* Measurements & measurement system

- measurand :- the thing which is measured
- Direct measurement
  - ↳ measured directly
  - ↳ measured by some method
  - ↳ not any conversion.

eg- Voltage

→ Primary m/s

- measured directly by looking into that particular factor only
- "Sight" → Brain.
- change in colour

→ Secondary m/s

- converts original measurement into a more comfortably readable measurand to human eyes

Pressure → change in length
measure secondary m/s

→ Tertiary m/s

conversion happens more than two times to measure easily.

Temp - $\boxed{\text{thermocouple}} \xrightarrow{\text{voltage}} \boxed{\text{Voltmeter}}$

→ change in length

Measurement system :-

The entirety of things / devices system used in measurement in an industry is known as measurement system.

Classification of instruments :-

1) **Absolute instrument**

→ deflection and physical constants.
→ need not be calibrated.
→ accurate

eg:- Tangent galvanometer
- current in a ckt
- direction of dc current

$$\boxed{i = \frac{2rH}{\mu_0 n}\tan\theta}$$

Secondary instrument

→ deflection is dicating on the instrument
→ directly measure the unknown qty.
→ Need calibration (using an absolute or std. instrument)
→ eg:- Ammeter or voltmeter, wattmeter

2) **Digital and analog instruments**

↳ o/p obtained in analog format signal
Analog ammeter & voltmeter

↳ discrete values
→ finite value
→ accurate
→ high cost
→ sensitivity ↑

3) **Manual or automatic instruments :-**

→ man help is required.
- traditional weight scale

→ electronic weight m/c

4) Self operated or power operated
↳ no external power supply
→ qty to be measured will activate the instrument
eg:- Ammeter

→ An external power supply is required
eg:- Electronic weigh machine

5) Indicating, Recording, Integrating and controlling instrument

Indicating:- Indicate the value to be measured → Ammeter, Voltmeter.

Recording:- Record the value of unknown qty.
→ graph
→ numerical value } eg:- ECG m/c

Integrating:- Summation of value of unknown qty from a particular time to time of m/s
eg:- Energy meter

controlling instrument:- controls the measured value by using some information obtained.
eg:- Electrical relays.

Process →
→ Mechanical, electrical, Electronics
- A mechanical process is there to measure the unknown quantity.
→ not Reliable due to wear and tear and high inertia.

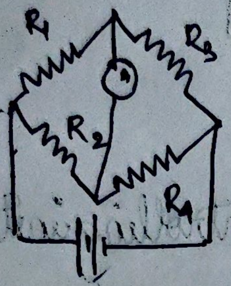
→ Electrical process:-
→ but mechanical system is there to display.
Ammeter.

→ Electronic
→ semiconductors
→ More accurate and reliable over other types of instruments
→ Electronic sensors

* Null type and deflection type instruments :-

→ Balancing to a null position in order to get a unknown quantity

↳ Deflection of a pointer shows the value of unknown qty.



* Callibration
Comparison of the perfomance of an instrument against a standard instrument.

↳ Need of callibration :-
→ accuracy and Tracability
→ fuel flow rate
→ Safety / Safe working of system
→ Compliance with Regulation
IMO, SOLAS.
→ For proper maintenance.

- Lab callibration, On-site callibration

Standards
- reference unit
- Primary standard
- National or International standards
Secondary standard.
Industry a c
Working standard
Regulation

Error in Instrument :-

- Deviation from true value
  Error = True value - Indicated value

Systematic errors

Regular reproducable error

- Instrumental error imperfections
  eg:- Null/zero position error
- Observer imperfection
  → parallax
  eg:- → stopwatch error
→ Careful observation, training
→ Theoretical observation
→ Environmental fluctuations
  eg:- Temp. may increase resistance
  Marine environmental conditions.
Use shield or cover, isolate.

Gross error

- due to observer
  eg:- Parallax error
- Incorrect recording

Random error

- Random reasons
- avg
- avg statistical analysis

* Perfomance characteristics of Instruments :-

Describe perfomance of an inst.

Static PC
→ will not change with time
→ change slowly time

Accuracy
Precision
Sensitivity
Reproducability
Repeatability
} desirable

Drift
Dead zone & Dead time
Threshold
Hysteresis
Resolution
Creep
} undesirable

Dynamic PC
→ change wrt time
→ speed of response & time of response
→ Lag X
→ Fidelity
→ Dynamic error X

i) Accuracy

- Closeness of the indicated value to the true value

$$\text{Accuracy} = A_i - A_t$$

$$\%\text{Accuracy} = \frac{A_i - A_t}{A_t} \times 100.$$

→ Express accuracy by

→ Accuracy as % of full scale reading

$\pm 0.1\%$ of full scale reading.

$50 \rightarrow 0.05$

$50V \rightarrow 50 \pm 0.05$

$25V \rightarrow 25 \pm 0.05.$

70 ± 0.05

Accuracy as % of true value

$\pm 0.1\%$ accuracy

$50 \rightarrow 50 \pm 0.05\ V$

$25 \rightarrow 25 \pm 0.025V$

$10 \rightarrow 10 \pm 0.01V$

Accuracy as % of scale span → Point accuracy

$\boxed{25 - 225}$

$225 - 25 = 200$

$\pm 0.1\%$ of scale span

$$\frac{0.1}{100} \times (225 - 25)$$

$$\frac{0.1}{100} \times 200 = \underline{\underline{0.2}} = \underline{\underline{\pm 0.2}}$$

ii) Precision:-

- Property of repeating the same measured value for multiple m/s.

or Consistency of the instrument in repeated measuring.

- Conformity of an instrument

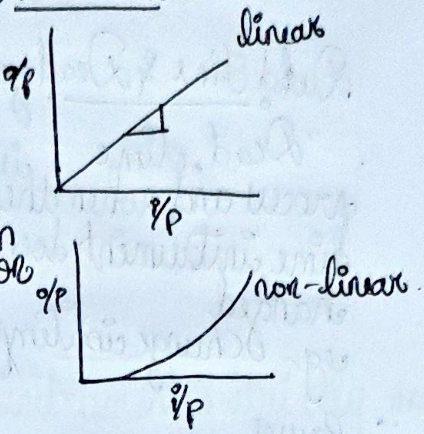
- Significant figures

$$\boxed{P = x_n - \left|\frac{x_n - \bar{x}_n}{x_n}\right|}$$

* Sensitivity

$$\boxed{S = \frac{\Delta A_{out}}{\Delta A_{in}} \times 100\ \%}$$

Linear calibn



Repeatability

Variation in instruments measurements taken on the same item under the same condition
ability of the instrument to repeat same reading under identical conditions

Reproducability

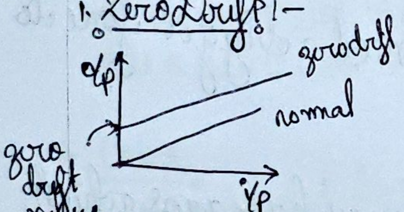
It is the ability of a measurement to be duplicated either by the same person or by someone else under slightly changed conditions
The degree to which repeated measurements taken under changed condition For example operator, time yields the same results which shows the reliability and consistency of the result.

~~Perfomance characteristics of Instruments~~

⇒ Drift

The slow change in the output of the instrument over time, when the input remain constant.

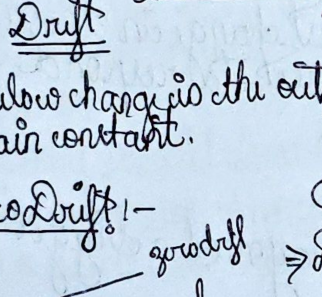
1. Zero Drift :-



Cause of drift
⇒ Due to stray EF/MF reduce due to ~~stage~~ shield
Mechanical variations :- Proper support
Wear & Tear :- Maintenance
Temperature change :- Proper temp regulation

If the whole calibration gradually shift due to various reasons, zero drift happens

2. Sensitivity drift



If there is a proportional change in the indication all along the upward scale is known as sensitivity drift
It defines the amount by which an instrument sensitivity of measurement varies when external conditions varies

8 Zonal drift

~~If the~~ The drift occurs only over a particular range of measurements is called zonal drift.

Dead Time & Dead zone

Dead time is the delay between when a change happens in the process and when the instrument starts to respond. During this time instrument does not show any indication that the value has changed.

eg:- Change in temperature takes time to show in instrument (furnace)

Causes

i) Sensor delay:-
Some instrument sensors take time to detect changes

ii) Transmission delays:-
Signal from the sensor may take time to reach the display unit.

iii) Process delay:-
Some instruments take time to process the sensored value before displaying it.

Dead zone (Dead band) is the smallest change in the measured value that does not cause the instrument to respond.

Causes:-

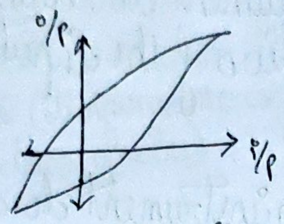
i) Friction:-
Moving parts inside a instrument may need a bigger force to start moving.

ii) Backlash:-
In mechanical systems, gears and linkage may have gaps which prevent small changes from being transmitted immediately.

iii) Noise filtering:-
Some instruments ignore small fluctuations to avoid noise.

* Threshold
If the input to an instrument is gradually increased from zero, it should reach a minimum value to actuate the measuring process. This minimum level of input is known as the threshold of that instrument

* Hysteresis



If the input measurement quantity to an instrument is strictly increased from a negative value,
The output varies in a particular manner and if the input is steadily decreased, the output varies in another manner. This known coincidence between these loading and unloading curves is referred to as hysteresis.

* Resolution
The smallest increment of a qty being measured which can be detected by an instrument with certainty.
If a non-zero input quantity is slowly increased output reading will not increase until some minimum change in the input take place. This minimum change which cause the instrument to show the a detectable output is called resolution

* Creep
Creep is caused by the time and instrument needed to adapt the change in applied input.

* Static error
- of an instrument is a numerical difference b/w the true value of a quantity and its value obtained by the instrument

$e_s = A_t - A_i$

$$\boxed{e_s = \frac{A_t - A_i}{A_t} \times 100 \%}$$

Dynamic Characteristic of Instrument

i) ~~Speed of~~ response:-

The rapidity with which an instrument or measurement system responds to the changes in the measured quantity.

ii) Time of response

- time required by an instrument to settle to ~~0~~ its final steady position after the application of the input.

iii) Measurement lag:-

The delay in the response of an instrument to a change in the measured ~~qu~~ value

→ Retardation type:- The response begins immediately after a change in measured quantity has occured

→ Time delay:- The response begins after a dead zone after the application of the input

Fidelity.

It is defined as the ability of the instrument to reproduce the output in the same form as input or degree to which a measurement system indicate the changes in measured quantity without any error

Dynamic error:-

Diff. b/w true value and of the quantity changing with time and the value indicated by the instrument if no static error is assumed

* Analysis of Error:-

Statistical Analysis of Errors:-

- Absolute error → $4V \pm 0.03\,V$
- Limitting error →
- Relative error → $4V \pm 0.00\,3\%$

true value or scale span

Uncertainity analysis:

$R = R(x_1, x_2, \ldots x_n)$ $\quad R = \frac{V}{I}$

$x_1 \rightarrow wR_1$

$x_2 \rightarrow wR_2$

$$\boxed{W_R = \left[\left(\frac{\partial R}{\partial x_1} w_{R_1}\right)^2 + \left(\frac{\partial R}{\partial x_2} w_{R_2}\right)^2 + \dots\right]^{1/2}}$$

⇒ w_{R_1}, w_{R_2} are the uncertainity of the measurements

⇒ W_R is the total uncertainity

Qn1- The resistance of a certain size of wire is given as $R = R_0[1+\alpha(T-20)]$ where $R_0 = 6\Omega \pm 0.3\%$ is the resistance at $20^\circ C$, and α is given $0.004^\circ C^{-1} \pm 1\%$ (temperature coefficient of resistance)
$T = 30 \pm 1^\circ C$ (temp of wire)

$x_1 = \alpha$
$x_2 = T$

$W_R = \dots$

$\frac{\partial R}{\partial x_1} = \frac{\partial R}{\partial \alpha} = R_0 T - R_0 20$

$R_0(T-20) \times 1 = 6(30-20)$

$= 60$

$R_0 + R_0 \alpha T - R_0 \alpha 20$

$\frac{1}{3\alpha} \times 100 = 3$

$\frac{\partial R}{\partial x_2} = \frac{\partial R}{\partial T} = R_0 \alpha \times = 6 \times 0.004$

0.024

Nominal R

$R = R_0[1+\alpha(T-20)]$

$= 6[1+(0.004(30-20)]$

1.04×6

6.24

$\frac{\partial R}{\partial R_0} = 1+\alpha(T-20)$

$= 1+0.004(30-20)$

$1+0.004 \times 10$

1.04

$\frac{1}{3} \times 100$
3

$w_{R_0} = 0.3\% \times 6$

$= 0.18$

$w_\alpha = 1\%$

0.0004×0.01

$w_T = 8.33\% \; 1^\circ C$

$\frac{4 \times 10^{-5}}{2}$

$\therefore w_R = \left[(1.04 \times 0.3)^2 + (1 \times 60)^2 + (8.33 \times 0.024)^2\right]^{1/2}$

$= 0.035043 84 + 0.024$

0.000576

0.00000576

$= \frac{0.0305}{6.24} \times 100 = 0.48\%$

Structural Analysis of Errors

1) Arithmetic mean

$x_i (x_1, x_2, \ldots x_n)$ $\quad A = \frac{x_1 + x_2 + \ldots + x_n}{n}$

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

2) Median

$x_1, x_2, \ldots x_n$

Median = central values x if n is odd.

$\frac{x_{m_1} + x_{m_2}}{2}$ if n is even

3) Deviation

$x_1, x_2, \ldots x_n$

mean $= x_m$ $\quad x_m = \frac{x_1 + x_2 + \ldots + x_n}{n}$

$d_i = x_i - x_m$

average of all deviations is zero

$d = \frac{1}{n}\sum_{i=1}^{n} d_i$

$\frac{1}{n}\sum_{i=1}^{n}(x_i - x_m)$ $\qquad |d| = \frac{1}{n}\sum_{i=1}^{n}|d_i|$

$= x_m - x_m = 0$

4) Standard deviation

RMS

$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - x_m)^2}$

5) Variance

$= \frac{1}{n}\sum_{i=1}^{n}(x_i - x_m)^2$

$GM = (x_1 x_2 \ldots x_n)^{1/2}$

Qn) The following readings of a certain physical length. Compute the mean, S.D, variance & avg of the absolute value of the deviation

5.30	4.83	5.81
5.73	5.45	5.75
6.77	6.09	
5.26	5.64	

Mean.

	$a_i - a_m$	$(a_i - a_m)^2$
5.30	0.313	0.097969
5.73	-0.117	0.013689
6.77	-1.157	1.338649
5.26	0.353	0.124609
4.33	1.283	1.646089
5.45	0.163	0.026569
6.09	-0.477	0.227529
5.64	-0.027	0.000729
5.81	-0.197	0.038809
5.75	-0.137	0.018769

$a_m = 5.613$

A moving point voltmeter has a uniform scale with 100 divisions. The full scale deviation is 200V and 1/10th of a scale deviation can be estimated with a fair degree of certainty. Determine the resolution of the instrument in volt.

$\frac{1}{10}$ th of scale deviation

Resolution =

one division $= \frac{200V}{100} = \underline{\underline{2V}}$.

Resolution gives 1/10th of scale deviation

$= \frac{1}{10} \times 2V = \underline{\underline{0.2V}}$

⇒ A digital voltmeter has a readout rate from 0 to 999 counts. Determine the resolution of the instrument in V, if the full scale reading is 9.999V.

$\frac{9.999}{999} = \frac{9.999}{9999} = \frac{1}{1000} = \underline{\underline{1mV}}$

⇒ A wattmeter having a range of 1000W has an error of ±1% of full scale deflection. If the true power is 100W, what would be the range of readings ~~(approx)~~

error = ±1% of 1000

= ±10 W   100 ± 10W

= $\underline{\underline{90 - 110 W}}$

* [If the error is ±1% of true value

C Range is 100W m/s

error of ±1% of 1000
±10

99-1010

working on thermal principle

⇒ A flowmeter of guaranteed accuracy of ±5% on full scale reading of $5\times10^{-6}\,m^3/s$. The flow measured by the meter is $2.5\times10^{-6}\,m^3/s$. Calculate the limitting error in percentage

±5% of $5\times10^{-6}\,m^3/s$

$$\text{error} = \frac{5}{100}\times5\times10^{-6} = 2.5\times10^{-6}\,m^3/s$$

Limitting error

Manufacturer specify the accuracy of the instrument as a percentage of full scale reading. This percentage is dicate the deviation from the specified value the deviation from the specified value of the measurand. This deviation is called limitting error or guaranteed error

$$A_i = A_s + \delta E$$

$$\text{The relative error} = e = \frac{\delta A}{A_s}$$

$$\delta A = e A_s$$

$$A_a = A_s \pm e A_s$$

$$A_a = A_s(1\pm e)$$

$$\text{Relative limitting error} = \left(\frac{A_a - A_s}{A_s}\right)\times100\%$$

$$\text{Relative error} = \frac{\delta A}{A_s} = \qquad \delta A = 5/100 \times 5\times10^{-6}$$

$$\text{val} = 0.25\times10^{-6}\,m^3/s$$

$$\delta A = e A_s$$

$$\frac{0.25}{2.5} = e$$

$$e = 0.1$$

$$\text{Relative error} = \pm0.1$$

$$2.5\times10^{-6}(1\pm0.1)\,m^3/s$$

A pressure gauge having a range of 1000 kN/m² has an error of ±1% of full scale deflection. If the true pressure is 100 kN/m², what would be the range of reading and if the error is specified as % of true value. Find the range of readings.

Determine the magnitude and limiting error in ohm and the % of resistance value for the series combination of following resistors (R_1 = 37 Ω ± 5%) R_2 = 75 Ω ± 5%, R_3 = 50 Ω ± 5%.

3) The resistance of a circuit is found by measuring current flowing and the power fed into the circuit. The relative limiting error in the measurement of power and current are ±1.5% and ±1% respectively. Find the limiting error in the measurement of resistance.

37 ± 5% 37.
75 ± 5% of 75.
50 ± 5% of 50.
= 162 ± ~~8~~ 5%
162 ± 8.1 Ω

The solution of unknown resistance from a wheatstone's bridge is

$R_4 = \frac{R_2 R_3}{R_1}$, R_1 = 100 ± 0.5% Ω
R_2 = 1000 ± 0.5% Ω
R_3 = 842 ± 0.5%

Determine the magnitude of unknown resistance and relative limiting error in

$R_4 = \frac{1000 \times 842}{100} = 8420\,\Omega$

% uncertainty = 0.5 + 0.5 + 0.5 = 1.5%

Limiting error = Absolute uncertainty 8420 × $\frac{1.5}{100}$ = ±126.3 Ω

R_4 = 8420 ± 126.3 Ω

8420 ± 1.5%

% limitting error:

$$\frac{\partial R_4}{R_4} = \pm\left(\frac{\partial R_1}{R_1} + \frac{\partial R_2}{R_2} + \frac{\partial R_3}{R_3}\right)$$

$$= \pm(0.5\% + 0.5 + 0.5)$$

$$= \pm(1.5\%)$$

$$= 8420 \pm 1.5\%$$

Q) The output power of a rotating shaft is measured by a dynamometer. The equation for output power is

$$P = \frac{2\pi \cdot F \cdot L \cdot N}{t \times 10^6} \text{ KW}.$$

F = force (N)
L = Length of torque arm (mm)
N = No. of revolutions
t = time for det rev

$F = \cancel{6.10 \cdot 0.02 kg}$ 4.58 ~~N = 1850 ± 1 rev~~
~~L = 400 ± 0.5 mm~~ $t = 60 \pm 0.1$ s

$L = 397 \pm 1.3$ mm
$N = 1202 \pm 1$ revolution
$t = 60 \pm 0.05$ s
$F = 0.46 \pm 0.002$

Determine the magnitude of power & limitting error in computed error:

$$P = 0.00365 \pm 2.352 \text{ KW}.$$

Q) Two resistors R_1 and R_2 are connected in series and other in parallel. The values of resistance are $R_1 = 100 \pm 0.1\,\Omega$, $R_2 = 50 \pm 0.03\,\Omega$. Calculate the uncertainity in resistance in both series and parallel.

$R_s = R_1 + R_2$

$= 150 \pm 0.13\,\Omega$

$$R_p = \frac{R_1 R_2}{R_1 + R_2} =$$

Limitting error — Cumulative error :-

The guarantee error is specified as ± 8V. Then for a measurement of 100V obtained from the instrument what would be the actual voltage or and what would be the relative error in percentage.

$A_s = 100V$

$A_a = 100 \pm 8A$

$100 \pm 3V$

$\Rightarrow \boxed{97V - 108V}$

If the error is given as ± 1% of full scale reading what would be the error

± 1% of full scale reading

$= \cancel{\pm 1\% \text{ of } 103V}$ 100 ± 8 V.

$\frac{8}{100}$

$y = x_1 + x_2$

$\frac{dy}{y} = \frac{d}{y}(x_1 + x_2)$

$= \frac{dx_1}{y} + \frac{dx_2}{y}$

$\frac{dy}{y} = \frac{x_1}{y} \cdot \frac{dx_1}{x_1} + \frac{x_2}{y} \frac{dx_2}{x_2}$

$\Rightarrow \frac{\delta y}{y} = \pm \left(\frac{x_1}{y} \cdot \frac{\delta x_1}{x_1} + \frac{x_2}{y} \frac{\delta x_2}{\delta x_2} \right)$

$y = x_1 - x_2$

$\frac{dy}{y} = \frac{d}{y}(x_1 - x_2)$

$= \frac{dx_1}{y} - \frac{dx_2}{y}$

$\pm \left(\frac{dy}{y} = \frac{x_1}{y} \cdot \frac{dx_1}{x_1} - \frac{x_2}{y} \cdot \frac{dx_2}{x_2} \right)$

$y = x_1 x_2$

$\log y = \log x_1 + \log x_2$

$\frac{1}{y} = \frac{1}{x_1} \frac{dx_1}{dy} + \frac{1}{x_2} \cdot \frac{dx_2}{dy}$

$\frac{dy}{y} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2}$

$\frac{dy}{y} = \pm \left(\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2} \right)$

$y = x_1^n$

$\log y = \log x_1^n$

$\log y = n \log x_1$

$\frac{1}{y} = n \cdot \frac{1}{x_1} \cdot \frac{\partial x_1}{\partial y}$

$\frac{\partial y}{y} = \frac{n \, \partial x_1}{x_1}$

$\frac{\partial y}{y} = \pm \left(n \frac{\delta x_1}{x_1} \right)$

Resistance of a circuit is found by measuring current flowing and the power fed into the circuit. Find the limiting error in the measurement of resistance.

limiting error is

$P = I^2 R$

$\frac{\delta P}{P} = 2\frac{\delta I}{I} + \frac{\delta R}{R}$

$= (2\times1) + 1.5$

$= 3.5\%$

$R = \frac{P}{I^2}$

$\frac{\delta P}{P} + 2\frac{\delta I}{I}$

$= 2\times1 + 1.5$

$= \pm 3.5\%$

~~Stead~~

Probable Error

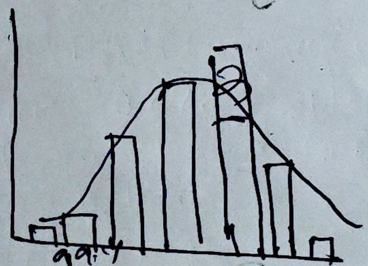
Statistical Analysis of Data

Multisample data → Different test conditions such as diff. test, test method, diff observer.

Single sample data → same test condition (diff time).

Statistical Analysis:-

99.7	1
99.8	3
99.9	10
100	20
100.1	12
100.2	4
100.3	1



→ Dispersion is the phenomena of ~~along~~ the scattering of data along the central value

→ Range (spread/scatter)

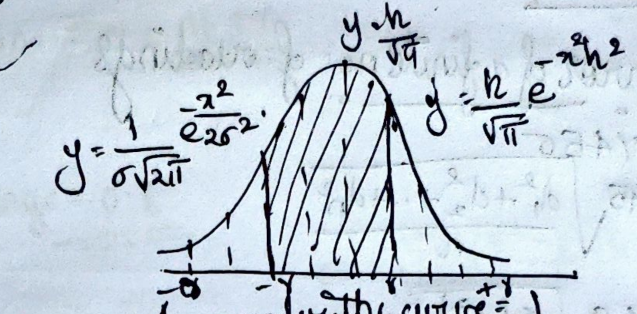
→ Normal distribution

→ The Gaussian law of probability states that the normal occurrence of deviations from average value ~~error~~ of an infinite no. of observation can be expressed as $\boxed{y = \frac{h}{\sqrt{\pi}} e^{-x^2h^2}}$

In this equation x is ~~mean~~ deviation from mean
y is no. of readings at any deviation x.
h - a constant called precision index

$\Rightarrow$ $$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

Then the deviations from the mean value are divided in terms of σ units so that the deviations are $x = \sigma, 2\sigma, 3\sigma, \ldots$



Area under the curve = 1

+ Precision index (h)
$\Rightarrow$ when $x = 0$ $y = \frac{h}{\sqrt{\pi}}$, the larger the value of h, the sharper is the curve
$\Rightarrow$ More precision as values are scattered towards central value

* Probable error :-
We have observed that the most probable or best value of a Gaussian distribution is obtained by taking arithmetic mean of various values of the variant and the confidence ie. the best value to the correctness of the sharpness of the distribution curve

Let us consider two points $-r$ and $+r$. ~~to~~ take as marked in fig. These points are located at the area bounded by the curve, the x axis and the ordinates erected from $-r$ to $+r$. is equal to half of the total area under the curve. That means half the deviations lies between ~~±~~ $x = \pm r$ and convenient measure of precision is the quantity r and this is called probable error.
~~Half of the observed values actually~~

If ~~A~~ determine r as the ~~[illegible]~~ value of n measurements and then makes an additional measurement. Then the chances are 50-50% that the value will lie between $-r$ and $+r$ ie the chances are even that any ~~es~~ one reading will have an error not greater than $\pm r$

$$y = \frac{h}{\sqrt{\pi}} \int_{-r}^{r} e^{-h^2 x^2} = \frac{1}{2}$$

$$r = \frac{0.4769}{h}$$

$x = \pm r$

$$\boxed{r = 0.67456}$$

Probable error of a finite no. of readings

$$r = 0.67456$$

$$r = 0.6745 \sqrt{\frac{d_1^2 + d_2^2 + \dots + d_n^2}{n}}$$

$$= 0.6745 \sqrt{\frac{\Sigma |d^2|}{n}}$$

For an infinite no. of deviation from the normal curve, n but for finite nos of deviations σ is replaced by standard deviation $S = \sqrt{\frac{d_1^2 + d_2^2 + \dots d_n^2}{n-1}}$

$$r = 0.6745 \sqrt{\frac{\Sigma d_i^2}{n-1}}$$

In the finite no. of readings, the average reading has a probable error of $r_m = 0.6745 \sqrt{\frac{\Sigma |d_i^2|}{n(n-1)}}$

If $n >>> 1$ $n - 1 \approx n$

$$\boxed{r_m = 0.6745 \frac{\sigma}{\sqrt{n}}}$$

* Standard deviation of mean

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

$$S.D \text{ of } S.D \quad \sigma_s = \frac{\sigma_m}{\sqrt{2n}} = \frac{\sigma_m}{\sqrt{2}}$$

The following 10 obs. were recorded when measuring a temperature

	d	d^2
417	0.27	0.0729
420	-0.03	0.0009
41.8	+0.17	0.0289
42.0	-0.03	0.0009
42.1	-0.13	0.0169
41.9	0.07	0.0049
42.0	-0.03	0.0009
41.9	0.07	0.0049
42.5	-0.53	0.2809
41.8	0.17	0.0289

The mean
S.D
Probable error of 1 reading
Probable error of mean
Range

$$\text{Mean} = \frac{\Sigma x}{n} = \underline{\underline{41.97}}$$

$$S.D = \sqrt{\frac{\Sigma d^2}{n}} = \frac{0.441}{10}$$

$$\sqrt{0.0441} = \underline{\underline{0.21}}$$

Range = $\underline{\underline{0.8}}$

$$S = \sqrt{\frac{\Sigma d^2}{n-1}} \text{ finit } < 20 = \sqrt{\frac{0.441}{(10-1)}} = \underline{\underline{0.22^\circ C}}$$

· In a test temperature is measured 100 times with variations in apparatus and procedure. After applying the correction, the results are

T °C	freq
397	1
398	3
399	12
400	23
401	37
402	16
403	4
404	2
405	2

i) Arithmetic mean
ii) Mean deviation
iii) S.D
iv) Probable error of 1 reading
v) Probable error of mean
vi) S.D of the S.P

		Tf	$\bar{d}$	d × f	d^2	fd^2
397	1	397	-3.78	-3.78	14.2884	14.288
398	3	1194	-2.78	-8.34	7.728	23.185
399	12	4788	-1.78	-21.36	3.168	38.020
400	23	9200	-0.78	-17.94	0.608	13.993
401	37	14837	+0.22	+8.14	0.048	1.708
402	16	6432	+1.22	+19.52	1.488	23.814
403	4	1612	+2.22	+8.88	4.928	19.714
404	2	808	+3.22	+6.44	10.368	20.737
405	2	810	+4.22	+8.44	17.808	35.618
		$\bar{x}$ = 400.78		0		$\Sigma fd^2 = 991.08$

$$D = \sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{191.08}{100}}$$

$$r_1 = 0.6745\ \sigma = 0.9324$$

$$r_m = \frac{r_1}{\sqrt{n}} = 0.093$$

$$\sigma_m = \frac{\sigma}{\sqrt{n}} = 0.1382$$

Probable error of combination of components

eg:- $R = \frac{V}{I}$

S.D =

$X = f(x_1, x_2, \ldots x_n)$

$$S.D\ \sigma_x = \sqrt{\left(\frac{\partial x}{\partial x_1}\right)^2 \sigma x_1^2 + \left(\frac{\partial x}{\partial x_2}\right)^2 \sigma x_2^2 + \left(\frac{\partial x}{\partial x_3}\right)^2 \sigma x_3^2}$$

$$r_x = \sqrt{\left(\frac{\partial x}{\partial x_1}\right)^2 r_{x_1}^2 + \left(\frac{\partial x}{\partial x_2}\right)^2 r_{x_2}^2 + \ldots \left(\frac{\partial x}{\partial x_n}\right)^2 r_{x_n}^2}$$

$r_{x_1}, r_{x_2}, \ldots$ are the probable error of component $x_1, x_2, \ldots x_n$

$\frac{\partial x}{\partial x_1} r_{x_1}$

Qn: We have a parallel circuit having two branches, the current in 1 branch is $I_1 = 100 \pm 2A$

$I_2 = 200 \pm 5A$

Determine the value of total current

$I = I_1 + I_2$

a) Considering the error in I_1 and I_2 as limiting error

b) Considering the errors as standard deviations

Fractional errors $I_1 = \frac{\delta I}{I} = \pm\left(\frac{I_1}{I} \cdot \frac{\delta I_1}{I_1} + \frac{I_2}{I} \cdot \frac{\delta I_2}{I_2}\right)$

$= \frac{\delta I_1}{I_1} = \frac{2}{100} = .002$; $\frac{\delta I_2}{I_2} = \frac{5}{200} = 0.025$

$$\frac{\delta I}{I} = \pm\left(\frac{100}{300} \times 0.02 + \frac{200}{300} \times 0.025\right)$$

$\rightarrow \pm 0.0233 \Rightarrow 2.33\%$

$I = 300 \pm 2.33\% \text{ of } 300$

$= 300(1 \pm 0.0233)$

$= \underline{300 \pm 7A}$

$$\sigma_I = \sqrt{\left(\frac{\partial I}{\partial I_1}\right)^2 \sigma_{I_1}^2 + \left(\frac{\partial I}{\partial I_2}\right)^2 \sigma_{I_2}^2}$$

$\sigma_{I_1} = 2 \quad \sigma_{I_2} = 5$

$\frac{\partial I}{\partial I_1} = 1 \quad \frac{\partial I}{\partial I_2} = 1$

$I = I_1 + I_2$

$$= \sqrt{1^2 \times 2^2 + 1^2 \times 5^2} = 5.38A$$

$$\underline{I = 300 \pm 5.38A}$$

* Module-II

Transducers

- converts a form of energy to others when it actuates

Electrical transducer

physical quantity non-electrical } electrical qty

Advantage :- electrical transducer

(ip device → sp device (signal processing) → op device ↓ read

- Easy signal processing →
- Mass inertia is less or negligible → es mass is less
- Miniaturization (IC chips)
→ Less friction
- Telemetry.

Transducer

→ Primary sensing element

→ Transduction element → electrical signal

Classification of Transducers

→ Based on transduction principle.

- Resistance
- Inductive
- Capacitive

} eg.

Resistance

~~Potentiometer~~ device

Positioning of the slider by an external force varies the resistance

		Application
Potentiometric device	resistance of a ckt changes	force, displacement
Resistance strain gauge	force/strain applied to strain gauge, its resistance changes	Force, Torque, Displacement.
Pirani gauge Hot wire meter	heating element resistance changes by convection cooling.	gas pressure gas flow

Thermometer Resistance hygrometer	-ve temp. coefficient	Humidity
	Resistance changes with moist. content	
Resistance thermometer	Temperature changes resistance (+ve temp. coefficient)	temperature, flow
Photoconductive cell	Incident light changes	Photo sensitive relay

* **Capacitive transducer**

Variable capacitive pressure gauge -	capacitance of a parallel plate capacitor change	Displacement pressure
Capacitor microphone	diaphragm changes	speech noise
Dielectric gauge	The variance in capacitance by changing dielectric medium	liquid level thickness

* **Inductive transduction**

Magnetic ckt transducer -	self induction or mutual induction by changing magnetic ckt	Pressure difference
Reluctance pickup	By changing reluctance	vibration
Differential transformer	change in position of core	position

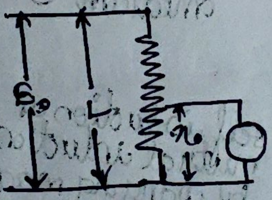
* Eddy current gauge	change in eddy current changes the inductance ~~change in m.f~~	force, displacement
Magnetostriction gauge	change in m.f	Force, torque, thickness

Hall effect pickup

<u>Tranducer</u>

i) Primary or Secondary tranducer
Bordentube pressure m/s system.

ii) Passive and Active tranducer
Passive tranducers derive power required for translation from an ext ~~auxilary power~~ externally powered tranducer
eg: - potentiometer



$E_o = \frac{x}{L} e_c$

The property of piezoelectric crystal is when a force is applied to it produce a voltage
The mass exert certain force due to the acceleration due to which voltage is generated
Force $\propto$ acceleration

Tranducers

Classification

→ Based on tranduction

→ Passive and Active

→ Analog and Digital transducers

Analog

Produce an analog o/p signal

↳ continuously varying with time

eg:- Potentiometer

Digital Trdr

digital o/p quities

→ shaft encoders | opaque and translucent | photo conducting and non-conducting

Transducers and inverse transducers

↳ Piezoelectric crystal

vol → much vibration

Resistive, Inductive, capacitive } Passive transducers

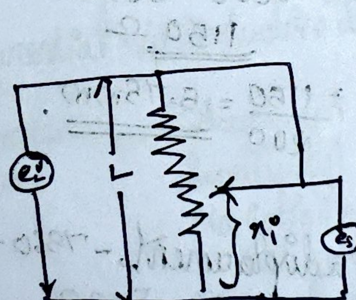
$R = \frac{\rho l}{A}$

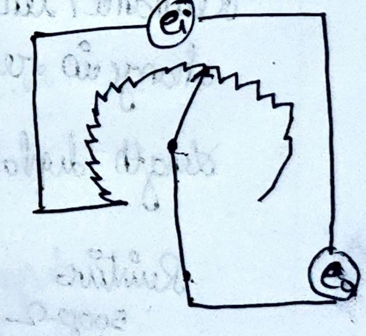
Resistive transducers

→ Environmental change, Appln of a physical qty } → change in resistance } NOTE

Eg

Potentiometer





Translational movement

Rotational movement

$e_o = \frac{x_i}{L} e_i$

$x_i = L \frac{e_o}{e_i}$

Advantages

- o/p sensitivity ↑
- efficiency
- less expense
- variable shapes
- Rugged construction
- Fast response

Disadvantages

- wear and tear
- life span - less

A linear resistance potentiometer is 50mm long and is uniformly wound with a wire having resistance of 10,000 Ω. Under normal conditions, the slider is at the centre of the potentiometer. Find the linear displacement when the resistance of the potentiometer is measured by a wheatstone's bridge.

i) 3850 Ω
ii) 7560 Ω

for the two displacements is the same direction and if it is possible to measure a minimum of 10 Ω resistance with the arrangement find the resolution of the potentiometer in mm.

At normal position Resistance 5000 Ω (Slider at the centre)

Resistance / length $= \frac{10000}{50} = 200\,\Omega/mm$

change in resistance $= 5000 - 3850 = 1150\,\Omega$

Length / displacement $= \frac{1150}{200} = 5.75mm$

Resistance

5000 Ω

While applying a displacement - 7560 Ω

Change in resistance $= 7560 - 5000 = 2560\,\Omega$

displacement $= \frac{2560}{200} = 12.8mm$

Resolution = Minimum measurable resistance multiplied with Ω/mm

$= 10 \times \frac{1}{200} = \frac{1}{20}$

Transducers

Inductive transducers

Self inductance.

Mutual inductance.

Eddy current

$L = \mu_0 N^2 \frac{l}{A}$

$= \mu_0 N^2 G$

Change in permeability

* Inductance Transducer :- (Inductive).

→ Change in self inductance

→ change in mutual inductance

→ Formation of eddy currents

change in self inductance :-

$L = \frac{N^2}{S}$ $\quad S = \frac{L}{\mu A}$ – Reluctance

$(A/l) = G$ geometric factor

$L = N^2 \mu (A/l)$

→ Number of turns
→ Geometric factor
→ Permeability of the medium

} Displacement.

$L \longrightarrow L + \Delta L$

ΔL → Differential o/p inductive transducer

$\left.\begin{matrix}(L+\Delta L)\\(L-\Delta L)\end{matrix}\right\}$ Difference. $2\Delta L$

Accuracy ↑
Sensitivity ↑
Variation with temp ↓
Vol. frequency ↓
external m-f effect

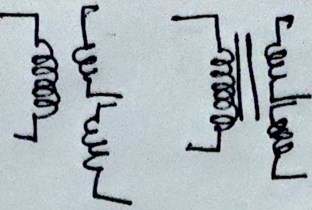
change in mutual inductance

$M = K\sqrt{L_1 L_2}$

M change ⟷ change in L

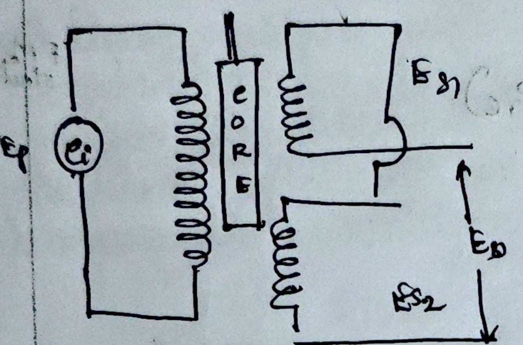
$L_1 + L_2 - 2M$ to $L_1 + L_2 + 2M$

Air cored }
Iron cored }



N

* Linear Variable Differential Transducer (LVDT)



$E_0 = E_{S1} - E_{S2}$

Advantages / D of LVDT

1) Very high range of measurement of displacement. It can measure from 1.3mm to
2) If you can incr full scale linearity we can measure from 0.003mm
3) It is a frictionless device and electrical isolation is another
4) It has immunity from external effects :-
5) The separation of LVDT core and coil permits the isolation of media such as pressurized, corrosive or harmful fluids from the coil assembly by a non-magnetic barrier interconnecting the core and coil.
5) High output and high sensitivity
6) Rugged construction, low power consumption

Disadvantages :-

i) For appreciable differential output relatively large differential output is required.

ii) Magnetic shield is required to avoid stray magnetic field

iii) Performance is affected by vibrations.

iv) It is operating at AC only (Dynamic response limited mechanically by the mass of the core and electrically by the applied voltage.

v) Temperature affects the performance of the transducer.

Application:-

→ Can be used to all applications where displacement varies from a fraction of a mm to a few cm

→ Converts mechanical quantity to electrical quantity

→ It act as a secondary transducer to measure force, weight and pressure etc.

The output of an LVDT is connected to a 5V voltmeter through an amplifier whose amplification factor is 250. An output of 2mV appears across the terminals of LVDT when the core moves through a distance of 0.5 mm. Calculate the sensitivity of the voltmeter. The scale can be read through milli voltmeter has 100 divisions. The resolution of voltmeter is 1/5 of a division. Calculate the resolution of voltmeter is 5

$S_{LVDT} = \frac{\text{o/p voltage}}{\text{i/p voltage}} = \frac{2\times10^{-3}}{0.5} = \underline{4\times10^{-3}\ V/mm}$

Sensitivity of instrument $= 250 \times 4\times10^{-3} = \cancel{10}\ \underline{1000\ mV/mm}$

$\underline{1V/mm}$

One scale division is $\frac{5}{100}$ is $\underline{50mV}$

Minimum volt that can be read on the voltmeter $= 50\times\frac{1}{5} = \underline{10\ mV}$

∴ Resolution of the instrument is $\frac{1}{1000} = \underline{10^{-3}\ mm}$