Module 2

ANALYSIS OF STRUCTURES

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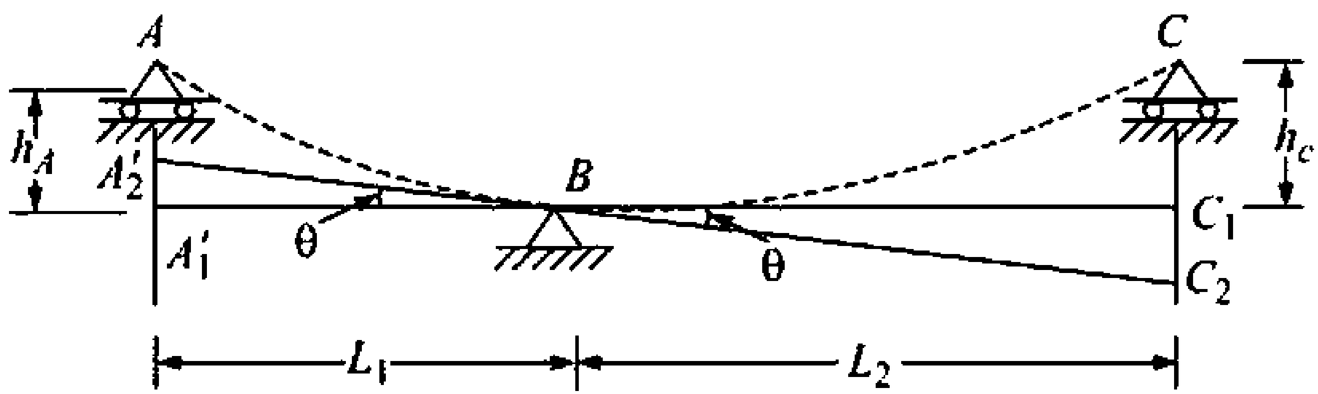
ANALYSIS OF CONTINOUS BEAMS

THREE MOMENT EQUATION

- The three moment equations express the relationship between the moments at three successive supports and the loading on the two spans between those three supports.

DERIVATION OF THREE MOMENT EQUATION

Let $A_1'BC_1$ be a horizontal line through B and $A_2'BC_2$ be the tangent to the elastic curve at B as shown in **Figure 1(a)**.



Let:

- A_1 = Area of free moment diagram in span AB
- A_2 = Area of free moment diagram in span BC
- a_1 = Distance of C.G. of A_1 from support A
- a_2 = Distance of C.G. of A_2 from support C

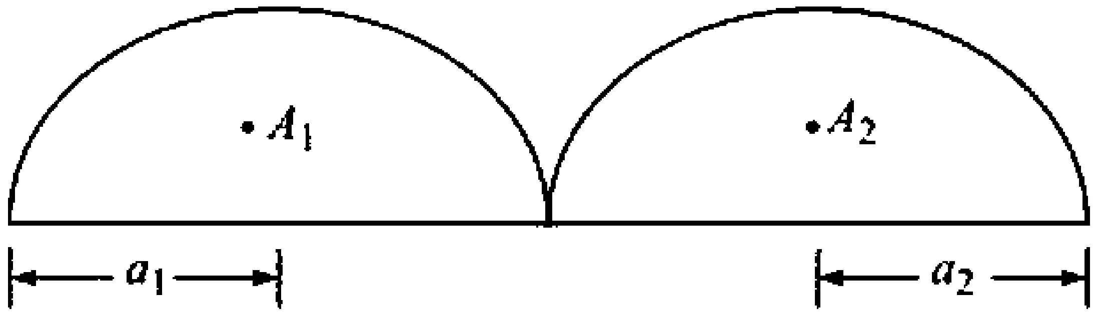


Figure 1(b) : Free moment diagrams on spans AB and BC.

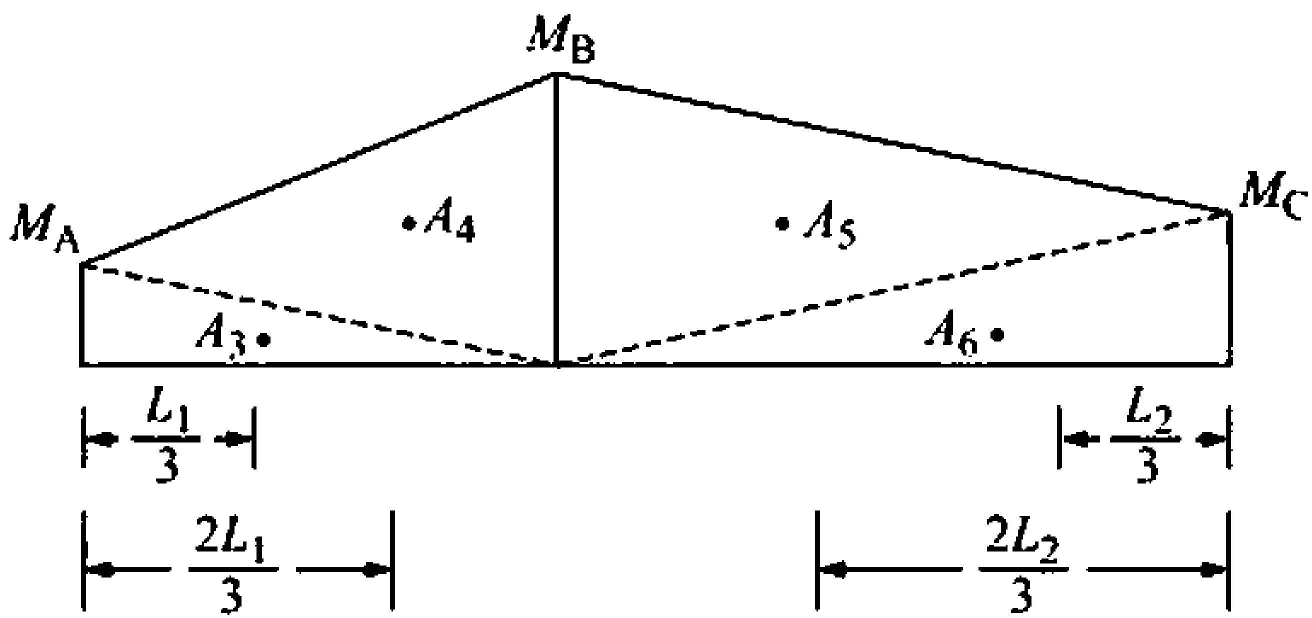
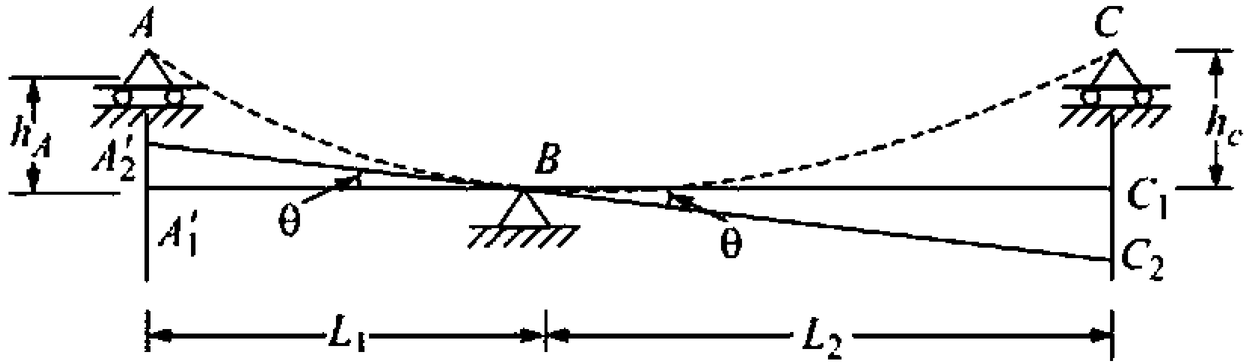


Figure 1(c): End moment diagram.



...(a)

Now from Figure 1 (a):

$$\frac{A_1'A_2'}{L_1} = \tan\theta = \frac{C_1C_2}{L_2}$$

But, $A_1'A_2' = h_A - AA_2'$

$= h_A -$ deflection of A from the tangent of B

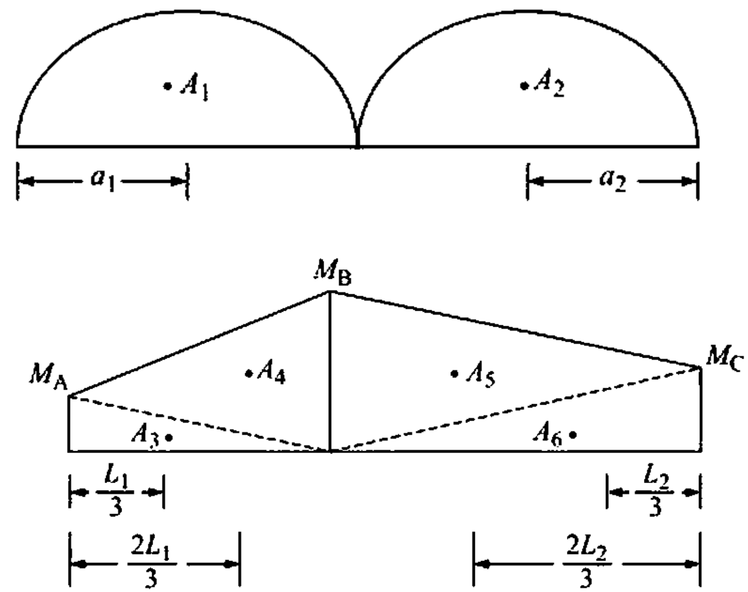
$= h_A -$ moment of $\left(\frac{M}{EI}\right)$ diagram between B and A about A

$$= h_A - \frac{1}{EI_1}\left[A_1a_1 + A_3 \times \frac{L_1}{3} + A_4 \times \frac{2L_1}{3}\right]$$

$$= h_A - \frac{1}{EI_1}\left[A_1a_1 + \frac{1}{2}M_A L_1 \times \frac{L_1}{3} + \frac{1}{2}M_B L_1 \times \frac{2L_1}{3}\right]$$

$$= h_A - \frac{1}{EI_1}\left[A_1a_1 + \frac{1}{6}M_A L_1^2 + \frac{M_B L_1^2}{3}\right]$$

$$= h_A - \frac{1}{6EI_1}\left[6A_1a_1 + M_A L_1^2 + 2M_B L_1^2\right]$$



...(b)

Similarly,

$$
\begin{aligned}
C_1C_2 &= CC_2 - h_c \\
&= \text{Deflection of } C \text{ from the tangent of } B - h_c \\
&= \text{Moment of } \left(\frac{M}{EI}\right) \text{ diagram between } B \text{ and } C \text{ about } C - h_c \\
&= \frac{1}{EI_2}\left[A_2a_2 + A_5 \times \frac{2L_2}{3} + A_6 \times \frac{L_2}{3}\right] - h_c \\
&= \frac{1}{EI_2}\left[A_2a_2 + \frac{1}{2}L_2M_B\left(\frac{2L_2}{3}\right) + \frac{1}{2}M_CL_2\left(\frac{L_2}{3}\right)\right] - h_c \\
&= \frac{1}{6EI_2}\left[6A_2a_2 + 2M_BL_2^2 + M_CL_2^2\right] - h_c \qquad \text{...}(c)
\end{aligned}
$$

Substituting Equations (b) and (c) in Equation (a), we get

$$
\frac{h_A}{L_1} - \left(\frac{1}{6EI_1L_1}\right)[6A_1a_1 + M_AL_1^2 + 2M_BL_1^2]
$$

$$
= \left(\frac{1}{6EI_2L_2}\right)[6A_2a_2 + 2M_BL_2^2 + M_CL_2^2] - \left(\frac{h_c}{L_2}\right)
$$

Multiplying it by $6E$ throughout and rearranging, we get

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right)$$

$$= -\frac{6A_1a_1}{I_1L_1} - \frac{6A_2a_2}{I_2L_2} + \frac{6Eh_A}{L_1} + \frac{6Eh_c}{L_2}$$

This equation is known as Three Moment Equation or Clapeyron's Theorem of three moments

…REFER CLASS NOTES….