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Department of Ship Technology
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Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406
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3 credits

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
Intact stability rules, Heel/ Load test.
Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

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   removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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Course Content. Module 3.

Earlier

Module 1. Stability Terms. 06 Lectures. Google forms test 10 Jan 25.

Module 2. Initial (Transverse) Stability. 07 lectures. Google forms test 29Feb25

Module 3. Large Angle Transverse Stability

Today

- 3.1 Large change in the attitude. Equivoluminal change. yCoB.

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Module 3. Large Angle Transverse Stability

- Study
  - Biran. 2nd Ed. Chapter 5. ***Statical Stability at Large Angles of Heel***
  - Semyonov. Chapter 4. ***Stability at Large Inclinations***

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3.1 Large change in attitude. Equivoluminal change.

- An equi-voluminal inclination is caused by a (first) moment about the axis of floatation. It can be caused by an external force (not through CoF) or a movement of a mass on-board.
- When there is an inclination, the CoB moves. The new CoB is determined by using the old CoB and the moment that caused the inclination. It can also be determined by considering the underwater volume.
- When the inclination is due to an external moment, there is a restoring couple, $\boldsymbol{\Delta}\ \boldsymbol{GZ}$, where $\boldsymbol{\Delta}$ is the displacement and $\boldsymbol{GZ}$ is the righting arm. The couple is also equal to the first moment of mass in the newly emerged and submerged volumes. This is used to find the change in the CoB. Therefore, the first moment of volume is of interest.

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3.1 Large change in attitude.
Three independent parameters to define the attitude.

- Read all the details in Semyonov. Determine the first moments for a ship of arbitrary shape. For the special case of a barge, the expressions should simplify to those in Biran.
- The water plane is defined by three parameters: The draft at midship, $T_{\otimes}$, the Heel angle, $\boldsymbol{\theta}$, and the Trim angle, $\boldsymbol{\psi}$ (psi). (see next slide). Note the difference in notation between Semyonov and Biran.
- To find GZ($T_{\otimes}$, $\boldsymbol{\theta}$, $\boldsymbol{\psi}$) the moments are needed for an arbitrary water plane and expressions are derived here.
- The global origin is on the keel at midship (Semyonov).

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3.1 Three independent parameters to define the attitude.

- Semyonov. Moments due to a small change in the attitude of the ship

14. DETERMINATION OF BUOYANCY OF A SHIP IMMERSED TO ANY WATERLINE. VLASOV'S INTEGRAL CURVES OF SECTIONAL AREAS

Consider a ship immersed to a waterline which is not parallel to the base plane XOY. Such a waterline will be referred to as "any waterline". Find the relations by the use of which it is possible to determine the displacement V and the co-ordinates of the centre of buoyancy of the ship x_c, y_c and z_c.

We first establish three parameters which define the position of any waterline in the system of co-ordinates adopted. The position of a waterline is uniquely defined by the following quantities (Fig. 15):

(1) by the height $T_{\otimes}$ of the point of intersection of the plane of any waterline and the axis OZ;

(2) by the angle ψ between the axis OX and the trace of the waterplane on the plane XOZ;

(3) by the angle θ between OY and the trace of the waterplane on the plane YOZ.

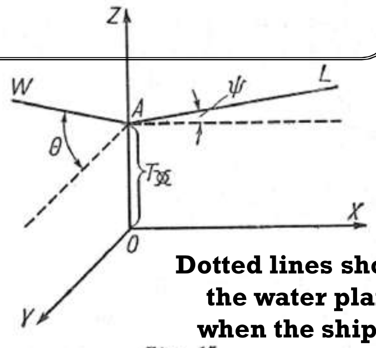

Fig. 15

Dotted lines show the water plane when the ship is on even keel

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3.1 Large change in attitude. Outline of procedure

1. Start with T_{∞} (draft), $\boldsymbol{\theta}$ (heel angle), and $\boldsymbol{\psi}$ (trim angle) as independent variables. Values of other parameters such as underwater volume are found using the independent variables.
2. Derive expressions for the change in volume and moments (about global coord axes) due to $\boldsymbol{\delta} T_{\infty}$, $\boldsymbol{\delta\theta}$, and $\boldsymbol{\delta\psi}$. The expressions are in terms of the first and second moments of the waterplane area about the global axes.
3. Express changes in volume and moments as total differentials
4. Use 2 and 3 to find partial derivatives of a) volume and b) first moments of volume in terms of first and second moments of the waterplane area about the global axes.

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Outline of procedure

5. Consider equivolume inclinations. Reduce the number of independent variables to two: $\boldsymbol{\delta\theta}$ and $\boldsymbol{\delta\psi}$.
6. Express the partial derivatives of moments in terms of second moments about floatation axes.
7. Find analytic expressions for change in CoB and the metacenter
8. Special case. Trim = 0.
9. Special case. Initial stability. $\boldsymbol{\theta} = \boldsymbol{\phi} = \mathbf{0}$.
10. Find the CoB and BM for zero Trim

T_{∞}

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Change in the underwater volume due to a change in the attitude

- Semyonov describes the procedure to find the change in the volume and the CoB and the metacentre
- First, we will focus on the derivation of an expression for the change in the volume
- Then, study the same slides for change in the CoB and the metacentre

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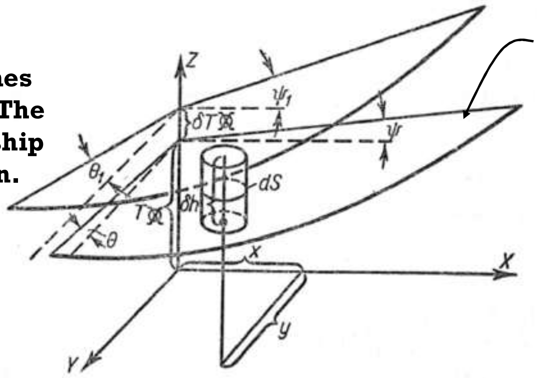
33. GENERAL CASE OF A SMALL CHANGE IN PARAMETERS DEFINING POSITION OF INCLINED WATERLINE Step #1 of Slide#9

Consider the general case when the parameters $T_{\text{ж}}$, ψ and θ which define the position of an inclined waterline are given small changes $\delta T_{\text{ж}}$, $\delta\psi$ and $\delta\theta$. Let us establish the manner in which the displacement and the static moments of the displacement with respect to the co-ordinate planes vary in this case.

Semyonov.

2 waterplanes are shown. The hull of the ship is not shown.



All integrals are over the inclined water plane <u>projected on the XOY plane</u>.

Fig. 44

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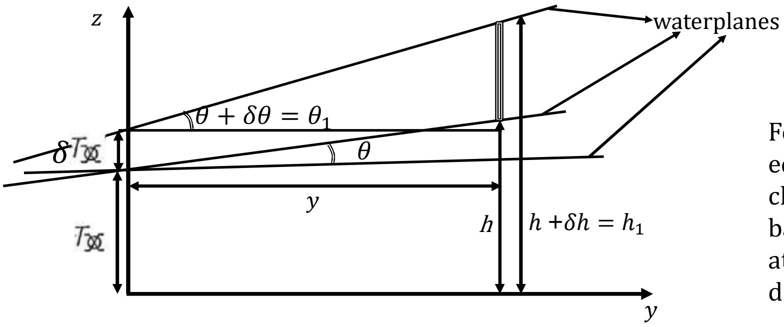
An arbitrary waterplane and a small change in the draft and the heel angle

- Arbitrary waterplane (T ψ, θ)
- After a small change (T + δT , $\psi + \delta\psi,\ \theta + \delta\theta$)



For a small equivoluminal change in a barge, the draft at amidships does not change

- $h = T + y\tan(\theta)$; $h + \delta h = T + \delta T + y\tan(\theta_1)$;
- $\delta h = \delta T + [y\tan(\theta_1) - y\tan(\theta)]$
- $\delta h = \delta h_T + \delta h_\theta$

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Small change in the heel angle

- $\delta h_\theta = y\tan(\theta_1) - y\tan(\theta) = y[\tan(\theta_1) - \tan(\theta)]$; $\theta_1 = \theta + \delta\theta$
- $\tan(\theta_1) = \frac{\tan(\theta)+\tan(\delta\theta)}{1-\tan(\theta)\tan(\delta\theta)} = \frac{\tan(\theta)+\delta\theta}{1-\tan(\theta)\delta\theta}$ by using $\tan(\delta\theta) \cong \delta\theta$
- $\tan(\theta_1) - \tan(\theta) = \frac{\tan(\theta)+\delta\theta}{1-\tan(\theta)\delta\theta}$ - $\tan(\theta) = \frac{\tan(\theta)+\delta\theta-\tan(\theta)+\tan^2(\theta)\delta\theta}{1-\tan(\theta)\delta\theta}$
- $\tan(\theta_1) - \tan(\theta) = \frac{\delta\theta+\tan^2(\theta)\delta\theta}{1-\tan(\theta)\delta\theta} \cong \delta\theta[1+\tan^2(\theta)] = \delta\theta\sec^2(\theta)$

  Under what condition is the denominator $\cong 1$?
- $\tan(\theta_1) - \tan(\theta) = \frac{\delta\theta}{\cos^2(\theta)}$
- $\delta h_\theta = y[\tan(\theta_1) - \tan(\theta)] = y\frac{\delta\theta}{\cos^2(\theta)}$. Used in Semyonov Eq. (33.1)
- Do a similar analysis for change in the trim angle. $\delta h_\psi = x\frac{\delta\psi}{\cos^2(\psi)}$

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Change in the height of the water column due to a change in the attitude

Suppose the displacement is changed by the quantity δV and the changes in static moments of the displacement or, which amounts to the same thing, the static moments of the volume δV are δM_{yz}, δM_{xz} and δM_{xy}. Determine the volume δV as the sum of the volumes of elementary prisms intercepted between the two inclined waterlines whose parameters differ by $\delta T_{\boxtimes}$, $\delta\psi$ and $\delta\theta$. The bases of these prisms dS represent the projections of the elements of area of the inclined waterplane on the plane XOY. The heights of these prisms δh are vertical and according to Fig. 44 are expressed as

Later, δV is made zero

XOY is the global reference plane

$$\delta h = \delta T_{\boxtimes} + x(\tan\psi_1 - \tan\psi) + y(\tan\theta_1 - \tan\theta).$$

The heel and trim angles and the draft change by small amounts.

Here

$$\psi_1 = \psi + \delta\psi \quad \text{and} \quad \theta_1 = \theta + \delta\theta$$

Heel angle θ

and therefore

$$\tan\psi_1 - \tan\psi = \frac{\delta\psi}{\cos^2\psi} \quad \text{and} \quad \tan\theta_1 - \tan\theta = \frac{\delta\theta}{\cos^2\theta}.$$

Derivation on previous slide

We have then the final expression

$$\delta h = \delta T_{\boxtimes} + x\frac{\delta\psi}{\cos^2\psi} + y\frac{\delta\theta}{\cos^2\theta}. \qquad (33.1)$$

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Change in Volume

- The original attitude (position) is ($T_{\boxtimes}$, ψ, θ) (draft, trim angle, heel angle)
- A point on the waterplane at (x, y) is at $z = T_{\boxtimes} + x\tan\psi + y\tan\theta. \quad (33.2)$
- After the change in attitude, a point on the new waterplane that is at (x, y) is at $z + \delta h$ where δh is defined in Eq. (33.1)
- Consider an elemental area dS on the waterplane when the attitude is ($T_{\boxtimes}$, $\psi = 0, \theta$=0). The volume of the elemental prism is $\delta h dS$.
- The volume that lies between the waterplanes for the original attitude and the slightly changed attitude is $\delta V = \int \delta h dS$ where the integral is over a waterplane that is parallel to the sea surface.
- Using δh in Eq. (33.1) in the above yields the expression for δV on the next slide

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Change in Volume and First Moments of Volume

S09#2

First, study the next few slides and understand change in volume. Next, study the same slides for change in first moments.

$$\delta h = \delta T_{\boxtimes} + x\frac{\delta\psi}{\cos^2\psi} + y\frac{\delta\theta}{\cos^2\theta}. \qquad (33.1)$$

The volume of the elementary prism is $\delta h dS$ and the static moments of this volume with respect to the co-ordinate planes are $x\delta h dS$, $y\delta h dS$ and $z\delta h dS$. Then, disregarding the small quantities of higher order, we can write

First moment

$$z = T_{\boxtimes} + x\tan\psi + y\tan\theta. \qquad (33.2)$$

Let us now set up expressions for δV, δM_{yz}, δM_{xz} and δM_{xy} taking into account relations (33.1) and (33.2).

dS is in the XOY plane

$$\delta V = \int_S \delta h dS = \delta T_{\boxtimes}\int_S dS + \frac{\delta\psi}{\cos^2\psi}\int_S x dS + \frac{\delta\theta}{\cos^2\theta}\int_S y dS;$$

δh see Eq. (33.1)

Second moment of area

$$\delta M_{yz} = \int_S x\delta h dS = \delta T_{\boxtimes}\int_S x dS + \frac{\delta\psi}{\cos^2\psi}\int_S x^2 dS + \frac{\delta\theta}{\cos^2\theta}\int_S xy dS;$$

x = dist between elemental volume and YZ plane

$$\delta M_{xz} = \int_S y\delta h dS = \delta T_{\boxtimes}\int_S y dS + \frac{\delta\psi}{\cos^2\psi}\int_S xy dS + \frac{\delta\theta}{\cos^2\theta}\int_S y^2 dS;$$

First moment of vol

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Change in First Moment of Volume about xy plane

S09#2

- The volume of the elemental prism is $\delta h dS$. The moment of this prism about the xy plane is $(z + \delta h/2)\delta h dS$. The moment of the entire volume between the original and new waterplanes (after neglecting $\delta h/2$) is
- $\delta M_{xy} = \int z\delta h dS = \int dS$ $\quad z = T_{\boxtimes} + x\tan\psi + y\tan\theta.$

$$\delta h = \delta T_{\boxtimes} + x\frac{\delta\psi}{\cos^2\psi} + y\frac{\delta\theta}{\cos^2\theta}$$

First moment of vol

$$\delta M_{xy} = \int_S z\delta h dS = \delta T_{\boxtimes}\left[T_{\boxtimes}\int_S dS + \tan\psi\int_S x dS + \tan\theta\int_S y dS\right] + \frac{\delta\psi}{\cos^2\psi}\left[T_{\boxtimes}\int_S x dS + \tan\psi\int_S x^2 dS + \tan\theta\int_S xy dS\right] + \frac{\delta\theta}{\cos^2\theta}\left[T_{\boxtimes}\int_S y dS + \tan\psi\int_S xy dS + \tan\theta\int_S y^2 dS\right].$$

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The integrals appearing in the formulas above represent the following well-known characteristics of the area of the projection of the inclined waterplane on the plane XOY:

(x_f, y_f) is the center of floatation in the global coords

$$\int_S dS = S \qquad \int_S x dS = x_f S \qquad \int_S y dS = y_f S;$$

$$\int_S x^2 dS = I_y \qquad \int_S y^2 dS = I_x \qquad \int_S xy dS = I_{xy}.$$

- From L14S17

$$\delta V = \int_S \delta h dS = \delta T_{\text{ж}} \int_S dS + \frac{\delta\psi}{\cos^2\psi}\int_S x dS + \frac{\delta\theta}{\cos^2\theta}\int_S y dS;$$

- So Taking into account these relations, we can write

$$\delta V = S\delta T_{\text{ж}} + \frac{Sx_f}{\cos^2\psi}\delta\psi + \frac{Sy_f}{\cos^2\theta}\delta\theta; \qquad (33.3)$$

- Similarly, substitution yields Eq. (33.4) to (33.6) on the next slide

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The integrals appearing in the formulas above represent the following well-known characteristics of the area of the projection of the inclined waterplane on the plane XOY:

(x_f, y_f) is the center of floatation in the global coords

$$\int_S dS = S \qquad \int_S x dS = x_f S \qquad \int_S y dS = y_f S;$$

$$\int_S x^2 dS = I_y \qquad \int_S y^2 dS = I_x \qquad \int_S xy dS = I_{xy}.$$

Taking into account these relations, we can write

$$\delta V = S\delta T_{\text{ж}} + \frac{Sx_f}{\cos^2\psi}\delta\psi + \frac{Sy_f}{\cos^2\theta}\delta\theta; \qquad (33.3)$$

$$\delta M_{yz} = Sx_f\delta T_{\text{ж}} + \frac{I_y}{\cos^2\psi}\delta\psi + \frac{I_{xy}}{\cos^2\theta}\delta\theta; \qquad (33.4)$$

$$\delta M_{xz} = Sy_f\delta T_{\text{ж}} + \frac{I_{xy}}{\cos^2\psi}\delta\psi + \frac{I_x}{\cos^2\theta}\delta\theta; \qquad (33.5)$$

$$\delta M_{xy} = (ST_{\text{ж}} + Sx_f \tan\varphi + Sy_f \tan\theta)\,\delta T_{\text{ж}} + \\ + (Sx_f T_{\text{ж}} + I_y \tan\psi + I_{xy}\tan\theta)\frac{\delta\psi}{\cos^2\psi} + \\ + (Sy_f T_{\text{ж}} + I_{xy}\tan\psi + I_x \tan\theta)\frac{\delta\theta}{\cos^2\theta}. \qquad (33.6)$$

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Step#3 in S09

On the other hand, δV, δM_{yz}, δM_{xz} and δM_{xy} may be represented as the total differentials

Taylor series for a fn of many variables

The volume changes if the heel or trim angle or draft change

$$\left.\begin{aligned}
\delta V &= \frac{\partial V}{\partial T_{\boxtimes}}\,\delta T_{\boxtimes} + \frac{\partial V}{\partial \psi}\,\delta\psi + \frac{\partial V}{\partial \theta}\,\delta\theta \\
\delta M_{yz} &= \frac{\partial M_{yz}}{\partial T_{\boxtimes}}\,\delta T_{\boxtimes} + \frac{\partial M_{yz}}{\partial \psi}\,\delta\psi + \frac{\partial M_{yz}}{\partial \theta}\,\delta\theta \\
\delta M_{xz} &= \frac{\partial M_{xz}}{\partial T_{\boxtimes}}\,\delta T_{\boxtimes} + \frac{\partial M_{xz}}{\partial \psi}\,\delta\psi + \frac{\partial M_{xz}}{\partial \theta}\,\delta\theta \\
\delta M_{xy} &= \frac{\partial M_{xy}}{\partial T_{\boxtimes}}\,\delta T_{\boxtimes} + \frac{\partial M_{xy}}{\partial \psi}\,\delta\psi + \frac{\partial M_{xy}}{\partial \theta}\,\delta\theta
\end{aligned}\right\} \quad (33.7)$$

Step#4

Comparing expressions (33.7) with expressions (33.3) through (33.6), we can set up the expressions for the partial derivatives

$$\boxed{\int_S x\,dS = x_f S}$$

$$\frac{\partial V}{\partial T_{\boxtimes}} = S; \quad \frac{\partial V}{\partial \psi} = \frac{Sx_f}{\cos^2\psi}; \quad \frac{\partial V}{\partial \theta} = \frac{Sy_f}{\cos^2\theta}; \quad (33.8)$$

$$\frac{\partial M_{yz}}{\partial T_{\boxtimes}} = Sx_f; \quad \frac{\partial M_{yz}}{\partial \psi} = \frac{I_y}{\cos^2\psi}; \quad \frac{\partial M_{yz}}{\partial \theta} = \frac{I_{xy}}{\cos^2\theta}; \quad (33.9)$$

$$\frac{\partial M_{xz}}{\partial T_{\boxtimes}} = Sy_f; \quad \frac{\partial M_{xz}}{\partial \psi} = \frac{I_{xy}}{\cos^2\psi}; \quad \frac{\partial M_{xz}}{\partial \theta} = \frac{I_x}{\cos^2\theta}; \quad (33.10)$$

Taylor Series Expansion

$$V(x_1, y_1, z_1) = V(x,y,z) + (x_1 - x)\frac{\partial V}{\partial x} + (y_1 - y)\frac{\partial V}{\partial y} + (z_1 - z)\frac{\partial V}{\partial z}$$

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Change in the volume by 2 methods

- Compare δV in Eqs. (33.3) and (33.7) to obtain Eq. (33.8)

$$\delta V = S\delta T_{\boxtimes} + \frac{Sx_f}{\cos^2\psi}\,\delta\psi + \frac{Sy_f}{\cos^2\theta}\,\delta\theta; \quad (33.3)$$

$$\delta V = \frac{\partial V}{\partial T_{\boxtimes}}\,\delta T_{\boxtimes} + \frac{\partial V}{\partial \psi}\,\delta\psi + \frac{\partial V}{\partial \theta}\,\delta\theta \quad (33.7)$$

$$\frac{\partial V}{\partial T_{\boxtimes}} = S; \quad \frac{\partial V}{\partial \psi} = \frac{Sx_f}{\cos^2\psi}; \quad \frac{\partial V}{\partial \theta} = \frac{Sy_f}{\cos^2\theta}; \quad (33.8)$$

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$$\left.\begin{aligned}\frac{\partial M_{xy}}{\partial T_{\otimes}} &= S\,(T_{\otimes} + x_f \tan\psi + y_f \tan\theta)\\ \frac{\partial M_{xy}}{\partial \psi} &= \frac{S x_f T_{\otimes}}{\cos^2\psi} + \frac{I_y \tan\psi}{\cos^2\psi} + \frac{I_{xy}\tan\theta}{\cos^2\psi}\\ \frac{\partial M_{xy}}{\partial \theta} &= \frac{S y_f T_{\otimes}}{\cos^2\theta} + \frac{I_{xy}\tan\psi}{\cos^2\theta} + \frac{I_x \tan\theta}{\cos^2\theta}.\end{aligned}\right\} \quad (33.11)$$

Step#5 in S09

Consider the special case of equivolume inclination. In this case $\delta V = 0$ and from (33.3) it follows that

$$\delta T_{\otimes} = -\frac{x_f}{\cos^2\psi}\,\delta\psi - \frac{y_f}{\cos^2\theta}\,\delta\theta. \quad (33.12)$$

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Wall-sided Barge

- Consider Eq. (33.12) in Semyonov for the special case of a cuboidal wall-sided barge. Consider equivoluminal pure heel and no trim. Using fundamentals for a wall-sided barge, the change in the draft should be zero.
- The CoF (Centre of Floatation) of a wall-sided barge is at x=y=0 (origin is on the keel at midship). Therefore, In Eq. (33.12), both the RHS terms are zero and the change in the draft is zero.
- Eq. (33.12) is thus verified for the special case of a wall-sided barge.

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Summary of equivolume change

- The draft at midship, T_M, the heel angle, $\boldsymbol{\theta}$, and the trim angle, $\boldsymbol{\psi}$ (psi) are primary or independent variables.
- They are used to derive an expression for the total differential of the underwater volume in terms of small changes in the independent variables:
  $\delta V = S\delta T_M + \frac{Sx_f}{\cos^2(\psi)}\delta\psi + \frac{Sy_f}{\cos^2(\theta)}\delta\theta$ Eq. (33.3) in Semyonov. $S =$ waterplane area. $(x_f, y_f) =$ CoF.
- For equivoluminal change in the attitude, $\delta V = 0$ and
  $$\delta T_M = -\frac{x_f}{\cos^2(\psi)}\delta\psi - \frac{y_f}{\cos^2(\theta)}\delta\theta \quad \text{Eq. (33.12) in Semyonov}$$
- For a cuboidal barge, $x_f = y_f = 0$. Therefore, $\delta T_M = 0$ for inclination in any plane and the equation for the general case is verified for the special case

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BM is perpendicular to the waterline

- The following quote is from Biran. 2nd Ed. Sec. 2.10. "As the tangent to the B-curve is parallel to the corresponding waterline, it follows that the buoyancy force is normal to the B-curve."
- Show that the tangent to the B-curve is parallel to the waterline for the special case of a wall-sided barge. (See the next slide).
- Then, as BM is perpendicular to the tangent, it follows that BM is perpendicular to the waterline

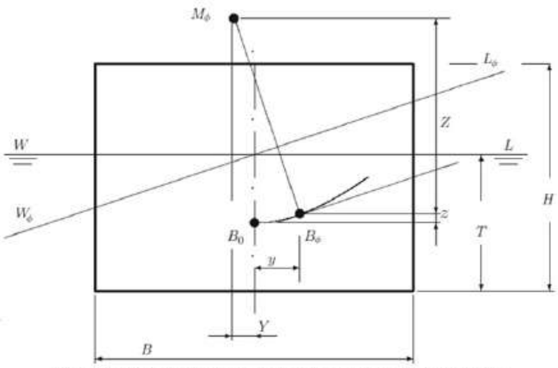

Figure 2.18 Centre of buoyancy and metacentre of simple barge

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Wall-Sided Barge

- The following equations are in Biran, Ship Hydrostatics and Stability. See L08.

$$y_B = \frac{B^2}{12}\tan(\phi) \quad (1) \qquad z_B = \frac{B^2}{24T}\tan^2(\phi) \quad (2)$$

$$z_B = \frac{6T}{B^2}y_B^2 \quad (3) \qquad \frac{dz_B}{dy_B} = \frac{12T}{B^2}y_B = \tan(\phi) \quad (4)$$

$$\frac{d^2z_B}{dy_B^2} = \frac{12T}{B^2} \quad (5) \qquad \overline{BM} = R = \frac{B^2}{12}\frac{1}{\cos^3(\phi)} \quad (6)$$ See L08S49

- $\frac{dz_B}{dy_B}$ is the slope of the tangent to the CoB curve. It is equal to $\tan(\phi)$ [See Eq. (4)]. Therefore, the tangent to the CoB is parallel to the waterline and the BM is perpendicular to the waterline.

$$R = \frac{(1+(dz/dy)^2)^{3/2}}{d^2z/dy^2} = \frac{B^2}{12T}\cdot\frac{1}{\cos^3\phi} \qquad (2.71)$$

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#6 in S09

General expression for $yCoB$. Change in the Moment.

- The same procedure is used to derive an expression for the change in the moment about the xz plane when there is a change in the attitude. From L14S20

$$\delta M_{xz} = Sy_f\delta T_M + \frac{I_{xy}}{\cos^2(\psi)}\delta\psi + \frac{I_x}{\cos^2(\theta)}\delta\theta.$$ Eq. (33.5) in Semyonov

- Substituting the equivolume condition, $\delta T_M = -\frac{x_f}{\cos^2(\psi)}\delta\psi - \frac{y_f}{\cos^2(\theta)}\delta\theta$, in Eq. (33.5) yields $\delta M_{xz} = \frac{I_{fxy}}{\cos^2(\psi)}\delta\psi + \frac{I_{fx}}{\cos^2(\theta)}\delta\theta$ which is Eq. (33.14) where the moments of inertia, I_{fxy} and I_{fx}, are found about the centre of floatation for the upright condition
- $I_{fxy} = I_{xy} - Sx_f y_f$. See the next slide also.
- For the special case of zero trim and only heel, this reduces to $\delta M_{xz} = \frac{I_{fx}}{\cos^2(\theta)}\delta\theta$ which is Eq. (39.2) in Semyonov. Use it to find $yCoB$.
- The MoI of an inclined waterplane is $I_{fx\theta}$ and Eq. 24.3 in Semyonov is $I_{fx} = I_{fx\theta}\cos^3(\theta)$ which is easily proved for a cuboidal barge. This is not needed here.

$$I_{fx} = I_{fx\theta}\cos^3\theta;$$

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General expression for $yCoB$. Change in the Moment.

- The change in the moment, δM_{xz}, is of interest as it is used to find the $yCoB$
- To find this change, the lever arm of the elemental volume is the distance to the xz plane which is = y coordinate. The moment is about the x axis.

Substituting (33.12) in (33.4), (33.5) and (33.6), we can write

$$\boxed{I_{fx} = I_x - Sy_f^2} \quad \delta M_{yz} = \frac{I_{fy}}{\cos^2\psi}\,\delta\psi + \frac{I_{fxy}}{\cos^2\theta}\,\delta\theta; \qquad (33.13)$$

$$\boxed{\delta M_{xz} = \frac{I_{fxy}}{\cos^2\psi}\,\delta\psi + \frac{I_{fx}}{\cos^2\theta}\,\delta\theta;} \qquad (33.14)$$

$$\delta M_{xy} = \left(\frac{I_{fy}\tan\psi}{\cos^2\psi} + \frac{I_{fxy}\tan\theta}{\cos^2\psi}\right)\delta\psi + \left(\frac{I_{fxy}\tan\psi}{\cos^2\theta} + \frac{I_{fx}\tan\theta}{\cos^2\theta}\right)\delta\theta. \qquad (33.15)$$

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General expression for $yCoB$. Verify for a wall-sided barge.

- When the ship starts at even keel (θ=0) and heels to a large angle θ, through an infinite number of infinitesimal changes, the $yCoB = y_\theta$ moves from 0 to y_θ where, using Eq. (39.2) on L14S28 and φ as a dummy variable for integration.

$$y_\theta = \frac{1}{V}\sum_\theta \delta M_{xz} = \frac{1}{V}\sum_\theta \frac{I_{fx}}{\cos^2(\theta)}\delta\theta = \frac{I_{fx}}{V}\int_0^\theta \frac{1}{\cos^2(\varphi)}d\varphi = BM\int_0^\theta \frac{1}{\cos^2(\varphi)}d\varphi$$

- $V =$ underwater volume. y_θ is expressed as an integral of the metacentric radius, BM, which is easy to find
- Use the Eq. for the general case and find the $yCoB$ for a wall-sided barge. Use $I_{fx} = \frac{LB^3}{12}$

$$y_\theta = \frac{LB^3}{12}\int_0^\theta \frac{1}{\cos^2(\varphi)}d\varphi = \frac{LB^3}{12LBT}\int_0^\theta \frac{1}{\cos^2(\varphi)}d\varphi = \frac{B^2}{12T}\tan(\theta)$$

- The above Eq. is derived in Biran. See L08. Check for the general expression is OK.
- Note that Semyonov uses θ for the heel angle and Biran uses ϕ.

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DEAR STUDENTS

Pay full attention during the lectures.

Questions are encouraged.

After classes, study for at least 2 hours every day.

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Module 2 is completed

A Google Forms test will be held on 29 Jan 2025

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29 Jan 2025

Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
Intact stability rules, Heel/ Load test.
Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

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removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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Course Content. Module 3.

Earlier

Module 1. Stability Terms. 06 Lectures. Google forms test 10 Jan 25.

Module 2. Initial (Transverse) Stability. 07 lectures. Google forms test 29Feb25

Module 3. Large Angle Transverse Stability

- 3.1 Large change in the attitude. Equivoluminal change. yCoB.

Today

- 3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.
- 3.3 Curve of Statical Stability

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3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.

- Semyonov pp 158

CHAPTER IV

Stability at Large Inclinations

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3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.

- Before we look at an intro to stability at large angles, recall the figures for stability at low angles. Are 2.9a, 2.9b and 2.9c stable?
- 2.9a and 2.9b are stable. 2.9c is unstable.

Basic Ship Hydrostatics 39

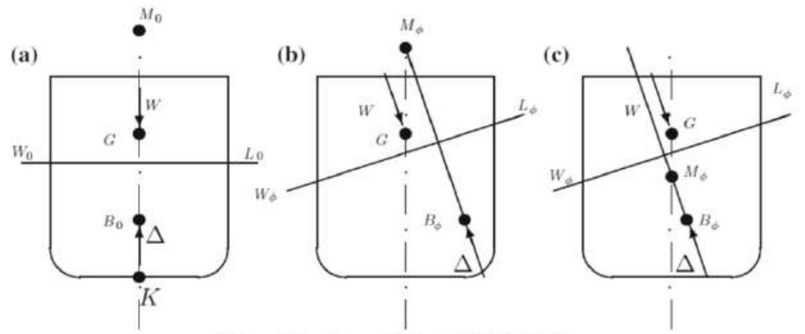

Figure 2.9 The condition of initial stability

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3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.

- Semyonov pp 158

See the next slide regarding "absence of a shift of the metacentre"

CHAPTER IV

Stability at Large Inclinations

Semyonov

38. GENERAL CONSIDERATION OF STABILITY AT LARGE INCLINATIONS

In the study of initial stability we have considered small inclinations of a ship. A reference to small inclinations implies such inclinations at which the metacentre does not move in the system of axes $OXYZ$ fixed in relation to the ship. The absence of a shift of the metacentre is a consequence of equality of moments of inertia of areas of equivolume waterplanes calculated about the axis of inclination. This assumption underlies the derivation of the metacentric formulas of stability and the latter underlie most of the other derivations of the previous chapter.

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3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.

- For ships with port-starboard symmetry, the metacentre lies on the centre-line for zero heel angle
- For non-zero heel angle
  - the metacentre does not lie on the centre-line.
  - For very small angles of heel, the metacentre lies very close to the centre-line but not on the centre-line
  - The statement in Semyonov pp 158 regarding "absence of shift" is incorrect (See L15S16). The metacentre is approximately on the centre-line for small angles of heel. It is not exactly on the centre-line for even small angles of heel.
  - The "absence of shift" is an approximation. Semyonov (translation) calls it an assumption.

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3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.

- Inclination in only the transverse plane (axis is longitudinal and is the fwd-aft centerline)

The previous chap deals with small angles of inclination

If the metacentre is slightly shifted at the inclination under consideration, all the derivations of the previous chapter may be applied without fear of obtaining serious errors in final results. In cases where this condition is not fulfilled there is a risk entailed in using the relations of the previous chapter. sin(Θ)≈ Θ

Most relations developed in this chapter may be used to solve various problems associated with inclination in the transverse plane but in the absence of inclination in the longitudinal plane, i. e., when $\theta \neq 0$ and $\psi = 0$. In this case, as has been ascertained in Sec. 18, the angle of heel

$$\theta_h = \theta.$$

In Semyonov
θ Heel
ψ Trim

It is this case that will be discussed in detail in the present chapter since it is most frequently encountered. The fact is that the angles ψ which in practice may occur along with the occurrence of the angle θ are so small that they may be neglected in most cases without introducing any error. The cases where the

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3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.

- For small angles of inclination, the righting lever is GZ = GM sin(θ) and Righting Couple = Displacement GM sin(θ) (see the next slide)
- There are several ways to derive $\overline{GZ}$ for large angles. For small angles, $\overline{GZ} = \overline{GM}\ \sin\theta$. For slightly larger heel and wall-sided barge, use Atwood's formula. Semyonov Eq. 24.7 is Righting moment = $\Delta\overline{GM}\ \sin\theta$

There are several possible ways of deriving the relations of this chapter. In the first instance it is possible, taking the metacentric formula of stability (24.7) as a basis, to obtain the expression for the correction to it to take account of the shift of the metacentre in the co-ordinate system $OXYZ$. Another way demands that the general expressions be found for the co-ordinates of the centre of buoyancy. By the use of these expressions it is possible to determine the lever of statical stability which, when multiplied by the weight, gives the righting moment. A third way implies the direct derivation of the expressions for two moments: the first one due to the immersed and emerged wedges and the second one due to the shift of the centre of gravity in relation to the vertical through the initial position of the centre of buoyancy. The righting moment is then obtained as the sum of these moments. Today, the 2nd method is presented.

To find M

1st Method. Add correction terms to the expression valid for small angles.

2nd Method. First find CoB curve. Then M and finally $\overline{GZ}$.

3rd Method. Directly find $\overline{GZ}$ from fundamentals.

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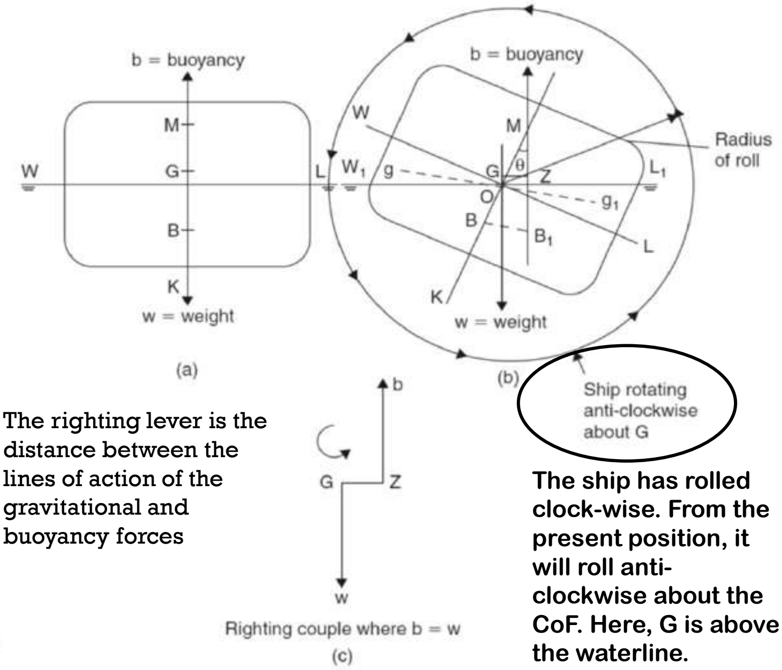

The righting lever is the distance between the lines of action of the gravitational and buoyancy forces

The ship has rolled clock-wise. From the present position, it will roll anti-clockwise about the CoF. Here, G is above the waterline.

Fig. 6.1 Stable equilibrium.

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General Wisdom!

- Paraphrase of Richard Feynman. Nobel Laureate in Physics.
- Author of “Surely you’re joking, Mr. Feynman!”
- If you can arrive at the same result using more than one method, you will be able to understand it much better.

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3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.

- Equations can be now be obtained for lines of a ship using numerical methods. Semyonov is an "old" book.

It is to be noted that whatever be the way of deriving formulas we necessarily obtain identical results since the underlying relation in the three cases is the relation between the moments of inertia of equivolume waterplanes about the longitudinal centroidal axes and the angle of heel θ. This relation cannot be expressed in analytic form as the lines of the ship are not representable by mathematical expressions. It is commonly given in the form of a diagram or a table.

In addition to those noted above, we can imagine various other ways of deriving the relation between the righting moment and the angle of heel but these ways cannot give anything new in principle because they will invariably be based on the use of one and the same underlying relation.

The relationship between the righting moment and angle of heel is of primary interest

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3.2 Large angle

- C_0 Centre of buoyancy when the angle of heel is 0
- C Centre of buoyancy when the angle of heel is φ
- C_1 Centre of buoyancy when the angle of heel is $\varphi + d\varphi$
- M Metacentre when the angle of heel is φ
- φ Angle of heel
- The coordinates of the center of buoyancy, C (Semyonov notation), are $(\boldsymbol{x_\theta}, \boldsymbol{y_\theta}, \boldsymbol{z_\theta})$
- θ = heel angle. φ = dummy variable for integration over heel angle
- The metacenter is M. It is not on y=0.
- Note the locations of the points C, C_1, C_0, M.
- $\boldsymbol{\theta}$ Theta. $\boldsymbol{\varphi}$ Phi. $\boldsymbol{\psi}$ Psi
- $\boldsymbol{\Delta}$ Delta. $\boldsymbol{\nabla}$ Del or Nabla

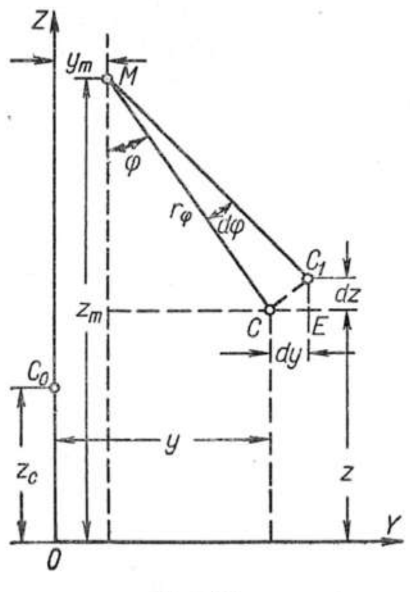

Fig. 54

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3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.

- Find CoB using $\overline{BM}$ = metacentric radius = I/V. Find M using CoB.

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Formulas (39.11) and (39.12) may be obtained in a different manner, from purely geometrical considerations. Let us first derive the expressions for infinitesimal displacements of the centre of buoyancy dy and dz corresponding to any values of the angle φ. Corresponding to an arbitrarily assigned angle of heel φ there is a position of the centre of buoyancy marked C in Fig. 54 and a position of the metacentre marked M. Let the ship be given an additional infinitesimal angle of inclination $d\varphi$. The centre of buoyancy is then shifted to a point C_1. Since the angle $d\varphi$ is small it may be taken that the point M is at the same time the metacentre for the angle of inclination $\varphi_1=\varphi+d\varphi$. The metacentric radius corresponding to the angles of inclination φ

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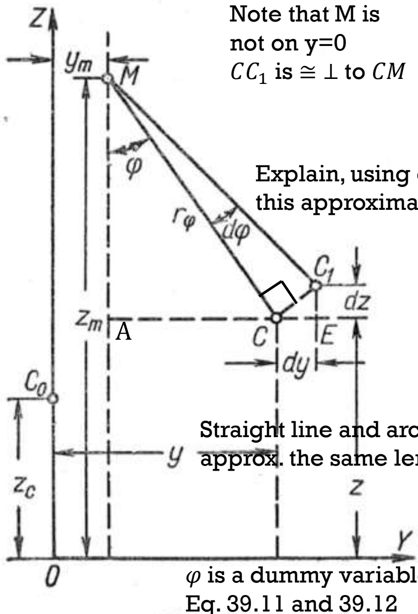

Note that M is not on y=0
CC_1 is $\cong \perp$ to CM

Straight line and arc have approx. the same length

φ is a dummy variable. See Eq. 39.11 and 39.12

Fig. 54

and φ_1 is represented in Fig. 54 by the segments $\overline{MC}=\overline{MC_1}$; denote them by r_φ.

Consider now ΔC_1CE (Fig. 54). Since the angle $d\varphi$ is small it may be taken that

Explain, using geometry, why this approximation is valid

$$\angle C_1CE \approx \varphi.$$

Then, examining the triangle, we find

$$dy=\overline{CE}=\overline{CC_1}\cos\varphi;$$
$$dz=\overline{EC_1}=\overline{CC_1}\sin\varphi.$$

On the other hand, we have

$$\overline{CC_1}\approx \breve{CC}_1=r_\varphi\, d\varphi$$

and after substitution we obtain

$$dy=r_\varphi \cos\varphi\, d\varphi; \quad (39.13)$$
$$dz=r_\varphi \sin\varphi\, d\varphi. \quad (39.14)$$

Integrating expressions (39.13)

Angle ACE = 180 deg.

Angle ACM = 90-φ.

Angle MCC$_1$ = 90 deg.

So, Angle CC$_1$E = φ

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3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.

- Coordinates of the metacenter

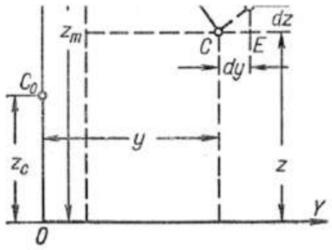

Fig. 54

$$dy = \overline{CE} = \overline{CC}_1 \cos\varphi;$$
$$dz = \overline{EC}_1 = \overline{CC}_1 \sin\varphi.$$

On the other hand, we have

$$\overline{CC}_1 \approx \breve{CC}_1 = r_\varphi\, d\varphi$$

and after substitution we obtain

$$dy = r_\varphi \cos\varphi\, d\varphi; \quad (39.13)$$
$$dz = r_\varphi \sin\varphi\, d\varphi. \quad (39.14)$$

Integrating expressions (39.13) and (39.14) from 0 to θ and taking into account that at $\theta = 0$ $y_\theta = 0$ and $z_\theta = z_c$, we arrive at expressions (39.11) and (39.12).

we can write

CoB is at y_θ, z_θ

r_φ can not be taken outside the integral because it depends on φ.

$$y_\theta = \int_0^\theta r_\varphi \cos\varphi\, d\varphi; \qquad (39.11)$$

$$z_\theta - z_c = \int_0^\theta r_\varphi \sin\varphi\, d\varphi. \qquad (39.12)$$

Here r_φ may be regarded as the transverse metacentric radius.

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3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.

- Coordinates of the metacenter using CoB

Fig. 54

Integrating expressions (39.13) and (39.14) from 0 to θ and taking into account that at $\theta = 0$ $y_\theta = 0$ and $z_\theta = z_c$, we arrive at expressions (39.11) and (39.12).

Directly from Fig. 54 we can obtain the expressions for the co-ordinates of the metacentre y_m and z_m

See Fig. 54 again to understand 39.15

$$y_m = y_\theta - r_\theta \sin\theta; \qquad (39.15)$$
$$z_m = z_\theta + r_\theta \cos\theta. \qquad (39.16)$$

Expressions (39.7), (39.8) and (39.9) define the co-ordinates x_θ, y_θ and z_θ of *the trajectory of the centre of buoyancy* corresponding to inclination in the transverse plane. The co-ordinates

Sec. 40 Lever of Statical Stability, Righting Moment 163

y_θ and z_θ define the curve of centres of buoyancy which represents the projection of the trajectory on the plane of inclination.

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3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.

- Step I. Find $\overline{BM}$ and use it to find CoB.
- Note the expressions for the $yCoB$ and the $zCoB$ on S17. Eq. (39.11 and 39.12) in Semyonov. It is relatively easy to find the metacentric radius for any attitude as only the Moment of Inertia of the waterplane and the underwater volume are needed. The metacentric radius, $\overline{BM}$, is then used to find the $yCoB$ and the $zCoB$.
- Step II. Use CoB and $\overline{BM}$ to find M.
- Use Semyonov Eqs. (39.15) and (39.16) to find the coordinates of M by using the CoB and $\overline{BM}$

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3.3 Curve of Statical Stability

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3.3 Curve of Statical Stability

- Biran. 2nd Edition. Compare righting arm with heeling arm to determine if the righting arm is sufficient.

5.3 The Curve of Statical Stability

The plot of the righting arm, $\overline{GZ}$, calculated from Eq. (5.2), as function of the heel angle, ϕ, at constant ∇ and $\overline{KG}$ values is called **curve of statical stability**. Such diagrams are used to evaluate the stability of the ship in a given loading condition. For a full appreciation it is necessary to compare the righting arm with the various **heeling arms** that can endanger stability. We discuss several models of heeling arms in Chapter 6. An example of statical-stability curve is shown in Figure 5.4; it is based on Table 5.1. The table can be calculated in an electronic spreadsheet, or in MATLAB as shown in Biran and Breiner (2002), Example 2.9. Slide #22

Let us identify some properties of the righting-arm curves. One important value is the maximum $\overline{GZ}$ value and the heel angle where this value occurs. For example, in Figure 5.4 the maximum righting-arm value is 1.009 m and the corresponding heel angle is 50°. Another important point is that in which the $\overline{GZ}$ curve crosses zero. The corresponding ϕ value is called **angle of vanishing stability**. In our example the righting-arm curve crosses zero at an angle greater than 90°, in a region outside the plot frame. The angle of vanishing stability can often occur at less than 90°, as shown, for example, in Figure 6.23. See Slide #23

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Important Characteristics of the Statical Stability Curve

1. Max value of $\overline{GZ}$
2. Angle at which the max of $\overline{GZ}$ occurs
3. Angle of vanishing stability at which $\phi > 0$ and $\overline{GZ} = 0$.
4. Slope of the $\overline{GZ}$ curve at the origin. It is $\overline{GM}$.
5. All IMO requirements

A very useful property refers to the tangent in the origin of the righting-arm curve. The slope of this tangent is given by:

Error in Biran.

$$\tan\alpha|_{\phi=0} = \frac{d[\overline{GM}\sin(\phi)]}{d\phi}$$

$$|\tan\alpha|_{\phi=0} = \left|\frac{d(\overline{GZ}\sin\phi)}{d\phi = 0}\right|_{\phi=0} = \frac{d\overline{GM}}{d\phi}\sin 0 + \overline{GM}_0\cos 0 = \overline{GM}_0 \qquad (5.4)$$

Equation (5.4) yields a simple rule for drawing the tangent:

In the curve of statical stability, at the heel angle 1 rad (approximately 57.3°) draw a vertical and measure on it a length equal to that of $\overline{GM}$. Draw a line from the origin of coordinates to the end of the measured segment. This line is tangent to the $\overline{GZ}$ curve.

From the triangle formed by the heel-angle axis, the vertical at 1 rad, and the tangent in origin, we find the slope of the line defined as above; it is equal to $\overline{GM}/1$, that is the same as yielded by Eq. (5.4). The tangent in the origin of the righting-arm curve should always appear in the curve of statical stability; it gives an immediate, visual indication of the $\overline{GM}$ magnitude, and it is a check of the correctness of the curve. We strongly recommend **not** to try the inverse operation, that is to "fit" a tangent to the curve and measure the resulting $\overline{GM}$ value. This would amount to graphic differentiation, a procedure that is neither accurate nor stable.

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3.3 Statical Stability Curve

- $\overline{GZ}$ curve passes through the origin. There is no righting lever when the ship is upright

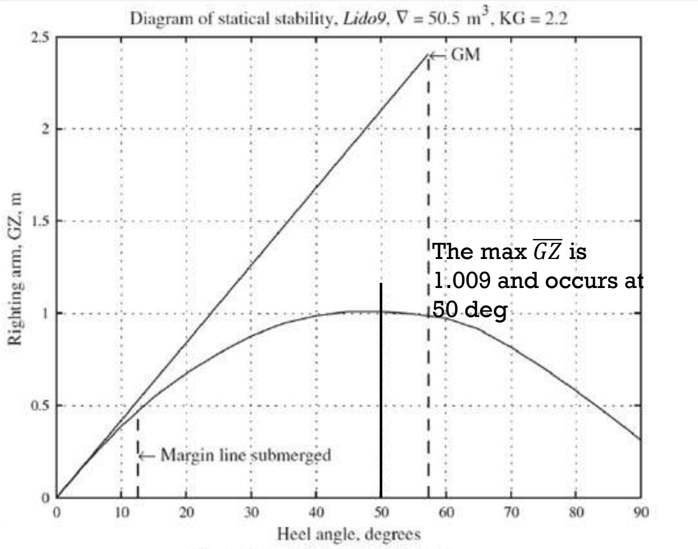

Figure 5.4 Statical-stability curve

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- Different types of statical stability curves

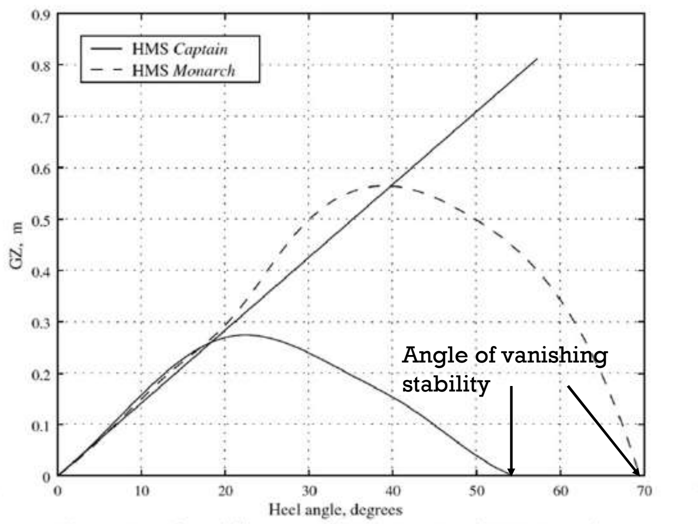

Figure 6.24 The stability curves of HMS *Captain* and HMS *Monarch*

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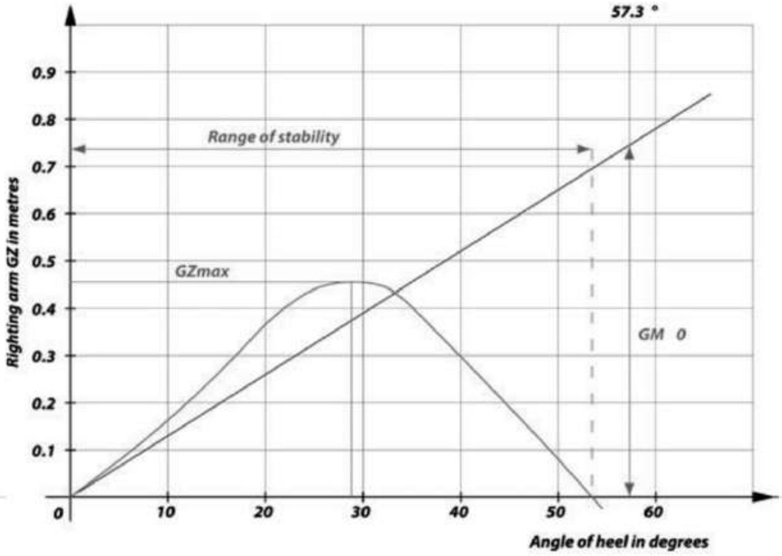
- The range of positive stability: Is the range in degrees between the upright equilibrium angle and angle of vanishing stability.



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3.3 Statical Stability Curve for +ve and –ve angles

- Heeling arm and righting arm for heel to port and starboard
- Note that $\overline{GZ}$ is negative when the ship heels to port. Couple = vector.

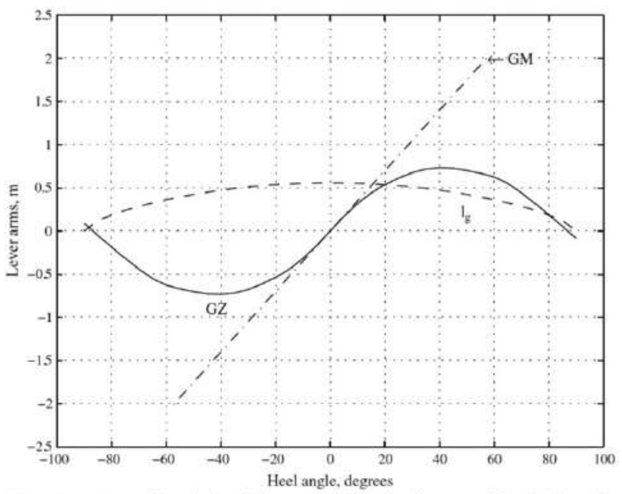

Figure 6.2 Curve of statical stability extended for heeling toward both ship sides

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Lester. Merchant Ship Stability.

- Find the value of $\overline{GM}$ and mark it in the $\overline{GZ}$ figure. Do NOT find $\overline{GM}$ by measuring the slope at the origin. Measurements are inaccurate.

FEATURES OF *GZ* CURVES

Initial slope and *GM*

A knowledge of the initial *GM* can be used to determine the slope of the origin of the *GZ* curve.

In *Figure 6.6, AD* is a line drawn as a tangent to the origin of the *GZ* curve. *AD* cuts an ordinate *DE* erected at 57.3° heel. *BC* is drawn close to the origin at angle θ.

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Slope at the origin of the $\overline{GZ}$ curve

Find the value of GM and mark it in the GZ figure.
Do NOT find GM by measuring the slope at the origin. Measurements are inaccurate.

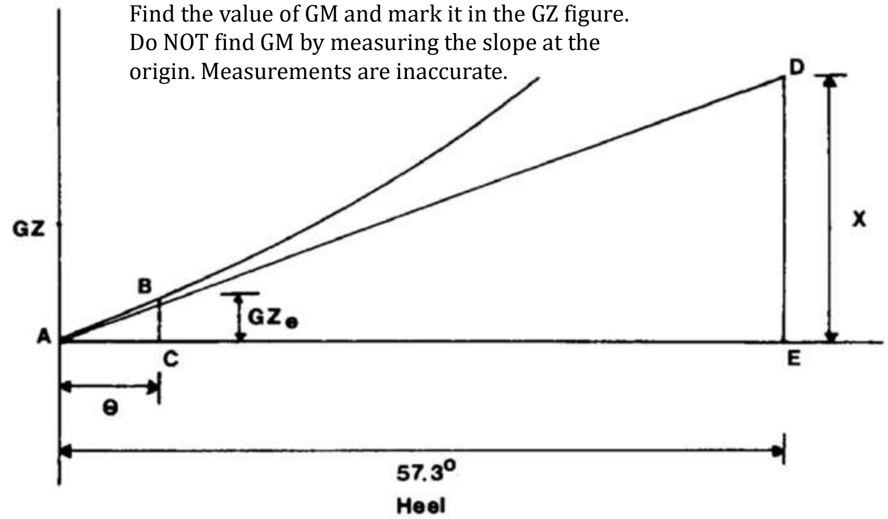

Figure 6.6 Relationship between GM and GZ curve

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Lester. Merchant Ship Stability.

Find the value of GM and mark it in the GZ figure.
Do NOT find GM by measuring the slope at the origin. Measurements are inaccurate.

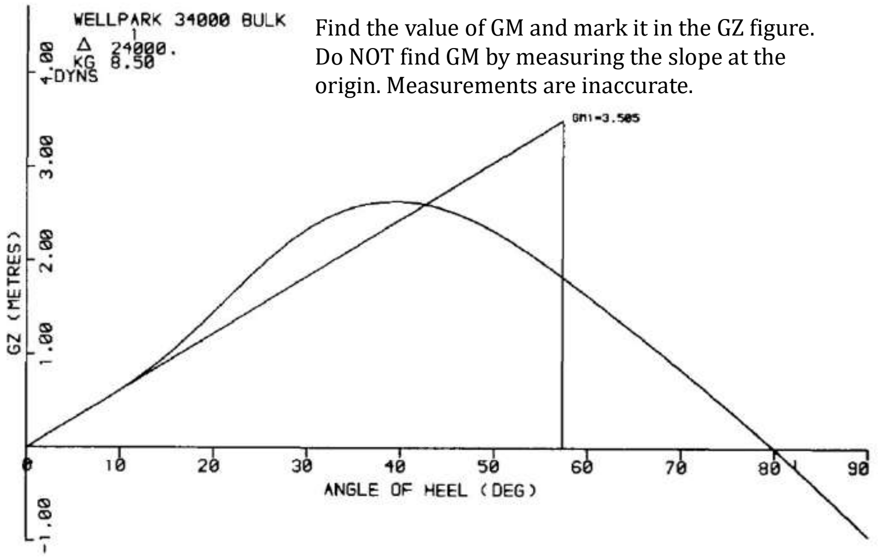

Figure 6.4 GZ curve of a typical stable vessel

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- For each angle of heel, estimate the CoB. The buoyancy force acts through the CoB and is perpendicular to the sea surface. Use this to estimate the coordinates of Z.
- Critically study the figures.

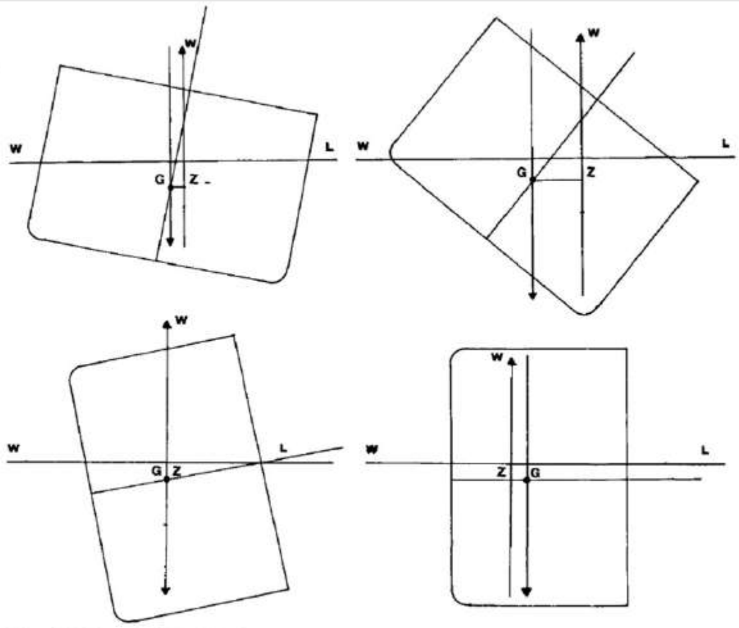

Figure 6.5 Change in GZ with heel

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GZ curve: assumed that G does not shift

Deck edge immersion is indicated by the point of flexure of the curve, although the exact point of flexure will depend upon sheer and position of superstructure.

Maximum GZ and the angle at which it occurs can be found by inspection. The range of stability can also be found by inspection.

It must be emphasized that only the early part of the curve up to say 40° heel can be regarded as giving a reasonable representation of the actual GZ value, as in practice at very large angles of heel it is probable that:

(a) Cargo will have shifted.
(b) Equipment will have broken loose.
(c) Water will have entered the vessel.

Thus making invalid the assumption that G does not shift.

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Glossary

- Angle of flooding – Downflooding angle related to intact stability is the angle of heel at which the lower edge of openings in the hull, superstructures or deckhouses that cannot be closed weathertight immerse.
- Air inlets to the Engine Room must be always open and the downflooding angle for the intact stability shall be calculated taking into account these openings. Some types of dangerous cargoes require continuous ventilation; in such cases it is necessary check the downflooding angle also for hold ventilation openings.

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Glossary

- Angle of list, list – A steady angle of heel created by forces within the ship. For example, when the ship is inclined due to her asymmetric construction, or by shifting a weight transversely within the ship. The list reduces the ship's stability. Therefore, it is essential to keep the ship upright at all times by a symmetrical distribution of masses.
- Angle of loll – The angle at which a ship with a negative initial metacentric height will lie at rest in still water. In a seaway, such a ship will oscillate between the angle of loll on SB and the one on PS. Depending upon external forces such as wind and waves a ship may suddenly flop over from PS to SB and then back again to PS. Such abrupt oscillation, different from a continuous roll, is characteristic for negative metacentric heights.
- An angle of loll can be corrected only by lowering the centre of gravity, not by moving loads transversely. This can be done by moving weight downwards, adding water ballast in double bottom tanks or removing weight above the ship vertical centre of gravity. Where empty ballast tanks are available these will afford the simplest means of lowering the ship's centre of gravity. The correct procedure is to add ballast on the low side of the ship. The first effect will be to increase the angle of heel and to cause a loss of stability due to the free surface of the water, but this effect is soon cancelled and the angle of heel will rapidly decrease.

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Glossary

- **Cross curves of stability** – Cross curves of stability is a set of curves from which the KN values for a set of constant heel-angle values at any particular displacement may be read. Thus, we have a curve for heel angle of 10o, next for 20o and so on. To find KN values for a given displacement volume it is necessary to draw a vertical line and read the values where this line crosses the curves. Nowadays KN values in tabulated forms are used instead of curves
- Curve of minimum operational metacentric height GM – During ship design stage many calculations are made in order to define minimum metacentric height GMMIN values, which would meet all intact and damage stability criteria. Based on these values a set of curves can be prepared for relevant draught range. The limiting envelope curve presents minimum operational metacentric heights which meet all stability criteria for draught range from lightship draught to maximum draught. To obtain accurate guidance for stability of a ship it is enough to calculate the actual metacentric height GM corrected for free surface effects and check whether it is higher or equal to the required GM_MIN value.

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- Curve of the maximum allowable vertical centre of gravity KG – According to SOLAS, the Master of the ship shall be supplied with reliable information to enable him assessment of the ship stability. Having the curve of the maximum allowable vertical centre of gravity versus draught is enough to calculate actual KG of the vessel and compare the result with the value allowable for a given draught.
- Curve of transverse metacentres – A curve showing the height of the transverse metacenter above base line corresponding to any displacement.
- Curve of vertical centres of buoyancy – The projection on the plane of inclination of the locus of the centre of buoyancy for varying inclinations with constant displacement.

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Glossary

- Dynamical stability – The dynamical stability of a ship at any inclination is defined as the work done in heeling the vessel to that inclination.
- Freeboard – Freeboard is the distance measured from the waterline to the upper edge of the deck plating at side of the freeboard deck amidships

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31 Jan 2025

Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406
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3 credits

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
   Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
   Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
   Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
   Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
   Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
   Intact stability rules, Heel/ Load test.
   Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

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removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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Course Content. Module 3.

Earlier

Module 1. Stability Terms. 06 Lectures. Google forms test 10 Jan 25.

Module 2. Initial (Transverse) Stability. 07 lectures. Google forms test 29Feb25

Module 3. Large Angle Transverse Stability

- 3.1 Large change in the attitude. Equivoluminal change. yCoB.
- 3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.
- 3.3 Curve of Statical Stability

Today

- 3.4 Effect of various factors on $\overline{GZ}$

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Effect of various factors on $\overline{GZ}$

Effect of form on GZ curve. Freeboard.

EFFECT OF FORM ON GZ CURVE

The GZ curves used to illustrate this section are based on computer outputs for the GZ curves of box shapes.

1. *Change in freeboard* (*Figures 6.15 and 6.16*)

Suppose a vessel has centre of gravity at G_0 and freeboard f_0, can have additional freeboard added to give freeboard f_1 with the draft remaining constant and the centre of gravity remaining at G_0. (G can remain fixed if weight is redistributed.)

Then if the vessel is heeled by an external force the initial shape of the curve will be unchanged. However, the angle of deck edge immersion will be delayed for the vessel with high freeboard. Thus the curve will continue to rise, until the larger angle of deck edge immersion. There is a considerable increase in max. GZ, the range of stability is increased and at large angles of heel the dynamical stability in increased. The improved stability at very large angles can be accounted for by considering the increased width of waterplane.

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Lester

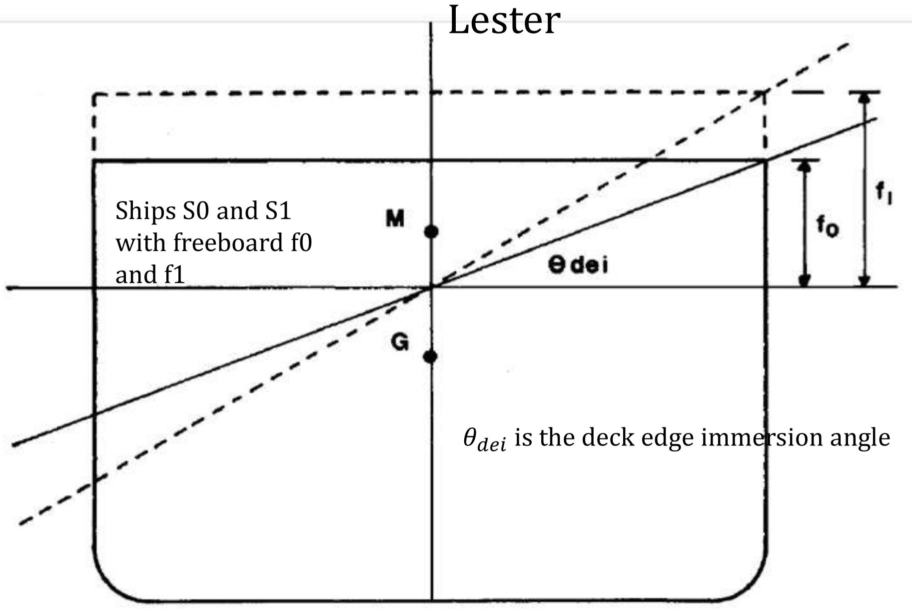

Figure 6.15 Effect of stability of increasing freeboard

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- There is no difference between the two GZ curves upto the angle of deck edge immersion of S0.
- Changing the freeboard has no effect on GM if the values of all the other parameters are the same

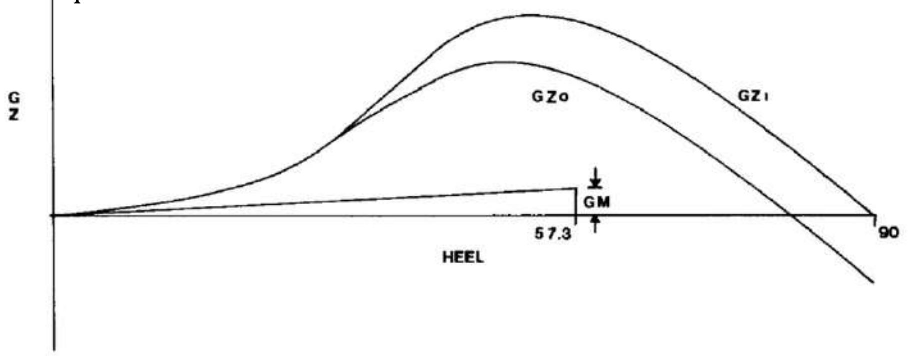

Figure 6.16 Effect on GZ curve of increasing freeboard

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Table 6.1

Effect of increased freeboard on Displacement, etc.	*Increased freeboard*	*Even super-structure*	*Uneven super-structure*	*Increased beam*	*Increased length*
Displacement	No change	No change	No change	Increase	Increase
GM	No change	No change	No change	Increase	No change
Deck edge immersion angle	Increased	Effective increase	Effective increase	Reduced	No change
Max. *GZ*	Increased	Increased	Increased	Small increase	No change
Range of stability	Increased	Increased	Increased	Possibly increased or decreased	No change
Righting moment	Increase at large angles	Increase at large angles	Increase at large angles	Increase	Increase
Dynamical stability to 40°	Increase	Increase	Increase	Increase	Increase
Trim	No change	Small change	Large change	No change	No change
Angle of flooding	Increased	Possible small reduction	Possible large reduction	Possible reduction	No change

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Effect of the superstructure on the GZ curve

2. *Superstructure* (*Figure 6.17*)
Watertight superstructure has a similar effect to increased freeboard, provided the superstructure is distributed equally about the centre of buoyancy, i.e. forecastle and poop or uniformly stowed timber deck cargo, etc.

However, if the superstructure is not uniform, i.e. offshore supply vessels, there will be a considerable shift of the centre of buoyancy when the superstructure enters the water causing the vessel to trim. As in the figure the trim will be towards the part of the vessel with the lower freeboard. This change of trim due to shift buoyancy as a vessel is heeled is called free trim.

All vessels will trim to some extent as they are heeled, in most cases the effect is ignored in presenting *GZ* curves. The use of computers makes the calculation of free trim practical and some programmes for calculating *GZ* take free trim into account.

- Poop. The name originates from the French word for stern, la poupe, from Latin puppis. Thus the poop deck is technically a stern deck, which in sailing ships was usually elevated as the roof of the stern or "after" cabin, also known as the "poop cabin".

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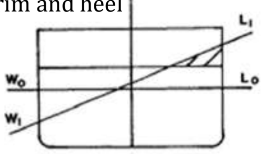
Effect of superstructure on stability

When the ship heels because of an external moment, the hatched forward superstructure area provides additional buoyancy. B moves forward and a restoring moment acts to correct the trim and heel

The draft aft < draft fwd



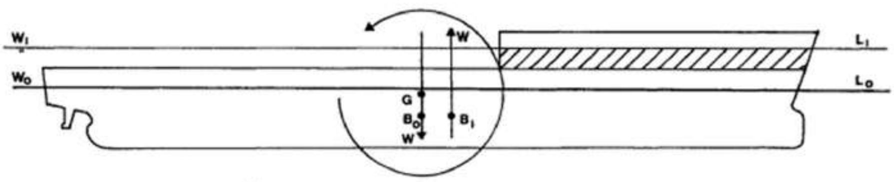


Figure 6.17 Effect of superstructure on stability

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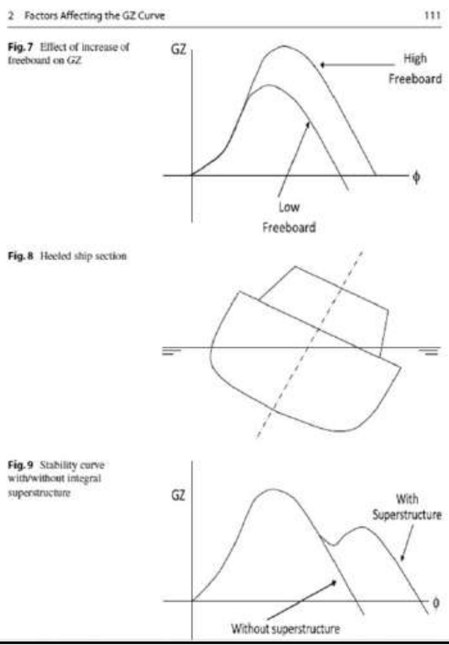
Effect of superstructure on stability

- Wilson. Basic Naval Architecture.



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Effect of the breadth on the GZ curve

3. *Increase in beam* (*Figures 6.18 and 6.19*)
Suppose a vessel with centre of gravity at G_0 and beam B_0 has its beam increased to B_1. In this case the centre of gravity can remain at G_0. However displacement must be increased. Since the width of the waterplane is increased, the inertia of the waterplane must be increased, hence the metacentre will rise to M_1. The initial slope of the GZ curve will be increased. However the angles of deck edge immersion will be earlier, and thereafter the slope of the curve will be reduced.

At large angles the waterplane is not greatly changed, there is little change in stability at these angles, the curves coincide at some very large angle. In the special case of a box shape the curves intersect at 90°; for most other vessels the point of intersection will be at some angle less than 90°.

Since displacement has been increased there will be an increase in righting moment.

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Effect of the breadth on the *GZ* curve

- Cuboidal ships S0 and S1 have length L, draft T, and zCoG = KG. S0 has breadth B0. S1 has breadth B1 = 1.1B0. If S0 has metacentric height GM0, find the metacentric height for S1.

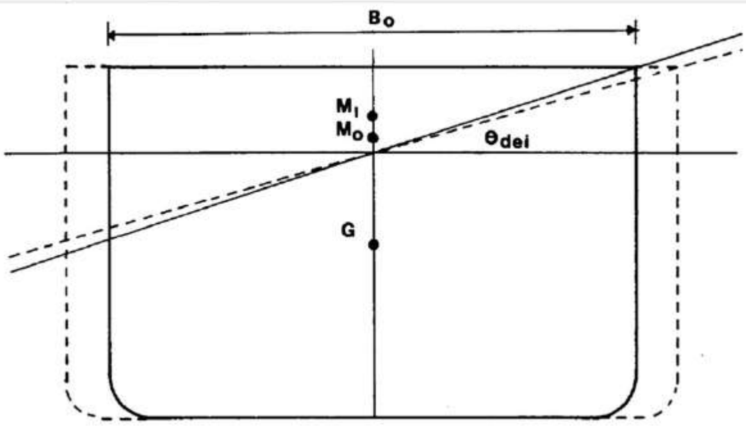

Figure 6.18 Effect on stability of increasing beam

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Effect of the breadth on the GZ curve

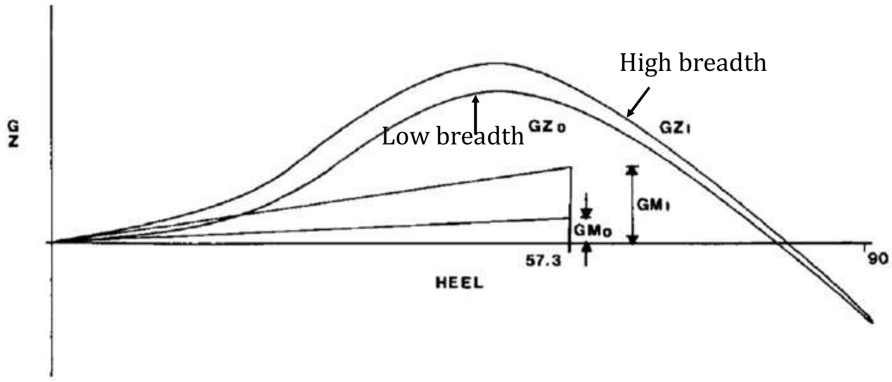

Figure 6.19 Effect on GZ curve of increasing beam

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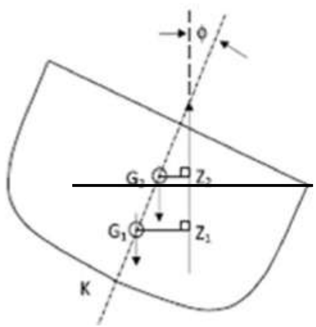
Effect of Length on GZ

4. *Length*

Length can be increased without altering the position of G and there will be no effect on the value of KM for box shapes and little effect for ship shapes. There will be no change in the angle of deck edge immersed, the shape of the GZ curve will be little changed. However, displacement must be increased thus increasing both righting moment and dynamical stability. The effects of changed form can be summarized in *Table 6.1*.

- Cuboidal ships S0 and S1 have breadth, B, draft T, and zCoG = KG. S0 has length L0. S1 has length L1 = 1.1L0. Find KM for both the ships.
- If S0 has metacentric height GM0, find the metacentric height for S1.

- See fig for deck immersion angle.
- Assignment. Explain why a change in the length will have no effect on the value of KM for box shapes and little effect for ship shapes.



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3.5c Effect of shifting weights on the GZ curve

- Earlier, we have studied the effect of shifting weights on the CoG and the CoB of the vessel.

EFFECT OF SHIFTING WEIGHTS WITHIN THE VESSEL

Vertical shift of weight

In *Figure 6.20* a vertical shift of weight has caused the centre of gravity to rise from G_0 to G_1. If the vessel is heeled to some angle θ then the value of the righting lever has been reduced from G_0Z_0 to G_1Z_1.

$$G_1Z_1 = G_0Z_0 - G_0X$$
$$G_0X = G_0G_1 \sin\theta$$
$$G_1Z_1 = G_0Z_0 - G_0G_1 \sin\theta$$

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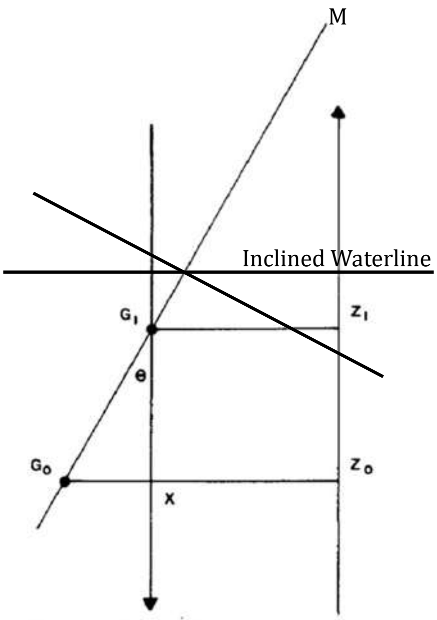

Figure 6.20 Effect on GZ of moving G vertically

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3.5c Effect of change in KG on GZ

and if the centre of gravity is lowered

$$G_1Z_1 = G_0Z_0 + G_0G_1 \sin\theta$$

The effect on stability is shown in *Figure 6.21*.

The rise in G therefore reduces the range of stability, the maximum GZ and reduces GM by G_0G_1.

In *Figure 6.22* the effect of $G_0G_1 > G_0M$ is shown. During the early part of the curve

$$G_0G_1 \sin\theta > G_0Z$$

hence the vessel is lolling. The vessel reaches the angle of loll when

$$G_0G_1 \sin\theta = G_0Z_0$$

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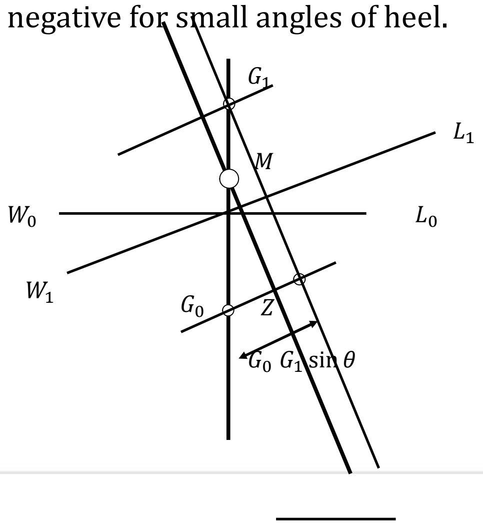
Effect of large increase in KG on GZ

- G_0 is below M and GZ is positive for small angles of heel.
- G_1 is above M and GZ is negative for small angles of heel.



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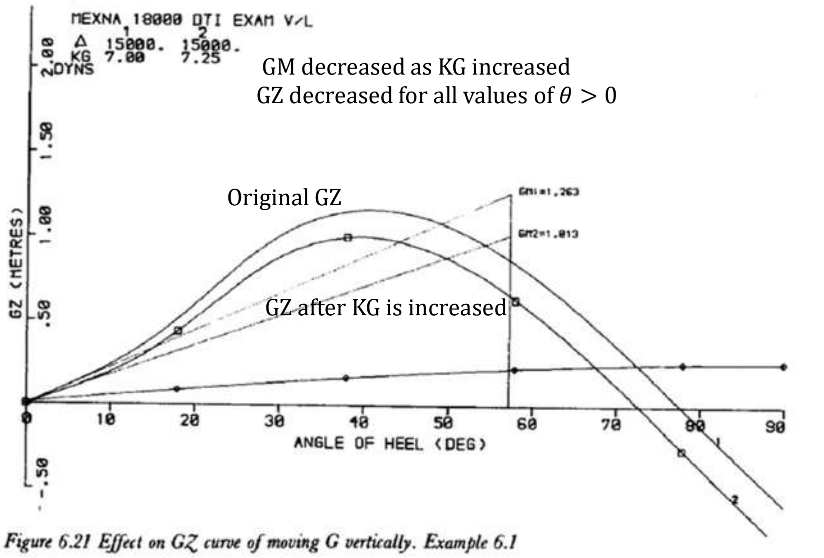

Figure 6.21 Effect on GZ curve of moving G vertically. Example 6.1

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Angle of Loll

In *Figure 6.22* the effect of $G_0G_1 > G_0M$ is shown. During the early part of the curve

$$G_0G_1 \sin \theta > G_0Z$$

hence the vessel is lolling. The vessel reaches the angle of loll when

$$G_0G_1 \sin \theta = G_0Z_0$$

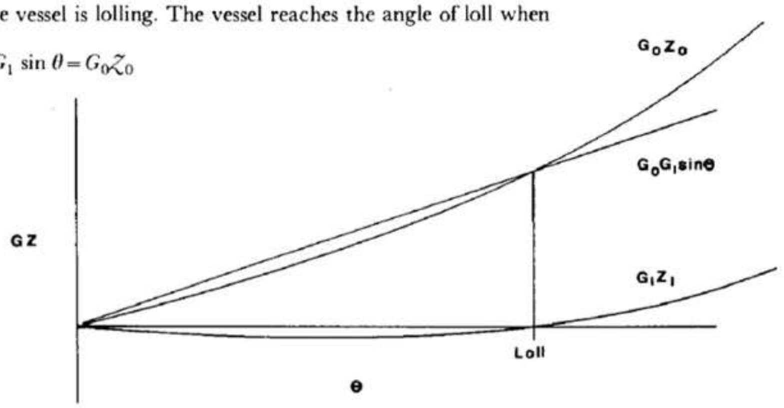

Figure 6.22 Shift of weight vertically causing instability

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Effect of Horizontal Shift of Weight

- See Fig. 6.23 in the next slide. The weight has moved to starboard.

Horizontal shift of weight

In *Figure 6.23* a horizontal shift of weight has caused the centre of gravity to move from G_0 to G_1. If the vessel is heeled to some angle θ then the value of the righting lever has been reduced from G_0Z_0 to G_1Z_1.

$$G_1Z_1 = G_0Z_0 - G_0X$$
$$G_0X = G_0G_1 \cos\theta$$
$$G_1Z_1 = G_0Z_0 - G_0G_1 \cos\theta$$

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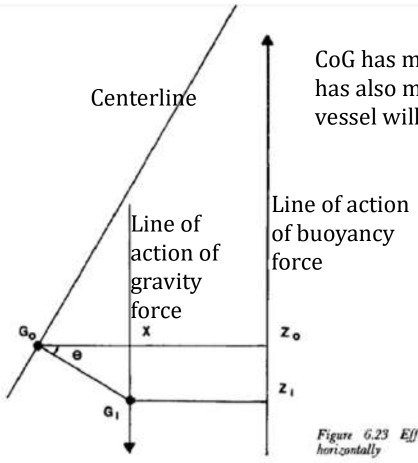

CoG has moved to starboard. So B has also moved to stbd. The vessel will list

Figure 6.23 *Effect on GZ of moving weight horizontally*

The effect on stability is shown in *Figure 6.24*. The value of GZ is reduced at all angles of heel. $\theta < 90$

The vessel moves to an angle of list where

$G_0G_1 \cos\theta = G_0Z_0$ Low side = side with higher draft = stbd

The range of stability on the low side will be reduced and dynamical stability will be reduced. The vertical separation between G and M when the vessel is upright is unchanged.

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The change in G causes a change in B and M. List occurs. B and M depend only on the underwater volume which depends on the weight distribution.

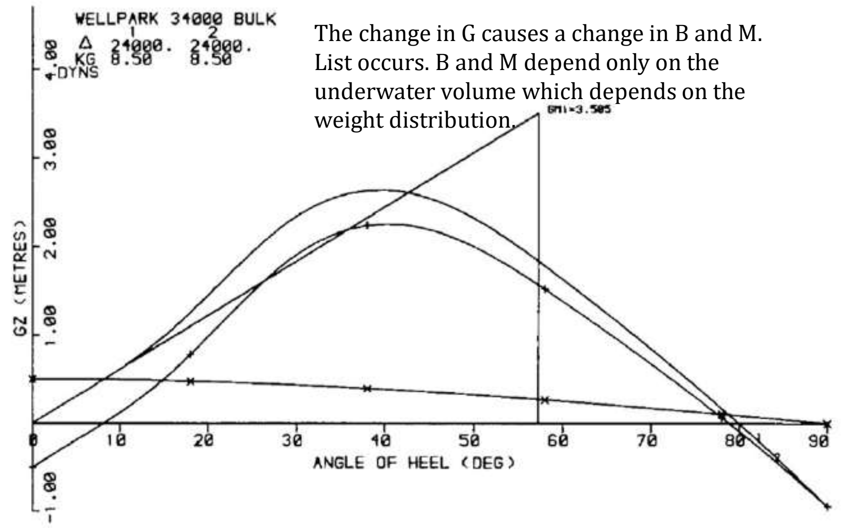

Figure 6.24 Effect on GZ curve of moving weight horizontally

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Effect of Shift of CG of ship on GZ Curve

- Where the CG is off the CL and the resulting statical stability curve is unsymmetrical, a plot of the stability curve for 90° each side of the upright gives a complete picture of the stability information
- The figure below shows the correction for both an upward and starboard movement of CG leading to such a plot
- The algebraic sum of the correction applied successively to the statical stability curve results in the final stability curve shown in the figure below and expressed as

GZ at an angle $\theta = G_0Z_0 - G_0G_1 \sin\theta - G_0G_3 \cos\theta$

In the next slide, note the asymmetry about $\theta = 0$ because of the horizontal shift in the weight

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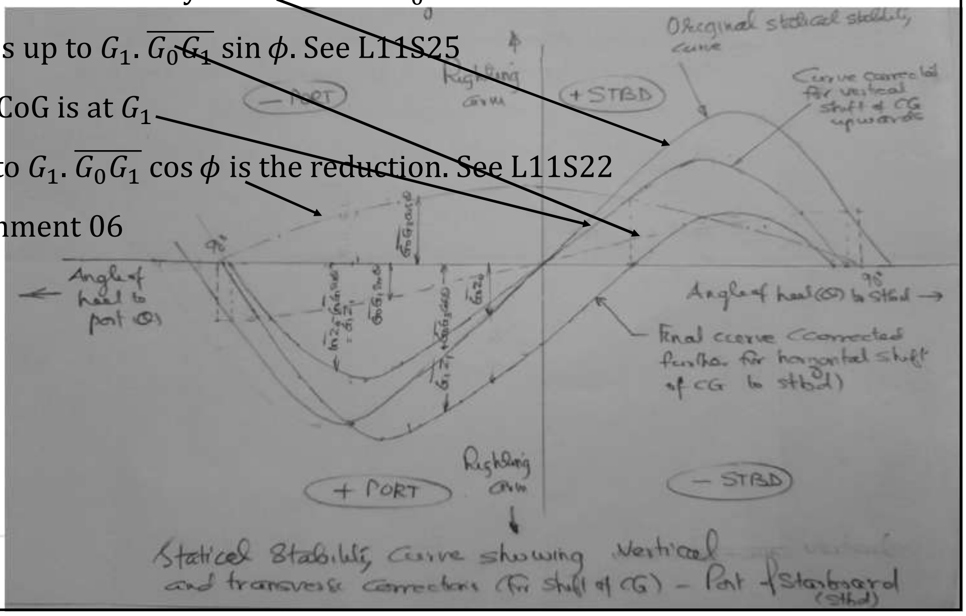
Effect of vertical and horizontal shift in CG

- Original statical stability curve. CoG at G_0.
- CoG moves up to G_1. $\overline{G_0G_1}\sin\phi$. See L11S25
- $\overline{GZ}$ when CoG is at G_1
- Hor shift to G_1. $\overline{G_0G_1}\cos\phi$ is the reduction. See L11S22
- See Assignment 06



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Lester. Example 6.1

Example 6.1 (*Figure 6.21*)

A vessel displacing 15 000 tonnes has *KG*, 7 m. Cargo is redistributed to cause *KG* to rise by 0.25 m. The values of *GZ* in the initial condition where

Heel	0	15	30	45	60	75	90	degrees
GZ	0.00	0.391	1.000	1.138	0.774	0.129	−0.584	m

2 Factors Affecting the GZ Curve 109

Fig. 3 Effects of increasing G on GZ

Wilson

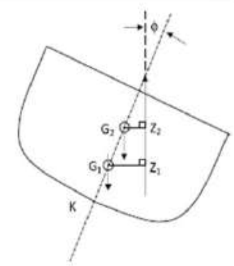
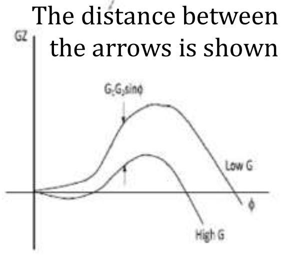


Fig. 4 Effect of position of G on GZ curve

Wilson

The distance between the arrows is shown



of the curve. If the shift upwards is large enough the vessel becomes unstable upright and an angle of loll arises (see Fig. 4).

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Lester. Example 6.1

Find for the initial and final condition
(a) The range of stability;
(b) The maximum GZ and the angle at which it occurs;
(c) The dynamical stability at 40°;
(d) An estimate of the GM.

$G_1Z_1 = G_0Z_0 - G_0G_1 \sin\theta$

Heel	0	15	30	45	60	75	90
G_0Z_0	0.000	0.391	1.000	1.138	0.774	0.129	−0.584 m
G_0G_1 sin heel	0.000	−0.064	−0.125	−0.177	−0.217	−0.242	−0.250 m
G_1Z_1	0.000	0.327	0.875	0.961	0.557	−0.113	−0.834 m

Row 2 = - $G_0\ G_1 \sin\theta$. Row3 = Row 1 + Row2.

G_0M_0 1.26 m Range 78°
G_1M_0 1.01 m Range 72°
Max. G_0Z_0 1.20 m at 42°
Max. G_1Z_1 1.00 m at 40°

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Effect of vertical shift of weight

Dynamical stability (initial)

Heel	*GZ*	*SM*	*F* (Area)
0	0.00	1	0
10	0.24	4	0.96
20	0.60	2	1.20
30	1.00	4	4.00
40	1.15	1	1.15
			7.31

$$\text{Area} = \frac{1}{3} \times h \times \sum F \text{ (Area)}$$
$$= \frac{1}{3} \times \frac{10}{57.3} \times 7.31 \text{ m radians}$$
$$= 0.43 \text{ in radians m}$$

Dynamical Stability = $W \times$ (Area under GZ curve)
= 15 000 × 0.43 tonne m
= 6450 tonne m

Dynamical stability (final)

Heel	*GZ*	*SM*	*F* (Area)
0	0.00	1	0.00
10	0.16	4	0.64
20	0.50	2	1.00
30	0.88	4	3.52
40	1.00	1	1.00
			6.16

$$\text{Area} = \frac{1}{3} \times h \times \sum F \text{ (Area)}$$
$$= \frac{1}{3} \times \frac{10}{57.3} \times 6.16 \text{ m radians}$$
$$= 0.36 \text{ m radians}$$

Dynamical stability = $W \times$ (Area under GZ curve)
= 15 000 × 0.36 tonne m
= ~~5700~~ 5400 tonne m.

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Effect of horizontal shift of weight

Example 6.2 (*Figure 6.25*)

A vessel displacing 12 000 tonnes has KG, 7.64 m. The values of GZ are as follows:

Heel	0	10	20	30	40	50	60	70	degrees
GZ	0	0.19	0.50	0.94	1.16	1.03	0.60	0.06	m

CG moves to stbd and not port

Cargo is redistributed so as to make the centre of gravity 0.13 m to port. Find the list and the dynamical stability from the list angle to 30°.

Heel	0	10	20	30	40	50	60	70
G_0Z_0	0	0.19	0.50	0.94	1.16	1.03	0.60	0.06
G_0G_1 cos	0.13	0.13	0.12	0.11	0.10	0.08	0.07	0.05
G_1Z_1	−0.13	0.06	0.38	0.83	1.06	0.95	0.53	0.01

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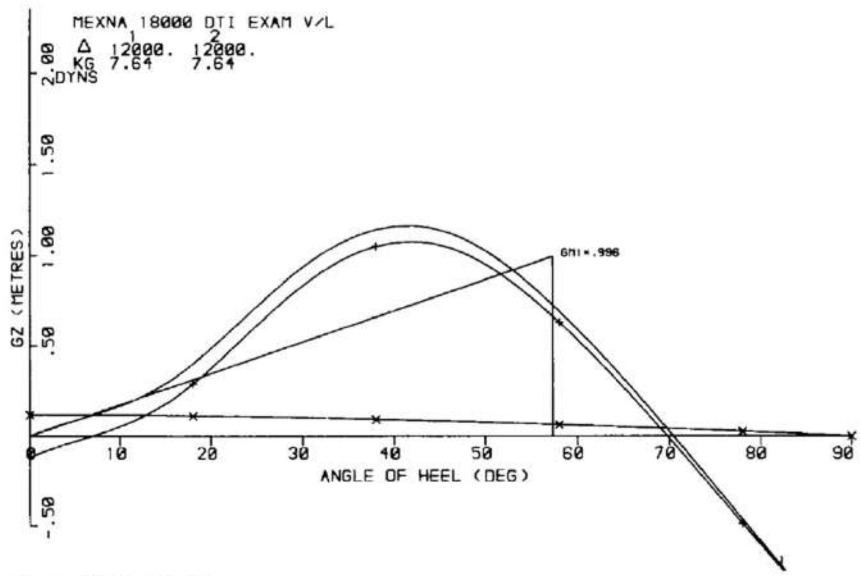

Figure 6.25 Example 6.2

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Ship at various angles of heel.
Illustrative figures.

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USNA: Heeling and Restoring couples

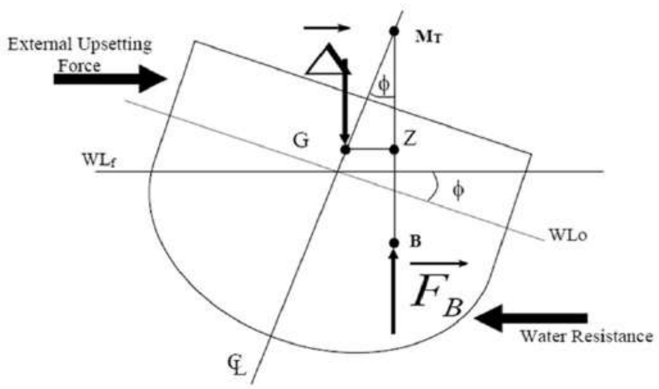

Figure 4.1 Heeled Ship due to an External Moment

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USNA: Curve of Intact Statical Stability

See Slide#36 and 37 for the attitude of the ship at Points A, B, C, D, and E

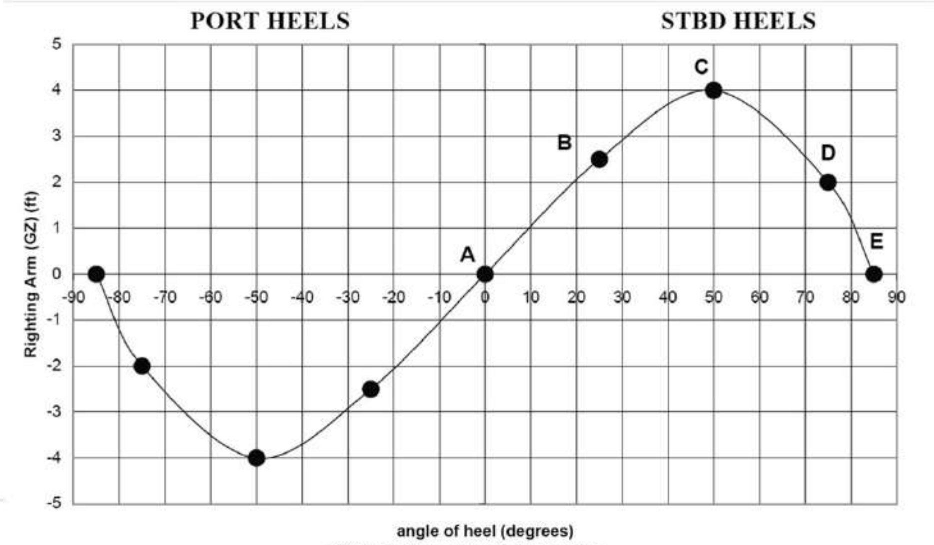

Figure 4.2 Curve of Intact Statical Stability

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USNA: Curve of Intact Statical Stability

M is shown to be on the centerline for all angles of heel. This is never the case.

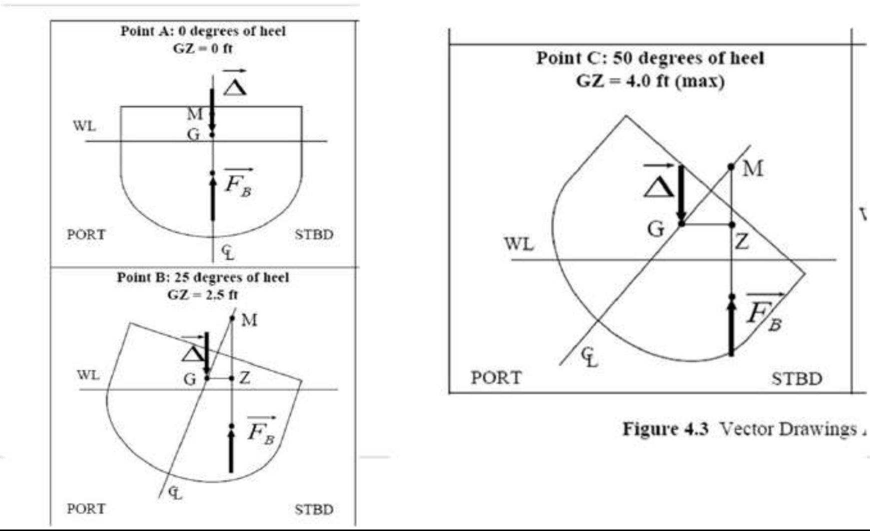

Figure 4.3 Vector Drawings

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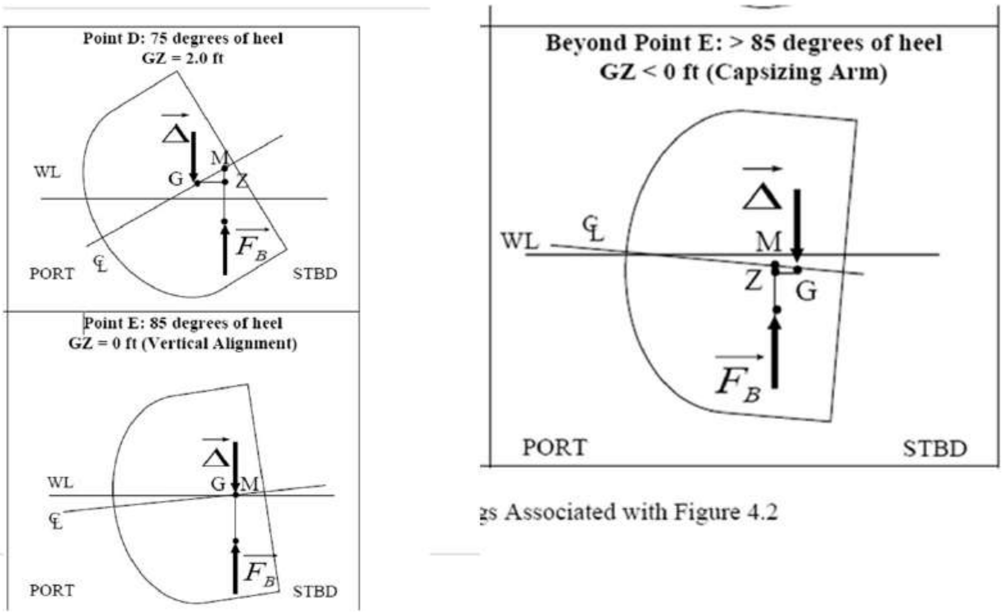
USNA: Curve of Intact Statical Stability



gs Associated with Figure 4.2

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Lester. Merchant Ship Stability.

Types of stability

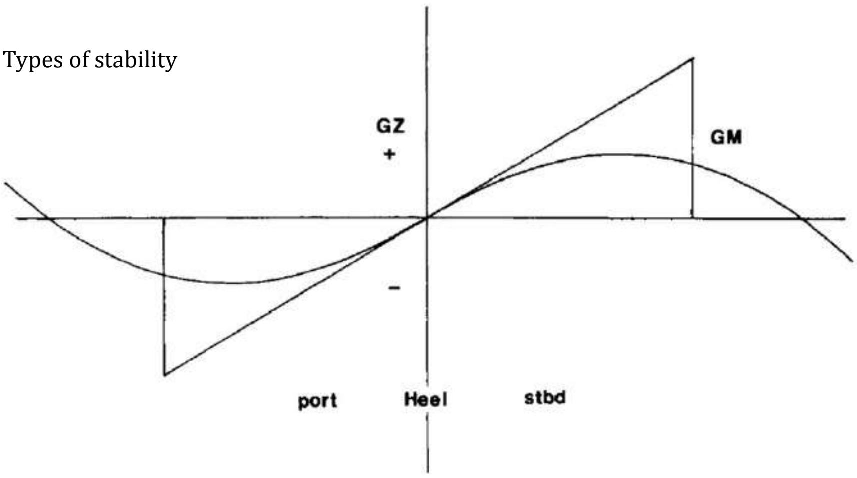

Figure 6.14(a) GZ curve to port and starboard of upright for stable vessel

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Lester. Merchant Ship Stability.

Types of stability

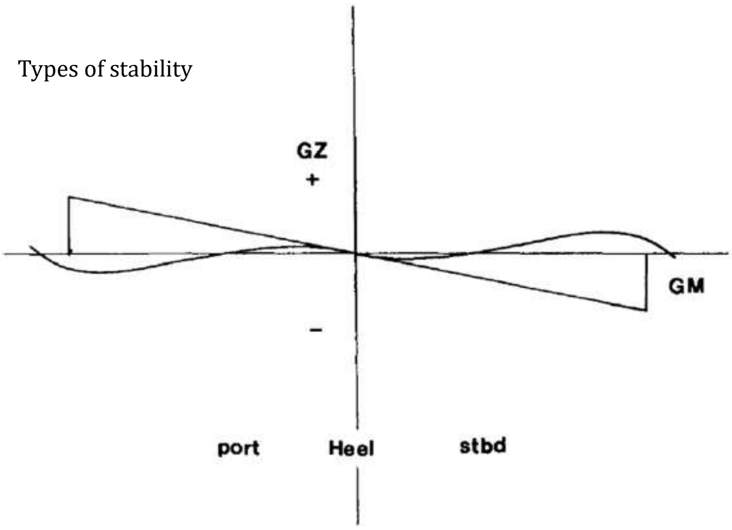

Figure 6.14(b) GZ curve to port and starboard for vessel in unstable equilibrium

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STABILITY AT LARGE ANGLES OF HEEL 135

Types of stability

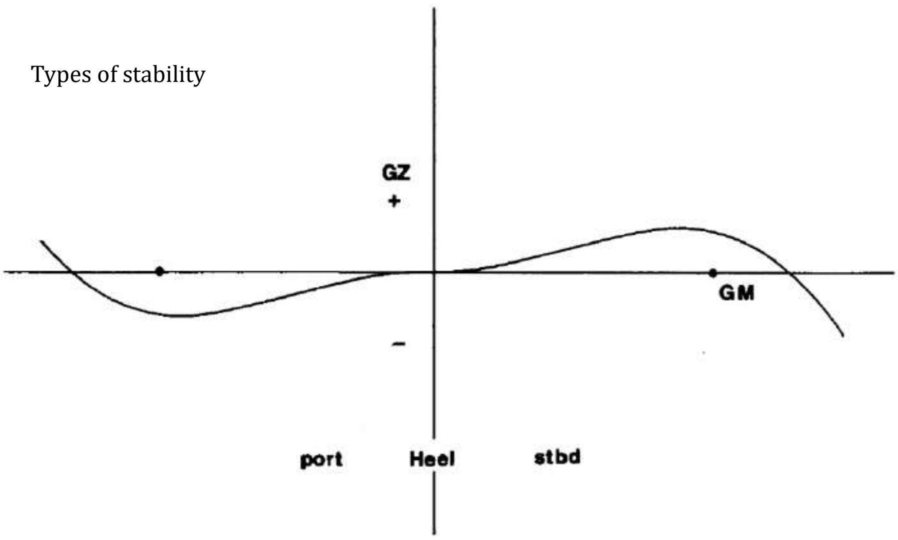

Figure 6.14(c) GZ curve to port and starboard for vessel in neutral equilibrium

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04 Feb 2025

Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406
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3 credits

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
   Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
   Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
   Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
   Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
   Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
   Intact stability rules, Heel/ Load test.
   Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

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removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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Course Content. Module 3.

Earlier

Module 1. Stability Terms. 06 Lectures. Google forms test 10 Jan 25.

Module 2. Initial (Transverse) Stability. 07 lectures. Google forms test 29Feb25

Module 3. Large Angle Transverse Stability

- 3.1 Large change in the attitude. Equivoluminal change. yCoB.
- 3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.
- 3.3 Curve of Statical Stability
- 3.4 Effect of various factors on $\overline{GZ}$

Today

- 3.5 IMO

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8.2 The IMO Code of Intact Stability

Biran

The *InterGovernmental Maritime Consultative Organization* was established in 1948 and was initially known as *IMCO*. The name was changed in 1982 to **IMO—International Maritime Organization.** The purpose of IMO is the intergovernmental cooperation in the development of regulations regarding shipping, maritime safety, maritime security, navigation, and the prevention of marine pollution from ships. IMO is a specialized agency of the United Nations; at the time of writing it has 170 Member States and three Associate Members. The headquarters are in London. A critical sketch of IMO's history can be found in Francescutto (2007).

ILLC = International Load Lines Convention

The main international instruments that address adequate buoyancy, subdivision (see Chapter 11 for this term) and stability are 'live' regulations that evolve with time and are made at IMO: the SOLAS convention, the ILLC and the IS code. Some provisions of the SOLAS convention are presented in Chapter 11, those of ILLC belong mainly to general ship design. In this chapter we refer to the IS code version issued in 2008 and entered into force the day of 1 July 2010 (see IMO, 2009). Part A of the code covers the **mandatory** criteria for 'ships and other marine vehicles of 24 m in length and above.' Part B of the code describes **recommendations** for particular sizes of certain types of ships and other marine vehicles not included in Part A, or recommendations intended to supplement the provisions of Part A for particular sizes or modes of operation. The 2008 IS Code applies to cargo ships and those carrying timber deck cargo, passenger ships, fishing vessels, special purpose ships, offshore supply vessels, mobile offshore drilling units (MODUs), pontoons, and containerships. Countries that adopted these regulations enforce them either directly, or by converting them into national ordinances.

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8.2.1 General Mandatory Criteria for Passenger and Cargo Ships

Part A of the code establishes the general criteria to be applied to all loading conditions taking into account the free-surface effect. There are two main criteria:

- the righting-arm curve ($\overline{GZ}$ curve);
- the severe wind and rolling criterion, known more as the **weather criterion**.

Area is important for dynamic stability

The required properties of the $\overline{GZ}$ $\boxed{\text{(in fact } \overline{GZ}_{eff}\text{)}}$ curve are:

1. The area under the righting-arm curve should not be less than 0.055 m rad up to 30°, and not less than 0.09 m rad up to 40° or the angle of downflooding if this angle is smaller than 40°. Additionally, the area under the righting-arm curve between 30° and 40°, or between 30° and the flooding angle, if this angle is less than 40°, should not be less than 0.03 m rad;
2. The righting arm, $\overline{GZ}$, shall be at least 0.2 m at an angle of heel equal to or greater than 30°;
3. The maximum righting arm should occur at an angle of heel not less than 25°. If this is not possible an equivalent criterion may be used with the approval of the Administration;
4. The initial metacentric height, $\overline{GM}_{eff}$, should not be less than 0.15 m.

- G_{eff} depends on free-surface effects and hanging loads
- The code uses frequently the terms **angle of flooding** and **downflooding**; they refer to the smallest angle of heel at which an opening in the hull, superstructure or deckhouse, that cannot be closed watertightly, submerges

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IMO General Mandatory Criteria for Passenger and Cargo Ships

The area under the righting arm curve $\overline{GZ}$

1a. Should not be less than 0.055 m rad from 0 to 30 deg

1b. Should not be less than 0.09 m rad from 0 to 40 deg

1c. Should not be less than 0.03 m rad from 30 to min(40 deg, flooding angle)

2. $\overline{GZ}$ should be at least 0.2 m at heel angle ≥ 30 deg
3. The max in the $\overline{GZ}$ should occur at heel angle ≥ 25 deg
4. $\overline{GM_{eff}} \geq 0.15$ m

RESOLUTION MSC.267(85)
(adopted on 4 December 2008)

ADOPTION OF THE INTERNATIONAL CODE ON INTACT STABILITY, 2008
(2008 IS CODE)

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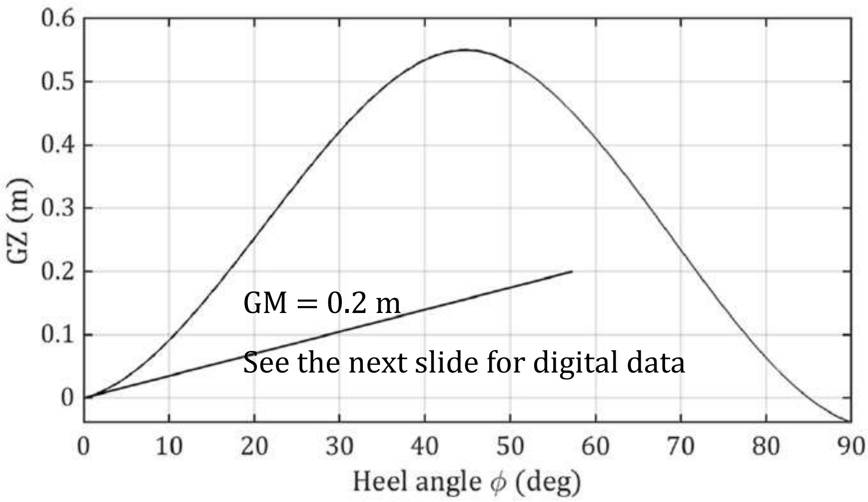

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Data for $\overline{GZ}$ curve on the previous slide

$\phi = [0 \quad 10 \quad 20 \quad 30 \quad 35 \quad 40 \quad 50 \quad 60 \quad 70 \quad 80 \quad 85 \quad 90]$

$\overline{GZ} = [0 \quad 0.0911 \quad 0.2526 \quad 0.4213 \quad 0.4887 \quad 0.5342 \quad 0.5308 \quad 0.4108 \quad 0.2342 \quad 0.0635 \quad 0 \quad -0.0386]$

Home Work. Find the area under the GZ curve from (1) 0 to 30 deg (2) 0 to 40 deg. (3) 30 to 40 deg. Are the IMO mandatory criteria met?

Ans. (1) 0.0951 m rad (2) 0.1797 m rad (3) 0.0847 m rad.

Yes. All the 3 criteria related to the question are met.

What is the range of stability? Ans. 85 deg.

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Biran. 2nd Ed.

- Do these curves satisfy the IMO requirements?
- At what angle is the GZ curve for HMS Captain maximum?
- What is the approximate area under the curve in m rad between
  (1) 0 and 30 deg?
  (2) 30 and 40 deg?
- 10 deg = 0.175 rad

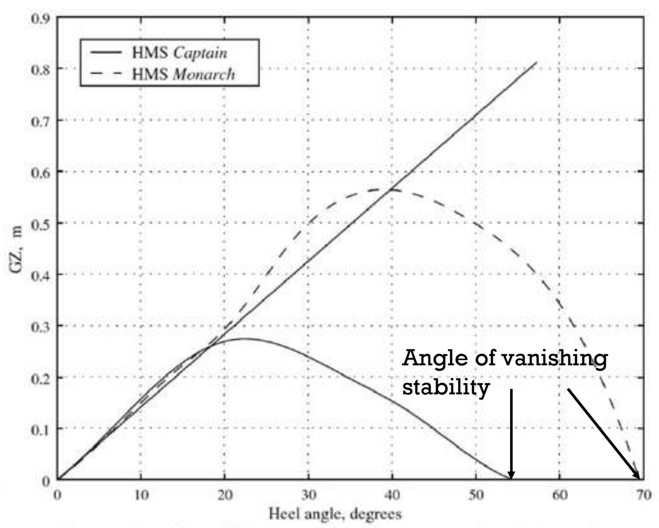

Figure 6.24 The stability curves of HMS *Captain* and HMS *Monarch*

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Glossary

- Angle of flooding – Downflooding angle related to intact stability is the angle of heel at which the lower edge of openings in the hull, superstructures or deckhouses that cannot be closed weathertight immerse.
- Air inlets to the Engine Room must be always open and the downflooding angle for the intact stability shall be calculated taking into account these openings. Some types of dangerous cargoes require continuous ventilation; in such cases it is necessary check the downflooding angle also for hold ventilation openings.

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Dynamic Stability and IMO

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Biran. Chap 6.6. Dynamical Stability

- Heeling moments can be caused by wind, by the centrifugal force in turning, by crowding of passengers on one side, by towing, or by the tension in the cable that links two vessels during operations at sea. Dividing a heeling moment by the displacement force we obtain a heeling arm. Heeling arms intersect the curve of statical stability at two points corresponding to angles of statical stability; only the first one is stable. See L17S14.
- Certain loads can reduce the stability and endanger the ship; they include laterally displaced loads, hanging loads, free-surfaces of liquids and shifting loads. When a ship is grounded or docked, and the water level descends, there is a critical point beyond which the sip capsizes.

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- ϕ_{st1} and ϕ_{st2} are static equilibrium points. If the heeling moment is applied very slowly, the ship will stop heeling at these points where the heeling moment = righting moment
- Only ϕ_{st1} is a stable point. If the ship is disturbed from this point, it will return to it.

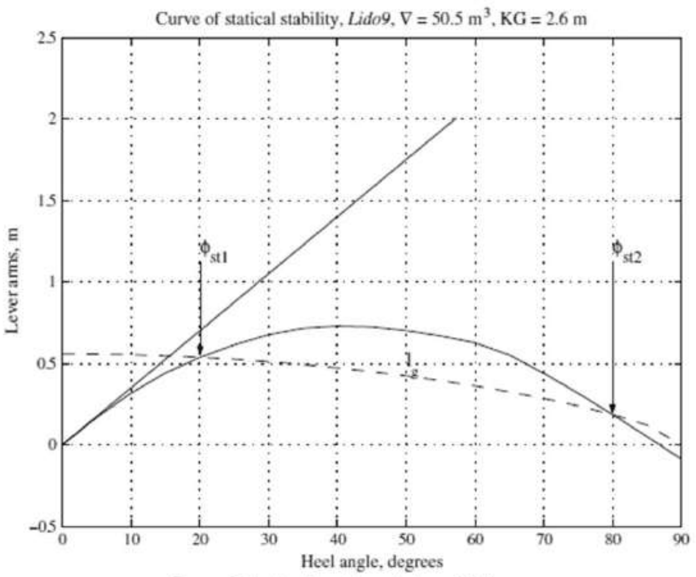

Figure 6.1 Angles of statical equilibrium

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Biran. Chap 6.6. Dynamical Stability

- Until now we assumed that the heeling moments are applied gradually and that inertial moments can be neglected. In brief, we studied statical stability. Heeling moments, however can appear, or increase suddenly. For example, wind speed is usually not constant, but fluctuates. Occasionally, sudden bursts of high intensity can occur; they are called gusts. As another example, losing a weight on one side of a ship can cause a sudden heeling moment that sends down the other side of the ship. In the latter cases we are interested in dynamical stability. It is no more sufficient to compare righting with heeling arms; <u>we must compare the energy of the heeling moment with the work done by the opposing righting moment</u>. It can be easily shown that the energy of the heeling moment is proportional to the area under the heeling-arm curve, and the work done by the righting moment is proportional to the area under the righting-arm curve. To prove this, let us remember that the work done by a force, F, which produces a motion from x1 to x2 is equal to …

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righting-arm curve. To prove this, let us remember that the work done by a force, F, which produces a motion from x_1 to x_2 is equal to

$$W = \int_{x_1}^{x_2} F dx \qquad (6.8)$$

If the path of the force F is an arc of circle of radius r, the length of the arc that subtends an angle $d\phi$ is $dx = r d\phi$. Substituting into Eq. (6.8) yields

$$W = \int_{\phi_1}^{\phi_2} F r d\phi = \int_{\phi_1}^{\phi_2} M d\phi \qquad (6.9)$$

The heeling moment continues to act as the ship heels from ϕ_1 to ϕ_2.

where M is a moment.

A ship subjected to a sudden heeling moment M_h, applied when the roll angle is ϕ_1, will reach for an instant an angle ϕ_2 up to which the energy of the heeling moment equals the work done by the righting moment, so that

Δ has units of mass. Force = mass times acc due to gravity

$$W = \int_{\phi_1}^{\phi_2} \frac{M_h}{g} d\phi = \int_{\phi_1}^{\phi_2} \overline{\Delta GZ} d\phi \qquad (6.10)$$

g has been brought from the RHS to the LHS

or

$$\int_{\phi_1}^{\phi_2} \frac{M_h}{g\Delta} d\phi = \int_{\phi_1}^{\phi_2} \overline{GZ} d\phi \qquad (6.11)$$

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Biran. Chap 6.6. Dynamical Stability

This condition is fulfilled in Figure 6.4 where the area under the heeling-arm curve is $A_2 + A_3$, and the area under the righting-arm curve is $A_1 + A_3$. As A_3 is common to both areas, the condition is reduced to $A_1 = A_2$. Moseley is quoted for having proposed the calculation of dynamical stability as early as 1850. It took several marine disasters and many years until the idea was accepted by the Naval-Architectural community.

In Figure 6.4 we marked with ϕ_{dyn} the maximum angle reached by the ship after being subjected to a gust of wind. An elegant way to find this angle is to calculate the areas under the curves as functions of the heel angle, ϕ, plot the resulting curves and find their points of intersection. The algorithm for calculating the integrals with variable upper limit is described in Section 3.4.

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- When the ship heels, the area that is exposed to the heeling force decreases and the heeling moment decreases
- Plot the righting and heeling areas. Find the point of intersection.

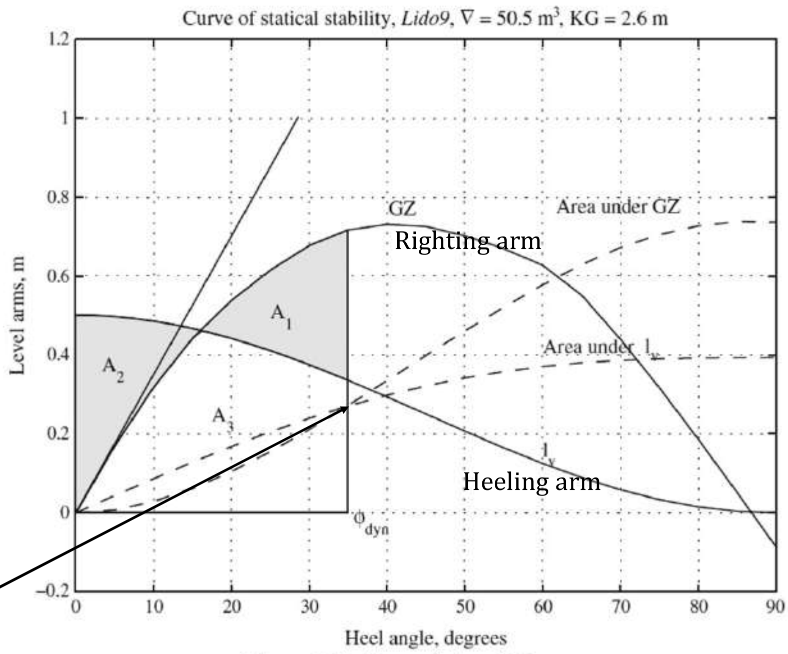

Figure 6.4 Dynamical stability

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Biran. Chap 6.6. Dynamical Stability

- See the next slide for Fig. 6.5

In Figure 6.4 we assumed that the gust of wind appeared when the ship was in an upright condition, that is $\phi_1 = 0$. As shown in Figure 6.5, the situation is less severe if $\phi_1 > 0$, and more dangerous if $\phi_1 < 0$. In both graphs the maximum dynamical angle is found by plotting the curve

$$\int_{\phi_1}^{\phi} \overline{GZ} d\phi - \int_{\phi_1}^{\phi} l_v d\phi$$

and looking for the point where it crosses zero. An analogy with a swing (or a pendulum) is illustrated in Figure 6.6. Many readers may have tried to accelerate a swing by pushing it

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ϕ_1 is the heel angle of the ship when the gust starts

- Top
  $\phi_1 > 0$
- Bottom
  $\phi_1 < 0$

Simple Models of Stability 137

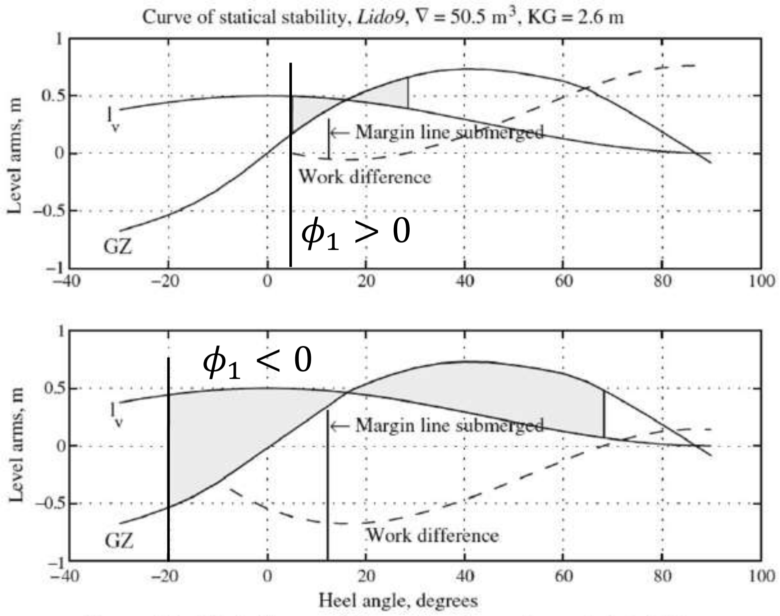

Figure 6.5 The influence of the roll angle on dynamical stability

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and looking for the point where it crosses zero. An analogy with a swing (or a pendulum) is illustrated in Figure 6.6. Many readers may have tried to accelerate a swing by pushing it periodically. Thus, they may know that a push given in position (a) sends the swing to an angle that is much larger than the angle achieved by pushing at position (b). Moreover, pushing the swing while it is in position (c) proves very difficult. The physical explanation is simple. In position (a) the energy transferred from the push is added to the potential energy accumulated by the swing, the latter energy acting to return the swing rightwards. In position (c) the potential energy accumulated by the swing tends to return it to position (b), opposing thus the energy impacted by the push. The influence of the roll angle on dynamical stability is taken into consideration by some stability regulations (see Chapter 8).

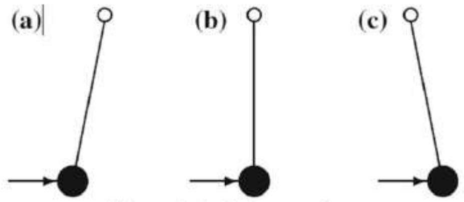

Figure 6.6 Swing analogy

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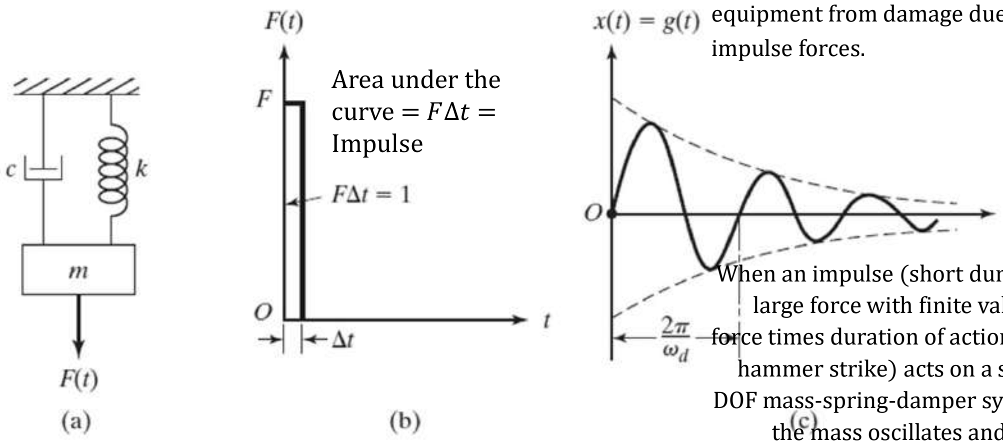
Response of a single degree of freedom system to an impulse

- S. S. Rao. Mechanical Vibrations. Chap. 4.5.1. When an impulse acts on a SDOF system, it responds as shown. A similar analysis can be done for a ship.

Stoppers are used to protect equipment from damage due to impulse forces.

Area under the curve $= F\Delta t =$ Impulse



When an impulse (short duration large force with finite value of force times duration of action. E.g. hammer strike) acts on a single DOF mass-spring-damper system, the mass oscillates and then comes to rest.

FIGURE 4.6 A single-degree-of-freedom system subjected to an impulse.

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Response of a single degree of freedom system to an impulse

- The general solution to the homogeneous, second order, ordinary differential equation has two coefficients, x_0 and $\dot{x}_0$, that are determined by using the initial conditions. x_0 = initial displacement. $\dot{x}_0$ = initial velocity.

We first consider the response of a single-degree-of-freedom system to an impulse excitation; this case is important in studying the response under more general excitations. Consider a viscously damped spring-mass system subjected to a unit impulse at $t = 0$, as shown in Figs. 4.6(a) and (b). For an underdamped system, the solution of the equation of motion

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{4.17}$$

is given by Eq. (2.72a) as

$$x(t) = e^{-\zeta\omega_n t}\left\{x_0 \cos \omega_d t + \frac{\dot{x}_0 + \zeta\omega_n x_0}{\omega_d} \sin \omega_d t\right\} \tag{4.18}$$

where

$$\zeta = \frac{c}{2m\omega_n} \tag{4.19}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} \tag{4.20}$$

$$\omega_n = \sqrt{\frac{k}{m}} \tag{4.21}$$

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Response of a single degree of freedom system to an impulse

- $x(t)$ is shown on L17S22

If the mass is at rest before the unit impulse is applied ($x = \dot{x} = 0$ for $t < 0$ or at $t = 0^-$), we obtain, from the impulse-momentum relation,

$$\text{Impulse} = \hat{f} = 1 = m\dot{x}(t=0) - m\dot{x}(t=0^-) = m\dot{x}_0 \tag{4.22}$$

Thus the initial conditions are given by

$$x(t=0) = x_0 = 0 \tag{4.23}$$

$$\dot{x}(t=0) = \dot{x}_0 = \frac{1}{m} \tag{4.24}$$

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Response of a ship to a gust of wind

- When a large wind force acts for a very short duration, or a load is dropped, an impulse acts on the ship. The oscillates and slowly comes to rest. Damping is provided by the sea and the energy that is input to the ship is dissipated. It is of interest to find the maximum angle of heel when the ship oscillates.

- When there is no wind and then a gust of wind blows for several seconds, the ship heels. Use Biran Figs. 6.4 and 6.5 to analyse the situation.

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Biran. 8.2.1. IMO Code. Weather criterion.

- See the figures on the next two slides. ℓ in Moore is Z in Biran.

The second criterion, applicable to cargo and passenger ships, is the weather criterion; it tests the ability of a ship to withstand the combined effects of beam wind and rolling. We explain this criterion with the help of Figure 8.1. First, the code assumes that the ship is subjected to a steady-wind heeling arm

When the ship is upright and subjected to a steady heeling arm, it will move to ϕ_0.

Calculate l_{w1}. Mark it on the y axis in the GZ curve. Find ϕ_0.

$$\ell_{w1} = \frac{PAZ}{1000g\Delta} \tag{8.1}$$

where $P = 504$ N m^{-2}, A is the projected lateral area of the ship and deck cargo above the waterline, in m^2, Z is the vertical distance from the centroid of A to the centre of the underwater lateral area, or approximately to half-draught, in m, Δ is the displacement mass, in t, and $g = 9.81$ m s^{-2}. Unlike the model developed in Section 6.3 (model used by the US Navy), IMO accepts the more severe assumption that the wind heeling arm does not decrease as the heel angle increases. The static angle caused by the wind arm l_{w1} is ϕ_0. It is assumed

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- IMO: l_{w1} is constant and does not decrease when heel angle increases
- At heel angle $= \phi_0$, the heeling arm $=$ $l_{w1} =$ righting arm

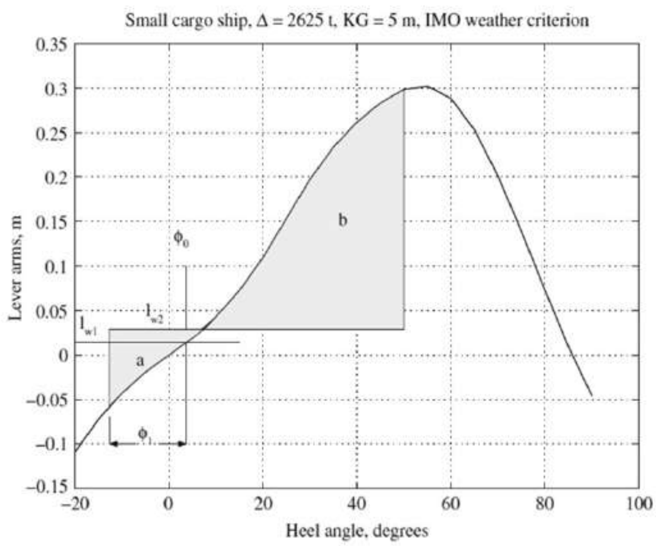

Figure 8.1 The IMO weather criterion

l_{w1}=0.1
$l_{w2} = 1.5 l_{w1}$

Plotting the curve of the arm ℓ_{w2} we distinguish the areas a and b. The area b is limited to the right at 50° or at the angle of flooding, whichever is the smaller. The area b should be equal to or greater than the area a. This provision refers to dynamical stability, as explained in Section 6.6.

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Moore. Intact Stability. SNAME 2010.

- ℓ in Moore is Z in Biran

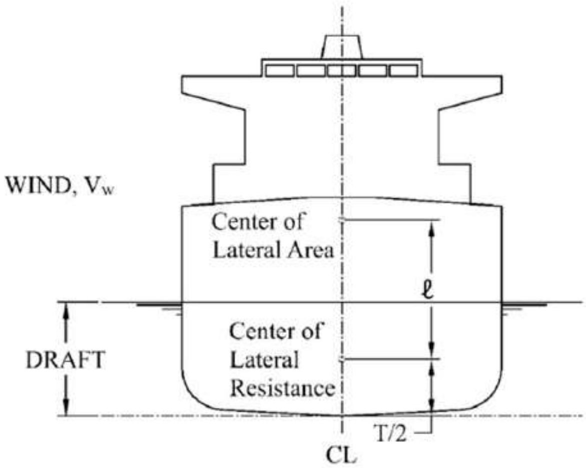

Fig. 55 Heeling effect of wind.

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that, due to wave action, the ship rolls from the angle ϕ_0 windward by an angle ϕ_1. The static angle of heel, ϕ_0, should not exceed 16° or 80% of the angle for deck immersion, which is less. Next, the code assumes that the ship is subjected to a gust of wind causing a heeling arm $\ell_{wt2} = 1.5\ell_{w1}$. The angle of roll is given by

See S14 for ϕ_1
See IMO for details of X_1 etc.

$$\phi_1 = 109kX_1X_2\sqrt{rs} \tag{8.2}$$

where ϕ_1 is measured in degrees, X_1 is a factor given in Table 2.3.4-1. of the code, X_2 is a factor given in Table 2.3.4-2. of the code, and k a factor defined as follows:

- $k = 1.0$ for round-bilge ships;
- $k = 0.7$ for a ship with sharp bilges;
- k as given by Table 2.3.4-3. of the code for a ship having bilge keels, a bar keel or both.

As commented in Section 6.13, by using the factor k, the IMO code considers indirectly the effect of damping on stability. More specifically, it acknowledges that sharp bilges, bilge keels, and bar keels reduce the roll amplitude. However, the code explicitly says that the angle of roll of ships with active anti-rolling devices should be determined without taking into account the operation of those devices, unless the Administration is satisfied by a proof that the system will be effective even in the event of a sudden power shutdown.

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DEAR STUDENTS

Questions are encouraged.

A Google Forms test will be held on 05 Feb 2025.
Portions: Only Module 2.

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11 Feb 2025
2nd IT 05 Feb 2025

Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
Intact stability rules, Heel/ Load test.
Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

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removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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Course Content. Module 3.

Earlier

Module 1. Stability Terms. 06 Lectures. Google forms test 10 Jan 25.

Module 2. Initial (Transverse) Stability. 07 lectures. Google forms test 29Feb25

Module 3. Large Angle Transverse Stability

- 3.1 Large change in the attitude. Equivoluminal change. yCoB.
- 3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.
- 3.3 Curve of Statical Stability
- 3.4 Effect of various factors on $\overline{GZ}$
- 3.5 IMO

Today

- 3.6 Prohaska method for calculating GZ curve
- 3.7 Cross-Curves of Stability

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3.6 Prohaska Method for calculating GZ curve

Sebastian, Guldhammer, and Yilmaz have been uploaded in Classroom.

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3.6 Prohaska Method

James Sebastian. Parametric prediction of the transverse dynamic stability of ships. MS Thesis. Naval Postgraduate School. Monterey, CA, USA, 1997.

- 1947 Prohaska method is explained well in this MS Thesis
- M is shown for the upright condition. It is not shown for the inclined condition.

C. RESIDUARY STABILITY THEORY

Prohaska (1947) introduced the concept of residuary stability whereby the righting arm could be expressed as the sum of two independent terms, one loading dominated and one hull form dominated. He suggested that

$$\boxed{\overline{GZ} = \overline{GM}\sin\phi + \overline{MS}} \qquad (10)$$

where MS represents the residuary stability of the hull form. It can be seen in Figure 5, which shows the stability of a ship at large angles of heel, that the movement of the pro-metacenter is the source of this additional term.

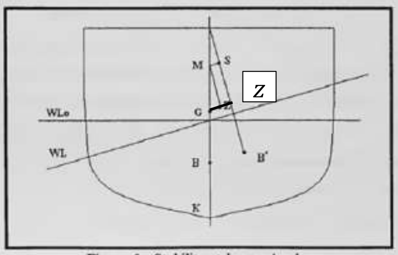

Figure 5. Stability at Large Angles

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Parametric prediction of the transverse dynamic stability of ships

- If $\overline{GZ} = \overline{GM}\sin\phi$ the amplitude or height is $\overline{GM}$

To facilitate non-dimensional plotting, he also introduced the residuary stability coefficient, C_{RS}, whereby

$$C_{RS} = \frac{MS}{BM_T} \tag{11}$$

Next slide: Fig. 6

Figure 6 shows curves of C_{RS} for various sized tankers and warships, calculated using General HydroStatics (GHS) software, for angles of heel up to 90 degrees. It can be readily seen that the positive C_{RS} at lower angles is what causes the GZ curve to have a steeper slope than the sine curve used in Equation (4). At higher angles, C_{RS} quickly becomes negative, drawing the GZ curve down toward the unstable region. The transition point between the lower and higher angles, although not easily discerned, corresponds to the point where the deck edge is immersed. In general, GZ can be thought of as getting its height from GM and its shape from C_{RS}.

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Parametric prediction of the transverse dynamic stability of ships

- In all the cases, C_{RS} is zero at zero heel angle and negative at large heel angle
- From Fig. 7, C_{RS} is independent of $\overline{KG}$

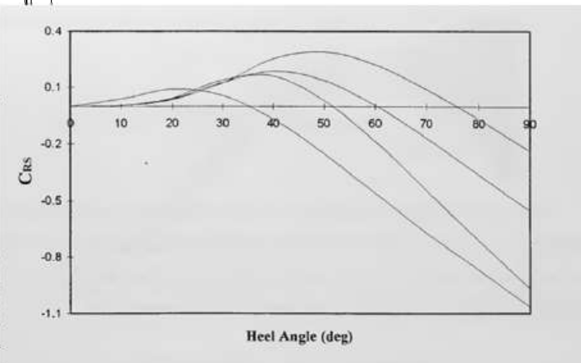

Figure 6. Residuary Stability Coefficient Curves

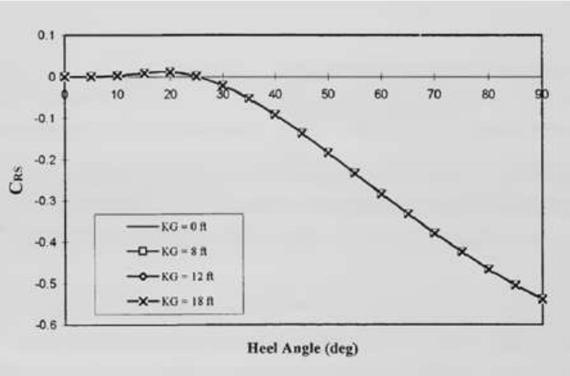

Figure 7. Residuary Stablitity Coefficient for Various KG

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Parametric prediction of the transverse dynamic stability of ships

- Fig. 7 is for various of $\overline{KG}$ but the same displacement and draft.
- If residuary stability is "merely hull form dominant", it will be weakly dependent on KG.

Residuary stability however, is not merely hull form "dominant;" it is entirely independent of the location of the center of gravity. This is illustrated in Figure 7, which shows the CRS curves for a fine-lined hull with varying heights of center of gravity (KG). While this is true, it can not be said that residuary stability is independent of loading all together. The extent of loading or total weight will determine the displacement, and hence the draft, of the vessel which greatly influences the residuary stability.

Prohaska's further research into residuary stability and the effects of hull form on transverse stability resulted in the broad conclusion that the ratios of beam to draft (B/T) and depth of hull (B/D) have a comparatively greater influence on transverse stability than do the coefficients of form (i.e. fullness parameters). His work included the analysis of C_{RS} for a series of 42 systematically varied hull forms, covering the range of fullness coefficients seen in the merchant ship fleet of the day. (Prohaska, 1951)

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1947 Prohaska method + 1974-75 Revision

- Read H. E. Guldhammer, "C_{RS} diagrams for design calculation of the stability of ships " J. Ocean Engineering Vol 6(6) pp 581-592 (1979).

THE "47-METHOD"

In 1947 Professor C. W. Prohaska published a paper, "Residuary Stability". Here the stability lever GZ was proposed divided as follows:

$$GZ = MS + GM \cdot \sin \varphi$$

as illustrated in Fig. 1.

The advantage of this is that GZ is split into a purely geometric part MS, and a part containing the familiar stability parameter GM. The stability data of a ship could now be

- $C_{RS}(\varphi, \frac{T}{B}, \frac{D_{11}}{B}, \delta, \beta)$

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C_{RS}-diagrams for design calculations of the stability of ships 583

M is the initial metacenter

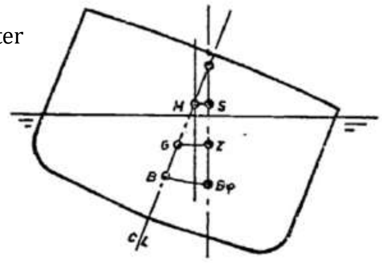

FIG. 1. Definition of the MS.

presented in a very practical way, making possible a direct reading of the size of the stability lever at any angle of heel, if only the displacement and the metacentric height GM were known, see Fig. 2.

However, the major part of the paper from 1947 described an approximate method to determine MS. The method was based on a large number of stability calculations from several shipyards and from different countries. The results were presented as diagrams of a coefficient on the "residuary stability lever" MS.

The "residuary stability coefficient" C_{RS} is defined as the MS made non-dimensional by division by BM, the metacentric radius, thus:

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1947 Prohaska method

The "residuary stability coefficient" C_{RS} is defined as the MS made non-dimensional by division by BM, the metacentric radius, thus:

$$C_{RS} = \frac{MS}{BM}.$$

The only parameters besides the heel φ are the draft ratio T/B and the depth ratio D_1/B here depth corrected for sheer thus:

$$D_1 = D + \frac{S_A + S_F}{6}.$$

- Use the symbols and equations in the following slides to calculate GZ

In spite of the very simple construction this diagram gave relatively excellent results so long as the ships examined were ordinary merchant ship types having not too fine lines, i.e. block coefficient ought to be larger than say 0.65.

At the Shipbuilding Department (now Department of Ocean Engineering) of the Technical University of Denmark, responsible for the 47-method, work on development of a method to cover finer shipforms was attempted using systematically varied forms.

The work appeared, however, much more difficult than expected and was suspended. After the introduction of electronic computation, the research was revived to include not only simplified forms but a new systematical series of "real ship forms".

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To use the 1947 Prohaska method

The following are needed to find $C_{RS}(\varphi, \frac{T}{B}, \frac{D_{11}}{B}, \delta, \beta)$

- GM = metacentric height
- BM = metacentric radius
- δ = Block coefficient = $\nabla/(LBT) = 0.5$ in Fig. 5
- β = midship coefficient = $A_x/(BT) = 0.75$ in Fig. 5
- T/B = draft / breadth. Fig. 5 x-axis.
- D_{11}/B = depth corrected for shear and erections/ breadth. Fig. 5. y-axis.
- Find $C_{RS} = MS/BM$ using Fig. 5. Use BM and find MS.
- $GZ = GM \sin(\theta) + MS$

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LIST OF SYMBOLS

L_{pp}	Length for calculation. Length between perpendiculars, but defined here as 0.97 length of waterline. For the merchant ship area the definition in the International Convention on Load Lines may be used. The stability lever is independent of L, but the definition influences the size of δ and α.
L_{WL}	Length at LWL
B	Breadth moulded at LWL
D	Depth moulded, from BL to lowest point of calculation deck at side
D_1	Depth corrected for sheer. $D_1 = D + \frac{(S_A + S_F)}{6}$
D_{11}	Depth corrected for sheer and erections. $D_{11} = D_1 + \Delta D'_H + \Delta D'_S$
ΔD_H	Basis correction for deck-houses. $D_H = \frac{\Sigma V}{L_{pp} \cdot B \cdot \alpha_D}$
$\Delta D'_H$	Correction for deckhouses at heel φ. $\Delta D'_H = k_H \cdot \Delta D_H$
ΔD_S	Basis correction for superstructures. $\Delta D_S = \frac{h_p \cdot l_p + h_f \cdot l_f}{L_{pp} \cdot B}$
$\Delta D'_S$	Correction for superstructures at heel φ. $\Delta D'_S = k_S \cdot \Delta D_S$
ΣV	Total volume of deck-houses considered
h_p, l_p	Height and length of poop } To be measured within
h_f, l_f	Height and length of forecastle } perpendiculars only
k_H	Correction factor for deck-houses. From Fig. 3
k_S	Correction factor for superstructures. From Fig. 4
BL	Moulded Baseline. Defined in the figures

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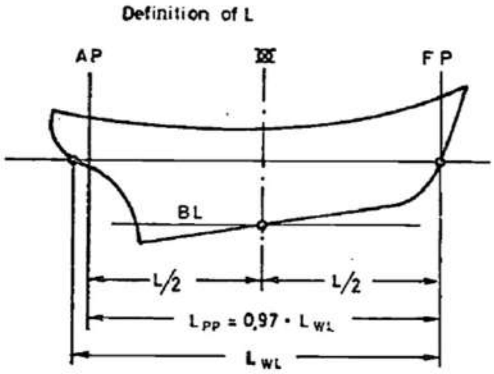
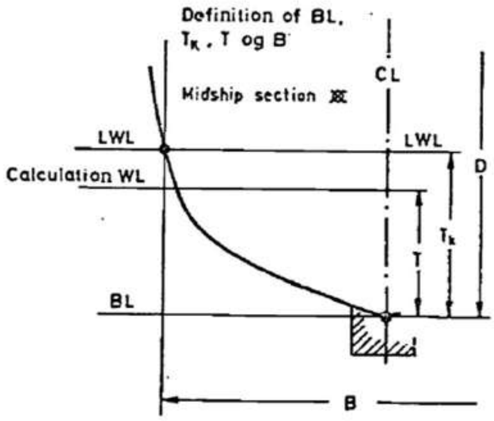
h_p, l_p	Height and length of poop	To be measured *within* perpendiculars only
h_f, l_f	Height and length of forecastle	
k_H	Correction factor for deck-houses. From Fig. 3	
k_S	Correction factor for superstructures. From Fig. 4	
BL	Moulded Baseline. Defined in the figures	





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1947 Prohaska method

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T	Draft to *BL* of calculation waterline
T_K	Draft to *BL* of *LWL*
$T_{1.5}$	Draft for calculating coefficients $= D_U 1.5$
LWL	Design load waterline
S_A	Sheer aft at side
S_F	Sheer forward at side
	Note The term "moulded" will be plain only in the case of steel ships. For wooden ships the above definitions should all be to outside of planking.
∇	Displacement volume
δ	Block coefficient $\frac{\nabla}{L_{pp} \cdot B \cdot T}$
δ_K	Block coefficient at $T = T_K$ (at *LWL*)
$\delta_{1.5}$	Block coefficient at $T = T_{1.5} = D_1/1.5$
α_K	Waterplane coefficient at $T = T_K$; $\alpha_K = \frac{A_{\nabla}}{L_{pp} \cdot B}$ (at *LWL*)
α_D	"Waterplane" coefficient of calculation deck
β	Midship section coefficient $= \frac{A_x}{B \cdot T}$
β_K	Midship section coefficient at $T = T_K$ (*LWL*)
$\beta_{1.5}$	Midship section coefficient at $T = T_{1.5} = D_1/1.5$
	Note The β is defined here by means of midship section area, and *not* by largest sectional area.
T/B	Draft-breadth ratio } Main parameters in the diagrams
D_{1l}/B	Depth-breadth ratio } Main parameters in the diagrams
φ	Angle of heel
GM	Metacentric height

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1947 Prohaska method

- D_{11} is the depth corrected for shear and erections

δ	Block coefficient $\frac{\nabla}{L_{pp} \cdot B \cdot T}$
δ_K	Block coefficient at $T = T_K$ (at LWL)
$\delta_{1.5}$	Block coefficient at $T = T_{1.5} = D_1/1.5$
α_K	Waterplane coefficient at $T = T_K$; $\alpha_K = \frac{A_W}{L_{pp} \cdot B}$ (at LWL)
α_D	"Waterplane" coefficient of calculation deck
β	Midship section coefficient $= \frac{A_x}{B \cdot T}$
β_K	Midship section coefficient at $T = T_K$ (LWL)
$\beta_{1.5}$	Midship section coefficient at $T = T_{1.5} = D_1/1.5$
Note	The β is defined here by means of midship section area, and *not* by largest sectional area.
T/B	Draft-breadth ratio
D_{11}/B	Depth-breadth ratio
φ	Angle of heel
GM	Metacentric height
BM	Metacentric radius
GZ	Stability lever
MS	Residuary stability lever
C_{RS}	Residuary stability coefficient
	$C_{RS} = MS/BM = C_{RS}' + m\,[(\beta_{1.5} - 0.75) - b\,(1 - \beta_{1.5})\,(\delta_{1.5} - 0.50)]$
C'_{RS}	Value of C_{RS} for standard conditions. From diagrams Fig. 6
m	Correcting factor for deviation of β from standard. From diagram Fig. 7
b	Correcting factor for deviation of δ from standard. Printed in calculation form.

T/B, D_{11}/B, φ } Main parameters in the diagrams

GZ, MS } $GZ = GM \cdot \sin\varphi + MS$

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Contour plots of C_{RS} by Prohaska

- Fig. 5 is for the special case of δ = Block coefficient = $\nabla/(LBT)$ = 0.5 and β = midship coefficient = $A_x/(BT)$ = 0.75. The heel angle is shown in big numbers. Contour lines of C_{RS} are shown in Fig. 5a.

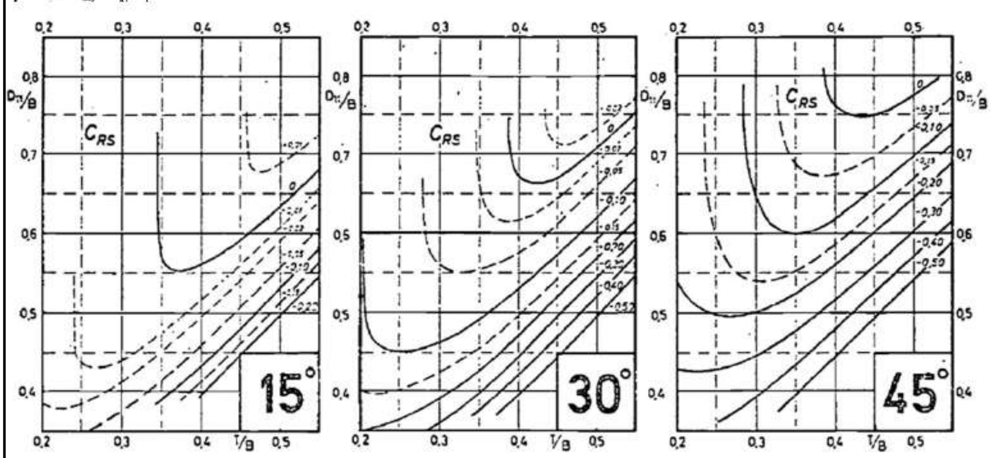

FIG. 5a.

For heel angle = 15 deg., T/B = 0.35, D_{11} /B = 0.57, C_{RS}=0.

For heel angle = 30 deg., T/B = 0.45, D_{11} /B = 0.65, C_{RS}=0.

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Contour plots of C_{RS} by Prohaska

- Fig. 5 is for the special case of δ = Block coefficient = $\nabla/(LBT)$ = 0.5 and β = midship coefficient = $A_x/(BT)$ = 0.75.

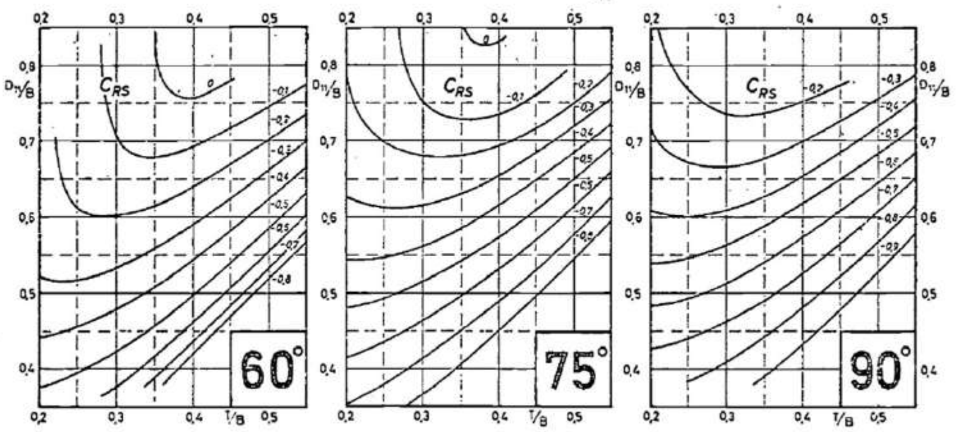

FIG. 5b.

FIG 5. (a) and (b). C_{RS}-diagrams, giving the value C_{RS}, for the coefficient at the standard conditions δ = 0.50 and β = 0.75.

For T/B = 0.45 and D_{11} /B = 0.6, draw the C_{RS} vs heel angle curve and use it to find GZ vs heel angle.

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- If δ ≠ 0.5 or $\beta \neq 0.75$, use Fig. 6 to find m and b. Then, use the Eq. on S22
- Fig. 6a is for b = 0.8. Fig. 6b is for b = 1.2.

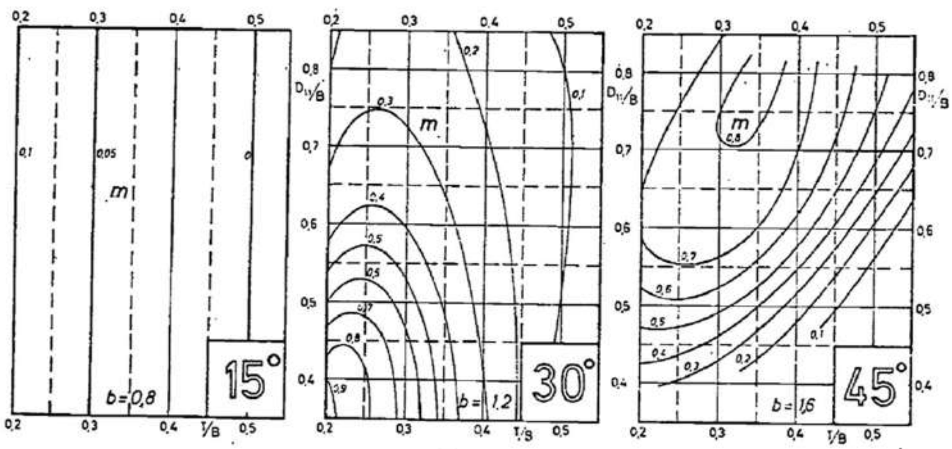

FIG. 6a.

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- If $\delta \neq 0.5$ or $\beta \neq 0.75$, use Fig. 6 to find m and b.

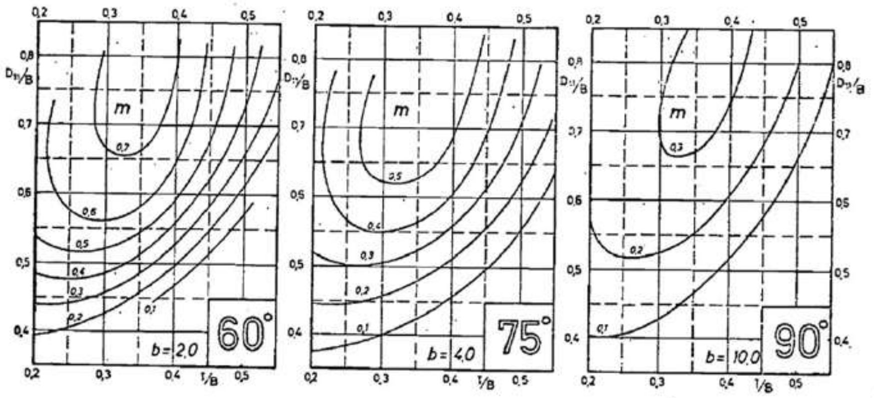

FIG. 6b.

FIG. 6. (a) and (b) m-diagrams, giving the value of the midship section correction coefficient m. The values of the fullness correction coefficient b are stated in the diagrams too. The coefficients m and b to be used by ships differing from standard values δ = 0.50 and β = 0.75.

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- Find C'_{RS} from Fig. 6. Then use this equation.

$$C_{RS} = C'_{RS} + m\,\{(\beta - 0.75) - b\,(1 - \beta)\,(\delta - 0.50)\},$$

where C'_{RS} is the value from Fig. 5, m is the correction coefficient for β from Fig. 6. The corresponding coefficient for the variation with δ is $-m.b.(1 - \beta)$, thus giving the above equation. The values of b are fixed for every φ and are stated on the m-diagrams, but will also be found printed in the calculation form.

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- This is the top half of a table.
- See the table and plot C_{RS} vs heel angle curve for T = 3 m and

C_{RS}-diagrams for design calculations of the stability of ships

(1)	L_{pp} = 22,8 m	(9)	δ_K = 0,387	Ship	I - 51
(2)	B = 6,7 m	(10)	α_K = 0,702	Type: Cutter	Erections: Foc'le, Deckhouse Hatch
(3)	D = 3,35 m	(11)	β_K = 0,618	Date. 7.10.75	Sign
(4)	T_K = 2,14 m	(12)	α_D = 0,847		
(5)	S_A = 0,35 m	(13)	ΣV = 62,1 m^3	(18) D_1/B = (7)/(2) = 0,529	(21) $T_{1,5}$ = (7)/1,5 = 2,36 m
(6)	S_F = 0,81	(14)	l_p = – m	(19) $\frac{(13)}{(1)\cdot(2)^2\cdot(12)}$ = 0,072	(22) $\delta_{1,5}$ = 0,506
(7)	$D_1 = D + (S_A + S_F)/6$ = (3) + $\frac{(5)+(6)}{6}$ = 3,54 m	(15)	h_p = – m		(23) $\beta_{1,5}$ = 0,680
		(16)	l_f = 5,13 m	(20) $\frac{(14)\cdot(15)+(16)\cdot(17)}{(1)\cdot(2)}$ = 0,073	(24) (1 − (23))((22) − 0,5) = 0,002
(8)	D_1/T_K = 1,655	(17)	h_f = 2,16 m		(25) (23) − 0,75 = −0,070

See note

(26) φ degr.	(27) T m	(28) T/B (27)/(2)	(29) k_H Diagr. Fig. 3	(30) $\Delta D_H/B$ (29)·(19)	(31) k_S Diagr. Fig. 4	(32) $\Delta D_S/B$ (31) (20)	(33) D_{11}/B (18)+(31)+(32)	(34) C_{RS} Diagr. Fig. 5	(35) b	(36) m Diagr. Fig. 6	(37) C_{RS} (34)·(35)(25)·(25)(24)	(38) Correction	(39) MS BM(37)(38) m
15°	2,00	0,298	BM:	2,294	0	0	0,529	−0,005	0,8	0,05	−0,01		−0,02
	3,00	0,448	m	1,550	0,10	0,007	0,536	−0,02		0,05	−0,02		−0,03
30°	2,00	0,298	0	0	0,19	0,014	0,543	−0,03	1,2	0,38	−0,06		−0,14
	3,00	0,448	0,20	0,014	0,34	0,025	0,568	−0,12		0,16	−0,13		−0,20

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- This is the bottom half of a table

45°	2,00	0,298	0,04	0,003	0,38	0,028	0,560	−0,10	1,6	0,70	−0,15		−0,34
	3,00	0,448	0,53	0,038	0,57	0,042	0,609	−0,20		0,41	−0,23		−0,36
60°	2,00	0,298	0,20	0,014	0,59	0,043	0,586	−0,21	2,0	0,64	−0,26		−0,60
	3,00	0,448	0,79	0,057	0,69	0,050	0,636	−0,28		0,42	−0,31		−0,48
75°	2,00	0,298	0,35	0,025	0,79	0,058	0,612	−0,29	4,0	0,52	−0,33		−0,76
	3,00	0,448	1,01	0,073	0,99	0,072	0,674	−0,33		0,38	−0,36		−0,56
90°	2,00	0,298	0,50	0,036	0,98	0,072	0,637	−0,34	10	0,28	−0,37		−0,85
	3,00	0,448	1,19	0,086	1,18	0,086	0,701	−0,37		0,22	−0,39		−0,60

NOTE: If $\delta_{1,5}$ and $\beta_{1,5}$ not available use: $\delta_{1,5} \approx \delta_K \frac{T_K}{T_{1,5}} + \alpha_K \frac{T_{1,5} - T_K}{T_{1,5}}$ and $\beta_{1,5} = \beta_K \frac{T_K}{T_{1,5}} + \frac{T_{1,5} - T_K}{T_{1,5}}$

FIG. 8. Sample calculation.

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- This is the full table. See Guldhammer. 1979. It has been uploaded in Classroom under Study Materials.

C_{RS}-diagrams for design calculations of the stability of ships 591

(1) Lpp = 22,8 m	(9) δ_K = 0,387	Ship: I-51	
(2) B = 6,7 m	(10) α_K = 0,702	Type: Cutter	Erections: Fo'c'le, Dackhouse, Hatch; Date: 7.10.75
(3) D = 3,35 m	(11) β_K = 0,618		
(4) T_K = 2,14 m	(12) α_D = 0,847		
(5) S_A = 0,35 m	(13) ΣV = 62,1 m³	(18) D_1/B = 0,529	(21) $T_{1,5}$ = 2,36 m
(6) S_F = 0,81	(14) l_p = – m	(19) = 0,072	(22) $\delta_{1,5}$ = 0,506
(7) $D_1 = D + (S_A + S_F)/6$ = 3,54 m	(15) h_p = – m		(23) $\beta_{1,5}$ = 0,680
	(16) l_1 = 5,13 m	(20) = 0,073	(24) = 0,002
(8) D_1/T_K = 1,655	(17) h_1 = 2,16 m		(25) –0,75 = –0,070

(26) φ degr.	(27) T m	(28) T/B	(29) k_H	(30) $\Delta D_H/B$	(31) k_S	(32) $\Delta D_S/B$	(33) D_H/B	(34) C_{RS}	(35) b	(36) m	(37) C_{RS}	(38) Correction	(39) MS m
15°	2,00	0,298	BM:	2,244	0	0	0,529	–0,005	0,8	0,05	–0,01		–0,02
	3,00	0,448	–	1,550	0,10	0,007	0,536	–0,02		0,05	–0,02		–0,03
30°	2,00	0,298	0	0	0,19	0,014	0,543	–0,03	1,2	0,38	–0,06		–0,14
	3,00	0,448	0,20	0,014	0,34	0,025	0,568	–0,12		0,16	–0,13		–0,20
45°	2,00	0,298	0,04	0,003	0,38	0,028	0,560	–0,10	1,6	0,70	–0,15		–0,34
	3,00	0,448	0,53	0,038	0,57	0,042	0,609	–0,20		0,41	–0,23		–0,36
60°	2,00	0,298	0,20	0,014	0,59	0,043	0,586	–0,21	2,0	0,64	–0,26		–0,60
	3,00	0,448	0,79	0,057	0,69	0,050	0,636	–0,21		0,42	–0,31		–0,48
75°	2,00	0,298	0,35	0,025	0,79	0,058	0,612	–0,29	4,0	0,52	–0,33		–0,76
	3,00	0,448	1,01	0,073	0,99	0,072	0,674	–0,33		0,38	–0,36		–0,56
90°	2,00	0,298	0,50	0,036	0,91	0,072	0,637	–0,34	10	0,28	–0,37		–0,85
	3,00	0,448	1,19	0,086	1,18	0,086	0,701	–0,37		0,22	–0,39		–0,60

Fig. 8. Sample calculation.

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3.7 Cross Curves of Stability

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3.6 Practical: Diagram of statical stability / Cross curves of stability. Krylov's Method

- Read Semyonov pages 223-229 and Appendix III (page 475)
- Prohaska's method for finding $\overline{GZ}$ is useful in the preliminary design stage as the linesplan is not known. Krylov's method is useful after the linesplan is known.

3. **Module III**
Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
~~Intact stability rules, Heel/ Load test.~~
Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Cross-Curves of Stability

- Biran
- The curve of statical stability is $\overline{GZ}(\phi)$ for a particular displacement and $\overline{KG}$
- Cross curves of stability are $l_k(\nabla, \phi)$
- $\overline{GZ}(\phi) = l_k(\nabla_0, \phi) - \overline{KG}\sin\phi$
- Cross-curves are used to find $\overline{GZ}(\phi)$ for a particular disp and $\overline{KG}$

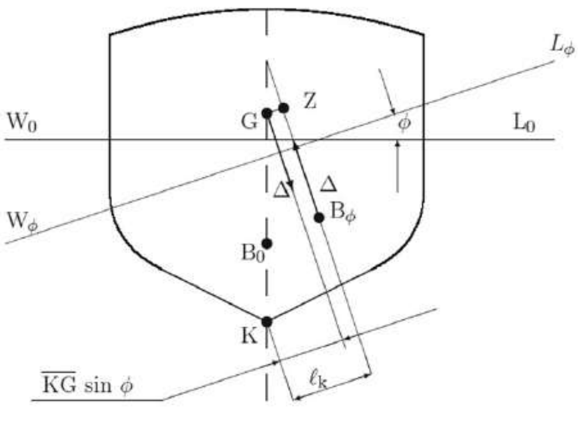

Figure 5.1 Definition of righting arm

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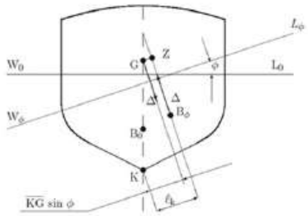
Cross-Curves of Stability



to the line of action of the buoyancy force intersects the latter line in Z. T and buoyancy produce a **righting moment** whose value is

$$M_R = \Delta \overline{GZ}$$

As Δ is a constant for all angles of heel, we can say that the righting moment is characterized by the **righting arm**, $\overline{GZ}$. From Figure 5.1 we write

$$\overline{GZ} = \ell_k - \overline{KG} \sin \phi \qquad (5.2)$$

For reasons to be explained several lines below, the distance ℓ_k is called **value of stability cross-curves**. This quantity results from hydrostatic calculations based on the ship lines. Such calculations are left today to the computer. The term $\overline{KG} \sin \phi$ depends on $\overline{KG}$, a quantity obtained from **weight calculations** as explained in Chapter 7. In European literature the term ℓ_k is often described as "lever arm of stability of form," while the term $\overline{KG} \sin \phi$ is called "lever arm of stability of weight."

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Cross-Curves of Stability

- Biran
- The curve of statical stability is $\overline{GZ}(\phi)$ for a particular displacement and $\overline{KG}$
- Cross curves of stability are $l_k(\nabla, \phi)$
- $\overline{GZ}(\phi) = l_k(\nabla_0, \phi) - \overline{KG} \sin \phi$
- Cross-curves are used to find $\overline{GZ}(\phi)$ for a particular disp and $\overline{KG}$

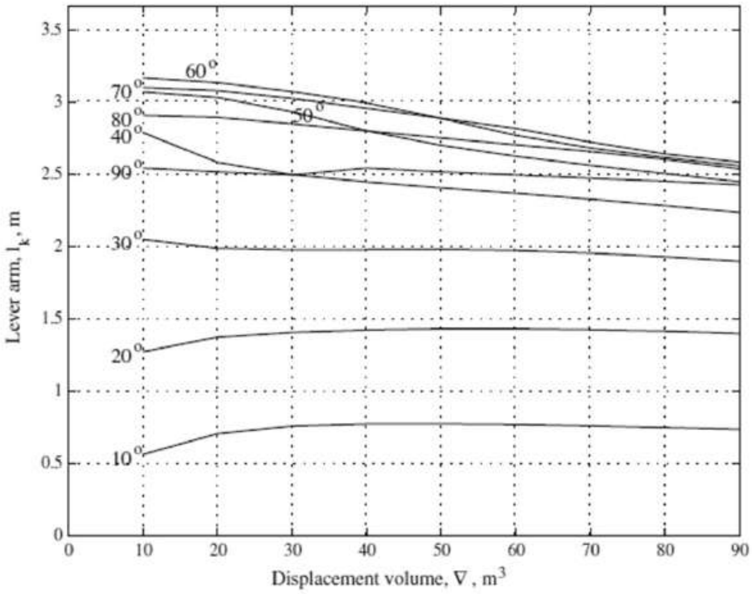

Figure 5.3 Cross-curves of stability of ship *Lido 9*

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- Semyonov
- Sec. 40
- Lever of statical stability and righting moment

The lever of statical stability will imply the projection of the righting lever on the plane of inclination which, in Fig. 55, is equal to the length of the perpendicular GK from the centre of gravity to the projection of the line of action of the buoyancy. In subsequent considerations the lever of statical stability will be denoted by l.

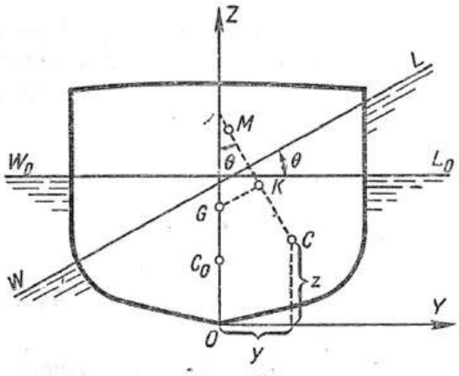

Fig. 55

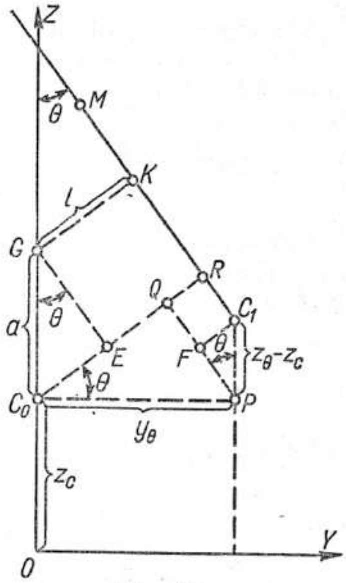

Fig. 56

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Semyonov. $\ell = \overline{GK}$.

- ℓ = Lever of statical stability = $\overline{GZ}$ in Biran
- $l_k(\nabla_0, \phi)$ is the lever of cross-curves stability in Biran
- $\overline{KB} = z_c$; CoB = (y_θ, z_θ); $\overline{GB} = a$
- For a particular θ, find the waterline using Krylov's method

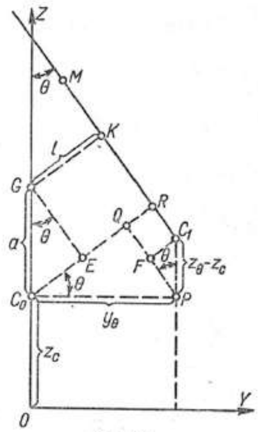

Fig. 56

Directly from Fig. 56 it follows that

$$l = \overline{GK} = \overline{C_0Q} + \overline{QR} - \overline{C_0E}.$$

From the same figure we have

$$\overline{C_0Q} = y_\theta \cos\theta; \quad \overline{QR} = (z_\theta - z_c)\sin\theta; \quad \overline{C_0E} = a\sin\theta.$$

Substituting the expressions derived above in the expression for l, we obtain

$$l = y_\theta \cos\theta + (z_\theta - z_c)\sin\theta - a\sin\theta. \qquad (40.1)$$

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12 Feb 2025

Stability of Ships

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
Intact stability rules, Heel/ Load test.
Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

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removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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Course Content. Module 3.

Earlier

Module 1. Stability Terms. 06 Lectures. Google forms test 10 Jan 25.

Module 2. Initial (Transverse) Stability. 07 lectures. Google forms test 29Feb25

Module 3. Large Angle Transverse Stability

- 3.1 Large change in the attitude. Equivoluminal change. yCoB.
- 3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.
- 3.3 Curve of Statical Stability
- 3.4 Effect of various factors on $\overline{GZ}$
- 3.5 IMO
- 3.6 Prohaska method for calculating GZ curve
- 3.7 Cross-Curves

Today

- 3.8 Regression analysis
- 3.9 Krylov's method

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3.8 Cross-Curves of Stability by Regression Analysis

- Group Assignment. Write a Matlab or Octave program to evaluate the expressions in this paper. Find KN (cross-curves) for your ship.

Marine Technology, Vol. 38, No. 2, April 2001, pp. 92–94

An Approximate Method for Cross Curves of Cargo Vessels

Hüseyin Yılmaz[1] and **Mesut Güner**[1]

In this study, a formula is presented to estimate cross curves of cargo vessels and to predict statical stability at the preliminary design stage of the vessel. The predictive technique is obtained by regression analysis of systematically varied cargo vessel series data. In order to achieve this procedure, some cargo vessel forms are generated using Series-60. The mathematical model in this predictive technique is constructed as a function of design parameters such as length, beam, depth, draft, and block coefficient. The prediction method developed in this work can also be used to determine the effect of specific hull form parameters and the load conditions on stability of cargo vessels. The present method is applied to a cargo vessel and then the results of the actual ship are compared with those of regression values.

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Yilmaz and Güner. Regression Analysis.

- The cross-curves of stability are first calculated using Naval Arch software for vessels with $5<L/B<7.5$ etc. See the snippet. T is the design draft. T_c is the draft for the cross-curve.

The parameter range of generated vessels is set as follows:

$$5.0 < L/B < 7.5$$
$$2.25 < B/T < 3.0$$
$$0.60 < C_B < 0.80$$
$$1.3 < D/T_C < 4.0$$

where D/T_C is the depth-draft ratio.
The actual cross-curve computations of the vessels are performed by a well-established stability software. The results obtained from the implementation of the software are used to establish approximated stability expression.

- The cross-curves are expressed as

$$KN = (a_1\Phi^1 + a_2\Phi^2 + a_3\Phi^3 + a_4\Phi^4 + a_5\Phi^5)\,B$$

- where Φ is the heel angle. The values of a_i are found using regression analysis (curve-fitting) by minimizing the error (difference) between the cross-curves computed using Naval Arch software and the "fit." The values of a_i depend on L/B, B/T, C_B , and D/T_C.

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Yilmaz and Güner. Regression Analysis.

The KN computed using Naval Arch software is in good agreement with the KN calculated using the eq. The coefficients in the eq. are obtained by using regression analysis which is aka curve-fitting.

Now, we can use the eq in the paper and don't need the Naval Arch software

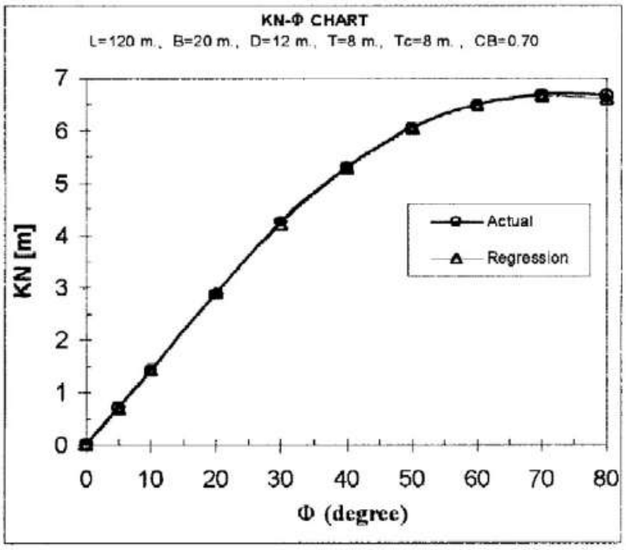

Fig. 2 Comparison of *KN*-Φ values

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3.9 Krylov's Method to find GZ

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Krylov's Method to find $\overline{GZ}$

- Use Krylov's method to find the waterline. The details are on the following slides.
- Use the waterline to find $\overline{BM}$.
- Use $\overline{BM}$ to find the CoB.
- Use Eq. (40.1) on L18S32 to find $\overline{GZ} = l$

Directly from Fig. 56 it follows that

$$l = \overline{GK} = \overline{C_0Q} + \overline{QR} - \overline{C_0E}.$$

From the same figure we have

$$\overline{C_0Q} = y_\theta \cos\theta; \quad \overline{QR} = (z_\theta - z_c)\sin\theta; \quad \overline{C_0E} = a\sin\theta.$$

Substituting the expressions derived above in the expression for l, we obtain

$$l = y_\theta \cos\theta + (z_\theta - z_c)\sin\theta - a\sin\theta. \qquad (40.1)$$

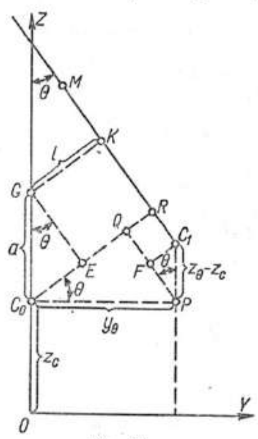

Fig. 56

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Krylov's Methods

- Semyonov.
- Methods to calculate the stability lever for various equivolume attitudes

53. PROCEDURE OF CALCULATING STABILITY LEVERS FOR EQUIVOLUME INCLINATIONS ELABORATED BY A. N. KRYLOV

A great number of various methods of calculating stability at large angles of heel may be found in the literature. We shall consider here the methods of calculation which are employed

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in naval architecture in the U.S.S.R. The development of these methods is due to A. N. Krylov.

In order to calculate the lever of statical stability by formula (40.1) it is necessary to know, as has been noted above, the relation $r(\theta)$ for equivolume inclinations. Thus it is necessary first to draw equivolume waterlines at equal angular intervals. We shall consider here two methods of drawing equivolume waterlines proposed by A. N. Krylov. Both methods are based on the drawing of an auxiliary waterline cutting off an approximately constant volume and on the subsequent determination of the distance between the auxiliary and equivolume waterlines.

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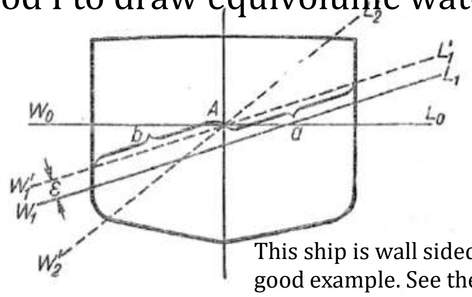
Krylov Method I to draw equivolume waterlines



This ship is wall sided for W_1L_1 and not a good example. See the next slide.

Fig. 94

A "diametral plane" is a plane that cuts through the center of a geometric shape, like a sphere or a cylinder, essentially dividing it into two equal halves along its diameter

Method I consists in drawing auxiliary waterlines for all angles of heel so that they all intersect along one straight line. In Fig. 94 the traces of these waterlines $W_1'L_1'$, $W_2'L_2'$, $W_3'L_3'$, etc., pass through a common point which is taken to be the intersection of the traces of the upright waterline and the diametral plane (point A). Thus it is assumed that there is a non-equivolume inclination about an axis which is projected into A.

After drawing an auxiliary waterline we calculate the volumes of the immersed wedge v_1 and the emerged wedge v_2. As a rule,

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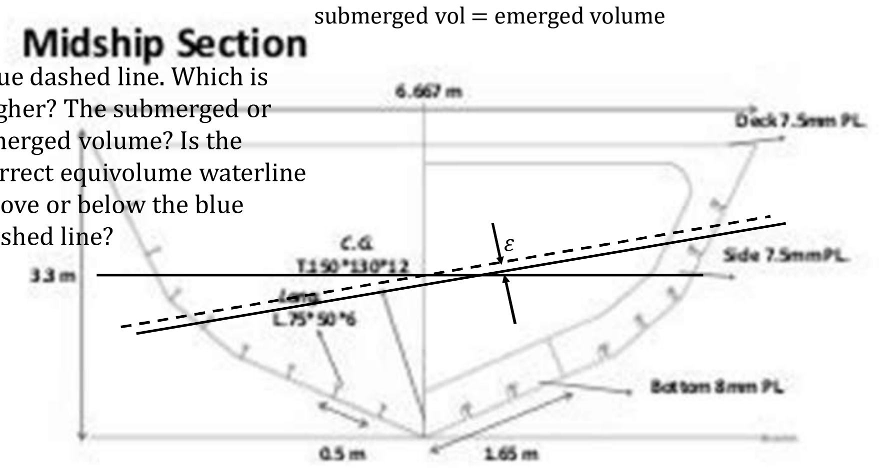
Krylov Method. Offshore Support Vessel

- Red line: even keel water line. Blue dashed line: auxiliary inclined water line. Blue line: actual inclined water line.

Blue line. It lies below the blue dashed line to make the submerged vol = emerged volume

Blue dashed line. Which is higher? The submerged or emerged volume? Is the correct equivolume waterline above or below the blue dashed line?



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Krylov Method 1 to find the Waterline

- The ship is on even keel and the underwater volume is $\nabla =$ Del or Nabla
- The ship heels to an angle ϕ. The underwater volume does not change. Find the new waterline.
- Draw an auxiliary waterline at an angle ϕ passing through the centerline. This is for a non-equivolume inclination by angle ϕ.
- Find the submerged volume v_1 and emerged volume v_2 and the difference. Let the inclined waterplane area be S_1. Let the projection of the inclined waterplane area on the XoY plane be S.
- Draw a new waterline to make the difference zero. It should be parallel to the auxiliary waterline and ε from it.
- Solve $\varepsilon S \approx v_1 - v_2$ to find ε. If it is positive, draw the new waterline below the aux waterline. This will reduce the submerged volume and increase the emerged volume.
- It is better to solve $\varepsilon S_1 \approx v_1 - v_2$. But Krylov prefers to use the projected area, S, in the early part of the derivation but switches back to the inclined area in Eq. (53.5)

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Method I consists in drawing auxiliary waterlines for all angles of heel so that they all intersect along one straight line. In Fig. 94 the traces of these waterlines $W_1'L_1'$, $W_2'L_2'$, $W_3'L_3'$, etc., pass through a common point which is taken to be the intersection of the traces of the upright waterline and the diametral plane (point A). Thus it is assumed that there is a non-equivolume inclination about an axis which is projected into A.

After drawing an auxiliary waterline we calculate the volumes of the immersed wedge v_1 and the emerged wedge v_2. As a rule, these volumes are not equal, hence the equivolume waterline passes at the same angle θ above or below the auxiliary waterline.

Thus the auxiliary waterline must be shifted through a distance ε to make it equivolume. In subsequent considerations the equivolume waterline will be referred to as the operating waterline and the formula will be derived by the use of which it is possible to calculate the distance ε between the auxiliary and operating waterlines. It is apparent that in order to pass to the operating waterline the auxiliary waterline must be shifted so that the volume of the layer intercepted between the waterlines

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Krylov Method 1

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is equal to the difference between the volumes of the wedges. This condition may approximately be written as

S = projected waterplane area. It is not symmetric about the centerline.

$$\varepsilon S = v_1 - v_2.$$

S_1 = inclined waterplane area

This relation is not exact since the volume of the layer intercepted between the waterlines may only approximately be taken to be equal to the volume of the cylinder with base S and height ε. From the relation above we can obtain

Instead of using S_1, Krylov used another method to improve the accuracy. See Slide#25.

$$\varepsilon = \frac{v_1 - v_2}{S}. \qquad (53.1)$$

Formula (33.3) makes it possible to calculate the change in volume δv for a small change in parameters of the inclined waterline δT_{M}, $\delta\psi$ and $\delta\theta$. In the present case the inclination is about the axis A (Fig. 94), hence the change occurs only in parameter θ. Then from formula (33.3) we get

Both S and y_f are projected. That is why a $\cos^2\theta$ term is there.

$$\delta v = \frac{S y_f}{\cos^2\theta}\,\delta\theta.$$

See L14S20 for (33.3).

- S is the upright waterplane area and is defined on the next slide
- Recall that for a wall-sided ship, $\delta v = 0$ as $y_f = 0$

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- M_x. M with 1 subscript is the first moment of the inclined WPA about the axis of flotation or inclination.
- M_{xz}. M with 2 subscripts is the first moment of the volume. See L14S17

In this formula S and y_f are the elements of the projection of the inclined waterline on the plane XOY. Then

$$\frac{S y_f}{\cos^2\theta} = M_x,$$

For $\theta = 0$, $M_x = S y_f$ which is correct.

with M_x being the static moment of the inclined waterplane about the axis of inclination. Thus we have

$$\delta v = M_x \delta\theta.$$

We can now write the expression for the difference $v_1 - v_2$

$$v_1 - v_2 = \int_0^\theta M_x\, d\theta. \qquad (53.2)$$

In this expression M_x is the current value of the static moment of inclined waterplanes about the axis of rotation. The static moment may be expressed as

M_x is a function of θ. a and b are functions of θ and x.

See a and b in Fig. 94 L19S11.

$$M_x = \frac{1}{2}\int_{-\frac{L}{2}}^{+\frac{L}{2}} (a^2 - b^2)\, dx, \qquad (53.3)$$

Moment for a strip = $adx\ (a/2)$

where a is the ordinate of the waterline reckoned from the axis of rotation on the immersed side, b is the same on the emerged side.

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Substituting (53.2) in (53.1), we can write

S is also a function of θ but it should not be taken inside the integral in Eq. (53.4). In Eq. (53.4), S is for a specific value of θ – see Eq. (53.5).

$$\varepsilon = \frac{1}{S}\int_0^\theta M_x d\theta, \qquad (53.4)$$

where the moments M_x are calculated by formula (53.3) and the area of the waterplane S is calculated by the formula

See a and b in Fig. 94 on L19S11 and use them to find S.

$$S = \int_{-\frac{L}{2}}^{+\frac{L}{2}} (a + b)\, dx. \qquad (53.5)$$

Method II consists in drawing auxiliary waterlines each time

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Krylov Method 1 for Cuboidal barge

1. For a cuboidal barge, y_f of the upright and inclined WPA is 0. The auxiliary waterline passes through the centerline and y_f.
2. $\delta v = \frac{S y_f}{\cos^2\theta}\delta\theta$ SeeL19S15. Note from L19S16 that S and y_f are the projections of the inclined waterplane area and yCoF, respectively, on the XOY plane. So δv=0.
3. $M_x = \frac{S y_f}{\cos^2\theta}$ is the first moment of the inclined WPA. L19S16. M_x =0
4. $\varepsilon = \frac{1}{S}\int_0^\theta M_x d\theta = 0$. So the auxiliary waterline is the actual waterline which is correct.

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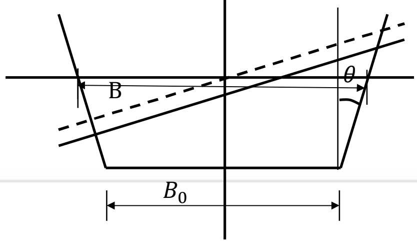
Krylov Method 1 for Trapezoidal barge

1. For a trapezoidal barge, y_f of the upright WPA is 0. The auxiliary waterline passes through the centerline and y_f of the upright WPA
2. $\delta v = \frac{S y_f}{\cos^2\theta}\delta\theta$ SeeL19S15. Note from L19S16 that S and y_f are the projections of the inclined waterplane area and yCoF, respectively, on the XOY plane. y_f of the inclined WPA is not zero. So $\delta v \neq 0$.
3. $M_x = \frac{S y_f}{\cos^2\theta}$ is the first moment of the inclined WPA. L19S16. $M_x \neq 0$
4. Find $v_1 - v_2 = \int_0^\theta M_x d\theta$
5. Find $v_1 - v_2$ using the areas of the triangles
6. Compare results in 4 and 5



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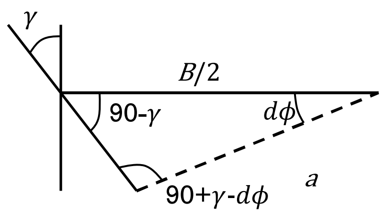
Krylov Method 1 for Trapezoidal barge

- Breadth at the upright waterline $= B$. Chine angle $= \gamma$.
- Emerged area.
- Length of the inclined waterline from the centerline to the hull $= a$.
- Area of the emerged triangle $= 0.5(B/2)\, a \sin d\phi$
- Law of sines. $a/\sin(90-\gamma) = (B/2)\,/\sin(90+\gamma\text{-}d\phi\,)$
- $a = \left(\frac{B}{2}\right)\frac{\cos\gamma}{\cos(\gamma - d\phi)}$
- Area_1 $= \frac{B^2}{8}\frac{\cos\gamma}{\cos(\gamma - d\phi)}\sin d\phi$



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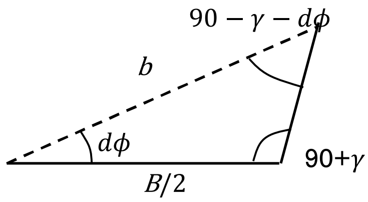
Krylov Method 1 for Trapezoidal barge

- Submerged area.
- Length of the inclined waterline from the centerline to the hull $= b$.
- Area of the emerged triangle $= 0.5(B/2)\ b \sin d\phi$
- Law of sines. $b / \sin(90+\gamma) = (B/2)\ / \sin(90-\gamma-d\phi\ )$
- $b = \left(\frac{B}{2}\right)\frac{\text{co}}{\cos(\gamma+d\phi)}$; Area_2$= \frac{B^2}{8}\frac{\cos\gamma}{\cos(\gamma+d\phi)}\sin d\phi$
- Diff in uw volume $= v_1 - v_2 =$ L (Area_1- Area_2)
- $= \frac{LB^2}{8}\cos\gamma \sin d\phi\ (\frac{1}{\cos(\gamma\text{-}d\phi)} - \frac{1}{\cos(\gamma+d\phi)})$
- Aux WPA $= S_1 = (a+b)L$
- Move the WPA down by $\varepsilon = \frac{v_1-v_2}{S_1} = \frac{B^2}{8(a+b)}\cos\gamma \sin d\phi\ (\frac{1}{\cos(\gamma-d\phi)} - \frac{1}{\cos(\gamma+d\phi)})$



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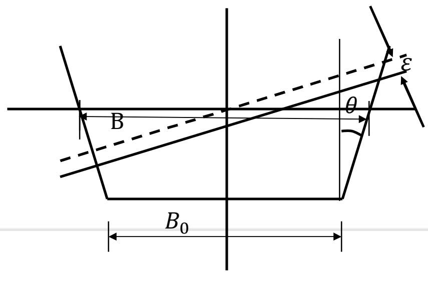
Upright and Inclined UW Volumes

- $(a+b) = \left(\frac{B}{2}\right)\frac{\cos}{\cos(\gamma-d\phi)} + \left(\frac{B}{2}\right)\frac{\text{co}}{\cos(\gamma+d\phi)} = \left(\frac{B}{2}\right)\cos\gamma\ (\frac{1}{\cos(\gamma-d\phi)} - \frac{1}{\cos(\gamma+d\phi)})$
- $\varepsilon = \frac{B^2}{8}\sin d\phi$
- Assignment. Find the draft at midships. Then, find the original and inclined uw volumes. Show that they are equal. Note that the breadth at the keel is B_0.



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19 Feb 2025

Stability of Ships

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
   Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
   Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
   Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
   Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
   Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
   Intact stability rules, Heel/ Load test.
   Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Stability of Ships. Course Content

4. **Module IV**
Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

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removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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Course Content. Module 3.

Earlier

Module 1. Stability Terms. 06 Lectures. Google forms test 10 Jan 25.

Module 2. Initial (Transverse) Stability. 07 lectures. Google forms test 29Feb25

Module 3. Large Angle Transverse Stability

- 3.1 Large change in the attitude. Equivoluminal change. yCoB.
- 3.2 Find CoB using $\overline{BM}$ = metacentric radius. Find M using CoB.
- 3.3 Curve of Statical Stability
- 3.4 Effect of various factors on $\overline{GZ}$
- 3.5 IMO
- 3.6 Prohaska method for calculating GZ curve
- 3.7 Cross-Curves
- 3.8 Regression analysis
- 3.9 Krylov’s method

Today

- 3.10 Heeling Moments

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3.10 Heeling Moments

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Compare the heeling and righting arms

- Biran

Heeling moments can be caused by wind, by the centrifugal force developed in turning, by transverse displacements of masses, by towing, or by the lateral pull developed in cables that connect two vessels during the transfer of loads at sea. In Chapter 5 we have shown that, when the ship heels at constant displacement, it is sufficient to consider the righting arm as an indicator of stability. Then, to assess the ship stability it is necessary to compare the righting arm with a **heeling arm**. According to the DIN-ISO standard, we note the heeling arm by the letter ℓ and indicate the nature of the righting arm by a subscript. To obtain a generic heeling arm, ℓ_g, corresponding to a generic heeling moment, M_g, we divide that moment by the ship weight

$$\ell_g = \frac{M_g}{g\Delta} \qquad (6.1)$$

where Δ is the displacement mass, and g, the acceleration due to gravity. In older practice it has

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- See the figures on the following slides

In Figure 6.1 we superimposed the curve of a generic heeling arm, ℓ_g, over the curve of the righting arm, $\overline{GZ}$. For almost all positive heeling angles shown in the plot the righting arm is

positive. We define the righting arm as positive if when the ship is heeled to starboard, the righting moment tends to return it toward port. In the same figure the heeling arm is also positive, meaning that the corresponding heeling moment tends to incline the ship toward starboard. What happens if the ship heels in the other direction, that is with the port side down? Let us extend the curve of statical stability by including negative heel angles, as in Figure 6.2. The righting arms corresponding to negative heel angles are negative. For a ship heeled toward port, the righting moment tends, indeed, to return the vessel toward starboard, therefore it has another sign than in the region of positive heel angles. The heeling moment, however, tends in general to heel the ship in the same direction as when the starboard is down and, therefore, it is positive. Summarizing, the righting-arm curve is symmetric about the origin, while the heeling-arm curves are symmetrical about the lever-arm axis.

anti-symmetric as shown in Fig. 6.2

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Heeling arm convention

- When the heeling moment makes the ship heel to stbd/port, the moment is acting along the +x/-x direction.
- The heeling arm is shown as positive for heel to stbd and port.

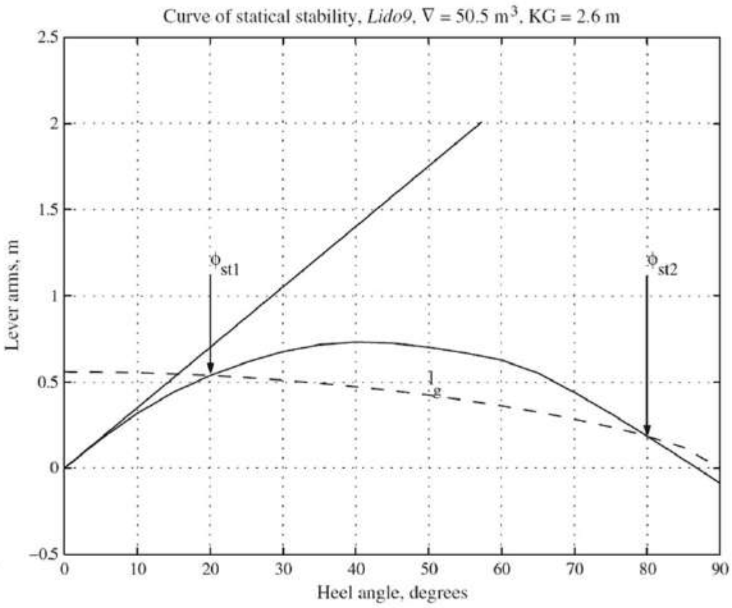

Figure 6.1 Angles of statical equilibrium

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Why is the righting arm curve anti-symmetric?

- When the ship heels to stbd, the righting couple is along the -x axis.
- When the ship heels to port, the righting couple is along the +x axis.
- This is indicated by using an anti-symmetric righting arm curve.

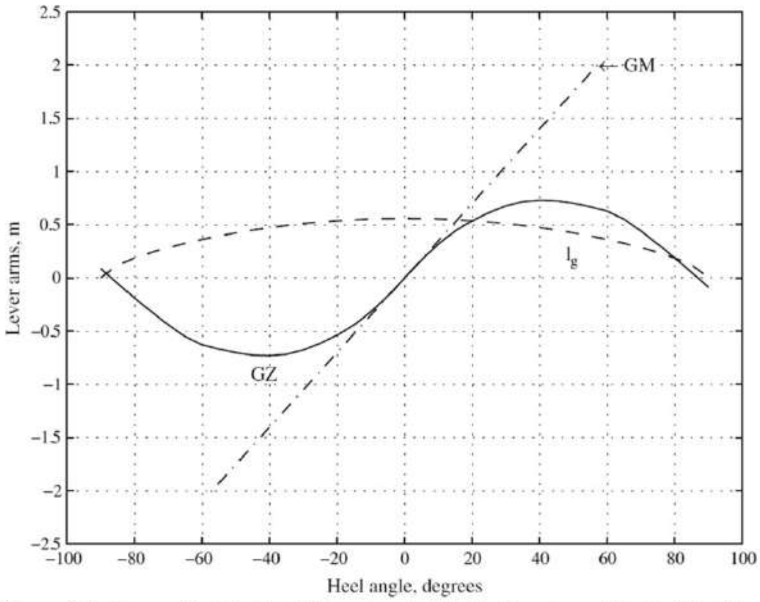

Figure 6.2 Curve of statical stability extended for heeling toward both ship sides

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Static stability: Heeling arm = Righting arm

We can appreciate the stability of a vessel by comparing the righting arm with the heeling arm as long as the heeling moment is applied gradually and inertia forces and moments can be neglected. When the heeling moment appears suddenly, as caused, by example, by a gust of wind, one has to compare the heeling energy with the work done by the righting moment. Such situations are discussed in the section on dynamical stability. In continuation we show how moving loads, solid, or liquid, endanger the ship stability, and we develop formulae for calculating the corresponding reduction of stability. Other situations in which the stability is endangered are those of grounding or positioning in dock. We show how to predict the moment in which those situations may become critical. This chapter also discusses the situations in which a ship sails with a negative metacentric height.

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Wind Heeling Arm

- See L17S28&29

Moore

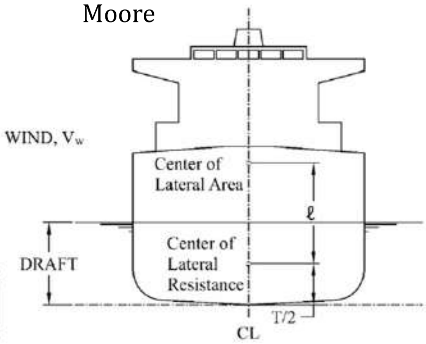

Fig. 55 Heeling effect of wind.

6.3 The Wind Heeling Arm

We use Figure 6.3 to develop a simple model of the heeling moment caused by a beam wind, that is a wind perpendicular to the centreline plane. In this situation the wind heeling arm is

Biran

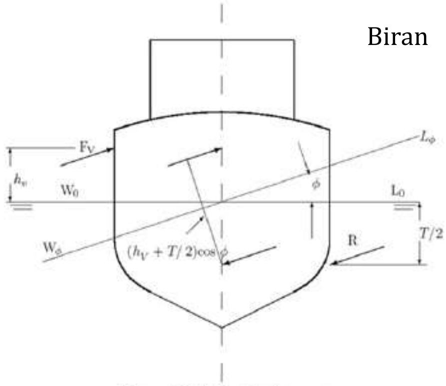

Figure 6.3 Wind heeling arm

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Heeling Arm in Turning

6.4 Heeling Arm in Turning

When a ship turns with a linear speed V, in a circle of radius R_{TC}, a centrifugal force, F_{TC}, develops; it acts in the centre of gravity, G, at a height $\overline{KG}$ above the baseline. From mechanics we know that

$$F_{TC} = \Delta \frac{V^2}{R_{TC}}$$

Under the influence of the force F_{TC} the ship tends to drift, a motion opposed by the water with a reaction R. To simplify calculations, we assume again that the water reaction acts at half-draught, that is at a height $T/2$ above the baseline. The two forces, F_{TC} and R, form a torque whose lever arm in upright condition is $(KG - T/2)$. For a heeling, flat ship this lever arm is proportional to $\cos\phi$. Dividing by the displacement force, we obtain the **heeling lever of the centrifugal force in turning circle**:

$$\ell_{TC} = \frac{1}{g}\frac{V^2}{R_{TC}}(\overline{KG} - T/2)\cos\phi \tag{6.4}$$

The speed V to be used in Eq. (6.4) is the speed in turning, smaller than the speed achieved when sailing on a straight-line path. The turning radius, R_{TC}, and the speed in turning, V, are not known in the first stages of ship design. If results of basin tests on a ship model, or of sea trials of the ship, or of a sister ship, are available, they should be substituted in Eq. (6.4). The

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Other Heeling Arms

6.5 Other Heeling Arms

A dangerous situation can arise if many passengers crowd on one side of the ship. There are two cases when passengers can do this: when attracted by a beautiful seascape, or when scared by some dangerous event. In the latter case passengers can also be tempted to go to upper decks. The resulting heeling arm can be calculated from

$$\ell_P = \frac{np}{\Delta}(y \cos\phi + z \sin\phi) \tag{6.7}$$

where n is the number of passengers, p, the average person mass, y, the transversal coordinate of the centre of gravity of the crowd, and z, the vertical translation of said centre. The second term between parentheses accounts for the virtual metacentric-height reduction. Wegner (1965) recommends to assume that up to seven passengers can crowd on a square metre, that the average mass of a passenger plus some luggage is 80 kg, and that the height of a passenger's centre of gravity above deck is 1.1 m. Similar values are prescribed by the

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Towing, Icing

skylights. Other heeling moments can occur when a tug tows a barge. The barge can drift and then the tension in the towing cable can be decomposed into two components, one parallel to the tug centreline, the other perpendicular to the first. The latter component can cause capsizing of the tug. The process is very fast and there may be no survivors. To avoid this situation tugs must be provided with quick-release mechanisms that free instantly the towing cable. Lateral forces also appear when fishing vessels tow nets or when two vessels are connected by cables during replenishment-at-sea operations. Special provisions are made in stability regulations for the situations mentioned above. **Icing** is a phenomenon known to ship crews sailing in very cold zones. The accumulation of ice has a double destabilizing effect: it raises the centre of gravity and it increases the sail area. The importance of ice formation should not be underestimated. For example, Arndt (1960a) cites cases in which blocks of ice 1 m thick developed on a poop deck, or walls of 60 cm of ice formed on the front surface of a bridge. Therefore, stability regulations take into account the effect of ice.

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Icing Heeling Arm

8.2.11 Icing

Part B, Chapter 6 of the code bears the title "Ice considerations." For any ship operating in areas where ice accretion is likely to occur, adversely affecting the stability, corresponding weight allowances should be included in the analysis of loading conditions. In the case of fishing vessels and cargo ships carrying timber on deck, the allowance for additional weight should be made for the arrival condition. The code specifies clearly, in a chart, the geographical areas in which ice accretion can occur. Most important examples are the regions of Iceland, the Baltic Sea, the north of North America, the Bering and Okhotsk Seas, the Tartary Strait, and the seas south of 60° S. The following values, prescribed for fishing vessels, illustrate the severity of the problem. Stability calculations should be carried on assuming ice accretion (this is the term used in the code) with the surface densities.

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- 30 kg m^{-2} on exposed weather decks and gangways;
- 7.5 kg m^{-2} for projected lateral areas on each side, above the waterplane.

The NES 109 standard also specifies criteria for checking stability under icing. A thickness of 150 mm should be assumed for all horizontal decks, with an ice density equal to 950 kg m^{-3}. Only the effect on displacement and $\overline{KG}$ should be considered, and not the effect on the sail area.

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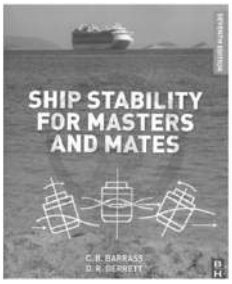
Icing



CHAPTER 32

Icing Allowances Plus Effects on Trim and Stability

In Arctic ocean conditions, the formation of ice on the upper structures of vessels can cause several problems (see Figures 32.1 and 32.2). Ice build-up can be formed from snowfall, sleet, blizzards, freezing fog, and sea spray in sub-zero temperatures. In the Arctic, the air temperatures can be as low as −40 °C in harbor and −30 °C at sea.

Icing allowances must be made for:

- Rise in G. Loss of transverse stability.
- Increase in weight. Increased draft due to increased weight.
- Loss of freeboard due to increased weight.
- Decrease in underkeel clearance.
- Contraction of steel due to temperature.
- Increased brittleness in steel structures.
- Nonsymmetrical formation of ice.
- Angle of list. Angle of loll.
- Change of trim.
- Impairment of maneuverability.

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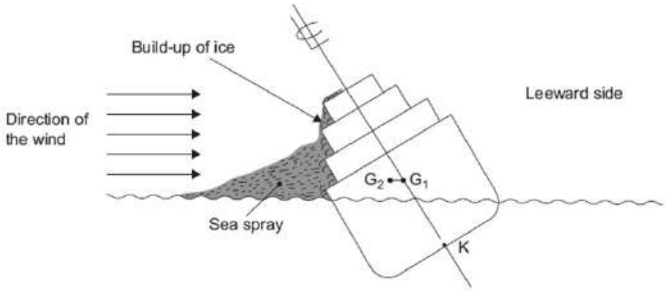

Figure 32.2: Asymmetrical Build-up of Ice, Causing an Angle of List.
G_1 moves to G_2.

- Reduction in forward speed.
- Increase in windage area on side of ship.

A 30 cm diameter of ice can form around a hawser or cable (see Figure 32.1). Blocks of ice 100 cm thick have been known to form on the poop deck of a ship in very cold weather zones. Walls of 60 cm of ice forming on the surface of a bridge front have been recorded. In ice-ridden waters the depth of ice may be up to 3 m.

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Effect of Icing on the Righting Arm (GZ)

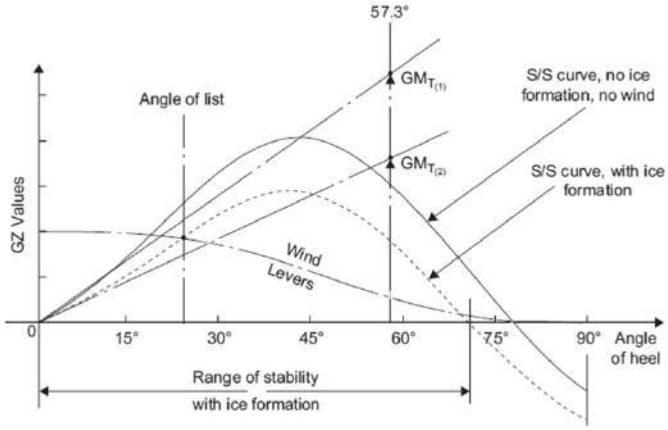

Figure 32.3:
Loss of Statical Stability Due to Wind and Formation of Ice.

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Large Angle of Heel: Numerical Example

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A cuboidal barge with angle of heel > DEI angle

- DEI = Deck-Edge-Immersion
- A water-tight cuboidal barge with length L, breadth, B, depth, D, and draft T, heels by a large angle, ϕ, and the deck-edge is immersed. Find the draft at midship and the waterplane area.
- See the figure on the next slide
- The uw volume before and after it heeled is LBT.
- The above water volume before and after it heeled is $LB(D-T)$
- The above water volume after it heeled is $0.5\ a\ a \tan\phi\ \ L = LB(D-T)$
- Alternatively, equate uw vol before and after heeling. $LBD\ - 0.5La^2 \tan\phi = LBT$
- So, $a^2 = 2B(D-T)\ /\tan\phi$. Use this to find a.

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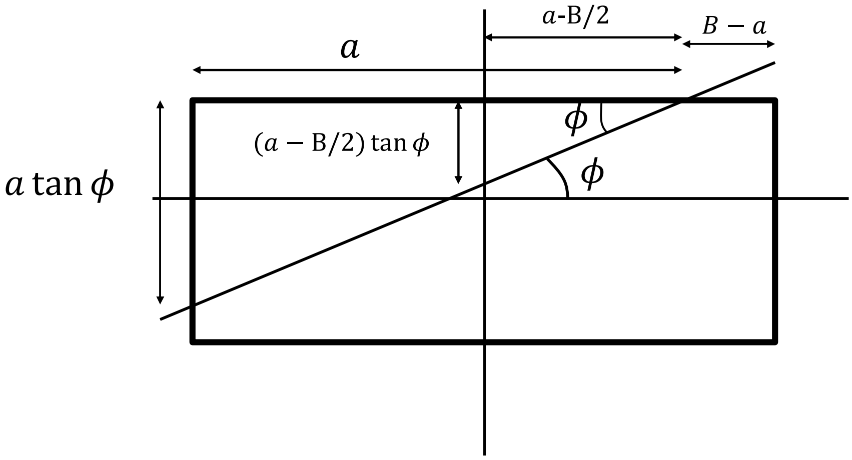
A cuboidal barge with angle of heel > DEI angle

- DEI = Deck-Edge-Immersion



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A cuboidal barge with angle of heel > DEI angle

- Draft at midship $= T_M$
- $T_M + \left(a - \frac{B}{2}\right)\tan\phi = D\,.\,T_M = D - \left(a - \frac{B}{2}\right)\tan\phi.$
- Note: $\phi = 90$ deg => $a = 0$ and $LDT_M = LBT$

- Breadth of the Waterplane Area $= B_\phi = a/\cos\phi$
- Or, find the hypotenuse: $B_\phi^2 = a^2(1+\tan^2\phi) = a^2(\sec^2\phi).\ B_\phi = a\sec\phi.$
- Waterplane Area $= LB_\phi = La\sec\phi.$
- $\overline{BM}_\phi = \frac{I}{V} = \frac{LB_\phi^3}{12\,LBT} = \frac{a^3\sec^3\phi}{12BT}$. Use this to find the CoB: see Assignment 07.

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Assignment L20

- A water-tight cuboidal barge has L = 10 m, B = 4 m, D = 3 m, T = 2 m. The CoG is on the centerline and $\overline{KG} = 1.5$ m. Draw the $\overline{BM}$, CoB, and the $\overline{GZ}$ curves for heel angle = 0 to 90 deg.
- For the upright condition, $\overline{KB} = 1$ m. $\overline{BM}_0 = \frac{B^2}{12T}$ =2/3 m. $\overline{KM} = 1.6667$ m. $\overline{GM} = 1.6667 – 1.5 = 0.16667$ m.
- See the next slide for $\overline{BM}$

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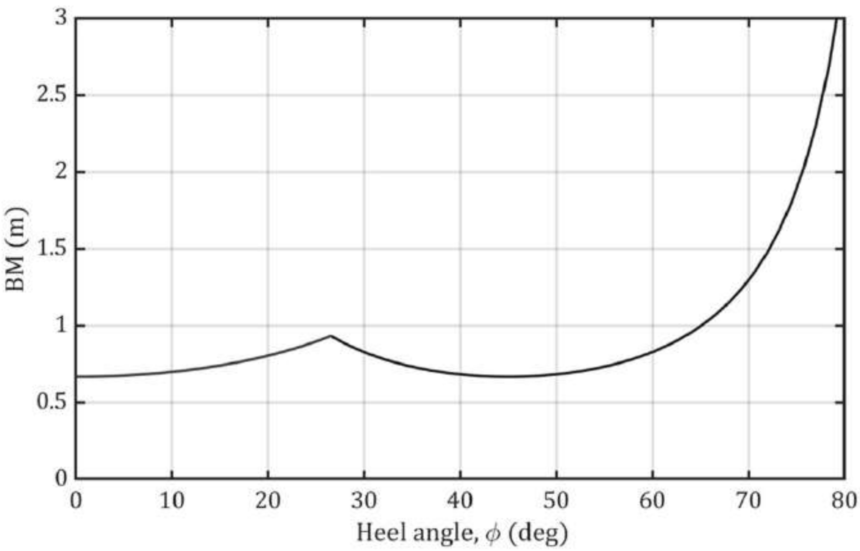

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