7.3) to find the trim. After it trims, the LCB will change.

displacement. For that draught we read the moment to change trim, MCT. We calculate the trim, in m, as

$$\text{trim} = T_F - T_A = \frac{\Delta(LCG - LCB)}{MCT} \qquad (7.3)$$

The trim angle is given by

$$\tan\theta = \frac{T_F - T_A}{L_{pp}} \qquad (7.4)$$

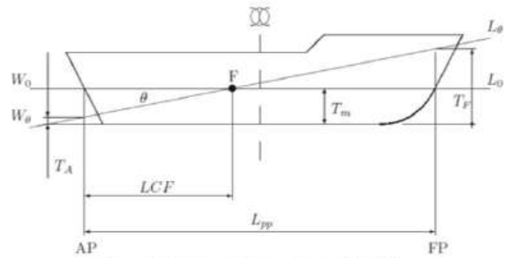

Figure 7.2 Finding the forward and aft draughts

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4.4 MCT by moving a mass on-board

- MCT = Moment to Change Trim by 1 m (or 1 cm)
- TPC = Tonnes Per Centimeter
- $\overline{GG_1}$ is parallel to movement of the mass on-board → The distance by which the CoG of the ship moved horizontally $\cong \overline{GM_L}\ \sin\theta$
- $\overline{GG_1} = \overline{GM_L}\ \sin\theta$
- $\tan\theta = \text{Trim}/LPP$
- B_1 should be higher than B

Moment $= \Delta\overline{GG_1} = \Delta\overline{GM_L}\ \sin\theta =$

MCT1m x Trim = MCT1m x $\tan\theta\ L_{PP}$

So, $\frac{\Delta\overline{GM_L}}{L_{PP}} \cong$ MCT1m

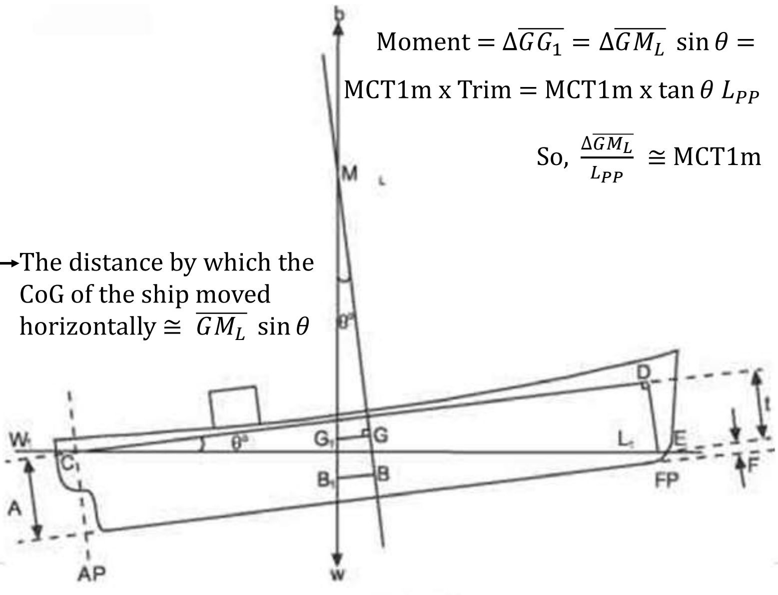

Fig. 15.13(c)

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4.4 MCT by moving a mass on-board

- Trim angle when the trim is 1 m ⟶ $\tan\theta = \frac{1}{L_{pp}}$ (4.13)

The notation θ for the angle of trim corresponds to the standards ISO 7463 and DIN 81209-1. At the angle of trim given by Eq. (4.13), the ~~displacement and buoyancy forces~~ original and final CoGs are separated by a distance $\overline{GM_L}\sin\theta$, where $\overline{GM_L}$ is the **longitudinal metacentric height** calculated as

$\overline{GG_1}=$

$$\overline{GM_L} = \overline{KB} + \overline{BM_L} - \overline{KG}$$

MCT

$\sin(\theta) \cong \tan(\theta)\ (\cong \theta) \cong \text{Trim}/L_{pp}$

The couple formed by the displacement and buoyancy forces is

When an external moment causes trim ⟶ Couple = $\Delta\overline{GM_L}\sin\theta \cong \Delta\overline{GM_L}\tan(\theta) \cong \Delta\overline{GM_L}\ \text{Trim}/L_{pp}$

For small angles of trim we assume $\tan\theta \approx \sin\theta$ and then the **moment to change trim by 1 m** is equal to

In the expression for moment, set Trim = 1m to find MCT1m

$$M_{CT} = \frac{\Delta\overline{GM_L}}{L_{pp}} \quad (4.14)$$

where M_{CT} is measured in t m/m, Δ in t, and $\overline{GM_L}$ and L_{pp}, in m. Although the SI unit is the metre, some design offices use the "moment to change trim by 1 cm." Then, the value of M_{CT} given by Eq. (4.14) should be divided by 100.

In the first design stages $\overline{KG}$ is not known. As $\overline{BM_L} \gg \overline{KB} - \overline{KG}$, we can assume the approximation $\overline{GM_L} \approx \overline{BM_L}$.

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MCT

- Use Fig. 4.2 to find $\overline{KM}_L$ (at T = 1.6 m). $\overline{KM}_L \cong 22.5$ m. Use that and $\overline{KG}$ to draw $\overline{GM}_L$ vs Draft.
- Then, find MCT1m
- If draft is fixed, disp in SW and FW are different

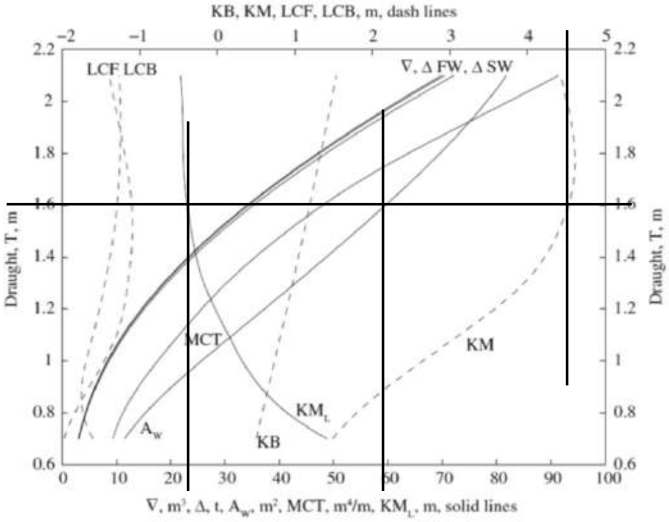

Figure 4.2 Hydrostatic curves of ship *Lido 9*

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4.5 Trim of a loaded ship. Biran 7.2.2

- The hydrostatic curves for a ship are known. The ship is loaded with cargo. Details of the lightship and the cargo are known. Find the trim.

1. Use the details of the load and find the displacement and the CoG
2. Use the hydrostatic curves or tables and the displacement and find the mean draft.
3. Use the hydrostatic curves and the draft and find the LCG, LCB, and approximate trimming moment
4. Use the hydrostatic curves and the draft and find $\overline{KM_L}$
5. Find $\overline{\mathrm{GM}} = \overline{KM_L}$ - $\overline{\mathrm{KG}}$. Use $\overline{\mathrm{GM}_L} \cong \overline{BM_L}$
6. Use the hydrostatic curves and find the MCT1m. Use it and find the trim and trim angle

- Details are on the next slide.
- Then …

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Given the load details, find the displacement and the CoG

1. List all the items on board the ship. Find the displacement.
2. Find the CoG of the ship.

- See Biran 7.2.2
- Displacement = 2625 t
- VCG = 13113.14/2625 = 5 m
- LCG =94186.54/2625 = 35.88 m
- Mean draft = 4.32 m

1	Small cargo ship, Homogeneous cargo, departure.					
2						
3	**Weight item**	**Mass**	**vcg**	**z-Moment**	**lcg**	**x-Moment**
4		t	m	tm	m	tm
5	Lightship	1247.66	5.93	7398.62	32.04	39975.03
6	Crew and effects	3.60	9.60	34.56	11.00	39.60
7	Provisons	5.00	7.30	36.50	3.50	17.50
8	Fuel oil	177.21	1.56	276.45	30.88	5472.24
9	Lubricating oil	4.50	4.65	20.93	8.45	38.03
10	Freshwater	103.09	4.61	475.24	27.19	2803.02
11	Ballast water			0.00		0.00
12	Cargo in hold	993.94	4.35	4323.64	42.62	42361.72
13	Cargo on deck			0.00		0.00
14	Fruit cargo	90.00	6.08	547.20	38.66	3479.40
15	**Full load**	2625.00	5.00	13113.14	35.88	94186.54
16						
17	Mean draught, m	4.32				
18	KM, m	5.16				
19	KG, m	5.00				
20	GM, m	0.16				
21	FS effect, m	0.04				
22	Effective GM, m	0.12				
23						

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4.5 Trim of a loaded ship. Biran 7.2.2

- A ship is loaded with cargo. Details of the lightship and the cargo are known. Find the trim.
- First, find the displacement. See L20S22. Then … do the following steps

1. The mean draught, T_m, corresponding to the calculated displacement, is read in the hydrostatic curves. See the next slide
2. The trimming moment is calculated as

$$M_{\text{trim}} = \Delta(LCG - LCB) \tag{7.1}$$

   where the LCB value corresponding to T_m is found in the hydrostatic curves. The moment to change trim, MCT, corresponding to T_m, is read from the hydrostatic curves and the trim is calculated as shown in Section 7.3. If the trim is small one can go to the next step, otherwise it is advisable to continue the calculations using the Bonjean curves or to resort to a computer programme.
3. The height of the metacentre above BL, $\overline{KM}$, corresponding to T_m, is read in the hydrostatic curves.
4. The metacentric height is calculated as

$$\overline{GM} = \overline{KM} - \overline{KG}$$

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- Displacement = 2625 t. Volume = 2625/1.025 = 2561 m^3. From the table below, draft is approx. 4.32 m.
- Next, do Step 2 on the previous slide. For details, see the next slide.

Table 6.6 Small cargo ship—partial hydrostatic data $L_{PP} = 75.40\ m$

Draught, T (m)	∇ (m^3)	$\overline{KM}$ (m)	Draught, T (m)	∇ (m^3)	$\overline{KM}$ (m)
2.00	993	6.75	4.32	2549	5.16
2.20	1118	6.39	4.40	2609	5.16
2.40	1243	6.09	4.60	2757	5.16
2.60	1377	5.83	4.80	2901	5.17
2.80	1504	5.63	5.00	3057	5.18
3.00	1640	5.48	5.20	3210	5.20
3.20	1776	5.37	5.40	3352	5.23
3.40	1907	5.28	5.60	3507	5.27
3.60	2045	5.24	5.80	3653	5.31
3.80	2189	5.20	5.96	3786	5.34
4.00	2322	5.18	6.00	3811	5.36
4.20	2471	5.17	6.20	3972	5.42

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4.5 Trim of a loaded ship. Biran 7.3

- Step 2. LCG is known from L20S22. LCG = 35.88 m. LCB is found from the table below. LCB = 0.291 m forward of midship = 75.40/2 + 0.291 = 37.991 m
- LCG ≠ LCB. So, the ship will trim.

Table 7.3 Small cargo ship—partial hydrostatic data, 2

Draught, T (m)	MCT (~~m~~)(t)	LCB from Midship (m)	LCF from Midship (m)	Draught, T (m)	MCT (~~m~~)(t)	LCB from Midship (m)	LCF from Midship (m)
2.00	2206	0.607	0.518	4.32	3223	0.291	−0.384
2.20	2296	0.600	0.460	4.40	3260	0.272	−0.430
2.40	2382	0.590	0.398	4.60	3336	0.225	−0.560
2.60	2470	0.575	0.330	4.80	3413	0.180	−0.698
2.80	2563	0.557	0.260	5.00	3485	0.131	−0.839
3.00	2645	0.537	0.190	5.20	3567	0.083	−0.960
3.20	2732	0.510	0.119	5.40	3639	0.033	−1.066
3.40	2824	0.480	0.041	5.60	3716	−0.018	−1.158
3.60	2906	0.442	−0.035	5.80	3793	−0.067	−1.231
3.80	2293	0.406	−0.017	5.96	3863	−0.108	−1.281
4.00	3085	0.360	−0.210	6.00	3880	−0.118	−1.293
4.20	3167	0.319	−0.314	6.20	3951	−0.167	−1.348

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4.5 Trim of a loaded ship

2. The trimming moment is calculated as

$$M_{\text{trim}} = \Delta(LCG - LCB) \quad (7.1)$$

where the LCB value corresponding to T_m is found in the hydrostatic curves. The moment to change trim, MCT, corresponding to T_m, is read from the hydrostatic curves and the trim is calculated as shown in Section 7.3. If the trim is small one can go to the next step, otherwise it is advisable to continue the calculations using the Bonjean curves or to resort to a computer programme.

- The CoG and the CoB should lie on the same vertical that is perpendicular to the waterline. If they don't, a moment occurs. Find the moment that acts by using the approximate Eq. (7.1). LCB is based on uniform draft. If the moment is non-zero, the LCB will change and a trim will occur.
- To find the trim, find MCT by using the table on the previous slide. MCT = 3223 t.

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4.5 Trim of a loaded ship

- Find the Trim and Trim Angle by using Eqs. (7.3) and (7.4)
- Trim = 2625 (35.88 – 37.99) / 3223 = -1.72 m

displacement. For that draught we read the moment to change trim, MCT. We calculate the trim, in m, as

$$\text{trim} = T_F - T_A = \frac{\Delta(LCG - LCB)}{MCT} \quad (7.3)$$

The trim angle is given by

$$\tan\theta = \frac{T_F - T_A}{L_{pp}} \quad (7.4)$$

perpendiculars is $L_{pp} = 75.40$ m. In the table LCB is measured from midship, positive forwards. As LCG is measured from AP, we calculate

$$75.40/2 + 0.291 = 37.99 \text{ m}$$

and the trim

$$\frac{\Delta(LCG - LCB)}{MCT} = \frac{2625(35.88 - 37.99)}{3223} = -1.72 \text{ m}$$

The ship is trimmed by the stern. In Table 7.3 LCF is measured from the midship, positive

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4.5 Trim of a loaded ship

- Do Step 3 on L20S23
- Find $\overline{KM}$ at the known draft using the data on L19S23. It is 5.16 m.
- Do Step 4 on L20S23. See L20S22 for $\overline{KG}$.
- Find the metacentric height. Find $\overline{GM_L} = \overline{KM_L} - \overline{KG}$.

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4.5 Trim of a loaded ship.

- Find the drafts at the forward and aft perpendiculars – given all the loading data and the hydrostatic info
- Trim has been found. Find the Trim Angle.

7.3 Trim

7.3.1 Finding the Trim and the Draughts at Perpendiculars

In Figure 7.2 we consider a ship initially on even keel; the corresponding waterline is W_0L_o. Let us assume that the ship trims reaching a new waterline, $W_\theta L_\theta$. If the trim angle, θ, is small (for normal loading conditions it is always small), the intersection line of the two waterlines, W_0L_o and $W_\theta L_\theta$, passes through the centre of flotation, F, of the initial waterplane. The midship draught of the ship on even keel, T_m, can be read in the hydrostatic curves at the intersection of the displacement curve and the vertical corresponding to the given displacement. For that draught we read the moment to change trim, MCT. We calculate the trim, in m, as

$$\text{trim} = T_F - T_A = \frac{\Delta(LCG - LCB)}{MCT} \tag{7.3}$$

The trim angle is given by

$$\tan\theta = \frac{T_F - T_A}{L_{pp}} \tag{7.4}$$

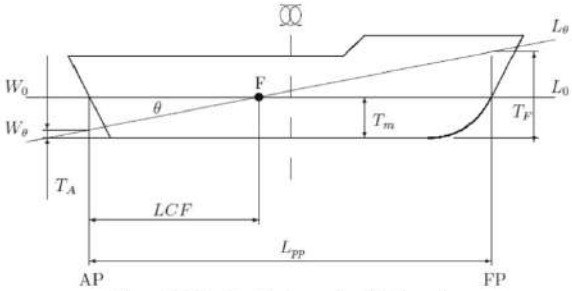

Figure 7.2 Finding the forward and aft draughts

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4.5 Trim of a loaded ship.

- Avoid approximations if the exact value can be found without much additional effort.
- Here, $T_m = T_{LCF}$ is an approximation
- See the numerical example where the draft at the CoF is found and used.

As the ship trims around a transversal axis passing through the centre of flotation F, we consider this point as fixed during trim and we assume that the draught of this point has the value T_m found above. From Figure 7.2 we see that

$$T_A = T_m - LCF \cdot \tan\theta = T_m - LCF \cdot \frac{\text{trim}}{L_{pp}} \tag{7.5}$$

and

For even keel $T_m = T_{LCF} = T_F$

$$T_F = \text{trim} + T_A = T_m + \text{trim}\left(1 - \frac{LCF}{L_{pp}}\right) \tag{7.6}$$

To give an example we consider again the loading case of the small cargo ship analysed in Section 7.2.2. In Tables 7.2 and 7.3 we find $T_m = 4.32$ m, $LCB = 0.291$ m, $LCF = -0.384$ m, and $MCT = 3223$ mtm^{-1}. We know that the length between perpendiculars is $L_{pp} = 75.40$ m. In the table LCB is measured from midship, positive forwards. As LCG is measured from AP, we calculate

$$75.40/2 + 0.291 = 37.99 \text{ m}$$

and the trim

$$\frac{\Delta(LCG - LCB)}{MCT} = \frac{2625(35.88 - 37.99)}{3223} = -1.72 \text{ m}$$

The ship is trimmed by the stern. In Table 7.3 LCF is measured from the midship, positive forward; the value measured from AP is

$$75.40/2 - 0.384 = 37.32 \text{ m}$$

and we calculate

$$T_A = 4.32 - 37.32\frac{-1.72}{75.4} = 5.17$$
$$T_F = -1.72 + 5.17 = 3.45$$

where the results are in m.

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Excercise

- Sketch a ship with length Lpp and a trim.
- The draft at the midship and the trim angle are known.
- The ship trims about the LCF.
- Use the sketch and derive expressions for the draft forward and draft aft.

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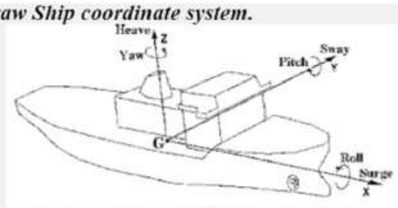
General Knowledge!

5. Draw Ship coordinate system.



6. Name the six degrees of freedom motions.

- *Surge*
- *Sway*
- *Heave*
- *Roll*
- *Pitch*
- *Yaw*

7. Name the translational motions of the ship.
The translational motions of the ship are Surge, Sway, Heave

8. Name the rotational motion of the ship.
The rotational motion of the ship are Roll , Pitch , Yaw

9. Name the inertial motion of the ship.
The inertial motion of the ship are Roll, Pitch, Heave

10. Name the non-inertial motion of the ship.
The non-inertial motion of the ship are Surge, Sway, Yaw.

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3.10 Heeling Moments

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28 Feb 2025

Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406
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3 credits

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
   Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
   Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
   Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
   Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
   Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
   Intact stability rules, Heel/ Load test.
   Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

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   removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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Longitudinal Stability

Completed

- 4.1 Trim and Pitch

Today

- 4.2 Longitudinal Metacenter
- 4.3 Longitudinal Center of Floatation
- 4.4 Moment to Change Trim (by 1 cm)
- 4.5 Trim of a loaded ship

Later

- 4.6 Trim and draft due to change in weight
- 4.7 Rules on draft and trim

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$\overline{GZ} \cong$ LCG – LCB for small pitch and trim angles

- A cuboidal barge of length, L, breadth, B, and draft, T, has zCoG = KG. It is on even keel. Then, it pitches (or trims due to a steady external moment) by 1 deg. Find the exact and approximate values of the righting lever arm.

See L21S16

- $\overline{KM} = \overline{KB} + \overline{BM_L} = \frac{T}{2} + \frac{L^2}{12T}$. $\overline{GM} = \overline{KM}$ - $\overline{KG}$.
- $\overline{GZ} = \overline{GM_L}\sin\theta = \left[\frac{T}{2} + \frac{L^2}{12T} - \overline{KG}\right]\sin\theta$.

$$M_{\text{trim}} = \Delta(LCG - LCB)$$

Biran Eq. (7.1)

- For small θ and $\frac{L^2}{12T} \gg \left(\frac{T}{2} - \overline{KG}\right)$, $\overline{GZ} \cong \frac{L^2}{12T}\theta$. More generally, $\overline{GZ} = \overline{GM_L}\theta \cong \overline{BM_L}\theta$
- LCG=0. LCB = xCoB = $\frac{L^2}{12T}\tan\theta \cong \overline{BM_L}\theta$. Using $\overline{GZ} \cong$ LCG – LCB, $\overline{GZ} \cong \frac{L^2}{12T}\theta$.
- For small pitch angles, it has been shown that the righting lever arm of a ship that pitches is approx. LCG – LCB and can be negative. Here, LCB is the xCoB after pitching.
- Use the same eq. for small trim due to a steady moment. Use the LCG before trim and LCB after trim. After trim, LCG will be on the same perpendicular to the WL as the LCB.

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4.2 Longitudinal Metacenter

- $\overline{BM_L} = I_L/V =$

$$\frac{BL^3/12}{LBT} = \frac{L^2}{12T}$$

https://www.youtube.com/watch?v=r-rQKU8jOGE

In the theory of moments of inertia the two axes for which we obtain the extreme values of moments of inertia are called **principal axes** and the corresponding moments, **principal moments of inertia**. When the waterplane area has an axis of symmetry, this axis is one of the principal axes; the other one is perpendicular to the first. The waterplane area of ships in upright condition has an axis of symmetry: the intersection of the waterplane and the centreline plane. The moment of inertia about this axis is the smallest one; it is used to calculate the **transverse metacentric radius**. The moment of inertia about the axis perpendicular in F to the centreline is the largest; it enters in the calculation of the **longitudinal metacentric radius**.

To give an idea of the relative orders of magnitude of the transverse and longitudinal metacentric radii, let us consider a parallelepipedic barge whose length is L, breadth, B, and draught, T. The volume of displacement equals $\nabla = LBT$. The transverse metacentric radius results from

$$\overline{BM} = \frac{LB^3/12}{LBT} = \frac{B^2}{12T}$$

The longitudinal metacentric radius is given by

$$\overline{BM_L} = \frac{BL^3/12}{LBT} = \frac{L^2}{12T}$$

The ratio of the two metacentric radii is

$$\frac{\overline{BM_L}}{\overline{BM}} = \left(\frac{L}{B}\right)^2$$

The length-breadth ratio ranges from 3.1, for some motor boats, to 10.5, for fast cruisers. Correspondingly, the ratio of the longitudinal to the transverse metacentric radius varies roughly between 10 and 110. As a rule of thumb, the longitudinal metacentric radius is of the same order of magnitude as the ship length.

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4.3 Longitudinal Center of Flotation

- Centre of flotation – The geometric center of the waterplane on which a vessel floats.
- LCF The longitudinal coordinate of the center of floatation
- A vessel trims, or rotates (about a transverse axis) through this point, when moved by a steady external force or a mass on-board is moved longitudinally

- The LCG of a ship is known. A steady external moment acts and the ship trims. LCG does not change. The new LCB is known. Find the trim.
- $\overline{GZ} \cong$ LCG – LCB. See L22S06
- To find the trim of a loaded ship, find the LCG and the upright LCB. Then use Eq. (7.3) to find the trim. After it trims, the LCB will change a little but that is neglected here.

displacement. For that draught we read the moment to change trim, *MCT*. We calculate the trim, in m, as

$$\text{trim} = T_F - T_A = \frac{\Delta(LCG - LCB)}{MCT} \qquad (7.3)$$

The trim angle is given by

$$\tan\theta = \frac{T_F - T_A}{L_{pp}} \qquad (7.4)$$

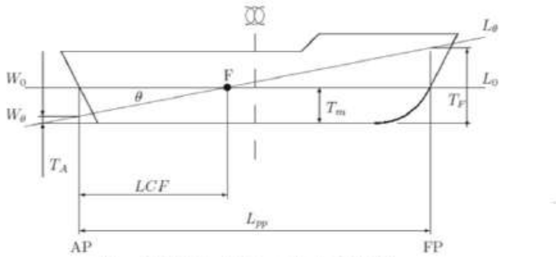

Figure 7.2 Finding the forward and aft draughts

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4.4 MCT by moving a mass on-board

- MCT = Moment to Change Trim by 1 m (or 1 cm)
- TPC = Tonnes Per Centimeter
- $\overline{GG_1}$ is parallel to movement of the mass on-board → The distance by which the CoG of the ship moved horizontally $\cong \overline{GM_L} \sin\theta$
- $\overline{GG_1} = \overline{GM_L} \sin\theta$
- $\tan\theta = \text{Trim}/LPP$
- In the fig, B_1 should be higher than B

Moment $= \Delta\overline{GG_1} = \Delta\overline{GM_L} \sin\theta =$ MCT1m x Trim = MCT1m x $\tan\theta\, L_{PP}$

So, $\frac{\Delta\overline{GM_L}}{L_{PP}} \cong$ MCT1m

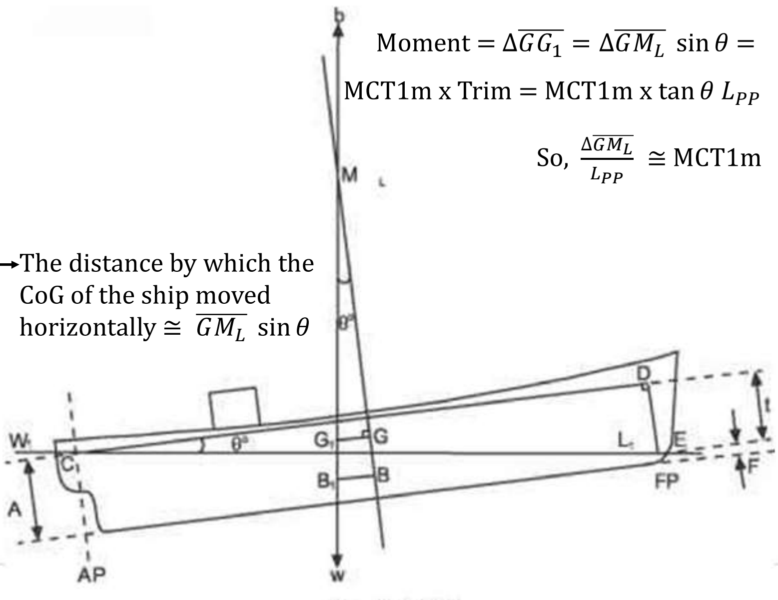

Fig. 15.13(c)

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4.4 MCT by moving a mass on-board

- Trim angle when the trim is 1 m → $\tan\theta = \dfrac{1}{L_{pp}}$ (4.13)

The notation θ for the angle of trim corresponds to the standards ISO 7463 and DIN 81209-1. At the angle of trim given by Eq. (4.13), the displacement and buoyancy forces are separated by a distance $\overline{GM_L}\sin\theta$, where $\overline{GM_L}$ is the **longitudinal metacentric height** calculated as

original and final CoGs

$\overline{GG_1} =$

$$\overline{GM_L} = \overline{KB} + \overline{BM_L} - \overline{KG}$$

$\sin(\theta) \cong \tan(\theta)\ (\cong \theta) \cong \text{Trim}/\ L_{pp}$

The couple formed by the displacement and buoyancy forces is

When an external steady moment causes trim

$$\text{Couple} = \Delta\overline{GM_L}\sin\theta \cong \Delta\overline{GM_L}\tan(\theta) \cong \Delta\overline{GM_L}\ \text{Trim}/L_{pp}$$

For small angles of trim we assume $\tan\theta \approx \sin\theta$ and then the **moment to change trim by 1 m** is equal to

In the expression for moment, set Trim = 1m to find MCT1m

$$M_{CT} = \frac{\Delta\overline{GM_L}}{L_{pp}} \qquad (4.14)$$

where M_{CT} is measured in t m/m, Δ in t, and $\overline{GM_L}$ and L_{pp}, in m. Although the SI unit is the metre, some design offices use the "moment to change trim by 1 cm." Then, the value of M_{CT} given by Eq. (4.14) should be divided by 100.

In the first design stages $\overline{KG}$ is not known. As $\overline{BM_L} \gg \overline{KB} - \overline{KG}$, we can assume the approximation $\overline{GM_L} \approx \overline{BM_L}$. See L22S06 where this approx. is used

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MCT

- Use Fig. 4.2 to find $\overline{\boldsymbol{KM}}_L$ (at T = 1.6 m). $\overline{\boldsymbol{KM}}_L \cong 22.5$ m. Use that and $\overline{KG}$ to draw $\overline{\boldsymbol{GM}}_L$ vs Draft.
- Then, find MCT1m
- If draft is fixed, disp in SW and FW are different

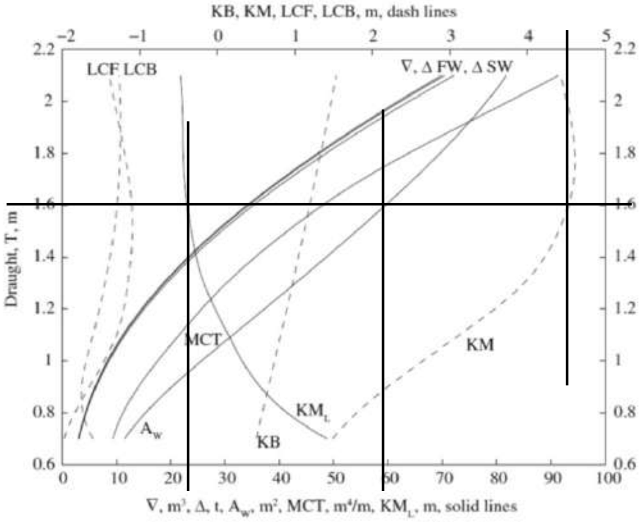

Figure 4.2 Hydrostatic curves of ship *Lido 9*

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4.5 Trim of a loaded ship. Biran 7.2.2

- The hydrostatic curves for a ship are known. The ship is loaded with cargo. Details of the lightship and the cargo are known. Find the trim.

1. Use the details of the load and find the displacement and the CoG
2. Use the hydrostatic curves or tables and the displacement and find the mean draft.
3. Use the hydrostatic curves and the draft and find the LCG, LCB, and approximate trimming moment
4. Use the hydrostatic curves and the draft and find $\overline{KM_L}$
5. Find $\overline{\mathrm{GM}} = \overline{KM_L}$ - $\overline{\mathrm{KG}}$. Use $\overline{GM_L} \cong \overline{BM_L}$
6. Use the hydrostatic curves and find the MCT1m. Use it and find the trim and trim angle

- Details are on the next slide.
- Then …

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Given the load details, find the displacement and the CoG

1. List all the items on board the ship. Find the displacement.
2. Find the CoG of the ship.

- See Biran 7.2.2
- Displacement = 2625 t
- VCG = 13113.14/2625 = 5 m
- LCG =94186.54/2625 = 35.88 m
- Mean draft = 4.32 m

1	Small cargo ship, Homogeneous cargo, departure.					
2						
3	**Weight item**	**Mass**	**vcg**	**z-Moment**	**lcg**	**x-Moment**
4		t	m	tm	m	tm
5	Lightship	1247.66	5.93	7398.62	32.04	39975.03
6	Crew and effects	3.60	9.60	34.56	11.00	39.60
7	Provisons	5.00	7.30	36.50	3.50	17.50
8	Fuel oil	177.21	1.56	276.45	30.88	5472.24
9	Lubricating oil	4.50	4.65	20.93	8.45	38.03
10	Freshwater	103.09	4.61	475.24	27.19	2803.02
11	Ballast water			0.00		0.00
12	Cargo in hold	993.94	4.35	4323.64	42.62	42361.72
13	Cargo on deck			0.00		0.00
14	Fruit cargo	90.00	6.08	547.20	38.66	3479.40
15	**Full load**	2625.00	5.00	13113.14	35.88	94186.54
16						
17	Mean draught, m	4.32				
18	KM, m	5.16				
19	KG, m	5.00				
20	GM, m	0.16				
21	FS effect, m	0.04				
22	Effective GM, m	0.12				
23						

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4.5 Trim of a loaded ship. Biran 7.2.2

- A ship is loaded with cargo. Details of the lightship and the cargo are known. Find the trim.
- First, find the displacement. See L22S12. Then … do the following steps

1. The mean draught, T_m, corresponding to the calculated displacement, is read in the hydrostatic curves. See the next slide
2. The trimming moment is calculated as

$$M_{\text{trim}} = \Delta(LCG - LCB) \tag{7.1}$$

where the LCB value corresponding to T_m is found in the hydrostatic curves. The moment to change trim, MCT, corresponding to T_m, is read from the hydrostatic curves and the trim is calculated as shown in Section 7.3. If the trim is small one can go to the next step, otherwise it is advisable to continue the calculations using the Bonjean curves or to resort to a computer programme.
3. The height of the metacentre above BL, $\overline{KM}$, corresponding to T_m, is read in the hydrostatic curves.
4. The metacentric height is calculated as

$$\overline{GM} = \overline{KM} - \overline{KG}$$

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- Displacement = 2625 t. Volume = 2625/1.025 = 2561 m^3. From the table below, draft is approx. 4.32 m.
- Next, do Step 2 on the previous slide. For details, see the next slide.

Table 6.6 Small cargo ship—partial hydrostatic data $L_{PP} = 75.40\ m$

Draught, T (m)	∇ (m^3)	$\overline{KM}$ (m)	Draught, T (m)	∇ (m^3)	$\overline{KM}$ (m)
2.00	993	6.75	4.32	2549	5.16
2.20	1118	6.39	4.40	2609	5.16
2.40	1243	6.09	4.60	2757	5.16
2.60	1377	5.83	4.80	2901	5.17
2.80	1504	5.63	5.00	3057	5.18
3.00	1640	5.48	5.20	3210	5.20
3.20	1776	5.37	5.40	3352	5.23
3.40	1907	5.28	5.60	3507	5.27
3.60	2045	5.24	5.80	3653	5.31
3.80	2189	5.20	5.96	3786	5.34
4.00	2322	5.18	6.00	3811	5.36
4.20	2471	5.17	6.20	3972	5.42

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4.5 Trim of a loaded ship. Biran 7.3

- Step 2. LCG is known from L22S12. LCG = 35.88 m. LCB is found from the table below. LCB = 0.291 m forward of midship = 75.40/2 + 0.291 = 37.991 m
- LCG ≠ LCB. So, the ship will trim.

Table 7.3 Small cargo ship—partial hydrostatic data, 2

Draught, *T* (m)	*MCT* (~~m~~)(t)	*LCB* from Midship (m)	*LCF* from Midship (m)	Draught, *T* (m)	*MCT* (~~m~~)(t)	*LCB* from Midship (m)	*LCF* from Midship (m)
2.00	2206	0.607	0.518	4.32	3223	0.291	−0.384
2.20	2296	0.600	0.460	4.40	3260	0.272	−0.430
2.40	2382	0.590	0.398	4.60	3336	0.225	−0.560
2.60	2470	0.575	0.330	4.80	3413	0.180	−0.698
2.80	2563	0.557	0.260	5.00	3485	0.131	−0.839
3.00	2645	0.537	0.190	5.20	3567	0.083	−0.960
3.20	2732	0.510	0.119	5.40	3639	0.033	−1.066
3.40	2824	0.480	0.041	5.60	3716	−0.018	−1.158
3.60	2906	0.442	−0.035	5.80	3793	−0.067	−1.231
3.80	2293	0.406	−0.017	5.96	3863	−0.108	−1.281
4.00	3085	0.360	−0.210	6.00	3880	−0.118	−1.293
4.20	3167	0.319	−0.314	6.20	3951	−0.167	−1.348

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4.5 Trim of a loaded ship

2. The trimming moment is calculated as

$$M_{\text{trim}} = \Delta(LCG - LCB) \tag{7.1}$$

where the *LCB* value corresponding to T_m is found in the hydrostatic curves. The moment to change trim, *MCT*, corresponding to T_m, is read from the hydrostatic curves and the trim is calculated as shown in Section 7.3. If the trim is small one can go to the next step, otherwise it is advisable to continue the calculations using the Bonjean curves or to resort to a computer programme.

- The CoG and the CoB should lie on the same vertical that is perpendicular to the waterline. If they don't, a moment occurs. Find the moment that acts by using the approximate Eq. (7.1). LCB is based on uniform draft. If the moment is non-zero, the LCB will change and a trim will occur.
- To find the trim, find MCT by using the table on the previous slide. MCT = 3223 t.

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4.5 Trim of a loaded ship

- Find the Trim and Trim Angle by using Eqs. (7.3) and (7.4)
- Trim = 2625 (35.88 – 37.99) / 3223 = -1.72 m

displacement. For that draught we read the moment to change trim, MCT. We calculate the trim, in m, as

$$\text{trim} = T_F - T_A = \frac{\Delta(LCG - LCB)}{MCT} \tag{7.3}$$

The trim angle is given by

$$\tan\theta = \frac{T_F - T_A}{L_{pp}} \tag{7.4}$$

perpendiculars is $L_{pp} = 75.40$ m. In the table LCB is measured from midship, positive forwards. As LCG is measured from AP, we calculate

$$75.40/2 + 0.291 = 37.99 \text{ m}$$

and the trim

$$\frac{\Delta(LCG - LCB)}{MCT} = \frac{2625(35.88 - 37.99)}{3223} = -1.72 \text{ m}$$

The ship is trimmed by the stern. In Table 7.3 LCF is measured from the midship, positive

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4.5 Trim of a loaded ship

- Do Step 3 on L22S11
- Find $\overline{KM_L}$ at the known draft using the hydrostatic data. Note that $\overline{KM_T}$ is shown on L22S14.
- Do Step 4 on L22S11. See L22S12 for $\overline{KG}$. It is 5 m.
- Find the metacentric height. Find $\overline{GM_L} = \overline{KM_L} - \overline{KG} = \overline{KM_L}$ - 5

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4.5 Trim of a loaded ship.

- Find the drafts at the forward and aft perpendiculars – given all the loading data and the hydrostatic info
- Trim has been found. Find the Trim Angle.
- Note that for $\theta \cong 0$, $\tan\theta \cong \theta$ where θ is in radians.

7.3 Trim

7.3.1 Finding the Trim and the Draughts at Perpendiculars

In Figure 7.2 we consider a ship initially on even keel; the corresponding waterline is W_0L_o. Let us assume that the ship trims reaching a new waterline, $W_\theta L_\theta$. If the trim angle, θ, is small (for normal loading conditions it is always small), the intersection line of the two waterlines, W_0L_o and $W_\theta L_\theta$, passes through the centre of flotation, F, of the initial waterplane. The midship draught of the ship on even keel, T_m, can be read in the hydrostatic curves at the intersection of the displacement curve and the vertical corresponding to the given displacement. For that draught we read the moment to change trim, MCT. We calculate the trim, in m, as

$$\text{trim} = T_F - T_A = \frac{\Delta(LCG - LCB)}{MCT} \quad (7.3)$$

The trim angle is given by

$$\tan\theta = \frac{T_F - T_A}{L_{pp}} \quad (7.4)$$

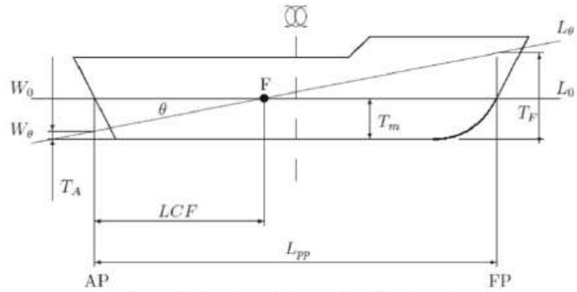

Figure 7.2 Finding the forward and aft draughts

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4.5 Trim of a loaded ship.

- Here, T_m = draft on even keel at all points including CoF
- See the numerical example where the draft at the CoF is found and used.
- Avoid approximations if the exact value can be found without much additional effort.

As the ship trims around a transversal axis passing through the centre of flotation F, we consider this point as fixed during trim and we assume that the draught of this point has the value T_m found above. From Figure 7.2 we see that

$$T_A = T_m - LCF \cdot \tan\theta = T_m - LCF \cdot \frac{\text{trim}}{L_{pp}} \quad (7.5)$$

For even keel $T_m = T_{LCF} = T_F$

and

$$T_F = \text{trim} + T_A = T_m + \text{trim}\left(1 - \frac{LCF}{L_{pp}}\right) \quad (7.6)$$

To give an example we consider again the loading case of the small cargo ship analysed in Section 7.2.2. In Tables 7.2 and 7.3 we find $T_m = 4.32$ m, $LCB = 0.291$ m, $LCF = -0.384$ m, and $MCT = 3223$ mtm^{-1}. We know that the length between perpendiculars is $L_{pp} = 75.40$ m. In the table LCB is measured from midship, positive forwards. As LCG is measured from AP, we calculate

$$75.40/2 + 0.291 = 37.99 \text{ m}$$

and the trim

$$\frac{\Delta(LCG - LCB)}{MCT} = \frac{2625(35.88 - 37.99)}{3223} = -1.72 \text{ m}$$

The ship is trimmed by the stern. In Table 7.3 LCF is measured from the midship, positive forward; the value measured from AP is

$$75.40/2 - 0.384 = 37.32 \text{ m}$$

and we calculate

$$T_A = 4.32 - 37.32\frac{-1.72}{75.4} = 5.17$$
$$T_F = -1.72 + 5.17 = 3.45$$

where the results are in m.

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Exercise

- Sketch a ship with length Lpp and a trim.
- The draft at the midship and the trim angle are known.
- The trimmed waterline passes through the LCF.
- Use the sketch and derive expressions for the draft forward and draft aft.

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4.6 Change in trim and draft due to change in weight

- Addition or removal of weight at some point on the ship

- Next Slide. Recall: TP1cm
- Then, Recall: What happens when a mass is moved
- Then, Numerical Example

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4.2.3 Derived Data

TPC. Addition or removal of weight

Let us suppose that we know the displacement, Δ_0, corresponding to a given draught, T_0, and we want to find by how many tonnes that displacement will change if the draught changes by δT, centimetres. Let the waterplane area be A_W m^2, and the water density, ρ_W t m^{-3}. For a small draught change we may neglect the slope of the shell (in other words we assume a wall-sided hull) and we write

$$\delta\Delta = \rho_W A_W \delta T$$

It is best to use SI units

If we measure Δ in tonnes, and δT in centimetres, we obtain

$$\delta\Delta = \rho_W A_w \frac{\delta T}{100} \qquad \cancel{\delta\Delta = \rho_W \frac{A_W}{\delta T}} \tag{4.11}$$

We call the quantity $\rho_W \frac{A_W}{100}$ **tonnes per centimetre immersion**, where, as explained previously, the *tonne* is a unit of mass, and use for it the notation **TPC**. In older, English-language books we find the notation $TP1$ as an acronym for **tonnes per inch** where the *ton* is a unit of weight. This quantity is calculated from an expression similar to Eq. (4.11), but adapted for English and American units. For SI units

$$TPC = \frac{A_W}{100} \times \rho_W \tag{4.12}$$

where ρ_W should be taken from the Appendix of Chapter 2. The problem posed above can be inverted: find the change in draught, δT, corresponding to a change of displacement, $\delta\Delta$. The obvious answer is

$$\delta T = \frac{\delta\Delta}{TPC}$$

The above calculations yield good approximations as long as the changes $\delta\Delta$, δT are small. In fact, Eq. (4.11) is a linearization of the relationship between displacement volume and waterplane area.

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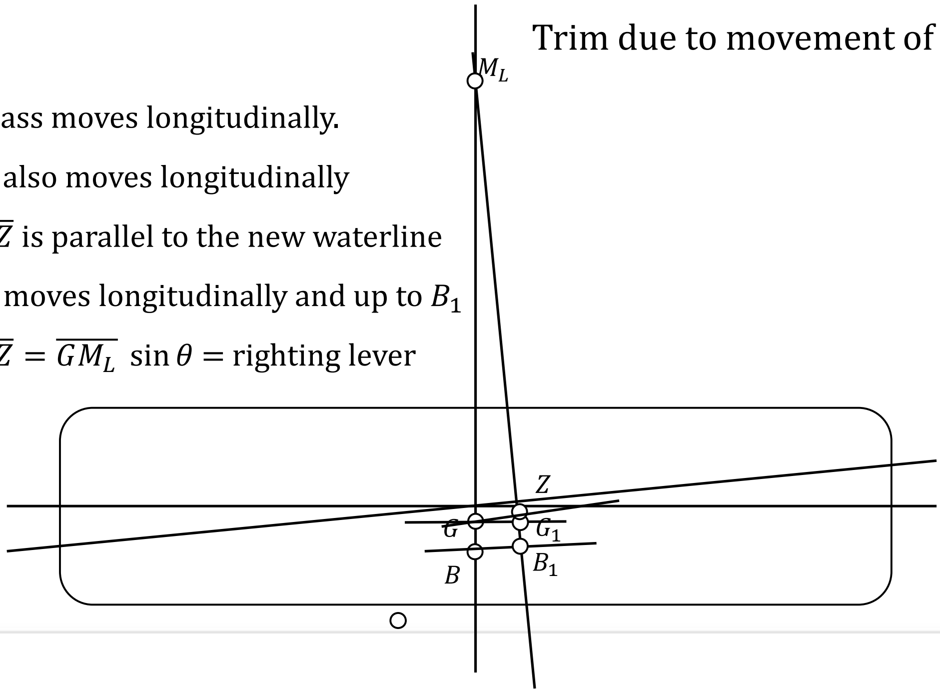
Trim due to movement of mass

- Mass moves longitudinally.
- G also moves longitudinally
- $\overline{GZ}$ is parallel to the new waterline
- B moves longitudinally and up to B_1
- $\overline{GZ} = \overline{GM_L}\ \sin\theta =$ righting lever



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- If an on-board mass is moved slowly to a new position, CoG and CoB will move and be on the same perpendicular to the waterline.
- But, will $LCG = LCB$? Generally, no, because VCG ≠ VCB
- Sketch a cuboidal barge. Show the upright waterline and an inclined waterline. KG is not equal to KB for the upright attitude. Show CoG and CoB for both the attitudes.
- Is $LCG = LCB$ in the trimmed condition? No.

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4.6 Change in trim and draft due to change in weight.
Tutorial 01. Loads and Trim.

- A cargo vessel has $L_{pp} = 100$ m, LCB at amidships, TP1cm = 20 t/cm, MCT1cm = 800 t.m/cm, LCF = 1 m aft of amidships, Draft fwd = 7.9 m and Draft aft = 8.1 m. The draft is to be changed to 8.4 m even keel. Where should masses be added or removed to achieve this?
- The origin is at the aft perpendicular. LCF = 49 m. The original trim = $\mathbb{T}_0 = T_F - T_A$ = -0.2 m. The original draft at the LCF = $T_{LCF} = T_A + \mathbb{T}_0$ LCF$/L_{pp}$ = 8.1 – 0.2(49)/100 = 8.002 m.
- Step 1. Find the effect of adding mass m_1(in tonne) at x_1. Step 1a. Find the effect of adding mass m_1 at the LCF. Step 1b. Find the effect of moving it to x_1.
- Step 1a. Parallel sinkage occurs when mass m_1 in tonnes is added at the LCF.
- Parallel Sinkage = $\Delta T = m_1$/ (100TP1cm) = m_1/2e3 where 100TP1cm = TP1m.

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4.6 Change in trim and draft due to change in weight.
Tutorial 01. Loads and Trim.

- Step 1b.
- Moment = $M = m_1(x_1 - \text{LCF})$. Change in trim = $\Delta\mathbb{T} = m_1(x_1 - \text{LCF})/(100\text{ MCT1cm})$ where 100 MCT1cm = MCT1m. Let the change in trim be $-\mathbb{T}_0$ = 0.2 m. Then, $m_1(x_1 - \text{LCF})$ = 0.2*100*800 tm = 1.6e4 tm. The change in the draft at the LCF = 0 m. After the moment is applied, the ship should be on even keel.
- Final draft at the LCF = Original draft at LCF + change in the draft at LCF due to parallel sinkage = 8.002 m + $m_1/2e3$. This should be equal to the desired final draft of 8.4 m. Therefore, 8.002 m + $m_1/2e3$ = 8.4. So, m_1 = (8.4-8.002)*2e3 = 796 tonnes.
- As m_1 = 796 t and $m_1(x_1 - \text{LCF})$= 1.6e4 tm, $(x_1 - \text{LCF}) = \frac{1.6e4}{796} = 20.1$ m.
- Add 796 tonnes at 20.1 m forward of the LCF to achieve even keel draft of 8.4 m.

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4.7 Rules on draft and trim

- Biran

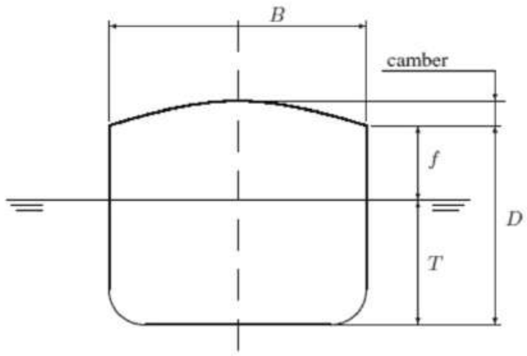

Figure 1.4 Breadth, depth, draught, and camber

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4.7 Rules on draft and trim

- Biran. Designed summer load line & other definitions

The **baseline**, shortly BL, is a line lying in the longitudinal plane of symmetry and parallel to the designed summer load waterline (see next paragraph for a definition). It appears as a horizontal in the lateral view of the hull surface. The baseline is used as the longitudinal axis, that is the x-axis of the system of coordinates in which hull points are defined. Therefore, it is recommended to place this line so that it passes through the lowest point of the hull surface. Then, all z-coordinates will be positive.

Before defining the dimensions of a ship we must choose a reference waterline. ISO 7462 recommends that this **load waterline** be the **designed summer load line**, that is the waterline up to which the ship can be loaded, in sea water, during summer when waves are lower than in winter. The qualifier "designed" means that this line was established in some design stage. In later design stages, or during operation, the load line may change. It would be very inconvenient to update this reference and change dimensions and coordinates; therefore, the "designed" datum line is kept even if no more exact. A notation older than ISO 7462 is DWL, an abbreviation for "Design Waterline."

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Definitions. Pay attention.

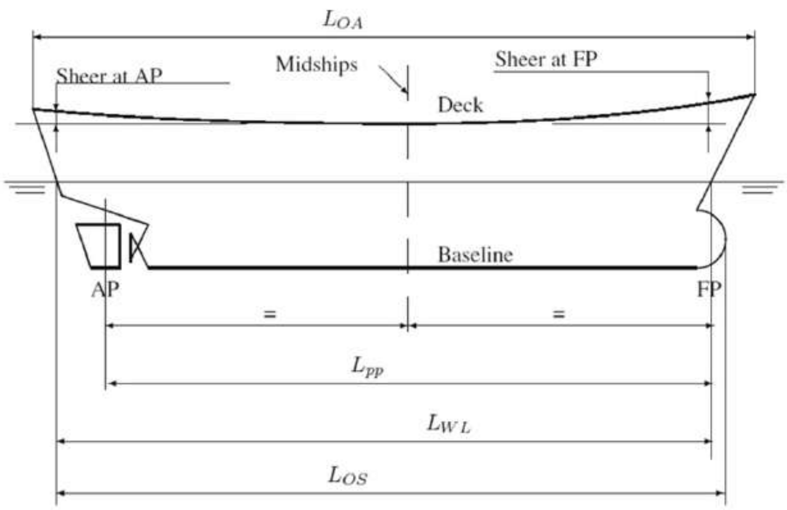

Figure 1.1 Length dimensions

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- At FP, steel is included.
- At AP, steel is not included.

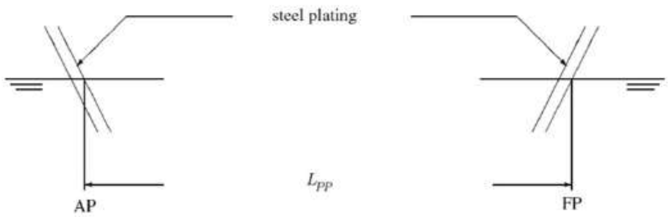

Figure 1.2 How to measure the length between perpendiculars

Definitions, Principal Dimensions 7

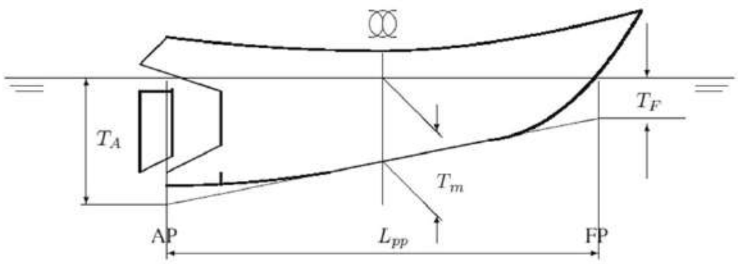

Figure 1.3 The case of a keel not parallel to the load line

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The **after perpendicular**, or **aft perpendicular**, noted AP, is a line drawn perpendicularly to the load line through the after side of the rudder post or through the axis of the rudder stock. The latter case is shown in Figures 1.1 and 1.3. For naval vessels, and today for some merchant ships, it is usual to place the AP at the intersection of the aftermost part of the moulded surface and the load line, as shown in Figure 1.2. The **forward perpendicular**, FP, is drawn perpendicularly to the load line through the intersection of the foreside of the stem with the load waterline. Mind the slight lack of consistency: while all moulded dimensions are measured to the moulded surface, the FP is drawn on the outer side of the stern. The distance between the after and the forward perpendicular, measured parallel to the load line, is called **length between perpendiculars** and its notation is L_{pp}. An older notation was LBP. We call **length overall**, L_{OA}, the length between the ship extremities. The **length overall submerged**, L_{OS}, is the maximum length of the submerged hull measured parallel to the designed load line.

We call **station** a point on the baseline, and the transverse section of the hull surface passing through that point. The station placed at half L_{pp} is called **midships**. It is usual to note the midship section by means of the symbol shown in Figure 1.5(a). In German literature we usually find the simplified form shown in Figure 1.5(b).

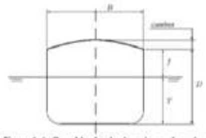

Figure 1.4 Breadth, depth, draught, and camber

The **moulded depth**, D, is the height above baseline of the intersection of the underside of the deck plate with the ship side (see Figure 1.4). When there are several decks, it is necessary to specify to which one refers the depth.

The **moulded draught**, T, is the vertical distance between the top of the keel to the designed summer load line, usually measured in the midships plane (see Figure 1.4). There may be appendages protruding below the keel, for example the sonar dome of a warship. Then, it is necessary to define an **extreme draught** that is the distance between the lowest point of the hull or of an appendage and the designed load line.

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4.7 Rules on draft and trim

Certain ships are designed with a keel that is not parallel to the load line. Some tugs and fishing vessels display this feature. To define the draughts associated with such a situation let us refer to Figure 1.3. We draw an auxiliary line that extends the keel afterward and forwards. The distance between the intersection of this auxiliary line with the aft perpendicular and the load line is called **aft draught** and is noted with T_A. Similarly, the distance between the load line and the intersection of the auxiliary line with the forward perpendicular is called **forward draught** and is noted with T_F. Then, the draught measured in the midship section is known as

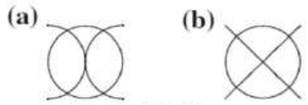

Figure 1.5 (a) Midships symbol in English literature and (b) midships symbol in German literature

midships draught and its symbol is T_M. The difference between depth and draught is called **freeboard**; in DIN 81209-1 it is noted by f.

The breadth of the waterplane in the midships section is called **moulded breadth** or **moulded beam** and we note it by B. The maximum breadth may occur in another section; for fast ships usually aft of midships. Also, the deck breadth may be larger than the moulded breadth.

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4.7 Rules on draft and trim

- Biran

The intact stability of new vessels of less than 15 m length that carry a combined load of passengers and cargo of less than 1000 kg is checked in an inclining experiment. The passengers, the crew without the skipper, and the cargo are transferred to one side of the ship, while the skipper may be assumed to stay at the steering position. Under these conditions the angle of heel shall not exceed 7°. For vessels with a watertight weather deck the freeboard shall be not less than 75 mm at any point. For open boats the freeboard to the top of the gunwale shall not be less than 250 mm at any point.

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4.7 Rules on draft and trim

- The Trim and Stability Book. Barrass. 6th Ed.

When a new ship is nearing completion, a Trim and Stability book is produced by the shipbuilder and presented to the shipowner. Shipboard officers will use this for the day to day operation of the vessel. In the Trim and Stability book is the following technical data:

1. General particulars of the ship and General Arrangement Plan.
2. Inclining experiment report and its results.
3. Capacity, VCG, LCG particulars for all holds, compartments, tanks etc.
4. Cross curves of stability. These may be GZ curves or KN curves.
5. Deadweight scale data. May be in diagram form or in tabular form.
6. Hydrostatic curves. May be in graphical form or in tabular form.
7. Example conditions of loading such as:

   Lightweight (empty vessel) condition.
   Full-loaded departure and arrival conditions.
   Heavy-ballast departure and arrival conditions.
   Medium-ballast departure and arrival conditions.
   Light-ballast departure and arrival conditions.

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Stringer and other Strakes

- A strake is the name given to each line of planking in a wooden vessel.[1] In modern ship construction it refers to the longitudinal run of plating covering the hull, deck and bulkhead structure. Certain specific strakes are uniquely identified:
- Keel: is a special strake of the Bottom plating extending from the centerplane outboard.
- Bottom: the Bottom Shell plate strakes extend from the Keel to the Bilge.
- Bilge: is the plating which transitions from the more-or-less horizontal Bottom Shell to the more-or-less vertical Side Shell and is generally curved. See also Chine (boating).
- Side: is the plating which extends from the Bilge strake(s) to the Shear strake.
- Shear: is a special strake of the Side plating. It is the strake that connects the Side Shell to the Strength Deck.
- Stringer: is a special strake of the Strength Deck plating. It is the strake that connects the Strength Deck to the Side Shell.
- Strength Deck: is a special deck. It is normally the uppermost continuous deck and forms the top flange of the hull girder.

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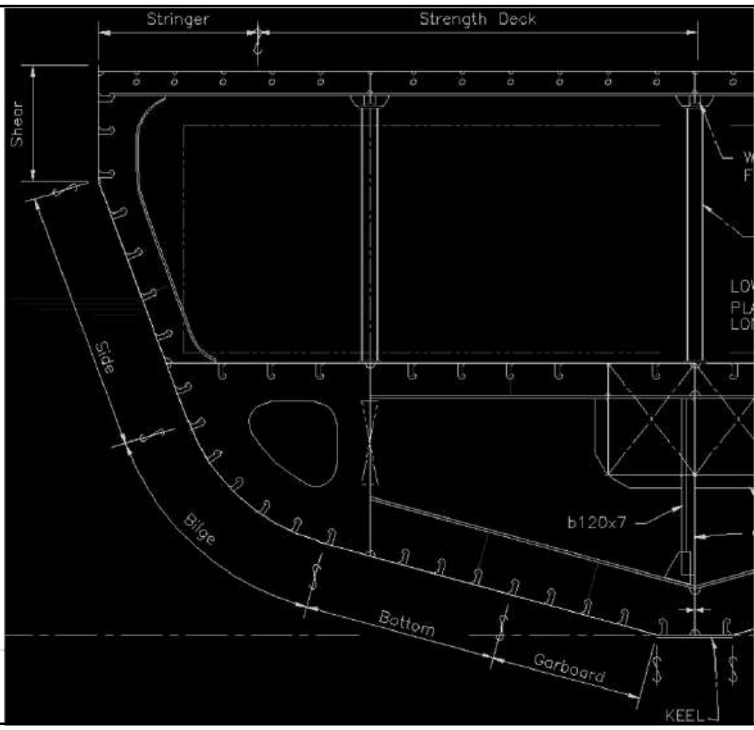
Stringer and other Strakes

https://en.wikipedia.org/wiki/Shell_plating



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4.7 Rules on draft and trim

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26.1.6 Depth for freeboard (D)

This is the moulded depth amidships, plus the thickness of the freeboard deck stringer plate, where fitted, plus $\frac{T(L-S)}{L}$ if the exposed freeboard deck is sheathed, where:

T is the mean thickness of the exposed sheathing clear of deck openings, and
S is the total length of superstructures.

The depth for freeboard (D) in a ship having a rounded gunwale with a radius greater than 4% of the breadth (B) or having topsides of unusual form is the depth for freeboard of a ship having a midship section with vertical topsides and with the same round of beam and area of topside section equal to that provided by the actual midship section.

26.1.8 Freeboard

The freeboard assigned is the distance measured vertically downwards amidships from the *upper edge* of the deck line to the *upper edge* of the related load line.

CLASS 2/1 STABILITY - SECTION 26 – Calculation and assignment of freeboard 347

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4.7 Rules on draft and trim

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26.1.9 Freeboard Deck

This is normally the uppermost continuous deck exposed to weather and sea, which has permanent means of closing all openings in the weather part thereof, and below which all openings in the sides of the ship are fitted with permanent means of watertight closing (figure 26.1). In a ship having a discontinuous freeboard deck, the lowest line of the exposed deck and the continuation of that line parallel to the upper part of the deck is taken as the freeboard deck (figure 26.2).

The owner may opt to designate a lower deck as the freeboard deck provided that it is a complete and permanent deck in a fore and aft direction at least between the machinery space and peak bulkheads and continuous athwartships (this is typical for a Ro-Ro vessel). In such cases that part of the hull that extends above the freeboard deck may be treated as superstructure for the purposes of calculation of freeboard (figure 26.2).

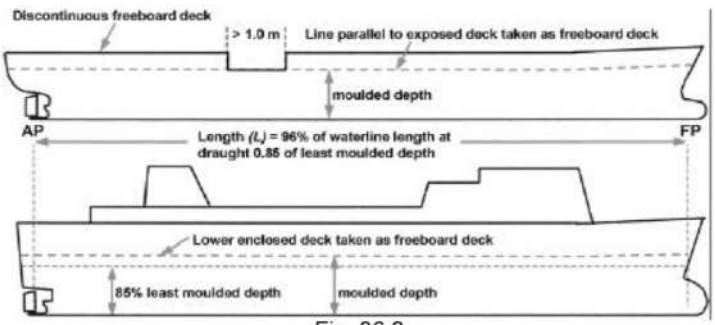

Fig. 26.2

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4.7 Trim and Stability Book

- The Trim and Stability Book. Barrass. 6th Ed.

For the arrival conditions, a ship should arrive at the end of the voyage (with cargo and/or passengers as per loaded departure conditions) with at least "10% stores and fuel remaining".

A mass of 75 kg should be assured for each passenger; but may be reduced to not less than 60 kg where this can be justified.

On each condition of loading there is a profile and plan view (at upper deck level usually). A colour scheme is adopted for each item of deadweight. Examples could be red for cargo, blue for fresh water, green for water ballast, brown for oil. Hatched lines for this Dwt distribution signify wing tanks P and S.

For each loaded condition, in the interests of safety, it is necessary to show:

Deadweight.
End draughts, thereby signifying a satisfactory and safe trim situation.

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4.7 Trim and Stability Book

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KG with no free surface effects (FSE), and (KG) with FSE taken into account.
Final transverse metacentric height (GM). This informs the officer if the ship is in stable, unstable or neutral equilibrium. It can also indicate if the ship's stability is approaching a dangerous state.
Total free surface effects of all slack tanks in this condition of loading.

A statical stability curve relevant to the actual loaded condition with the important characteristics clearly indicated. For each S/S curve it is important to observe the following:

Maximum GZ and the angle of heel at which it occurs.
Range of stability.
Area enclosed from zero degrees to thirty degrees (A1) and the area enclosed from thirty degrees to forty degrees (A2) as shown in Figure 46.1.

Next slide

Shear force and bending moment curves, with upper limit lines clearly superimposed as shown in Figure 46.2.

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4.7 Rules on draft and trim

- The Trim and Stability Book. Barrass. 6th Ed.

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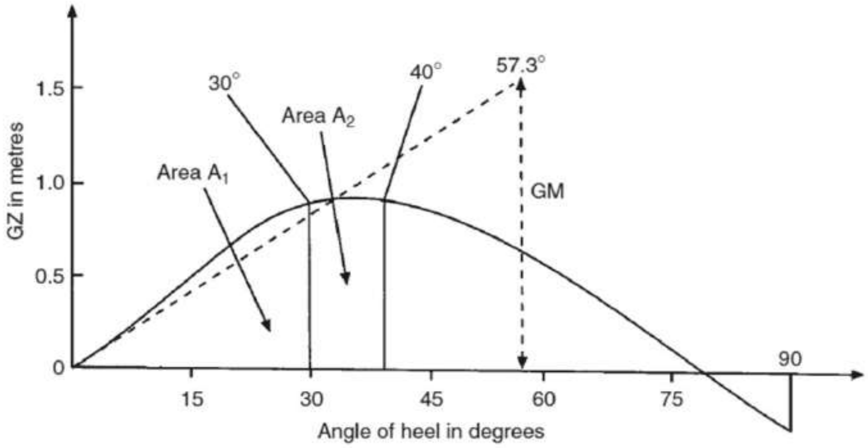

Fig. 46.1 Enclosed areas on a statical stability curve.

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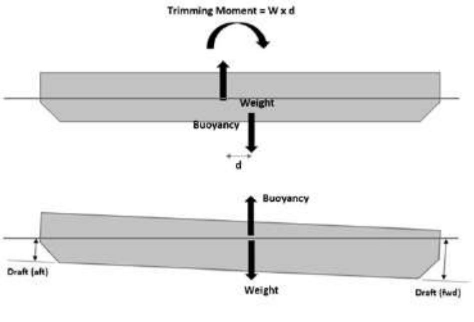
4.7 Rules on draft and trim

- TheNavalArch.com



- **We can see that in the final condition the draft at fwd is more than the draft at the aft.**
- **The trim (in m) is given by the difference in the drafts fwd and aft. In the above case, it will be 'trim by fwd'.**
- **In degrees, the trim is given by**

Trim (degrees) = $\tan^{-1}$[(draft fwd – draft aft)/Length of ship]

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4.7 Rules on draft and trim

- Excessive trim by stern will cause:
  - damage to the bottom plate of bow by waves;
  - enlarged bridge blind area;
  - ship maneuverability deterioration; and
  - ship speed ability deterioration.

- Excessive trim by bow will cause:
  - easy shipping of water on fore deck;
  - easy exposure to the water surface of propeller blades and rudder
  - plates, resulting in propeller driving or racing as ship is pitching and heaving;
  - ship maneuverability deterioration; and
  - ship speed ability deterioration.

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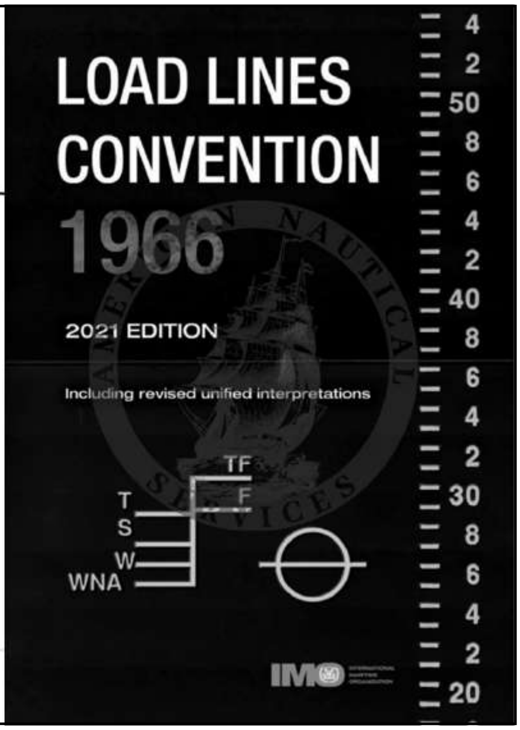
Load Lines Convention 2021

INTERNATIONAL ASSOCIATION OF CLASSIFICATION SOCIETIES

Interpretations of the

International Convention on Load Lines, 1966



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Intl Convention Load Lines

Chapter III - Freeboards

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TheNavalArch.com

- Regulation 27. Types of Ships
- The approach to freeboard calculation (as laid out in ICLL 66) is to first calculate the standard freeboard that is attributable to the vessel. The ICLL 66 provides standard tables for the minimum freeboard needed, depending on the length and type of ship. There are two types of ships recognized:
- Type A – tankers and other liquid carriers with high integrity of the main deck
- Type B – those which are not Type A

Tabular Freeboard (Regulation 28 of ICLL)

LL18
(1968)
(Rev.1
July 2008)

Freeboard tables (Regulation 28)

(a) Type A ships

(i) Freeboards for Type A ships with lengths between 365 m and 400 m shall be determined by the following formula:

$$f = 221 + 16{,}10L - 0{,}02L^2$$

where f is the freeboard in mm
L is the length as defined in Regulation 3(1).

(ii) Freeboards for Type A ships with lengths of 400 m and above shall be the constant value, 3460 mm.

(b) Type B ships

(i) Freeboards for Type B ships with lengths between 365 m and 400 m shall be determined by the following formula:

$$f = -587 + 23L - 0{,}0188L^2$$

where f is the freeboard in mm
L is the length as defined in Regulation 3(1).

(ii) Freeboards for Type B ships with lengths of 400 m and above shall be the constant value, 5605 mm.

https://thenavalarch.com/the-why-and-how-of-freeboard-calculation-of-a-ship/

- TheNavalArch.com

Table A Freeboard table for Type «A» ships

Length of ship (m)	*Freeboard (mm)*
24	200
25	208
26	217
27	225
28	233
29	242
30	250
31	258
32	267
33	275
34	283
35	292
36	300
37	308
38	316
39	325
40	334
41	344
42	354
43	364
44	374

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https://thenavalarch.com/the-why-and-how-of-freeboard-calculation-of-a-ship/

Table B Freeboard table for Type «B» ships

Length of ship (m)	*Freeboard (mm)*
24	200
25	208
26	217
27	225
28	233
29	242
30	250
31	258
32	267
33	275
34	283
35	292
36	300
37	308
38	316
39	325

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Freeboard Calculations

- Freeboard Calculation – considerations
- The calculation of freeboard is carried out as per the International Convention on Load Lines, 1966, as Amended by the Protocol of 1988 – Annex I – Regulations for Determining Load Lines.
- The basic purpose while assigning the right freeboard is to ensure that the vessel has sufficient reserve buoyancy.
- Size of the vessel – a bigger vessel needs higher reserve buoyancy, and thus its freeboard shall be higher than a smaller vessel

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Freeboard Calculations

- Structural integrity of the vessel – A vessel with many large hatch openings on the deck is likely to have more compromises on the 'intactness' of its structure. Comparatively, a vessel with high integrity of the main deck – vessels carrying liquid cargoes/tankers – will be more intact and will have a lower need for reserve buoyancy.
- Other factors that affect reserve buoyancy – any other factor that affects reserve buoyancy shall have to be taken into account
- If there is superstructure that can contribute to reserve buoyancy, then the freeboard can be reduced
- If there's additional sheer in the vessel, then the reserve buoyancy goes up and freeboard can be reduced

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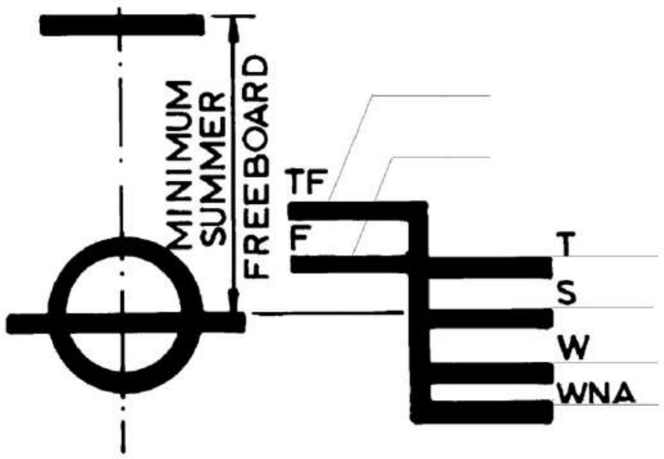
https://thenavalarch.com/the-why-and-how-of-freeboard-calculation-of-a-ship/



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https://thenavalarch.com/the-why-and-how-of-freeboard-calculation-of-a-ship/

- From the above process, the Summer freeboard of the vessel can be calculated. The summer freeboard is used to calculate the summer loadline of the vessel. The Summer Loadline is marked on the vessel as a 'Loadline Mark'. It is indicated by the upper edge of the line which passes through the centre of the ring and also by a line marked S.
- Freeboard first. Loadline next.

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https://thenavalarch.com/the-why-and-how-of-freeboard-calculation-of-a-ship/

There are multiple other loadlines of the vessel, depending on its area of operation. For sailing ships, only the Fresh Water and Winter North Atlantic loadlines are to be shown, apart from Summer Loadline. These additional lines are:

- The Winter Load Line indicated by the upper edge of a line marked W.
- The Winter North Atlantic Load Line indicated by the upper edge of a line marked WNA.
- The Tropical Load Line indicated by the upper edge of a line marked T.
- The Fresh Water Load Line in summer indicated by the upper edge of a line marked F. The Fresh Water Load Line in summer is marked abaft the vertical line. The difference between the Fresh Water Load Line in summer and the Summer Load Line is the allowance to be made for loading in fresh water at the other load lines.
- The Tropical Fresh Water Load Line indicated by the upper edge of a line marked TF, and marked abaft the vertical line.

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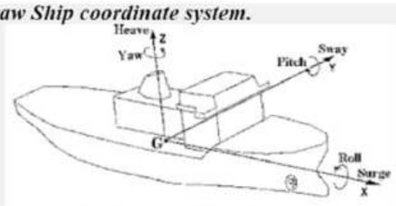
General Knowledge!

- Module 4 is completed

5. Draw Ship coordinate system.



6. Name the six degrees of freedom motions.

- *Surge*
- *Sway*
- *Heave*
- *Roll*
- *Pitch*
- *Yaw*

7. Name the translational motions of the ship.

The translational motions of the ship are Surge, Sway, Heave

8. Name the rotational motion of the ship.

The rotational motion of the ship are Roll , Pitch , Yaw

9. Name the inertial motion of the ship.

The inertial motion of the ship are Roll, Pitch, Heave

10. Name the non-inertial motion of the ship.

The non-inertial motion of the ship are Surge, Sway, Yaw.

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Module 4 is completed

Read books, magazines, and journals

and become outstanding Naval Architects

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05 Mar 2025

Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
   Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
   Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
   Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
   Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
   Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
   Intact stability rules, Heel/ Load test.
   Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

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   removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.
5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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Longitudinal Stability

Completed

- 4.1 Trim and Pitch
- 4.2 Longitudinal Metacenter
- 4.3 Longitudinal Center of Floatation
- 4.4 Moment to Change Trim (by 1 cm)
- 4.5 Trim of a loaded ship

Today

- 4.6 Change in draft and trim due to change in weight
- 4.7 Rules on draft and trim

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4.6 Change in trim and draft due to change in weight

- Addition or removal of weight at some point on the ship
- Next Slide. Recall: TP1cm
- Then, Recall: What happens when a mass is moved
- Then, Numerical Example

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4.2.3 Derived Data

TPC. Addition or removal of weight

Let us suppose that we know the displacement, Δ_0, corresponding to a given draught, T_0, and we want to find by how many tonnes that displacement will change if the draught changes by δT, centimetres. Let the waterplane area be A_W m^2, and the water density, ρ_W t m^{-3}. For a small draught change we may neglect the slope of the shell (in other words we assume a wall-sided hull) and we write

$$\delta\Delta = \rho_W A_W \delta T$$

It is best to use SI units

If we measure Δ in tonnes, and δT in centimetres, we obtain

$$\delta\Delta = \rho_W A_w \frac{\delta T}{100} \qquad \cancel{\delta\Delta = \rho_W \frac{A_W}{\delta T}} \qquad (4.11)$$

We call the quantity $\rho_W \frac{A_W}{100}$ **tonnes per centimetre immersion**, where, as explained previously, the *tonne* is a unit of mass, and use for it the notation **TPC**. In older, English-language books we find the notation TPI as an acronym for **tonnes per inch** where the *ton* is a unit of weight. This quantity is calculated from an expression similar to Eq. (4.11), but adapted for English and American units. For SI units

$$TPC = \frac{A_W}{100} \times \rho_W \qquad (4.12)$$

where ρ_W should be taken from the Appendix of Chapter 2. The problem posed above can be inverted: find the change in draught, δT, corresponding to a change of displacement, $\delta\Delta$. The obvious answer is

$$\delta T = \frac{\delta\Delta}{TPC}$$

The above calculations yield good approximations as long as the changes $\delta\Delta$, δT are small. In fact, Eq. (4.11) is a linearization of the relationship between displacement volume and waterplane area.

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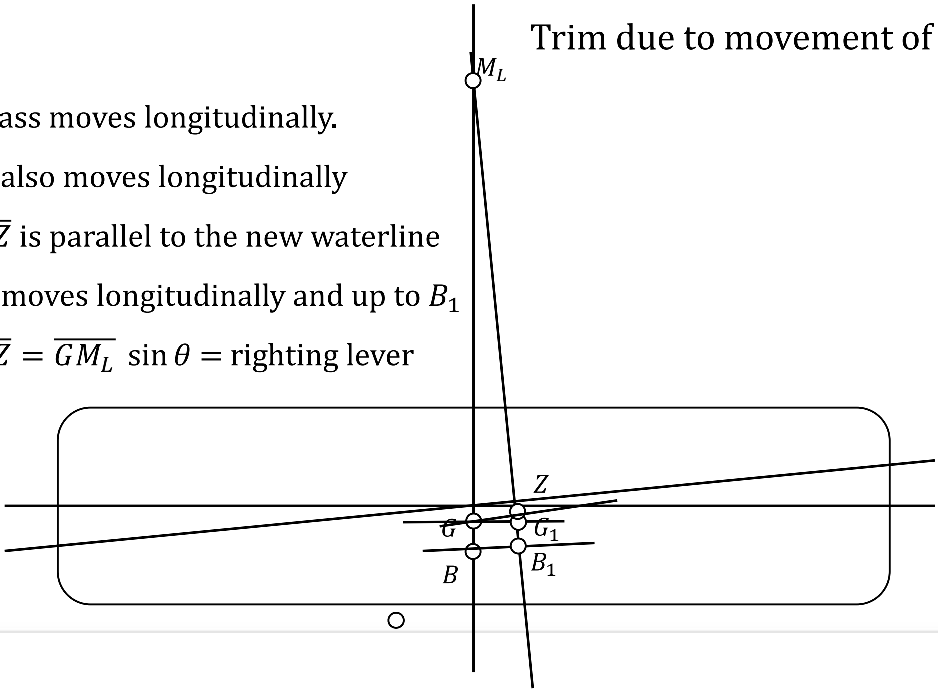
Trim due to movement of mass

- Mass moves longitudinally.
- G also moves longitudinally
- $\overline{GZ}$ is parallel to the new waterline
- B moves longitudinally and up to B_1
- $\overline{GZ} = \overline{GM_L}\ \sin\theta =$ righting lever



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Upright and Trimmed Ships

- When the ship is upright, $LCG = LCB$. CoG and CoB need not be at midship but will be on the centerline. The line through CoG and CoB is normal to the waterline and passes through the metacenter.
- If an on-board mass is moved slowly to a new position, CoG and CoB will move and be on the same perpendicular to the new waterline.
- But, will $LCG = LCB$? Not exactly because VCG ≠ VCB. Approximately, yes, for longitudinal movement of mass.
- Sketch a cuboidal barge. Show the upright waterline and an inclined waterline. KG is not equal to KB for the upright attitude. Show CoG and CoB for both the attitudes.
- Is $LCG = LCB$ when it is (a) upright (b) trimmed condition? (a) Yes (b) No

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4.6 Change in trim and draft due to change in weight. Tutorial 01. Loads and Trim.

- A cargo vessel has $L_{pp} = 100$ m, LCB at amidships, TP1cm = 12 t/cm ($A_W =$ TP1cm*100/1.025 = 1170.73 m^2) MCT1cm = 80 t.m/cm [for a cuboidal barge, MCT1m $\cong \rho LBT\left(\frac{L^2}{12T}\right)\left(\frac{1}{L}\right) = \rho\frac{L^2B}{12}$], LCF = 2 m aft of amidships, Draft fwd = 7.9 m and Draft aft = 8.1 m. The draft is to be changed to 8.4 m even keel. Where should masses be added or removed to achieve this?
- The origin is at the aft perpendicular. LCF = 48 m. The original trim = $\mathbb{T}_0 = T_F - T_A$ = -0.2 m. The original draft at the LCF = $T_{LCF} = T_A + \mathbb{T}_0$ LCF/L_{pp} = 8.1 – 0.2(48)/100 = 8.004 m.
- Step 1. Find the effect of adding mass m_1(in tonne) at x_1. Step 1a. Find the effect of adding mass m_1 at the LCF. Step 1b. Find the effect of moving it to x_1.
- Step 1a. Parallel sinkage occurs when mass m_1 in tonnes is added at the LCF.
- Parallel Sinkage = $\Delta T = m_1$/ (100TP1cm) = $m_1/1200$ where 100TP1cm = TP1m.

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4.6 Change in trim and draft due to change in weight. Tutorial 01. Loads and Trim.

- Step 1b.
- Moment = $M = m_1(x_1 - \text{LCF})$. Change in trim = $\Delta\mathbb{T} = m_1(x_1 - \text{LCF})/(100\ \text{MCT1cm})$ where 100 MCT1cm = MCT1m. Let the change in trim be $\Delta\mathbb{T} = -\mathbb{T}_0$ = 0.2 m. Then, $m_1(x_1 - \text{LCF})$ = 0.2*100*80 tm = 1600 tm. The change in the draft at the LCF = 0 m. After the moment is applied, the ship should be on even keel.
- Final draft at the LCF = Original draft at LCF + change in the draft at LCF due to parallel sinkage = 8.004 m + $m_1/1200$. This should be equal to the desired final draft of 8.4 m. Therefore, 8.004 m + $m_1/1200$ = 8.4. So, m_1 = (8.4-8.004)*1200 = 475.2 tonnes.
- As m_1 = 475.2 t and $m_1(x_1 - \text{LCF})$= 1.6e3 tm, $(x_1 - \text{LCF}) = \frac{1.6e3}{475.2}$ = 3.367 m.
- Add 475.2 tonnes at 3.367 m forward of the LCF to achieve even keel draft of 8.4 m.

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4.7 Rules on draft and trim

- Biran. Note the definition of the freeboard, f.

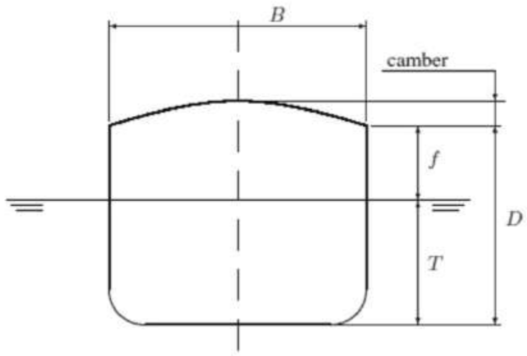

Figure 1.4 Breadth, depth, draught, and camber

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4.7 Rules on draft and trim

- Biran. Designed summer load line & other definitions

The **baseline**, shortly BL, is a line lying in the longitudinal plane of symmetry and parallel to the designed summer load waterline (see next paragraph for a definition). It appears as a horizontal in the lateral view of the hull surface. The baseline is used as the longitudinal axis, that is the x-axis of the system of coordinates in which hull points are defined. Therefore, it is recommended to place this line so that it passes through the lowest point of the hull surface. Then, all z-coordinates will be positive.

Before defining the dimensions of a ship we must choose a reference waterline. ISO 7462 recommends that this **load waterline** be the **designed summer load line**, that is the waterline up to which the ship can be loaded, in sea water, during summer when waves are lower than in winter. The qualifier "designed" means that this line was established in some design stage. In later design stages, or during operation, the load line may change. It would be very inconvenient to update this reference and change dimensions and coordinates; therefore, the "designed" datum line is kept even if no more exact. A notation older than ISO 7462 is DWL, an abbreviation for "Design Waterline."

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Definitions. Pay attention.

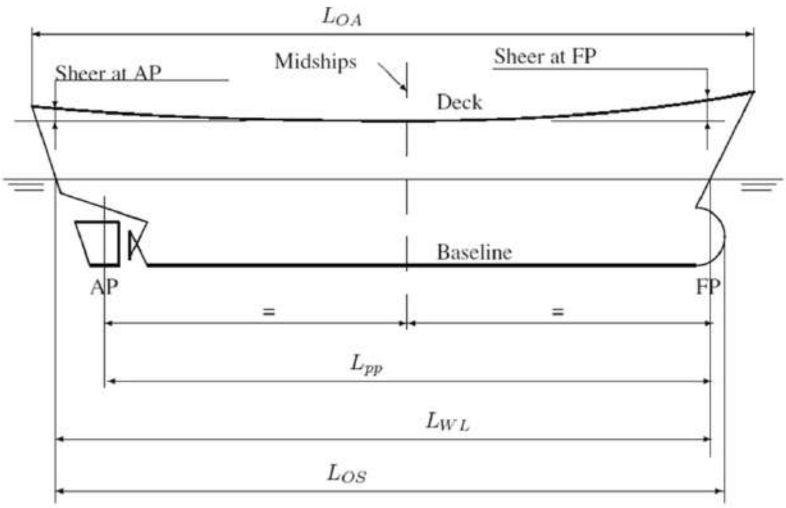

Figure 1.1 Length dimensions

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- At FP, steel is included.
- At AP, steel is not included.

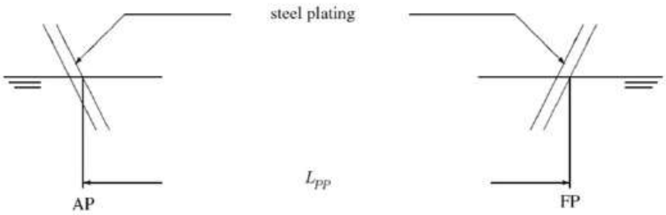

Figure 1.2 How to measure the length between perpendiculars

Definitions, Principal Dimensions 7

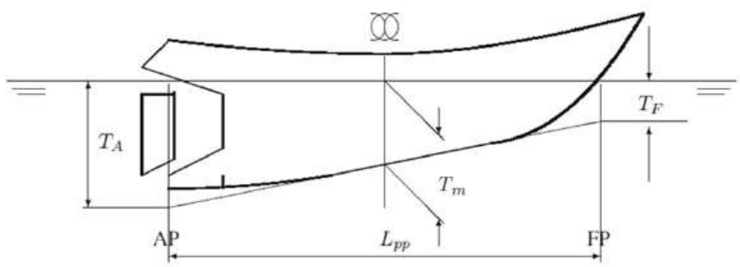

Figure 1.3 The case of a keel not parallel to the load line

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The **after perpendicular**, or **aft perpendicular**, noted AP, is a line drawn perpendicularly to the load line through the after side of the rudder post or through the axis of the rudder stock. The latter case is shown in Figures 1.1 and 1.3. For naval vessels, and today for some merchant ships, it is usual to place the AP at the intersection of the aftermost part of the moulded surface and the load line, as shown in Figure 1.2. The **forward perpendicular**, FP, is drawn perpendicularly to the load line through the intersection of the foreside of the stem with the load waterline. Mind the slight lack of consistency: while all moulded dimensions are measured to the moulded surface, the FP is drawn on the outer side of the stem. The distance between the after and the forward perpendicular, measured parallel to the load line, is called **length between perpendiculars** and its notation is L_{pp}. An older notation was LBP. We call **length overall**, L_{OA}, the length between the ship extremities. The **length overall submerged**, L_{OS}, is the maximum length of the submerged hull measured parallel to the designed load line.

We call **station** a point on the baseline, and the transverse section of the hull surface passing through that point. The station placed at half L_{pp} is called **midships**. It is usual to note the midship section by means of the symbol shown in Figure 1.5(a). In German literature we usually find the simplified form shown in Figure 1.5(b).

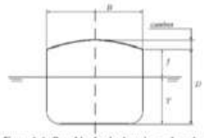

Figure 1.4 Breadth, depth, draught, and camber

The **moulded depth**, D, is the height above baseline of the intersection of the underside of the deck plate with the ship side (see Figure 1.4). When there are several decks, it is necessary to specify to which one refers the depth.

The **moulded draught**, T, is the vertical distance between the top of the keel to the designed summer load line, usually measured in the midships plane (see Figure 1.4). There may be appendages protruding below the keel, for example the sonar dome of a warship. Then, it is necessary to define an **extreme draught** that is the distance between the lowest point of the hull or of an appendage and the designed load line.

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4.7 Rules on draft and trim

Certain ships are designed with a keel that is not parallel to the load line. Some tugs and fishing vessels display this feature. To define the draughts associated with such a situation let us refer to Figure 1.3. We draw an auxiliary line that extends the keel afterward and forwards. The distance between the intersection of this auxiliary line with the aft perpendicular and the load line is called **aft draught** and is noted with T_A. Similarly, the distance between the load line and the intersection of the auxiliary line with the forward perpendicular is called **forward draught** and is noted with T_F. Then, the draught measured in the midship section is known as

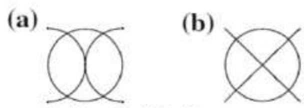

Figure 1.5 (a) Midships symbol in English literature and (b) midships symbol in German literature

midships draught and its symbol is T_M. The difference between depth and draught is called **freeboard**; in DIN 81209-1 it is noted by f.

The breadth of the waterplane in the midships section is called **moulded breadth** or **moulded beam** and we note it by B. The maximum breadth may occur in another section; for fast ships usually aft of midships. Also, the deck breadth may be larger than the moulded breadth.

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4.7 Rules on draft and trim

- Biran

The intact stability of new vessels of less than 15 m length that carry a combined load of passengers and cargo of less than 1000 kg is checked in an inclining experiment. The passengers, the crew without the skipper, and the cargo are transferred to one side of the ship, while the skipper may be assumed to stay at the steering position. Under these conditions the angle of heel shall not exceed 7°. For vessels with a watertight weather deck the freeboard shall be not less than 75 mm at any point. For open boats the freeboard to the top of the gunwale shall not be less than 250 mm at any point.

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Stringer and other Strakes

- A strake is the name given to each line of planking in a wooden vessel.[1] In modern ship construction it refers to the longitudinal run of plating covering the hull, deck and bulkhead structure. Certain specific strakes are uniquely identified:
- Keel: is a special strake of the Bottom plating extending from the centerplane outboard.
- Bottom: the Bottom Shell plate strakes extend from the Keel to the Bilge.
- Bilge: is the plating which transitions from the more-or-less horizontal Bottom Shell to the more-or-less vertical Side Shell and is generally curved. See also Chine (boating).
- Side: is the plating which extends from the Bilge strake(s) to the Shear strake.
- Shear: is a special strake of the Side plating. It is the strake that connects the Side Shell to the Strength Deck.
- Stringer: is a special strake of the Strength Deck plating. It is the strake that connects the Strength Deck to the Side Shell.
- Strength Deck: is a special deck. It is normally the uppermost continuous deck and forms the top flange of the hull girder.

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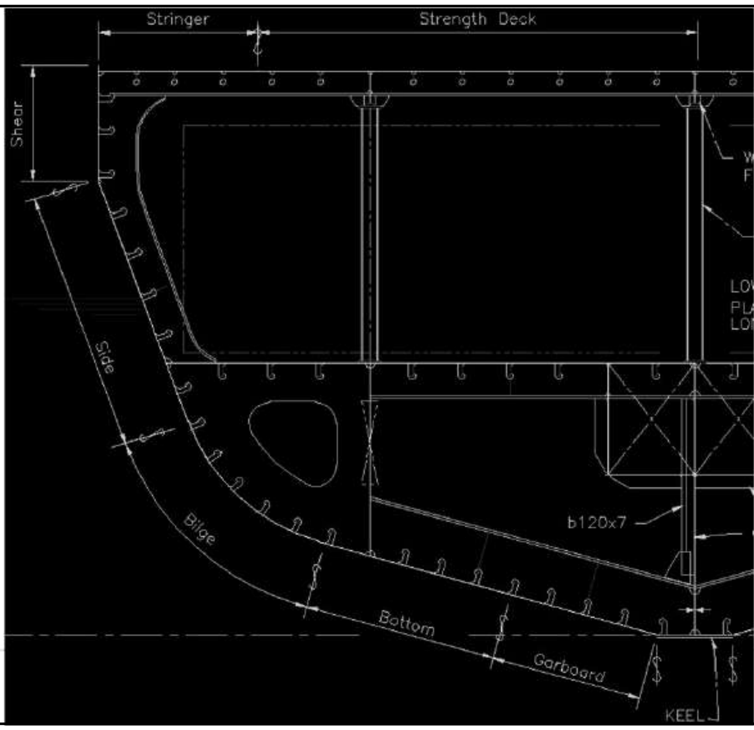
Stringer and other Strakes

https://en.wikipedia.org/wiki/Shell_plating



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4.7 Rules on draft and trim

26.1.6 Depth for freeboard (D)

This is the moulded depth amidships, plus the thickness of the freeboard deck stringer plate, where fitted, plus $\frac{T(L-S)}{L}$ if the exposed freeboard deck is sheathed, where:

T is the mean thickness of the exposed sheathing clear of deck openings, and
S is the total length of superstructures.

The depth for freeboard (D) in a ship having a rounded gunwale with a radius greater than 4% of the breadth (B) or having topsides of unusual form is the depth for freeboard of a ship having a midship section with vertical topsides and with the same round of beam and area of topside section equal to that provided by the actual midship section.

26.1.8 Freeboard

The freeboard assigned is the distance measured vertically downwards amidships from the *upper edge* of the deck line to the *upper edge* of the related load line.

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4.7 Rules on draft and trim

26.1.9 Freeboard Deck

This is normally the uppermost continuous deck exposed to weather and sea, which has permanent means of closing all openings in the weather part thereof, and below which all openings in the sides of the ship are fitted with permanent means of watertight closing (figure 26.1). In a ship having a discontinuous freeboard deck, the lowest line of the exposed deck and the continuation of that line parallel to the upper part of the deck is taken as the freeboard deck (figure 26.2).

The owner may opt to designate a lower deck as the freeboard deck provided that it is a complete and permanent deck in a fore and aft direction at least between the machinery space and peak bulkheads and continuous athwartships (this is typical for a Ro-Ro vessel). In such cases that part of the hull that extends above the freeboard deck may be treated as superstructure for the purposes of calculation of freeboard (figure 26.2).

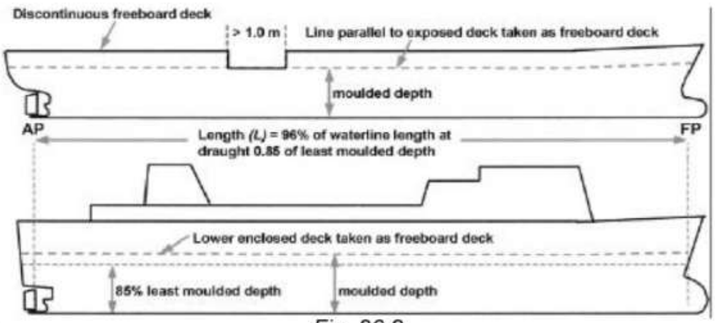

Fig. 26.2

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4.7 Rules on draft and trim

- The Trim and Stability Book. Barrass. 6th Ed.

When a new ship is nearing completion, a Trim and Stability book is produced by the shipbuilder and presented to the shipowner. Shipboard officers will use this for the day to day operation of the vessel. In the Trim and Stability book is the following technical data:

1. General particulars of the ship and General Arrangement Plan.
2. Inclining experiment report and its results.
3. Capacity, VCG, LCG particulars for all holds, compartments, tanks etc.
4. Cross curves of stability. These may be GZ curves or KN curves.
5. Deadweight scale data. May be in diagram form or in tabular form.
6. Hydrostatic curves. May be in graphical form or in tabular form.
7. Example conditions of loading such as:

   Lightweight (empty vessel) condition.
   Full-loaded departure and arrival conditions.
   Heavy-ballast departure and arrival conditions.
   Medium-ballast departure and arrival conditions.
   Light-ballast departure and arrival conditions.

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4.7 Trim and Stability Book

- The Trim and Stability Book. Barrass. 6^{th} Ed.

For the arrival conditions, a ship should arrive at the end of the voyage (with cargo and/or passengers as per loaded departure conditions) with at least "10% stores and fuel remaining".

A mass of 75 kg should be assured for each passenger; but may be reduced to not less than 60 kg where this can be justified.

On each condition of loading there is a profile and plan view (at upper deck level usually). A colour scheme is adopted for each item of deadweight. Examples could be red for cargo, blue for fresh water, green for water ballast, brown for oil. Hatched lines for this Dwt distribution signify wing tanks P and S.

For each loaded condition, in the interests of safety, it is necessary to show:

Deadweight.
End draughts, thereby signifying a satisfactory and safe trim situation.

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4.7 Trim and Stability Book

- The Trim and Stability Book. Barrass. 6^{th} Ed.

KG with no free surface effects (FSE), and (KG) with FSE taken into account.
Final transverse metacentric height (GM). This informs the officer if the ship is in stable, unstable or neutral equilibrium. It can also indicate if the ship's stability is approaching a dangerous state.
Total free surface effects of all slack tanks in this condition of loading.

A statical stability curve relevant to the actual loaded condition with the important characteristics clearly indicated. For each S/S curve it is important to observe the following:

Maximum GZ and the angle of heel at which it occurs.
Range of stability.
Area enclosed from zero degrees to thirty degrees (A1) and the area enclosed from thirty degrees to forty degrees (A2) as shown in Figure 46.1. Next slide
Shear force and bending moment curves, with upper limit lines clearly superimposed as shown in Figure 46.2.

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4.7 Rules on draft and trim

- The Trim and Stability Book. Barrass. 6th Ed.

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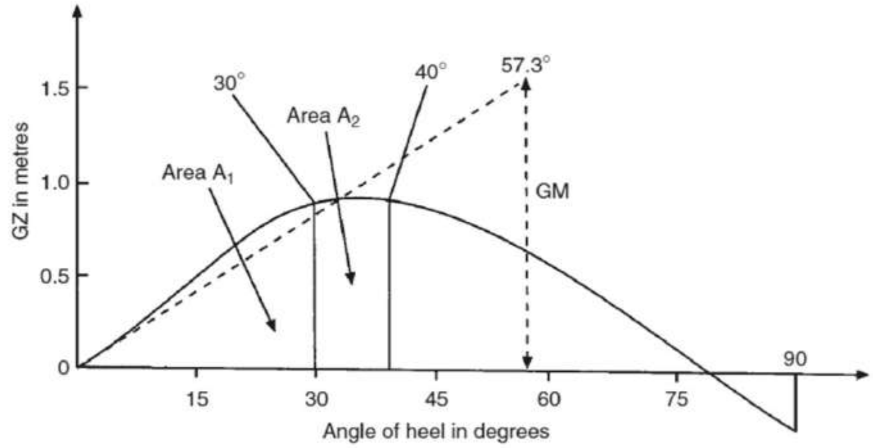

Fig. 46.1 Enclosed areas on a statical stability curve.

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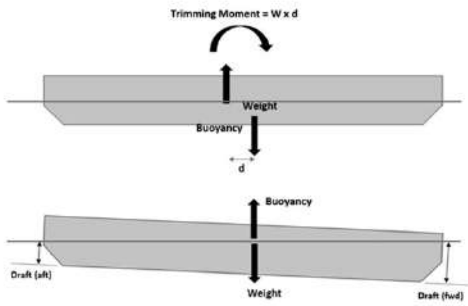
4.7 Rules on draft and trim

- TheNavalArch.com



- In the final condition the draft at fwd is more than the draft at the aft.
- The trim (in m) is given by the difference in the drafts fwd and aft. In the above case, it will be 'trim by fwd'.
- Trim Angle = $\tan^{-1}$[(draft fwd – draft aft)/Length of ship]

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4.7 Effect of excessive trim

- Excessive trim by stern will cause:
  - damage to the bottom plate of bow by waves;
  - enlarged bridge blind area;
  - ship maneuverability deterioration; and
  - ship speed ability deterioration.

- Excessive trim by bow will cause:
  - easy shipping of water on fore deck;
  - easy exposure to the water surface of propeller blades and rudder
  - plates, resulting in propeller driving or racing as ship is pitching and heaving;
  - ship maneuverability deterioration; and
  - ship speed ability deterioration.

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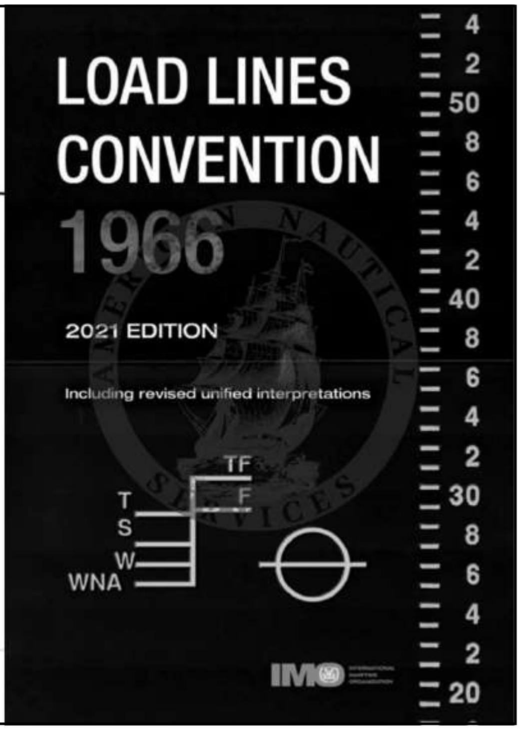
Load Lines Convention 2021

INTERNATIONAL ASSOCIATION OF CLASSIFICATION SOCIETIES

Interpretations of the

International Convention on Load Lines, 1966



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Intl Convention Load Lines

Chapter III - Freeboards

Regulation 27 - Types of ships

Regulation 28 - Freeboard tables

Regulation 29 - Correction to the freeboard for ships under 100 m in length

Regulation 30 - Correction for block coefficient

Regulation 31 - Correction for depth

Regulation 32 - Correction for position of deck line

Regulation 32-1 - Correction for recess in freeboard deck

Regulation 33 - Standard height of superstructure

Regulation 34 - Length of superstructure

Regulation 35 - Effective length of superstructure

Regulation 36 - Trunks

Regulation 37 - Deduction for superstructures and trunks

Regulation 38 - Sheer

Regulation 39 - Minimum bow height and reserve buoyancy

Regulation 40 - Minimum freeboards

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TheNavalArch.com

- Regulation 27. Types of Ships
- The approach to freeboard calculation (as laid out in ICLL 66) is to first calculate the standard freeboard that is attributable to the vessel. The ICLL 66 provides standard tables for the minimum freeboard needed, depending on the length and type of ship. There are two types of ships recognized:
- Type A – tankers and other liquid carriers with high integrity of the main deck
- Type B – those which are not Type A

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Tabular Freeboard (Regulation 28 of ICLL)

LL18
(1968)
(Rev.1
July 2008)

Freeboard tables (Regulation 28)

(a) Type A ships

(i) Freeboards for Type A ships with lengths between 365 m and 400 m shall be determined by the following formula:

$$f = 221 + 16{,}10L - 0{,}02L^2$$

where f is the freeboard in mm
L is the length as defined in Regulation 3(1).

(ii) Freeboards for Type A ships with lengths of 400 m and above shall be the constant value, 3460 mm.

What is the freeboard of a Type A ship with length 450 m?

- Ans. 3460 mm

(b) Type B ships

(i) Freeboards for Type B ships with lengths between 365 m and 400 m shall be determined by the following formula:

$$f = -587 + 23L - 0{,}0188L^2$$

where f is the freeboard in mm
L is the length as defined in Regulation 3(1).

(ii) Freeboards for Type B ships with lengths of 400 m and above shall be the constant value, 5605 mm.

What is the freeboard of a Type B ship with length 370 m?

- Ans. 5350 mm

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https://thenavalarch.com/the-why-and-how-of-freeboard-calculation-of-a-ship/

- TheNavalArch.com

Table A Freeboard table for Type «A» ships

Length of ship (m)	*Freeboard (mm)*
24	200
25	208
26	217
27	225
28	233
29	242
30	250
31	258
32	267
33	275
34	283
35	292
36	300
37	308
38	316
39	325
40	334
41	344
42	354
43	364
44	374

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https://thenavalarch.com/the-why-and-how-of-freeboard-calculation-of-a-ship/

Table B Freeboard table for Type «B» ships

Length of ship (m)	*Freeboard (mm)*
24	200
25	208
26	217
27	225
28	233
29	242
30	250
31	258
32	267
33	275
34	283
35	292
36	300
37	308
38	316
39	325

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Freeboard Calculations

- Freeboard Calculation – considerations
- The calculation of freeboard is carried out as per the International Convention on Load Lines, 1966, as Amended by the Protocol of 1988 – Annex I – Regulations for Determining Load Lines.
- The basic purpose while assigning the right freeboard is to ensure that the vessel has sufficient reserve buoyancy.
- Size of the vessel – a bigger vessel needs higher reserve buoyancy, and thus its freeboard shall be higher than a smaller vessel

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Freeboard Calculations

- Structural integrity of the vessel – A vessel with many large hatch openings on the deck is likely to have more compromises on the 'intactness' of its structure. Comparatively, a vessel with high integrity of the main deck – vessels carrying liquid cargoes/tankers – will be more intact and will have a lower need for reserve buoyancy.
- Any other factor that affects reserve buoyancy shall have to be taken into account
- If there is superstructure that can contribute to reserve buoyancy, then the freeboard can be reduced
- If there's additional sheer in the vessel, then the reserve buoyancy goes up and freeboard can be reduced

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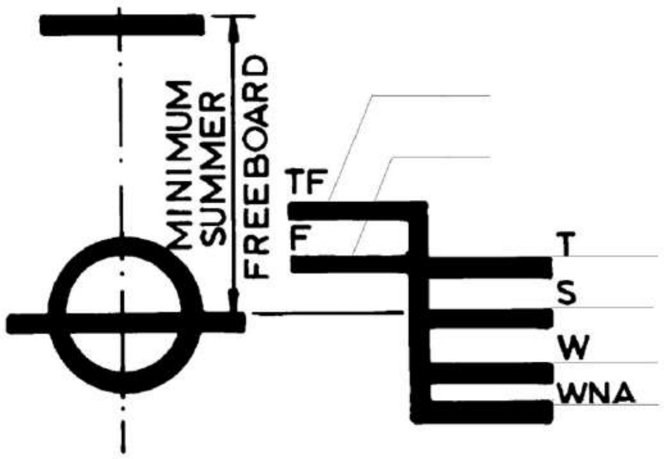

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https://thenavalarch.com/the-why-and-how-of-freeboard-calculation-of-a-ship/

- From the above process, the Summer freeboard of the vessel can be calculated. The summer freeboard is used to calculate the summer loadline of the vessel. The Summer Loadline is marked on the vessel as a 'Loadline Mark'. It is indicated by the upper edge of the line which passes through the centre of the ring and also by a line marked S.
- Freeboard first. Loadline next.

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There are multiple other loadlines of the vessel, depending on its area of operation. For sailing ships, only the Fresh Water and Winter North Atlantic loadlines are to be shown, apart from Summer Loadline. These additional lines are:

- The Winter Load Line indicated by the upper edge of a line marked W.
- The Winter North Atlantic Load Line indicated by the upper edge of a line marked WNA.
- The Tropical Load Line indicated by the upper edge of a line marked T.
- The Fresh Water Load Line in summer indicated by the upper edge of a line marked F. The Fresh Water Load Line in summer is marked abaft the vertical line. The difference between the Fresh Water Load Line in summer and the Summer Load Line is the allowance to be made for loading in fresh water at the other load lines.
- The Tropical Fresh Water Load Line indicated by the upper edge of a line marked TF, and marked abaft the vertical line.

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Module 4 is completed

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and become outstanding Naval Architects

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25 Mar 2025

Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

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4.8 Exact attitude of a cuboidal barge when a mass is moved

- A cuboidal barge of displacement Δ=8000 t has length L =100 m and breadth B =20 m. It is upright in fresh water. KG = 4.8 m. (a) Find the TP1cm and MCT1m. (b) m = 1000 t on board the barge is moved forward by d =5 m. Find the change in the xCoG and the xCoB. (c) Then, the mass is moved up by 1 m. Find the change in the zCoG and the zCoB.
- T = Draft = $\frac{\Delta}{\rho LB}$ = 8000/(1*100*20) = 4 m
- TP1cm = WPA * Density / 100 = 100*20*1/100 = 20 t/cm
- %MCT1m; Trim1 = 1; Vol_wedge = 0.5 * (Trim1/2) * Breadth * Length/2
- CoG_wedge = (2/3)*Length; MCT1m_exact = Vol_wedge * CoG_wedge * Density;
- When the mass moves forward, the CoG moves from G to G1. Then, the CoB moves forward by BB1 and up by B1B2 as the barge trims by angle θ. G1B2 is perpendicular to the new waterline and makes an angle θ with the vertical.

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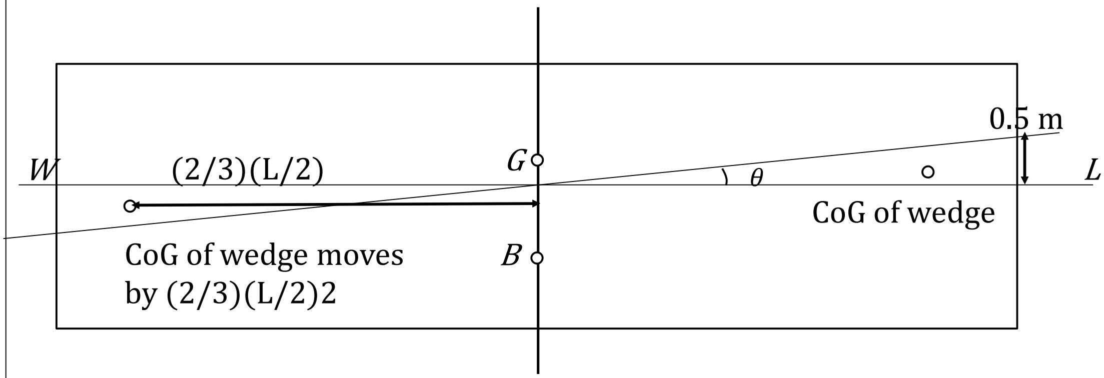
MCT1m

When the ship trims, a wedge moves from the aft to the forward.



- %MCT1m; Trim = 1; Vol_wedge = 0.5 * (Trim/2) * Breadth * Length/2 = 250
- CoG_wedge moves by = (2/3)*Length;
- MCT1m_exact = Vol_wedge * CoG_wedge * Density;

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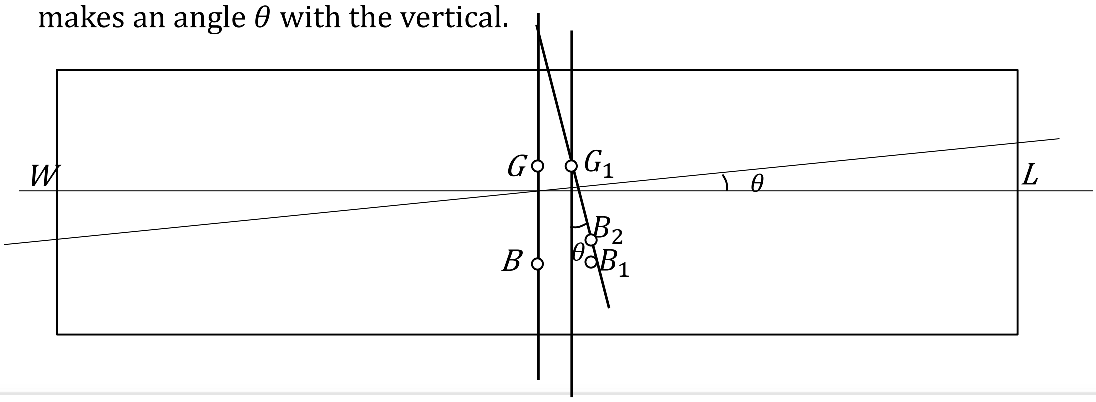
Mass moves forward

- When the mass moves forward, the CoG moves horizontally from G to G1. Then, the CoB moves horizontally forward by BB1 and up by B1B2 as the barge trims by angle θ. G1B2 is perpendicular to the new waterline and makes an angle θ with the vertical.



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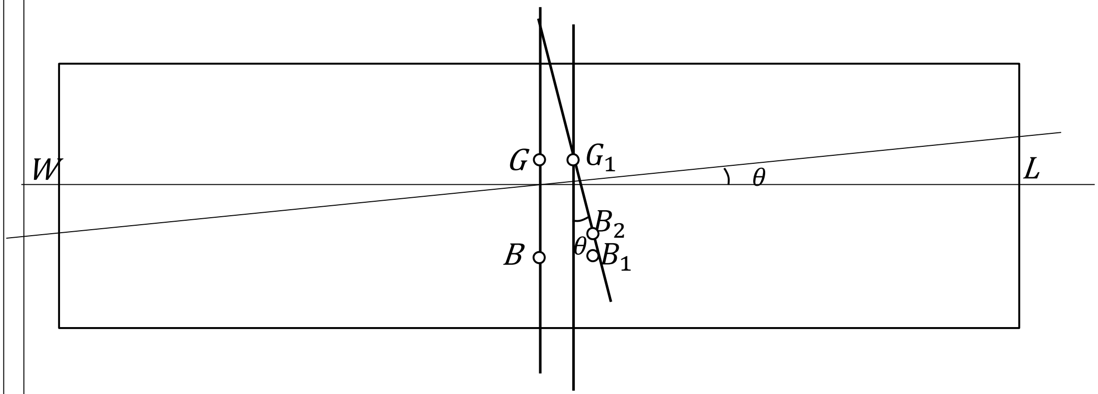
Trim due to forward movement of a mass

$GG_1 = \frac{md}{\Delta}$; $BB_1 = \frac{L^2}{12T}\tan\theta$; $B_1B_2 = \frac{L^2}{24T}\tan^2\theta$; Let $\alpha = \frac{L^2}{12T}$



$x = \tan\theta = \frac{BB_1 - GG_1}{GB - B_1B_2}$; $x = \frac{\alpha x - GG_1}{GB - 0.5\alpha x^2}$; $0.5\alpha x^3 + (\alpha - GB)x - GG_1 = 0$

Cubic equation. 3 solutions. Expressions are known.

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Exact Analysis. MATLAB

```
Q on S2. a) Length = 100; Breadth = 20;
WPA = Length * Breadth; Density = 1; % tonne/m^3
Displacement = 8000; % tonne
UWVolume = Displacement/Density
Draft = UWVolume / WPA
TP1cm = WPA * Density / 100
%MCT1m
Trim1 = 1
Vol_wedge = 0.5 * (Trim1/2) * Breadth * Length/2
CoG_wedge = (2/3)*Length
MCT1m_exact = Vol_wedge * CoG_wedge * Density
```

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UWVolume = 8000

Draft = 4

TP1cm = 20

Trim1 = 1

Vol_wedge = 250

CoG_wedge = 66.6667

BML = 208.3333

KG = 4.8000

MCT1m_exact = 1.66667e+04 t.

GML = KML – KG = KB+BKL-KG = 2+208.3333-4.8 = 205.5333

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See the Question on S2. Find MCT1m_approx

% Righting moment = Disp * GML * sin(x). If trim = 1 m, righting moment = MCT1m and tan(x) = 1/Length. Use sin(x) ≅ tan(x)=1/Length. Then, righting moment = MCT1m ≅ Disp * GML /Length

```
BML = Length^2/(12*Draft)
KB = Draft/2;
KG = Draft * 1.2
KML = KB + BML;
GML = KML - KG;  = 205.5333
MCT1m_approx = Displacement * GML/Length;
fprintf(' MCT1m_exact = %10.5e t. MCT1m_approx = %10.5e m \n', MCT1m_exact, MCT1m_approx)
```

MCT1m_exact = 1.66667e+04 t. MCT1m_approx = 1.64427e+04 m

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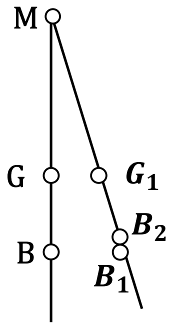
See the Question on S2. Part b.




```
% 1000 t on board is moved forward by 5 m
Mass = 1000;
Moment_extra = Mass*5
% Change in the xCoG
xCoG_change = Moment_extra/Displacement
GG1 = xCoG_change

% Method 1. Approximate analysis. Assume that the M does not move
% tan(theta) = GG1/GML = BB1/(BML-B1B2).
% This is similar to the equation in Barrass and Derrett 7th Ed
Chap 14 p140
tan_theta = GG1/GML = 0.625/205.5333 = 0.0030409
BB1 = BML * tan_theta = 208.3333 * 0.0030409 = 0.6335
theta_approx = atand(tan_theta)
```

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Moment_extra = 5000
xCoG_change = 0.6250
GG1 = 0.6250
tan_theta = 0.0030

BB1 = 0.6335
theta_approx = 0.174228439 deg

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```
% Method 2
% If the xCoG moves forward, the ship will trim and xCoB will move
forward until CoB is below the CoG. Assume that it trims by theta and
solve for
% x = tan(theta)
% tan(theta) = (BB1-GG1)/(BG-B1B2)

alpha = Length^2/(12*Draft)
% solve 0.5*alpha*x^3 +(alpha-BG)*x - GG1 = 0
BG = KG - KB
% in the next line, x is a variable. X is the solution to the eq.
% 0 is the initial guess for X
X = fzero(@(x) 0.5*alpha*x^3 +(alpha-BG)*x - GG1, 0)
theta_exact = atand(X)

xCoB_Shift = alpha*X

fprintf(' Trim angle. Approx = %13.8e deg. Exact = %13.8e deg
\n',theta_approx, theta_exact)
xCoB_xCoG_diff = xCoB_Shift - xCoG_change
```

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alpha = 208.3333

BG = 2.8000

X = 0.003040855032549

theta_exact = 0.174227622 deg

xCoB_Shift = 0.6335

Trim angle. Approx = 1.74228439e-01 deg. Exact = 1.74227622e-01 deg

xCoB_xCoG_diff = 0.0085

zCoG_change = 0.1250

zCoB_change = 0

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```
% Move the mass up by 1 m
zCoG_change = Mass * 1/Displacement
zCoB_change = 0
```

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Cubic equation. 3 roots. MATLAB.

```
% Solution to a cubic equation. ax^3 + bx^2 + cx + d = 0
clear; close all; clc;
example = 1
if example == 1
    % The roots are 1, 2, and 3
    a = 1; b = -6; c = 11; d = -6;
elseif example == 2
    % (x-1)^3 = x^3 - 3x^2 + 3x – 1
    a = 1; b = -3; c = 3; d = -1;
end
p = [a b c d];
% Method 1. Use the MATLAB function roots. Then use the function polyval to evaluate the cubic eq at the roots.
roots_1 = roots([a b c d])
check1(1) = polyval(p,roots_1(1));
check1(2) = polyval(p,roots_1(2));
check1(3) = polyval(p,roots_1(3));
if max(abs(check1)) < 1e-13
    disp('On substitution, it is found that all 3 values of roots_1 satisfy the polynomial eq.’)
else
    check1
    disp(' The values of check1 are shown above. They should be zero. Aborted.’)
    return
end
```

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Cubic equation. 3 roots. MATLAB.

```
% Method 2. See Abramowitz and Stegun. Handbook of Mathematical Functions. Sec. 3.8.2
% If b = 0, then a2 = 0 and q = a1/3 and r = -a0/2 and this special case has to be handled separately
a2 = b/a; a1 = c/a; a0 = d/a;
q = a1/3 - a2^2/9;
r = (a1*a2 - 3*a0)/6 - a2^3/27;
s1 = (r + (q^3+r^2)^0.5)^(1/3);
s2 = (r - (q^3+r^2)^0.5)^(1/3);
roots_2(1) = (s1+s2) - a2/3;
roots_2(2) = -(s1+s2)/2 - a2/3 + 1i*sqrt(3)*(s1-s2)/2;
roots_2(3) = -(s1+s2)/2 - a2/3 - 1i*sqrt(3)*(s1-s2)/2;
roots_2
check2(1) = polyval(p,roots_2(1));
check2(2) = polyval(p,roots_2(2));
check2(3) = polyval(p,roots_2(3));
if max(abs(check2)) < 1e-13
    disp('On substitution, it is found that all 3 values of roots_2 satisfy the polynomial eq.')
else
    check2
    disp(' The values of check2 are shown above. They should be zero. Aborted.')
    return
end
return
```

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Read books, magazines, and journals

and become outstanding Naval Architects

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Apr25