Static equillibrium.

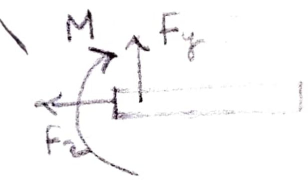
* $\vec{a} = 0$

$\Sigma \vec{F} = 0$, $\Sigma F\hat{i} + \Sigma F\hat{j} = 0$

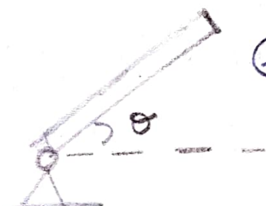
Support reactions

① Fixed support.

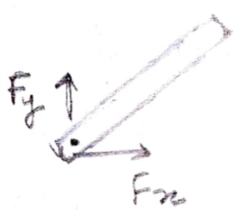
All 3 F_x, F_y & M are support reactions.



② Hinge/Pin



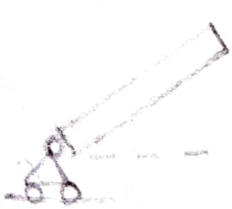
2 F_x & F_y are support reactions



③ Roller

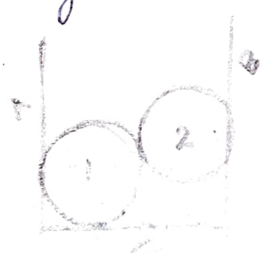
1 support reaction F_y.

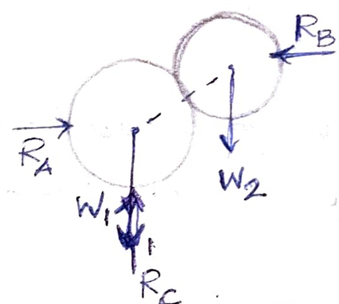
④ Rocker

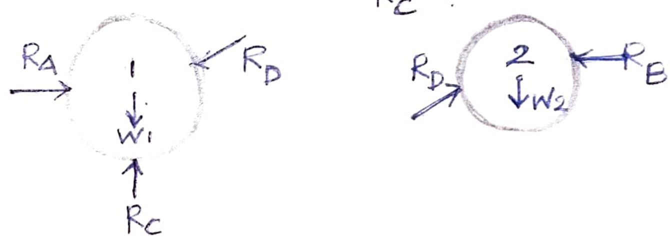


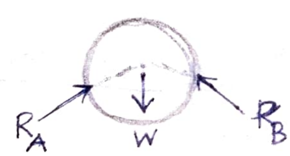
1 support reaction F_y

Q. Consider two spherical balls of Weights W_1, W_2, resting inside a hollow cylinder.



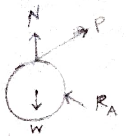




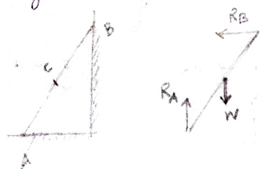


sphere in V shaped groove

Drum rolled along horizontal comes accross a stop obstacle.



A uniform ladder leaning against a smooth wall.



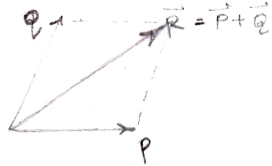
→ Equal forces: Two forces are said to be equal if they have magnitude & direction even if their points of application aren't the same.

→ Equivalent forces: Two forces are said to be equivalent if they produce the same effect on a rigid body. Based on a specific effect

Mostly in mechanics we are concerned with forces having an equivalent effect on a rigid body rather than equal forces.
The resultant of 2 forces acting on a body, is thus sense is an equivalent force

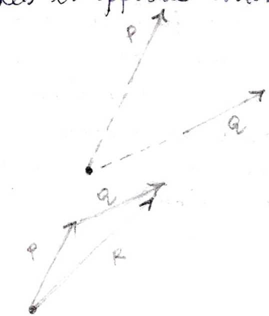
→ Parallelogram law

If two forces acting at a point are represented in magnitude & direction by adjacent sides of a parallelogram then the diagonal passing through their point of intersection represent the resultant in both mag & direction.



→ Law of Triangular forces

If two forces acting at a point are represented by 2 sides of a Δ, taken in order, then their sum / resultant is represented by 3^{rd} side taken in opposite order

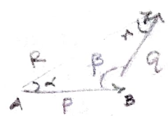


The sum or resultant of 2 forces P & Q acting at O, can be constructed by a Δ, such that, forces P & Q are represented by the 2 sides of this Δ, taken in order, the closing side taken in opposite order then represent the sum or resultant $\vec{P} + \vec{Q}$

$|\vec{R}| = \sqrt{|P|^2 + |Q|^2}$, is mag of resultant $\vec{R}$ if ⊥

Magnitude of $\vec{R}$ can be determined by measuring length of $\vec{R}$ (graphically) or (Trigonometrically) if included angle β between forces $\vec{P}$ & $\vec{Q}$ is known, by relation

$$|\vec{R}|^2 = P^2 + Q^2 + 2PQ\cos\theta$$

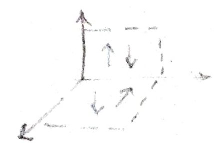


here, $R^2 = P^2 + Q^2 - 2PQ\cos\beta$

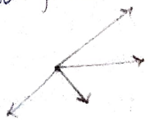
If 3 forces acting at a single point are in eqm, then each of forces is directly proportional to sin (angle btw remaining 2 forces)

$$\frac{P}{\sin\gamma} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\beta}$$

When a no: of forces lay in the same plane, they are called coplanar forces



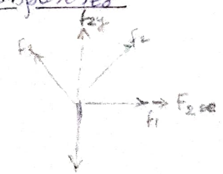
Concurrent forces: All forces whose lines of action pass through a common point.



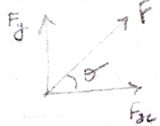
Law of polygon of forces

If a number of coplanar forces acting at a point such that they can be represented in magnitude & direction by sides of a polygon taken in order, then resultant is represented in mag & dir by closing side of polygon taken in opposite order.

→ **Force resultants & force components**



Resolving force components along specific axes

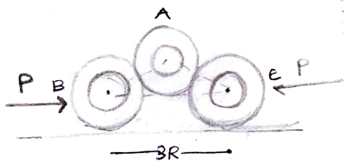


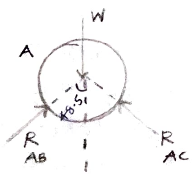
$F = F_x + F_y$

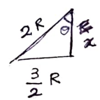
$= \sqrt{F_x^2 + F_y^2}$

$\theta = \tan^{-1} \frac{F_y}{F_x}$

Q Draw the freebody diagram of the water pipes each of weight W & radius r supported as shown in fig. (Assume all contact surfaces to be smooth)

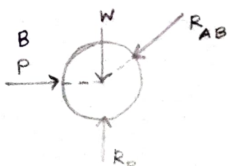


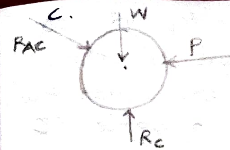




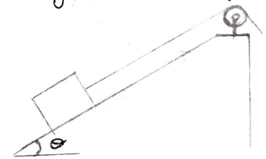
$\sin\theta = \frac{3}{4}$

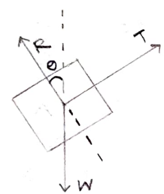
$\theta = 48.5$



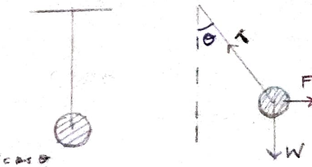


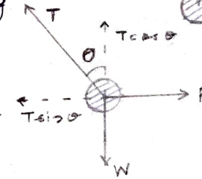
Q A block of weight W is supported on a smooth inclined plane by applying force P through a cord which passes over a smooth frictionless pulley.





Q. A spherical ball of weight 75 N is attached to a string & is suspended from the ceiling. Find tension in string if a horizontal force is applied to the ball as shown. Determine ∠ string makes with the vertical & also the tension in the string (F = 150 N.)





$T\cos\theta = W = 75$

$T\sin\theta = F = 150$

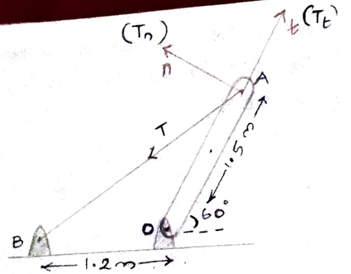
$\tan\theta = 2$

$\theta = \tan^{-1}(2) = 63.435°$

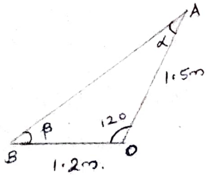
$150 = T \times \sin(63.43)$

$T = 168.53\,N$

Q The ~~table~~ ~~AB~~ prevents bar OA from rotating clockwise about pivot point O. If the cable tension is 750 N determine the 'n & t' components of this force acting on point A of the bar (normal & tangential)



Q The guy cables AB & CA are attached to the top of a transmission tower. The tension in cable AB is 8 kN. ~~Require~~ Calculate the required tension T in cable AB such that net effect of the 2 cable tensions is a downward force at A. Determine mag R of downward force.



(Resultant's idea)

$AB^2 = (1.2)^2 + (1.5)^2 - 2(1.2)(1.5)\cos 120°$

$AB = 2.34\ m$

$T_n = T\sin\alpha$

$T_t = T\cos\alpha$

$\frac{\sin\alpha}{1.2} = \frac{\sin 120}{2.34} = \frac{\sqrt{3}}{2\times 2.34}$

$\sin\alpha = \frac{\sqrt{3}\times 1.2}{2\times 2.34} = \cancel{0.769}$

$\alpha = 26.3°$

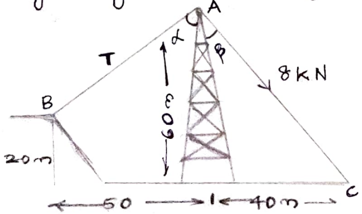
$\cancel{\alpha = 50.26°}$

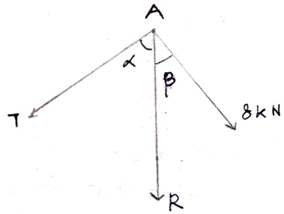
$T_n = T\sin\alpha = 750 \times \cancel{26} \sin(26.3)$

$= 333\ N$

$T_t = T\cos\alpha = 750 \times 0.896$

$= 672.36$ //





$T\sin\alpha = 8kN.\sin\beta$

$\tan\alpha = \frac{50}{40} \Rightarrow 51.3° \leftarrow \alpha$

$\tan\beta = \frac{40}{60} \Rightarrow 33.6° \leftarrow \beta$

$T\sin\alpha = 8kN.\sin\beta$

$T = \frac{8000 \times 0.553}{0.78}$

$= 5.68\ kN$

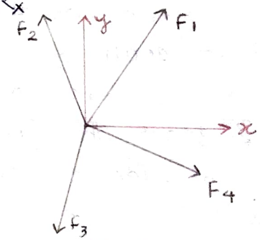
$R = T\cos\alpha + 8kN\cos\beta$

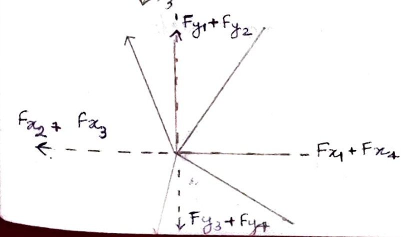
$(5.68 \times 0.625 + 8kN \times 0.832)\ kN$

$= 10.2\ kN$

→ Resultant of several concurrent coplanar sources

Consider No. of coplanar force F_1, F_2, F_3, F_4 by using analytical method, horizontal components can be added to a single force R_x

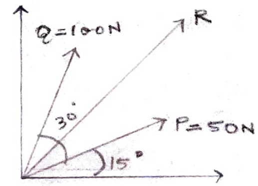




$R_x = \Sigma F_x = F_{x1}+F_{x2}+F_{x3}+F_{x4}$

$R_y = \Sigma F_y$

Q 2 forces act at a point as shown. Determine magnitude & direction of resultant choosing X, Y axes as shown & resolving forces P & Q along the axes.



$R_x = P_x + Q_x$

$= P\cos\alpha + Q\cos\beta$

$= 50\cos 15 + 100\cos 45$

$= 50\times 0.965 + 100\times 0.707$

$= 118.95\ N$

$R_y = P_y + Q_y$

$= P\sin\alpha + Q\sin\beta$

$= 50\sin 15 + 100\sin 45$

$= 12.94 + 70.7$

$= 83.64$

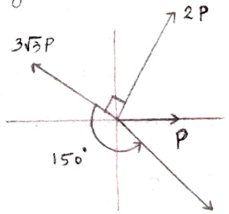
$R = \sqrt{F_x^2+F_y^2} = 145.46\ N$

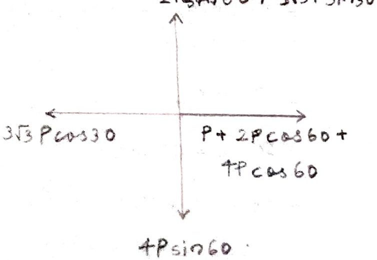
Angle made by R

$= \tan^{-1}\left(\frac{R_y}{R_x}\right)$

$= \tan^{-1}\left(\frac{83.6}{119}\right) = 35^\circ$

Q Find the magnitude & direction of resultant R of the 4 concurrent forces acting as shown in the fig.





ΣF_x

$= P\cos 0 + 2P\cos 60 + 3\sqrt{3}P\cos 150 + 4P\cos 300$

$= P + P - \frac{9}{2}P + \cancel{2\sqrt{3}P}\ 2P$

$= \frac{-P}{2}$

$\Sigma F_y = P\sin 0 + 2P\sin 60 + 3\sqrt{3}P\sin 150 + 4P\sin 300$

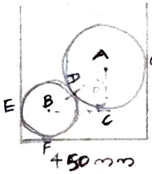
$= \frac{\sqrt{3}}{2}P$

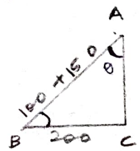
$R = \sqrt{\frac{3P^2}{4} + \frac{P^2}{4}} = P$

$\alpha = \tan^{-1}\frac{\sqrt{3}}{-1} = \tan^{-1}(-\sqrt{3})$

$= 120^\circ$

Q 2 cylinders A & B are at rest in a box as shown. A has a diameter of 300 mm & weighs 1200 N. B has a diameter of 200 mm & weighs 360 N. The box is 450 mm wide at the bottom. Find the reactions at supporting surface.



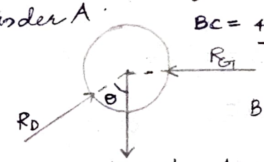


BC = 450 − (100+150)

BC = 200 mm

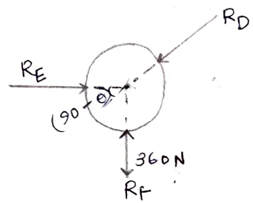
$AC = \sqrt{250^2 - 200^2}$

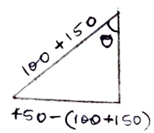
cylinder A.



~~ΣFx = 0~~

cylinder B





$\theta = \sin^{-1}\left(\frac{200}{250}\right)$

$\theta = \sin^{-1}\left(\frac{4}{5}\right)$

$= 53.06^\circ$

For cylinder A

$\Sigma F_x = 0$

$R_G - R_D\cos(90-\theta) = 0$

$\Sigma F_y = 0$

$R_D\sin(90-\theta) - 1200 = 0$

$R_D = \frac{1200}{\cos\theta} = 1993.9\ N$

$R_G = R_D\cos(90-\theta)$

$= R_D\sin\theta$

$= 1993.9 \times \sin 53.06$

$= 1593.6\ N$

For cylinder B

$\Sigma F_x = 0$

$R_E - R_D\cos(90-\theta) = 0$

$R_E = R_D \frac{\cancel{\cos\theta}}{\sin\theta} = \cancel{1200}\ 1600\ N$

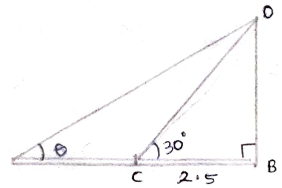
$\Sigma F_y = 0$

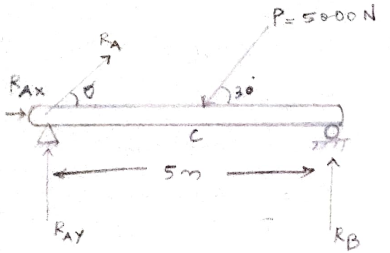
~~$R_D\sin\theta + R_F - 360 = 0$~~

$R_D\sin(90-\theta) + R_F - 360 = 0$

$R_F = 1200 + 360 = 1560$

Q A force of P = 5000 N, is applied at the center C of the beam AB of length 5m. Find the reactions at the hinge & roller supports.





$\frac{1}{2} = \tan 30 = \frac{OB}{CB} = \cancel{\frac{OB \times 2}{5}}$ $\frac{1}{\sqrt{3}} = \frac{OB\,2}{5}$

$\cancel{\frac{1}{2} = \frac{2OB}{5}}$

$\cancel{OB = \frac{5}{4}}$

$OB = \frac{5}{2\sqrt{3}}$

$\tan\theta = \frac{OB}{5} = \frac{1}{2\sqrt{3}}$

$\theta = 16.102$

$\Sigma F_x = 0$

$R_{AX} - P\cos 30 = 0$

$\frac{P\sqrt{3}}{2} = R_{AX} = 5000 \times \frac{\sqrt{3}}{2}$

$= 4330.12\ N$

$\Sigma F_y = 0$

$R_{AY} + R_B - P\sin\theta = 0$

$R_{AY} + R_B = 5000 \times \frac{1}{2}$

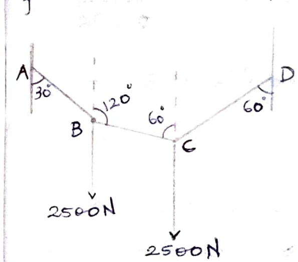
$\tan\theta = \frac{R_{AY}}{R_{AX}} = \frac{1}{2\sqrt{3}}$

$R_{AY} = \frac{R_{AX}}{2\sqrt{3}} = \frac{4330.12}{2\sqrt{3}} = 1250N$

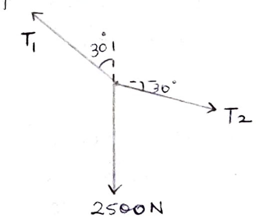
$R_B = 2500 - R_{AY}$

$= 1250\ N$

Q Two equal loads of 2500N are supported by flexible strings ABCD at points B & C. Find the tensions in the portions AB, BC & CD.



At point B



$\Sigma F_x = 0$

$T_1 \sin 30 - T_2 \cos 30 = 0$

$\frac{T_1}{2} = \frac{T_2\sqrt{3}}{2}$

$T_1 = \sqrt{3}\, T_2$

$\Sigma F_y = 0$

$T_1 \cos 30 = T_2 \sin 30 + 2500$

$\frac{T_1\sqrt{3}}{2} = \frac{T_2}{2} + 2500$

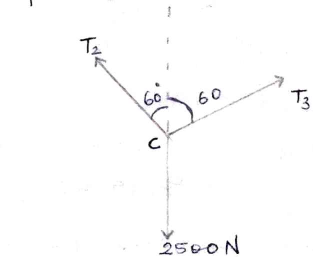
$T_1\sqrt{3} = T_2 + 5000$

$3 T_2 = T_2 + 5000$

$T_2 = \frac{5000}{2} = 2500$

$T_1 = \sqrt{3}\, T_2 = 4330N$

At point C

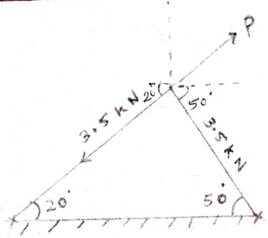


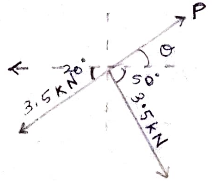
$\Sigma F_x = 0$

$T_2 \sin 30 - T_3 \sin 30 = 0$

$T_2 = T_3 = 2500\ N$

Q. Two ropes are tied together at C. If maximum permissible tension in each rope is 3.5 kN. What is max force P that can be applied & in what direction





$\Sigma f_x = 0$

$3.5\,kN \cos 20 - 3.5\,kN \cos 50 - P\cos\theta = 0$

$P\cos\theta = 3.5\,kN(\cos 20 - \cos 50)$

$= 3.5\,kN(0.939 - 0.642)$

$= 3.5\,kN(0.296)$

$\Sigma f_y = 0$

$P\sin\theta = 3.5\,kN(\sin 20 + \sin 50)$

$= 3.5\,kN(1.108)$

$P = \sqrt{(P\sin\theta)^2 + (P\cos\theta)^2}$

$= 3.5\,kN\sqrt{(0.296)^2 + (1.108)^2}$

$= 3.5\,kN\sqrt{1.315}$

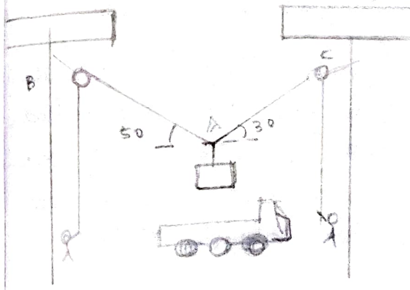
$= 3.5\,kN \times 1.146$

$= 4\,kN$

$\dfrac{P\sin\theta}{P\cos\theta} = \tan\theta = \dfrac{1.108}{0.296}$

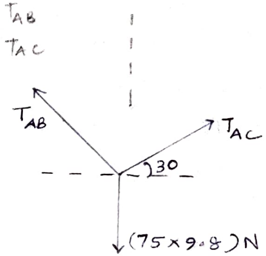
$\Rightarrow \theta = \tan^{-1}(3.743) = 75°$

Q Consider 75 kg crate shown in diagram lying between 2 building & now being lifted onto a truck which will remove it. The crate is supported by a vertical cable which is joined at A to two ropes which passes over pullies attached to the buildings at B & C. Find the tension in each of the ropes AB & AC



647 T_{AB}

480 T_{AC}



$\Sigma f_x = 0$

$T_{AB}\cos 50 = T_{AC}\cos 30$

$T_{AB} = T_{AC} \times 1.347$

$\Sigma f_y = 0$

$T_{AB}\sin 50 + T_{AC}\sin 30 = 75 \times 9.8$

$T_{AC} \times 1.347 \times \sin 50 + T_{AC}\sin 30 = 735$

$T_{AC}(1.032 + 0.5) = 735$

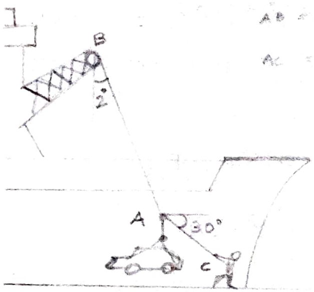
$T_{AC} = 480\,N$

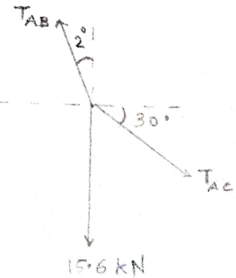
$T_{AB} = T_{AC} \times 1.347 = 647\,N$

Q In ship unloading op. a 15.6 kN automobile is supported by a cable. A rope is tied to the cable at A & pulled in order to center the automobile over its intended position. The angle between cable & the vertical is 2°. While the angle between the rope & horizontal is 30°. What is the tension in the rope

AB = 15.93 kN

AC = 0.64 kN





→ Mome $\Sigma F_x = 0$

$T_{AB} \sin 2 = T_{AC} \cos 30$

$T_{AC} = \frac{T_{AB} \sin 2 \times 2}{\sqrt{3}}$

$T_{AC} = 0.04\, T_{AB}$ — ①

$\Sigma F_y = 0$

$T_{AB} \cos 2 = 15.6\,kN + \frac{T_{AC}}{2}$

$T_{AB} \cos 2 = 15.6\,kN + \frac{T_{AB} \times 0.04}{2}$

$T_{AB} (\cos 2 - 0.02) = 15.6\,kN$

$T_{AB} = \frac{15.6\,kN}{0.98} = 15.93\,kN$

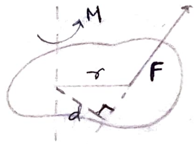
$T_{AC} = 0.04 \times T_{AB}$

$= 0.64\,kN$

→ Moment of a force

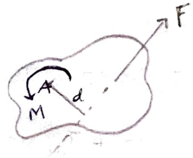
In addition to the tendency of a force to move a body in the direction of application it can also tend to rotate a body about its axis

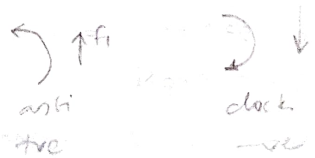
⇒ Moment about a point is equal to the force & the ⊥r distance of the point from line of action ~~from~~ of the force



$M = Fd$

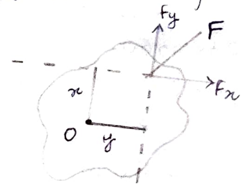
Right hand thumb rule





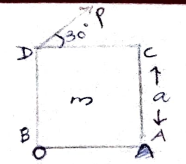
Varignon's theorem

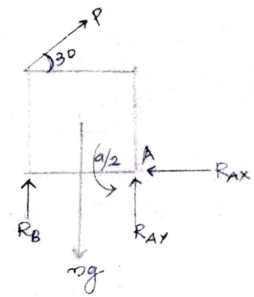
The moment of a force about any point is equal to the sum of moments of the components of the force about the same point



$M = F.d = x F_x - y F_y$

Q A square block of wood of mass M is hinged at A & rests on a roller at D its pulled by means of a string attached at D & inclined at an angle 30° with the horizontal. Determine the force P, which should be applied to the string to just lift the block off the roller



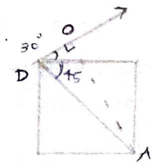


roller ൽ നിന്ന് lip off ചെയ്യുന്ന time ലെ case നോക്കുമ്പോൾ, ആ moment -ൽ $R_B = 0$ ആയി consider ചെയ്യുന്നു.

* Applying concepts of moments

$\Sigma M_A = 0$ $\qquad R_B = 0$

mg കാരണമുണ്ടാകുന്ന moment +ve ആയി എടുത്തു. കാരണം about A, anticlockwise



Force × ⊥r distance

$\sin(30+45) = \frac{OA}{AD}$

$\sin 75 \times AD = OA = 1.366\,a$

$$mg \times \frac{a}{2} - P \times OA = 0$$

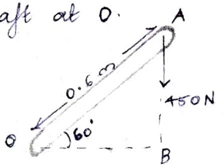
$$mg \times \frac{a}{2} = P \times 1.366a$$

$$P = \frac{mg}{2 \times 1.366}$$

$$= \frac{mg}{2.732}$$

$$= \underline{0.366\, mg}$$

Q 450 N vertical force is applied to the end of lever which is attached to a shaft at O.



Determine

a) Moment of the 450 N force about O

b) Horizontal force applied at A which creates the same moment about A.

c) The smallest force applied at A which creates the same moment about O.

d) How far from the shaft 1100 N vertical force must act to create the same moment about O.

a) $\cancel{\sin}\cos 60 = \frac{AB}{\cancel{OB}}\ \frac{OB}{OA}$

$\cancel{\frac{1}{2} =}$

$OB = OA \times \frac{1}{2} = 0.3$

Moment $= 450 \times 0.3$

$= \underline{135\ Nm}$.

$AB = OA \times \sin 60$

$= 0.6 \times \frac{\sqrt{3}}{2}$.

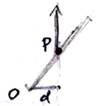
b) Moment $= F \times AB$

$135 = F \times 0.6 \times \frac{\sqrt{3}}{2}$

$F = \underline{260\,N}$

c) $135 = F \times 0.6$

$F = 225\,N$



d)

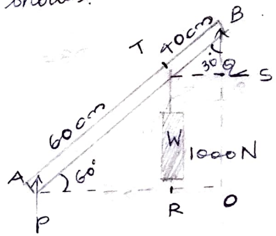
$135 = 1100 \times d$

$d = 0.123$

$\cos 60 = \frac{d}{OP}$

$OP = 2 \times 0.123 = \underline{0.246m}$

Q A (prismatic) weightless wooden beam AB, with an ext weight of 1000N hanging from the beam is supported at the 2 ends as shown.



Find the load shared by each support.

$\Sigma M_A = 0 \Rightarrow \quad Q \times OP = 1000 \times PR$

$Q \times \frac{100}{2} = 1000 \times \frac{60}{2}$.

$Q = 600\,N$ or

$\Sigma F_y = 0,\ P + Q = 1000$

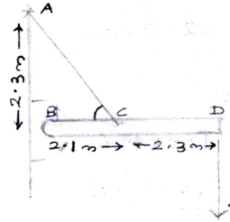
$\Sigma M_B = 0 \Rightarrow$

$\cancel{\times TS} \qquad \cancel{\times OP}$

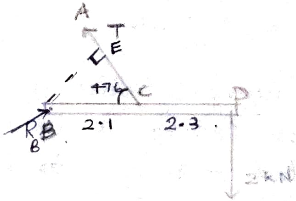
$1000 \times \frac{40}{2} = P \times \frac{100}{2}$

So, $P = 400\,N$

Q A weightless beam BCD is held with the help of a tie rod AC.



Determine the reaction at B & tension in tie rod AC



$\angle ACB = ?$

$\tan \angle ACB = \frac{2.3}{2.1} = 47.602°$.

$\sin 47.6 = \frac{BE}{2.1}$

$BE = 1.55m \Leftarrow \perp r$ distance

M_A,

$T \times 1.55 = 2\,kN \times 4.4$

$T = \underline{5.677\ kN}$

$\Sigma F_y = 0$

$R_{By} + T\sin 47.6 = 2\,kN$

$R_{By} = 2\,kN - T\sin 47.6$

$= -2.192\,kN$

$\Sigma F_x = 0$

$R_{Bx} = T\cos 47.6$

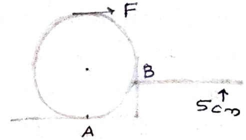
$= \underline{3.828\,kN}$

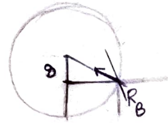
$R = \sqrt{(2.192)^2 + (3.828)^2}$

$= \sqrt{4.8 + 14.65}$

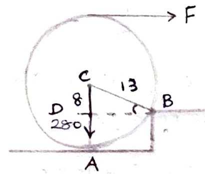
$= \underline{4.4\,kN}$

Q A heavy cylinder of mass 280 kg is to be pulled over a curb of hieght 5 cm, by a horizontal force F by means of a rope around a cylinder. Determine the magnitude of pull for impending motion over the curb while the radius of cylinder is 13 cm





DB =



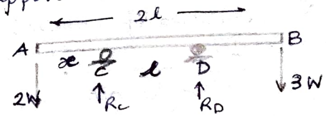
$\Sigma M_B = 0$

$DB = \sqrt{13^2 - 8^2} = 10.246$

$(280 \times 9.81) \times 10.246 = F \times (13+8)$

$F = \underline{1340.17\,N}$

Q A bar AB of length $2l$ of negligible weight rests on 2 roller supports C & D placed at a distance l apart. The bar supports 2 vertical weights as shown. For the reactions at the supports to be equal find the distance x, of the end A of the bar from the support C



$R_C = R_D$ (given)

$\Sigma F_y = 0$

$R_C + R_D = 2W + 3W$

$2R_C = 5W$

$R_C = R_D = \underline{\frac{5}{2}W}$

$\Sigma M_C = 0 \quad 2Wx + R_D l = 3W(2l - x)$

$+2W \times x = -R_D l + 3W(2l - x)$

$+2Wx = -\frac{5}{2}Wl + 3W(2l - x)$

$2x = -\frac{5}{2}l + 6l - 3x$

~~$5x = \frac{17l}{2}$~~

~~$x = 1.7l$~~

$5x = 6l - \frac{5}{2}l$

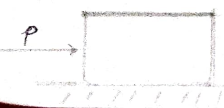
$= \frac{7}{10}l = \underline{0.7l}$

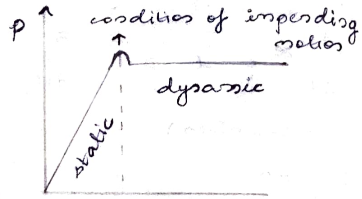
Coulomb friction is that friction which occur b/w bodies having dry contact surfaces. Major cause of dry friction is believed to be the microscopic roughness of surfaces of contact. Interlocking microscopic protuberences oppose the relative motion between the surfaces.

A smooth surface can only support a normal force, on the other hand, rough surface in addition can support a force tangent to the contact surface.

— Law's of Coulombs — friction

When a force P is applied to an idealised block





In this plot, the force P is shown to draw from the highest or limitting value to a lower value which is constant with time

This latter constant value is independent of the velocity of the object. The condition corresponding to the maximum value is termed as impending slippage or impending motion.

While carrying out exp on blocks tending to move without rotation or actually moving without rotation, on flat surfaces, Coulomb in 1781 presented some conclusion which are applicable at condition of impending slipage

For block problems,

The total force of friction that can be developed is independent ~~of~~ magnitude of area of contact.

For low relative velocities between sliding objects, the frictional force is practically independent of velocity.

The total frictional force that can be developed is proportional to normal force transmitted accross the surface of contact.

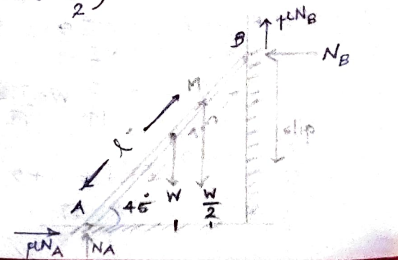
$$f \propto N$$

$$f = \mu N$$

This equation is valid only at conditions of impending slippage or while the body is slipping.

We have dynamic friction μ_k & static friction μ_s.

Q A 7m long ladder rests against a vertical wall with which makes an $\angle 45°$ with the floor. If a man whose weight is half of that of the ladder, climbs it then, at what distance along the ladder will he be when the ladder is about to slip (μ b/w ladder & wall is $\frac{1}{3}$ & that b/w ladder & ground is $\frac{1}{2}$)



$\Sigma F_x = 0$

$\mu N_A - N_B = 0$

$\mu N_A = N_B$

$\frac{N_A}{2} = N_B$ — ①

$\Sigma F_y = 0$

$N_A + \mu N_B = W + \frac{W}{2}$

$N_A + \frac{1}{3} \times \frac{N_A}{2} = \frac{3}{2} W$

$\frac{7}{6} N_A = \frac{3}{2} W$

$N_A = \frac{9W}{7}$

$N_B = \frac{9}{14} W$

$\Sigma M_A = 0$

$W \times 3.5\cos 45 + \frac{W}{2}\cos(45)\, l$

$= \mu N_B \frac{7}{\sqrt{2}} + N_B \frac{7}{\sqrt{2}}$

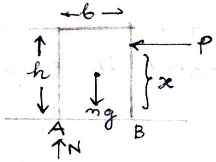
$\frac{7}{2}\frac{3.5}{\sqrt{2}} W + \frac{W}{2\sqrt{2}} l = \frac{1}{3}\frac{7}{\sqrt{2}}\frac{9}{14} W + \frac{9}{14} W \times \frac{7}{\sqrt{2}}$

$\cancel{3.5 + \frac{l}{2} = 3 + 9}$ $\quad 7 + l = 3 + 9$

$\cancel{\frac{l}{2} = \frac{17}{2}}$ $\quad \cancel{l = 17}$

$l = 5m$

Q. A rectangular block of mass M rests on a floor (μ b/w block & floor). What's the highest position for horizontal force P that would permit it to just move the block, without tipping.



$\cancel{mg \times \frac{b}{2} = (P - \mu mg) \cdot x}$

$\cancel{x = \frac{mg\, b}{2(P - \mu mg)}}$

$\Sigma F_x = 0$

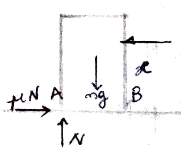
$P = \mu N$

$P \times x = mg \times \frac{b}{2}$

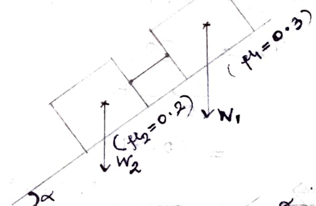
$\mu N \quad x = mg \times \frac{b}{2}$

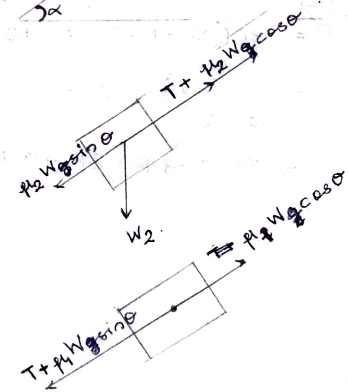
$\mu (mg)\, x = mg \times \frac{b}{2}$

$x = \frac{b}{2\mu}$



Q. Two blocks of weight $W_1 = 50N$ & $W_2 = 50N$, rest on a rough inclined plane & is connected by a string as shown. The coefficients of friction b/w inclined plane & the weights W_1 & W_2 are $\mu_1 = 0.3$, & $\mu_2 = 0.2$ respectively. Find the inclination of the plane for which slipping will impend.





$\cancel{T + W_2 \mu_2 \cos\theta = \mu_2 W_2 \sin\theta}$

$\cancel{T + \mu_1 W_1 \sin\theta = \mu_1 W_1 \cos\theta}$

$\cancel{W \mu_2 \cos\theta - \mu_2 W \sin\theta \cos\theta}$

$\cancel{= \mu_1 W \cos\theta - \mu_1 W \sin\theta}$

$\cancel{\sin\theta(\mu_2 + \mu_1) = \cos\theta(\mu_1 + \mu_2)}$

$f_1 = \mu_1 \cdot N = 0.3 \times 50\cos\alpha$
$= 15\cos\alpha$

$f_2 = \mu_2 \cdot N = 0.2 \times 50 = 10\cos\alpha$

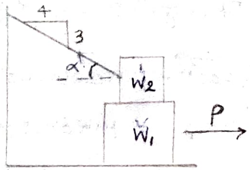
$\Sigma F_x = 0$

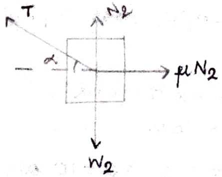
$50\sin\alpha + 50\sin\alpha = 15\cos\alpha + 10\cos\alpha$

$\tan\alpha = \frac{1}{4} \Rightarrow \alpha = 14.1$

Q. A block of weight $W_1 = 200N$ rests on a horizontal surface & supports on top of it another block W_2 of 50N. The block W_2 is attached with vertical wall by a string AB. Find the amount of horizontal force P applied to lower block necessary to impend slip. The μ for all ~~surface~~

contigous surfaces is 0.3.





$\Sigma f_x = 0,\ \mu N_2 = T\cos\alpha$

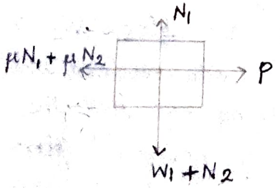
$T = \dfrac{\mu N_2}{\cos\alpha}$

$\Sigma f_y = 0,\ W_2 = T\sin\alpha + N_2$

$T = \dfrac{W_2 - N_2}{\sin\alpha}$

$\dfrac{\mu(W_2)}{\cos\alpha} = \dfrac{W_2 - N_2}{\sin\alpha}$ —— ①

~~$\tan\alpha$~~



$\Sigma f_x = 0,\quad P = \mu(N_1 + N_2)$ —— ②

$\Sigma f_y = 0\quad W_1 = N_1 - N_2$

~~$W_1 = N_1 = 200N$~~

from eq ①

$\tan\alpha = \dfrac{W_2 - N_2}{\mu N_2}$

~~$\dfrac{3}{4} = \dfrac{50 - N_2}{\frac{3}{10}\times 50}$~~

~~$\dfrac{15\times 3}{4} = 50 - N_2$~~

~~$N_2 = 50 - \dfrac{45}{4}$~~

$N_2(\mu\tan\alpha + 1) = W_2$

$N_2 = \dfrac{50}{\frac{3}{10}\times\frac{3}{4} + 1}$

$= 40.81\ N$

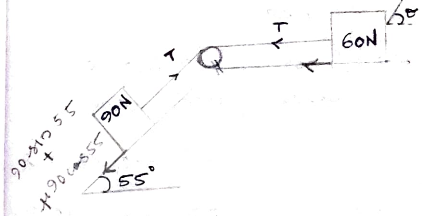
② → $N_1 = W_1 + N_2 = 240.81$

$P = \mu(N_1 + N_2)$

$= \dfrac{3}{10}(240.81 + 40.81)$

$=$ ~~72.243 N~~ $84.49\ N$

Q Determine the least value of P to cause motion to impend rightwards in the figure. Assume pulley to be frictionless, μ for all contigous surfaces be 0.2.



$T = 90\sin 55 + 90\cos 55 \times \mu$

$T + \mu(60 - P\sin\theta) = P\cos\theta$

$90\sin 55 + 90\cos 55\,\mu = P\cos\theta + \mu P\sin\theta - 60\mu$

$P\left(\cos\theta + \frac{2}{10}\sin\theta\right) = 96.048$

$= 94.1831$

↓

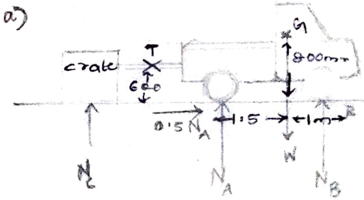
maximise

Q A truck has a mass of 12262 N & a centre of mass at G. Determine the greatest load it can pull if, the truck has rear wheel drive while the front wheel are free to roll.

b) The truck has 4 wheel drive.

The coefficient of static frictio between wheel and ground $\mu_s = 0.5$ & between the crate & ground is $\mu_s' = 0.4$

a)



$\Sigma M_B = 0$

$W\times 1 - N_A(2.5) + T\times 600 = 0$

T = 2.5

W = 6.97 kN

b)

15.3 kN

6131.5 N

Distributed loading

Reduction of a simple distributed loading

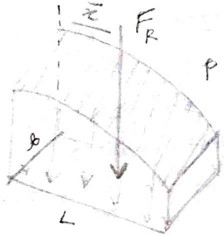
A body may be subjected to loading thats distributed over a surface

eg. Pressure inside a water filled tank

These are all distributed loadings & pressure exerted ~~under~~ on all points on the surface, indicates the intensity of loading.

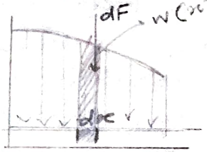
Uniform loading along a single axis

Consider a beam or plate that has a const width and subjected to pressure loading that varies only along x axis.



This loading can be described by the fn $P = P(x)$ N/m^2

$$W(x) = p(x) \cdot b$$



To find magnitude of the resultant force we use F_R which is equivalent to the sum of all forces in the system. Since there is infinite no: of parallel forces, dF acting on the beam, we can use integration over its length to find the force.

$$dF = w(x) \cdot dx = dA$$

$$F_R = \int w(x) \cdot dx = \int dA = A$$

The magnitude of F_R is equal to the total area A under the loading diagram

Location of resultant force

$\bar{x}$ of the line of action of F_R can be determined by equating moments of force resultant & parallel force distribution about point O

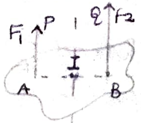
$$-F_R \bar{x} = -\int_L x \cdot w(x)\, dx$$

$$\bar{x} = \frac{\int x \cdot w(x)\, dx}{F_R}$$

$$\bar{x} = \frac{\int x \cdot dA}{\int_A dA}$$

Parallel forces in a plane

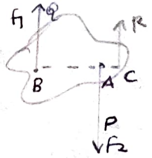
Resultant of 2 || forces acting in same direction



The resultant of 2 like || forces acts parallel to them & its position is such that, it divides joining the distance between their points of application in the ratio which is inversely proportional to their magnitude

$$\frac{P}{Q} = \frac{IB}{IA}$$

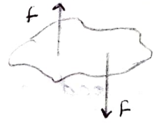
Resultant of 2 || forces acting in opposite direction



In the case of 2 unlike || forces, the resultant lies outside the line joining the points of action of two forces & on the same side as that of the larger force

$$\frac{P}{Q} = \frac{BC}{AC}$$

Two equal parallel forces acting in opposite directions



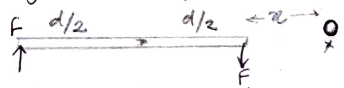
A system of 2 equal || forces acting in opposite direction cannot be replaced by a single force. In such a case, 2 forces form a couple which has the tendency to rotate the body. The resultant force of the couple is 0, hence the body doesnt translate but only rotate. The ⊥r distance between lines of action of 2 forces is termed as the arm of the couple

→ Moment of a couple

The rotational tendancy of the couple is measured by its moment. The moment of couple is the product of either 1 of the forces forming the couple & arm of the couple

2 couples acting in a plane can be in eqm, if moments are equal in magnitude & opposite in direction.

eg. A bar of length 'd' hinged at midpoint



Moment about O ⇒

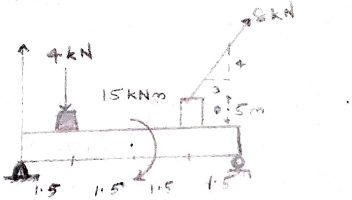
$$-(F\times(d+x)) + Fx = M$$

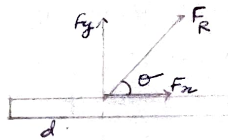
$$Fd = M$$

The magnitude of couple is independent of reference location Always equal to force × dist.

Force system resultants

Q In the figure shows replace the force & couple-moment system acting on the beam by a eq. resultant force & find where its line of action intersect the beam measured from point O





$$\Sigma F_x = 8\times\frac{3}{5} =$$

$$\Sigma F_y = 8\times\frac{4}{5} - 4$$

$$F_R = \sqrt{F_x^2 + F_y^2} = 5.37\text{kN}$$

$$\tan\theta = \frac{F_y}{F_x} \Rightarrow \theta = 26.6^\circ$$

$\Sigma M_O = \cancel{0}\ F_R d$

$-4\times1.5 - 15 \cancel{\#} 8\times\frac{3}{5}(0.5)$

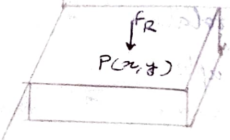
$+8\times\frac{4}{5}(4.5)$

$= F_{Ry} d$

$5.4 = F_R \sin\theta . d$

$d = \frac{5.4}{5.37\times\sin(26.6)}$

$= \underline{\underline{2.246 m}}$



$\Sigma M_O \Rightarrow$ about X axis

$F_R\times y = 500\times0 + 600\times0 + 100\times5 - 400\times10$

$-1400\, y = -3500$

$y = \frac{-3500}{-1400}$

$= \underline{\underline{2.5\ m}}$

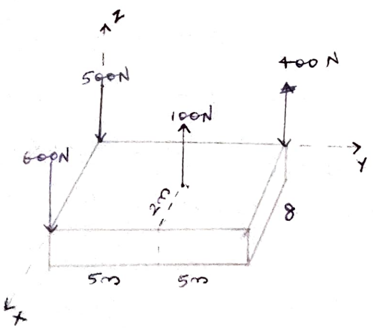
Q The slab shown in fig is subjected to 4 parallel force. Determine mag & direction of resultant force equivalent to given force system & locate its point of application on the slab

ΣM_O about Y axis

$F_R\times x = 100\times6 - 600\times8$

$x = \frac{-4200}{1400}$

$= \underline{\underline{3m}}$



$F_R = -500 - 400 - 600 + 100$

$= -1400 N$

Module 3

Centre of gravity (CG)

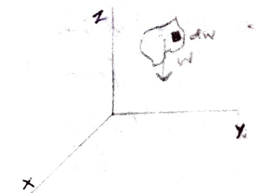
A body is composed of a infinite no. of particles of differential size. And so, if the body is located within a gravitational field ~~with~~ each of these particles will have a weight dW

These elemental weights will form an approx parallel force system & the resultant of this system is the total weight of the body which passes through a single point called Centre of gravity CG.

The weight of body W, is equal to the sum of weight of all its particles

$W = \int dW.$

The location of center of gravity (CG) measured from Y axis



$\bar{x} W = \int x\, dW$

* $\bar{x} = \frac{\int x . dW}{\int dW}$

* $\bar{y} = \frac{\int y . dW}{\int dW}$

* $\bar{z} = \frac{\int z\, dW}{\int dW}$

Centre of Mass of a body

Inorder to study the dynamic response or accelerated motion of the body, its important to locate the (CoM) of the body.

This location can be determined by substituting

$W = mg$

$dW = g\,dm$

* $\bar{x} = \dfrac{\int x\,dm}{\int dm}$

* $\bar{y} = \dfrac{\int y\,dm}{\int dm}$

* $\bar{z} = \dfrac{\int z\,dm}{\int dm}$

→ Centroid of volume

If the body is made of a homogeneous material, then its density is a const

* $\bar{x} = \dfrac{\int x.dv}{\int dv}$

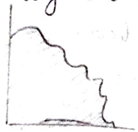
* $\bar{y} = \dfrac{\int y\,dv}{\int dv}$

* $\bar{z} = \dfrac{\int z.dv}{\int dv}$

→ Centroid of area

If an area lye in x-y plane & is bounded by the curve

$y = f(x)$



Then its centroid will be in this plane & can be determined from

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA}$$

→ Centroid of a line

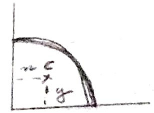
$$\bar{x} = \frac{\int_L x.dL}{\int dL}$$

Q Find the centroid of a plane uniform wire of length L

$$\bar{x} = \frac{\int_0^L x.dL}{\int dL} = \frac{\int_0^L x\,dx}{\int dx}$$

$$= \left.\frac{x^2}{2x}\right]_0^L = \left.\frac{x}{2}\right]_0^L = \frac{L}{2}$$

Q Determine the centroid of the quater circle shown in fig, where the radius is R



$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\int x\,dA}{A}$$

$x^2 + y^2 = R^2$

$y = R^2 - x^2$

$y = \sqrt{R^2 - x^2}$

$dA = \int y.dx$

$= \sqrt{R^2 - x^2}.dx$

$$\frac{\int x\,dA}{A} = \frac{\int x\sqrt{R^2 - x^2}\,dx}{A}$$

$= \cancel{x \sqrt{R^2 - x^2}}$

$= \dfrac{4r}{3\pi}$

Centroids of various shapes of areas.

	Area	$\bar{x}$	$\bar{y}$
Rectangle (b×d)	b×d	$\frac{b}{2}$	$\frac{d}{2}$
Triangle (right)	$\frac{1}{2}bh$	$\frac{b}{3}$	$\frac{h}{3}$
Isosceles triangle	$\frac{1}{2}bh$	$\frac{b}{2}$	$\frac{h}{2}$
Quater circle	$\frac{1}{4}\pi r^2$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircle	$\frac{1}{2}\pi r^2$	0	$\frac{4r}{3\pi}$

→ Centroid of composite plane figure

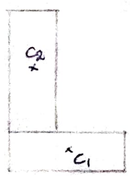
Composite area is one which is made up of several peices or components that represents familiar geometric shapes.

To find the centroid of the given area, divide the area into dif. component parts

Centroid $x_c = \dfrac{\Sigma A_i x_i}{\Sigma \Delta A_i}$

$y_c = \dfrac{\Sigma A_i y_i}{\Sigma \Delta A_i}$

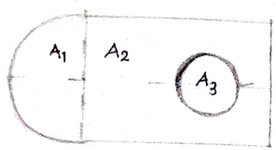
eg. For an L section



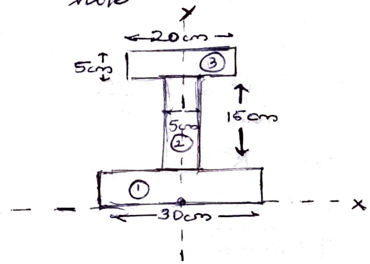
$x_c = \dfrac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$

$y_c = \dfrac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

eg. For hatched area



		$\bar{x}$	A	$\bar{x}A$
A_1	Semi circle	−	+	−
A_2	Rectangle	+	+	+
A_3	circular hole	+	−	−



	Area	x_c	y_c
①	30x5	0	2.5
②	15x5	0	12.5
③	20x5	0	22.5

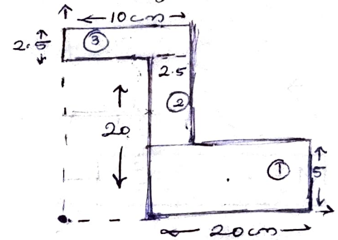
$X_c = 0$

$Y_c = \dfrac{\Sigma A_i y_i}{\Sigma A}$

$= \dfrac{30\times5\times\frac{5}{2} + 15\times5\times\frac{25}{2} + \frac{45}{2}\times20\times5}{30\times5 + 15\times5 + 20\times5}$

$= 10.96$

Q. Find the centroid of the cross sectional area as shown in figure.



	Area	x_c	y_c
① →	20x5	7.5+10	2.5
② →	2.5x15	7.5+1.25	5+7.5
③ →	2.5x10	5	20+1.25

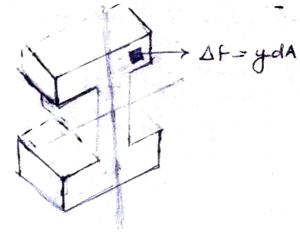
$x_c = \dfrac{20\times5\times17.5 + 2.5\times15\times8.75 + 2.5\times10\times5}{20\times5 + 2.5\times15 + 2.5\times10}$

$= \dfrac{2203.125}{162.5} = \underline{\underline{13.557\text{cm}}}$

$y_c = \dfrac{20\times5\times2.5 + 2.5\times15\times12.5 + 2.5\times10\times21.25}{162.5}$

$= \underline{\underline{7.69\text{ cm}}}$

→ Moment of Inertia of Area / Second moment



The magnitude of resultant R of the elemental forces ΔF, which act over the entire section is given by

$$Q_x = \int y \cdot dA$$

Its the first moment $Q_{(x)}$ of the section about the X axis

$\Rightarrow \bar{y}A = 0$

The system of forces ΔF or $Q_{(x)}$ reduces to a couple

$\Delta M_x = y \cdot \Delta F$

$= y \cdot Q_{(x)}$

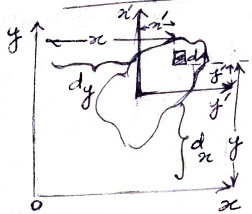
$= y \cdot \int y\, dA$

$= \int y^2 dA$

$\boxed{I = \int y^2 dA}$ is called the 2^{nd} moment or moment of inertia wrt X axis

$$\boxed{\begin{aligned} I_x &= \int y^2 dA \\ I_y &= \int x^2 dA \end{aligned}}$$

→ Parallel axis theorem



xy is the rectangular coordinate axis through any point O in the plane of figure of area A.

x'-y' is the corresponding parallel axis thround the centroid c ~~hav~~ of the area A. This axis through the centroid of an area is called the centroidal axis

Moment of Inertia of the area A, about the X axis

$$I_x = \int y^2 dA$$

$$= \int (y' + d_x)^2 dA$$

$$= \int (y'^2 + 2y'd_x + d_x^2) dA$$

$$= \int y'^2 dA + 2\int y' d_x dA + \int d_x^2 dA$$

* d_x is the ⊥r distance b/w the axis x & x'

$\int y' dA$ is I^{st} moment of area

∴ its 0. ∴ $2\int d_x y' dA = 0$

⇒

$$M = \int y'^2 dA + \int d_x^2 dA$$

$$\boxed{M_{par} = I'_x + A(d_x^2)} \longrightarrow *$$

similarly

$$M_{par} = I'_y + A(d_y^2)$$

I'_x or $\bar{I}_x$ is the moment of inertia of an area about its centroidal axis.

Thus, the moment of inertia of an area wrt any axis in its plane is equal to the moment of inertia of the area wrt a parallel centroidal axis + the product of the area & square of the distance between the two axes.

$$\cancel{I = \bar{I} + (Ad)}$$

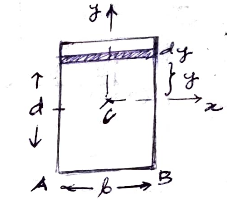
$$I = \bar{I} + Ad^2$$

Q Find the moment of inertia of a rectangular cross section

a) About its centroidal axis

b) About the base AB.

a)



$$I_x = \int y^2 dA$$

$$= \int_{-d/2}^{d/2} y^2 \cdot b \cdot dy$$

$$= \left.\frac{y^3}{3} b\right]_{-d/2}^{d/2}$$

$$= \frac{\left(\frac{d^3}{8} + \frac{d^3}{8}\right) b}{3}$$

$$= \frac{d^3 b}{12}$$

Moment of inertia abt centroidal axis of rectangle

similarly $I_y = \frac{db^3}{12}$.

x axis d നെ cut ചെയ്യുന്നു. അതുകൊണ്ട്, I_x എടുക്കുമ്പോൾ d ന് cube value.

b) Moment of inertia about AB ⇒

$$I_{AB} = I_x + A(dis)^2$$

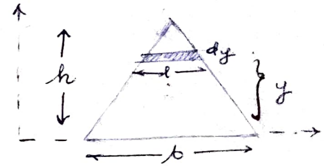
$$= I_x + bd \cdot \left(\frac{d}{2}\right)^2$$

$$= I_x + \frac{d^3 b}{4}$$

$$= \frac{d^3 b}{12} + \frac{d^3 b}{4}$$

$$= \frac{d^3 b}{3}$$

Q Determine the moment of inertia of a triangle wrt its base.



$$I_x = \int y^2 (dA)$$

$$\frac{l}{b} = \frac{h-y}{h}$$

$$= \int y^2 l \cdot dy$$

$$l = \frac{b(h-y)}{h}$$

$$= \int y^2 \left(\frac{b(h-y)}{h}\right) dy$$

$$= \frac{b}{h} \int y^2 (h-y) dy$$

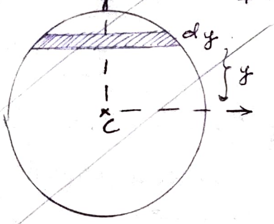
$$= \frac{b}{h} \times h \left.\frac{y^3}{3}\right]_0^h - \frac{b}{h} \times \frac{y^4}{4}$$

$$= \frac{b}{h}\left(\frac{h^4}{3} - \frac{h^4}{4}\right)$$

$$= \frac{b}{h}\left(\frac{h^4}{12}\right)$$

$$= \frac{bh^3}{12} \qquad [L^4]$$

H.W Circular area about centroidal axis $\left(\frac{\pi r^4}{4}\right)$



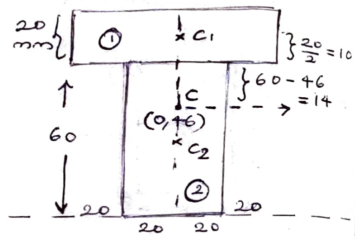
$$I_x = \int y^2 dA$$

$$= \int_{-r}^{r} y^2 \pi r \, dy$$

$$= \pi r \left[\frac{y^3}{3}\right]_{-r}^{r}$$

$$= \pi r. ($$

Q. Determine the moment of inertia of the area of a T section as shown wrt to the centroidal X axis.



$$\bar{x} = \frac{(20\times80\times0) + (40\times(60\times0))}{20\times80 + 40\times60}$$

$$= 0$$

$$\bar{y} = \frac{(40\times60\times30) + (20\times80\times70)}{(20\times80 + 40\times60)}$$

$$= 46\text{mm}$$

Moment of inertia of

① $\Rightarrow \frac{bd^3}{12} = I_{x①}$

$$\Rightarrow \frac{80\times20^3}{12} =$$

$$\bar{I}_{x①} = I_{x①} + A(cc_1)$$

$$= \frac{80\times20^3}{12} + 20\times80\times(24)^2$$

$$= 975\times10^3$$

$$I_{x(2)} = \frac{bd^3}{12} = \frac{40\times60^3}{12}$$

$$\bar{I}_{x(2)} = I_{x(2)} + A(cc_2)$$

$$= \frac{40\times60^3}{12} + (40\times60)16^2$$

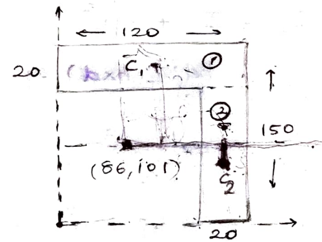
$$= \cancel{758,400}$$

$$1334400$$

$$\bar{I}_{x_1} + \bar{I}_{x_2} = 2309\times10^3 \text{ mm}^4$$

$$= 2.3\times10^6 \text{ mm}^4$$

Q. Find moment of inertia of about area of L section about centroidal X axis.



$$x_c = \frac{(20\times130\times110) + (20\times120\times60)}{(20\times130) + (20\times120)}$$

$$= \frac{430000}{5000} = 86$$

$$y_c = \frac{(20\times130\times\frac{130}{2}) + (20\times120\times140)}{5000}$$

$$= 101$$

$$I_{x_1} = \bar{I}_{x_1} + A_1 d_1^2$$

$$= \frac{bd^3}{12} + A_1 d_1^2$$

$$= \frac{120\times20^3}{12} + (20\times120)39^2$$

$$= 3.73\times10^6 \text{ mm}^4$$

$$I_{x_2} = \bar{I}_{x_2} + A_2 d^2$$

$$= \frac{bd^3}{12} + A_2 d_2^2$$

$$= \frac{20\times130^3}{12} + 20\times130\times36^2$$

$$= 7.03\times10^6 \text{ mm}^4$$

$$I_x' = I_{x_1} + I_{x_2}$$

$$= 10.76\times10^6 \text{ mm}^4$$

$$I_{y_1} = \bar{I}_{y_1} + A_1 d_1^2$$

$$= \frac{bd^3}{12} + A_1 d_1^2$$

$$= \frac{20\times120^3}{12} + (20\times120)26^2$$

$$= 4.5\times10^6$$

$I_{y_2} = \bar{I}_{y_2} + A_2 d_2^2$

$= \frac{bd^3}{12} + A_2 d_2^2$

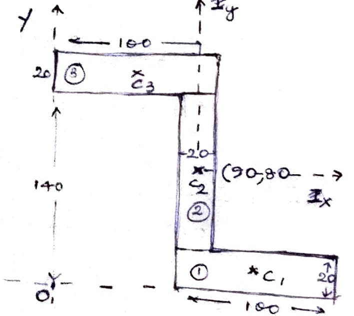
$= \frac{130 \times 20^3}{12} + 130 \times 20 \times 24^2$

$= 1.58 \times 10^6$

$I_y = (1.58 + 4.5) \times 10^6$

$= 6.08 \times 10^6 \, mm^4$

Q Determine the moment of Inertia of the Z section about the centroidal x_0 & y_0 ~~direc~~ axis.



Area	x_c	y_c	
20×100	130	10	← ①
20×120	90	80	← ②
20×100	50	150	← ③

$x_c = \frac{(100\times20\times130) + (20\times120\times90) + (20\times100\times50)}{(20\times100) + (20\times120) + (20 ...}$

$= \frac{57600}{6400}$

$= \underline{\underline{90}}$

$y_c = \frac{(100\times20\times10) + (20\times120\times80) + (20\times100\times150)}{6400}$

$= \underline{\underline{80}}$

Centroid is at c_2 itself (90,80)

$I_{x_1} = \bar{I}_{x_1} + A_1 d_1^2$

$= \frac{bd^3}{12} + (b \times d)\, d_1^2$

$= \frac{100 \times 20^3}{12} + (20 \times 100) 70^2$

$= 9.867 \times 10^6 \, mm^4$

$I_{x_2} = \frac{bd^3}{12}$

$= \frac{20 \times 120^3}{12}$

$= \cancel{4.147}\ 2.88 \times 10^6 \, mm^4$

$I_{x_3} = \frac{bd^3}{12} + A_3 d_3^2$

$= \frac{100 \times 20^3}{12} + (100 \times 20)\, 70^2$

$= 9.867 \times 10^6 \, mm^4$

$I_x = I_{x_1} + I_{x_2} + I_{x_3}$

$= \underline{\underline{22.613 \times 10^6 \, mm^4}}$

$I_{y_1} = \bar{I}_{y_1} + A_1 d_1^2$

$= \frac{bd^3}{12} + A_1 d_1^2$

$= \frac{20 \times 100^3}{12} + (20 \times 100)\, 40^2$

$= 4.867 \times 10^6 \, mm^4$

$I_{y_2} = \bar{I}_{y_2} + A_2 d_2^2$

$= \frac{bd^3}{12} + A_2 d_2^2$

$= \frac{120 \times 20^3}{12} + 0$

$= 0.08 \times 10^6 \, mm^4$

$I_{y_3} = \bar{I}_{y_3} + A_3 d_3^2$

$= \frac{bd^3}{12} + A_3 d_3^2$

$= \frac{20 \times 100^3}{12} + (20 \times 100)\, 40^2$

$= 4.867 \times 10^6$

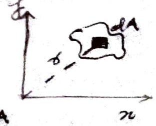
$I_y = I_{y_1} + I_{y_2} + I_{y_3}$

$= \underline{\underline{9.814 \times 10^6 \, mm^4}}$

Polar moment of inertia (J)

The moment of inertia of an area of a plane figure wrt an axis ⊥r to the XY plane & passing through a pole (Z axis) is called as Polar Moment of Inertia

$J_0 = \int r^2 \, dA$



$= \int (x^2 + y^2)\, dA$

$= \int x^2 dA + \int y^2 dA$

$\boxed{J_0 = I_x + I_y}$

The polar moment of inertia of a given area can be computed from the rectangular moments of inertia I_x & I_y of the area if these quantities are already known.

VIBRATION

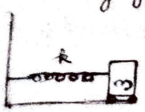
Oscillations about an eqm point (can either be periodic or random). It can be a particle, a body or a system of connected bodies

There are 2 types
free vibration & forced vibration

free vibration : caused by initial source which is removed so that structure vibrates without any forces acting on it. {short initial vib.}

forced vibration : Case where excitation is permanent and is continuously applied to the structure {continuous excitation}
eg. Drilling, washing machines

A case of free vibration

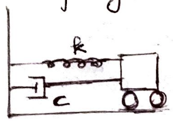


$\boxed{mx'' + kx = 0}$

(without damping)

For free vibration with damping,

(dashpot or damper)

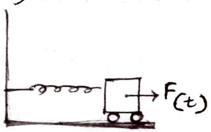


$\boxed{mx'' + cx' + kx = 0}$

For forced vibrations

i) without damping,

$F = P_o \sin\theta$



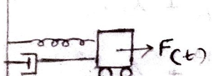
wrt time

$F_{(t)} = P_o \sin(\omega t)$

$\boxed{mx'' + kx = F_{(t)}}$

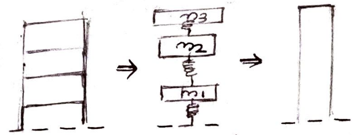
$mx'' + kx = P_o \sin(\omega t)$

ii) with damping



$\boxed{mx'' + cx' + kx = F_{(t)}}$

SDOF : Single Degree of Freedom.



Lumped mass concept

Application of polar moment of inertia

Problems concerning the ~~problems~~ torsion of cylindrical shafts & problems dealing with rotation of slabs.

→ Radius of Gyration

Consider an area A which has a moment of inertia I_x wrt X axis. Imagine that we concentrate this area into a thin strip parallel to the X axis

If the area A thus concentrated into a strip is to have the same moment of inertia wrt the X axis, then the strip should be placed k_x from X axis, where k is described by radius of gyration

$I_x = \int y^2 dA$
$= k_x^2 A$

$k_x = \sqrt{\frac{I_x}{A}}$

similarly,

$I_y = \int x^2 dA$
$= k_y^2 A$

$k_y = \sqrt{\frac{I_y}{A}}$

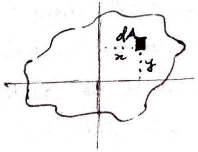
$J_o = \int r^2 dA = \int k_o^2 dA$

radius of gyration, $k_o = \sqrt{\frac{J_o}{A}}$

$k_o^2 = k_x^2 + k_y^2$

→ Product of Inertia

In certain problems involving unsymmetrical cross sections & in calculation of moments of inertia about the rotated axis



$I_{xy} = \int xy \, dA$

where x, y are coordinates of elemental area dA. The quantity I_{xy} is called product of inertia of area A wrt XY axes

Unlike moment of inertia which is +ve for +ve area, the product of inertia may be +ve, −ve or zero

→ Rotation of Axes

$$I'_x = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I'_y = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

Rectilinear Motion

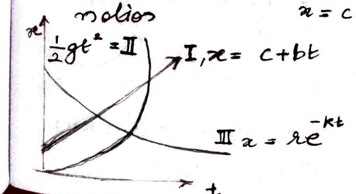
Kinematics is concerned only with space - time relationship of a given motion of the body & is not concerned about the forces that cause of motion

Kinetics is concerned with find the kind of motion of a body or a system under the action of given forces.

→ Rectilinear motion.

displacement (dif forms of movement of the particle about the X axis)

i) Uniform rectilinear motion $x = c + bt$



ii) freely falling body

$$x = \frac{1}{2}gt^2$$

iii) displacement decreases exponentially with time

$$x = xe^{-kt}$$

$$\text{Acceleration} = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v.\frac{dv}{dx}$$

① $v = u + at$

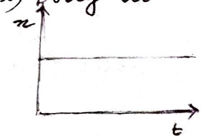
② $s = ut + \frac{1}{2}at^2$

③ $v^2 = u^2 + 2as$

→ Graphical representation

I. Position time graph

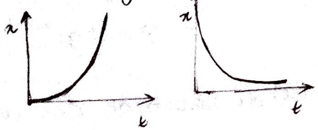
i) Body at rest



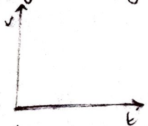
$V = 0$
$x = \text{const}$

ii) Uniform motion

iii) Non uniform motion

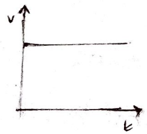


② Velocity time graph

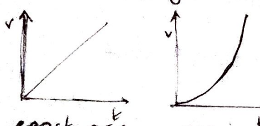


i) Body at rest

ii) uniform motion   v const



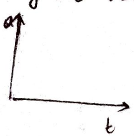
iii) Non uniform motion



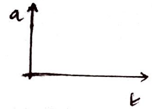
const acc   varying acc

③ Acceleration time graphs

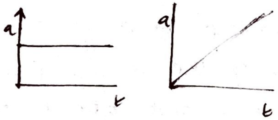
i) Body at rest

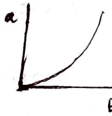


ii) uniform motion



iii) Non - uniform



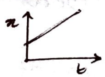


Area under acceleration time plot gives change in velocity

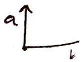
$\Delta V = \int a \cdot dt$

→ Different patterns of motion

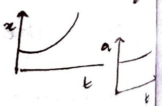
I $x = c + bt$



$V = \frac{dx}{dt} = b, a = 0$

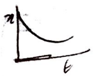


II $x = c + bt^2$



$V = \frac{dx}{dt} = 2bt, a = 2b$

III $x = c + be^{-kt}$



$V = \frac{dx}{dt} = -e^{-kt}(kb)$

$a = +k^2be^{-kt}$

Q The rectilinear motion of a particle is defined by the displacement - time eqn as

$x = x_0 + V_0t + \frac{1}{2}at^2$

find displacement & velocity at time $t = 2s$ while x_0 250 mm, $V_0 = 125$ mm/s & $a = 0.5$ m/s^2.

at $t = 0 \Rightarrow x = x_0 = 250$ mm

at $t = 2 \Rightarrow x = 250 + 125 \times 2 + \frac{1}{2} \times 0.5 \times 1000 \times 4$

$x_{(2)} - x_{(0)} = 1.25$ m

$V = \frac{dx}{dt} = V_0 + at$

$V_{(2)} = 125 + 0.5 \times 1000 \times 2$

$= 1.125$ m/s

Q The velocity - time relation of a moving particle is given by $x_0 = \frac{1}{2}ct^2$, $c = 2.4$ m/s^2. Determine displacement of particle at 3s if there was no initial displacement.

$V = \frac{1}{2}ct^2$

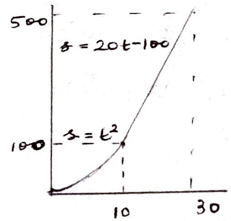
$x = \int_0^3 \frac{1}{2}ct^2\, dt$

$= \frac{1}{2}c \times \frac{t^3}{3}\Big]_0^3$

$= \frac{1}{2}c \times \frac{3^3}{3}$

$= \frac{9}{2} \times \frac{24}{10} = 10.8$ m

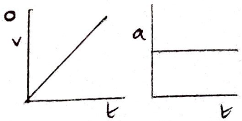
Q. A bicycle moves along a straight road such that its position is described by graph in the fig. Construct a-t & v-t graph



$t = 0 \rightarrow 10$

$V = 2t$

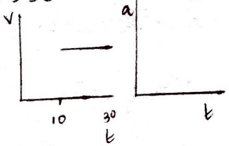
$a = 2$

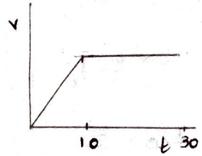


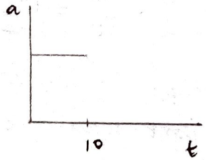
for $t = 10 \to 30$

$V = 20$

$a = 0$





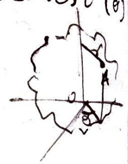


Rigid body is a system of particles for which the distances of particles remain unchanged

RIGID BODIES

Kinematics of rotation

* Angular displacement (θ) :



As the body rotates, angle of ~~I~~ rotation θ varies with time

→ Angular velocity (ω)

Angle of rotation per sec

If t_1 & t_2 denotes 2 successive instance of time & θ_1 & θ_2 the corresponding angle of rotation

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

→ Instantaneous angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

When a body rotates ~~till~~ about a fixed axis, each point in it at a distance r from the ~~point of~~ axis of rotation. It describes a circle of radius R.

If Δs is a small displacement of point (A) around this ~~cycle~~ circle during ~~at~~ interval ~~time~~ of time ΔT,

angle of rotation of body

Then velocity of point

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{r\Delta\theta}{\Delta t}$$

$r\theta$ or $r\omega$ = ~~$r\dot\theta$~~

→ Angular acceleration (α)

The time rate of change of angular velocity of a rotating body ~~is~~ called angular acceleration wrt time

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

$$= \frac{d\omega}{dt}$$

$$= \omega' \to \theta$$

$a_n = \frac{v^2}{r}$

$$a = \frac{d}{dt}(v) = r\omega' = r\alpha$$

$$a_n = \frac{v^2}{r} =$$

$$a_n = \frac{v^2}{r} = \frac{(r\omega)^2}{r}$$

$$= \omega^2 r \text{ or } \theta'^2 r$$

Equations defining the rotation of a rigid body about a fixed axis

Angular velocity $= \frac{d\theta}{dt} = \omega$

Angular acceleration $= \frac{d\omega}{dt} = \alpha$

$= \omega' \text{ or } \theta''$

i) $\omega = \omega_0 + \alpha_c t$

ii) $\theta = \omega_0 t + \frac{1}{2}\alpha_c t^2 + \theta_0$

iii) $\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

$v : \omega$

$u : \omega_0$

$s : (\theta - \theta_0)$

$a : \alpha_c$.

$$I = \int_m r^2 dm$$

$$= \int_V r^2 \rho \, dV$$

$$I\theta'' = M$$

$$M = \int r^2 dm \times \theta''$$

for tortion pendulum

$$\omega = \sqrt{\frac{cg}{k^2}}$$

$I = mk^2$

c: dis from pivot to centre of mass