* Structural integity, under water acoustics (How sound propogates under water for navigation), stability, hydrodynamics (Resistance on hull) manuevarability of ship (ship responds to diffrent types of wave motions) all can be said as roles of fluid mechanics.

⇒

PROPERTIES *

1) Density of the fluid $\rho = \frac{m}{V}$

For water :- $\rho_w = 1000 \ kg \ m^{-3} \ / \ 1 \ kg \ mm^{-3}$

2) specific weight :- Also called weight density (γ)

$$\gamma = \frac{Weight}{Volume} = \frac{W}{V} \quad (Nm^{-3}).$$

$$= \frac{mg}{V} = \rho g.$$

$$\boxed{\gamma = \rho g}$$

3) Specific Volume :- inverse of density.

$$SV = \frac{V}{M} = \frac{1}{\rho} \quad (m^3 kg^{-1})$$

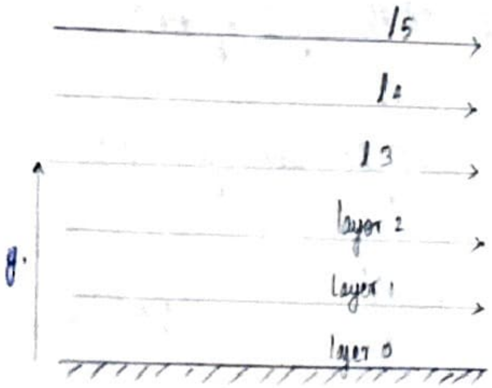
4) Specific gravity :- Measure of how much a fluid heavier than water. OR ratio of weight density of a fluid to the weight density of a std. fluid. For liquids std fluid is water, for gases std fluid is air.

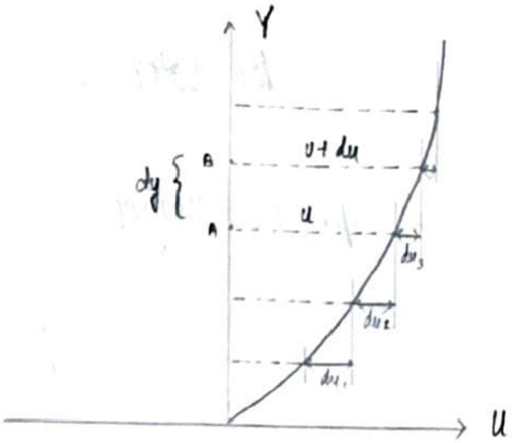
$$S = \frac{\gamma_F}{\gamma_w}$$

- If $S = 13.6$ for Hg, then $\rho_{Hg} = 13.6$ times that of water $= 13600\ kg\,m^{-3}$.

5) Viscocity :- How easily fluid can flow or how thick the fluid is.

Property of a fluid which offers resistance to the movement of one layer of fluid over the adjacent layer.





As per newtons experiments :- shear stress formed is prop. to vel. grad.

$$\tau \propto \frac{du}{dy}.$$ as du decreases, τ also decreases.

$\propto$ velocity gradient or rate of shear strain. or rate of shear deformation.

Newtons law of viscosity.

$$\boxed{\tau = \mu \frac{du}{dy}} \Rightarrow \mu = \frac{\tau}{(du/dy)} = \frac{\tau}{\text{velocity gradient}}$$

μ = Coeff. of visocity / viscosity / Coeff. of dynamic viscosity.

ie fom the relation, It is defined as the shear stress required to produce unit velocity gradient.

Unit :- $\eta = \frac{N}{m^2} \frac{m}{(m/t)} = \frac{Ns}{m^2}$ = Pascal . second

CGS UNIT :- dyne s / cm^2 1 dyne = 10^{-5} N

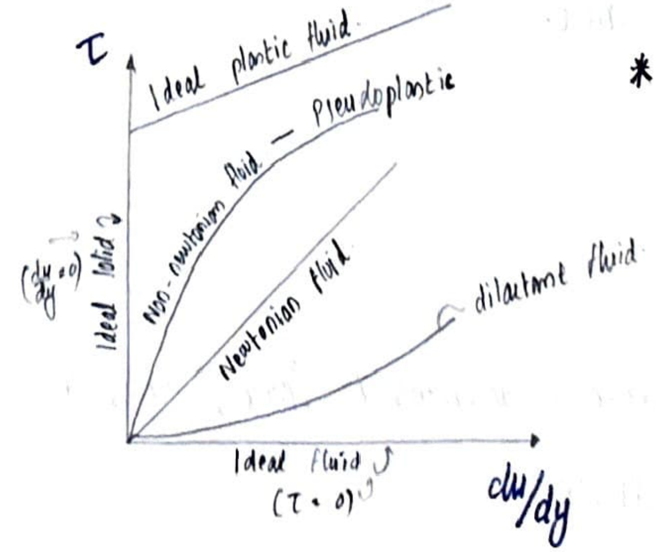
→ KINEMATIC VISCOCITY :- Ratio of μ and ρ. (ν)

$\nu = \frac{\mu}{\rho}$ → unit = m^2/second.
= cm^2/second (stoke)

⊙ Statement of newtons law of viscocity :-

The shear stress on a fluid element layer is directly proportional to rate of shear strain ($\tau \propto \frac{du}{dy}$). Fluids which obey this law is known as newtonian fluid and other fluids are called non-newtonian fluids.

TYPES OF FLUIDS



* Ideal fluid :- Zero shear stress. (imaginary). All fluids existing in nature are real fluids which offers some form of resistance to the movement of fluid layers. All real fluids are known as viscous fluids.

* Ideal plastic fluid :- A fluid in which shear stress is more than the yield value, and shear stress (τ) is prop. to rate of shear strain. Is know as an ideal plastic fluid.

Also known as BINGHAM PLASTIC

① Power law model :- $\tau = m \left(\frac{du}{dy}\right)^n$ m: Flow behavior index (Pseudo n<1)
n: Flow consistency index (dilat n>1)

* Compressibility : Resiprocal of bulk modulous (K)

$$C = 1/K = \frac{\Delta V/V}{\sigma}$$ = Volumetric change occur per stress.

- Compressibility of water → $B_w = 2\times10^6\ kN/m^2$
Compressibility of air → $B_A = 101\ KNm^{-2}$
bulk modul

$\frac{C_w}{C_A} = \frac{2\times10^6}{101} \approx \frac{200\times10^0}{1.01} \approx 1.99\times10^4$

$C_w = 1.99\times10^4\ C_A$

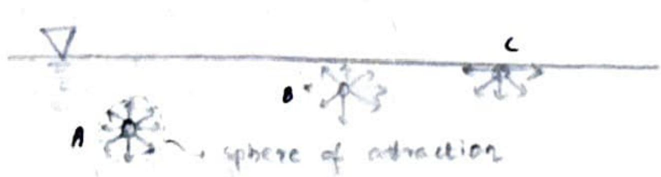
Hence higher the value of ~~Compressibility~~, higher stress is required to produce unit volumetric strain. Hence higher incompressibility.

SURFACE TENSION

It is the property of fluids to

* behave like a streched membrane
* minimize the surface area.

Reason :



At the free surface, all the molecules experience a down word force, thus a free surface act's like a very thin film under tension.

Surface tension is expressed as force per unit length or surface energy per unit area.

$$ST = \frac{\text{Force}}{\text{Length}} \quad \text{or} \quad \frac{\text{Surface energy}}{\text{Area}} \qquad [\text{unit } N/m]$$

① <u>S·T on a liquid droplet</u>

Considering the eqm,

Force due to ST = Pressure force exerted from the other half

$$\sigma \cdot \pi d = P \cdot \frac{\pi d^2}{4} \Rightarrow \boxed{\sigma = \frac{Pd}{4}}$$

② <u>ST on a hollow bubble</u> *

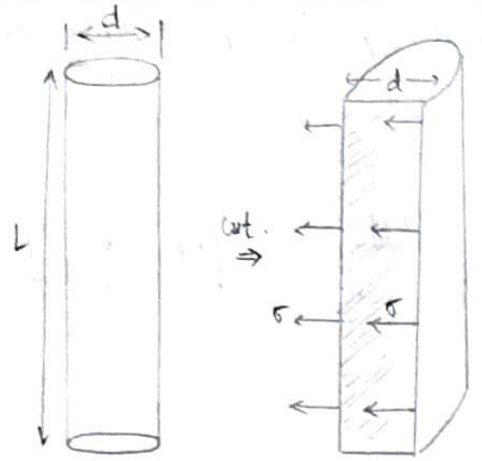
Similar to the above derivation, there are 2 surface's in contact with air.

ie, $P \cdot \frac{\pi d^2}{4} = 2 \times [\sigma \cdot \pi d] \Rightarrow \boxed{\sigma = \frac{Pd}{8}}$

P = air pressure intencity from the other half.

③ <u>S·T on a water jet</u>

Since longitudinal stress is higher, (Concept of hoop stress) →

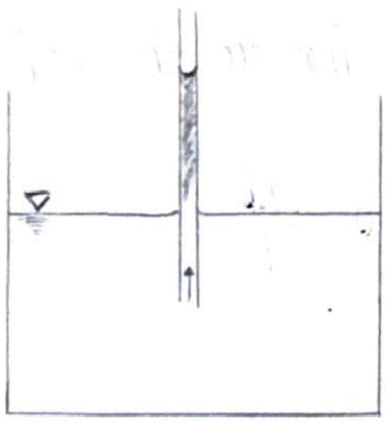


A small cylindrical portion from jet.

$$\sigma \times 2L = P \cdot L d$$

$$\boxed{\sigma = \frac{Pd}{2}}$$

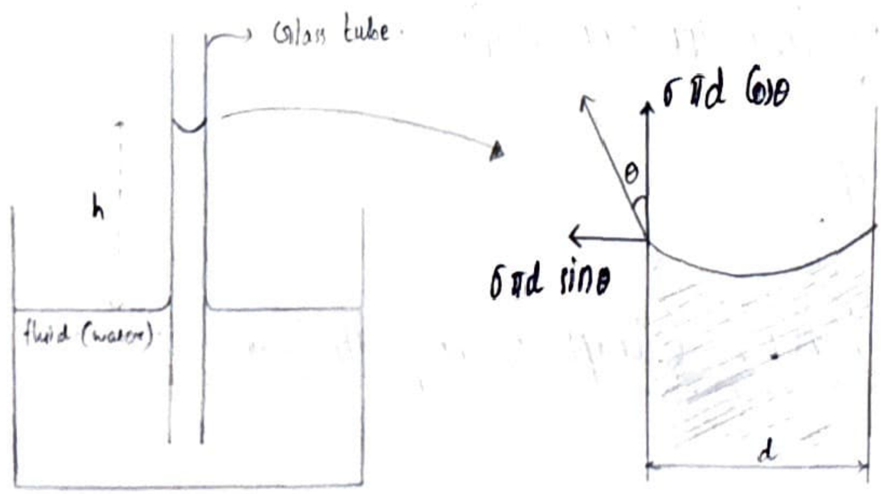
Capilarity :- The phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in liquid.



When the adhesive force b/w the fluid and material of tube is greater than the cohesive forces between water molecules

* The capilarity depends on the

* density
* specific weight
* diameter of tube
* surface tension liquid.



σ = surf. tension per unit meter.

Capilarity : wt. of liquid Column of ht. "h" = m. g

$$= \rho V g = \rho A h . g$$

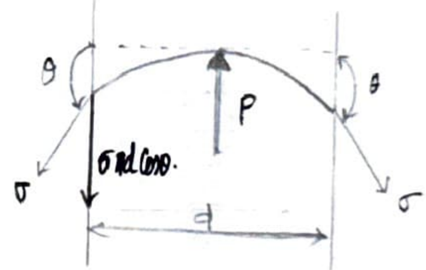
$$= \rho \pi d^2/4 \cdot h g . \longrightarrow ①$$

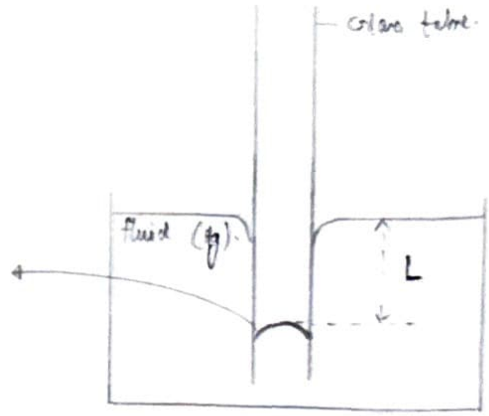
Vertical Component of S.T force = $\sigma . (\pi d) . \cos\theta \longrightarrow ②$

(∵ watersurface is curved @ edge)

$$① = ② \rightarrow \boxed{h = \frac{4\sigma \cos\theta}{\rho g d}}$$

Capillary fall :-





Considering eq^n : S.T force component on vertical direction : $\sigma . (\pi d) . \cos\theta$

Hydrostatic pressure acting upwards = intensity of pressure @ a depth h . A

$$= h . \rho g . (\pi d^2/4) .$$

$$\sigma \pi d \cos\theta = h \rho g \frac{\pi d^2}{4} \Rightarrow \boxed{h = \frac{4\sigma \cos\theta}{\rho g d}}$$

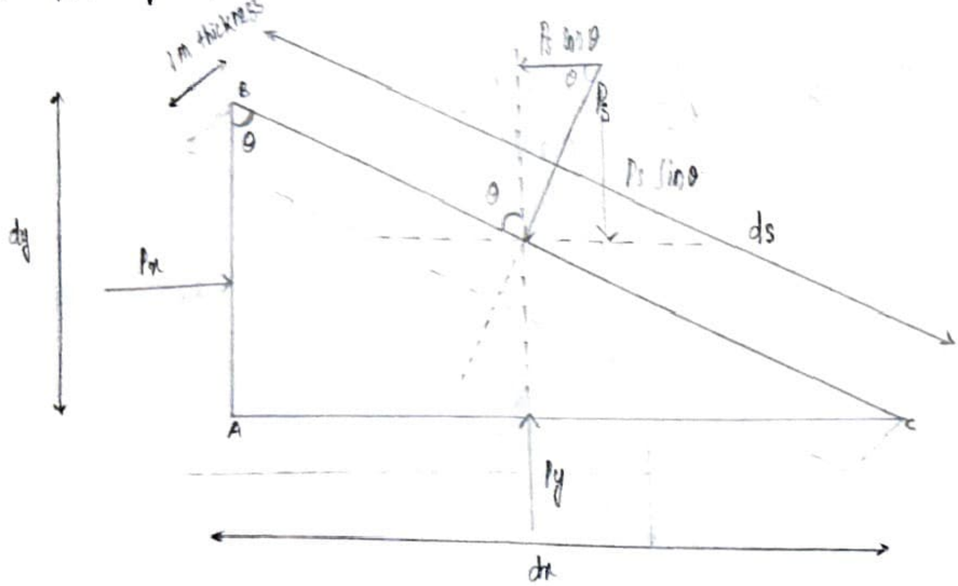
Similar expression

① θ between Hg & glass tube = 128°.

Pascal's Law :- It states that intensity of pressure @ a point in static fluid is equal in all directions.

- Fluid element : A tiny hypothetical packet of fluid that contains a very small mass of fluid. This element is so small that we can approximate it as a point. For fluid at rest, force exerted will be always $\perp^r$ to the surface.

→ Consider a prism or wedge type element, around a particular point in a static fluid point.



Resolving forces in x direction :- $P_x \times dy \times 1 - P_s \cdot ds \cdot 1 \cos\theta = 0$,

Here $\cos\theta = dy/ds \Rightarrow dy = ds \cos\theta$, $dx = ds \sin\theta$

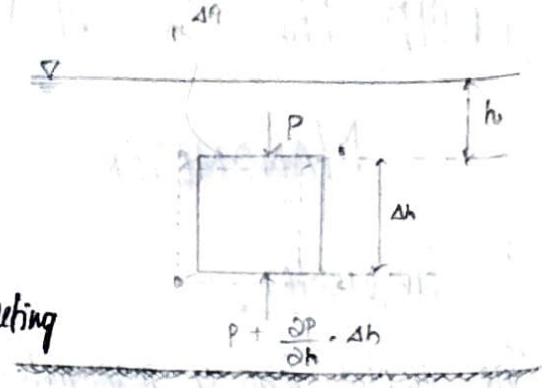
ie we get $P_x\, dy = P_s\, dy \Rightarrow \boxed{P_x = P_s}$

Resolving forces in y-direction :- $P_y\, dx \cdot 1 = P_s\, ds \cdot 1 \cdot \sin\theta \Rightarrow P_y\, dx = P_s\, dx$

$$\boxed{P_y = P_s}$$

ie $P_x = P_y = P_z$ ⇝ Pascal's law.

Hydrostatic law :- The pressure at any point in a fluid at rest is obtained by hydrostatic law. It state's that the rate of increase of pressure in a vertically downward direction must be equal to specific weight of the fluid at that point.



⇒ Since forces on AD & BC cancel's ;

for eqⁿ ;

If ΔA is the area on which the pressure force is acting

taking vertical dirⁿ :

$$\rho V = \rho(A\,\Delta h)$$

$$P\Delta A - \left[P + \frac{\partial P}{\partial h} \cdot \Delta h\right]\Delta A + mg = 0$$

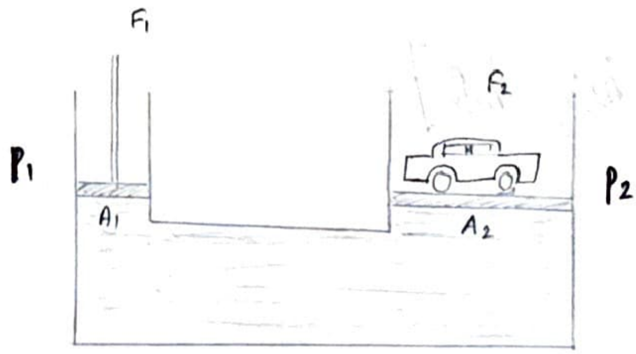
$$P\Delta A - P\Delta A - \frac{\partial P}{\partial h} \cdot \Delta h\,\Delta A + \rho\Delta A\,\Delta h\,g = 0$$

$$\frac{\partial P}{\partial h}\,\Delta h \cdot \Delta A = \rho\,\Delta h \cdot \Delta A\,g \Rightarrow \boxed{\frac{\partial P}{\partial h} = \rho g = \gamma}$$

Hydrostatic law.

on integrating : $\int_0^P \partial P = \int_0^h \partial h \cdot \gamma \Rightarrow P = \gamma h = h\rho g$ * $\boxed{P = h\rho g.}$

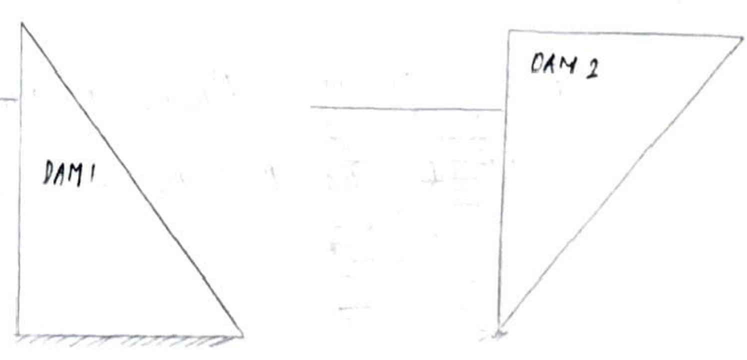
- Hydrolic lift :- From pascals law, pressure applied on a fluid will transmit equally.



Since $P_1 = P_2$

$$\Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \boxed{\begin{array}{c} F_2 = \frac{A_2}{A_1} \cdot F_1 \\ F_2 > F_1 \end{array}}$$

● uni- an :-



which dam is reliable ?

⇒ ①, fluid pres. is higher @ bottom, hence strength of dam ① is higher than dam ②.

FLUID PRESSURE MEASURMENT *

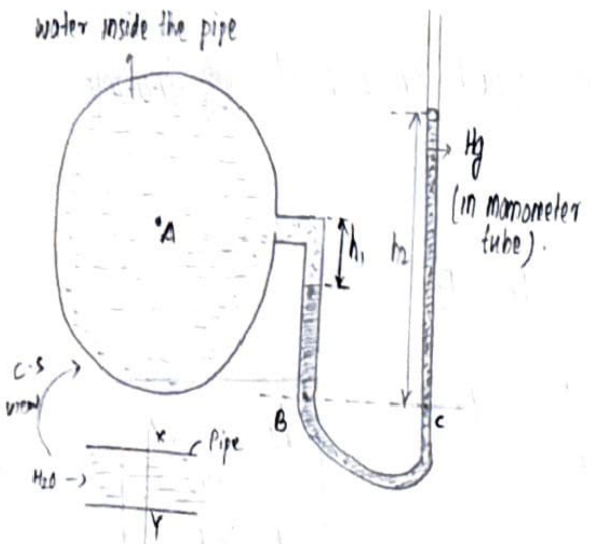
MANOMETER

① Piezometer *

The simplest form of manometer is piezometer. It is a single vertical tube open @ top inserted in to a pipe or a tank.

* Limitation: The hight of the tube limit's the pressure that can be measured. In such Cases, we can use U-tube manometer.

② U-tube manometer



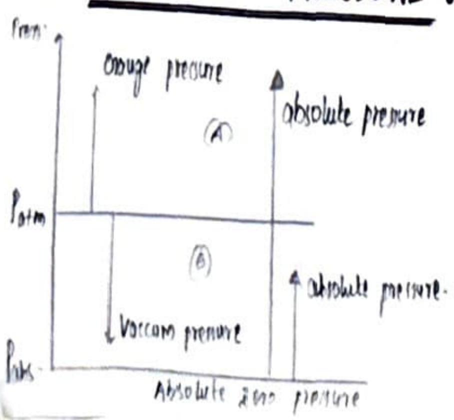
$P_B = \rho_w g h_1 + P_A$

$P_C = \rho_{Hg} \cdot g h_2$

} Both are equal.

$$\boxed{P_A = g\left[h_2 \rho_{Hg} - h_1 \rho_w\right]}$$

③ TYPES OF PRESSURE :-



- Gauge pressure : Atmo. pressure is taken as datum / reference and measured above the atmo. pressure level.

- Vacuum pressure : Atmospheric pressure is reference and measured below atmospheric pressure.

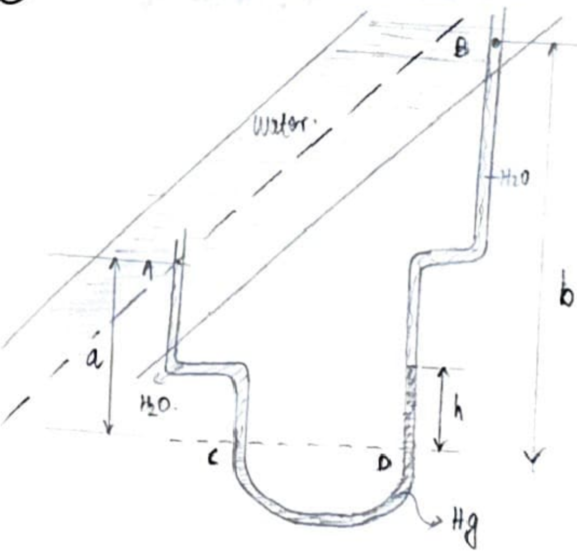
- Absolute pressure is measured with reference to absolute zero pressure. It can be either below or above the atmospheric level.

$P_{abs} = P_{atm} + P_{gauge}$ (@ A) $\longrightarrow$ $P_{vac} = P_{atm} - P_{abs}$ (@ B)

MEASURMENT OF DIFFERENTIAL PRESSURE

→ Pressure diffrence b/w 2 points.

(III) Differential U-tube manometer.



$P_C = P_A + \rho_w a g$

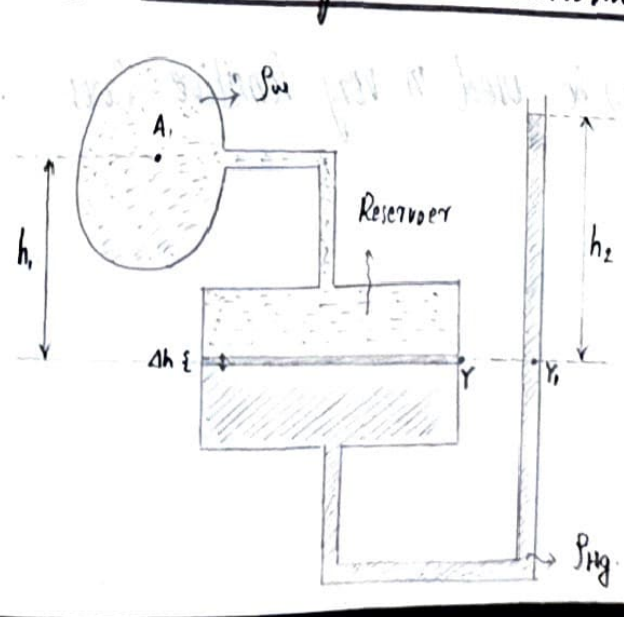
$P_D = P_B + \rho_w g(b-h) + \rho_{Hg} \cdot hg$

$P_C = P_D$ (PASCALS LAW)

$P_A - P_B = \rho_w g(b-h) + \rho_{Hg} hg - \rho_w ag$

$= g\left[\rho_w(b-h) + \rho_{Hg} h - ag\right]$

(IV) Vertical single Column manometer



* A U-tube manometer is fitted on a reservoer.

* Why a large reservoer is provided on right part

→ Since it is a large measurment, the variation Δh is small and due to this, h_1 is constant. ie we only have to measure h_2.

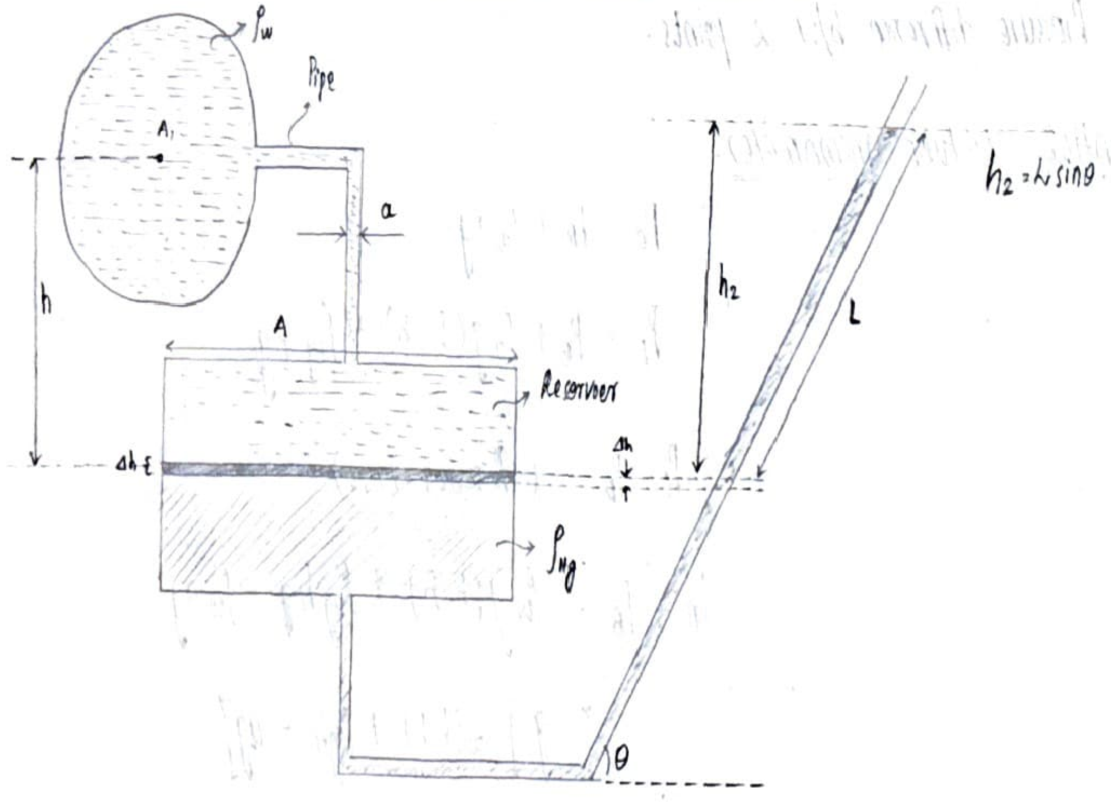
Pressure at left = Pressure at right ⇒ $\rho_w g (h_1 + \Delta h) + P_A = \rho_{Hg} g (h_2 + \Delta h)$

If $\Delta h \approx 0$ (negligible)

$P_A = \rho_{Hg} g h_2 - \rho_w \cdot g \cdot h_1$ ⇒ $\boxed{P_A = g[\rho_{Hg} h_2 - \rho_w h_1]}$

* The area of the reservoir is very large compared to the area of pipe $[A_R \approx 100 A_P]$
  Hence h_1 is a known quantity. only unknown is h_2.

(V) INCLINED SINGLE COLUMN MANOMETER

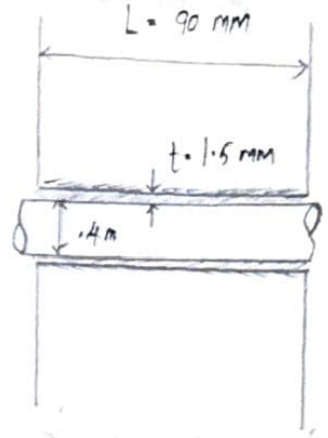


* $\boxed{P_A = \rho_{Hg} g (L \sin\theta) - \rho_w g h_1}$ (since $\Delta h \approx 0$).
* The fluid indicator can move freely. So it can be used in very sensitive cases.

Qn :- The dynamic viscosity of an oil used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m & rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 90 mm. The oil film thickness = 1.5 mm.

P =

Ans :- $\mu = 6$ poise, $D = .4\,m$, $N = 190$ rpm, $L = 90$ mm, $t = 1.5$ mm



Shear stress = $\tau = \mu \, du/dy$.

$P = \frac{2\pi N T}{60}$, T = Torque = Force × distance
= [Shear stress × Area] × distance

① Tangential velocity = $u = \frac{\pi D N}{60}$ and $du = V - 0$ (at point of contact, $V = 0$).

$dy = 1.5$ mm.

ie $T = \mu \frac{du}{dy} \cdot A \cdot \left[D/2\right] = \mu \cdot \left(\frac{\pi D N}{60} \cdot \frac{1}{t}\right) \cdot (\pi D \cdot L) \cdot \frac{D}{2}$

1 poise = $.1\ Ns\,m^{-2}$

A = Circumfrential area.

$= .6 \times \frac{\pi \times .4 \times 190}{60} \times \frac{1}{1.5 \times 10^{-3}} \times \pi \times .4 \times 90 \times 10^{-3} \times \frac{.4}{2}$

$\frac{190}{60} = \frac{a}{1}$

$= \frac{6480 \cdot 778}{180} \approx 36$

1 m = 100 cm = 1000 mm

10^3.

$P = \frac{2\pi \times 190 \times 36}{60} = 2\pi \times 3.1667 \times 36 = 716.29$ W.

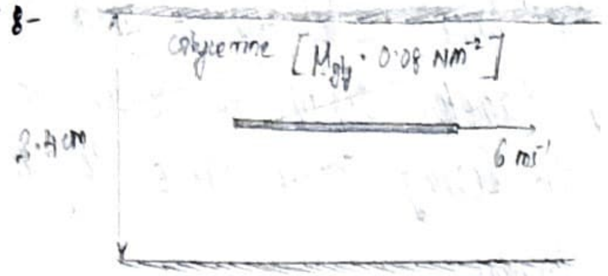
Qn: 2 large plane surfaces 2.4 cm apart. The space b/w the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area .5 m² b/w the two large plane surfaces, at a speed of 0.6 m s⁻¹ if

i) the thin plate is in the middle of 2 plane surfaces

ii) the thin plate is @ a dist. of 0.8 cm from one of plane surfaces.

$\mu_{gly} = 8.1 \times 10^{-1}\ Ns\,m^{-2}$

Ans 8- Force comes due to shear stress.



(1) @ middle 8- $F = F_1 + F_2$ ($F_1 = F_2$ here)

$= 2F$

$= 2 \times \mu \dfrac{du}{dy} \cdot A$

$= 2 \times .08 \times \dfrac{.6}{1.2 \times 10^{-2}} \times .5 = \underline{\underline{40\ N}}$

$du =$ Velocity @ surf − Vel. @ bound.

$= 6 - 0 = \underline{6\ ms^{-1}}$

$dy = 1.2\ cm = \underline{\underline{1.2 \times 10^{-2}\ m}}$

1) $F = F_1 + F_2 = \dfrac{.08 \times .6}{.8 \times 10^{-2}} \times .5 + \dfrac{.08 \times .6}{1.6 \times 10^{-2}} \times .5 = 80 \times [.5 + 60 \times .5] = \underline{45}$

Qn: Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is restricted to 2 mm.

Ans 8- Given σ = Surf. tension = .073575 N/m

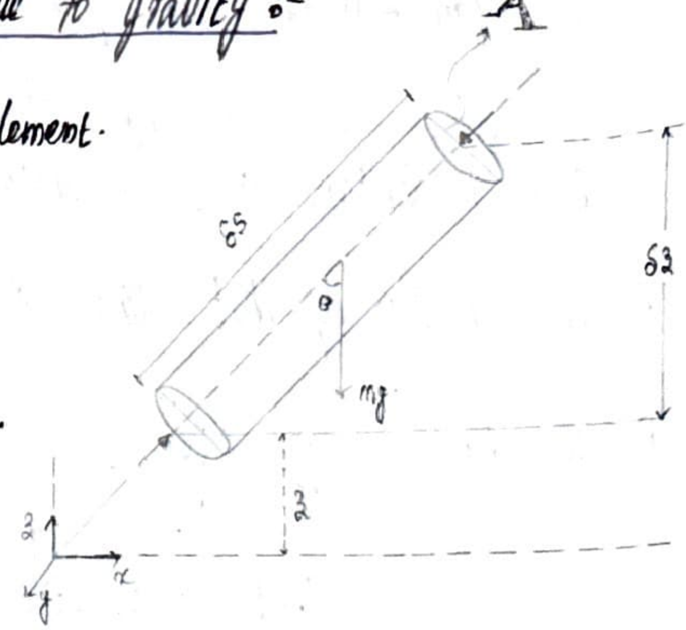
$h = \dfrac{4\sigma \cos\theta}{\rho g d}$ $\Rightarrow$ $d = \dfrac{4\sigma \cos\theta}{\rho g h} = \dfrac{4 \times .073575 \times \cos 0}{1 \times 9.8 \times 2 \times 10^{-3}} = \underline{15.015\ mm}$

STATE'S OF FLUID - STATIC FLUID

For static fluid, force is always acting $\perp^r$ to the surface [pressure force]. Shear stress only acts when there is relative motion b/w fluid elements. However fluid statics can be extended to case's where the fluid is moving as a whole such that all the particles are @ rest w.r.t each other.

Book by Douglas.

① General Equation for pressure variation due to gravity :-



Let A be the cross sectional area of the fluid element.

p = Pressure along acting on the lower end

p+δp = Pressure on the other end

Taking eq^n of forces in "s" direction (axial dir^n).

$$mass = \rho V = \rho \cdot A \delta s$$

$$pA - (p + \delta p)A - mg\cos\theta = 0$$

$$\delta p A = \rho A \delta s \cdot g\cos\theta .$$

We should not consider the forces due to the surrounding fluids that is acting on the circumfrence which acts normal to the C.S.A

$$\boxed{\delta p = -\rho g\, \delta s \cos\theta}$$

in differential form, it can be written as :- $\boxed{\dfrac{dp}{ds} = -\rho g\cos\theta}$

above eq^n gives :- (i) At horizontal plane

$$dp/ds = \partial p/\partial x = \partial p/\partial y = -\rho g \cos 90 = \underline{0}$$

From above eq^n it is clear that in a static fluid, pressure is constant everywhere in a horizontal plane. ie free surface always remain horizont

(ii) For vertical plane $(\theta = 0°)$

$$\partial p/\partial s = \partial p/\partial z = -\rho g \quad \text{(A constant variation)}.$$

For a particular horizontal plane ; $\frac{\partial p}{\partial z}$ is constant.

ie ρg = Constant.

If "g" is Constant ⇒ ρ is Constant on a horizontal plane.

⊙ We can also say that since $\partial p/\partial x = \frac{\partial p}{\partial y} = 0$ (no variation), $\frac{\partial p}{\partial z}$ can be replaced by $\frac{dP}{dz} = -\rho g$

⊙ we have $\frac{dp}{dz} = -\rho g$ ⇒ $\int_{P_1}^{P_2} dp = -\int_{z_1}^{z_2} \rho g \, dz$. (Applying proper limit)

$P_2 - P_1 = -(z_2 - z_1)\rho g$. OR $\boxed{P_2 - P_1 = -\int_{z_1}^{z_2} \rho g \, dz.}$

⊙ CONDITIONS OF EQM UNDER GRAVITY OBTAINED FROM GENERAL EQNS-

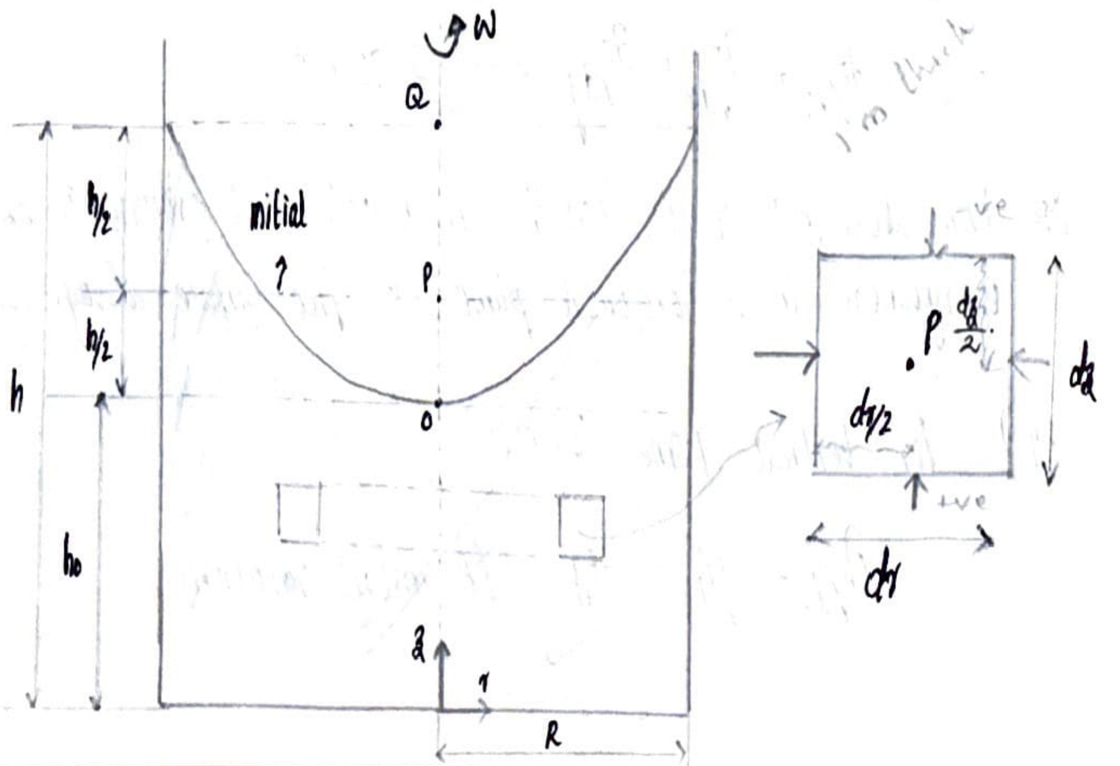
(I) The pressure @ all points on a horizontal plane is Constant.

(II) Density at all points on a horizontal plane is Constant.

(III) Change of pressure with elevation is given by $\frac{dp}{dz} = -\rho g$

FLUID ROTATION ABOUT A VERTICAL AXIS

A K Jain book



* When a fluid is rotated about a vertical ~~speed~~ axis with const. angular velocity every particles move like a solid. This type of flow is known as forced vortex flow. eg: Rotating a bucket containing water in which water rises nearsides and depreses near axis

* In case of a free vortex flow, velocity varies inversly with distance from the axis. Eg: washbasin in which initially it will be plugged and then flows.

→ Let P be the pressure at the centre of ring element.

Assuming P is decresing towards left and increasing towards right. also p is decrining towards bottom & increasing towards top.

→ Top : $P + \frac{\partial P}{\partial z} \cdot \frac{dz}{2}$ Right : $P + \frac{\partial P}{\partial r} \frac{dr}{2} \cdot (dz \times 1)$

Bottom : $P - \frac{\partial P}{\partial z} \cdot \frac{dz}{2}$ left : $P - \frac{\partial P}{\partial r} \cdot \frac{dr}{2} \cdot (dr \cdot 1)$

Cosider a ring element of radius $= r$ and cross section $dz \cdot dr$. Take a unit circumfrential length. Applying Newtons 2^{nd} law,

weight of ring element : $W = mg = \rho v g = (\rho g)(dr \cdot dz \cdot 1) = (\rho g)\, dr\, dz$.

$\sum F_z = a_z \cdot M$ (a_z here is zero). $a_z = 0$.

> wt of fluid elem

$\Rightarrow -\left[P + \frac{\partial P}{\partial z} \frac{dz}{2}\right] \cdot dr + \left(P - \frac{\partial P}{\partial z} \frac{dz}{2}\right) \cdot dr - \rho \times \frac{dr \cdot dz \cdot 1}{m} \cdot g = M \cdot 0$

$\therefore \frac{\partial P}{\partial z} = -\gamma$

Applying Newtons 2^{nd} law in r dirn :- Here $a_r =$ Centripetal accl^n $= -r\omega^2$

$a_r = -r\omega^2$ (-ve because it acts in -ve r. dir^n).

$$-\left(p+\frac{\partial p}{\partial r}\frac{dr}{2}\right)\cdot dz + \left(p-\frac{\partial p}{\partial r}\cdot\frac{dr}{2}\right)\cdot dz = \rho\, dr\, dz\cdot(-r\omega^2).$$

$$\Rightarrow \quad \frac{\partial p}{\partial z} = \gamma\left[\frac{r\omega^2}{g}\right]. \qquad (\rho = \gamma/g)$$

Pressure change dp (considering

$$dp = \frac{\partial p}{\partial r}\cdot dr + \frac{\partial p}{\partial z}\cdot dz$$

$$dp = \frac{\gamma}{g}(r\omega^2)\, dr - \gamma\, dz$$

For surface of constant pressure, $dp = 0 \Rightarrow \frac{\gamma}{g}(r\omega^2)\, dr - \gamma\, dz = 0$

$$\frac{dz}{dr} = \frac{r\omega^2}{g}$$

on integrating :- $\int dz = \int \frac{r\omega^2}{g}\cdot dr$

$$z = \frac{\omega^2}{2g}\cdot r^2 + C \quad \text{(Parabolic Surface).}$$

$\longrightarrow$ (1)

Free surface @ Centre, $r=0$

Std eq^n of parabola (vertical) $\Rightarrow$ $y = ax^2+bx+c$, if it is symmetrical about y-axis, ($x=0$ axis)

$$y = ax^2 + c \quad , \text{ Here } z = r^2\left(\frac{\omega^2}{2g}\right) + h_0$$

From eq^n (1), it is evident that free surface and surf. of const. pressure are paraboloids of revolution. For paraboloids, it volume = ½ volume of the circumscribing cylinder. ie mathematically OP = PQ.

* This indicates that liquid is depressed in the centre by the same distance as it rises at the sides.

Thus, pressure (P) at any distance (radial) is given by hydrostatic law,

$$P = h\rho g = \gamma h$$

Here $P = \gamma z = \gamma \left[\frac{r^2 w^2}{2g} + h_0 \right]$

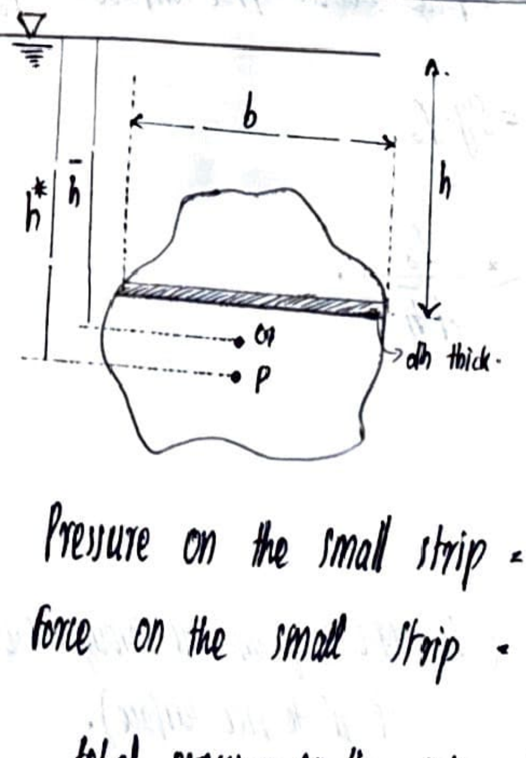
HYDROSTATIC FORCES ON SURFACES

Total pressure :-

The fluid pressure always acts ⊥ to the surface. "Total pressure" is defined as a force excerted by the static fluid on a surface either plane or curved.

"Centre of pressure" is the point of application of the resultant of the total pressure on the surface in contact.

ⓐ C.O.P on vertical surface immerced in fluid :-



To find out the total pressure, we need to devide the entire surface in to small parallel strips and then integrate force on a small strip.

* Pressure on the small strip = $\rho g h$ $\qquad b \cdot dh = dA$

Force on the small strip = $P \times A = h\rho g \cdot (b\, dh) = dF$

total pressure on the entire surface = $\int \rho g h \cdot b\, dh = \int \rho g h \cdot dA$

$$= \rho g \int h \cdot dA$$

where $h \cdot dA$ is the moment of small strip about free surface & $\int h \cdot dA$ is the moment of entire area about free surface.

Also $\int dA \cdot h = A \cdot \bar{h}$ ($\bar{h}$ is centre of gravity).

$$\boxed{\text{Total pressure of the vertical surface immersed in fluid} = \rho g \cdot A\bar{h}}$$

Position of C.O.P on vertical surface :-

C.O.P (h^*) is calculated by principal of moment, which states that moment of resultant force about an axis is equal to sum of moments of the component forces. ~~about~~

Moment of resultant force about free surf. $= F \cdot h^*$

" " component " " " " $= dF \cdot h$

Sum of moment of the component forces $= \int dF \cdot h = \int (\rho g h \cdot b\,dh) \cdot h$

$= \rho g \int h^2 dA = \rho g \cdot I_0$

Also $\int h^2 dA$ is the moment of inertia of the entire surface about free surface. $= I_0$

By principle of moments $\Rightarrow$ $F \cdot h^* = \rho g \cdot I_0$

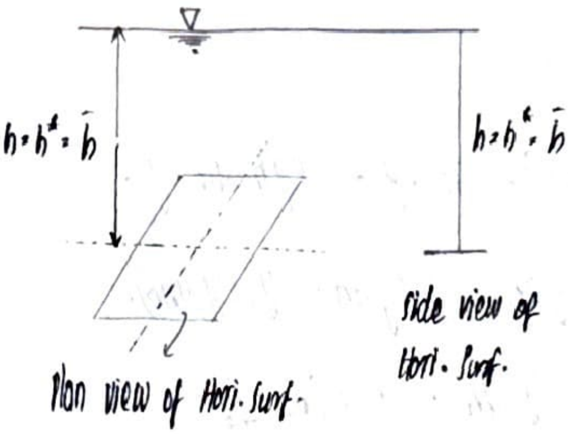
$$\Rightarrow h^* = \frac{\rho g \cdot I_0}{F} = \frac{\rho g \cdot I_0}{\rho g \cdot A\bar{h}} = \frac{I_0}{A\bar{h}}.$$

Position of C.O.P $= h^* = \dfrac{I_0}{A\bar{h}}$

From parallel axis theorem : $I_0 = I_G + A\bar{h}^2$ (I_G = M.O.I of an axis through C.O.G & $\parallel^{el}$ to free surface).

$$\boxed{h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}}$$

(b) Horizontal Surface Immersed in Fluid.



Pressure $= h\rho g$

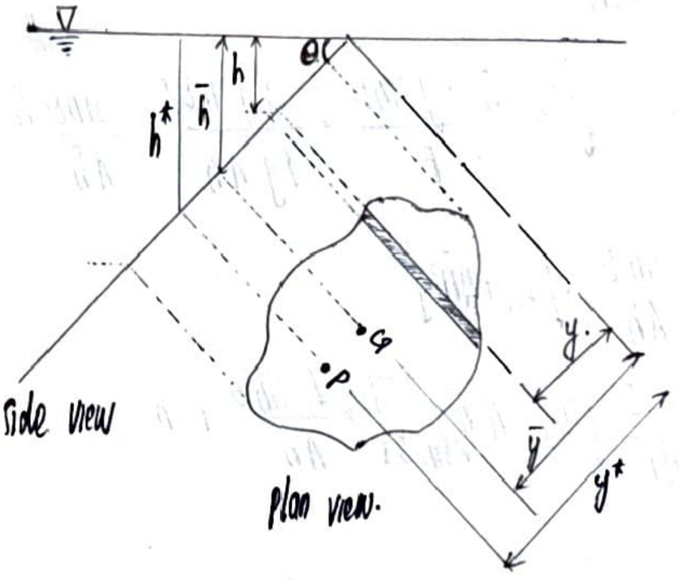
Total pressure $= h\rho g \cdot A$

If $h = h^* = \bar{h}$, then $Ah^* = A\bar{h}$

$$\boxed{\text{Total pressure} = \rho g \cdot A\bar{h}}$$

* Note: C.O.P = dist. of immersion ($h = h^*$).

(c) INCLINED PLANE SURFACE IMMERSED IN LIQUID



From similar triangles

$$\sin\theta = \frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*}$$

Total pressure, F on the entire surf :-

$$F = \int \rho g \cdot h\,dA$$

$$= \int \rho g [y \sin\theta]\, dA$$

ie $F = \int \rho g \sin\theta \cdot y\,dA = \rho g \sin\theta \int y\,dA$ [$\int y\,dA$ = moment of entire area about Y-Y axis]

Also $\int y\,dA = A \cdot \bar{y}$

Hence, $F = \rho g \sin\theta \cdot A\bar{y} = \rho g A \cdot [\bar{y}\sin\theta] = \rho g A \bar{h}$ $\Rightarrow$ $\boxed{F = \rho g A \bar{h}}$

Position of C.O.P (h^*) :-

Here we are using principle of moment.

Moment of resultant force about Y-Y axis = $F \cdot y^*$

Moment of dF about Y-Y axis (dF = force on small strip) = $dF \cdot y = \rho g h \cdot dA \cdot y$.

Sum of moments of component forces = $\int \rho g h \cdot y\,dA = \int \rho g\,dA \cdot (y)(y \sin\theta)$

$= \rho g \sin\theta \int y^2 dA = \rho g \sin\theta\, I_o$

$\int y^2 dA$ = M.O.I of the small strip about Y-Y axis & $\int y^2 dA$ = M.O.I of entire area about Y

$\int y^2 dA = I_o$, then

Hence $F \cdot y^* = \rho g \sin\theta \cdot I_o \Rightarrow y^* = \frac{I_o \rho g \sin\theta}{F} = \frac{\cancel{\rho g} \sin\theta I_o}{\cancel{\rho g} A \bar{h}} = \frac{\sin\theta\, I_o}{A\bar{h}}$

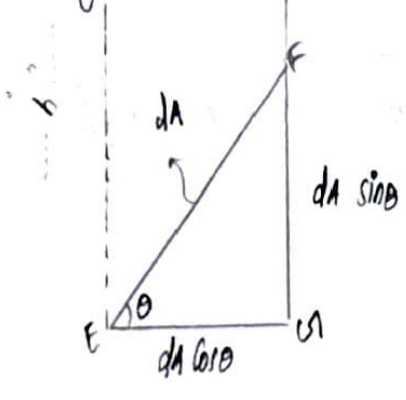
Also, $y^* = \frac{h^*}{\sin\theta} \Rightarrow h^* = \frac{\sin^2\theta \cdot I_o}{A\bar{h}} = \frac{\sin^2\theta}{A\bar{h}} \cdot \left[I_G + A\bar{y}^2\right]$

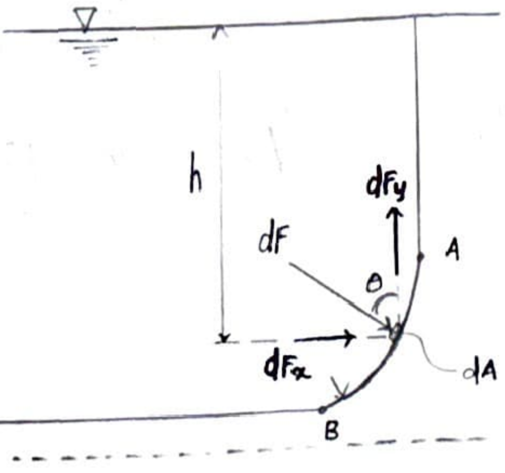
$= \frac{\sin^2\theta}{A\bar{h}} \left[I_G + A\bar{h}^2/\sin^2\theta\right] = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$

$$\boxed{h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}}$$

(d) CURVED SURFACE IMMERCED IN LIQUID

Enlarged view of dA :-





Integrating $\rho gh \cdot dA$ term is not possible directly here since the direction of forces varies from point to point along the curved surfaces. Thus ~~along the curved~~ surface resolve dF in to dF_x & dF_y.

$$dF_x = dF \sin\theta$$
$$dF_y = dF \cos\theta$$

Pressure on $dA = \rho gh$

Pressure force on $dA = (\rho gh) \cdot dA$

$\Rightarrow \quad dF_x = dF \sin\theta = (\rho gh \cdot dA) \sin\theta \qquad \Rightarrow \quad dF_y = (\rho gh \cdot dA) \cos\theta$

Now integrating dF_x & dF_y

$\int dF_x = F_x = \int \rho gh \,(dA \sin\theta)$, simly $\int dF_y = F_y = \int \rho gh \, dA \cos\theta$

Hence $dF_x = \rho gh \,(dA \sin\theta)$ represents the pressure force acting on the projected area of dA. Thus $F_x = \int \rho gh \, dA \sin\theta$, represents the "total pressure" on the projected area of the entire curved surface on the vertical plane.

* $dA \cos\theta \cdot E\theta$ represents the horizontal projection of dA, hence $h \cdot dA \cos\theta$ means the volume of liquid contains elementory area dA. ie $\int h \cdot dA \cos\theta$ is the total volume contains b/w projected area of curved surf. up to the free surface. ie

$F_y = \int \rho gh \cdot dA \cos\theta$ is the total weight supported up to free surface.