Dr. D. D. Ebenezer

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Department of Ship Technology
Cochin University of Science & Technology

02 Dec 2024

Stability of Ships

B. Tech. NA&SB. 2023-27. 48th Batch. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

Dr. D. D. Ebenezer
Adjunct Faculty
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Jan-Apr 2024

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Dr. D. D. Ebenezer

- 1983. B. Tech. Naval Architecture. IIT Madras. Gold Medalist.
- 1986. M. S. Ocean Engineering. Univ. Rhode Island, USA.
- 1990. Ph. D. Ocean Engineering. Univ. Rhode Island, USA.
- 1990-2020. Naval Physical and Oceanographic Lab., DRDO, Ministry of Defence. Retired as Scientist H = Vice Admiral in the Indian Navy
- 2012 – Present. Associate Editor, The Journal of the Acoustical Society of America (Only one in India). https://asa.scitation.org/jas/info/editors
- 2021-Present. CUSAT.
- 2021-Present. Chairman of the Hydro-Vibro-Acoustics Panel of the Naval Research Board, DRDO

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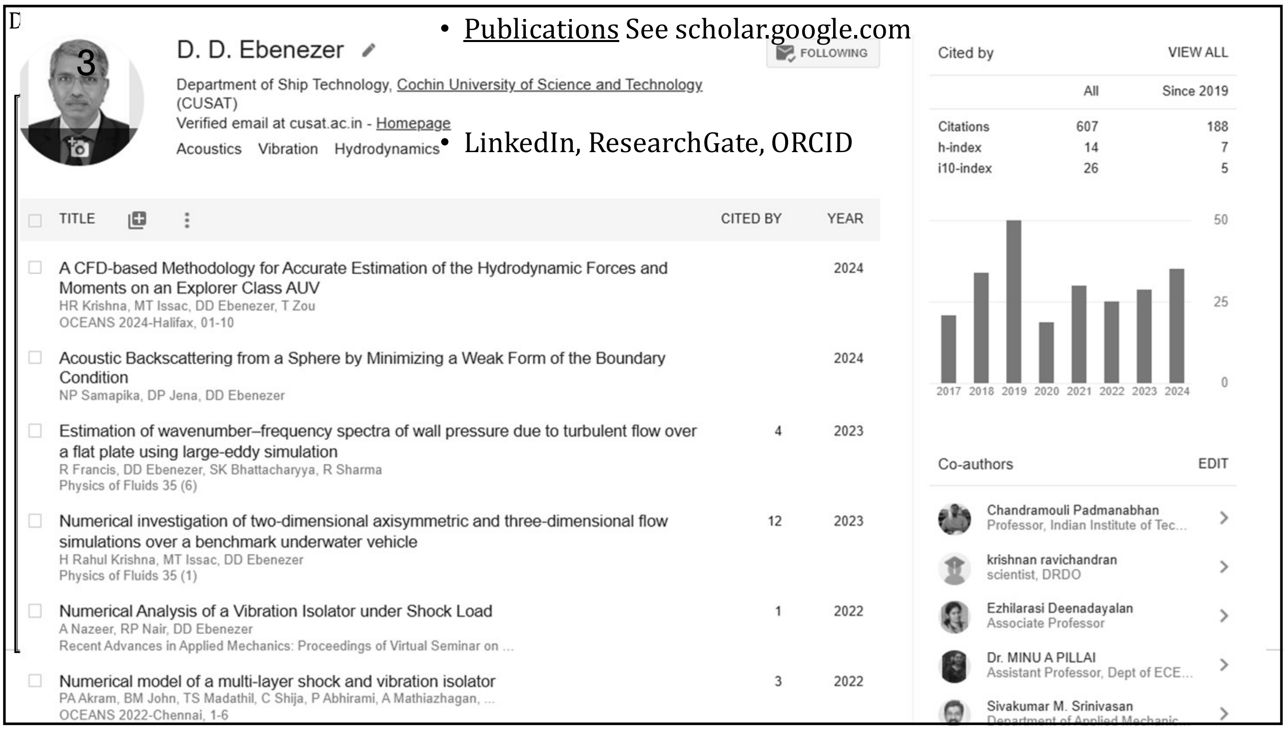
- Publications See scholar.google.com
- LinkedIn, ResearchGate, ORCID



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Publications while at CUSAT

1. HR Krishna, MT Issac, DD Ebenezer, T Zou. A CFD-based Methodology for Accurate Estimation of the Hydrodynamic Forces and Moments on an Explorer Class AUV. OCEANS 2024-Halifax, 01-10. 2024
2. NP Samapika, DP Jena, DD Ebenezer. Acoustic Backscattering from a Sphere by Minimizing a Weak Form of the Boundary Condition. Intl Congress Sound and Vibration. Amsterdam. 2024
3. R Francis, DD Ebenezer, SK Bhattacharyya, R Sharma. Estimation of wavenumber–frequency spectra of wall pressure due to turbulent flow over a flat plate using large-eddy simulation. Physics of Fluids 35 (6) 2023
4. H Rahul Krishna, MT Issac, DD Ebenezer. Numerical investigation of two-dimensional axisymmetric and three-dimensional flow simulations over a benchmark underwater vehicle. Physics of Fluids 35 (1) 2023
5. A Nazeer, RP Nair, DD Ebenezer. Numerical Analysis of a Vibration Isolator under Shock Load
6. Recent Advances in Applied Mechanics: Proceedings of Virtual Seminar on … 2022
7. PA Akram, BM John, TS Madathil, C Shija, P Abhirami, A Mathiazhagan, DD Ebenezer. Numerical model of a multi-layer shock and vibration isolator. OCEANS 2022-Chennai, 1-6 2022.
8. C Banann, RP Nair, DD Ebenezer. Numerical Simulation of Strain-Rate Effect of Al Circular Tube Under Dynamic Loading Conditions. Composite Materials for Extreme Loading: Proceedings of the Indo-Korean … 2021.

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Accepted for Presentation at OSICON 2025

1. H. Rahul Krishna, M. T. Issac, D. D. Ebenezer, Ting Zou, Neil Bose and Bethany Randell. NUMERICAL ANALYSIS OF THE HULL-PROPELLER INTERACTIONS FOR AN EXPLORER CLASS AUTONOMOUS UNDERWATER VEHICLE.
2. Richi Rajan, Manoj T. Issac and D. D. Ebenezer. NUMERICAL INVESTIGATION OF HYDRODYNAMIC PERFORMANCE AND RADIATED NOISE OF DTMB 4119 PROPELLER.
3. Angela Susan Tonio, Manoj T. Isaac, and D. D. Ebenezer. DETERMINATION OF THE HYDRODYNAMIC COEFFICIENTS USING THE TURNING CIRCLE PATH OF A SURFACE SHIP.
4. S. Sruthy Raj, Beena Mary John and D. D. Ebenezer. CALM WATER RESISTANCE OF THE JAPAN BULK CARRIER USING THE $k - \omega\ SST$ TURBULENCE MODEL
5. Achsah Belinda Arby, U. R. Harikrishnan, T. Shakkir, G. A. Surya Teja, Manoj T Issac, D. D. Ebenezer. CFD ANALYSIS OF TORQUE OF SHIP RUDDERS WITH FLAT PLATE AND NACA 0020 SECTIONS.
6. Razmia P Ali, P. K. Satheesh Babu, and D. D. Ebenezer. THE EFFECT OF THE MESH AND TURBULENCE MODEL ON THE SIMULATION OF A SHIP MOVING AT A UNIFORM SPEED.

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Funded Research at CUSAT

- 2021. Kerala State Council for Science Technology and Environment. Student Project. Engineering Stream. Hydrodynamic analysis of a submarine launched towed body with fins. 6 months. Rs. 10,000/-
- 2022. KSCSTE. ETP10000710. CFD analysis of a self propelled AUV with a ducted propeller. 3 years. Rs. 20,65,800/-
- 2023. Cochin Shipyard Limited. Basic Research on Marine Hydrodynamics using Free CFD software “OpenFOAM”. 14 months. Rs. 27,57,679 + 18% GST = Rs. 32.5 lakhs
- 2024 01. SeaTech Integrated Technology, Mumbai. CFD analysis of a fin.
- 2024 05. SeaTech Integrated Technology, Mumbai. CFD analysis of a fin.
- 2025. Cochin Shipyard Limited. Basic Research on Optimisation of Displacement Ship Hullsusing Shipflow (Basic+Design) and OpenFOAM. Rs. 40 lakhs.

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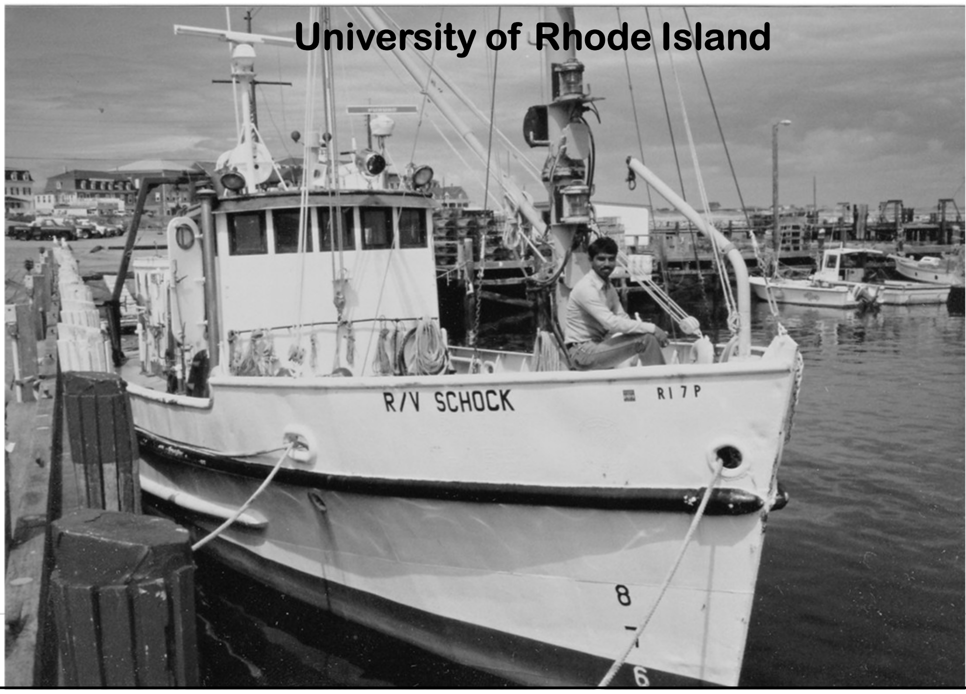

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R.V. *Edson Schock*
65-foot OE research vessel
URI

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NPOL Research Vessel Sagardhwani

Disp: 2,050 long tons

Length: 85.1 m

Breadth: 12.8 m

Propulsion:

2 × diesel engines

3,860 hp sustained

Speed: 16 knots

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DDE at NPOL

- Designed and developed many types of sonar transducers for Naval ships and submarines
- Designed new features in Sonar system
- Dived in submarines and sailed in ships

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Stability of Ships

14 weeks of Lecture

5 Modules

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4th Semester. B. Tech. Ship Technology.

4th SEMESTER

Code	Subject	Hrs/Week				Credit	Marks		
		L	T	P	Total		Internal Exam	University Exam	Total
20-215-0401	Mathematics IV	3	1	-	4	3	100	100	200
20-215-0402	Fluid Mechanics II	3	1	-	4	3	100	100	200
20-215-0403	Design of Machine Elements	2	2	-	4	3	100	100	200
20-215-0404	Analysis of Structures	3	1	-	4	3	100	100	200
20-215-0405	Material Science	3	1	-	4	3	100	100	200
20-215-0406	Stability of Ships	3	1	-	4	3	100	100	200
20-215-0407	Language Lab	-	-	2	2	1	50	-	50
20-215-0408	Material Testing Lab	-	-	4	4	1	50	-	50
	Total	17	7	6	30	20	700	600	1300

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Download and read the syllabus.

https://shiptech.cusat.ac.in/course-details.php?id=11&slug=btech

https://drive.google.com/file/d/1ApUoucu0GbjpkAtfq39PsCSoZLPUosvv/view

The ships you design and build should be stable even in the high seas. Watch this video.

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Stability is VERY Important

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20-215-0406 STABILITY OF SHIPS

Course Description: This course is designed to offer the students an understanding of the concept of stability of ships, which include transverse stability, longitudinal stability and damage stability.

20-215-0406	Stability of Ships	Category	L	T	P	Credit	Year of Induction
		PCC	3	1	-	3	2020

Pre-requisites: Nil

Course Objectives: The objective of the course is to provide the learners an understanding of the theory and calculation of intact and damage stability of shipsand to equip them with a practical knowledge for conducting inclining experiments and preparing stability booklets.

Course Outcomes: After the completion of the course the students will be able to:

CO 1	Understand the concept of static equilibrium and stability of floating body and the effects on transverse stability due to various external and internal factors.
CO 2	Explain the transverse stability of ships, for small and large angles of inclination.
CO 3	Discern the purpose of an inclining experiment and the procedures involved in it.
CO 4	Compare the cross curves of stability and generate a specific loading condition, perform trim and stability calculation for that loading condition and check with IMO intact stability criteria.
CO 6	Estimate trim of a ship resulting from addition, removal, and shifting of weights and due to change in density of water

Stability of Ships.

Course Description, Objectives, and Outcomes.

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Stability of Ships. Course Description & Objectives

20-215-0406 STABILITY OF SHIPS

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Pre-requisites: Nil

Course Objectives: The objective of the course is to provide the learners an understanding of the theory and calculation of intact and damage stability of shipsand to equip them with a practical knowledge for conducting inclining experiments and preparing stability booklets.

- DDE can be contacted at any time by WhatsApp

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Stability of Ships. Course Outcomes.

- Students are taught to think and not just memorise
- Make sure that you understand the details and will be able to apply the lessons learnt
- Students should keep track of what they are learning and map the lectures to the outcomes

- Students are encouraged to contact DDE at any time through WhatsApp or email or mobile

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Course and Program Outcomes

- See the downloadable B.Tech. Syllabus in the DoST website for the Program Outcomes

Mapping of Course Outcomes against Program Outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	1	1									
CO 2	3	2	1	1								
CO 3	3	3	1	1	1							1
CO 4	3	3	1	1	1							
CO 5	3	3	1	1	1							

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Assessment

Assessment Pattern:

Bloom's Category	Continuous Assessment Tests		End Semester Examination
	1	2	
Remember	10	10	15
Understand	20	20	35
Apply	20	20	50
Analyse			
Evaluate			

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- Note that greater weight is given to Application

Mark distribution

Total Marks	CIE	ESE	ESE Duration
200	100	100	3 hours

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Attendance and End-Sem Pattern

- Get 10 easy marks. Attend all the classes. You lose more than just attendance by being absent
  - 2018 Syllabus. Marks for attendance.
  - 75% needed to write end-sem exam

% of attendance	marks awarded
96-100	10
91 – 95	8
86 – 90	6
81 – 85	4
76 – 80	2
below 76	0

Mark distribution

Total Marks	CIE	ESE	ESE Duration
200	100	100	3 hours

Continuous Internal Evaluation Pattern:

Attendance ⟷ 10 marks
Continuous Assessment : 40 marks
Internal Tests : 50 marks

End Semester Examination Pattern:

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 5 questions with 1 question from each module, having 4 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 16 marks.

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Earlier Courses – No Pre-requisites

- Intro to Naval Arch. Study these modules again.

2. **Module II**
   Introduction to ship geometry
   Some physical fundamentals - Archimedes principle, laws of floatation stability and trim. The ship's form-main dimensions, lines plan, coefficients and their meaning, Fairing process and table of offsets; Hydrostatic particulars &Bonjean Curves: - (Volume of Displacement/ Displacement, Centre of Buoyancy, Centre of Floatation, KMT And BMT Metacentric Radius, TPC 1cm, MCT 1cm, Form Coefficients (C_B, C_P, C_M and C_W), LCF)

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3. **Module III**
   Introduction to Bonjean and hydrostatic curves
   Integration rules: - Trapezoidal rule; Simpson's rules, 6 ordinate rules; Tchebycheff's rule; Areas, volumes and moments Bonjean calculations and curves, sectional area curves. Hydrostatic calculations and curves.

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Stability of Ships. Course Content. Exam question paper will be based on this.

Course Content:

1. **Module I**
   Stability terms. Potential energy. Equilibrium. Weight displacement and Volume displacement; Change of density, FWA, DWA. Equi-volume inclinations, shift of CoB due to inclinations, CoB curve in lateral plane, (*initial*) metacentre, metacentric radius, metacentric height; metacentre at large angles of inclinations, pro-metacentre. CoG, righting moment and lever; Statical, metacentric, residuary, form and weight stabilities. Surface of flotation, curve of flotation. Derivation of BM = I / V.

2. **Module II**
   Initial (*transverse*) stability: GM_0, GZ at small angles of inclinations, Wall sided ships. Sinkage and stability due to addition, removal and shift (transverse and vertical) of weight, suspended weights and free surface of liquids; Inclining Experiment; stability while docking and grounding; Stiff/ Tender ship.

3. **Module III**
   Large angle (*transverse*) stability: Diagram of statical stability (GZ curve), characteristics of GZ curve, effect of form, shift of G and super structure on GZ curve, static equilibrium criteria, Methods of calculating GZ curve (Prohaska, Krylov and from ship form), Cross curves of stability.
   Dynamical stability, diagram of dynamical stability, dynamic stability criteria.
   Moments due to wind, shift of Cargo and passengers, turning and non-symmetric accumulation of ice.
   Intact stability rules, Heel/ Load test.
   Practical: Diagram of statical stability / Cross curves of stability (Krylov's method).

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Stability of Ships. Course Content

4. **Module IV**
   Longitudinal Stability: Trim, longitudinal metacentre, longitudinal centre of flotation, moment to change trim, trimming moment, change of trim and drafts due to addition,

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removal and longitudinal shift of weight, trim and draft change due to change of density. Rules on draft and trim.

5. **Module V**
   Damage stability: Bilging, Surface and volume permeability; Sinkage, heel, change of trim and drafts due to bilging of midship, side and end compartments.

Practical: Floodable length calculation and subdivision of ship. Stability in waves,

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References

References:

I bought Ref. 1 for Rs. 7.20 using prize money in 1981!

1) V. Semyonov-Tyan-Shansky, "Statics and Dynamics of the Ship", Peace publishers, Moscow, 2004.
2) Derret, "Ship Stability for Masters and Mates", Butterworth-Heinemann., 1999.
3) Capt. A. R. Lester, "Merchant Ship Stability", Butterworths, 1985.
4) Adrian Biran, "Ship Hydrostatics and Stability", Elsevier. 2013.
5) Kemp & Young, "Ship Stability, notes and examples", Butterworth-Heinemann., 2001.
6) J. Anthony Hind, "Stability and Trim of Fishing Vessels", Fishing news books Ltd., 1989.
7) . H. Subramanian, "Ship Stability", Nut shell series book, Vijay publications, Mumbai, 2009.
8) J. Klinkert, H. W. White, "Nautical Calculations Explained", Routledge & Kegan Paul, London, 1969.
9) International Maritime Organisation (IMO), "SOLAS", 2017.
10) Colin.S.Moore, J.R. Paulling, Principles of Naval Architecture Series: Intact stability, SNAME,New Jersey, 2010.
11) E. A. Stokoe, Reed's "Naval Architecture for Marine Engineers", 2003.
12) W. Muckle, "Naval Architecture for Marine Engineers", 2004.
13) R. Munro Smith, "Ships and Naval Architecture", Institution of Marine Engineers, 1977.
14) R. Munro Smith, "Notes and Examples in Naval Architecture", E.Arnold, 1965.
15) R. Munro-Smith, "Elements of Ship Design", Marine Media Management Ltd., 1975.
16) K. J. Rawson, E. C. Tupper, "Basic Ship Theory", Butterworth-Heinemann, 2001.
17) E. C. Tupper, "Introduction to Naval Architecture", Butterworth-Heinemann, 2013.
18) Andrew McCance Robb, "Theory of Naval Architecture", Charlse Griffin.1952
19) Thomas C. Gillmer and Bruce Johnson, "Introduction to Naval Architecture".ments, IE& FN 1982.

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MODULE 1. EQUILIBRIUM OF SHIPS

1.1 Conditions for static equilibrium of a floating body
1.1.1 Archimedes Principle. Hydrostatic pressure distribution on floating cuboids and bodies of arbitrary shape
1.1.2 Stevin's Law
1.2 Small changes from equilibrium position
1.2.1 Three types of equilibrium
1.2.2 Small vertical change in the position
1.2.3Small inclination
1.2.3.1 Bouguer's Metacentre
1.2.3.2 Euler's Theorem and the axis of inclination
1.2.3.3 Centre of flotation
1.3 Change in the density
1.3.1 Fresh water allowance
1.3.2 Dock water allowance
1.3.3 Plimsoll line
1.4 Change in the centre of gravity
1.4.1 Movement of a mass on-board
1.4.2 Addition or removal of a mass
1.5 Change in the centre of buoyancy
1.5.1 Addition or removal of a small mass
1.5.2 Addition or removal of a medium mass
1.5.3 Large change in Addition or removal of a large mass
1.5.4 Small change in the inclination
1.5.5 Large change in the inclination
1.5.6 Curve of centres of buoyancy
1.6 Change in the metacentre
1.6.1 Metacentric radius. BM = I/V
1.6.2 Metacentric Evolute
1.6.3 Metacentric height

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The Big Picture

- The ship has to operate in a variety of conditions
  - Light and Heavy loads
  - Calm and Rough seas
  - Intact and Damaged hull
- Design for stability in all the operating conditions

The ships you design and build should be stable even in the high seas. Watch this video. See the link in classroom.

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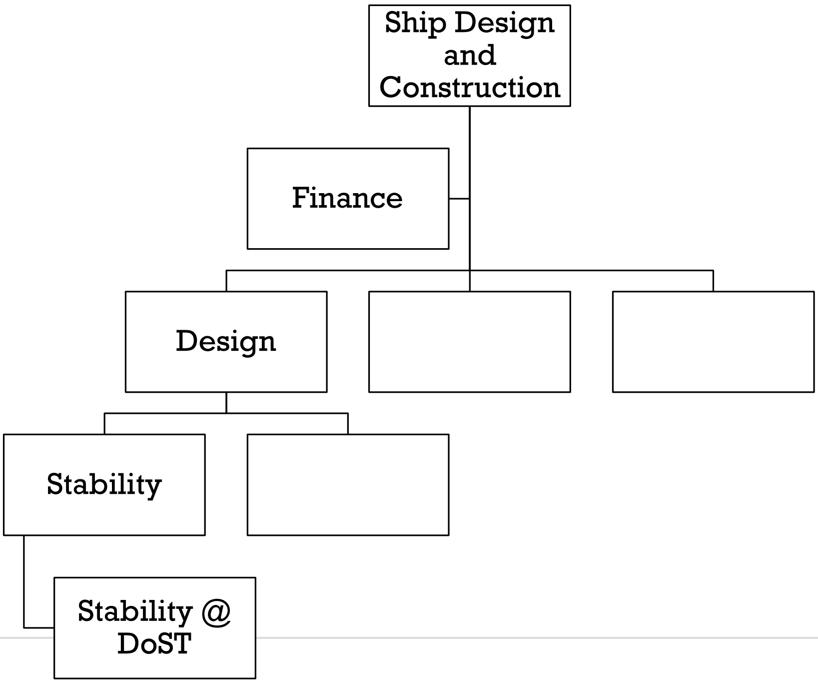

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Naval Architecture Terminology

Quiz. Define

- LBP Length Between Perpendiculars
- LOA Length Over All
- Lightship Weight
- Molded Breadth

  Distance from port to starboard measured between inside edges of plating
- LOS
  - Length Overall Submerged

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LBP

- Biran

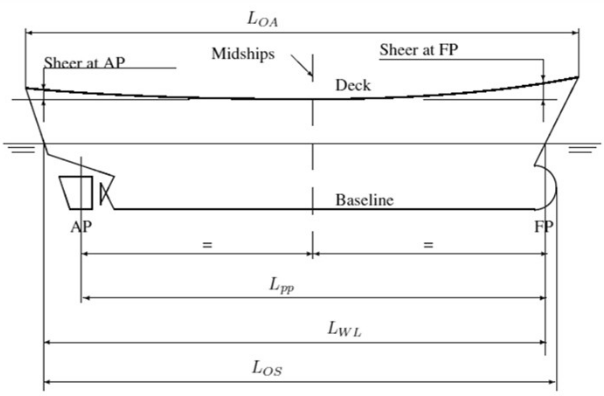

Figure 1.1 Length dimensions

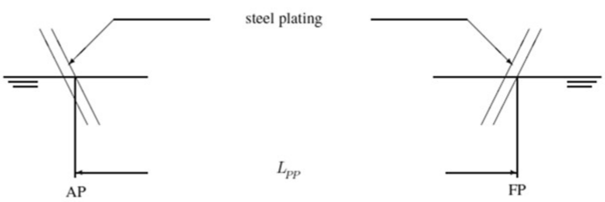

Figure 1.2 How to measure the length between perpendiculars

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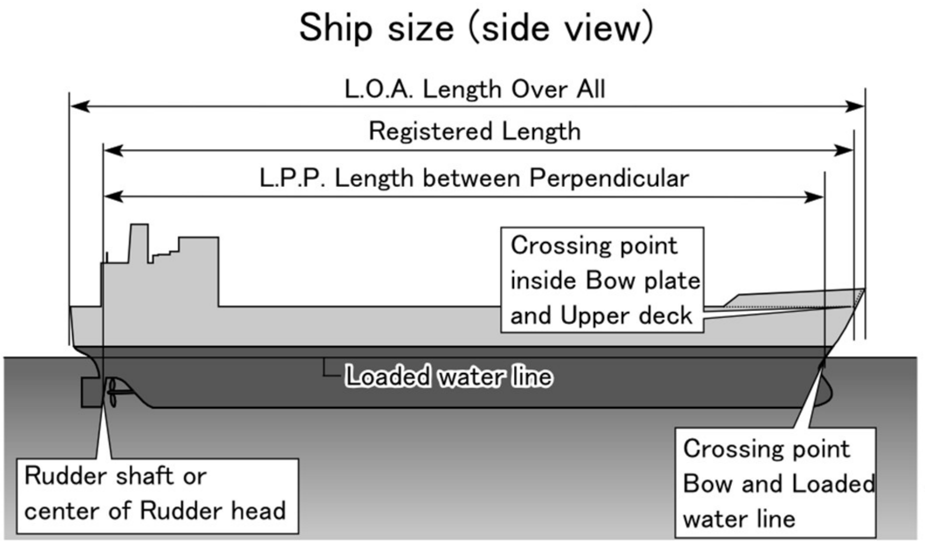
Wikipedia



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Naval Architecture Terminology

- https://www.themaritimesite.com/a-guide-to-understanding-ship-weight-and-tonnage-measurements/
- https://marinersgalaxy.com/ship-dimensions-terminology-and/

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Wärtsilä Encyclopedia of Marine Technology

- over 3000 terms related to the marine industry
- shipbuilding trends that are moving the industry forward
- 45 MB

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Today. Module 1.

Today

1.1 Conditions for static equilibrium of a floating body

1.1.1 Archimedes Principle. Hydrostatic pressure distribution on floating cuboids and bodies of arbitrary shape

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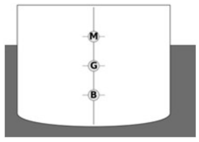
Review of Stability terms



- Barrass and Derrett. Ship Stability for Masters and Mates. Chap 6.

Recapitulation

1. The centre of gravity of a body 'G' is the point through which the force of gravity is considered to act vertically downwards with a force equal to the weight of the body. KG is VCG of the ship. **What is K?**
2. The centre of buoyancy 'B' is the point through which the force of buoyancy is considered to act vertically upwards with a force equal to the weight of water displaced. It is the centre of gravity of the underwater volume. KB is VCB of the ship.
3. To float at rest in still water, a vessel must displace her own weight of water, and the centre of gravity must be in the same vertical line as the centre of buoyancy.
4. KM = KB + BM. Also KM = KG + GM. **M is the metacenter. Define it.**

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Stability terms

- Heel and List

Definitions

1. *Heel*. A ship is said to be heeled when she is inclined by an external force. For example, when the ship is inclined by the action of the waves or wind.
2. *List*. A ship is said to be listed when she is inclined by forces within the ship. For example, when the ship is inclined by shifting a weight transversely within the ship. This is a fixed angle of heel.

- Port and Starboard

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Assumptions

For most of this course, the treatment of stability is based on the following assumptions:

- 1. the water is incompressible (there are no incompressible materials)
- 2. viscosity plays no role (resistance depends on viscosity)
- 3. surface tension plays no role (important for short wave-length waves)
- 4. the water surface is plane (stability in waves is gaining importance)
- 5. the floating bodies are perfectly rigid (sonar domes are thinner and more flexible than the hull of a ship or submarine)

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Read books, magazines, and journals

and become outstanding Naval Architects

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Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406
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Assignment 01

- The equation for the Wigley hull is $y = \pm\frac{B}{2}\left[1-\left(\frac{2x}{L}\right)^2\right]\left[1-\left(\frac{2z}{T}\right)^2\right]$ where $-\frac{L}{2} \le x \le \frac{L}{2}, -T \le z \le 0$. The origin is on the centerline, at midship, and on the sea surface.
- The length, L, of a Wigley oil tanker is 176 m. The breadth, B, is 32.2 m. The depth, D, is 18.2 m. The draft, T, is 11 m.
- https://iopscience.iop.org/article/10.1088/1742-6596/1985/1/012018/pdf
- Plot the waterlines at 0, 0.5, 1, 2, 3, … 11. Find the waterplane areas.
- Plot the cross-sections at $x = 0, \frac{L}{10}, \frac{2L}{10}, \frac{3L}{10}, \dots \frac{L}{2}$. Find the cross-sectional areas.
- Find the underwater volume of the Wigley oil tanker using a) the waterplane areas and b) the cross-sectional areas.

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MODULE 1. EQUILIBRIUM OF SHIPS

1.1 Conditions for static equilibrium of a floating body
1.1.1 Archimedes Principle. Hydrostatic pressure distribution on floating cuboids and bodies of arbitrary shape
1.1.2 Stevin's Law
1.2 Small changes from equilibrium position
1.2.1 Three types of equilibrium
1.2.2 Small vertical change in the position
1.2.3 Small inclination
1.2.3.1 Bouguer's Metacentre
1.2.3.2 Euler's Theorem and the axis of inclination
1.2.3.3 Centre of flotation
1.3 Change in the density
1.3.1 Fresh water allowance
1.3.2 Dock water allowance
1.3.3 Plimsoll line
1.4 Change in the centre of gravity
1.4.1 Movement of a mass on-board
1.4.2 Addition or removal of a mass
1.5 Change in the centre of buoyancy
1.5.1 Addition or removal of a small mass
1.5.2 Addition or removal of a medium mass
1.5.3 Large change in Addition or removal of a large mass
1.5.4 Small change in the inclination
1.5.5 Large change in the inclination
1.5.6 Curve of centres of buoyancy
1.6 Change in the metacentre
1.6.1 Metacentric radius. BM = I/V
1.6.2 Metacentric Evolute
1.6.3 Metacentric height

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1.1 Conditions for static equilibrium of a floating body

Today

1.1 Conditions for static equilibrium of a floating body

1.1.1 Archimedes Principle. Hydrostatic pressure distribution on floating cuboids and bodies of arbitrary shape

1.1.2 Stevin's Law

1.2 Small changes from equilibrium position

1.2.1 Three types of equilibrium

1.2.2 Small vertical change in the position

1.2.3Small inclination

1.2.3.1 Bouguer's Metacentre

1.2.3.2 Euler's Theorem and the axis of inclination

1.2.3.3 Centre of flotation

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1.1.1 Archimedes Principle. Hydrostatic pressure distribution on floating cuboids and bodies of arbitrary shape

- A body that is partially or fully immersed in a fluid is subjected to an upwards force equal to the weight of the fluid displaced.
- The above statement is known as Archimedes' principle. One legend has it that Archimedes (Greek, lived in Syracuse—Sicily—between 287 and 212 BC) discovered this law while taking a bath and that he was so happy that he ran naked in the streets shouting "I have found" (in Greek "Heureka," see entry "eureka" in Merriam-Webster, 1991). The legend may be nice, but it is most probably not true. What is certain is that Archimedes used his principle to assess the amount of gold in gold-silver alloys.
- Mass and Weight
- Mass is the amount of matter in an object. It is measured in kilograms (kg).
- Mass is different from weight. Mass is constant, while weight changes with gravity.
- <u>For floating bodies: the mass of the floating body = mass of liquid displaced</u>

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1.1.1 Archimedes Principle. Hydrostatic pressure distribution on floating cuboids and bodies of arbitrary shape

- Figure on next slide
- We consider a simple-form solid as shown in Figure 2.1; it is a parallelepiped whose horizontal, rectangular cross-section has the sides B and L. We consider the body immersed to the draught T . Let us call the top face 1, the bottom face 2, and number the vertical faces with 3–6. Figure 2.1b shows the diagrams of the liquid pressures acting on faces 4 and 6. To obtain the absolute pressure we must add the force due to the atmospheric pressure p0.

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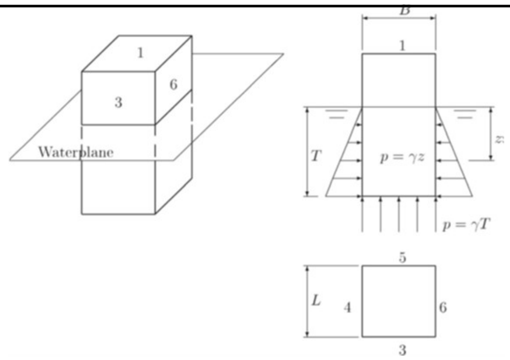
Hydrostatic Forces



- **Figure 2.1 Hydrostatic forces on a body with a simple geometrical form**

Assuming that forces are positive in a rightwards direction, and adding the force due to the atmospheric pressure, we obtain

Force = $\int$ Pressure dS

$$F_4 = L\int_0^T \gamma z\,dz + p_0LT = \frac{1}{2}\gamma LT^2 + p_0LT \tag{2.1}$$

Similarly, the force on face 6 is

$$F_6 = -L\int_0^T \gamma z\,dz - p_0LT = -\frac{1}{2}\gamma LT^2 - p_0LT \tag{2.2}$$

As the force on face 6 is equal and opposed to that on face 4 we conclude that the two forces cancel each other.

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Archimedes' Principle. Hydrostatic Forces

Proceeding in the same way we find that the force on face 6, F_6, is equal and opposed to the force on face 4. The sum of the two forces is zero. In continuation we find that the forces on faces 3 and 5 cancel one another. The only forces that remain are those on the bottom and on the top face, that is faces 2 and 1. The force on the top face is due only to atmospheric pressure and equals

$$F_1 = -p_0LB \tag{2.5}$$

and the force on the bottom,

$$F_2 = p_0LB + \gamma LBT \tag{2.6}$$

The resultant of F_1 and F_2 is an upwards force given by

$$F = F_2 + F_1 = \gamma LBT + p_0LB - p_0LT = \gamma LBT \tag{2.7}$$

The product LBT is actually the volume of the immersed body. Then, the force F given by Eq. (2.7) is the weight of the volume of liquid displaced by the immersed body. This verifies Archimedes' principle for the solid considered in this subsection.

We saw above that the atmospheric pressure does not play a role in the derivation of Archimedes' principle. Neither does it play any role in most other problems we are going to treat in this book; therefore, we shall ignore it in future.

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Line of action of Buoyancy Force

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- The line of action of the buoyancy force is important. It is perpendicular to the sea surface. It passes through the centre of buoyancy.
- Zoom of the underwater portion

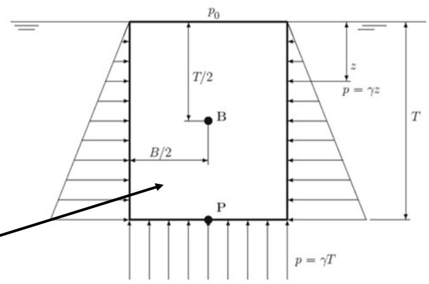

Figure 2.2 **Zoom of** Figure 2.1

Let us consider in Figure 2.2 a "zoom" of Figure 2.1. It is natural to see the resultant of the forces of pressure as applied at the point **P** situated in the centroid of face 2. The meaning of this sentence is that, for any coordinate planes, the moment of the force γLBT applied at the point **P** equals the integral of the moments of pressures. In the same figure, the point **B** is the *centre of volume* of the solid. If our solid would be made of a homogeneous material, the point **B** would be its *centre of gravity*. We see that **P** is situated exactly under **B**, but at double

draught. As a vector can be moved along its line of action, without changing its moments, it is commonly admitted that the force γLBT is applied in the point **B**. A frequent statement is: the force exercised by the liquid is applied in the centre of the displaced volume. The correct statement should be: "We can consider that the force exercised by the liquid is applied in the centre of the displaced volume." The force γLBT is called **buoyancy force**.

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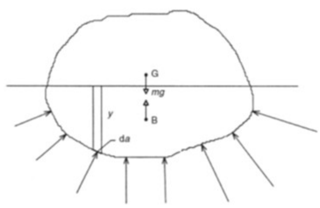
Buoyancy force

- The pressure on the surface is p. It is a scalar and varies with position but is the product of only the depth and ρg
- It acts on a differential area $d\sigma$. Area is a vector with direction along the normal to the area, n. In ship problems, n is often the inward pointing vector.
- $pd\sigma$ is a force. The component of the force in the z or upward direction is $pd\sigma \ \cos(n, z)$. (n, z) is the angle between the normal and the z direction.
- The integral of the component of the elemental force in the z direction is the buoyancy force.



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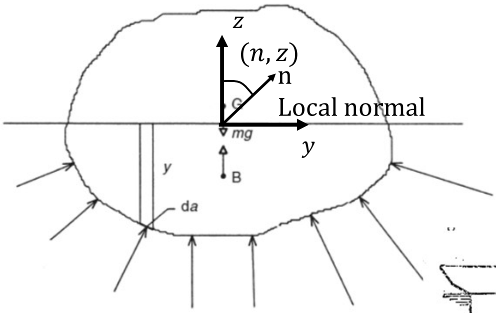
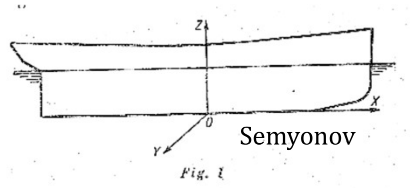
Equilibrium. Gravitational Stability

- A cylindrical log with an arbitrary cross-section and unit length
  - A circular cylinder is a special case



- (n, z) is the angle between the local normal to the surface, n, and the z axis.
- p acts along the local normal to the surface
- $d\sigma$ is the elemental area on which the pressure is acting
- $p \cos(n, z)$ is the component of the force in the upward direction



- Semyonov: Buoyancy force
- Note the axes. In a right handed coordinate system, draw the X axis
- Semyonov uses a left handed coordinate system which WAS common in Naval Architecture.
- Write an expression for the buoyancy force in terms of the hydrostatic pressure

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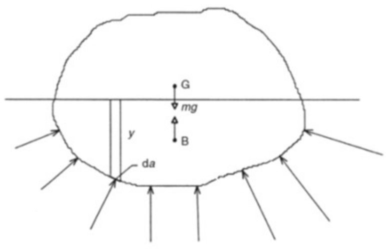
Semyonov: Buoyancy force

- The first Ref book is by Semyonov
- A cylindrical log with an arbitrary cross-section and unit length
  - A circular cylinder is a special case.
  - What is the net force parallel to the sea surface?
  - Zero. Explain why.
  - The attitude (draft, heel and trim angles) of the floating body will change such that all the net forces and moments are zero and it is in static equilibrium.



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Semyonov: Buoyancy force

- The ship is stationary and in a equilibrium position. So, the resultant forces and moments are zero.

$T - z$ is the distance between the point of interest and the sea-surface.

4. BUOYANCY AND CONDITIONS OF EQUILIBRIUM OF A FLOATING SHIP

The immersed surface of a stationary ship is acted upon at each point by the water pressure directed along the inward normal to this surface. The magnitude of the pressure p is expressed, as is known from hydrostatics, by the following formula:

$$p = p_0 + \gamma (T - z), \tag{4.1}$$

where p_0 is the atmospheric pressure, γ the specific weight of the water, T the draught of the ship, z the co-ordinate of the surface point under consideration.

The pressures exerted by the fluid on the immersed surface of the ship are reduced to the resultant Q passing through the centroid of the immersed volume.

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Semyonov: Buoyancy force

The force acting on an element of area $d\sigma$ of the surface σ of a submerged body is $pd\sigma$. A set of these forces, as is known from solid statics, can be reduced to a force Q and a couple with moment M. The force and couple in question may be written in projections as

$$Q_x = \int_\sigma p \cos(n, x)\, d\sigma; \quad Q_y = \int_\sigma p \cos(n, y)\, d\sigma; \tag{4.2}$$

Buoyancy force = Q_z $= \int p \cos(n, z) d\sigma$

$$Q_z = \int_\sigma p \cos(n, z)\, d\sigma;$$

3 forces

The forces of gravity and buoyancy form a couple. They are equal in magnitude and opposite in direction.

- Here, n is the inward normal.
- The forces Q_x and Q_y are zero even when the ship is inclined and there is no symmetry in the underwater volume. Otherwise, the ship will move in the plane of the sea surface.
- Q_z is equal to the weight of the ship.

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Semyonov: Buoyancy Forces

Sec. 4 *Buoyancy and Conditions of Equilibrium* *17*

integrals appearing in formulas (4.2) and (4.3) are zero

$$\int_\sigma p\cos(n,\ x)\,d\sigma=0;\quad \int_\sigma p\cos(n,\ y)\,d\sigma=0;\quad \int_\sigma pz\cos(n,\ y)\,d\sigma=0;$$

Port-stbd symmetry of the underwater vol is <u>not</u> necessary

Thus

$$Q_x=Q_y=M_z=0; \tag{4.4}$$

$$Q_z=Q=\int_\sigma p\cos(n,\ z)\,d\sigma; \tag{4.5}$$

Most ships do not have fwd-aft symmetry

All integrals that have $p\cos(n,z)$ in the integrand are non-zero. Others are zero bcos of the symmetry of the <u>projected</u> areas

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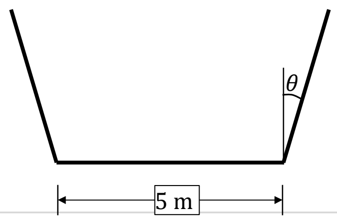
Assignment

- Q1. A barge with a uniform trapezoidal cross-section is floating on even keel. The length is 50 m. The draft is 1 m. $\theta=10$ deg. Find the buoyancy force by
  (1) using the underwater volume
  (2) integrating the hydrostatic pressure acting on the hull.
- See CLASSROOM for the exact question.



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Assignment

- Q1. A circular cylinder of length L with flat ends is floating on water with its axis on the sea surface. The radius and draft are equal to a. The density of water is ρ.
- 1. What is the density of the cylinder?
- 2. What is the weight of the water displaced?
- 3. Integrate the hydrostatic pressure acting on the cylinder and find the buoyancy force on the cylinder.
- See CLASSROOM for the exact question.

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Extra Reading: Archimedes' Principle

- Biran. Chap. 2.2.2. General case. Floating and submerged bodies
- To find the buoyancy force on a curved surface, integrate the pressure over the area. Find the total buoyancy force on a fully submerged circular cylinder of length L with flat ends. The radius is a. The axis of the cylinder is at $2a$ below the surface.

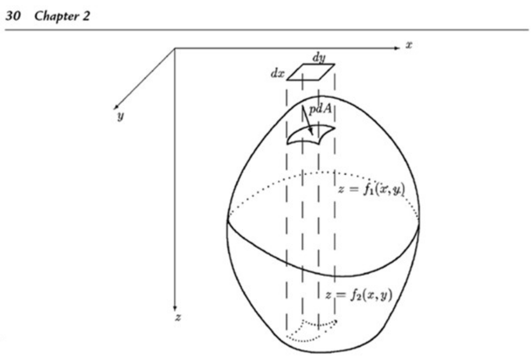

Figure 2.4 Archimedes' principle—vertical force

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Newton's Laws

1. An object at rest remains at rest, and an object in motion remains in motion at constant speed and in a straight line, unless acted on by an unbalanced force.
2. The acceleration of an object depends on the mass of the object and the amount of force applied. $F = ma$.
3. Whenever one object exerts a force on another object, the second object exerts an equal and opposite force on the first.

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Equilibrium of a Floating Body

- A body is said to be in static equilibrium if it is not subjected to accelerations. Newton's second law shows that this happens if the sum of all forces acting on that body is zero and the sum of the moments of those forces is also zero. Two forces always act on a floating body: the weight of that body and the buoyancy force.
- The first condition for equilibrium, that is the one regarding the sum of forces, is expressed as Archimedes' principle. The second condition, regarding the sum of moments, is stated as Stevin's law.
- Before considering Stevin's Law, consider simple examples of forces and moments.

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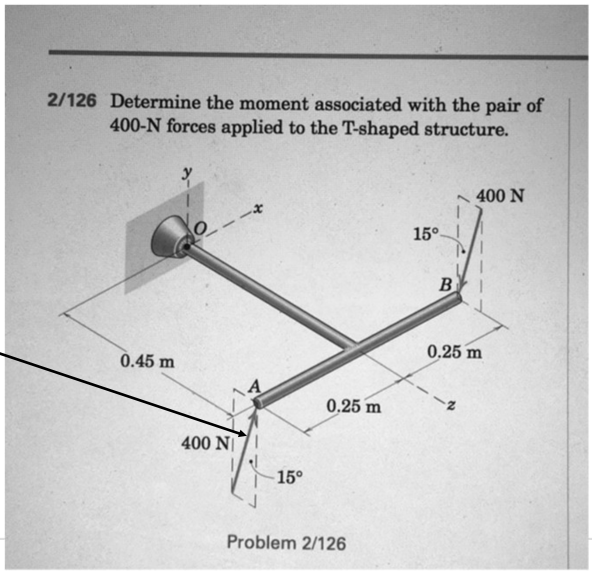
Couples

- The net force is zero as the forces are equal and opposite. A couple acts.
- The forces are resolved in the 3 directions, (x, y, z).
- n is the direction in which the force is acting. $(n, y) = 15$ deg. is the angle between the force and the y axis. $|\vec{F}|\cos(n, y)$ is the force along the y direction. Moment $\vec{M} = \vec{r} \times \vec{F}$.
- Lever arm length = 0.5 m.
- Answer: M = 51.8 ĵ - 193.2 k̂ N.m
- 400*sind(15)*0.5 = 51.8
- 400*cosd(15)*0.5 = 193.2


2/126 Determine the moment associated with the pair of 400-N forces applied to the T-shaped structure.

Problem 2/126

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Shames. Moment about an axis.

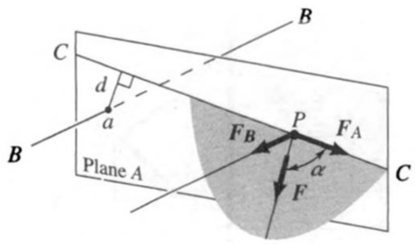

Figure 3.13. Formulating the moment about an axis B–B.

B-B is ⊥ to Plane A. Grey region is on a plane that is ⊥ to Plane A. F lies on it. F_A is ⊥ to the B axis. F_B is parallel to the B-B axis.

- Moments are vectors
- Moments about an axis are scalars

To compute the moment (or torque) of a force $\boldsymbol{F}$ in a plane perpendicular to plane A about an axis B–B (Fig. 3.13), we pass any plane A perpendicular to the axis. This plane cuts B–B at a and the line of action of force $\boldsymbol{F}$ at some point P. The force $\boldsymbol{F}$ is then projected to form a rectangular component F_B along a line at P normal to plane A and thus parallel to B–B, as shown in the diagram. The intersection of plane A with the plane of forces F_B and $\boldsymbol{F}$ (the latter plane is shown shaded and is a plane through $\boldsymbol{F}$ and perpendicular to plane A) gives a direction C–C along which the other rectangular component of $\boldsymbol{F}$, denoted as F_A, can be projected.[4] The moment of $\boldsymbol{F}$ about the line B–B is then defined as the scalar representation of the moment of F_A about point a with a magnitude equal to $F_A d$—a problem discussed at the beginning of the previous section (Case A). Thus in accordance with the definition, the component F_B, which is parallel to the axis B–B, contributes no moment about the axis, and we may say:

$$\text{Moment about axis } B\text{–}B = (F_A)(d) = |\boldsymbol{F}|(\cos\alpha)(d)$$

with an appropriate sign. The moment about an axis clearly is a scalar, even though this moment is associated with a particular axis that has a distinct direction. The situa-

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- Force acts at (x, y, z)
- The moment about each axis has two components
- I. Shames. Engineering Mechanics - Statics and Dynamics. 4th Ed.
- See books.google.com

distance is z. Using the right-hand-screw rule for ascertaining the sense of each of the moments, we can say:

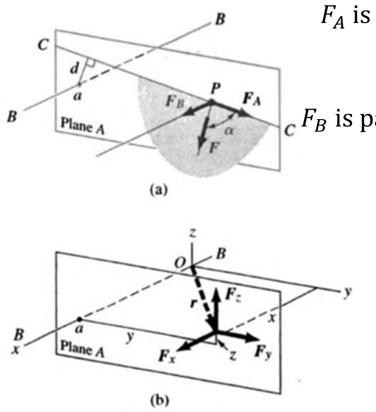
$$\text{moment about } x \text{ axis} = (yF_z - zF_y) \qquad (3.10)$$

Were we to take moments of $\boldsymbol{F}$ about the origin O, we would get (see Eq. 3.8)

$$\begin{aligned} \boldsymbol{M} &= M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k} = \boldsymbol{r} \times \boldsymbol{F} \\ &= (yF_z - zF_y)\boldsymbol{i} + (zF_x - xF_z)\boldsymbol{j} + (xF_y - yF_x)\boldsymbol{k} \end{aligned} \qquad (3.11)$$

Cross Product

$$\begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{matrix}$$



F_A is $\perp$ to the B axis

F_B is parallel to the B axis

Figure 3.16. Moment about an axis.

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Semyonov: Buoyancy Moments

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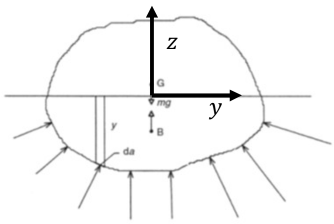
- The net effect of the pressure is equivalent to 3 forces and 3 moments
- The Moment of a force is a measure of its tendency to cause a body to rotate about a specific point or axis.
- The moment about the z axis is due to the forces acting along the x and y axes multiplied by the y and x lever arms, respectively. Note the signs. Moment $\vec{M} = \vec{r} \times \vec{F}$.



- The moment arm is the PERPENDICULAR distance between the line of action of the force and the center of moments.
- Moment about the x axis causes heel

$$\left.\begin{aligned} M_x &= \int_\sigma p\,[y\cos(n,\ z) - z\cos(n,\ y)]\,d\sigma \\ M_y &= \int_\sigma p\,[z\cos(n,\ x) - x\cos(n,\ z)]\,d\sigma \\ M_z &= \int_\sigma p\,[x\cos(n,\ y) - y\cos(n,\ x)]\,d\sigma \end{aligned}\right\} \text{3 moments} \qquad (4.3)$$

Here n is the inward normal.

Since p is a function of z only and σ is a closed surface (including the one resting on the free surface of the water) some of the

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Semyonov: Buoyancy Forces and Moments

Sec. 4 *Buoyancy and Conditions of Equilibrium* 17

integrals appearing in formulas (4.2) and (4.3) are zero

$$\int_\sigma p\cos(n,\ x)\,d\sigma=0;\quad \int_\sigma p\cos(n,\ y)\,d\sigma=0;\quad \int_\sigma pz\cos(n,\ y)\,d\sigma=0;$$

$$\int_\sigma pz\cos(n,\ x)\,d\sigma=0;\quad \int_\sigma px\cos(n,\ y)\,d\sigma=0;\quad \int_\sigma py\cos(n,\ x)\,d\sigma=0.$$

Thus

$$Q_x=Q_y=M_z=0; \tag{4.4}$$

$$Q_z=Q=\int_\sigma p\cos(n,\ z)\,d\sigma; \tag{4.5}$$

$$M_x=\int_\sigma py\cos(n,\ z)\,d\sigma; \tag{4.6}$$

$$M_y=-\int_\sigma px\cos(n,\ z)\,d\sigma. \tag{4.7}$$

All integrals that have $p\cos(n,z)$ in the integrand are non-zero. Others are zero bcos of the symmetry of the projected areas

Port-stbd symmetry of the underwater vol is not necessary

Most ships do not have fwd-aft symmetry

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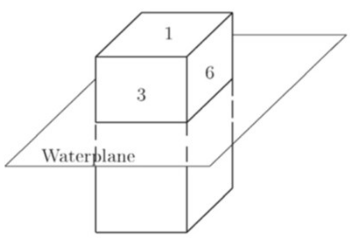
1.1.2 Stevin's Law

- What is the attitude of a block of wood or a barge floating in water?



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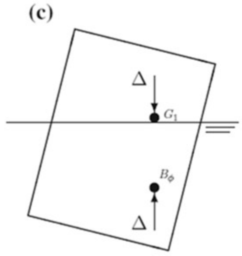

- Can Archimedes' principle be used to find the attitude?
- NO. Use Stevin's Law to find the attitude.

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Stevin's Law

Consider 3 different horizontal lines lying on the sea surface and parallel to the normal to the screen. They are below G, above B, and midway between B and G, respectively. In Fig. 2.7b, the moments about these lines, due to G and B, are all equal as G and B are equal and opposite and form a couple.

2.3.2 Moments

The moments are zero about any axis

In this subsection we discuss the second condition of equilibrium of a floating body: the sum of the moments of all forces acting on it must be zero. This condition is fulfilled in Figure 2.7a where the *centre of gravity*, G, and the centre of buoyancy, B, of the floating body are on the same **vertical** line. The weight of the body and the buoyancy force are equal—that is Δ—opposed, and act along the same line. The sum of their moments about any reference is zero.

Let us assume that the centre of gravity moves in the same plane, to a new position, G_1 (Figure 2.7b). The sum of the moments is no more zero; it causes a clockwise inclination of the body, by an angle ϕ. A volume submerges at right, another volume emerges at left. The

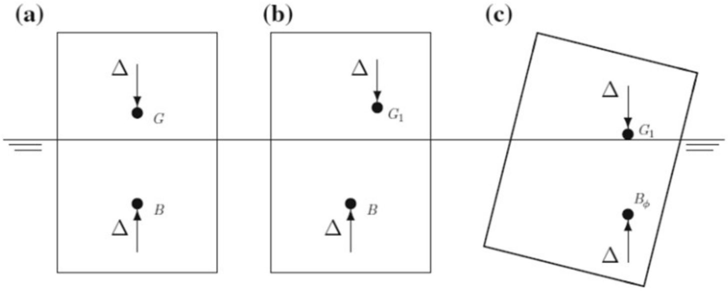

Figure 2.7 Stevin's Law, 1

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Stevin's Law

The gravitational and buoyancy forces are equal. So, they form a couple if their lines of action are not the same as in Fig. 2.7b.

result is that the centre of buoyancy moves to the right, to a new point that we mark by B_ϕ. The floating body will find a position of equilibrium when the two points G_1 and B_ϕ will be on the same vertical line. This situation is shown in Figure 2.7c.

There is a possibility of redrawing Figure 2.7 so that all situations are shown in one figure. To do this, instead of showing the body inclined clockwise by an angle ϕ, and keeping the waterline constant, we keep the position of the body constant and draw the waterline inclined counterclockwise by the angle ϕ. Thus, in Figure 2.8 the waterline corresponding to the initial position is W_0L_0. The weight force, equal to Δ, acts through the initial centre of gravity, G_0; it is vertical, that is perpendicular to the waterline W_0L_0. The buoyancy force, also equal to Δ, acts through the initial centre of buoyancy, B_0: it is vertical, that is perpendicular to the initial waterline.

We assume now that the centre of gravity moves to a new position, G_1. The floating body rotates in the same direction, by an angle ϕ, until it reaches a position of equilibrium in which the new waterline is $W_\phi L_\phi$. The new centre of buoyancy is B_ϕ. The line connecting G_1 and B_ϕ is vertical, that is perpendicular to the waterline $W_\phi L_\phi$. The weight and the buoyancy force act along this line.

It should be "centre of gravity and centre of buoyancy."
Biran sometimes uses "centre of gravity of the buoyancy force" instead of "centre of buoyancy force"

Thus, in the case of a floating body, the second condition of equilibrium is satisfied if the centre of gravity and the centre of gravity are on the same vertical line. This condition is attributed to Simon Stevin (Simon of Bruges, Flanders, 1548–1620). Stevin is perhaps better known for other studies, among them one on decimal fractions that helped to establish the

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Stevin's Law

- The gravitational and buoyancy forces are equal and opposite. If the CoG is moved, a couple acts on the ship. Then, the CoB moves to make the couple zero. The lines of action of G and B are the same in 3D space when the ship is in equilibrium.

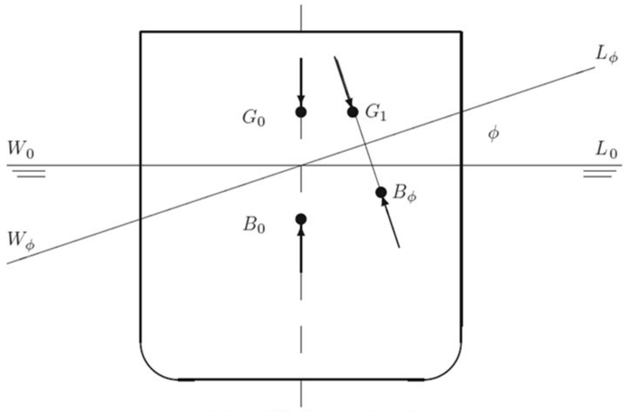

Figure 2.8 Stevin's Law, 2

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Application of Stevin's Law

- A homogeneous cuboid is floating in water. The length, breadth, and draft are L, B, and T, respectively. The CoG is shifted transversely by TCG. Find the attitude of the cuboid.
- Special case: The density of the cuboid is half the density of the water. Therefore, the CoG is on the centreline and half the cuboid is underwater.
- About what axis will the cuboid incline? Later, we will answer this question formally using Euler's theorem. Now, assume, intuitively, that it will incline about the centreline.
- TCB $= B^2 \tan\phi/(12\,T)$. Transverse change in the CoB due to heel by ϕ
- VCB $= B^2 \tan^2\phi/(24\,T)$. Vertical change in the CoB due to heel by ϕ.
- Later, we will prove the above two equations for a wall-sided ship.

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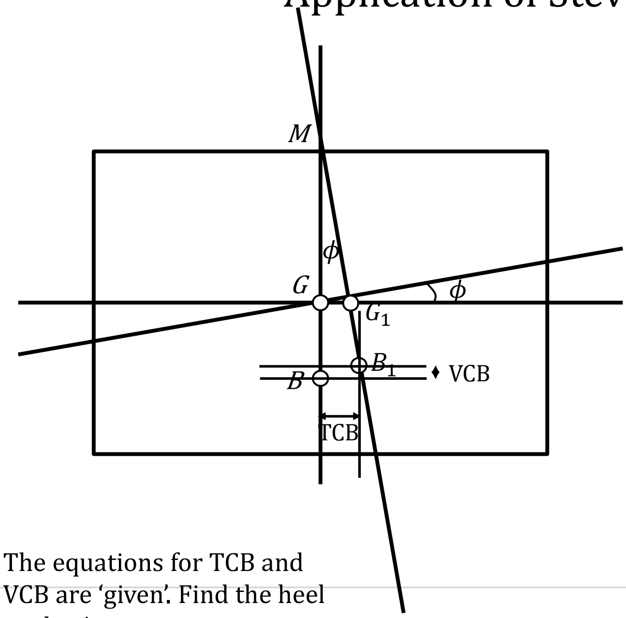
Application of Stevin's Law



- Original location of CoG
- CoG after it moves
- Original location of CoB
- CoB after it moves

Assume that the cuboid lists by ϕ

Draw the perpendiculars to the original and final waterlines. They meet at the metacentre, M.

The changes in the transverse and vertical CoBs are

- TCB $= B^2 \tan\phi/(12\,T)$
- VCB $= B^2 \tan^2\phi/(24\,T)$

The equations for TCB and VCB are 'given'. Find the heel angle, ϕ.

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Application of Stevin's Law

- $\tan\phi = \frac{TCB - TCG}{BG -} = \frac{B^2 \tan\phi/(12\,T) - \mathrm{TCG}}{BG - B^2 \tan^2\phi/(24\,T)}$
- This equation can be solved to find $\tan\phi$. It is a cubic eq. Those who are interested can solve it and show it to me.
- If ϕ is small ($<$ 10 deg), we can find an approximate solution
- $\tan\phi = \frac{TCB -}{BG - VC} \cong \frac{B^2 \tan\ \ /(12\,T) - \mathrm{TCG}}{BG}$
- $\tan\phi \cong \frac{\mathrm{TCG}}{B^2 \tan\phi\ /(12\,T) - BG}$

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10 Dec 2024

Stability of Ships

B. Tech. NA&SB. 2021-25. 20-215-0406

Department of Ship Technology
CUSAT, Kochi 682022

3 credits

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MODULE 1. EQUILIBRIUM OF SHIPS

1.1 Conditions for static equilibrium of a floating body

1.1.1 Archimedes Principle. Hydrostatic pressure distribution on floating cuboids and bodies of arbitrary shape

1.1.2 Stevin's Law

1.2 Small changes from equilibrium position

1.2.1 Three types of equilibrium

1.2.2 Small vertical change in the position

1.2.3 Small inclination

1.2.3.1 Bouguer's Metacentre

1.2.3.2 Euler's Theorem and the axis of inclination

1.2.3.3 Centre of flotation

1.3 Change in the density

1.3.1 Fresh water allowance

1.3.2 Dock water allowance

1.3.3 Plimsoll line

1.4 Change in the centre of gravity

1.4.1 Movement of a mass on-board

1.4.2 Addition or removal of a mass

1.5 Change in the centre of buoyancy

1.5.1 Addition or removal of a small mass

1.5.2 Addition or removal of a medium mass

1.5.3 Large change in Addition or removal of a large mass

1.5.4 Small change in the inclination

1.5.5 Large change in the inclination

1.5.6 Curve of centres of buoyancy

1.6 Change in the metacentre

1.6.1 Metacentric radius. $BM = I/V$

1.6.2 Metacentric Evolute

1.6.3 Metacentric height

1.1 Conditions for static equilibrium of a floating body

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1.1.1 Archimedes Principle. Hydrostatic pressure distribution on floating cuboids and bodies of arbitrary shape

1.1.2 Stevin's Law

Today

1.2 Small changes from equilibrium position

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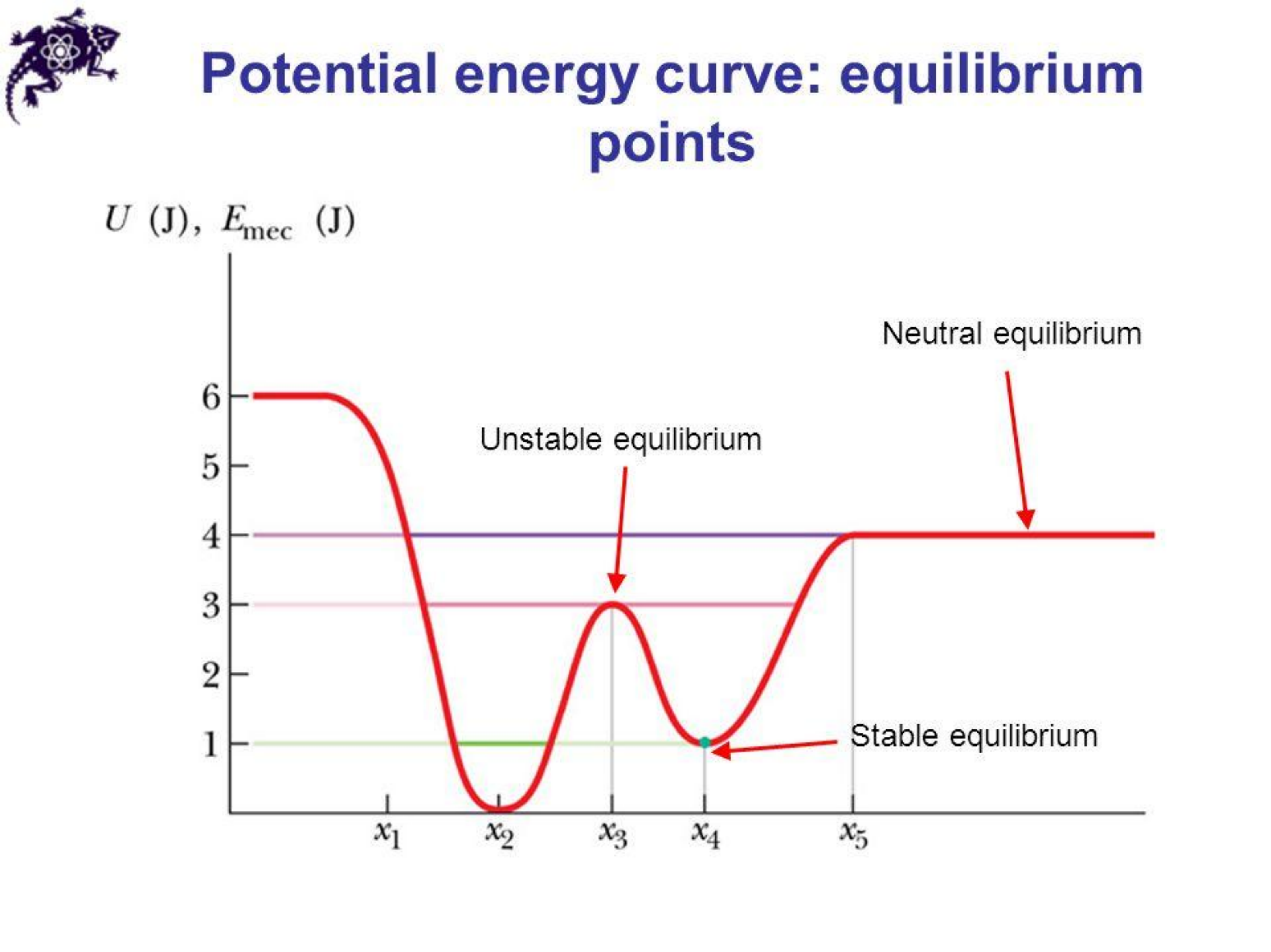
1.2.3.1 Bouguer's Metacentre

1.2.3.2 Euler's Theorem and the axis of inclination

1.2.3.3 Centre of flotation

1.2.1 Three types of equilibrium



1.2.1 Equilibrium. Gravitational Stability

- In general, a rigid body is considered to be in a state of equilibrium when the resultants of all forces and moments acting on the body are zero. In dealing with static floating body stability, we are interested in that state of equilibrium associated with the floating body, upright and at rest in a still liquid. In this case the resultant of all gravity forces (weights) acting downward, and the resultant of the buoyancy forces, acting upward on the body, are of equal magnitude and are applied in the same vertical line.
- When the net force and moment are zero, there is static equilibrium that may be stable, unstable, or neutral.
- Colin S. Moore, Intact Stability, Principles of Naval Architecture Series, 2010.

1.2.2 Small vertical change in the position

- (a) Stable equilibrium. When a downward force acts for a short time on a body that has sufficient freeboard and is initially at equilibrium, the body will undergo damped heaving motion. It oscillates and the draft and CoB also oscillate. The oscillations are damped by the water and the ship returns to its equilibrium position. All ships will be in stable equilibrium when a small vertical change in the position is considered.

Stable Equilibrium for small vertical movements

Next slide

As an example let us consider the body shown in Figure 2.1. If this body floats freely at the surface we conclude from Eq. (2.17) that the total volume is larger than the weight divided by the specific gravity of the fluid. This body floats in stable equilibrium as to draught. To show this let us imagine that some force causes it to move downwards so that its draught increases by the quantity δT. Archimedes' principle tells us that a new force, $\gamma LB\delta T$, appears and that it is directed upwards. Suppose now that the cause that moved the body downwards decreases slowly. Then, the force $\gamma LB\delta T$ returns the body to its initial position. In fact, as the body moves (slowly) upwards, δT decreases until it becomes zero and then the motion ceases. If the force that drove the body downwards ceases abruptly, the body oscillates around its initial position and, if damping forces are active—they always exist in nature—the body will eventually come to rest in its initial position.

Next, we assume that some force moved the body upwards so that its draught decreases by δT. A force $-\gamma LB\,\delta T$ appears now and it is directed downwards. Therefore, if the body is released slowly it will descend until $\delta T = 0$. This completes the proof that the body floating freely **at the surface** is in stable equilibrium with regard to its draught. We mention "with regard to draught" because, as shown in the next section, the body may be unstable with regard to heel.

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Stable Equilibrium for small vertical movements

- See previous slide

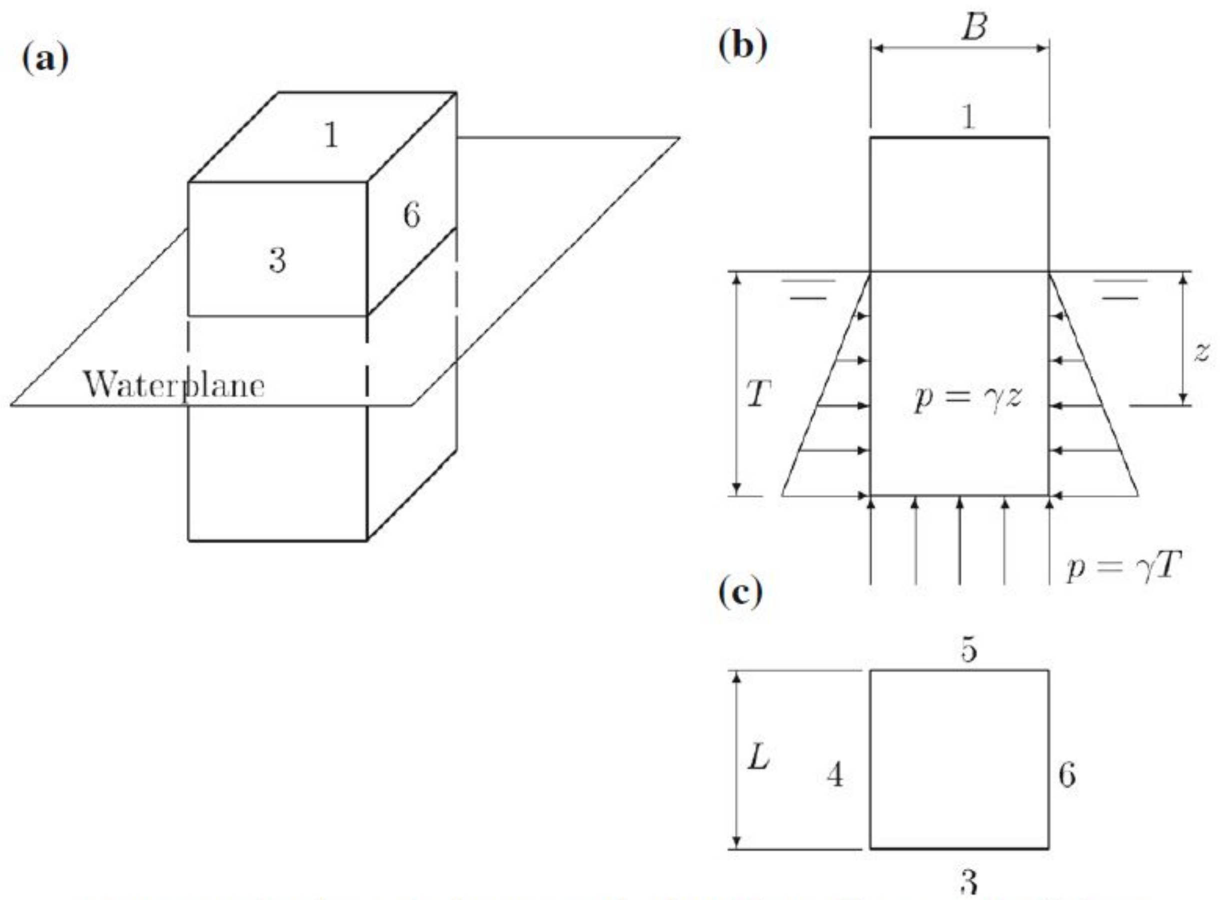


Figure 2.1 Hydrostatic forces on a body with simple geometrical form

Dec24 – Apr25

1.2.2 Small vertical change in the position

- (b) Neutral Equilibrium. A ship cannot be in neutral equilibrium with respect to vertical displacement. If the draft increases, the buoyancy force exceeds the gravitational force and there is a restoring force that causes it to move towards the equilibrium position. It is in neutral equilibrium with respect to horizontal displacement as there are no restoring forces when a gust of wind acts on it.
- (c) Unstable Equilibrium. If a floating body has zero freeboard and there are openings above the waterline it is in unstable vertical equilibrium. If it is displaced from its original position (draft) it will sink. If it has a very small freeboard, it is stable for "small" vertical displacements but unstable for larger displacements.

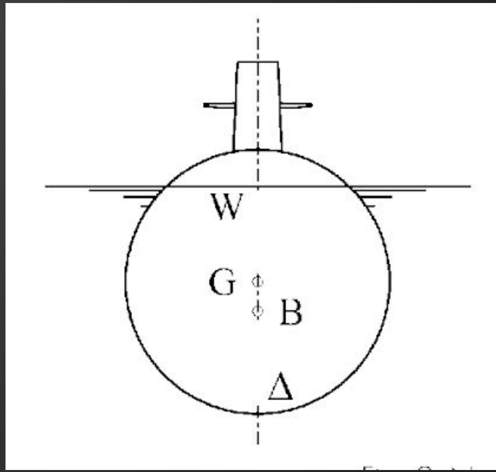
1.2.3 Small inclination

- (a) Stable equilibrium. If a floating body, initially at equilibrium, is disturbed by an external moment, there will be a change in its angular attitude. If upon removal of the external moment, the body returns to its original position, it is said to have been in stable equilibrium and to have positive stability.
- The ship inclines to port or starboard under the action of small forces due to winds and waves. When the external force ceases, the ship returns to its equilibrium position. For very large forces, the equilibrium may be unstable.

1.2.3 Small inclination

(b) Neutral equilibrium. If, on the other hand, a floating body that assumes a displaced inclination because of an external moment remains in that displaced position when the external moment is removed, the body is said to have been in neutral equilibrium and has neutral stability. A floating cylindrical homogeneous log (with a sail) would be in neutral equilibrium (if the sail does not touch water).

When there is an external force, the CoG of the body does not change if everything is secured. In the fig, the CoG and the CoB do not change due to a small inclination of the submarine. There are no restoring forces to bring it back to the upright position



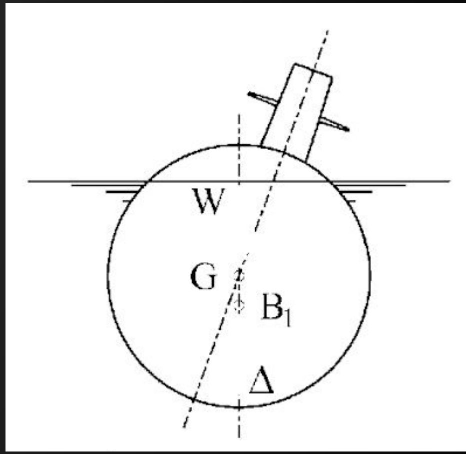


Fig. 2 Neutral equilibrium of floating body.
Moore. Intact Stability.

1.2.3 Small inclination

- (c) Unstable Equilibrium. If, for a floating body displaced from its original angular attitude, the displacement continues to increase in the same direction after the moment is removed, it is said to have been in unstable equilibrium and was initially unstable. Note that there may be a situation in which the body is stable with respect to "small" displacements and unstable with respect to larger displacements from the equilibrium position.
- This is a very common situation for a ship, and we will consider cases of stability at small angles of heel (initial stability) and at large angles separately.

1.2.3.1 Bouguer's Metacentre

2.5 Initial Stability

Figure 2.9a is a transverse section through a ship in *upright condition*, that is unheeled. If this section passes through the centre of buoyancy, B, we know from Stevin's law that it contains the centre of gravity, G. The waterline is W_0L_0. The weight force, W, acts through the centre

- Ship is upright. A section perpendicular to the sea surface, through CoB, is taken. It will have CoG also. Otherwise a moment will act.

GZ = distance between lines of action of gravitational and buoyancy forces. GZ(ϕ) = 0.

Basic Ship Hydrostatics **39**

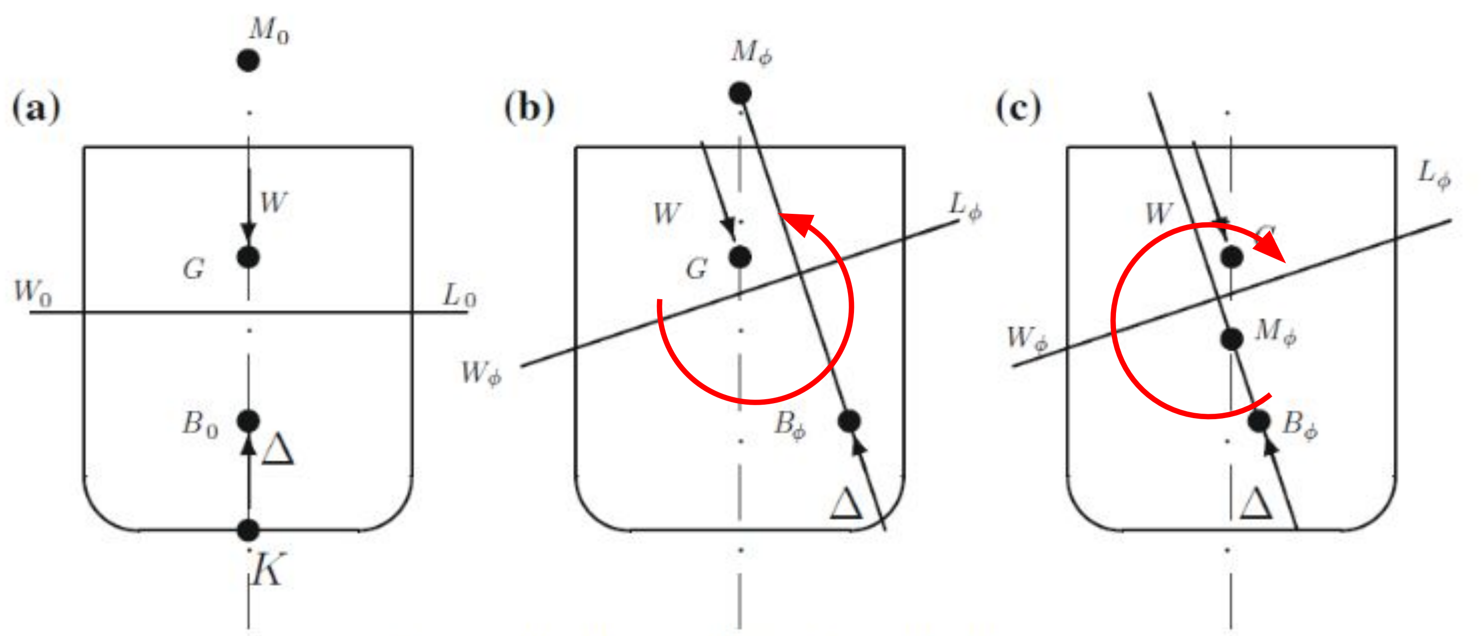


Figure 2.9 The condition of initial stability

1.2.3.1 Bouguer's Metacentre. Initial Stability.

- A ship with initial neutral equilibrium: The slope of the GZ vs ϕ curve is zero at the origin. Stable: slope > 0. Unstable: slope<0.

of gravity, G; the buoyancy force, Δ, through the centre of buoyancy, B_0. The forces W and Δ are equal and collinear and the ship is in an equilibrium condition. Let the ship heel to the starboard with an angle ϕ. For reasons that will become clear in Section 2.8, we assume that the *heel angle is small*. As previously explained, we leave the ship as she is and draw the waterline as inclined to port, with the same angle ϕ. This is done in Figure 2.9b where the new waterline is $W_\phi L_\phi$. If the weights are fixed, as they should be, the centre of gravity remains in the same position, G. Because a volume submerges at starboard, and an equal volume emerges at port, the centre of buoyancy moves to starboard, to a new position, B_ϕ. Both forces W and Δ are vertical, that is perpendicular to the waterline $W_\phi L_\phi$. These two forces form a moment that tends to return the ship toward port, that is to her initial condition. We say that the ship is stable.

- GZ = distance between lines of action of gravitational and buoyancy forces

Figure 2.9c also shows the ship heeled toward starboard with an angle ϕ. In the situation shown in this figure the moment of the two forces W and Δ heels the ship further toward starboard. We say that the ship is unstable.

1.2.3.1 Bouguer's Metacentre. Initial Stability.

The difference between the situations in Figure 2.9b and c can be described elegantly by the concept of **metacentre**. This abstract notion was introduced by Pierre Bouguer (French, 1698–1758) in 1746, in his *Traité du Navire*. Around the same time Euler was also looking for a criterion of stability. He recognized Bouguer's priority. The fascinating story of the quest for a stability criterion can be read in Ferreiro (2010). Let us refer again to Figure 2.9b. For a ship, the dot-point line is the trace of the port-to-starboard symmetry plane, that is the *centreline*. More generally, for any floating body, the dot-point line is the *line of action of the buoyancy force before heeling*. The new line of action of the buoyancy force passes through the new centre of buoyancy and is perpendicular to $W_\phi L_\phi$. The two lines intersect in the point M_ϕ.

Only for inclination from the upright position. More details later.

Bouguer called this point *metacentre*. Remember, this definition holds for small heel angles only.

We can see now the difference between the two heeled situations shown in Figure 2.9:

- in (b) the metacentre is situated *above the centre of gravity*, G;
- in (c) the metacentre is situated *below the centre of gravity*, G.

We conclude that *the equilibrium of the floating body is stable if the metacentre is situated above the centre of gravity*.

GZ vs angle of heel. Ship in stable equilibrium.

- The curve is anti-symmetric about the origin.
- A ship with initial stable equilibrium: The slope of the GZ vs ϕ curve is > zero at the origin.

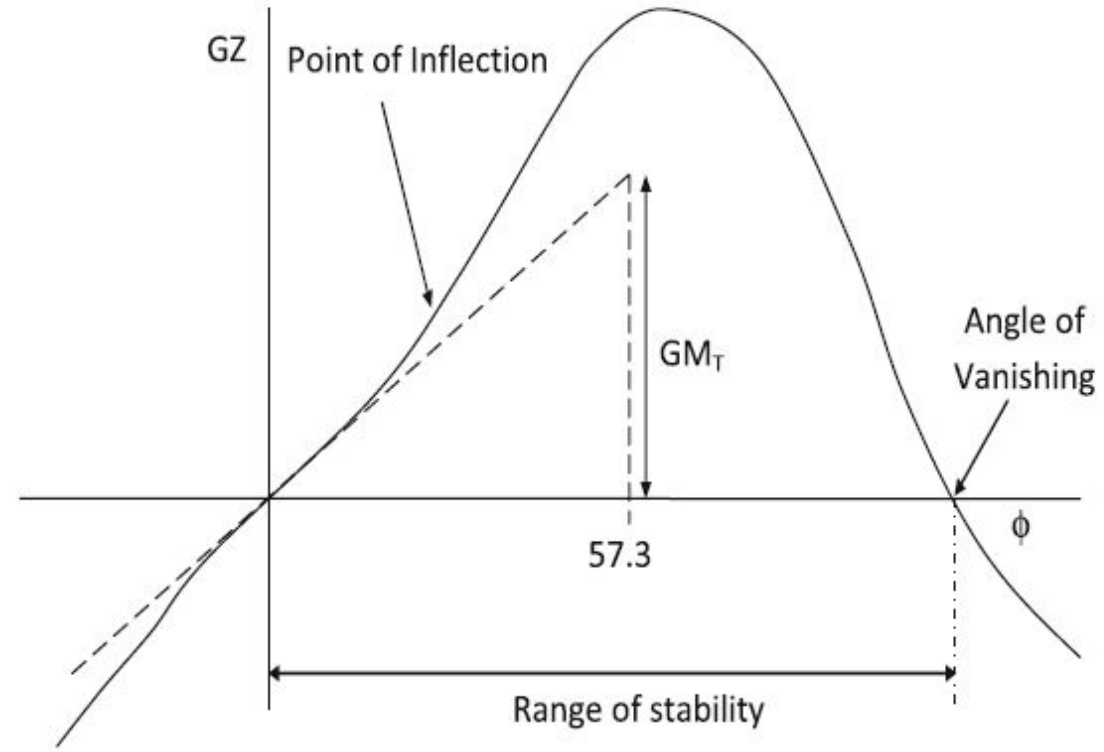
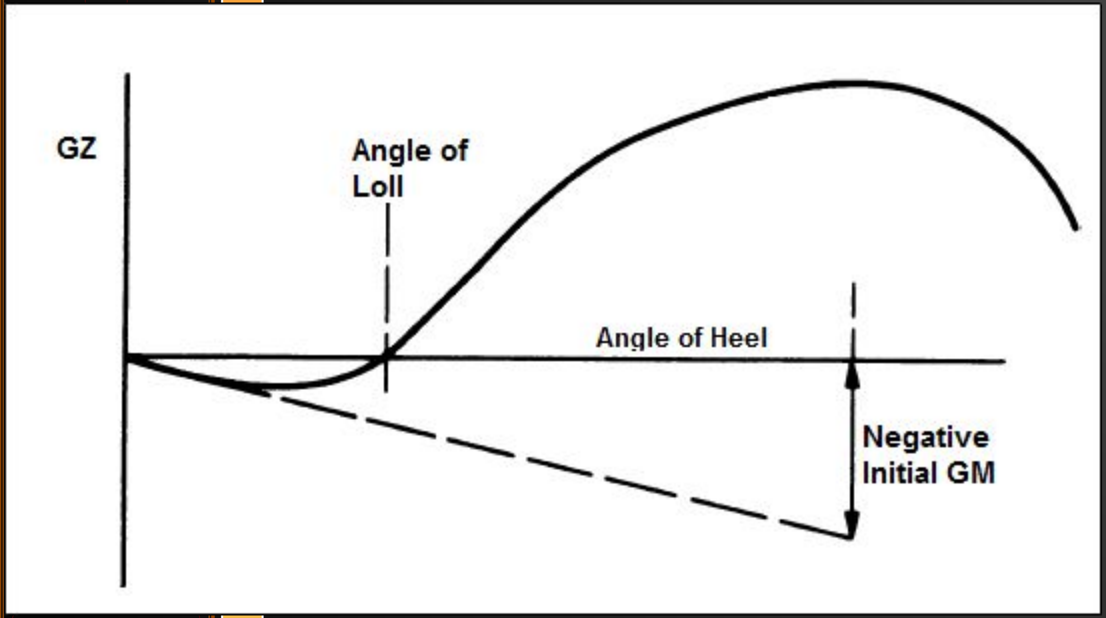


Fig. 2 Righting moment curve

GZ vs angle of heel. Ship in unstable equilibrium.

- Lester. The initial GM is negative. When the ship heels a little, it does not return to the upright position. The couple acting on it pushes it further away from the upright position till it reaches the loll angle. If it heels beyond the loll angle, the couple acts to bring it back to the loll angle.



GZ vs angle of heel. Ship in neutral equilibrium.

- A ship with initial neutral equilibrium: The slope of the GZ vs ϕ curve is zero at the origin. The initial GM is zero.
- Lester

1.2.3.2 Euler's Theorem and the axis of inclination

- Semyonov

Axis of Equivolume Inclination. Euler's Theorem.

- Biran

2.8.1 *A Theorem on the Axis of Inclination*

Let us assume that the initial waterplane of the body shown in Figure 2.11 is W_0L_0. Next we consider the same body inclined by a *small* angle ϕ, such that the new waterplane is $W_\phi L_\phi$. The weight of the body does not change; therefore, also the submerged volume does not change. If so, the volume of the "wedge" that submerges at right, between the two planes

W_0L_0 and $W_\phi L_\phi$, equals the volume of the wedge that emerges at left, between the same two planes. Let us express this mathematically. We take the intersection of the two planes as the x-axis. This is the *axis of inclination*.

This x axis is not the global x axis. It is the axis of inclination.

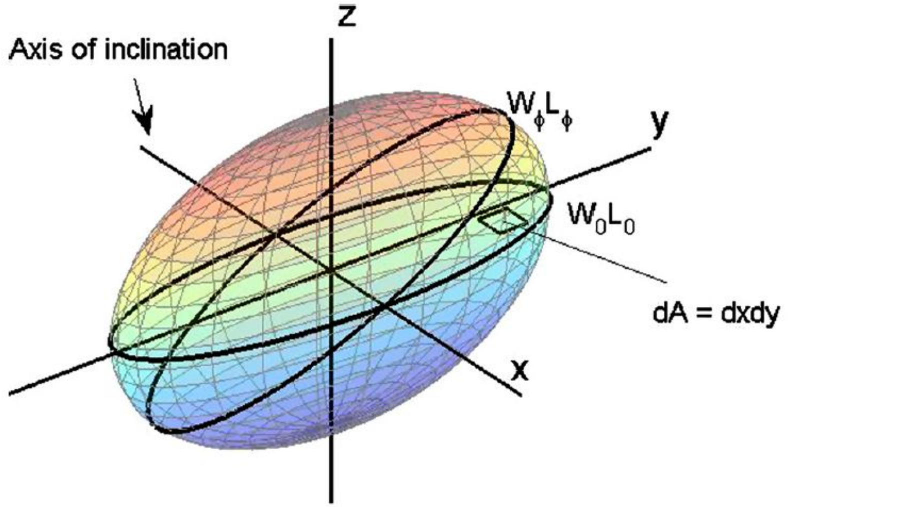


Figure 2.11 Euler's theorem on the axis of inclination.

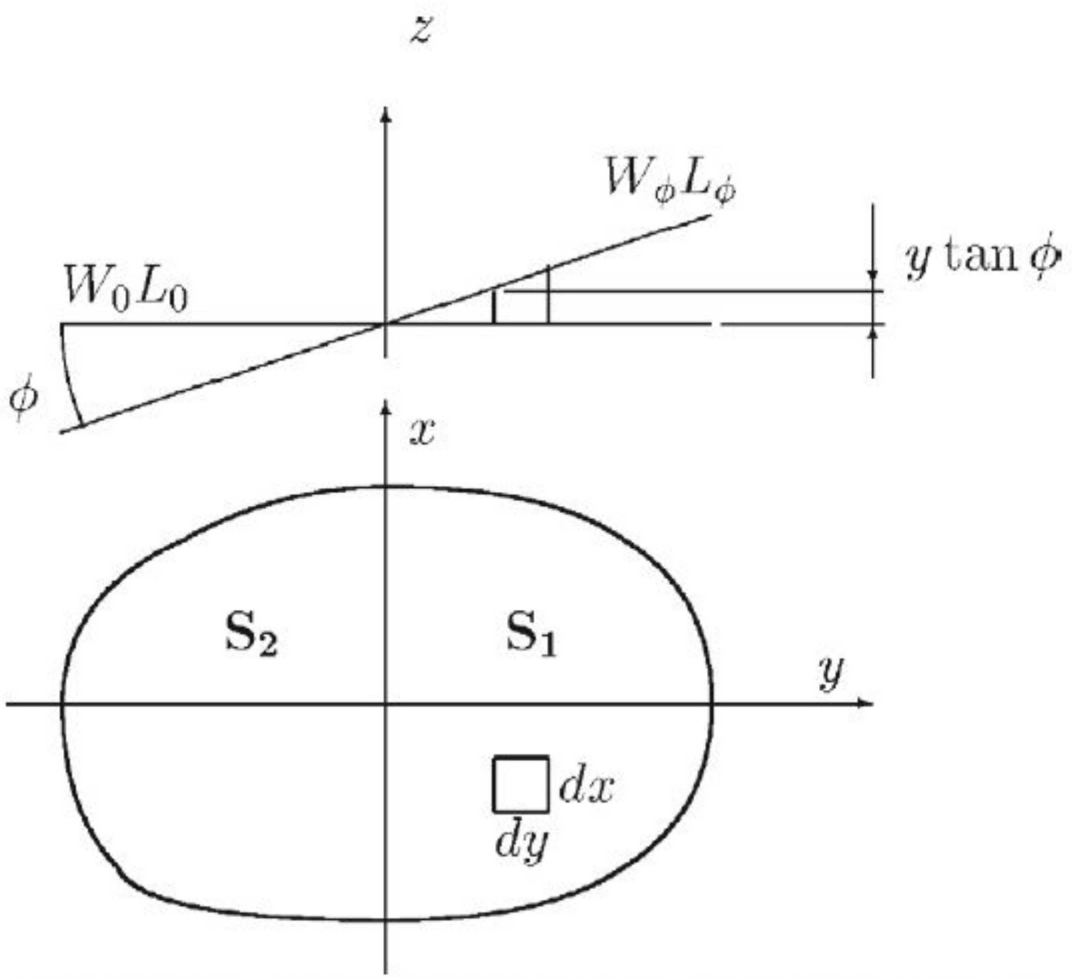


Figure 2.12 Euler's theorem on the axis of inclination, 2

$W_0 L_0$ and $W_\phi L_\phi$, equals the volume of the wedge that emerges at left, between the same two planes. Let us express this mathematically. We take the intersection of the two planes as the x-axis. This is the *axis of inclination*.

Equivolume Inclination

As shown in Figure 2.12, an element of volume situated at a distance y from the axis of inclination has the height $y \tan \phi$. If the base of this element of volume is $dA = dxdy$, the volume is $y \tan \phi \, dxdy$. Let the area of the waterplane W_0L_0 at the right of the axis of intersection be $\mathbf{S_1}$; that at the left, $\mathbf{S_2}$. Then, the volume that submerges is

At this point in the derivation, we do not know where the axis of intersection lies. It is to be found.

$$V_1 = \int \int_{\mathbf{S_1}} y \tan \phi \, dxdy \tag{2.24}$$

and the volume that emerges,

$$V_2 = - \int \int_{\mathbf{S_2}} y \tan \phi \, dxdy \tag{2.25}$$

By assuming a small heel angle, ϕ, we can consider the submerging and emerging volumes as wall sided and write Eqs. (2.24) and (2.25) as we did.

The condition for constant volume is

$$V_1 = V_2$$

1.2.3.2 Euler's Theorem and the axis of inclination

The condition for constant volume is

$$V_1 = V_2$$

Combining this with Eqs. (2.24) and (2.25) yields

$$\int\int_{\mathbf{S_1}} y \tan\phi \, dx dy = -\int\int_{\mathbf{S_2}} y \tan\phi \, dx dy \tag{2.26}$$

and, finally

$$\int\int_{\mathbf{S}} y \, dx dy = 0 \tag{2.27}$$

where $\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}$ is the whole waterplane. In words, the first moment of the waterplane area, with respect with the axis of inclination, is zero. This happens only if the axis of inclination passes through the *centroid* of the waterplane area. We remind the reader that the coordinates of the centroid of an area A are defined by

$$x_C = \frac{\int\int_A x \, dx dy}{\int\int_A dx dy}, \quad y_C = \frac{\int\int_A y \, dx dy}{\int\int_A dx dy}$$

Or, as the *Webster's Ninth Collegiate Dictionary* puts it, "corresponds to the centre of mass of

0 x 9.25 in

1.2.3.3 Centre of flotation

- Centre of Flotation

Or, as the *Webster's Ninth Collegiate Dictionary* puts it, "corresponds to the centre of mass of a thin plate of uniform thickness." The centroid of the waterplane area is known as **centre of flotation** and is noted by F.

A statement of this property is:

> *Let the initial waterplane of a floating body be W_0L_0. After an inclination, at constant volume of displacement, with an angle ϕ, the new waterplane is $W_\phi L_\phi$. The intersection of the two waterplanes is the axis of inclination. If the angle of inclination tends to zero, the axis of inclination tends to a straight line passing through the centroid of the waterplane area.*

In practice, this property holds if the angle of inclination is sufficiently small. For heeling of a vessel, this can mean a few degrees, 5° for some forms, even 15° for others. If the inclination is the trimming of an intact vessel, the angles are usually small enough and this property always holds. The property also holds for larger heel angles if the floating body is **wall sided**. This is the name given to floating bodies whose surface includes a cylinder (in the broader geometrical sense), with generators perpendicular to the initial waterplane. An illustration of such a case is given in Example 2.5. In French and Italian, for example, the term used for wall-sided bodies is *cylindrical floating bodies*.

The cylinder is floating with its axis along the normal to the waterplane are

The term used in some languages, such as French, Italian, or Spanish, for an axis passing through the centroid of an area is **barycentric axis**. This term is economic and we shall use it whenever it will help us to express ideas more concisely.

A consequence of Euler's Theorem. Centre of Flotation.

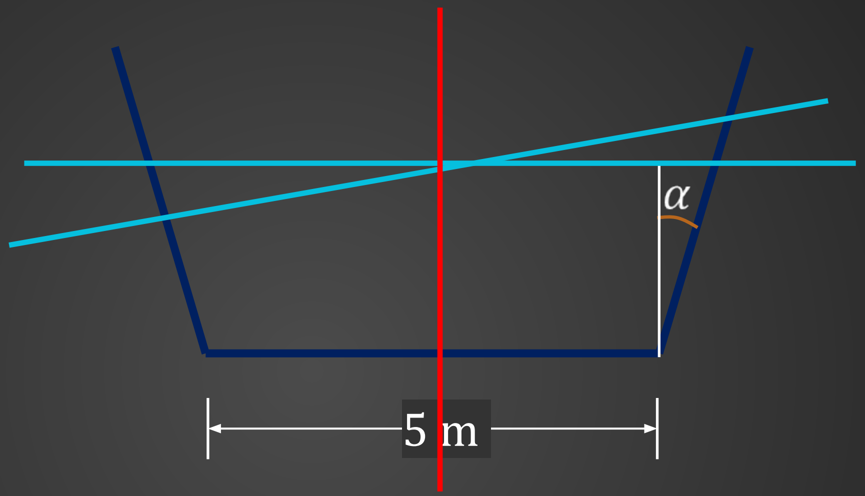
- Semyonov

Thus the line of intersection of two equivolume waterlines, the angle between them being infinitesimally small, must pass through the centroids of both waterlines. This is Euler's theorem. We have proved it for heel but it can be proved for an equivolume inclination about any axis lying in the waterplane.

Let us formulate a consequence of Euler's theorem which is its generalization. *If a floating ship is turned through an infinitesimal angle about any axis passing through the centroid of the waterplane, the volume of the immersed portion of the ship is unchanged.*

- This consequence is strictly valid only for wall-sided ships

Inclination of ships that are not wall-sided



- Euler’s theorem is valid only for an inclination angle that is infinitesimally small. Consider a barge with a trapezoidal cross-section. The centroid of the upright waterline is on the centerline. When there is an equivolume heel by a few degrees, the waterline is as shown in the figure. The centroid of the inclined waterline does not lie on the centerline of the barge because the submerged volume is equal to the emerged volume. The barge in the figure is not wall-sided. Later, we will use Krylov’s method to find the new waterline.

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1.1.2 Stevin's Law
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1.2.2 Small vertical change in the position
1.2.3 Small inclination
1.2.3.1 Bouguer's Metacentre
1.2.3.2 Euler's Theorem and the axis of inclination
1.2.3.3 Centre of flotation

Today

1.3 Change in the density
1.3.1 Fresh water allowance
1.3.2 Dock water allowance
1.3.3 Plimsoll line
1.4 Change in the centre of gravity
1.4.1 Movement of a mass on-board
1.4.2 Addition or removal of a mass
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1.5.3 Large change in Addition or removal of a large mass
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1.6 Change in the metacentre
1.6.1 Metacentric radius. BM = I/V
1.6.2 Metacentric Evolute
1.6.3 Metacentric height

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1.3 Change in the density
Tonnes and TPC

- 1 metric ton (or tonne) = 1000 kg
- 1 long ton = 2240 lb = 1016.05 kg
- Use the correct case for units. It is kg and not Kg.
- TPI = Ton per inch = The number of long tons to be added to a ship to increase its draft at the Longitudinal Centre of Floatation by 1 inch
- TPC = Tonnes per cm … see the next slide

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1.3 Change in the density. Biran: TPC and TPI

- TP1cm is introduced to make it easy to find out the load to be added or removed to change the draft by 1 cm.
- It is used when the density changes also.

4.2.3 Derived Data

ρ_W is in t/m^3. Ton has units of mass

Let us suppose that we know the displacement, Δ_0, corresponding to a given draught, T_0, and we want to find by how many tonnes that displacement will change if the draught changes by δT, centimetres. Let the waterplane area be A_W m^2, and the water density, ρ_W t m^{-3}. For a small draught change we may neglect the slope of the shell (in other words we assume a wall-sided hull) and we write

$$\delta\Delta = \rho_W A_W \delta T$$

$\delta\Delta = \rho_W A_W \delta T$ in SI units

If we measure Δ in tonnes, and δT in centimetres, we obtain

Δ **in t,** ρ_W in t/m^3, δT in cm
$$\delta\Delta = \rho_W \frac{A_W \boxed{\delta T}}{\boxed{100}} \qquad (4.11)$$

Note error in Biran Eq. 4.11. Corrected here.

We call the quantity $\rho_W \frac{A_W}{100}$ **tonnes per centimetre immersion**, where, as explained previously, the *tonne* is a unit of mass, and use for it the notation **TPC**. In older, English-language books we find the notation TPI as an acronym for **tonnes per inch** where the *ton* is a unit of weight. This quantity is calculated from an expression similar to Eq. (4.11), but adapted for English and American units. For SI units

$$TPC = \frac{A_W}{100} \times \rho_W \qquad (4.12)$$

where ρ_W should be taken from the Appendix of Chapter 2. The problem posed above can be inverted: find the change in draught, δT, corresponding to a change of displacement, $\delta\Delta$. The obvious answer is

$$\delta T = \frac{\delta\Delta}{TPC}$$

The above calculations yield good approximations as long as the changes $\delta\Delta$, δT are small. In fact, Eq. (4.11) is a linearization of the relationship between displacement volume and waterplane area.

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1.3 Change in Density

- What is the difference between mass and weight? On the earth, g is a function of latitude and longitude.
- If the ship goes to a place where the g is different but the density is the same, the draft will … complete the sentence
- not change.

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1.3 Change of Density

- When the ship moves to water with different density …
- Δ = Delta = Displacement in tonne. ∇ = Nabla = Del = Volume
- w = density in t/m^3 = ρ_W in Biran

there is a parallel rise. What happens if the weight of the ship remains the same and the density of the water in which it is floating is changed? Let us examine, at first, a body of displacement Δ floating in water of density w_1 which passes into water of lower density w_2. It will sink deeper because the water is less buoyant. The weight and buoyancy have not changed because nothing has been added to the body or taken away.

$$\therefore\ \Delta = \nabla_1 w_1 = \nabla_2 w_2$$

$$\frac{\nabla_1}{\nabla_2} = \frac{w_2}{w_1}$$

Units for mass density = t/m^3. $\frac{w_2}{w_1} = \frac{\rho_{w2}}{\rho_{w1}}$

$\nabla_1 = \nabla_2 \frac{w_2}{w_1}$ Use this in the next slide

Rawson and Tupper. Be careful about the notation in the book where Δ is weight and not mass, and w is weight density

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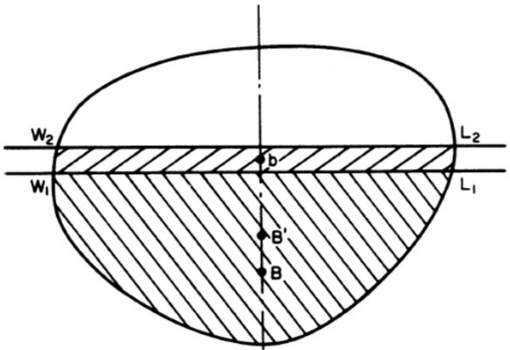
1.3 Change of Density

- Find the increase in the draft
- Find the increase in the Center of Buoyancy due to addition of a layer
- Later, we will see the effect of the movement of a wedge due to inclination
- w = density in t/m^3 = ρ_W in Biran
- When the draft changes, the CoB will also change.

The volume of the layer $= \nabla_2 - \nabla_1 = \delta\nabla$

$$\delta\nabla = \nabla_2 - \nabla_1 = \nabla_2 - \nabla_2 \frac{w_2}{w_1} = \nabla_2\left(1 - \frac{w_2}{w_1}\right)$$



The notation in Rawson and Tupper is confusing. Use the notation used here.

Fig. 3.9

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1.3 Change in the density and draft

- Change in the underwater volume $= \delta\nabla = \nabla_2 - \nabla_1 = \nabla_2 - \nabla_2 \frac{w_2}{w_1} = \nabla_2\left(1 - \frac{w_2}{w_1}\right)$
- Mass of layer = Change in vol * mass density $= \nabla_2\left(1 - \frac{w_2}{w_1}\right) w_2 = \Delta\left(1 - \frac{w_2}{w_1}\right)$ where the displacement, $\Delta = \nabla_1 w_1 = \nabla_2 w_2$, has not changed
- Thickness of layer ≈ Change in vol / Waterplane Area
- Books for seamen (mates) relate the thickness of the new layer to the TP1cm but this is only for convenience in calculating the thickness
- Use TP1cm = Waterplane Area * density/100 $= A_W w/100$; $A_W w = 100$ TP1cm
- Thickness of layer ≈ (Mass of layer/ w_2) / $A_W = \frac{\text{Mass of layer}}{A_W w_2} = \frac{\Delta\left(1-\frac{w_2}{w_1}\right)}{100 \text{ TP1cm}}$

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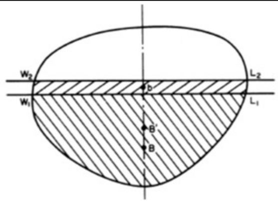
1.3 Change in the density and CoB



- The CoB of the original volume depends on only the lines-plan of the underwater volume. It does not depend on the density of water.
- Take moments about the CoB of the original volume
- Take moments of the original volume and the added volume:
- $\nabla_1 0 + (\nabla_2 - \nabla_1)\overline{Bb} = \nabla_2 \overline{BB^1}$

Taking first moments of the volume about B the original centre of buoyancy,

$$\overline{BB^1} = \frac{(\nabla_2 - \nabla_1)\overline{Bb}}{\nabla_2}$$

$$= \left(1 - \frac{w_2}{w_1}\right)\overline{Bb}$$

B is the original CoB
B^1 is the new CoB
b is the CoB of the layer

- Rearranging yields the change in the CoB
- $\overline{BB^1} = \frac{(\nabla_2 - \nabla_1)\overline{Bb}}{\nabla_2} = \left(1 - \frac{\nabla_1}{\nabla_2}\right)\overline{Bb} = \left(1 - \frac{w_2}{w_1}\right)\overline{Bb}$
- To find the change in the CoB, it is necessary to know the original CoB

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1.3.1 Fresh Water Allowance

- The Fresh Water Allowance is the amount (the number of millimetres) by which the mean draft changes when a ship passes from salt water to fresh water, or vice versa, whilst floating at the loaded draft.
- What are the densities of fresh, dock, and sea water?
- Fresh water 1000 kg/m^3
- The density of sea water changes with the location and time. A value of 1025 kg/m^3 is often used in calculations.

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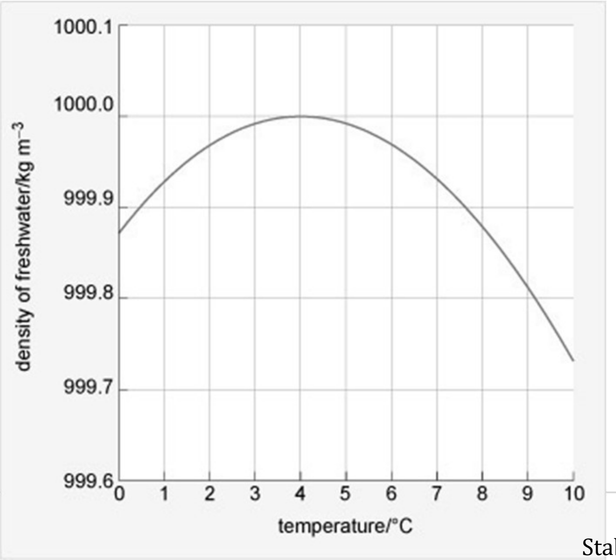
1.3.1 Fresh Water Allowance

- Density of Fresh Water. At what temp is the density maximum?
- 4 deg C



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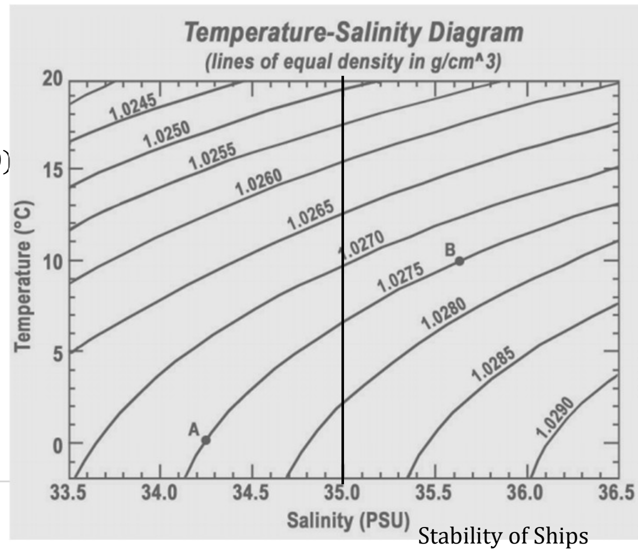
1.3.1 Fresh Water Allowance

- Density of sea water. For fixed salinity, density decreases when temperature increases
- PSU = practical salinity unit
  = g/kg
  = equivalent to per thousand or (o/00)
- Typical salinity is 35 PSU
- If the salinity is 35 PSU, at what temp is the density 1.025 t/m^3?



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1.3.1 FWA

Prove that FWA (in mm) = Displacement (in tonnes)/(4*TPC)

- The volume of water displaced by a ship at sea is V.
- In fresh water, it displaces an additional volume, v.

The total volume of fresh water displaced is then V + v.

$$\begin{aligned}
\text{Mass} &= \text{Volume} \times \text{Density}\\
\therefore\ \text{Mass of SW displaced} &= 1025\,V\\
\text{and mass of FW displaced} &= 1000\,(V + v)\\
\text{but mass of FW displaced} &= \text{Mass of SW displaced}\\
\therefore\ 1000\,(V + v) &= 1025\,V\\
1000\,V + 1000\,v &= 1025\,V\\
1000\,v &= 25\,V\\
v &= V/40
\end{aligned}$$

- Continued

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1.3.1 FWA

Prove that FWA (in mm) = Displacement (in tonnes)/(4*TPC)

- Additional volume in FW = $\frac{V}{40} = v$ = FWA (in mm) A_w / 1000.
- For TP1cm formula in the next line, area is in m^2 and density is in t/m^3
- $\text{TPC}_s = A_w\, \rho_s /100$. Therefore, $A_w = \text{TPC}_s\, 100\, /\, \rho_s$
- $A_w = \frac{\text{TPC}_s\, 100}{\rho_s} = \frac{V}{40}\, \frac{1000}{\text{FWA}}$.
- FWA = $\frac{V\rho_s}{4}\, \frac{1}{\text{TPC}_s}$
- $W = V\rho_s$ (tonne)
- Therefore, FWA = $\frac{W}{4\ \text{TPC}_s}$

Now let w be the mass of salt water in volume v, in tonnes and let W be the mass of salt water in volume V, in tonnes,

$$\begin{aligned}
\therefore\ w &= W/40\\
\text{but } w &= \frac{\text{FWA}}{10} \times \text{TPC}\\
\frac{\text{FWA}}{10} \times \text{TPC} &= W/40
\end{aligned}$$

TPC = TPC_s = TP1cm in salt water

or

$$\text{FWA} = \frac{W}{4 \times \text{TPC}}\ \text{mm}$$

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1.3.1 Fresh Water Allowance

- An ocean going ship enters a fresh water river. How does this affect the draft?
- What does not change? What changes?
  - The mass and weight of the ship do not change
  - Therefore, the buoyancy force on the ship does not change
  - The density of the water decreases
  - Therefore, the volume of water displaced increases

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1.3.2 DWA definition

- Dock Water Allowance is similar to FWA and is the amount in millimetres by which the ships mean draft changes when a vessel moves between a salt water and dock water. Dock water is the water whose density is neither that of fresh water or salt water but in-between the two. The Relative Density (RD) of dock water is between 1.0 and 1.025. DWA depends on the density of the dock water.
- After the ship loads at a dock and moves to the sea where the density is higher, the draft decreases.
- Dock (port) water density in ports is published (on the internet)
- Chennai 1018 to 1025 kg/m^3
- Kochi 1020 kg/m^3

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1.3.2 DWA

- Find the change in the draft, x, when a ship moves from sea water of density ρ_s to dock water of density ρ_1
- $V\rho_s = (V+v)\rho_1$; $\frac{(V+v)}{V} = \frac{\rho_s}{\rho_1}$; Subtract 1 from both sides. $\frac{v}{V} = \frac{\rho_s-\rho_1}{\rho_1} = \frac{A_w x}{V}$; $x \propto (\rho_s - \rho_1)/\rho_1$
- $DWA = x_{dock} \propto (\rho_s - \rho_{dock})/\rho_{dock}$; FWA $= x_{fresh} \propto (\rho_s - \rho_{fresh})/\rho_{fresh}$
- DWA/FWA $= \boxed{\frac{\rho_s-\rho_{dock}}{(\rho_s-\rho_{fresh})}}\frac{\rho_{fresh}}{\rho_{dock}}$. The equation below, from Barrass and Derrett is only approximately right.

Barrass and Derrett. Ship Stability. Ed. 6. Page 39.

Let x = The Dock Water Allowance
Let ρ_{DW} = Density of the dock water

Dock water density is usually between 1000 and 1025 kg/m^3

Then

Note the salt water and fresh water densities used in the formula below: 1025 and 1000 kg/m^3.

$$\frac{\text{x mm}}{\text{FWA mm}} \cong \frac{1025 - \rho_{DW}}{1025 - 1000}$$

$$\text{Dock Water Allowance} = \frac{\text{FWA}(1025 - \rho_{DW})}{25}$$

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Barrass and Derrett Eq is Approximate

A cuboidal barge has length 40 m, breadth 5 m, and draft 1 m at sea where the density is 1025 kg/m^3.

(1) Find the change in the draft when it is in fresh water of density $\rho_{Fresh} = 1000$ kg/m^3.

(2) Find the change in the draft when it is in dock water of density $\rho_{Dock} = 1020$ kg/m^3.

Answer. The displacement = volume*density is the same. Therefore,

(1) T_{Fresh} = 40*5*1*1025/(40*5*1000) = 1.025 m. FWA = 25 mm.

(2) T_{Dock} = 40*5*1*1025/(40*5*1020) = 1.0049 m. DWA = 4.9 mm.

Note that Barrass and Derrett Eq. on L04S17 is only approximate. Using their Eq.

(1025- ρ_{Dock})/(1025- ρ_{Fresh}) = (1025-1020)/(1025-1000) = 5/25 ≅ DWA/FWA

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- Barrass and Derrett. Ship Stability. 6th Ed. Page 284.
- In this example, B&D use their approximate eq. on L04S17. Use the exact equation for DWA and repeat the calculations.

A ship is floating in water of relative density 1.015. The present displacement is 12 000 tonnes, KG 7.7 m, KM 8.6 m. The present drafts are F 8.25 m, A 8.65 m, and the present freeboard amidships is 1.06 m. The Summer draft is 8.53 m and the Summer freeboard is 1.02 m FWA 160 mm TPC 20. Assuming that the KM is constant, find the amount of cargo (KG 10.0 m) which can be loaded for the ship to proceed to sea at the loaded Summer draft. Also find the amount of the hog or sag and the initial GM on departure.

Summer freeboard	1.02 m	Present mean freeboard	1.06 m
Summer draft	+ 8.53 m	Depth Mld	9.55 m
Depth Mld	9.55 m	Present draft amidships	8.49 m
		Average of drafts F and A	8.45 m
		Ship is sagged by	0.04 m

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- Approx Eq.
- Exact Eq. DWA/FWA = $\frac{\rho_s - \rho_{dock}}{(\rho_s - \rho_{fresh})} \frac{\rho_{fresh}}{\rho_{dock}}$
- Is this Eq. exact?

$$\text{Dock water allowance (DWA)} = \frac{(1025 - \rho_{DW})}{25} \times \text{FWA} = \frac{10}{25} \times 160 = 64\text{ mm}$$
$$= 0.064\text{ m}$$

$$\text{TPC in dock water} = \frac{RD_{DW}}{RD_{SW}} \times TPC_{SW} = \frac{1.015}{1.025} \times 20$$
$$= 19.8\text{ tonnes}$$

Summer freeboard = 1.020 m
DWA = 0.064 m
Min. permissible freeboard = 0.956 m
Present freeboard = 1.060 m
Mean sinkage = 0.104 m or 10.4 cm
Cargo to load = Sinkage × TPC_{DW} = 10.4 × 19.8
Cargo to load = 205.92 tonnes

$$GG_1 = \frac{w \times d}{W + w} = \frac{205.92 \times (10 - 7.7)}{12\,000 + 205.92} = \frac{473.62}{12\,205.92}$$

∴ Rise of G = 0.039 m
Present GM (8.6 − 7.7) = 0.900 m
GM on departure = 0.861 m

and ship has a sag of 0.04 m.

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1.3.3 Plimsoll line

- Fig. 1.3 is on Slide 22.

Figure 1.3 shows a ship's load line marks. The center of the disk is at a distance below the deck line equal to the ship's statutory freeboard. Then 540 mm forward of the disk is a vertical line 25 mm thick, with horizontal lines measuring 230 × 25 mm on each side of it. The upper edge of the one marked 'S' is in line with the horizontal line through the disk and indicates the draft to which the ship may be loaded when floating in salt water in a Summer Zone. Above this line and pointing aft is another line marked 'F', the upper edge of which indicates the draft to which the ship may be loaded when floating in fresh water in a Summer Zone. If loaded to this draft in fresh water the ship will automatically rise to 'S' when she passes into salt water. The perpendicular distance in millimeters between the upper edges of these two lines is therefore the ship's Fresh Water Allowance.

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Load Line Marks

- Barrass and Derrett. 7th Edition. Page 8.

- Memorize the details

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8 Chapter 1

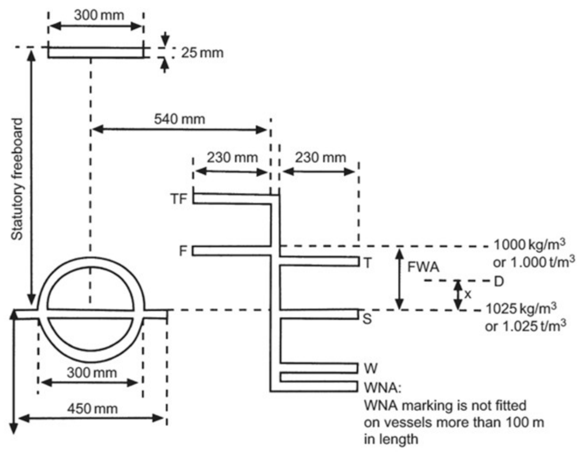

Figure 1.3

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1.3.3 Plimsoll Line and DWA

- When the ship is loading in dock water that is of density ρ_d where $\rho_s > \rho_d > \rho_f$, the top of the appropriate load line (summer, winter, or winter North Atlantic) on the Plimsoll mark may be submerged by such a distance, S, that she will automatically rise to the appropriate load line when in the open sea.

When the ship is loading in dock water that is of a density between these two limits, 'S' may be submerged such a distance that she will automatically rise to 'S' when the open sea and salt

water is reached. The distance by which 'S' can be submerged, called the *Dock Water Allowance*, is found in practice by simple proportion as follows:

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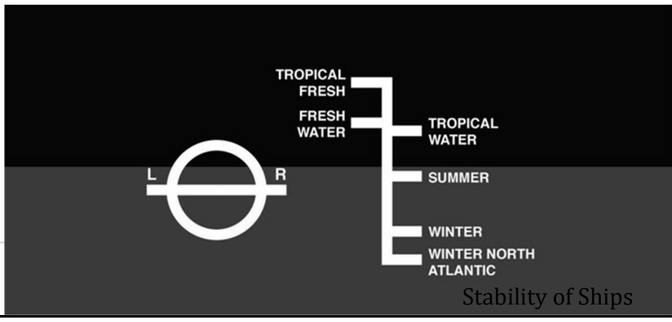
Incorrect Plimsoll Line Mark

- This figure in https://intlreg.org/2019/11/30/do-you-know-what-plimsoll-lines-on-ships-are/ is not a good one. The lines are not aligned.
- Barrass and Derrett. 7th Ed. Page 8. The upper edge of the line marked 'Summer' should be in line with the horizontal line through the disk and indicates the draft to which the ship may be loaded when floating in salt water in a Summer Zone.



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MODULE 1. EQUILIBRIUM OF SHIPS

1.1 Conditions for static equilibrium of a floating body
1.1.1 Archimedes Principle. Hydrostatic pressure distribution on floating cuboids and bodies of arbitrary shape
1.1.2 Stevin's Law
1.2 Small changes from equilibrium position
1.2.1 Three types of equilibrium
1.2.2 Small vertical change in the position
1.2.3 Small inclination
1.2.3.1 Bouguer's Metacentre
1.2.3.2 Euler's Theorem and the axis of inclination
1.2.3.3 Centre of flotation
1.3 Change in the density
1.3.1 Fresh water allowance
1.3.2 Dock water allowance
1.3.3 Plimsoll line

Today

1.4 Change in the centre of gravity
1.4.1 Movement of a mass on-board
1.4.2 Addition or removal of a mass
1.5 Change in the centre of buoyancy
1.5.1 Addition or removal of a small mass
1.5.2 Addition or removal of a medium mass
1.5.3 Large change in Addition or removal of a large mass
1.5.4 Small change in the inclination
1.5.5 Large change in the inclination
1.5.6 Curve of centres of buoyancy
1.6 Change in the metacentre
1.6.1 Metacentric radius. BM = I/V
1.6.2 Metacentric Evolute
1.6.3 Metacentric height

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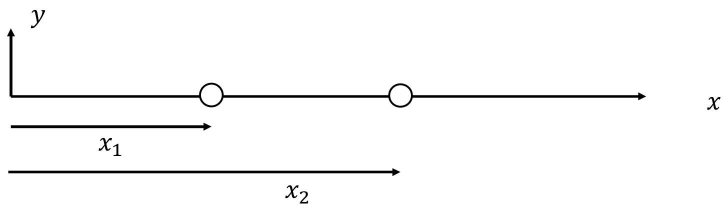
1.4 Centre of Gravity

- Consider masses, $m_1, m_2, m_3, \dots m_N$, with CoGs at $\bar{x}_i, i = 1, 2, 3, \dots N$.
- The CoG is at $\bar{x}_G = \frac{\sum_{i=1}^{N} m_i \bar{x}_i}{\sum_{i=1}^{N} m_i}$



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1.4.1 Movement of mass on board. Change in the CoG.

- The mass m_i is at x_i, $i = 1, 2$. The CoG is at $x_G = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$. No matter where the origin is, the CoG is at the same point with reference to the two masses.
- For example, if the two masses are equal, the CoG is midway between them.
- If the masses are m and $2m$, the CoG lies on the straight line joining the two masses and the distance between it and m is twice the distance between it and $2m$.
- m_2 is moved by Δx. Find the change in the CoG.

2.7 A Lemma on Moving Volumes or Masses

Figure 2.10 shows a system of two masses, m_1 and m_2. Let the x-coordinate of the mass m_1 be x_1; that of the mass m_2, x_2. The centre of gravity of the system is G and its x-coordinate is given by

$$x_G = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} \tag{2.22}$$

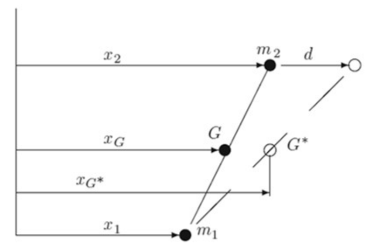

Figure 2.10 Moving a mass in a system of masses

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1.4.1 Movement of mass on board. Change in the CoG.

This will be used to find the CoB of an inclined ship

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Let us move the mass m_2 a distance d in the x-direction. The new centre of gravity is G^* and its x-coordinate,

Typo in Biran. Corrected.

$$x_G^* = \frac{x_1 m_1 + (x_2 + d)m_2}{m_1 + m_2} = x_G + \frac{dm_2}{m_1 + m_2} \quad (2.23)$$

The product dm_2 is the change of moment caused by the translation of the mass m_2. The centre of gravity of the system moved a distance equal to the change of moment divided by the total mass of the system. A formal statement of this Lemma is:

Given a system of masses, if one of its components is moved in a certain direction, the centre of gravity of the system moves in the same direction, a distance equal to the change of moment divided by the total mass.

A similar Lemma holds for a system of volumes in which one of them is moved to a new position. The reader is invited to solve Exercise 2.6 and prove the Lemma for a three-dimensional system of masses.

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1.4.1 Movement of mass on board. 3D case.

- The mass m_i is at (x_i, y_i, z_i), $i = 1, 2, 3, \ldots N$. The mass m_1 is moved to $(x_1 + \Delta x_1, y_1 + \Delta y_1, z_1 + \Delta z_1)$. Find the original CoG, the new CoG, and the change in the CoG.
- The original CoG is at (x_G, y_G, z_G). $x_G = \frac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i}$. $y_G = \frac{\sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i}$. $z_G = \frac{\sum_{i=1}^{N} m_i z_i}{\sum_{i=1}^{N} m_i}$.
- The new CoG is at $(x_G + \Delta x_G, y_G + \Delta y_G, z_G + \Delta z_G)$
- $x_G + \Delta x_G = \frac{m_1 \Delta x_1 + \sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i}$. $y_G + \Delta y_G = \frac{m_1 \Delta y_1 + \sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i}$. $z_G + \Delta z_G = \frac{m_1 \Delta z_1 + \sum_{i=1}^{N} m_i z_i}{\sum_{i=1}^{N} m_i}$
- The change in the CoG is $\Delta x_G = \frac{m_1 \Delta x_1}{\sum_{i=1}^{N} m_i}$. $\Delta y_G = \frac{m_1 \Delta y_1}{\sum_{i=1}^{N} m_i}$. $\Delta z_G = \frac{m_1 \Delta z_1}{\sum_{i=1}^{N} m_i}$
- To find the change in the CoG, it is NOT necessary to know the original CoG.
- The change in the CoG is in the direction in which the body moved.

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Stability of Ships

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1.4.2 Addition and removal of mass on board. 3D case.

- The mass m_i is at (x_i, y_i, z_i), $i = 1, 2, 3, \ldots N$. The mass m_{N+1} is added at $(x_{N+1}, y_{N+1}, z_{N+1})$. Find the change in the CoG.
- The original total mass is $\sum_{i=1}^{N} m_i = M_N$. The new total mass is $\sum_{i=1}^{N+1} m_i = M_{N+1} = M_N + m_{N+1}$
- The original CoG is at (x_G, y_G, z_G). $x_G = \frac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i}$. $y_G = \frac{\sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i}$. $z_G = \frac{\sum_{i=1}^{N} m_i z_i}{\sum_{i=1}^{N} m_i}$.
- The new CoG is at $(x_G + \Delta x_G, y_G + \Delta y_G, z_G + \Delta z_G)$
- $x_G + \Delta x_G = \frac{m_{N+1} x_{N+1} + \sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N+1} m_i} = \frac{M_N x_G + m_{N+1} x_{N+1}}{M_{N+1}}$; $y_G + \Delta y_G = \frac{M_N x_G + m_{N+1} y_{N+1}}{M_N + m_{N+1}}$.

  $z_G + \Delta z_G = \frac{M_N x_G + m_{N+1} z_{N+1}}{M_{N+1}}$
- What is the CoG if the nth mass is removed?

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Class Assignment

- The initial and final (x, y, z) coordinates of 5 masses are shown in the table below. Find the change in the x and y coordinates of the Centre of Gravity.

Mass (kg)	5	4	3	2	1
Initial (m)	(0,1,2)	(1,2,3)	(2,3,4)	(4,5,6)	(7,8,9)
Final (m)	(0,2,1)	(1,2,5)	(3,3,5)	(4,5,4)	(7,0,0)

- Total mass = 5+4+3+2+1 = 15 kg
- Change in the x coordinate = 3*1/15 = 1/5
- Change in the y coordinate = (5*1 -8*1)/15 = -3/15 = -1/5

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Effect of adding or removing a mass on the GM

- If a mass is added to a ship above the CoG of the ship, the CoG will move up and the GM will decrease.
- If a mass is added to a ship below the CoG of the ship, the CoG will move down and the GM will increase.

- If a mass that is above the CoG is removed from a ship, the CoG will move down and the GM will increase.
- If a mass that is below the CoG is removed from a ship, the CoG will move up and the GM will decrease.

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MODULE 1. EQUILIBRIUM OF SHIPS

1.1 Conditions for static equilibrium of a floating body
1.1.1 Archimedes Principle. Hydrostatic pressure distribution on floating cuboids and bodies of arbitrary shape
1.1.2 Stevin's Law
1.2 Small changes from equilibrium position
1.2.1 Three types of equilibrium
1.2.2 Small vertical change in the position
1.2.3 Small inclination
1.2.3.1 Bouguer's Metacentre
1.2.3.2 Euler's Theorem and the axis of inclination
1.2.3.3 Centre of flotation
1.3 Change in the density
1.3.1 Fresh water allowance
1.3.2 Dock water allowance
1.3.3 Plimsoll line
1.4 Change in the centre of gravity
1.4.1 Movement of a mass on-board
1.4.2 Addition or removal of a mass

Today
1.5 Change in the centre of buoyancy
1.5.1 Addition or removal of a small mass
1.5.2 Addition or removal of a medium mass
1.5.3 Large change in Addition or removal of a large mass
1.5.4 Small change in the inclination
1.5.5 Large change in the inclination
1.5.6 Curve of centres of buoyancy
1.6 Change in the metacentre
1.6.1 Metacentric radius. BM = I/V
1.6.2 Metacentric Evolute
1.6.3 Metacentric height

No heel or trim in 1.5.1 to 1.5.3

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1.2.3.3 Centre of Flotation and Axis of Inclination

RECALL!

- Biran pp 43
- The centroid of the waterplane area is known as the Center of Flotation

A statement of the property proven above is

> Let the initial waterplane of a floating body be W_0L_0. After an inclination, at constant volume of displacement, with an angle 0, the new waterplane is $W_\phi L_\phi$. The intersection of the two waterplanes is the axis of inclination. If the angle of inclination tends to zero, the axis of inclination tends to a straight line passing through the centroid of the waterplane area.

In practice, this property holds if the angle of inclination is sufficiently small. For heeling of a vessel, this can mean a few degrees, 5° for some forms, even 15° for others. If the inclination is the trimming of an intact vessel, the angles are usually small enough and this property always holds. The property also holds for larger heel angles if the floating body is **wall sided.** This is the name given

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1.5.1 Addition or Removal of Mass. Change in Disp

- Semyonov, Sec. 11, deals with the case of a mass added on-board a ship such that there is no heel or trim but there is a change in the displacement, the CoG, the draft, and the CoB. The analysis starts with the assumption that there is no heel or trim and the proof for the point at which the mass should be placed is in L05S34 and 35.

Sec. 11 Alteration in Draught Due to Weight Addition 39

11. ALTERATION IN DRAUGHT DUE TO ADDITION AND REMOVAL OF A SMALL AND LARGE WEIGHT. CONDITION FOR ABSENCE OF HEEL AND TRIM

In this section we shall consider the addition of a weight aboard ship which does not cause the ship to heel or to trim.

After adding a weight P the displacement of the ship is increased to give

$$D_1 = D + P. \tag{11.1}$$

In order that the first condition of equilibrium (see Sec. 4) should not be violated the corresponding increase in the volume of displacement is rendered necessary. Before weight addition the first condition of equilibrium is

$$D = \gamma V. \tag{11.2}$$

After weight addition it becomes

$$D_1 = \gamma V_1. \tag{11.3}$$

- If there is a heel or trim, consider the angle of inclination. Here, the angle is zero.

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Addition or Removal of Mass. Change in Disp.

- A weight, P, is added to a ship. γ = specific weight of water. Semyonov p 16

If we deduct (11.2) from (11.3) and take into account (11.1) we can obtain

$$P = \gamma (V_1 - V). \tag{11.4}$$

Here $(V_1 - V)$ is the increase in the volume of displacement which neutralizes the addition of the weight P. If the weight P is removed from the ship, P should be taken with the minus sign. The difference $(V_1 - V)$ is then also negative and shows the extent to which the volume of displacement has been reduced.

In both cases the change in volume of displacement can be effected only through a change in draught. In the first case the draught is increased while in the second case it is reduced.

- The analysis, so far, does not tell us whether or not the ship will trim or heel.
- If a mass is removed, use the equations with a negative value of P.

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1.5.1 Addition or Removal of Mass. CoG.

- To find the new CoG, moments can be taken about an arbitrary point
- Option 1. Take moments about the global origin for the ship
- Option 2. Take moments about the CoG just before the mass is added or removed.
- Option 1 is always useful. Option 2 is useful to illustrate a special case.

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Addition or Removal of Mass. CoG.

- It is easier here to take moments about the old CoG.
- Old CoG = (x_g, y_g, z_g). Denominator = new total weight = D+P

The change in weight of the ship may be accompanied by a change in co-ordinates of the centre of gravity. If the co-ordinates of the centre of gravity of the added weight P are x_p, y_p and z_p, the changes in co-ordinates of the centre of gravity of the ship in accordance with the theorem of moments are given by

$$\left.\begin{aligned}\delta x_g = \frac{P}{D+P}(x_p - x_g);\quad \delta y_g = \frac{P}{D+P}\boxed{(y_p - y_g)}\\ \delta z_g = \frac{P}{D+P}(z_p - z_g).\end{aligned}\right\} \qquad (11.5)$$

From expressions (11.5) it is seen that the position of the centre of gravity of the ship is unchanged if the centre of gravity of the added weight coincides with the centre of gravity of the ship.

The co-ordinates of the centre of gravity of the ship after weight addition are

$$x_{g_1} = x_g + \delta x_g;\ y_{g_1} = y_g + \delta y_g;\ z_{g_1} = z_g + \delta z_g. \qquad (11.6)$$

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Will the Center of Buoyancy change?

- From Eq. (11.5), it is seen that the addition or removal of a mass at the CoG (all 3 coords) does not change the CoG.
- Next: Is it possible to add or remove a mass such that none of the 3 coordinates of the CoB will change?
- NO. Why?

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List of Symbols. Semyonov

- $\boldsymbol{p}$ small mass (weight in Semyonov) that is added to the ship
- $(\boldsymbol{x_c}, \boldsymbol{y_c}, \boldsymbol{z_c})$ Coordinates of the CoB
- $(\boldsymbol{dx_c}, \boldsymbol{dy_c}, \boldsymbol{dz_c})$ Change in the CoB
- $(\boldsymbol{x_f}, \boldsymbol{y_f}, \boldsymbol{z_f})$ Coords of the C of Floatation
- $\boldsymbol{\gamma}$ Density of water (ρg in Semyonov)
- $\boldsymbol{S}$ Waterplane area
- $\boldsymbol{V}$ Volume displaced by the ship
- $\boldsymbol{D}$ total mass of the ship before the addition

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Will the Center of Buoyancy change?

- The addition or removal of a mass at the CoG (all 3 coords) does not change any of the coordinates of the CoG.
- Is it possible to add or remove a mass such that none of the 3 coordinates of the CoB will change?
- NO. Why?
- Because the zCoB of the added or removed "surface" layer can not be the same as the old CoB which lies within the submerged volume.
- But, mass can be added at $\boldsymbol{x_f}$ without changing the trim and the xCoB will not change if $(\boldsymbol{x_f} - \boldsymbol{x_c}) = 0$. Mass can be added at $\boldsymbol{y_f}$ without changing the heel and the yCoB will not change if $(\boldsymbol{y_f} - \boldsymbol{y_c}) = 0$. Example: cuboidal and trapezoidal barges. Proof is in today's lecture.

Thus the addition or removal of a weight from the ship does not always lead to a change in position of her centre of gravity but always entails a change in position of her centre of buoyancy.

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1.5 Change in the CoB. 5 cases

No change in heel or trim for cases 1 to 3

1. Infinitesimally small mass (added mass << displacement of the ship)
2. Moderately small mass (less than 5% of the displacement)
3. Arbitrary mass (more than 5% of the displacement)

No change in displacement for cases 4 to 5 (equivolume)

4. Small change in the inclination
5. Large change in the inclination

- For CoG, the equations are simple and all 3 cases (small, moderately small, and arbitrary) are analyzed in the same way because the mass is “lumped”. For CoB, ...
- Read Semyonov Chapter 2. Section 11.

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1.5.1 Infinitesimally Small Mass. CoB

The alteration in the position of the centre of buoyancy cannot be expressed in all 3 cases by simple formulas similar to relations (11.5). It is therefore necessary to deal separately with the addition of a small and a large weight.

Consider first the addition of an infinitesimally small weight dD. To do this we differentiate expression (11.2)

$$\boxed{D = \gamma V.}\ \boxed{(11.2)} \qquad dD = \gamma dV. \qquad (11.7)$$

Taking into account (8.3) we can obtain from (11.7) the formula for the change in draught S = Waterplane area

$$\boxed{\frac{dV}{dz} = S. \quad (8.3)} \qquad dz = \frac{dD}{\gamma S}. \qquad \gamma \int S dz = \mathrm{D} \quad (11.8)$$

The formulas for the changes in co-ordinates of the centre of buoyancy can be derived from expressions (9.9) and (10.6)

Take moments about CoB (x_c, y_c, z_c) or any other point

$$dx_c = \frac{dV}{V}(x_f - x_c); \qquad (11.9)$$

$$dz_c = \frac{dV}{V}\boxed{(z_f - z_c)} \qquad (11.10)$$

- Draw a graph of V vs z. Find the slope. It is equal to the waterplane area.
- As there is no heel or trim, the CoB of the added layer is at the CoF. See the next slide.

- $(x_f, y_f, z_f = T =$ draft) is the center of buoyancy of the added thin layer of water. Indirectly, this means that there is no change in the draft.

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1.5.1 Infinitesimally small mass. Change in CoB

- To find the change in the CoB, take moments about the global origin
- $Vx_c + dVx_f = (V + dV)\,(x_c + dx_c)$
- Expand RHS and neglect products of small terms: $dVdx_c$
- Cancelling terms, $dVx_f = Vdx_c + x_c dV$
- Rearranging, $dx_c = \frac{dV}{V}(x_f - x_c)$
- The above Eq. is Semyonov (11.9)
- Similarly, $dy_c = \frac{dV}{V}(y_f - y_c)$. But $y_f = y_c$ as there is no heel or trim & $dy_c = 0$
- Similarly, $dz_c = \frac{dV}{V}(z_f - z_c)$

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Change in CoB in terms of Displacement

$$dx_c = \frac{dV}{V}(x_f - x_c); \qquad (11.9)$$

$y_c = y_f = 0$

$$dz_c = \frac{dV}{V}(z - z_c). \qquad (11.10)$$

The change in co-ordinate y_c is obviously zero in so far as the symmetry of the volume of the immersed portion of the ship with respect to the diametral plane is not violated as a consequence of the change in draught.

Using expressions (11.2) and (11.7) we can obtain

$D = \gamma V$	(11.2)

$$dx_c = \frac{dD}{D}(x_f - x_c); \qquad (11.11)$$

$$dz_c = \frac{dD}{D}(z - z_c). \qquad (11.12)$$

- Semyonov says that we should use 11.2 and 11.7. The displacement is proportional to the underwater volume. So, 11.11 follows from 11.9.

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1.5.2. Addition of a Moderately Small Mass. Draft.

Sec. 11 Alteration in Draught Due to Weight Addition 41

In practice it is necessary to deal with the addition and removal of a weight which is not infinitesimally small but finite in magnitude and may be treated only conditionally as small. In order to derive simple formulas for the changes in draught and co-ordinates of the centre of buoyancy it is sufficient for the present case to make an assumption that the sides of the ship are vertical in way of the waterline. On the basis of this assumption we may treat the change in volume

$$\delta V = V_1 - V$$

See Fig in next slide

as a cylinder with base S and height δT (Fig. 11). Then

$$\delta V = S\delta T.$$

Now, on the basis of relation (11.4), denoting the added weight by p, we can write

$$p = \gamma S \delta T. \qquad (11.14)$$

If p is the added weight, the RHS has $\gamma = \rho g$. If p is the added mass, the RHS has only ρ.

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Addition of a Moderately Small Mass. Draft

Solving (11.14) for δT we obtain

Accurate if S(z) = constant. Wall sided.

$$\delta T = \frac{p}{\gamma S}. \qquad (11.15)$$

But, S(z) monotonically increases with z in most ships.

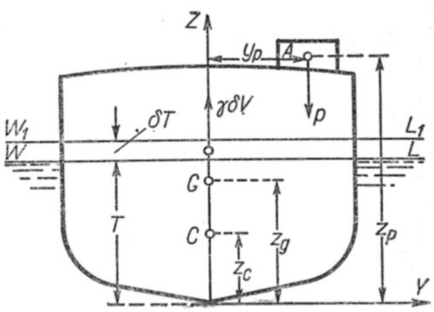
$T_1 >$ actual draft in a trapezoidal barge

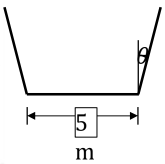
This formula is absolutely accurate if the sides of the ship are vertical.

Modern ships have flaring sides at the ends and sometimes in the midship region as well. Hence formula (11.15) is liable to overestimate the change in draught when the weight is added and underestimate it when the weight is removed. Thus in all cases the draught after weight addition, calculated by the use of formula (11.16),

$$T_1 = T + \delta T \qquad (11.16)$$

is somewhat greater than the true one.





According to the assumption

Fig. 11

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Addition of a Moderately Small Mass. CoB

According to the assumption made above the volume δV represents a cylinder with base S and height δT. It may therefore be taken that the centroid of the volume δV lies on the same vertical as the centroid of the area S of the initial waterline at a distance $\frac{\delta T}{2}$ from the initial waterplane (Figs. 11 and 12). Hence the co-ordinates of the centroid of the volume δV in the co-ordinate system adopted are x_f, 0 and $T + \frac{\delta T}{2}$.

Fig. 11

- For the infinitesimally small mass case, $(x_f, y_f, z = T)$ is the center of buoyancy of the added infinitesimally thin layer of water
- For the moderately small mass case, $(x_f, y_f, T + \delta T/2)$ is the center of buoyancy of the added moderately thin layer of water

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Set up equations for the moments of the volumes with respect to the transverse and horizontal planes through the centre of buoyancy (point C in Figs. 11 and 12)

$$\delta V\,(x_f - x_c) = (V + \delta V)\,\delta x_c; \tag{11.17}$$

$$\delta V\left(T + \frac{\delta T}{2} - z_c\right) = (V + \delta V)\,\delta z_c. \tag{11.18}$$

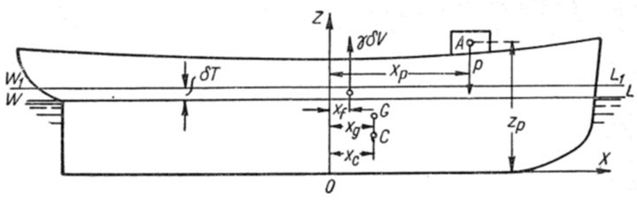

Fig. 12

- For this ship, if the draft increases from T to $T + \delta T$, the shape of the central buttock line shows that x_f moves aft.

Note the D+p in the denominator for the moderately small mass case

Solving equations (11.17) and (11.18) for δx_c and δz_c and multiplying δV and V by γ we obtain

D is in the denominator for the infinitesimally small

$$\delta x_c = \frac{p}{D + p}(x_f - x_c); \tag{11.19}$$

$$\delta z_c = \frac{p}{D + p}\left(T + \frac{\delta T}{2} - z_c\right). \tag{11.20}$$

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Addition of a Moderately Small Mass. CoB

If in expressions (11.19) and (11.20) p is considered to be an infinitesimal quantity, it becomes possible to neglect p in comparison with D and $\frac{\delta T}{2}$ in comparison with T. Then formulas (11.19) and (11.20) differ in no way from formulas (11.11) and (11.12).

- $\boldsymbol{dx_c = \frac{p}{D+p}(x_f - x_c)}$ (11.19)
- If $\boldsymbol{D \gg p}$, $\boldsymbol{D + p \cong D}$. This is equivalent to neglecting products of small terms: $\boldsymbol{dV dx_c}$
- Then, $\boldsymbol{dx_c = \frac{p}{D}(x_f - x_c)}$ (11.11)
- It is always good to check a derivation for limiting cases to see if it yields the same results as those obtained in a simple way. For e.g. quadratic eq.

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What is a Moderately Small Mass?

- Semyonov's book is more than 50 years old. As modern computational tools are available, use the approximation of moderately small mass only if it is less than 5% of the displacement. Semyonov says 15 to 20%.

when using the formulas derived above. In most cases these formulas may be used with sufficient accuracy for practical purposes if the weight p amounts to not more than 15 to 20 per cent of the weight of the ship D.

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1.5.3 Addition of Arbitrary Mass

- Semyonov. Chapter 2. Download from Classroom

Consider the case of the addition of a large weight understanding by the latter such a weight for which the formulas derived above do not ensure the desired accuracy. In this case there is no possibility of deriving simple formulas for the changes in draught and

co-ordinates of the centre of buoyancy. It is therefore necessary to make use of the relations $V(z)$, $x_c(z)$ and $z_c(z)$ which are at our disposal, i. e., of the curve of displacement and the curves of longitudinal and vertical centres of buoyancy, representing them all in one drawing (Fig. 13). Laying off the added weight P, to the scale adopted, on the horizontal axis from the point corresponding to

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one drawing (Fig. 13). Laying off the added weight P, to the scale adopted, on the horizontal axis from the point corresponding to

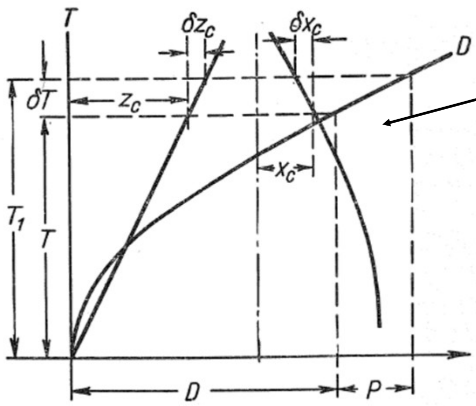

Fig. 13

the original displacement D, we obtain the change in draught on the vertical axis as the difference

$$\delta T = T_1 - T.$$

This figure has graphs of Displacement, z_c and x_c vs the draft, T.

The origin for the graphs of D and z_c is not the same as the origin for the graph of x_c as δx_c can be negative.

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1.5.3 Method to find new T and CoB

- In the fig, mark the point $(\boldsymbol{D}, \boldsymbol{T})$.
- Use $\boldsymbol{T}$ and find $\boldsymbol{x_c}$ and $\boldsymbol{z_c}$.
- Use $\boldsymbol{D}+\boldsymbol{p}$ and find $\boldsymbol{T_1}$. Find $\boldsymbol{T_1}-\boldsymbol{T}= \boldsymbol{\delta T}$.
- Use $\boldsymbol{T_1}$ and find $\boldsymbol{x_c}+\boldsymbol{\delta}\, \boldsymbol{x_c}$ and $\boldsymbol{z_c}+\boldsymbol{\delta z_c}$

Then from the draughts T_1 and T we find the changes in co-ordinates of the centre of buoyancy as the differences

$$\delta x_c = x_{c_1} - x_c; \qquad \delta z_c = z_{c_1} - z_c.$$

If the weight is removed from the ship, P should be laid off in the diagram in the opposite direction. The changes in co-ordinates of the centre of gravity of the ship can be calculated by formulas (11.5).

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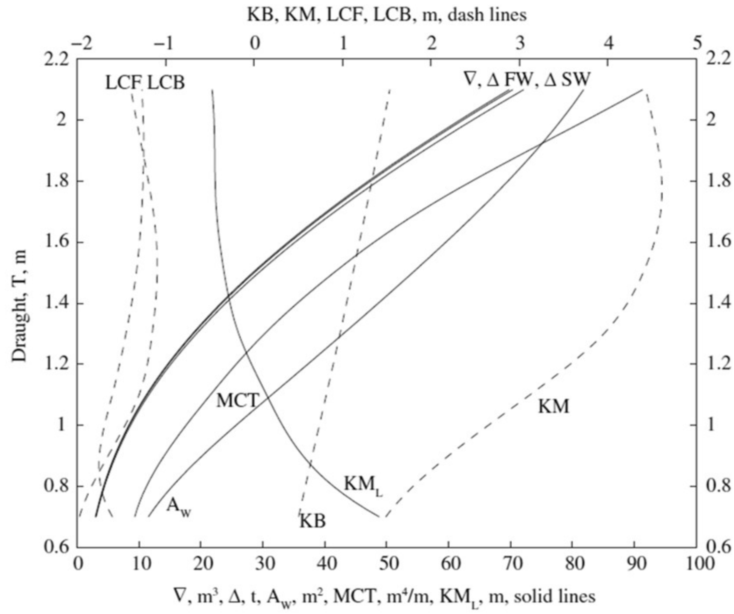

Figure 4.2 Hydrostatic curves of ship *Lido 9*

Exercise in curve fitting

- $\nabla(T=0)=0$
- Find an equation for $\nabla(T)$
- Use the digital data in Table 4.2 (next slide)
- Let $\nabla \cong aT^2+bT$. Find a and b for Lido 9 using the data in Biran Fig. 4.2 and Table 4.2. Use (a) T = 0.7 and 1.1 m (b) T = 1.7 and 2.1 m.

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- Use Matlab, Excel or Google Sheets. Draw a graph of T vs ∇. Fit polynomial curves of various orders. Find the coefficients of the fitted curve.

Table 4.2 Hydrostatic data of ship *Lido 9*

Data	Units	Draught							
	m	*0.700*	*0.900*	*1.100*	*1.300*	*1.500*	*1.700*	*1.900*	*2.100*
Trim difference (by head > 0)	m	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Volume of displacement	m^3	2.998	6.090	11.212	18.669	28.379	40.314	54.197	69.825
LCB fwd of midship	m	−1.599	−1.747	−1.600	−1.446	−1.329	−1.268	−1.246	−1.266
KB	m	0.506	0.660	0.819	0.973	1.120	1.263	1.401	1.536
Waterline area	m^2	11.529	20.221	31.449	42.998	54.183	64.708	74.088	81.810
LCF	m	−1.973	−1.648	−1.298	−1.150	−1.092	−1.137	−1.259	−1.388
Long mom of inertia	m^4	144.830	218.207	334.093	469.420	642.827	857.657	1129.524	1416.003
Moment to change trim	m^4/m	9.344	14.078	21.554	30.285	41.473	55.333	72.872	91.355
Transverse mom of inertia	m^4	2.950	9.364	25.814	55.665	93.061	134.428	171.925	201.990
Longitudinal, *KM*	m	48.813	36.491	30.615	26.117	23.772	22.538	22.242	21.815
Transverse, *KM*	m	1.490	2.198	3.121	3.955	4.400	4.598	4.574	4.429
Block coefficient, *CB*	–	0.110	0.126	0.149	0.177	0.216	0.261	0.301	0.342
Waterline coefficient, *CW*	–	0.296	0.377	0.461	0.531	0.620	0.712	0.783	0.841
Midship coefficient, *CM*	–	0.069	0.124	0.172	0.220	0.280	0.344	0.398	0.444
Prismatic coefficient, *CP*	–	–	–	0.870	0.807	0.773	0.758	0.758	0.770

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Addition of Moderately Small Mass. Condition for no heel or trim.

- Before the addition of the mass, the CoG and CoB lie on the same vertical line that is perpendicular to the sea-surface
- If the change in the xCoG = change in the xCoB and the change in the yCoG = change in the yCoB, the heel and trim will not change

Let us now find the conditions of weight addition under which the ship is neither heeled nor trimmed. The second condition of equilibrium (4.19) for a ship floating in the upright position after weight addition may be written as

Stevin's Law

$$x_{g_1} = x_{c_1} \quad \text{and} \quad y_{g_1} = y_{c_1} = 0.$$

Since before weight addition we have the equalities

$$x_g = x_c \quad \text{and} \quad y_g = 0$$

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Addition of Moderately Small Mass. Condition for no heel or trim.

44 *Buoyancy* Ch. II

the following relations must hold:

$$\delta x_g = \delta x_c \text{ and } \delta y_g = 0.$$

Making substitutions of (11.5) and (11.19) and taking into account that $\delta y_c = 0$, after cancellations and transformations we obtain

$$x_p = x_f \quad \text{and} \quad y_p = 0. \qquad (11.22)$$

This result can be reached if we consider the addition of an infinitesimally small weight.

Condition (11.22) so obtained may be formulated as follows: *in order that the ship be neither heeled nor trimmed as a consequence of the addition or removal of a small weight, the centre of gravity of the weight must be located on the same vertical as the centroid of the waterplane.*

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Addition of Moderately Small Mass. Condition for no heel or trim.

- for addition of any mass

$$\left.\begin{array}{l} \delta x_g = \frac{P}{D+P}(x_p - x_g); \quad \delta y_g = \frac{P}{D+P}\boxed{(y_p - y_g)} \\ \delta z_g = \frac{P}{D+P}(z_p - z_g). \end{array}\right\} \qquad (11.5)$$

- Addition of moderately small mass
  - $\boldsymbol{dx_c} = \frac{\boldsymbol{p}}{\boldsymbol{D+p}}(\boldsymbol{x_f} - \boldsymbol{x_c})$ (11.19)
- For no trim, $\delta \boldsymbol{x}_g = \boldsymbol{dx_c}$. This implies $(\boldsymbol{x}_p - \boldsymbol{x}_g) = (\boldsymbol{x_f} - \boldsymbol{x_c})$. But $\boldsymbol{x}_g = \boldsymbol{x}_c$. Therefore, there will be no trim if $\boldsymbol{x}_p = \boldsymbol{x}_f$.
- For no heel, $\delta y_g = \boldsymbol{d}y_c$. This implies $(y_p - y_g) = (y_f - y_c)$. But $y_g = y_c$. Therefore, there will be no heel if $y_p = y_f$. In most cases, $y_p = 0$.

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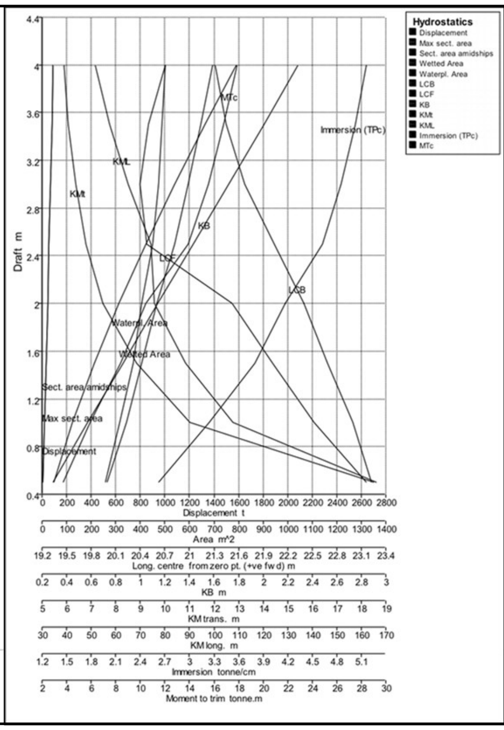
See the next slide also



Hydrostatic Curves

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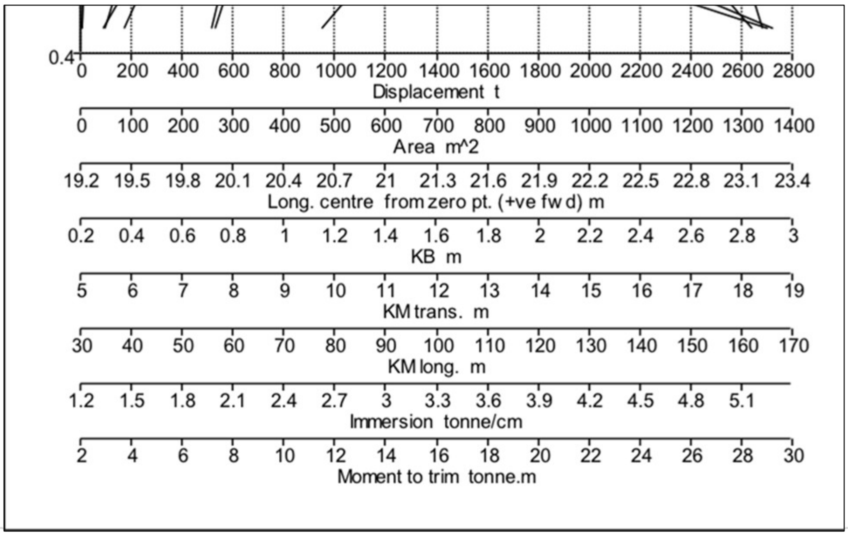
Hydrostatic Curves. Multiple Scales



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Stability of Ships

B. Tech. NA&SB. 2023-27. 20-215-0406
Department of Ship Technology
CUSAT, Kochi 682022
3 credits

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MODULE 1. EQUILIBRIUM OF SHIPS

1.1 Conditions for static equilibrium of a floating body
1.1.1 Archimedes Principle. Hydrostatic pressure distribution on floating cuboids and bodies of arbitrary shape
1.1.2 Stevin's Law
1.2 Small changes from equilibrium position
1.2.1 Three types of equilibrium
1.2.2 Small vertical change in the position
1.2.3 Small inclination
1.2.3.1 Bouguer's Metacentre
1.2.3.2 Euler's Theorem and the axis of inclination
1.2.3.3 Centre of flotation
1.3 Change in the density
1.3.1 Fresh water allowance
1.3.2 Dock water allowance
1.3.3 Plimsoll line
1.4 Change in the centre of gravity
1.4.1 Movement of a mass on-board
1.4.2 Addition or removal of a mass

Today

1.5 Change in the centre of buoyancy
1.5.1 Addition or removal of a small mass
1.5.2 Addition or removal of a medium mass
1.5.3 Large change in Addition or removal of a large mass
1.5.4 Change in the inclination
1.5.5 Curve of centres of buoyancy
1.6 Change in the metacentre
1.6.1 Metacentric radius. BM = I/V
1.6.2 Metacentric Evolute
1.6.3 Metacentric height

No heel or trim in 1.5.1 to 1.5.3

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1.5 Change in the CoB. 4 cases

No change in heel or trim for cases 1 to 3

1. Infinitesimally small mass (added mass << displacement of the ship)
2. Moderately small mass (less than 5% of the displacement)
3. Arbitrary mass (more than 5% of the displacement)

No change in displacement for case 4 (equivolume)

4. Change in the inclination

- For CoG, the equations are simple and all 3 cases (small, moderately small, and arbitrary) are analyzed in the same way because the mass is “lumped”. For CoB, ...
- Read Semyonov Chapter 2. Section 11.

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1.5.4 Change in the CoB due to inclination

20. SHIFT OF CENTRE OF BUOYANCY WITH SMALL EQUIVOLUME INCLINATION. SURFACE OF CENTRES OF BUOYANCY. CURVE OF CENTRES OF BUOYANCY

Semyonov

For an equivolume inclination of the ship the magnitude of the immersed volume remains unchanged but its form changes since instead of the emerged portions of the hull other portions become immersed which have the same volume but other outlines and another disposition. The centre of buoyancy is the centroid of the immersed volume, and therefore its position depends on the form of the immersed portion. Hence the position of the centre of buoyancy varies with equivolume inclination due to the change in form of the volume.

The following theorem which is a direct consequence of the theorem of moments is known from the course on theoretical mechanics. If one of the bodies constituting a system moves in any direction, the centre of gravity of the whole system moves in the same direction parallel to the shift of the centre of gravity of that body. The shift of the centre of gravity of the system and the shift of the centre of gravity of the shifted body are in the inverse ratio of their

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weights. This theorem is proved for weights but it can be applied to volumes.

Let us apply this theorem to equivolume inclinations of the ship. In effect, the appearance of the immersed and emerged wedges equal in volume may be treated as the transference of a certain portion of the total volume V. The volume δv is always transferred in the direction of inclination; so, when the ship is heeled to starboard the volume δv is transferred from port to starboard and when the ship is trimmed by the head it is transferred from

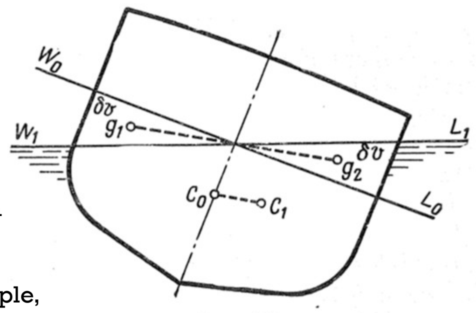

Fig. 23

Show the proof for this theorem for the special case of 2 masses m_1 and m_2.

The centroid of a triangle is at mean value of the coordinates of the vertices. For example, $x_c = \frac{1}{3}(x_1 + x_2 + x_3)$

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1.5.4 Change in the CoB due to inclination

aft to forward. Hence, in accordance with the theorem above, the centre of buoyancy is also shifted in the direction of inclination. In Fig. 23 the centroids of the volumes of the emerged and immersed wedges are denoted by g_1 and g_2. The direction of the shift of the centroid of the volume δv is coincident with the line g_1g_2 and the amount of the shift is equal to the segment $\overline{g_1g_2}$. If the shift of the centre of buoyancy be denoted by $\overline{C_0C_1}$ in Fig. 23, then using the foregoing theorem we can write

Discuss vector notation for this eq

$$C_0C_1 \parallel g_1g_2; \quad \overline{C_0C_1} = \overline{g_1g_2}\,\frac{\delta v}{V}. \tag{20.1}$$

Thus we have found the general conditions of the shift of the centre of buoyancy with equivolume inclination (20.1).

Rewrite the second expression as

$$V\,\overline{C_0C_1} = \delta v\,\overline{g_1g_2}. \tag{20.2}$$

The left-hand side of equality (20.2) represents *the increment of static moment of the volume* with respect to the plane perpendicular to the line of shift g_1g_2 and the right-hand side represents *the moment of transference of the wedge of volume* δv.

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The attitude of a ship

- The attitude of a ship (floating body) is fully described by specifying
  1. Displacement
  2. Trim angle
  3. Heel angle
- There are other alternative descriptions based on other triplets. For example, displacement can be replaced with draft at the origin. See the next slide.

- Standard Notation: Trim angle = θ (theta). Heel angle = ϕ (phi). Semyonov uses old notation.

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The attitude of a ship

- An alternative way of defining the attitude of a ship. From Semyonov.
- The attitude is specified by giving the values of
  1. Draft at the origin
  2. Trim angle
  3. Heel angle
- Note that θ (theta) is the heel angle and ψ is the trim angle in this old notation.

14. DETERMINATION OF BUOYANCY OF A SHIP IMMERSED TO ANY WATERLINE. VLASOV'S INTEGRAL CURVES OF SECTIONAL AREAS

Consider a ship immersed to a waterline which is not parallel to the base plane XOY. Such a waterline will be referred to as "any waterline". Find the relations by the use of which it is possible to determine the displacement V and the co-ordinates of the centre of buoyancy of the ship x_c, y_c and z_c.

We first establish three parameters which define the position of any waterline in the system of co-ordinates adopted. The position of a waterline is uniquely defined by the following quantities (Fig. 15):

(1) by the height $T_{ж}$ of the point of intersection of the plane of any waterline and the axis OZ;

(2) by the angle ψ between the axis OX and the trace of the waterplane on the plane XOZ;

(3) by the angle θ between OY and the trace of the waterplane on the plane YOZ.

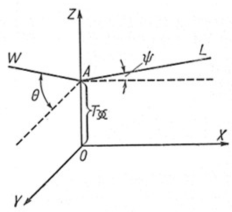

Fig. 15

It is to be noted that in practice the angles ψ and θ can be determined as follows. If the draught forward T_f and the draught aft T_a are determined from the marks of immersion of a ship floating at any waterline WL, we can find the angle ψ using the expression

$$\tan \psi = \frac{T_f - T_a}{L_0}.$$

Here L_0 is the distance between the marks of immersion at stem and stern.

If there are immersion marks placed at the sides of the ship in the midship plane, taking them and designating the draughts on port and starboard sides by T_P and T_S respectively we obtain

$$\tan \theta = \frac{T_S - T_P}{B},$$

where B is the breadth of the ship at amidships. This formula is valid provided the waterline is not outside the straight-sided portion of the ship.

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Heel, Trim, and CoB

- In general, $\overline{g_1 g_2}$ does not lie on a transverse cross-section.
- A mass is on the CoF. If it is moved transversely, the ship will heel but not trim. If it is moved longitudinally, the ship will trim but not heel. In the general case, both heel and trim will occur.
- Heel and trim take place about the centre of flotation. The axis of inclination passes through the CoF.
- For every attitude of a ship, there is ONE corresponding CoB
- In general, for pure equivolume heel of a ship, the CoB will move both transversely and longitudinally because the CoBs of the submerged and emerged volumes are not on the same transverse plane for a non-wall-sided ship.
- The line joining all the CoBs (trajectory of the CoBs) as a ship is made to undergo pure heel does not lie on a plane.
- Draw the C surface (surface of the centres of buoyancy) of a barge with uniform cross-section.

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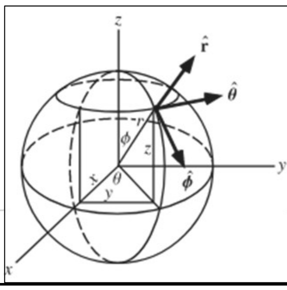
1.5.5 Surface of Centers of Buoyancy

- A. B. Biran, Ship Hydrostatics and Stability. Chap. 2.1
- To each position of a floating body, there corresponds one CoB.
- For each axis about which a floating body inclines, there corresponds one Metacentre. Example: transverse and longitudinal metacenters

To each position of a floating body correspond one centre of buoyancy and one ~~metacentre~~. Each position of the floating body is defined by three parameters, for instance the triple {*displacement*, *angle of heel*, *angle of trim*}; we call them the **parameters of the floating condition.** If we keep two parameters constant and let one vary, the centre of buoyancy travels along a curve and the metacentre along another. If only one parameter is kept constant and two vary, the centre of buoyancy and the metacentre generate two surfaces. In this chapter we shall briefly show what happens when the displacement is constant. The discussion of the case in which only one angle (that is, either heel or trim) varies leads to the concept of **metacentric evolute.**

Students should review the spherical coordinate system and use it to explain inclination



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1.5.5 Surface of Centers of Buoyancy

- Equivolume but not limited to small angles

So far we have considered infinitesimal equivolume inclinations of the ship and the associated shifts of the centre of buoyancy. We shall now touch upon some problems associated with the shift of the centre of buoyancy when equivolume inclinations are not restricted to small angles. Considering all possible equivolume inclinations of the ship with no limitation imposed on the direction of the axis of inclination or the magnitude of the angle of inclination we come to the conclusion that all centres of buoyancy corresponding to all possible positions of the ship, with the constant volume of the immersed portion, lie on a closed surface which is the locus of centres of buoyancy for a given equivolume inclination. This surface is known in the ship theory as *the surface of centres of buoyancy*, or *the C surface*. This is for a fixed displacement

When the ship is inclined in a particular plane the centre of buoyancy is shifted not only over the C surface but along a particular curve lying on this surface. This curve is termed *the trajectory of centres of buoyancy*, or *the C trajectory*. The C trajectory is in general a curve of double curvature. As it does not lie in one plane

The projection of the C trajectory on the plane of inclination is termed *the curve of centres of buoyancy*, or *the C curve*. This curve is obviously plane. Students to discuss

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1.5.5 CoB curve in the horizontal plane

Aalto University School of Engineering

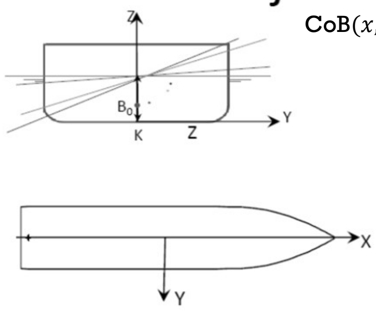
Aalto University

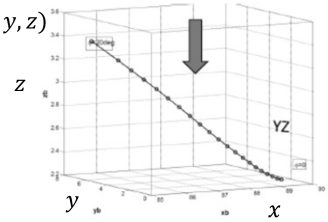
Projection of the curve

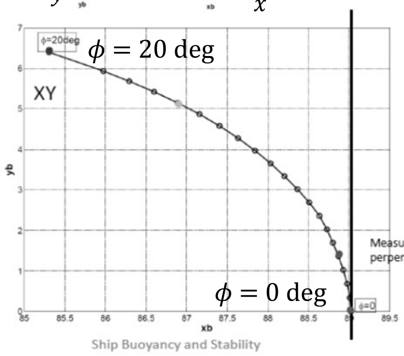
Trim angle = 0
Heel angle varies

CoB(x, y, z)

B=B(∇_0, ϕ, 0)
ϕ ∈ [0°, 20°]







Projection on the horizontal plane XY

The curve of the center of buoyancy for finite angles comes out from the YZ plane

02/11/2017 Maria Acanfora Ship Buoyancy and Stability 25

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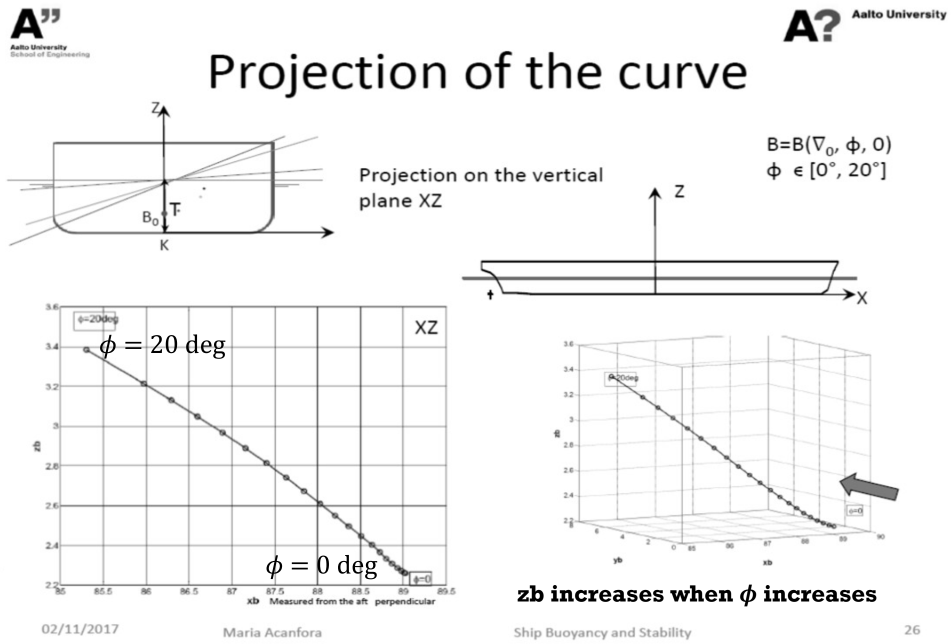
1.5.5 CoB curve in vertical plane



In the fig at the bottom-left, the axes are xb and zb. The ends of the blue line correspond to $\phi = 0$ and 20 deg.

Which end corresponds to 0 deg?

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1.5.5 Surface of Centers of Buoyancy

- The CoB and the M are shown for various angles of heel. Each BM line is perpendicular to the associated waterline.
- Learn to draw this figure.

66 Chapter 2

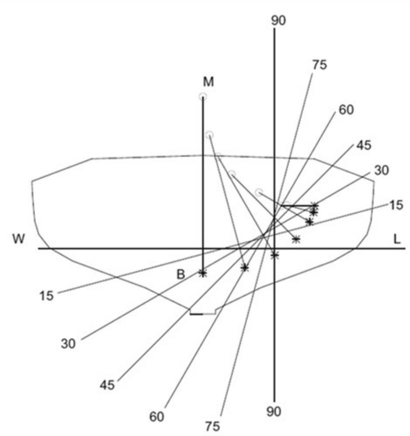

Figure 2.23 B and M curves of vessel *Lido 9*

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1.6 Change in the metacentre

- Biran
- The transverse metacenter is shown for 2 special cases. The longitudinal metacenter is at a different point.

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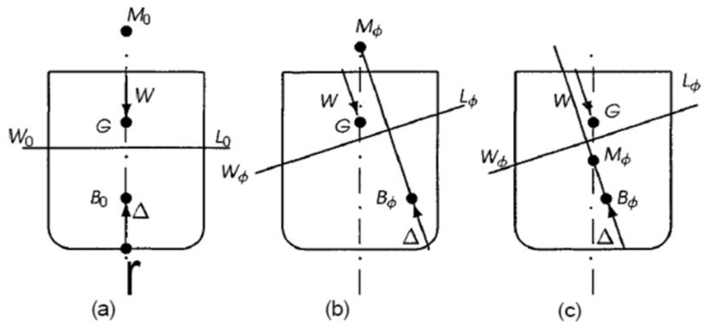

Figure 2.9 The condition of initial stability

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1.6 Change in the metacentre

- Biran. pp 39.
  https://en.wikipedia.org/wiki/Pierre_Bouguer

For his contributions of overwhelming importance, Bouguer was sometimes described as 'the father of naval architecture' (quotation in Stoot, 1959). It must be emphasized here that the definition of the metacentre is not connected at all with the form of a ship. Therefore, the fact that in the above figures the metacentre is the intersection of the new line of action of the buoyancy force and the centreline is true only for symmetrical hulls heeled from the upright condition. For a general floating body we can reformulate the definition as follows:

> Let us consider a floating body and its centre of buoyancy B_ϕ. Let the line of action of the buoyancy force be R. If the body changes its inclination by an angle $\delta\phi$, the centre of buoyancy changes its position to $B_{\phi+\delta\phi}$ and the new line of action of the buoyancy force will be, say, S When $\delta\phi$ tends to zero, the intersection of the lines R and S tends to a point that we call metacentre.

Readers familiar with elementary differential geometry will recognize that, defined as above, the metacentre is the **the centre of curvature of the curve of centres of buoyancy.** The notion of curvature is defined in Chapter 13.

The metacentre does not depend on the underwater hull form. It depends on only the underwater volume and the moment of inertia of the waterplane area. This will be proved later.

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1.6.1 Metacentric Radius = BM = I/V

- Outline of the proof

1) The half-breadth of the ship is y and the angle of heel is θ
2) Write expressions for the elemental area and volume of the submerged wedge
3) Find the moment of the elemental submerged volume about the centre-line
4) Total change in moment = 2*moment of submerged volume because an equal volume has emerged out of the water
5) Express the change in moment as the product of
   1) the 2^{nd} moment of inertia of the water-plane area, I, and the angle of heel, θ
   2) the product of the distance $\overline{gg_1}$ moved by the CoB of the wedge and the submerged volume, v.
   3) the product of the distance by which the CoB of the ship moves $\overline{BB_1}$ and the underwater volume of the ship, V.

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1.6.1 Prove that BM = I/V

6) Express the change in the CoB of the ship as the product of the metacentric radius, $\overline{BM}$, and the angle of heel θ.
7) Use the equations to show that $\overline{BM} = I/V$ by eliminating the other variables

The details are shown in the following slides

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1.6.1 BM = I/V

- Barrass and Derrett

Calculating KB, BM and metacentric diagrams 107

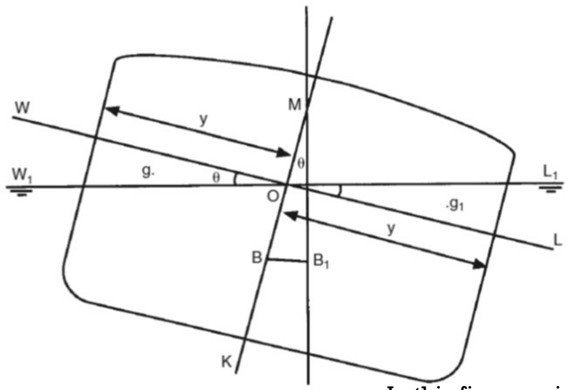

Fig. 12.3(a)

In this figure, y is the half-breadth of the ship

Since θ is a small angle then arc WW_1 = arc LL_1 = θy

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1.6.1 BM = I/V

Since θ is a small angle then arc WW_1 = arc LL_1
$= \theta y$

The area of a triangle = half the base x height

Also:

$$\text{Area of wedge } WOW_1 = \text{Area of wedge } LOL_1 = \tfrac{1}{2}\theta y^2$$

Consider an elementary wedge of longitudinal length dx as in Figure 12.3(b).

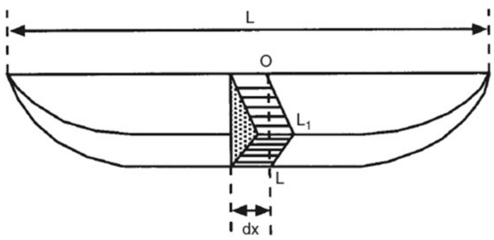

Fig. 12.3(b)

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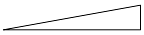
1.6.1 BM = I/V

What are the coordinates of the centroid of the right angle triangle? The origin is at the Left corner.



Ans. $\left(\frac{2y}{3}, \frac{y\theta}{3}\right)$

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$$\text{The volume of this wedge} = \tfrac{1}{2}\theta y^2\,dx$$

$$\text{The moment of the wedge about the centre line} = \tfrac{1}{2}\theta y^2\,dx \times \tfrac{2}{3}y = \tfrac{1}{3}\theta y^3\,dx$$

$$\text{The total moment of both wedges about the centre line} = \tfrac{2}{3}\theta y^3\,dx$$

$$\text{The sum of the moments of all such wedges} = \int_O^L \tfrac{2}{3}\theta y^3\,dx = \theta\int_O^L \tfrac{2}{3} y^3\,dx$$

$u =$ distance from centre-line to $\mathrm{d}u\mathrm{d}x$

$$I = \int_0^L 2\int_0^y u^2 \mathrm{d}u\,\mathrm{d}x = \int_0^L 2y^3/3\,\mathrm{d}x$$

But

$$\int_O^L \tfrac{2}{3} y^3\,dx = \left.\begin{array}{l}\text{The second moment}\\ \text{of the water-plane}\\ \text{area about the ship's centre line}\end{array}\right\} = I$$

$$\therefore\ \text{The sum of the moments of the wedges} = I \times \theta$$

$$\text{But the sum of the moments} = v \times gg_1$$

where v is the volume of the immersed or emerged wedge.

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Why does BM depend on the 2nd moment of inertia?

- Elemental area $\mathrm{d}u\ \mathrm{d}x$. Distance to center-line $= u$
- 2nd moment of elemental area$= u^2\mathrm{d}u\ \mathrm{d}x$

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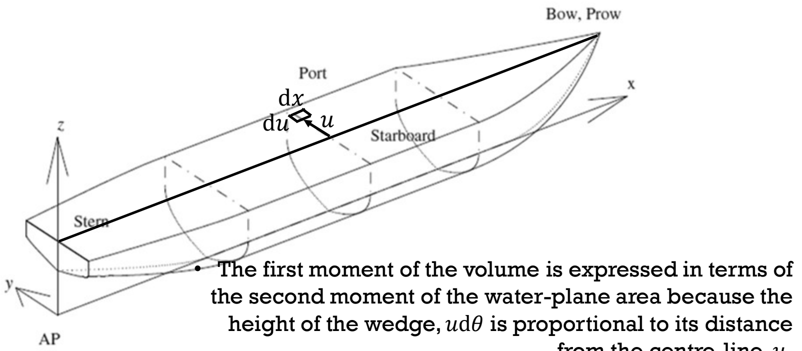

- The first moment of the volume is expressed in terms of the second moment of the water-plane area because the height of the wedge, $u\mathrm{d}\theta$ is proportional to its distance from the centre-line, u.

Figure 1.6 System of coordinates recommended by DIN 81209-1

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1.6.1 BM = I/V

- All the ideas in this proof are very important. Understand them well. Memorize the proof.
- $V\overline{BB_1} = v\overline{gg_1} = I\theta =$ change in the moment due to shifting of the underwater volume

where v is the volume of the immersed or emerged wedge.

$$\therefore\ I \times \theta = v \times gg_1 \quad (1)$$

or

$$I = \frac{v \times gg_1}{\theta} \quad (2)$$

Now:

$$BB_1 = \frac{v \times gg_1}{V} \quad (3)$$

and

$$BB_1 = BM \times \theta \quad (4)$$

Use (3) & (4)

$$\therefore\ BM \times \theta = \frac{v \times gg_1}{V} \quad (5)$$

or

Use (5). Cross multiply

$$BM \times V = \frac{v \times gg_1}{\theta} \quad (6)$$

Substituting in (I) above

Use (2) & (6)

$$BM \times V = I \quad (7)$$

$$\therefore\ BM = \frac{I}{V} \quad (8)$$

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1.6.2 Metacentric radius

- The initial waterplane is $\boldsymbol{W_0 L_0}$
- The ship inclines about some arbitrary axis and the new waterplane is $\boldsymbol{W_\phi L_\phi}$. Equi-volume displacement occurs.
- For eg., it can heel AND trim, as shown in the figures
- BM is the metacentric radius
- For small angles, B oscillates with M as centre

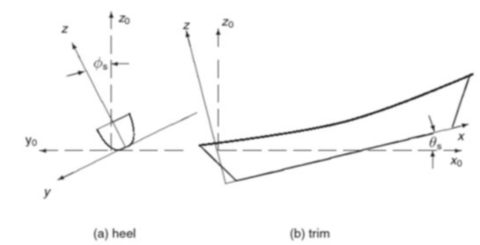

Figure 1.7 Heel and trim

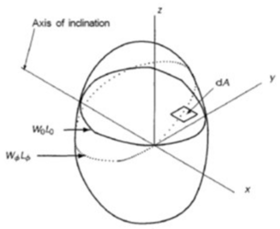

Figure 2.11 Euler's theorem on the axis of inclination

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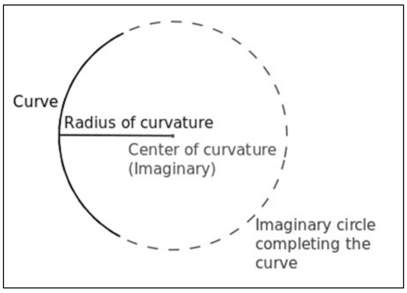
1.6.2 Metacentric radius



$$R = \left| \frac{\left(1 + y'^2\right)^{\frac{3}{2}}}{y''} \right|, \qquad \text{where} \quad y' = \frac{dy}{dx}, \quad y'' = \frac{d^2y}{dx^2},$$

- Note that if $y' \ll 1, R \cong \left|\frac{1}{y''}\right|$
- The equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Find the radius of curvature of the ellipse at (a,0) and (0,b).

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1.6.2 Metacentric evolute

- The CoB and the M are shown for various angles of heel. Each BM line is perpendicular to the associated waterline.
- Learn to draw this figure.
- The metacentric evolute is the curve that passes through all the metacentres for various angles of heel

66 Chapter 2

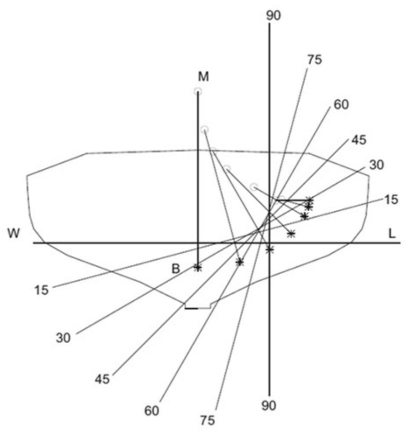

Figure 2.23 B and M curves of vessel *Lido 9*

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Metacenter

- By convention, the transverse metacenter is known simply as the metacenter; and the longitudinal metacenter is known as the longitudinal metacenter
- The metacenter depends only on the axis of inclination and the geometry of the waterplane area and the underwater volume
- For left-right symmetric ships, the transverse metacenter is the metacenter when the axis of inclination is the X axis that lies at the center of the ship in the forward-aft direction

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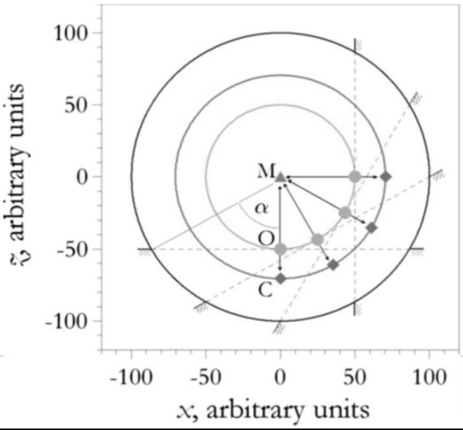
J. Mégel and J. Kliava. Metacenter and ship stability. 2010. Am. J. Physics.

- Fig. 4. Buoyancy, flotation, and metacentric curves for a floating circle (cylinder) of radius R=100 in arbitrary units of length. The immersed part is delimited by a chord making the central angle 2α =120°. The heel angles are ϕ =0, 30, 60, and 90°. The diamonds, circles, and triangle indicate the locations of C, O, and M, respectively. The metacentric curve is reduced to a point.
- M = Metacenter
- C = Center of buoyancy
- O = Center of floatation
- 4 waterlines are shown in Fig. 4. For each waterline, find out C, O, and M. Check if your ans are correct by using Fig. 4.



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Re-examination of centre of buoyancy curve and its evolute for rectangular cross section, Part 1: Swallowtail discontinuity bounds

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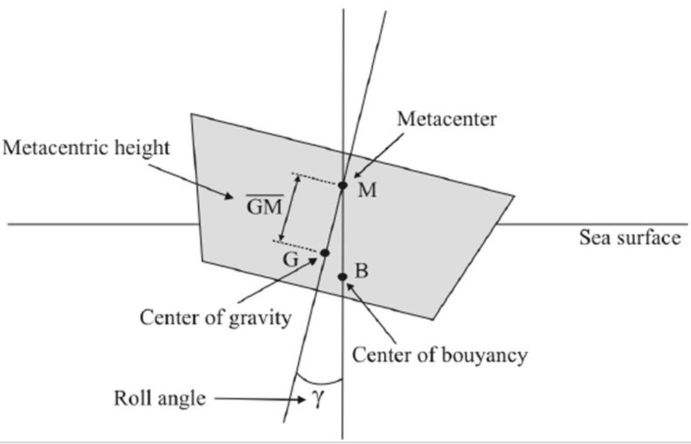
1.6.3 Metacentric Height

- GM = metacentric height



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