STABILITY

Basic assumptions:

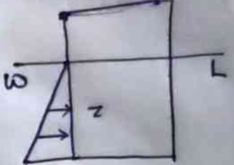
1) Water is incompressible.
2) Ideal fluid. (No viscosity)
3) No surface tension. (Negligible compared to order of size).
4) Water surface is plane. (Not true, but allows us to derive basic eqns).
5) Floating bodies are perfectly rigid. (Again not true, but allows us to work with concentrated loads).

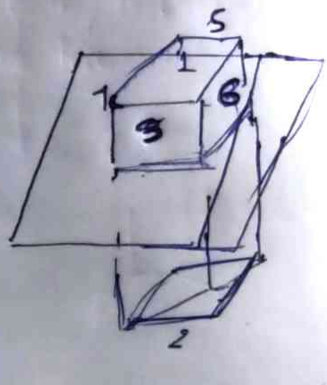
★ Density of sea water depends on:
1) Temperature 2) Salinity

Archimedes principle - 'Any object, wholly/partially immersed in a fluid, is buoyed up by a force equal to weight of the fluid displaced by the object.

$P_{abs} = P_{atm} + P_{gauge}$

$P_{gauge} = \gamma z.$





$F_4 = L\int_0^T \gamma z \cdot dz + P_0 LT$ (atm pressure)

$F_6 = -L\int_0^T \gamma z \cdot dz - P_0 LT$

$= \frac{1}{2}\gamma LT + \underline{\underline{P_0 LT}}$

$|F_6| = -|F_4|$, $|F_5| = -|F_3|$

$F_1 = P_0 LB$

$F_2 = -P_0 LB - \gamma TLB.$

Resultant $= F_1 + F_2 = -\gamma T * LB$

$= \underline{\underline{-\gamma V_{submerged}}}$

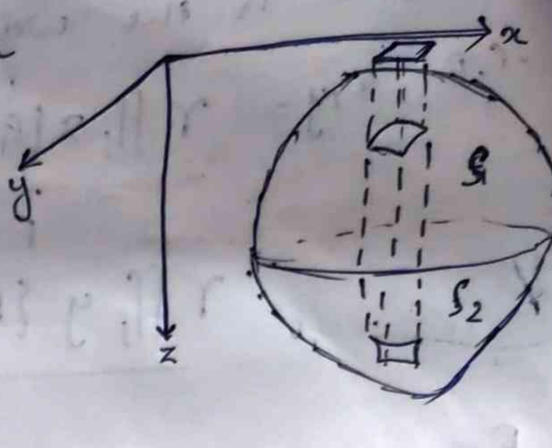
$= \underline{\underline{-W \text{ submerged}}}$

Simply, $F_4 = -$ Area of pressure triangle * Length of side $-$ ($\underline{\underline{P_{atm} \times \text{Area of side}}}$)

General case:

Mathematical expression for a sphere with centre (a,b,c) & radius r:

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2.$$



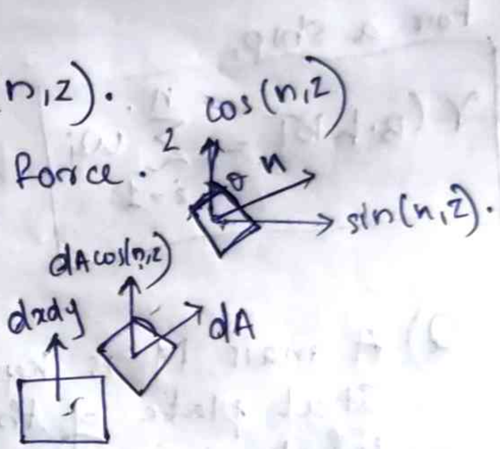
$z = f(x,y)$

$S_1 \longrightarrow z = f_1(x,y)$

$S_2 \longrightarrow z = f_2(x,y)$.

$dF = (\gamma \cdot z)\cdot dA = \gamma f_1(x,y)\cdot dA \cos(n,z)$.

$\cos(n,z) \longrightarrow$ vertical component of force.



$F = \int \gamma f_1(x,y)\cos(n,z)\, dA$.

$\cos(n,z)\cdot dA = dxdy$

$\therefore F_{S_1} = \int \gamma f_1(x,y)\cdot dxdy$.

$F_{S_2} = \gamma \int_{S_2} f_2(x,y)\, dxdy$.

$F = \gamma\left[f_1(x,y) - f_2(x,y)\right] dxdy$.

$=$ difference in z coordinate at a particular (x,y).

$\therefore F_V = \gamma \iint_S \{f_1(x,y) - f_2(x,y)\}\, dxdy$

yields the volume of the submerged body.

$F_H = -\gamma z\, dxdy = 0$.

Resultant of Horizontal components $= 0$.

* If it wasn't zero, we would have obtained a force propulsion force.

Total force $F = F_V + F_H = \gamma \iint_S \{ f_1(x,y) - f_2(x,y) \} \, dx\, dy$

which is the weight of liquid displaced.

C.B $x, y = \dfrac{\gamma \iint_S x \{ f_1(x,y) - f_2(x,y) \} \, dx\, dy}{F}, \quad \dfrac{\gamma \iint_S y \{ f_1(x,y) - f_2(x,y) \} \, dx\, dy}{F}$

For a ship,

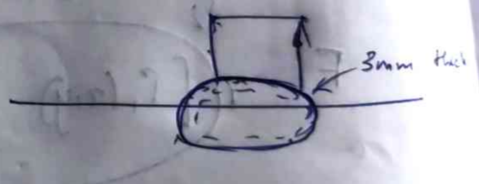
$$\boxed{\gamma C_B \cdot L B T = \sum_{i=1}^{n} W_i}$$

$W_i \to$ light ship.

Q) A mass M is kept over a buoy which is having a Steel plate of thickness 3mm floating in the water. What should be the diameter, for the whole thing not to sink.

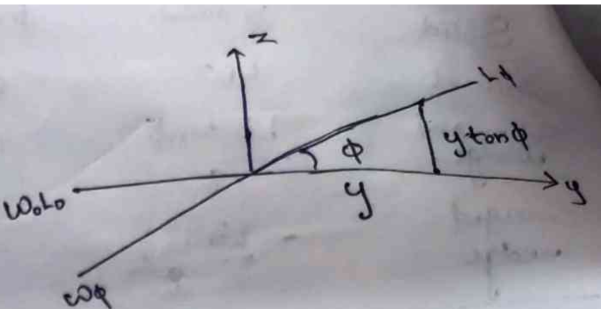
Acc. to archimedes principle,

Total weight = Weight of the fluid displaced.



$$\rho_s \left[\frac{1}{2} \frac{4\pi}{3} \left(r_0^3 - (r_0 - 3\times10^{-3})^3 \right) \right] + M = \rho_w \cdot \frac{1}{2} \frac{4\pi}{3} r_0^3 .$$

Angle of Inclination



$$V_1 = \int z\,dx\,dy$$
$$= \int y\tan\phi\,dx\,dy$$
$$V_2 = -\int y\tan\phi\,dx\,dy \text{ (submerged)}.$$
$$V_1 = V_2 \implies 2\tan\phi \int y\,dx\,dy = 0$$
$$\implies \int y\,dx\,dy = 0$$

$$y_{centroid} = \frac{\int y\,dx\,dy}{\int dx\,dy} \implies y_c = 0. \implies \text{The body heels through the centre of flotation.}$$

Metacentric radius : $BM = \frac{I_T}{\nabla}$.

$I_T \rightarrow$ transverse moment of inertia of area of waterplane.

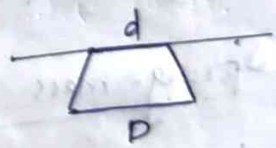
★ For a submarine, there is no waterline $(I_T = 0)$

$\therefore \boxed{BM = 0}$

Stevin's law

Pressure at any point within a fluid is proportional to the depth of that point. $\boxed{P = P_0 + \rho g h}$

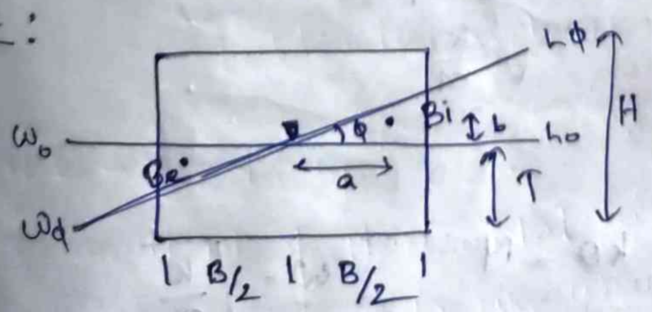
★ Centroid of a Trapezium given by $\boxed{\frac{D+2d}{D+d}}$



$$\overline{KB} = \frac{T}{3} \cdot \frac{D+2d}{D+d}$$

$T \rightarrow$ draft

Note :



B_i = Centre of Buoyancy of water immersed.

$$a = \frac{2}{3}\cdot\frac{B}{2} = \frac{B}{3}$$
$$b = \frac{B}{3}\tan\phi$$

$V = \frac{1}{2}\frac{B}{2}\tan\phi \cdot \frac{B}{2}$

Solid	Volume	tcb	Moment
Initial	LBT	0	0
Submerged wedge	$\frac{LB^2\tan\phi}{8}$	$B/3$	$\frac{LB^3\tan\phi}{3\cdot 8}$
Emerged wedge	$-\frac{LB^3\tan\phi}{8}$	$-B/3$	$\frac{LB^3\tan\phi}{3\cdot 8}$
Total	LBT		$\frac{LB^3\tan\phi}{12}$

$$\underset{(y)}{tcb} = \frac{\text{Total moment}}{\text{Total volume}} = \frac{LB^3\tan\phi}{LBT\times 12} = \frac{B^2\tan\phi}{12T}$$

|||ly $$\underset{(z)}{vcb} = \frac{B^2\tan^2\phi}{24T}$$

$$\boxed{z = \frac{6T}{B^2}\cdot y^2}$$ eqn of a parabola.

Taking slope, $\frac{dz}{dy} = \frac{dz/d\phi}{dy/d\phi}$
$= \tan\phi$

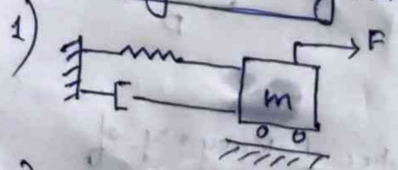
$$\boxed{y_B = \frac{I}{\nabla}\tan\phi}$$

Conditions for metacentre to be fixed:

1. $\tan\theta = \theta$ $(\theta = 0, 0.15, 0.2 \dots)$
2. Volume submerged = Volume emerged.
3. Waterplane must not change drastically.

* $KM_L \sim KM_T \times$ length (app.x)

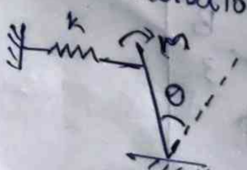
Spring-mass system

1) 

$ma + \omega + kx = F$

$\Rightarrow \omega = \sqrt{\frac{k}{m}}$

2) For a rotational system,



$$I\frac{d^2\theta}{dt^2} + k\theta = M$$

$$\omega = \sqrt{\frac{k}{I}}$$

$$\text{Roll period } (T) = \frac{2\pi k}{\sqrt{g \cdot GM}}$$

$k \to$ transverse radius of gyration.
$g \to$ acc. due to gravity.
$\to$ [how ship's mass is distributed relative to the axis of roll]
often approximated as fraction of ship's beam. (B).

$k \propto B$ $[k = (0.35 - 0.45) * B]$

$\therefore \boxed{T \propto \frac{B}{\sqrt{GM}}}$

$$\boxed{\begin{gathered} g\,\Delta GM \sin\theta \longleftrightarrow k\theta \\ k \to g\Delta GM \end{gathered}}$$

Free surface effect

Effect due to a free liquid surface in a tank inside the ship.

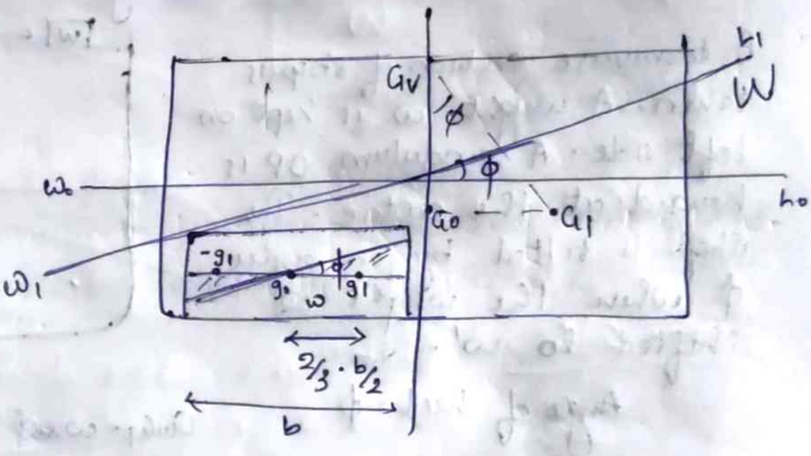
$$G_0G_1 = \frac{w \times (g_0g_1)}{W}$$

$w = V \times \rho$

$V = \frac{1}{2} \times \frac{b}{2} \times \frac{b}{2} \tan\phi \cdot l$

$\therefore w = \frac{lb^2}{8}\phi \cdot \rho_i$

$2g_0g_1 = \frac{b}{3} \times 2 = \frac{2b}{3}$



Note – Shifting of weight is from $-g_1$ to g_1.
hence $d = 2g_0g_1$

Moment of Inertia of the tank.

$$\therefore G_0G_1 = \frac{2b}{3} \times \frac{lb^2}{8}\phi\,\rho_i \cdot \frac{1}{W} = \frac{lb^3}{12}\phi\,\rho_i \cdot \frac{1}{W} = \frac{I\phi\rho_i}{W}$$

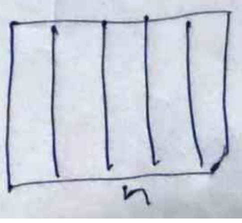
$\frac{G_0G_1}{G_0G_V} = \tan\phi = \phi$; $\boxed{G_0G_V = \frac{I\rho_i}{W}}$

$G_V \to$ Vertical / virtual centre of gravity.

★ Due to the tank, there is an extra loss of stability. Hence, the tank should be divided to n compartments to minimize the loss of stability.

$$I_n = \frac{l(b/n)^3}{12} \times n = \frac{lb^3}{12n^2}$$

Longitudinal Subdivision.



For transverse subdivision, $I_n = \frac{l/n \, b^3}{12} \times n = \frac{I}{n^2}$

- Hence there is no use in transverse subdivision.

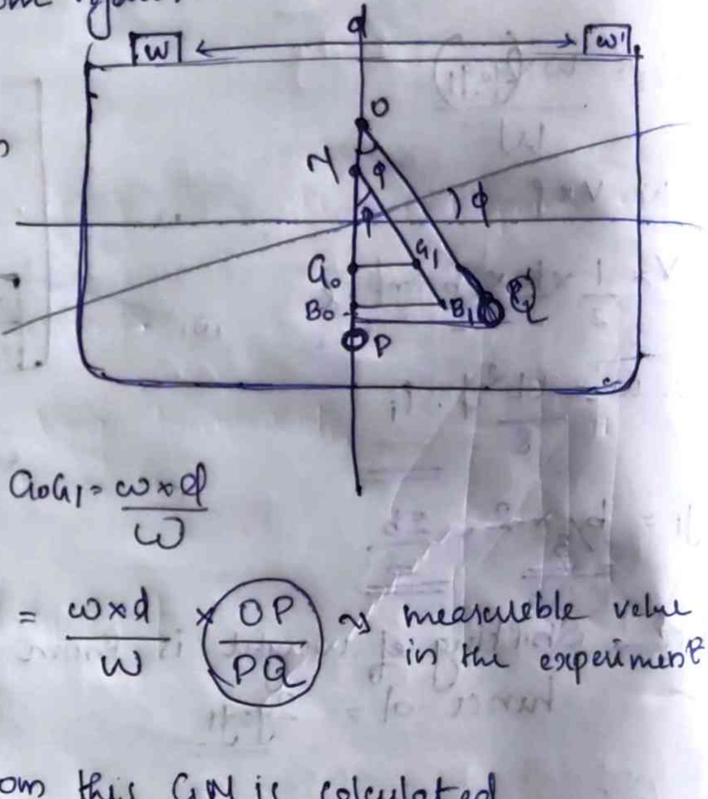
Fluid GM / Fluid KG :→ GM / KG in a ship due to the presence of free surface effect. (when heeling occurs).

Inclining experiment

Purpose: To calculate KG. (What we actually calculate is GM).

$KG = KM - GM$.

↳ Comes from hydrostatic data.



A transverse section of ship is taken. A weight w is kept on left side. A pendulum OP is hanged at the centre. The ship is tilted by an angle ϕ when the weight w shifted to w'.

Angle of heel $= \phi$

$G_0G_1 = \frac{w \times d}{W}$

$\frac{G_0M}{G_0G_1} = \frac{OP}{PQ} \Rightarrow G_0M = \frac{w \times d}{W} \times \frac{OP}{PQ}$ → measurable value in the experiment.

[From property of similar triangles].

From this GM is calculated.

Note – * The water in which the experiment is done, should be calm (no waves, winds, currents etc).

* Slackened moorings.
* All tanks should be emptied or filled.
* All movable weights should be secured / removed.

Loads adversely affecting stability:

1) Shifting of weights.
2) Hanging loads.
3) Free surface effect.
4) High speed manoeuvring
5) Grounding / Docking.

Dry Docking

Types:

1) Floating dry dock

Pontoon with sponsons on both sides. Pumping is done with help of ballast pumps. Consists of:

a) keel blocks b) side blocks c) side sponsons d) Rails for cranes.

2) Excavated dock

Same as floating dock, difference bieng it has a closing door at the opening. Sloped slightly towards the opening. Ballast pumps are located near door itself.

3) Patent slip.

- Used for small ships of length 140m or less.
- Jack up system which brings out ship above water surface.

4) Ship lift.

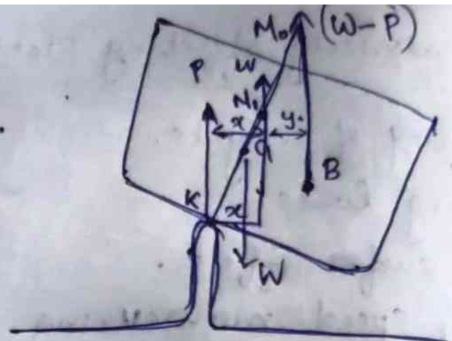
Similiar to patent slip, for ships having length upto 125m. Cradles move below, then the ship is lifted by winches.

* Critical draft - Minimum draft at which the ship's GM becomes negative. (loss of stability).

∴ Dry docks, should be designed such that the clamps get attached to the ship before its draft goes below critical draft.

- Critical instant — The moment at which the ship's bottom touches the keel block.

Normal force due to grounding = P.
Net force acting through B = (W − P).
Total force upward = P + W − P = W



- Due to grounding there is a reduction in metacentre from M_0 to M_1.

$(W-P)y = Px$ — ①

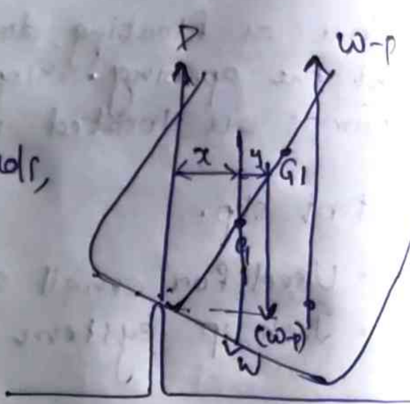
$x = KM_1 \sin\phi \qquad y = M_0M_1 \sin\phi$

① $\longrightarrow (W-P)\,M_0M_1 = P \cdot KM_1$

$$\boxed{M_0M_1 = \frac{P \times KM_0}{W}}$$

P → reaction force
1) Either a buoyant force acting on a different point other than centre of buoyancy.
2) Or the negative displacement.

Case 2 : Net force acting downwards,
There is a shift in G.



$Wy = Px$

$x = KG_1 \sin\phi \qquad y = G_0G_1 \sin\phi$

$$\Longrightarrow \boxed{G_0G_1 = \frac{P \times G_0}{W-P}}$$

Note — When combining trimming with docking, due to trim by aft (in most cases)

t = change of trim = $\dfrac{\text{Moment causing trim}}{MCTC}$

$= \dfrac{P \times l}{MCTC}$

$l \longrightarrow$ distance b/w Aft perpendicular and centre of flotation.